Computer Algebra Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

1-Algebraic-functions/1.2-Trinomial-products/1.2.3-General/46-1.2.3.2-d-x-^m-a+b-x^n+c-x^-2-n-^p

Nasser M. Abbasi

December 9, 2023 Compiled on December 9, 2023 at 5:19am

Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [664]. This is test number [46].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (664)	0.00 (0)
Mathematica	99.70 (662)	0.30 (2)
Fricas	80.57 (535)	19.43 (129)
Maple	74.70 (496)	25.30 (168)
Giac	65.66 (436)	34.34 (228)
Mupad	54.22 (360)	45.78 (304)
Maxima	45.48 (302)	54.52 (362)
Sympy	38.25 (254)	61.75 (410)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

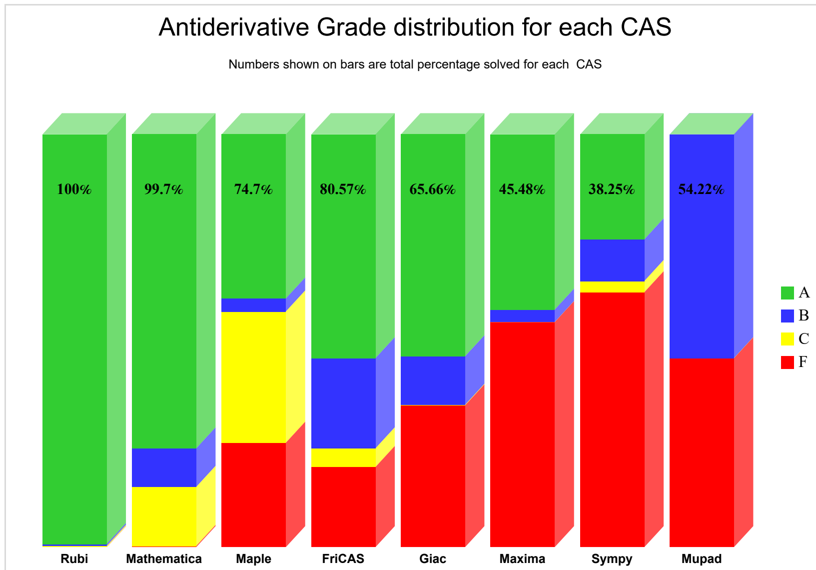
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

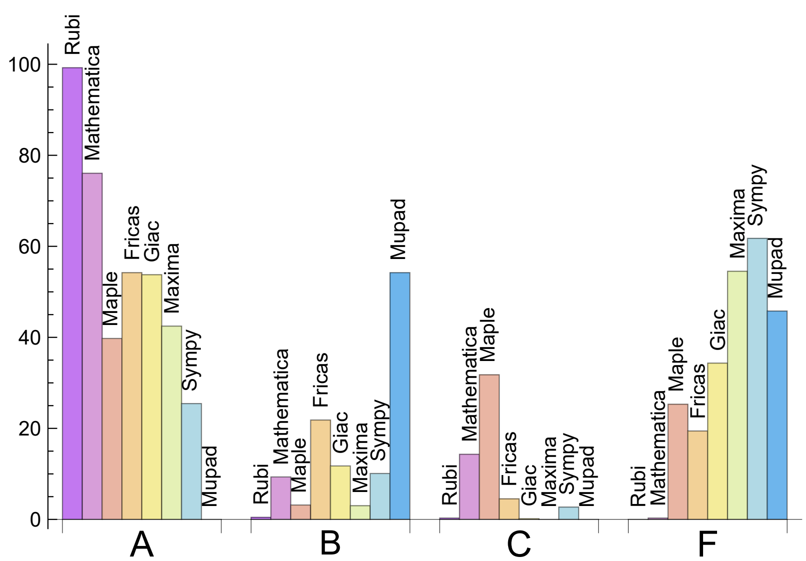
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.548	0.151	0.301	0.000
Mathematica	76.054	9.337	14.307	0.301
Fricas	54.217	21.837	4.518	19.428
Giac	53.765	11.747	0.151	34.337
Maxima	42.470	3.012	0.000	54.518
Maple	39.759	3.163	31.777	25.301
Sympy	25.452	10.090	2.711	61.747
Mupad	0.000	54.217	0.000	45.783

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	2	100.00	0.00	0.00
Fricas	129	66.67	10.85	22.48
Maple	168	100.00	0.00	0.00
Giac	228	96.05	1.32	2.63
Mupad	304	0.00	100.00	0.00
Maxima	362	77.07	0.55	22.38
Sympy	410	76.34	23.17	0.49

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.23
Rubi	0.31
Fricas	0.31
Giac	0.35
Mathematica	0.71
Maple	1.54
Sympy	4.33
Mupad	6.55

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	81.13	0.80	55.50	0.76
Maple	138.26	1.00	62.00	0.66
Rubi	138.30	0.93	107.00	1.00
Mathematica	154.11	1.09	89.50	0.94
Giac	316.85	1.83	96.00	0.83
Sympy	511.19	3.80	79.00	1.00
Fricas	691.01	3.14	131.00	1.26
Mupad	1652.88	7.04	119.00	0.99

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

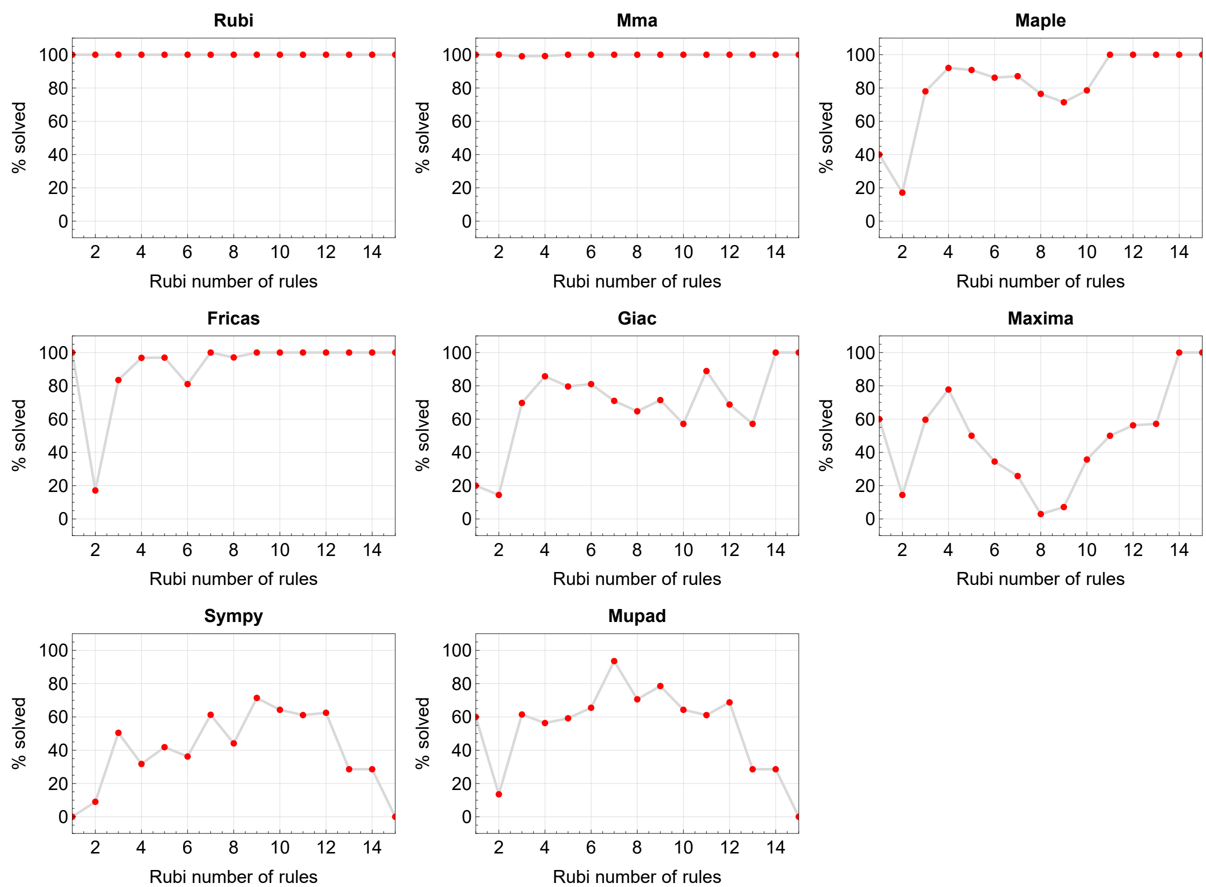


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

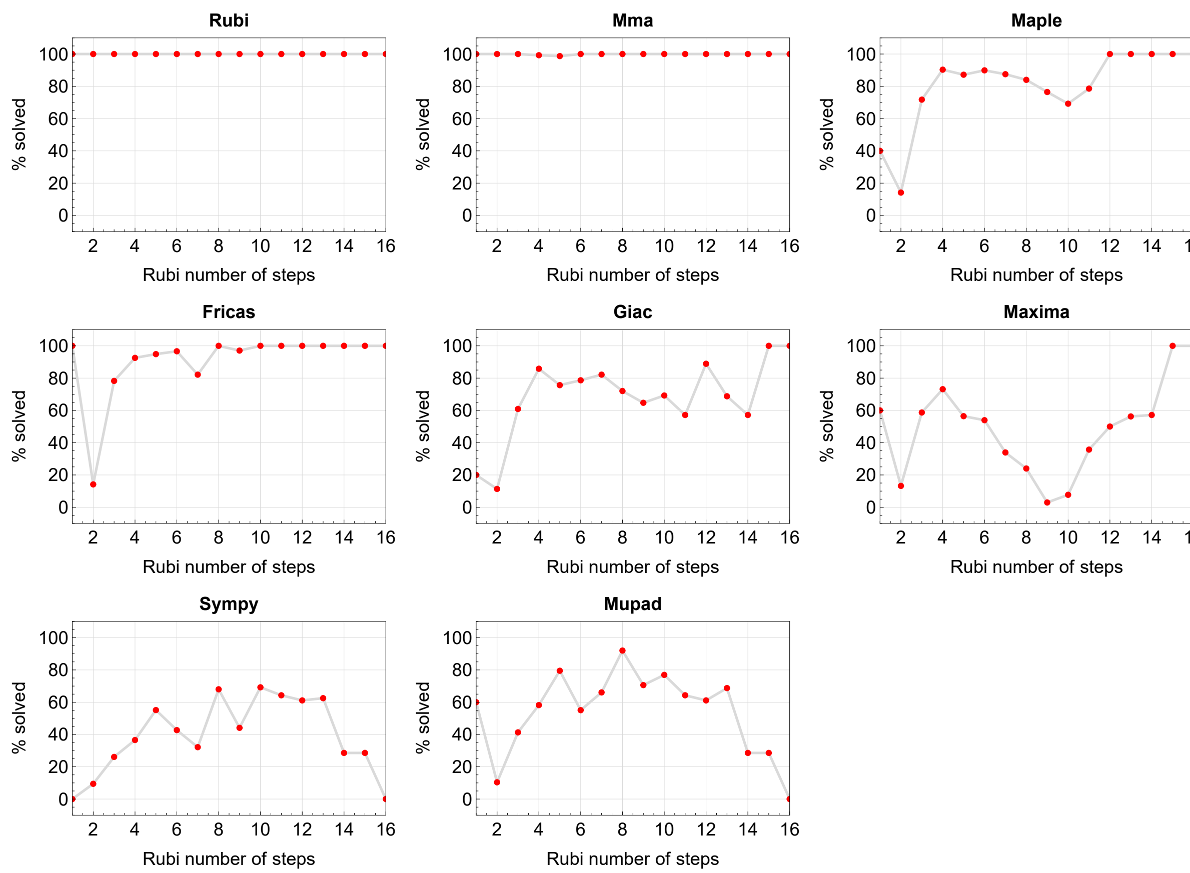


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

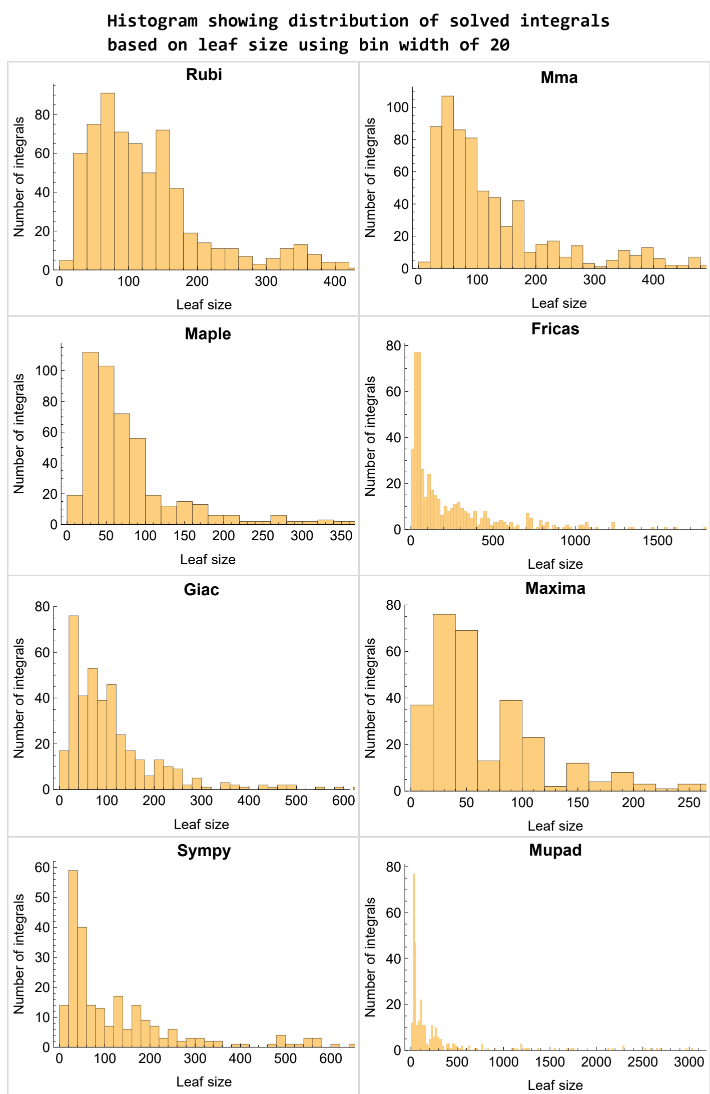


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

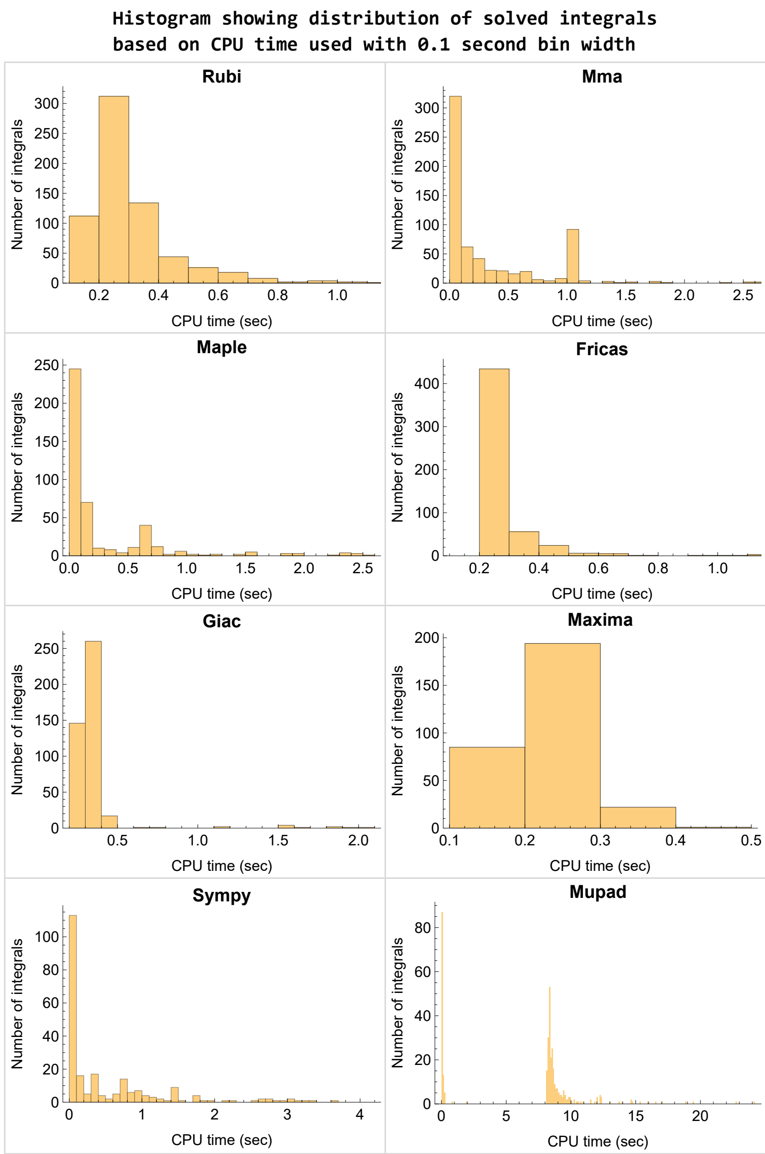


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

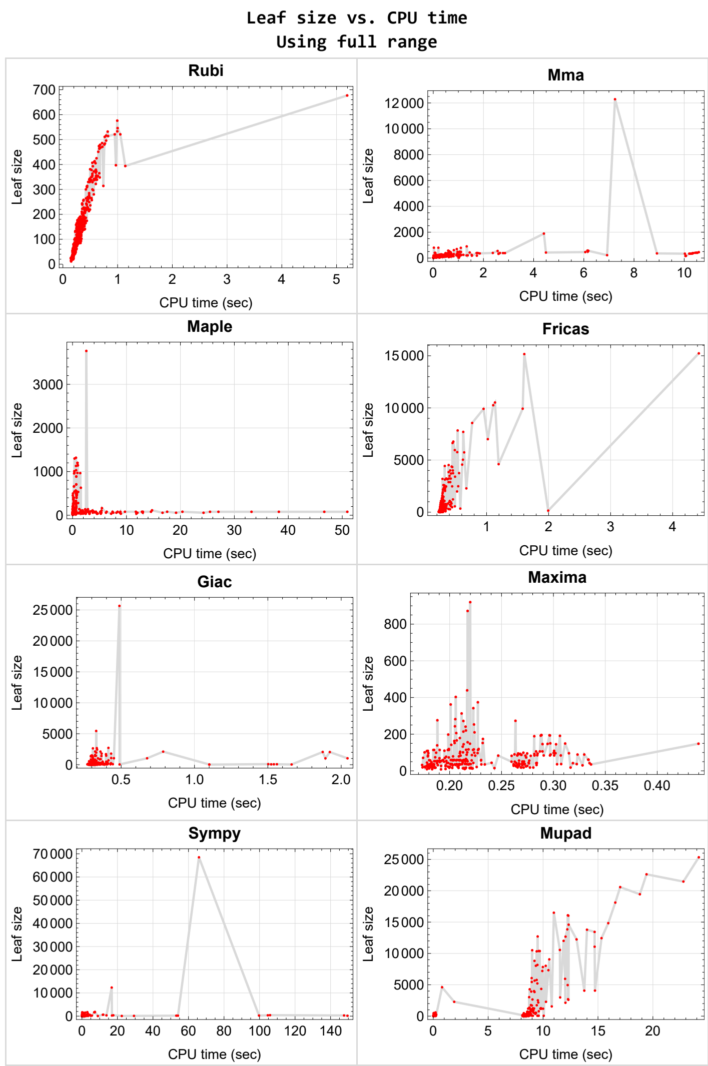


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{663, 664\}$

Mathematica $\{132, 250, 256, 309, 327, 347, 367, 385, 602\}$

Maple $\{6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 39, 42, 45, 48, 52, 55, 58, 61, 64, 67, 76, 79, 82, 85, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 465, 515, 522, 548, 596, 597, 598\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

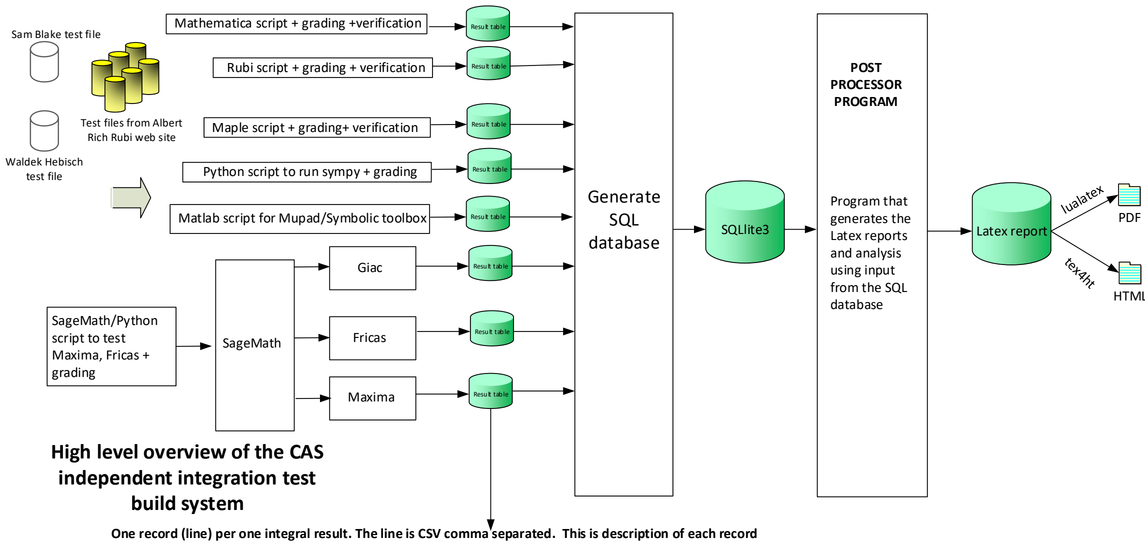
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form "{n,n,..}" which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

Nasser M. Abbasi
June 27, 2023
Design.vsdx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664 }

B grade { 154 }

C grade { 176, 478 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 13, 14, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 101, 102, 104, 105, 107, 109, 110, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 133, 134, 135, 136, 137, 138, 139, 140, 151, 152, 153, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 171, 174, 183, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 234, 235, 236, 237, 238, 239, 240, 241, 242, 248, 249, 253, 254, 255, 256, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 310, 311, 312, 313, 314, 315, 328, 330, 332, 337, 339, 342, 345, 348, 349, 350, 352, 355, 368, 369, 370, 371, 372, 373, 374, 376, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 562, 563, 565, 566, 567, 572, 579, 582, 583, 584, 585, 586, 587, 588, 593, 596, 597, 598, 599, 604, 606, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 663 }

B grade { 9, 12, 15, 30, 39, 61, 76, 100, 103, 106, 108, 111, 154, 196, 197, 198, 199, 200, 214, 215, 216, 217, 218, 232, 233, 243, 244, 245, 246, 247, 252, 564, 568, 569, 570, 571, 573, 574, 575, 576, 577, 578, 580, 581, 589, 590, 591, 592, 594, 595, 600, 601, 602, 603, 605, 607, 608, 609, 610, 611, 612, 664 }

C grade { 132, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 170, 172, 173, 175, 176, 177, 178, 179, 180, 181, 182, 184, 250, 251, 257, 258, 309, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 329, 331, 333, 334, 335, 336, 338, 340, 341, 343, 344, 346, 347, 351, 353, 354, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 375, 377, 378, 379, 380, 381, 382, 383, 384, 385, 409, 410, 411, 457, 458, 478, 500, 501, 502, 503, 504, 505, 546, 547, 559, 560, 561 }

F normal fail { 661, 662 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 1, 2, 3, 4, 5, 7, 8, 10, 11, 13, 14, 16, 17, 19, 20, 22, 23, 25, 26, 28, 29, 31, 32, 34, 35, 36, 37, 38, 40, 41, 43, 44, 46, 47, 49, 50, 51, 53, 54, 56, 57, 59, 60, 62, 63, 65, 66, 68, 69, 70, 71, 72, 73, 74, 75, 77, 78, 80, 81, 83, 84, 86, 87, 118, 119, 120, 125, 126, 127, 130, 138, 139, 140, 141, 142, 151, 152, 153, 155, 156, 157, 171, 174, 177, 180, 183, 237, 238, 249, 272, 273, 274, 275, 276, 277, 278, 279, 280, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 310, 312, 314, 316, 317, 318, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 339, 342, 345, 348, 349, 350, 352, 354, 355, 356, 368, 370, 372, 374, 376, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 435, 436, 437, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 459, 460, 461, 462, 463, 464, 466, 467, 468, 469, 470, 471, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 502, 503, 508, 509, 510, 511, 512, 513, 514, 516, 517, 518, 519, 520, 521, 523, 524, 525, 526, 527, 528, 533, 540, 544, 572, 579 }

B grade { 154, 248, 433, 434, 438, 549, 550, 551, 552, 553, 554, 555, 565, 607, 608, 609, 610, 611, 612, 663, 664 }

C grade { 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 39, 42, 45, 48, 52, 55, 58, 61, 64, 67, 76, 79, 82, 85, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 143, 144, 145, 146, 147, 148, 149, 150, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 172, 173, 175, 176, 178, 179, 181, 182, 184, 281, 282, 283, 284, 285, 286, 287, 288, 289, 311, 313, 315, 319, 320, 321, 322, 323, 324, 325, 326, 338, 340, 341, 343, 344, 346, 351, 353, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 369, 371, 373, 375, 377, 378, 379, 380, 381, 382, 383, 384, 396, 397, 398, 399, 400, 401, 402, 403, 404, 456, 457, 458, 465, 500, 501, 504, 505, 515, 522, 548, 556, 557, 558, 559, 560, 561, 596, 597, 598, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660 }

F normal fail { 121, 122, 123, 124, 128, 129, 131, 132, 133, 134, 135, 136, 137, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 239, 240, 241, 242, 243, 244, 245, 246, 247, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 290, 309, 327, 347, 367, 385, 472, 473, 474, 475, 476, 477, 478, 506, 507, 529, 530, 531, 532, 534, 535, 536, 537, 538, 539, 541, 542, 543, 545, 546, 547, 562, 563, 564, 566, 567, 568, 569, 570, 571, 573, 574, 575, 576, 577, 578, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 599, 600, 601, 602, 603, 604, 605, 606, 661, 662 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 125, 126, 127, 130, 138, 139, 140, 141, 142, 151, 152, 153, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 234, 235, 236, 237, 238, 240, 241, 242, 249, 272, 273, 274, 275, 276, 277, 278, 279, 280, 291, 292, 294, 296, 298, 310, 312, 314, 316, 318, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 339, 342, 345, 348, 349, 350, 352, 354, 355, 356, 368, 370, 374, 376, 378, 379, 380, 381, 382, 384, 386, 388, 392, 394, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 455, 460, 462, 463, 464, 465, 466, 467, 468, 469, 473, 474, 475, 478, 479, 491, 493, 494, 495, 496, 497, 498, 499, 501, 502, 503, 504, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 533, 540, 543, 544, 545, 546, 547, 548, 549, 550, 551, 553, 554, 555, 565, 572, 579, 586, 616, 617, 639, 641, 642 }

B grade { 82, 143, 144, 145, 146, 147, 148, 149, 150, 154, 239, 248, 293, 295, 297, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 311, 313, 315, 317, 319, 320, 321, 322, 323, 324, 325, 326, 369, 371, 372, 373, 375, 377, 383, 387, 389, 390, 391, 393, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 454, 456, 457, 458, 470, 471, 552, 556, 557, 558, 559, 560, 561, 593, 596, 597, 598, 607, 608, 609, 610, 611, 612, 613, 614, 615, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 640, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 663, 664 }

C grade { 176, 281, 282, 283, 284, 285, 286, 287, 288, 289, 338, 340, 341, 343, 344, 346, 351, 353, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 500, 505 }

F normal fail { 121, 122, 123, 124, 128, 129, 131, 132, 133, 134, 135, 136, 137, 196, 197, 198, 199, 200, 214, 215, 216, 217, 218, 229, 230, 231, 232, 233, 243, 244, 245, 246, 247, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 290, 309, 327, 347, 367, 385, 476, 477, 529, 530, 531, 532, 534, 535, 536, 537, 538, 539, 541, 542, 562, 563, 564, 566, 567, 599, 600, 601, 606, 661, 662 }

F(-1) timedout fail { 459, 461, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 492 }

F(-2) exception fail { 472, 568, 569, 570, 571, 573, 574, 575, 576, 577, 578, 580, 581, 582, 583, 584, 585, 587, 588, 589, 590, 591, 592, 594, 595, 602, 603, 604, 605 }

2.1.5 Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 43, 44, 46, 47, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 80, 81, 83, 84, 86, 87, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 125, 126, 127, 130, 151, 152, 153, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 171, 174, 177, 180, 183, 248, 249, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 348, 350, 352, 354, 356, 368, 370, 372, 374, 376, 386, 387, 388, 392, 393, 394, 395, 405, 406, 407, 408, 409, 410, 440, 441, 442, 443, 444, 445, 446, 447, 448, 455, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 473, 474, 475, 479, 480, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 506, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 533, 540, 548, 596, 597, 598 }

B grade { 9, 21, 42, 45, 48, 79, 82, 85, 88, 154, 389, 391, 607, 608, 609, 610, 611, 612, 663, 664 }

C grade { }

F normal fail { 121, 122, 123, 124, 128, 129, 131, 132, 133, 134, 135, 136, 137, 143, 144, 145, 146, 147, 148, 149, 150, 170, 172, 173, 175, 176, 178, 179, 181, 182, 184, 196, 197, 198, 199, 200, 214, 215, 216, 217, 218, 229, 230, 231, 232, 233, 243, 244, 245, 246, 247, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 290, 309, 311, 313, 315, 317, 319, 320, 321, 322, 323, 324, 325, 326, 327, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 349, 351, 353, 355, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 369, 371, 373, 375, 377, 378, 379, 380, 381, 382, 383, 384, 385, 396, 397, 398, 399, 400, 401, 402, 403, 404, 411, 449, 450, 451, 452, 453, 454, 456, 457, 458, 459, 472, 476, 477, 478, 481, 500, 501, 502, 503, 504, 505, 507, 529, 530, 531, 532, 534, 535, 536, 537, 538, 539, 541, 542, 543, 544, 545, 546, 547, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 599, 600, 601, 602, 603, 604, 605, 606, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662 }

F(-1) timedout fail { 390, 572 }

F(-2) exception fail { 138, 139, 140, 141, 142, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 234, 235, 236, 237, 238, 239, 240, 241, 242, 310, 312, 314, 316, 318, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439 }

2.1.6 Giac

A grade { 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 120, 126, 127, 130, 138, 139, 140, 141, 142, 151, 152, 153, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 171, 174, 177, 180, 183, 188, 189, 204, 205, 222, 237, 238, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 310, 312, 314, 316, 318, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 348, 350, 351, 352, 353, 354, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 386, 387, 388, 390, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 453, 455, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 475, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 496, 500, 501, 502, 515, 516, 517, 518, 523, 524, 525, 552, 614, 616, 619, 622, 624, 628, 637, 639, 641, 663, 664 }

B grade { 45, 61, 82, 118, 119, 125, 154, 170, 172, 173, 175, 178, 179, 181, 223, 248, 249, 311, 313, 315, 317, 349, 355, 389, 391, 454, 456, 473, 474, 522, 558, 596, 597, 598, 607, 608, 609, 610, 611, 612, 613, 615, 617, 618, 620, 621, 623, 625, 626, 627, 629, 630, 631, 632, 633, 634, 635, 636, 638, 640, 642, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660 }

C grade { 176 }

F normal fail { 1, 2, 121, 122, 123, 124, 128, 129, 131, 132, 133, 134, 135, 136, 137, 143, 144, 145, 146, 147, 148, 149, 150, 185, 186, 187, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 239, 240, 241, 242, 243, 244, 245, 246, 247, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 290, 309, 319, 320, 321, 322, 323, 324, 325, 326, 327, 347, 367, 385, 457, 458, 472, 476, 477, 478, 493, 494, 495, 497, 498, 499, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 519, 520, 521, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 553, 554, 555, 556, 557, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 599, 600, 601, 602, 603, 604, 605, 606, 661, 662 }

F(-1) timedout fail { 449, 450, 479 }

F(-2) exception fail { 182, 184, 451, 452, 459, 643 }

2.1.7 Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 9, 12, 15, 16, 17, 18, 19, 20, 21, 22, 27, 30, 43, 44, 45, 46, 47, 48, 49, 61, 80, 81, 82, 83, 84, 85, 86, 87, 88, 91, 94, 97, 100, 108, 111, 125, 126, 127, 130, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 204, 205, 222, 223, 224, 225, 236, 237, 238, 248, 249, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 451, 452, 455, 456, 457, 458, 460, 465, 468, 469, 470, 471, 473, 474, 475, 478, 485, 496, 552, 565, 596, 597, 598, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 663, 664 }

C grade { }

F normal fail { }

F(-1) timedout fail { 7, 8, 10, 11, 13, 14, 23, 24, 25, 26, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 89, 90, 92, 93, 95, 96, 98, 99, 101, 102, 103, 104, 105, 106, 107, 109, 110, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 128, 129, 131, 132, 133, 134, 135, 136, 137, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 239, 240, 241, 242, 243, 244, 245, 246, 247, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 290, 309, 327, 347, 367, 385, 449, 450, 453, 454, 459, 461, 462, 463, 464, 466, 467, 472, 476, 477, 479, 480, 481, 482, 483, 484, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 599, 600, 601, 602, 603, 604, 605, 606, 661, 662 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 5, 143, 144, 145, 146, 147, 148, 149, 151, 152, 153, 154, 155, 156, 157, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 311, 313, 315, 317, 322, 323, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 339, 342, 345, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 386, 388, 390, 392, 394, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 440, 441, 442, 443, 444, 445, 446, 447, 448, 456, 457, 460, 461, 462, 463, 464, 465, 466, 493, 613, 615, 618, 620, 638, 640, 643, 645 }

B grade { 138, 139, 140, 141, 248, 249, 310, 312, 314, 316, 387, 389, 391, 393, 395, 412, 413, 414, 415, 416, 417, 418, 422, 423, 424, 425, 426, 427, 431, 432, 433, 434, 435, 436, 437, 494, 495, 496, 497, 544, 596, 597, 598, 607, 608, 609, 610, 611, 612, 614, 616, 617, 621, 622, 624, 631, 633, 639, 641, 642, 646, 647, 649, 655, 657, 663, 664 }

C grade { 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 338, 340, 341, 343, 344, 346 }

F normal fail { 1, 2, 3, 4, 9, 10, 11, 12, 13, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 252, 253, 254, 255, 271, 290, 327, 347, 367, 385, 449, 450, 451, 452, 453, 454, 455, 459, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 481, 482, 483, 484, 485, 486, 487, 488, 490, 491, 500, 501, 502, 503, 504, 505, 506, 507, 510, 511, 512, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 546, 547, 556, 557, 558, 559, 560, 561, 562, 563, 564, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 599, 602, 603, 604, 605, 661, 662 }

F(-1) timedout fail { 6, 7, 8, 14, 15, 16, 17, 18, 19, 20, 21, 22, 142, 150, 250, 251, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 309, 318, 319, 320, 321, 324, 325, 326, 419, 420, 421, 428, 429, 430, 438, 439, 458, 480, 489, 492, 508, 509, 513, 514, 543, 545, 548, 549, 550, 551, 552, 553, 554, 555, 565, 566, 567, 600, 601, 606, 619, 623, 625, 626, 627, 628, 629, 630, 632, 634, 635, 636, 637, 644, 648, 650, 651, 652, 653, 654, 656, 658, 659, 660 }

F(-2) exception fail { 498, 499 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	35	39	46	53	0	0	40
N.S.	1	1.00	0.67	0.75	0.88	1.02	0.00	0.00	0.77
time (sec)	N/A	0.174	0.049	0.773	0.213	0.259	0.000	0.000	8.335

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	29	14	28	0	0	29
N.S.	1	1.00	1.00	1.16	0.56	1.12	0.00	0.00	1.16
time (sec)	N/A	0.144	0.007	0.634	0.213	0.269	0.000	0.000	8.181

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	17	21	0	14	21
N.S.	1	1.00	1.00	0.96	0.74	0.91	0.00	0.61	0.91
time (sec)	N/A	0.142	0.010	0.582	0.221	0.270	0.000	0.316	8.144

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	83	46	41	38	54	0	52	51
N.S.	1	1.08	0.60	0.53	0.49	0.70	0.00	0.68	0.66
time (sec)	N/A	0.219	0.255	0.585	0.215	0.282	0.000	0.334	8.277

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	53	48	36	37	46	48	38	51
N.S.	1	1.10	1.00	0.75	0.77	0.96	1.00	0.79	1.06
time (sec)	N/A	0.213	0.019	0.419	0.307	0.273	0.071	0.301	0.054

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	79	71	39	31	83	13	0	23	59
N.S.	1	0.90	0.49	0.39	1.05	0.16	0.00	0.29	0.75
time (sec)	N/A	0.208	1.020	0.187	0.225	0.285	0.000	0.299	8.222

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	49	39	36	13	13	0	29	0
N.S.	1	0.62	0.49	0.46	0.16	0.16	0.00	0.37	0.00
time (sec)	N/A	0.205	1.007	4.595	0.222	0.283	0.000	0.298	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	49	39	36	13	13	0	29	0
N.S.	1	0.62	0.49	0.46	0.16	0.16	0.00	0.37	0.00
time (sec)	N/A	0.197	1.011	3.115	0.222	0.274	0.000	0.305	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	B	A	F	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	36	36	88	23	52	13	0	22	33
N.S.	1	1.00	2.44	0.64	1.44	0.36	0.00	0.61	0.92
time (sec)	N/A	0.176	0.327	0.093	0.217	0.255	0.000	0.305	8.399

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	49	39	36	13	13	0	29	0
N.S.	1	0.62	0.49	0.46	0.16	0.16	0.00	0.37	0.00
time (sec)	N/A	0.191	1.007	1.931	0.215	0.269	0.000	0.314	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	50	36	33	10	10	0	20	0
N.S.	1	0.68	0.49	0.45	0.14	0.14	0.00	0.27	0.00
time (sec)	N/A	0.185	0.006	1.556	0.215	0.257	0.000	0.291	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	A	A	F	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	75	45	454	26	96	11	0	28	109
N.S.	1	0.60	6.05	0.35	1.28	0.15	0.00	0.37	1.45
time (sec)	N/A	0.195	0.692	0.062	0.210	0.256	0.000	0.304	8.371

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	47	38	36	14	14	0	29	0
N.S.	1	0.61	0.49	0.47	0.18	0.18	0.00	0.38	0.00
time (sec)	N/A	0.196	1.010	2.390	0.215	0.254	0.000	0.315	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	44	37	34	15	15	0	26	0
N.S.	1	0.59	0.50	0.46	0.20	0.20	0.00	0.35	0.00
time (sec)	N/A	0.193	1.007	3.159	0.220	0.255	0.000	0.300	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	75	45	178	28	99	17	0	43	112
N.S.	1	0.60	2.37	0.37	1.32	0.23	0.00	0.57	1.49
time (sec)	N/A	0.198	0.220	0.074	0.224	0.260	0.000	0.289	8.356

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	47	37	34	13	13	0	30	33
N.S.	1	0.61	0.48	0.44	0.17	0.17	0.00	0.39	0.43
time (sec)	N/A	0.201	1.012	4.524	0.214	0.253	0.000	0.293	8.230

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	49	39	35	15	15	0	31	35
N.S.	1	0.62	0.49	0.44	0.19	0.19	0.00	0.39	0.44
time (sec)	N/A	0.197	1.009	5.245	0.220	0.258	0.000	0.320	8.169

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	79	39	37	22	86	13	0	30	33
N.S.	1	0.49	0.47	0.28	1.09	0.16	0.00	0.38	0.42
time (sec)	N/A	0.185	1.011	0.056	0.221	0.256	0.000	0.319	8.173

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	49	39	35	15	15	0	31	35
N.S.	1	0.62	0.49	0.44	0.19	0.19	0.00	0.39	0.44
time (sec)	N/A	0.192	1.008	7.162	0.221	0.285	0.000	0.297	8.172

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	49	39	35	15	15	0	31	35
N.S.	1	0.62	0.49	0.44	0.19	0.19	0.00	0.39	0.44
time (sec)	N/A	0.197	1.010	8.955	0.219	0.248	0.000	0.296	8.271

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	79	49	39	24	117	15	0	31	35
N.S.	1	0.62	0.49	0.30	1.48	0.19	0.00	0.39	0.44
time (sec)	N/A	0.197	1.013	0.066	0.218	0.240	0.000	0.306	8.136

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	49	39	35	15	15	0	31	35
N.S.	1	0.62	0.49	0.44	0.19	0.19	0.00	0.39	0.44
time (sec)	N/A	0.197	1.014	11.827	0.243	0.239	0.000	0.330	8.206

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	75	61	58	35	35	0	67	0
N.S.	1	0.45	0.37	0.35	0.21	0.21	0.00	0.40	0.00
time (sec)	N/A	0.223	1.017	8.139	0.234	0.252	0.000	0.287	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	119	78	113	42	114	35	0	67	0
N.S.	1	0.66	0.95	0.35	0.96	0.29	0.00	0.56	0.00
time (sec)	N/A	0.229	0.515	0.118	0.227	0.252	0.000	0.311	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	75	61	58	35	35	0	67	0
N.S.	1	0.45	0.37	0.35	0.21	0.21	0.00	0.40	0.00
time (sec)	N/A	0.222	1.015	6.202	0.218	0.264	0.000	0.288	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	75	61	58	35	35	0	67	0
N.S.	1	0.45	0.37	0.35	0.21	0.21	0.00	0.40	0.00
time (sec)	N/A	0.220	1.019	5.336	0.218	0.254	0.000	0.324	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	78	73	113	31	83	35	0	45	46
N.S.	1	0.94	1.45	0.40	1.06	0.45	0.00	0.58	0.59
time (sec)	N/A	0.209	0.466	0.100	0.247	0.253	0.000	0.306	8.319

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	75	61	58	35	35	0	67	0
N.S.	1	0.45	0.37	0.35	0.21	0.21	0.00	0.40	0.00
time (sec)	N/A	0.211	1.015	3.764	0.231	0.248	0.000	0.284	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	75	61	58	35	35	0	67	0
N.S.	1	0.45	0.37	0.35	0.21	0.21	0.00	0.40	0.00
time (sec)	N/A	0.221	1.014	3.145	0.209	0.259	0.000	0.308	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	A	A	F	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	36	38	112	23	52	35	0	44	36
N.S.	1	1.06	3.11	0.64	1.44	0.97	0.00	1.22	1.00
time (sec)	N/A	0.176	0.709	0.087	0.212	0.249	0.000	0.308	8.485

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	75	61	58	35	35	0	67	0
N.S.	1	0.45	0.37	0.35	0.21	0.21	0.00	0.40	0.00
time (sec)	N/A	0.215	1.014	1.858	0.205	0.274	0.000	0.312	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	78	59	56	32	32	0	64	0
N.S.	1	0.48	0.36	0.35	0.20	0.20	0.00	0.40	0.00
time (sec)	N/A	0.220	1.012	1.524	0.206	0.261	0.000	0.302	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	160	74	60	54	152	32	0	65	0
N.S.	1	0.46	0.38	0.34	0.95	0.20	0.00	0.41	0.00
time (sec)	N/A	0.221	1.016	0.083	0.205	0.270	0.000	0.311	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	73	61	58	37	37	0	67	0
N.S.	1	0.44	0.37	0.35	0.22	0.22	0.00	0.41	0.00
time (sec)	N/A	0.218	1.014	2.433	0.220	0.266	0.000	0.329	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	71	61	58	37	37	0	65	0
N.S.	1	0.44	0.37	0.36	0.23	0.23	0.00	0.40	0.00
time (sec)	N/A	0.220	1.016	3.132	0.202	0.261	0.000	0.312	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	73	62	59	156	38	0	85	0
N.S.	1	0.45	0.39	0.37	0.97	0.24	0.00	0.53	0.00
time (sec)	N/A	0.225	1.015	0.109	0.217	0.262	0.000	0.286	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	73	61	58	37	37	0	69	0
N.S.	1	0.44	0.37	0.35	0.22	0.22	0.00	0.42	0.00
time (sec)	N/A	0.221	1.013	4.434	0.212	0.240	0.000	0.300	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	71	61	58	37	37	0	68	0
N.S.	1	0.44	0.37	0.36	0.23	0.23	0.00	0.42	0.00
time (sec)	N/A	0.219	1.015	5.162	0.217	0.237	0.000	0.319	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	162	72	612	59	220	39	0	86	0
N.S.	1	0.44	3.78	0.36	1.36	0.24	0.00	0.53	0.00
time (sec)	N/A	0.224	0.820	0.091	0.216	0.238	0.000	0.287	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	73	61	58	37	37	0	70	0
N.S.	1	0.44	0.37	0.35	0.22	0.22	0.00	0.42	0.00
time (sec)	N/A	0.215	1.012	7.541	0.219	0.258	0.000	0.343	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	70	61	58	37	37	0	67	0
N.S.	1	0.43	0.38	0.36	0.23	0.23	0.00	0.41	0.00
time (sec)	N/A	0.212	1.013	8.781	0.201	0.263	0.000	0.299	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	161	74	266	52	253	39	0	85	0
N.S.	1	0.46	1.65	0.32	1.57	0.24	0.00	0.53	0.00
time (sec)	N/A	0.222	0.309	0.089	0.223	0.263	0.000	0.408	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	73	61	57	37	37	0	69	151
N.S.	1	0.44	0.37	0.35	0.22	0.22	0.00	0.42	0.92
time (sec)	N/A	0.219	1.012	12.025	0.216	0.258	0.000	0.318	8.227

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	75	61	57	37	37	0	69	151
N.S.	1	0.45	0.37	0.34	0.22	0.22	0.00	0.41	0.90
time (sec)	N/A	0.219	1.016	13.024	0.207	0.262	0.000	0.281	8.253

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	41	41	59	41	148	35	0	68	151
N.S.	1	1.00	1.44	1.00	3.61	0.85	0.00	1.66	3.68
time (sec)	N/A	0.179	1.011	0.087	0.220	0.269	0.000	0.291	8.223

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	75	61	57	37	37	0	69	151
N.S.	1	0.45	0.37	0.34	0.22	0.22	0.00	0.41	0.90
time (sec)	N/A	0.219	1.013	16.682	0.209	0.257	0.000	0.330	8.163

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	75	61	57	37	37	0	69	151
N.S.	1	0.45	0.37	0.34	0.22	0.22	0.00	0.41	0.90
time (sec)	N/A	0.220	1.014	19.193	0.219	0.255	0.000	0.329	8.118

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	84	75	61	44	179	37	0	69	151
N.S.	1	0.89	0.73	0.52	2.13	0.44	0.00	0.82	1.80
time (sec)	N/A	0.200	1.012	0.097	0.210	0.261	0.000	0.394	8.198

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	75	61	57	37	37	0	69	151
N.S.	1	0.45	0.37	0.34	0.22	0.22	0.00	0.41	0.90
time (sec)	N/A	0.219	1.014	24.284	0.201	0.276	0.000	0.285	8.278

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	101	83	80	57	57	0	105	0
N.S.	1	0.40	0.33	0.31	0.22	0.22	0.00	0.41	0.00
time (sec)	N/A	0.246	1.023	14.543	0.206	0.281	0.000	0.292	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	101	83	80	57	57	0	105	0
N.S.	1	0.40	0.33	0.31	0.22	0.22	0.00	0.41	0.00
time (sec)	N/A	0.243	1.022	13.043	0.209	0.269	0.000	0.292	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	160	107	135	53	145	57	0	105	0
N.S.	1	0.67	0.84	0.33	0.91	0.36	0.00	0.66	0.00
time (sec)	N/A	0.262	0.809	0.281	0.213	0.263	0.000	0.301	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	101	83	80	57	57	0	105	0
N.S.	1	0.40	0.33	0.31	0.22	0.22	0.00	0.41	0.00
time (sec)	N/A	0.244	1.021	9.722	0.203	0.263	0.000	0.283	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	101	83	80	57	57	0	105	0
N.S.	1	0.40	0.33	0.31	0.22	0.22	0.00	0.41	0.00
time (sec)	N/A	0.242	1.016	8.354	0.221	0.251	0.000	0.284	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	119	88	135	42	114	57	0	105	0
N.S.	1	0.74	1.13	0.35	0.96	0.48	0.00	0.88	0.00
time (sec)	N/A	0.238	0.751	0.172	0.223	0.241	0.000	0.293	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	101	83	80	57	57	0	105	0
N.S.	1	0.40	0.33	0.31	0.22	0.22	0.00	0.41	0.00
time (sec)	N/A	0.238	1.018	6.322	0.210	0.253	0.000	0.322	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	101	83	80	57	57	0	105	0
N.S.	1	0.40	0.33	0.31	0.22	0.22	0.00	0.41	0.00
time (sec)	N/A	0.241	1.016	5.226	0.203	0.243	0.000	0.284	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	78	73	135	31	83	57	0	67	0
N.S.	1	0.94	1.73	0.40	1.06	0.73	0.00	0.86	0.00
time (sec)	N/A	0.208	0.693	0.110	0.211	0.248	0.000	0.285	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	101	83	80	57	57	0	105	0
N.S.	1	0.40	0.33	0.31	0.22	0.22	0.00	0.41	0.00
time (sec)	N/A	0.237	1.018	3.750	0.230	0.257	0.000	0.289	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	98	83	80	56	56	0	104	0
N.S.	1	0.39	0.33	0.32	0.22	0.22	0.00	0.41	0.00
time (sec)	N/A	0.238	1.017	2.852	0.212	0.265	0.000	0.295	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	A	A	F	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	36	38	82	23	52	57	0	66	36
N.S.	1	1.06	2.28	0.64	1.44	1.58	0.00	1.83	1.00
time (sec)	N/A	0.178	1.017	0.105	0.215	0.285	0.000	0.321	8.214

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	98	83	80	56	56	0	104	0
N.S.	1	0.39	0.33	0.32	0.22	0.22	0.00	0.41	0.00
time (sec)	N/A	0.233	1.019	1.913	0.212	0.285	0.000	0.293	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	101	81	78	53	53	0	101	0
N.S.	1	0.41	0.33	0.32	0.21	0.21	0.00	0.41	0.00
time (sec)	N/A	0.246	1.014	1.470	0.192	0.255	0.000	0.297	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	251	98	82	75	206	55	0	104	0
N.S.	1	0.39	0.33	0.30	0.82	0.22	0.00	0.41	0.00
time (sec)	N/A	0.235	1.019	0.082	0.206	0.274	0.000	0.282	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	97	83	80	59	59	0	105	0
N.S.	1	0.39	0.33	0.32	0.24	0.24	0.00	0.42	0.00
time (sec)	N/A	0.246	1.018	2.378	0.202	0.289	0.000	0.306	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	97	83	80	59	59	0	103	0
N.S.	1	0.39	0.33	0.32	0.24	0.24	0.00	0.41	0.00
time (sec)	N/A	0.242	1.017	2.814	0.217	0.294	0.000	0.292	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	252	97	85	81	214	61	0	124	0
N.S.	1	0.38	0.34	0.32	0.85	0.24	0.00	0.49	0.00
time (sec)	N/A	0.245	1.018	0.105	0.213	0.274	0.000	0.314	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	95	83	80	59	59	0	107	0
N.S.	1	0.38	0.33	0.32	0.24	0.24	0.00	0.43	0.00
time (sec)	N/A	0.243	1.019	4.334	0.201	0.258	0.000	0.304	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	97	83	80	59	59	0	106	0
N.S.	1	0.39	0.33	0.32	0.24	0.24	0.00	0.42	0.00
time (sec)	N/A	0.246	1.017	5.160	0.214	0.284	0.000	0.297	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	99	276	82	282	61	0	126	0
N.S.	1	0.39	1.10	0.33	1.12	0.24	0.00	0.50	0.00
time (sec)	N/A	0.242	0.804	0.122	0.205	0.289	0.000	0.299	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	94	83	80	59	59	0	107	0
N.S.	1	0.38	0.33	0.32	0.24	0.24	0.00	0.43	0.00
time (sec)	N/A	0.242	1.018	7.413	0.203	0.286	0.000	0.290	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	93	83	80	59	59	0	105	0
N.S.	1	0.38	0.34	0.32	0.24	0.24	0.00	0.43	0.00
time (sec)	N/A	0.244	1.016	8.876	0.210	0.262	0.000	0.285	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	99	279	82	313	61	0	127	0
N.S.	1	0.39	1.11	0.33	1.24	0.24	0.00	0.50	0.00
time (sec)	N/A	0.245	0.748	0.223	0.212	0.268	0.000	0.284	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	99	83	80	59	59	0	108	0
N.S.	1	0.39	0.33	0.32	0.23	0.23	0.00	0.43	0.00
time (sec)	N/A	0.249	1.017	11.737	0.203	0.268	0.000	0.294	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	93	83	80	59	59	0	106	0
N.S.	1	0.38	0.34	0.32	0.24	0.24	0.00	0.43	0.00
time (sec)	N/A	0.245	1.012	12.872	0.214	0.261	0.000	0.301	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	252	96	696	81	342	61	0	125	0
N.S.	1	0.38	2.76	0.32	1.36	0.24	0.00	0.50	0.00
time (sec)	N/A	0.246	0.938	0.326	0.223	0.277	0.000	0.290	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	99	83	80	59	59	0	108	0
N.S.	1	0.39	0.33	0.32	0.23	0.23	0.00	0.43	0.00
time (sec)	N/A	0.241	1.016	17.542	0.203	0.275	0.000	0.287	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	94	83	80	59	59	0	105	0
N.S.	1	0.38	0.33	0.32	0.24	0.24	0.00	0.42	0.00
time (sec)	N/A	0.240	1.015	20.374	0.205	0.272	0.000	0.315	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	251	98	268	72	374	61	0	123	0
N.S.	1	0.39	1.07	0.29	1.49	0.24	0.00	0.49	0.00
time (sec)	N/A	0.252	0.495	0.632	0.227	0.265	0.000	0.296	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	97	83	79	59	59	0	107	231
N.S.	1	0.39	0.33	0.31	0.24	0.24	0.00	0.43	0.92
time (sec)	N/A	0.246	1.018	25.488	0.210	0.254	0.000	0.311	8.348

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	99	83	79	59	59	0	107	231
N.S.	1	0.39	0.33	0.31	0.23	0.23	0.00	0.42	0.91
time (sec)	N/A	0.239	1.018	27.049	0.204	0.266	0.000	0.317	8.362

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	41	41	81	58	210	57	0	106	231
N.S.	1	1.00	1.98	1.41	5.12	1.39	0.00	2.59	5.63
time (sec)	N/A	0.182	1.016	1.203	0.224	0.276	0.000	0.295	8.367

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	99	83	79	59	59	0	107	231
N.S.	1	0.39	0.33	0.31	0.23	0.23	0.00	0.42	0.91
time (sec)	N/A	0.245	1.018	33.178	0.214	0.262	0.000	0.298	8.346

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	101	83	79	59	59	0	107	231
N.S.	1	0.40	0.33	0.31	0.23	0.23	0.00	0.42	0.91
time (sec)	N/A	0.240	1.015	38.250	0.230	0.272	0.000	0.300	8.448

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	84	75	83	68	241	59	0	107	231
N.S.	1	0.89	0.99	0.81	2.87	0.70	0.00	1.27	2.75
time (sec)	N/A	0.199	1.015	2.323	0.215	0.257	0.000	0.293	8.360

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	101	83	79	59	59	0	107	231
N.S.	1	0.40	0.33	0.31	0.23	0.23	0.00	0.42	0.91
time (sec)	N/A	0.241	1.013	46.670	0.207	0.265	0.000	0.300	8.343

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	101	83	79	59	59	0	107	231
N.S.	1	0.40	0.33	0.31	0.23	0.23	0.00	0.42	0.91
time (sec)	N/A	0.250	1.016	50.938	0.208	0.281	0.000	0.293	8.340

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	128	103	83	66	272	59	0	107	231
N.S.	1	0.80	0.65	0.52	2.12	0.46	0.00	0.84	1.80
time (sec)	N/A	0.216	1.025	4.142	0.213	0.264	0.000	0.296	8.352

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	240	163	131	77	109	123	0	146	0
N.S.	1	0.68	0.55	0.32	0.45	0.51	0.00	0.61	0.00
time (sec)	N/A	0.343	1.042	4.101	0.301	0.259	0.000	0.296	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	235	153	128	74	106	106	0	143	0
N.S.	1	0.65	0.54	0.31	0.45	0.45	0.00	0.61	0.00
time (sec)	N/A	0.332	1.026	3.208	0.283	0.257	0.000	0.301	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	44	44	51	22	15	13	0	22	33
N.S.	1	1.00	1.16	0.50	0.34	0.30	0.00	0.50	0.75
time (sec)	N/A	0.186	0.110	0.335	0.214	0.264	0.000	0.309	8.425

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	202	146	109	47	98	304	0	124	0
N.S.	1	0.72	0.54	0.23	0.49	1.50	0.00	0.61	0.00
time (sec)	N/A	0.313	1.025	2.265	0.300	0.278	0.000	0.303	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	202	143	109	47	98	299	0	122	0
N.S.	1	0.71	0.54	0.23	0.49	1.48	0.00	0.60	0.00
time (sec)	N/A	0.313	1.016	1.876	0.301	0.264	0.000	0.289	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	80	55	134	31	43	18	0	32	48
N.S.	1	0.69	1.68	0.39	0.54	0.22	0.00	0.40	0.60
time (sec)	N/A	0.193	0.171	0.447	0.223	0.245	0.000	0.305	8.571

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	238	161	133	93	106	103	0	131	0
N.S.	1	0.68	0.56	0.39	0.45	0.43	0.00	0.55	0.00
time (sec)	N/A	0.331	1.024	2.737	0.287	0.276	0.000	0.310	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	243	158	140	94	106	143	0	125	0
N.S.	1	0.65	0.58	0.39	0.44	0.59	0.00	0.51	0.00
time (sec)	N/A	0.322	1.025	3.569	0.289	0.275	0.000	0.304	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	125	65	175	44	73	33	0	50	75
N.S.	1	0.52	1.40	0.35	0.58	0.26	0.00	0.40	0.60
time (sec)	N/A	0.222	0.179	0.472	0.199	0.269	0.000	0.284	8.516

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	280	200	235	89	149	512	0	185	0
N.S.	1	0.71	0.84	0.32	0.53	1.83	0.00	0.66	0.00
time (sec)	N/A	0.366	1.050	3.856	0.311	0.277	0.000	0.312	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	276	191	235	87	146	503	0	176	0
N.S.	1	0.69	0.85	0.32	0.53	1.82	0.00	0.64	0.00
time (sec)	N/A	0.365	1.042	3.245	0.289	0.280	0.000	0.307	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	A	A	F	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	38	38	143	23	16	26	0	24	34
N.S.	1	1.00	3.76	0.61	0.42	0.68	0.00	0.63	0.89
time (sec)	N/A	0.174	0.351	0.059	0.209	0.256	0.000	0.285	8.237

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	277	200	237	90	147	514	0	176	0
N.S.	1	0.72	0.86	0.32	0.53	1.86	0.00	0.64	0.00
time (sec)	N/A	0.365	1.055	1.934	0.294	0.299	0.000	0.302	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	286	207	235	88	145	499	0	177	0
N.S.	1	0.72	0.82	0.31	0.51	1.74	0.00	0.62	0.00
time (sec)	N/A	0.378	1.049	1.527	0.303	0.285	0.000	0.310	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	147	84	790	70	88	90	0	87	0
N.S.	1	0.57	5.37	0.48	0.60	0.61	0.00	0.59	0.00
time (sec)	N/A	0.242	1.099	0.104	0.216	0.259	0.000	0.317	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	316	215	260	115	148	201	0	201	0
N.S.	1	0.68	0.82	0.36	0.47	0.64	0.00	0.64	0.00
time (sec)	N/A	0.403	1.060	2.417	0.440	0.284	0.000	0.299	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	316	212	266	116	150	242	0	184	0
N.S.	1	0.67	0.84	0.37	0.47	0.77	0.00	0.58	0.00
time (sec)	N/A	0.402	1.067	2.796	0.297	0.271	0.000	0.296	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	188	98	901	92	117	119	0	120	0
N.S.	1	0.52	4.79	0.49	0.62	0.63	0.00	0.64	0.00
time (sec)	N/A	0.255	1.333	0.116	0.223	0.253	0.000	0.299	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	359	243	218	105	195	723	0	205	0
N.S.	1	0.68	0.61	0.29	0.54	2.01	0.00	0.57	0.00
time (sec)	N/A	0.437	1.094	5.373	0.288	0.300	0.000	0.316	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	A	A	F	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	78	73	232	31	43	58	0	32	42
N.S.	1	0.94	2.97	0.40	0.55	0.74	0.00	0.41	0.54
time (sec)	N/A	0.211	0.500	0.071	0.197	0.249	0.000	0.312	8.387

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	368	254	229	109	195	734	0	207	0
N.S.	1	0.69	0.62	0.30	0.53	1.99	0.00	0.56	0.00
time (sec)	N/A	0.456	1.106	3.703	0.289	0.276	0.000	0.325	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	360	241	221	107	193	723	0	199	0
N.S.	1	0.67	0.61	0.30	0.54	2.01	0.00	0.55	0.00
time (sec)	N/A	0.433	1.094	2.812	0.297	0.286	0.000	0.294	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	A	A	F	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	38	38	267	23	16	48	0	24	34
N.S.	1	1.00	7.03	0.61	0.42	1.26	0.00	0.63	0.89
time (sec)	N/A	0.177	0.589	0.059	0.203	0.256	0.000	0.329	8.368

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	359	254	219	112	191	734	0	198	0
N.S.	1	0.71	0.61	0.31	0.53	2.04	0.00	0.55	0.00
time (sec)	N/A	0.441	1.087	1.869	0.306	0.299	0.000	0.349	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	364	269	211	110	189	719	0	199	0
N.S.	1	0.74	0.58	0.30	0.52	1.98	0.00	0.55	0.00
time (sec)	N/A	0.456	1.087	1.534	0.282	0.279	0.000	0.327	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	223	116	96	92	132	178	0	109	0
N.S.	1	0.52	0.43	0.41	0.59	0.80	0.00	0.49	0.00
time (sec)	N/A	0.273	1.032	0.117	0.213	0.271	0.000	0.321	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	398	269	242	137	192	311	0	223	0
N.S.	1	0.68	0.61	0.34	0.48	0.78	0.00	0.56	0.00
time (sec)	N/A	0.484	1.095	2.342	0.288	0.276	0.000	0.322	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	398	266	234	138	194	352	0	215	0
N.S.	1	0.67	0.59	0.35	0.49	0.88	0.00	0.54	0.00
time (sec)	N/A	0.469	1.113	2.863	0.296	0.277	0.000	0.321	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	269	132	119	114	163	207	0	143	0
N.S.	1	0.49	0.44	0.42	0.61	0.77	0.00	0.53	0.00
time (sec)	N/A	0.294	1.036	0.135	0.204	0.256	0.000	0.314	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	159	111	453	243	369	0	900	0
N.S.	1	0.51	0.35	1.45	0.78	1.18	0.00	2.88	0.00
time (sec)	N/A	0.325	0.080	0.044	0.205	0.272	0.000	0.360	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	113	131	199	119	159	0	384	0
N.S.	1	0.55	0.64	0.97	0.58	0.78	0.00	1.87	0.00
time (sec)	N/A	0.272	0.061	0.031	0.228	0.265	0.000	0.311	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	67	53	56	35	35	0	83	0
N.S.	1	0.69	0.55	0.58	0.36	0.36	0.00	0.86	0.00
time (sec)	N/A	0.222	0.022	0.025	0.211	0.273	0.000	0.290	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	73	73	62	0	0	0	0	0	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.204	0.022	0.000	0.000	0.000	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	73	73	60	0	0	0	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.204	0.019	0.000	0.000	0.000	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	73	73	60	0	0	0	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.200	0.046	0.000	0.000	0.000	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	77	77	66	0	0	0	0	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.203	0.038	0.000	0.000	0.000	0.000	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	163	110	150	115	163	0	375	207
N.S.	1	0.95	0.64	0.87	0.67	0.95	0.00	2.18	1.20
time (sec)	N/A	0.308	0.085	0.273	0.216	0.269	0.000	0.311	8.388

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	128	77	96	79	108	0	235	137
N.S.	1	0.98	0.59	0.74	0.61	0.83	0.00	1.81	1.05
time (sec)	N/A	0.275	0.059	0.160	0.203	0.247	0.000	0.286	8.363

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	83	51	58	54	70	0	132	85
N.S.	1	0.99	0.61	0.69	0.64	0.83	0.00	1.57	1.01
time (sec)	N/A	0.221	0.047	0.114	0.197	0.272	0.000	0.321	8.311

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	60	60	51	0	0	0	0	0	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.187	0.054	0.000	0.000	0.000	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	60	60	51	0	0	0	0	0	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.188	0.045	0.000	0.000	0.000	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	45	32	31	30	37	0	58	46
N.S.	1	1.10	0.78	0.76	0.73	0.90	0.00	1.41	1.12
time (sec)	N/A	0.178	0.030	0.067	0.203	0.255	0.000	0.316	8.362

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	60	51	0	0	0	0	0	0
N.S.	1	1.03	0.88	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.179	0.048	0.000	0.000	0.000	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	53	55	211	0	0	0	0	0	0
N.S.	1	1.04	3.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.176	0.159	0.000	0.000	0.000	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	63	63	54	0	0	0	0	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.201	0.085	0.000	0.000	0.000	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	58	49	0	0	0	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.186	0.065	0.000	0.000	0.000	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	60	60	51	0	0	0	0	0	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.184	0.061	0.000	0.000	0.000	0.000	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	67	55	0	0	0	0	0	0
N.S.	1	1.05	0.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.203	0.037	0.000	0.000	0.000	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	60	60	51	0	0	0	0	0	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.180	0.061	0.000	0.000	0.000	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	80	78	83	0	254	316	75	1758
N.S.	1	0.99	0.96	1.02	0.00	3.14	3.90	0.93	21.70
time (sec)	N/A	0.248	0.041	0.099	0.000	0.285	1.718	0.399	8.891

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	64	62	60	0	197	223	59	1199
N.S.	1	1.02	0.98	0.95	0.00	3.13	3.54	0.94	19.03
time (sec)	N/A	0.227	0.016	0.072	0.000	0.268	0.841	0.427	8.642

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	42	37	0	129	131	36	174
N.S.	1	1.00	1.11	0.97	0.00	3.39	3.45	0.95	4.58
time (sec)	N/A	0.181	0.009	0.050	0.000	0.284	0.378	0.359	8.245

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	73	66	65	0	223	253	66	1362
N.S.	1	1.06	0.96	0.94	0.00	3.23	3.67	0.96	19.74
time (sec)	N/A	0.244	0.018	0.071	0.000	0.288	16.912	0.359	8.766

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	95	92	85	0	293	0	93	4281
N.S.	1	1.07	1.03	0.96	0.00	3.29	0.00	1.04	48.10
time (sec)	N/A	0.293	0.022	0.087	0.000	0.307	0.000	0.365	8.782

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	636	546	70	61	0	3402	279	0	4069
N.S.	1	0.86	0.11	0.10	0.00	5.35	0.44	0.00	6.40
time (sec)	N/A	0.989	0.023	0.501	0.000	0.492	147.579	0.000	14.726

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	631	521	70	59	0	2882	196	0	2280
N.S.	1	0.83	0.11	0.09	0.00	4.57	0.31	0.00	3.61
time (sec)	N/A	0.946	0.023	0.042	0.000	0.415	53.135	0.000	1.908

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	558	516	44	43	0	2314	175	0	2695
N.S.	1	0.92	0.08	0.08	0.00	4.15	0.31	0.00	4.83
time (sec)	N/A	0.783	0.015	0.045	0.000	0.309	1.451	0.000	12.275

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	558	496	42	43	0	1542	122	0	2129
N.S.	1	0.89	0.08	0.08	0.00	2.76	0.22	0.00	3.82
time (sec)	N/A	0.767	0.014	0.042	0.000	0.287	0.935	0.000	12.036

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	558	511	43	41	0	1798	158	0	1543
N.S.	1	0.92	0.08	0.07	0.00	3.22	0.28	0.00	2.77
time (sec)	N/A	0.763	0.014	0.042	0.000	0.290	0.748	0.000	10.791

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	558	484	45	40	0	2206	155	0	2597
N.S.	1	0.87	0.08	0.07	0.00	3.95	0.28	0.00	4.65
time (sec)	N/A	0.760	0.020	0.039	0.000	0.328	3.641	0.000	12.305

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	610	532	71	61	0	3225	252	0	2978
N.S.	1	0.87	0.12	0.10	0.00	5.29	0.41	0.00	4.88
time (sec)	N/A	0.828	0.025	0.069	0.000	0.393	2.216	0.000	11.555

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	612	514	75	62	0	3225	0	0	4063
N.S.	1	0.84	0.12	0.10	0.00	5.27	0.00	0.00	6.64
time (sec)	N/A	0.817	0.023	0.070	0.000	0.422	0.000	0.000	13.755

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	37	35	28	27	27	29	29	27
N.S.	1	1.06	1.00	0.80	0.77	0.77	0.83	0.83	0.77
time (sec)	N/A	0.191	0.005	0.074	0.192	0.247	0.057	0.321	0.029

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	23	22	22	22	24	22
N.S.	1	1.00	1.00	0.82	0.79	0.79	0.79	0.86	0.79
time (sec)	N/A	0.180	0.005	0.048	0.200	0.248	0.053	0.293	0.027

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	25	21	18	17	17	15	19	17
N.S.	1	1.19	1.00	0.86	0.81	0.81	0.71	0.90	0.81
time (sec)	N/A	0.177	0.005	0.046	0.216	0.237	0.046	0.296	0.030

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	B	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	25	21	18	17	17	15	19	16
N.S.	1	2.50	2.10	1.80	1.70	1.70	1.50	1.90	1.60
time (sec)	N/A	0.175	0.004	0.042	0.200	0.252	0.047	0.300	0.213

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	33	27	22	23	21	20	24	21
N.S.	1	1.22	1.00	0.81	0.85	0.78	0.74	0.89	0.78
time (sec)	N/A	0.186	0.007	0.056	0.208	0.246	0.064	0.335	8.393

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	40	34	27	28	35	29	36	26
N.S.	1	1.18	1.00	0.79	0.82	1.03	0.85	1.06	0.76
time (sec)	N/A	0.201	0.005	0.067	0.209	0.252	0.077	0.293	8.292

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	47	41	33	35	40	34	41	32
N.S.	1	1.15	1.00	0.80	0.85	0.98	0.83	1.00	0.78
time (sec)	N/A	0.203	0.005	0.070	0.202	0.242	0.090	0.288	0.024

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	141	118	68	94	102	144	96	124
N.S.	1	1.14	0.95	0.55	0.76	0.82	1.16	0.77	1.00
time (sec)	N/A	0.401	0.047	0.074	0.285	0.265	0.334	0.326	0.141

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	139	114	62	92	90	129	94	119
N.S.	1	1.14	0.93	0.51	0.75	0.74	1.06	0.77	0.98
time (sec)	N/A	0.381	0.019	0.061	0.282	0.253	0.328	0.310	0.136

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	135	111	63	89	99	134	91	118
N.S.	1	1.13	0.93	0.53	0.75	0.83	1.13	0.76	0.99
time (sec)	N/A	0.349	0.027	0.059	0.297	0.269	0.321	0.304	0.104

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	129	111	53	85	88	126	87	104
N.S.	1	1.14	0.98	0.47	0.75	0.78	1.12	0.77	0.92
time (sec)	N/A	0.342	0.016	0.056	0.284	0.265	0.315	0.315	0.091

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	129	107	60	84	106	134	86	114
N.S.	1	1.15	0.96	0.54	0.75	0.95	1.20	0.77	1.02
time (sec)	N/A	0.321	0.016	0.056	0.301	0.273	0.300	0.311	8.350

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	129	106	54	84	102	110	86	113
N.S.	1	1.15	0.95	0.48	0.75	0.91	0.98	0.77	1.01
time (sec)	N/A	0.322	0.015	0.065	0.291	0.249	0.298	0.303	8.366

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	128	108	56	84	84	119	86	113
N.S.	1	1.14	0.96	0.50	0.75	0.75	1.06	0.77	1.01
time (sec)	N/A	0.318	0.017	0.051	0.297	0.252	1.207	0.304	8.479

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	128	107	58	84	124	124	86	110
N.S.	1	1.14	0.96	0.52	0.75	1.11	1.11	0.77	0.98
time (sec)	N/A	0.316	0.015	0.055	0.307	0.253	1.151	0.376	0.127

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	140	118	61	89	117	139	91	119
N.S.	1	1.18	0.99	0.51	0.75	0.98	1.17	0.76	1.00
time (sec)	N/A	0.348	0.027	0.070	0.330	0.268	1.167	0.302	8.386

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	140	113	59	89	126	128	91	118
N.S.	1	1.18	0.95	0.50	0.75	1.06	1.08	0.76	0.99
time (sec)	N/A	0.349	0.036	0.066	0.323	0.262	1.041	0.313	8.460

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	152	118	67	96	112	141	98	124
N.S.	1	1.21	0.94	0.53	0.76	0.89	1.12	0.78	0.98
time (sec)	N/A	0.386	0.029	0.072	0.315	0.252	1.197	0.317	0.111

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	152	118	65	96	153	136	98	121
N.S.	1	1.21	0.94	0.52	0.76	1.21	1.08	0.78	0.96
time (sec)	N/A	0.387	0.041	0.079	0.302	0.254	1.035	0.304	8.495

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	412	350	59	44	0	300	26	641	320
N.S.	1	0.85	0.14	0.11	0.00	0.73	0.06	1.56	0.78
time (sec)	N/A	0.623	0.010	0.043	0.000	0.273	0.091	0.332	8.731

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	40	39	33	32	32	37	32	34
N.S.	1	1.03	1.00	0.85	0.82	0.82	0.95	0.82	0.87
time (sec)	N/A	0.208	0.009	0.044	0.284	0.249	0.054	0.298	0.030

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	411	367	41	40	0	295	26	827	304
N.S.	1	0.89	0.10	0.10	0.00	0.72	0.06	2.01	0.74
time (sec)	N/A	0.594	0.014	0.035	0.000	0.276	0.079	0.322	8.612

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	411	349	39	40	0	261	24	640	327
N.S.	1	0.85	0.09	0.10	0.00	0.64	0.06	1.56	0.80
time (sec)	N/A	0.506	0.007	0.037	0.000	0.269	0.075	0.328	8.678

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	19	18	18	27	18	20
N.S.	1	1.00	1.00	0.83	0.78	0.78	1.17	0.78	0.87
time (sec)	N/A	0.169	0.007	0.036	0.277	0.252	0.050	0.318	0.023

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	375	355	40	38	0	295	26	815	304
N.S.	1	0.95	0.11	0.10	0.00	0.79	0.07	2.17	0.81
time (sec)	N/A	0.521	0.008	0.035	0.000	0.282	0.077	0.308	8.524

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	C	C	F	C	A	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	337	42	37	0	285	20	632	327
N.S.	1	1.81	0.23	0.20	0.00	1.53	0.11	3.40	1.76
time (sec)	N/A	0.496	0.016	0.030	0.000	0.267	0.083	0.298	8.506

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	44	55	33	38	34	41	35	36
N.S.	1	1.07	1.34	0.80	0.93	0.83	1.00	0.85	0.88
time (sec)	N/A	0.217	0.012	0.052	0.272	0.250	0.065	0.302	0.032

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	416	372	61	35	0	255	24	829	286
N.S.	1	0.89	0.15	0.08	0.00	0.61	0.06	1.99	0.69
time (sec)	N/A	0.553	0.011	0.058	0.000	0.267	0.086	0.307	8.545

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	418	356	65	38	0	337	31	645	324
N.S.	1	0.85	0.16	0.09	0.00	0.81	0.07	1.54	0.78
time (sec)	N/A	0.527	0.010	0.053	0.000	0.247	0.089	0.312	8.811

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	49	51	38	43	51	48	45	41
N.S.	1	1.02	1.06	0.79	0.90	1.06	1.00	0.94	0.85
time (sec)	N/A	0.227	0.011	0.055	0.284	0.252	0.075	0.297	0.039

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	423	379	54	46	0	338	39	839	318
N.S.	1	0.90	0.13	0.11	0.00	0.80	0.09	1.98	0.75
time (sec)	N/A	0.582	0.013	0.059	0.000	0.270	0.096	0.293	8.722

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	381	339	38	33	0	299	24	0	513
N.S.	1	0.89	0.10	0.09	0.00	0.78	0.06	0.00	1.35
time (sec)	N/A	0.602	0.007	0.041	0.000	0.279	0.065	0.000	9.360

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	19	18	18	27	18	20
N.S.	1	1.00	1.00	0.83	0.78	0.78	1.17	0.78	0.87
time (sec)	N/A	0.169	0.010	0.036	0.281	0.252	0.047	0.385	0.030

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	399	351	37	36	0	261	24	0	351
N.S.	1	0.88	0.09	0.09	0.00	0.65	0.06	0.00	0.88
time (sec)	N/A	0.523	0.007	0.038	0.000	0.257	0.062	0.000	9.294

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	231	238	206	0	0	451	0	0	543
N.S.	1	1.03	0.89	0.00	0.00	1.95	0.00	0.00	2.35
time (sec)	N/A	0.426	0.436	0.000	0.000	0.280	0.000	0.000	9.329

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	171	180	166	0	0	367	0	0	315
N.S.	1	1.05	0.97	0.00	0.00	2.15	0.00	0.00	1.84
time (sec)	N/A	0.329	0.306	0.000	0.000	0.288	0.000	0.000	8.694

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	153	163	132	0	0	303	0	0	193
N.S.	1	1.07	0.86	0.00	0.00	1.98	0.00	0.00	1.26
time (sec)	N/A	0.307	0.242	0.000	0.000	0.266	0.000	0.000	8.356

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	108	119	101	0	0	237	0	96	87
N.S.	1	1.10	0.94	0.00	0.00	2.19	0.00	0.89	0.81
time (sec)	N/A	0.248	0.219	0.000	0.000	0.279	0.000	0.308	8.389

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	83	87	87	0	0	197	0	74	72
N.S.	1	1.05	1.05	0.00	0.00	2.37	0.00	0.89	0.87
time (sec)	N/A	0.214	0.294	0.000	0.000	0.264	0.000	0.327	8.489

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	109	109	105	0	0	566	0	0	88
N.S.	1	1.00	0.96	0.00	0.00	5.19	0.00	0.00	0.81
time (sec)	N/A	0.290	0.174	0.000	0.000	0.288	0.000	0.000	8.366

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	112	115	107	0	0	601	0	0	91
N.S.	1	1.03	0.96	0.00	0.00	5.37	0.00	0.00	0.81
time (sec)	N/A	0.293	0.168	0.000	0.000	0.279	0.000	0.000	8.567

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	88	92	91	0	0	215	0	0	0
N.S.	1	1.05	1.03	0.00	0.00	2.44	0.00	0.00	0.00
time (sec)	N/A	0.234	0.233	0.000	0.000	0.270	0.000	0.000	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	116	127	108	0	0	259	0	0	0
N.S.	1	1.09	0.93	0.00	0.00	2.23	0.00	0.00	0.00
time (sec)	N/A	0.266	0.400	0.000	0.000	0.298	0.000	0.000	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	161	171	141	0	0	325	0	0	0
N.S.	1	1.06	0.88	0.00	0.00	2.02	0.00	0.00	0.00
time (sec)	N/A	0.328	0.510	0.000	0.000	0.315	0.000	0.000	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	199	216	176	0	0	389	0	0	0
N.S.	1	1.09	0.88	0.00	0.00	1.95	0.00	0.00	0.00
time (sec)	N/A	0.406	0.676	0.000	0.000	0.337	0.000	0.000	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	140	140	358	0	0	0	0	0	0
N.S.	1	1.00	2.56	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.339	8.914	0.000	0.000	0.000	0.000	0.000	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	140	140	337	0	0	0	0	0	0
N.S.	1	1.00	2.41	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.261	10.026	0.000	0.000	0.000	0.000	0.000	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	135	135	335	0	0	0	0	0	0
N.S.	1	1.00	2.48	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.247	10.235	0.000	0.000	0.000	0.000	0.000	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	138	138	340	0	0	0	0	0	0
N.S.	1	1.00	2.46	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.290	10.262	0.000	0.000	0.000	0.000	0.000	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	140	140	340	0	0	0	0	0	0
N.S.	1	1.00	2.43	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.287	10.209	0.000	0.000	0.000	0.000	0.000	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	293	285	289	0	0	641	0	0	0
N.S.	1	0.97	0.99	0.00	0.00	2.19	0.00	0.00	0.00
time (sec)	N/A	0.472	0.932	0.000	0.000	0.302	0.000	0.000	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	223	227	220	0	0	535	0	0	0
N.S.	1	1.02	0.99	0.00	0.00	2.40	0.00	0.00	0.00
time (sec)	N/A	0.385	0.664	0.000	0.000	0.279	0.000	0.000	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	204	210	194	0	0	451	0	0	0
N.S.	1	1.03	0.95	0.00	0.00	2.21	0.00	0.00	0.00
time (sec)	N/A	0.351	0.515	0.000	0.000	0.277	0.000	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	150	166	140	0	0	361	0	170	223
N.S.	1	1.11	0.93	0.00	0.00	2.41	0.00	1.13	1.49
time (sec)	N/A	0.293	0.372	0.000	0.000	0.279	0.000	0.349	8.571

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	124	134	114	0	0	297	0	133	115
N.S.	1	1.08	0.92	0.00	0.00	2.40	0.00	1.07	0.93
time (sec)	N/A	0.252	0.641	0.000	0.000	0.256	0.000	0.354	8.525

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	155	169	143	0	0	727	0	0	0
N.S.	1	1.09	0.92	0.00	0.00	4.69	0.00	0.00	0.00
time (sec)	N/A	0.377	0.513	0.000	0.000	0.347	0.000	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	150	157	131	0	0	713	0	0	0
N.S.	1	1.05	0.87	0.00	0.00	4.75	0.00	0.00	0.00
time (sec)	N/A	0.372	0.459	0.000	0.000	0.350	0.000	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	151	159	131	0	0	713	0	0	0
N.S.	1	1.05	0.87	0.00	0.00	4.72	0.00	0.00	0.00
time (sec)	N/A	0.358	0.509	0.000	0.000	0.322	0.000	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	163	177	148	0	0	771	0	0	0
N.S.	1	1.09	0.91	0.00	0.00	4.73	0.00	0.00	0.00
time (sec)	N/A	0.379	0.670	0.000	0.000	0.328	0.000	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	133	143	118	0	0	319	0	0	0
N.S.	1	1.08	0.89	0.00	0.00	2.40	0.00	0.00	0.00
time (sec)	N/A	0.277	0.638	0.000	0.000	0.304	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	162	178	160	0	0	383	0	0	0
N.S.	1	1.10	0.99	0.00	0.00	2.36	0.00	0.00	0.00
time (sec)	N/A	0.321	0.974	0.000	0.000	0.354	0.000	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	216	222	201	0	0	473	0	0	0
N.S.	1	1.03	0.93	0.00	0.00	2.19	0.00	0.00	0.00
time (sec)	N/A	0.385	1.329	0.000	0.000	0.445	0.000	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	255	267	244	0	0	557	0	0	0
N.S.	1	1.05	0.96	0.00	0.00	2.18	0.00	0.00	0.00
time (sec)	N/A	0.460	1.533	0.000	0.000	0.488	0.000	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	141	141	453	0	0	0	0	0	0
N.S.	1	1.00	3.21	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.300	10.583	0.000	0.000	0.000	0.000	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	141	141	410	0	0	0	0	0	0
N.S.	1	1.00	2.91	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.275	10.503	0.000	0.000	0.000	0.000	0.000	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	136	136	408	0	0	0	0	0	0
N.S.	1	1.00	3.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.278	10.480	0.000	0.000	0.000	0.000	0.000	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	139	139	379	0	0	0	0	0	0
N.S.	1	1.00	2.73	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.292	10.354	0.000	0.000	0.000	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	141	141	379	0	0	0	0	0	0
N.S.	1	1.00	2.69	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.304	10.353	0.000	0.000	0.000	0.000	0.000	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	171	191	138	0	0	303	0	0	0
N.S.	1	1.12	0.81	0.00	0.00	1.77	0.00	0.00	0.00
time (sec)	N/A	0.380	0.275	0.000	0.000	0.275	0.000	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	121	133	101	0	0	241	0	0	0
N.S.	1	1.10	0.83	0.00	0.00	1.99	0.00	0.00	0.00
time (sec)	N/A	0.283	0.228	0.000	0.000	0.268	0.000	0.000	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	104	114	91	0	0	203	0	0	0
N.S.	1	1.10	0.88	0.00	0.00	1.95	0.00	0.00	0.00
time (sec)	N/A	0.253	0.185	0.000	0.000	0.275	0.000	0.000	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	68	69	68	0	0	161	0	59	55
N.S.	1	1.01	1.00	0.00	0.00	2.37	0.00	0.87	0.81
time (sec)	N/A	0.208	0.131	0.000	0.000	0.267	0.000	0.310	8.530

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	43	43	41	0	0	118	0	74	34
N.S.	1	1.00	0.95	0.00	0.00	2.74	0.00	1.72	0.79
time (sec)	N/A	0.180	0.079	0.000	0.000	0.262	0.000	0.316	8.622

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	44	44	44	0	0	124	0	0	36
N.S.	1	1.00	1.00	0.00	0.00	2.82	0.00	0.00	0.82
time (sec)	N/A	0.185	0.078	0.000	0.000	0.255	0.000	0.000	8.483

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	72	74	72	0	0	179	0	0	56
N.S.	1	1.03	1.00	0.00	0.00	2.49	0.00	0.00	0.78
time (sec)	N/A	0.214	0.149	0.000	0.000	0.260	0.000	0.000	8.507

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	108	118	91	0	0	221	0	0	0
N.S.	1	1.09	0.84	0.00	0.00	2.05	0.00	0.00	0.00
time (sec)	N/A	0.268	0.235	0.000	0.000	0.274	0.000	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	145	163	110	0	0	263	0	0	0
N.S.	1	1.12	0.76	0.00	0.00	1.81	0.00	0.00	0.00
time (sec)	N/A	0.347	0.375	0.000	0.000	0.282	0.000	0.000	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	192	218	141	0	0	327	0	0	0
N.S.	1	1.14	0.73	0.00	0.00	1.70	0.00	0.00	0.00
time (sec)	N/A	0.432	0.497	0.000	0.000	0.313	0.000	0.000	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	140	140	168	0	0	0	0	0	0
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.305	10.069	0.000	0.000	0.000	0.000	0.000	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	140	140	168	0	0	0	0	0	0
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.273	10.064	0.000	0.000	0.000	0.000	0.000	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	135	135	163	0	0	0	0	0	0
N.S.	1	1.00	1.21	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.249	10.051	0.000	0.000	0.000	0.000	0.000	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	138	138	343	0	0	0	0	0	0
N.S.	1	1.00	2.49	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.292	10.228	0.000	0.000	0.000	0.000	0.000	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	140	140	342	0	0	0	0	0	0
N.S.	1	1.00	2.44	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.291	10.204	0.000	0.000	0.000	0.000	0.000	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	195	206	170	0	0	591	0	0	0
N.S.	1	1.06	0.87	0.00	0.00	3.03	0.00	0.00	0.00
time (sec)	N/A	0.401	0.651	0.000	0.000	0.309	0.000	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	137	150	131	0	0	459	0	0	0
N.S.	1	1.09	0.96	0.00	0.00	3.35	0.00	0.00	0.00
time (sec)	N/A	0.297	0.427	0.000	0.000	0.305	0.000	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	120	130	95	0	0	387	0	0	84
N.S.	1	1.08	0.79	0.00	0.00	3.22	0.00	0.00	0.70
time (sec)	N/A	0.271	0.388	0.000	0.000	0.283	0.000	0.000	8.611

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	38	0	68	0	45	38
N.S.	1	1.00	1.00	0.97	0.00	1.74	0.00	1.15	0.97
time (sec)	N/A	0.177	0.239	4.661	0.000	0.265	0.000	0.489	8.649

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	37	0	67	0	45	37
N.S.	1	1.00	1.00	0.97	0.00	1.76	0.00	1.18	0.97
time (sec)	N/A	0.172	0.257	2.474	0.000	0.266	0.000	0.401	8.681

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	92	92	91	0	0	389	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	4.23	0.00	0.00	0.00
time (sec)	N/A	0.240	0.423	0.000	0.000	0.297	0.000	0.000	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	142	155	125	0	0	485	0	0	0
N.S.	1	1.09	0.88	0.00	0.00	3.42	0.00	0.00	0.00
time (sec)	N/A	0.326	0.505	0.000	0.000	0.306	0.000	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	198	209	166	0	0	615	0	0	0
N.S.	1	1.06	0.84	0.00	0.00	3.11	0.00	0.00	0.00
time (sec)	N/A	0.411	0.663	0.000	0.000	0.353	0.000	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	256	265	210	0	0	705	0	0	0
N.S.	1	1.04	0.82	0.00	0.00	2.75	0.00	0.00	0.00
time (sec)	N/A	0.524	0.920	0.000	0.000	0.411	0.000	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	143	143	340	0	0	0	0	0	0
N.S.	1	1.00	2.38	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.301	10.307	0.000	0.000	0.000	0.000	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	143	143	362	0	0	0	0	0	0
N.S.	1	1.00	2.53	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.269	10.317	0.000	0.000	0.000	0.000	0.000	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	138	138	359	0	0	0	0	0	0
N.S.	1	1.00	2.60	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.262	10.336	0.000	0.000	0.000	0.000	0.000	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	141	141	407	0	0	0	0	0	0
N.S.	1	1.00	2.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.295	10.525	0.000	0.000	0.000	0.000	0.000	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	143	143	405	0	0	0	0	0	0
N.S.	1	1.00	2.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.293	10.437	0.000	0.000	0.000	0.000	0.000	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	70	301	110	241	1459	449	260
N.S.	1	1.00	0.69	2.98	1.09	2.39	14.45	4.45	2.57
time (sec)	N/A	0.254	0.617	0.194	0.195	0.263	0.823	0.325	8.493

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	35	51	50	71	299	119	89
N.S.	1	1.00	0.67	0.98	0.96	1.37	5.75	2.29	1.71
time (sec)	N/A	0.199	0.049	0.059	0.195	0.244	0.373	0.336	8.346

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	173	173	84	0	0	0	0	0	0
N.S.	1	1.00	0.49	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.318	0.245	0.000	0.000	0.000	0.000	0.000	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	315	322	78	0	0	0	0	0	0
N.S.	1	1.02	0.25	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.592	0.378	0.000	0.000	0.000	0.000	0.000	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	158	158	357	0	0	0	0	0	0
N.S.	1	1.00	2.26	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.312	1.854	0.000	0.000	0.000	0.000	0.000	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	157	157	181	0	0	0	0	0	0
N.S.	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.302	1.541	0.000	0.000	0.000	0.000	0.000	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	157	157	181	0	0	0	0	0	0
N.S.	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.314	1.737	0.000	0.000	0.000	0.000	0.000	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	160	160	221	0	0	0	0	0	0
N.S.	1	1.00	1.38	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.313	6.927	0.000	0.000	0.000	0.000	0.000	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	155	155	179	0	0	0	0	0	0
N.S.	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.300	0.259	0.000	0.000	0.000	0.000	0.000	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	224	222	162	0	0	0	0	0	0
N.S.	1	0.99	0.72	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.341	0.331	0.000	0.000	0.000	0.000	0.000	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	161	162	162	0	0	0	0	0	0
N.S.	1	1.01	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.253	0.270	0.000	0.000	0.000	0.000	0.000	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	130	130	138	0	0	0	0	0	0
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.219	0.150	0.000	0.000	0.000	0.000	0.000	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	138	138	166	0	0	0	0	0	0
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.269	0.403	0.000	0.000	0.000	0.000	0.000	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	138	138	166	0	0	0	0	0	0
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.277	0.426	0.000	0.000	0.000	0.000	0.000	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	138	138	166	0	0	0	0	0	0
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.258	0.415	0.000	0.000	0.000	0.000	0.000	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	133	133	161	0	0	0	0	0	0
N.S.	1	1.00	1.21	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.256	0.135	0.000	0.000	0.000	0.000	0.000	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	157	157	157	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.311	0.217	0.000	0.000	0.000	0.000	0.000	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	136	136	164	0	0	0	0	0	0
N.S.	1	1.00	1.21	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.275	0.407	0.000	0.000	0.000	0.000	0.000	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	138	138	166	0	0	0	0	0	0
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.272	0.424	0.000	0.000	0.000	0.000	0.000	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	164	162	162	0	0	0	0	0	0
N.S.	1	0.99	0.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.290	0.275	0.000	0.000	0.000	0.000	0.000	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	138	138	166	0	0	0	0	0	0
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.273	0.422	0.000	0.000	0.000	0.000	0.000	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	138	138	166	0	0	0	0	0	0
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.272	0.412	0.000	0.000	0.000	0.000	0.000	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	168	166	164	0	0	0	0	0	0
N.S.	1	0.99	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.293	0.311	0.000	0.000	0.000	0.000	0.000	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	32	32	34	0	0	0	0	0	0
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.165	0.031	0.000	0.000	0.000	0.000	0.000	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	33	24	25	24	31	22	24	25
N.S.	1	1.10	0.80	0.83	0.80	1.03	0.73	0.80	0.83
time (sec)	N/A	0.173	0.018	0.070	0.274	0.250	0.052	0.289	0.026

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	18	18	19	18	23	15	18	18
N.S.	1	0.82	0.82	0.86	0.82	1.05	0.68	0.82	0.82
time (sec)	N/A	0.170	0.006	0.036	0.184	0.249	0.036	0.295	8.299

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	27	23	20	19	24	15	19	21
N.S.	1	1.17	1.00	0.87	0.83	1.04	0.65	0.83	0.91
time (sec)	N/A	0.165	0.008	0.062	0.269	0.242	0.044	0.314	8.309

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	9	9	8	9	11
N.S.	1	1.00	1.00	0.91	0.82	0.82	0.73	0.82	1.00
time (sec)	N/A	0.145	0.004	0.030	0.178	0.242	0.037	0.289	0.011

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	27	20	20	19	23	15	19	20
N.S.	1	1.17	0.87	0.87	0.83	1.00	0.65	0.83	0.87
time (sec)	N/A	0.158	0.006	0.052	0.266	0.244	0.044	0.292	0.015

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	21	24	32	19	29	20
N.S.	1	1.00	1.00	0.88	1.00	1.33	0.79	1.21	0.83
time (sec)	N/A	0.174	0.011	0.053	0.180	0.265	0.051	0.303	8.317

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	35	30	25	25	31	26	25	25
N.S.	1	1.17	1.00	0.83	0.83	1.03	0.87	0.83	0.83
time (sec)	N/A	0.170	0.014	0.081	0.264	0.246	0.066	0.291	0.022

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	33	44	31	33	31
N.S.	1	1.00	1.00	0.85	1.00	1.33	0.94	1.00	0.94
time (sec)	N/A	0.181	0.011	0.060	0.179	0.237	0.065	0.323	0.027

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	38	33	28	30	36	29	31	30
N.S.	1	1.03	0.89	0.76	0.81	0.97	0.78	0.84	0.81
time (sec)	N/A	0.182	0.012	0.103	0.267	0.244	0.081	0.304	0.027

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	110	94	34	83	106	90	83	45
N.S.	1	1.06	0.90	0.33	0.80	1.02	0.87	0.80	0.43
time (sec)	N/A	0.312	0.052	0.055	0.273	0.253	0.070	0.308	8.288

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	109	93	35	84	103	90	84	47
N.S.	1	1.10	0.94	0.35	0.85	1.04	0.91	0.85	0.47
time (sec)	N/A	0.313	0.048	0.056	0.282	0.263	0.069	0.316	8.271

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	107	90	33	82	97	82	82	45
N.S.	1	1.10	0.93	0.34	0.85	1.00	0.85	0.85	0.46
time (sec)	N/A	0.298	0.041	0.054	0.260	0.271	0.067	0.296	0.045

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	109	92	35	84	99	83	84	46
N.S.	1	1.10	0.93	0.35	0.85	1.00	0.84	0.85	0.46
time (sec)	N/A	0.299	0.034	0.049	0.265	0.260	0.070	0.296	0.024

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	107	91	33	82	101	88	82	44
N.S.	1	1.10	0.94	0.34	0.85	1.04	0.91	0.85	0.45
time (sec)	N/A	0.287	0.033	0.048	0.278	0.249	0.071	0.307	8.150

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	114	98	41	88	112	97	88	49
N.S.	1	1.08	0.92	0.39	0.83	1.06	0.92	0.83	0.46
time (sec)	N/A	0.302	0.053	0.078	0.269	0.251	0.086	0.312	8.118

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	116	96	39	90	122	99	87	51
N.S.	1	1.09	0.91	0.37	0.85	1.15	0.93	0.82	0.48
time (sec)	N/A	0.305	0.051	0.082	0.262	0.253	0.087	0.285	8.261

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	119	103	44	95	127	102	96	55
N.S.	1	1.05	0.91	0.39	0.84	1.12	0.90	0.85	0.49
time (sec)	N/A	0.320	0.052	0.084	0.275	0.278	0.095	0.285	0.053

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	123	101	42	95	127	102	94	55
N.S.	1	1.09	0.89	0.37	0.84	1.12	0.90	0.83	0.49
time (sec)	N/A	0.318	0.052	0.090	0.266	0.266	0.096	0.327	0.058

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	30	30	32	0	0	0	0	0	0
N.S.	1	1.00	1.07	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.162	0.132	0.000	0.000	0.000	0.000	0.000	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	35	39	35	34	46	34	35	26
N.S.	1	1.09	1.22	1.09	1.06	1.44	1.06	1.09	0.81
time (sec)	N/A	0.171	0.026	0.050	0.180	0.241	0.053	0.286	0.026

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	22	22	19	18	23	15	19	20
N.S.	1	0.85	0.85	0.73	0.69	0.88	0.58	0.73	0.77
time (sec)	N/A	0.170	0.007	0.045	0.186	0.233	0.037	0.325	8.248

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	29	33	30	29	40	26	30	21
N.S.	1	1.16	1.32	1.20	1.16	1.60	1.04	1.20	0.84
time (sec)	N/A	0.161	0.010	0.050	0.181	0.248	0.051	0.300	8.439

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	11	10	9	9	8	9	11
N.S.	1	1.00	0.85	0.77	0.69	0.69	0.62	0.69	0.85
time (sec)	N/A	0.148	0.003	0.036	0.183	0.232	0.031	0.325	0.013

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	29	33	30	29	40	26	30	21
N.S.	1	1.16	1.32	1.20	1.16	1.60	1.04	1.20	0.84
time (sec)	N/A	0.161	0.007	0.048	0.182	0.286	0.044	0.303	0.019

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	26	21	24	32	19	30	22
N.S.	1	1.00	0.93	0.75	0.86	1.14	0.68	1.07	0.79
time (sec)	N/A	0.174	0.008	0.060	0.180	0.247	0.054	0.308	0.036

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	35	41	36	37	54	36	38	26
N.S.	1	1.09	1.28	1.12	1.16	1.69	1.12	1.19	0.81
time (sec)	N/A	0.171	0.015	0.063	0.185	0.234	0.071	0.319	0.027

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	35	35	32	35	50	29	36	32
N.S.	1	0.95	0.95	0.86	0.95	1.35	0.78	0.97	0.86
time (sec)	N/A	0.185	0.010	0.067	0.185	0.231	0.064	0.337	0.030

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	42	49	41	42	59	41	42	32
N.S.	1	1.08	1.26	1.05	1.08	1.51	1.05	1.08	0.82
time (sec)	N/A	0.180	0.011	0.069	0.188	0.234	0.086	0.302	0.029

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	37	38	29	28	49	32	30	26
N.S.	1	1.09	1.12	0.85	0.82	1.44	0.94	0.88	0.76
time (sec)	N/A	0.183	0.014	0.097	0.262	0.253	0.063	0.310	8.372

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	34	35	30	29	46	32	31	23
N.S.	1	1.17	1.21	1.03	1.00	1.59	1.10	1.07	0.79
time (sec)	N/A	0.168	0.011	0.090	0.265	0.260	0.064	0.310	0.018

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	32	31	28	27	43	26	29	21
N.S.	1	1.19	1.15	1.04	1.00	1.59	0.96	1.07	0.78
time (sec)	N/A	0.165	0.010	0.090	0.275	0.243	0.059	0.284	0.018

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	34	33	30	29	45	27	31	23
N.S.	1	1.17	1.14	1.03	1.00	1.55	0.93	1.07	0.79
time (sec)	N/A	0.169	0.009	0.088	0.262	0.244	0.066	0.289	0.017

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	32	33	28	27	44	31	29	21
N.S.	1	1.19	1.22	1.04	1.00	1.63	1.15	1.07	0.78
time (sec)	N/A	0.159	0.008	0.094	0.266	0.247	0.066	0.293	0.017

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	39	40	36	35	55	37	37	26
N.S.	1	1.08	1.11	1.00	0.97	1.53	1.03	1.03	0.72
time (sec)	N/A	0.179	0.016	0.109	0.272	0.240	0.090	0.323	0.025

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	41	38	36	37	63	39	34	28
N.S.	1	1.14	1.06	1.00	1.03	1.75	1.08	0.94	0.78
time (sec)	N/A	0.181	0.015	0.123	0.322	0.246	0.085	0.331	8.219

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	46	51	41	42	68	44	43	34
N.S.	1	1.07	1.19	0.95	0.98	1.58	1.02	1.00	0.79
time (sec)	N/A	0.195	0.016	0.126	0.319	0.246	0.091	0.299	0.026

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	48	43	41	42	68	44	41	34
N.S.	1	1.12	1.00	0.95	0.98	1.58	1.02	0.95	0.79
time (sec)	N/A	0.193	0.014	0.134	0.270	0.261	0.095	0.288	0.026

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	163	163	82	0	0	0	0	0	0
N.S.	1	1.00	0.50	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.262	0.259	0.000	0.000	0.000	0.000	0.000	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	80	78	83	0	254	316	75	3916
N.S.	1	0.99	0.96	1.02	0.00	3.14	3.90	0.93	48.35
time (sec)	N/A	0.250	0.037	0.085	0.000	0.279	2.700	1.562	8.922

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	193	210	139	0	1071	134	2043	5659
N.S.	1	1.01	1.09	0.72	0.00	5.58	0.70	10.64	29.47
time (sec)	N/A	0.371	0.086	0.097	0.000	0.265	2.808	1.923	9.231

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	64	62	60	0	197	223	59	2654
N.S.	1	1.02	0.98	0.95	0.00	3.13	3.54	0.94	42.13
time (sec)	N/A	0.231	0.018	0.067	0.000	0.265	1.503	1.523	9.035

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	166	171	105	0	567	76	1034	1220
N.S.	1	1.04	1.08	0.66	0.00	3.57	0.48	6.50	7.67
time (sec)	N/A	0.284	0.058	0.077	0.000	0.261	1.433	2.043	9.181

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	42	37	0	129	131	36	260
N.S.	1	1.00	1.11	0.97	0.00	3.39	3.45	0.95	6.84
time (sec)	N/A	0.182	0.008	0.047	0.000	0.260	0.442	1.662	8.206

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	158	133	99	0	619	88	1028	1105
N.S.	1	1.03	0.86	0.64	0.00	4.02	0.57	6.68	7.18
time (sec)	N/A	0.261	0.053	0.071	0.000	0.276	2.166	1.893	8.841

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	73	66	66	0	223	253	68	1690
N.S.	1	1.06	0.96	0.96	0.00	3.23	3.67	0.99	24.49
time (sec)	N/A	0.245	0.019	0.080	0.000	0.283	99.709	1.541	8.977

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	180	75	159	0	1134	153	2055	5451
N.S.	1	0.98	0.41	0.86	0.00	6.16	0.83	11.17	29.62
time (sec)	N/A	0.328	0.024	0.110	0.000	0.275	149.527	1.874	8.971

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	95	92	84	0	293	0	94	8817
N.S.	1	1.07	1.03	0.94	0.00	3.29	0.00	1.06	99.07
time (sec)	N/A	0.296	0.023	0.100	0.000	0.388	0.000	1.503	9.218

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	381	344	70	63	0	7003	0	0	12709
N.S.	1	0.90	0.18	0.17	0.00	18.38	0.00	0.00	33.36
time (sec)	N/A	0.590	0.028	0.104	0.000	1.020	0.000	0.000	9.507

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	376	337	70	59	0	4001	0	0	10382
N.S.	1	0.90	0.19	0.16	0.00	10.64	0.00	0.00	27.61
time (sec)	N/A	0.520	0.027	0.048	0.000	0.414	0.000	0.000	9.700

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	325	314	44	43	0	4433	0	0	8033
N.S.	1	0.97	0.14	0.13	0.00	13.64	0.00	0.00	24.72
time (sec)	N/A	0.449	0.018	0.044	0.000	0.326	0.000	0.000	9.344

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	325	312	42	43	0	2141	126	0	8169
N.S.	1	0.96	0.13	0.13	0.00	6.59	0.39	0.00	25.14
time (sec)	N/A	0.446	0.017	0.050	0.000	0.297	3.227	0.000	9.486

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	315	309	43	43	0	3193	172	0	6067
N.S.	1	0.98	0.14	0.14	0.00	10.14	0.55	0.00	19.26
time (sec)	N/A	0.449	0.017	0.050	0.000	0.289	3.034	0.000	8.960

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	315	300	45	40	0	3125	0	0	10337
N.S.	1	0.95	0.14	0.13	0.00	9.92	0.00	0.00	32.82
time (sec)	N/A	0.422	0.023	0.042	0.000	0.329	0.000	0.000	9.418

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	363	330	71	63	0	5758	0	0	10509
N.S.	1	0.91	0.20	0.17	0.00	15.86	0.00	0.00	28.95
time (sec)	N/A	0.488	0.025	0.094	0.000	0.527	0.000	0.000	9.010

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	365	330	75	62	0	5030	0	0	16497
N.S.	1	0.90	0.21	0.17	0.00	13.78	0.00	0.00	45.20
time (sec)	N/A	0.468	0.029	0.083	0.000	0.619	0.000	0.000	10.981

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	127	133	488	0	0	0	0	0	0
N.S.	1	1.05	3.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.249	1.057	0.000	0.000	0.000	0.000	0.000	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	42	44	36	35	35	42	35	37
N.S.	1	0.95	1.00	0.82	0.80	0.80	0.95	0.80	0.84
time (sec)	N/A	0.200	0.012	0.096	0.336	0.246	0.079	0.354	0.028

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	50	98	43	42	40	51	42	43
N.S.	1	0.93	1.81	0.80	0.78	0.74	0.94	0.78	0.80
time (sec)	N/A	0.213	0.140	0.097	0.335	0.228	0.064	0.299	0.029

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	39	37	31	30	30	37	30	32
N.S.	1	1.05	1.00	0.84	0.81	0.81	1.00	0.81	0.86
time (sec)	N/A	0.201	0.009	0.078	0.328	0.244	0.061	0.300	0.024

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	83	94	62	61	61	76	61	51
N.S.	1	1.11	1.25	0.83	0.81	0.81	1.01	0.81	0.68
time (sec)	N/A	0.282	0.089	0.088	0.334	0.250	0.097	0.316	0.058

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	19	18	18	26	18	17
N.S.	1	1.00	1.00	0.83	0.78	0.78	1.13	0.78	0.74
time (sec)	N/A	0.167	0.009	0.071	0.317	0.235	0.051	0.302	0.038

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	83	79	62	61	61	76	61	51
N.S.	1	1.11	1.05	0.83	0.81	0.81	1.01	0.81	0.68
time (sec)	N/A	0.258	0.038	0.076	0.333	0.243	0.094	0.292	8.156

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	43	133	31	36	32	41	36	34
N.S.	1	1.10	3.41	0.79	0.92	0.82	1.05	0.92	0.87
time (sec)	N/A	0.215	0.049	0.099	0.304	0.241	0.078	0.307	8.198

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	52	100	44	42	45	53	42	43
N.S.	1	0.96	1.85	0.81	0.78	0.83	0.98	0.78	0.80
time (sec)	N/A	0.219	0.030	0.117	0.283	0.238	0.075	0.332	0.029

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	50	136	38	41	49	48	46	41
N.S.	1	1.04	2.83	0.79	0.85	1.02	1.00	0.96	0.85
time (sec)	N/A	0.227	0.043	0.125	0.276	0.235	0.088	0.295	0.040

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	93	107	69	73	84	88	73	62
N.S.	1	1.04	1.20	0.78	0.82	0.94	0.99	0.82	0.70
time (sec)	N/A	0.328	0.059	0.133	0.274	0.251	0.118	0.319	0.023

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	172	139	92	0	210	192	109	100
N.S.	1	1.22	0.99	0.65	0.00	1.49	1.36	0.77	0.71
time (sec)	N/A	0.394	0.197	0.111	0.000	0.248	0.384	0.304	8.363

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	92	68	67	0	70	82	66	38
N.S.	1	1.05	0.77	0.76	0.00	0.80	0.93	0.75	0.43
time (sec)	N/A	0.283	0.018	0.104	0.000	0.261	0.077	0.306	8.423

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	160	135	89	0	197	197	108	99
N.S.	1	1.14	0.96	0.64	0.00	1.41	1.41	0.77	0.71
time (sec)	N/A	0.381	0.118	0.106	0.000	0.261	0.380	0.297	0.039

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	171	135	89	0	209	214	108	97
N.S.	1	1.22	0.96	0.64	0.00	1.49	1.53	0.77	0.69
time (sec)	N/A	0.361	0.108	0.094	0.000	0.256	0.389	0.306	8.309

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	92	68	67	0	70	82	66	40
N.S.	1	1.05	0.77	0.76	0.00	0.80	0.93	0.75	0.45
time (sec)	N/A	0.273	0.013	0.089	0.000	0.251	0.091	0.339	0.023

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	165	140	96	0	209	218	113	102
N.S.	1	1.14	0.97	0.66	0.00	1.44	1.50	0.78	0.70
time (sec)	N/A	0.415	0.176	0.127	0.000	0.272	0.388	0.341	0.030

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	178	148	92	0	237	197	113	104
N.S.	1	1.21	1.01	0.63	0.00	1.61	1.34	0.77	0.71
time (sec)	N/A	0.376	0.223	0.123	0.000	0.263	0.390	0.334	0.017

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	102	95	75	0	90	94	100	52
N.S.	1	1.04	0.97	0.77	0.00	0.92	0.96	1.02	0.53
time (sec)	N/A	0.323	0.031	0.138	0.000	0.270	0.112	0.337	0.021

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	174	171	96	0	232	209	120	110
N.S.	1	1.13	1.11	0.62	0.00	1.51	1.36	0.78	0.71
time (sec)	N/A	0.428	0.248	0.143	0.000	0.267	0.387	0.327	0.019

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	127	133	79	0	0	0	0	0	0
N.S.	1	1.05	0.62	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.231	0.084	0.000	0.000	0.000	0.000	0.000	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	43	46	38	37	37	42	37	39
N.S.	1	0.93	1.00	0.83	0.80	0.80	0.91	0.80	0.85
time (sec)	N/A	0.203	0.009	0.066	0.267	0.239	0.066	0.341	0.030

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	55	44	0	47	48	99	29
N.S.	1	1.00	0.96	0.77	0.00	0.82	0.84	1.74	0.51
time (sec)	N/A	0.219	0.012	0.067	0.000	0.236	0.058	0.341	8.263

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	40	39	33	32	32	37	32	34
N.S.	1	1.03	1.00	0.85	0.82	0.82	0.95	0.82	0.87
time (sec)	N/A	0.202	0.008	0.054	0.265	0.238	0.063	0.297	8.227

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	90	98	32	0	153	70	76	53
N.S.	1	1.10	1.20	0.39	0.00	1.87	0.85	0.93	0.65
time (sec)	N/A	0.288	0.098	0.063	0.000	0.277	0.108	0.300	0.031

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	19	18	18	26	18	17
N.S.	1	1.00	1.00	0.83	0.78	0.78	1.13	0.78	0.74
time (sec)	N/A	0.165	0.006	0.053	0.269	0.232	0.057	0.303	8.213

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	101	83	32	0	165	70	64	53
N.S.	1	1.23	1.01	0.39	0.00	2.01	0.85	0.78	0.65
time (sec)	N/A	0.274	0.045	0.068	0.000	0.245	0.102	0.303	0.029

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	44	55	33	38	34	41	38	36
N.S.	1	1.07	1.34	0.80	0.93	0.83	1.00	0.93	0.88
time (sec)	N/A	0.220	0.014	0.073	0.270	0.234	0.084	0.306	8.175

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	59	55	44	0	50	49	99	29
N.S.	1	1.04	0.96	0.77	0.00	0.88	0.86	1.74	0.51
time (sec)	N/A	0.219	0.017	0.081	0.000	0.237	0.069	0.335	8.152

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	49	51	38	43	51	48	48	41
N.S.	1	1.02	1.06	0.79	0.90	1.06	1.00	1.00	0.85
time (sec)	N/A	0.223	0.015	0.086	0.264	0.248	0.085	0.315	0.038

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	102	56	46	0	174	83	56	63
N.S.	1	1.06	0.58	0.48	0.00	1.81	0.86	0.58	0.66
time (sec)	N/A	0.334	0.016	0.090	0.000	0.246	0.125	0.324	0.032

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	356	348	59	44	0	418	26	254	209
N.S.	1	0.98	0.17	0.12	0.00	1.17	0.07	0.71	0.59
time (sec)	N/A	0.655	0.023	0.063	0.000	0.264	1.487	0.329	0.104

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	413	41	32	0	101	165	205	53
N.S.	1	1.50	0.15	0.12	0.00	0.37	0.60	0.75	0.19
time (sec)	N/A	0.588	0.012	0.061	0.000	0.240	0.098	0.320	0.057

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	347	363	39	40	0	393	24	253	474
N.S.	1	1.05	0.11	0.12	0.00	1.13	0.07	0.73	1.37
time (sec)	N/A	0.523	0.013	0.062	0.000	0.260	1.478	0.371	8.246

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	355	407	40	40	0	545	26	253	286
N.S.	1	1.15	0.11	0.11	0.00	1.54	0.07	0.71	0.81
time (sec)	N/A	0.531	0.011	0.062	0.000	0.271	1.442	0.362	0.048

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	411	42	30	0	101	165	205	53
N.S.	1	1.49	0.15	0.11	0.00	0.37	0.60	0.75	0.19
time (sec)	N/A	0.567	0.012	0.054	0.000	0.248	0.099	0.317	0.024

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	360	365	61	40	0	555	29	258	253
N.S.	1	1.01	0.17	0.11	0.00	1.54	0.08	0.72	0.70
time (sec)	N/A	0.656	0.016	0.096	0.000	0.257	1.467	0.310	8.313

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	370	354	65	38	0	443	31	258	213
N.S.	1	0.96	0.18	0.10	0.00	1.20	0.08	0.70	0.58
time (sec)	N/A	0.627	0.014	0.089	0.000	0.264	1.491	0.314	8.346

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	287	425	54	43	0	124	182	217	63
N.S.	1	1.48	0.19	0.15	0.00	0.43	0.63	0.76	0.22
time (sec)	N/A	0.609	0.015	0.086	0.000	0.249	0.131	0.353	8.342

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	377	377	54	44	0	428	37	265	486
N.S.	1	1.00	0.14	0.12	0.00	1.14	0.10	0.70	1.29
time (sec)	N/A	0.589	0.016	0.104	0.000	0.255	1.462	0.319	0.038

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	117	117	79	0	0	0	0	0	0
N.S.	1	1.00	0.68	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.260	0.091	0.000	0.000	0.000	0.000	0.000	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	57	38	50	62	60	50	64
N.S.	1	1.00	0.92	0.61	0.81	1.00	0.97	0.81	1.03
time (sec)	N/A	0.242	0.029	0.074	0.263	0.233	0.067	0.376	0.080

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	102	97	42	0	152	54	66	130
N.S.	1	1.13	1.08	0.47	0.00	1.69	0.60	0.73	1.44
time (sec)	N/A	0.249	0.113	0.103	0.000	0.275	0.111	0.347	8.387

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	59	53	33	45	56	53	45	59
N.S.	1	1.07	0.96	0.60	0.82	1.02	0.96	0.82	1.07
time (sec)	N/A	0.241	0.018	0.071	0.276	0.247	0.060	0.334	8.420

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	99	75	34	0	143	49	47	117
N.S.	1	1.22	0.93	0.42	0.00	1.77	0.60	0.58	1.44
time (sec)	N/A	0.209	0.034	0.083	0.000	0.264	0.094	0.329	0.079

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	46	38	19	31	43	42	31	30
N.S.	1	2.00	1.65	0.83	1.35	1.87	1.83	1.35	1.30
time (sec)	N/A	0.193	0.009	0.056	0.270	0.253	0.049	0.333	0.060

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	80	74	34	0	159	49	41	125
N.S.	1	1.07	0.99	0.45	0.00	2.12	0.65	0.55	1.67
time (sec)	N/A	0.190	0.027	0.081	0.000	0.250	0.095	0.341	0.032

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	74	55	35	51	58	58	51	42
N.S.	1	1.30	0.96	0.61	0.89	1.02	1.02	0.89	0.74
time (sec)	N/A	0.251	0.024	0.074	0.270	0.235	0.074	0.331	8.366

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	104	65	42	0	171	56	68	130
N.S.	1	1.17	0.73	0.47	0.00	1.92	0.63	0.76	1.46
time (sec)	N/A	0.236	0.017	0.092	0.000	0.250	0.112	0.337	0.034

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	85	60	42	56	76	65	63	49
N.S.	1	1.29	0.91	0.64	0.85	1.15	0.98	0.95	0.74
time (sec)	N/A	0.312	0.024	0.088	0.267	0.244	0.089	0.328	8.265

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	111	73	48	0	190	65	77	136
N.S.	1	1.14	0.75	0.49	0.00	1.96	0.67	0.79	1.40
time (sec)	N/A	0.253	0.015	0.106	0.000	0.247	0.132	0.375	0.080

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	460	480	58	46	0	422	29	240	216
N.S.	1	1.04	0.13	0.10	0.00	0.92	0.06	0.52	0.47
time (sec)	N/A	0.750	0.017	0.060	0.000	0.267	0.926	0.370	8.307

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	431	463	41	40	0	361	26	239	149
N.S.	1	1.07	0.10	0.09	0.00	0.84	0.06	0.55	0.35
time (sec)	N/A	0.651	0.012	0.063	0.000	0.271	0.932	0.366	8.334

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	451	479	39	40	0	373	24	239	454
N.S.	1	1.06	0.09	0.09	0.00	0.83	0.05	0.53	1.01
time (sec)	N/A	0.668	0.013	0.061	0.000	0.257	0.864	0.376	0.125

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	427	451	40	40	0	525	26	239	275
N.S.	1	1.06	0.09	0.09	0.00	1.23	0.06	0.56	0.64
time (sec)	N/A	0.639	0.010	0.064	0.000	0.250	0.885	0.431	8.286

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	414	466	42	37	0	309	26	239	403
N.S.	1	1.13	0.10	0.09	0.00	0.75	0.06	0.58	0.97
time (sec)	N/A	0.663	0.011	0.059	0.000	0.260	0.915	0.327	0.045

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	416	468	61	42	0	559	32	244	292
N.S.	1	1.12	0.15	0.10	0.00	1.34	0.08	0.59	0.70
time (sec)	N/A	0.686	0.017	0.092	0.000	0.258	0.940	0.378	8.254

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	466	486	65	40	0	451	34	244	492
N.S.	1	1.04	0.14	0.09	0.00	0.97	0.07	0.52	1.06
time (sec)	N/A	0.714	0.016	0.098	0.000	0.267	0.961	0.373	0.118

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	117	117	575	0	0	0	0	0	0
N.S.	1	1.00	4.91	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.242	0.595	0.000	0.000	0.000	0.000	0.000	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	56	38	50	62	58	53	64
N.S.	1	1.00	0.90	0.61	0.81	1.00	0.94	0.85	1.03
time (sec)	N/A	0.232	0.215	0.050	0.260	0.235	0.067	0.332	8.450

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	102	103	67	92	114	170	97	90
N.S.	1	1.13	1.14	0.74	1.02	1.27	1.89	1.08	1.00
time (sec)	N/A	0.230	0.148	0.069	0.268	0.254	0.202	0.341	0.059

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	59	53	33	45	57	53	48	59
N.S.	1	1.07	0.96	0.60	0.82	1.04	0.96	0.87	1.07
time (sec)	N/A	0.224	0.050	0.045	0.271	0.238	0.058	0.319	0.059

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	99	91	62	87	109	165	92	77
N.S.	1	1.22	1.12	0.77	1.07	1.35	2.04	1.14	0.95
time (sec)	N/A	0.214	0.068	0.067	0.266	0.256	0.195	0.324	8.372

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	46	38	19	0	43	42	33	30
N.S.	1	2.00	1.65	0.83	0.00	1.87	1.83	1.43	1.30
time (sec)	N/A	0.195	0.024	0.043	0.000	0.248	0.057	0.336	8.636

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	80	91	62	87	107	165	92	83
N.S.	1	1.07	1.21	0.83	1.16	1.43	2.20	1.23	1.11
time (sec)	N/A	0.190	0.050	0.043	0.269	0.246	0.190	0.321	8.561

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	71	55	64	51	59	58	54	42
N.S.	1	1.25	0.96	1.12	0.89	1.04	1.02	0.95	0.74
time (sec)	N/A	0.226	0.050	0.061	0.272	0.241	0.079	0.299	0.242

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	104	103	67	92	125	172	97	88
N.S.	1	1.17	1.16	0.75	1.03	1.40	1.93	1.09	0.99
time (sec)	N/A	0.226	0.081	0.076	0.268	0.242	0.221	0.321	0.039

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	78	61	71	56	76	66	66	49
N.S.	1	1.18	0.92	1.08	0.85	1.15	1.00	1.00	0.74
time (sec)	N/A	0.248	0.063	0.069	0.277	0.242	0.090	0.338	8.589

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	111	111	72	99	130	199	104	95
N.S.	1	1.14	1.14	0.74	1.02	1.34	2.05	1.07	0.98
time (sec)	N/A	0.256	0.103	0.080	0.271	0.248	0.242	0.333	8.585

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	178	160	69	0	318	58	148	246
N.S.	1	1.05	0.94	0.41	0.00	1.87	0.34	0.87	1.45
time (sec)	N/A	0.296	0.344	0.121	0.000	0.253	0.735	0.370	8.567

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	185	160	68	0	261	53	147	147
N.S.	1	1.11	0.96	0.41	0.00	1.56	0.32	0.88	0.88
time (sec)	N/A	0.303	0.184	0.085	0.000	0.260	0.732	0.369	0.124

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	177	132	60	0	269	49	147	269
N.S.	1	1.02	0.76	0.35	0.00	1.55	0.28	0.85	1.55
time (sec)	N/A	0.261	0.173	0.097	0.000	0.277	0.711	0.368	0.114

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	173	131	64	0	285	53	147	269
N.S.	1	1.19	0.90	0.44	0.00	1.97	0.37	1.01	1.86
time (sec)	N/A	0.256	0.047	0.081	0.000	0.281	0.729	0.345	0.045

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	164	160	68	0	261	53	147	245
N.S.	1	0.97	0.95	0.40	0.00	1.54	0.31	0.87	1.45
time (sec)	N/A	0.246	0.131	0.077	0.000	0.274	0.734	0.332	8.377

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	190	174	73	0	327	63	152	250
N.S.	1	1.10	1.01	0.42	0.00	1.90	0.37	0.88	1.45
time (sec)	N/A	0.293	0.241	0.100	0.000	0.280	0.751	0.356	8.311

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	184	166	73	0	347	63	152	268
N.S.	1	1.01	0.91	0.40	0.00	1.91	0.35	0.84	1.47
time (sec)	N/A	0.281	0.211	0.104	0.000	0.280	0.753	0.358	8.533

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	197	189	79	0	324	73	159	257
N.S.	1	1.14	1.09	0.46	0.00	1.87	0.42	0.92	1.49
time (sec)	N/A	0.364	0.226	0.108	0.000	0.264	0.775	0.393	8.328

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	189	79	0	352	70	159	291
N.S.	1	1.00	1.00	0.42	0.00	1.86	0.37	0.84	1.54
time (sec)	N/A	0.316	0.242	0.117	0.000	0.282	0.803	0.348	8.385

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	19	21	18	17	17	15	17	16
N.S.	1	0.90	1.00	0.86	0.81	0.81	0.71	0.81	0.76
time (sec)	N/A	0.184	0.009	0.048	0.180	0.245	0.047	0.296	0.041

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	22	26	23	22	22	19	22	22
N.S.	1	0.85	1.00	0.88	0.85	0.85	0.73	0.85	0.85
time (sec)	N/A	0.187	0.010	0.042	0.179	0.248	0.061	0.306	8.318

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	39	37	31	30	30	37	30	32
N.S.	1	1.05	1.00	0.84	0.81	0.81	1.00	0.81	0.86
time (sec)	N/A	0.209	0.023	0.055	0.263	0.260	0.067	1.101	8.396

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	19	18	18	27	18	20
N.S.	1	1.00	1.00	0.83	0.78	0.78	1.17	0.78	0.87
time (sec)	N/A	0.168	0.012	0.036	0.271	0.271	0.056	1.101	8.422

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	43	197	31	36	32	41	33	34
N.S.	1	1.10	5.05	0.79	0.92	0.82	1.05	0.85	0.87
time (sec)	N/A	0.213	0.048	0.098	0.262	0.264	0.078	0.300	0.037

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	50	208	38	41	49	48	45	41
N.S.	1	1.04	4.33	0.79	0.85	1.02	1.00	0.94	0.85
time (sec)	N/A	0.225	0.047	0.089	0.267	0.287	0.091	0.288	8.305

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	43	197	31	0	32	41	33	34
N.S.	1	1.10	5.05	0.79	0.00	0.82	1.05	0.85	0.87
time (sec)	N/A	0.218	0.021	0.063	0.000	0.278	0.080	0.285	0.019

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	140	164	0	466	605	145	183
N.S.	1	1.00	0.95	1.12	0.00	3.17	4.12	0.99	1.24
time (sec)	N/A	0.334	0.122	0.155	0.000	0.290	0.735	0.277	0.095

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	112	128	0	383	498	113	151
N.S.	1	1.00	0.95	1.08	0.00	3.25	4.22	0.96	1.28
time (sec)	N/A	0.299	0.066	0.123	0.000	0.312	0.602	0.283	8.289

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	84	98	0	297	381	86	112
N.S.	1	1.00	0.94	1.10	0.00	3.34	4.28	0.97	1.26
time (sec)	N/A	0.265	0.073	0.077	0.000	0.285	0.467	0.307	8.318

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	73	75	0	235	306	67	172
N.S.	1	1.00	1.04	1.07	0.00	3.36	4.37	0.96	2.46
time (sec)	N/A	0.240	0.046	0.044	0.000	0.294	0.336	0.294	0.082

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	57	56	0	185	216	55	112
N.S.	1	1.00	1.02	1.00	0.00	3.30	3.86	0.98	2.00
time (sec)	N/A	0.216	0.023	0.037	0.000	0.292	0.165	0.309	8.390

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	38	35	0	120	124	34	46
N.S.	1	1.00	1.06	0.97	0.00	3.33	3.44	0.94	1.28
time (sec)	N/A	0.183	0.007	0.037	0.000	0.297	0.107	0.286	0.026

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	63	61	61	0	211	564	62	213
N.S.	1	1.02	0.98	0.98	0.00	3.40	9.10	1.00	3.44
time (sec)	N/A	0.242	0.052	0.069	0.000	0.291	4.462	0.324	0.273

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	85	77	81	0	269	0	79	339
N.S.	1	1.05	0.95	1.00	0.00	3.32	0.00	0.98	4.19
time (sec)	N/A	0.295	0.066	0.056	0.000	0.295	0.000	0.302	8.619

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	113	102	128	0	358	0	105	447
N.S.	1	1.09	0.98	1.23	0.00	3.44	0.00	1.01	4.30
time (sec)	N/A	0.340	0.096	0.121	0.000	0.315	0.000	0.310	8.586

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	140	131	157	0	445	0	136	524
N.S.	1	1.02	0.96	1.15	0.00	3.25	0.00	0.99	3.82
time (sec)	N/A	0.384	0.076	0.119	0.000	0.328	0.000	0.298	8.599

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	186	163	238	0	1029	1012	188	382
N.S.	1	0.95	0.83	1.21	0.00	5.25	5.16	0.96	1.95
time (sec)	N/A	0.402	0.183	0.174	0.000	0.292	1.347	0.330	8.686

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	154	132	198	0	837	842	161	261
N.S.	1	1.03	0.88	1.32	0.00	5.58	5.61	1.07	1.74
time (sec)	N/A	0.359	0.128	0.159	0.000	0.298	1.072	0.280	8.471

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	125	109	169	0	635	729	125	279
N.S.	1	1.10	0.96	1.48	0.00	5.57	6.39	1.10	2.45
time (sec)	N/A	0.314	0.100	0.106	0.000	0.316	0.758	0.297	8.523

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	81	97	0	387	280	88	135
N.S.	1	1.00	1.14	1.37	0.00	5.45	3.94	1.24	1.90
time (sec)	N/A	0.225	0.069	0.075	0.000	0.277	0.317	0.288	8.310

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	69	70	0	338	253	76	110
N.S.	1	1.00	1.05	1.06	0.00	5.12	3.83	1.15	1.67
time (sec)	N/A	0.221	0.052	0.068	0.000	0.296	0.294	0.293	0.058

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	70	68	0	341	265	76	119
N.S.	1	1.00	1.06	1.03	0.00	5.17	4.02	1.15	1.80
time (sec)	N/A	0.218	0.050	0.066	0.000	0.275	0.309	0.303	0.048

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	139	107	177	0	781	0	126	620
N.S.	1	1.29	0.99	1.64	0.00	7.23	0.00	1.17	5.74
time (sec)	N/A	0.359	0.122	0.132	0.000	0.343	0.000	0.297	8.703

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	173	131	205	0	975	0	171	775
N.S.	1	1.17	0.89	1.39	0.00	6.59	0.00	1.16	5.24
time (sec)	N/A	0.411	0.177	0.089	0.000	0.401	0.000	0.308	8.847

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	213	175	255	0	1226	0	229	914
N.S.	1	1.05	0.87	1.26	0.00	6.07	0.00	1.13	4.52
time (sec)	N/A	0.458	0.259	0.184	0.000	0.458	0.000	0.314	9.119

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	257	260	401	0	1926	1714	282	705
N.S.	1	1.08	1.09	1.68	0.00	8.09	7.20	1.18	2.96
time (sec)	N/A	0.519	0.238	0.200	0.000	0.294	3.038	0.330	8.593

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	221	221	357	0	1603	1510	245	620
N.S.	1	1.16	1.16	1.88	0.00	8.44	7.95	1.29	3.26
time (sec)	N/A	0.447	0.197	0.154	0.000	0.287	1.868	0.295	8.820

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	122	174	260	0	953	547	202	343
N.S.	1	1.10	1.57	2.34	0.00	8.59	4.93	1.82	3.09
time (sec)	N/A	0.273	0.118	0.099	0.000	0.292	0.822	0.308	0.114

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	118	126	223	0	872	513	163	271
N.S.	1	1.10	1.18	2.08	0.00	8.15	4.79	1.52	2.53
time (sec)	N/A	0.267	0.138	0.096	0.000	0.286	0.663	0.286	8.205

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	129	131	210	0	887	570	154	313
N.S.	1	1.12	1.14	1.83	0.00	7.71	4.96	1.34	2.72
time (sec)	N/A	0.287	0.092	0.106	0.000	0.280	0.755	0.287	8.410

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	115	102	118	0	788	481	135	253
N.S.	1	1.12	0.99	1.15	0.00	7.65	4.67	1.31	2.46
time (sec)	N/A	0.259	0.064	0.092	0.000	0.286	0.604	0.289	8.392

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	112	97	116	0	785	474	136	285
N.S.	1	1.09	0.94	1.13	0.00	7.62	4.60	1.32	2.77
time (sec)	N/A	0.255	0.069	0.093	0.000	0.263	0.697	0.297	8.492

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	236	178	352	0	1985	0	239	1089
N.S.	1	1.28	0.96	1.90	0.00	10.73	0.00	1.29	5.89
time (sec)	N/A	0.489	0.233	0.185	0.000	0.524	0.000	0.293	8.990

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	280	221	404	0	2280	0	309	1255
N.S.	1	1.17	0.92	1.69	0.00	9.54	0.00	1.29	5.25
time (sec)	N/A	0.551	0.259	0.133	0.000	0.674	0.000	0.298	9.079

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	42	40	27	30	30	34	32	26
N.S.	1	1.05	1.00	0.68	0.75	0.75	0.85	0.80	0.65
time (sec)	N/A	0.198	0.006	0.034	0.183	0.249	0.060	0.295	0.025

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	35	33	22	25	25	27	27	21
N.S.	1	1.06	1.00	0.67	0.76	0.76	0.82	0.82	0.64
time (sec)	N/A	0.192	0.004	0.033	0.184	0.231	0.059	0.307	8.287

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	28	26	17	20	20	20	22	16
N.S.	1	1.08	1.00	0.65	0.77	0.77	0.77	0.85	0.62
time (sec)	N/A	0.184	0.004	0.031	0.182	0.245	0.060	0.279	0.042

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	23	21	14	17	17	17	19	13
N.S.	1	1.10	1.00	0.67	0.81	0.81	0.81	0.90	0.62
time (sec)	N/A	0.187	0.004	0.027	0.186	0.248	0.066	0.282	0.036

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	25	21	14	17	17	15	19	8
N.S.	1	1.09	0.91	0.61	0.74	0.74	0.65	0.83	0.35
time (sec)	N/A	0.189	0.003	0.031	0.185	0.248	0.046	0.307	8.486

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	29	27	18	21	21	24	24	17
N.S.	1	1.07	1.00	0.67	0.78	0.78	0.89	0.89	0.63
time (sec)	N/A	0.189	0.014	0.042	0.226	0.298	0.066	0.289	0.056

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	36	34	27	26	30	31	29	22
N.S.	1	1.06	1.00	0.79	0.76	0.88	0.91	0.85	0.65
time (sec)	N/A	0.198	0.004	0.048	0.196	0.254	0.072	0.275	0.025

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	43	41	31	31	39	36	34	26
N.S.	1	1.05	1.00	0.76	0.76	0.95	0.88	0.83	0.63
time (sec)	N/A	0.199	0.005	0.047	0.189	0.261	0.075	0.323	8.552

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	50	48	36	36	44	41	39	32
N.S.	1	1.04	1.00	0.75	0.75	0.92	0.85	0.81	0.67
time (sec)	N/A	0.203	0.006	0.050	0.181	0.275	0.087	0.310	0.027

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	219	208	249	0	959	0	0	0
N.S.	1	1.07	1.02	1.22	0.00	4.70	0.00	0.00	0.00
time (sec)	N/A	0.488	1.375	0.147	0.000	0.426	0.000	0.000	0.000

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	150	158	179	0	709	0	0	0
N.S.	1	1.03	1.09	1.23	0.00	4.89	0.00	0.00	0.00
time (sec)	N/A	0.366	0.584	0.067	0.000	0.323	0.000	0.000	0.000

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	107	131	121	0	590	0	0	100
N.S.	1	1.02	1.25	1.15	0.00	5.62	0.00	0.00	0.95
time (sec)	N/A	0.284	0.194	0.038	0.000	0.320	0.000	0.000	8.525

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	89	88	0	171	0	0	53
N.S.	1	1.00	1.33	1.31	0.00	2.55	0.00	0.00	0.79
time (sec)	N/A	0.213	0.153	0.048	0.000	0.275	0.000	0.000	8.578

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	148	140	197	0	465	0	231	0
N.S.	1	1.11	1.05	1.48	0.00	3.50	0.00	1.74	0.00
time (sec)	N/A	0.323	0.461	0.083	0.000	0.310	0.000	0.410	0.000

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	249	246	376	0	1081	0	499	0
N.S.	1	1.13	1.12	1.71	0.00	4.91	0.00	2.27	0.00
time (sec)	N/A	0.469	1.199	0.151	0.000	0.407	0.000	0.374	0.000

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	46	32	40	8	8	0	29	134
N.S.	1	0.63	0.44	0.55	0.11	0.11	0.00	0.40	1.84
time (sec)	N/A	0.187	1.016	0.069	0.192	0.251	0.000	0.318	8.388

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	183	202	57	0	1059	129	2109	3026
N.S.	1	1.02	1.13	0.32	0.00	5.92	0.72	11.78	16.91
time (sec)	N/A	0.373	0.110	0.048	0.000	0.277	1.233	0.786	8.733

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	631	521	70	59	0	2882	196	0	2280
N.S.	1	0.83	0.11	0.09	0.00	4.57	0.31	0.00	3.61
time (sec)	N/A	1.039	0.041	0.059	0.000	0.388	54.067	0.000	10.265

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	376	337	70	59	0	4001	0	0	10382
N.S.	1	0.90	0.19	0.16	0.00	10.64	0.00	0.00	27.61
time (sec)	N/A	0.560	0.040	0.047	0.000	0.456	0.000	0.000	9.601

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	111	105	84	0	0	0	0	0
N.S.	1	1.05	0.99	0.79	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.291	0.204	0.032	0.000	0.000	0.000	0.000	0.000

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	47	55	50	54	53	51	49	44
N.S.	1	1.18	1.38	1.25	1.35	1.32	1.28	1.22	1.10
time (sec)	N/A	0.194	0.030	0.079	0.188	0.285	0.162	0.282	0.027

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	76	50	41	23	0	124	45	0
N.S.	1	1.01	0.67	0.55	0.31	0.00	1.65	0.60	0.00
time (sec)	N/A	0.204	0.028	0.091	0.198	0.000	0.397	0.289	0.000

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	107	117	98	114	84	233	140	0
N.S.	1	0.78	0.85	0.72	0.83	0.61	1.70	1.02	0.00
time (sec)	N/A	0.250	0.057	0.075	0.190	0.262	5.616	0.309	0.000

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	107	93	76	114	61	204	102	0
N.S.	1	0.78	0.68	0.55	0.83	0.45	1.49	0.74	0.00
time (sec)	N/A	0.245	0.022	0.020	0.185	0.284	1.431	0.295	0.000

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	93	67	54	114	32	175	64	0
N.S.	1	0.68	0.49	0.39	0.83	0.23	1.28	0.47	0.00
time (sec)	N/A	0.229	0.018	0.023	0.199	0.258	0.616	0.292	0.000

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	88	59	43	42	114	10	146	26	71
N.S.	1	0.67	0.49	0.48	1.30	0.11	1.66	0.30	0.81
time (sec)	N/A	0.187	0.008	0.086	0.184	0.245	0.478	0.277	8.736

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	85	65	52	36	33	170	61	0
N.S.	1	0.58	0.44	0.35	0.24	0.22	1.16	0.41	0.00
time (sec)	N/A	0.227	0.023	0.041	0.183	0.262	0.432	0.319	0.000

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	99	72	81	55	113	0	64	0
N.S.	1	0.76	0.55	0.62	0.42	0.87	0.00	0.49	0.00
time (sec)	N/A	0.239	0.029	0.115	0.190	0.270	0.000	0.292	0.000

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	105	56	43	53	136	0	43	53
N.S.	1	0.78	0.41	0.32	0.39	1.01	0.00	0.32	0.39
time (sec)	N/A	0.243	0.055	0.263	0.188	0.278	0.000	0.314	9.067

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	105	56	43	53	209	0	43	53
N.S.	1	0.77	0.41	0.31	0.39	1.53	0.00	0.31	0.39
time (sec)	N/A	0.239	0.052	0.493	0.186	0.338	0.000	0.291	9.485

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	105	56	43	53	275	0	43	53
N.S.	1	0.77	0.41	0.31	0.39	2.01	0.00	0.31	0.39
time (sec)	N/A	0.242	0.055	0.779	0.185	0.410	0.000	0.289	9.598

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	105	56	43	53	343	0	43	53
N.S.	1	0.77	0.41	0.31	0.39	2.50	0.00	0.31	0.39
time (sec)	N/A	0.242	0.054	1.184	0.191	0.577	0.000	0.290	10.060

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	77	77	68	0	0	0	0	0	0
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.244	0.139	0.000	0.000	0.000	0.000	0.000	0.000

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	468	330	207	0	362	579	0	1564	777
N.S.	1	0.71	0.44	0.00	0.77	1.24	0.00	3.34	1.66
time (sec)	N/A	0.460	0.308	0.000	0.201	0.425	0.000	0.323	9.607

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	315	239	143	0	198	297	0	745	390
N.S.	1	0.76	0.45	0.00	0.63	0.94	0.00	2.37	1.24
time (sec)	N/A	0.359	0.240	0.000	0.199	0.330	0.000	0.317	8.760

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	142	138	83	0	77	110	0	229	138
N.S.	1	0.97	0.58	0.00	0.54	0.77	0.00	1.61	0.97
time (sec)	N/A	0.269	0.137	0.000	0.197	0.302	0.000	0.341	8.439

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	69	69	58	0	0	0	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.214	0.106	0.000	0.000	0.000	0.000	0.000	0.000

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	75	75	61	0	0	0	0	0	0
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.215	0.104	0.000	0.000	0.000	0.000	0.000	0.000

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	C	F	F	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	146	162	101	0	0	82	0	0	69
N.S.	1	1.11	0.69	0.00	0.00	0.56	0.00	0.00	0.47
time (sec)	N/A	0.308	0.333	0.000	0.000	0.338	0.000	0.000	8.602

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	112	93	103	114	147	0	0	0
N.S.	1	0.64	0.53	0.59	0.65	0.84	0.00	0.00	0.00
time (sec)	N/A	0.257	0.093	0.256	0.198	1.994	0.000	0.000	0.000

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	152	121	163	119	0	0	105	0
N.S.	1	0.57	0.45	0.61	0.44	0.00	0.00	0.39	0.00
time (sec)	N/A	0.292	0.129	5.449	0.199	0.000	0.000	0.304	0.000

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	88	66	65	0	0	0	80	0
N.S.	1	0.49	0.37	0.36	0.00	0.00	0.00	0.45	0.00
time (sec)	N/A	0.242	0.046	0.170	0.000	0.000	0.000	0.443	0.000

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	391	148	125	113	79	0	0	173	0
N.S.	1	0.38	0.32	0.29	0.20	0.00	0.00	0.44	0.00
time (sec)	N/A	0.294	0.070	0.126	0.209	0.000	0.000	0.325	0.000

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	120	99	91	57	0	0	128	0
N.S.	1	0.41	0.34	0.31	0.20	0.00	0.00	0.44	0.00
time (sec)	N/A	0.262	0.060	0.036	0.193	0.000	0.000	0.331	0.000

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	91	75	69	30	0	0	79	0
N.S.	1	0.48	0.40	0.37	0.16	0.00	0.00	0.42	0.00
time (sec)	N/A	0.229	0.044	0.030	0.196	0.000	0.000	0.312	0.000

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	59	49	47	10	0	0	34	39
N.S.	1	0.67	0.56	0.53	0.11	0.00	0.00	0.39	0.44
time (sec)	N/A	0.194	0.024	0.023	0.196	0.000	0.000	0.306	8.479

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	96	86	78	44	0	0	77	0
N.S.	1	0.51	0.45	0.41	0.23	0.00	0.00	0.41	0.00
time (sec)	N/A	0.257	0.040	0.026	0.240	0.000	0.000	0.337	0.000

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	300	139	126	141	97	0	0	127	0
N.S.	1	0.46	0.42	0.47	0.32	0.00	0.00	0.42	0.00
time (sec)	N/A	0.290	0.097	0.038	0.227	0.000	0.000	0.312	0.000

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	410	181	152	199	139	0	0	147	0
N.S.	1	0.44	0.37	0.49	0.34	0.00	0.00	0.36	0.00
time (sec)	N/A	0.344	0.096	0.037	0.188	0.000	0.000	0.310	0.000

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	118	98	91	57	0	0	126	0
N.S.	1	0.41	0.34	0.31	0.20	0.00	0.00	0.44	0.00
time (sec)	N/A	0.262	0.057	0.153	0.181	0.000	0.000	0.310	0.000

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	119	101	91	52	0	0	125	0
N.S.	1	0.41	0.35	0.31	0.18	0.00	0.00	0.43	0.00
time (sec)	N/A	0.257	0.062	0.136	0.188	0.000	0.000	0.314	0.000

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	139	98	141	99	302	0	84	0
N.S.	1	0.63	0.44	0.64	0.45	1.36	0.00	0.38	0.00
time (sec)	N/A	0.274	0.088	1.248	0.187	0.390	0.000	0.314	0.000

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	391	150	124	117	79	0	0	172	0
N.S.	1	0.38	0.32	0.30	0.20	0.00	0.00	0.44	0.00
time (sec)	N/A	0.292	0.067	0.058	0.179	0.000	0.000	0.427	0.000

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	41	38	47	45	38	63	0	0
N.S.	1	0.89	0.83	1.02	0.98	0.83	1.37	0.00	0.00
time (sec)	N/A	0.187	0.051	0.663	0.178	0.271	22.442	0.000	0.000

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	26	27	31	32	24	48	0	0
N.S.	1	0.93	0.96	1.11	1.14	0.86	1.71	0.00	0.00
time (sec)	N/A	0.178	0.043	0.650	0.173	0.270	8.957	0.000	0.000

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	18	19	15	46	0	0
N.S.	1	1.00	1.00	1.20	1.27	1.00	3.07	0.00	0.00
time (sec)	N/A	0.144	0.024	0.627	0.174	0.258	1.745	0.000	0.000

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	26	25	26	27	22	54	25	20
N.S.	1	1.13	1.09	1.13	1.17	0.96	2.35	1.09	0.87
time (sec)	N/A	0.155	0.037	0.642	0.182	0.272	1.744	0.276	8.710

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	54	48	58	58	59	100	0	0
N.S.	1	0.95	0.84	1.02	1.02	1.04	1.75	0.00	0.00
time (sec)	N/A	0.203	0.078	0.625	0.180	0.287	29.315	0.000	0.000

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	70	63	75	71	72	0	0	0
N.S.	1	0.92	0.83	0.99	0.93	0.95	0.00	0.00	0.00
time (sec)	N/A	0.208	0.091	0.615	0.174	0.276	0.000	0.000	0.000

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	84	76	90	84	85	0	0	0
N.S.	1	0.90	0.82	0.97	0.90	0.91	0.00	0.00	0.00
time (sec)	N/A	0.220	0.103	0.628	0.179	0.264	0.000	0.000	0.000

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	259	34	54	0	270	0	203	0
N.S.	1	1.10	0.14	0.23	0.00	1.14	0.00	0.86	0.00
time (sec)	N/A	0.448	0.040	0.691	0.000	0.266	0.000	0.321	0.000

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	34	54	0	212	0	136	0
N.S.	1	1.00	0.21	0.34	0.00	1.32	0.00	0.85	0.00
time (sec)	N/A	0.331	0.039	0.672	0.000	0.277	0.000	0.287	0.000

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	48	32	79	0	151	0	38	0
N.S.	1	0.96	0.64	1.58	0.00	3.02	0.00	0.76	0.00
time (sec)	N/A	0.177	0.035	0.664	0.000	0.266	0.000	0.297	0.000

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	70	34	97	0	161	0	0	0
N.S.	1	1.03	0.50	1.43	0.00	2.37	0.00	0.00	0.00
time (sec)	N/A	0.203	0.041	0.670	0.000	0.265	0.000	0.000	0.000

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	178	34	73	0	171	0	0	0
N.S.	1	1.01	0.19	0.41	0.00	0.97	0.00	0.00	0.00
time (sec)	N/A	0.344	0.039	0.671	0.000	0.266	0.000	0.000	0.000

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	277	34	73	0	249	0	0	0
N.S.	1	1.10	0.13	0.29	0.00	0.99	0.00	0.00	0.00
time (sec)	N/A	0.465	0.038	0.688	0.000	0.276	0.000	0.000	0.000

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	37	37	38	0	43	59	0	0	0
N.S.	1	1.00	1.03	0.00	1.16	1.59	0.00	0.00	0.00
time (sec)	N/A	0.175	0.039	0.000	0.199	0.259	0.000	0.000	0.000

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	38	38	43	0	0	59	0	0	0
N.S.	1	1.00	1.13	0.00	0.00	1.55	0.00	0.00	0.00
time (sec)	N/A	0.175	0.084	0.000	0.000	0.263	0.000	0.000	0.000

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	77	96	208	74	74	0	0	0
N.S.	1	0.69	0.86	1.86	0.66	0.66	0.00	0.00	0.00
time (sec)	N/A	0.218	0.042	0.113	0.186	0.261	0.000	0.000	0.000

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	77	70	135	48	48	0	0	0
N.S.	1	0.69	0.62	1.21	0.43	0.43	0.00	0.00	0.00
time (sec)	N/A	0.213	0.024	0.047	0.190	0.259	0.000	0.000	0.000

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	75	44	64	22	22	0	0	0
N.S.	1	0.76	0.44	0.65	0.22	0.22	0.00	0.00	0.00
time (sec)	N/A	0.211	0.013	0.041	0.185	0.267	0.000	0.000	0.000

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	78	47	71	32	24	0	0	0
N.S.	1	0.87	0.52	0.79	0.36	0.27	0.00	0.00	0.00
time (sec)	N/A	0.211	0.025	0.060	0.179	0.264	0.000	0.000	0.000

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	75	40	37	41	41	0	0	0
N.S.	1	1.56	0.83	0.77	0.85	0.85	0.00	0.00	0.00
time (sec)	N/A	0.207	0.019	0.039	0.182	0.257	0.000	0.000	0.000

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	77	40	37	69	69	0	0	0
N.S.	1	0.88	0.45	0.42	0.78	0.78	0.00	0.00	0.00
time (sec)	N/A	0.212	0.055	0.047	0.190	0.266	0.000	0.000	0.000

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	77	40	37	97	97	0	0	0
N.S.	1	0.88	0.45	0.42	1.10	1.10	0.00	0.00	0.00
time (sec)	N/A	0.205	0.059	0.050	0.180	0.254	0.000	0.000	0.000

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	108	77	55	99	47	57	0	173	0
N.S.	1	0.71	0.51	0.92	0.44	0.53	0.00	1.60	0.00
time (sec)	N/A	0.242	0.032	0.070	0.183	0.300	0.000	0.283	0.000

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	62	46	61	25	28	0	53	0
N.S.	1	0.67	0.49	0.66	0.27	0.30	0.00	0.57	0.00
time (sec)	N/A	0.215	0.021	0.034	0.179	0.266	0.000	0.290	0.000

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	62	46	61	25	28	0	53	0
N.S.	1	0.67	0.49	0.66	0.27	0.30	0.00	0.57	0.00
time (sec)	N/A	0.214	0.023	0.029	0.177	0.288	0.000	0.270	0.000

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	57	39	56	19	20	0	25	0
N.S.	1	0.65	0.44	0.64	0.22	0.23	0.00	0.28	0.00
time (sec)	N/A	0.200	0.010	0.028	0.177	0.261	0.000	0.279	0.000

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	54	38	54	13	15	0	0	0
N.S.	1	0.64	0.45	0.64	0.15	0.18	0.00	0.00	0.00
time (sec)	N/A	0.212	0.013	0.035	0.180	0.267	0.000	0.000	0.000

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	63	42	61	22	23	0	0	0
N.S.	1	0.67	0.45	0.65	0.23	0.24	0.00	0.00	0.00
time (sec)	N/A	0.218	0.025	0.033	0.182	0.284	0.000	0.000	0.000

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	65	47	61	22	23	0	0	0
N.S.	1	0.68	0.49	0.64	0.23	0.24	0.00	0.00	0.00
time (sec)	N/A	0.218	0.023	0.034	0.176	0.268	0.000	0.000	0.000

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	238	137	90	499	276	390	0	2719	0
N.S.	1	0.58	0.38	2.10	1.16	1.64	0.00	11.42	0.00
time (sec)	N/A	0.299	0.086	0.086	0.188	0.275	0.000	0.413	0.000

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	111	123	146	108	144	0	292	0
N.S.	1	0.52	0.58	0.69	0.51	0.68	0.00	1.38	0.00
time (sec)	N/A	0.264	0.063	0.030	0.178	0.289	0.000	0.310	0.000

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	110	124	145	109	145	0	292	0
N.S.	1	0.52	0.59	0.69	0.52	0.69	0.00	1.38	0.00
time (sec)	N/A	0.260	0.059	0.031	0.179	0.278	0.000	0.311	0.000

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	105	122	138	101	130	0	263	0
N.S.	1	0.51	0.59	0.67	0.49	0.63	0.00	1.28	0.00
time (sec)	N/A	0.248	0.058	0.029	0.176	0.275	0.000	0.297	0.000

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	89	67	127	43	44	0	0	0
N.S.	1	0.45	0.34	0.65	0.22	0.22	0.00	0.00	0.00
time (sec)	N/A	0.242	0.024	0.035	0.270	0.275	0.000	0.000	0.000

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	111	124	147	101	131	0	0	0
N.S.	1	0.52	0.58	0.69	0.48	0.62	0.00	0.00	0.00
time (sec)	N/A	0.271	0.073	0.043	0.217	0.272	0.000	0.000	0.000

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	117	124	145	101	134	0	0	0
N.S.	1	0.54	0.57	0.67	0.46	0.61	0.00	0.00	0.00
time (sec)	N/A	0.273	0.070	0.045	0.213	0.289	0.000	0.000	0.000

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	76	83	62	0	0	0	0	0	0
N.S.	1	1.09	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.224	0.027	0.000	0.000	0.000	0.000	0.000	0.000

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	71	53	0	0	0	0	0	0
N.S.	1	1.11	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.202	0.041	0.000	0.000	0.000	0.000	0.000	0.000

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	71	53	0	0	0	0	0	0
N.S.	1	1.11	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.198	0.015	0.000	0.000	0.000	0.000	0.000	0.000

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	55	62	44	0	0	0	0	0	0
N.S.	1	1.13	0.80	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.183	0.015	0.000	0.000	0.000	0.000	0.000	0.000

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	64	45	66	27	22	0	0	0
N.S.	1	0.75	0.53	0.78	0.32	0.26	0.00	0.00	0.00
time (sec)	N/A	0.199	0.021	0.036	0.185	0.263	0.000	0.000	0.000

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	65	72	51	0	0	0	0	0	0
N.S.	1	1.11	0.78	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.197	0.015	0.000	0.000	0.000	0.000	0.000	0.000

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	67	74	53	0	0	0	0	0	0
N.S.	1	1.10	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.200	0.016	0.000	0.000	0.000	0.000	0.000	0.000

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	76	85	61	0	0	0	0	0	0
N.S.	1	1.12	0.80	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.211	0.022	0.000	0.000	0.000	0.000	0.000	0.000

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	73	55	0	0	0	0	0	0
N.S.	1	1.14	0.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.198	0.042	0.000	0.000	0.000	0.000	0.000	0.000

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	73	55	0	0	0	0	0	0
N.S.	1	1.14	0.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.192	0.016	0.000	0.000	0.000	0.000	0.000	0.000

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	57	64	46	0	0	0	0	0	0
N.S.	1	1.12	0.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.184	0.014	0.000	0.000	0.000	0.000	0.000	0.000

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	95	79	104	70	106	0	0	0
N.S.	1	0.60	0.50	0.65	0.44	0.67	0.00	0.00	0.00
time (sec)	N/A	0.255	0.036	0.038	0.195	0.300	0.000	0.000	0.000

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	65	74	53	0	0	0	0	0	0
N.S.	1	1.14	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.202	0.017	0.000	0.000	0.000	0.000	0.000	0.000

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	67	76	55	0	0	0	0	0	0
N.S.	1	1.13	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.199	0.015	0.000	0.000	0.000	0.000	0.000	0.000

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	52	52	58	0	0	79	0	0	0
N.S.	1	1.00	1.12	0.00	0.00	1.52	0.00	0.00	0.00
time (sec)	N/A	0.191	0.058	0.000	0.000	0.271	0.000	0.000	0.000

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	32	51	0	45	160	0	0
N.S.	1	1.00	0.74	1.19	0.00	1.05	3.72	0.00	0.00
time (sec)	N/A	0.182	0.111	0.960	0.000	0.275	4.979	0.000	0.000

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	130	129	80	0	0	103	0	0	0
N.S.	1	0.99	0.62	0.00	0.00	0.79	0.00	0.00	0.00
time (sec)	N/A	0.261	0.144	0.000	0.000	0.280	0.000	0.000	0.000

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	102	99	59	0	0	82	0	0	0
N.S.	1	0.97	0.58	0.00	0.00	0.80	0.00	0.00	0.00
time (sec)	N/A	0.238	0.112	0.000	0.000	0.263	0.000	0.000	0.000

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	117	127	75	0	0	165	0	0	0
N.S.	1	1.09	0.64	0.00	0.00	1.41	0.00	0.00	0.00
time (sec)	N/A	0.277	0.096	0.000	0.000	0.266	0.000	0.000	0.000

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	103	87	54	113	59	78	0	0	0
N.S.	1	0.84	0.52	1.10	0.57	0.76	0.00	0.00	0.00
time (sec)	N/A	0.227	0.069	14.742	0.194	0.252	0.000	0.000	0.000

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	103	97	973	0	353	0	0	0
N.S.	1	0.93	0.87	8.77	0.00	3.18	0.00	0.00	0.00
time (sec)	N/A	0.280	0.245	0.351	0.000	0.269	0.000	0.000	0.000

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	82	82	664	0	285	0	0	0
N.S.	1	0.94	0.94	7.63	0.00	3.28	0.00	0.00	0.00
time (sec)	N/A	0.255	0.169	0.287	0.000	0.273	0.000	0.000	0.000

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	66	66	402	0	231	0	0	0
N.S.	1	0.97	0.97	5.91	0.00	3.40	0.00	0.00	0.00
time (sec)	N/A	0.231	0.111	0.243	0.000	0.258	0.000	0.000	0.000

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	43	113	0	159	0	39	39
N.S.	1	1.00	1.10	2.90	0.00	4.08	0.00	1.00	1.00
time (sec)	N/A	0.185	0.067	0.198	0.000	0.266	0.000	0.275	8.383

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	99	90	658	0	333	0	0	0
N.S.	1	1.01	0.92	6.71	0.00	3.40	0.00	0.00	0.00
time (sec)	N/A	0.306	0.216	0.253	0.000	0.283	0.000	0.000	0.000

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	129	115	958	0	429	0	0	0
N.S.	1	1.02	0.91	7.60	0.00	3.40	0.00	0.00	0.00
time (sec)	N/A	0.348	0.357	0.292	0.000	0.292	0.000	0.000	0.000

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	158	147	1300	0	522	0	0	0
N.S.	1	0.96	0.90	7.93	0.00	3.18	0.00	0.00	0.00
time (sec)	N/A	0.394	0.317	0.373	0.000	0.279	0.000	0.000	0.000

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	353	329	340	280	0	3481	0	0	0
N.S.	1	0.93	0.96	0.79	0.00	9.86	0.00	0.00	0.00
time (sec)	N/A	0.543	0.703	1.036	0.000	0.403	0.000	0.000	0.000

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	610	533	526	260	0	2461	0	0	0
N.S.	1	0.87	0.86	0.43	0.00	4.03	0.00	0.00	0.00
time (sec)	N/A	0.959	0.639	0.700	0.000	0.347	0.000	0.000	0.000

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	167	145	114	0	801	0	1037	0
N.S.	1	0.99	0.86	0.67	0.00	4.74	0.00	6.14	0.00
time (sec)	N/A	0.294	0.294	0.369	0.000	0.289	0.000	0.677	0.000

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	206	127	268	0	1229	0	0	0
N.S.	1	1.00	0.62	1.31	0.00	6.00	0.00	0.00	0.00
time (sec)	N/A	0.389	0.224	0.530	0.000	0.298	0.000	0.000	0.000

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	699	576	127	534	0	3155	0	0	0
N.S.	1	0.82	0.18	0.76	0.00	4.51	0.00	0.00	0.00
time (sec)	N/A	0.989	0.176	1.044	0.000	0.369	0.000	0.000	0.000

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	414	372	127	630	0	4375	0	0	0
N.S.	1	0.90	0.31	1.52	0.00	10.57	0.00	0.00	0.00
time (sec)	N/A	0.624	0.170	1.543	0.000	0.436	0.000	0.000	0.000

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	140	149	265	0	0	0	0	0	0
N.S.	1	1.06	1.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.270	0.549	0.000	0.000	0.000	0.000	0.000	0.000

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	136	144	263	0	0	0	0	0	0
N.S.	1	1.06	1.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.248	0.487	0.000	0.000	0.000	0.000	0.000	0.000

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	124	133	261	0	0	0	0	0	0
N.S.	1	1.07	2.10	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.226	0.456	0.000	0.000	0.000	0.000	0.000	0.000

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	75	72	397	0	259	0	0	224
N.S.	1	1.01	0.97	5.36	0.00	3.50	0.00	0.00	3.03
time (sec)	N/A	0.252	0.158	0.192	0.000	0.285	0.000	0.000	8.493

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	142	151	240	0	0	0	0	0	0
N.S.	1	1.06	1.69	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.251	0.375	0.000	0.000	0.000	0.000	0.000	0.000

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	140	150	258	0	0	0	0	0	0
N.S.	1	1.07	1.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.254	0.387	0.000	0.000	0.000	0.000	0.000	0.000

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	148	148	365	0	0	0	0	0	0
N.S.	1	1.00	2.47	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.348	0.609	0.000	0.000	0.000	0.000	0.000	0.000

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	148	148	366	0	0	0	0	0	0
N.S.	1	1.00	2.47	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.317	0.566	0.000	0.000	0.000	0.000	0.000	0.000

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	148	148	364	0	0	0	0	0	0
N.S.	1	1.00	2.46	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.287	0.509	0.000	0.000	0.000	0.000	0.000	0.000

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	139	139	351	0	0	0	0	0	0
N.S.	1	1.00	2.53	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.265	0.512	0.000	0.000	0.000	0.000	0.000	0.000

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	115	113	125	0	658	0	0	0
N.S.	1	0.97	0.95	1.05	0.00	5.53	0.00	0.00	0.00
time (sec)	N/A	0.292	0.280	0.115	0.000	0.289	0.000	0.000	0.000

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	149	149	365	0	0	0	0	0	0
N.S.	1	1.00	2.45	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.317	0.482	0.000	0.000	0.000	0.000	0.000	0.000

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	151	151	365	0	0	0	0	0	0
N.S.	1	1.00	2.42	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.317	0.492	0.000	0.000	0.000	0.000	0.000	0.000

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	149	149	469	0	0	0	0	0	0
N.S.	1	1.00	3.15	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.318	1.105	0.000	0.000	0.000	0.000	0.000	0.000

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	149	149	475	0	0	0	0	0	0
N.S.	1	1.00	3.19	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.318	1.050	0.000	0.000	0.000	0.000	0.000	0.000

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	149	149	471	0	0	0	0	0	0
N.S.	1	1.00	3.16	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.291	1.054	0.000	0.000	0.000	0.000	0.000	0.000

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	140	140	466	0	0	0	0	0	0
N.S.	1	1.00	3.33	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.298	1.056	0.000	0.000	0.000	0.000	0.000	0.000

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	177	159	209	0	827	0	0	0
N.S.	1	1.02	0.92	1.21	0.00	4.78	0.00	0.00	0.00
time (sec)	N/A	0.385	0.627	0.088	0.000	0.434	0.000	0.000	0.000

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	150	150	477	0	0	0	0	0	0
N.S.	1	1.00	3.18	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.329	0.975	0.000	0.000	0.000	0.000	0.000	0.000

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	152	152	471	0	0	0	0	0	0
N.S.	1	1.00	3.10	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.315	0.971	0.000	0.000	0.000	0.000	0.000	0.000

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	148	148	175	0	0	0	0	0	0
N.S.	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.329	0.205	0.000	0.000	0.000	0.000	0.000	0.000

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	148	148	175	0	0	0	0	0	0
N.S.	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.315	0.162	0.000	0.000	0.000	0.000	0.000	0.000

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	148	148	175	0	0	0	0	0	0
N.S.	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.288	0.157	0.000	0.000	0.000	0.000	0.000	0.000

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	139	139	166	0	0	0	0	0	0
N.S.	1	1.00	1.19	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.261	0.121	0.000	0.000	0.000	0.000	0.000	0.000

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	47	47	46	0	0	148	0	0	0
N.S.	1	1.00	0.98	0.00	0.00	3.15	0.00	0.00	0.00
time (sec)	N/A	0.189	0.136	0.000	0.000	0.252	0.000	0.000	0.000

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	149	149	173	0	0	0	0	0	0
N.S.	1	1.00	1.16	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.319	0.183	0.000	0.000	0.000	0.000	0.000	0.000

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	151	151	175	0	0	0	0	0	0
N.S.	1	1.00	1.16	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.321	0.169	0.000	0.000	0.000	0.000	0.000	0.000

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	151	151	398	0	0	0	0	0	0
N.S.	1	1.00	2.64	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.324	0.652	0.000	0.000	0.000	0.000	0.000	0.000

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	151	151	398	0	0	0	0	0	0
N.S.	1	1.00	2.64	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.319	0.678	0.000	0.000	0.000	0.000	0.000	0.000

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	151	151	398	0	0	0	0	0	0
N.S.	1	1.00	2.64	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.289	0.626	0.000	0.000	0.000	0.000	0.000	0.000

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	142	142	384	0	0	0	0	0	0
N.S.	1	1.00	2.70	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.271	0.703	0.000	0.000	0.000	0.000	0.000	0.000

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	98	96	98	0	0	449	0	0	0
N.S.	1	0.98	1.00	0.00	0.00	4.58	0.00	0.00	0.00
time (sec)	N/A	0.248	0.655	0.000	0.000	0.338	0.000	0.000	0.000

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	152	152	395	0	0	0	0	0	0
N.S.	1	1.00	2.60	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.325	0.603	0.000	0.000	0.000	0.000	0.000	0.000

Problem 595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	154	154	399	0	0	0	0	0	0
N.S.	1	1.00	2.59	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.326	0.609	0.000	0.000	0.000	0.000	0.000	0.000

Problem 596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	182	182	137	3765	273	2303	68491	25656	1734
N.S.	1	1.00	0.75	20.69	1.50	12.65	376.32	140.97	9.53
time (sec)	N/A	0.379	0.560	2.555	0.263	0.330	65.900	0.488	9.427

Problem 597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	117	117	86	1032	152	706	12323	5454	543
N.S.	1	1.00	0.74	8.82	1.30	6.03	105.32	46.62	4.64
time (sec)	N/A	0.278	0.162	0.671	0.232	0.308	16.765	0.330	9.138

Problem 598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	58	58	41	172	65	142	1096	557	83
N.S.	1	1.00	0.71	2.97	1.12	2.45	18.90	9.60	1.43
time (sec)	N/A	0.203	0.097	0.160	0.216	0.283	3.362	0.281	8.933

Problem 599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	175	175	307	0	0	0	0	0	0
N.S.	1	1.00	1.75	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.313	0.819	0.000	0.000	0.000	0.000	0.000	0.000

Problem 600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	328	314	1890	0	0	0	0	0	0
N.S.	1	0.96	5.76	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.730	4.409	0.000	0.000	0.000	0.000	0.000	0.000

Problem 601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	615	677	12289	0	0	0	0	0	0
N.S.	1	1.10	19.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	5.186	7.243	0.000	0.000	0.000	0.000	0.000	0.000

Problem 602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	161	161	545	0	0	0	0	0	0
N.S.	1	1.00	3.39	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.355	2.564	0.000	0.000	0.000	0.000	0.000	0.000

Problem 603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	160	160	388	0	0	0	0	0	0
N.S.	1	1.00	2.42	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.335	0.708	0.000	0.000	0.000	0.000	0.000	0.000

Problem 604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	160	160	183	0	0	0	0	0	0
N.S.	1	1.00	1.14	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.335	0.367	0.000	0.000	0.000	0.000	0.000	0.000

Problem 605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	163	163	428	0	0	0	0	0	0
N.S.	1	1.00	2.63	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.340	1.452	0.000	0.000	0.000	0.000	0.000	0.000

Problem 606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	158	158	181	0	0	0	0	0	0
N.S.	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.315	0.477	0.000	0.000	0.000	0.000	0.000	0.000

Problem 607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	41	150	162	142	142	178	160	141
N.S.	1	0.89	3.26	3.52	3.09	3.09	3.87	3.48	3.07
time (sec)	N/A	0.194	0.047	0.601	0.202	0.268	0.035	0.275	0.054

Problem 608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	81	401	494	403	403	559	475	383
N.S.	1	0.91	4.51	5.55	4.53	4.53	6.28	5.34	4.30
time (sec)	N/A	0.266	0.082	0.627	0.206	0.254	0.072	0.296	8.885

Problem 609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	121	797	1130	872	872	1314	1079	777
N.S.	1	0.88	5.78	8.19	6.32	6.32	9.52	7.82	5.63
time (sec)	N/A	0.307	0.203	0.641	0.217	0.260	0.130	0.307	9.022

Problem 610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	44	154	179	166	166	240	204	164
N.S.	1	0.80	2.80	3.25	3.02	3.02	4.36	3.71	2.98
time (sec)	N/A	0.198	0.019	0.629	0.207	0.265	0.046	0.301	0.046

Problem 611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	84	405	566	439	439	722	597	419
N.S.	1	0.81	3.89	5.44	4.22	4.22	6.94	5.74	4.03
time (sec)	N/A	0.250	0.059	0.597	0.217	0.256	0.071	0.299	0.123

Problem 612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	124	801	1318	920	920	1654	1330	825
N.S.	1	0.78	5.04	8.29	5.79	5.79	10.40	8.36	5.19
time (sec)	N/A	0.282	0.025	0.638	0.220	0.261	0.148	0.306	8.905

Problem 613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	199	219	158	0	1231	178	1315	3988
N.S.	1	1.03	1.13	0.82	0.00	6.38	0.92	6.81	20.66
time (sec)	N/A	0.365	0.106	0.595	0.000	0.285	1.735	0.299	9.178

Problem 614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	79	77	151	0	434	280	126	278
N.S.	1	0.98	0.95	1.86	0.00	5.36	3.46	1.56	3.43
time (sec)	N/A	0.256	0.037	0.635	0.000	0.293	0.921	0.299	8.885

Problem 615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	170	175	140	0	703	104	1403	590
N.S.	1	1.04	1.07	0.85	0.00	4.29	0.63	8.55	3.60
time (sec)	N/A	0.288	0.063	0.592	0.000	0.255	0.726	0.294	0.213

Problem 616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	46	129	0	272	168	52	61
N.S.	1	1.00	1.07	3.00	0.00	6.33	3.91	1.21	1.42
time (sec)	N/A	0.200	0.013	0.083	0.000	0.284	0.566	0.290	8.683

Problem 617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	92	128	155	0	468	320	280	2173
N.S.	1	0.98	1.36	1.65	0.00	4.98	3.40	2.98	23.12
time (sec)	N/A	0.278	0.062	0.633	0.000	0.277	13.829	0.357	9.564

Problem 618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	188	206	168	0	1339	211	932	3844
N.S.	1	0.96	1.06	0.86	0.00	6.87	1.08	4.78	19.71
time (sec)	N/A	0.331	0.232	0.635	0.000	0.279	2.956	0.307	9.223

Problem 619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	118	154	213	0	810	0	101	4950
N.S.	1	0.98	1.27	1.76	0.00	6.69	0.00	0.83	40.91
time (sec)	N/A	0.329	0.096	0.664	0.000	0.295	0.000	0.300	12.240

Problem 620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	223	235	188	0	2044	347	1347	5214
N.S.	1	1.00	1.05	0.84	0.00	9.12	1.55	6.01	23.28
time (sec)	N/A	0.429	0.133	0.702	0.000	0.331	104.662	0.304	9.481

Problem 621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	255	263	323	0	2454	573	1408	7327
N.S.	1	0.94	0.97	1.20	0.00	9.09	2.12	5.21	27.14
time (sec)	N/A	0.397	0.304	0.630	0.000	0.321	11.780	0.366	10.483

Problem 622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	95	100	276	0	1021	495	162	427
N.S.	1	0.98	1.03	2.85	0.00	10.53	5.10	1.67	4.40
time (sec)	N/A	0.252	0.084	0.655	0.000	0.310	2.651	0.304	8.604

Problem 623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	241	247	319	0	2474	0	1416	7200
N.S.	1	0.95	0.97	1.26	0.00	9.74	0.00	5.57	28.35
time (sec)	N/A	0.363	0.615	0.615	0.000	0.345	0.000	0.322	9.932

Problem 624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	95	98	270	0	1042	495	163	417
N.S.	1	0.97	1.00	2.76	0.00	10.63	5.05	1.66	4.26
time (sec)	N/A	0.245	0.089	0.169	0.000	0.300	2.521	0.295	8.619

Problem 625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	264	271	364	0	3228	0	1460	9056
N.S.	1	0.88	0.91	1.22	0.00	10.80	0.00	4.88	30.29
time (sec)	N/A	0.432	0.546	0.134	0.000	0.350	0.000	0.297	10.568

Problem 626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	184	235	399	0	2476	0	467	11072
N.S.	1	1.14	1.45	2.46	0.00	15.28	0.00	2.88	68.35
time (sec)	N/A	0.397	0.261	0.737	0.000	0.445	0.000	0.401	14.695

Problem 627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	348	339	339	441	0	4330	0	879	10556
N.S.	1	0.97	0.97	1.27	0.00	12.44	0.00	2.53	30.33
time (sec)	N/A	0.630	0.961	0.727	0.000	0.447	0.000	0.353	11.530

Problem 628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	226	284	462	0	4562	0	219	12436
N.S.	1	1.06	1.33	2.17	0.00	21.42	0.00	1.03	58.38
time (sec)	N/A	0.452	0.328	0.750	0.000	0.607	0.000	0.292	15.322

Problem 629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	408	394	384	489	0	5734	0	2122	12239
N.S.	1	0.97	0.94	1.20	0.00	14.05	0.00	5.20	30.00
time (sec)	N/A	1.137	1.781	0.781	0.000	0.639	0.000	0.359	13.051

Problem 630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	341	340	328	704	0	6633	0	1802	12677
N.S.	1	1.00	0.96	2.06	0.00	19.45	0.00	5.28	37.18
time (sec)	N/A	0.490	2.600	0.693	0.000	0.452	0.000	0.367	12.046

Problem 631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	160	146	544	0	3739	1671	349	1182
N.S.	1	1.07	0.97	3.63	0.00	24.93	11.14	2.33	7.88
time (sec)	N/A	0.295	0.127	0.727	0.000	0.460	7.147	0.345	10.000

Problem 632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	363	378	382	885	0	7701	0	2447	14584
N.S.	1	1.04	1.05	2.44	0.00	21.21	0.00	6.74	40.18
time (sec)	N/A	0.537	2.857	0.689	0.000	0.624	0.000	0.336	12.329

Problem 633	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	157	147	541	0	3708	1646	349	1157
N.S.	1	1.03	0.97	3.56	0.00	24.39	10.83	2.30	7.61
time (sec)	N/A	0.292	0.119	0.352	0.000	0.426	6.932	0.312	9.812

Problem 634	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	437	405	424	1010	0	8554	0	2641	16086
N.S.	1	0.93	0.97	2.31	0.00	19.57	0.00	6.04	36.81
time (sec)	N/A	0.600	4.490	0.328	0.000	0.768	0.000	0.307	12.255

Problem 635	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	297	391	966	0	9908	0	1045	19440
N.S.	1	1.16	1.53	3.79	0.00	38.85	0.00	4.10	76.24
time (sec)	N/A	0.547	2.658	1.426	0.000	0.951	0.000	0.451	18.815

Problem 636	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	484	471	560	1197	0	10260	0	1458	18112
N.S.	1	0.97	1.16	2.47	0.00	21.20	0.00	3.01	37.42
time (sec)	N/A	0.746	6.181	0.902	0.000	1.107	0.000	0.398	16.577

Problem 637	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	325	353	491	1141	0	15165	0	379	21465
N.S.	1	1.09	1.51	3.51	0.00	46.66	0.00	1.17	66.05
time (sec)	N/A	0.601	6.158	0.943	0.000	1.608	0.000	0.318	22.778

Problem 638	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	222	162	0	1346	219	1366	4605
N.S.	1	1.00	1.10	0.80	0.00	6.66	1.08	6.76	22.80
time (sec)	N/A	0.361	0.097	0.593	0.000	0.290	1.930	0.303	0.803

Problem 639	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	82	80	154	0	446	332	158	287
N.S.	1	0.94	0.92	1.77	0.00	5.13	3.82	1.82	3.30
time (sec)	N/A	0.263	0.028	0.616	0.000	0.273	1.010	0.292	0.258

Problem 640	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	173	178	143	0	799	124	1443	683
N.S.	1	1.02	1.05	0.84	0.00	4.70	0.73	8.49	4.02
time (sec)	N/A	0.292	0.073	0.593	0.000	0.285	0.778	0.328	8.550

Problem 641	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	47	130	0	274	189	61	477
N.S.	1	1.00	1.07	2.95	0.00	6.23	4.30	1.39	10.84
time (sec)	N/A	0.202	0.011	0.076	0.000	0.262	0.594	0.386	0.119

Problem 642	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	95	131	173	0	474	348	291	2520
N.S.	1	0.92	1.27	1.68	0.00	4.60	3.38	2.83	24.47
time (sec)	N/A	0.282	0.055	0.662	0.000	0.312	17.677	0.400	9.526

Problem 643	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	191	209	172	0	1477	258	0	4339
N.S.	1	0.94	1.02	0.84	0.00	7.24	1.26	0.00	21.27
time (sec)	N/A	0.331	0.236	0.665	0.000	0.306	3.196	0.000	9.834

Problem 644	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	121	157	217	0	828	0	356	5947
N.S.	1	0.91	1.18	1.63	0.00	6.23	0.00	2.68	44.71
time (sec)	N/A	0.327	0.091	0.668	0.000	0.378	0.000	0.406	12.005

Problem 645	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	226	238	192	0	2212	411	1353	5771
N.S.	1	0.96	1.01	0.81	0.00	9.37	1.74	5.73	24.45
time (sec)	N/A	0.421	0.133	0.696	0.000	0.302	105.919	0.294	9.417

Problem 646	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	279	258	266	327	0	2578	641	1474	8025
N.S.	1	0.92	0.95	1.17	0.00	9.24	2.30	5.28	28.76
time (sec)	N/A	0.396	0.283	0.625	0.000	0.319	11.975	0.302	10.242

Problem 647	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	98	103	280	0	1077	556	202	460
N.S.	1	0.95	1.00	2.72	0.00	10.46	5.40	1.96	4.47
time (sec)	N/A	0.260	0.083	0.685	0.000	0.322	2.768	0.294	8.600

Problem 648	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	244	250	323	0	2600	0	1482	7835
N.S.	1	0.93	0.95	1.23	0.00	9.89	0.00	5.63	29.79
time (sec)	N/A	0.357	0.599	0.630	0.000	0.304	0.000	0.299	9.982

Problem 649	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	96	99	272	0	1066	525	202	442
N.S.	1	0.98	1.01	2.78	0.00	10.88	5.36	2.06	4.51
time (sec)	N/A	0.246	0.079	0.181	0.000	0.298	2.607	0.299	8.549

Problem 650	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	187	238	403	0	2486	0	489	13434
N.S.	1	1.07	1.37	2.32	0.00	14.29	0.00	2.81	77.21
time (sec)	N/A	0.396	0.277	0.740	0.000	0.543	0.000	0.398	14.694

Problem 651	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	360	342	342	445	0	4520	0	1031	12008
N.S.	1	0.95	0.95	1.24	0.00	12.56	0.00	2.86	33.36
time (sec)	N/A	0.617	0.973	0.753	0.000	0.389	0.000	0.342	11.869

Problem 652	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	229	287	466	0	4604	0	705	14830
N.S.	1	1.00	1.26	2.04	0.00	20.19	0.00	3.09	65.04
time (sec)	N/A	0.442	0.306	0.810	0.000	1.194	0.000	0.414	15.937

Problem 653	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	423	397	387	493	0	5954	0	2137	13781
N.S.	1	0.94	0.91	1.17	0.00	14.08	0.00	5.05	32.58
time (sec)	N/A	1.031	1.727	0.801	0.000	0.488	0.000	0.323	13.999

Problem 654	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	353	343	331	708	0	6770	0	1958	13840
N.S.	1	0.97	0.94	2.01	0.00	19.18	0.00	5.55	39.21
time (sec)	N/A	0.495	2.605	0.691	0.000	0.464	0.000	0.330	12.207

Problem 655	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	163	149	548	0	3843	1794	431	1267
N.S.	1	1.03	0.94	3.45	0.00	24.17	11.28	2.71	7.97
time (sec)	N/A	0.310	0.147	0.737	0.000	0.388	7.323	0.330	9.802

Problem 656	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	375	381	385	889	0	7838	0	2679	16025
N.S.	1	1.02	1.03	2.37	0.00	20.90	0.00	7.14	42.73
time (sec)	N/A	0.537	2.782	0.698	0.000	0.534	0.000	0.334	12.303

Problem 657	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	158	148	543	0	3748	1707	429	1199
N.S.	1	1.03	0.97	3.55	0.00	24.50	11.16	2.80	7.84
time (sec)	N/A	0.292	0.115	0.379	0.000	0.556	7.207	0.327	9.757

Problem 658	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	300	394	970	0	9926	0	1077	22621
N.S.	1	1.11	1.46	3.59	0.00	36.76	0.00	3.99	83.78
time (sec)	N/A	0.545	2.373	0.904	0.000	1.582	0.000	0.432	19.421

Problem 659	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	499	474	575	1201	0	10518	0	1704	20580
N.S.	1	0.95	1.15	2.41	0.00	21.08	0.00	3.41	41.24
time (sec)	N/A	0.698	6.144	0.925	0.000	1.136	0.000	0.396	17.022

Problem 660	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	356	461	1145	0	15231	0	1791	25334
N.S.	1	1.04	1.34	3.34	0.00	44.41	0.00	5.22	73.86
time (sec)	N/A	0.590	6.058	0.999	0.000	4.422	0.000	0.432	24.183

Problem 661	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	340	338	0	0	0	0	0	0	0
N.S.	1	0.99	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.502	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 662	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	398	393	0	0	0	0	0	0	0
N.S.	1	0.99	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.544	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 663	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	104	104	104	107	28	29
N.S.	1	1.00	1.00	3.06	3.06	3.06	3.15	0.82	0.85
time (sec)	N/A	0.187	0.016	0.561	0.210	0.267	0.042	0.353	8.503

Problem 664	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	48	188	174	174	174	187	46	46
N.S.	1	0.86	3.36	3.11	3.11	3.11	3.34	0.82	0.82
time (sec)	N/A	0.224	0.011	0.587	0.232	0.236	0.068	0.325	0.084

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [158] had the largest ratio of [.8750000000000000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.00	15	0.133
2	A	1	1	1.00	15	0.067
3	A	1	1	1.00	15	0.067
4	A	3	3	1.08	15	0.200
5	A	10	9	1.10	11	0.818
6	A	5	4	0.90	26	0.154
7	A	4	4	0.62	26	0.154
8	A	4	4	0.62	26	0.154
9	A	4	3	1.00	26	0.115
10	A	4	4	0.62	24	0.167
11	A	2	2	0.68	22	0.091
12	A	4	4	0.60	26	0.154
13	A	4	4	0.61	26	0.154
14	A	4	4	0.59	26	0.154
15	A	4	4	0.60	26	0.154
16	A	4	4	0.61	26	0.154
17	A	4	4	0.62	26	0.154
18	A	5	4	0.49	26	0.154
19	A	4	4	0.62	26	0.154
20	A	4	4	0.62	26	0.154
21	A	4	4	0.62	26	0.154
22	A	4	4	0.62	26	0.154

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	4	4	0.45	26	0.154
24	A	6	5	0.66	26	0.192
25	A	4	4	0.45	26	0.154
26	A	4	4	0.45	26	0.154
27	A	5	4	0.94	26	0.154
28	A	4	4	0.45	26	0.154
29	A	4	4	0.45	26	0.154
30	A	4	3	1.06	26	0.115
31	A	4	4	0.45	24	0.167
32	A	3	3	0.48	22	0.136
33	A	6	5	0.46	26	0.192
34	A	4	4	0.44	26	0.154
35	A	4	4	0.44	26	0.154
36	A	6	5	0.45	26	0.192
37	A	4	4	0.44	26	0.154
38	A	4	4	0.44	26	0.154
39	A	6	5	0.44	26	0.192
40	A	4	4	0.44	26	0.154
41	A	4	4	0.43	26	0.154
42	A	6	5	0.46	26	0.192
43	A	4	4	0.44	26	0.154
44	A	4	4	0.45	26	0.154
45	A	3	3	1.00	26	0.115
46	A	4	4	0.45	26	0.154
47	A	4	4	0.45	26	0.154
48	A	6	5	0.89	26	0.192
49	A	4	4	0.45	26	0.154
50	A	4	4	0.40	26	0.154
51	A	4	4	0.40	26	0.154
52	A	6	5	0.67	26	0.192
53	A	4	4	0.40	26	0.154
54	A	4	4	0.40	26	0.154
55	A	6	5	0.74	26	0.192
56	A	4	4	0.40	26	0.154

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
57	A	4	4	0.40	26	0.154
58	A	5	4	0.94	26	0.154
59	A	4	4	0.40	26	0.154
60	A	4	4	0.39	26	0.154
61	A	4	3	1.06	26	0.115
62	A	4	4	0.39	24	0.167
63	A	3	3	0.41	22	0.136
64	A	6	5	0.39	26	0.192
65	A	4	4	0.39	26	0.154
66	A	4	4	0.39	26	0.154
67	A	6	5	0.38	26	0.192
68	A	4	4	0.38	26	0.154
69	A	4	4	0.39	26	0.154
70	A	6	5	0.39	26	0.192
71	A	4	4	0.38	26	0.154
72	A	4	4	0.38	26	0.154
73	A	6	5	0.39	26	0.192
74	A	4	4	0.39	26	0.154
75	A	4	4	0.38	26	0.154
76	A	6	5	0.38	26	0.192
77	A	4	4	0.39	26	0.154
78	A	4	4	0.38	26	0.154
79	A	6	5	0.39	26	0.192
80	A	4	4	0.39	26	0.154
81	A	4	4	0.39	26	0.154
82	A	3	3	1.00	26	0.115
83	A	4	4	0.39	26	0.154
84	A	4	4	0.40	26	0.154
85	A	6	5	0.89	26	0.192
86	A	4	4	0.40	26	0.154
87	A	4	4	0.40	26	0.154
88	A	7	6	0.80	26	0.231
89	A	12	11	0.68	26	0.423
90	A	12	11	0.65	26	0.423

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
91	A	4	3	1.00	26	0.115
92	A	11	10	0.72	24	0.417
93	A	11	10	0.71	22	0.455
94	A	7	6	0.69	26	0.231
95	A	12	11	0.68	26	0.423
96	A	12	11	0.65	26	0.423
97	A	6	5	0.52	26	0.192
98	A	13	12	0.71	26	0.462
99	A	13	12	0.69	26	0.462
100	A	3	2	1.00	26	0.077
101	A	13	12	0.72	24	0.500
102	A	13	12	0.72	22	0.545
103	A	6	5	0.57	26	0.192
104	A	14	13	0.68	26	0.500
105	A	14	13	0.67	26	0.500
106	A	6	5	0.52	26	0.192
107	A	15	14	0.68	26	0.538
108	A	4	3	0.94	26	0.115
109	A	15	14	0.69	26	0.538
110	A	15	14	0.67	26	0.538
111	A	3	2	1.00	26	0.077
112	A	15	14	0.71	24	0.583
113	A	15	14	0.74	22	0.636
114	A	6	5	0.52	26	0.192
115	A	16	15	0.68	26	0.577
116	A	16	15	0.67	26	0.577
117	A	6	5	0.49	26	0.192
118	A	4	4	0.51	28	0.143
119	A	4	4	0.55	28	0.143
120	A	4	4	0.69	28	0.143
121	A	3	3	1.00	28	0.107
122	A	3	3	1.00	28	0.107
123	A	3	3	1.00	28	0.107
124	A	2	2	1.00	26	0.077

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
125	A	5	4	0.95	24	0.167
126	A	5	4	0.98	24	0.167
127	A	5	4	0.99	24	0.167
128	A	2	2	1.00	24	0.083
129	A	2	2	1.00	24	0.083
130	A	4	3	1.10	24	0.125
131	A	2	2	1.03	22	0.091
132	A	2	2	1.04	20	0.100
133	A	4	3	1.00	24	0.125
134	A	2	2	1.00	24	0.083
135	A	2	2	1.00	24	0.083
136	A	4	3	1.05	24	0.125
137	A	2	2	1.00	24	0.083
138	A	4	3	0.99	18	0.167
139	A	6	5	1.02	18	0.278
140	A	4	3	1.00	18	0.167
141	A	8	7	1.06	18	0.389
142	A	6	5	1.07	18	0.278
143	A	13	12	0.86	18	0.667
144	A	12	11	0.83	18	0.611
145	A	11	10	0.92	18	0.556
146	A	11	10	0.89	18	0.556
147	A	11	10	0.92	16	0.625
148	A	11	10	0.87	14	0.714
149	A	13	12	0.87	18	0.667
150	A	13	12	0.84	18	0.667
151	A	4	3	1.06	16	0.188
152	A	4	3	1.00	16	0.188
153	A	4	3	1.19	16	0.188
154	B	4	3	2.50	16	0.188
155	A	4	3	1.22	16	0.188
156	A	4	3	1.18	16	0.188
157	A	4	3	1.15	16	0.188
158	A	15	14	1.14	16	0.875

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
159	A	14	13	1.14	16	0.812
160	A	13	12	1.13	16	0.750
161	A	12	11	1.14	16	0.688
162	A	11	10	1.15	16	0.625
163	A	11	10	1.15	16	0.625
164	A	11	10	1.14	14	0.714
165	A	11	10	1.14	12	0.833
166	A	13	12	1.18	16	0.750
167	A	13	12	1.18	16	0.750
168	A	14	13	1.21	16	0.812
169	A	15	14	1.21	16	0.875
170	A	10	9	0.85	16	0.562
171	A	7	6	1.03	16	0.375
172	A	9	8	0.89	16	0.500
173	A	9	8	0.85	16	0.500
174	A	4	3	1.00	16	0.188
175	A	9	8	0.95	14	0.571
176	C	9	8	1.81	12	0.667
177	A	8	7	1.07	16	0.438
178	A	10	9	0.89	16	0.562
179	A	11	10	0.85	16	0.625
180	A	5	4	1.02	16	0.250
181	A	12	11	0.90	16	0.688
182	A	9	8	0.89	10	0.800
183	A	4	3	1.00	14	0.214
184	A	9	8	0.88	14	0.571
185	A	10	9	1.03	20	0.450
186	A	8	7	1.05	20	0.350
187	A	8	7	1.07	20	0.350
188	A	6	5	1.10	20	0.250
189	A	5	4	1.05	20	0.200
190	A	9	8	1.00	20	0.400
191	A	8	7	1.03	20	0.350
192	A	5	4	1.05	20	0.200

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
193	A	6	5	1.09	20	0.250
194	A	8	7	1.06	20	0.350
195	A	10	9	1.09	20	0.450
196	A	2	2	1.00	20	0.100
197	A	2	2	1.00	18	0.111
198	A	2	2	1.00	16	0.125
199	A	2	2	1.00	20	0.100
200	A	2	2	1.00	20	0.100
201	A	11	10	0.97	20	0.500
202	A	9	8	1.02	20	0.400
203	A	9	8	1.03	20	0.400
204	A	7	6	1.11	20	0.300
205	A	6	5	1.08	20	0.250
206	A	11	10	1.09	20	0.500
207	A	11	10	1.05	20	0.500
208	A	10	9	1.05	20	0.450
209	A	10	9	1.09	20	0.450
210	A	6	5	1.08	20	0.250
211	A	7	6	1.10	20	0.300
212	A	9	8	1.03	20	0.400
213	A	11	10	1.05	20	0.500
214	A	2	2	1.00	20	0.100
215	A	2	2	1.00	18	0.111
216	A	2	2	1.00	16	0.125
217	A	2	2	1.00	20	0.100
218	A	2	2	1.00	20	0.100
219	A	9	8	1.12	20	0.400
220	A	7	6	1.10	20	0.300
221	A	7	6	1.10	20	0.300
222	A	5	4	1.01	20	0.200
223	A	4	3	1.00	20	0.150
224	A	4	3	1.00	20	0.150
225	A	5	4	1.03	20	0.200
226	A	7	6	1.09	20	0.300

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
227	A	9	8	1.12	20	0.400
228	A	11	10	1.14	20	0.500
229	A	2	2	1.00	20	0.100
230	A	2	2	1.00	18	0.111
231	A	2	2	1.00	16	0.125
232	A	2	2	1.00	20	0.100
233	A	2	2	1.00	20	0.100
234	A	9	8	1.06	20	0.400
235	A	7	6	1.09	20	0.300
236	A	6	5	1.08	20	0.250
237	A	3	2	1.00	20	0.100
238	A	3	2	1.00	20	0.100
239	A	6	5	1.00	20	0.250
240	A	7	6	1.09	20	0.300
241	A	9	8	1.06	20	0.400
242	A	11	10	1.04	20	0.500
243	A	2	2	1.00	20	0.100
244	A	2	2	1.00	18	0.111
245	A	2	2	1.00	16	0.125
246	A	2	2	1.00	20	0.100
247	A	2	2	1.00	20	0.100
248	A	2	2	1.00	20	0.100
249	A	2	2	1.00	18	0.111
250	A	3	3	1.00	20	0.150
251	A	5	5	1.02	20	0.250
252	A	2	2	1.00	22	0.091
253	A	2	2	1.00	22	0.091
254	A	2	2	1.00	22	0.091
255	A	2	2	1.00	22	0.091
256	A	2	2	1.00	20	0.100
257	A	6	5	0.99	18	0.278
258	A	4	3	1.01	18	0.167
259	A	3	2	1.00	18	0.111
260	A	2	2	1.00	18	0.111

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
261	A	2	2	1.00	18	0.111
262	A	2	2	1.00	16	0.125
263	A	2	2	1.00	14	0.143
264	A	4	3	1.00	18	0.167
265	A	2	2	1.00	18	0.111
266	A	2	2	1.00	18	0.111
267	A	4	3	0.99	18	0.167
268	A	2	2	1.00	18	0.111
269	A	2	2	1.00	18	0.111
270	A	4	3	0.99	18	0.167
271	A	2	2	1.00	16	0.125
272	A	6	5	1.10	16	0.312
273	A	5	4	0.82	16	0.250
274	A	5	4	1.17	16	0.250
275	A	2	2	1.00	16	0.125
276	A	5	4	1.17	14	0.286
277	A	5	4	1.00	16	0.250
278	A	6	5	1.17	16	0.312
279	A	5	4	1.00	16	0.250
280	A	7	6	1.03	16	0.375
281	A	12	11	1.06	16	0.688
282	A	11	10	1.10	16	0.625
283	A	11	10	1.10	16	0.625
284	A	11	10	1.10	16	0.625
285	A	11	10	1.10	12	0.833
286	A	12	11	1.08	16	0.688
287	A	12	11	1.09	16	0.688
288	A	13	12	1.05	16	0.750
289	A	13	12	1.09	16	0.750
290	A	2	2	1.00	16	0.125
291	A	6	5	1.09	16	0.312
292	A	5	4	0.85	16	0.250
293	A	5	4	1.16	16	0.250
294	A	2	2	1.00	16	0.125

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
295	A	5	4	1.16	14	0.286
296	A	5	4	1.00	16	0.250
297	A	6	5	1.09	16	0.312
298	A	5	4	0.95	16	0.250
299	A	7	6	1.08	16	0.375
300	A	6	6	1.09	16	0.375
301	A	5	5	1.17	16	0.312
302	A	5	5	1.19	16	0.312
303	A	5	5	1.17	16	0.312
304	A	5	5	1.19	12	0.417
305	A	6	6	1.08	16	0.375
306	A	6	6	1.14	16	0.375
307	A	7	7	1.07	16	0.438
308	A	7	7	1.12	16	0.438
309	A	3	3	1.00	18	0.167
310	A	4	3	0.99	18	0.167
311	A	5	4	1.01	18	0.222
312	A	6	5	1.02	18	0.278
313	A	4	3	1.04	18	0.167
314	A	4	3	1.00	18	0.167
315	A	4	3	1.03	16	0.188
316	A	8	7	1.06	18	0.389
317	A	6	5	0.98	18	0.278
318	A	6	5	1.07	18	0.278
319	A	7	7	0.90	18	0.389
320	A	5	5	0.90	18	0.278
321	A	5	5	0.97	18	0.278
322	A	4	4	0.96	18	0.222
323	A	5	5	0.98	18	0.278
324	A	4	4	0.95	14	0.286
325	A	7	7	0.91	18	0.389
326	A	6	6	0.90	18	0.333
327	A	3	3	1.05	14	0.214
328	A	4	3	0.95	14	0.214

Continued on next page

2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
329	A	6	5	0.93	14	0.357
330	A	6	5	1.05	14	0.357
331	A	9	8	1.11	14	0.571
332	A	4	3	1.00	14	0.214
333	A	8	7	1.11	12	0.583
334	A	8	7	1.10	14	0.500
335	A	7	6	0.96	14	0.429
336	A	6	5	1.04	14	0.357
337	A	12	11	1.04	14	0.786
338	A	9	8	1.22	14	0.571
339	A	8	7	1.05	14	0.500
340	A	9	8	1.14	14	0.571
341	A	8	7	1.22	14	0.500
342	A	8	7	1.05	10	0.700
343	A	11	10	1.14	14	0.714
344	A	10	9	1.21	14	0.643
345	A	11	10	1.04	14	0.714
346	A	13	12	1.13	14	0.857
347	A	3	3	1.05	16	0.188
348	A	4	3	0.93	16	0.188
349	A	6	5	1.00	16	0.312
350	A	7	6	1.03	16	0.375
351	A	9	8	1.10	16	0.500
352	A	4	3	1.00	16	0.188
353	A	8	7	1.23	14	0.500
354	A	8	7	1.07	16	0.438
355	A	6	5	1.04	16	0.312
356	A	5	4	1.02	16	0.250
357	A	12	11	1.06	16	0.688
358	A	10	9	0.98	16	0.562
359	A	8	7	1.50	16	0.438
360	A	9	8	1.05	16	0.500
361	A	8	7	1.15	16	0.438
362	A	8	7	1.49	12	0.583

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
363	A	12	11	1.01	16	0.688
364	A	11	10	0.96	16	0.625
365	A	11	10	1.48	16	0.625
366	A	13	12	1.00	16	0.750
367	A	3	3	1.00	16	0.188
368	A	4	3	1.00	16	0.188
369	A	5	4	1.13	16	0.250
370	A	4	3	1.07	16	0.188
371	A	4	3	1.22	16	0.188
372	A	4	3	2.00	16	0.188
373	A	4	3	1.07	14	0.214
374	A	4	3	1.30	16	0.188
375	A	6	5	1.17	16	0.312
376	A	4	3	1.29	16	0.188
377	A	7	6	1.14	16	0.375
378	A	11	10	1.04	16	0.625
379	A	10	9	1.07	16	0.562
380	A	10	9	1.06	16	0.562
381	A	10	9	1.06	16	0.562
382	A	10	9	1.13	12	0.750
383	A	12	11	1.12	16	0.688
384	A	12	11	1.04	16	0.688
385	A	3	3	1.00	16	0.188
386	A	4	3	1.00	16	0.188
387	A	5	4	1.13	16	0.250
388	A	4	3	1.07	16	0.188
389	A	4	3	1.22	16	0.188
390	A	4	3	2.00	16	0.188
391	A	4	3	1.07	14	0.214
392	A	4	3	1.25	16	0.188
393	A	5	4	1.17	16	0.250
394	A	4	3	1.18	16	0.188
395	A	8	7	1.14	16	0.438
396	A	5	5	1.05	16	0.312

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
397	A	5	5	1.11	16	0.312
398	A	4	4	1.02	16	0.250
399	A	5	5	1.19	16	0.312
400	A	4	4	0.97	12	0.333
401	A	6	6	1.10	16	0.375
402	A	6	6	1.01	16	0.375
403	A	9	9	1.14	16	0.562
404	A	8	8	1.00	16	0.500
405	A	4	3	0.90	16	0.188
406	A	4	3	0.85	16	0.188
407	A	6	5	1.05	14	0.357
408	A	4	3	1.00	14	0.214
409	A	8	7	1.10	14	0.500
410	A	6	5	1.04	14	0.357
411	A	9	8	1.10	10	0.800
412	A	3	3	1.00	18	0.167
413	A	3	3	1.00	18	0.167
414	A	3	3	1.00	16	0.188
415	A	3	3	1.00	14	0.214
416	A	6	5	1.00	18	0.278
417	A	4	3	1.00	18	0.167
418	A	8	7	1.02	18	0.389
419	A	5	5	1.05	18	0.278
420	A	5	5	1.09	18	0.278
421	A	5	5	1.02	18	0.278
422	A	4	4	0.95	16	0.250
423	A	5	5	1.03	14	0.357
424	A	4	4	1.10	18	0.222
425	A	5	4	1.00	18	0.222
426	A	5	4	1.00	18	0.222
427	A	5	4	1.00	18	0.222
428	A	5	5	1.29	18	0.278
429	A	5	5	1.17	18	0.278
430	A	5	5	1.05	18	0.278

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
431	A	7	7	1.08	14	0.500
432	A	6	6	1.16	18	0.333
433	A	6	5	1.10	18	0.278
434	A	6	5	1.10	18	0.278
435	A	7	6	1.12	18	0.333
436	A	6	5	1.12	18	0.278
437	A	6	5	1.09	18	0.278
438	A	7	7	1.28	18	0.389
439	A	7	7	1.17	18	0.389
440	A	3	3	1.05	18	0.167
441	A	3	3	1.06	16	0.188
442	A	3	3	1.08	14	0.214
443	A	3	3	1.10	18	0.167
444	A	4	3	1.09	18	0.167
445	A	3	3	1.07	18	0.167
446	A	3	3	1.06	18	0.167
447	A	3	3	1.05	18	0.167
448	A	3	3	1.04	18	0.167
449	A	13	12	1.07	16	0.750
450	A	11	10	1.03	16	0.625
451	A	8	7	1.02	16	0.438
452	A	5	4	1.00	16	0.250
453	A	7	6	1.11	16	0.375
454	A	9	8	1.13	16	0.500
455	A	2	2	0.63	22	0.091
456	A	4	4	1.02	14	0.286
457	A	13	12	0.83	14	0.857
458	A	6	6	0.90	14	0.429
459	A	9	8	1.05	20	0.400
460	A	6	5	1.18	23	0.217
461	A	5	4	1.01	22	0.182
462	A	6	5	0.78	26	0.192
463	A	6	5	0.78	26	0.192
464	A	6	5	0.68	26	0.192

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
465	A	2	2	0.67	26	0.077
466	A	6	5	0.58	26	0.192
467	A	6	5	0.76	26	0.192
468	A	6	5	0.78	26	0.192
469	A	6	5	0.77	26	0.192
470	A	6	5	0.77	26	0.192
471	A	6	5	0.77	26	0.192
472	A	5	4	1.00	30	0.133
473	A	5	4	0.71	28	0.143
474	A	5	4	0.76	26	0.154
475	A	5	4	0.97	24	0.167
476	A	4	3	1.00	28	0.107
477	A	4	3	1.00	28	0.107
478	C	1	1	1.11	77	0.013
479	A	6	5	0.64	26	0.192
480	A	6	5	0.57	26	0.192
481	A	7	6	0.49	24	0.250
482	A	7	6	0.38	26	0.231
483	A	7	6	0.41	26	0.231
484	A	7	6	0.48	26	0.231
485	A	2	2	0.67	26	0.077
486	A	7	6	0.51	26	0.231
487	A	7	6	0.46	26	0.231
488	A	7	6	0.44	26	0.231
489	A	7	6	0.41	26	0.231
490	A	7	6	0.41	26	0.231
491	A	6	5	0.63	26	0.192
492	A	7	6	0.38	26	0.231
493	A	5	4	0.89	23	0.174
494	A	5	4	0.93	23	0.174
495	A	2	2	1.00	23	0.087
496	A	6	5	1.13	21	0.238
497	A	5	4	0.95	23	0.174
498	A	5	4	0.92	23	0.174

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
499	A	5	4	0.90	23	0.174
500	A	12	11	1.10	25	0.440
501	A	12	11	1.00	25	0.440
502	A	6	5	0.96	25	0.200
503	A	7	6	1.03	25	0.240
504	A	14	13	1.01	25	0.520
505	A	14	13	1.10	25	0.520
506	A	1	1	1.00	26	0.038
507	A	1	1	1.00	28	0.036
508	A	5	4	0.69	32	0.125
509	A	5	4	0.69	32	0.125
510	A	5	4	0.76	32	0.125
511	A	5	4	0.87	32	0.125
512	A	4	3	1.56	32	0.094
513	A	4	3	0.88	32	0.094
514	A	4	3	0.88	32	0.094
515	A	3	3	0.71	30	0.100
516	A	3	3	0.67	28	0.107
517	A	3	3	0.67	26	0.115
518	A	2	2	0.65	24	0.083
519	A	3	3	0.64	28	0.107
520	A	3	3	0.67	28	0.107
521	A	3	3	0.68	28	0.107
522	A	3	3	0.58	30	0.100
523	A	3	3	0.52	28	0.107
524	A	3	3	0.52	26	0.115
525	A	3	3	0.51	24	0.125
526	A	6	5	0.45	28	0.179
527	A	3	3	0.52	28	0.107
528	A	3	3	0.54	28	0.107
529	A	2	2	1.09	30	0.067
530	A	2	2	1.11	28	0.071
531	A	2	2	1.11	26	0.077
532	A	2	2	1.13	24	0.083

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
533	A	7	6	0.75	28	0.214
534	A	2	2	1.11	28	0.071
535	A	2	2	1.10	28	0.071
536	A	2	2	1.12	30	0.067
537	A	2	2	1.14	28	0.071
538	A	2	2	1.14	26	0.077
539	A	2	2	1.12	24	0.083
540	A	6	5	0.60	28	0.179
541	A	2	2	1.14	28	0.071
542	A	2	2	1.13	28	0.071
543	A	2	2	1.00	36	0.056
544	A	2	2	1.00	33	0.061
545	A	3	3	0.99	34	0.088
546	A	3	3	0.97	33	0.091
547	A	3	3	1.09	35	0.086
548	A	5	4	0.84	30	0.133
549	A	4	3	0.93	24	0.125
550	A	4	3	0.94	24	0.125
551	A	6	5	0.97	24	0.208
552	A	4	3	1.00	22	0.136
553	A	6	5	1.01	24	0.208
554	A	6	5	1.02	24	0.208
555	A	6	5	0.96	24	0.208
556	A	6	5	0.93	26	0.192
557	A	12	11	0.87	26	0.423
558	A	4	3	0.99	26	0.115
559	A	6	5	1.00	26	0.192
560	A	14	13	0.82	26	0.500
561	A	8	7	0.90	26	0.269
562	A	2	2	1.06	20	0.100
563	A	2	2	1.06	18	0.111
564	A	2	2	1.07	16	0.125
565	A	8	7	1.01	20	0.350
566	A	2	2	1.06	20	0.100

Continued on next page

2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
567	A	2	2	1.07	20	0.100
568	A	2	2	1.00	22	0.091
569	A	2	2	1.00	22	0.091
570	A	2	2	1.00	20	0.100
571	A	2	2	1.00	18	0.111
572	A	9	8	0.97	22	0.364
573	A	2	2	1.00	22	0.091
574	A	2	2	1.00	22	0.091
575	A	2	2	1.00	22	0.091
576	A	2	2	1.00	22	0.091
577	A	2	2	1.00	20	0.100
578	A	2	2	1.00	18	0.111
579	A	11	10	1.02	22	0.455
580	A	2	2	1.00	22	0.091
581	A	2	2	1.00	22	0.091
582	A	2	2	1.00	22	0.091
583	A	2	2	1.00	22	0.091
584	A	2	2	1.00	20	0.100
585	A	2	2	1.00	18	0.111
586	A	4	3	1.00	22	0.136
587	A	2	2	1.00	22	0.091
588	A	2	2	1.00	22	0.091
589	A	2	2	1.00	22	0.091
590	A	2	2	1.00	22	0.091
591	A	2	2	1.00	20	0.100
592	A	2	2	1.00	18	0.111
593	A	6	5	0.98	22	0.227
594	A	2	2	1.00	22	0.091
595	A	2	2	1.00	22	0.091
596	A	2	2	1.00	22	0.091
597	A	2	2	1.00	22	0.091
598	A	2	2	1.00	20	0.100
599	A	2	2	1.00	22	0.091
600	A	4	4	0.96	22	0.182

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
601	A	6	6	1.10	22	0.273
602	A	2	2	1.00	24	0.083
603	A	2	2	1.00	24	0.083
604	A	2	2	1.00	24	0.083
605	A	2	2	1.00	24	0.083
606	A	2	2	1.00	22	0.091
607	A	4	3	0.89	28	0.107
608	A	5	4	0.91	30	0.133
609	A	5	4	0.88	30	0.133
610	A	4	3	0.80	31	0.097
611	A	5	4	0.81	33	0.121
612	A	5	4	0.78	33	0.121
613	A	5	4	1.03	30	0.133
614	A	7	6	0.98	30	0.200
615	A	4	3	1.04	30	0.100
616	A	5	4	1.00	28	0.143
617	A	9	8	0.98	30	0.267
618	A	6	5	0.96	30	0.167
619	A	7	6	0.98	30	0.200
620	A	7	6	1.00	30	0.200
621	A	5	4	0.94	30	0.133
622	A	6	5	0.98	30	0.167
623	A	5	4	0.95	30	0.133
624	A	6	5	0.97	28	0.179
625	A	6	5	0.88	22	0.227
626	A	7	6	1.14	30	0.200
627	A	7	6	0.97	30	0.200
628	A	7	6	1.06	30	0.200
629	A	9	8	0.97	30	0.267
630	A	7	6	1.00	30	0.200
631	A	7	6	1.07	30	0.200
632	A	7	6	1.04	30	0.200
633	A	7	6	1.03	28	0.214
634	A	8	7	0.93	22	0.318

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
635	A	9	8	1.16	30	0.267
636	A	9	8	0.97	30	0.267
637	A	9	8	1.09	30	0.267
638	A	5	4	1.00	33	0.121
639	A	7	6	0.94	33	0.182
640	A	4	3	1.02	33	0.091
641	A	5	4	1.00	31	0.129
642	A	9	8	0.92	33	0.242
643	A	6	5	0.94	33	0.152
644	A	7	6	0.91	33	0.182
645	A	7	6	0.96	33	0.182
646	A	5	4	0.92	33	0.121
647	A	6	5	0.95	33	0.152
648	A	5	4	0.93	33	0.121
649	A	6	5	0.98	31	0.161
650	A	7	6	1.07	33	0.182
651	A	7	6	0.95	33	0.182
652	A	7	6	1.00	33	0.182
653	A	9	8	0.94	33	0.242
654	A	7	6	0.97	33	0.182
655	A	7	6	1.03	33	0.182
656	A	7	6	1.02	33	0.182
657	A	7	6	1.03	31	0.194
658	A	9	8	1.11	33	0.242
659	A	9	8	0.95	33	0.242
660	A	9	8	1.04	33	0.242
661	A	5	4	0.99	26	0.154
662	A	4	3	0.99	28	0.107
663	A	4	3	1.00	24	0.125
664	A	5	4	0.86	26	0.154

CHAPTER 3

Listing of integrals

3.1 $\int (ax^3+bx^6)^{5/3}\,dx$

3.1.1 Optimal result

Integrand size = 15, antiderivative size = 52

$$\int (ax^3+bx^6)^{5/3}\,dx = -\frac{3a(ax^3+bx^6)^{8/3}}{88b^2x^8}+\frac{(ax^3+bx^6)^{8/3}}{11bx^5}$$

output
```
-3/88*a*(b*x^6+a*x^3)^(8/3)/b^2/x^8+1/11*(b*x^6+a*x^3)^(8/3)/b/x^5
```

3.1.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.67

$$\int (ax^3+bx^6)^{5/3}\,dx = \frac{(x^3(a+bx^3))^{8/3}\,(-3a+8bx^3)}{88b^2x^8}$$

input
```
Integrate[(a*x^3 + b*x^6)^(5/3),x]
```

output
```
((x^3*(a + b*x^3))^(8/3)*(-3*a + 8*b*x^3))/(88*b^2*x^8)
```

3.1.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1908, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax^3+bx^6\right)^{5/3} dx$$

$$\Big\downarrow \text{ 1908}$$

$$\frac{\left(ax^3+bx^6\right)^{8/3}}{11bx^5}-\frac{3a\int \frac{\left(bx^6+ax^3\right)^{5/3}}{x^3}dx}{11b}$$

$$\Big\downarrow \text{ 1920}$$

$$\frac{\left(ax^3+bx^6\right)^{8/3}}{11bx^5}-\frac{3a\left(ax^3+bx^6\right)^{8/3}}{88b^2x^8}$$

input
```
Int[(a*x^3 + b*x^6)^(5/3),x]
```

output
```
(-3*a*(a*x^3 + b*x^6)^(8/3))/(88*b^2*x^8) + (a*x^3 + b*x^6)^(8/3)/(11*b*x^
5)
```

3.1.3.1 Defintions of rubi rules used

rule 1908
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

3.1.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.75

method	result	size
gosper	$-\frac{(b\,x^3+a)(-8b\,x^3+3a)(b\,x^6+a\,x^3)^{\frac{5}{3}}}{88b^2x^5}$	39
trager	$-\frac{(-8b^3x^9-13b^2x^6a-2a^2b\,x^3+3a^3)(b\,x^6+a\,x^3)^{\frac{2}{3}}}{88b^2x^2}$	54
risch	$-\frac{(x^3(b\,x^3+a))^{\frac{2}{3}}(-8b^3x^9-13b^2x^6a-2a^2b\,x^3+3a^3)}{88x^2b^2}$	54

input
```
int((b*x^6+a*x^3)^(5/3),x,method=_RETURNVERBOSE)
```

output
```
-1/88*(b*x^3+a)*(-8*b*x^3+3*a)*(b*x^6+a*x^3)^(5/3)/b^2/x^5
```

3.1.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.02

$$\int (ax^3+bx^6)^{5/3}\,dx = \frac{(8\,b^3x^9+13\,ab^2x^6+2\,a^2bx^3-3\,a^3)(bx^6+ax^3)^{\frac{2}{3}}}{88\,b^2x^2}$$

input
```
integrate((b*x^6+a*x^3)^(5/3),x, algorithm="fricas")
```

output
```
1/88*(8*b^3*x^9 + 13*a*b^2*x^6 + 2*a^2*b*x^3 - 3*a^3)*(b*x^6 + a*x^3)^(2/3
)/(b^2*x^2)
```

3.1.6 Sympy [F]

$$\int (ax^3+bx^6)^{5/3}\,dx = \int (ax^3+bx^6)^{\frac{5}{3}}\,dx$$

input
```
integrate((b*x**6+a*x**3)**(5/3),x)
```

output
```
Integral((a*x**3 + b*x**6)**(5/3), x)
```

3.1.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.88

$$\int \left(ax^3+bx^6\right)^{5/3}\,dx=\frac{(8\,b^3x^9+13\,ab^2x^6+2\,a^2bx^3-3\,a^3)(bx^3+a)^{\frac{2}{3}}}{88\,b^2}$$

input
```
integrate((b*x^6+a*x^3)^(5/3),x, algorithm="maxima")
```

output
```
1/88*(8*b^3*x^9 + 13*a*b^2*x^6 + 2*a^2*b*x^3 - 3*a^3)*(b*x^3 + a)^(2/3)/b^
2
```

3.1.8 Giac [F]

$$\int \left(ax^3+bx^6\right)^{5/3}\,dx=\int \left(bx^6+ax^3\right)^{\frac{5}{3}}\,dx$$

input
```
integrate((b*x^6+a*x^3)^(5/3),x, algorithm="giac")
```

output
```
integrate((b*x^6 + a*x^3)^(5/3), x)
```

3.1.9 Mupad [B] (verification not implemented)

Time = 8.33 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.77

$$\int \left(ax^3+bx^6\right)^{5/3}\,dx=-\frac{\left(b\,x^3+a\right)^2\,\left(b\,x^6+a\,x^3\right)^{2/3}\,\left(3\,a-8\,b\,x^3\right)}{88\,b^2\,x^2}$$

input
```
int((a*x^3 + b*x^6)^(5/3),x)
```

output
```
-((a + b*x^3)^2*(a*x^3 + b*x^6)^(2/3)*(3*a - 8*b*x^3))/(88*b^2*x^2)
```

3.2 $\int (ax^3 + bx^6)^{2/3} dx$

3.2.1 Optimal result

Integrand size = 15, antiderivative size = 25

$$\int (ax^3 + bx^6)^{2/3} dx = \frac{(ax^3 + bx^6)^{5/3}}{5bx^5}$$

output
```
1/5*(b*x^6+a*x^3)^(5/3)/b/x^5
```

3.2.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (ax^3 + bx^6)^{2/3} dx = \frac{(x^3(a + bx^3))^{5/3}}{5bx^5}$$

input
```
Integrate[(a*x^3 + b*x^6)^(2/3),x]
```

output
```
(x^3*(a + b*x^3))^(5/3)/(5*b*x^5)
```

3.2.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1906}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax^3+bx^6\right)^{2/3}\,dx$$

$$\Big\downarrow \text{1906}$$

$$\frac{\left(ax^3+bx^6\right)^{5/3}}{5bx^5}$$

input
```
Int[(a*x^3 + b*x^6)^(2/3),x]
```

output
```
(a*x^3 + b*x^6)^(5/3)/(5*b*x^5)
```

3.2.3.1 Defintions of rubi rules used

rule 1906
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

3.2.4 Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

method	result	size
gosper	$\frac{(b\,x^3+a)\left(b\,x^6+a\,x^3\right)^{\frac{2}{3}}}{5b\,x^2}$	29
trager	$\frac{(b\,x^3+a)\left(b\,x^6+a\,x^3\right)^{\frac{2}{3}}}{5b\,x^2}$	29
risch	$\frac{\left(x^3\,(b\,x^3+a)\right)^{\frac{2}{3}}(b\,x^3+a)}{5x^2b}$	29

input
```
int((b*x^6+a*x^3)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
1/5*(b*x^3+a)/b/x^2*(b*x^6+a*x^3)^(2/3)
```

3.2.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

$$\int \left(ax^3+bx^6\right)^{2/3} dx = \frac{(bx^6+ax^3)^{\frac{2}{3}}(bx^3+a)}{5\,bx^2}$$

input
```
integrate((b*x^6+a*x^3)^(2/3),x, algorithm="fricas")
```

output
```
1/5*(b*x^6 + a*x^3)^(2/3)*(b*x^3 + a)/(b*x^2)
```

3.2.6 Sympy [F]

$$\int \left(ax^3+bx^6\right)^{2/3} dx = \int \left(ax^3+bx^6\right)^{\frac{2}{3}} dx$$

input
```
integrate((b*x**6+a*x**3)**(2/3),x)
```

output
```
Integral((a*x**3 + b*x**6)**(2/3), x)
```

3.2.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.56

$$\int \left(ax^3+bx^6\right)^{2/3} dx = \frac{(bx^3+a)^{\frac{5}{3}}}{5\,b}$$

input
```
integrate((b*x^6+a*x^3)^(2/3),x, algorithm="maxima")
```

output
```
1/5*(b*x^3 + a)^(5/3)/b
```

3.2.8 Giac [F]

$$\int \left(ax^3+bx^6\right)^{2/3} dx = \int \left(bx^6+ax^3\right)^{\frac{2}{3}} dx$$

input
```
integrate((b*x^6+a*x^3)^(2/3),x, algorithm="giac")
```

output
```
integrate((b*x^6 + a*x^3)^(2/3), x)
```

3.2.9 Mupad [B] (verification not implemented)

Time = 8.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \left(ax^3+bx^6\right)^{2/3} dx = \frac{\left(\frac{a}{5\,b}+\frac{x^3}{5}\right)\,\left(b\,x^6+a\,x^3\right)^{2/3}}{x^2}$$

input
```
int((a*x^3 + b*x^6)^(2/3),x)
```

output
```
((a/(5*b) + x^3/5)*(a*x^3 + b*x^6)^(2/3))/x^2
```

3.3 $\int \frac{1}{(ax^3+bx^6)^{2/3}} dx$

3.3.1 Optimal result

Integrand size = 15, antiderivative size = 23

$$\int \frac{1}{(ax^3+bx^6)^{2/3}} dx = -\frac{\sqrt[3]{ax^3+bx^6}}{ax^2}$$

output
```
-(b*x^6+a*x^3)^(1/3)/a/x^2
```

3.3.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ax^3+bx^6)^{2/3}} dx = -\frac{\sqrt[3]{x^3(a+bx^3)}}{ax^2}$$

input
```
Integrate[(a*x^3 + b*x^6)^(-2/3),x]
```

output
```
-((x^3*(a + b*x^3))^(1/3)/(a*x^2))
```

3.3.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1906}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax^3+bx^6)^{2/3}}\,dx$$

$$\Big\downarrow \text{ 1906}$$

$$-\frac{\sqrt[3]{ax^3+bx^6}}{ax^2}$$

input
```
Int[(a*x^3 + b*x^6)^(-2/3),x]
```

output
```
-((a*x^3 + b*x^6)^(1/3)/(a*x^2))
```

3.3.3.1 Defintions of rubi rules used

rule 1906
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

3.3.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

method	result	size
trager	$-\frac{(b\,x^6+a\,x^3)^{\frac{1}{3}}}{a\,x^2}$	22
pseudoelliptic	$-\frac{(x^3(b\,x^3+a))^{\frac{1}{3}}}{a\,x^2}$	22
gosper	$-\frac{x(b\,x^3+a)}{a(b\,x^6+a\,x^3)^{\frac{2}{3}}}$	27
risch	$-\frac{x(b\,x^3+a)}{(x^3(b\,x^3+a))^{\frac{2}{3}}\,a}$	27

input
```
int(1/(b*x^6+a*x^3)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
-(b*x^6+a*x^3)^(1/3)/a/x^2
```

3.3.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{(ax^3+bx^6)^{2/3}}\,dx = -\frac{(bx^6+ax^3)^{\frac{1}{3}}}{ax^2}$$

input
```
integrate(1/(b*x^6+a*x^3)^(2/3),x, algorithm="fricas")
```

output
```
-(b*x^6 + a*x^3)^(1/3)/(a*x^2)
```

3.3.6 Sympy [F]

$$\int \frac{1}{(ax^3+bx^6)^{2/3}}\,dx = \int \frac{1}{(ax^3+bx^6)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(b*x**6+a*x**3)**(2/3),x)
```

output
```
Integral((a*x**3 + b*x**6)**(-2/3), x)
```

3.3.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{1}{(ax^3+bx^6)^{2/3}}\,dx = -\frac{(bx^3+a)^{\frac{1}{3}}}{ax}$$

input
```
integrate(1/(b*x^6+a*x^3)^(2/3),x, algorithm="maxima")
```

output
```
-(b*x^3 + a)^(1/3)/(a*x)
```

3.3.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.61

$$\int \frac{1}{(ax^3+bx^6)^{2/3}}\,dx = -\frac{\left(b+\frac{a}{x^3}\right)^{\frac{1}{3}}}{a}$$

input
```
integrate(1/(b*x^6+a*x^3)^(2/3),x, algorithm="giac")
```

output
```
-(b + a/x^3)^(1/3)/a
```

3.3.9 Mupad [B] (verification not implemented)

Time = 8.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{(ax^3+bx^6)^{2/3}}\,dx = -\frac{\left(b\,x^6+a\,x^3\right)^{1/3}}{a\,x^2}$$

input
```
int(1/(a*x^3 + b*x^6)^(2/3),x)
```

output
```
-(a*x^3 + b*x^6)^(1/3)/(a*x^2)
```

3.4 $\int \frac{1}{(ax^3+bx^6)^{5/3}} dx$

3.4.1 Optimal result

Integrand size = 15, antiderivative size = 77

$$\int \frac{1}{(ax^3+bx^6)^{5/3}} dx = \frac{1}{2ax^2(ax^3+bx^6)^{2/3}} - \frac{3\sqrt[3]{ax^3+bx^6}}{4a^2x^5} + \frac{9b\sqrt[3]{ax^3+bx^6}}{4a^3x^2}$$

output
```
1/2/a/x^2/(b*x^6+a*x^3)^(2/3)-3/4*(b*x^6+a*x^3)^(1/3)/a^2/x^5+9/4*b*(b*x^6
+a*x^3)^(1/3)/a^3/x^2
```

3.4.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.60

$$\int \frac{1}{(ax^3+bx^6)^{5/3}} dx = \frac{-a^2+6abx^3+9b^2x^6}{4a^3x^2(x^3(a+bx^3))^{2/3}}$$

input
```
Integrate[(a*x^3 + b*x^6)^(-5/3),x]
```

output
```
(-a^2 + 6*a*b*x^3 + 9*b^2*x^6)/(4*a^3*x^2*(x^3*(a + b*x^3))^(2/3))
```

3.4.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1907, 1922, 1906}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax^3+bx^6)^{5/3}}\,dx$$

$$\Big\downarrow \text{ 1907}$$

$$\frac{3\int \frac{1}{x^3(bx^6+ax^3)^{2/3}}dx}{a}+\frac{1}{2ax^2(ax^3+bx^6)^{2/3}}$$

$$\Big\downarrow \text{ 1922}$$

$$\frac{3\left(-\frac{3b\int \frac{1}{(bx^6+ax^3)^{2/3}}dx}{4a}-\frac{\sqrt[3]{ax^3+bx^6}}{4ax^5}\right)}{a}+\frac{1}{2ax^2(ax^3+bx^6)^{2/3}}$$

$$\Big\downarrow \text{ 1906}$$

$$\frac{3\left(\frac{3b\sqrt[3]{ax^3+bx^6}}{4a^2x^2}-\frac{\sqrt[3]{ax^3+bx^6}}{4ax^5}\right)}{a}+\frac{1}{2ax^2(ax^3+bx^6)^{2/3}}$$

input
```
Int[(a*x^3 + b*x^6)^(-5/3),x]
```

output
```
1/(2*a*x^2*(a*x^3 + b*x^6)^(2/3)) + (3*(-1/4*(a*x^3 + b*x^6)^(1/3)/(a*x^5)
 + (3*b*(a*x^3 + b*x^6)^(1/3))/(4*a^2*x^2)))/a
```

3.4.3.1 Defintions of rubi rules used

rule 1906
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

rule 1907
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[-(a*x^j +
 b*x^n)^(p + 1)/(a*(n - j)*(p + 1)*x^(j - 1)), x] + Simp[(n*p + n - j + 1)/
(a*(n - j)*(p + 1))   Int[(a*x^j + b*x^n)^(p + 1)/x^j, x], x] /; FreeQ[{a,
b, j, n}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j +
1)/(n - j)], 0] && LtQ[p, -1]
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

3.4.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.53

method	result	size
pseudoelliptic	$-\frac{-9b^{2}x^{6}-6ab\,x^{3}+a^{2}}{4x^{2}\left(x^{3}\left(b\,x^{3}+a\right)\right)^{\frac{2}{3}}a^{3}}$	41
gosper	$-\frac{x\left(b\,x^{3}+a\right)\left(-9b^{2}x^{6}-6ab\,x^{3}+a^{2}\right)}{4a^{3}\left(b\,x^{6}+a\,x^{3}\right)^{\frac{5}{3}}}$	46
trager	$-\frac{\left(-9b^{2}x^{6}-6ab\,x^{3}+a^{2}\right)\left(b\,x^{6}+a\,x^{3}\right)^{\frac{1}{3}}}{4\left(b\,x^{3}+a\right)x^{5}a^{3}}$	50
risch	$-\frac{\left(b\,x^{3}+a\right)\left(-7b\,x^{3}+a\right)}{4a^{3}x^{2}\left(x^{3}\left(b\,x^{3}+a\right)\right)^{\frac{2}{3}}}+\frac{b^{2}x^{4}}{2a^{3}\left(x^{3}\left(b\,x^{3}+a\right)\right)^{\frac{2}{3}}}$	62

input
```
int(1/(b*x^6+a*x^3)^(5/3),x,method=_RETURNVERBOSE)
```

output
```
-1/4/x^2*(-9*b^2*x^6-6*a*b*x^3+a^2)/(x^3*(b*x^3+a))^(2/3)/a^3
```

3.4.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.70

$$\int \frac{1}{(ax^3+bx^6)^{5/3}}\,dx = \frac{(9\,b^2x^6+6\,abx^3-a^2)(bx^6+ax^3)^{\frac{1}{3}}}{4\,(a^3bx^8+a^4x^5)}$$

input
```
integrate(1/(b*x^6+a*x^3)^(5/3),x, algorithm="fricas")
```

output
```
1/4*(9*b^2*x^6 + 6*a*b*x^3 - a^2)*(b*x^6 + a*x^3)^(1/3)/(a^3*b*x^8 + a^4*x
^5)
```

3.4.6 Sympy [F]

$$\int \frac{1}{(ax^3+bx^6)^{5/3}}\,dx = \int \frac{1}{(ax^3+bx^6)^{\frac{5}{3}}}\,dx$$

input
```
integrate(1/(b*x**6+a*x**3)**(5/3),x)
```

output
```
Integral((a*x**3 + b*x**6)**(-5/3), x)
```

3.4.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.49

$$\int \frac{1}{(ax^3+bx^6)^{5/3}}\,dx = \frac{9\,b^2x^6+6\,abx^3-a^2}{4\,(bx^3+a)^{\frac{2}{3}}a^3x^4}$$

input
```
integrate(1/(b*x^6+a*x^3)^(5/3),x, algorithm="maxima")
```

output
```
1/4*(9*b^2*x^6 + 6*a*b*x^3 - a^2)/((b*x^3 + a)^(2/3)*a^3*x^4)
```

3.4.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.68

$$\int \frac{1}{(ax^3+bx^6)^{5/3}}\,dx = \frac{b^2}{2\,a^3\left(b+\frac{a}{x^3}\right)^{\frac{2}{3}}} - \frac{a^9\left(b+\frac{a}{x^3}\right)^{\frac{4}{3}} - 8\,a^9\left(b+\frac{a}{x^3}\right)^{\frac{1}{3}}b}{4\,a^{12}}$$

input
```
integrate(1/(b*x^6+a*x^3)^(5/3),x, algorithm="giac")
```

output
```
1/2*b^2/(a^3*(b + a/x^3)^(2/3)) - 1/4*(a^9*(b + a/x^3)^(4/3) - 8*a^9*(b +
a/x^3)^(1/3)*b)/a^12
```

3.4.9 Mupad [B] (verification not implemented)

Time = 8.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.66

$$\int \frac{1}{(ax^3+bx^6)^{5/3}}\,dx = \frac{\left(b\,x^6+a\,x^3\right)^{1/3}\left(-a^2+6\,a\,b\,x^3+9\,b^2\,x^6\right)}{4\,a^3\,x^5\,\left(b\,x^3+a\right)}$$

input
```
int(1/(a*x^3 + b*x^6)^(5/3),x)
```

output
```
((a*x^3 + b*x^6)^(1/3)*(9*b^2*x^6 - a^2 + 6*a*b*x^3))/(4*a^3*x^5*(a + b*x^
3))
```

3.5 $\int \frac{1}{-x^3+x^6}\,dx$

3.5.1 Optimal result

Integrand size = 11, antiderivative size = 48

$$\int \frac{1}{-x^3+x^6}\,dx=\frac{1}{2x^2}-\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{3}\log(1-x)-\frac{1}{6}\log\left(1+x+x^2\right)$$

output
```
1/2/x^2+1/3*ln(1-x)-1/6*ln(x^2+x+1)-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2
)
```

3.5.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{1}{-x^3+x^6}\,dx=\frac{1}{2x^2}-\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{3}\log(1-x)-\frac{1}{6}\log\left(1+x+x^2\right)$$

input
```
Integrate[(-x^3 + x^6)^(-1),x]
```

output
```
1/(2*x^2) - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x]/3 - Log[1 + x +
 x^2]/6
```

3.5.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.818, Rules used = {2026, 847, 750, 16, 25, 1142, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6-x^3} \, dx$$

$$\Big\downarrow \text{2026}$$

$$\int \frac{1}{x^3 (x^3-1)}dx$$

$$\Big\downarrow \text{847}$$

$$\int \frac{1}{x^3-1}dx+\frac{1}{2x^2}$$

$$\Big\downarrow \text{750}$$

$$\frac{1}{3}\int -\frac{x+2}{x^2+x+1}dx+\frac{1}{3}\int \frac{1}{x-1}dx+\frac{1}{2x^2}$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{3}\int -\frac{x+2}{x^2+x+1}dx+\frac{1}{2x^2}+\frac{1}{3}\log(1-x)$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{3}\int \frac{x+2}{x^2+x+1}dx+\frac{1}{2x^2}+\frac{1}{3}\log(1-x)$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{3}\left(-\frac{3}{2}\int \frac{1}{x^2+x+1}dx-\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx\right)+\frac{1}{2x^2}+\frac{1}{3}\log(1-x)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{3}\left(3\int \frac{1}{-(2x+1)^2-3}d(2x+1)-\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx\right)+\frac{1}{2x^2}+\frac{1}{3}\log(1-x)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{3}\left(-\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx-\sqrt{3}\arctan\left(\frac{2x+1}{\sqrt{3}}\right)\right)+\frac{1}{2x^2}+\frac{1}{3}\log(1-x)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{3}\left(-\sqrt{3}\arctan\left(\frac{2x+1}{\sqrt{3}}\right)-\frac{1}{2}\log\left(x^2+x+1\right)\right)+\frac{1}{2x^2}+\frac{1}{3}\log(1-x)$$

input
```
Int[(-x^3 + x^6)^(-1),x]
```

output
```
1/(2*x^2) + Log[1 - x]/3 + (-(Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]]) - Log[1 +
 x + x^2]/2)/3
```

3.5.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

3.5.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.75

method	result	size
risch	$\frac{1}{2x^2}+\frac{\ln(x-1)}{3}-\frac{\ln(x^2+x+1)}{6}-\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x+\frac{1}{2}\right)\sqrt{3}}{3}\right)}{3}$	36
default	$\frac{1}{2x^2}-\frac{\ln(x^2+x+1)}{6}-\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}+\frac{\ln(x-1)}{3}$	38
meijerg	$-\frac{(-1)^{\frac{2}{3}}\left(\frac{3(-1)^{\frac{1}{3}}}{2x^2}+\frac{x(-1)^{\frac{1}{3}}\left(\ln\left(1-\left(x^3\right)^{\frac{1}{3}}\right)-\frac{\ln\left(1+\left(x^3\right)^{\frac{1}{3}}+\left(x^3\right)^{\frac{2}{3}}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(x^3\right)^{\frac{1}{3}}}{2+\left(x^3\right)^{\frac{1}{3}}}\right)\right)}{\left(x^3\right)^{\frac{1}{3}}}\right)}{3}$	78

input
```
int(1/(x^6-x^3),x,method=_RETURNVERBOSE)
```

output
```
1/2/x^2+1/3*ln(x-1)-1/6*ln(x^2+x+1)-1/3*3^(1/2)*arctan(2/3*(x+1/2)*3^(1/2)
)
```

3.5.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

$$\int \frac{1}{-x^3+x^6}\,dx = -\frac{2\sqrt{3}x^2\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+x^2\log\left(x^2+x+1\right)-2x^2\log\left(x-1\right)-3}{6x^2}$$

input
```
integrate(1/(x^6-x^3),x, algorithm="fricas")
```

output
```
-1/6*(2*sqrt(3)*x^2*arctan(1/3*sqrt(3)*(2*x + 1)) + x^2*log(x^2 + x + 1) -
 2*x^2*log(x - 1) - 3)/x^2
```

3.5.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{1}{-x^3+x^6}\,dx = \frac{\log\left(x-1\right)}{3}-\frac{\log\left(x^2+x+1\right)}{6}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}+\frac{1}{2x^2}$$

input
```
integrate(1/(x**6-x**3),x)
```

output
```
log(x - 1)/3 - log(x**2 + x + 1)/6 - sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/
3)/3 + 1/(2*x**2)
```

3.5.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.77

$$\int \frac{1}{-x^3+x^6}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{2x^2}-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{3}\log\left(x-1\right)$$

input
```
integrate(1/(x^6-x^3),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/2/x^2 - 1/6*log(x^2 + x + 1
) + 1/3*log(x - 1)
```

3.5.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.79

$$\int \frac{1}{-x^3+x^6}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{2x^2}-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{3}\log\left(|x-1|\right)$$

input
```
integrate(1/(x^6-x^3),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/2/x^2 - 1/6*log(x^2 + x + 1
) + 1/3*log(abs(x - 1))
```

3.5.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int \frac{1}{-x^3+x^6}\,dx = \frac{\ln(x-1)}{3}+\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right) - \ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)+\frac{1}{2x^2}$$

input
```
int(-1/(x^3 - x^6),x)
```

output
```
log(x - 1)/3 + log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/6 - 1/6) - log(
x + (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/6 + 1/6) + 1/(2*x^2)
```

3.6 $\int x^5\sqrt{a^2+2abx^3+b^2x^6}\,dx$

3.6.1 Optimal result

Integrand size = 26, antiderivative size = 79

$$\int x^5\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{ax^6\sqrt{a^2+2abx^3+b^2x^6}}{6(a+bx^3)}+\frac{bx^9\sqrt{a^2+2abx^3+b^2x^6}}{9(a+bx^3)}$$

output
```
1/6*a*x^6*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/9*b*x^9*((b*x^3+a)^2)^(1/2)/(b*x
^3+a)
```

3.6.2 Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.49

$$\int x^5\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{\sqrt{(a+bx^3)^2}(3ax^6+2bx^9)}{18(a+bx^3)}$$

input
```
Integrate[x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

output
```
(Sqrt[(a + b*x^3)^2]*(3*a*x^6 + 2*b*x^9))/(18*(a + b*x^3))
```

3.6.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1693, 1100, 1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \sqrt{a^2+2abx^3+b^2x^6} \, dx$$

$$\Big\downarrow \ 1693$$

$$\frac{1}{3} \int x^3 \sqrt{b^2x^6+2abx^3+a^2}dx^3$$

$$\Big\downarrow \ 1100$$

$$\frac{1}{3} \left(\frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{3b^2}-\frac{a \int \sqrt{b^2x^6+2abx^3+a^2}dx^3}{b}\right)$$

$$\Big\downarrow \ 1079$$

$$\frac{1}{3} \left(\frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{3b^2}-\frac{a\sqrt{a^2+2abx^3+b^2x^6} \int \left(b^2x^3+ab\right) dx^3}{b^2 \left(a+bx^3\right)}\right)$$

$$\Big\downarrow \ 17$$

$$\frac{1}{3} \left(\frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{3b^2}-\frac{a\left(a+bx^3\right) \sqrt{a^2+2abx^3+b^2x^6}}{2b^2}\right)$$

input
```
Int[x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

output
```
(-1/2*(a*(a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/b^2 + (a^2 + 2*a*b*x
^3 + b^2*x^6)^(3/2)/(3*b^2))/3
```

3.6.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1100
```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.6.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.39

method	result	size
pseudoelliptic	$-\frac{\operatorname{csgn}\left(b\,x^{3}+a\right)\left(b\,x^{3}+a\right)^{2}\left(-2b\,x^{3}+a\right)}{18b^{2}}$	31
gosper	$\frac{x^{6}\left(2b\,x^{3}+3a\right)\sqrt{\left(b\,x^{3}+a\right)^{2}}}{18b\,x^{3}+18a}$	36
default	$\frac{x^{6}\left(2b\,x^{3}+3a\right)\sqrt{\left(b\,x^{3}+a\right)^{2}}}{18b\,x^{3}+18a}$	36
risch	$\frac{a\,x^{6}\sqrt{\left(b\,x^{3}+a\right)^{2}}}{6b\,x^{3}+6a}+\frac{b\,x^{9}\sqrt{\left(b\,x^{3}+a\right)^{2}}}{9b\,x^{3}+9a}$	54

input
```
int(x^5*((b*x^3+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/18*csgn(b*x^3+a)*(b*x^3+a)^2*(-2*b*x^3+a)/b^2
```

3.6.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.16

$$\int x^5\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{1}{9}\,bx^9+\frac{1}{6}\,ax^6$$

input
```
integrate(x^5*((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/9*b*x^9 + 1/6*a*x^6
```

3.6.6 Sympy [F(-1)]

Timed out.

$$\int x^5\sqrt{a^2+2abx^3+b^2x^6}\,dx=\text{Timed out}$$

input
```
integrate(x**5*((b*x**3+a)**2)**(1/2),x)
```

output
```
Timed out
```

3.6.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.05

$$\int x^5\sqrt{a^2+2abx^3+b^2x^6}\,dx=-\frac{\sqrt{b^2x^6+2\,abx^3+a^2}ax^3}{6\,b}-\frac{\sqrt{b^2x^6+2\,abx^3+a^2}a^2}{6\,b^2}+\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}}{9\,b^2}$$

input
```
integrate(x^5*((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a*x^3/b - 1/6*sqrt(b^2*x^6 + 2*a*b*x^
3 + a^2)*a^2/b^2 + 1/9*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/b^2
```

3.6.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.29

$$\int x^5\sqrt{a^2+2abx^3+b^2x^6}\,dx = \frac{1}{18}\left(2\,bx^9+3\,ax^6\right)\mathrm{sgn}\left(bx^3+a\right)$$

input
```
integrate(x^5*((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/18*(2*b*x^9 + 3*a*x^6)*sgn(b*x^3 + a)
```

3.6.9 Mupad [B] (verification not implemented)

Time = 8.22 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.75

$$\int x^5\sqrt{a^2+2abx^3+b^2x^6}\,dx = \frac{\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}\,\left(8\,b^2\,\left(a^2+b^2\,x^6\right)-12\,a^2\,b^2+4\,a\,b^3\,x^3\right)}{72\,b^4}$$

input
```
int(x^5*((a + b*x^3)^2)^(1/2),x)
```

output
```
((a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2)*(8*b^2*(a^2 + b^2*x^6) - 12*a^2*b^2 + 4
*a*b^3*x^3))/(72*b^4)
```

3.7 $\int x^4\sqrt{a^2+2abx^3+b^2x^6}\,dx$

3.7.1 Optimal result

Integrand size = 26, antiderivative size = 79

$$\int x^4\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{ax^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}+\frac{bx^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}$$

output
```
1/5*a*x^5*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/8*b*x^8*((b*x^3+a)^2)^(1/2)/(b*x
^3+a)
```

3.7.2 Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.49

$$\int x^4\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{\sqrt{\left(a+bx^3\right)^2}\left(8ax^5+5bx^8\right)}{40\left(a+bx^3\right)}$$

input
```
Integrate[x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

output
```
(Sqrt[(a + b*x^3)^2]*(8*a*x^5 + 5*b*x^8))/(40*(a + b*x^3))
```

3.7.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.62, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\sqrt{a^2+2abx^3+b^2x^6}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int bx^4\left(bx^3+a\right)dx}{b\left(a+bx^3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^4\left(bx^3+a\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(bx^7+ax^4\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(\frac{ax^5}{5}+\frac{bx^8}{8}\right)}{a+bx^3}$$

input
```
Int[x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*((a*x^5)/5 + (b*x^8)/8))/(a + b*x^3)
```

3.7.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.7.4 Maple [A] (verified)

Time = 4.60 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.46

method	result	size
gosper	$\frac{x^{5}\left(5 b \,x^{3}+8 a \right) \sqrt{\left(b \,x^{3}+a \right)^{2}}}{40 b \,x^{3}+40 a}$	36
default	$\frac{x^{5}\left(5 b \,x^{3}+8 a \right) \sqrt{\left(b \,x^{3}+a \right)^{2}}}{40 b \,x^{3}+40 a}$	36
risch	$\frac{a \,x^{5} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{5 b \,x^{3}+5 a}+\frac{b \,x^{8} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{8 b \,x^{3}+8 a}$	54

input
```
int(x^4*((b*x^3+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/40*x^5*(5*b*x^3+8*a)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

3.7.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.16

$$\int x^4\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{1}{8}\,bx^8+\frac{1}{5}\,ax^5$$

input
```
integrate(x^4*((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/8*b*x^8 + 1/5*a*x^5
```

3.7.6 Sympy [F(-1)]

Timed out.

$$\int x^4\sqrt{a^2+2abx^3+b^2x^6}\,dx=\text{Timed out}$$

input
```
integrate(x**4*((b*x**3+a)**2)**(1/2),x)
```

output
```
Timed out
```

3.7.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.16

$$\int x^4\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{1}{8}\,bx^8+\frac{1}{5}\,ax^5$$

input
```
integrate(x^4*((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/8*b*x^8 + 1/5*a*x^5
```

3.7.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.37

$$\int x^4\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{1}{8}\,bx^8\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{5}\,ax^5\mathrm{sgn}\left(bx^3+a\right)$$

input
```
integrate(x^4*((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/8*b*x^8*sgn(b*x^3 + a) + 1/5*a*x^5*sgn(b*x^3 + a)
```

3.7.9 Mupad [F(-1)]

Timed out.

$$\int x^4\sqrt{a^2+2abx^3+b^2x^6}\,dx=\int x^4\,\sqrt{{\left(b\,x^3+a\right)}^2}\,dx$$

input
```
int(x^4*((a + b*x^3)^2)^(1/2),x)
```

output
```
int(x^4*((a + b*x^3)^2)^(1/2), x)
```

3.8 $\int x^3\sqrt{a^2+2abx^3+b^2x^6}\,dx$

3.8.1 Optimal result

Integrand size = 26, antiderivative size = 79

$$\int x^3\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{ax^4\sqrt{a^2+2abx^3+b^2x^6}}{4(a+bx^3)}+\frac{bx^7\sqrt{a^2+2abx^3+b^2x^6}}{7(a+bx^3)}$$

output
```
1/4*a*x^4*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/7*b*x^7*((b*x^3+a)^2)^(1/2)/(b*x
^3+a)
```

3.8.2 Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.49

$$\int x^3\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{\sqrt{(a+bx^3)^2}(7ax^4+4bx^7)}{28(a+bx^3)}$$

input
```
Integrate[x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

output
```
(Sqrt[(a + b*x^3)^2]*(7*a*x^4 + 4*b*x^7))/(28*(a + b*x^3))
```

3.8.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.62, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{a^2+2abx^3+b^2x^6}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int bx^3\left(bx^3+a\right)dx}{b\left(a+bx^3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^3\left(bx^3+a\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(bx^6+ax^3\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(\frac{ax^4}{4}+\frac{bx^7}{7}\right)}{a+bx^3}$$

input
```
Int[x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*((a*x^4)/4 + (b*x^7)/7))/(a + b*x^3)
```

3.8.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.8.4 Maple [A] (verified)

Time = 3.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.46

method	result	size
gosper	$\frac{x^{4} \left(4 b \,x^{3}+7 a \right) \sqrt{\left(b \,x^{3}+a \right)^{2}}}{28 b \,x^{3}+28 a}$	36
default	$\frac{x^{4} \left(4 b \,x^{3}+7 a \right) \sqrt{\left(b \,x^{3}+a \right)^{2}}}{28 b \,x^{3}+28 a}$	36
risch	$\frac{a \,x^{4} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{4 b \,x^{3}+4 a}+\frac{b \,x^{7} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{7 b \,x^{3}+7 a}$	54

input
```
int(x^3*((b*x^3+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/28*x^4*(4*b*x^3+7*a)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

3.8.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.16

$$\int x^3\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{1}{7}\,bx^7+\frac{1}{4}\,ax^4$$

input
```
integrate(x^3*((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/7*b*x^7 + 1/4*a*x^4
```

3.8.6 Sympy [F(-1)]

Timed out.

$$\int x^3\sqrt{a^2+2abx^3+b^2x^6}\,dx=\text{Timed out}$$

input
```
integrate(x**3*((b*x**3+a)**2)**(1/2),x)
```

output
```
Timed out
```

3.8.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.16

$$\int x^3\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{1}{7}\,bx^7+\frac{1}{4}\,ax^4$$

input
```
integrate(x^3*((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/7*b*x^7 + 1/4*a*x^4
```

3.8.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.37

$$\int x^3\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{1}{7}\,bx^7\text{sgn}\left(bx^3+a\right)+\frac{1}{4}\,ax^4\text{sgn}\left(bx^3+a\right)$$

input
```
integrate(x^3*((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/7*b*x^7*sgn(b*x^3 + a) + 1/4*a*x^4*sgn(b*x^3 + a)
```

3.8.9 Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{a^2+2abx^3+b^2x^6}\,dx=\int x^3\,\sqrt{(b\,x^3+a)^2}\,dx$$

input
```
int(x^3*((a + b*x^3)^2)^(1/2),x)
```

output
```
int(x^3*((a + b*x^3)^2)^(1/2), x)
```

3.9 $\int x^2\sqrt{a^2+2abx^3+b^2x^6}\,dx$

3.9.1 Optimal result

Integrand size = 26, antiderivative size = 36

$$\int x^2\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}}{6b}$$

output
```
1/6*(b*x^3+a)*((b*x^3+a)^2)^(1/2)/b
```

3.9.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 88 vs. 2(36) = 72.

Time = 0.33 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.44

$$\int x^2\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{x^3(2a+bx^3)\left(\sqrt{a^2}bx^3+a\left(\sqrt{a^2}-\sqrt{(a+bx^3)^2}\right)\right)}{-6a^2-6abx^3+6\sqrt{a^2}\sqrt{(a+bx^3)^2}}$$

input
```
Integrate[x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

output
```
(x^3*(2*a + b*x^3)*(Sqrt[a^2]*b*x^3 + a*(Sqrt[a^2] - Sqrt[(a + b*x^3)^2]))
)/(-6*a^2 - 6*a*b*x^3 + 6*Sqrt[a^2]*Sqrt[(a + b*x^3)^2])
```

3.9.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1690, 1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{a^2+2abx^3+b^2x^6}\,dx$$

$$\Big\downarrow \; 1690$$

$$\frac{1}{3}\int \sqrt{b^2x^6+2abx^3+a^2}dx^3$$

$$\Big\downarrow \; 1079$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(b^2x^3+ab\right)dx^3}{3b\left(a+bx^3\right)}$$

$$\Big\downarrow \; 17$$

$$\frac{\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}{6b}$$

input
```
Int[x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

output
```
((a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(6*b)
```

3.9.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

3.9.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.64

method	result	size
pseudoelliptic	$\frac{(b\,x^3+a)^2\,\mathrm{csgn}(b\,x^3+a)}{6b}$	23
default	$\frac{(b\,x^3+a)\sqrt{(b\,x^3+a)^2}}{6b}$	24
risch	$\frac{(b\,x^3+a)\sqrt{(b\,x^3+a)^2}}{6b}$	24
gosper	$\frac{x^3\,(b\,x^3+2a)\sqrt{(b\,x^3+a)^2}}{6b\,x^3+6a}$	35

input
```
int(x^2*((b*x^3+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(b*x^3+a)^2*csgn(b*x^3+a)/b
```

3.9.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.36

$$\int x^2\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{1}{6}\,bx^6+\frac{1}{3}\,ax^3$$

input
```
integrate(x^2*((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/6*b*x^6 + 1/3*a*x^3
```

3.9.6 Sympy [F]

$$\int x^2\sqrt{a^2+2abx^3+b^2x^6}\,dx=\int x^2\sqrt{(a+bx^3)^2}\,dx$$

input
```
integrate(x**2*((b*x**3+a)**2)**(1/2),x)
```

output
```
Integral(x**2*sqrt((a + b*x**3)**2), x)
```

3.9.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(23) = 46$.

Time = 0.22 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.44

$$\int x^2\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{1}{6}\sqrt{b^2x^6+2\,abx^3+a^2}x^3+\frac{\sqrt{b^2x^6+2\,abx^3+a^2}a}{6\,b}$$

input
```
integrate(x^2*((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*x^3 + 1/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a
^2)*a/b
```

3.9.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.61

$$\int x^2\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{1}{6}\left(bx^6+2\,ax^3\right)\operatorname{sgn}\left(bx^3+a\right)$$

input
```
integrate(x^2*((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/6*(b*x^6 + 2*a*x^3)*sgn(b*x^3 + a)
```

3.9.9 Mupad [B] (verification not implemented)

Time = 8.40 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

$$\int x^2\sqrt{a^2+2abx^3+b^2x^6}\,dx = \left(\frac{a}{6\,b}+\frac{x^3}{6}\right)\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}$$

input
```
int(x^2*((a + b*x^3)^2)^(1/2),x)
```

output
```
(a/(6*b) + x^3/6)*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2)
```

3.10 $\int x\sqrt{a^2+2abx^3+b^2x^6}\,dx$

3.10.1 Optimal result

Integrand size = 24, antiderivative size = 79

$$\int x\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{ax^2\sqrt{a^2+2abx^3+b^2x^6}}{2(a+bx^3)}+\frac{bx^5\sqrt{a^2+2abx^3+b^2x^6}}{5(a+bx^3)}$$

output
```
1/2*a*x^2*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/5*b*x^5*((b*x^3+a)^2)^(1/2)/(b*x
^3+a)
```

3.10.2 Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.49

$$\int x\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{\sqrt{(a+bx^3)^2}(5ax^2+2bx^5)}{10(a+bx^3)}$$

input
```
Integrate[x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

output
```
(Sqrt[(a + b*x^3)^2]*(5*a*x^2 + 2*b*x^5))/(10*(a + b*x^3))
```

3.10.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.62, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{a^2+2abx^3+b^2x^6}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int bx\left(bx^3+a\right)dx}{b\left(a+bx^3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int x\left(bx^3+a\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(bx^4+ax\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(\frac{ax^2}{2}+\frac{bx^5}{5}\right)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

input
```
Int[x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

output
```
(((a*x^2)/2 + (b*x^5)/5)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3)
```

3.10.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.10.4 Maple [A] (verified)

Time = 1.93 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.46

method	result	size
gosper	$\frac{x^{2}\left(2 b \,x^{3}+5 a \right) \sqrt{\left(b \,x^{3}+a \right)^{2}}}{10 b \,x^{3}+10 a}$	36
default	$\frac{x^{2}\left(2 b \,x^{3}+5 a \right) \sqrt{\left(b \,x^{3}+a \right)^{2}}}{10 b \,x^{3}+10 a}$	36
risch	$\frac{a \,x^{2} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{2 b \,x^{3}+2 a}+\frac{b \,x^{5} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{5 b \,x^{3}+5 a}$	54

input
```
int(x*((b*x^3+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/10*x^2*(2*b*x^3+5*a)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

3.10.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.16

$$\int x\sqrt{a^2+2abx^3+b^2x^6}\,dx = \frac{1}{5}\,bx^5+\frac{1}{2}\,ax^2$$

input
```
integrate(x*((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/5*b*x^5 + 1/2*a*x^2
```

3.10.6 Sympy [F]

$$\int x\sqrt{a^2+2abx^3+b^2x^6}\,dx=\int x\sqrt{(a+bx^3)^2}\,dx$$

input
```
integrate(x*((b*x**3+a)**2)**(1/2),x)
```

output
```
Integral(x*sqrt((a + b*x**3)**2), x)
```

3.10.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.16

$$\int x\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{1}{5}\,bx^5+\frac{1}{2}\,ax^2$$

input
```
integrate(x*((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/5*b*x^5 + 1/2*a*x^2
```

3.10.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.37

$$\int x\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{1}{5}\,bx^5\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{2}\,ax^2\mathrm{sgn}\left(bx^3+a\right)$$

input
```
integrate(x*((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/5*b*x^5*sgn(b*x^3 + a) + 1/2*a*x^2*sgn(b*x^3 + a)
```

3.10.9 Mupad [F(-1)]

Timed out.

$$\int x\sqrt{a^2+2abx^3+b^2x^6}\,dx=\int x\sqrt{(b\,x^3+a)^2}\,dx$$

input
```
int(x*((a + b*x^3)^2)^(1/2),x)
```

output
```
int(x*((a + b*x^3)^2)^(1/2), x)
```

3.11 $\int \sqrt{a^2+2abx^3+b^2x^6}\,dx$

3.11.1 Optimal result

Integrand size = 22, antiderivative size = 74

$$\int \sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{ax\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{bx^4\sqrt{a^2+2abx^3+b^2x^6}}{4\,(a+bx^3)}$$

output
```
a*x*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/4*b*x^4*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

3.11.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.49

$$\int \sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{\sqrt{(a+bx^3)^2}(4ax+bx^4)}{4\,(a+bx^3)}$$

input
```
Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

output
```
(Sqrt[(a + b*x^3)^2]*(4*a*x + b*x^4))/(4*(a + b*x^3))
```

3.11.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.68, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1384, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a^2+2abx^3+b^2x^6}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(b^2x^3+ab\right)dx}{b\left(a+bx^3\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(abx+\frac{b^2x^4}{4}\right)\sqrt{a^2+2abx^3+b^2x^6}}{b\left(a+bx^3\right)}$$

input
```
Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

output
```
((a*b*x + (b^2*x^4)/4)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(b*(a + b*x^3))
```

3.11.3.1 Defintions of rubi rules used

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.11.4 Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.45

method	result	size
gosper	$\frac{x(b\,x^3+4a)\sqrt{(b\,x^3+a)^2}}{4b\,x^3+4a}$	33
default	$\frac{x(b\,x^3+4a)\sqrt{(b\,x^3+a)^2}}{4b\,x^3+4a}$	33
risch	$\frac{ax\sqrt{(b\,x^3+a)^2}}{b\,x^3+a}+\frac{b\,x^4\sqrt{(b\,x^3+a)^2}}{4b\,x^3+4a}$	51

input
```
int(((b*x^3+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*x*(b*x^3+4*a)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

3.11.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.14

$$\int \sqrt{a^2+2abx^3+b^2x^6}\,dx = \frac{1}{4}\,bx^4+ax$$

input
```
integrate(((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/4*b*x^4 + a*x
```

3.11.6 Sympy [F]

$$\int \sqrt{a^2+2abx^3+b^2x^6}\,dx = \int \sqrt{(a+bx^3)^2}\,dx$$

input
```
integrate(((b*x**3+a)**2)**(1/2),x)
```

output
```
Integral(sqrt((a + b*x**3)**2), x)
```

3.11.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.14

$$\int \sqrt{a^2 + 2abx^3 + b^2x^6}\,dx = \frac{1}{4}\,bx^4 + ax$$

input
```
integrate(((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/4*b*x^4 + a*x
```

3.11.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.27

$$\int \sqrt{a^2 + 2abx^3 + b^2x^6}\,dx = \frac{1}{4}\,\left(bx^4 + 4\,ax\right)\mathrm{sgn}\left(bx^3 + a\right)$$

input
```
integrate(((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/4*(b*x^4 + 4*a*x)*sgn(b*x^3 + a)
```

3.11.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a^2 + 2abx^3 + b^2x^6}\,dx = \int \sqrt{{\left(b\,x^3 + a\right)}^2}\,dx$$

input
```
int(((a + b*x^3)^2)^(1/2),x)
```

output
```
int(((a + b*x^3)^2)^(1/2), x)
```

3.12 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x}\,dx$

3.12.1 Optimal result

Integrand size = 26, antiderivative size = 75

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x}\,dx=\frac{bx^3\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}+\frac{a\sqrt{a^2+2abx^3+b^2x^6}\log(x)}{a+bx^3}$$

output
```
1/3*b*x^3*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+a*ln(x)*((b*x^3+a)^2)^(1/2)/(b*x^3
+a)
```

3.12.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 454 vs. 2(75) = 150.

Time = 0.69 (sec) , antiderivative size = 454, normalized size of antiderivative = 6.05

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x}\,dx$$
$$=\frac{-2a\sqrt{a^2}bx^3-2\sqrt{a^2}b^2x^6+2abx^3\sqrt{\left(a+bx^3\right)^2}-2a\left(a^2+abx^3-\sqrt{a^2}\sqrt{\left(a+bx^3\right)^2}\right)\operatorname{arctanh}\left(\frac{bx^3}{\sqrt{a^2}-\sqrt{(a+bx}}\right.}{}$$

input
```
Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x,x]
```

output
```
(-2*a*Sqrt[a^2]*b*x^3 - 2*Sqrt[a^2]*b^2*x^6 + 2*a*b*x^3*Sqrt[(a + b*x^3)^2
] - 2*a*(a^2 + a*b*x^3 - Sqrt[a^2]*Sqrt[(a + b*x^3)^2])*ArcTanh[(b*x^3)/(S
qrt[a^2] - Sqrt[(a + b*x^3)^2])] - 2*((a^2)^(3/2) + a*Sqrt[a^2]*b*x^3 - a^
2*Sqrt[(a + b*x^3)^2])*Log[x^3] + (a^2)^(3/2)*Log[Sqrt[a^2] - b*x^3 - Sqrt
[(a + b*x^3)^2]] + a*Sqrt[a^2]*b*x^3*Log[Sqrt[a^2] - b*x^3 - Sqrt[(a + b*x
^3)^2]] - a^2*Sqrt[(a + b*x^3)^2]*Log[Sqrt[a^2] - b*x^3 - Sqrt[(a + b*x^3)
^2]] + (a^2)^(3/2)*Log[Sqrt[a^2] + b*x^3 - Sqrt[(a + b*x^3)^2]] + a*Sqrt[a
^2]*b*x^3*Log[Sqrt[a^2] + b*x^3 - Sqrt[(a + b*x^3)^2]] - a^2*Sqrt[(a + b*x
^3)^2]*Log[Sqrt[a^2] + b*x^3 - Sqrt[(a + b*x^3)^2]])/(6*(a^2 + a*b*x^3 - S
qrt[a^2]*Sqrt[(a + b*x^3)^2]))
```

3.12.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.60, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x}dx$$
$$\Big\downarrow \text{ 1384}$$
$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b(bx^3+a)}{x}dx}{b(a+bx^3)}$$
$$\Big\downarrow \text{ 27}$$
$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{bx^3+a}{x}dx}{a+bx^3}$$
$$\Big\downarrow \text{ 802}$$
$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(bx^2+\frac{a}{x}\right)dx}{a+bx^3}$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(a\log(x)+\frac{bx^3}{3}\right)}{a+bx^3}$$

input
```
Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*((b*x^3)/3 + a*Log[x]))/(a + b*x^3)
```

3.12.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.12.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.35

method	result	size
pseudoelliptic	$\frac{\operatorname{csgn}\left(b\,x^{3}+a\right)\left(b\,x^{3}+a+a\ln\left(b\,x^{3}\right)\right)}{3}$	26
default	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(b\,x^{3}+3a\ln\left(x\right)\right)}{3b\,x^{3}+3a}$	34
risch	$\frac{b\,x^{3}\sqrt{\left(b\,x^{3}+a\right)^{2}}}{3b\,x^{3}+3a}+\frac{a\ln\left(x\right)\sqrt{\left(b\,x^{3}+a\right)^{2}}}{b\,x^{3}+a}$	52

input
```
int(((b*x^3+a)^2)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/3*csgn(b*x^3+a)*(b*x^3+a+a*ln(b*x^3))
```

3.12.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.15

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x}\,dx = \frac{1}{3}\,bx^3 + a\log\left(x\right)$$

input
```
integrate(((b*x^3+a)^2)^(1/2)/x,x, algorithm="fricas")
```

output
```
1/3*b*x^3 + a*log(x)
```

3.12.6 Sympy [F]

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x}\,dx = \int \frac{\sqrt{(a+bx^3)^2}}{x}\,dx$$

input
```
integrate(((b*x**3+a)**2)**(1/2)/x,x)
```

output
```
Integral(sqrt((a + b*x**3)**2)/x, x)
```

3.12.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.28

$$\begin{aligned}\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x}\,dx &= \frac{1}{3}\left(-1\right)^{2\,b^2x^3+2\,ab}a\log\left(2\,b^2x^3+2\,ab\right)\\ &\quad-\frac{1}{3}\left(-1\right)^{2\,abx^3+2\,a^2}a\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right)\\ &\quad+\frac{1}{3}\sqrt{b^2x^6+2\,abx^3+a^2}\end{aligned}$$

input
```
integrate(((b*x^3+a)^2)^(1/2)/x,x, algorithm="maxima")
```

output
```
1/3*(-1)^(2*b^2*x^3 + 2*a*b)*a*log(2*b^2*x^3 + 2*a*b) - 1/3*(-1)^(2*a*b*x^
3 + 2*a^2)*a*log(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x))) + 1/3*sqrt(b^2*x^6 +
 2*a*b*x^3 + a^2)
```

3.12.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.37

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x}\,dx = \frac{1}{3}\,bx^3\mathrm{sgn}\left(bx^3+a\right)+a\log\left(|x|\right)\mathrm{sgn}\left(bx^3+a\right)$$

input
```
integrate(((b*x^3+a)^2)^(1/2)/x,x, algorithm="giac")
```

output
```
1/3*b*x^3*sgn(b*x^3 + a) + a*log(abs(x))*sgn(b*x^3 + a)
```

3.12.9 Mupad [B] (verification not implemented)

Time = 8.37 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.45

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x}\,dx = \frac{\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{3} - \frac{\ln\left(a\,b+\frac{a^2}{x^3}+\frac{\sqrt{a^2}\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{x^3}\right)\sqrt{a^2}}{3} + \frac{a\,b\,\ln\left(a\,b+\sqrt{{\left(b\,x^3+a\right)}^2}\,\sqrt{b^2}+b^2\,x^3\right)}{3\,\sqrt{b^2}}$$

input
```
int(((a + b*x^3)^2)^(1/2)/x,x)
```

output
```
(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2)/3 - (log(a*b + a^2/x^3 + ((a^2)^(1/2)*(a
^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/x^3)*(a^2)^(1/2))/3 + (a*b*log(a*b + ((a
+ b*x^3)^2)^(1/2)*(b^2)^(1/2) + b^2*x^3))/(3*(b^2)^(1/2))
```

3.13 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^2} dx$

3.13.1 Optimal result

Integrand size = 26, antiderivative size = 77

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^2} dx = -\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{x(a+bx^3)} + \frac{bx^2\sqrt{a^2+2abx^3+b^2x^6}}{2(a+bx^3)}$$

output
```
-a*((b*x^3+a)^2)^(1/2)/x/(b*x^3+a)+1/2*b*x^2*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

3.13.2 Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.49

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^2} dx = \frac{(-2a+bx^3)\sqrt{(a+bx^3)^2}}{2x(a+bx^3)}$$

input
```
Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^2,x]
```

output
```
((-2*a + b*x^3)*Sqrt[(a + b*x^3)^2])/(2*x*(a + b*x^3))
```

3.13.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.61, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^2} dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b(bx^3+a)}{x^2}dx}{b(a+bx^3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{bx^3+a}{x^2}dx}{a+bx^3}$$

$$\Big\downarrow \text{802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a}{x^2}+bx\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(\frac{bx^2}{2}-\frac{a}{x}\right)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

input
```
Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^2,x]
```

output
```
((-(a/x) + (b*x^2)/2)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3)
```

3.13.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.13.4 Maple [A] (verified)

Time = 2.39 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.47

method	result	size
gosper	$-\frac{\left(-b\,x^{3}+2 a\right) \sqrt{\left(b\,x^{3}+a\right)^{2}}}{2 x \left(b\,x^{3}+a\right)}$	36
default	$-\frac{\left(-b\,x^{3}+2 a\right) \sqrt{\left(b\,x^{3}+a\right)^{2}}}{2 x \left(b\,x^{3}+a\right)}$	36
risch	$-\frac{a \sqrt{\left(b\,x^{3}+a\right)^{2}}}{x \left(b\,x^{3}+a\right)}+\frac{b\,x^{2} \sqrt{\left(b\,x^{3}+a\right)^{2}}}{2 b\,x^{3}+2 a}$	54

input
```
int(((b*x^3+a)^2)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-b*x^3+2*a)*((b*x^3+a)^2)^(1/2)/x/(b*x^3+a)
```

3.13.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.18

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^2}\,dx = \frac{bx^3-2\,a}{2\,x}$$

input
```
integrate(((b*x^3+a)^2)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
1/2*(b*x^3 - 2*a)/x
```

3.13.6 Sympy [F]

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^2}\,dx=\int \frac{\sqrt{(a+bx^3)^2}}{x^2}\,dx$$

input
```
integrate(((b*x**3+a)**2)**(1/2)/x**2,x)
```

output
```
Integral(sqrt((a + b*x**3)**2)/x**2, x)
```

3.13.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.18

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^2}\,dx=\frac{bx^3-2\,a}{2\,x}$$

input
```
integrate(((b*x^3+a)^2)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
1/2*(b*x^3 - 2*a)/x
```

3.13.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.38

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^2}\,dx=\frac{1}{2}\,bx^2\mathrm{sgn}\left(bx^3+a\right)-\frac{a\mathrm{sgn}\left(bx^3+a\right)}{x}$$

input
```
integrate(((b*x^3+a)^2)^(1/2)/x^2,x, algorithm="giac")
```

output
```
1/2*b*x^2*sgn(b*x^3 + a) - a*sgn(b*x^3 + a)/x
```

3.13.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^2}\,dx=\int \frac{\sqrt{{\left(b\,x^3+a\right)}^2}}{x^2}\,dx$$

input
```
int(((a + b*x^3)^2)^(1/2)/x^2,x)
```

output
```
int(((a + b*x^3)^2)^(1/2)/x^2, x)
```

3.14 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^3}\,dx$

3.14.1 Optimal result

Integrand size = 26, antiderivative size = 74

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^3}\,dx = -\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}+\frac{bx\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

output
```
-1/2*a*((b*x^3+a)^2)^(1/2)/x^2/(b*x^3+a)+b*x*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

3.14.2 Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.50

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^3}\,dx = -\frac{\left(a-2bx^3\right)\sqrt{\left(a+bx^3\right)^2}}{2x^2\left(a+bx^3\right)}$$

input
```
Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^3,x]
```

output
```
-1/2*((a - 2*b*x^3)*Sqrt[(a + b*x^3)^2])/(x^2*(a + b*x^3))
```

3.14.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.59, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^3}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b(bx^3+a)}{x^3}dx}{b(a+bx^3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{bx^3+a}{x^3}dx}{a+bx^3}$$

$$\Big\downarrow \text{802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a}{x^3}+b\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(bx-\frac{a}{2x^2}\right)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

input
```
Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^3,x]
```

output
```
((-1/2*a/x^2 + b*x)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3)
```

3.14.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.14.4 Maple [A] (verified)

Time = 3.16 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.46

method	result	size
gosper	$-\frac{\left(-2 b \,x^{3}+a \right) \sqrt{\left(b \,x^{3}+a \right)^{2}}}{2 x^{2} \left(b \,x^{3}+a \right)}$	34
default	$-\frac{\left(-2 b \,x^{3}+a \right) \sqrt{\left(b \,x^{3}+a \right)^{2}}}{2 x^{2} \left(b \,x^{3}+a \right)}$	34
risch	$-\frac{a \sqrt{\left(b \,x^{3}+a \right)^{2}}}{2 x^{2} \left(b \,x^{3}+a \right)}+\frac{b x \sqrt{\left(b \,x^{3}+a \right)^{2}}}{b \,x^{3}+a}$	51

input
```
int(((b*x^3+a)^2)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-2*b*x^3+a)*((b*x^3+a)^2)^(1/2)/x^2/(b*x^3+a)
```

3.14.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.20

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^3}\,dx=\frac{2\,bx^3-a}{2\,x^2}$$

input
```
integrate(((b*x^3+a)^2)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
1/2*(2*b*x^3 - a)/x^2
```

3.14.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^3}\,dx = \text{Timed out}$$

input
```
integrate(((b*x**3+a)**2)**(1/2)/x**3,x)
```

output
```
Timed out
```

3.14.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.20

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^3}\,dx = \frac{2\,bx^3-a}{2\,x^2}$$

input
```
integrate(((b*x^3+a)^2)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
1/2*(2*b*x^3 - a)/x^2
```

3.14.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.35

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^3}\,dx = bx\text{sgn}\left(bx^3+a\right) - \frac{a\text{sgn}(bx^3+a)}{2\,x^2}$$

input
```
integrate(((b*x^3+a)^2)^(1/2)/x^3,x, algorithm="giac")
```

output
```
b*x*sgn(b*x^3 + a) - 1/2*a*sgn(b*x^3 + a)/x^2
```

3.14.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^3}\,dx = \int \frac{\sqrt{\left(b\,x^3 + a\right)^2}}{x^3}\,dx$$

input
```
int(((a + b*x^3)^2)^(1/2)/x^3,x)
```

output
```
int(((a + b*x^3)^2)^(1/2)/x^3, x)
```

3.15 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^4}\,dx$

3.15.1 Optimal result

Integrand size = 26, antiderivative size = 75

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^4}\,dx = -\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\,(a+bx^3)} + \frac{b\sqrt{a^2+2abx^3+b^2x^6}\log(x)}{a+bx^3}$$

output
```
-1/3*a*((b*x^3+a)^2)^(1/2)/x^3/(b*x^3+a)+b*ln(x)*((b*x^3+a)^2)^(1/2)/(b*x^
3+a)
```

3.15.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 178 vs. 2(75) = 150.

Time = 0.22 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.37

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^4}\,dx$$
$$= \frac{a\sqrt{a^2} - a\sqrt{(a+bx^3)^2} - 2abx^3\operatorname{arctanh}\left(\frac{bx^3}{\sqrt{a^2}-\sqrt{(a+bx^3)^2}}\right) - 2\sqrt{a^2}bx^3\log\left(x^3\right) + \sqrt{a^2}bx^3\log\Bigg(a\bigg(\sqrt{a^2} - bx^3}{6ax^3}$$

input
```
Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^4,x]
```

output
```
(a*Sqrt[a^2] - a*Sqrt[(a + b*x^3)^2] - 2*a*b*x^3*ArcTanh[(b*x^3)/(Sqrt[a^2
] - Sqrt[(a + b*x^3)^2])] - 2*Sqrt[a^2]*b*x^3*Log[x^3] + Sqrt[a^2]*b*x^3*L
og[a*(Sqrt[a^2] - b*x^3 - Sqrt[(a + b*x^3)^2])] + Sqrt[a^2]*b*x^3*Log[a*(S
qrt[a^2] + b*x^3 - Sqrt[(a + b*x^3)^2])])/(6*a*x^3)
```

3.15.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.60, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^4}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b(bx^3+a)}{x^4}dx}{b(a+bx^3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{bx^3+a}{x^4}dx}{a+bx^3}$$

$$\Big\downarrow \text{802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a}{x^4}+\frac{b}{x}\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(b\log(x)-\frac{a}{3x^3}\right)}{a+bx^3}$$

input
```
Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^4,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-1/3*a/x^3 + b*Log[x]))/(a + b*x^3)
```

3.15.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.15.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.37

method	result	size
pseudoelliptic	$-\frac{\operatorname{csgn}\left(b\,x^{3}+a\right)\left(-\ln\left(b\,x^{3}\right) b\,x^{3}+a\right)}{3 x^{3}}$	28
default	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(3 b \ln\left(x\right) x^{3}-a\right)}{3 x^{3} \left(b\,x^{3}+a\right)}$	38
risch	$-\frac{a \sqrt{\left(b\,x^{3}+a\right)^{2}}}{3 x^{3} \left(b\,x^{3}+a\right)}+\frac{b \ln\left(x\right) \sqrt{\left(b\,x^{3}+a\right)^{2}}}{b\,x^{3}+a}$	52

input
```
int(((b*x^3+a)^2)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*csgn(b*x^3+a)*(-ln(b*x^3)*b*x^3+a)/x^3
```

3.15.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.23

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^4}\,dx = \frac{3\,bx^3\log\left(x\right)-a}{3\,x^3}$$

input
```
integrate(((b*x^3+a)^2)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
1/3*(3*b*x^3*log(x) - a)/x^3
```

3.15.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^4}\,dx = \text{Timed out}$$

input
```
integrate(((b*x**3+a)**2)**(1/2)/x**4,x)
```

output
```
Timed out
```

3.15.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.32

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^4}\,dx = \frac{1}{3}\left(-1\right)^{2\,b^2x^3+2\,ab}b\log\left(2\,b^2x^3+2\,ab\right) - \frac{1}{3}\left(-1\right)^{2\,abx^3+2\,a^2}b\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right) - \frac{\sqrt{b^2x^6+2\,abx^3+a^2}}{3\,x^3}$$

input
```
integrate(((b*x^3+a)^2)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
1/3*(-1)^(2*b^2*x^3 + 2*a*b)*b*log(2*b^2*x^3 + 2*a*b) - 1/3*(-1)^(2*a*b*x^
3 + 2*a^2)*b*log(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x))) - 1/3*sqrt(b^2*x^6 +
 2*a*b*x^3 + a^2)/x^3
```

3.15.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.57

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^4}\,dx = b\log\left(|x|\right)\operatorname{sgn}\left(bx^3+a\right) - \frac{bx^3\operatorname{sgn}(bx^3+a)+a\operatorname{sgn}(bx^3+a)}{3\,x^3}$$

input
```
integrate(((b*x^3+a)^2)^(1/2)/x^4,x, algorithm="giac")
```

output
```
b*log(abs(x))*sgn(b*x^3 + a) - 1/3*(b*x^3*sgn(b*x^3 + a) + a*sgn(b*x^3 + a
))/x^3
```

3.15.9 Mupad [B] (verification not implemented)

Time = 8.36 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.49

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^4}\,dx = \frac{\ln\left(a\,b+\sqrt{\left(b\,x^3+a\right)^2}\,\sqrt{b^2}+b^2\,x^3\right)\,\sqrt{b^2}}{3} - \frac{\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{3\,x^3} - \frac{a\,b\,\ln\left(a\,b+\frac{a^2}{x^3}+\frac{\sqrt{a^2}\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{x^3}\right)}{3\,\sqrt{a^2}}$$

input
```
int(((a + b*x^3)^2)^(1/2)/x^4,x)
```

output
```
(log(a*b + ((a + b*x^3)^2)^(1/2)*(b^2)^(1/2) + b^2*x^3)*(b^2)^(1/2))/3 - (
a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2)/(3*x^3) - (a*b*log(a*b + a^2/x^3 + ((a^2)
^(1/2)*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/x^3))/(3*(a^2)^(1/2))
```

3.16 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^5}\,dx$

3.16.1 Optimal result

Integrand size = 26, antiderivative size = 77

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^5}\,dx = -\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)} - \frac{b\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}$$

output
```
-1/4*a*((b*x^3+a)^2)^(1/2)/x^4/(b*x^3+a)-b*((b*x^3+a)^2)^(1/2)/x/(b*x^3+a)
```

3.16.2 Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.48

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^5}\,dx = -\frac{\sqrt{\left(a+bx^3\right)^2}\left(a+4bx^3\right)}{4x^4\left(a+bx^3\right)}$$

input
```
Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^5,x]
```

output
```
-1/4*(Sqrt[(a + b*x^3)^2]*(a + 4*b*x^3))/(x^4*(a + b*x^3))
```

3.16.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.61, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^5}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b(bx^3+a)}{x^5}dx}{b(a+bx^3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{bx^3+a}{x^5}dx}{a+bx^3}$$

$$\Big\downarrow \text{802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a}{x^5}+\frac{b}{x^2}\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(-\frac{a}{4x^4}-\frac{b}{x}\right)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

input
```
Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^5,x]
```

output
```
((-1/4*a/x^4 - b/x)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3)
```

3.16.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.16.4 Maple [A] (verified)

Time = 4.52 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.44

method	result	size
gosper	$-\frac{(4b\,x^3+a)\sqrt{(b\,x^3+a)^2}}{4(b\,x^3+a)x^4}$	34
default	$-\frac{(4b\,x^3+a)\sqrt{(b\,x^3+a)^2}}{4(b\,x^3+a)x^4}$	34
risch	$\frac{\left(-b\,x^3-\frac{a}{4}\right)\sqrt{(b\,x^3+a)^2}}{x^4(b\,x^3+a)}$	35

input
```
int(((b*x^3+a)^2)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(4*b*x^3+a)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)/x^4
```

3.16.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.17

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^5}\,dx=-\frac{4\,bx^3+a}{4\,x^4}$$

input
```
integrate(((b*x^3+a)^2)^(1/2)/x^5,x, algorithm="fricas")
```

output
```
-1/4*(4*b*x^3 + a)/x^4
```

3.16.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^5}\,dx = \text{Timed out}$$

input
```
integrate(((b*x**3+a)**2)**(1/2)/x**5,x)
```

output
```
Timed out
```

3.16.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.17

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^5}\,dx = -\frac{4\,bx^3+a}{4\,x^4}$$

input
```
integrate(((b*x^3+a)^2)^(1/2)/x^5,x, algorithm="maxima")
```

output
```
-1/4*(4*b*x^3 + a)/x^4
```

3.16.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.39

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^5}\,dx = -\frac{4\,bx^3\mathrm{sgn}(bx^3+a)+a\mathrm{sgn}(bx^3+a)}{4\,x^4}$$

input
```
integrate(((b*x^3+a)^2)^(1/2)/x^5,x, algorithm="giac")
```

output
```
-1/4*(4*b*x^3*sgn(b*x^3 + a) + a*sgn(b*x^3 + a))/x^4
```

3.16.9 Mupad [B] (verification not implemented)

Time = 8.23 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.43

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^5}\,dx = -\frac{(4\,b\,x^3 + a)\,\sqrt{(b\,x^3 + a)^2}}{4\,x^4\,(b\,x^3 + a)}$$

input
```
int(((a + b*x^3)^2)^(1/2)/x^5,x)
```

output
```
-((a + 4*b*x^3)*((a + b*x^3)^2)^(1/2))/(4*x^4*(a + b*x^3))
```

3.17 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^6}\,dx$

3.17.1 Optimal result

Integrand size = 26, antiderivative size = 79

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^6}\,dx=-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}$$

output
```
-1/5*a*((b*x^3+a)^2)^(1/2)/x^5/(b*x^3+a)-1/2*b*((b*x^3+a)^2)^(1/2)/x^2/(b*
x^3+a)
```

3.17.2 Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.49

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^6}\,dx=-\frac{\sqrt{\left(a+bx^3\right)^2}\left(2a+5bx^3\right)}{10x^5\left(a+bx^3\right)}$$

input
```
Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^6,x]
```

output
```
-1/10*(Sqrt[(a + b*x^3)^2]*(2*a + 5*b*x^3))/(x^5*(a + b*x^3))
```

3.17.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.62, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^6}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b(bx^3+a)}{x^6}dx}{b(a+bx^3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{bx^3+a}{x^6}dx}{a+bx^3}$$

$$\Big\downarrow \text{802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a}{x^6}+\frac{b}{x^3}\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(-\frac{a}{5x^5}-\frac{b}{2x^2}\right)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

input
```
Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^6,x]
```

output
```
((-1/5*a/x^5 - b/(2*x^2))*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3)
```

3.17.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.17.4 Maple [A] (verified)

Time = 5.24 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.44

method	result	size
risch	$\frac{\left(-\frac{b\,x^{3}}{2}-\frac{a}{5}\right)\sqrt{\left(b\,x^{3}+a\right)^{2}}}{x^{5}\left(b\,x^{3}+a\right)}$	35
gosper	$-\frac{\left(5b\,x^{3}+2a\right)\sqrt{\left(b\,x^{3}+a\right)^{2}}}{10\left(b\,x^{3}+a\right)x^{5}}$	36
default	$-\frac{\left(5b\,x^{3}+2a\right)\sqrt{\left(b\,x^{3}+a\right)^{2}}}{10\left(b\,x^{3}+a\right)x^{5}}$	36

input
```
int(((b*x^3+a)^2)^(1/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
1/x^5*(-1/2*b*x^3-1/5*a)/(b*x^3+a)*((b*x^3+a)^2)^(1/2)
```

3.17.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.19

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^6}\,dx=-\frac{5\,bx^3+2\,a}{10\,x^5}$$

input
```
integrate(((b*x^3+a)^2)^(1/2)/x^6,x, algorithm="fricas")
```

output
```
-1/10*(5*b*x^3 + 2*a)/x^5
```

3.17.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^6}\,dx = \text{Timed out}$$

input
```
integrate(((b*x**3+a)**2)**(1/2)/x**6,x)
```

output
```
Timed out
```

3.17.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.19

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^6}\,dx = -\frac{5\,bx^3 + 2\,a}{10\,x^5}$$

input
```
integrate(((b*x^3+a)^2)^(1/2)/x^6,x, algorithm="maxima")
```

output
```
-1/10*(5*b*x^3 + 2*a)/x^5
```

3.17.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.39

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^6}\,dx = -\frac{5\,bx^3 \text{sgn}(bx^3 + a) + 2\,a \text{sgn}(bx^3 + a)}{10\,x^5}$$

input
```
integrate(((b*x^3+a)^2)^(1/2)/x^6,x, algorithm="giac")
```

output
```
-1/10*(5*b*x^3*sgn(b*x^3 + a) + 2*a*sgn(b*x^3 + a))/x^5
```

3.17.9 Mupad [B] (verification not implemented)

Time = 8.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.44

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^6}\,dx = -\frac{(5\,b\,x^3 + 2\,a)\,\sqrt{(b\,x^3 + a)^2}}{10\,x^5\,(b\,x^3 + a)}$$

input
```
int(((a + b*x^3)^2)^(1/2)/x^6,x)
```

output
```
-((2*a + 5*b*x^3)*((a + b*x^3)^2)^(1/2))/(10*x^5*(a + b*x^3))
```

3.18 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^7} dx$

3.18.1 Optimal result

Integrand size = 26, antiderivative size = 79

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^7} \, dx = -\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{6x^6(a+bx^3)} - \frac{b\sqrt{a^2+2abx^3+b^2x^6}}{3x^3(a+bx^3)}$$

output
```
-1/6*a*((b*x^3+a)^2)^(1/2)/x^6/(b*x^3+a)-1/3*b*((b*x^3+a)^2)^(1/2)/x^3/(b*
x^3+a)
```

3.18.2 Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.47

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^7} \, dx = -\frac{\sqrt{(a+bx^3)^2}(a+2bx^3)}{6x^6(a+bx^3)}$$

input
```
Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^7,x]
```

output
```
-1/6*(Sqrt[(a + b*x^3)^2]*(a + 2*b*x^3))/(x^6*(a + b*x^3))
```

3.18.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.49, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1693, 1102, 27, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^7} \, dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int \frac{\sqrt{b^2x^6+2abx^3+a^2}}{x^9}dx^3$$

$$\Big\downarrow \text{1102}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b(bx^3+a)}{x^9}dx^3}{3b\left(a+bx^3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{bx^3+a}{x^9}dx^3}{3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{48}$$

$$-\frac{\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}{6ax^6}$$

input
```
Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^7,x]
```

output
```
-1/6*((a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a*x^6)
```

3.18.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.18.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.28

method	result	size
pseudoelliptic	$-\frac{\operatorname{csgn}\left(b\,x^{3}+a\right)\left(2 b\,x^{3}+a\right)}{6 x^{6}}$	22
gosper	$-\frac{\left(2 b\,x^{3}+a\right)\sqrt{\left(b\,x^{3}+a\right)^{2}}}{6 x^{6} \left(b\,x^{3}+a\right)}$	34
default	$-\frac{\left(2 b\,x^{3}+a\right)\sqrt{\left(b\,x^{3}+a\right)^{2}}}{6 x^{6} \left(b\,x^{3}+a\right)}$	34
risch	$\frac{\left(-\frac{b\,x^{3}}{3}-\frac{a}{6}\right)\sqrt{\left(b\,x^{3}+a\right)^{2}}}{x^{6} \left(b\,x^{3}+a\right)}$	35

input
```
int(((b*x^3+a)^2)^(1/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*csgn(b*x^3+a)*(2*b*x^3+a)/x^6
```

3.18.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.16

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^7}\,dx = -\frac{2\,bx^3+a}{6\,x^6}$$

input
```
integrate(((b*x^3+a)^2)^(1/2)/x^7,x, algorithm="fricas")
```

output
```
-1/6*(2*b*x^3 + a)/x^6
```

3.18.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^7} \, dx = \text{Timed out}$$

input
```
integrate(((b*x**3+a)**2)**(1/2)/x**7,x)
```

output
```
Timed out
```

3.18.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^7} \, dx = \frac{\sqrt{b^2x^6 + 2\,abx^3 + a^2}b^2}{6\,a^2} + \frac{\sqrt{b^2x^6 + 2\,abx^3 + a^2}b}{6\,ax^3} - \frac{\left(b^2x^6 + 2\,abx^3 + a^2\right)^{\frac{3}{2}}}{6\,a^2x^6}$$

input
```
integrate(((b*x^3+a)^2)^(1/2)/x^7,x, algorithm="maxima")
```

output
```
1/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^2/a^2 + 1/6*sqrt(b^2*x^6 + 2*a*b*x^3
 + a^2)*b/(a*x^3) - 1/6*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/(a^2*x^6)
```

3.18.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.38

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^7} \, dx = -\frac{2\,bx^3\text{sgn}(bx^3 + a) + a\text{sgn}(bx^3 + a)}{6\,x^6}$$

input
```
integrate(((b*x^3+a)^2)^(1/2)/x^7,x, algorithm="giac")
```

output
```
-1/6*(2*b*x^3*sgn(b*x^3 + a) + a*sgn(b*x^3 + a))/x^6
```

3.18.9 Mupad [B] (verification not implemented)

Time = 8.17 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.42

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^7}\, dx = -\frac{(2\,b\,x^3 + a)\,\sqrt{(b\,x^3 + a)^2}}{6\,x^6\,(b\,x^3 + a)}$$

input
```
int(((a + b*x^3)^2)^(1/2)/x^7,x)
```

output
```
-((a + 2*b*x^3)*((a + b*x^3)^2)^(1/2))/(6*x^6*(a + b*x^3))
```

3.19 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^8}\,dx$

3.19.1 Optimal result

Integrand size = 26, antiderivative size = 79

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^8}\,dx=-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\,(a+bx^3)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\,(a+bx^3)}$$

output
```
-1/7*a*((b*x^3+a)^2)^(1/2)/x^7/(b*x^3+a)-1/4*b*((b*x^3+a)^2)^(1/2)/x^4/(b*
x^3+a)
```

3.19.2 Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.49

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^8}\,dx=-\frac{\sqrt{(a+bx^3)^2}(4a+7bx^3)}{28x^7\,(a+bx^3)}$$

input
```
Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^8,x]
```

output
```
-1/28*(Sqrt[(a + b*x^3)^2]*(4*a + 7*b*x^3))/(x^7*(a + b*x^3))
```

3.19.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.62, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^8}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b(bx^3+a)}{x^8}dx}{b\left(a+bx^3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{bx^3+a}{x^8}dx}{a+bx^3}$$

$$\Big\downarrow \text{802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a}{x^8}+\frac{b}{x^5}\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(-\frac{a}{7x^7}-\frac{b}{4x^4}\right)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

input
```
Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^8,x]
```

output
```
((-1/7*a/x^7 - b/(4*x^4))*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3)
```

3.19.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.19.4 Maple [A] (verified)

Time = 7.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.44

method	result	size
risch	$\frac{\left(-\frac{b\,x^{3}}{4}-\frac{a}{7}\right)\sqrt{\left(b\,x^{3}+a\right)^{2}}}{x^{7}\left(b\,x^{3}+a\right)}$	35
gosper	$-\frac{\left(7b\,x^{3}+4a\right)\sqrt{\left(b\,x^{3}+a\right)^{2}}}{28x^{7}\left(b\,x^{3}+a\right)}$	36
default	$-\frac{\left(7b\,x^{3}+4a\right)\sqrt{\left(b\,x^{3}+a\right)^{2}}}{28x^{7}\left(b\,x^{3}+a\right)}$	36

input
```
int(((b*x^3+a)^2)^(1/2)/x^8,x,method=_RETURNVERBOSE)
```

output
```
1/x^7*(-1/4*b*x^3-1/7*a)/(b*x^3+a)*((b*x^3+a)^2)^(1/2)
```

3.19.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.19

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^8}\,dx=-\frac{7\,bx^3+4\,a}{28\,x^7}$$

input
```
integrate(((b*x^3+a)^2)^(1/2)/x^8,x, algorithm="fricas")
```

output
```
-1/28*(7*b*x^3 + 4*a)/x^7
```

3.19.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^8} \, dx = \text{Timed out}$$

input
```
integrate(((b*x**3+a)**2)**(1/2)/x**8,x)
```

output
```
Timed out
```

3.19.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.19

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^8} \, dx = -\frac{7\,bx^3 + 4\,a}{28\,x^7}$$

input
```
integrate(((b*x^3+a)^2)^(1/2)/x^8,x, algorithm="maxima")
```

output
```
-1/28*(7*b*x^3 + 4*a)/x^7
```

3.19.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.39

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^8} \, dx = -\frac{7\,bx^3 \text{sgn}(bx^3 + a) + 4\,a\text{sgn}(bx^3 + a)}{28\,x^7}$$

input
```
integrate(((b*x^3+a)^2)^(1/2)/x^8,x, algorithm="giac")
```

output
```
-1/28*(7*b*x^3*sgn(b*x^3 + a) + 4*a*sgn(b*x^3 + a))/x^7
```

3.19.9 Mupad [B] (verification not implemented)

Time = 8.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.44

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^8}\,dx = -\frac{(7\,b\,x^3+4\,a)\,\sqrt{(b\,x^3+a)^2}}{28\,x^7\,(b\,x^3+a)}$$

input
```
int(((a + b*x^3)^2)^(1/2)/x^8,x)
```

output
```
-((4*a + 7*b*x^3)*((a + b*x^3)^2)^(1/2))/(28*x^7*(a + b*x^3))
```

3.20 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^9}\,dx$

3.20.1 Optimal result

Integrand size = 26, antiderivative size = 79

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^9}\,dx=-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{8x^8(a+bx^3)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{5x^5(a+bx^3)}$$

output
```
-1/8*a*((b*x^3+a)^2)^(1/2)/x^8/(b*x^3+a)-1/5*b*((b*x^3+a)^2)^(1/2)/x^5/(b*
x^3+a)
```

3.20.2 Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.49

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^9}\,dx=-\frac{\sqrt{(a+bx^3)^2}(5a+8bx^3)}{40x^8(a+bx^3)}$$

input
```
Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^9,x]
```

output
```
-1/40*(Sqrt[(a + b*x^3)^2]*(5*a + 8*b*x^3))/(x^8*(a + b*x^3))
```

3.20.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.62, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^9}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b(bx^3+a)}{x^9}dx}{b\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{bx^3+a}{x^9}dx}{a+bx^3}$$

$$\Big\downarrow \text{ 802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a}{x^9}+\frac{b}{x^6}\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\left(-\frac{a}{8x^8}-\frac{b}{5x^5}\right)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

input
```
Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^9,x]
```

output
```
((-1/8*a/x^8 - b/(5*x^5))*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3)
```

3.20.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.20.4 Maple [A] (verified)

Time = 8.96 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.44

method	result	size
risch	$\frac{\left(-\frac{b\,x^{3}}{5}-\frac{a}{8}\right)\sqrt{\left(b\,x^{3}+a\right)^{2}}}{x^{8}\left(b\,x^{3}+a\right)}$	35
gosper	$-\frac{\left(8b\,x^{3}+5a\right)\sqrt{\left(b\,x^{3}+a\right)^{2}}}{40x^{8}\left(b\,x^{3}+a\right)}$	36
default	$-\frac{\left(8b\,x^{3}+5a\right)\sqrt{\left(b\,x^{3}+a\right)^{2}}}{40x^{8}\left(b\,x^{3}+a\right)}$	36

input
```
int(((b*x^3+a)^2)^(1/2)/x^9,x,method=_RETURNVERBOSE)
```

output
```
1/x^8*(-1/5*b*x^3-1/8*a)/(b*x^3+a)*((b*x^3+a)^2)^(1/2)
```

3.20.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.19

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^9}\,dx = -\frac{8\,bx^3+5\,a}{40\,x^8}$$

input
```
integrate(((b*x^3+a)^2)^(1/2)/x^9,x, algorithm="fricas")
```

output
```
-1/40*(8*b*x^3 + 5*a)/x^8
```

3.20.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^9}\,dx = \text{Timed out}$$

input
```
integrate(((b*x**3+a)**2)**(1/2)/x**9,x)
```

output
```
Timed out
```

3.20.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.19

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^9}\,dx = -\frac{8\,bx^3+5\,a}{40\,x^8}$$

input
```
integrate(((b*x^3+a)^2)^(1/2)/x^9,x, algorithm="maxima")
```

output
```
-1/40*(8*b*x^3 + 5*a)/x^8
```

3.20.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.39

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^9}\,dx = -\frac{8\,bx^3\text{sgn}(bx^3+a)+5\,a\text{sgn}(bx^3+a)}{40\,x^8}$$

input
```
integrate(((b*x^3+a)^2)^(1/2)/x^9,x, algorithm="giac")
```

output
```
-1/40*(8*b*x^3*sgn(b*x^3 + a) + 5*a*sgn(b*x^3 + a))/x^8
```

3.20.9 Mupad [B] (verification not implemented)

Time = 8.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.44

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^9}\,dx=-\frac{\left(8\,b\,x^3+5\,a\right)\,\sqrt{{\left(b\,x^3+a\right)}^2}}{40\,x^8\,\left(b\,x^3+a\right)}$$

input
```
int(((a + b*x^3)^2)^(1/2)/x^9,x)
```

output
```
-((5*a + 8*b*x^3)*((a + b*x^3)^2)^(1/2))/(40*x^8*(a + b*x^3))
```

3.21 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^{10}}\,dx$

3.21.1 Optimal result

Integrand size = 26, antiderivative size = 79

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^{10}}\,dx=-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{9x^9\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{6x^6\left(a+bx^3\right)}$$

output
```
-1/9*a*((b*x^3+a)^2)^(1/2)/x^9/(b*x^3+a)-1/6*b*((b*x^3+a)^2)^(1/2)/x^6/(b*
x^3+a)
```

3.21.2 Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.49

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^{10}}\,dx=-\frac{\sqrt{\left(a+bx^3\right)^2}\left(2a+3bx^3\right)}{18x^9\left(a+bx^3\right)}$$

input
```
Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^10,x]
```

output
```
-1/18*(Sqrt[(a + b*x^3)^2]*(2*a + 3*b*x^3))/(x^9*(a + b*x^3))
```

3.21.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.62, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^{10}}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b(bx^3+a)}{x^{10}}dx}{b\left(a+bx^3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{bx^3+a}{x^{10}}dx}{a+bx^3}$$

$$\Big\downarrow \text{802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a}{x^{10}}+\frac{b}{x^7}\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(-\frac{a}{9x^9}-\frac{b}{6x^6}\right)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

input
```
Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^10,x]
```

output
```
((-1/9*a/x^9 - b/(6*x^6))*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3)
```

3.21.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.21.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.30

method	result	size
pseudoelliptic	$-\frac{\operatorname{csgn}\left(b\,x^{3}+a\right)\left(3 b\,x^{3}+2 a\right)}{18 x^{9}}$	24
risch	$\frac{\left(-\frac{b\,x^{3}}{6}-\frac{a}{9}\right)\sqrt{\left(b\,x^{3}+a\right)^{2}}}{x^{9}\left(b\,x^{3}+a\right)}$	35
gosper	$-\frac{\left(3 b\,x^{3}+2 a\right)\sqrt{\left(b\,x^{3}+a\right)^{2}}}{18 x^{9}\left(b\,x^{3}+a\right)}$	36
default	$-\frac{\left(3 b\,x^{3}+2 a\right)\sqrt{\left(b\,x^{3}+a\right)^{2}}}{18 x^{9}\left(b\,x^{3}+a\right)}$	36

input
```
int(((b*x^3+a)^2)^(1/2)/x^10,x,method=_RETURNVERBOSE)
```

output
```
-1/18*csgn(b*x^3+a)*(3*b*x^3+2*a)/x^9
```

3.21.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.19

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^{10}}\,dx = -\frac{3\,bx^3 + 2\,a}{18\,x^9}$$

input
```
integrate(((b*x^3+a)^2)^(1/2)/x^10,x, algorithm="fricas")
```

output
```
-1/18*(3*b*x^3 + 2*a)/x^9
```

3.21.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^{10}}\,dx = \text{Timed out}$$

input
```
integrate(((b*x**3+a)**2)**(1/2)/x**10,x)
```

output
```
Timed out
```

3.21.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(53) = 106$.

Time = 0.22 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.48

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^{10}}\,dx = -\frac{\sqrt{b^2x^6+2\,abx^3+a^2}b^3}{6\,a^3} - \frac{\sqrt{b^2x^6+2\,abx^3+a^2}b^2}{6\,a^2x^3} + \frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{3}{2}}b}{6\,a^3x^6} - \frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{3}{2}}}{9\,a^2x^9}$$

input
```
integrate(((b*x^3+a)^2)^(1/2)/x^10,x, algorithm="maxima")
```

output
```
-1/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^3/a^3 - 1/6*sqrt(b^2*x^6 + 2*a*b*x^
3 + a^2)*b^2/(a^2*x^3) + 1/6*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*b/(a^3*x^6)
 - 1/9*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/(a^2*x^9)
```

3.21.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.39

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^{10}}\,dx = -\frac{3\,bx^3\text{sgn}\left(bx^3+a\right)+2\,a\text{sgn}\left(bx^3+a\right)}{18\,x^9}$$

input
```
integrate(((b*x^3+a)^2)^(1/2)/x^10,x, algorithm="giac")
```

output
```
-1/18*(3*b*x^3*sgn(b*x^3 + a) + 2*a*sgn(b*x^3 + a))/x^9
```

3.21.9 Mupad [B] (verification not implemented)

Time = 8.14 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.44

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^{10}}\,dx = -\frac{(3\,b\,x^3 + 2\,a)\,\sqrt{(b\,x^3 + a)^2}}{18\,x^9\,(b\,x^3 + a)}$$

input
```
int(((a + b*x^3)^2)^(1/2)/x^10,x)
```

output
```
-((2*a + 3*b*x^3)*((a + b*x^3)^2)^(1/2))/(18*x^9*(a + b*x^3))
```

3.22 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^{11}}\,dx$

3.22.1 Optimal result

Integrand size = 26, antiderivative size = 79

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^{11}}\,dx = -\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}$$

output
```
-1/10*a*((b*x^3+a)^2)^(1/2)/x^10/(b*x^3+a)-1/7*b*((b*x^3+a)^2)^(1/2)/x^7/(
b*x^3+a)
```

3.22.2 Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.49

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^{11}}\,dx = -\frac{\sqrt{\left(a+bx^3\right)^2}\left(7a+10bx^3\right)}{70x^{10}\left(a+bx^3\right)}$$

input
```
Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^11,x]
```

output
```
-1/70*(Sqrt[(a + b*x^3)^2]*(7*a + 10*b*x^3))/(x^10*(a + b*x^3))
```

3.22.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.62, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^{11}}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b(bx^3+a)}{x^{11}}dx}{b\,(a+bx^3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{bx^3+a}{x^{11}}dx}{a+bx^3}$$

$$\Big\downarrow \text{802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a}{x^{11}}+\frac{b}{x^8}\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(-\frac{a}{10x^{10}}-\frac{b}{7x^7}\right)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

input
```
Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^11,x]
```

output
```
((-1/10*a/x^10 - b/(7*x^7))*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3)
```

3.22.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.22.4 Maple [A] (verified)

Time = 11.83 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.44

method	result	size
risch	$\frac{\left(-\frac{b\,x^{3}}{7}-\frac{a}{10}\right)\sqrt{\left(b\,x^{3}+a\right)^{2}}}{x^{10}\left(b\,x^{3}+a\right)}$	35
gosper	$-\frac{\left(10 b\,x^{3}+7 a\right)\sqrt{\left(b\,x^{3}+a\right)^{2}}}{70 x^{10}\left(b\,x^{3}+a\right)}$	36
default	$-\frac{\left(10 b\,x^{3}+7 a\right)\sqrt{\left(b\,x^{3}+a\right)^{2}}}{70 x^{10}\left(b\,x^{3}+a\right)}$	36

input
```
int(((b*x^3+a)^2)^(1/2)/x^11,x,method=_RETURNVERBOSE)
```

output
```
1/x^10*(-1/7*b*x^3-1/10*a)/(b*x^3+a)*((b*x^3+a)^2)^(1/2)
```

3.22.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.19

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^{11}}\,dx=-\frac{10\,bx^3+7\,a}{70\,x^{10}}$$

input
```
integrate(((b*x^3+a)^2)^(1/2)/x^11,x, algorithm="fricas")
```

output
```
-1/70*(10*b*x^3 + 7*a)/x^10
```

3.22.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^{11}}\,dx = \text{Timed out}$$

input
```
integrate(((b*x**3+a)**2)**(1/2)/x**11,x)
```

output
```
Timed out
```

3.22.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.19

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^{11}}\,dx = -\frac{10\,bx^3+7\,a}{70\,x^{10}}$$

input
```
integrate(((b*x^3+a)^2)^(1/2)/x^11,x, algorithm="maxima")
```

output
```
-1/70*(10*b*x^3 + 7*a)/x^10
```

3.22.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.39

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^{11}}\,dx = -\frac{10\,bx^3\text{sgn}(bx^3+a)+7\,a\text{sgn}(bx^3+a)}{70\,x^{10}}$$

input
```
integrate(((b*x^3+a)^2)^(1/2)/x^11,x, algorithm="giac")
```

output
```
-1/70*(10*b*x^3*sgn(b*x^3 + a) + 7*a*sgn(b*x^3 + a))/x^10
```

3.22.9 Mupad [B] (verification not implemented)

Time = 8.21 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.44

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^{11}}\,dx = -\frac{(10\,b\,x^3+7\,a)\,\sqrt{(b\,x^3+a)^2}}{70\,x^{10}\,(b\,x^3+a)}$$

input
```
int(((a + b*x^3)^2)^(1/2)/x^11,x)
```

output
```
-((7*a + 10*b*x^3)*((a + b*x^3)^2)^(1/2))/(70*x^10*(a + b*x^3))
```

3.23 $\int x^9(a^2 + 2abx^3 + b^2x^6)^{3/2}\, dx$

3.23.1 Optimal result

Integrand size = 26, antiderivative size = 167

$$\int x^9 \left(a^2+2abx^3+b^2x^6\right)^{3/2}\, dx = \frac{a^3x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10\left(a+bx^3\right)} + \frac{3a^2bx^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)} + \frac{3ab^2x^{16}\sqrt{a^2+2abx^3+b^2x^6}}{16\left(a+bx^3\right)} + \frac{b^3x^{19}\sqrt{a^2+2abx^3+b^2x^6}}{19\left(a+bx^3\right)}$$

output
```
1/10*a^3*x^10*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+3/13*a^2*b*x^13*((b*x^3+a)^2)^
(1/2)/(b*x^3+a)+3/16*a*b^2*x^16*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/19*b^3*x^1
9*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

3.23.2 Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.37

$$\int x^9 \left(a^2+2abx^3+b^2x^6\right)^{3/2}\, dx = \frac{x^{10}\sqrt{\left(a+bx^3\right)^2}\left(1976a^3+4560a^2bx^3+3705ab^2x^6+1040b^3x^9\right)}{19760\left(a+bx^3\right)}$$

input
```
Integrate[x^9*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

output
```
(x^10*Sqrt[(a + b*x^3)^2]*(1976*a^3 + 4560*a^2*b*x^3 + 3705*a*b^2*x^6 + 10
40*b^3*x^9))/(19760*(a + b*x^3))
```

3.23.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.45, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^9 \left(a^2+2abx^3+b^2x^6\right)^{3/2} \, dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int b^3x^9\left(bx^3+a\right)^3dx}{b^3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^9\left(bx^3+a\right)^3dx}{a+bx^3}$$

$$\Big\downarrow \text{ 802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(b^3x^{18}+3ab^2x^{15}+3a^2bx^{12}+a^3x^9\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(\frac{a^3x^{10}}{10}+\frac{3}{13}a^2bx^{13}+\frac{3}{16}ab^2x^{16}+\frac{b^3x^{19}}{19}\right)}{a+bx^3}$$

input
```
Int[x^9*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*((a^3*x^10)/10 + (3*a^2*b*x^13)/13 + (3*a
*b^2*x^16)/16 + (b^3*x^19)/19))/(a + b*x^3)
```

3.23.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.23.4 Maple [A] (verified)

Time = 8.14 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.35

method	result	size
gosper	$\frac{x^{10}\left(1040 b^{3} x^{9}+3705 b^{2} x^{6} a+4560 a^{2} b \,x^{3}+1976 a^{3}\right) \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{3}{2}}}{19760 \left(b \,x^{3}+a \right)^{3}}$	58
default	$\frac{x^{10}\left(1040 b^{3} x^{9}+3705 b^{2} x^{6} a+4560 a^{2} b \,x^{3}+1976 a^{3}\right) \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{3}{2}}}{19760 \left(b \,x^{3}+a \right)^{3}}$	58
risch	$\frac{a^{3} x^{10} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{10 b \,x^{3}+10 a}+\frac{3 a^{2} b \,x^{13} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{13 \left(b \,x^{3}+a \right)}+\frac{3 a \,b^{2} x^{16} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{16 \left(b \,x^{3}+a \right)}+\frac{b^{3} x^{19} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{19 b \,x^{3}+19 a}$	116

input
```
int(x^9*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/19760*x^10*(1040*b^3*x^9+3705*a*b^2*x^6+4560*a^2*b*x^3+1976*a^3)*((b*x^3
+a)^2)^(3/2)/(b*x^3+a)^3
```

3.23.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.21

$$\int x^9 \left(a^2 + 2abx^3 + b^2x^6\right)^{3/2} \, dx = \frac{1}{19} b^3 x^{19} + \frac{3}{16} a b^2 x^{16} + \frac{3}{13} a^2 b x^{13} + \frac{1}{10} a^3 x^{10}$$

input
```
integrate(x^9*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

output
```
1/19*b^3*x^19 + 3/16*a*b^2*x^16 + 3/13*a^2*b*x^13 + 1/10*a^3*x^10
```

3.23.6 Sympy [F]

$$\int x^9 \left(a^2 + 2abx^3 + b^2x^6\right)^{3/2} \, dx = \int x^9 \left(\left(a + bx^3\right)^2\right)^{\frac{3}{2}} \, dx$$

input
```
integrate(x**9*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

output
```
Integral(x**9*((a + b*x**3)**2)**(3/2), x)
```

3.23.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.21

$$\int x^9 \left(a^2 + 2abx^3 + b^2x^6\right)^{3/2} \, dx = \frac{1}{19} b^3 x^{19} + \frac{3}{16} a b^2 x^{16} + \frac{3}{13} a^2 b x^{13} + \frac{1}{10} a^3 x^{10}$$

input
```
integrate(x^9*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

output
```
1/19*b^3*x^19 + 3/16*a*b^2*x^16 + 3/13*a^2*b*x^13 + 1/10*a^3*x^10
```

3.23.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.40

$$\int x^9 \left(a^2+2 a b x^3+b^2 x^6\right)^{3/2} \, dx = \frac{1}{19} b^3 x^{19} \operatorname{sgn}\left(b x^3+a\right) + \frac{3}{16} a b^2 x^{16} \operatorname{sgn}\left(b x^3+a\right) + \frac{3}{13} a^2 b x^{13} \operatorname{sgn}\left(b x^3+a\right) + \frac{1}{10} a^3 x^{10} \operatorname{sgn}\left(b x^3+a\right)$$

input
```
integrate(x^9*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

output
```
1/19*b^3*x^19*sgn(b*x^3 + a) + 3/16*a*b^2*x^16*sgn(b*x^3 + a) + 3/13*a^2*b
*x^13*sgn(b*x^3 + a) + 1/10*a^3*x^10*sgn(b*x^3 + a)
```

3.23.9 Mupad [F(-1)]

Timed out.

$$\int x^9 \left(a^2+2 a b x^3+b^2 x^6\right)^{3/2} \, dx = \int x^9 \,{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{3/2} \,d x$$

input
```
int(x^9*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

output
```
int(x^9*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

3.24 $\int x^8(a^2+2abx^3+b^2x^6)^{3/2}\,dx$

3.24.1 Optimal result

Integrand size = 26, antiderivative size = 119

$$\int x^8\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{a^2(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}}{12b^3}-\frac{2a(a+bx^3)^4\sqrt{a^2+2abx^3+b^2x^6}}{15b^3}+\frac{(a+bx^3)^5\sqrt{a^2+2abx^3+b^2x^6}}{18b^3}$$

output
```
1/12*a^2*(b*x^3+a)^3*((b*x^3+a)^2)^(1/2)/b^3-2/15*a*(b*x^3+a)^4*((b*x^3+a)
^2)^(1/2)/b^3+1/18*(b*x^3+a)^5*((b*x^3+a)^2)^(1/2)/b^3
```

3.24.2 Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.95

$$\int x^8\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{x^9(20a^3+45a^2bx^3+36ab^2x^6+10b^3x^9)\left(\sqrt{a^2}bx^3+a\left(\sqrt{a^2}-\sqrt{(a+bx^3)^2}\right)\right)}{180\left(-a^2-abx^3+\sqrt{a^2}\sqrt{(a+bx^3)^2}\right)}$$

input
```
Integrate[x^8*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

output
```
(x^9*(20*a^3 + 45*a^2*b*x^3 + 36*a*b^2*x^6 + 10*b^3*x^9)*(Sqrt[a^2]*b*x^3
+ a*(Sqrt[a^2] - Sqrt[(a + b*x^3)^2])))/(180*(-a^2 - a*b*x^3 + Sqrt[a^2]*S
qrt[(a + b*x^3)^2]))
```

3.24.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.66, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1384, 27, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8 \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int b^3x^8\left(bx^3+a\right)^3dx}{b^3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^8\left(bx^3+a\right)^3dx}{a+bx^3}$$

$$\Big\downarrow \text{ 798}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^6\left(bx^3+a\right)^3dx^3}{3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(b^3x^{15}+3ab^2x^{12}+3a^2bx^9+a^3x^6\right)dx^3}{3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(\frac{a^3x^9}{3}+\frac{3}{4}a^2bx^{12}+\frac{3}{5}ab^2x^{15}+\frac{b^3x^{18}}{6}\right)}{3\left(a+bx^3\right)}$$

input
```
Int[x^8*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*((a^3*x^9)/3 + (3*a^2*b*x^12)/4 + (3*a*b^
2*x^15)/5 + (b^3*x^18)/6))/(3*(a + b*x^3))
```

3.24.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.24.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.35

method	result	size
pseudoelliptic	$\frac{\operatorname{csgn}\left(b\,x^{3}+a\right)\left(b\,x^{3}+a\right)^{4}\left(10b^{2}x^{6}-4ab\,x^{3}+a^{2}\right)}{180b^{3}}$	42
gosper	$\frac{x^{9}\left(10b^{3}x^{9}+36b^{2}x^{6}a+45a^{2}b\,x^{3}+20a^{3}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{180\left(b\,x^{3}+a\right)^{3}}$	58
default	$\frac{x^{9}\left(10b^{3}x^{9}+36b^{2}x^{6}a+45a^{2}b\,x^{3}+20a^{3}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{180\left(b\,x^{3}+a\right)^{3}}$	58
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,a^{3}x^{9}}{9b\,x^{3}+9a}+\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,a^{2}b\,x^{12}}{4b\,x^{3}+4a}+\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,b^{2}a\,x^{15}}{5b\,x^{3}+5a}+\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,b^{3}x^{18}}{18b\,x^{3}+18a}$	116

input
```
int(x^8*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/180*csgn(b*x^3+a)*(b*x^3+a)^4*(10*b^2*x^6-4*a*b*x^3+a^2)/b^3
```

3.24.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.29

$$\int x^8 (a^2 + 2abx^3 + b^2x^6)^{3/2} \, dx = \frac{1}{18} b^3x^{18} + \frac{1}{5} ab^2x^{15} + \frac{1}{4} a^2bx^{12} + \frac{1}{9} a^3x^9$$

input
```
integrate(x^8*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

output
```
1/18*b^3*x^18 + 1/5*a*b^2*x^15 + 1/4*a^2*b*x^12 + 1/9*a^3*x^9
```

3.24.6 Sympy [F]

$$\int x^8 (a^2 + 2abx^3 + b^2x^6)^{3/2} \, dx = \int x^8 \left((a + bx^3)^2 \right)^{\frac{3}{2}} \, dx$$

input
```
integrate(x**8*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

output
```
Integral(x**8*((a + b*x**3)**2)**(3/2), x)
```

3.24.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.96

$$\int x^8 (a^2 + 2abx^3 + b^2x^6)^{3/2} \, dx = \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{3}{2}} a^2x^3}{12\,b^2} + \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{5}{2}} x^3}{18\,b^2} + \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{3}{2}} a^3}{12\,b^3} - \frac{7\,(b^2x^6 + 2\,abx^3 + a^2)^{\frac{5}{2}} a}{90\,b^3}$$

input
```
integrate(x^8*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

output
```
1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*a^2*x^3/b^2 + 1/18*(b^2*x^6 + 2*a*b
*x^3 + a^2)^(5/2)*x^3/b^2 + 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*a^3/b^3
 - 7/90*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*a/b^3
```

3.24.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.56

$$\int x^8 \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx = \frac{1}{18}b^3x^{18}\text{sgn}\left(bx^3+a\right) + \frac{1}{5}ab^2x^{15}\text{sgn}\left(bx^3+a\right) + \frac{1}{4}a^2bx^{12}\text{sgn}\left(bx^3+a\right) + \frac{1}{9}a^3x^9\text{sgn}\left(bx^3+a\right)$$

input
```
integrate(x^8*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

output
```
1/18*b^3*x^18*sgn(b*x^3 + a) + 1/5*a*b^2*x^15*sgn(b*x^3 + a) + 1/4*a^2*b*x
^12*sgn(b*x^3 + a) + 1/9*a^3*x^9*sgn(b*x^3 + a)
```

3.24.9 Mupad [F(-1)]

Timed out.

$$\int x^8 \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx = \int x^8 \left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{3/2} dx$$

input
```
int(x^8*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

output
```
int(x^8*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

3.25 $\int x^7(a^2+2abx^3+b^2x^6)^{3/2}\,dx$

3.25.1 Optimal result

Integrand size = 26, antiderivative size = 167

$$\int x^7\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{a^3x^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{3a^2bx^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{3ab^2x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14\left(a+bx^3\right)}+\frac{b^3x^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17\left(a+bx^3\right)}$$

output
```
1/8*a^3*x^8*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+3/11*a^2*b*x^11*((b*x^3+a)^2)^(1
/2)/(b*x^3+a)+3/14*a*b^2*x^14*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/17*b^3*x^17*
((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

3.25.2 Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.37

$$\int x^7\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{x^8\sqrt{(a+bx^3)^2}(1309a^3+2856a^2bx^3+2244ab^2x^6+616b^3x^9)}{10472\left(a+bx^3\right)}$$

input
```
Integrate[x^7*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

output
```
(x^8*Sqrt[(a + b*x^3)^2]*(1309*a^3 + 2856*a^2*b*x^3 + 2244*a*b^2*x^6 + 616
*b^3*x^9))/(10472*(a + b*x^3))
```

3.25.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.45, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7 \left(a^2+2abx^3+b^2x^6\right)^{3/2} \, dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6} \int b^3x^7 \left(bx^3+a\right)^3 dx}{b^3 \left(a+bx^3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6} \int x^7 \left(bx^3+a\right)^3 dx}{a+bx^3}$$

$$\Big\downarrow \text{802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6} \int \left(b^3x^{16}+3ab^2x^{13}+3a^2bx^{10}+a^3x^7\right) dx}{a+bx^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6} \left(\frac{a^3x^8}{8}+\frac{3}{11}a^2bx^{11}+\frac{3}{14}ab^2x^{14}+\frac{b^3x^{17}}{17}\right)}{a+bx^3}$$

input
```
Int[x^7*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*((a^3*x^8)/8 + (3*a^2*b*x^11)/11 + (3*a*b
^2*x^14)/14 + (b^3*x^17)/17))/(a + b*x^3)
```

3.25.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.25.4 Maple [A] (verified)

Time = 6.20 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.35

method	result	size
gosper	$\frac{x^{8}\left(616 b^{3} x^{9}+2244 b^{2} x^{6} a+2856 a^{2} b \,x^{3}+1309 a^{3}\right) \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{3}{2}}}{10472 \left(b \,x^{3}+a \right)^{3}}$	58
default	$\frac{x^{8}\left(616 b^{3} x^{9}+2244 b^{2} x^{6} a+2856 a^{2} b \,x^{3}+1309 a^{3}\right) \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{3}{2}}}{10472 \left(b \,x^{3}+a \right)^{3}}$	58
risch	$\frac{a^{3} x^{8} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{8 b \,x^{3}+8 a}+\frac{3 a^{2} b \,x^{11} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{11 \left(b \,x^{3}+a \right)}+\frac{3 a \,b^{2} x^{14} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{14 \left(b \,x^{3}+a \right)}+\frac{b^{3} x^{17} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{17 b \,x^{3}+17 a}$	116

input
```
int(x^7*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/10472*x^8*(616*b^3*x^9+2244*a*b^2*x^6+2856*a^2*b*x^3+1309*a^3)*((b*x^3+a
)^2)^(3/2)/(b*x^3+a)^3
```

3.25.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.21

$$\int x^7 \left(a^2+2abx^3+b^2x^6\right)^{3/2} \, dx = \frac{1}{17}\,b^3x^{17}+\frac{3}{14}\,ab^2x^{14}+\frac{3}{11}\,a^2bx^{11}+\frac{1}{8}\,a^3x^8$$

input
```
integrate(x^7*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

output
```
1/17*b^3*x^17 + 3/14*a*b^2*x^14 + 3/11*a^2*b*x^11 + 1/8*a^3*x^8
```

3.25.6 Sympy [F]

$$\int x^7 \left(a^2+2abx^3+b^2x^6\right)^{3/2} \, dx = \int x^7 \left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}} \, dx$$

input
```
integrate(x**7*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

output
```
Integral(x**7*((a + b*x**3)**2)**(3/2), x)
```

3.25.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.21

$$\int x^7 \left(a^2+2abx^3+b^2x^6\right)^{3/2} \, dx = \frac{1}{17}\,b^3x^{17}+\frac{3}{14}\,ab^2x^{14}+\frac{3}{11}\,a^2bx^{11}+\frac{1}{8}\,a^3x^8$$

input
```
integrate(x^7*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

output
```
1/17*b^3*x^17 + 3/14*a*b^2*x^14 + 3/11*a^2*b*x^11 + 1/8*a^3*x^8
```

3.25.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.40

$$\int x^7\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{1}{17}b^3x^{17}\mathrm{sgn}\left(bx^3+a\right)+\frac{3}{14}ab^2x^{14}\mathrm{sgn}\left(bx^3+a\right)+\frac{3}{11}a^2bx^{11}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{8}a^3x^8\mathrm{sgn}\left(bx^3+a\right)$$

input
```
integrate(x^7*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

output
```
1/17*b^3*x^17*sgn(b*x^3 + a) + 3/14*a*b^2*x^14*sgn(b*x^3 + a) + 3/11*a^2*b
*x^11*sgn(b*x^3 + a) + 1/8*a^3*x^8*sgn(b*x^3 + a)
```

3.25.9 Mupad [F(-1)]

Timed out.

$$\int x^7\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\int x^7\,\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{3/2}\,dx$$

input
```
int(x^7*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

output
```
int(x^7*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

3.26 $\int x^6(a^2+2abx^3+b^2x^6)^{3/2}\,dx$

3.26.1 Optimal result

Integrand size = 26, antiderivative size = 167

$$\int x^6\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{a^3x^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{3a^2bx^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10\left(a+bx^3\right)}+\frac{3ab^2x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}+\frac{b^3x^{16}\sqrt{a^2+2abx^3+b^2x^6}}{16\left(a+bx^3\right)}$$

output
```
1/7*a^3*x^7*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+3/10*a^2*b*x^10*((b*x^3+a)^2)^(1
/2)/(b*x^3+a)+3/13*a*b^2*x^13*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/16*b^3*x^16*
((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

3.26.2 Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.37

$$\int x^6\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{x^7\sqrt{(a+bx^3)^2}(1040a^3+2184a^2bx^3+1680ab^2x^6+455b^3x^9)}{7280\left(a+bx^3\right)}$$

input
```
Integrate[x^6*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

output
```
(x^7*Sqrt[(a + b*x^3)^2]*(1040*a^3 + 2184*a^2*b*x^3 + 1680*a*b^2*x^6 + 455
*b^3*x^9))/(7280*(a + b*x^3))
```

3.26.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.45, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6 \left(a^2+2abx^3+b^2x^6\right)^{3/2} \, dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6} \int b^3x^6\left(bx^3+a\right)^3dx}{b^3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6} \int x^6\left(bx^3+a\right)^3dx}{a+bx^3}$$

$$\Big\downarrow \text{ 802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6} \int \left(b^3x^{15}+3ab^2x^{12}+3a^2bx^9+a^3x^6\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(\frac{a^3x^7}{7}+\frac{3}{10}a^2bx^{10}+\frac{3}{13}ab^2x^{13}+\frac{b^3x^{16}}{16}\right)}{a+bx^3}$$

input
```
Int[x^6*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*((a^3*x^7)/7 + (3*a^2*b*x^10)/10 + (3*a*b
^2*x^13)/13 + (b^3*x^16)/16))/(a + b*x^3)
```

3.26.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.26.4 Maple [A] (verified)

Time = 5.34 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.35

method	result	size
gosper	$\frac{x^{7} \left(455 b^{3} x^{9}+1680 b^{2} x^{6} a+2184 a^{2} b \,x^{3}+1040 a^{3}\right) \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{3}{2}}}{7280 \left(b \,x^{3}+a \right)^{3}}$	58
default	$\frac{x^{7} \left(455 b^{3} x^{9}+1680 b^{2} x^{6} a+2184 a^{2} b \,x^{3}+1040 a^{3}\right) \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{3}{2}}}{7280 \left(b \,x^{3}+a \right)^{3}}$	58
risch	$\frac{a^{3} x^{7} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{7 b \,x^{3}+7 a}+\frac{3 a^{2} b \,x^{10} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{10 \left(b \,x^{3}+a \right)}+\frac{3 a \,b^{2} x^{13} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{13 \left(b \,x^{3}+a \right)}+\frac{b^{3} x^{16} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{16 b \,x^{3}+16 a}$	116

input
```
int(x^6*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/7280*x^7*(455*b^3*x^9+1680*a*b^2*x^6+2184*a^2*b*x^3+1040*a^3)*((b*x^3+a)
^2)^(3/2)/(b*x^3+a)^3
```

3.26.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.21

$$\int x^6 \left(a^2+2abx^3+b^2x^6\right)^{3/2} \, dx = \frac{1}{16} b^3 x^{16} + \frac{3}{13} a b^2 x^{13} + \frac{3}{10} a^2 b x^{10} + \frac{1}{7} a^3 x^7$$

input
```
integrate(x^6*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

output
```
1/16*b^3*x^16 + 3/13*a*b^2*x^13 + 3/10*a^2*b*x^10 + 1/7*a^3*x^7
```

3.26.6 Sympy [F]

$$\int x^6 \left(a^2+2abx^3+b^2x^6\right)^{3/2} \, dx = \int x^6 \left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}} \, dx$$

input
```
integrate(x**6*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

output
```
Integral(x**6*((a + b*x**3)**2)**(3/2), x)
```

3.26.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.21

$$\int x^6 \left(a^2+2abx^3+b^2x^6\right)^{3/2} \, dx = \frac{1}{16} b^3 x^{16} + \frac{3}{13} a b^2 x^{13} + \frac{3}{10} a^2 b x^{10} + \frac{1}{7} a^3 x^7$$

input
```
integrate(x^6*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

output
```
1/16*b^3*x^16 + 3/13*a*b^2*x^13 + 3/10*a^2*b*x^10 + 1/7*a^3*x^7
```

3.26.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.40

$$\int x^6 (a^2 + 2abx^3 + b^2x^6)^{3/2} \, dx = \frac{1}{16} b^3 x^{16} \mathrm{sgn}(bx^3 + a) + \frac{3}{13} ab^2 x^{13} \mathrm{sgn}(bx^3 + a) + \frac{3}{10} a^2 b x^{10} \mathrm{sgn}(bx^3 + a) + \frac{1}{7} a^3 x^7 \mathrm{sgn}(bx^3 + a)$$

input
```
integrate(x^6*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

output
```
1/16*b^3*x^16*sgn(b*x^3 + a) + 3/13*a*b^2*x^13*sgn(b*x^3 + a) + 3/10*a^2*b
*x^10*sgn(b*x^3 + a) + 1/7*a^3*x^7*sgn(b*x^3 + a)
```

3.26.9 Mupad [F(-1)]

Timed out.

$$\int x^6 (a^2 + 2abx^3 + b^2x^6)^{3/2} \, dx = \int x^6 \left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^{3/2} dx$$

input
```
int(x^6*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

output
```
int(x^6*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

3.27 $\int x^5(a^2+2abx^3+b^2x^6)^{3/2}\,dx$

3.27.1 Optimal result

Integrand size = 26, antiderivative size = 78

$$\int x^5\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=-\frac{a(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}}{12b^2}+\frac{(a+bx^3)^4\sqrt{a^2+2abx^3+b^2x^6}}{15b^2}$$

output
```
-1/12*a*(b*x^3+a)^3*((b*x^3+a)^2)^(1/2)/b^2+1/15*(b*x^3+a)^4*((b*x^3+a)^2)
^(1/2)/b^2
```

3.27.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.45

$$\int x^5\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{x^6(10a^3+20a^2bx^3+15ab^2x^6+4b^3x^9)\left(\sqrt{a^2}bx^3+a\left(\sqrt{a^2}-\sqrt{(a+bx^3)^2}\right)\right)}{60\left(-a^2-abx^3+\sqrt{a^2}\sqrt{(a+bx^3)^2}\right)}$$

input
```
Integrate[x^5*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

output
```
(x^6*(10*a^3 + 20*a^2*b*x^3 + 15*a*b^2*x^6 + 4*b^3*x^9)*(Sqrt[a^2]*b*x^3 +
 a*(Sqrt[a^2] - Sqrt[(a + b*x^3)^2])))/(60*(-a^2 - a*b*x^3 + Sqrt[a^2]*Sqr
t[(a + b*x^3)^2]))
```

3.27.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1693, 1100, 1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx$$

$$\Big\downarrow \text{ 1693}$$

$$\frac{1}{3}\int x^3 \left(b^2x^6+2abx^3+a^2\right)^{3/2} dx^3$$

$$\Big\downarrow \text{ 1100}$$

$$\frac{1}{3}\left(\frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{5b^2}-\frac{a\int \left(b^2x^6+2abx^3+a^2\right)^{3/2} dx^3}{b}\right)$$

$$\Big\downarrow \text{ 1079}$$

$$\frac{1}{3}\left(\frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{5b^2}-\frac{a\sqrt{a^2+2abx^3+b^2x^6}\int \left(b^2x^3+ab\right)^3 dx^3}{b^4\left(a+bx^3\right)}\right)$$

$$\Big\downarrow \text{ 17}$$

$$\frac{1}{3}\left(\frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{5b^2}-\frac{a\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}{4b^2}\right)$$

input
```
Int[x^5*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

output
```
(-1/4*(a*(a + b*x^3)^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/b^2 + (a^2 + 2*a*b
*x^3 + b^2*x^6)^(5/2)/(5*b^2))/3
```

3.27.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1100
```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.27.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.40

method	result	size
pseudoelliptic	$-\frac{\operatorname{csgn}\left(b\,x^{3}+a\right)\left(b\,x^{3}+a\right)^{4}\left(-4b\,x^{3}+a\right)}{60b^{2}}$	31
gosper	$\frac{x^{6}\left(4b^{3}x^{9}+15b^{2}x^{6}a+20a^{2}b\,x^{3}+10a^{3}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{60\left(b\,x^{3}+a\right)^{3}}$	58
default	$\frac{x^{6}\left(4b^{3}x^{9}+15b^{2}x^{6}a+20a^{2}b\,x^{3}+10a^{3}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{60\left(b\,x^{3}+a\right)^{3}}$	58
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,a^{3}x^{6}}{6b\,x^{3}+6a}+\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,a^{2}b\,x^{9}}{3b\,x^{3}+3a}+\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,b^{2}a\,x^{12}}{4b\,x^{3}+4a}+\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,b^{3}x^{15}}{15b\,x^{3}+15a}$	116

input
```
int(x^5*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/60*csgn(b*x^3+a)*(b*x^3+a)^4*(-4*b*x^3+a)/b^2
```

3.27.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.45

$$\int x^5 \left(a^2+2abx^3+b^2x^6\right)^{3/2} \, dx = \frac{1}{15}\,b^3x^{15}+\frac{1}{4}\,ab^2x^{12}+\frac{1}{3}\,a^2bx^9+\frac{1}{6}\,a^3x^6$$

input
```
integrate(x^5*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

output
```
1/15*b^3*x^15 + 1/4*a*b^2*x^12 + 1/3*a^2*b*x^9 + 1/6*a^3*x^6
```

3.27.6 Sympy [F]

$$\int x^5 \left(a^2+2abx^3+b^2x^6\right)^{3/2} \, dx = \int x^5 \left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}} \, dx$$

input
```
integrate(x**5*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

output
```
Integral(x**5*((a + b*x**3)**2)**(3/2), x)
```

3.27.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.06

$$\int x^5 \left(a^2+2abx^3+b^2x^6\right)^{3/2} \, dx = -\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{3}{2}}ax^3}{12\,b} - \frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{3}{2}}a^2}{12\,b^2} + \frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{5}{2}}}{15\,b^2}$$

input
```
integrate(x^5*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*a*x^3/b - 1/12*(b^2*x^6 + 2*a*b*x^
3 + a^2)^(3/2)*a^2/b^2 + 1/15*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/b^2
```

3.27.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.58

$$\int x^5 \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx = \frac{1}{60} \left(4 b^3 x^{15}+15 a b^2 x^{12}+20 a^2 b x^9+10 a^3 x^6\right) \mathrm{sgn}\left(b x^3+a\right)$$

input
```
integrate(x^5*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

output
```
1/60*(4*b^3*x^15 + 15*a*b^2*x^12 + 20*a^2*b*x^9 + 10*a^3*x^6)*sgn(b*x^3 +
a)
```

3.27.9 Mupad [B] (verification not implemented)

Time = 8.32 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.59

$$\int x^5 \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx = \frac{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{3/2}\,\left(-a^2+3\,a\,b\,x^3+4\,b^2\,x^6\right)}{60\,b^2}$$

input
```
int(x^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

output
```
((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)*(4*b^2*x^6 - a^2 + 3*a*b*x^3))/(60*b^2)
```

3.28 $\int x^4(a^2 + 2abx^3 + b^2x^6)^{3/2}\,dx$

3.28.1 Optimal result

Integrand size = 26, antiderivative size = 167

$$\int x^4\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{a^3x^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}+\frac{3a^2bx^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{3ab^2x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{b^3x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14\left(a+bx^3\right)}$$

output
```
1/5*a^3*x^5*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+3/8*a^2*b*x^8*((b*x^3+a)^2)^(1/2
)/(b*x^3+a)+3/11*a*b^2*x^11*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/14*b^3*x^14*((
b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

3.28.2 Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.37

$$\int x^4\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{x^5\sqrt{(a+bx^3)^2}(616a^3+1155a^2bx^3+840ab^2x^6+220b^3x^9)}{3080\left(a+bx^3\right)}$$

input
```
Integrate[x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

output
```
(x^5*Sqrt[(a + b*x^3)^2]*(616*a^3 + 1155*a^2*b*x^3 + 840*a*b^2*x^6 + 220*b
^3*x^9))/(3080*(a + b*x^3))
```

3.28.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.45, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6} \int b^3x^4\left(bx^3+a\right)^3dx}{b^3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6} \int x^4\left(bx^3+a\right)^3dx}{a+bx^3}$$

$$\Big\downarrow \text{802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6} \int \left(b^3x^{13}+3ab^2x^{10}+3a^2bx^7+a^3x^4\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(\frac{a^3x^5}{5}+\frac{3}{8}a^2bx^8+\frac{3}{11}ab^2x^{11}+\frac{b^3x^{14}}{14}\right)}{a+bx^3}$$

input
```
Int[x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*((a^3*x^5)/5 + (3*a^2*b*x^8)/8 + (3*a*b^2
*x^11)/11 + (b^3*x^14)/14))/(a + b*x^3)
```

3.28.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.28.4 Maple [A] (verified)

Time = 3.76 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.35

method	result	size
gosper	$\frac{x^{5}\left(220 b^{3} x^{9}+840 b^{2} x^{6} a+1155 a^{2} b\,x^{3}+616 a^{3}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{3080\left(b\,x^{3}+a\right)^{3}}$	58
default	$\frac{x^{5}\left(220 b^{3} x^{9}+840 b^{2} x^{6} a+1155 a^{2} b\,x^{3}+616 a^{3}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{3080\left(b\,x^{3}+a\right)^{3}}$	58
risch	$\frac{a^{3} x^{5} \sqrt{\left(b\,x^{3}+a\right)^{2}}}{5 b\,x^{3}+5 a}+\frac{3 a^{2} b\,x^{8} \sqrt{\left(b\,x^{3}+a\right)^{2}}}{8\left(b\,x^{3}+a\right)}+\frac{3 a\,b^{2} x^{11} \sqrt{\left(b\,x^{3}+a\right)^{2}}}{11\left(b\,x^{3}+a\right)}+\frac{b^{3} x^{14} \sqrt{\left(b\,x^{3}+a\right)^{2}}}{14 b\,x^{3}+14 a}$	116

input
```
int(x^4*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3080*x^5*(220*b^3*x^9+840*a*b^2*x^6+1155*a^2*b*x^3+616*a^3)*((b*x^3+a)^2
)^(3/2)/(b*x^3+a)^3
```

3.28.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.21

$$\int x^4 \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx = \frac{1}{14}b^3x^{14}+\frac{3}{11}ab^2x^{11}+\frac{3}{8}a^2bx^8+\frac{1}{5}a^3x^5$$

input
```
integrate(x^4*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

output
```
1/14*b^3*x^14 + 3/11*a*b^2*x^11 + 3/8*a^2*b*x^8 + 1/5*a^3*x^5
```

3.28.6 Sympy [F]

$$\int x^4 \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx = \int x^4 \left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}} dx$$

input
```
integrate(x**4*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

output
```
Integral(x**4*((a + b*x**3)**2)**(3/2), x)
```

3.28.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.21

$$\int x^4 \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx = \frac{1}{14}b^3x^{14}+\frac{3}{11}ab^2x^{11}+\frac{3}{8}a^2bx^8+\frac{1}{5}a^3x^5$$

input
```
integrate(x^4*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

output
```
1/14*b^3*x^14 + 3/11*a*b^2*x^11 + 3/8*a^2*b*x^8 + 1/5*a^3*x^5
```

3.28.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.40

$$\int x^4 (a^2 + 2abx^3 + b^2x^6)^{3/2} \, dx = \frac{1}{14} b^3 x^{14} \mathrm{sgn}\left(bx^3 + a\right) + \frac{3}{11} ab^2 x^{11} \mathrm{sgn}\left(bx^3 + a\right) + \frac{3}{8} a^2 bx^8 \mathrm{sgn}\left(bx^3 + a\right) + \frac{1}{5} a^3 x^5 \mathrm{sgn}\left(bx^3 + a\right)$$

input
```
integrate(x^4*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

output
```
1/14*b^3*x^14*sgn(b*x^3 + a) + 3/11*a*b^2*x^11*sgn(b*x^3 + a) + 3/8*a^2*b*
x^8*sgn(b*x^3 + a) + 1/5*a^3*x^5*sgn(b*x^3 + a)
```

3.28.9 Mupad [F(-1)]

Timed out.

$$\int x^4 (a^2 + 2abx^3 + b^2x^6)^{3/2} \, dx = \int x^4 \left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^{3/2} \, dx$$

input
```
int(x^4*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

output
```
int(x^4*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

3.29 $\int x^3(a^2+2abx^3+b^2x^6)^{3/2}\,dx$

3.29.1 Optimal result

Integrand size = 26, antiderivative size = 167

$$\int x^3\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{a^3x^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{3a^2bx^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{3ab^2x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10\left(a+bx^3\right)}+\frac{b^3x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}$$

output
```
1/4*a^3*x^4*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+3/7*a^2*b*x^7*((b*x^3+a)^2)^(1/2
)/(b*x^3+a)+3/10*a*b^2*x^10*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/13*b^3*x^13*((
b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

3.29.2 Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.37

$$\int x^3\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{x^4\sqrt{\left(a+bx^3\right)^2}(455a^3+780a^2bx^3+546ab^2x^6+140b^3x^9)}{1820\left(a+bx^3\right)}$$

input
```
Integrate[x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

output
```
(x^4*Sqrt[(a + b*x^3)^2]*(455*a^3 + 780*a^2*b*x^3 + 546*a*b^2*x^6 + 140*b^
3*x^9))/(1820*(a + b*x^3))
```

3.29.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.45, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a^2+2abx^3+b^2x^6\right)^{3/2} \, dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6} \int b^3x^3 \left(bx^3+a\right)^3dx}{b^3 \left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6} \int x^3 \left(bx^3+a\right)^3dx}{a+bx^3}$$

$$\Big\downarrow \text{ 802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6} \int \left(b^3x^{12}+3ab^2x^9+3a^2bx^6+a^3x^3\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6} \left(\frac{a^3x^4}{4}+\frac{3}{7}a^2bx^7+\frac{3}{10}ab^2x^{10}+\frac{b^3x^{13}}{13}\right)}{a+bx^3}$$

input
```
Int[x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*((a^3*x^4)/4 + (3*a^2*b*x^7)/7 + (3*a*b^2
*x^10)/10 + (b^3*x^13)/13))/(a + b*x^3)
```

3.29.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.29.4 Maple [A] (verified)

Time = 3.14 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.35

method	result	size
gosper	$\frac{x^{4}\left(140 b^{3} x^{9}+546 b^{2} x^{6} a+780 a^{2} b\,x^{3}+455 a^{3}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{1820 \left(b\,x^{3}+a\right)^{3}}$	58
default	$\frac{x^{4}\left(140 b^{3} x^{9}+546 b^{2} x^{6} a+780 a^{2} b\,x^{3}+455 a^{3}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{1820 \left(b\,x^{3}+a\right)^{3}}$	58
risch	$\frac{a^{3} x^{4} \sqrt{\left(b\,x^{3}+a\right)^{2}}}{4 b\,x^{3}+4 a}+\frac{3 a^{2} b\,x^{7} \sqrt{\left(b\,x^{3}+a\right)^{2}}}{7 \left(b\,x^{3}+a\right)}+\frac{3 a\,b^{2} x^{10} \sqrt{\left(b\,x^{3}+a\right)^{2}}}{10 \left(b\,x^{3}+a\right)}+\frac{b^{3} x^{13} \sqrt{\left(b\,x^{3}+a\right)^{2}}}{13 b\,x^{3}+13 a}$	116

input
```
int(x^3*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/1820*x^4*(140*b^3*x^9+546*a*b^2*x^6+780*a^2*b*x^3+455*a^3)*((b*x^3+a)^2)
^(3/2)/(b*x^3+a)^3
```

3.29.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.21

$$\int x^3 \left(a^2 + 2abx^3 + b^2x^6\right)^{3/2} \, dx = \frac{1}{13} b^3x^{13} + \frac{3}{10} ab^2x^{10} + \frac{3}{7} a^2bx^7 + \frac{1}{4} a^3x^4$$

input
```
integrate(x^3*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

output
```
1/13*b^3*x^13 + 3/10*a*b^2*x^10 + 3/7*a^2*b*x^7 + 1/4*a^3*x^4
```

3.29.6 Sympy [F]

$$\int x^3 \left(a^2 + 2abx^3 + b^2x^6\right)^{3/2} \, dx = \int x^3 \left(\left(a + bx^3\right)^2\right)^{\frac{3}{2}} \, dx$$

input
```
integrate(x**3*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

output
```
Integral(x**3*((a + b*x**3)**2)**(3/2), x)
```

3.29.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.21

$$\int x^3 \left(a^2 + 2abx^3 + b^2x^6\right)^{3/2} \, dx = \frac{1}{13} b^3x^{13} + \frac{3}{10} ab^2x^{10} + \frac{3}{7} a^2bx^7 + \frac{1}{4} a^3x^4$$

input
```
integrate(x^3*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

output
```
1/13*b^3*x^13 + 3/10*a*b^2*x^10 + 3/7*a^2*b*x^7 + 1/4*a^3*x^4
```

3.29.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.40

$$\int x^3(a^2+2abx^3+b^2x^6)^{3/2}\,dx=\frac{1}{13}\,b^3x^{13}\mathrm{sgn}\left(bx^3+a\right)+\frac{3}{10}\,ab^2x^{10}\mathrm{sgn}\left(bx^3+a\right)+\frac{3}{7}\,a^2bx^7\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{4}\,a^3x^4\mathrm{sgn}\left(bx^3+a\right)$$

input
```
integrate(x^3*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

output
```
1/13*b^3*x^13*sgn(b*x^3 + a) + 3/10*a*b^2*x^10*sgn(b*x^3 + a) + 3/7*a^2*b*
x^7*sgn(b*x^3 + a) + 1/4*a^3*x^4*sgn(b*x^3 + a)
```

3.29.9 Mupad [F(-1)]

Timed out.

$$\int x^3(a^2+2abx^3+b^2x^6)^{3/2}\,dx=\int x^3\,\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{3/2}\,dx$$

input
```
int(x^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

output
```
int(x^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

3.30 $\int x^2(a^2+2abx^3+b^2x^6)^{3/2}\,dx$

3.30.1 Optimal result

Integrand size = 26, antiderivative size = 36

$$\int x^2\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{(a+bx^3)\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{12b}$$

output
```
1/12*(b*x^3+a)*(b^2*x^6+2*a*b*x^3+a^2)^(3/2)/b
```

3.30.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 112 vs. 2(36) = 72.

Time = 0.71 (sec) , antiderivative size = 112, normalized size of antiderivative = 3.11

$$\int x^2\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{x^3(4a^3+6a^2bx^3+4ab^2x^6+b^3x^9)\left(\sqrt{a^2}bx^3+a\left(\sqrt{a^2}-\sqrt{(a+bx^3)^2}\right)\right)}{12\left(-a^2-abx^3+\sqrt{a^2}\sqrt{(a+bx^3)^2}\right)}$$

input
```
Integrate[x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

output
```
(x^3*(4*a^3 + 6*a^2*b*x^3 + 4*a*b^2*x^6 + b^3*x^9)*(Sqrt[a^2]*b*x^3 + a*(S
qrt[a^2] - Sqrt[(a + b*x^3)^2])))/(12*(-a^2 - a*b*x^3 + Sqrt[a^2]*Sqrt[(a
+ b*x^3)^2]))
```

3.30.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1690, 1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a^2+2abx^3+b^2x^6\right)^{3/2}dx$$

$$\Big\downarrow \ 1690$$

$$\frac{1}{3}\int \left(b^2x^6+2abx^3+a^2\right)^{3/2}dx^3$$

$$\Big\downarrow \ 1079$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(b^2x^3+ab\right)^3dx^3}{3b^3\left(a+bx^3\right)}$$

$$\Big\downarrow \ 17$$

$$\frac{\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}{12b}$$

input
```
Int[x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

output
```
((a + b*x^3)^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(12*b)
```

3.30.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

3.30.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.64

method	result	size
pseudoelliptic	$\frac{\left(b\,x^{3}+a\right)^{4} \operatorname{csgn}\left(b\,x^{3}+a\right)}{12 b}$	23
default	$\frac{\left(b\,x^{3}+a\right) \left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{12 b}$	24
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(b\,x^{3}+a\right)^{3}}{12 b}$	26
gosper	$\frac{x^{3} \left(b^{3} x^{9}+4 b^{2} x^{6} a+6 a^{2} b\,x^{3}+4 a^{3}\right) \left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{12 \left(b\,x^{3}+a\right)^{3}}$	57

input
```
int(x^2*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/12*(b*x^3+a)^4*csgn(b*x^3+a)/b
```

3.30.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

$$\int x^2 \left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx = \frac{1}{12}\,b^3x^{12}+\frac{1}{3}\,ab^2x^9+\frac{1}{2}\,a^2bx^6+\frac{1}{3}\,a^3x^3$$

input
```
integrate(x^2*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

output
```
1/12*b^3*x^12 + 1/3*a*b^2*x^9 + 1/2*a^2*b*x^6 + 1/3*a^3*x^3
```

3.30.6 Sympy [F]

$$\int x^2\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\int x^2\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x**2*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

output
```
Integral(x**2*((a + b*x**3)**2)**(3/2), x)
```

3.30.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.44

$$\int x^2\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{1}{12}\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{3}{2}}x^3+\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{3}{2}}a}{12\,b}$$

input
```
integrate(x^2*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

output
```
1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*x^3 + 1/12*(b^2*x^6 + 2*a*b*x^3 + a
^2)^(3/2)*a/b
```

3.30.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.22

$$\int x^2\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{1}{12}\left(2\left(bx^6+2\,ax^3\right)a^2+\left(bx^6+2\,ax^3\right)^2b\right)\mathrm{sgn}\left(bx^3+a\right)$$

input
```
integrate(x^2*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

output
```
1/12*(2*(b*x^6 + 2*a*x^3)*a^2 + (b*x^6 + 2*a*x^3)^2*b)*sgn(b*x^3 + a)
```

3.30.9 Mupad [B] (verification not implemented)

Time = 8.48 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int x^2 \left(a^2 + 2abx^3 + b^2x^6\right)^{3/2} \, dx = \frac{\left(b^2\,x^3 + a\,b\right)\,\left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^{3/2}}{12\,b^2}$$

input
```
int(x^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

output
```
((a*b + b^2*x^3)*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2))/(12*b^2)
```

3.31 $\int x(a^2+2abx^3+b^2x^6)^{3/2}\,dx$

3.31.1 Optimal result

Integrand size = 24, antiderivative size = 167

$$\int x\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{a^3x^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{3a^2bx^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}+\frac{3ab^2x^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{b^3x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}$$

output
```
1/2*a^3*x^2*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+3/5*a^2*b*x^5*((b*x^3+a)^2)^(1/2
)/(b*x^3+a)+3/8*a*b^2*x^8*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/11*b^3*x^11*((b*
x^3+a)^2)^(1/2)/(b*x^3+a)
```

3.31.2 Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.37

$$\int x\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{x^2\sqrt{\left(a+bx^3\right)^2}\left(220a^3+264a^2bx^3+165ab^2x^6+40b^3x^9\right)}{440\left(a+bx^3\right)}$$

input
```
Integrate[x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

output
```
(x^2*Sqrt[(a + b*x^3)^2]*(220*a^3 + 264*a^2*b*x^3 + 165*a*b^2*x^6 + 40*b^3
*x^9))/(440*(a + b*x^3))
```

3.31.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.45, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a^2+2abx^3+b^2x^6\right)^{3/2}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int b^3x\left(bx^3+a\right)^3dx}{b^3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int x\left(bx^3+a\right)^3dx}{a+bx^3}$$

$$\Big\downarrow \text{802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(b^3x^{10}+3ab^2x^7+3a^2bx^4+a^3x\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(\frac{a^3x^2}{2}+\frac{3}{5}a^2bx^5+\frac{3}{8}ab^2x^8+\frac{b^3x^{11}}{11}\right)}{a+bx^3}$$

input
```
Int[x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*((a^3*x^2)/2 + (3*a^2*b*x^5)/5 + (3*a*b^2
*x^8)/8 + (b^3*x^11)/11))/(a + b*x^3)
```

3.31.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.31.4 Maple [A] (verified)

Time = 1.86 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.35

method	result	size
gosper	$\frac{x^{2}\left(40 b^{3} x^{9}+165 b^{2} x^{6} a+264 a^{2} b \,x^{3}+220 a^{3}\right)\left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{3}{2}}}{440 \left(b \,x^{3}+a \right)^{3}}$	58
default	$\frac{x^{2}\left(40 b^{3} x^{9}+165 b^{2} x^{6} a+264 a^{2} b \,x^{3}+220 a^{3}\right)\left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{3}{2}}}{440 \left(b \,x^{3}+a \right)^{3}}$	58
risch	$\frac{a^{3} x^{2} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{2 b \,x^{3}+2 a}+\frac{3 a^{2} b \,x^{5} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{5 \left(b \,x^{3}+a \right)}+\frac{3 a \,b^{2} x^{8} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{8 \left(b \,x^{3}+a \right)}+\frac{b^{3} x^{11} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{11 b \,x^{3}+11 a}$	116

input
```
int(x*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/440*x^2*(40*b^3*x^9+165*a*b^2*x^6+264*a^2*b*x^3+220*a^3)*((b*x^3+a)^2)^(
3/2)/(b*x^3+a)^3
```

3.31.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.21

$$\int x\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{1}{11}\,b^3x^{11}+\frac{3}{8}\,ab^2x^8+\frac{3}{5}\,a^2bx^5+\frac{1}{2}\,a^3x^2$$

input
```
integrate(x*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

output
```
1/11*b^3*x^11 + 3/8*a*b^2*x^8 + 3/5*a^2*b*x^5 + 1/2*a^3*x^2
```

3.31.6 Sympy [F]

$$\int x\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\int x\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

output
```
Integral(x*((a + b*x**3)**2)**(3/2), x)
```

3.31.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.21

$$\int x\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{1}{11}\,b^3x^{11}+\frac{3}{8}\,ab^2x^8+\frac{3}{5}\,a^2bx^5+\frac{1}{2}\,a^3x^2$$

input
```
integrate(x*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

output
```
1/11*b^3*x^11 + 3/8*a*b^2*x^8 + 3/5*a^2*b*x^5 + 1/2*a^3*x^2
```

3.31.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.40

$$\int x\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{1}{11}\,b^3x^{11}\mathrm{sgn}\left(bx^3+a\right)+\frac{3}{8}\,ab^2x^8\mathrm{sgn}\left(bx^3+a\right)+\frac{3}{5}\,a^2bx^5\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{2}\,a^3x^2\mathrm{sgn}\left(bx^3+a\right)$$

input
```
integrate(x*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

output
```
1/11*b^3*x^11*sgn(b*x^3 + a) + 3/8*a*b^2*x^8*sgn(b*x^3 + a) + 3/5*a^2*b*x^
5*sgn(b*x^3 + a) + 1/2*a^3*x^2*sgn(b*x^3 + a)
```

3.31.9 Mupad [F(-1)]

Timed out.

$$\int x\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\int x\,\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{3/2}\,dx$$

input
```
int(x*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

output
```
int(x*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

3.32 $\int (a^2+2abx^3+b^2x^6)^{3/2}\,dx$

3.32.1 Optimal result

Integrand size = 22, antiderivative size = 162

$$\int (a^2+2abx^3+b^2x^6)^{3/2}\,dx = \frac{a^3x(a^2+2abx^3+b^2x^6)^{3/2}}{(a+bx^3)^3} + \frac{3a^2bx^4(a^2+2abx^3+b^2x^6)^{3/2}}{4(a+bx^3)^3} + \frac{3ab^2x^7(a^2+2abx^3+b^2x^6)^{3/2}}{7(a+bx^3)^3} + \frac{b^3x^{10}(a^2+2abx^3+b^2x^6)^{3/2}}{10(a+bx^3)^3}$$

output
```
a^3*x*(b^2*x^6+2*a*b*x^3+a^2)^(3/2)/(b*x^3+a)^3+3/4*a^2*b*x^4*(b^2*x^6+2*a
*b*x^3+a^2)^(3/2)/(b*x^3+a)^3+3/7*a*b^2*x^7*(b^2*x^6+2*a*b*x^3+a^2)^(3/2)/
(b*x^3+a)^3+1/10*b^3*x^10*(b^2*x^6+2*a*b*x^3+a^2)^(3/2)/(b*x^3+a)^3
```

3.32.2 Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.36

$$\int (a^2+2abx^3+b^2x^6)^{3/2}\,dx = \frac{x\sqrt{(a+bx^3)^2}(140a^3+105a^2bx^3+60ab^2x^6+14b^3x^9)}{140(a+bx^3)}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

output
```
(x*Sqrt[(a + b*x^3)^2]*(140*a^3 + 105*a^2*b*x^3 + 60*a*b^2*x^6 + 14*b^3*x^
9))/(140*(a + b*x^3))
```

3.32.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.48, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1384, 747, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx$$

$$\Big\downarrow \ 1384$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(b^2x^3+ab\right)^3dx}{b^3\left(a+bx^3\right)}$$

$$\Big\downarrow \ 747$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(b^6x^9+3ab^5x^6+3a^2b^4x^3+a^3b^3\right)dx}{b^3\left(a+bx^3\right)}$$

$$\Big\downarrow \ 2009$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(a^3b^3x+\frac{3}{4}a^2b^4x^4+\frac{3}{7}ab^5x^7+\frac{b^6x^{10}}{10}\right)}{b^3\left(a+bx^3\right)}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(a^3*b^3*x + (3*a^2*b^4*x^4)/4 + (3*a*b^5
*x^7)/7 + (b^6*x^10)/10))/(b^3*(a + b*x^3))
```

3.32.3.1 Defintions of rubi rules used

rule 747
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.32.4 Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.35

method	result	size
gosper	$\frac{x\left(14 b^{3} x^{9}+60 b^{2} x^{6} a+105 a^{2} b\,x^{3}+140 a^{3}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{140\left(b\,x^{3}+a\right)^{3}}$	56
default	$\frac{x\left(14 b^{3} x^{9}+60 b^{2} x^{6} a+105 a^{2} b\,x^{3}+140 a^{3}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{140\left(b\,x^{3}+a\right)^{3}}$	56
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\, b^{3} x^{10}}{10 b\,x^{3}+10 a}+\frac{3 \sqrt{\left(b\,x^{3}+a\right)^{2}}\, a\,b^{2} x^{7}}{7\left(b\,x^{3}+a\right)}+\frac{3 \sqrt{\left(b\,x^{3}+a\right)^{2}}\, a^{2} b\,x^{4}}{4\left(b\,x^{3}+a\right)}+\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\, a^{3} x}{b\,x^{3}+a}$	113

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/140*x*(14*b^3*x^9+60*a*b^2*x^6+105*a^2*b*x^3+140*a^3)*((b*x^3+a)^2)^(3/2
)/(b*x^3+a)^3
```

3.32.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.20

$$\int \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx = \frac{1}{10}b^3x^{10}+\frac{3}{7}ab^2x^7+\frac{3}{4}a^2bx^4+a^3x$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

output
```
1/10*b^3*x^10 + 3/7*a*b^2*x^7 + 3/4*a^2*b*x^4 + a^3*x
```

3.32.6 Sympy [F]

$$\int \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx = \int \left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}} dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

output
```
Integral((a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2), x)
```

3.32.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.20

$$\int \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx = \frac{1}{10}b^3x^{10}+\frac{3}{7}ab^2x^7+\frac{3}{4}a^2bx^4+a^3x$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

output
```
1/10*b^3*x^10 + 3/7*a*b^2*x^7 + 3/4*a^2*b*x^4 + a^3*x
```

3.32.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.40

$$\int \left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{1}{10}\,b^3x^{10}\text{sgn}\left(bx^3+a\right)+\frac{3}{7}\,ab^2x^7\text{sgn}\left(bx^3+a\right)+\frac{3}{4}\,a^2bx^4\text{sgn}\left(bx^3+a\right)+a^3x\text{sgn}\left(bx^3+a\right)$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

output
```
1/10*b^3*x^10*sgn(b*x^3 + a) + 3/7*a*b^2*x^7*sgn(b*x^3 + a) + 3/4*a^2*b*x^
4*sgn(b*x^3 + a) + a^3*x*sgn(b*x^3 + a)
```

3.32.9 Mupad [F(-1)]

Timed out.

$$\int \left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\int {\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{3/2}\,dx$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

output
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

3.33 $\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x} dx$

3.33.1 Optimal result

Integrand size = 26, antiderivative size = 160

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x} dx = \frac{a^2bx^3\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3} + \frac{ab^2x^6\sqrt{a^2+2abx^3+b^2x^6}}{2(a+bx^3)} + \frac{b^3x^9\sqrt{a^2+2abx^3+b^2x^6}}{9(a+bx^3)} + \frac{a^3\sqrt{a^2+2abx^3+b^2x^6}\log(x)}{a+bx^3}$$

output
```
a^2*b*x^3*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/2*a*b^2*x^6*((b*x^3+a)^2)^(1/2)/
(b*x^3+a)+1/9*b^3*x^9*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+a^3*ln(x)*((b*x^3+a)^2
)^(1/2)/(b*x^3+a)
```

3.33.2 Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.38

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x} dx = \frac{\sqrt{(a+bx^3)^2}(bx^3(18a^2+9abx^3+2b^2x^6)+18a^3\log(x))}{18(a+bx^3)}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x,x]
```

output
```
(Sqrt[(a + b*x^3)^2]*(b*x^3*(18*a^2 + 9*a*b*x^3 + 2*b^2*x^6) + 18*a^3*Log[
x]))/(18*(a + b*x^3))
```

3.33.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.46, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1384, 27, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x}dx$$

$$\Big\downarrow \ 1384$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b^3\left(bx^3+a\right)^3}{x}dx}{b^3\left(a+bx^3\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(bx^3+a\right)^3}{x}dx}{a+bx^3}$$

$$\Big\downarrow \ 798$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(bx^3+a\right)^3}{x^3}dx^3}{3\left(a+bx^3\right)}$$

$$\Big\downarrow \ 49$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(b^3x^6+3ab^2x^3+3a^2b+\frac{a^3}{x^3}\right)dx^3}{3\left(a+bx^3\right)}$$

$$\Big\downarrow \ 2009$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(a^3\log\left(x^3\right)+3a^2bx^3+\frac{3}{2}ab^2x^6+\frac{b^3x^9}{3}\right)}{3\left(a+bx^3\right)}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(3*a^2*b*x^3 + (3*a*b^2*x^6)/2 + (b^3*x^9
)/3 + a^3*Log[x^3]))/(3*(a + b*x^3))
```

3.33.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.33.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.34

method	result	size
pseudoelliptic	$\frac{\operatorname{csgn}\left(b\,x^{3}+a\right) \left(2 b^{3} x^{9}+9 b^{2} x^{6} a+18 a^{2} b\,x^{3}+6 a^{3} \ln \left(b\,x^{3}\right)+11 a^{3}\right)}{18}$	54
default	$\frac{\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}} \left(2 b^{3} x^{9}+9 b^{2} x^{6} a+18 a^{2} b\,x^{3}+18 a^{3} \ln \left(x\right)\right)}{18 \left(b\,x^{3}+a\right)^{3}}$	57
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\, b \left(\frac{1}{9} b^{2} x^{9}+\frac{1}{2} a b\,x^{6}+a^{2} x^{3}\right)}{b\,x^{3}+a}+\frac{a^{3} \ln \left(x\right) \sqrt{\left(b\,x^{3}+a\right)^{2}}}{b\,x^{3}+a}$	73

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/18*csgn(b*x^3+a)*(2*b^3*x^9+9*b^2*x^6*a+18*a^2*b*x^3+6*a^3*ln(b*x^3)+11*
a^3)
```

3.33.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.20

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x}\,dx = \frac{1}{9}\,b^3x^9 + \frac{1}{2}\,ab^2x^6 + a^2bx^3 + a^3\log(x)$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x,x, algorithm="fricas")
```

output
```
1/9*b^3*x^9 + 1/2*a*b^2*x^6 + a^2*b*x^3 + a^3*log(x)
```

3.33.6 Sympy [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x}\,dx = \int \frac{\left((a+bx^3)^2\right)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x,x)
```

output
```
Integral(((a + b*x**3)**2)**(3/2)/x, x)
```

3.33.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.95

$$\begin{aligned}\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x}\,dx &= \frac{1}{6}\,\sqrt{b^2x^6+2\,abx^3+a^2}abx^3 \\ &+ \frac{1}{3}\,(-1)^{2\,b^2x^3+2\,ab}\,a^3\log\left(2\,b^2x^3+2\,ab\right) - \frac{1}{3}\,(-1)^{2\,abx^3+2\,a^2}\,a^3\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right) \\ &+ \frac{1}{2}\,\sqrt{b^2x^6+2\,abx^3+a^2}a^2 + \frac{1}{9}\,\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{3}{2}}\end{aligned}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x,x, algorithm="maxima")
```

output
```
1/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a*b*x^3 + 1/3*(-1)^(2*b^2*x^3 + 2*a*b)
*a^3*log(2*b^2*x^3 + 2*a*b) - 1/3*(-1)^(2*a*b*x^3 + 2*a^2)*a^3*log(2*a*b*x
/abs(x) + 2*a^2/(x^2*abs(x))) + 1/2*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a^2 +
1/9*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)
```

3.33.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.41

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x}\,dx = \frac{1}{9}\,b^3x^9\text{sgn}\left(bx^3+a\right) + \frac{1}{2}\,ab^2x^6\text{sgn}\left(bx^3+a\right) + a^2bx^3\text{sgn}\left(bx^3+a\right) + a^3\log\left(|x|\right)\text{sgn}\left(bx^3+a\right)$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x,x, algorithm="giac")
```

output
```
1/9*b^3*x^9*sgn(b*x^3 + a) + 1/2*a*b^2*x^6*sgn(b*x^3 + a) + a^2*b*x^3*sgn(
b*x^3 + a) + a^3*log(abs(x))*sgn(b*x^3 + a)
```

3.33.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x}\,dx = \int \frac{(a^2+2\,a\,b\,x^3+b^2\,x^6)^{3/2}}{x}\,dx$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x,x)
```

output
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x, x)
```

3.34 $\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^2} dx$

3.34.1 Optimal result

Integrand size = 26, antiderivative size = 165

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^2} dx = -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{x(a+bx^3)} + \frac{3a^2bx^2\sqrt{a^2+2abx^3+b^2x^6}}{2(a+bx^3)} + \frac{3ab^2x^5\sqrt{a^2+2abx^3+b^2x^6}}{5(a+bx^3)} + \frac{b^3x^8\sqrt{a^2+2abx^3+b^2x^6}}{8(a+bx^3)}$$

output
```
-a^3*((b*x^3+a)^2)^(1/2)/x/(b*x^3+a)+3/2*a^2*b*x^2*((b*x^3+a)^2)^(1/2)/(b*
x^3+a)+3/5*a*b^2*x^5*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/8*b^3*x^8*((b*x^3+a)^
2)^(1/2)/(b*x^3+a)
```

3.34.2 Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.37

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^2} dx = \frac{\sqrt{(a+bx^3)^2}(-40a^3+60a^2bx^3+24ab^2x^6+5b^3x^9)}{40x(a+bx^3)}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^2,x]
```

output
```
(Sqrt[(a + b*x^3)^2]*(-40*a^3 + 60*a^2*b*x^3 + 24*a*b^2*x^6 + 5*b^3*x^9))/
(40*x*(a + b*x^3))
```

3.34.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.44, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^2}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b^3\left(bx^3+a\right)^3}{x^2}dx}{b^3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(bx^3+a\right)^3}{x^2}dx}{a+bx^3}$$

$$\Big\downarrow \text{ 802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(b^3x^7+3ab^2x^4+3a^2bx+\frac{a^3}{x^2}\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(-\frac{a^3}{x}+\frac{3}{2}a^2bx^2+\frac{3}{5}ab^2x^5+\frac{b^3x^8}{8}\right)}{a+bx^3}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^2,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-(a^3/x) + (3*a^2*b*x^2)/2 + (3*a*b^2*x^
5)/5 + (b^3*x^8)/8))/(a + b*x^3)
```

3.34.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.34.4 Maple [A] (verified)

Time = 2.43 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.35

method	result	size
gosper	$-\frac{\left(-5 b^{3} x^{9}-24 b^{2} x^{6} a-60 a^{2} b\,x^{3}+40 a^{3}\right) \left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{40 x \left(b\,x^{3}+a\right)^{3}}$	58
default	$-\frac{\left(-5 b^{3} x^{9}-24 b^{2} x^{6} a-60 a^{2} b\,x^{3}+40 a^{3}\right) \left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{40 x \left(b\,x^{3}+a\right)^{3}}$	58
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\, b \left(\frac{1}{8} b^{2} x^{8}+\frac{3}{5} a b\,x^{5}+\frac{3}{2} a^{2} x^{2}\right)}{b\,x^{3}+a}-\frac{a^{3} \sqrt{\left(b\,x^{3}+a\right)^{2}}}{x \left(b\,x^{3}+a\right)}$	76

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/40*(-5*b^3*x^9-24*a*b^2*x^6-60*a^2*b*x^3+40*a^3)*((b*x^3+a)^2)^(3/2)/x/
(b*x^3+a)^3
```

3.34.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.22

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^2} dx = \frac{5 b^3x^9 + 24 ab^2x^6 + 60 a^2bx^3 - 40 a^3}{40 x}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^2,x, algorithm="fricas")
```

output
```
1/40*(5*b^3*x^9 + 24*a*b^2*x^6 + 60*a^2*b*x^3 - 40*a^3)/x
```

3.34.6 Sympy [F]

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^2} dx = \int \frac{\left((a + bx^3)^2\right)^{\frac{3}{2}}}{x^2} dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**2,x)
```

output
```
Integral(((a + b*x**3)**2)**(3/2)/x**2, x)
```

3.34.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.22

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^2} dx = \frac{5 b^3x^9 + 24 ab^2x^6 + 60 a^2bx^3 - 40 a^3}{40 x}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^2,x, algorithm="maxima")
```

output
```
1/40*(5*b^3*x^9 + 24*a*b^2*x^6 + 60*a^2*b*x^3 - 40*a^3)/x
```

3.34.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.41

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^2}\,dx = \frac{1}{8}\,b^3x^8\mathrm{sgn}\left(bx^3+a\right) + \frac{3}{5}\,ab^2x^5\mathrm{sgn}\left(bx^3+a\right) + \frac{3}{2}\,a^2bx^2\mathrm{sgn}\left(bx^3+a\right) - \frac{a^3\mathrm{sgn}\left(bx^3+a\right)}{x}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^2,x, algorithm="giac")
```

output
```
1/8*b^3*x^8*sgn(b*x^3 + a) + 3/5*a*b^2*x^5*sgn(b*x^3 + a) + 3/2*a^2*b*x^2*
sgn(b*x^3 + a) - a^3*sgn(b*x^3 + a)/x
```

3.34.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^2}\,dx = \int \frac{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{3/2}}{x^2}\,dx$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^2,x)
```

output
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^2, x)
```

3.35 $\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^3}\,dx$

3.35.1 Optimal result

Integrand size = 26, antiderivative size = 163

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^3}\,dx = -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{2x^2(a+bx^3)}+\frac{3a^2bx\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{3ab^2x^4\sqrt{a^2+2abx^3+b^2x^6}}{4(a+bx^3)}+\frac{b^3x^7\sqrt{a^2+2abx^3+b^2x^6}}{7(a+bx^3)}$$

output
```
-1/2*a^3*((b*x^3+a)^2)^(1/2)/x^2/(b*x^3+a)+3*a^2*b*x*((b*x^3+a)^2)^(1/2)/(
b*x^3+a)+3/4*a*b^2*x^4*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/7*b^3*x^7*((b*x^3+a
)^2)^(1/2)/(b*x^3+a)
```

3.35.2 Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.37

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^3}\,dx = \frac{\sqrt{(a+bx^3)^2}(-14a^3+84a^2bx^3+21ab^2x^6+4b^3x^9)}{28x^2(a+bx^3)}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^3,x]
```

output
```
(Sqrt[(a + b*x^3)^2]*(-14*a^3 + 84*a^2*b*x^3 + 21*a*b^2*x^6 + 4*b^3*x^9))/
(28*x^2*(a + b*x^3))
```

3.35.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.44, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^3}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b^3\left(bx^3+a\right)^3}{x^3}dx}{b^3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(bx^3+a\right)^3}{x^3}dx}{a+bx^3}$$

$$\Big\downarrow \text{ 802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(b^3x^6+3ab^2x^3+3a^2b+\frac{a^3}{x^3}\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(-\frac{a^3}{2x^2}+3a^2bx+\frac{3}{4}ab^2x^4+\frac{b^3x^7}{7}\right)}{a+bx^3}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^3,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-1/2*a^3/x^2 + 3*a^2*b*x + (3*a*b^2*x^4)
/4 + (b^3*x^7)/7))/(a + b*x^3)
```

3.35.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.35.4 Maple [A] (verified)

Time = 3.13 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.36

method	result	size
gosper	$-\frac{\left(-4 b^{3} x^{9}-21 b^{2} x^{6} a-84 a^{2} b \,x^{3}+14 a^{3}\right) \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{3}{2}}}{28 x^{2} \left(b \,x^{3}+a \right)^{3}}$	58
default	$-\frac{\left(-4 b^{3} x^{9}-21 b^{2} x^{6} a-84 a^{2} b \,x^{3}+14 a^{3}\right) \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{3}{2}}}{28 x^{2} \left(b \,x^{3}+a \right)^{3}}$	58
risch	$\frac{\sqrt{\left(b \,x^{3}+a \right)^{2}}\, b \left(\frac{1}{7} b^{2} x^{7}+\frac{3}{4} a b \,x^{4}+3 a^{2} x \right)}{b \,x^{3}+a}-\frac{a^{3} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{2 x^{2} \left(b \,x^{3}+a \right)}$	74

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/28*(-4*b^3*x^9-21*a*b^2*x^6-84*a^2*b*x^3+14*a^3)*((b*x^3+a)^2)^(3/2)/x^
2/(b*x^3+a)^3
```

3.35.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.23

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^3} \, dx = \frac{4\, b^3x^9 + 21\, ab^2x^6 + 84\, a^2bx^3 - 14\, a^3}{28\, x^2}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^3,x, algorithm="fricas")
```

output
```
1/28*(4*b^3*x^9 + 21*a*b^2*x^6 + 84*a^2*b*x^3 - 14*a^3)/x^2
```

3.35.6 Sympy [F]

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^3} \, dx = \int \frac{\left((a + bx^3)^2\right)^{\frac{3}{2}}}{x^3} \, dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**3,x)
```

output
```
Integral(((a + b*x**3)**2)**(3/2)/x**3, x)
```

3.35.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.23

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^3} \, dx = \frac{4\, b^3x^9 + 21\, ab^2x^6 + 84\, a^2bx^3 - 14\, a^3}{28\, x^2}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^3,x, algorithm="maxima")
```

output
```
1/28*(4*b^3*x^9 + 21*a*b^2*x^6 + 84*a^2*b*x^3 - 14*a^3)/x^2
```

3.35.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.40

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^3}\,dx = \frac{1}{7}\,b^3x^7\mathrm{sgn}\left(bx^3+a\right) + \frac{3}{4}\,ab^2x^4\mathrm{sgn}\left(bx^3+a\right) + 3\,a^2bx\mathrm{sgn}\left(bx^3+a\right) - \frac{a^3\mathrm{sgn}(bx^3+a)}{2\,x^2}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^3,x, algorithm="giac")
```

output
```
1/7*b^3*x^7*sgn(b*x^3 + a) + 3/4*a*b^2*x^4*sgn(b*x^3 + a) + 3*a^2*b*x*sgn(
b*x^3 + a) - 1/2*a^3*sgn(b*x^3 + a)/x^2
```

3.35.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^3}\,dx = \int \frac{(a^2+2\,a\,b\,x^3+b^2\,x^6)^{3/2}}{x^3}\,dx$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^3,x)
```

output
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^3, x)
```

3.36 $\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^4} dx$

3.36.1 Optimal result

Integrand size = 26, antiderivative size = 161

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^4} dx = -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{3x^3(a+bx^3)} + \frac{ab^2x^3\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3} + \frac{b^3x^6\sqrt{a^2+2abx^3+b^2x^6}}{6(a+bx^3)} + \frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}\log(x)}{a+bx^3}$$

output
```
-1/3*a^3*((b*x^3+a)^2)^(1/2)/x^3/(b*x^3+a)+a*b^2*x^3*((b*x^3+a)^2)^(1/2)/(
b*x^3+a)+1/6*b^3*x^6*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+3*a^2*b*ln(x)*((b*x^3+a
)^2)^(1/2)/(b*x^3+a)
```

3.36.2 Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.39

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^4} dx = \frac{\sqrt{(a+bx^3)^2}(-2a^3+6ab^2x^6+b^3x^9+18a^2bx^3\log(x))}{6x^3(a+bx^3)}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^4,x]
```

output
```
(Sqrt[(a + b*x^3)^2]*(-2*a^3 + 6*a*b^2*x^6 + b^3*x^9 + 18*a^2*b*x^3*Log[x]
))/(6*x^3*(a + b*x^3))
```

3.36.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.45, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1384, 27, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^4}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b^3\left(bx^3+a\right)^3}{x^4}dx}{b^3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(bx^3+a\right)^3}{x^4}dx}{a+bx^3}$$

$$\Big\downarrow \text{798}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(bx^3+a\right)^3}{x^6}dx^3}{3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{49}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^3}{x^6}+\frac{3ba^2}{x^3}+3b^2a+b^3x^3\right)dx^3}{3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(-\frac{a^3}{x^3}+3a^2b\log\left(x^3\right)+3ab^2x^3+\frac{b^3x^6}{2}\right)}{3\left(a+bx^3\right)}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^4,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-(a^3/x^3) + 3*a*b^2*x^3 + (b^3*x^6)/2 +
 3*a^2*b*Log[x^3]))/(3*(a + b*x^3))
```

3.36.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.36.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.37

method	result	size
default	$\frac{\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}\left(b^{3}x^{9}+6b^{2}x^{6}a+18a^{2}b\ln\left(x\right)x^{3}-2a^{3}\right)}{6x^{3}\left(b\,x^{3}+a\right)^{3}}$	59
pseudoelliptic	$-\frac{\operatorname{csgn}\left(b\,x^{3}+a\right)\left(-\frac{b^{3}x^{9}}{2}-3b^{2}x^{6}a-3\ln\left(b\,x^{3}\right)a^{2}b\,x^{3}-\frac{5a^{2}b\,x^{3}}{2}+a^{3}\right)}{3x^{3}}$	59
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,b\left(b\,x^{3}+3a\right)^{2}}{6b\,x^{3}+6a}-\frac{a^{3}\sqrt{\left(b\,x^{3}+a\right)^{2}}}{3x^{3}\left(b\,x^{3}+a\right)}+\frac{3a^{2}b\ln\left(x\right)\sqrt{\left(b\,x^{3}+a\right)^{2}}}{b\,x^{3}+a}$	92

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/6*((b*x^3+a)^2)^(3/2)*(b^3*x^9+6*b^2*x^6*a+18*a^2*b*ln(x)*x^3-2*a^3)/x^3
/(b*x^3+a)^3
```

3.36.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.24

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^4}\,dx=\frac{b^3x^9+6\,ab^2x^6+18\,a^2bx^3\log(x)-2\,a^3}{6\,x^3}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^4,x, algorithm="fricas")
```

output
```
1/6*(b^3*x^9 + 6*a*b^2*x^6 + 18*a^2*b*x^3*log(x) - 2*a^3)/x^3
```

3.36.6 Sympy [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^4}\,dx=\int \frac{\left((a+bx^3)^2\right)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**4,x)
```

output
```
Integral(((a + b*x**3)**2)**(3/2)/x**4, x)
```

3.36.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.97

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^4}\,dx=\frac{1}{2}\sqrt{b^2x^6+2\,abx^3+a^2}b^2x^3 + (-1)^{2\,b^2x^3+2\,ab}a^2b\log\left(2\,b^2x^3+2\,ab\right)-(-1)^{2\,abx^3+2\,a^2}a^2b\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right) + \frac{3}{2}\sqrt{b^2x^6+2\,abx^3+a^2}ab-\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}}{3\,x^3}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^4,x, algorithm="maxima")
```

output
```
1/2*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^2*x^3 + (-1)^(2*b^2*x^3 + 2*a*b)*a^2
*b*log(2*b^2*x^3 + 2*a*b) - (-1)^(2*a*b*x^3 + 2*a^2)*a^2*b*log(2*a*b*x/abs
(x) + 2*a^2/(x^2*abs(x))) + 3/2*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a*b - 1/3*
(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/x^3
```

3.36.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.53

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^4}\,dx = \frac{1}{6}\,b^3x^6\text{sgn}\left(bx^3+a\right)+ab^2x^3\text{sgn}\left(bx^3+a\right) + 3\,a^2b\log\left(|x|\right)\text{sgn}\left(bx^3+a\right)-\frac{3\,a^2bx^3\text{sgn}(bx^3+a)+a^3\text{sgn}(bx^3+a)}{3\,x^3}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^4,x, algorithm="giac")
```

output
```
1/6*b^3*x^6*sgn(b*x^3 + a) + a*b^2*x^3*sgn(b*x^3 + a) + 3*a^2*b*log(abs(x)
)*sgn(b*x^3 + a) - 1/3*(3*a^2*b*x^3*sgn(b*x^3 + a) + a^3*sgn(b*x^3 + a))/x
^3
```

3.36.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^4}\,dx = \int \frac{(a^2+2\,a\,b\,x^3+b^2\,x^6)^{3/2}}{x^4}\,dx$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^4,x)
```

output
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^4, x)
```

3.37 $\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^5} \, dx$

3.37.1 Optimal result

Integrand size = 26, antiderivative size = 165

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^5} \, dx = -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{4x^4(a+bx^3)} - \frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{x(a+bx^3)} + \frac{3ab^2x^2\sqrt{a^2+2abx^3+b^2x^6}}{2(a+bx^3)} + \frac{b^3x^5\sqrt{a^2+2abx^3+b^2x^6}}{5(a+bx^3)}$$

output
```
-1/4*a^3*((b*x^3+a)^2)^(1/2)/x^4/(b*x^3+a)-3*a^2*b*((b*x^3+a)^2)^(1/2)/x/(
b*x^3+a)+3/2*a*b^2*x^2*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/5*b^3*x^5*((b*x^3+a
)^2)^(1/2)/(b*x^3+a)
```

3.37.2 Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.37

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^5} \, dx = \frac{\sqrt{(a+bx^3)^2}(-5a^3-60a^2bx^3+30ab^2x^6+4b^3x^9)}{20x^4(a+bx^3)}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^5,x]
```

output
```
(Sqrt[(a + b*x^3)^2]*(-5*a^3 - 60*a^2*b*x^3 + 30*a*b^2*x^6 + 4*b^3*x^9))/(
20*x^4*(a + b*x^3))
```

3.37.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.44, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^5}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b^3\left(bx^3+a\right)^3}{x^5}dx}{b^3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(bx^3+a\right)^3}{x^5}dx}{a+bx^3}$$

$$\Big\downarrow \text{802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(b^3x^4+3ab^2x+\frac{3a^2b}{x^2}+\frac{a^3}{x^5}\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(-\frac{a^3}{4x^4}-\frac{3a^2b}{x}+\frac{3}{2}ab^2x^2+\frac{b^3x^5}{5}\right)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^5,x]
```

output
```
((-1/4*a^3/x^4 - (3*a^2*b)/x + (3*a*b^2*x^2)/2 + (b^3*x^5)/5)*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(a + b*x^3)
```

3.37.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.37.4 Maple [A] (verified)

Time = 4.43 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.35

method	result	size
gosper	$-\frac{\left(-4 b^{3} x^{9}-30 b^{2} x^{6} a+60 a^{2} b \,x^{3}+5 a^{3}\right) \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{3}{2}}}{20 \left(b \,x^{3}+a \right)^{3} x^{4}}$	58
default	$-\frac{\left(-4 b^{3} x^{9}-30 b^{2} x^{6} a+60 a^{2} b \,x^{3}+5 a^{3}\right) \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{3}{2}}}{20 \left(b \,x^{3}+a \right)^{3} x^{4}}$	58
risch	$\frac{\sqrt{\left(b \,x^{3}+a \right)^{2}}\, b^{2} \left(\frac{1}{5} b \,x^{5}+\frac{3}{2} a \,x^{2}\right)}{b \,x^{3}+a}+\frac{\sqrt{\left(b \,x^{3}+a \right)^{2}}\, \left(-3 a^{2} b \,x^{3}-\frac{1}{4} a^{3}\right)}{\left(b \,x^{3}+a \right) x^{4}}$	78

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/20*(-4*b^3*x^9-30*a*b^2*x^6+60*a^2*b*x^3+5*a^3)*((b*x^3+a)^2)^(3/2)/(b*
x^3+a)^3/x^4
```

3.37.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.22

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^5}\,dx = \frac{4\,b^3x^9 + 30\,ab^2x^6 - 60\,a^2bx^3 - 5\,a^3}{20\,x^4}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^5,x, algorithm="fricas")
```

output
```
1/20*(4*b^3*x^9 + 30*a*b^2*x^6 - 60*a^2*b*x^3 - 5*a^3)/x^4
```

3.37.6 Sympy [F]

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^5}\,dx = \int \frac{\left((a + bx^3)^2\right)^{\frac{3}{2}}}{x^5}\,dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**5,x)
```

output
```
Integral(((a + b*x**3)**2)**(3/2)/x**5, x)
```

3.37.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.22

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^5}\,dx = \frac{4\,b^3x^9 + 30\,ab^2x^6 - 60\,a^2bx^3 - 5\,a^3}{20\,x^4}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^5,x, algorithm="maxima")
```

output
```
1/20*(4*b^3*x^9 + 30*a*b^2*x^6 - 60*a^2*b*x^3 - 5*a^3)/x^4
```

3.37.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.42

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^5}\,dx = \frac{1}{5}\,b^3x^5\mathrm{sgn}\left(bx^3+a\right) + \frac{3}{2}\,ab^2x^2\mathrm{sgn}\left(bx^3+a\right) - \frac{12\,a^2bx^3\mathrm{sgn}\left(bx^3+a\right)+a^3\mathrm{sgn}\left(bx^3+a\right)}{4\,x^4}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^5,x, algorithm="giac")
```

output
```
1/5*b^3*x^5*sgn(b*x^3 + a) + 3/2*a*b^2*x^2*sgn(b*x^3 + a) - 1/4*(12*a^2*b*
x^3*sgn(b*x^3 + a) + a^3*sgn(b*x^3 + a))/x^4
```

3.37.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^5}\,dx = \int \frac{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{3/2}}{x^5}\,dx$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^5,x)
```

output
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^5, x)
```

3.38 $\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^6} dx$

3.38.1 Optimal result

Integrand size = 26, antiderivative size = 163

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^6} dx = -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{5x^5(a+bx^3)} - \frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{2x^2(a+bx^3)} + \frac{3ab^2x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3} + \frac{b^3x^4\sqrt{a^2+2abx^3+b^2x^6}}{4(a+bx^3)}$$

output
```
-1/5*a^3*((b*x^3+a)^2)^(1/2)/x^5/(b*x^3+a)-3/2*a^2*b*((b*x^3+a)^2)^(1/2)/x
^2/(b*x^3+a)+3*a*b^2*x*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/4*b^3*x^4*((b*x^3+a
)^2)^(1/2)/(b*x^3+a)
```

3.38.2 Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.37

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^6} dx = \frac{\sqrt{(a+bx^3)^2}(-4a^3-30a^2bx^3+60ab^2x^6+5b^3x^9)}{20x^5(a+bx^3)}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^6,x]
```

output
```
(Sqrt[(a + b*x^3)^2]*(-4*a^3 - 30*a^2*b*x^3 + 60*a*b^2*x^6 + 5*b^3*x^9))/(
20*x^5*(a + b*x^3))
```

3.38.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.44, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^6}dx$$
$$\Big\downarrow \text{1384}$$
$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b^3\left(bx^3+a\right)^3}{x^6}dx}{b^3\left(a+bx^3\right)}$$
$$\Big\downarrow \text{27}$$
$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(bx^3+a\right)^3}{x^6}dx}{a+bx^3}$$
$$\Big\downarrow \text{802}$$
$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^3}{x^6}+\frac{3ba^2}{x^3}+3b^2a+b^3x^3\right)dx}{a+bx^3}$$
$$\Big\downarrow \text{2009}$$
$$\frac{\left(-\frac{a^3}{5x^5}-\frac{3a^2b}{2x^2}+3ab^2x+\frac{b^3x^4}{4}\right)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^6,x]
```

output
```
((-1/5*a^3/x^5 - (3*a^2*b)/(2*x^2) + 3*a*b^2*x + (b^3*x^4)/4)*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(a + b*x^3)
```

3.38.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.38.4 Maple [A] (verified)

Time = 5.16 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.36

method	result	size
gosper	$-\frac{\left(-5 b^{3} x^{9}-60 b^{2} x^{6} a+30 a^{2} b\,x^{3}+4 a^{3}\right) \left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{20 \left(b\,x^{3}+a\right)^{3} x^{5}}$	58
default	$-\frac{\left(-5 b^{3} x^{9}-60 b^{2} x^{6} a+30 a^{2} b\,x^{3}+4 a^{3}\right) \left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{20 \left(b\,x^{3}+a\right)^{3} x^{5}}$	58
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\, b^{2} \left(\frac{1}{4} b\,x^{4}+3 a x \right)}{b\,x^{3}+a}+\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\, \left(-\frac{3}{2} a^{2} b\,x^{3}-\frac{1}{5} a^{3}\right)}{\left(b\,x^{3}+a\right) x^{5}}$	76

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/20*(-5*b^3*x^9-60*a*b^2*x^6+30*a^2*b*x^3+4*a^3)*((b*x^3+a)^2)^(3/2)/(b*
x^3+a)^3/x^5
```

3.38.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.23

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^6}\,dx = \frac{5\,b^3x^9+60\,ab^2x^6-30\,a^2bx^3-4\,a^3}{20\,x^5}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^6,x, algorithm="fricas")
```

output
```
1/20*(5*b^3*x^9 + 60*a*b^2*x^6 - 30*a^2*b*x^3 - 4*a^3)/x^5
```

3.38.6 Sympy [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^6}\,dx = \int \frac{\left((a+bx^3)^2\right)^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**6,x)
```

output
```
Integral(((a + b*x**3)**2)**(3/2)/x**6, x)
```

3.38.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.23

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^6}\,dx = \frac{5\,b^3x^9+60\,ab^2x^6-30\,a^2bx^3-4\,a^3}{20\,x^5}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^6,x, algorithm="maxima")
```

output
```
1/20*(5*b^3*x^9 + 60*a*b^2*x^6 - 30*a^2*b*x^3 - 4*a^3)/x^5
```

3.38.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.42

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^6}\,dx = \frac{1}{4}b^3x^4\mathrm{sgn}\left(bx^3+a\right) + 3\,ab^2x\mathrm{sgn}\left(bx^3+a\right) - \frac{15\,a^2bx^3\mathrm{sgn}(bx^3+a)+2\,a^3\mathrm{sgn}(bx^3+a)}{10\,x^5}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^6,x, algorithm="giac")
```

output
```
1/4*b^3*x^4*sgn(b*x^3 + a) + 3*a*b^2*x*sgn(b*x^3 + a) - 1/10*(15*a^2*b*x^3
*sgn(b*x^3 + a) + 2*a^3*sgn(b*x^3 + a))/x^5
```

3.38.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^6}\,dx = \int \frac{(a^2+2\,a\,b\,x^3+b^2\,x^6)^{3/2}}{x^6}\,dx$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^6,x)
```

output
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^6, x)
```

3.39 $\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^7} dx$

3.39.1 Optimal result

Integrand size = 26, antiderivative size = 162

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^7} dx = -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{6x^6(a+bx^3)} - \frac{a^2b\sqrt{a^2+2abx^3+b^2x^6}}{x^3(a+bx^3)} + \frac{b^3x^3\sqrt{a^2+2abx^3+b^2x^6}}{3(a+bx^3)} + \frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}\log(x)}{a+bx^3}$$

output
```
-1/6*a^3*((b*x^3+a)^2)^(1/2)/x^6/(b*x^3+a)-a^2*b*((b*x^3+a)^2)^(1/2)/x^3/(
b*x^3+a)+1/3*b^3*x^3*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+3*a*b^2*ln(x)*((b*x^3+a
)^2)^(1/2)/(b*x^3+a)
```

3.39.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 612 vs. 2(162) = 324.

Time = 0.82 (sec) , antiderivative size = 612, normalized size of antiderivative = 3.78

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^7} dx = \frac{4a^4\sqrt{a^2}+28a^3\sqrt{a^2}bx^3+35(a^2)^{3/2}b^2x^6+3a\sqrt{a^2}b^3x^9-8\sqrt{a^2}b^4x^{12}-4a^4\sqrt{(}}{}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^7,x]
```

output
```
(4*a^4*Sqrt[a^2] + 28*a^3*Sqrt[a^2]*b*x^3 + 35*(a^2)^(3/2)*b^2*x^6 + 3*a*S
qrt[a^2]*b^3*x^9 - 8*Sqrt[a^2]*b^4*x^12 - 4*a^4*Sqrt[(a + b*x^3)^2] - 24*a
^3*b*x^3*Sqrt[(a + b*x^3)^2] - 11*a^2*b^2*x^6*Sqrt[(a + b*x^3)^2] + 8*a*b^
3*x^9*Sqrt[(a + b*x^3)^2] - 24*a*b^2*x^6*(a^2 + a*b*x^3 - Sqrt[a^2]*Sqrt[(
a + b*x^3)^2])*ArcTanh[(b*x^3)/(Sqrt[a^2] - Sqrt[(a + b*x^3)^2])] - 24*b^2
*x^6*((a^2)^(3/2) + a*Sqrt[a^2]*b*x^3 - a^2*Sqrt[(a + b*x^3)^2])*Log[x^3]
+ 12*(a^2)^(3/2)*b^2*x^6*Log[Sqrt[a^2] - b*x^3 - Sqrt[(a + b*x^3)^2]] + 12
*a*Sqrt[a^2]*b^3*x^9*Log[Sqrt[a^2] - b*x^3 - Sqrt[(a + b*x^3)^2]] - 12*a^2
*b^2*x^6*Sqrt[(a + b*x^3)^2]*Log[Sqrt[a^2] - b*x^3 - Sqrt[(a + b*x^3)^2]]
+ 12*(a^2)^(3/2)*b^2*x^6*Log[Sqrt[a^2] + b*x^3 - Sqrt[(a + b*x^3)^2]] + 12
*a*Sqrt[a^2]*b^3*x^9*Log[Sqrt[a^2] + b*x^3 - Sqrt[(a + b*x^3)^2]] - 12*a^2
*b^2*x^6*Sqrt[(a + b*x^3)^2]*Log[Sqrt[a^2] + b*x^3 - Sqrt[(a + b*x^3)^2]])
/(24*x^6*(a^2 + a*b*x^3 - Sqrt[a^2]*Sqrt[(a + b*x^3)^2]))
```

3.39.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.44, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1384, 27, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^7}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b^3\left(bx^3+a\right)^3}{x^7}dx}{b^3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(bx^3+a\right)^3}{x^7}dx}{a+bx^3}$$

$$\Big\downarrow \text{ 798}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(bx^3+a\right)^3}{x^9}dx^3}{3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(\frac{a^3}{x^9}+\frac{3ba^2}{x^6}+\frac{3b^2a}{x^3}+b^3\right)dx^3}{3(a+bx^3)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(-\frac{a^3}{2x^6}-\frac{3a^2b}{x^3}+3ab^2\log\left(x^3\right)+b^3x^3\right)}{3(a+bx^3)}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^7,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-1/2*a^3/x^6 - (3*a^2*b)/x^3 + b^3*x^3 +
 3*a*b^2*Log[x^3]))/(3*(a + b*x^3))
```

3.39.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.39. $\int\frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^7}\,dx$

3.39.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.36

method	result	size
pseudoelliptic	$-\frac{\operatorname{csgn}\left(b\,x^{3}+a\right)\left(-2 b^{3} x^{9}-6 \ln \left(b\,x^{3}\right) a\,b^{2} x^{6}-2 b^{2} x^{6} a+6 a^{2} b\,x^{3}+a^{3}\right)}{6 x^{6}}$	59
default	$\frac{\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}\left(2 b^{3} x^{9}+18 b^{2} a \ln \left(x\right) x^{6}-6 a^{2} b\,x^{3}-a^{3}\right)}{6 \left(b\,x^{3}+a\right)^{3} x^{6}}$	60
risch	$\frac{b^{3} x^{3} \sqrt{\left(b\,x^{3}+a\right)^{2}}}{3 b\,x^{3}+3 a}+\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(-a^{2} b\,x^{3}-\frac{1}{6} a^{3}\right)}{\left(b\,x^{3}+a\right) x^{6}}+\frac{3 a\,b^{2} \ln \left(x\right) \sqrt{\left(b\,x^{3}+a\right)^{2}}}{b\,x^{3}+a}$	97

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*csgn(b*x^3+a)*(-2*b^3*x^9-6*ln(b*x^3)*a*b^2*x^6-2*b^2*x^6*a+6*a^2*b*x
^3+a^3)/x^6
```

3.39.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.24

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^7}\,dx=\frac{2\,b^3x^9+18\,ab^2x^6\log\left(x\right)-6\,a^2bx^3-a^3}{6\,x^6}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^7,x, algorithm="fricas")
```

output
```
1/6*(2*b^3*x^9 + 18*a*b^2*x^6*log(x) - 6*a^2*b*x^3 - a^3)/x^6
```

3.39.6 Sympy [F]

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^7}\,dx=\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}{x^7}\,dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**7,x)
```

output
```
Integral(((a + b*x**3)**2)**(3/2)/x**7, x)
```

3.39.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.36

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^7}\,dx = \frac{\sqrt{b^2x^6+2\,abx^3+a^2}b^3x^3}{2\,a} + (-1)^{2\,b^2x^3+2\,ab}ab^2\log\left(2\,b^2x^3+2\,ab\right) - (-1)^{2\,abx^3+2\,a^2}ab^2\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right) + \frac{3}{2}\sqrt{b^2x^6+2\,abx^3+a^2}b^2 + \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}b^2}{6\,a^2} - \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}b}{6\,ax^3} - \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}}{6\,a^2x^6}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^7,x, algorithm="maxima")
```

output
```
1/2*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^3*x^3/a + (-1)^(2*b^2*x^3 + 2*a*b)*a
*b^2*log(2*b^2*x^3 + 2*a*b) - (-1)^(2*a*b*x^3 + 2*a^2)*a*b^2*log(2*a*b*x/a
bs(x) + 2*a^2/(x^2*abs(x))) + 3/2*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^2 + 1/
6*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*b^2/a^2 - 1/6*(b^2*x^6 + 2*a*b*x^3 + a
^2)^(3/2)*b/(a*x^3) - 1/6*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/(a^2*x^6)
```

3.39.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.53

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^7}\,dx = \frac{1}{3}\,b^3x^3\mathrm{sgn}\left(bx^3+a\right) + 3\,ab^2\log\left(|x|\right)\mathrm{sgn}\left(bx^3+a\right) - \frac{9\,ab^2x^6\mathrm{sgn}(bx^3+a) + 6\,a^2bx^3\mathrm{sgn}(bx^3+a) + a^3\mathrm{sgn}(bx^3+a)}{6\,x^6}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^7,x, algorithm="giac")
```

output
```
1/3*b^3*x^3*sgn(b*x^3 + a) + 3*a*b^2*log(abs(x))*sgn(b*x^3 + a) - 1/6*(9*a
*b^2*x^6*sgn(b*x^3 + a) + 6*a^2*b*x^3*sgn(b*x^3 + a) + a^3*sgn(b*x^3 + a))
/x^6
```

3.39.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^7} \, dx = \int \frac{(a^2 + 2\,a\,b\,x^3 + b^2\,x^6)^{3/2}}{x^7} \, dx$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^7,x)
```

output
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^7, x)
```

3.40 $\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^8} dx$

3.40.1 Optimal result

Integrand size = 26, antiderivative size = 165

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^8} dx = -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{7x^7(a+bx^3)} - \frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{4x^4(a+bx^3)} - \frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{x(a+bx^3)} + \frac{b^3x^2\sqrt{a^2+2abx^3+b^2x^6}}{2(a+bx^3)}$$

output
```
-1/7*a^3*((b*x^3+a)^2)^(1/2)/x^7/(b*x^3+a)-3/4*a^2*b*((b*x^3+a)^2)^(1/2)/x
^4/(b*x^3+a)-3*a*b^2*((b*x^3+a)^2)^(1/2)/x/(b*x^3+a)+1/2*b^3*x^2*((b*x^3+a
)^2)^(1/2)/(b*x^3+a)
```

3.40.2 Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.37

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^8} dx = -\frac{\sqrt{(a+bx^3)^2}(4a^3+21a^2bx^3+84ab^2x^6-14b^3x^9)}{28x^7(a+bx^3)}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^8,x]
```

output
```
-1/28*(Sqrt[(a + b*x^3)^2]*(4*a^3 + 21*a^2*b*x^3 + 84*a*b^2*x^6 - 14*b^3*x
^9))/(x^7*(a + b*x^3))
```

3.40.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.44, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^8}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b^3\left(bx^3+a\right)^3}{x^8}dx}{b^3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(bx^3+a\right)^3}{x^8}dx}{a+bx^3}$$

$$\Big\downarrow \text{ 802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^3}{x^8}+\frac{3ba^2}{x^5}+\frac{3b^2a}{x^2}+b^3x\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\left(-\frac{a^3}{7x^7}-\frac{3a^2b}{4x^4}-\frac{3ab^2}{x}+\frac{b^3x^2}{2}\right)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^8,x]
```

output
```
((-1/7*a^3/x^7 - (3*a^2*b)/(4*x^4) - (3*a*b^2)/x + (b^3*x^2)/2)*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(a + b*x^3)
```

3.40.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.40.4 Maple [A] (verified)

Time = 7.54 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.35

method	result	size
gosper	$-\frac{\left(-14 b^{3} x^{9}+84 b^{2} x^{6} a+21 a^{2} b \,x^{3}+4 a^{3}\right) \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{3}{2}}}{28 \left(b \,x^{3}+a \right)^{3} x^{7}}$	58
default	$-\frac{\left(-14 b^{3} x^{9}+84 b^{2} x^{6} a+21 a^{2} b \,x^{3}+4 a^{3}\right) \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{3}{2}}}{28 \left(b \,x^{3}+a \right)^{3} x^{7}}$	58
risch	$\frac{b^{3} x^{2} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{2 b \,x^{3}+2 a}+\frac{\sqrt{\left(b \,x^{3}+a \right)^{2}}\, \left(-3 b^{2} x^{6} a -\frac{3}{4} a^{2} b \,x^{3}-\frac{1}{7} a^{3}\right)}{\left(b \,x^{3}+a \right) x^{7}}$	78

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/28*(-14*b^3*x^9+84*a*b^2*x^6+21*a^2*b*x^3+4*a^3)*((b*x^3+a)^2)^(3/2)/(b
*x^3+a)^3/x^7
```

3.40.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.22

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^8}\,dx = \frac{14\,b^3x^9 - 84\,ab^2x^6 - 21\,a^2bx^3 - 4\,a^3}{28\,x^7}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^8,x, algorithm="fricas")
```

output
```
1/28*(14*b^3*x^9 - 84*a*b^2*x^6 - 21*a^2*b*x^3 - 4*a^3)/x^7
```

3.40.6 Sympy [F]

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^8}\,dx = \int \frac{\left((a + bx^3)^2\right)^{\frac{3}{2}}}{x^8}\,dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**8,x)
```

output
```
Integral(((a + b*x**3)**2)**(3/2)/x**8, x)
```

3.40.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.22

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^8}\,dx = \frac{14\,b^3x^9 - 84\,ab^2x^6 - 21\,a^2bx^3 - 4\,a^3}{28\,x^7}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^8,x, algorithm="maxima")
```

output
```
1/28*(14*b^3*x^9 - 84*a*b^2*x^6 - 21*a^2*b*x^3 - 4*a^3)/x^7
```

3.40.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.42

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^8}\,dx = \frac{1}{2}\,b^3x^2\mathrm{sgn}\left(bx^3+a\right) - \frac{84\,ab^2x^6\mathrm{sgn}(bx^3+a)+21\,a^2bx^3\mathrm{sgn}(bx^3+a)+4\,a^3\mathrm{sgn}(bx^3+a)}{28\,x^7}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^8,x, algorithm="giac")
```

output
```
1/2*b^3*x^2*sgn(b*x^3 + a) - 1/28*(84*a*b^2*x^6*sgn(b*x^3 + a) + 21*a^2*b*
x^3*sgn(b*x^3 + a) + 4*a^3*sgn(b*x^3 + a))/x^7
```

3.40.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^8}\,dx = \int \frac{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{3/2}}{x^8}\,dx$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^8,x)
```

output
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^8, x)
```

3.41 $\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^9}\,dx$

3.41.1 Optimal result

Integrand size = 26, antiderivative size = 162

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^9}\,dx = -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{8x^8(a+bx^3)} - \frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{5x^5(a+bx^3)} - \frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{2x^2(a+bx^3)} + \frac{b^3x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

output
```
-1/8*a^3*((b*x^3+a)^2)^(1/2)/x^8/(b*x^3+a)-3/5*a^2*b*((b*x^3+a)^2)^(1/2)/x
^5/(b*x^3+a)-3/2*a*b^2*((b*x^3+a)^2)^(1/2)/x^2/(b*x^3+a)+b^3*x*((b*x^3+a)^
2)^(1/2)/(b*x^3+a)
```

3.41.2 Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.38

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^9}\,dx = -\frac{\sqrt{(a+bx^3)^2}(5a^3+24a^2bx^3+60ab^2x^6-40b^3x^9)}{40x^8(a+bx^3)}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^9,x]
```

output
```
-1/40*(Sqrt[(a + b*x^3)^2]*(5*a^3 + 24*a^2*b*x^3 + 60*a*b^2*x^6 - 40*b^3*x
^9))/(x^8*(a + b*x^3))
```

3.41.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.43, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^9}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b^3\left(bx^3+a\right)^3}{x^9}dx}{b^3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(bx^3+a\right)^3}{x^9}dx}{a+bx^3}$$

$$\Big\downarrow \text{802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^3}{x^9}+\frac{3ba^2}{x^6}+\frac{3b^2a}{x^3}+b^3\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(-\frac{a^3}{8x^8}-\frac{3a^2b}{5x^5}-\frac{3ab^2}{2x^2}+b^3x\right)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^9,x]
```

output
```
((-1/8*a^3/x^8 - (3*a^2*b)/(5*x^5) - (3*a*b^2)/(2*x^2) + b^3*x)*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(a + b*x^3)
```

3.41.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.41.4 Maple [A] (verified)

Time = 8.78 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.36

method	result	size
gosper	$-\frac{\left(-40 b^{3} x^{9}+60 b^{2} x^{6} a+24 a^{2} b\,x^{3}+5 a^{3}\right) \left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{40 \left(b\,x^{3}+a\right)^{3} x^{8}}$	58
default	$-\frac{\left(-40 b^{3} x^{9}+60 b^{2} x^{6} a+24 a^{2} b\,x^{3}+5 a^{3}\right) \left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{40 \left(b\,x^{3}+a\right)^{3} x^{8}}$	58
risch	$\frac{b^{3} x \sqrt{\left(b\,x^{3}+a\right)^{2}}}{b\,x^{3}+a}+\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\, \left(-\frac{3}{2} b^{2} x^{6} a-\frac{3}{5} a^{2} b\,x^{3}-\frac{1}{8} a^{3}\right)}{\left(b\,x^{3}+a\right) x^{8}}$	75

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/40*(-40*b^3*x^9+60*a*b^2*x^6+24*a^2*b*x^3+5*a^3)*((b*x^3+a)^2)^(3/2)/(b
*x^3+a)^3/x^8
```

3.41.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.23

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^9}\,dx = \frac{40\,b^3x^9-60\,ab^2x^6-24\,a^2bx^3-5\,a^3}{40\,x^8}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^9,x, algorithm="fricas")
```

output
```
1/40*(40*b^3*x^9 - 60*a*b^2*x^6 - 24*a^2*b*x^3 - 5*a^3)/x^8
```

3.41.6 Sympy [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^9}\,dx = \int \frac{\left((a+bx^3)^2\right)^{\frac{3}{2}}}{x^9}\,dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**9,x)
```

output
```
Integral(((a + b*x**3)**2)**(3/2)/x**9, x)
```

3.41.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.23

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^9}\,dx = \frac{40\,b^3x^9-60\,ab^2x^6-24\,a^2bx^3-5\,a^3}{40\,x^8}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^9,x, algorithm="maxima")
```

output
```
1/40*(40*b^3*x^9 - 60*a*b^2*x^6 - 24*a^2*b*x^3 - 5*a^3)/x^8
```

3.41.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.41

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^9}\,dx = b^3x\mathrm{sgn}(bx^3+a) - \frac{60\,ab^2x^6\mathrm{sgn}(bx^3+a)+24\,a^2bx^3\mathrm{sgn}(bx^3+a)+5\,a^3\mathrm{sgn}(bx^3+a)}{40\,x^8}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^9,x, algorithm="giac")
```

output
```
b^3*x*sgn(b*x^3 + a) - 1/40*(60*a*b^2*x^6*sgn(b*x^3 + a) + 24*a^2*b*x^3*sg
n(b*x^3 + a) + 5*a^3*sgn(b*x^3 + a))/x^8
```

3.41.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^9}\,dx = \int \frac{(a^2+2\,a\,b\,x^3+b^2\,x^6)^{3/2}}{x^9}\,dx$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^9,x)
```

output
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^9, x)
```

3.42 $\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{10}}\,dx$

3.42.1 Optimal result

Integrand size = 26, antiderivative size = 161

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{10}}\,dx = -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{9x^9(a+bx^3)}-\frac{a^2b\sqrt{a^2+2abx^3+b^2x^6}}{2x^6(a+bx^3)} -\frac{ab^2\sqrt{a^2+2abx^3+b^2x^6}}{x^3(a+bx^3)}+\frac{b^3\sqrt{a^2+2abx^3+b^2x^6}\log(x)}{a+bx^3}$$

output
```
-1/9*a^3*((b*x^3+a)^2)^(1/2)/x^9/(b*x^3+a)-1/2*a^2*b*((b*x^3+a)^2)^(1/2)/x
^6/(b*x^3+a)-a*b^2*((b*x^3+a)^2)^(1/2)/x^3/(b*x^3+a)+b^3*ln(x)*((b*x^3+a)^
2)^(1/2)/(b*x^3+a)
```

3.42.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.65

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{10}}\,dx = \frac{2a^3\sqrt{a^2}+9(a^2)^{3/2}bx^3+18a\sqrt{a^2}b^2x^6-2a^3\sqrt{(a+bx^3)^2}-7a^2bx^3\sqrt{(a+bx^3)^2}}{}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^10,x]
```

output
```
(2*a^3*Sqrt[a^2] + 9*(a^2)^(3/2)*b*x^3 + 18*a*Sqrt[a^2]*b^2*x^6 - 2*a^3*Sq
rt[(a + b*x^3)^2] - 7*a^2*b*x^3*Sqrt[(a + b*x^3)^2] - 11*a*b^2*x^6*Sqrt[(a
 + b*x^3)^2] - 12*a*b^3*x^9*ArcTanh[(b*x^3)/(Sqrt[a^2] - Sqrt[(a + b*x^3)^
2])] - 12*Sqrt[a^2]*b^3*x^9*Log[x^3] + 6*Sqrt[a^2]*b^3*x^9*Log[a*(Sqrt[a^2
] - b*x^3 - Sqrt[(a + b*x^3)^2])] + 6*Sqrt[a^2]*b^3*x^9*Log[a*(Sqrt[a^2] +
 b*x^3 - Sqrt[(a + b*x^3)^2])])/(36*a*x^9)
```

3.42.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.46, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1384, 27, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{10}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b^3\left(bx^3+a\right)^3}{x^{10}}dx}{b^3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(bx^3+a\right)^3}{x^{10}}dx}{a+bx^3}$$

$$\Big\downarrow \text{798}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(bx^3+a\right)^3}{x^{12}}dx^3}{3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{49}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^3}{x^{12}}+\frac{3ba^2}{x^9}+\frac{3b^2a}{x^6}+\frac{b^3}{x^3}\right)dx^3}{3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(-\frac{a^3}{3x^9}-\frac{3a^2b}{2x^6}-\frac{3ab^2}{x^3}+b^3\log\left(x^3\right)\right)}{3\left(a+bx^3\right)}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^10,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-1/3*a^3/x^9 - (3*a^2*b)/(2*x^6) - (3*a*
b^2)/x^3 + b^3*Log[x^3]))/(3*(a + b*x^3))
```

3.42.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.42.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.32

method	result	size
pseudoelliptic	$\frac{\operatorname{csgn}(b\,x^3+a)\left(6\ln\left(b\,x^3\right)b^3x^9-18b^2x^6a-9a^2b\,x^3-2a^3\right)}{18x^9}$	52
default	$\frac{\left(\left(b\,x^3+a\right)^2\right)^{\frac{3}{2}}\left(18b^3\ln\left(x\right)x^9-18b^2x^6a-9a^2b\,x^3-2a^3\right)}{18\left(b\,x^3+a\right)^3x^9}$	60
risch	$\frac{\sqrt{\left(b\,x^3+a\right)^2}\left(-b^2x^6a-\frac{1}{2}a^2b\,x^3-\frac{1}{9}a^3\right)}{\left(b\,x^3+a\right)x^9}+\frac{b^3\ln\left(x\right)\sqrt{\left(b\,x^3+a\right)^2}}{b\,x^3+a}$	76

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^10,x,method=_RETURNVERBOSE)
```

output
```
1/18*csgn(b*x^3+a)*(6*ln(b*x^3)*b^3*x^9-18*b^2*x^6*a-9*a^2*b*x^3-2*a^3)/x^
9
```

3.42.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.24

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{10}}\,dx=\frac{18\,b^3x^9\log\left(x\right)-18\,ab^2x^6-9\,a^2bx^3-2\,a^3}{18\,x^9}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^10,x, algorithm="fricas")
```

output
```
1/18*(18*b^3*x^9*log(x) - 18*a*b^2*x^6 - 9*a^2*b*x^3 - 2*a^3)/x^9
```

3.42.6 Sympy [F]

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{10}}\,dx=\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}{x^{10}}\,dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**10,x)
```

output
```
Integral(((a + b*x**3)**2)**(3/2)/x**10, x)
```

3.42.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(113) = 226$.

Time = 0.22 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.57

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{10}}\,dx = \frac{\sqrt{b^2x^6+2\,abx^3+a^2}b^4x^3}{6\,a^2} + \frac{1}{3}\,(-1)^{2\,b^2x^3+2\,ab}\,b^3\log\left(2\,b^2x^3+2\,ab\right) - \frac{1}{3}\,(-1)^{2\,abx^3+2\,a^2}\,b^3\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right) + \frac{\sqrt{b^2x^6+2\,abx^3+a^2}b^3}{2\,a} - \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}b^3}{18\,a^3} - \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}b^2}{6\,a^2x^3} + \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}b}{18\,a^3x^6} - \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}}{9\,a^2x^9}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^10,x, algorithm="maxima")
```

output
```
1/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^4*x^3/a^2 + 1/3*(-1)^(2*b^2*x^3 + 2*
a*b)*b^3*log(2*b^2*x^3 + 2*a*b) - 1/3*(-1)^(2*a*b*x^3 + 2*a^2)*b^3*log(2*a
*b*x/abs(x) + 2*a^2/(x^2*abs(x))) + 1/2*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^
3/a - 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*b^3/a^3 - 1/6*(b^2*x^6 + 2*a*
b*x^3 + a^2)^(3/2)*b^2/(a^2*x^3) + 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*
b/(a^3*x^6) - 1/9*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/(a^2*x^9)
```

3.42.8 Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.53

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{10}}\,dx = b^3\log\left(|x|\right)\operatorname{sgn}\left(bx^3+a\right) - \frac{11\,b^3x^9\operatorname{sgn}(bx^3+a)+18\,ab^2x^6\operatorname{sgn}(bx^3+a)+9\,a^2bx^3\operatorname{sgn}(bx^3+a)+2\,a^3\operatorname{sgn}(bx^3+a)}{18\,x^9}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^10,x, algorithm="giac")
```

output
```
b^3*log(abs(x))*sgn(b*x^3 + a) - 1/18*(11*b^3*x^9*sgn(b*x^3 + a) + 18*a*b^
2*x^6*sgn(b*x^3 + a) + 9*a^2*b*x^3*sgn(b*x^3 + a) + 2*a^3*sgn(b*x^3 + a))/
x^9
```

3.42.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{10}}\,dx = \int \frac{(a^2+2\,a\,b\,x^3+b^2\,x^6)^{3/2}}{x^{10}}\,dx$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^10,x)
```

output
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^10, x)
```

3.43 $\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{11}}\,dx$

3.43.1 Optimal result

Integrand size = 26, antiderivative size = 165

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{11}}\,dx = -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}(a+bx^3)} - \frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{7x^7(a+bx^3)} - \frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{4x^4(a+bx^3)} - \frac{b^3\sqrt{a^2+2abx^3+b^2x^6}}{x(a+bx^3)}$$

output
```
-1/10*a^3*((b*x^3+a)^2)^(1/2)/x^10/(b*x^3+a)-3/7*a^2*b*((b*x^3+a)^2)^(1/2)
/x^7/(b*x^3+a)-3/4*a*b^2*((b*x^3+a)^2)^(1/2)/x^4/(b*x^3+a)-b^3*((b*x^3+a)^
2)^(1/2)/x/(b*x^3+a)
```

3.43.2 Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.37

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{11}}\,dx = -\frac{\sqrt{(a+bx^3)^2}(14a^3+60a^2bx^3+105ab^2x^6+140b^3x^9)}{140x^{10}(a+bx^3)}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^11,x]
```

output
```
-1/140*(Sqrt[(a + b*x^3)^2]*(14*a^3 + 60*a^2*b*x^3 + 105*a*b^2*x^6 + 140*b
^3*x^9))/(x^10*(a + b*x^3))
```

3.43.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.44, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{11}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b^3\left(bx^3+a\right)^3}{x^{11}}dx}{b^3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(bx^3+a\right)^3}{x^{11}}dx}{a+bx^3}$$

$$\Big\downarrow \text{802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^3}{x^{11}}+\frac{3ba^2}{x^8}+\frac{3b^2a}{x^5}+\frac{b^3}{x^2}\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(-\frac{a^3}{10x^{10}}-\frac{3a^2b}{7x^7}-\frac{3ab^2}{4x^4}-\frac{b^3}{x}\right)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^11,x]
```

output
```
((-1/10*a^3/x^10 - (3*a^2*b)/(7*x^7) - (3*a*b^2)/(4*x^4) - b^3/x)*Sqrt[a^2
 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3)
```

3.43.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.43.4 Maple [A] (verified)

Time = 12.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.35

method	result	size
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(-b^{3} x^{9}-\frac{3}{4} b^{2} x^{6} a-\frac{3}{7} a^{2} b\,x^{3}-\frac{1}{10} a^{3}\right)}{\left(b\,x^{3}+a\right) x^{10}}$	57
gosper	$-\frac{\left(140 b^{3} x^{9}+105 b^{2} x^{6} a+60 a^{2} b\,x^{3}+14 a^{3}\right) \left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{140 x^{10} \left(b\,x^{3}+a\right)^{3}}$	58
default	$-\frac{\left(140 b^{3} x^{9}+105 b^{2} x^{6} a+60 a^{2} b\,x^{3}+14 a^{3}\right) \left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{140 x^{10} \left(b\,x^{3}+a\right)^{3}}$	58

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^11,x,method=_RETURNVERBOSE)
```

output
```
((b*x^3+a)^2)^(1/2)/(b*x^3+a)*(-b^3*x^9-3/4*b^2*x^6*a-3/7*a^2*b*x^3-1/10*a
^3)/x^10
```

3.43.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.22

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^{11}}\,dx = -\frac{140\,b^3x^9 + 105\,ab^2x^6 + 60\,a^2bx^3 + 14\,a^3}{140\,x^{10}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^11,x, algorithm="fricas")
```

output
```
-1/140*(140*b^3*x^9 + 105*a*b^2*x^6 + 60*a^2*b*x^3 + 14*a^3)/x^10
```

3.43.6 Sympy [F]

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^{11}}\,dx = \int \frac{\left((a + bx^3)^2\right)^{\frac{3}{2}}}{x^{11}}\,dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**11,x)
```

output
```
Integral(((a + b*x**3)**2)**(3/2)/x**11, x)
```

3.43.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.22

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^{11}}\,dx = -\frac{140\,b^3x^9 + 105\,ab^2x^6 + 60\,a^2bx^3 + 14\,a^3}{140\,x^{10}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^11,x, algorithm="maxima")
```

output
```
-1/140*(140*b^3*x^9 + 105*a*b^2*x^6 + 60*a^2*b*x^3 + 14*a^3)/x^10
```

3.43.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.42

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{11}}\,dx = -\frac{140\,b^3x^9\mathrm{sgn}(bx^3+a)+105\,ab^2x^6\mathrm{sgn}(bx^3+a)+60\,a^2bx^3\mathrm{sgn}(bx^3+a)+14\,a^3\mathrm{sgn}(bx^3+a)}{140\,x^{10}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^11,x, algorithm="giac")
```

output
```
-1/140*(140*b^3*x^9*sgn(b*x^3 + a) + 105*a*b^2*x^6*sgn(b*x^3 + a) + 60*a^2
*b*x^3*sgn(b*x^3 + a) + 14*a^3*sgn(b*x^3 + a))/x^10
```

3.43.9 Mupad [B] (verification not implemented)

Time = 8.23 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.92

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{11}}\,dx = -\frac{a^3\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{10\,x^{10}\,\left(b\,x^3+a\right)}-\frac{b^3\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{x\,\left(b\,x^3+a\right)} -\frac{3\,a\,b^2\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{4\,x^4\,\left(b\,x^3+a\right)}-\frac{3\,a^2\,b\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{7\,x^7\,\left(b\,x^3+a\right)}$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^11,x)
```

output
```
- (a^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(10*x^10*(a + b*x^3)) - (b^3*(a^
2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(x*(a + b*x^3)) - (3*a*b^2*(a^2 + b^2*x^6
+ 2*a*b*x^3)^(1/2))/(4*x^4*(a + b*x^3)) - (3*a^2*b*(a^2 + b^2*x^6 + 2*a*b*
x^3)^(1/2))/(7*x^7*(a + b*x^3))
```

3.44 $\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{12}}\,dx$

3.44.1 Optimal result

Integrand size = 26, antiderivative size = 167

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{12}}\,dx = -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}(a+bx^3)} - \frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{8x^8(a+bx^3)} - \frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{5x^5(a+bx^3)} - \frac{b^3\sqrt{a^2+2abx^3+b^2x^6}}{2x^2(a+bx^3)}$$

output
```
-1/11*a^3*((b*x^3+a)^2)^(1/2)/x^11/(b*x^3+a)-3/8*a^2*b*((b*x^3+a)^2)^(1/2)
/x^8/(b*x^3+a)-3/5*a*b^2*((b*x^3+a)^2)^(1/2)/x^5/(b*x^3+a)-1/2*b^3*((b*x^3
+a)^2)^(1/2)/x^2/(b*x^3+a)
```

3.44.2 Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.37

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{12}}\,dx = -\frac{\sqrt{(a+bx^3)^2}(40a^3+165a^2bx^3+264ab^2x^6+220b^3x^9)}{440x^{11}(a+bx^3)}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^12,x]
```

output
```
-1/440*(Sqrt[(a + b*x^3)^2]*(40*a^3 + 165*a^2*b*x^3 + 264*a*b^2*x^6 + 220*
b^3*x^9))/(x^11*(a + b*x^3))
```

3.44.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.45, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{12}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b^3\left(bx^3+a\right)^3}{x^{12}}dx}{b^3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(bx^3+a\right)^3}{x^{12}}dx}{a+bx^3}$$

$$\Big\downarrow \text{802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^3}{x^{12}}+\frac{3ba^2}{x^9}+\frac{3b^2a}{x^6}+\frac{b^3}{x^3}\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(-\frac{a^3}{11x^{11}}-\frac{3a^2b}{8x^8}-\frac{3ab^2}{5x^5}-\frac{b^3}{2x^2}\right)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^12,x]
```

output
```
((-1/11*a^3/x^11 - (3*a^2*b)/(8*x^8) - (3*a*b^2)/(5*x^5) - b^3/(2*x^2))*Sq
rt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3)
```

3.44.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.44.4 Maple [A] (verified)

Time = 13.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.34

method	result	size
risch	$\frac{\sqrt{(b\,x^3+a)^2}\,\left(-\frac{1}{2}b^3x^9-\frac{3}{5}b^2x^6a-\frac{3}{8}a^2b\,x^3-\frac{1}{11}a^3\right)}{(b\,x^3+a)x^{11}}$	57
gosper	$-\frac{(220b^3x^9+264b^2x^6a+165a^2b\,x^3+40a^3)\left((b\,x^3+a)^2\right)^{\frac{3}{2}}}{440x^{11}(b\,x^3+a)^3}$	58
default	$-\frac{(220b^3x^9+264b^2x^6a+165a^2b\,x^3+40a^3)\left((b\,x^3+a)^2\right)^{\frac{3}{2}}}{440x^{11}(b\,x^3+a)^3}$	58

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^12,x,method=_RETURNVERBOSE)
```

output
```
((b*x^3+a)^2)^(1/2)/(b*x^3+a)*(-1/2*b^3*x^9-3/5*b^2*x^6*a-3/8*a^2*b*x^3-1/
11*a^3)/x^11
```

3.44.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.22

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{12}}\,dx = -\frac{220\,b^3x^9+264\,ab^2x^6+165\,a^2bx^3+40\,a^3}{440\,x^{11}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^12,x, algorithm="fricas")
```

output
```
-1/440*(220*b^3*x^9 + 264*a*b^2*x^6 + 165*a^2*b*x^3 + 40*a^3)/x^11
```

3.44.6 Sympy [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{12}}\,dx = \int \frac{\left((a+bx^3)^2\right)^{\frac{3}{2}}}{x^{12}}\,dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**12,x)
```

output
```
Integral(((a + b*x**3)**2)**(3/2)/x**12, x)
```

3.44.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.22

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{12}}\,dx = -\frac{220\,b^3x^9+264\,ab^2x^6+165\,a^2bx^3+40\,a^3}{440\,x^{11}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^12,x, algorithm="maxima")
```

output
```
-1/440*(220*b^3*x^9 + 264*a*b^2*x^6 + 165*a^2*b*x^3 + 40*a^3)/x^11
```

3.44.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.41

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{12}}\,dx = -\frac{220\,b^3x^9\mathrm{sgn}(bx^3+a)+264\,ab^2x^6\mathrm{sgn}(bx^3+a)+165\,a^2bx^3\mathrm{sgn}(bx^3+a)+40\,a^3\mathrm{sgn}(bx^3+a)}{440\,x^{11}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^12,x, algorithm="giac")
```

output
```
-1/440*(220*b^3*x^9*sgn(b*x^3 + a) + 264*a*b^2*x^6*sgn(b*x^3 + a) + 165*a^
2*b*x^3*sgn(b*x^3 + a) + 40*a^3*sgn(b*x^3 + a))/x^11
```

3.44.9 Mupad [B] (verification not implemented)

Time = 8.25 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.90

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{12}}\,dx = -\frac{a^3\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{11\,x^{11}\,(b\,x^3+a)} - \frac{b^3\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{2\,x^2\,(b\,x^3+a)} - \frac{3\,a\,b^2\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{5\,x^5\,(b\,x^3+a)} - \frac{3\,a^2\,b\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{8\,x^8\,(b\,x^3+a)}$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^12,x)
```

output
```
- (a^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(11*x^11*(a + b*x^3)) - (b^3*(a^
2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(2*x^2*(a + b*x^3)) - (3*a*b^2*(a^2 + b^2*
x^6 + 2*a*b*x^3)^(1/2))/(5*x^5*(a + b*x^3)) - (3*a^2*b*(a^2 + b^2*x^6 + 2*
a*b*x^3)^(1/2))/(8*x^8*(a + b*x^3))
```

3.45 $\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{13}} \, dx$

3.45.1 Optimal result

Integrand size = 26, antiderivative size = 41

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{13}} \, dx = -\frac{(a+bx^3)^3 \sqrt{a^2+2abx^3+b^2x^6}}{12ax^{12}}$$

output
```
-1/12*(b*x^3+a)^3*((b*x^3+a)^2)^(1/2)/a/x^12
```

3.45.2 Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.44

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{13}} \, dx = -\frac{\sqrt{(a+bx^3)^2}(a^3+4a^2bx^3+6ab^2x^6+4b^3x^9)}{12x^{12}(a+bx^3)}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^13,x]
```

output
```
-1/12*(Sqrt[(a + b*x^3)^2]*(a^3 + 4*a^2*b*x^3 + 6*a*b^2*x^6 + 4*b^3*x^9))/
(x^12*(a + b*x^3))
```

3.45.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1384, 27, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{13}}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b^3\left(bx^3+a\right)^3}{x^{13}}dx}{b^3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(bx^3+a\right)^3}{x^{13}}dx}{a+bx^3}$$

$$\Big\downarrow \text{ 796}$$

$$-\frac{\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}{12ax^{12}}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^13,x]
```

output
```
-1/12*((a + b*x^3)^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a*x^12)
```

3.45.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

3.45.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

method	result	size
pseudoelliptic	$-\frac{\operatorname{csgn}\left(b\,x^{3}+a\right)\left(2 b\,x^{3}+a\right)\left(2 b^{2} x^{6}+2 a b\,x^{3}+a^{2}\right)}{12 x^{12}}$	41
gosper	$-\frac{\left(4 b^{3} x^{9}+6 b^{2} x^{6} a+4 a^{2} b\,x^{3}+a^{3}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{12 x^{12} \left(b\,x^{3}+a\right)^{3}}$	56
default	$-\frac{\left(4 b^{3} x^{9}+6 b^{2} x^{6} a+4 a^{2} b\,x^{3}+a^{3}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{12 x^{12} \left(b\,x^{3}+a\right)^{3}}$	56
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(-\frac{1}{3} b^{3} x^{9}-\frac{1}{2} b^{2} x^{6} a-\frac{1}{3} a^{2} b\,x^{3}-\frac{1}{12} a^{3}\right)}{\left(b\,x^{3}+a\right) x^{12}}$	57

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^13,x,method=_RETURNVERBOSE)
```

output
```
-1/12*csgn(b*x^3+a)*(2*b*x^3+a)*(2*b^2*x^6+2*a*b*x^3+a^2)/x^12
```

3.45.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{13}}\,dx=-\frac{4\,b^3x^9+6\,ab^2x^6+4\,a^2bx^3+a^3}{12\,x^{12}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^13,x, algorithm="fricas")
```

output
```
-1/12*(4*b^3*x^9 + 6*a*b^2*x^6 + 4*a^2*b*x^3 + a^3)/x^12
```

3.45.6 Sympy [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{13}}\,dx = \int \frac{\left((a+bx^3)^2\right)^{\frac{3}{2}}}{x^{13}}\,dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**13,x)
```

output
```
Integral(((a + b*x**3)**2)**(3/2)/x**13, x)
```

3.45.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(28) = 56$.

Time = 0.22 (sec) , antiderivative size = 148, normalized size of antiderivative = 3.61

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{13}}\,dx = \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}b^4}{12\,a^4} + \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}b^3}{12\,a^3x^3} - \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}b^2}{12\,a^4x^6} + \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}b}{12\,a^3x^9} - \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}}{12\,a^2x^{12}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^13,x, algorithm="maxima")
```

output
```
1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*b^4/a^4 + 1/12*(b^2*x^6 + 2*a*b*x^3
 + a^2)^(3/2)*b^3/(a^3*x^3) - 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^2/(
a^4*x^6) + 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b/(a^3*x^9) - 1/12*(b^2*
x^6 + 2*a*b*x^3 + a^2)^(5/2)/(a^2*x^12)
```

3.45.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(28) = 56$.

Time = 0.29 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.66

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{13}}\,dx = -\frac{4\,b^3x^9\mathrm{sgn}(bx^3+a)+6\,ab^2x^6\mathrm{sgn}(bx^3+a)+4\,a^2bx^3\mathrm{sgn}(bx^3+a)+a^3\mathrm{sgn}(bx^3+a)}{12\,x^{12}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^13,x, algorithm="giac")
```

output
```
-1/12*(4*b^3*x^9*sgn(b*x^3 + a) + 6*a*b^2*x^6*sgn(b*x^3 + a) + 4*a^2*b*x^3
*sgn(b*x^3 + a) + a^3*sgn(b*x^3 + a))/x^12
```

3.45.9 Mupad [B] (verification not implemented)

Time = 8.22 (sec) , antiderivative size = 151, normalized size of antiderivative = 3.68

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{13}}\,dx = -\frac{a^3\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{12\,x^{12}\,\left(b\,x^3+a\right)} - \frac{b^3\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{3\,x^3\,\left(b\,x^3+a\right)} - \frac{a\,b^2\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{2\,x^6\,\left(b\,x^3+a\right)} - \frac{a^2\,b\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{3\,x^9\,\left(b\,x^3+a\right)}$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^13,x)
```

output
```
- (a^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(12*x^12*(a + b*x^3)) - (b^3*(a^
2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(3*x^3*(a + b*x^3)) - (a*b^2*(a^2 + b^2*x^
6 + 2*a*b*x^3)^(1/2))/(2*x^6*(a + b*x^3)) - (a^2*b*(a^2 + b^2*x^6 + 2*a*b*
x^3)^(1/2))/(3*x^9*(a + b*x^3))
```

3.46 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{14}}\,dx$

3.46.1 Optimal result

Integrand size = 26, antiderivative size = 167

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{14}}\,dx = -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}\left(a+bx^3\right)} - \frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}\left(a+bx^3\right)} - \frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)} - \frac{b^3\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}$$

output
```
-1/13*a^3*((b*x^3+a)^2)^(1/2)/x^13/(b*x^3+a)-3/10*a^2*b*((b*x^3+a)^2)^(1/2
)/x^10/(b*x^3+a)-3/7*a*b^2*((b*x^3+a)^2)^(1/2)/x^7/(b*x^3+a)-1/4*b^3*((b*x
^3+a)^2)^(1/2)/x^4/(b*x^3+a)
```

3.46.2 Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.37

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{14}}\,dx = -\frac{\sqrt{\left(a+bx^3\right)^2}\left(140a^3+546a^2bx^3+780ab^2x^6+455b^3x^9\right)}{1820x^{13}\left(a+bx^3\right)}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^14,x]
```

output
```
-1/1820*(Sqrt[(a + b*x^3)^2]*(140*a^3 + 546*a^2*b*x^3 + 780*a*b^2*x^6 + 45
5*b^3*x^9))/(x^13*(a + b*x^3))
```

3.46.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.45, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{14}}dx$$

$$\Big\downarrow \ 1384$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b^3\left(bx^3+a\right)^3}{x^{14}}dx}{b^3\left(a+bx^3\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(bx^3+a\right)^3}{x^{14}}dx}{a+bx^3}$$

$$\Big\downarrow \ 802$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^3}{x^{14}}+\frac{3ba^2}{x^{11}}+\frac{3b^2a}{x^8}+\frac{b^3}{x^5}\right)dx}{a+bx^3}$$

$$\Big\downarrow \ 2009$$

$$\frac{\left(-\frac{a^3}{13x^{13}}-\frac{3a^2b}{10x^{10}}-\frac{3ab^2}{7x^7}-\frac{b^3}{4x^4}\right)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^14,x]
```

output
```
((-1/13*a^3/x^13 - (3*a^2*b)/(10*x^10) - (3*a*b^2)/(7*x^7) - b^3/(4*x^4))*
Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3)
```

3.46.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.46.4 Maple [A] (verified)

Time = 16.68 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.34

method	result	size
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(-\frac{1}{4}b^{3}x^{9}-\frac{3}{7}b^{2}x^{6}a-\frac{3}{10}a^{2}b\,x^{3}-\frac{1}{13}a^{3}\right)}{\left(b\,x^{3}+a\right)x^{13}}$	57
gosper	$-\frac{\left(455b^{3}x^{9}+780b^{2}x^{6}a+546a^{2}b\,x^{3}+140a^{3}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{1820x^{13}\left(b\,x^{3}+a\right)^{3}}$	58
default	$-\frac{\left(455b^{3}x^{9}+780b^{2}x^{6}a+546a^{2}b\,x^{3}+140a^{3}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{1820x^{13}\left(b\,x^{3}+a\right)^{3}}$	58

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^14,x,method=_RETURNVERBOSE)
```

output
```
((b*x^3+a)^2)^(1/2)/(b*x^3+a)*(-1/4*b^3*x^9-3/7*b^2*x^6*a-3/10*a^2*b*x^3-1
/13*a^3)/x^13
```

3.46.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.22

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^{14}}\,dx = -\frac{455\,b^3x^9 + 780\,ab^2x^6 + 546\,a^2bx^3 + 140\,a^3}{1820\,x^{13}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^14,x, algorithm="fricas")
```

output
```
-1/1820*(455*b^3*x^9 + 780*a*b^2*x^6 + 546*a^2*b*x^3 + 140*a^3)/x^13
```

3.46.6 Sympy [F]

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^{14}}\,dx = \int \frac{\left((a + bx^3)^2\right)^{\frac{3}{2}}}{x^{14}}\,dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**14,x)
```

output
```
Integral(((a + b*x**3)**2)**(3/2)/x**14, x)
```

3.46.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.22

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^{14}}\,dx = -\frac{455\,b^3x^9 + 780\,ab^2x^6 + 546\,a^2bx^3 + 140\,a^3}{1820\,x^{13}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^14,x, algorithm="maxima")
```

output
```
-1/1820*(455*b^3*x^9 + 780*a*b^2*x^6 + 546*a^2*b*x^3 + 140*a^3)/x^13
```

3.46.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.41

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{14}}\,dx = -\frac{455\,b^3x^9\mathrm{sgn}(bx^3+a)+780\,ab^2x^6\mathrm{sgn}(bx^3+a)+546\,a^2bx^3\mathrm{sgn}(bx^3+a)+140\,a^3\mathrm{sgn}(bx^3+a)}{1820\,x^{13}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^14,x, algorithm="giac")
```

output
```
-1/1820*(455*b^3*x^9*sgn(b*x^3 + a) + 780*a*b^2*x^6*sgn(b*x^3 + a) + 546*a
^2*b*x^3*sgn(b*x^3 + a) + 140*a^3*sgn(b*x^3 + a))/x^13
```

3.46.9 Mupad [B] (verification not implemented)

Time = 8.16 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.90

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{14}}\,dx = -\frac{a^3\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{13\,x^{13}\,(b\,x^3+a)} - \frac{b^3\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{4\,x^4\,(b\,x^3+a)} - \frac{3\,a\,b^2\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{7\,x^7\,(b\,x^3+a)} - \frac{3\,a^2\,b\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{10\,x^{10}\,(b\,x^3+a)}$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^14,x)
```

output
```
- (a^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(13*x^13*(a + b*x^3)) - (b^3*(a^
2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(4*x^4*(a + b*x^3)) - (3*a*b^2*(a^2 + b^2*
x^6 + 2*a*b*x^3)^(1/2))/(7*x^7*(a + b*x^3)) - (3*a^2*b*(a^2 + b^2*x^6 + 2*
a*b*x^3)^(1/2))/(10*x^10*(a + b*x^3))
```

3.47 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{15}}\,dx$

3.47.1 Optimal result

Integrand size = 26, antiderivative size = 167

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{15}}\,dx=-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{14x^{14}\left(a+bx^3\right)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}\left(a+bx^3\right)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\left(a+bx^3\right)}-\frac{b^3\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}$$

output
```
-1/14*a^3*((b*x^3+a)^2)^(1/2)/x^14/(b*x^3+a)-3/11*a^2*b*((b*x^3+a)^2)^(1/2
)/x^11/(b*x^3+a)-3/8*a*b^2*((b*x^3+a)^2)^(1/2)/x^8/(b*x^3+a)-1/5*b^3*((b*x
^3+a)^2)^(1/2)/x^5/(b*x^3+a)
```

3.47.2 Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.37

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{15}}\,dx=-\frac{\sqrt{\left(a+bx^3\right)^2}\left(220a^3+840a^2bx^3+1155ab^2x^6+616b^3x^9\right)}{3080x^{14}\left(a+bx^3\right)}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^15,x]
```

output
```
-1/3080*(Sqrt[(a + b*x^3)^2]*(220*a^3 + 840*a^2*b*x^3 + 1155*a*b^2*x^6 + 6
16*b^3*x^9))/(x^14*(a + b*x^3))
```

3.47.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.45, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{15}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b^3\left(bx^3+a\right)^3}{x^{15}}dx}{b^3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(bx^3+a\right)^3}{x^{15}}dx}{a+bx^3}$$

$$\Big\downarrow \text{802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^3}{x^{15}}+\frac{3ba^2}{x^{12}}+\frac{3b^2a}{x^9}+\frac{b^3}{x^6}\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(-\frac{a^3}{14x^{14}}-\frac{3a^2b}{11x^{11}}-\frac{3ab^2}{8x^8}-\frac{b^3}{5x^5}\right)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^15,x]
```

output
```
((-1/14*a^3/x^14 - (3*a^2*b)/(11*x^11) - (3*a*b^2)/(8*x^8) - b^3/(5*x^5))*
Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3)
```

3.47.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.47.4 Maple [A] (verified)

Time = 19.19 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.34

method	result	size
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(-\frac{1}{14}a^{3}-\frac{3}{11}a^{2}b\,x^{3}-\frac{3}{8}b^{2}x^{6}a-\frac{1}{5}b^{3}x^{9}\right)}{\left(b\,x^{3}+a\right)x^{14}}$	57
gosper	$-\frac{\left(616b^{3}x^{9}+1155b^{2}x^{6}a+840a^{2}b\,x^{3}+220a^{3}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{3080x^{14}\left(b\,x^{3}+a\right)^{3}}$	58
default	$-\frac{\left(616b^{3}x^{9}+1155b^{2}x^{6}a+840a^{2}b\,x^{3}+220a^{3}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{3080x^{14}\left(b\,x^{3}+a\right)^{3}}$	58

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^15,x,method=_RETURNVERBOSE)
```

output
```
((b*x^3+a)^2)^(1/2)/(b*x^3+a)*(-1/14*a^3-3/11*a^2*b*x^3-3/8*b^2*x^6*a-1/5*
b^3*x^9)/x^14
```

3.47.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.22

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^{15}}\,dx = -\frac{616\,b^3x^9 + 1155\,ab^2x^6 + 840\,a^2bx^3 + 220\,a^3}{3080\,x^{14}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^15,x, algorithm="fricas")
```

output
```
-1/3080*(616*b^3*x^9 + 1155*a*b^2*x^6 + 840*a^2*b*x^3 + 220*a^3)/x^14
```

3.47.6 Sympy [F]

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^{15}}\,dx = \int \frac{\left((a + bx^3)^2\right)^{\frac{3}{2}}}{x^{15}}\,dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**15,x)
```

output
```
Integral(((a + b*x**3)**2)**(3/2)/x**15, x)
```

3.47.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.22

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^{15}}\,dx = -\frac{616\,b^3x^9 + 1155\,ab^2x^6 + 840\,a^2bx^3 + 220\,a^3}{3080\,x^{14}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^15,x, algorithm="maxima")
```

output
```
-1/3080*(616*b^3*x^9 + 1155*a*b^2*x^6 + 840*a^2*b*x^3 + 220*a^3)/x^14
```

3.47.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.41

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{15}}\,dx = -\frac{616\,b^3x^9\operatorname{sgn}(bx^3+a)+1155\,ab^2x^6\operatorname{sgn}(bx^3+a)+840\,a^2bx^3\operatorname{sgn}(bx^3+a)+220\,a^3\operatorname{sgn}(bx^3+a)}{3080\,x^{14}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^15,x, algorithm="giac")
```

output
```
-1/3080*(616*b^3*x^9*sgn(b*x^3 + a) + 1155*a*b^2*x^6*sgn(b*x^3 + a) + 840*
a^2*b*x^3*sgn(b*x^3 + a) + 220*a^3*sgn(b*x^3 + a))/x^14
```

3.47.9 Mupad [B] (verification not implemented)

Time = 8.12 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.90

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{15}}\,dx = -\frac{a^3\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{14\,x^{14}\,\left(b\,x^3+a\right)} - \frac{b^3\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{5\,x^5\,\left(b\,x^3+a\right)} - \frac{3\,a\,b^2\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{8\,x^8\,\left(b\,x^3+a\right)} - \frac{3\,a^2\,b\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{11\,x^{11}\,\left(b\,x^3+a\right)}$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^15,x)
```

output
```
- (a^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(14*x^14*(a + b*x^3)) - (b^3*(a^
2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(5*x^5*(a + b*x^3)) - (3*a*b^2*(a^2 + b^2*
x^6 + 2*a*b*x^3)^(1/2))/(8*x^8*(a + b*x^3)) - (3*a^2*b*(a^2 + b^2*x^6 + 2*
a*b*x^3)^(1/2))/(11*x^11*(a + b*x^3))
```

3.48 $\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{16}}\,dx$

3.48.1 Optimal result

Integrand size = 26, antiderivative size = 84

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{16}}\,dx = -\frac{(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}}{15ax^{15}} + \frac{b(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}}{60a^2x^{12}}$$

output
```
-1/15*(b*x^3+a)^3*((b*x^3+a)^2)^(1/2)/a/x^15+1/60*b*(b*x^3+a)^3*((b*x^3+a)
^2)^(1/2)/a^2/x^12
```

3.48.2 Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.73

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{16}}\,dx = -\frac{\sqrt{(a+bx^3)^2}(4a^3+15a^2bx^3+20ab^2x^6+10b^3x^9)}{60x^{15}(a+bx^3)}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^16,x]
```

output
```
-1/60*(Sqrt[(a + b*x^3)^2]*(4*a^3 + 15*a^2*b*x^3 + 20*a*b^2*x^6 + 10*b^3*x
^9))/(x^15*(a + b*x^3))
```

3.48.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.89, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1384, 27, 798, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{16}}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b^3\left(bx^3+a\right)^3}{x^{16}}dx}{b^3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(bx^3+a\right)^3}{x^{16}}dx}{a+bx^3}$$

$$\Big\downarrow \text{ 798}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(bx^3+a\right)^3}{x^{18}}dx^3}{3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 55}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(-\frac{b\int \frac{\left(bx^3+a\right)^3}{x^{15}}dx^3}{5a}-\frac{\left(a+bx^3\right)^4}{5ax^{15}}\right)}{3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 48}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(\frac{b\left(a+bx^3\right)^4}{20a^2x^{12}}-\frac{\left(a+bx^3\right)^4}{5ax^{15}}\right)}{3\left(a+bx^3\right)}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^16,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-1/5*(a + b*x^3)^4/(a*x^15) + (b*(a + b*
x^3)^4)/(20*a^2*x^12)))/(3*(a + b*x^3))
```

3.48. $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{16}}dx$

3.48.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

3.48. $\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{16}}\,dx$

3.48.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.10 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.52

method	result	size
pseudoelliptic	$-\frac{\left(\frac{5}{2}b^{3}x^{9}+5b^{2}x^{6}a+\frac{15}{4}a^{2}b\,x^{3}+a^{3}\right)\operatorname{csgn}\left(b\,x^{3}+a\right)}{15x^{15}}$	44
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(-\frac{1}{15}a^{3}-\frac{1}{4}a^{2}b\,x^{3}-\frac{1}{3}b^{2}x^{6}a-\frac{1}{6}b^{3}x^{9}\right)}{\left(b\,x^{3}+a\right)x^{15}}$	57
gosper	$-\frac{\left(10b^{3}x^{9}+20b^{2}x^{6}a+15a^{2}b\,x^{3}+4a^{3}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{60x^{15}\left(b\,x^{3}+a\right)^{3}}$	58
default	$-\frac{\left(10b^{3}x^{9}+20b^{2}x^{6}a+15a^{2}b\,x^{3}+4a^{3}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{60x^{15}\left(b\,x^{3}+a\right)^{3}}$	58

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^16,x,method=_RETURNVERBOSE)
```

output
```
-1/15*(5/2*b^3*x^9+5*b^2*x^6*a+15/4*a^2*b*x^3+a^3)*csgn(b*x^3+a)/x^15
```

3.48.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.44

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{16}}\,dx=-\frac{10\,b^3x^9+20\,ab^2x^6+15\,a^2bx^3+4\,a^3}{60\,x^{15}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^16,x, algorithm="fricas")
```

output
```
-1/60*(10*b^3*x^9 + 20*a*b^2*x^6 + 15*a^2*b*x^3 + 4*a^3)/x^15
```

3.48.6 Sympy [F]

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^{16}}\, dx = \int \frac{\left((a + bx^3)^2\right)^{\frac{3}{2}}}{x^{16}}\, dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**16,x)
```

output
```
Integral(((a + b*x**3)**2)**(3/2)/x**16, x)
```

3.48.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. $2(58) = 116$.

Time = 0.21 (sec) , antiderivative size = 179, normalized size of antiderivative = 2.13

$$\begin{aligned}\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^{16}}\, dx = &-\frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{3}{2}}b^5}{12\,a^5} \\ &- \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{3}{2}}b^4}{12\,a^4x^3} + \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{5}{2}}b^3}{12\,a^5x^6} \\ &- \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{5}{2}}b^2}{12\,a^4x^9} + \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{5}{2}}b}{12\,a^3x^{12}} - \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{5}{2}}}{15\,a^2x^{15}}\end{aligned}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^16,x, algorithm="maxima")
```

output
```
-1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*b^5/a^5 - 1/12*(b^2*x^6 + 2*a*b*x^
3 + a^2)^(3/2)*b^4/(a^4*x^3) + 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^3/
(a^5*x^6) - 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^2/(a^4*x^9) + 1/12*(b
^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b/(a^3*x^12) - 1/15*(b^2*x^6 + 2*a*b*x^3 +
 a^2)^(5/2)/(a^2*x^15)
```

3.48.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.82

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{16}}\,dx = -\frac{10\,b^3x^9\mathrm{sgn}(bx^3+a)+20\,ab^2x^6\mathrm{sgn}(bx^3+a)+15\,a^2bx^3\mathrm{sgn}(bx^3+a)+4\,a^3\mathrm{sgn}(bx^3+a)}{60\,x^{15}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^16,x, algorithm="giac")
```

output
```
-1/60*(10*b^3*x^9*sgn(b*x^3 + a) + 20*a*b^2*x^6*sgn(b*x^3 + a) + 15*a^2*b*
x^3*sgn(b*x^3 + a) + 4*a^3*sgn(b*x^3 + a))/x^15
```

3.48.9 Mupad [B] (verification not implemented)

Time = 8.20 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.80

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{16}}\,dx = -\frac{a^3\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{15\,x^{15}\,(b\,x^3+a)} - \frac{b^3\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{6\,x^6\,(b\,x^3+a)} - \frac{a\,b^2\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{3\,x^9\,(b\,x^3+a)} - \frac{a^2\,b\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{4\,x^{12}\,(b\,x^3+a)}$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^16,x)
```

output
```
- (a^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(15*x^15*(a + b*x^3)) - (b^3*(a^
2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(6*x^6*(a + b*x^3)) - (a*b^2*(a^2 + b^2*x^
6 + 2*a*b*x^3)^(1/2))/(3*x^9*(a + b*x^3)) - (a^2*b*(a^2 + b^2*x^6 + 2*a*b*
x^3)^(1/2))/(4*x^12*(a + b*x^3))
```

3.49 $\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{17}}\,dx$

3.49.1 Optimal result

Integrand size = 26, antiderivative size = 167

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{17}}\,dx = -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{16x^{16}(a+bx^3)} - \frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}(a+bx^3)} - \frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}(a+bx^3)} - \frac{b^3\sqrt{a^2+2abx^3+b^2x^6}}{7x^7(a+bx^3)}$$

output
```
-1/16*a^3*((b*x^3+a)^2)^(1/2)/x^16/(b*x^3+a)-3/13*a^2*b*((b*x^3+a)^2)^(1/2
)/x^13/(b*x^3+a)-3/10*a*b^2*((b*x^3+a)^2)^(1/2)/x^10/(b*x^3+a)-1/7*b^3*((b
*x^3+a)^2)^(1/2)/x^7/(b*x^3+a)
```

3.49.2 Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.37

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{17}}\,dx = -\frac{\sqrt{(a+bx^3)^2}(455a^3+1680a^2bx^3+2184ab^2x^6+1040b^3x^9)}{7280x^{16}(a+bx^3)}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^17,x]
```

output
```
-1/7280*(Sqrt[(a + b*x^3)^2]*(455*a^3 + 1680*a^2*b*x^3 + 2184*a*b^2*x^6 +
1040*b^3*x^9))/(x^16*(a + b*x^3))
```

3.49.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.45, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{17}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b^3\left(bx^3+a\right)^3}{x^{17}}dx}{b^3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(bx^3+a\right)^3}{x^{17}}dx}{a+bx^3}$$

$$\Big\downarrow \text{802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^3}{x^{17}}+\frac{3ba^2}{x^{14}}+\frac{3b^2a}{x^{11}}+\frac{b^3}{x^8}\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(-\frac{a^3}{16x^{16}}-\frac{3a^2b}{13x^{13}}-\frac{3ab^2}{10x^{10}}-\frac{b^3}{7x^7}\right)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^17,x]
```

output
```
((-1/16*a^3/x^16 - (3*a^2*b)/(13*x^13) - (3*a*b^2)/(10*x^10) - b^3/(7*x^7)
)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3)
```

3.49.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.49.4 Maple [A] (verified)

Time = 24.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.34

method	result	size
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(-\frac{1}{16}a^{3}-\frac{3}{13}a^{2}b\,x^{3}-\frac{3}{10}b^{2}x^{6}a-\frac{1}{7}b^{3}x^{9}\right)}{\left(b\,x^{3}+a\right)x^{16}}$	57
gosper	$-\frac{\left(1040b^{3}x^{9}+2184b^{2}x^{6}a+1680a^{2}b\,x^{3}+455a^{3}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{7280x^{16}\left(b\,x^{3}+a\right)^{3}}$	58
default	$-\frac{\left(1040b^{3}x^{9}+2184b^{2}x^{6}a+1680a^{2}b\,x^{3}+455a^{3}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{7280x^{16}\left(b\,x^{3}+a\right)^{3}}$	58

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^17,x,method=_RETURNVERBOSE)
```

output
```
((b*x^3+a)^2)^(1/2)/(b*x^3+a)*(-1/16*a^3-3/13*a^2*b*x^3-3/10*b^2*x^6*a-1/7
*b^3*x^9)/x^16
```

3.49.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.22

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{17}}\,dx = -\frac{1040\,b^3x^9+2184\,ab^2x^6+1680\,a^2bx^3+455\,a^3}{7280\,x^{16}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^17,x, algorithm="fricas")
```

output
```
-1/7280*(1040*b^3*x^9 + 2184*a*b^2*x^6 + 1680*a^2*b*x^3 + 455*a^3)/x^16
```

3.49.6 Sympy [F]

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{17}}\,dx = \int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}{x^{17}}\,dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**17,x)
```

output
```
Integral(((a + b*x**3)**2)**(3/2)/x**17, x)
```

3.49.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.22

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{17}}\,dx = -\frac{1040\,b^3x^9+2184\,ab^2x^6+1680\,a^2bx^3+455\,a^3}{7280\,x^{16}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^17,x, algorithm="maxima")
```

output
```
-1/7280*(1040*b^3*x^9 + 2184*a*b^2*x^6 + 1680*a^2*b*x^3 + 455*a^3)/x^16
```

3.49.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.41

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{17}}\,dx = -\frac{1040\,b^3x^9\operatorname{sgn}(bx^3+a)+2184\,ab^2x^6\operatorname{sgn}(bx^3+a)+1680\,a^2bx^3\operatorname{sgn}(bx^3+a)+455\,a^3\operatorname{sgn}(bx^3+a)}{7280\,x^{16}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^17,x, algorithm="giac")
```

output
```
-1/7280*(1040*b^3*x^9*sgn(b*x^3 + a) + 2184*a*b^2*x^6*sgn(b*x^3 + a) + 168
0*a^2*b*x^3*sgn(b*x^3 + a) + 455*a^3*sgn(b*x^3 + a))/x^16
```

3.49.9 Mupad [B] (verification not implemented)

Time = 8.28 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.90

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{17}}\,dx = -\frac{a^3\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{16\,x^{16}\,(b\,x^3+a)} - \frac{b^3\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{7\,x^7\,(b\,x^3+a)} - \frac{3\,a\,b^2\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{10\,x^{10}\,(b\,x^3+a)} - \frac{3\,a^2\,b\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{13\,x^{13}\,(b\,x^3+a)}$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^17,x)
```

output
```
- (a^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(16*x^16*(a + b*x^3)) - (b^3*(a^
2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(7*x^7*(a + b*x^3)) - (3*a*b^2*(a^2 + b^2*
x^6 + 2*a*b*x^3)^(1/2))/(10*x^10*(a + b*x^3)) - (3*a^2*b*(a^2 + b^2*x^6 +
2*a*b*x^3)^(1/2))/(13*x^13*(a + b*x^3))
```

3.50 $\int x^{13}(a^2+2abx^3+b^2x^6)^{5/2}\,dx$

3.50.1 Optimal result

Integrand size = 26, antiderivative size = 255

$$\int x^{13}\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{a^5x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14\left(a+bx^3\right)}+\frac{5a^4bx^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17\left(a+bx^3\right)}+\frac{a^3b^2x^{20}\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{10a^2b^3x^{23}\sqrt{a^2+2abx^3+b^2x^6}}{23\left(a+bx^3\right)}+\frac{5ab^4x^{26}\sqrt{a^2+2abx^3+b^2x^6}}{26\left(a+bx^3\right)}+\frac{b^5x^{29}\sqrt{a^2+2abx^3+b^2x^6}}{29\left(a+bx^3\right)}$$

output

```
1/14*a^5*x^14*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/17*a^4*b*x^17*((b*x^3+a)^2)^
(1/2)/(b*x^3+a)+1/2*a^3*b^2*x^20*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+10/23*a^2*b
^3*x^23*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/26*a*b^4*x^26*((b*x^3+a)^2)^(1/2)/
(b*x^3+a)+1/29*b^5*x^29*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

3.50.2 Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int x^{13}\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{x^{14}\sqrt{(a+bx^3)^2}(147407a^5+606970a^4bx^3+1031849a^3b^2x^6+897260a^2b^3x^9+396865ab^4x^{12}}{2063698\,(a+bx^3)}$$

input
```
Integrate[x^13*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
(x^14*Sqrt[(a + b*x^3)^2]*(147407*a^5 + 606970*a^4*b*x^3 + 1031849*a^3*b^2
*x^6 + 897260*a^2*b^3*x^9 + 396865*a*b^4*x^12 + 71162*b^5*x^15))/(2063698*
(a + b*x^3))
```

3.50.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.40, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{13}\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx$$

$$\Big\downarrow \ 1384$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int b^5x^{13}\left(bx^3+a\right)^5dx}{b^5\left(a+bx^3\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^{13}\left(bx^3+a\right)^5dx}{a+bx^3}$$

$$\Big\downarrow \ 802$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(b^5x^{28}+5ab^4x^{25}+10a^2b^3x^{22}+10a^3b^2x^{19}+5a^4bx^{16}+a^5x^{13}\right)dx}{a+bx^3}$$

$$\Big\downarrow \ 2009$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(\frac{a^5x^{14}}{14}+\frac{5}{17}a^4bx^{17}+\frac{1}{2}a^3b^2x^{20}+\frac{10}{23}a^2b^3x^{23}+\frac{5}{26}ab^4x^{26}+\frac{b^5x^{29}}{29}\right)}{a+bx^3}$$

input
```
Int[x^13*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*((a^5*x^14)/14 + (5*a^4*b*x^17)/17 + (a^3
*b^2*x^20)/2 + (10*a^2*b^3*x^23)/23 + (5*a*b^4*x^26)/26 + (b^5*x^29)/29))/
(a + b*x^3)
```

3.50.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.50.4 Maple [A] (verified)

Time = 14.54 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.31

method	result
gosper	$\frac{x^{14}(71162b^5x^{15}+396865a\,b^4x^{12}+897260a^2b^3x^9+1031849a^3b^2x^6+606970a^4b\,x^3+147407a^5)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{2063698(b\,x^3+a)^5}$
default	$\frac{x^{14}(71162b^5x^{15}+396865a\,b^4x^{12}+897260a^2b^3x^9+1031849a^3b^2x^6+606970a^4b\,x^3+147407a^5)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{2063698(b\,x^3+a)^5}$
risch	$\frac{a^5x^{14}\sqrt{(b\,x^3+a)^2}}{14b\,x^3+14a}+\frac{5a^4b\,x^{17}\sqrt{(b\,x^3+a)^2}}{17(b\,x^3+a)}+\frac{a^3b^2x^{20}\sqrt{(b\,x^3+a)^2}}{2b\,x^3+2a}+\frac{10a^2b^3x^{23}\sqrt{(b\,x^3+a)^2}}{23(b\,x^3+a)}+\frac{5a\,b^4x^{26}\sqrt{(b\,x^3+a)^2}}{26(b\,x^3+a)}+\frac{b^5x^{29}}{29}$

input
```
int(x^13*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/2063698*x^14*(71162*b^5*x^15+396865*a*b^4*x^12+897260*a^2*b^3*x^9+103184
9*a^3*b^2*x^6+606970*a^4*b*x^3+147407*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

3.50.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.22

$$\int x^{13}\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{1}{29}\,b^5x^{29}+\frac{5}{26}\,ab^4x^{26}+\frac{10}{23}\,a^2b^3x^{23}+\frac{1}{2}\,a^3b^2x^{20}+\frac{5}{17}\,a^4bx^{17}+\frac{1}{14}\,a^5x^{14}$$

input
```
integrate(x^13*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

output
```
1/29*b^5*x^29 + 5/26*a*b^4*x^26 + 10/23*a^2*b^3*x^23 + 1/2*a^3*b^2*x^20 +
5/17*a^4*b*x^17 + 1/14*a^5*x^14
```

3.50.6 Sympy [F]

$$\int x^{13}\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\int x^{13}\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(x**13*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

output
```
Integral(x**13*((a + b*x**3)**2)**(5/2), x)
```

3.50.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.22

$$\int x^{13}\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{1}{29}b^5x^{29}+\frac{5}{26}ab^4x^{26}+\frac{10}{23}a^2b^3x^{23}+\frac{1}{2}a^3b^2x^{20}+\frac{5}{17}a^4bx^{17}+\frac{1}{14}a^5x^{14}$$

input
```
integrate(x^13*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

output
```
1/29*b^5*x^29 + 5/26*a*b^4*x^26 + 10/23*a^2*b^3*x^23 + 1/2*a^3*b^2*x^20 +
5/17*a^4*b*x^17 + 1/14*a^5*x^14
```

3.50.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.41

$$\int x^{13}\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{1}{29}b^5x^{29}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{26}ab^4x^{26}\mathrm{sgn}\left(bx^3+a\right)+\frac{10}{23}a^2b^3x^{23}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{2}a^3b^2x^{20}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{17}a^4bx^{17}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{14}a^5x^{14}\mathrm{sgn}\left(bx^3+a\right)$$

input
```
integrate(x^13*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

output
```
1/29*b^5*x^29*sgn(b*x^3 + a) + 5/26*a*b^4*x^26*sgn(b*x^3 + a) + 10/23*a^2*
b^3*x^23*sgn(b*x^3 + a) + 1/2*a^3*b^2*x^20*sgn(b*x^3 + a) + 5/17*a^4*b*x^1
7*sgn(b*x^3 + a) + 1/14*a^5*x^14*sgn(b*x^3 + a)
```

3.50.9 Mupad [F(-1)]

Timed out.

$$\int x^{13}\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\int x^{13}\,{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{5/2}\,dx$$

input
```
int(x^13*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

output
```
int(x^13*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

3.51 $\int x^{12}(a^2+2abx^3+b^2x^6)^{5/2}\,dx$

3.51.1 Optimal result

Integrand size = 26, antiderivative size = 255

$$\int x^{12}\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{a^5x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\,(a+bx^3)}+\frac{5a^4bx^{16}\sqrt{a^2+2abx^3+b^2x^6}}{16\,(a+bx^3)}+\frac{10a^3b^2x^{19}\sqrt{a^2+2abx^3+b^2x^6}}{19\,(a+bx^3)}+\frac{5a^2b^3x^{22}\sqrt{a^2+2abx^3+b^2x^6}}{11\,(a+bx^3)}+\frac{ab^4x^{25}\sqrt{a^2+2abx^3+b^2x^6}}{5\,(a+bx^3)}+\frac{b^5x^{28}\sqrt{a^2+2abx^3+b^2x^6}}{28\,(a+bx^3)}$$

output
```
1/13*a^5*x^13*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/16*a^4*b*x^16*((b*x^3+a)^2)^
(1/2)/(b*x^3+a)+10/19*a^3*b^2*x^19*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/11*a^2*
b^3*x^22*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/5*a*b^4*x^25*((b*x^3+a)^2)^(1/2)/
(b*x^3+a)+1/28*b^5*x^28*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

3.51.2 Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int x^{12}\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{x^{13}\sqrt{(a+bx^3)^2}(117040a^5+475475a^4bx^3+800800a^3b^2x^6+691600a^2b^3x^9+304304ab^4x^{12}}{1521520\,(a+bx^3)}$$

input
```
Integrate[x^12*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
(x^13*Sqrt[(a + b*x^3)^2]*(117040*a^5 + 475475*a^4*b*x^3 + 800800*a^3*b^2*
x^6 + 691600*a^2*b^3*x^9 + 304304*a*b^4*x^12 + 54340*b^5*x^15))/(1521520*(
a + b*x^3))
```

3.51.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.40, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{12}\left(a^2+2abx^3+b^2x^6\right)^{5/2}dx$$

$$\Big\downarrow \ 1384$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int b^5x^{12}\left(bx^3+a\right)^5dx}{b^5\left(a+bx^3\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^{12}\left(bx^3+a\right)^5dx}{a+bx^3}$$

$$\Big\downarrow \ 802$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(b^5x^{27}+5ab^4x^{24}+10a^2b^3x^{21}+10a^3b^2x^{18}+5a^4bx^{15}+a^5x^{12}\right)dx}{a+bx^3}$$

$$\Big\downarrow \ 2009$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(\frac{a^5x^{13}}{13}+\frac{5}{16}a^4bx^{16}+\frac{10}{19}a^3b^2x^{19}+\frac{5}{11}a^2b^3x^{22}+\frac{1}{5}ab^4x^{25}+\frac{b^5x^{28}}{28}\right)}{a+bx^3}$$

input
```
Int[x^12*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*((a^5*x^13)/13 + (5*a^4*b*x^16)/16 + (10*
a^3*b^2*x^19)/19 + (5*a^2*b^3*x^22)/11 + (a*b^4*x^25)/5 + (b^5*x^28)/28))/
(a + b*x^3)
```

3.51.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.51.4 Maple [A] (verified)

Time = 13.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.31

method	result
gosper	$\frac{x^{13}\left(54340b^5x^{15}+304304a\,b^4x^{12}+691600a^2b^3x^9+800800a^3b^2x^6+475475a^4b\,x^3+117040a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}}{1521520\left(b\,x^3+a\right)^5}$
default	$\frac{x^{13}\left(54340b^5x^{15}+304304a\,b^4x^{12}+691600a^2b^3x^9+800800a^3b^2x^6+475475a^4b\,x^3+117040a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}}{1521520\left(b\,x^3+a\right)^5}$
risch	$\frac{a^5x^{13}\sqrt{\left(b\,x^3+a\right)^2}}{13b\,x^3+13a}+\frac{5a^4b\,x^{16}\sqrt{\left(b\,x^3+a\right)^2}}{16\left(b\,x^3+a\right)}+\frac{10a^3b^2x^{19}\sqrt{\left(b\,x^3+a\right)^2}}{19\left(b\,x^3+a\right)}+\frac{5a^2b^3x^{22}\sqrt{\left(b\,x^3+a\right)^2}}{11\left(b\,x^3+a\right)}+\frac{a\,b^4x^{25}\sqrt{\left(b\,x^3+a\right)^2}}{5b\,x^3+5a}+\frac{b^5x^{28}}{28}$

input
```
int(x^12*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/1521520*x^13*(54340*b^5*x^15+304304*a*b^4*x^12+691600*a^2*b^3*x^9+800800
*a^3*b^2*x^6+475475*a^4*b*x^3+117040*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

3.51.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.22

$$\int x^{12}\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{1}{28}\,b^5x^{28}+\frac{1}{5}\,ab^4x^{25}+\frac{5}{11}\,a^2b^3x^{22}+\frac{10}{19}\,a^3b^2x^{19}+\frac{5}{16}\,a^4bx^{16}+\frac{1}{13}\,a^5x^{13}$$

input
```
integrate(x^12*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

output
```
1/28*b^5*x^28 + 1/5*a*b^4*x^25 + 5/11*a^2*b^3*x^22 + 10/19*a^3*b^2*x^19 +
5/16*a^4*b*x^16 + 1/13*a^5*x^13
```

3.51.6 Sympy [F]

$$\int x^{12}\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\int x^{12}\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(x**12*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

output
```
Integral(x**12*((a + b*x**3)**2)**(5/2), x)
```

3.51.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.22

$$\int x^{12}\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{1}{28}\,b^5x^{28}+\frac{1}{5}\,ab^4x^{25}+\frac{5}{11}\,a^2b^3x^{22}+\frac{10}{19}\,a^3b^2x^{19}+\frac{5}{16}\,a^4bx^{16}+\frac{1}{13}\,a^5x^{13}$$

input
```
integrate(x^12*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

output
```
1/28*b^5*x^28 + 1/5*a*b^4*x^25 + 5/11*a^2*b^3*x^22 + 10/19*a^3*b^2*x^19 +
5/16*a^4*b*x^16 + 1/13*a^5*x^13
```

3.51.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.41

$$\int x^{12}\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{1}{28}\,b^5x^{28}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{5}\,ab^4x^{25}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{11}\,a^2b^3x^{22}\mathrm{sgn}\left(bx^3+a\right)+\frac{10}{19}\,a^3b^2x^{19}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{16}\,a^4bx^{16}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{13}\,a^5x^{13}\mathrm{sgn}\left(bx^3+a\right)$$

input
```
integrate(x^12*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

output
```
1/28*b^5*x^28*sgn(b*x^3 + a) + 1/5*a*b^4*x^25*sgn(b*x^3 + a) + 5/11*a^2*b^
3*x^22*sgn(b*x^3 + a) + 10/19*a^3*b^2*x^19*sgn(b*x^3 + a) + 5/16*a^4*b*x^1
6*sgn(b*x^3 + a) + 1/13*a^5*x^13*sgn(b*x^3 + a)
```

3.51.9 Mupad [F(-1)]

Timed out.

$$\int x^{12}\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\int x^{12}\,{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{5/2}\,dx$$

input
```
int(x^12*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

output
```
int(x^12*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

3.52 $\int x^{11}(a^2+2abx^3+b^2x^6)^{5/2}\,dx$

3.52.1 Optimal result

Integrand size = 26, antiderivative size = 160

$$\int x^{11}\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=-\frac{a^3(a+bx^3)^5\sqrt{a^2+2abx^3+b^2x^6}}{18b^4}+\frac{a^2(a+bx^3)^6\sqrt{a^2+2abx^3+b^2x^6}}{7b^4}-\frac{a(a+bx^3)^7\sqrt{a^2+2abx^3+b^2x^6}}{8b^4}+\frac{(a+bx^3)^8\sqrt{a^2+2abx^3+b^2x^6}}{27b^4}$$

output
```
-1/18*a^3*(b*x^3+a)^5*((b*x^3+a)^2)^(1/2)/b^4+1/7*a^2*(b*x^3+a)^6*((b*x^3+
a)^2)^(1/2)/b^4-1/8*a*(b*x^3+a)^7*((b*x^3+a)^2)^(1/2)/b^4+1/27*(b*x^3+a)^8
*((b*x^3+a)^2)^(1/2)/b^4
```

3.52.2 Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.84

$$\int x^{11}\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{x^{12}(126a^5+504a^4bx^3+840a^3b^2x^6+720a^2b^3x^9+315ab^4x^{12}+56b^5x^{15})\left(\sqrt{a^2}bx^3+a\left(\sqrt{a^2}\right.\right.}{1512\left(-a^2-abx^3+\sqrt{a^2}\sqrt{(a+bx^3)^2}\right)}$$

input
```
Integrate[x^11*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
(x^12*(126*a^5 + 504*a^4*b*x^3 + 840*a^3*b^2*x^6 + 720*a^2*b^3*x^9 + 315*a
*b^4*x^12 + 56*b^5*x^15)*(Sqrt[a^2]*b*x^3 + a*(Sqrt[a^2] - Sqrt[(a + b*x^3
)^2])))/(1512*(-a^2 - a*b*x^3 + Sqrt[a^2]*Sqrt[(a + b*x^3)^2]))
```

3.52.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.67, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1384, 27, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{11} \left(a^2+2abx^3+b^2x^6\right)^{5/2} \, dx$$

$$\Big\downarrow \ 1384$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6} \int b^5 x^{11} \left(bx^3+a\right)^5 dx}{b^5 \left(a+bx^3\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6} \int x^{11} \left(bx^3+a\right)^5 dx}{a+bx^3}$$

$$\Big\downarrow \ 798$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6} \int x^9 \left(bx^3+a\right)^5 dx^3}{3 \left(a+bx^3\right)}$$

$$\Big\downarrow \ 49$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6} \int \left(\frac{\left(bx^3+a\right)^8}{b^3}-\frac{3a\left(bx^3+a\right)^7}{b^3}+\frac{3a^2\left(bx^3+a\right)^6}{b^3}-\frac{a^3\left(bx^3+a\right)^5}{b^3}\right) dx^3}{3 \left(a+bx^3\right)}$$

$$\Big\downarrow \ 2009$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6} \left(-\frac{a^3\left(a+bx^3\right)^6}{6b^4}+\frac{3a^2\left(a+bx^3\right)^7}{7b^4}+\frac{\left(a+bx^3\right)^9}{9b^4}-\frac{3a\left(a+bx^3\right)^8}{8b^4}\right)}{3 \left(a+bx^3\right)}$$

input
```
Int[x^11*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-1/6*(a^3*(a + b*x^3)^6)/b^4 + (3*a^2*(a
 + b*x^3)^7)/(7*b^4) - (3*a*(a + b*x^3)^8)/(8*b^4) + (a + b*x^3)^9/(9*b^4)
))/(3*(a + b*x^3))
```

3.52.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.52.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.33

method	result
pseudoelliptic	$-\frac{\operatorname{csgn}(b\,x^3+a)\,(b\,x^3+a)^6\,(-56b^3x^9+21b^2x^6a-6a^2b\,x^3+a^3)}{1512b^4}$
gosper	$\frac{x^{12}(56b^5x^{15}+315a\,b^4x^{12}+720a^2b^3x^9+840a^3b^2x^6+504a^4b\,x^3+126a^5)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{1512(b\,x^3+a)^5}$
default	$\frac{x^{12}(56b^5x^{15}+315a\,b^4x^{12}+720a^2b^3x^9+840a^3b^2x^6+504a^4b\,x^3+126a^5)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{1512(b\,x^3+a)^5}$
risch	$\frac{5\sqrt{(b\,x^3+a)^2}\,b^4a\,x^{24}}{24(b\,x^3+a)}+\frac{\sqrt{(b\,x^3+a)^2}\,b^5x^{27}}{27b\,x^3+27a}+\frac{5\sqrt{(b\,x^3+a)^2}\,a^3b^2x^{18}}{9(b\,x^3+a)}+\frac{10\sqrt{(b\,x^3+a)^2}\,a^2b^3x^{21}}{21(b\,x^3+a)}+\frac{\sqrt{(b\,x^3+a)^2}\,a^5x^{12}}{12b\,x^3+12a}$

input
```
int(x^11*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/1512*csgn(b*x^3+a)*(b*x^3+a)^6*(-56*b^3*x^9+21*a*b^2*x^6-6*a^2*b*x^3+a^
3)/b^4
```

3.52.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.36

$$\int x^{11}\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{1}{27}\,b^5x^{27}+\frac{5}{24}\,ab^4x^{24}+\frac{10}{21}\,a^2b^3x^{21}+\frac{5}{9}\,a^3b^2x^{18}+\frac{1}{3}\,a^4bx^{15}+\frac{1}{12}\,a^5x^{12}$$

input
```
integrate(x^11*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

output
```
1/27*b^5*x^27 + 5/24*a*b^4*x^24 + 10/21*a^2*b^3*x^21 + 5/9*a^3*b^2*x^18 +
1/3*a^4*b*x^15 + 1/12*a^5*x^12
```

3.52.6 Sympy [F]

$$\int x^{11}\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\int x^{11}\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(x**11*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

output
```
Integral(x**11*((a + b*x**3)**2)**(5/2), x)
```

3.52.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.91

$$\int x^{11}\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{7}{2}}x^6}{27\,b^2}-\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{5}{2}}a^3x^3}{18\,b^3}-\frac{11\,\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{7}{2}}ax^3}{216\,b^3}-\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{5}{2}}a^4}{18\,b^4}+\frac{83\,\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{7}{2}}a^2}{1512\,b^4}$$

input
```
integrate(x^11*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

output
```
1/27*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*x^6/b^2 - 1/18*(b^2*x^6 + 2*a*b*x^3
 + a^2)^(5/2)*a^3*x^3/b^3 - 11/216*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*a*x^3
/b^3 - 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*a^4/b^4 + 83/1512*(b^2*x^6 +
 2*a*b*x^3 + a^2)^(7/2)*a^2/b^4
```

3.52.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.66

$$\int x^{11}\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{1}{27}\,b^5x^{27}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{24}\,ab^4x^{24}\mathrm{sgn}\left(bx^3+a\right)+\frac{10}{21}\,a^2b^3x^{21}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{9}\,a^3b^2x^{18}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{3}\,a^4bx^{15}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{12}\,a^5x^{12}\mathrm{sgn}\left(bx^3+a\right)$$

input
```
integrate(x^11*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

output
```
1/27*b^5*x^27*sgn(b*x^3 + a) + 5/24*a*b^4*x^24*sgn(b*x^3 + a) + 10/21*a^2*
b^3*x^21*sgn(b*x^3 + a) + 5/9*a^3*b^2*x^18*sgn(b*x^3 + a) + 1/3*a^4*b*x^15
*sgn(b*x^3 + a) + 1/12*a^5*x^12*sgn(b*x^3 + a)
```

3.52.9 Mupad [F(-1)]

Timed out.

$$\int x^{11}\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\int x^{11}\,\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}\,dx$$

input
```
int(x^11*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

output
```
int(x^11*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

3.53 $\int x^{10}(a^2+2abx^3+b^2x^6)^{5/2}\,dx$

3.53.1 Optimal result

Integrand size = 26, antiderivative size = 255

$$\int x^{10}\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{a^5x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{5a^4bx^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14\left(a+bx^3\right)}+\frac{10a^3b^2x^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17\left(a+bx^3\right)}+\frac{a^2b^3x^{20}\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{5ab^4x^{23}\sqrt{a^2+2abx^3+b^2x^6}}{23\left(a+bx^3\right)}+\frac{b^5x^{26}\sqrt{a^2+2abx^3+b^2x^6}}{26\left(a+bx^3\right)}$$

output
```
1/11*a^5*x^11*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/14*a^4*b*x^14*((b*x^3+a)^2)^
(1/2)/(b*x^3+a)+10/17*a^3*b^2*x^17*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/2*a^2*b
^3*x^20*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/23*a*b^4*x^23*((b*x^3+a)^2)^(1/2)/
(b*x^3+a)+1/26*b^5*x^26*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

3.53.2 Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int x^{10}\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{x^{11}\sqrt{(a+bx^3)^2}(71162a^5+279565a^4bx^3+460460a^3b^2x^6+391391a^2b^3x^9+170170ab^4x^{12}+$$
$$782782\,(a+bx^3)$$

input
```
Integrate[x^10*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
(x^11*Sqrt[(a + b*x^3)^2]*(71162*a^5 + 279565*a^4*b*x^3 + 460460*a^3*b^2*x
^6 + 391391*a^2*b^3*x^9 + 170170*a*b^4*x^12 + 30107*b^5*x^15))/(782782*(a
+ b*x^3))
```

3.53.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.40, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{10}\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int b^5x^{10}\left(bx^3+a\right)^5dx}{b^5\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^{10}\left(bx^3+a\right)^5dx}{a+bx^3}$$

$$\Big\downarrow \text{ 802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(b^5x^{25}+5ab^4x^{22}+10a^2b^3x^{19}+10a^3b^2x^{16}+5a^4bx^{13}+a^5x^{10}\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(\frac{a^5x^{11}}{11}+\frac{5}{14}a^4bx^{14}+\frac{10}{17}a^3b^2x^{17}+\frac{1}{2}a^2b^3x^{20}+\frac{5}{23}ab^4x^{23}+\frac{b^5x^{26}}{26}\right)}{a+bx^3}$$

input
```
Int[x^10*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*((a^5*x^11)/11 + (5*a^4*b*x^14)/14 + (10*
a^3*b^2*x^17)/17 + (a^2*b^3*x^20)/2 + (5*a*b^4*x^23)/23 + (b^5*x^26)/26))/
(a + b*x^3)
```

3.53.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.53.4 Maple [A] (verified)

Time = 9.72 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.31

method	result
gosper	$\frac{x^{11}\left(30107b^{5}x^{15}+170170a\,b^{4}x^{12}+391391a^{2}b^{3}x^{9}+460460a^{3}b^{2}x^{6}+279565a^{4}b\,x^{3}+71162a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{782782\left(b\,x^{3}+a\right)^{5}}$
default	$\frac{x^{11}\left(30107b^{5}x^{15}+170170a\,b^{4}x^{12}+391391a^{2}b^{3}x^{9}+460460a^{3}b^{2}x^{6}+279565a^{4}b\,x^{3}+71162a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{782782\left(b\,x^{3}+a\right)^{5}}$
risch	$\frac{a^{5}x^{11}\sqrt{\left(b\,x^{3}+a\right)^{2}}}{11b\,x^{3}+11a}+\frac{5a^{4}b\,x^{14}\sqrt{\left(b\,x^{3}+a\right)^{2}}}{14\left(b\,x^{3}+a\right)}+\frac{10a^{3}b^{2}x^{17}\sqrt{\left(b\,x^{3}+a\right)^{2}}}{17\left(b\,x^{3}+a\right)}+\frac{a^{2}b^{3}x^{20}\sqrt{\left(b\,x^{3}+a\right)^{2}}}{2b\,x^{3}+2a}+\frac{5a\,b^{4}x^{23}\sqrt{\left(b\,x^{3}+a\right)^{2}}}{23\left(b\,x^{3}+a\right)}+$ $b^{5}x^{26}$ … / 26…

input
```
int(x^10*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/782782*x^11*(30107*b^5*x^15+170170*a*b^4*x^12+391391*a^2*b^3*x^9+460460*
a^3*b^2*x^6+279565*a^4*b*x^3+71162*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

3.53.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.22

$$\int x^{10}\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{1}{26}\,b^5x^{26}+\frac{5}{23}\,ab^4x^{23}+\frac{1}{2}\,a^2b^3x^{20}+\frac{10}{17}\,a^3b^2x^{17}+\frac{5}{14}\,a^4bx^{14}+\frac{1}{11}\,a^5x^{11}$$

input
```
integrate(x^10*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

output
```
1/26*b^5*x^26 + 5/23*a*b^4*x^23 + 1/2*a^2*b^3*x^20 + 10/17*a^3*b^2*x^17 +
5/14*a^4*b*x^14 + 1/11*a^5*x^11
```

3.53.6 Sympy [F]

$$\int x^{10}\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\int x^{10}\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(x**10*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

output
```
Integral(x**10*((a + b*x**3)**2)**(5/2), x)
```

3.53.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.22

$$\int x^{10}\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{1}{26}\,b^5x^{26}+\frac{5}{23}\,ab^4x^{23}+\frac{1}{2}\,a^2b^3x^{20}+\frac{10}{17}\,a^3b^2x^{17}+\frac{5}{14}\,a^4bx^{14}+\frac{1}{11}\,a^5x^{11}$$

input
```
integrate(x^10*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

output
```
1/26*b^5*x^26 + 5/23*a*b^4*x^23 + 1/2*a^2*b^3*x^20 + 10/17*a^3*b^2*x^17 +
5/14*a^4*b*x^14 + 1/11*a^5*x^11
```

3.53.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.41

$$\int x^{10}\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{1}{26}\,b^5x^{26}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{23}\,ab^4x^{23}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{2}\,a^2b^3x^{20}\mathrm{sgn}\left(bx^3+a\right)+\frac{10}{17}\,a^3b^2x^{17}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{14}\,a^4bx^{14}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{11}\,a^5x^{11}\mathrm{sgn}\left(bx^3+a\right)$$

input
```
integrate(x^10*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

output
```
1/26*b^5*x^26*sgn(b*x^3 + a) + 5/23*a*b^4*x^23*sgn(b*x^3 + a) + 1/2*a^2*b^
3*x^20*sgn(b*x^3 + a) + 10/17*a^3*b^2*x^17*sgn(b*x^3 + a) + 5/14*a^4*b*x^1
4*sgn(b*x^3 + a) + 1/11*a^5*x^11*sgn(b*x^3 + a)
```

3.53.9 Mupad [F(-1)]

Timed out.

$$\int x^{10}\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\int x^{10}\,{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{5/2}\,dx$$

input
```
int(x^10*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

output
```
int(x^10*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

3.54 $\int x^9(a^2+2abx^3+b^2x^6)^{5/2}\,dx$

3.54.1 Optimal result

Integrand size = 26, antiderivative size = 255

$$\int x^9\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{a^5x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10\,(a+bx^3)}+\frac{5a^4bx^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\,(a+bx^3)}+\frac{5a^3b^2x^{16}\sqrt{a^2+2abx^3+b^2x^6}}{8\,(a+bx^3)}+\frac{10a^2b^3x^{19}\sqrt{a^2+2abx^3+b^2x^6}}{19\,(a+bx^3)}+\frac{5ab^4x^{22}\sqrt{a^2+2abx^3+b^2x^6}}{22\,(a+bx^3)}+\frac{b^5x^{25}\sqrt{a^2+2abx^3+b^2x^6}}{25\,(a+bx^3)}$$

output
```
1/10*a^5*x^10*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/13*a^4*b*x^13*((b*x^3+a)^2)^
(1/2)/(b*x^3+a)+5/8*a^3*b^2*x^16*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+10/19*a^2*b
^3*x^19*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/22*a*b^4*x^22*((b*x^3+a)^2)^(1/2)/
(b*x^3+a)+1/25*b^5*x^25*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

3.54.2 Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int x^9\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{x^{10}\sqrt{(a+bx^3)^2}(54340a^5+209000a^4bx^3+339625a^3b^2x^6+286000a^2b^3x^9+123500ab^4x^{12}+}{543400\,(a+bx^3)}$$

input
```
Integrate[x^9*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
(x^10*Sqrt[(a + b*x^3)^2]*(54340*a^5 + 209000*a^4*b*x^3 + 339625*a^3*b^2*x
^6 + 286000*a^2*b^3*x^9 + 123500*a*b^4*x^12 + 21736*b^5*x^15))/(543400*(a
+ b*x^3))
```

3.54.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.40, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^9\left(a^2+2abx^3+b^2x^6\right)^{5/2}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int b^5x^9\left(bx^3+a\right)^5dx}{b^5\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^9\left(bx^3+a\right)^5dx}{a+bx^3}$$

$$\Big\downarrow \text{ 802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(b^5x^{24}+5ab^4x^{21}+10a^2b^3x^{18}+10a^3b^2x^{15}+5a^4bx^{12}+a^5x^9\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(\frac{a^5x^{10}}{10}+\frac{5}{13}a^4bx^{13}+\frac{5}{8}a^3b^2x^{16}+\frac{10}{19}a^2b^3x^{19}+\frac{5}{22}ab^4x^{22}+\frac{b^5x^{25}}{25}\right)}{a+bx^3}$$

input
```
Int[x^9*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*((a^5*x^10)/10 + (5*a^4*b*x^13)/13 + (5*a
^3*b^2*x^16)/8 + (10*a^2*b^3*x^19)/19 + (5*a*b^4*x^22)/22 + (b^5*x^25)/25)
)/(a + b*x^3)
```

3.54.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.54.4 Maple [A] (verified)

Time = 8.35 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.31

method	result
gosper	$\frac{x^{10}\left(21736b^{5}x^{15}+123500a\,b^{4}x^{12}+286000a^{2}b^{3}x^{9}+339625a^{3}b^{2}x^{6}+209000a^{4}b\,x^{3}+54340a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{543400\left(b\,x^{3}+a\right)^{5}}$
default	$\frac{x^{10}\left(21736b^{5}x^{15}+123500a\,b^{4}x^{12}+286000a^{2}b^{3}x^{9}+339625a^{3}b^{2}x^{6}+209000a^{4}b\,x^{3}+54340a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{543400\left(b\,x^{3}+a\right)^{5}}$
risch	$\frac{a^{5}x^{10}\sqrt{\left(b\,x^{3}+a\right)^{2}}}{10b\,x^{3}+10a}+\frac{5a^{4}b\,x^{13}\sqrt{\left(b\,x^{3}+a\right)^{2}}}{13\left(b\,x^{3}+a\right)}+\frac{5a^{3}b^{2}x^{16}\sqrt{\left(b\,x^{3}+a\right)^{2}}}{8\left(b\,x^{3}+a\right)}+\frac{10a^{2}b^{3}x^{19}\sqrt{\left(b\,x^{3}+a\right)^{2}}}{19\left(b\,x^{3}+a\right)}+\frac{5a\,b^{4}x^{22}\sqrt{\left(b\,x^{3}+a\right)^{2}}}{22\left(b\,x^{3}+a\right)}+\frac{b^{5}x^{2}}{2}$

input
```
int(x^9*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/543400*x^10*(21736*b^5*x^15+123500*a*b^4*x^12+286000*a^2*b^3*x^9+339625*
a^3*b^2*x^6+209000*a^4*b*x^3+54340*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

3.54.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.22

$$\int x^9\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{1}{25}\,b^5x^{25}+\frac{5}{22}\,ab^4x^{22}+\frac{10}{19}\,a^2b^3x^{19}+\frac{5}{8}\,a^3b^2x^{16}+\frac{5}{13}\,a^4bx^{13}+\frac{1}{10}\,a^5x^{10}$$

input
```
integrate(x^9*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

output
```
1/25*b^5*x^25 + 5/22*a*b^4*x^22 + 10/19*a^2*b^3*x^19 + 5/8*a^3*b^2*x^16 +
5/13*a^4*b*x^13 + 1/10*a^5*x^10
```

3.54.6 Sympy [F]

$$\int x^9\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\int x^9\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(x**9*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

output
```
Integral(x**9*((a + b*x**3)**2)**(5/2), x)
```

3.54.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.22

$$\int x^9\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{1}{25}\,b^5x^{25}+\frac{5}{22}\,ab^4x^{22}+\frac{10}{19}\,a^2b^3x^{19}+\frac{5}{8}\,a^3b^2x^{16}+\frac{5}{13}\,a^4bx^{13}+\frac{1}{10}\,a^5x^{10}$$

input
```
integrate(x^9*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

output
```
1/25*b^5*x^25 + 5/22*a*b^4*x^22 + 10/19*a^2*b^3*x^19 + 5/8*a^3*b^2*x^16 +
5/13*a^4*b*x^13 + 1/10*a^5*x^10
```

3.54.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.41

$$\int x^9\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{1}{25}\,b^5x^{25}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{22}\,ab^4x^{22}\mathrm{sgn}\left(bx^3+a\right)+\frac{10}{19}\,a^2b^3x^{19}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{8}\,a^3b^2x^{16}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{13}\,a^4bx^{13}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{10}\,a^5x^{10}\mathrm{sgn}\left(bx^3+a\right)$$

input
```
integrate(x^9*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

output
```
1/25*b^5*x^25*sgn(b*x^3 + a) + 5/22*a*b^4*x^22*sgn(b*x^3 + a) + 10/19*a^2*
b^3*x^19*sgn(b*x^3 + a) + 5/8*a^3*b^2*x^16*sgn(b*x^3 + a) + 5/13*a^4*b*x^1
3*sgn(b*x^3 + a) + 1/10*a^5*x^10*sgn(b*x^3 + a)
```

3.54.9 Mupad [F(-1)]

Timed out.

$$\int x^9\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\int x^9\,\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}\,dx$$

input
```
int(x^9*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

output
```
int(x^9*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

3.55 $\int x^8(a^2+2abx^3+b^2x^6)^{5/2}\,dx$

3.55.1 Optimal result

Integrand size = 26, antiderivative size = 119

$$\int x^8\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{a^2(a+bx^3)^5\sqrt{a^2+2abx^3+b^2x^6}}{18b^3}-\frac{2a(a+bx^3)^6\sqrt{a^2+2abx^3+b^2x^6}}{21b^3}+\frac{(a+bx^3)^7\sqrt{a^2+2abx^3+b^2x^6}}{24b^3}$$

output
```
1/18*a^2*(b*x^3+a)^5*((b*x^3+a)^2)^(1/2)/b^3-2/21*a*(b*x^3+a)^6*((b*x^3+a)
^2)^(1/2)/b^3+1/24*(b*x^3+a)^7*((b*x^3+a)^2)^(1/2)/b^3
```

3.55.2 Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.13

$$\int x^8\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{x^9(56a^5+210a^4bx^3+336a^3b^2x^6+280a^2b^3x^9+120ab^4x^{12}+21b^5x^{15})\left(\sqrt{a^2}bx^3+a\left(\sqrt{a^2}-\right.\right.}{504\left(-a^2-abx^3+\sqrt{a^2}\sqrt{(a+bx^3)^2}\right)}$$

input
```
Integrate[x^8*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
(x^9*(56*a^5 + 210*a^4*b*x^3 + 336*a^3*b^2*x^6 + 280*a^2*b^3*x^9 + 120*a*b
^4*x^12 + 21*b^5*x^15)*(Sqrt[a^2]*b*x^3 + a*(Sqrt[a^2] - Sqrt[(a + b*x^3)^
2])))/(504*(-a^2 - a*b*x^3 + Sqrt[a^2]*Sqrt[(a + b*x^3)^2]))
```

3.55.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.74, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1384, 27, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8 \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6} \int b^5x^8\left(bx^3+a\right)^5dx}{b^5\left(a+bx^3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6} \int x^8\left(bx^3+a\right)^5dx}{a+bx^3}$$

$$\Big\downarrow \text{798}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6} \int x^6\left(bx^3+a\right)^5dx^3}{3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{49}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6} \int \left(\frac{\left(bx^3+a\right)^7}{b^2}-\frac{2a\left(bx^3+a\right)^6}{b^2}+\frac{a^2\left(bx^3+a\right)^5}{b^2}\right)dx^3}{3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(\frac{a^2\left(a+bx^3\right)^6}{6b^3}+\frac{\left(a+bx^3\right)^8}{8b^3}-\frac{2a\left(a+bx^3\right)^7}{7b^3}\right)}{3\left(a+bx^3\right)}$$

input
```
Int[x^8*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*((a^2*(a + b*x^3)^6)/(6*b^3) - (2*a*(a +
b*x^3)^7)/(7*b^3) + (a + b*x^3)^8/(8*b^3)))/(3*(a + b*x^3))
```

3.55.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.55. $\int x^8(a^2+2abx^3+b^2x^6)^{5/2}\,dx$

3.55.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.35

method	result
pseudoelliptic	$\frac{\operatorname{csgn}\left(b\,x^{3}+a\right)\left(b\,x^{3}+a\right)^{6}\left(21 b^{2} x^{6}-6 a b\,x^{3}+a^{2}\right)}{504 b^{3}}$
gosper	$\frac{x^{9}\left(21 b^{5} x^{15}+120 a\,b^{4} x^{12}+280 a^{2} b^{3} x^{9}+336 a^{3} b^{2} x^{6}+210 a^{4} b\,x^{3}+56 a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{504\left(b\,x^{3}+a\right)^{5}}$
default	$\frac{x^{9}\left(21 b^{5} x^{15}+120 a\,b^{4} x^{12}+280 a^{2} b^{3} x^{9}+336 a^{3} b^{2} x^{6}+210 a^{4} b\,x^{3}+56 a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{504\left(b\,x^{3}+a\right)^{5}}$
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\, b^{5} x^{24}}{24 b\,x^{3}+24 a}+\frac{5 \sqrt{\left(b\,x^{3}+a\right)^{2}}\, b^{4} a\,x^{21}}{21\left(b\,x^{3}+a\right)}+\frac{5 \sqrt{\left(b\,x^{3}+a\right)^{2}}\, a^{2} b^{3} x^{18}}{9\left(b\,x^{3}+a\right)}+\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\, a^{5} x^{9}}{9 b\,x^{3}+9 a}+\frac{5 \sqrt{\left(b\,x^{3}+a\right)^{2}}\, b\,a^{4} x^{12}}{12\left(b\,x^{3}+a\right)}+$

input
```
int(x^8*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/504*csgn(b*x^3+a)*(b*x^3+a)^6*(21*b^2*x^6-6*a*b*x^3+a^2)/b^3
```

3.55.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.48

$$\int x^8\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{1}{24}\,b^5x^{24}+\frac{5}{21}\,ab^4x^{21}+\frac{5}{9}\,a^2b^3x^{18}+\frac{2}{3}\,a^3b^2x^{15}+\frac{5}{12}\,a^4bx^{12}+\frac{1}{9}\,a^5x^9$$

input
```
integrate(x^8*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

output
```
1/24*b^5*x^24 + 5/21*a*b^4*x^21 + 5/9*a^2*b^3*x^18 + 2/3*a^3*b^2*x^15 + 5/
12*a^4*b*x^12 + 1/9*a^5*x^9
```

3.55.6 Sympy [F]

$$\int x^8 \left(a^2+2abx^3+b^2x^6\right)^{5/2} \, dx = \int x^8 \left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}} \, dx$$

input
```
integrate(x**8*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

output
```
Integral(x**8*((a + b*x**3)**2)**(5/2), x)
```

3.55.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.96

$$\int x^8 \left(a^2+2abx^3+b^2x^6\right)^{5/2} \, dx = \frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{5}{2}}a^2x^3}{18\,b^2} + \frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{7}{2}}x^3}{24\,b^2} + \frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{5}{2}}a^3}{18\,b^3} - \frac{3\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{7}{2}}a}{56\,b^3}$$

input
```
integrate(x^8*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

output
```
1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*a^2*x^3/b^2 + 1/24*(b^2*x^6 + 2*a*b
*x^3 + a^2)^(7/2)*x^3/b^2 + 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*a^3/b^3
 - 3/56*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*a/b^3
```

3.55.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.88

$$\int x^8 \left(a^2+2abx^3+b^2x^6\right)^{5/2} \, dx = \frac{1}{24}\,b^5x^{24}\mathrm{sgn}\left(bx^3+a\right) + \frac{5}{21}\,ab^4x^{21}\mathrm{sgn}\left(bx^3+a\right) + \frac{5}{9}\,a^2b^3x^{18}\mathrm{sgn}\left(bx^3+a\right) + \frac{2}{3}\,a^3b^2x^{15}\mathrm{sgn}\left(bx^3+a\right) + \frac{5}{12}\,a^4bx^{12}\mathrm{sgn}\left(bx^3+a\right) + \frac{1}{9}\,a^5x^9\mathrm{sgn}\left(bx^3+a\right)$$

input
```
integrate(x^8*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

output
```
1/24*b^5*x^24*sgn(b*x^3 + a) + 5/21*a*b^4*x^21*sgn(b*x^3 + a) + 5/9*a^2*b^
3*x^18*sgn(b*x^3 + a) + 2/3*a^3*b^2*x^15*sgn(b*x^3 + a) + 5/12*a^4*b*x^12*
sgn(b*x^3 + a) + 1/9*a^5*x^9*sgn(b*x^3 + a)
```

3.55.9 Mupad [F(-1)]

Timed out.

$$\int x^8(a^2+2abx^3+b^2x^6)^{5/2}\,dx=\int x^8\,{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{5/2}\,dx$$

input
```
int(x^8*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

output
```
int(x^8*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

3.56 $\int x^7(a^2+2abx^3+b^2x^6)^{5/2}\,dx$

3.56.1 Optimal result

Integrand size = 26, antiderivative size = 255

$$\int x^7\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{a^5x^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{5a^4bx^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{5a^3b^2x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{10a^2b^3x^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17\left(a+bx^3\right)}+\frac{ab^4x^{20}\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{b^5x^{23}\sqrt{a^2+2abx^3+b^2x^6}}{23\left(a+bx^3\right)}$$

output
```
1/8*a^5*x^8*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/11*a^4*b*x^11*((b*x^3+a)^2)^(1
/2)/(b*x^3+a)+5/7*a^3*b^2*x^14*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+10/17*a^2*b^3
*x^17*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/4*a*b^4*x^20*((b*x^3+a)^2)^(1/2)/(b*
x^3+a)+1/23*b^5*x^23*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

3.56.2 Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int x^7\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{x^8\sqrt{(a+bx^3)^2}(30107a^5+109480a^4bx^3+172040a^3b^2x^6+141680a^2b^3x^9+60214ab^4x^{12}+1}{240856\,(a+bx^3)}$$

input
```
Integrate[x^7*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
(x^8*Sqrt[(a + b*x^3)^2]*(30107*a^5 + 109480*a^4*b*x^3 + 172040*a^3*b^2*x^
6 + 141680*a^2*b^3*x^9 + 60214*a*b^4*x^12 + 10472*b^5*x^15))/(240856*(a +
b*x^3))
```

3.56.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.40, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int b^5x^7\left(bx^3+a\right)^5dx}{b^5\left(a+bx^3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^7\left(bx^3+a\right)^5dx}{a+bx^3}$$

$$\Big\downarrow \text{802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(b^5x^{22}+5ab^4x^{19}+10a^2b^3x^{16}+10a^3b^2x^{13}+5a^4bx^{10}+a^5x^7\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(\frac{a^5x^8}{8}+\frac{5}{11}a^4bx^{11}+\frac{5}{7}a^3b^2x^{14}+\frac{10}{17}a^2b^3x^{17}+\frac{1}{4}ab^4x^{20}+\frac{b^5x^{23}}{23}\right)}{a+bx^3}$$

input
```
Int[x^7*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*((a^5*x^8)/8 + (5*a^4*b*x^11)/11 + (5*a^3
*b^2*x^14)/7 + (10*a^2*b^3*x^17)/17 + (a*b^4*x^20)/4 + (b^5*x^23)/23))/(a
+ b*x^3)
```

3.56.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.56.4 Maple [A] (verified)

Time = 6.32 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.31

method	result
gosper	$\frac{x^8(10472b^5x^{15}+60214a\,b^4x^{12}+141680a^2b^3x^9+172040a^3b^2x^6+109480a^4b\,x^3+30107a^5)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{240856(b\,x^3+a)^5}$
default	$\frac{x^8(10472b^5x^{15}+60214a\,b^4x^{12}+141680a^2b^3x^9+172040a^3b^2x^6+109480a^4b\,x^3+30107a^5)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{240856(b\,x^3+a)^5}$
risch	$\frac{a^5x^8\sqrt{(b\,x^3+a)^2}}{8b\,x^3+8a}+\frac{5a^4b\,x^{11}\sqrt{(b\,x^3+a)^2}}{11(b\,x^3+a)}+\frac{5a^3b^2x^{14}\sqrt{(b\,x^3+a)^2}}{7(b\,x^3+a)}+\frac{10a^2b^3x^{17}\sqrt{(b\,x^3+a)^2}}{17(b\,x^3+a)}+\frac{a\,b^4x^{20}\sqrt{(b\,x^3+a)^2}}{4b\,x^3+4a}+\frac{b^5x^{23}}{23b}$

input
```
int(x^7*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/240856*x^8*(10472*b^5*x^15+60214*a*b^4*x^12+141680*a^2*b^3*x^9+172040*a^
3*b^2*x^6+109480*a^4*b*x^3+30107*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

3.56.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.22

$$\int x^7\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{1}{23}\,b^5x^{23}+\frac{1}{4}\,ab^4x^{20}+\frac{10}{17}\,a^2b^3x^{17}+\frac{5}{7}\,a^3b^2x^{14}+\frac{5}{11}\,a^4bx^{11}+\frac{1}{8}\,a^5x^8$$

input
```
integrate(x^7*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

output
```
1/23*b^5*x^23 + 1/4*a*b^4*x^20 + 10/17*a^2*b^3*x^17 + 5/7*a^3*b^2*x^14 + 5
/11*a^4*b*x^11 + 1/8*a^5*x^8
```

3.56.6 Sympy [F]

$$\int x^7\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\int x^7\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(x**7*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

output
```
Integral(x**7*((a + b*x**3)**2)**(5/2), x)
```

3.56.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.22

$$\int x^7 (a^2 + 2abx^3 + b^2x^6)^{5/2} \, dx = \frac{1}{23} b^5 x^{23} + \frac{1}{4} ab^4 x^{20} + \frac{10}{17} a^2 b^3 x^{17} + \frac{5}{7} a^3 b^2 x^{14} + \frac{5}{11} a^4 b x^{11} + \frac{1}{8} a^5 x^8$$

input
```
integrate(x^7*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

output
```
1/23*b^5*x^23 + 1/4*a*b^4*x^20 + 10/17*a^2*b^3*x^17 + 5/7*a^3*b^2*x^14 + 5
/11*a^4*b*x^11 + 1/8*a^5*x^8
```

3.56.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.41

$$\int x^7 (a^2 + 2abx^3 + b^2x^6)^{5/2} \, dx = \frac{1}{23} b^5 x^{23} \text{sgn}(bx^3 + a) + \frac{1}{4} ab^4 x^{20} \text{sgn}(bx^3 + a) + \frac{10}{17} a^2 b^3 x^{17} \text{sgn}(bx^3 + a) + \frac{5}{7} a^3 b^2 x^{14} \text{sgn}(bx^3 + a) + \frac{5}{11} a^4 b x^{11} \text{sgn}(bx^3 + a) + \frac{1}{8} a^5 x^8 \text{sgn}(bx^3 + a)$$

input
```
integrate(x^7*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

output
```
1/23*b^5*x^23*sgn(b*x^3 + a) + 1/4*a*b^4*x^20*sgn(b*x^3 + a) + 10/17*a^2*b
^3*x^17*sgn(b*x^3 + a) + 5/7*a^3*b^2*x^14*sgn(b*x^3 + a) + 5/11*a^4*b*x^11
*sgn(b*x^3 + a) + 1/8*a^5*x^8*sgn(b*x^3 + a)
```

3.56.9 Mupad [F(-1)]

Timed out.

$$\int x^7 (a^2 + 2abx^3 + b^2x^6)^{5/2} \, dx = \int x^7 \left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^{5/2} dx$$

input
```
int(x^7*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

output
```
int(x^7*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

3.57 $\int x^6(a^2 + 2abx^3 + b^2x^6)^{5/2}\,dx$

3.57.1 Optimal result

Integrand size = 26, antiderivative size = 255

$$\int x^6\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{a^5x^7\sqrt{a^2+2abx^3+b^2x^6}}{7\,(a+bx^3)}+\frac{a^4bx^{10}\sqrt{a^2+2abx^3+b^2x^6}}{2\,(a+bx^3)}+\frac{10a^3b^2x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\,(a+bx^3)}+\frac{5a^2b^3x^{16}\sqrt{a^2+2abx^3+b^2x^6}}{8\,(a+bx^3)}+\frac{5ab^4x^{19}\sqrt{a^2+2abx^3+b^2x^6}}{19\,(a+bx^3)}+\frac{b^5x^{22}\sqrt{a^2+2abx^3+b^2x^6}}{22\,(a+bx^3)}$$

output
```
1/7*a^5*x^7*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/2*a^4*b*x^10*((b*x^3+a)^2)^(1/
2)/(b*x^3+a)+10/13*a^3*b^2*x^13*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/8*a^2*b^3*
x^16*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/19*a*b^4*x^19*((b*x^3+a)^2)^(1/2)/(b*
x^3+a)+1/22*b^5*x^22*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

3.57.2 Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int x^6\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{x^7\sqrt{(a+bx^3)^2}(21736a^5+76076a^4bx^3+117040a^3b^2x^6+95095a^2b^3x^9+40040ab^4x^{12}+691}{152152\,(a+bx^3)}$$

input
```
Integrate[x^6*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
(x^7*Sqrt[(a + b*x^3)^2]*(21736*a^5 + 76076*a^4*b*x^3 + 117040*a^3*b^2*x^6
 + 95095*a^2*b^3*x^9 + 40040*a*b^4*x^12 + 6916*b^5*x^15))/(152152*(a + b*x
^3))
```

3.57.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.40, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6 \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx$$

$$\Big\downarrow \ 1384$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int b^5x^6\left(bx^3+a\right)^5dx}{b^5\left(a+bx^3\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^6\left(bx^3+a\right)^5dx}{a+bx^3}$$

$$\Big\downarrow \ 802$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(b^5x^{21}+5ab^4x^{18}+10a^2b^3x^{15}+10a^3b^2x^{12}+5a^4bx^9+a^5x^6\right)dx}{a+bx^3}$$

$$\Big\downarrow \ 2009$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(\frac{a^5x^7}{7}+\frac{1}{2}a^4bx^{10}+\frac{10}{13}a^3b^2x^{13}+\frac{5}{8}a^2b^3x^{16}+\frac{5}{19}ab^4x^{19}+\frac{b^5x^{22}}{22}\right)}{a+bx^3}$$

input
```
Int[x^6*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*((a^5*x^7)/7 + (a^4*b*x^10)/2 + (10*a^3*b
^2*x^13)/13 + (5*a^2*b^3*x^16)/8 + (5*a*b^4*x^19)/19 + (b^5*x^22)/22))/(a
+ b*x^3)
```

3.57.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.57.4 Maple [A] (verified)

Time = 5.23 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.31

method	result
gosper	$\frac{x^{7}\left(6916 b^{5} x^{15}+40040 a\,b^{4} x^{12}+95095 a^{2} b^{3} x^{9}+117040 a^{3} b^{2} x^{6}+76076 a^{4} b\,x^{3}+21736 a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{152152 \left(b\,x^{3}+a\right)^{5}}$
default	$\frac{x^{7}\left(6916 b^{5} x^{15}+40040 a\,b^{4} x^{12}+95095 a^{2} b^{3} x^{9}+117040 a^{3} b^{2} x^{6}+76076 a^{4} b\,x^{3}+21736 a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{152152 \left(b\,x^{3}+a\right)^{5}}$
risch	$\frac{a^{5} x^{7} \sqrt{\left(b\,x^{3}+a\right)^{2}}}{7 b\,x^{3}+7 a}+\frac{a^{4} b\,x^{10} \sqrt{\left(b\,x^{3}+a\right)^{2}}}{2 b\,x^{3}+2 a}+\frac{10 a^{3} b^{2} x^{13} \sqrt{\left(b\,x^{3}+a\right)^{2}}}{13 \left(b\,x^{3}+a\right)}+\frac{5 a^{2} b^{3} x^{16} \sqrt{\left(b\,x^{3}+a\right)^{2}}}{8 \left(b\,x^{3}+a\right)}+\frac{5 a\,b^{4} x^{19} \sqrt{\left(b\,x^{3}+a\right)^{2}}}{19 \left(b\,x^{3}+a\right)}+\frac{b^{5} x^{22}}{22b}$

input
```
int(x^6*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/152152*x^7*(6916*b^5*x^15+40040*a*b^4*x^12+95095*a^2*b^3*x^9+117040*a^3*
b^2*x^6+76076*a^4*b*x^3+21736*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

3.57.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.22

$$\int x^6 \left(a^2 + 2abx^3 + b^2x^6\right)^{5/2} \, dx = \frac{1}{22} b^5 x^{22} + \frac{5}{19} a b^4 x^{19} + \frac{5}{8} a^2 b^3 x^{16} + \frac{10}{13} a^3 b^2 x^{13} + \frac{1}{2} a^4 b x^{10} + \frac{1}{7} a^5 x^7$$

input
```
integrate(x^6*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

output
```
1/22*b^5*x^22 + 5/19*a*b^4*x^19 + 5/8*a^2*b^3*x^16 + 10/13*a^3*b^2*x^13 +
1/2*a^4*b*x^10 + 1/7*a^5*x^7
```

3.57.6 Sympy [F]

$$\int x^6 \left(a^2 + 2abx^3 + b^2x^6\right)^{5/2} \, dx = \int x^6 \left(\left(a + bx^3\right)^2\right)^{\frac{5}{2}} \, dx$$

input
```
integrate(x**6*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

output
```
Integral(x**6*((a + b*x**3)**2)**(5/2), x)
```

3.57.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.22

$$\int x^6 \left(a^2 + 2abx^3 + b^2x^6\right)^{5/2} \, dx = \frac{1}{22} b^5 x^{22} + \frac{5}{19} a b^4 x^{19} + \frac{5}{8} a^2 b^3 x^{16} + \frac{10}{13} a^3 b^2 x^{13} + \frac{1}{2} a^4 b x^{10} + \frac{1}{7} a^5 x^7$$

input
```
integrate(x^6*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

output
```
1/22*b^5*x^22 + 5/19*a*b^4*x^19 + 5/8*a^2*b^3*x^16 + 10/13*a^3*b^2*x^13 +
1/2*a^4*b*x^10 + 1/7*a^5*x^7
```

3.57.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.41

$$\int x^6 \left(a^2+2abx^3+b^2x^6\right)^{5/2} \, dx = \frac{1}{22} b^5 x^{22} \mathrm{sgn}\left(bx^3+a\right) + \frac{5}{19} ab^4 x^{19} \mathrm{sgn}\left(bx^3+a\right) + \frac{5}{8} a^2 b^3 x^{16} \mathrm{sgn}\left(bx^3+a\right) + \frac{10}{13} a^3 b^2 x^{13} \mathrm{sgn}\left(bx^3+a\right) + \frac{1}{2} a^4 b x^{10} \mathrm{sgn}\left(bx^3+a\right) + \frac{1}{7} a^5 x^7 \mathrm{sgn}\left(bx^3+a\right)$$

input
```
integrate(x^6*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

output
```
1/22*b^5*x^22*sgn(b*x^3 + a) + 5/19*a*b^4*x^19*sgn(b*x^3 + a) + 5/8*a^2*b^
3*x^16*sgn(b*x^3 + a) + 10/13*a^3*b^2*x^13*sgn(b*x^3 + a) + 1/2*a^4*b*x^10
*sgn(b*x^3 + a) + 1/7*a^5*x^7*sgn(b*x^3 + a)
```

3.57.9 Mupad [F(-1)]

Timed out.

$$\int x^6 \left(a^2+2abx^3+b^2x^6\right)^{5/2} \, dx = \int x^6 \, \left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2} \,dx$$

input
```
int(x^6*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

output
```
int(x^6*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

3.58 $\int x^5(a^2+2abx^3+b^2x^6)^{5/2}\,dx$

3.58.1 Optimal result

Integrand size = 26, antiderivative size = 78

$$\int x^5\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=-\frac{a(a+bx^3)^5\sqrt{a^2+2abx^3+b^2x^6}}{18b^2}+\frac{(a+bx^3)^6\sqrt{a^2+2abx^3+b^2x^6}}{21b^2}$$

output
```
-1/18*a*(b*x^3+a)^5*((b*x^3+a)^2)^(1/2)/b^2+1/21*(b*x^3+a)^6*((b*x^3+a)^2)
^(1/2)/b^2
```

3.58.2 Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.73

$$\int x^5\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{x^6(21a^5+70a^4bx^3+105a^3b^2x^6+84a^2b^3x^9+35ab^4x^{12}+6b^5x^{15})\left(\sqrt{a^2}bx^3+a\left(\sqrt{a^2}-\sqrt{(a\right.\right.}{126\left(-a^2-abx^3+\sqrt{a^2}\sqrt{(a+bx^3)^2}\right)}$$

input
```
Integrate[x^5*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
(x^6*(21*a^5 + 70*a^4*b*x^3 + 105*a^3*b^2*x^6 + 84*a^2*b^3*x^9 + 35*a*b^4*
x^12 + 6*b^5*x^15)*(Sqrt[a^2]*b*x^3 + a*(Sqrt[a^2] - Sqrt[(a + b*x^3)^2]))
)/(126*(-a^2 - a*b*x^3 + Sqrt[a^2]*Sqrt[(a + b*x^3)^2]))
```

3.58.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1693, 1100, 1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx$$

$$\Big\downarrow \ 1693$$

$$\frac{1}{3}\int x^3 \left(b^2x^6+2abx^3+a^2\right)^{5/2} dx^3$$

$$\Big\downarrow \ 1100$$

$$\frac{1}{3}\left(\frac{\left(a^2+2abx^3+b^2x^6\right)^{7/2}}{7b^2}-\frac{a\int \left(b^2x^6+2abx^3+a^2\right)^{5/2} dx^3}{b}\right)$$

$$\Big\downarrow \ 1079$$

$$\frac{1}{3}\left(\frac{\left(a^2+2abx^3+b^2x^6\right)^{7/2}}{7b^2}-\frac{a\sqrt{a^2+2abx^3+b^2x^6}\int \left(b^2x^3+ab\right)^5 dx^3}{b^6\left(a+bx^3\right)}\right)$$

$$\Big\downarrow \ 17$$

$$\frac{1}{3}\left(\frac{\left(a^2+2abx^3+b^2x^6\right)^{7/2}}{7b^2}-\frac{a\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{6b^2}\right)$$

input
```
Int[x^5*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
(-1/6*(a*(a + b*x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/b^2 + (a^2 + 2*a*b
*x^3 + b^2*x^6)^(7/2)/(7*b^2))/3
```

3.58.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1100
```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.58.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.40

method	result
pseudoelliptic	$-\frac{\operatorname{csgn}\left(b\,x^{3}+a\right)\left(b\,x^{3}+a\right)^{6}\left(-6b\,x^{3}+a\right)}{126b^{2}}$
gosper	$\frac{x^{6}\left(6b^{5}x^{15}+35a\,b^{4}x^{12}+84a^{2}b^{3}x^{9}+105a^{3}b^{2}x^{6}+70a^{4}b\,x^{3}+21a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{126\left(b\,x^{3}+a\right)^{5}}$
default	$\frac{x^{6}\left(6b^{5}x^{15}+35a\,b^{4}x^{12}+84a^{2}b^{3}x^{9}+105a^{3}b^{2}x^{6}+70a^{4}b\,x^{3}+21a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{126\left(b\,x^{3}+a\right)^{5}}$
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,a^{5}x^{6}}{6b\,x^{3}+6a}+\frac{5\sqrt{\left(b\,x^{3}+a\right)^{2}}\,b\,a^{4}x^{9}}{9\left(b\,x^{3}+a\right)}+\frac{5\sqrt{\left(b\,x^{3}+a\right)^{2}}\,a^{3}b^{2}x^{12}}{6\left(b\,x^{3}+a\right)}+\frac{2\sqrt{\left(b\,x^{3}+a\right)^{2}}\,a^{2}b^{3}x^{15}}{3\left(b\,x^{3}+a\right)}+\frac{5\sqrt{\left(b\,x^{3}+a\right)^{2}}\,b^{4}a\,x^{18}}{18\left(b\,x^{3}+a\right)}$

input
```
int(x^5*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/126*csgn(b*x^3+a)*(b*x^3+a)^6*(-6*b*x^3+a)/b^2
```

3.58.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.73

$$\int x^5 \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx = \frac{1}{21}b^5x^{21}+\frac{5}{18}ab^4x^{18} + \frac{2}{3}a^2b^3x^{15}+\frac{5}{6}a^3b^2x^{12}+\frac{5}{9}a^4bx^9+\frac{1}{6}a^5x^6$$

input
```
integrate(x^5*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

output
```
1/21*b^5*x^21 + 5/18*a*b^4*x^18 + 2/3*a^2*b^3*x^15 + 5/6*a^3*b^2*x^12 + 5/
9*a^4*b*x^9 + 1/6*a^5*x^6
```

3.58.6 Sympy [F]

$$\int x^5 \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx = \int x^5 \left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}} dx$$

input
```
integrate(x**5*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

output
```
Integral(x**5*((a + b*x**3)**2)**(5/2), x)
```

3.58.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.06

$$\int x^5 \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx = -\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{5}{2}}ax^3}{18\,b} - \frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{5}{2}}a^2}{18\,b^2} + \frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{7}{2}}}{21\,b^2}$$

input
```
integrate(x^5*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*a*x^3/b - 1/18*(b^2*x^6 + 2*a*b*x^
3 + a^2)^(5/2)*a^2/b^2 + 1/21*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)/b^2
```

3.58.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.86

$$\int x^5\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{1}{126}\left(6\,b^5x^{21}+35\,ab^4x^{18}+84\,a^2b^3x^{15}+105\,a^3b^2x^{12}+70\,a^4bx^9+21\,a^5x^6\right)\operatorname{sgn}\left(bx^3+a\right)$$

input
```
integrate(x^5*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

output
```
1/126*(6*b^5*x^21 + 35*a*b^4*x^18 + 84*a^2*b^3*x^15 + 105*a^3*b^2*x^12 + 7
0*a^4*b*x^9 + 21*a^5*x^6)*sgn(b*x^3 + a)
```

3.58.9 Mupad [F(-1)]

Timed out.

$$\int x^5\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\int x^5\,\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}\,dx$$

input
```
int(x^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

output
```
int(x^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

3.59 $\int x^4 (a^2 + 2abx^3 + b^2x^6)^{5/2} \, dx$

3.59.1 Optimal result

Integrand size = 26, antiderivative size = 255

$$\int x^4 \left(a^2 + 2abx^3 + b^2x^6\right)^{5/2} \, dx = \frac{a^5x^5\sqrt{a^2 + 2abx^3 + b^2x^6}}{5(a + bx^3)} + \frac{5a^4bx^8\sqrt{a^2 + 2abx^3 + b^2x^6}}{8(a + bx^3)} + \frac{10a^3b^2x^{11}\sqrt{a^2 + 2abx^3 + b^2x^6}}{11(a + bx^3)} + \frac{5a^2b^3x^{14}\sqrt{a^2 + 2abx^3 + b^2x^6}}{7(a + bx^3)} + \frac{5ab^4x^{17}\sqrt{a^2 + 2abx^3 + b^2x^6}}{17(a + bx^3)} + \frac{b^5x^{20}\sqrt{a^2 + 2abx^3 + b^2x^6}}{20(a + bx^3)}$$

output

```
1/5*a^5*x^5*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/8*a^4*b*x^8*((b*x^3+a)^2)^(1/2
)/(b*x^3+a)+10/11*a^3*b^2*x^11*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/7*a^2*b^3*x
^14*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/17*a*b^4*x^17*((b*x^3+a)^2)^(1/2)/(b*x
^3+a)+1/20*b^5*x^20*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

3.59.2 Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int x^4\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{x^5\sqrt{(a+bx^3)^2}\left(10472a^5+32725a^4bx^3+47600a^3b^2x^6+37400a^2b^3x^9+15400ab^4x^{12}+2618\ldots\right.}{52360\left(a+bx^3\right)}$$

input
```
Integrate[x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
(x^5*Sqrt[(a + b*x^3)^2]*(10472*a^5 + 32725*a^4*b*x^3 + 47600*a^3*b^2*x^6
+ 37400*a^2*b^3*x^9 + 15400*a*b^4*x^12 + 2618*b^5*x^15))/(52360*(a + b*x^3
))
```

3.59.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.40, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\left(a^2+2abx^3+b^2x^6\right)^{5/2}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int b^5x^4\left(bx^3+a\right)^5dx}{b^5\left(a+bx^3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^4\left(bx^3+a\right)^5dx}{a+bx^3}$$

$$\Big\downarrow \text{802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(b^5x^{19}+5ab^4x^{16}+10a^2b^3x^{13}+10a^3b^2x^{10}+5a^4bx^7+a^5x^4\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(\frac{a^5x^5}{5}+\frac{5}{8}a^4bx^8+\frac{10}{11}a^3b^2x^{11}+\frac{5}{7}a^2b^3x^{14}+\frac{5}{17}ab^4x^{17}+\frac{b^5x^{20}}{20}\right)}{a+bx^3}$$

input
```
Int[x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*((a^5*x^5)/5 + (5*a^4*b*x^8)/8 + (10*a^3*
b^2*x^11)/11 + (5*a^2*b^3*x^14)/7 + (5*a*b^4*x^17)/17 + (b^5*x^20)/20))/(a
 + b*x^3)
```

3.59.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.59.4 Maple [A] (verified)

Time = 3.75 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.31

method	result
gosper	$\frac{x^{5}\left(2618 b^{5} x^{15}+15400 a\,b^{4} x^{12}+37400 a^{2} b^{3} x^{9}+47600 a^{3} b^{2} x^{6}+32725 a^{4} b\,x^{3}+10472 a^{5}\right) \left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{52360 \left(b\,x^{3}+a\right)^{5}}$
default	$\frac{x^{5}\left(2618 b^{5} x^{15}+15400 a\,b^{4} x^{12}+37400 a^{2} b^{3} x^{9}+47600 a^{3} b^{2} x^{6}+32725 a^{4} b\,x^{3}+10472 a^{5}\right) \left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{52360 \left(b\,x^{3}+a\right)^{5}}$
risch	$\frac{a^{5} x^{5} \sqrt{\left(b\,x^{3}+a\right)^{2}}}{5 b\,x^{3}+5 a}+\frac{5 a^{4} b\,x^{8} \sqrt{\left(b\,x^{3}+a\right)^{2}}}{8 \left(b\,x^{3}+a\right)}+\frac{10 a^{3} b^{2} x^{11} \sqrt{\left(b\,x^{3}+a\right)^{2}}}{11 \left(b\,x^{3}+a\right)}+\frac{5 a^{2} b^{3} x^{14} \sqrt{\left(b\,x^{3}+a\right)^{2}}}{7 \left(b\,x^{3}+a\right)}+\frac{5 a\,b^{4} x^{17} \sqrt{\left(b\,x^{3}+a\right)^{2}}}{17 \left(b\,x^{3}+a\right)}+\frac{b^{5} x^{20}}{20 b}$

input
```
int(x^4*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/52360*x^5*(2618*b^5*x^15+15400*a*b^4*x^12+37400*a^2*b^3*x^9+47600*a^3*b^
2*x^6+32725*a^4*b*x^3+10472*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

3.59.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.22

$$\int x^4 \left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx = \frac{1}{20}\,b^5x^{20}+\frac{5}{17}\,ab^4x^{17} + \frac{5}{7}\,a^2b^3x^{14}+\frac{10}{11}\,a^3b^2x^{11}+\frac{5}{8}\,a^4bx^8+\frac{1}{5}\,a^5x^5$$

input
```
integrate(x^4*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

output
```
1/20*b^5*x^20 + 5/17*a*b^4*x^17 + 5/7*a^2*b^3*x^14 + 10/11*a^3*b^2*x^11 +
5/8*a^4*b*x^8 + 1/5*a^5*x^5
```

3.59.6 Sympy [F]

$$\int x^4 \left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx = \int x^4 \left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(x**4*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

output
```
Integral(x**4*((a + b*x**3)**2)**(5/2), x)
```

3.59.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.22

$$\int x^4 (a^2 + 2abx^3 + b^2x^6)^{5/2} dx = \frac{1}{20} b^5 x^{20} + \frac{5}{17} ab^4 x^{17} + \frac{5}{7} a^2 b^3 x^{14} + \frac{10}{11} a^3 b^2 x^{11} + \frac{5}{8} a^4 b x^8 + \frac{1}{5} a^5 x^5$$

input
```
integrate(x^4*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

output
```
1/20*b^5*x^20 + 5/17*a*b^4*x^17 + 5/7*a^2*b^3*x^14 + 10/11*a^3*b^2*x^11 +
5/8*a^4*b*x^8 + 1/5*a^5*x^5
```

3.59.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.41

$$\int x^4 (a^2 + 2abx^3 + b^2x^6)^{5/2} dx = \frac{1}{20} b^5 x^{20} \operatorname{sgn}(bx^3 + a) + \frac{5}{17} ab^4 x^{17} \operatorname{sgn}(bx^3 + a) + \frac{5}{7} a^2 b^3 x^{14} \operatorname{sgn}(bx^3 + a) + \frac{10}{11} a^3 b^2 x^{11} \operatorname{sgn}(bx^3 + a) + \frac{5}{8} a^4 b x^8 \operatorname{sgn}(bx^3 + a) + \frac{1}{5} a^5 x^5 \operatorname{sgn}(bx^3 + a)$$

input
```
integrate(x^4*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

output
```
1/20*b^5*x^20*sgn(b*x^3 + a) + 5/17*a*b^4*x^17*sgn(b*x^3 + a) + 5/7*a^2*b^
3*x^14*sgn(b*x^3 + a) + 10/11*a^3*b^2*x^11*sgn(b*x^3 + a) + 5/8*a^4*b*x^8*
sgn(b*x^3 + a) + 1/5*a^5*x^5*sgn(b*x^3 + a)
```

3.59.9 Mupad [F(-1)]

Timed out.

$$\int x^4 (a^2 + 2abx^3 + b^2x^6)^{5/2} dx = \int x^4 \left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^{5/2} dx$$

input
```
int(x^4*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

output
```
int(x^4*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

3.60 $\int x^3(a^2+2abx^3+b^2x^6)^{5/2}\,dx$

3.60.1 Optimal result

Integrand size = 26, antiderivative size = 252

$$\int x^3\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{a^5x^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{5a^4bx^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{a^3b^2x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{10a^2b^3x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}+\frac{5ab^4x^{16}\sqrt{a^2+2abx^3+b^2x^6}}{16\left(a+bx^3\right)}+\frac{b^5x^{19}\sqrt{a^2+2abx^3+b^2x^6}}{19\left(a+bx^3\right)}$$

output
```
1/4*a^5*x^4*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/7*a^4*b*x^7*((b*x^3+a)^2)^(1/2
)/(b*x^3+a)+a^3*b^2*x^10*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+10/13*a^2*b^3*x^13*
((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/16*a*b^4*x^16*((b*x^3+a)^2)^(1/2)/(b*x^3+a
)+1/19*b^5*x^19*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

3.60.2 Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int x^3 \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx = \frac{x^4\sqrt{(a+bx^3)^2}(6916a^5+19760a^4bx^3+27664a^3b^2x^6+21280a^2b^3x^9+8645ab^4x^{12}+1456b^5x}{27664\,(a+bx^3)}$$

input
```
Integrate[x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
(x^4*Sqrt[(a + b*x^3)^2]*(6916*a^5 + 19760*a^4*b*x^3 + 27664*a^3*b^2*x^6 +
 21280*a^2*b^3*x^9 + 8645*a*b^4*x^12 + 1456*b^5*x^15))/(27664*(a + b*x^3))
```

3.60.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.39, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx$$

$$\Big\downarrow \; 1384$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int b^5x^3\left(bx^3+a\right)^5dx}{b^5\,(a+bx^3)}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^3\left(bx^3+a\right)^5dx}{a+bx^3}$$

$$\Big\downarrow \; 802$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(b^5x^{18}+5ab^4x^{15}+10a^2b^3x^{12}+10a^3b^2x^9+5a^4bx^6+a^5x^3\right)dx}{a+bx^3}$$

$$\Big\downarrow \; 2009$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(\frac{a^5x^4}{4}+\frac{5}{7}a^4bx^7+a^3b^2x^{10}+\frac{10}{13}a^2b^3x^{13}+\frac{5}{16}ab^4x^{16}+\frac{b^5x^{19}}{19}\right)}{a+bx^3}$$

input
```
Int[x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*((a^5*x^4)/4 + (5*a^4*b*x^7)/7 + a^3*b^2*
x^10 + (10*a^2*b^3*x^13)/13 + (5*a*b^4*x^16)/16 + (b^5*x^19)/19))/(a + b*x
^3)
```

3.60.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.60.4 Maple [A] (verified)

Time = 2.85 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.32

method	result
gosper	$\frac{x^4(1456b^5x^{15}+8645a\,b^4x^{12}+21280a^2b^3x^9+27664a^3b^2x^6+19760a^4b\,x^3+6916a^5)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{27664(b\,x^3+a)^5}$
default	$\frac{x^4(1456b^5x^{15}+8645a\,b^4x^{12}+21280a^2b^3x^9+27664a^3b^2x^6+19760a^4b\,x^3+6916a^5)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{27664(b\,x^3+a)^5}$
risch	$\frac{a^5x^4\sqrt{(b\,x^3+a)^2}}{4b\,x^3+4a}+\frac{5a^4b\,x^7\sqrt{(b\,x^3+a)^2}}{7(b\,x^3+a)}+\frac{a^3b^2x^{10}\sqrt{(b\,x^3+a)^2}}{b\,x^3+a}+\frac{10a^2b^3x^{13}\sqrt{(b\,x^3+a)^2}}{13(b\,x^3+a)}+\frac{5a\,b^4x^{16}\sqrt{(b\,x^3+a)^2}}{16(b\,x^3+a)}+\frac{b^5x^{19}\sqrt{}}{19b\,x}$

input
```
int(x^3*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/27664*x^4*(1456*b^5*x^15+8645*a*b^4*x^12+21280*a^2*b^3*x^9+27664*a^3*b^2
*x^6+19760*a^4*b*x^3+6916*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

3.60.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.22

$$\int x^3\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{1}{19}\,b^5x^{19}+\frac{5}{16}\,ab^4x^{16}+\frac{10}{13}\,a^2b^3x^{13}+a^3b^2x^{10}+\frac{5}{7}\,a^4bx^7+\frac{1}{4}\,a^5x^4$$

input
```
integrate(x^3*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

output
```
1/19*b^5*x^19 + 5/16*a*b^4*x^16 + 10/13*a^2*b^3*x^13 + a^3*b^2*x^10 + 5/7*
a^4*b*x^7 + 1/4*a^5*x^4
```

3.60.6 Sympy [F]

$$\int x^3\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\int x^3\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(x**3*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

output
```
Integral(x**3*((a + b*x**3)**2)**(5/2), x)
```

3.60.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.22

$$\int x^3\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{1}{19}\,b^5x^{19}+\frac{5}{16}\,ab^4x^{16}+\frac{10}{13}\,a^2b^3x^{13}+a^3b^2x^{10}+\frac{5}{7}\,a^4bx^7+\frac{1}{4}\,a^5x^4$$

input
```
integrate(x^3*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

output
```
1/19*b^5*x^19 + 5/16*a*b^4*x^16 + 10/13*a^2*b^3*x^13 + a^3*b^2*x^10 + 5/7*
a^4*b*x^7 + 1/4*a^5*x^4
```

3.60.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.41

$$\int x^3\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{1}{19}\,b^5x^{19}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{16}\,ab^4x^{16}\mathrm{sgn}\left(bx^3+a\right)+\frac{10}{13}\,a^2b^3x^{13}\mathrm{sgn}\left(bx^3+a\right)+a^3b^2x^{10}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{7}\,a^4bx^7\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{4}\,a^5x^4\mathrm{sgn}\left(bx^3+a\right)$$

input
```
integrate(x^3*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

output
```
1/19*b^5*x^19*sgn(b*x^3 + a) + 5/16*a*b^4*x^16*sgn(b*x^3 + a) + 10/13*a^2*
b^3*x^13*sgn(b*x^3 + a) + a^3*b^2*x^10*sgn(b*x^3 + a) + 5/7*a^4*b*x^7*sgn(
b*x^3 + a) + 1/4*a^5*x^4*sgn(b*x^3 + a)
```

3.60.9 Mupad [F(-1)]

Timed out.

$$\int x^3\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\int x^3\,{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{5/2}\,dx$$

input
```
int(x^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

output
```
int(x^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

3.61 $\int x^2(a^2 + 2abx^3 + b^2x^6)^{5/2}\,dx$

3.61.1 Optimal result

Integrand size = 26, antiderivative size = 36

$$\int x^2(a^2 + 2abx^3 + b^2x^6)^{5/2}\,dx = \frac{(a+bx^3)(a^2+2abx^3+b^2x^6)^{5/2}}{18b}$$

output
```
1/18*(b*x^3+a)*(b^2*x^6+2*a*b*x^3+a^2)^(5/2)/b
```

3.61.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 82 vs. 2(36) = 72.

Time = 1.02 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.28

$$\int x^2(a^2 + 2abx^3 + b^2x^6)^{5/2}\,dx = \frac{x^3\sqrt{(a+bx^3)^2}(6a^5+15a^4bx^3+20a^3b^2x^6+15a^2b^3x^9+6ab^4x^{12}+b^5x^{15})}{18(a+bx^3)}$$

input
```
Integrate[x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
(x^3*Sqrt[(a + b*x^3)^2]*(6*a^5 + 15*a^4*b*x^3 + 20*a^3*b^2*x^6 + 15*a^2*b
^3*x^9 + 6*a*b^4*x^12 + b^5*x^15))/(18*(a + b*x^3))
```

3.61.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1690, 1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx$$

$$\Big\downarrow \text{ 1690}$$

$$\frac{1}{3}\int \left(b^2x^6+2abx^3+a^2\right)^{5/2} dx^3$$

$$\Big\downarrow \text{ 1079}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(b^2x^3+ab\right)^5 dx^3}{3b^5\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 17}$$

$$\frac{\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{18b}$$

input
```
Int[x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
((a + b*x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(18*b)
```

3.61.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

3.61.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.64

method	result	size
pseudoelliptic	$\frac{\left(b\,x^{3}+a\right)^{6} \operatorname{csgn}\left(b\,x^{3}+a\right)}{18 b}$	23
default	$\frac{\left(b\,x^{3}+a\right) \left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{18 b}$	24
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(b\,x^{3}+a\right)^{5}}{18 b}$	26
gosper	$\frac{x^{3} \left(b^{5} x^{15}+6 a\,b^{4} x^{12}+15 a^{2} b^{3} x^{9}+20 a^{3} b^{2} x^{6}+15 a^{4} b\,x^{3}+6 a^{5}\right) \left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{18 \left(b\,x^{3}+a\right)^{5}}$	79

input
```
int(x^2*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/18*(b*x^3+a)^6*csgn(b*x^3+a)/b
```

3.61.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.58

$$\int x^2 \left(a^2+2abx^3+b^2x^6\right)^{5/2} \, dx = \frac{1}{18} \, b^5 x^{18} + \frac{1}{3} \, a b^4 x^{15} + \frac{5}{6} \, a^2 b^3 x^{12} + \frac{10}{9} \, a^3 b^2 x^9 + \frac{5}{6} \, a^4 b x^6 + \frac{1}{3} \, a^5 x^3$$

input
```
integrate(x^2*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

output
```
1/18*b^5*x^18 + 1/3*a*b^4*x^15 + 5/6*a^2*b^3*x^12 + 10/9*a^3*b^2*x^9 + 5/6
*a^4*b*x^6 + 1/3*a^5*x^3
```

3.61.6 Sympy [F]

$$\int x^2\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\int x^2\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(x**2*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

output
```
Integral(x**2*((a + b*x**3)**2)**(5/2), x)
```

3.61.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.44

$$\int x^2\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{1}{18}\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{5}{2}}x^3+\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{5}{2}}a}{18\,b}$$

input
```
integrate(x^2*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

output
```
1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*x^3 + 1/18*(b^2*x^6 + 2*a*b*x^3 + a
^2)^(5/2)*a/b
```

3.61.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(32) = 64.

Time = 0.32 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.83

$$\int x^2\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{1}{18}\left(3\left(bx^6+2\,ax^3\right)a^4+3\left(bx^6+2\,ax^3\right)^2a^2b+\left(bx^6+2\,ax^3\right)^3b^2\right)\operatorname{sgn}\left(bx^3+a\right)$$

input
```
integrate(x^2*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

output
```
1/18*(3*(b*x^6 + 2*a*x^3)*a^4 + 3*(b*x^6 + 2*a*x^3)^2*a^2*b + (b*x^6 + 2*a
*x^3)^3*b^2)*sgn(b*x^3 + a)
```

3.61.9 Mupad [B] (verification not implemented)

Time = 8.21 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int x^2\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{\left(b^2\,x^3+a\,b\right)\,\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}}{18\,b^2}$$

input
```
int(x^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

output
```
((a*b + b^2*x^3)*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2))/(18*b^2)
```

3.62 $\int x(a^2+2abx^3+b^2x^6)^{5/2}\,dx$

3.62.1 Optimal result

Integrand size = 24, antiderivative size = 252

$$\int x\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{a^5x^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{a^4bx^5\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{5a^3b^2x^8\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{10a^2b^3x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{5ab^4x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14\left(a+bx^3\right)}+\frac{b^5x^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17\left(a+bx^3\right)}$$

output
```
1/2*a^5*x^2*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+a^4*b*x^5*((b*x^3+a)^2)^(1/2)/(b
*x^3+a)+5/4*a^3*b^2*x^8*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+10/11*a^2*b^3*x^11*(
(b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/14*a*b^4*x^14*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
+1/17*b^5*x^17*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

3.62.2 Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int x\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{x^2\sqrt{\left(a+bx^3\right)^2}\left(2618a^5+5236a^4bx^3+6545a^3b^2x^6+4760a^2b^3x^9+1870ab^4x^{12}+308b^5x^{15}\right)}{5236\left(a+bx^3\right)}$$

input
```
Integrate[x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
(x^2*Sqrt[(a + b*x^3)^2]*(2618*a^5 + 5236*a^4*b*x^3 + 6545*a^3*b^2*x^6 + 4
760*a^2*b^3*x^9 + 1870*a*b^4*x^12 + 308*b^5*x^15))/(5236*(a + b*x^3))
```

3.62.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.39, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a^2+2abx^3+b^2x^6\right)^{5/2}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int b^5x\left(bx^3+a\right)^5dx}{b^5\left(a+bx^3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int x\left(bx^3+a\right)^5dx}{a+bx^3}$$

$$\Big\downarrow \text{802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(b^5x^{16}+5ab^4x^{13}+10a^2b^3x^{10}+10a^3b^2x^7+5a^4bx^4+a^5x\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(\frac{a^5x^2}{2}+a^4bx^5+\frac{5}{4}a^3b^2x^8+\frac{10}{11}a^2b^3x^{11}+\frac{5}{14}ab^4x^{14}+\frac{b^5x^{17}}{17}\right)}{a+bx^3}$$

input
```
Int[x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*((a^5*x^2)/2 + a^4*b*x^5 + (5*a^3*b^2*x^8
)/4 + (10*a^2*b^3*x^11)/11 + (5*a*b^4*x^14)/14 + (b^5*x^17)/17))/(a + b*x^
3)
```

3.62.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.62.4 Maple [A] (verified)

Time = 1.91 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.32

method	result
gosper	$\frac{x^{2} \left(308 b^{5} x^{15}+1870 a \,b^{4} x^{12}+4760 a^{2} b^{3} x^{9}+6545 a^{3} b^{2} x^{6}+5236 a^{4} b \,x^{3}+2618 a^{5}\right) \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{5}{2}}}{5236 \left(b \,x^{3}+a \right)^{5}}$
default	$\frac{x^{2} \left(308 b^{5} x^{15}+1870 a \,b^{4} x^{12}+4760 a^{2} b^{3} x^{9}+6545 a^{3} b^{2} x^{6}+5236 a^{4} b \,x^{3}+2618 a^{5}\right) \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{5}{2}}}{5236 \left(b \,x^{3}+a \right)^{5}}$
risch	$\frac{a^{5} x^{2} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{2 b \,x^{3}+2 a}+\frac{a^{4} b \,x^{5} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{b \,x^{3}+a}+\frac{5 a^{3} b^{2} x^{8} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{4 \left(b \,x^{3}+a \right)}+\frac{10 a^{2} b^{3} x^{11} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{11 \left(b \,x^{3}+a \right)}+\frac{5 a \,b^{4} x^{14} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{14 \left(b \,x^{3}+a \right)}+\frac{b^{5} x^{17} \sqrt{(}}{17 b x^{3}}$

input
```
int(x*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/5236*x^2*(308*b^5*x^15+1870*a*b^4*x^12+4760*a^2*b^3*x^9+6545*a^3*b^2*x^6
+5236*a^4*b*x^3+2618*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

3.62.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.22

$$\int x(a^2+2abx^3+b^2x^6)^{5/2}\,dx=\frac{1}{17}\,b^5x^{17}+\frac{5}{14}\,ab^4x^{14}+\frac{10}{11}\,a^2b^3x^{11}+\frac{5}{4}\,a^3b^2x^8+a^4bx^5+\frac{1}{2}\,a^5x^2$$

input
```
integrate(x*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

output
```
1/17*b^5*x^17 + 5/14*a*b^4*x^14 + 10/11*a^2*b^3*x^11 + 5/4*a^3*b^2*x^8 + a
^4*b*x^5 + 1/2*a^5*x^2
```

3.62.6 Sympy [F]

$$\int x(a^2+2abx^3+b^2x^6)^{5/2}\,dx=\int x\Big(\big(a+bx^3\big)^2\Big)^{\frac{5}{2}}\,dx$$

input
```
integrate(x*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

output
```
Integral(x*((a + b*x**3)**2)**(5/2), x)
```

3.62.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.22

$$\int x(a^2+2abx^3+b^2x^6)^{5/2}\,dx=\frac{1}{17}\,b^5x^{17}+\frac{5}{14}\,ab^4x^{14}+\frac{10}{11}\,a^2b^3x^{11}+\frac{5}{4}\,a^3b^2x^8+a^4bx^5+\frac{1}{2}\,a^5x^2$$

input
```
integrate(x*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

output
```
1/17*b^5*x^17 + 5/14*a*b^4*x^14 + 10/11*a^2*b^3*x^11 + 5/4*a^3*b^2*x^8 + a
^4*b*x^5 + 1/2*a^5*x^2
```

3.62.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.41

$$\int x(a^2+2abx^3+b^2x^6)^{5/2}\,dx = \frac{1}{17}b^5x^{17}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{14}ab^4x^{14}\mathrm{sgn}\left(bx^3+a\right)+\frac{10}{11}a^2b^3x^{11}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{4}a^3b^2x^8\mathrm{sgn}\left(bx^3+a\right)+a^4bx^5\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{2}a^5x^2\mathrm{sgn}\left(bx^3+a\right)$$

input
```
integrate(x*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

output
```
1/17*b^5*x^17*sgn(b*x^3 + a) + 5/14*a*b^4*x^14*sgn(b*x^3 + a) + 10/11*a^2*
b^3*x^11*sgn(b*x^3 + a) + 5/4*a^3*b^2*x^8*sgn(b*x^3 + a) + a^4*b*x^5*sgn(b
*x^3 + a) + 1/2*a^5*x^2*sgn(b*x^3 + a)
```

3.62.9 Mupad [F(-1)]

Timed out.

$$\int x(a^2+2abx^3+b^2x^6)^{5/2}\,dx = \int x\,\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}\,dx$$

input
```
int(x*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

output
```
int(x*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

3.63 $\int \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx$

3.63.1 Optimal result

Integrand size = 22, antiderivative size = 247

$$\int \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx = \frac{a^5x(a^2+2abx^3+b^2x^6)^{5/2}}{(a+bx^3)^5} + \frac{5a^4bx^4(a^2+2abx^3+b^2x^6)^{5/2}}{4\,(a+bx^3)^5} + \frac{10a^3b^2x^7(a^2+2abx^3+b^2x^6)^{5/2}}{7\,(a+bx^3)^5} + \frac{a^2b^3x^{10}(a^2+2abx^3+b^2x^6)^{5/2}}{(a+bx^3)^5} + \frac{5ab^4x^{13}(a^2+2abx^3+b^2x^6)^{5/2}}{13\,(a+bx^3)^5} + \frac{b^5x^{16}(a^2+2abx^3+b^2x^6)^{5/2}}{16\,(a+bx^3)^5}$$

output
```
a^5*x*(b^2*x^6+2*a*b*x^3+a^2)^(5/2)/(b*x^3+a)^5+5/4*a^4*b*x^4*(b^2*x^6+2*a
*b*x^3+a^2)^(5/2)/(b*x^3+a)^5+10/7*a^3*b^2*x^7*(b^2*x^6+2*a*b*x^3+a^2)^(5/
2)/(b*x^3+a)^5+a^2*b^3*x^10*(b^2*x^6+2*a*b*x^3+a^2)^(5/2)/(b*x^3+a)^5+5/13
*a*b^4*x^13*(b^2*x^6+2*a*b*x^3+a^2)^(5/2)/(b*x^3+a)^5+1/16*b^5*x^16*(b^2*x
^6+2*a*b*x^3+a^2)^(5/2)/(b*x^3+a)^5
```

3.63.2 Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.33

$$\int \left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{x\sqrt{(a+bx^3)^2}(1456a^5+1820a^4bx^3+2080a^3b^2x^6+1456a^2b^3x^9+560ab^4x^{12}+91b^5x^{15})}{1456\,(a+bx^3)}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
(x*Sqrt[(a + b*x^3)^2]*(1456*a^5 + 1820*a^4*b*x^3 + 2080*a^3*b^2*x^6 + 145
6*a^2*b^3*x^9 + 560*a*b^4*x^12 + 91*b^5*x^15))/(1456*(a + b*x^3))
```

3.63.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.41, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1384, 747, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx$$
$$\Big\downarrow \text{1384}$$
$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(b^2x^3+ab\right)^5dx}{b^5\left(a+bx^3\right)}$$
$$\Big\downarrow \text{747}$$
$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(b^{10}x^{15}+5ab^9x^{12}+10a^2b^8x^9+10a^3b^7x^6+5a^4b^6x^3+a^5b^5\right)dx}{b^5\left(a+bx^3\right)}$$
$$\Big\downarrow \text{2009}$$
$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(a^5b^5x+\frac{5}{4}a^4b^6x^4+\frac{10}{7}a^3b^7x^7+a^2b^8x^{10}+\frac{5}{13}ab^9x^{13}+\frac{b^{10}x^{16}}{16}\right)}{b^5\left(a+bx^3\right)}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(a^5*b^5*x + (5*a^4*b^6*x^4)/4 + (10*a^3*
b^7*x^7)/7 + a^2*b^8*x^10 + (5*a*b^9*x^13)/13 + (b^10*x^16)/16))/(b^5*(a +
 b*x^3))
```

3.63.3.1 Defintions of rubi rules used

rule 747
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.63.4 Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.32

method	result
gosper	$\frac{x\left(91b^{5}x^{15}+560a\,b^{4}x^{12}+1456a^{2}b^{3}x^{9}+2080a^{3}b^{2}x^{6}+1820a^{4}b\,x^{3}+1456a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{1456\left(b\,x^{3}+a\right)^{5}}$
default	$\frac{x\left(91b^{5}x^{15}+560a\,b^{4}x^{12}+1456a^{2}b^{3}x^{9}+2080a^{3}b^{2}x^{6}+1820a^{4}b\,x^{3}+1456a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{1456\left(b\,x^{3}+a\right)^{5}}$
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,a^{5}x}{b\,x^{3}+a}+\frac{5\sqrt{\left(b\,x^{3}+a\right)^{2}}\,b\,a^{4}x^{4}}{4\left(b\,x^{3}+a\right)}+\frac{10\sqrt{\left(b\,x^{3}+a\right)^{2}}\,a^{3}b^{2}x^{7}}{7\left(b\,x^{3}+a\right)}+\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,a^{2}b^{3}x^{10}}{b\,x^{3}+a}+\frac{5\sqrt{\left(b\,x^{3}+a\right)^{2}}\,b^{4}a\,x^{13}}{13\left(b\,x^{3}+a\right)}+\frac{\sqrt{\left(b\,x^{3}\right.}}{16b\,x}$

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/1456*x*(91*b^5*x^15+560*a*b^4*x^12+1456*a^2*b^3*x^9+2080*a^3*b^2*x^6+182
0*a^4*b*x^3+1456*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

3.63.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.21

$$\int \left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{1}{16}\,b^5x^{16}+\frac{5}{13}\,ab^4x^{13}+a^2b^3x^{10}+\frac{10}{7}\,a^3b^2x^7+\frac{5}{4}\,a^4bx^4+a^5x$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

output
```
1/16*b^5*x^16 + 5/13*a*b^4*x^13 + a^2*b^3*x^10 + 10/7*a^3*b^2*x^7 + 5/4*a^
4*b*x^4 + a^5*x
```

3.63.6 Sympy [F]

$$\int \left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\int \left(a^2+2abx^3+b^2x^6\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

output
```
Integral((a**2 + 2*a*b*x**3 + b**2*x**6)**(5/2), x)
```

3.63.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.21

$$\int \left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{1}{16}\,b^5x^{16}+\frac{5}{13}\,ab^4x^{13}+a^2b^3x^{10}+\frac{10}{7}\,a^3b^2x^7+\frac{5}{4}\,a^4bx^4+a^5x$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

output
```
1/16*b^5*x^16 + 5/13*a*b^4*x^13 + a^2*b^3*x^10 + 10/7*a^3*b^2*x^7 + 5/4*a^
4*b*x^4 + a^5*x
```

3.63.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.41

$$\int \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx = \frac{1}{16}b^5x^{16}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{13}ab^4x^{13}\mathrm{sgn}\left(bx^3+a\right) + a^2b^3x^{10}\mathrm{sgn}\left(bx^3+a\right)+\frac{10}{7}a^3b^2x^7\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{4}a^4bx^4\mathrm{sgn}\left(bx^3+a\right)+a^5x\mathrm{sgn}\left(bx^3+a\right)$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

output
```
1/16*b^5*x^16*sgn(b*x^3 + a) + 5/13*a*b^4*x^13*sgn(b*x^3 + a) + a^2*b^3*x^
10*sgn(b*x^3 + a) + 10/7*a^3*b^2*x^7*sgn(b*x^3 + a) + 5/4*a^4*b*x^4*sgn(b*
x^3 + a) + a^5*x*sgn(b*x^3 + a)
```

3.63.9 Mupad [F(-1)]

Timed out.

$$\int \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx = \int \left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2} dx$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

output
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

3.64 $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x}\,dx$

3.64.1 Optimal result

Integrand size = 26, antiderivative size = 251

$$\begin{aligned}\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x}\,dx =& \frac{5a^4bx^3\sqrt{a^2+2abx^3+b^2x^6}}{3(a+bx^3)} \\ &+ \frac{5a^3b^2x^6\sqrt{a^2+2abx^3+b^2x^6}}{3(a+bx^3)} + \frac{10a^2b^3x^9\sqrt{a^2+2abx^3+b^2x^6}}{9(a+bx^3)} \\ &+ \frac{5ab^4x^{12}\sqrt{a^2+2abx^3+b^2x^6}}{12(a+bx^3)} + \frac{b^5x^{15}\sqrt{a^2+2abx^3+b^2x^6}}{15(a+bx^3)} \\ &+ \frac{a^5\sqrt{a^2+2abx^3+b^2x^6}\log(x)}{a+bx^3}\end{aligned}$$

output
```
5/3*a^4*b*x^3*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/3*a^3*b^2*x^6*((b*x^3+a)^2)^
(1/2)/(b*x^3+a)+10/9*a^2*b^3*x^9*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/12*a*b^4*
x^12*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/15*b^5*x^15*((b*x^3+a)^2)^(1/2)/(b*x^
3+a)+a^5*ln(x)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

3.64.2 Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.33

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x}\,dx = \frac{\sqrt{(a+bx^3)^2}(bx^3(300a^4+300a^3bx^3+200a^2b^2x^6+75ab^3x^9+12b^4x^{12})+18}{180\,(a+bx^3)}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x,x]
```

output
```
(Sqrt[(a + b*x^3)^2]*(b*x^3*(300*a^4 + 300*a^3*b*x^3 + 200*a^2*b^2*x^6 + 7
5*a*b^3*x^9 + 12*b^4*x^12) + 180*a^5*Log[x]))/(180*(a + b*x^3))
```

3.64.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.39, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1384, 27, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x}dx$$

$$\Big\downarrow \; 1384$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b^5(bx^3+a)^5}{x}dx}{b^5\,(a+bx^3)}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(bx^3+a)^5}{x}dx}{a+bx^3}$$

$$\Big\downarrow \; 798$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(bx^3+a)^5}{x^3}dx^3}{3\,(a+bx^3)}$$

$$\Big\downarrow \; 49$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(b^5x^{12}+5ab^4x^9+10a^2b^3x^6+10a^3b^2x^3+5a^4b+\frac{a^5}{x^3}\right)dx^3}{3\,(a+bx^3)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(a^5\log\left(x^3\right)+5a^4bx^3+5a^3b^2x^6+\frac{10}{3}a^2b^3x^9+\frac{5}{4}ab^4x^{12}+\frac{b^5x^{15}}{5}\right)}{3\left(a+bx^3\right)}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(5*a^4*b*x^3 + 5*a^3*b^2*x^6 + (10*a^2*b^
3*x^9)/3 + (5*a*b^4*x^12)/4 + (b^5*x^15)/5 + a^5*Log[x^3]))/(3*(a + b*x^3)
)
```

3.64.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.64.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.30

method	result	size
pseudoelliptic	$\frac{\operatorname{csgn}\left(b\,x^{3}+a\right)\left(\frac{b^{5} x^{15}}{5}+\frac{5 a\,b^{4} x^{12}}{4}+\frac{10 a^{2} b^{3} x^{9}}{3}+5 a^{3} b^{2} x^{6}+5 a^{4} b\,x^{3}+a^{5} \ln\left(b\,x^{3}\right)+\frac{137 a^{5}}{60}\right)}{3}$	75
default	$\frac{\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}} \left(12 b^{5} x^{15}+75 a\,b^{4} x^{12}+200 a^{2} b^{3} x^{9}+300 a^{3} b^{2} x^{6}+300 a^{4} b\,x^{3}+180 a^{5} \ln\left(x\right)\right)}{180 \left(b\,x^{3}+a\right)^{5}}$	79
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\, b\left(\frac{1}{15} b^{4} x^{15}+\frac{5}{12} a\,b^{3} x^{12}+\frac{10}{9} a^{2} b^{2} x^{9}+\frac{5}{3} a^{3} b\,x^{6}+\frac{5}{3} a^{4} x^{3}\right)}{b\,x^{3}+a}+\frac{a^{5} \ln\left(x\right) \sqrt{\left(b\,x^{3}+a\right)^{2}}}{b\,x^{3}+a}$	96

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/3*csgn(b*x^3+a)*(1/5*b^5*x^15+5/4*a*b^4*x^12+10/3*a^2*b^3*x^9+5*a^3*b^2*
x^6+5*a^4*b*x^3+a^5*ln(b*x^3)+137/60*a^5)
```

3.64.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.22

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x}\,dx = \frac{1}{15}\,b^5x^{15}+\frac{5}{12}\,ab^4x^{12} + \frac{10}{9}\,a^2b^3x^9+\frac{5}{3}\,a^3b^2x^6+\frac{5}{3}\,a^4bx^3+a^5\log\left(x\right)$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x,x, algorithm="fricas")
```

output
```
1/15*b^5*x^15 + 5/12*a*b^4*x^12 + 10/9*a^2*b^3*x^9 + 5/3*a^3*b^2*x^6 + 5/3
*a^4*b*x^3 + a^5*log(x)
```

3.64.6 Sympy [F]

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x}\, dx = \int \frac{\left((a + bx^3)^2\right)^{\frac{5}{2}}}{x}\, dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x,x)
```

output
```
Integral(((a + b*x**3)**2)**(5/2)/x, x)
```

3.64.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.82

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x}\, dx = \frac{1}{6}\sqrt{b^2x^6 + 2\,abx^3 + a^2}a^3bx^3 + \frac{1}{3}\left(-1\right)^{2\,b^2x^3 + 2\,ab}a^5\log\left(2\,b^2x^3 + 2\,ab\right) - \frac{1}{3}\left(-1\right)^{2\,abx^3 + 2\,a^2}a^5\log\left(\frac{2\,abx}{|x|} + \frac{2\,a^2}{x^2|x|}\right) + \frac{1}{12}\left(b^2x^6 + 2\,abx^3 + a^2\right)^{\frac{3}{2}}abx^3 + \frac{1}{2}\sqrt{b^2x^6 + 2\,abx^3 + a^2}a^4 + \frac{7}{36}\left(b^2x^6 + 2\,abx^3 + a^2\right)^{\frac{3}{2}}a^2 + \frac{1}{15}\left(b^2x^6 + 2\,abx^3 + a^2\right)^{\frac{5}{2}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x,x, algorithm="maxima")
```

output
```
1/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a^3*b*x^3 + 1/3*(-1)^(2*b^2*x^3 + 2*a*
b)*a^5*log(2*b^2*x^3 + 2*a*b) - 1/3*(-1)^(2*a*b*x^3 + 2*a^2)*a^5*log(2*a*b
*x/abs(x) + 2*a^2/(x^2*abs(x))) + 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*a
*b*x^3 + 1/2*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a^4 + 7/36*(b^2*x^6 + 2*a*b*x
^3 + a^2)^(3/2)*a^2 + 1/15*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)
```

3.64.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.41

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x}\,dx = \frac{1}{15}b^5x^{15}\operatorname{sgn}(bx^3 + a) + \frac{5}{12}ab^4x^{12}\operatorname{sgn}(bx^3 + a) + \frac{10}{9}a^2b^3x^9\operatorname{sgn}(bx^3 + a) + \frac{5}{3}a^3b^2x^6\operatorname{sgn}(bx^3 + a) + \frac{5}{3}a^4bx^3\operatorname{sgn}(bx^3 + a) + a^5\log(|x|)\operatorname{sgn}(bx^3 + a)$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x,x, algorithm="giac")
```

output
```
1/15*b^5*x^15*sgn(b*x^3 + a) + 5/12*a*b^4*x^12*sgn(b*x^3 + a) + 10/9*a^2*b
^3*x^9*sgn(b*x^3 + a) + 5/3*a^3*b^2*x^6*sgn(b*x^3 + a) + 5/3*a^4*b*x^3*sgn
(b*x^3 + a) + a^5*log(abs(x))*sgn(b*x^3 + a)
```

3.64.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x}\,dx = \int \frac{(a^2 + 2\,a\,b\,x^3 + b^2\,x^6)^{5/2}}{x}\,dx$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x,x)
```

output
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x, x)
```

3.65 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^2}\,dx$

3.65.1 Optimal result

Integrand size = 26, antiderivative size = 251

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^2}\,dx=-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}+\frac{5a^4bx^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{2a^3b^2x^5\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{5a^2b^3x^8\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{5ab^4x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{b^5x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14\left(a+bx^3\right)}$$

output
```
-a^5*((b*x^3+a)^2)^(1/2)/x/(b*x^3+a)+5/2*a^4*b*x^2*((b*x^3+a)^2)^(1/2)/(b*
x^3+a)+2*a^3*b^2*x^5*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/4*a^2*b^3*x^8*((b*x^3
+a)^2)^(1/2)/(b*x^3+a)+5/11*a*b^4*x^11*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/14*
b^5*x^14*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

3.65.2 Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^2}\,dx=\frac{\sqrt{\left(a+bx^3\right)^2}\left(-308a^5+770a^4bx^3+616a^3b^2x^6+385a^2b^3x^9+140ab^4x^{12}+2\right.}{308x\left(a+bx^3\right)}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^2,x]
```

output
```
(Sqrt[(a + b*x^3)^2]*(-308*a^5 + 770*a^4*b*x^3 + 616*a^3*b^2*x^6 + 385*a^2
*b^3*x^9 + 140*a*b^4*x^12 + 22*b^5*x^15))/(308*x*(a + b*x^3))
```

3.65.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.39, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^2}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b^5\left(bx^3+a\right)^5}{x^2}dx}{b^5\left(a+bx^3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(bx^3+a\right)^5}{x^2}dx}{a+bx^3}$$

$$\Big\downarrow \text{802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(b^5x^{13}+5ab^4x^{10}+10a^2b^3x^7+10a^3b^2x^4+5a^4bx+\frac{a^5}{x^2}\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(-\frac{a^5}{x}+\frac{5}{2}a^4bx^2+2a^3b^2x^5+\frac{5}{4}a^2b^3x^8+\frac{5}{11}ab^4x^{11}+\frac{b^5x^{14}}{14}\right)}{a+bx^3}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^2,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-(a^5/x) + (5*a^4*b*x^2)/2 + 2*a^3*b^2*x
^5 + (5*a^2*b^3*x^8)/4 + (5*a*b^4*x^11)/11 + (b^5*x^14)/14))/(a + b*x^3)
```

3.65.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.65.4 Maple [A] (verified)

Time = 2.38 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.32

method	result	size
gosper	$-\frac{\left(-22 b^{5} x^{15}-140 a\,b^{4} x^{12}-385 a^{2} b^{3} x^{9}-616 a^{3} b^{2} x^{6}-770 a^{4} b\,x^{3}+308 a^{5}\right) \left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{308 x \left(b\,x^{3}+a\right)^{5}}$	80
default	$-\frac{\left(-22 b^{5} x^{15}-140 a\,b^{4} x^{12}-385 a^{2} b^{3} x^{9}-616 a^{3} b^{2} x^{6}-770 a^{4} b\,x^{3}+308 a^{5}\right) \left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{308 x \left(b\,x^{3}+a\right)^{5}}$	80
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\, b \left(\frac{1}{14} b^{4} x^{14}+\frac{5}{11} a\,b^{3} x^{11}+\frac{5}{4} a^{2} b^{2} x^{8}+2 a^{3} b\,x^{5}+\frac{5}{2} a^{4} x^{2}\right)}{b\,x^{3}+a}-\frac{a^{5} \sqrt{\left(b\,x^{3}+a\right)^{2}}}{x \left(b\,x^{3}+a\right)}$	98

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/308*(-22*b^5*x^15-140*a*b^4*x^12-385*a^2*b^3*x^9-616*a^3*b^2*x^6-770*a^
4*b*x^3+308*a^5)*((b*x^3+a)^2)^(5/2)/x/(b*x^3+a)^5
```

3.65.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.24

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^2}\,dx = \frac{22\,b^5x^{15}+140\,ab^4x^{12}+385\,a^2b^3x^9+616\,a^3b^2x^6+770\,a^4bx^3-308\,a^5}{308\,x}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^2,x, algorithm="fricas")
```

output
```
1/308*(22*b^5*x^15 + 140*a*b^4*x^12 + 385*a^2*b^3*x^9 + 616*a^3*b^2*x^6 +
770*a^4*b*x^3 - 308*a^5)/x
```

3.65.6 Sympy [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^2}\,dx = \int \frac{\left((a+bx^3)^2\right)^{\frac{5}{2}}}{x^2}\,dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**2,x)
```

output
```
Integral(((a + b*x**3)**2)**(5/2)/x**2, x)
```

3.65.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.24

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^2}\,dx = \frac{22\,b^5x^{15}+140\,ab^4x^{12}+385\,a^2b^3x^9+616\,a^3b^2x^6+770\,a^4bx^3-308\,a^5}{308\,x}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^2,x, algorithm="maxima")
```

output
```
1/308*(22*b^5*x^15 + 140*a*b^4*x^12 + 385*a^2*b^3*x^9 + 616*a^3*b^2*x^6 +
770*a^4*b*x^3 - 308*a^5)/x
```

3.65.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.42

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^2}\,dx = \frac{1}{14}\,b^5x^{14}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{11}\,ab^4x^{11}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{4}\,a^2b^3x^8\mathrm{sgn}\left(bx^3+a\right)+2\,a^3b^2x^5\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{2}\,a^4bx^2\mathrm{sgn}\left(bx^3+a\right)-\frac{a^5\mathrm{sgn}\left(bx^3+a\right)}{x}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^2,x, algorithm="giac")
```

output
```
1/14*b^5*x^14*sgn(b*x^3 + a) + 5/11*a*b^4*x^11*sgn(b*x^3 + a) + 5/4*a^2*b^
3*x^8*sgn(b*x^3 + a) + 2*a^3*b^2*x^5*sgn(b*x^3 + a) + 5/2*a^4*b*x^2*sgn(b*
x^3 + a) - a^5*sgn(b*x^3 + a)/x
```

3.65.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^2}\,dx = \int \frac{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}}{x^2}\,dx$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^2,x)
```

output
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^2, x)
```

3.66 $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^3} dx$

3.66.1 Optimal result

Integrand size = 26, antiderivative size = 251

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^3} dx = -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{2x^2(a+bx^3)} + \frac{5a^4bx\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3} + \frac{5a^3b^2x^4\sqrt{a^2+2abx^3+b^2x^6}}{2(a+bx^3)} + \frac{10a^2b^3x^7\sqrt{a^2+2abx^3+b^2x^6}}{7(a+bx^3)} + \frac{ab^4x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{2(a+bx^3)} + \frac{b^5x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13(a+bx^3)}$$

output
```
-1/2*a^5*((b*x^3+a)^2)^(1/2)/x^2/(b*x^3+a)+5*a^4*b*x*((b*x^3+a)^2)^(1/2)/(
b*x^3+a)+5/2*a^3*b^2*x^4*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+10/7*a^2*b^3*x^7*((
b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/2*a*b^4*x^10*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1
/13*b^5*x^13*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

3.66.2 Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^3} dx = \frac{\sqrt{(a+bx^3)^2}(-91a^5+910a^4bx^3+455a^3b^2x^6+260a^2b^3x^9+91ab^4x^{12}+14b}{182x^2(a+bx^3)}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^3,x]
```

output
```
(Sqrt[(a + b*x^3)^2]*(-91*a^5 + 910*a^4*b*x^3 + 455*a^3*b^2*x^6 + 260*a^2*
b^3*x^9 + 91*a*b^4*x^12 + 14*b^5*x^15))/(182*x^2*(a + b*x^3))
```

3.66.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.39, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^3}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b^5\left(bx^3+a\right)^5}{x^3}dx}{b^5\left(a+bx^3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(bx^3+a\right)^5}{x^3}dx}{a+bx^3}$$

$$\Big\downarrow \text{802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(b^5x^{12}+5ab^4x^9+10a^2b^3x^6+10a^3b^2x^3+5a^4b+\frac{a^5}{x^3}\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(-\frac{a^5}{2x^2}+5a^4bx+\frac{5}{2}a^3b^2x^4+\frac{10}{7}a^2b^3x^7+\frac{1}{2}ab^4x^{10}+\frac{b^5x^{13}}{13}\right)}{a+bx^3}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^3,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-1/2*a^5/x^2 + 5*a^4*b*x + (5*a^3*b^2*x^
4)/2 + (10*a^2*b^3*x^7)/7 + (a*b^4*x^10)/2 + (b^5*x^13)/13))/(a + b*x^3)
```

3.66.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.66.4 Maple [A] (verified)

Time = 2.81 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.32

method	result	size
gosper	$-\frac{\left(-14 b^{5} x^{15}-91 a\,b^{4} x^{12}-260 a^{2} b^{3} x^{9}-455 a^{3} b^{2} x^{6}-910 a^{4} b\,x^{3}+91 a^{5}\right) \left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{182 x^{2} \left(b\,x^{3}+a\right)^{5}}$	80
default	$-\frac{\left(-14 b^{5} x^{15}-91 a\,b^{4} x^{12}-260 a^{2} b^{3} x^{9}-455 a^{3} b^{2} x^{6}-910 a^{4} b\,x^{3}+91 a^{5}\right) \left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{182 x^{2} \left(b\,x^{3}+a\right)^{5}}$	80
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\, b \left(\frac{1}{13} b^{4} x^{13}+\frac{1}{2} a\,b^{3} x^{10}+\frac{10}{7} a^{2} b^{2} x^{7}+\frac{5}{2} a^{3} b\,x^{4}+5 a^{4} x\right)}{b\,x^{3}+a}-\frac{a^{5} \sqrt{\left(b\,x^{3}+a\right)^{2}}}{2 x^{2} \left(b\,x^{3}+a\right)}$	96

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/182*(-14*b^5*x^15-91*a*b^4*x^12-260*a^2*b^3*x^9-455*a^3*b^2*x^6-910*a^4
*b*x^3+91*a^5)*((b*x^3+a)^2)^(5/2)/x^2/(b*x^3+a)^5
```

3.66.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.24

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^3}\,dx = \frac{14\,b^5x^{15}+91\,ab^4x^{12}+260\,a^2b^3x^9+455\,a^3b^2x^6+910\,a^4bx^3-91\,a^5}{182\,x^2}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^3,x, algorithm="fricas")
```

output
```
1/182*(14*b^5*x^15 + 91*a*b^4*x^12 + 260*a^2*b^3*x^9 + 455*a^3*b^2*x^6 + 9
10*a^4*b*x^3 - 91*a^5)/x^2
```

3.66.6 Sympy [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^3}\,dx = \int \frac{\left((a+bx^3)^2\right)^{\frac{5}{2}}}{x^3}\,dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**3,x)
```

output
```
Integral(((a + b*x**3)**2)**(5/2)/x**3, x)
```

3.66.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.24

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^3}\,dx = \frac{14\,b^5x^{15}+91\,ab^4x^{12}+260\,a^2b^3x^9+455\,a^3b^2x^6+910\,a^4bx^3-91\,a^5}{182\,x^2}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^3,x, algorithm="maxima")
```

output
```
1/182*(14*b^5*x^15 + 91*a*b^4*x^12 + 260*a^2*b^3*x^9 + 455*a^3*b^2*x^6 + 9
10*a^4*b*x^3 - 91*a^5)/x^2
```

3.66.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.41

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^3}\,dx = \frac{1}{13}\,b^5x^{13}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{2}\,ab^4x^{10}\mathrm{sgn}\left(bx^3+a\right)+\frac{10}{7}\,a^2b^3x^7\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{2}\,a^3b^2x^4\mathrm{sgn}\left(bx^3+a\right)+5\,a^4bx\mathrm{sgn}\left(bx^3+a\right)-\frac{a^5\mathrm{sgn}(bx^3+a)}{2\,x^2}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^3,x, algorithm="giac")
```

output
```
1/13*b^5*x^13*sgn(b*x^3 + a) + 1/2*a*b^4*x^10*sgn(b*x^3 + a) + 10/7*a^2*b^
3*x^7*sgn(b*x^3 + a) + 5/2*a^3*b^2*x^4*sgn(b*x^3 + a) + 5*a^4*b*x*sgn(b*x^
3 + a) - 1/2*a^5*sgn(b*x^3 + a)/x^2
```

3.66.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^3}\,dx = \int \frac{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}}{x^3}\,dx$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^3,x)
```

output
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^3, x)
```

3.67 $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^4} dx$

3.67.1 Optimal result

Integrand size = 26, antiderivative size = 252

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^4} dx = -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{3x^3(a+bx^3)} + \frac{10a^3b^2x^3\sqrt{a^2+2abx^3+b^2x^6}}{3(a+bx^3)} + \frac{5a^2b^3x^6\sqrt{a^2+2abx^3+b^2x^6}}{3(a+bx^3)} + \frac{5ab^4x^9\sqrt{a^2+2abx^3+b^2x^6}}{9(a+bx^3)} + \frac{b^5x^{12}\sqrt{a^2+2abx^3+b^2x^6}}{12(a+bx^3)} + \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}\log(x)}{a+bx^3}$$

output
```
-1/3*a^5*((b*x^3+a)^2)^(1/2)/x^3/(b*x^3+a)+10/3*a^3*b^2*x^3*((b*x^3+a)^2)^
(1/2)/(b*x^3+a)+5/3*a^2*b^3*x^6*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/9*a*b^4*x^
9*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/12*b^5*x^12*((b*x^3+a)^2)^(1/2)/(b*x^3+a
)+5*a^4*b*ln(x)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

3.67.2 Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.34

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^4}\,dx = \frac{\sqrt{(a+bx^3)^2}(-12a^5+120a^3b^2x^6+60a^2b^3x^9+20ab^4x^{12}+3b^5x^{15}+180a^4b}{36x^3\,(a+bx^3)}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^4,x]
```

output
```
(Sqrt[(a + b*x^3)^2]*(-12*a^5 + 120*a^3*b^2*x^6 + 60*a^2*b^3*x^9 + 20*a*b^
4*x^12 + 3*b^5*x^15 + 180*a^4*b*x^3*Log[x]))/(36*x^3*(a + b*x^3))
```

3.67.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.38, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1384, 27, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^4}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b^5(bx^3+a)^5}{x^4}dx}{b^5\,(a+bx^3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(bx^3+a)^5}{x^4}dx}{a+bx^3}$$

$$\Big\downarrow \text{798}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(bx^3+a)^5}{x^6}dx^3}{3\,(a+bx^3)}$$

$$\Big\downarrow \text{49}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(b^5x^9+5ab^4x^6+10a^2b^3x^3+10a^3b^2+\frac{5a^4b}{x^3}+\frac{a^5}{x^6}\right)dx^3}{3\,(a+bx^3)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(-\frac{a^5}{x^3}+5a^4b\log\left(x^3\right)+10a^3b^2x^3+5a^2b^3x^6+\frac{5}{3}ab^4x^9+\frac{b^5x^{12}}{4}\right)}{3\left(a+bx^3\right)}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^4,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-(a^5/x^3) + 10*a^3*b^2*x^3 + 5*a^2*b^3*
x^6 + (5*a*b^4*x^9)/3 + (b^5*x^12)/4 + 5*a^4*b*Log[x^3]))/(3*(a + b*x^3))
```

3.67.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.67.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.32

method	result	size
pseudoelliptic	$-\frac{\operatorname{csgn}(b\,x^{3}+a)\left(-\frac{b^{5} x^{15}}{4}-\frac{5 a\,b^{4} x^{12}}{3}-5 a^{2} b^{3} x^{9}-10 a^{3} b^{2} x^{6}-5 \ln(b\,x^{3}) a^{4} b\,x^{3}-\frac{77 a^{4} b\,x^{3}}{12}+a^{5}\right)}{3 x^{3}}$	81
default	$\frac{\left((b\,x^{3}+a)^{2}\right)^{\frac{5}{2}} (3 b^{5} x^{15}+20 a\,b^{4} x^{12}+60 a^{2} b^{3} x^{9}+120 a^{3} b^{2} x^{6}+180 b\,a^{4} \ln(x) x^{3}-12 a^{5})}{36 x^{3} (b\,x^{3}+a)^{5}}$	82
risch	$\frac{\sqrt{(b\,x^{3}+a)^{2}}\, b^{2} (\frac{1}{12} b^{3} x^{12}+\frac{5}{9} a\,b^{2} x^{9}+\frac{5}{3} a^{2} b\,x^{6}+\frac{10}{3} a^{3} x^{3})}{b\,x^{3}+a}-\frac{a^{5} \sqrt{(b\,x^{3}+a)^{2}}}{3 x^{3} (b\,x^{3}+a)}+\frac{5 a^{4} b \ln(x) \sqrt{(b\,x^{3}+a)^{2}}}{b\,x^{3}+a}$	117

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*csgn(b*x^3+a)*(-1/4*b^5*x^15-5/3*a*b^4*x^12-5*a^2*b^3*x^9-10*a^3*b^2*
x^6-5*ln(b*x^3)*a^4*b*x^3-77/12*a^4*b*x^3+a^5)/x^3
```

3.67.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.24

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^4}\,dx = \frac{3\,b^5x^{15}+20\,ab^4x^{12}+60\,a^2b^3x^9+120\,a^3b^2x^6+180\,a^4bx^3\log(x)-12\,a^5}{36\,x^3}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^4,x, algorithm="fricas")
```

output
```
1/36*(3*b^5*x^15 + 20*a*b^4*x^12 + 60*a^2*b^3*x^9 + 120*a^3*b^2*x^6 + 180*
a^4*b*x^3*log(x) - 12*a^5)/x^3
```

3.67.6 Sympy [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^4}\,dx=\int \frac{\left((a+bx^3)^2\right)^{\frac{5}{2}}}{x^4}\,dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**4,x)
```

output
```
Integral(((a + b*x**3)**2)**(5/2)/x**4, x)
```

3.67.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.85

$$\begin{aligned}\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^4}\,dx &= \frac{5}{6}\sqrt{b^2x^6+2\,abx^3+a^2}a^2b^2x^3 \\ &\quad + \frac{5}{3}\,(-1)^{2\,b^2x^3+2\,ab}\,a^4b\log\left(2\,b^2x^3+2\,ab\right)-\frac{5}{3}\,(-1)^{2\,abx^3+2\,a^2}\,a^4b\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right) \\ &\quad + \frac{5}{12}\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{3}{2}}b^2x^3+\frac{5}{2}\sqrt{b^2x^6+2\,abx^3+a^2}a^3b \\ &\quad + \frac{35}{36}\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{3}{2}}ab-\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}}{3\,x^3}\end{aligned}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^4,x, algorithm="maxima")
```

output
```
5/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a^2*b^2*x^3 + 5/3*(-1)^(2*b^2*x^3 + 2*
a*b)*a^4*b*log(2*b^2*x^3 + 2*a*b) - 5/3*(-1)^(2*a*b*x^3 + 2*a^2)*a^4*b*log
(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x))) + 5/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(
3/2)*b^2*x^3 + 5/2*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a^3*b + 35/36*(b^2*x^6
+ 2*a*b*x^3 + a^2)^(3/2)*a*b - 1/3*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^3
```

3.67.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.49

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^4}\,dx = \frac{1}{12}\,b^5x^{12}\mathrm{sgn}\left(bx^3+a\right) + \frac{5}{9}\,ab^4x^9\mathrm{sgn}\left(bx^3+a\right) + \frac{5}{3}\,a^2b^3x^6\mathrm{sgn}\left(bx^3+a\right) + \frac{10}{3}\,a^3b^2x^3\mathrm{sgn}\left(bx^3+a\right) + 5\,a^4b\log\left(|x|\right)\mathrm{sgn}\left(bx^3+a\right) - \frac{5\,a^4bx^3\mathrm{sgn}(bx^3+a)+a^5\mathrm{sgn}(bx^3+a)}{3\,x^3}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^4,x, algorithm="giac")
```

output
```
1/12*b^5*x^12*sgn(b*x^3 + a) + 5/9*a*b^4*x^9*sgn(b*x^3 + a) + 5/3*a^2*b^3*
x^6*sgn(b*x^3 + a) + 10/3*a^3*b^2*x^3*sgn(b*x^3 + a) + 5*a^4*b*log(abs(x))
*sgn(b*x^3 + a) - 1/3*(5*a^4*b*x^3*sgn(b*x^3 + a) + a^5*sgn(b*x^3 + a))/x^
3
```

3.67.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^4}\,dx = \int \frac{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}}{x^4}\,dx$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^4,x)
```

output
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^4, x)
```

3.68 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^5}\,dx$

3.68.1 Optimal result

Integrand size = 26, antiderivative size = 249

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^5}\,dx = -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)} + \frac{5a^3b^2x^2\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3} + \frac{2a^2b^3x^5\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3} + \frac{5ab^4x^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)} + \frac{b^5x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}$$

output
```
-1/4*a^5*((b*x^3+a)^2)^(1/2)/x^4/(b*x^3+a)-5*a^4*b*((b*x^3+a)^2)^(1/2)/x/(
b*x^3+a)+5*a^3*b^2*x^2*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+2*a^2*b^3*x^5*((b*x^3
+a)^2)^(1/2)/(b*x^3+a)+5/8*a*b^4*x^8*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/11*b^
5*x^11*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

3.68.2 Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^5}\,dx = \frac{\sqrt{\left(a+bx^3\right)^2}\left(-22a^5-440a^4bx^3+440a^3b^2x^6+176a^2b^3x^9+55ab^4x^{12}+8b^5\ldots\right.}{88x^4\left(a+bx^3\right)}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^5,x]
```

output
```
(Sqrt[(a + b*x^3)^2]*(-22*a^5 - 440*a^4*b*x^3 + 440*a^3*b^2*x^6 + 176*a^2*
b^3*x^9 + 55*a*b^4*x^12 + 8*b^5*x^15))/(88*x^4*(a + b*x^3))
```

3.68.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.38, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^5}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b^5\left(bx^3+a\right)^5}{x^5}dx}{b^5\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(bx^3+a\right)^5}{x^5}dx}{a+bx^3}$$

$$\Big\downarrow \text{ 802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(b^5x^{10}+5ab^4x^7+10a^2b^3x^4+10a^3b^2x+\frac{5a^4b}{x^2}+\frac{a^5}{x^5}\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(-\frac{a^5}{4x^4}-\frac{5a^4b}{x}+5a^3b^2x^2+2a^2b^3x^5+\frac{5}{8}ab^4x^8+\frac{b^5x^{11}}{11}\right)}{a+bx^3}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^5,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-1/4*a^5/x^4 - (5*a^4*b)/x + 5*a^3*b^2*x
^2 + 2*a^2*b^3*x^5 + (5*a*b^4*x^8)/8 + (b^5*x^11)/11))/(a + b*x^3)
```

3.68.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.68.4 Maple [A] (verified)

Time = 4.33 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.32

method	result	size
gosper	$-\frac{\left(-8b^{5}x^{15}-55a\,b^{4}x^{12}-176a^{2}b^{3}x^{9}-440a^{3}b^{2}x^{6}+440a^{4}b\,x^{3}+22a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{88x^{4}\left(b\,x^{3}+a\right)^{5}}$	80
default	$-\frac{\left(-8b^{5}x^{15}-55a\,b^{4}x^{12}-176a^{2}b^{3}x^{9}-440a^{3}b^{2}x^{6}+440a^{4}b\,x^{3}+22a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{88x^{4}\left(b\,x^{3}+a\right)^{5}}$	80
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,b^{2}\left(\frac{1}{11}b^{3}x^{11}+\frac{5}{8}a\,b^{2}x^{8}+2a^{2}b\,x^{5}+5a^{3}x^{2}\right)}{b\,x^{3}+a}+\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(-5a^{4}b\,x^{3}-\frac{1}{4}a^{5}\right)}{\left(b\,x^{3}+a\right)x^{4}}$	100

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/88*(-8*b^5*x^15-55*a*b^4*x^12-176*a^2*b^3*x^9-440*a^3*b^2*x^6+440*a^4*b
*x^3+22*a^5)*((b*x^3+a)^2)^(5/2)/x^4/(b*x^3+a)^5
```

3.68.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.24

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^5}\,dx = \frac{8\,b^5x^{15} + 55\,ab^4x^{12} + 176\,a^2b^3x^9 + 440\,a^3b^2x^6 - 440\,a^4bx^3 - 22\,a^5}{88\,x^4}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^5,x, algorithm="fricas")
```

output
```
1/88*(8*b^5*x^15 + 55*a*b^4*x^12 + 176*a^2*b^3*x^9 + 440*a^3*b^2*x^6 - 440
*a^4*b*x^3 - 22*a^5)/x^4
```

3.68.6 Sympy [F]

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^5}\,dx = \int \frac{\left((a+bx^3)^2\right)^{\frac{5}{2}}}{x^5}\,dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**5,x)
```

output
```
Integral(((a + b*x**3)**2)**(5/2)/x**5, x)
```

3.68.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.24

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^5}\,dx = \frac{8\,b^5x^{15} + 55\,ab^4x^{12} + 176\,a^2b^3x^9 + 440\,a^3b^2x^6 - 440\,a^4bx^3 - 22\,a^5}{88\,x^4}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^5,x, algorithm="maxima")
```

output
```
1/88*(8*b^5*x^15 + 55*a*b^4*x^12 + 176*a^2*b^3*x^9 + 440*a^3*b^2*x^6 - 440
*a^4*b*x^3 - 22*a^5)/x^4
```

3.68.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.43

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^5}\,dx = \frac{1}{11}\,b^5x^{11}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{8}\,ab^4x^8\mathrm{sgn}\left(bx^3+a\right) + 2\,a^2b^3x^5\mathrm{sgn}\left(bx^3+a\right)+5\,a^3b^2x^2\mathrm{sgn}\left(bx^3+a\right)-\frac{20\,a^4bx^3\mathrm{sgn}(bx^3+a)+a^5\mathrm{sgn}(bx^3+a)}{4\,x^4}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^5,x, algorithm="giac")
```

output
```
1/11*b^5*x^11*sgn(b*x^3 + a) + 5/8*a*b^4*x^8*sgn(b*x^3 + a) + 2*a^2*b^3*x^
5*sgn(b*x^3 + a) + 5*a^3*b^2*x^2*sgn(b*x^3 + a) - 1/4*(20*a^4*b*x^3*sgn(b*
x^3 + a) + a^5*sgn(b*x^3 + a))/x^4
```

3.68.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^5}\,dx = \int \frac{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}}{x^5}\,dx$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^5,x)
```

output
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^5, x)
```

3.69 $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^6}\,dx$

3.69.1 Optimal result

Integrand size = 26, antiderivative size = 251

$$\begin{aligned}\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^6}\,dx &= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{5x^5(a+bx^3)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{2x^2(a+bx^3)}\\ &+\frac{10a^3b^2x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{5a^2b^3x^4\sqrt{a^2+2abx^3+b^2x^6}}{2(a+bx^3)}\\ &+\frac{5ab^4x^7\sqrt{a^2+2abx^3+b^2x^6}}{7(a+bx^3)}+\frac{b^5x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10(a+bx^3)}\end{aligned}$$

output
```
-1/5*a^5*((b*x^3+a)^2)^(1/2)/x^5/(b*x^3+a)-5/2*a^4*b*((b*x^3+a)^2)^(1/2)/x
^2/(b*x^3+a)+10*a^3*b^2*x*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/2*a^2*b^3*x^4*((
b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/7*a*b^4*x^7*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/
10*b^5*x^10*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

3.69.2 Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^6}\,dx = \frac{\sqrt{(a+bx^3)^2}(-14a^5-175a^4bx^3+700a^3b^2x^6+175a^2b^3x^9+50ab^4x^{12}+7b^5}{70x^5(a+bx^3)}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^6,x]
```

output
```
(Sqrt[(a + b*x^3)^2]*(-14*a^5 - 175*a^4*b*x^3 + 700*a^3*b^2*x^6 + 175*a^2*
b^3*x^9 + 50*a*b^4*x^12 + 7*b^5*x^15))/(70*x^5*(a + b*x^3))
```

3.69.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.39, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^6}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b^5\left(bx^3+a\right)^5}{x^6}dx}{b^5\left(a+bx^3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(bx^3+a\right)^5}{x^6}dx}{a+bx^3}$$

$$\Big\downarrow \text{802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(b^5x^9+5ab^4x^6+10a^2b^3x^3+10a^3b^2+\frac{5a^4b}{x^3}+\frac{a^5}{x^6}\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(-\frac{a^5}{5x^5}-\frac{5a^4b}{2x^2}+10a^3b^2x+\frac{5}{2}a^2b^3x^4+\frac{5}{7}ab^4x^7+\frac{b^5x^{10}}{10}\right)}{a+bx^3}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^6,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-1/5*a^5/x^5 - (5*a^4*b)/(2*x^2) + 10*a^
3*b^2*x + (5*a^2*b^3*x^4)/2 + (5*a*b^4*x^7)/7 + (b^5*x^10)/10))/(a + b*x^3
)
```

3.69.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.69.4 Maple [A] (verified)

Time = 5.16 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.32

method	result	size
gosper	$-\frac{\left(-7 b^{5} x^{15}-50 a \,b^{4} x^{12}-175 a^{2} b^{3} x^{9}-700 a^{3} b^{2} x^{6}+175 a^{4} b \,x^{3}+14 a^{5}\right) \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{5}{2}}}{70 x^{5} \left(b \,x^{3}+a \right)^{5}}$	80
default	$-\frac{\left(-7 b^{5} x^{15}-50 a \,b^{4} x^{12}-175 a^{2} b^{3} x^{9}-700 a^{3} b^{2} x^{6}+175 a^{4} b \,x^{3}+14 a^{5}\right) \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{5}{2}}}{70 x^{5} \left(b \,x^{3}+a \right)^{5}}$	80
risch	$\frac{\sqrt{\left(b \,x^{3}+a \right)^{2}}\, b^{2} \left(\frac{1}{10} b^{3} x^{10}+\frac{5}{7} a \,b^{2} x^{7}+\frac{5}{2} a^{2} b \,x^{4}+10 a^{3} x \right)}{b \,x^{3}+a}+\frac{\sqrt{\left(b \,x^{3}+a \right)^{2}}\, \left(-\frac{5}{2} a^{4} b \,x^{3}-\frac{1}{5} a^{5}\right)}{\left(b \,x^{3}+a \right) x^{5}}$	98

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/70*(-7*b^5*x^15-50*a*b^4*x^12-175*a^2*b^3*x^9-700*a^3*b^2*x^6+175*a^4*b
*x^3+14*a^5)*((b*x^3+a)^2)^(5/2)/x^5/(b*x^3+a)^5
```

3.69.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.24

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^6}\,dx = \frac{7\,b^5x^{15}+50\,ab^4x^{12}+175\,a^2b^3x^9+700\,a^3b^2x^6-175\,a^4bx^3-14\,a^5}{70\,x^5}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^6,x, algorithm="fricas")
```

output
```
1/70*(7*b^5*x^15 + 50*a*b^4*x^12 + 175*a^2*b^3*x^9 + 700*a^3*b^2*x^6 - 175
*a^4*b*x^3 - 14*a^5)/x^5
```

3.69.6 Sympy [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^6}\,dx = \int \frac{\left((a+bx^3)^2\right)^{\frac{5}{2}}}{x^6}\,dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**6,x)
```

output
```
Integral(((a + b*x**3)**2)**(5/2)/x**6, x)
```

3.69.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.24

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^6}\,dx = \frac{7\,b^5x^{15}+50\,ab^4x^{12}+175\,a^2b^3x^9+700\,a^3b^2x^6-175\,a^4bx^3-14\,a^5}{70\,x^5}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^6,x, algorithm="maxima")
```

output
```
1/70*(7*b^5*x^15 + 50*a*b^4*x^12 + 175*a^2*b^3*x^9 + 700*a^3*b^2*x^6 - 175
*a^4*b*x^3 - 14*a^5)/x^5
```

3.69.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.42

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^6}\,dx = \frac{1}{10}b^5x^{10}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{7}ab^4x^7\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{2}a^2b^3x^4\mathrm{sgn}\left(bx^3+a\right)+10\,a^3b^2x\mathrm{sgn}\left(bx^3+a\right)-\frac{25\,a^4bx^3\mathrm{sgn}\left(bx^3+a\right)+2\,a^5\mathrm{sgn}\left(bx^3+a\right)}{10\,x^5}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^6,x, algorithm="giac")
```

output
```
1/10*b^5*x^10*sgn(b*x^3 + a) + 5/7*a*b^4*x^7*sgn(b*x^3 + a) + 5/2*a^2*b^3*
x^4*sgn(b*x^3 + a) + 10*a^3*b^2*x*sgn(b*x^3 + a) - 1/10*(25*a^4*b*x^3*sgn(
b*x^3 + a) + 2*a^5*sgn(b*x^3 + a))/x^5
```

3.69.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^6}\,dx = \int \frac{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}}{x^6}\,dx$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^6,x)
```

output
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^6, x)
```

3.70 $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^7} dx$

3.70.1 Optimal result

Integrand size = 26, antiderivative size = 252

$$\begin{aligned}\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^7}\,dx &= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{6x^6(a+bx^3)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{3x^3(a+bx^3)} \\ &+ \frac{10a^2b^3x^3\sqrt{a^2+2abx^3+b^2x^6}}{3(a+bx^3)} + \frac{5ab^4x^6\sqrt{a^2+2abx^3+b^2x^6}}{6(a+bx^3)} \\ &+ \frac{b^5x^9\sqrt{a^2+2abx^3+b^2x^6}}{9(a+bx^3)} + \frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}\log(x)}{a+bx^3}\end{aligned}$$

output
```
-1/6*a^5*((b*x^3+a)^2)^(1/2)/x^6/(b*x^3+a)-5/3*a^4*b*((b*x^3+a)^2)^(1/2)/x
^3/(b*x^3+a)+10/3*a^2*b^3*x^3*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/6*a*b^4*x^6*
((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/9*b^5*x^9*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+10
*a^3*b^2*ln(x)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

3.70.2 Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.10

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^7}\,dx = \frac{1}{3}\left(\frac{(12a^5+120a^4bx^3+57a^3b^2x^6-240a^2b^3x^9-60ab^4x^{12}-8b^5x^{15})\left(\sqrt{a^2}bx\right.}{24x^6\left(a^2+abx^3-\sqrt{a^2}\sqrt{(a+bx^3)^2}\right)}\right.$$

$$-10a^3b^2\text{arctanh}\left(\frac{bx^3}{\sqrt{a^2}-\sqrt{(a+bx^3)^2}}\right)$$

$$-10(a^2)^{3/2}b^2\log(x^3)+5(a^2)^{3/2}b^2\log\left(\sqrt{a^2}-bx^3-\sqrt{(a+bx^3)^2}\right)+5(a^2)^{3/2}b^2\log\left(\sqrt{a^2}+bx^3-\sqrt{(a+bx^3\right.$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^7,x]
```

output
```
(((12*a^5 + 120*a^4*b*x^3 + 57*a^3*b^2*x^6 - 240*a^2*b^3*x^9 - 60*a*b^4*x^
12 - 8*b^5*x^15)*(Sqrt[a^2]*b*x^3 + a*(Sqrt[a^2] - Sqrt[(a + b*x^3)^2])))/
(24*x^6*(a^2 + a*b*x^3 - Sqrt[a^2]*Sqrt[(a + b*x^3)^2])) - 10*a^3*b^2*ArcT
anh[(b*x^3)/(Sqrt[a^2] - Sqrt[(a + b*x^3)^2])] - 10*(a^2)^(3/2)*b^2*Log[x^
3] + 5*(a^2)^(3/2)*b^2*Log[Sqrt[a^2] - b*x^3 - Sqrt[(a + b*x^3)^2]] + 5*(a
^2)^(3/2)*b^2*Log[Sqrt[a^2] + b*x^3 - Sqrt[(a + b*x^3)^2]])/3
```

3.70.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.39, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1384, 27, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^7}\,dx$$

$$\Big\downarrow \; 1384$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b^5(bx^3+a)^5}{x^7}dx}{b^5(a+bx^3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(bx^3+a)^5}{x^7}dx}{a+bx^3}$$

$$\Big\downarrow \text{798}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(bx^3+a)^5}{x^9}dx^3}{3\,(a+bx^3)}$$

$$\Big\downarrow \text{49}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(b^5x^6+5ab^4x^3+10a^2b^3+\frac{10a^3b^2}{x^3}+\frac{5a^4b}{x^6}+\frac{a^5}{x^9}\right)dx^3}{3\,(a+bx^3)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(-\frac{a^5}{2x^6}-\frac{5a^4b}{x^3}+10a^3b^2\log\left(x^3\right)+10a^2b^3x^3+\frac{5}{2}ab^4x^6+\frac{b^5x^9}{3}\right)}{3\,(a+bx^3)}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^7,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-1/2*a^5/x^6 - (5*a^4*b)/x^3 + 10*a^2*b^
3*x^3 + (5*a*b^4*x^6)/2 + (b^5*x^9)/3 + 10*a^3*b^2*Log[x^3]))/(3*(a + b*x^
3))
```

3.70.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

3.70. $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^7}\,dx$

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.70.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.33

method	result	size
default	$\frac{\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}\left(2b^{5}x^{15}+15a\,b^{4}x^{12}+60a^{2}b^{3}x^{9}+180a^{3}b^{2}\ln\left(x\right)x^{6}-30a^{4}b\,x^{3}-3a^{5}\right)}{18\left(b\,x^{3}+a\right)^{5}x^{6}}$	82
pseudoelliptic	$-\frac{\operatorname{csgn}\left(b\,x^{3}+a\right)\left(-\frac{2b^{5}x^{15}}{3}-5a\,b^{4}x^{12}-20a^{2}b^{3}x^{9}-20\ln\left(b\,x^{3}\right)a^{3}b^{2}x^{6}-\frac{47a^{3}b^{2}x^{6}}{3}+10a^{4}b\,x^{3}+a^{5}\right)}{6x^{6}}$	83
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,b^{3}\left(\frac{1}{9}b^{2}x^{9}+\frac{5}{6}ab\,x^{6}+\frac{10}{3}a^{2}x^{3}\right)}{b\,x^{3}+a}+\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(-\frac{5}{3}a^{4}b\,x^{3}-\frac{1}{6}a^{5}\right)}{\left(b\,x^{3}+a\right)x^{6}}+\frac{10a^{3}b^{2}\ln\left(x\right)\sqrt{\left(b\,x^{3}+a\right)^{2}}}{b\,x^{3}+a}$	119

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
1/18*((b*x^3+a)^2)^(5/2)*(2*b^5*x^15+15*a*b^4*x^12+60*a^2*b^3*x^9+180*a^3*
b^2*ln(x)*x^6-30*a^4*b*x^3-3*a^5)/(b*x^3+a)^5/x^6
```

3.70.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.24

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^7}\,dx=\frac{2\,b^5x^{15}+15\,ab^4x^{12}+60\,a^2b^3x^9+180\,a^3b^2x^6\log\left(x\right)-30\,a^4bx^3-3\,a^5}{18\,x^6}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^7,x, algorithm="fricas")
```

output
```
1/18*(2*b^5*x^15 + 15*a*b^4*x^12 + 60*a^2*b^3*x^9 + 180*a^3*b^2*x^6*log(x)
 - 30*a^4*b*x^3 - 3*a^5)/x^6
```

3.70.6 Sympy [F]

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^7}\, dx = \int \frac{\left((a + bx^3)^2\right)^{\frac{5}{2}}}{x^7}\, dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**7,x)
```

output
```
Integral(((a + b*x**3)**2)**(5/2)/x**7, x)
```

3.70.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.12

$$\begin{aligned}\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^7}\, dx &= \frac{5}{3}\sqrt{b^2x^6 + 2\,abx^3 + a^2}ab^3x^3 \\ &+ \frac{10}{3}\,(-1)^{2\,b^2x^3 + 2\,ab}\,a^3b^2\log\left(2\,b^2x^3 + 2\,ab\right) \\ &- \frac{10}{3}\,(-1)^{2\,abx^3 + 2\,a^2}\,a^3b^2\log\left(\frac{2\,abx}{|x|} + \frac{2\,a^2}{x^2|x|}\right) + \frac{5\,(b^2x^6 + 2\,abx^3 + a^2)^{\frac{3}{2}}b^3x^3}{6\,a} \\ &+ 5\sqrt{b^2x^6 + 2\,abx^3 + a^2}a^2b^2 + \frac{35}{18}\left(b^2x^6 + 2\,abx^3 + a^2\right)^{\frac{3}{2}}b^2 \\ &+ \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{5}{2}}b^2}{6\,a^2} - \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{5}{2}}b}{2\,ax^3} - \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{7}{2}}}{6\,a^2x^6}\end{aligned}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^7,x, algorithm="maxima")
```

output
```
5/3*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a*b^3*x^3 + 10/3*(-1)^(2*b^2*x^3 + 2*a
*b)*a^3*b^2*log(2*b^2*x^3 + 2*a*b) - 10/3*(-1)^(2*a*b*x^3 + 2*a^2)*a^3*b^2
*log(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x))) + 5/6*(b^2*x^6 + 2*a*b*x^3 + a^2
)^(3/2)*b^3*x^3/a + 5*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a^2*b^2 + 35/18*(b^2
*x^6 + 2*a*b*x^3 + a^2)^(3/2)*b^2 + 1/6*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*
b^2/a^2 - 1/2*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b/(a*x^3) - 1/6*(b^2*x^6 +
 2*a*b*x^3 + a^2)^(7/2)/(a^2*x^6)
```

3.70.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.50

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^7}\,dx = \frac{1}{9}b^5x^9\operatorname{sgn}(bx^3+a) + \frac{5}{6}ab^4x^6\operatorname{sgn}(bx^3+a) + \frac{10}{3}a^2b^3x^3\operatorname{sgn}(bx^3+a) + 10\,a^3b^2\log(|x|)\operatorname{sgn}(bx^3+a) - \frac{30\,a^3b^2x^6\operatorname{sgn}(bx^3+a) + 10\,a^4bx^3\operatorname{sgn}(bx^3+a) + a^5\operatorname{sgn}(bx^3+a)}{6\,x^6}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^7,x, algorithm="giac")
```

output
```
1/9*b^5*x^9*sgn(b*x^3 + a) + 5/6*a*b^4*x^6*sgn(b*x^3 + a) + 10/3*a^2*b^3*x
^3*sgn(b*x^3 + a) + 10*a^3*b^2*log(abs(x))*sgn(b*x^3 + a) - 1/6*(30*a^3*b^
2*x^6*sgn(b*x^3 + a) + 10*a^4*b*x^3*sgn(b*x^3 + a) + a^5*sgn(b*x^3 + a))/x
^6
```

3.70.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^7}\,dx = \int \frac{(a^2+2\,a\,b\,x^3+b^2\,x^6)^{5/2}}{x^7}\,dx$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^7,x)
```

output
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^7, x)
```

3.71 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^8}\,dx$

3.71.1 Optimal result

Integrand size = 26, antiderivative size = 248

$$\begin{aligned}\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^8}\,dx = & -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)} \\ & -\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}+\frac{5a^2b^3x^2\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3} \\ & +\frac{ab^4x^5\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{b^5x^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}\end{aligned}$$

output
```
-1/7*a^5*((b*x^3+a)^2)^(1/2)/x^7/(b*x^3+a)-5/4*a^4*b*((b*x^3+a)^2)^(1/2)/x
^4/(b*x^3+a)-10*a^3*b^2*((b*x^3+a)^2)^(1/2)/x/(b*x^3+a)+5*a^2*b^3*x^2*((b*
x^3+a)^2)^(1/2)/(b*x^3+a)+a*b^4*x^5*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/8*b^5*
x^8*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

3.71.2 Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^8}\,dx = \frac{\sqrt{\left(a+bx^3\right)^2}\left(-8a^5-70a^4bx^3-560a^3b^2x^6+280a^2b^3x^9+56ab^4x^{12}+7b^5x^{15}\right.}{56x^7\left(a+bx^3\right)}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^8,x]
```

output
```
(Sqrt[(a + b*x^3)^2]*(-8*a^5 - 70*a^4*b*x^3 - 560*a^3*b^2*x^6 + 280*a^2*b^
3*x^9 + 56*a*b^4*x^12 + 7*b^5*x^15))/(56*x^7*(a + b*x^3))
```

3.71.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.38, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^8}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b^5\left(bx^3+a\right)^5}{x^8}dx}{b^5\left(a+bx^3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(bx^3+a\right)^5}{x^8}dx}{a+bx^3}$$

$$\Big\downarrow \text{802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(b^5x^7+5ab^4x^4+10a^2b^3x+\frac{10a^3b^2}{x^2}+\frac{5a^4b}{x^5}+\frac{a^5}{x^8}\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(-\frac{a^5}{7x^7}-\frac{5a^4b}{4x^4}-\frac{10a^3b^2}{x}+5a^2b^3x^2+ab^4x^5+\frac{b^5x^8}{8}\right)}{a+bx^3}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^8,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-1/7*a^5/x^7 - (5*a^4*b)/(4*x^4) - (10*a
^3*b^2)/x + 5*a^2*b^3*x^2 + a*b^4*x^5 + (b^5*x^8)/8))/(a + b*x^3)
```

3.71.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.71.4 Maple [A] (verified)

Time = 7.41 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.32

method	result	size
gosper	$-\frac{\left(-7 b^{5} x^{15}-56 a\,b^{4} x^{12}-280 a^{2} b^{3} x^{9}+560 a^{3} b^{2} x^{6}+70 a^{4} b\,x^{3}+8 a^{5}\right) \left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{56 \left(b\,x^{3}+a\right)^{5} x^{7}}$	80
default	$-\frac{\left(-7 b^{5} x^{15}-56 a\,b^{4} x^{12}-280 a^{2} b^{3} x^{9}+560 a^{3} b^{2} x^{6}+70 a^{4} b\,x^{3}+8 a^{5}\right) \left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{56 \left(b\,x^{3}+a\right)^{5} x^{7}}$	80
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\, b^{3} \left(\frac{1}{8} b^{2} x^{8}+a b\,x^{5}+5 a^{2} x^{2}\right)}{b\,x^{3}+a}+\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\, \left(-10 a^{3} b^{2} x^{6}-\frac{5}{4} a^{4} b\,x^{3}-\frac{1}{7} a^{5}\right)}{\left(b\,x^{3}+a\right) x^{7}}$	99

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/56*(-7*b^5*x^15-56*a*b^4*x^12-280*a^2*b^3*x^9+560*a^3*b^2*x^6+70*a^4*b*
x^3+8*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5/x^7
```

3.71.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.24

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^8}\,dx = \frac{7\,b^5x^{15}+56\,ab^4x^{12}+280\,a^2b^3x^9-560\,a^3b^2x^6-70\,a^4bx^3-8\,a^5}{56\,x^7}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^8,x, algorithm="fricas")
```

output
```
1/56*(7*b^5*x^15 + 56*a*b^4*x^12 + 280*a^2*b^3*x^9 - 560*a^3*b^2*x^6 - 70*
a^4*b*x^3 - 8*a^5)/x^7
```

3.71.6 Sympy [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^8}\,dx = \int \frac{\left((a+bx^3)^2\right)^{\frac{5}{2}}}{x^8}\,dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**8,x)
```

output
```
Integral(((a + b*x**3)**2)**(5/2)/x**8, x)
```

3.71.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.24

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^8}\,dx = \frac{7\,b^5x^{15}+56\,ab^4x^{12}+280\,a^2b^3x^9-560\,a^3b^2x^6-70\,a^4bx^3-8\,a^5}{56\,x^7}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^8,x, algorithm="maxima")
```

output
```
1/56*(7*b^5*x^15 + 56*a*b^4*x^12 + 280*a^2*b^3*x^9 - 560*a^3*b^2*x^6 - 70*
a^4*b*x^3 - 8*a^5)/x^7
```

3.71.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.43

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^8}\,dx = \frac{1}{8}b^5x^8\operatorname{sgn}(bx^3+a) + ab^4x^5\operatorname{sgn}(bx^3+a) + 5\,a^2b^3x^2\operatorname{sgn}(bx^3+a) - \frac{280\,a^3b^2x^6\operatorname{sgn}(bx^3+a) + 35\,a^4bx^3\operatorname{sgn}(bx^3+a) + 4\,a^5\operatorname{sgn}(bx^3+a)}{28\,x^7}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^8,x, algorithm="giac")
```

output
```
1/8*b^5*x^8*sgn(b*x^3 + a) + a*b^4*x^5*sgn(b*x^3 + a) + 5*a^2*b^3*x^2*sgn(
b*x^3 + a) - 1/28*(280*a^3*b^2*x^6*sgn(b*x^3 + a) + 35*a^4*b*x^3*sgn(b*x^3
 + a) + 4*a^5*sgn(b*x^3 + a))/x^7
```

3.71.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^8}\,dx = \int \frac{(a^2+2\,a\,b\,x^3+b^2\,x^6)^{5/2}}{x^8}\,dx$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^8,x)
```

output
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^8, x)
```

3.72 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^9}\,dx$

3.72.1 Optimal result

Integrand size = 26, antiderivative size = 247

$$\begin{aligned}\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^9}\,dx &= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\left(a+bx^3\right)}-\frac{a^4b\sqrt{a^2+2abx^3+b^2x^6}}{x^5\left(a+bx^3\right)}\\ &-\frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{x^2\left(a+bx^3\right)}+\frac{10a^2b^3x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}\\ &+\frac{5ab^4x^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{b^5x^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}\end{aligned}$$

output
```
-1/8*a^5*((b*x^3+a)^2)^(1/2)/x^8/(b*x^3+a)-a^4*b*((b*x^3+a)^2)^(1/2)/x^5/(
b*x^3+a)-5*a^3*b^2*((b*x^3+a)^2)^(1/2)/x^2/(b*x^3+a)+10*a^2*b^3*x*((b*x^3+
a)^2)^(1/2)/(b*x^3+a)+5/4*a*b^4*x^4*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/7*b^5*
x^7*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

3.72.2 Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.34

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^9}\,dx = \frac{\sqrt{\left(a+bx^3\right)^2}\left(-7a^5-56a^4bx^3-280a^3b^2x^6+560a^2b^3x^9+70ab^4x^{12}+8b^5x^{15}\right.}{56x^8\left(a+bx^3\right)}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^9,x]
```

output
```
(Sqrt[(a + b*x^3)^2]*(-7*a^5 - 56*a^4*b*x^3 - 280*a^3*b^2*x^6 + 560*a^2*b^
3*x^9 + 70*a*b^4*x^12 + 8*b^5*x^15))/(56*x^8*(a + b*x^3))
```

3.72.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.38, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^9}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b^5\left(bx^3+a\right)^5}{x^9}dx}{b^5\left(a+bx^3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(bx^3+a\right)^5}{x^9}dx}{a+bx^3}$$

$$\Big\downarrow \text{802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(b^5x^6+5ab^4x^3+10a^2b^3+\frac{10a^3b^2}{x^3}+\frac{5a^4b}{x^6}+\frac{a^5}{x^9}\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(-\frac{a^5}{8x^8}-\frac{a^4b}{x^5}-\frac{5a^3b^2}{x^2}+10a^2b^3x+\frac{5}{4}ab^4x^4+\frac{b^5x^7}{7}\right)}{a+bx^3}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^9,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-1/8*a^5/x^8 - (a^4*b)/x^5 - (5*a^3*b^2)
/x^2 + 10*a^2*b^3*x + (5*a*b^4*x^4)/4 + (b^5*x^7)/7))/(a + b*x^3)
```

3.72.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.72.4 Maple [A] (verified)

Time = 8.88 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.32

method	result	size
gosper	$-\frac{\left(-8 b^{5} x^{15}-70 a\, b^{4} x^{12}-560 a^{2} b^{3} x^{9}+280 a^{3} b^{2} x^{6}+56 a^{4} b\, x^{3}+7 a^{5}\right) \left(\left(b\, x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{56 \left(b\, x^{3}+a\right)^{5} x^{8}}$	80
default	$-\frac{\left(-8 b^{5} x^{15}-70 a\, b^{4} x^{12}-560 a^{2} b^{3} x^{9}+280 a^{3} b^{2} x^{6}+56 a^{4} b\, x^{3}+7 a^{5}\right) \left(\left(b\, x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{56 \left(b\, x^{3}+a\right)^{5} x^{8}}$	80
risch	$\frac{\sqrt{\left(b\, x^{3}+a\right)^{2}}\, b^{3} \left(\frac{1}{7} b^{2} x^{7}+\frac{5}{4} a b\, x^{4}+10 a^{2} x\right)}{b\, x^{3}+a}+\frac{\sqrt{\left(b\, x^{3}+a\right)^{2}}\, \left(-5 a^{3} b^{2} x^{6}-a^{4} b\, x^{3}-\frac{1}{8} a^{5}\right)}{\left(b\, x^{3}+a\right) x^{8}}$	98

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/56*(-8*b^5*x^15-70*a*b^4*x^12-560*a^2*b^3*x^9+280*a^3*b^2*x^6+56*a^4*b*
x^3+7*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5/x^8
```

3.72.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.24

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^9}\,dx = \frac{8\,b^5x^{15}+70\,ab^4x^{12}+560\,a^2b^3x^9-280\,a^3b^2x^6-56\,a^4bx^3-7\,a^5}{56\,x^8}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^9,x, algorithm="fricas")
```

output
```
1/56*(8*b^5*x^15 + 70*a*b^4*x^12 + 560*a^2*b^3*x^9 - 280*a^3*b^2*x^6 - 56*
a^4*b*x^3 - 7*a^5)/x^8
```

3.72.6 Sympy [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^9}\,dx = \int \frac{\left((a+bx^3)^2\right)^{\frac{5}{2}}}{x^9}\,dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**9,x)
```

output
```
Integral(((a + b*x**3)**2)**(5/2)/x**9, x)
```

3.72.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.24

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^9}\,dx = \frac{8\,b^5x^{15}+70\,ab^4x^{12}+560\,a^2b^3x^9-280\,a^3b^2x^6-56\,a^4bx^3-7\,a^5}{56\,x^8}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^9,x, algorithm="maxima")
```

output
```
1/56*(8*b^5*x^15 + 70*a*b^4*x^12 + 560*a^2*b^3*x^9 - 280*a^3*b^2*x^6 - 56*
a^4*b*x^3 - 7*a^5)/x^8
```

3.72.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.43

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^9}\,dx = \frac{1}{7}b^5x^7\mathrm{sgn}(bx^3+a) + \frac{5}{4}ab^4x^4\mathrm{sgn}(bx^3+a) + 10\,a^2b^3x\mathrm{sgn}(bx^3+a) - \frac{40\,a^3b^2x^6\mathrm{sgn}(bx^3+a) + 8\,a^4bx^3\mathrm{sgn}(bx^3+a) + a^5\mathrm{sgn}(bx^3+a)}{8\,x^8}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^9,x, algorithm="giac")
```

output
```
1/7*b^5*x^7*sgn(b*x^3 + a) + 5/4*a*b^4*x^4*sgn(b*x^3 + a) + 10*a^2*b^3*x*s
gn(b*x^3 + a) - 1/8*(40*a^3*b^2*x^6*sgn(b*x^3 + a) + 8*a^4*b*x^3*sgn(b*x^3
 + a) + a^5*sgn(b*x^3 + a))/x^8
```

3.72.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^9}\,dx = \int \frac{(a^2+2\,a\,b\,x^3+b^2\,x^6)^{5/2}}{x^9}\,dx$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^9,x)
```

output
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^9, x)
```

3.73 $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{10}}\,dx$

3.73.1 Optimal result

Integrand size = 26, antiderivative size = 252

$$\begin{aligned}\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{10}}\,dx &= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{9x^9(a+bx^3)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{6x^6(a+bx^3)}\\ &\quad-\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{3x^3(a+bx^3)}+\frac{5ab^4x^3\sqrt{a^2+2abx^3+b^2x^6}}{3(a+bx^3)}\\ &\quad+\frac{b^5x^6\sqrt{a^2+2abx^3+b^2x^6}}{6(a+bx^3)}+\frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}\log(x)}{a+bx^3}\end{aligned}$$

output
```
-1/9*a^5*((b*x^3+a)^2)^(1/2)/x^9/(b*x^3+a)-5/6*a^4*b*((b*x^3+a)^2)^(1/2)/x
^6/(b*x^3+a)-10/3*a^3*b^2*((b*x^3+a)^2)^(1/2)/x^3/(b*x^3+a)+5/3*a*b^4*x^3*
((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/6*b^5*x^6*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+10
*a^2*b^3*ln(x)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

3.73.2 Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.11

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{10}}\,dx = \frac{1}{3}\left(\frac{(4a^5+30a^4bx^3+120a^3b^2x^6+53a^2b^3x^9-60ab^4x^{12}-6b^5x^{15})\left(\sqrt{a^2}bx^3+\right.}{12x^9\left(a^2+abx^3-\sqrt{a^2}\sqrt{(a+bx^3)^2}\right)}\right.$$
$$-10a^2b^3\text{arctanh}\left(\frac{bx^3}{\sqrt{a^2}-\sqrt{(a+bx^3)^2}}\right)-10a\sqrt{a^2}b^3\log\left(x^3\right)$$
$$\left.+5a\sqrt{a^2}b^3\log\left(\sqrt{a^2}-bx^3-\sqrt{(a+bx^3)^2}\right)+5a\sqrt{a^2}b^3\log\left(\sqrt{a^2}+bx^3-\sqrt{(a+bx^3)^2}\right)\right)$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^10,x]
```

output
```
(((4*a^5 + 30*a^4*b*x^3 + 120*a^3*b^2*x^6 + 53*a^2*b^3*x^9 - 60*a*b^4*x^12
 - 6*b^5*x^15)*(Sqrt[a^2]*b*x^3 + a*(Sqrt[a^2] - Sqrt[(a + b*x^3)^2])))/(1
2*x^9*(a^2 + a*b*x^3 - Sqrt[a^2]*Sqrt[(a + b*x^3)^2])) - 10*a^2*b^3*ArcTan
h[(b*x^3)/(Sqrt[a^2] - Sqrt[(a + b*x^3)^2])] - 10*a*Sqrt[a^2]*b^3*Log[x^3]
 + 5*a*Sqrt[a^2]*b^3*Log[Sqrt[a^2] - b*x^3 - Sqrt[(a + b*x^3)^2]] + 5*a*Sq
rt[a^2]*b^3*Log[Sqrt[a^2] + b*x^3 - Sqrt[(a + b*x^3)^2]])/3
```

3.73.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.39, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1384, 27, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{10}}dx$$
$$\Big\downarrow \text{ 1384}$$
$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b^5(bx^3+a)^5}{x^{10}}dx}{b^5(a+bx^3)}$$

$$\Big\downarrow \color{blue}{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(bx^3+a)^5}{x^{10}}dx}{a+bx^3}$$

$$\Big\downarrow \color{blue}{798}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(bx^3+a)^5}{x^{12}}dx^3}{3(a+bx^3)}$$

$$\Big\downarrow \color{blue}{49}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^5}{x^{12}}+\frac{5ba^4}{x^9}+\frac{10b^2a^3}{x^6}+\frac{10b^3a^2}{x^3}+5b^4a+b^5x^3\right)dx^3}{3(a+bx^3)}$$

$$\Big\downarrow \color{blue}{2009}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(-\frac{a^5}{3x^9}-\frac{5a^4b}{2x^6}-\frac{10a^3b^2}{x^3}+10a^2b^3\log\left(x^3\right)+5ab^4x^3+\frac{b^5x^6}{2}\right)}{3(a+bx^3)}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^10,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-1/3*a^5/x^9 - (5*a^4*b)/(2*x^6) - (10*a
^3*b^2)/x^3 + 5*a*b^4*x^3 + (b^5*x^6)/2 + 10*a^2*b^3*Log[x^3]))/(3*(a + b*
x^3))
```

3.73.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

3.73. $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{10}}dx$

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.73.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.33

method	result	size
default	$\frac{\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}} \left(3 b^{5} x^{15}+30 a\,b^{4} x^{12}+180 a^{2} b^{3} \ln \left(x \right) x^{9}-60 a^{3} b^{2} x^{6}-15 a^{4} b\,x^{3}-2 a^{5}\right)}{18 \left(b\,x^{3}+a \right)^{5} x^{9}}$	82
pseudoelliptic	$-\frac{\operatorname{csgn}\left(b\,x^{3}+a \right) \left(-\frac{3 b^{5} x^{15}}{2}-15 a\,b^{4} x^{12}-30 \ln \left(b\,x^{3}\right) a^{2} b^{3} x^{9}-\frac{27 a^{2} b^{3} x^{9}}{2}+30 a^{3} b^{2} x^{6}+\frac{15 a^{4} b\,x^{3}}{2}+a^{5}\right)}{9 x^{9}}$	83
risch	$\frac{\sqrt{\left(b\,x^{3}+a \right)^{2}}\, b^{3} \left(b\,x^{3}+5 a \right)^{2}}{6 b\,x^{3}+6 a}+\frac{\sqrt{\left(b\,x^{3}+a \right)^{2}}\, \left(-\frac{10}{3} a^{3} b^{2} x^{6}-\frac{5}{6} a^{4} b\,x^{3}-\frac{1}{9} a^{5}\right)}{\left(b\,x^{3}+a \right) x^{9}}+\frac{10 a^{2} b^{3} \ln \left(x \right) \sqrt{\left(b\,x^{3}+a \right)^{2}}}{b\,x^{3}+a}$	118

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^10,x,method=_RETURNVERBOSE)
```

output
```
1/18*((b*x^3+a)^2)^(5/2)*(3*b^5*x^15+30*a*b^4*x^12+180*a^2*b^3*ln(x)*x^9-6
0*a^3*b^2*x^6-15*a^4*b*x^3-2*a^5)/(b*x^3+a)^5/x^9
```

3.73.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.24

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{10}}\,dx = \frac{3\,b^5x^{15}+30\,ab^4x^{12}+180\,a^2b^3x^9\log\left(x\right)-60\,a^3b^2x^6-15\,a^4bx^3-2\,a^5}{18\,x^9}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^10,x, algorithm="fricas")
```

output
```
1/18*(3*b^5*x^15 + 30*a*b^4*x^12 + 180*a^2*b^3*x^9*log(x) - 60*a^3*b^2*x^6
 - 15*a^4*b*x^3 - 2*a^5)/x^9
```

3.73.6 Sympy [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{10}}\,dx = \int \frac{\left((a+bx^3)^2\right)^{\frac{5}{2}}}{x^{10}}\,dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**10,x)
```

output
```
Integral(((a + b*x**3)**2)**(5/2)/x**10, x)
```

3.73.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.24

$$\begin{aligned}\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{10}}\,dx &= \frac{5}{3}\sqrt{b^2x^6+2\,abx^3+a^2}b^4x^3 \\ &+ \frac{10}{3}(-1)^{2\,b^2x^3+2\,ab}\,a^2b^3\log\left(2\,b^2x^3+2\,ab\right) \\ &- \frac{10}{3}(-1)^{2\,abx^3+2\,a^2}\,a^2b^3\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right)+\frac{5\,(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}b^4x^3}{6\,a^2} \\ &+ 5\sqrt{b^2x^6+2\,abx^3+a^2}ab^3+\frac{35\,(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}b^3}{18\,a}+\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}b^3}{18\,a^3} \\ &- \frac{11\,(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}b^2}{18\,a^2x^3}-\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{7}{2}}b}{18\,a^3x^6}-\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{7}{2}}}{9\,a^2x^9}\end{aligned}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^10,x, algorithm="maxima")
```

output
```
5/3*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^4*x^3 + 10/3*(-1)^(2*b^2*x^3 + 2*a*b
)*a^2*b^3*log(2*b^2*x^3 + 2*a*b) - 10/3*(-1)^(2*a*b*x^3 + 2*a^2)*a^2*b^3*l
og(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x))) + 5/6*(b^2*x^6 + 2*a*b*x^3 + a^2)^
(3/2)*b^4*x^3/a^2 + 5*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a*b^3 + 35/18*(b^2*x
^6 + 2*a*b*x^3 + a^2)^(3/2)*b^3/a + 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)
*b^3/a^3 - 11/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^2/(a^2*x^3) - 1/18*(b
^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b/(a^3*x^6) - 1/9*(b^2*x^6 + 2*a*b*x^3 + a
^2)^(7/2)/(a^2*x^9)
```

3.73.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.50

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{10}}\,dx = \frac{1}{6}\,b^5x^6\text{sgn}\left(bx^3+a\right) + \frac{5}{3}\,ab^4x^3\text{sgn}\left(bx^3+a\right) + 10\,a^2b^3\log\left(|x|\right)\text{sgn}\left(bx^3+a\right) - \frac{110\,a^2b^3x^9\text{sgn}(bx^3+a) + 60\,a^3b^2x^6\text{sgn}(bx^3+a) + 15\,a^4bx^3\text{sgn}(bx^3+a) + 2\,a^5\text{sgn}(bx^3+a)}{18\,x^9}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^10,x, algorithm="giac")
```

output
```
1/6*b^5*x^6*sgn(b*x^3 + a) + 5/3*a*b^4*x^3*sgn(b*x^3 + a) + 10*a^2*b^3*log
(abs(x))*sgn(b*x^3 + a) - 1/18*(110*a^2*b^3*x^9*sgn(b*x^3 + a) + 60*a^3*b^
2*x^6*sgn(b*x^3 + a) + 15*a^4*b*x^3*sgn(b*x^3 + a) + 2*a^5*sgn(b*x^3 + a))
/x^9
```

3.73.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{10}}\,dx = \int \frac{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}}{x^{10}}\,dx$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^10,x)
```

output
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^10, x)
```

3.74 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{11}}\,dx$

3.74.1 Optimal result

Integrand size = 26, antiderivative size = 253

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{11}}\,dx = -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}\,(a+bx^3)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\,(a+bx^3)}-\frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{2x^4\,(a+bx^3)}-\frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{x\,(a+bx^3)}+\frac{5ab^4x^2\sqrt{a^2+2abx^3+b^2x^6}}{2\,(a+bx^3)}+\frac{b^5x^5\sqrt{a^2+2abx^3+b^2x^6}}{5\,(a+bx^3)}$$

output
```
-1/10*a^5*((b*x^3+a)^2)^(1/2)/x^10/(b*x^3+a)-5/7*a^4*b*((b*x^3+a)^2)^(1/2)
/x^7/(b*x^3+a)-5/2*a^3*b^2*((b*x^3+a)^2)^(1/2)/x^4/(b*x^3+a)-10*a^2*b^3*((
b*x^3+a)^2)^(1/2)/x/(b*x^3+a)+5/2*a*b^4*x^2*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+
1/5*b^5*x^5*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

3.74.2 Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{11}}\,dx = -\frac{\sqrt{(a+bx^3)^2}\left(7a^5+50a^4bx^3+175a^3b^2x^6+700a^2b^3x^9-175ab^4x^{12}-14b^5x^{15}\right)}{70x^{10}\,(a+bx^3)}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^11,x]
```

output
```
-1/70*(Sqrt[(a + b*x^3)^2]*(7*a^5 + 50*a^4*b*x^3 + 175*a^3*b^2*x^6 + 700*a
^2*b^3*x^9 - 175*a*b^4*x^12 - 14*b^5*x^15))/(x^10*(a + b*x^3))
```

3.74.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.39, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{11}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b^5\left(bx^3+a\right)^5}{x^{11}}dx}{b^5\left(a+bx^3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(bx^3+a\right)^5}{x^{11}}dx}{a+bx^3}$$

$$\Big\downarrow \text{802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^5}{x^{11}}+\frac{5ba^4}{x^8}+\frac{10b^2a^3}{x^5}+\frac{10b^3a^2}{x^2}+5b^4xa+b^5x^4\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(-\frac{a^5}{10x^{10}}-\frac{5a^4b}{7x^7}-\frac{5a^3b^2}{2x^4}-\frac{10a^2b^3}{x}+\frac{5}{2}ab^4x^2+\frac{b^5x^5}{5}\right)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^11,x]
```

output
```
((-1/10*a^5/x^10 - (5*a^4*b)/(7*x^7) - (5*a^3*b^2)/(2*x^4) - (10*a^2*b^3)/
x + (5*a*b^4*x^2)/2 + (b^5*x^5)/5)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b
*x^3)
```

3.74.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.74.4 Maple [A] (verified)

Time = 11.74 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.32

method	result	size
gosper	$-\frac{\left(-14 b^{5} x^{15}-175 a \,b^{4} x^{12}+700 a^{2} b^{3} x^{9}+175 a^{3} b^{2} x^{6}+50 a^{4} b \,x^{3}+7 a^{5}\right) \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{5}{2}}}{70 \left(b \,x^{3}+a \right)^{5} x^{10}}$	80
default	$-\frac{\left(-14 b^{5} x^{15}-175 a \,b^{4} x^{12}+700 a^{2} b^{3} x^{9}+175 a^{3} b^{2} x^{6}+50 a^{4} b \,x^{3}+7 a^{5}\right) \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{5}{2}}}{70 \left(b \,x^{3}+a \right)^{5} x^{10}}$	80
risch	$\frac{\sqrt{\left(b \,x^{3}+a \right)^{2}}\, b^{4} \left(\frac{1}{5} b \,x^{5}+\frac{5}{2} a \,x^{2}\right)}{b \,x^{3}+a}+\frac{\sqrt{\left(b \,x^{3}+a \right)^{2}}\, \left(-10 a^{2} b^{3} x^{9}-\frac{5}{2} a^{3} b^{2} x^{6}-\frac{5}{7} a^{4} b \,x^{3}-\frac{1}{10} a^{5}\right)}{\left(b \,x^{3}+a \right) x^{10}}$	100

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^11,x,method=_RETURNVERBOSE)
```

output
```
-1/70*(-14*b^5*x^15-175*a*b^4*x^12+700*a^2*b^3*x^9+175*a^3*b^2*x^6+50*a^4*
b*x^3+7*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5/x^10
```

3.74.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.23

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{11}}\,dx = \frac{14\,b^5x^{15}+175\,ab^4x^{12}-700\,a^2b^3x^9-175\,a^3b^2x^6-50\,a^4bx^3-7\,a^5}{70\,x^{10}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^11,x, algorithm="fricas")
```

output
```
1/70*(14*b^5*x^15 + 175*a*b^4*x^12 - 700*a^2*b^3*x^9 - 175*a^3*b^2*x^6 - 5
0*a^4*b*x^3 - 7*a^5)/x^10
```

3.74.6 Sympy [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{11}}\,dx = \int \frac{\left((a+bx^3)^2\right)^{\frac{5}{2}}}{x^{11}}\,dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**11,x)
```

output
```
Integral(((a + b*x**3)**2)**(5/2)/x**11, x)
```

3.74.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.23

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{11}}\,dx = \frac{14\,b^5x^{15}+175\,ab^4x^{12}-700\,a^2b^3x^9-175\,a^3b^2x^6-50\,a^4bx^3-7\,a^5}{70\,x^{10}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^11,x, algorithm="maxima")
```

output
```
1/70*(14*b^5*x^15 + 175*a*b^4*x^12 - 700*a^2*b^3*x^9 - 175*a^3*b^2*x^6 - 5
0*a^4*b*x^3 - 7*a^5)/x^10
```

3.74.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.43

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{11}}\,dx = \frac{1}{5}\,b^5x^5\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{2}\,ab^4x^2\mathrm{sgn}\left(bx^3+a\right) - \frac{700\,a^2b^3x^9\mathrm{sgn}(bx^3+a)+175\,a^3b^2x^6\mathrm{sgn}(bx^3+a)+50\,a^4bx^3\mathrm{sgn}(bx^3+a)+7\,a^5\mathrm{sgn}(bx^3+a)}{70\,x^{10}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^11,x, algorithm="giac")
```

output
```
1/5*b^5*x^5*sgn(b*x^3 + a) + 5/2*a*b^4*x^2*sgn(b*x^3 + a) - 1/70*(700*a^2*
b^3*x^9*sgn(b*x^3 + a) + 175*a^3*b^2*x^6*sgn(b*x^3 + a) + 50*a^4*b*x^3*sgn
(b*x^3 + a) + 7*a^5*sgn(b*x^3 + a))/x^10
```

3.74.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{11}}\,dx = \int \frac{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}}{x^{11}}\,dx$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^11,x)
```

output
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^11, x)
```

3.75 $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{12}}\,dx$

3.75.1 Optimal result

Integrand size = 26, antiderivative size = 247

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{12}}\,dx = -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}(a+bx^3)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{8x^8(a+bx^3)} - \frac{2a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{x^5(a+bx^3)} - \frac{5a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{x^2(a+bx^3)} + \frac{5ab^4x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3} + \frac{b^5x^4\sqrt{a^2+2abx^3+b^2x^6}}{4(a+bx^3)}$$

output
```
-1/11*a^5*((b*x^3+a)^2)^(1/2)/x^11/(b*x^3+a)-5/8*a^4*b*((b*x^3+a)^2)^(1/2)
/x^8/(b*x^3+a)-2*a^3*b^2*((b*x^3+a)^2)^(1/2)/x^5/(b*x^3+a)-5*a^2*b^3*((b*x
^3+a)^2)^(1/2)/x^2/(b*x^3+a)+5*a*b^4*x*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/4*b
^5*x^4*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

3.75.2 Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.34

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{12}}\,dx = -\frac{\sqrt{(a+bx^3)^2}(8a^5+55a^4bx^3+176a^3b^2x^6+440a^2b^3x^9-440ab^4x^{12}-22b^5x^{15})}{88x^{11}(a+bx^3)}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^12,x]
```

output
```
-1/88*(Sqrt[(a + b*x^3)^2]*(8*a^5 + 55*a^4*b*x^3 + 176*a^3*b^2*x^6 + 440*a
^2*b^3*x^9 - 440*a*b^4*x^12 - 22*b^5*x^15))/(x^11*(a + b*x^3))
```

3.75.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.38, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{12}}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b^5\left(bx^3+a\right)^5}{x^{12}}dx}{b^5\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(bx^3+a\right)^5}{x^{12}}dx}{a+bx^3}$$

$$\Big\downarrow \text{ 802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^5}{x^{12}}+\frac{5ba^4}{x^9}+\frac{10b^2a^3}{x^6}+\frac{10b^3a^2}{x^3}+5b^4a+b^5x^3\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\left(-\frac{a^5}{11x^{11}}-\frac{5a^4b}{8x^8}-\frac{2a^3b^2}{x^5}-\frac{5a^2b^3}{x^2}+5ab^4x+\frac{b^5x^4}{4}\right)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^12,x]
```

output
```
((-1/11*a^5/x^11 - (5*a^4*b)/(8*x^8) - (2*a^3*b^2)/x^5 - (5*a^2*b^3)/x^2 +
 5*a*b^4*x + (b^5*x^4)/4)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3)
```

3.75.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.75.4 Maple [A] (verified)

Time = 12.87 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.32

method	result	size
gosper	$-\frac{\left(-22 b^{5} x^{15}-440 a\,b^{4} x^{12}+440 a^{2} b^{3} x^{9}+176 a^{3} b^{2} x^{6}+55 a^{4} b\,x^{3}+8 a^{5}\right) \left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{88 \left(b\,x^{3}+a\right)^{5} x^{11}}$	80
default	$-\frac{\left(-22 b^{5} x^{15}-440 a\,b^{4} x^{12}+440 a^{2} b^{3} x^{9}+176 a^{3} b^{2} x^{6}+55 a^{4} b\,x^{3}+8 a^{5}\right) \left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{88 \left(b\,x^{3}+a\right)^{5} x^{11}}$	80
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\, b^{4} \left(\frac{1}{4} b\,x^{4}+5 a x \right)}{b\,x^{3}+a}+\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\, \left(-5 a^{2} b^{3} x^{9}-2 a^{3} b^{2} x^{6}-\frac{5}{8} a^{4} b\,x^{3}-\frac{1}{11} a^{5}\right)}{\left(b\,x^{3}+a\right) x^{11}}$	98

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^12,x,method=_RETURNVERBOSE)
```

output
```
-1/88*(-22*b^5*x^15-440*a*b^4*x^12+440*a^2*b^3*x^9+176*a^3*b^2*x^6+55*a^4*
b*x^3+8*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5/x^11
```

3.75.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.24

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{12}}\,dx = \frac{22\,b^5x^{15}+440\,ab^4x^{12}-440\,a^2b^3x^9-176\,a^3b^2x^6-55\,a^4bx^3-8\,a^5}{88\,x^{11}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^12,x, algorithm="fricas")
```

output
```
1/88*(22*b^5*x^15 + 440*a*b^4*x^12 - 440*a^2*b^3*x^9 - 176*a^3*b^2*x^6 - 5
5*a^4*b*x^3 - 8*a^5)/x^11
```

3.75.6 Sympy [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{12}}\,dx = \int \frac{\left((a+bx^3)^2\right)^{\frac{5}{2}}}{x^{12}}\,dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**12,x)
```

output
```
Integral(((a + b*x**3)**2)**(5/2)/x**12, x)
```

3.75.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.24

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{12}}\,dx = \frac{22\,b^5x^{15}+440\,ab^4x^{12}-440\,a^2b^3x^9-176\,a^3b^2x^6-55\,a^4bx^3-8\,a^5}{88\,x^{11}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^12,x, algorithm="maxima")
```

output
```
1/88*(22*b^5*x^15 + 440*a*b^4*x^12 - 440*a^2*b^3*x^9 - 176*a^3*b^2*x^6 - 5
5*a^4*b*x^3 - 8*a^5)/x^11
```

3.75.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.43

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{12}}\,dx = \frac{1}{4}\,b^5x^4\mathrm{sgn}\left(bx^3+a\right)+5\,ab^4x\mathrm{sgn}\left(bx^3+a\right) - \frac{440\,a^2b^3x^9\mathrm{sgn}(bx^3+a)+176\,a^3b^2x^6\mathrm{sgn}(bx^3+a)+55\,a^4bx^3\mathrm{sgn}(bx^3+a)+8\,a^5\mathrm{sgn}(bx^3+a)}{88\,x^{11}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^12,x, algorithm="giac")
```

output
```
1/4*b^5*x^4*sgn(b*x^3 + a) + 5*a*b^4*x*sgn(b*x^3 + a) - 1/88*(440*a^2*b^3*
x^9*sgn(b*x^3 + a) + 176*a^3*b^2*x^6*sgn(b*x^3 + a) + 55*a^4*b*x^3*sgn(b*x
^3 + a) + 8*a^5*sgn(b*x^3 + a))/x^11
```

3.75.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{12}}\,dx = \int \frac{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}}{x^{12}}\,dx$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^12,x)
```

output
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^12, x)
```

3.76 $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{13}}\,dx$

3.76.1 Optimal result

Integrand size = 26, antiderivative size = 252

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{13}}\,dx = -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{12x^{12}(a+bx^3)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{9x^9(a+bx^3)} - \frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{3x^6(a+bx^3)} - \frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{3x^3(a+bx^3)} + \frac{b^5x^3\sqrt{a^2+2abx^3+b^2x^6}}{3(a+bx^3)} + \frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}\log(x)}{a+bx^3}$$

output
```
-1/12*a^5*((b*x^3+a)^2)^(1/2)/x^12/(b*x^3+a)-5/9*a^4*b*((b*x^3+a)^2)^(1/2)
/x^9/(b*x^3+a)-5/3*a^3*b^2*((b*x^3+a)^2)^(1/2)/x^6/(b*x^3+a)-10/3*a^2*b^3*
((b*x^3+a)^2)^(1/2)/x^3/(b*x^3+a)+1/3*b^5*x^3*((b*x^3+a)^2)^(1/2)/(b*x^3+a
)+5*a*b^4*ln(x)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

3.76.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 696 vs. $2(252) = 504$.

Time = 0.94 (sec) , antiderivative size = 696, normalized size of antiderivative = 2.76

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{13}}\,dx = \frac{48a^6\sqrt{a^2}+368a^5\sqrt{a^2}bx^3+1280a^4\sqrt{a^2}b^2x^6+2880a^3\sqrt{a^2}b^3x^9+2677(a^2)^{3/2}}{}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^13,x]
```

output
```
(48*a^6*Sqrt[a^2] + 368*a^5*Sqrt[a^2]*b*x^3 + 1280*a^4*Sqrt[a^2]*b^2*x^6 +
 2880*a^3*Sqrt[a^2]*b^3*x^9 + 2677*(a^2)^(3/2)*b^4*x^12 + 565*a*Sqrt[a^2]*
b^5*x^15 - 192*Sqrt[a^2]*b^6*x^18 - 48*a^6*Sqrt[(a + b*x^3)^2] - 320*a^5*b
*x^3*Sqrt[(a + b*x^3)^2] - 960*a^4*b^2*x^6*Sqrt[(a + b*x^3)^2] - 1920*a^3*
b^3*x^9*Sqrt[(a + b*x^3)^2] - 757*a^2*b^4*x^12*Sqrt[(a + b*x^3)^2] + 192*a
*b^5*x^15*Sqrt[(a + b*x^3)^2] - 960*a*b^4*x^12*(a^2 + a*b*x^3 - Sqrt[a^2]*
Sqrt[(a + b*x^3)^2])*ArcTanh[(b*x^3)/(Sqrt[a^2] - Sqrt[(a + b*x^3)^2])] -
960*b^4*x^12*((a^2)^(3/2) + a*Sqrt[a^2]*b*x^3 - a^2*Sqrt[(a + b*x^3)^2])*L
og[x^3] + 480*(a^2)^(3/2)*b^4*x^12*Log[Sqrt[a^2] - b*x^3 - Sqrt[(a + b*x^3
)^2]] + 480*a*Sqrt[a^2]*b^5*x^15*Log[Sqrt[a^2] - b*x^3 - Sqrt[(a + b*x^3)^
2]] - 480*a^2*b^4*x^12*Sqrt[(a + b*x^3)^2]*Log[Sqrt[a^2] - b*x^3 - Sqrt[(a
 + b*x^3)^2]] + 480*(a^2)^(3/2)*b^4*x^12*Log[Sqrt[a^2] + b*x^3 - Sqrt[(a +
 b*x^3)^2]] + 480*a*Sqrt[a^2]*b^5*x^15*Log[Sqrt[a^2] + b*x^3 - Sqrt[(a + b
*x^3)^2]] - 480*a^2*b^4*x^12*Sqrt[(a + b*x^3)^2]*Log[Sqrt[a^2] + b*x^3 - S
qrt[(a + b*x^3)^2]])/(576*x^12*(a^2 + a*b*x^3 - Sqrt[a^2]*Sqrt[(a + b*x^3)
^2]))
```

3.76.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.38, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1384, 27, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{13}}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b^5\left(bx^3+a\right)^5}{x^{13}}dx}{b^5\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(bx^3+a\right)^5}{x^{13}}dx}{a+bx^3}$$

$$\Big\downarrow \text{ 798}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int\frac{(bx^3+a)^5}{x^{15}}dx^3}{3(a+bx^3)}$$

$$\Big\downarrow \text{49}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(\frac{a^5}{x^{15}}+\frac{5ba^4}{x^{12}}+\frac{10b^2a^3}{x^9}+\frac{10b^3a^2}{x^6}+\frac{5b^4a}{x^3}+b^5\right)dx^3}{3(a+bx^3)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(-\frac{a^5}{4x^{12}}-\frac{5a^4b}{3x^9}-\frac{5a^3b^2}{x^6}-\frac{10a^2b^3}{x^3}+5ab^4\log\left(x^3\right)+b^5x^3\right)}{3(a+bx^3)}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^13,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-1/4*a^5/x^12 - (5*a^4*b)/(3*x^9) - (5*a
^3*b^2)/x^6 - (10*a^2*b^3)/x^3 + b^5*x^3 + 5*a*b^4*Log[x^3]))/(3*(a + b*x^
3))
```

3.76.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.76.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.33 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.32

method	result	size
pseudoelliptic	$-\frac{\left(-4b^{5}x^{15}-20\ln\left(b\,x^{3}\right)a\,b^{4}x^{12}-4a\,b^{4}x^{12}+40a^{2}b^{3}x^{9}+20a^{3}b^{2}x^{6}+\frac{20a^{4}b\,x^{3}}{3}+a^{5}\right)\operatorname{csgn}\left(b\,x^{3}+a\right)}{12x^{12}}$	81
default	$\frac{\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}\left(12b^{5}x^{15}+180b^{4}a\ln\left(x\right)x^{12}-120a^{2}b^{3}x^{9}-60a^{3}b^{2}x^{6}-20a^{4}b\,x^{3}-3a^{5}\right)}{36\left(b\,x^{3}+a\right)^{5}x^{12}}$	82
risch	$\frac{b^{5}x^{3}\sqrt{\left(b\,x^{3}+a\right)^{2}}}{3b\,x^{3}+3a}+\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(-\frac{10}{3}a^{2}b^{3}x^{9}-\frac{5}{3}a^{3}b^{2}x^{6}-\frac{5}{9}a^{4}b\,x^{3}-\frac{1}{12}a^{5}\right)}{\left(b\,x^{3}+a\right)x^{12}}+\frac{5a\,b^{4}\ln\left(x\right)\sqrt{\left(b\,x^{3}+a\right)^{2}}}{b\,x^{3}+a}$	119

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^13,x,method=_RETURNVERBOSE)
```

output
```
-1/12*(-4*b^5*x^15-20*ln(b*x^3)*a*b^4*x^12-4*a*b^4*x^12+40*a^2*b^3*x^9+20*
a^3*b^2*x^6+20/3*a^4*b*x^3+a^5)*csgn(b*x^3+a)/x^12
```

3.76.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.24

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{13}}\,dx = \frac{12\,b^5x^{15}+180\,ab^4x^{12}\log\left(x\right)-120\,a^2b^3x^9-60\,a^3b^2x^6-20\,a^4bx^3-3\,a^5}{36\,x^{12}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^13,x, algorithm="fricas")
```

output
```
1/36*(12*b^5*x^15 + 180*a*b^4*x^12*log(x) - 120*a^2*b^3*x^9 - 60*a^3*b^2*x
^6 - 20*a^4*b*x^3 - 3*a^5)/x^12
```

3.76.6 Sympy [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{13}}\,dx = \int \frac{\left((a+bx^3)^2\right)^{\frac{5}{2}}}{x^{13}}\,dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**13,x)
```

output
```
Integral(((a + b*x**3)**2)**(5/2)/x**13, x)
```

3.76.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.36

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{13}}\,dx = \frac{5\sqrt{b^2x^6+2\,abx^3+a^2}b^5x^3}{6\,a} + \frac{5}{3}(-1)^{2\,b^2x^3+2\,ab}\,ab^4\log\left(2\,b^2x^3+2\,ab\right) - \frac{5}{3}(-1)^{2\,abx^3+2\,a^2}\,ab^4\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right) + \frac{5\,(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}b^5x^3}{12\,a^3} + \frac{5}{2}\sqrt{b^2x^6+2\,abx^3+a^2}b^4 + \frac{35\,(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}b^4}{36\,a^2} + \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}b^4}{9\,a^4} - \frac{2\,(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}b^3}{9\,a^3x^3} - \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{7}{2}}b^2}{9\,a^4x^6} + \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{7}{2}}b}{36\,a^3x^9} - \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{7}{2}}}{12\,a^2x^{12}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^13,x, algorithm="maxima")
```

output
```
5/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^5*x^3/a + 5/3*(-1)^(2*b^2*x^3 + 2*a*
b)*a*b^4*log(2*b^2*x^3 + 2*a*b) - 5/3*(-1)^(2*a*b*x^3 + 2*a^2)*a*b^4*log(2
*a*b*x/abs(x) + 2*a^2/(x^2*abs(x))) + 5/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/
2)*b^5*x^3/a^3 + 5/2*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^4 + 35/36*(b^2*x^6
+ 2*a*b*x^3 + a^2)^(3/2)*b^4/a^2 + 1/9*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b
^4/a^4 - 2/9*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^3/(a^3*x^3) - 1/9*(b^2*x^
6 + 2*a*b*x^3 + a^2)^(7/2)*b^2/(a^4*x^6) + 1/36*(b^2*x^6 + 2*a*b*x^3 + a^2
)^(7/2)*b/(a^3*x^9) - 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)/(a^2*x^12)
```

3.76.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.50

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{13}}\,dx = \frac{1}{3}\,b^5x^3\mathrm{sgn}\left(bx^3+a\right)+5\,ab^4\log\left(|x|\right)\mathrm{sgn}\left(bx^3+a\right) - \frac{125\,ab^4x^{12}\mathrm{sgn}(bx^3+a)+120\,a^2b^3x^9\mathrm{sgn}(bx^3+a)+60\,a^3b^2x^6\mathrm{sgn}(bx^3+a)+20\,a^4bx^3\mathrm{sgn}(bx^3+a)+3\,a^5\mathrm{s}}{36\,x^{12}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^13,x, algorithm="giac")
```

output
```
1/3*b^5*x^3*sgn(b*x^3 + a) + 5*a*b^4*log(abs(x))*sgn(b*x^3 + a) - 1/36*(12
5*a*b^4*x^12*sgn(b*x^3 + a) + 120*a^2*b^3*x^9*sgn(b*x^3 + a) + 60*a^3*b^2*
x^6*sgn(b*x^3 + a) + 20*a^4*b*x^3*sgn(b*x^3 + a) + 3*a^5*sgn(b*x^3 + a))/x
^12
```

3.76.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{13}}\,dx = \int \frac{(a^2+2\,a\,b\,x^3+b^2\,x^6)^{5/2}}{x^{13}}\,dx$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^13,x)
```

output
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^13, x)
```

3.77 $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{14}}\,dx$

3.77.1 Optimal result

Integrand size = 26, antiderivative size = 253

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{14}}\,dx = -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}(a+bx^3)} - \frac{a^4b\sqrt{a^2+2abx^3+b^2x^6}}{2x^{10}(a+bx^3)} - \frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{7x^7(a+bx^3)} - \frac{5a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{2x^4(a+bx^3)} - \frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{x(a+bx^3)} + \frac{b^5x^2\sqrt{a^2+2abx^3+b^2x^6}}{2(a+bx^3)}$$

output

```
-1/13*a^5*((b*x^3+a)^2)^(1/2)/x^13/(b*x^3+a)-1/2*a^4*b*((b*x^3+a)^2)^(1/2)
/x^10/(b*x^3+a)-10/7*a^3*b^2*((b*x^3+a)^2)^(1/2)/x^7/(b*x^3+a)-5/2*a^2*b^3
*((b*x^3+a)^2)^(1/2)/x^4/(b*x^3+a)-5*a*b^4*((b*x^3+a)^2)^(1/2)/x/(b*x^3+a)
+1/2*b^5*x^2*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

3.77.2 Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{14}}\,dx = -\frac{\sqrt{(a+bx^3)^2}\left(14a^5+91a^4bx^3+260a^3b^2x^6+455a^2b^3x^9+910ab^4x^{12}-91b^5x^{15}\right)}{182x^{13}(a+bx^3)}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^14,x]
```

output
```
-1/182*(Sqrt[(a + b*x^3)^2]*(14*a^5 + 91*a^4*b*x^3 + 260*a^3*b^2*x^6 + 455
*a^2*b^3*x^9 + 910*a*b^4*x^12 - 91*b^5*x^15))/(x^13*(a + b*x^3))
```

3.77.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.39, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{14}}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b^5\left(bx^3+a\right)^5}{x^{14}}dx}{b^5\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(bx^3+a\right)^5}{x^{14}}dx}{a+bx^3}$$

$$\Big\downarrow \text{ 802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^5}{x^{14}}+\frac{5ba^4}{x^{11}}+\frac{10b^2a^3}{x^8}+\frac{10b^3a^2}{x^5}+\frac{5b^4a}{x^2}+b^5x\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\left(-\frac{a^5}{13x^{13}}-\frac{a^4b}{2x^{10}}-\frac{10a^3b^2}{7x^7}-\frac{5a^2b^3}{2x^4}-\frac{5ab^4}{x}+\frac{b^5x^2}{2}\right)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^14,x]
```

output
```
((-1/13*a^5/x^13 - (a^4*b)/(2*x^10) - (10*a^3*b^2)/(7*x^7) - (5*a^2*b^3)/(
2*x^4) - (5*a*b^4)/x + (b^5*x^2)/2)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a +
b*x^3)
```

3.77.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.77.4 Maple [A] (verified)

Time = 17.54 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.32

method	result	size
gosper	$-\frac{\left(-91 b^{5} x^{15}+910 a\,b^{4} x^{12}+455 a^{2} b^{3} x^{9}+260 a^{3} b^{2} x^{6}+91 a^{4} b\,x^{3}+14 a^{5}\right) \left(\left(b\,x^{3}+a \right)^{2}\right)^{\frac{5}{2}}}{182 \left(b\,x^{3}+a \right)^{5} x^{13}}$	80
default	$-\frac{\left(-91 b^{5} x^{15}+910 a\,b^{4} x^{12}+455 a^{2} b^{3} x^{9}+260 a^{3} b^{2} x^{6}+91 a^{4} b\,x^{3}+14 a^{5}\right) \left(\left(b\,x^{3}+a \right)^{2}\right)^{\frac{5}{2}}}{182 \left(b\,x^{3}+a \right)^{5} x^{13}}$	80
risch	$\frac{b^{5} x^{2} \sqrt{\left(b\,x^{3}+a \right)^{2}}}{2 b\,x^{3}+2 a}+\frac{\sqrt{\left(b\,x^{3}+a \right)^{2}}\, \left(-5 a\,b^{4} x^{12}-\frac{5}{2} a^{2} b^{3} x^{9}-\frac{10}{7} a^{3} b^{2} x^{6}-\frac{1}{2} a^{4} b\,x^{3}-\frac{1}{13} a^{5}\right)}{\left(b\,x^{3}+a \right) x^{13}}$	100

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^14,x,method=_RETURNVERBOSE)
```

output
```
-1/182*(-91*b^5*x^15+910*a*b^4*x^12+455*a^2*b^3*x^9+260*a^3*b^2*x^6+91*a^4
*b*x^3+14*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5/x^13
```

3.77.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.23

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{14}}\,dx = \frac{91\,b^5x^{15}-910\,ab^4x^{12}-455\,a^2b^3x^9-260\,a^3b^2x^6-91\,a^4bx^3-14\,a^5}{182\,x^{13}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^14,x, algorithm="fricas")
```

output
```
1/182*(91*b^5*x^15 - 910*a*b^4*x^12 - 455*a^2*b^3*x^9 - 260*a^3*b^2*x^6 -
91*a^4*b*x^3 - 14*a^5)/x^13
```

3.77.6 Sympy [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{14}}\,dx = \int \frac{\left((a+bx^3)^2\right)^{\frac{5}{2}}}{x^{14}}\,dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**14,x)
```

output
```
Integral(((a + b*x**3)**2)**(5/2)/x**14, x)
```

3.77.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.23

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{14}}\,dx = \frac{91\,b^5x^{15}-910\,ab^4x^{12}-455\,a^2b^3x^9-260\,a^3b^2x^6-91\,a^4bx^3-14\,a^5}{182\,x^{13}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^14,x, algorithm="maxima")
```

output
```
1/182*(91*b^5*x^15 - 910*a*b^4*x^12 - 455*a^2*b^3*x^9 - 260*a^3*b^2*x^6 -
91*a^4*b*x^3 - 14*a^5)/x^13
```

3.77.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.43

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{14}}\,dx = \frac{1}{2}\,b^5x^2\mathrm{sgn}\left(bx^3+a\right) - \frac{910\,ab^4x^{12}\mathrm{sgn}(bx^3+a)+455\,a^2b^3x^9\mathrm{sgn}(bx^3+a)+260\,a^3b^2x^6\mathrm{sgn}(bx^3+a)+91\,a^4bx^3\mathrm{sgn}(bx^3+a)+14\,a}{182\,x^{13}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^14,x, algorithm="giac")
```

output
```
1/2*b^5*x^2*sgn(b*x^3 + a) - 1/182*(910*a*b^4*x^12*sgn(b*x^3 + a) + 455*a^
2*b^3*x^9*sgn(b*x^3 + a) + 260*a^3*b^2*x^6*sgn(b*x^3 + a) + 91*a^4*b*x^3*s
gn(b*x^3 + a) + 14*a^5*sgn(b*x^3 + a))/x^13
```

3.77.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{14}}\,dx = \int \frac{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}}{x^{14}}\,dx$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^14,x)
```

output
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^14, x)
```

3.78 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{15}}\,dx$

3.78.1 Optimal result

Integrand size = 26, antiderivative size = 248

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{15}}\,dx = -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{14x^{14}\left(a+bx^3\right)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}\left(a+bx^3\right)} - \frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{4x^8\left(a+bx^3\right)} - \frac{2a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{x^5\left(a+bx^3\right)} - \frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)} + \frac{b^5x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

output
```
-1/14*a^5*((b*x^3+a)^2)^(1/2)/x^14/(b*x^3+a)-5/11*a^4*b*((b*x^3+a)^2)^(1/2
)/x^11/(b*x^3+a)-5/4*a^3*b^2*((b*x^3+a)^2)^(1/2)/x^8/(b*x^3+a)-2*a^2*b^3*(
(b*x^3+a)^2)^(1/2)/x^5/(b*x^3+a)-5/2*a*b^4*((b*x^3+a)^2)^(1/2)/x^2/(b*x^3+
a)+b^5*x*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

3.78.2 Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{15}}\,dx = -\frac{\sqrt{\left(a+bx^3\right)^2}\left(22a^5+140a^4bx^3+385a^3b^2x^6+616a^2b^3x^9+770ab^4x^{12}-308b^5x^{15}\right)}{308x^{14}\left(a+bx^3\right)}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^15,x]
```

output
```
-1/308*(Sqrt[(a + b*x^3)^2]*(22*a^5 + 140*a^4*b*x^3 + 385*a^3*b^2*x^6 + 61
6*a^2*b^3*x^9 + 770*a*b^4*x^12 - 308*b^5*x^15))/(x^14*(a + b*x^3))
```

3.78.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.38, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{15}}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b^5\left(bx^3+a\right)^5}{x^{15}}dx}{b^5\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(bx^3+a\right)^5}{x^{15}}dx}{a+bx^3}$$

$$\Big\downarrow \text{ 802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^5}{x^{15}}+\frac{5ba^4}{x^{12}}+\frac{10b^2a^3}{x^9}+\frac{10b^3a^2}{x^6}+\frac{5b^4a}{x^3}+b^5\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\left(-\frac{a^5}{14x^{14}}-\frac{5a^4b}{11x^{11}}-\frac{5a^3b^2}{4x^8}-\frac{2a^2b^3}{x^5}-\frac{5ab^4}{2x^2}+b^5x\right)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^15,x]
```

output
```
((-1/14*a^5/x^14 - (5*a^4*b)/(11*x^11) - (5*a^3*b^2)/(4*x^8) - (2*a^2*b^3)
/x^5 - (5*a*b^4)/(2*x^2) + b^5*x)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*
x^3)
```

3.78.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.78.4 Maple [A] (verified)

Time = 20.37 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.32

method	result	size
gosper	$-\frac{\left(-308 b^{5} x^{15}+770 a \,b^{4} x^{12}+616 a^{2} b^{3} x^{9}+385 a^{3} b^{2} x^{6}+140 a^{4} b \,x^{3}+22 a^{5}\right) \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{5}{2}}}{308 x^{14} \left(b \,x^{3}+a \right)^{5}}$	80
default	$-\frac{\left(-308 b^{5} x^{15}+770 a \,b^{4} x^{12}+616 a^{2} b^{3} x^{9}+385 a^{3} b^{2} x^{6}+140 a^{4} b \,x^{3}+22 a^{5}\right) \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{5}{2}}}{308 x^{14} \left(b \,x^{3}+a \right)^{5}}$	80
risch	$\frac{b^{5} x \sqrt{\left(b \,x^{3}+a \right)^{2}}}{b \,x^{3}+a}+\frac{\sqrt{\left(b \,x^{3}+a \right)^{2}}\, \left(-\frac{5}{2} a \,b^{4} x^{12}-2 a^{2} b^{3} x^{9}-\frac{5}{4} a^{3} b^{2} x^{6}-\frac{5}{11} a^{4} b \,x^{3}-\frac{1}{14} a^{5}\right)}{\left(b \,x^{3}+a \right) x^{14}}$	97

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^15,x,method=_RETURNVERBOSE)
```

output
```
-1/308*(-308*b^5*x^15+770*a*b^4*x^12+616*a^2*b^3*x^9+385*a^3*b^2*x^6+140*a
^4*b*x^3+22*a^5)*((b*x^3+a)^2)^(5/2)/x^14/(b*x^3+a)^5
```

3.78.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.24

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{15}} dx = \frac{308 b^5x^{15} - 770 ab^4x^{12} - 616 a^2b^3x^9 - 385 a^3b^2x^6 - 140 a^4bx^3 - 22 a^5}{308 x^{14}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^15,x, algorithm="fricas")
```

output
```
1/308*(308*b^5*x^15 - 770*a*b^4*x^12 - 616*a^2*b^3*x^9 - 385*a^3*b^2*x^6 -
 140*a^4*b*x^3 - 22*a^5)/x^14
```

3.78.6 Sympy [F]

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{15}} dx = \int \frac{\left((a + bx^3)^2\right)^{\frac{5}{2}}}{x^{15}} dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**15,x)
```

output
```
Integral(((a + b*x**3)**2)**(5/2)/x**15, x)
```

3.78.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.24

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{15}} dx = \frac{308 b^5x^{15} - 770 ab^4x^{12} - 616 a^2b^3x^9 - 385 a^3b^2x^6 - 140 a^4bx^3 - 22 a^5}{308 x^{14}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^15,x, algorithm="maxima")
```

output
```
1/308*(308*b^5*x^15 - 770*a*b^4*x^12 - 616*a^2*b^3*x^9 - 385*a^3*b^2*x^6 -
 140*a^4*b*x^3 - 22*a^5)/x^14
```

3.78.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.42

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{15}}\,dx = b^5x\,\mathrm{sgn}(bx^3+a) - \frac{770\,ab^4x^{12}\mathrm{sgn}(bx^3+a)+616\,a^2b^3x^9\mathrm{sgn}(bx^3+a)+385\,a^3b^2x^6\mathrm{sgn}(bx^3+a)+140\,a^4bx^3\mathrm{sgn}(bx^3+a)+22}{308\,x^{14}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^15,x, algorithm="giac")
```

output
```
b^5*x*sgn(b*x^3 + a) - 1/308*(770*a*b^4*x^12*sgn(b*x^3 + a) + 616*a^2*b^3*
x^9*sgn(b*x^3 + a) + 385*a^3*b^2*x^6*sgn(b*x^3 + a) + 140*a^4*b*x^3*sgn(b*
x^3 + a) + 22*a^5*sgn(b*x^3 + a))/x^14
```

3.78.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{15}}\,dx = \int \frac{(a^2+2\,a\,b\,x^3+b^2\,x^6)^{5/2}}{x^{15}}\,dx$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^15,x)
```

output
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^15, x)
```

3.79 $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{16}}\,dx$

3.79.1 Optimal result

Integrand size = 26, antiderivative size = 251

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{16}}\,dx = -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{15x^{15}(a+bx^3)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{12x^{12}(a+bx^3)} - \frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{9x^9(a+bx^3)} - \frac{5a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{3x^6(a+bx^3)} - \frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{3x^3(a+bx^3)} + \frac{b^5\sqrt{a^2+2abx^3+b^2x^6}\log(x)}{a+bx^3}$$

output
```
-1/15*a^5*((b*x^3+a)^2)^(1/2)/x^15/(b*x^3+a)-5/12*a^4*b*((b*x^3+a)^2)^(1/2
)/x^12/(b*x^3+a)-10/9*a^3*b^2*((b*x^3+a)^2)^(1/2)/x^9/(b*x^3+a)-5/3*a^2*b^
3*((b*x^3+a)^2)^(1/2)/x^6/(b*x^3+a)-5/3*a*b^4*((b*x^3+a)^2)^(1/2)/x^3/(b*x
^3+a)+b^5*ln(x)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

3.79.2 Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.07

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{16}}\,dx = \frac{1}{360}\left(-\frac{\sqrt{(a+bx^3)^2}(12a^4+63a^3bx^3+137a^2b^2x^6+163ab^3x^9+137b^4x^{12})}{x^{15}} + \frac{\sqrt{a^2}(12a^4+75a^3bx^3+200a^2b^2x^6+300ab^3x^9+300b^4x^{12})}{x^{15}} - 120b^5\operatorname{arctanh}\left(\frac{bx^3}{\sqrt{a^2}-\sqrt{(a+bx^3)^2}}\right) - \frac{120\sqrt{a^2}b^5\log(x^3)}{a} + \frac{60\sqrt{a^2}b^5\log\left(a\left(\sqrt{a^2}-bx^3-\sqrt{(a+bx^3)^2}\right)\right)}{a} + \frac{60\sqrt{a^2}b^5\log\left(a\left(\sqrt{a^2}+bx^3-\sqrt{(a+bx^3)^2}\right)\right)}{a}\right)$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^16,x]
```

output
```
(-((Sqrt[(a + b*x^3)^2]*(12*a^4 + 63*a^3*b*x^3 + 137*a^2*b^2*x^6 + 163*a*b
^3*x^9 + 137*b^4*x^12))/x^15) + (Sqrt[a^2]*(12*a^4 + 75*a^3*b*x^3 + 200*a^
2*b^2*x^6 + 300*a*b^3*x^9 + 300*b^4*x^12))/x^15 - 120*b^5*ArcTanh[(b*x^3)/
(Sqrt[a^2] - Sqrt[(a + b*x^3)^2])] - (120*Sqrt[a^2]*b^5*Log[x^3])/a + (60*
Sqrt[a^2]*b^5*Log[a*(Sqrt[a^2] - b*x^3 - Sqrt[(a + b*x^3)^2])])/a + (60*Sq
rt[a^2]*b^5*Log[a*(Sqrt[a^2] + b*x^3 - Sqrt[(a + b*x^3)^2])])/a)/360
```

3.79.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.39, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1384, 27, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{16}}\,dx$$

$$\Big\downarrow \; 1384$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b^5(bx^3+a)^5}{x^{16}}dx}{b^5(a+bx^3)}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(bx^3+a)^5}{x^{16}}dx}{a+bx^3}$$

$$\Big\downarrow \; 798$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(bx^3+a)^5}{x^{18}}dx^3}{3(a+bx^3)}$$

$$\Big\downarrow \; 49$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^5}{x^{18}}+\frac{5ba^4}{x^{15}}+\frac{10b^2a^3}{x^{12}}+\frac{10b^3a^2}{x^9}+\frac{5b^4a}{x^6}+\frac{b^5}{x^3}\right)dx^3}{3(a+bx^3)}$$

$$\Big\downarrow \; 2009$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(-\frac{a^5}{5x^{15}}-\frac{5a^4b}{4x^{12}}-\frac{10a^3b^2}{3x^9}-\frac{5a^2b^3}{x^6}-\frac{5ab^4}{x^3}+b^5\log(x^3)\right)}{3(a+bx^3)}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^16,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-1/5*a^5/x^15 - (5*a^4*b)/(4*x^12) - (10
*a^3*b^2)/(3*x^9) - (5*a^2*b^3)/x^6 - (5*a*b^4)/x^3 + b^5*Log[x^3]))/(3*(a
 + b*x^3))
```

3.79.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.79.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.63 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.29

method	result	size
pseudoelliptic	$-\frac{\operatorname{csgn}\left(b\,x^{3}+a\right)\left(-5\ln\left(b\,x^{3}\right)b^{5}x^{15}+a\left(25b^{4}x^{12}+25a\,b^{3}x^{9}+\frac{50}{3}a^{2}b^{2}x^{6}+\frac{25}{4}a^{3}b\,x^{3}+a^{4}\right)\right)}{15x^{15}}$	72
default	$\frac{\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}\left(180b^{5}\ln\left(x\right)x^{15}-300a\,b^{4}x^{12}-300a^{2}b^{3}x^{9}-200a^{3}b^{2}x^{6}-75a^{4}b\,x^{3}-12a^{5}\right)}{180\left(b\,x^{3}+a\right)^{5}x^{15}}$	82
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(-\frac{1}{15}a^{5}-\frac{5}{12}a^{4}b\,x^{3}-\frac{10}{9}a^{3}b^{2}x^{6}-\frac{5}{3}a^{2}b^{3}x^{9}-\frac{5}{3}a\,b^{4}x^{12}\right)}{\left(b\,x^{3}+a\right)x^{15}}+\frac{b^{5}\ln\left(x\right)\sqrt{\left(b\,x^{3}+a\right)^{2}}}{b\,x^{3}+a}$	98

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^16,x,method=_RETURNVERBOSE)
```

output
```
-1/15*csgn(b*x^3+a)*(-5*ln(b*x^3)*b^5*x^15+a*(25*b^4*x^12+25*a*b^3*x^9+50/
3*a^2*b^2*x^6+25/4*a^3*b*x^3+a^4))/x^15
```

3.79.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.24

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{16}}\,dx = \frac{180\,b^5x^{15}\log(x) - 300\,ab^4x^{12} - 300\,a^2b^3x^9 - 200\,a^3b^2x^6 - 75\,a^4bx^3 - 12\,a^5}{180\,x^{15}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^16,x, algorithm="fricas")
```

output
```
1/180*(180*b^5*x^15*log(x) - 300*a*b^4*x^12 - 300*a^2*b^3*x^9 - 200*a^3*b^
2*x^6 - 75*a^4*b*x^3 - 12*a^5)/x^15
```

3.79.6 Sympy [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{16}}\,dx = \int \frac{\left((a+bx^3)^2\right)^{\frac{5}{2}}}{x^{16}}\,dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**16,x)
```

output
```
Integral(((a + b*x**3)**2)**(5/2)/x**16, x)
```

3.79.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 374 vs. 2(175) = 350.

Time = 0.23 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.49

$$\begin{aligned}\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{16}}\,dx &= \frac{\sqrt{b^2x^6+2\,abx^3+a^2}b^6x^3}{6\,a^2} \\ &+ \frac{1}{3}\,(-1)^{2\,b^2x^3+2\,ab}\,b^5\log\left(2\,b^2x^3+2\,ab\right) - \frac{1}{3}\,(-1)^{2\,abx^3+2\,a^2}\,b^5\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right) \\ &+ \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}b^6x^3}{12\,a^4} + \frac{\sqrt{b^2x^6+2\,abx^3+a^2}b^5}{2\,a} + \frac{7\,(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}b^5}{36\,a^3} \\ &- \frac{2\,(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}b^5}{45\,a^5} - \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}b^4}{9\,a^4x^3} + \frac{2\,(b^2x^6+2\,abx^3+a^2)^{\frac{7}{2}}b^3}{45\,a^5x^6} \\ &- \frac{11\,(b^2x^6+2\,abx^3+a^2)^{\frac{7}{2}}b^2}{180\,a^4x^9} + \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{7}{2}}b}{20\,a^3x^{12}} - \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{7}{2}}}{15\,a^2x^{15}}\end{aligned}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^16,x, algorithm="maxima")
```

output
```
1/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^6*x^3/a^2 + 1/3*(-1)^(2*b^2*x^3 + 2*
a*b)*b^5*log(2*b^2*x^3 + 2*a*b) - 1/3*(-1)^(2*a*b*x^3 + 2*a^2)*b^5*log(2*a
*b*x/abs(x) + 2*a^2/(x^2*abs(x))) + 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)
*b^6*x^3/a^4 + 1/2*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^5/a + 7/36*(b^2*x^6 +
 2*a*b*x^3 + a^2)^(3/2)*b^5/a^3 - 2/45*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b
^5/a^5 - 1/9*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^4/(a^4*x^3) + 2/45*(b^2*x
^6 + 2*a*b*x^3 + a^2)^(7/2)*b^3/(a^5*x^6) - 11/180*(b^2*x^6 + 2*a*b*x^3 +
a^2)^(7/2)*b^2/(a^4*x^9) + 1/20*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b/(a^3*x
^12) - 1/15*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)/(a^2*x^15)
```

3.79.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.49

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{16}}\,dx = b^5\log\left(|x|\right)\operatorname{sgn}\left(bx^3+a\right) - \frac{137\,b^5x^{15}\operatorname{sgn}(bx^3+a)+300\,ab^4x^{12}\operatorname{sgn}(bx^3+a)+300\,a^2b^3x^9\operatorname{sgn}(bx^3+a)+200\,a^3b^2x^6\operatorname{sgn}(bx^3+a)+75}{180\,x^{15}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^16,x, algorithm="giac")
```

output
```
b^5*log(abs(x))*sgn(b*x^3 + a) - 1/180*(137*b^5*x^15*sgn(b*x^3 + a) + 300*
a*b^4*x^12*sgn(b*x^3 + a) + 300*a^2*b^3*x^9*sgn(b*x^3 + a) + 200*a^3*b^2*x
^6*sgn(b*x^3 + a) + 75*a^4*b*x^3*sgn(b*x^3 + a) + 12*a^5*sgn(b*x^3 + a))/x
^15
```

3.79.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{16}}\,dx = \int \frac{(a^2+2\,a\,b\,x^3+b^2\,x^6)^{5/2}}{x^{16}}\,dx$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^16,x)
```

output
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^16, x)
```

3.80 $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{17}}\,dx$

3.80.1 Optimal result

Integrand size = 26, antiderivative size = 251

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{17}}\,dx = -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{16x^{16}(a+bx^3)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}(a+bx^3)} - \frac{a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{x^{10}(a+bx^3)} - \frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{7x^7(a+bx^3)} - \frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{4x^4(a+bx^3)} - \frac{b^5\sqrt{a^2+2abx^3+b^2x^6}}{x(a+bx^3)}$$

output
```
-1/16*a^5*((b*x^3+a)^2)^(1/2)/x^16/(b*x^3+a)-5/13*a^4*b*((b*x^3+a)^2)^(1/2
)/x^13/(b*x^3+a)-a^3*b^2*((b*x^3+a)^2)^(1/2)/x^10/(b*x^3+a)-10/7*a^2*b^3*(
(b*x^3+a)^2)^(1/2)/x^7/(b*x^3+a)-5/4*a*b^4*((b*x^3+a)^2)^(1/2)/x^4/(b*x^3+
a)-b^5*((b*x^3+a)^2)^(1/2)/x/(b*x^3+a)
```

3.80.2 Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{17}}\,dx = -\frac{\sqrt{(a+bx^3)^2}(91a^5+560a^4bx^3+1456a^3b^2x^6+2080a^2b^3x^9+1820ab^4x^{12}+1456b^5x^{15})}{1456x^{16}(a+bx^3)}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^17,x]
```

output
```
-1/1456*(Sqrt[(a + b*x^3)^2]*(91*a^5 + 560*a^4*b*x^3 + 1456*a^3*b^2*x^6 +
2080*a^2*b^3*x^9 + 1820*a*b^4*x^12 + 1456*b^5*x^15))/(x^16*(a + b*x^3))
```

3.80.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.39, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{17}}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b^5\left(bx^3+a\right)^5}{x^{17}}dx}{b^5\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(bx^3+a\right)^5}{x^{17}}dx}{a+bx^3}$$

$$\Big\downarrow \text{ 802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^5}{x^{17}}+\frac{5ba^4}{x^{14}}+\frac{10b^2a^3}{x^{11}}+\frac{10b^3a^2}{x^8}+\frac{5b^4a}{x^5}+\frac{b^5}{x^2}\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\left(-\frac{a^5}{16x^{16}}-\frac{5a^4b}{13x^{13}}-\frac{a^3b^2}{x^{10}}-\frac{10a^2b^3}{7x^7}-\frac{5ab^4}{4x^4}-\frac{b^5}{x}\right)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^17,x]
```

output
```
((-1/16*a^5/x^16 - (5*a^4*b)/(13*x^13) - (a^3*b^2)/x^10 - (10*a^2*b^3)/(7*
x^7) - (5*a*b^4)/(4*x^4) - b^5/x)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*
x^3)
```

3.80.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.80.4 Maple [A] (verified)

Time = 25.49 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.31

method	result	size
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(-\frac{1}{16}a^{5}-\frac{5}{13}a^{4}b\,x^{3}-a^{3}b^{2}x^{6}-\frac{10}{7}a^{2}b^{3}x^{9}-\frac{5}{4}a\,b^{4}x^{12}-b^{5}x^{15}\right)}{\left(b\,x^{3}+a\right)x^{16}}$	79
gosper	$-\frac{\left(1456b^{5}x^{15}+1820a\,b^{4}x^{12}+2080a^{2}b^{3}x^{9}+1456a^{3}b^{2}x^{6}+560a^{4}b\,x^{3}+91a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{1456x^{16}\left(b\,x^{3}+a\right)^{5}}$	80
default	$-\frac{\left(1456b^{5}x^{15}+1820a\,b^{4}x^{12}+2080a^{2}b^{3}x^{9}+1456a^{3}b^{2}x^{6}+560a^{4}b\,x^{3}+91a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{1456x^{16}\left(b\,x^{3}+a\right)^{5}}$	80

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^17,x,method=_RETURNVERBOSE)
```

output
```
((b*x^3+a)^2)^(1/2)/(b*x^3+a)*(-1/16*a^5-5/13*a^4*b*x^3-a^3*b^2*x^6-10/7*a
^2*b^3*x^9-5/4*a*b^4*x^12-b^5*x^15)/x^16
```

3.80.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.24

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{17}}\,dx = -\frac{1456\,b^5x^{15}+1820\,ab^4x^{12}+2080\,a^2b^3x^9+1456\,a^3b^2x^6+560\,a^4bx^3+91\,a^5}{1456\,x^{16}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^17,x, algorithm="fricas")
```

output
```
-1/1456*(1456*b^5*x^15 + 1820*a*b^4*x^12 + 2080*a^2*b^3*x^9 + 1456*a^3*b^2
*x^6 + 560*a^4*b*x^3 + 91*a^5)/x^16
```

3.80.6 Sympy [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{17}}\,dx = \int \frac{\left((a+bx^3)^2\right)^{\frac{5}{2}}}{x^{17}}\,dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**17,x)
```

output
```
Integral(((a + b*x**3)**2)**(5/2)/x**17, x)
```

3.80.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.24

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{17}}\,dx = -\frac{1456\,b^5x^{15}+1820\,ab^4x^{12}+2080\,a^2b^3x^9+1456\,a^3b^2x^6+560\,a^4bx^3+91\,a^5}{1456\,x^{16}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^17,x, algorithm="maxima")
```

output
```
-1/1456*(1456*b^5*x^15 + 1820*a*b^4*x^12 + 2080*a^2*b^3*x^9 + 1456*a^3*b^2
*x^6 + 560*a^4*b*x^3 + 91*a^5)/x^16
```

3.80.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.43

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{17}}\,dx =$$
$$-\frac{1456\,b^5x^{15}\mathrm{sgn}\left(bx^3+a\right)+1820\,ab^4x^{12}\mathrm{sgn}\left(bx^3+a\right)+2080\,a^2b^3x^9\mathrm{sgn}\left(bx^3+a\right)+1456\,a^3b^2x^6\mathrm{sgn}\left(bx^3+a\right)+}{1456\,x^{16}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^17,x, algorithm="giac")
```

output
```
-1/1456*(1456*b^5*x^15*sgn(b*x^3 + a) + 1820*a*b^4*x^12*sgn(b*x^3 + a) + 2
080*a^2*b^3*x^9*sgn(b*x^3 + a) + 1456*a^3*b^2*x^6*sgn(b*x^3 + a) + 560*a^4
*b*x^3*sgn(b*x^3 + a) + 91*a^5*sgn(b*x^3 + a))/x^16
```

3.80.9 Mupad [B] (verification not implemented)

Time = 8.35 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.92

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{17}}\,dx = -\frac{a^5\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{16\,x^{16}\,\left(b\,x^3+a\right)}-\frac{b^5\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{x\,\left(b\,x^3+a\right)}$$
$$-\frac{5\,a\,b^4\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{4\,x^4\,\left(b\,x^3+a\right)}-\frac{5\,a^4\,b\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{13\,x^{13}\,\left(b\,x^3+a\right)}$$
$$-\frac{10\,a^2\,b^3\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{7\,x^7\,\left(b\,x^3+a\right)}-\frac{a^3\,b^2\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{x^{10}\,\left(b\,x^3+a\right)}$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^17,x)
```

output
```
- (a^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(16*x^16*(a + b*x^3)) - (b^5*(a^
2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(x*(a + b*x^3)) - (5*a*b^4*(a^2 + b^2*x^6
+ 2*a*b*x^3)^(1/2))/(4*x^4*(a + b*x^3)) - (5*a^4*b*(a^2 + b^2*x^6 + 2*a*b*
x^3)^(1/2))/(13*x^13*(a + b*x^3)) - (10*a^2*b^3*(a^2 + b^2*x^6 + 2*a*b*x^3
)^(1/2))/(7*x^7*(a + b*x^3)) - (a^3*b^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))
/(x^10*(a + b*x^3))
```

3.81 $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{18}}\,dx$

3.81.1 Optimal result

Integrand size = 26, antiderivative size = 253

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{18}}\,dx = -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{17x^{17}(a+bx^3)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{14x^{14}(a+bx^3)} - \frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}(a+bx^3)} - \frac{5a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{4x^8(a+bx^3)} - \frac{ab^4\sqrt{a^2+2abx^3+b^2x^6}}{x^5(a+bx^3)} - \frac{b^5\sqrt{a^2+2abx^3+b^2x^6}}{2x^2(a+bx^3)}$$

output
```
-1/17*a^5*((b*x^3+a)^2)^(1/2)/x^17/(b*x^3+a)-5/14*a^4*b*((b*x^3+a)^2)^(1/2
)/x^14/(b*x^3+a)-10/11*a^3*b^2*((b*x^3+a)^2)^(1/2)/x^11/(b*x^3+a)-5/4*a^2*
b^3*((b*x^3+a)^2)^(1/2)/x^8/(b*x^3+a)-a*b^4*((b*x^3+a)^2)^(1/2)/x^5/(b*x^3
+a)-1/2*b^5*((b*x^3+a)^2)^(1/2)/x^2/(b*x^3+a)
```

3.81.2 Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{18}}\,dx = -\frac{\sqrt{(a+bx^3)^2}(308a^5+1870a^4bx^3+4760a^3b^2x^6+6545a^2b^3x^9+5236ab^4x^{12}+2618b^5x^{15})}{5236x^{17}(a+bx^3)}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^18,x]
```

output
```
-1/5236*(Sqrt[(a + b*x^3)^2]*(308*a^5 + 1870*a^4*b*x^3 + 4760*a^3*b^2*x^6
+ 6545*a^2*b^3*x^9 + 5236*a*b^4*x^12 + 2618*b^5*x^15))/(x^17*(a + b*x^3))
```

3.81.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.39, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{18}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b^5\left(bx^3+a\right)^5}{x^{18}}dx}{b^5\left(a+bx^3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(bx^3+a\right)^5}{x^{18}}dx}{a+bx^3}$$

$$\Big\downarrow \text{802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^5}{x^{18}}+\frac{5ba^4}{x^{15}}+\frac{10b^2a^3}{x^{12}}+\frac{10b^3a^2}{x^9}+\frac{5b^4a}{x^6}+\frac{b^5}{x^3}\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(-\frac{a^5}{17x^{17}}-\frac{5a^4b}{14x^{14}}-\frac{10a^3b^2}{11x^{11}}-\frac{5a^2b^3}{4x^8}-\frac{ab^4}{x^5}-\frac{b^5}{2x^2}\right)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^18,x]
```

output
```
((-1/17*a^5/x^17 - (5*a^4*b)/(14*x^14) - (10*a^3*b^2)/(11*x^11) - (5*a^2*b
^3)/(4*x^8) - (a*b^4)/x^5 - b^5/(2*x^2))*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/
(a + b*x^3)
```

3.81.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.81.4 Maple [A] (verified)

Time = 27.05 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.31

method	result	size
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(-\frac{1}{17}a^{5}-\frac{5}{14}a^{4}b\,x^{3}-\frac{10}{11}a^{3}b^{2}x^{6}-\frac{5}{4}a^{2}b^{3}x^{9}-a\,b^{4}x^{12}-\frac{1}{2}b^{5}x^{15}\right)}{\left(b\,x^{3}+a\right)x^{17}}$	79
gosper	$-\frac{\left(2618b^{5}x^{15}+5236a\,b^{4}x^{12}+6545a^{2}b^{3}x^{9}+4760a^{3}b^{2}x^{6}+1870a^{4}b\,x^{3}+308a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{5236x^{17}\left(b\,x^{3}+a\right)^{5}}$	80
default	$-\frac{\left(2618b^{5}x^{15}+5236a\,b^{4}x^{12}+6545a^{2}b^{3}x^{9}+4760a^{3}b^{2}x^{6}+1870a^{4}b\,x^{3}+308a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{5236x^{17}\left(b\,x^{3}+a\right)^{5}}$	80

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^18,x,method=_RETURNVERBOSE)
```

output
```
((b*x^3+a)^2)^(1/2)/(b*x^3+a)*(-1/17*a^5-5/14*a^4*b*x^3-10/11*a^3*b^2*x^6-
5/4*a^2*b^3*x^9-a*b^4*x^12-1/2*b^5*x^15)/x^17
```

3.81.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.23

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{18}}\,dx = -\frac{2618\,b^5x^{15}+5236\,ab^4x^{12}+6545\,a^2b^3x^9+4760\,a^3b^2x^6+1870\,a^4bx^3+308\,a^5}{5236\,x^{17}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^18,x, algorithm="fricas")
```

output
```
-1/5236*(2618*b^5*x^15 + 5236*a*b^4*x^12 + 6545*a^2*b^3*x^9 + 4760*a^3*b^2
*x^6 + 1870*a^4*b*x^3 + 308*a^5)/x^17
```

3.81.6 Sympy [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{18}}\,dx = \int \frac{\left((a+bx^3)^2\right)^{\frac{5}{2}}}{x^{18}}\,dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**18,x)
```

output
```
Integral(((a + b*x**3)**2)**(5/2)/x**18, x)
```

3.81.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.23

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{18}}\,dx = -\frac{2618\,b^5x^{15}+5236\,ab^4x^{12}+6545\,a^2b^3x^9+4760\,a^3b^2x^6+1870\,a^4bx^3+308\,a^5}{5236\,x^{17}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^18,x, algorithm="maxima")
```

output
```
-1/5236*(2618*b^5*x^15 + 5236*a*b^4*x^12 + 6545*a^2*b^3*x^9 + 4760*a^3*b^2
*x^6 + 1870*a^4*b*x^3 + 308*a^5)/x^17
```

3.81.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.42

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{18}}\,dx =$$
$$-\frac{2618\,b^5x^{15}\mathrm{sgn}(bx^3+a)+5236\,ab^4x^{12}\mathrm{sgn}(bx^3+a)+6545\,a^2b^3x^9\mathrm{sgn}(bx^3+a)+4760\,a^3b^2x^6\mathrm{sgn}(bx^3+a)+}{5236\,x^{17}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^18,x, algorithm="giac")
```

output
```
-1/5236*(2618*b^5*x^15*sgn(b*x^3 + a) + 5236*a*b^4*x^12*sgn(b*x^3 + a) + 6
545*a^2*b^3*x^9*sgn(b*x^3 + a) + 4760*a^3*b^2*x^6*sgn(b*x^3 + a) + 1870*a^
4*b*x^3*sgn(b*x^3 + a) + 308*a^5*sgn(b*x^3 + a))/x^17
```

3.81.9 Mupad [B] (verification not implemented)

Time = 8.36 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.91

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{18}}\,dx = -\frac{a^5\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{17\,x^{17}\,\left(b\,x^3+a\right)}-\frac{b^5\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{2\,x^2\,\left(b\,x^3+a\right)} - \frac{a\,b^4\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{x^5\,\left(b\,x^3+a\right)}-\frac{5\,a^4\,b\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{14\,x^{14}\,\left(b\,x^3+a\right)} - \frac{5\,a^2\,b^3\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{4\,x^8\,\left(b\,x^3+a\right)}-\frac{10\,a^3\,b^2\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{11\,x^{11}\,\left(b\,x^3+a\right)}$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^18,x)
```

output
```
- (a^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(17*x^17*(a + b*x^3)) - (b^5*(a^
2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(2*x^2*(a + b*x^3)) - (a*b^4*(a^2 + b^2*x^
6 + 2*a*b*x^3)^(1/2))/(x^5*(a + b*x^3)) - (5*a^4*b*(a^2 + b^2*x^6 + 2*a*b*
x^3)^(1/2))/(14*x^14*(a + b*x^3)) - (5*a^2*b^3*(a^2 + b^2*x^6 + 2*a*b*x^3)
^(1/2))/(4*x^8*(a + b*x^3)) - (10*a^3*b^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2
))/(11*x^11*(a + b*x^3))
```

3.82 $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{19}}\,dx$

3.82.1 Optimal result

Integrand size = 26, antiderivative size = 41

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{19}}\,dx = -\frac{(a+bx^3)^5\sqrt{a^2+2abx^3+b^2x^6}}{18ax^{18}}$$

output
```
-1/18*(b*x^3+a)^5*((b*x^3+a)^2)^(1/2)/a/x^18
```

3.82.2 Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.98

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{19}}\,dx = -\frac{\sqrt{(a+bx^3)^2}(a^5+6a^4bx^3+15a^3b^2x^6+20a^2b^3x^9+15ab^4x^{12}+6b^5x^{15})}{18x^{18}(a+bx^3)}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^19,x]
```

output
```
-1/18*(Sqrt[(a + b*x^3)^2]*(a^5 + 6*a^4*b*x^3 + 15*a^3*b^2*x^6 + 20*a^2*b^
3*x^9 + 15*a*b^4*x^12 + 6*b^5*x^15))/(x^18*(a + b*x^3))
```

3.82.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1384, 27, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{19}}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b^5\left(bx^3+a\right)^5}{x^{19}}dx}{b^5\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(bx^3+a\right)^5}{x^{19}}dx}{a+bx^3}$$

$$\Big\downarrow \text{ 796}$$

$$-\frac{\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{18ax^{18}}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^19,x]
```

output
```
-1/18*((a + b*x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a*x^18)
```

3.82.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

3.82.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 1.20 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.41

method	result	size
pseudoelliptic	$-\frac{\operatorname{csgn}\left(b\,x^{3}+a\right)\left(2 b\,x^{3}+a\right)\left(b^{2} x^{6}+a b\,x^{3}+a^{2}\right)\left(3 b^{2} x^{6}+3 a b\,x^{3}+a^{2}\right)}{18 x^{18}}$	58
gosper	$-\frac{\left(6 b^{5} x^{15}+15 a\,b^{4} x^{12}+20 a^{2} b^{3} x^{9}+15 a^{3} b^{2} x^{6}+6 a^{4} b\,x^{3}+a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{18 x^{18} \left(b\,x^{3}+a\right)^{5}}$	78
default	$-\frac{\left(6 b^{5} x^{15}+15 a\,b^{4} x^{12}+20 a^{2} b^{3} x^{9}+15 a^{3} b^{2} x^{6}+6 a^{4} b\,x^{3}+a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{18 x^{18} \left(b\,x^{3}+a\right)^{5}}$	78
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(-\frac{1}{18} a^{5}-\frac{1}{3} a^{4} b\,x^{3}-\frac{5}{6} a^{3} b^{2} x^{6}-\frac{10}{9} a^{2} b^{3} x^{9}-\frac{5}{6} a\,b^{4} x^{12}-\frac{1}{3} b^{5} x^{15}\right)}{\left(b\,x^{3}+a\right) x^{18}}$	79

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^19,x,method=_RETURNVERBOSE)
```

output
```
-1/18*csgn(b*x^3+a)*(2*b*x^3+a)*(b^2*x^6+a*b*x^3+a^2)*(3*b^2*x^6+3*a*b*x^3
+a^2)/x^18
```

3.82.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(28) = 56.

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.39

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{19}}\,dx=-\frac{6\,b^5x^{15}+15\,ab^4x^{12}+20\,a^2b^3x^9+15\,a^3b^2x^6+6\,a^4bx^3+a^5}{18\,x^{18}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^19,x, algorithm="fricas")
```

output
```
-1/18*(6*b^5*x^15 + 15*a*b^4*x^12 + 20*a^2*b^3*x^9 + 15*a^3*b^2*x^6 + 6*a^
4*b*x^3 + a^5)/x^18
```

3.82.6 Sympy [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{19}}\,dx = \int \frac{\left((a+bx^3)^2\right)^{\frac{5}{2}}}{x^{19}}\,dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**19,x)
```

output
```
Integral(((a + b*x**3)**2)**(5/2)/x**19, x)
```

3.82.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. 2(28) = 56.

Time = 0.22 (sec) , antiderivative size = 210, normalized size of antiderivative = 5.12

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{19}}\,dx = \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}b^6}{18\,a^6} + \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}b^5}{18\,a^5x^3} - \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{7}{2}}b^4}{18\,a^6x^6} + \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{7}{2}}b^3}{18\,a^5x^9} - \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{7}{2}}b^2}{18\,a^4x^{12}} + \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{7}{2}}b}{18\,a^3x^{15}} - \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{7}{2}}}{18\,a^2x^{18}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^19,x, algorithm="maxima")
```

output
```
1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^6/a^6 + 1/18*(b^2*x^6 + 2*a*b*x^3
 + a^2)^(5/2)*b^5/(a^5*x^3) - 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b^4/(
a^6*x^6) + 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b^3/(a^5*x^9) - 1/18*(b^
2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b^2/(a^4*x^12) + 1/18*(b^2*x^6 + 2*a*b*x^3
+ a^2)^(7/2)*b/(a^3*x^15) - 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)/(a^2*x^
18)
```

3.82.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. 2(28) = 56.

Time = 0.30 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.59

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{19}}\,dx = -\frac{6\,b^5x^{15}\mathrm{sgn}(bx^3+a)+15\,ab^4x^{12}\mathrm{sgn}(bx^3+a)+20\,a^2b^3x^9\mathrm{sgn}(bx^3+a)+15\,a^3b^2x^6\mathrm{sgn}(bx^3+a)+6\,a^4bx^3\mathrm{sg}}{18\,x^{18}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^19,x, algorithm="giac")
```

output
```
-1/18*(6*b^5*x^15*sgn(b*x^3 + a) + 15*a*b^4*x^12*sgn(b*x^3 + a) + 20*a^2*b
^3*x^9*sgn(b*x^3 + a) + 15*a^3*b^2*x^6*sgn(b*x^3 + a) + 6*a^4*b*x^3*sgn(b*
x^3 + a) + a^5*sgn(b*x^3 + a))/x^18
```

3.82.9 Mupad [B] (verification not implemented)

Time = 8.37 (sec) , antiderivative size = 231, normalized size of antiderivative = 5.63

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{19}}\,dx = -\frac{a^5\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{18\,x^{18}\,(b\,x^3+a)} - \frac{b^5\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{3\,x^3\,(b\,x^3+a)} - \frac{5\,a\,b^4\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{6\,x^6\,(b\,x^3+a)} - \frac{a^4\,b\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{3\,x^{15}\,(b\,x^3+a)} - \frac{10\,a^2\,b^3\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{9\,x^9\,(b\,x^3+a)} - \frac{5\,a^3\,b^2\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{6\,x^{12}\,(b\,x^3+a)}$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^19,x)
```

output
```
- (a^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(18*x^18*(a + b*x^3)) - (b^5*(a^
2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(3*x^3*(a + b*x^3)) - (5*a*b^4*(a^2 + b^2*
x^6 + 2*a*b*x^3)^(1/2))/(6*x^6*(a + b*x^3)) - (a^4*b*(a^2 + b^2*x^6 + 2*a*
b*x^3)^(1/2))/(3*x^15*(a + b*x^3)) - (10*a^2*b^3*(a^2 + b^2*x^6 + 2*a*b*x^
3)^(1/2))/(9*x^9*(a + b*x^3)) - (5*a^3*b^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/
2))/(6*x^12*(a + b*x^3))
```

3.83 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{20}}\,dx$

3.83.1 Optimal result

Integrand size = 26, antiderivative size = 253

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{20}}\,dx=-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{19x^{19}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{16x^{16}\left(a+bx^3\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}\left(a+bx^3\right)}-\frac{a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{x^{10}\left(a+bx^3\right)}-\frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{b^5\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}$$

output
```
-1/19*a^5*((b*x^3+a)^2)^(1/2)/x^19/(b*x^3+a)-5/16*a^4*b*((b*x^3+a)^2)^(1/2
)/x^16/(b*x^3+a)-10/13*a^3*b^2*((b*x^3+a)^2)^(1/2)/x^13/(b*x^3+a)-a^2*b^3*
((b*x^3+a)^2)^(1/2)/x^10/(b*x^3+a)-5/7*a*b^4*((b*x^3+a)^2)^(1/2)/x^7/(b*x^
3+a)-1/4*b^5*((b*x^3+a)^2)^(1/2)/x^4/(b*x^3+a)
```

3.83.2 Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{20}}\,dx=-\frac{\sqrt{\left(a+bx^3\right)^2}\left(1456a^5+8645a^4bx^3+21280a^3b^2x^6+27664a^2b^3x^9+19760ab^4x^{12}+6916b^5x^{15}\right)}{27664x^{19}\left(a+bx^3\right)}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^20,x]
```

output
```
-1/27664*(Sqrt[(a + b*x^3)^2]*(1456*a^5 + 8645*a^4*b*x^3 + 21280*a^3*b^2*x
^6 + 27664*a^2*b^3*x^9 + 19760*a*b^4*x^12 + 6916*b^5*x^15))/(x^19*(a + b*x
^3))
```

3.83.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.39, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{20}}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b^5\left(bx^3+a\right)^5}{x^{20}}dx}{b^5\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(bx^3+a\right)^5}{x^{20}}dx}{a+bx^3}$$

$$\Big\downarrow \text{ 802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^5}{x^{20}}+\frac{5ba^4}{x^{17}}+\frac{10b^2a^3}{x^{14}}+\frac{10b^3a^2}{x^{11}}+\frac{5b^4a}{x^8}+\frac{b^5}{x^5}\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\left(-\frac{a^5}{19x^{19}}-\frac{5a^4b}{16x^{16}}-\frac{10a^3b^2}{13x^{13}}-\frac{a^2b^3}{x^{10}}-\frac{5ab^4}{7x^7}-\frac{b^5}{4x^4}\right)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^20,x]
```

output
```
((-1/19*a^5/x^19 - (5*a^4*b)/(16*x^16) - (10*a^3*b^2)/(13*x^13) - (a^2*b^3
)/x^10 - (5*a*b^4)/(7*x^7) - b^5/(4*x^4))*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
/(a + b*x^3)
```

3.83.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.83.4 Maple [A] (verified)

Time = 33.18 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.31

method	result	size
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(-\frac{1}{19}a^{5}-\frac{5}{16}a^{4}b\,x^{3}-\frac{10}{13}a^{3}b^{2}x^{6}-a^{2}b^{3}x^{9}-\frac{5}{7}a\,b^{4}x^{12}-\frac{1}{4}b^{5}x^{15}\right)}{\left(b\,x^{3}+a\right)x^{19}}$	79
gosper	$-\frac{\left(6916b^{5}x^{15}+19760a\,b^{4}x^{12}+27664a^{2}b^{3}x^{9}+21280a^{3}b^{2}x^{6}+8645a^{4}b\,x^{3}+1456a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{27664x^{19}\left(b\,x^{3}+a\right)^{5}}$	80
default	$-\frac{\left(6916b^{5}x^{15}+19760a\,b^{4}x^{12}+27664a^{2}b^{3}x^{9}+21280a^{3}b^{2}x^{6}+8645a^{4}b\,x^{3}+1456a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{27664x^{19}\left(b\,x^{3}+a\right)^{5}}$	80

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^20,x,method=_RETURNVERBOSE)
```

output
```
((b*x^3+a)^2)^(1/2)/(b*x^3+a)*(-1/19*a^5-5/16*a^4*b*x^3-10/13*a^3*b^2*x^6-
a^2*b^3*x^9-5/7*a*b^4*x^12-1/4*b^5*x^15)/x^19
```

3.83.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.23

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{20}} \, dx = -\frac{6916 \, b^5 x^{15} + 19760 \, ab^4 x^{12} + 27664 \, a^2 b^3 x^9 + 21280 \, a^3 b^2 x^6 + 8645 \, a^4 b x^3 + 1456 \, a^5}{27664 \, x^{19}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^20,x, algorithm="fricas")
```

output
```
-1/27664*(6916*b^5*x^15 + 19760*a*b^4*x^12 + 27664*a^2*b^3*x^9 + 21280*a^3
*b^2*x^6 + 8645*a^4*b*x^3 + 1456*a^5)/x^19
```

3.83.6 Sympy [F]

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{20}} \, dx = \int \frac{\left((a + bx^3)^2\right)^{\frac{5}{2}}}{x^{20}} \, dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**20,x)
```

output
```
Integral(((a + b*x**3)**2)**(5/2)/x**20, x)
```

3.83.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.23

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{20}} \, dx = -\frac{6916 \, b^5 x^{15} + 19760 \, ab^4 x^{12} + 27664 \, a^2 b^3 x^9 + 21280 \, a^3 b^2 x^6 + 8645 \, a^4 b x^3 + 1456 \, a^5}{27664 \, x^{19}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^20,x, algorithm="maxima")
```

output
```
-1/27664*(6916*b^5*x^15 + 19760*a*b^4*x^12 + 27664*a^2*b^3*x^9 + 21280*a^3
*b^2*x^6 + 8645*a^4*b*x^3 + 1456*a^5)/x^19
```

3.83.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.42

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{20}}\,dx =$$
$$-\frac{6916\,b^5x^{15}\text{sgn}(bx^3+a) + 19760\,ab^4x^{12}\text{sgn}(bx^3+a) + 27664\,a^2b^3x^9\text{sgn}(bx^3+a) + 21280\,a^3b^2x^6\text{sgn}(bx^3+}{27664\,x^{19}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^20,x, algorithm="giac")
```

output
```
-1/27664*(6916*b^5*x^15*sgn(b*x^3 + a) + 19760*a*b^4*x^12*sgn(b*x^3 + a) +
 27664*a^2*b^3*x^9*sgn(b*x^3 + a) + 21280*a^3*b^2*x^6*sgn(b*x^3 + a) + 864
5*a^4*b*x^3*sgn(b*x^3 + a) + 1456*a^5*sgn(b*x^3 + a))/x^19
```

3.83.9 Mupad [B] (verification not implemented)

Time = 8.35 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.91

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{20}}\,dx = -\frac{a^5\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{19\,x^{19}\,\left(b\,x^3+a\right)} - \frac{b^5\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{4\,x^4\,\left(b\,x^3+a\right)} - \frac{5\,a\,b^4\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{7\,x^7\,\left(b\,x^3+a\right)} - \frac{5\,a^4\,b\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{16\,x^{16}\,\left(b\,x^3+a\right)} - \frac{a^2\,b^3\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{x^{10}\,\left(b\,x^3+a\right)} - \frac{10\,a^3\,b^2\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{13\,x^{13}\,\left(b\,x^3+a\right)}$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^20,x)
```

output
```
- (a^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(19*x^19*(a + b*x^3)) - (b^5*(a^
2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(4*x^4*(a + b*x^3)) - (5*a*b^4*(a^2 + b^2*
x^6 + 2*a*b*x^3)^(1/2))/(7*x^7*(a + b*x^3)) - (5*a^4*b*(a^2 + b^2*x^6 + 2*
a*b*x^3)^(1/2))/(16*x^16*(a + b*x^3)) - (a^2*b^3*(a^2 + b^2*x^6 + 2*a*b*x^
3)^(1/2))/(x^10*(a + b*x^3)) - (10*a^3*b^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/
2))/(13*x^13*(a + b*x^3))
```

3.83. $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{20}}\,dx$

3.84 $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{21}}\,dx$

3.84.1 Optimal result

Integrand size = 26, antiderivative size = 255

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{21}}\,dx = -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{20x^{20}(a+bx^3)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{17x^{17}(a+bx^3)} - \frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{7x^{14}(a+bx^3)} - \frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}(a+bx^3)} - \frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{8x^8(a+bx^3)} - \frac{b^5\sqrt{a^2+2abx^3+b^2x^6}}{5x^5(a+bx^3)}$$

output
```
-1/20*a^5*((b*x^3+a)^2)^(1/2)/x^20/(b*x^3+a)-5/17*a^4*b*((b*x^3+a)^2)^(1/2
)/x^17/(b*x^3+a)-5/7*a^3*b^2*((b*x^3+a)^2)^(1/2)/x^14/(b*x^3+a)-10/11*a^2*
b^3*((b*x^3+a)^2)^(1/2)/x^11/(b*x^3+a)-5/8*a*b^4*((b*x^3+a)^2)^(1/2)/x^8/(
b*x^3+a)-1/5*b^5*((b*x^3+a)^2)^(1/2)/x^5/(b*x^3+a)
```

3.84.2 Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{21}}\,dx = -\frac{\sqrt{(a+bx^3)^2}(2618a^5+15400a^4bx^3+37400a^3b^2x^6+47600a^2b^3x^9+32725ab^4x^{12}+10472b^5x^{15})}{52360x^{20}(a+bx^3)}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^21,x]
```

output
```
-1/52360*(Sqrt[(a + b*x^3)^2]*(2618*a^5 + 15400*a^4*b*x^3 + 37400*a^3*b^2*
x^6 + 47600*a^2*b^3*x^9 + 32725*a*b^4*x^12 + 10472*b^5*x^15))/(x^20*(a + b
*x^3))
```

3.84.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.40, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{21}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b^5\left(bx^3+a\right)^5}{x^{21}}dx}{b^5\left(a+bx^3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(bx^3+a\right)^5}{x^{21}}dx}{a+bx^3}$$

$$\Big\downarrow \text{802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^5}{x^{21}}+\frac{5ba^4}{x^{18}}+\frac{10b^2a^3}{x^{15}}+\frac{10b^3a^2}{x^{12}}+\frac{5b^4a}{x^9}+\frac{b^5}{x^6}\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(-\frac{a^5}{20x^{20}}-\frac{5a^4b}{17x^{17}}-\frac{5a^3b^2}{7x^{14}}-\frac{10a^2b^3}{11x^{11}}-\frac{5ab^4}{8x^8}-\frac{b^5}{5x^5}\right)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^21,x]
```

output
```
((-1/20*a^5/x^20 - (5*a^4*b)/(17*x^17) - (5*a^3*b^2)/(7*x^14) - (10*a^2*b^
3)/(11*x^11) - (5*a*b^4)/(8*x^8) - b^5/(5*x^5))*Sqrt[a^2 + 2*a*b*x^3 + b^2
*x^6])/(a + b*x^3)
```

3.84.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.84.4 Maple [A] (verified)

Time = 38.25 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.31

method	result	size
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(-\frac{1}{20}a^{5}-\frac{5}{17}a^{4}b\,x^{3}-\frac{5}{7}a^{3}b^{2}x^{6}-\frac{10}{11}a^{2}b^{3}x^{9}-\frac{5}{8}a\,b^{4}x^{12}-\frac{1}{5}b^{5}x^{15}\right)}{\left(b\,x^{3}+a\right)x^{20}}$	79
gosper	$-\frac{\left(10472b^{5}x^{15}+32725a\,b^{4}x^{12}+47600a^{2}b^{3}x^{9}+37400a^{3}b^{2}x^{6}+15400a^{4}b\,x^{3}+2618a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{52360x^{20}\left(b\,x^{3}+a\right)^{5}}$	80
default	$-\frac{\left(10472b^{5}x^{15}+32725a\,b^{4}x^{12}+47600a^{2}b^{3}x^{9}+37400a^{3}b^{2}x^{6}+15400a^{4}b\,x^{3}+2618a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{52360x^{20}\left(b\,x^{3}+a\right)^{5}}$	80

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^21,x,method=_RETURNVERBOSE)
```

output
```
((b*x^3+a)^2)^(1/2)/(b*x^3+a)*(-1/20*a^5-5/17*a^4*b*x^3-5/7*a^3*b^2*x^6-10
/11*a^2*b^3*x^9-5/8*a*b^4*x^12-1/5*b^5*x^15)/x^20
```

3.84.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.23

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{21}}\,dx = -\frac{10472\,b^5x^{15}+32725\,ab^4x^{12}+47600\,a^2b^3x^9+37400\,a^3b^2x^6+15400\,a^4bx^3+2618\,a^5}{52360\,x^{20}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^21,x, algorithm="fricas")
```

output
```
-1/52360*(10472*b^5*x^15 + 32725*a*b^4*x^12 + 47600*a^2*b^3*x^9 + 37400*a^
3*b^2*x^6 + 15400*a^4*b*x^3 + 2618*a^5)/x^20
```

3.84.6 Sympy [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{21}}\,dx = \int \frac{\left((a+bx^3)^2\right)^{\frac{5}{2}}}{x^{21}}\,dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**21,x)
```

output
```
Integral(((a + b*x**3)**2)**(5/2)/x**21, x)
```

3.84.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.23

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{21}}\,dx = -\frac{10472\,b^5x^{15}+32725\,ab^4x^{12}+47600\,a^2b^3x^9+37400\,a^3b^2x^6+15400\,a^4bx^3+2618\,a^5}{52360\,x^{20}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^21,x, algorithm="maxima")
```

output
```
-1/52360*(10472*b^5*x^15 + 32725*a*b^4*x^12 + 47600*a^2*b^3*x^9 + 37400*a^
3*b^2*x^6 + 15400*a^4*b*x^3 + 2618*a^5)/x^20
```

3.84.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.42

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{21}}\,dx = -\frac{10472\,b^5x^{15}\mathrm{sgn}(bx^3+a)+32725\,ab^4x^{12}\mathrm{sgn}(bx^3+a)+47600\,a^2b^3x^9\mathrm{sgn}(bx^3+a)+37400\,a^3b^2x^6\mathrm{sgn}(bx^3}{52360\,x^{20}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^21,x, algorithm="giac")
```

output
```
-1/52360*(10472*b^5*x^15*sgn(b*x^3 + a) + 32725*a*b^4*x^12*sgn(b*x^3 + a)
+ 47600*a^2*b^3*x^9*sgn(b*x^3 + a) + 37400*a^3*b^2*x^6*sgn(b*x^3 + a) + 15
400*a^4*b*x^3*sgn(b*x^3 + a) + 2618*a^5*sgn(b*x^3 + a))/x^20
```

3.84.9 Mupad [B] (verification not implemented)

Time = 8.45 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.91

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{21}}\,dx = -\frac{a^5\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{20\,x^{20}\,\left(b\,x^3+a\right)}-\frac{b^5\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{5\,x^5\,\left(b\,x^3+a\right)} -\frac{5\,a\,b^4\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{8\,x^8\,\left(b\,x^3+a\right)}-\frac{5\,a^4\,b\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{17\,x^{17}\,\left(b\,x^3+a\right)} -\frac{10\,a^2\,b^3\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{11\,x^{11}\,\left(b\,x^3+a\right)}-\frac{5\,a^3\,b^2\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{7\,x^{14}\,\left(b\,x^3+a\right)}$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^21,x)
```

output
```
- (a^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(20*x^20*(a + b*x^3)) - (b^5*(a^
2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(5*x^5*(a + b*x^3)) - (5*a*b^4*(a^2 + b^2*
x^6 + 2*a*b*x^3)^(1/2))/(8*x^8*(a + b*x^3)) - (5*a^4*b*(a^2 + b^2*x^6 + 2*
a*b*x^3)^(1/2))/(17*x^17*(a + b*x^3)) - (10*a^2*b^3*(a^2 + b^2*x^6 + 2*a*b
*x^3)^(1/2))/(11*x^11*(a + b*x^3)) - (5*a^3*b^2*(a^2 + b^2*x^6 + 2*a*b*x^3
)^(1/2))/(7*x^14*(a + b*x^3))
```

3.84. $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{21}}\,dx$

3.85 $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{22}}\,dx$

3.85.1 Optimal result

Integrand size = 26, antiderivative size = 84

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{22}}\,dx = -\frac{(a+bx^3)^5\sqrt{a^2+2abx^3+b^2x^6}}{21ax^{21}} + \frac{b(a+bx^3)^5\sqrt{a^2+2abx^3+b^2x^6}}{126a^2x^{18}}$$

output
```
-1/21*(b*x^3+a)^5*((b*x^3+a)^2)^(1/2)/a/x^21+1/126*b*(b*x^3+a)^5*((b*x^3+a
)^2)^(1/2)/a^2/x^18
```

3.85.2 Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.99

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{22}}\,dx = -\frac{\sqrt{(a+bx^3)^2}(6a^5+35a^4bx^3+84a^3b^2x^6+105a^2b^3x^9+70ab^4x^{12}+21b^5x^{15})}{126x^{21}(a+bx^3)}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^22,x]
```

output
```
-1/126*(Sqrt[(a + b*x^3)^2]*(6*a^5 + 35*a^4*b*x^3 + 84*a^3*b^2*x^6 + 105*a
^2*b^3*x^9 + 70*a*b^4*x^12 + 21*b^5*x^15))/(x^21*(a + b*x^3))
```

3.85.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.89, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1384, 27, 798, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{22}} \, dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6} \int \frac{b^5\left(bx^3+a\right)^5}{x^{22}}dx}{b^5\left(a+bx^3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6} \int \frac{\left(bx^3+a\right)^5}{x^{22}}dx}{a+bx^3}$$

$$\Big\downarrow \text{798}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6} \int \frac{\left(bx^3+a\right)^5}{x^{24}}dx^3}{3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{55}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(-\frac{b\int \frac{\left(bx^3+a\right)^5}{x^{21}}dx^3}{7a}-\frac{\left(a+bx^3\right)^6}{7ax^{21}}\right)}{3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{48}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(\frac{b\left(a+bx^3\right)^6}{42a^2x^{18}}-\frac{\left(a+bx^3\right)^6}{7ax^{21}}\right)}{3\left(a+bx^3\right)}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^22,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-1/7*(a + b*x^3)^6/(a*x^21) + (b*(a + b*
x^3)^6)/(42*a^2*x^18)))/(3*(a + b*x^3))
```

3.85.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

3.85. $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{22}} \, dx$

3.85.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 2.32 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.81

method	result	size
pseudoelliptic	$-\frac{\operatorname{csgn}\left(b\,x^{3}+a\right)\left(21 b^{5} x^{15}+70 a\,b^{4} x^{12}+105 a^{2} b^{3} x^{9}+84 a^{3} b^{2} x^{6}+35 a^{4} b\,x^{3}+6 a^{5}\right)}{126 x^{21}}$	68
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(-\frac{1}{21} a^{5}-\frac{5}{18} a^{4} b\,x^{3}-\frac{2}{3} a^{3} b^{2} x^{6}-\frac{5}{6} a^{2} b^{3} x^{9}-\frac{5}{9} a\,b^{4} x^{12}-\frac{1}{6} b^{5} x^{15}\right)}{\left(b\,x^{3}+a\right) x^{21}}$	79
gosper	$-\frac{\left(21 b^{5} x^{15}+70 a\,b^{4} x^{12}+105 a^{2} b^{3} x^{9}+84 a^{3} b^{2} x^{6}+35 a^{4} b\,x^{3}+6 a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{126 x^{21} \left(b\,x^{3}+a\right)^{5}}$	80
default	$-\frac{\left(21 b^{5} x^{15}+70 a\,b^{4} x^{12}+105 a^{2} b^{3} x^{9}+84 a^{3} b^{2} x^{6}+35 a^{4} b\,x^{3}+6 a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{126 x^{21} \left(b\,x^{3}+a\right)^{5}}$	80

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^22,x,method=_RETURNVERBOSE)
```

output
```
-1/126*csgn(b*x^3+a)*(21*b^5*x^15+70*a*b^4*x^12+105*a^2*b^3*x^9+84*a^3*b^2
*x^6+35*a^4*b*x^3+6*a^5)/x^21
```

3.85.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.70

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{22}}\,dx = -\frac{21\,b^5x^{15}+70\,ab^4x^{12}+105\,a^2b^3x^9+84\,a^3b^2x^6+35\,a^4bx^3+6\,a^5}{126\,x^{21}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^22,x, algorithm="fricas")
```

output
```
-1/126*(21*b^5*x^15 + 70*a*b^4*x^12 + 105*a^2*b^3*x^9 + 84*a^3*b^2*x^6 + 3
5*a^4*b*x^3 + 6*a^5)/x^21
```

3.85.6 Sympy [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{22}}\,dx=\int \frac{\left((a+bx^3)^2\right)^{\frac{5}{2}}}{x^{22}}\,dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**22,x)
```

output
```
Integral(((a + b*x**3)**2)**(5/2)/x**22, x)
```

3.85.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. 2(58) = 116.

Time = 0.22 (sec) , antiderivative size = 241, normalized size of antiderivative = 2.87

$$\begin{aligned}\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{22}}\,dx=&-\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}b^7}{18\,a^7}-\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}b^6}{18\,a^6x^3}\\&+\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{7}{2}}b^5}{18\,a^7x^6}-\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{7}{2}}b^4}{18\,a^6x^9}+\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{7}{2}}b^3}{18\,a^5x^{12}}\\&-\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{7}{2}}b^2}{18\,a^4x^{15}}+\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{7}{2}}b}{18\,a^3x^{18}}-\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{7}{2}}}{21\,a^2x^{21}}\end{aligned}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^22,x, algorithm="maxima")
```

output
```
-1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^7/a^7 - 1/18*(b^2*x^6 + 2*a*b*x^
3 + a^2)^(5/2)*b^6/(a^6*x^3) + 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b^5/
(a^7*x^6) - 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b^4/(a^6*x^9) + 1/18*(b
^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b^3/(a^5*x^12) - 1/18*(b^2*x^6 + 2*a*b*x^3
 + a^2)^(7/2)*b^2/(a^4*x^15) + 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b/(a
^3*x^18) - 1/21*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)/(a^2*x^21)
```

3.85.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.27

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{22}}\,dx = -\frac{21\,b^5x^{15}\mathrm{sgn}(bx^3+a)+70\,ab^4x^{12}\mathrm{sgn}(bx^3+a)+105\,a^2b^3x^9\mathrm{sgn}(bx^3+a)+84\,a^3b^2x^6\mathrm{sgn}(bx^3+a)+35\,a^4b}{126\,x^{21}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^22,x, algorithm="giac")
```

output
```
-1/126*(21*b^5*x^15*sgn(b*x^3 + a) + 70*a*b^4*x^12*sgn(b*x^3 + a) + 105*a^
2*b^3*x^9*sgn(b*x^3 + a) + 84*a^3*b^2*x^6*sgn(b*x^3 + a) + 35*a^4*b*x^3*sg
n(b*x^3 + a) + 6*a^5*sgn(b*x^3 + a))/x^21
```

3.85.9 Mupad [B] (verification not implemented)

Time = 8.36 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.75

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{22}}\,dx = -\frac{a^5\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{21\,x^{21}\,\left(b\,x^3+a\right)} - \frac{b^5\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{6\,x^6\,\left(b\,x^3+a\right)} - \frac{5\,a\,b^4\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{9\,x^9\,\left(b\,x^3+a\right)} - \frac{5\,a^4\,b\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{18\,x^{18}\,\left(b\,x^3+a\right)} - \frac{5\,a^2\,b^3\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{6\,x^{12}\,\left(b\,x^3+a\right)} - \frac{2\,a^3\,b^2\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{3\,x^{15}\,\left(b\,x^3+a\right)}$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^22,x)
```

output
```
- (a^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(21*x^21*(a + b*x^3)) - (b^5*(a^
2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(6*x^6*(a + b*x^3)) - (5*a*b^4*(a^2 + b^2*
x^6 + 2*a*b*x^3)^(1/2))/(9*x^9*(a + b*x^3)) - (5*a^4*b*(a^2 + b^2*x^6 + 2*
a*b*x^3)^(1/2))/(18*x^18*(a + b*x^3)) - (5*a^2*b^3*(a^2 + b^2*x^6 + 2*a*b*
x^3)^(1/2))/(6*x^12*(a + b*x^3)) - (2*a^3*b^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^
(1/2))/(3*x^15*(a + b*x^3))
```

3.86 $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{23}}\,dx$

3.86.1 Optimal result

Integrand size = 26, antiderivative size = 255

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{23}}\,dx = -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{22x^{22}(a+bx^3)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{19x^{19}(a+bx^3)} - \frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{8x^{16}(a+bx^3)} - \frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}(a+bx^3)} - \frac{ab^4\sqrt{a^2+2abx^3+b^2x^6}}{2x^{10}(a+bx^3)} - \frac{b^5\sqrt{a^2+2abx^3+b^2x^6}}{7x^7(a+bx^3)}$$

output
```
-1/22*a^5*((b*x^3+a)^2)^(1/2)/x^22/(b*x^3+a)-5/19*a^4*b*((b*x^3+a)^2)^(1/2
)/x^19/(b*x^3+a)-5/8*a^3*b^2*((b*x^3+a)^2)^(1/2)/x^16/(b*x^3+a)-10/13*a^2*
b^3*((b*x^3+a)^2)^(1/2)/x^13/(b*x^3+a)-1/2*a*b^4*((b*x^3+a)^2)^(1/2)/x^10/
(b*x^3+a)-1/7*b^5*((b*x^3+a)^2)^(1/2)/x^7/(b*x^3+a)
```

3.86.2 Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{23}}\,dx = -\frac{\sqrt{(a+bx^3)^2}(6916a^5+40040a^4bx^3+95095a^3b^2x^6+117040a^2b^3x^9+76076ab^4x^{12}+21736b^5x^{15})}{152152x^{22}(a+bx^3)}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^23,x]
```

output
```
-1/152152*(Sqrt[(a + b*x^3)^2]*(6916*a^5 + 40040*a^4*b*x^3 + 95095*a^3*b^2
*x^6 + 117040*a^2*b^3*x^9 + 76076*a*b^4*x^12 + 21736*b^5*x^15))/(x^22*(a +
 b*x^3))
```

3.86.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.40, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{23}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b^5\left(bx^3+a\right)^5}{x^{23}}dx}{b^5\left(a+bx^3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(bx^3+a\right)^5}{x^{23}}dx}{a+bx^3}$$

$$\Big\downarrow \text{802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^5}{x^{23}}+\frac{5ba^4}{x^{20}}+\frac{10b^2a^3}{x^{17}}+\frac{10b^3a^2}{x^{14}}+\frac{5b^4a}{x^{11}}+\frac{b^5}{x^8}\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(-\frac{a^5}{22x^{22}}-\frac{5a^4b}{19x^{19}}-\frac{5a^3b^2}{8x^{16}}-\frac{10a^2b^3}{13x^{13}}-\frac{ab^4}{2x^{10}}-\frac{b^5}{7x^7}\right)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^23,x]
```

output
```
((-1/22*a^5/x^22 - (5*a^4*b)/(19*x^19) - (5*a^3*b^2)/(8*x^16) - (10*a^2*b^
3)/(13*x^13) - (a*b^4)/(2*x^10) - b^5/(7*x^7))*Sqrt[a^2 + 2*a*b*x^3 + b^2*
x^6])/(a + b*x^3)
```

3.86.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.86.4 Maple [A] (verified)

Time = 46.67 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.31

method	result	size
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(-\frac{1}{22}a^{5}-\frac{5}{19}a^{4}b\,x^{3}-\frac{5}{8}a^{3}b^{2}x^{6}-\frac{10}{13}a^{2}b^{3}x^{9}-\frac{1}{2}a\,b^{4}x^{12}-\frac{1}{7}b^{5}x^{15}\right)}{\left(b\,x^{3}+a\right)x^{22}}$	79
gosper	$-\frac{\left(21736b^{5}x^{15}+76076a\,b^{4}x^{12}+117040a^{2}b^{3}x^{9}+95095a^{3}b^{2}x^{6}+40040a^{4}b\,x^{3}+6916a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{152152x^{22}\left(b\,x^{3}+a\right)^{5}}$	80
default	$-\frac{\left(21736b^{5}x^{15}+76076a\,b^{4}x^{12}+117040a^{2}b^{3}x^{9}+95095a^{3}b^{2}x^{6}+40040a^{4}b\,x^{3}+6916a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{152152x^{22}\left(b\,x^{3}+a\right)^{5}}$	80

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^23,x,method=_RETURNVERBOSE)
```

output
```
((b*x^3+a)^2)^(1/2)/(b*x^3+a)*(-1/22*a^5-5/19*a^4*b*x^3-5/8*a^3*b^2*x^6-10
/13*a^2*b^3*x^9-1/2*a*b^4*x^12-1/7*b^5*x^15)/x^22
```

3.86.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.23

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{23}}\,dx = -\frac{21736\,b^5x^{15}+76076\,ab^4x^{12}+117040\,a^2b^3x^9+95095\,a^3b^2x^6+40040\,a^4bx^3+6916\,a^5}{152152\,x^{22}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^23,x, algorithm="fricas")
```

output
```
-1/152152*(21736*b^5*x^15 + 76076*a*b^4*x^12 + 117040*a^2*b^3*x^9 + 95095*
a^3*b^2*x^6 + 40040*a^4*b*x^3 + 6916*a^5)/x^22
```

3.86.6 Sympy [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{23}}\,dx = \int \frac{\left((a+bx^3)^2\right)^{\frac{5}{2}}}{x^{23}}\,dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**23,x)
```

output
```
Integral(((a + b*x**3)**2)**(5/2)/x**23, x)
```

3.86.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.23

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{23}}\,dx = -\frac{21736\,b^5x^{15}+76076\,ab^4x^{12}+117040\,a^2b^3x^9+95095\,a^3b^2x^6+40040\,a^4bx^3+6916\,a^5}{152152\,x^{22}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^23,x, algorithm="maxima")
```

output
```
-1/152152*(21736*b^5*x^15 + 76076*a*b^4*x^12 + 117040*a^2*b^3*x^9 + 95095*
a^3*b^2*x^6 + 40040*a^4*b*x^3 + 6916*a^5)/x^22
```

3.86.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.42

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{23}}\,dx = -\frac{21736\,b^5x^{15}\mathrm{sgn}(bx^3+a)+76076\,ab^4x^{12}\mathrm{sgn}(bx^3+a)+117040\,a^2b^3x^9\mathrm{sgn}(bx^3+a)+95095\,a^3b^2x^6\mathrm{sgn}(bx^3}{152152\,x^{22}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^23,x, algorithm="giac")
```

output
```
-1/152152*(21736*b^5*x^15*sgn(b*x^3 + a) + 76076*a*b^4*x^12*sgn(b*x^3 + a)
 + 117040*a^2*b^3*x^9*sgn(b*x^3 + a) + 95095*a^3*b^2*x^6*sgn(b*x^3 + a) +
40040*a^4*b*x^3*sgn(b*x^3 + a) + 6916*a^5*sgn(b*x^3 + a))/x^22
```

3.86.9 Mupad [B] (verification not implemented)

Time = 8.34 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.91

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{23}}\,dx = -\frac{a^5\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{22\,x^{22}\,\left(b\,x^3+a\right)} - \frac{b^5\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{7\,x^7\,\left(b\,x^3+a\right)} - \frac{a\,b^4\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{2\,x^{10}\,\left(b\,x^3+a\right)} - \frac{5\,a^4\,b\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{19\,x^{19}\,\left(b\,x^3+a\right)} - \frac{10\,a^2\,b^3\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{13\,x^{13}\,\left(b\,x^3+a\right)} - \frac{5\,a^3\,b^2\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{8\,x^{16}\,\left(b\,x^3+a\right)}$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^23,x)
```

output
```
- (a^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(22*x^22*(a + b*x^3)) - (b^5*(a^
2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(7*x^7*(a + b*x^3)) - (a*b^4*(a^2 + b^2*x^
6 + 2*a*b*x^3)^(1/2))/(2*x^10*(a + b*x^3)) - (5*a^4*b*(a^2 + b^2*x^6 + 2*a
*b*x^3)^(1/2))/(19*x^19*(a + b*x^3)) - (10*a^2*b^3*(a^2 + b^2*x^6 + 2*a*b*
x^3)^(1/2))/(13*x^13*(a + b*x^3)) - (5*a^3*b^2*(a^2 + b^2*x^6 + 2*a*b*x^3)
^(1/2))/(8*x^16*(a + b*x^3))
```

3.86. $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{23}}\,dx$

3.87 $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{24}}\,dx$

3.87.1 Optimal result

Integrand size = 26, antiderivative size = 255

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{24}}\,dx = -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{23x^{23}(a+bx^3)} - \frac{a^4b\sqrt{a^2+2abx^3+b^2x^6}}{4x^{20}(a+bx^3)} - \frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{17x^{17}(a+bx^3)} - \frac{5a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{7x^{14}(a+bx^3)} - \frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}(a+bx^3)} - \frac{b^5\sqrt{a^2+2abx^3+b^2x^6}}{8x^8(a+bx^3)}$$

output
```
-1/23*a^5*((b*x^3+a)^2)^(1/2)/x^23/(b*x^3+a)-1/4*a^4*b*((b*x^3+a)^2)^(1/2)
/x^20/(b*x^3+a)-10/17*a^3*b^2*((b*x^3+a)^2)^(1/2)/x^17/(b*x^3+a)-5/7*a^2*b
^3*((b*x^3+a)^2)^(1/2)/x^14/(b*x^3+a)-5/11*a*b^4*((b*x^3+a)^2)^(1/2)/x^11/
(b*x^3+a)-1/8*b^5*((b*x^3+a)^2)^(1/2)/x^8/(b*x^3+a)
```

3.87.2 Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{24}}\,dx = -\frac{\sqrt{(a+bx^3)^2}(10472a^5+60214a^4bx^3+141680a^3b^2x^6+172040a^2b^3x^9+109480ab^4x^{12}+30107b^5x^{15})}{240856x^{23}(a+bx^3)}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^24,x]
```

output
```
-1/240856*(Sqrt[(a + b*x^3)^2]*(10472*a^5 + 60214*a^4*b*x^3 + 141680*a^3*b
^2*x^6 + 172040*a^2*b^3*x^9 + 109480*a*b^4*x^12 + 30107*b^5*x^15))/(x^23*(
a + b*x^3))
```

3.87.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.40, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{24}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b^5\left(bx^3+a\right)^5}{x^{24}}dx}{b^5\left(a+bx^3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(bx^3+a\right)^5}{x^{24}}dx}{a+bx^3}$$

$$\Big\downarrow \text{802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^5}{x^{24}}+\frac{5ba^4}{x^{21}}+\frac{10b^2a^3}{x^{18}}+\frac{10b^3a^2}{x^{15}}+\frac{5b^4a}{x^{12}}+\frac{b^5}{x^9}\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(-\frac{a^5}{23x^{23}}-\frac{a^4b}{4x^{20}}-\frac{10a^3b^2}{17x^{17}}-\frac{5a^2b^3}{7x^{14}}-\frac{5ab^4}{11x^{11}}-\frac{b^5}{8x^8}\right)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^24,x]
```

output
```
((-1/23*a^5/x^23 - (a^4*b)/(4*x^20) - (10*a^3*b^2)/(17*x^17) - (5*a^2*b^3)
/(7*x^14) - (5*a*b^4)/(11*x^11) - b^5/(8*x^8))*Sqrt[a^2 + 2*a*b*x^3 + b^2*
x^6])/(a + b*x^3)
```

3.87.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.87.4 Maple [A] (verified)

Time = 50.94 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.31

method	result	size
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(-\frac{1}{23}a^{5}-\frac{1}{4}a^{4}b\,x^{3}-\frac{10}{17}a^{3}b^{2}x^{6}-\frac{5}{7}a^{2}b^{3}x^{9}-\frac{5}{11}a\,b^{4}x^{12}-\frac{1}{8}b^{5}x^{15}\right)}{\left(b\,x^{3}+a\right)x^{23}}$	79
gosper	$-\frac{\left(30107b^{5}x^{15}+109480a\,b^{4}x^{12}+172040a^{2}b^{3}x^{9}+141680a^{3}b^{2}x^{6}+60214a^{4}b\,x^{3}+10472a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{240856x^{23}\left(b\,x^{3}+a\right)^{5}}$	80
default	$-\frac{\left(30107b^{5}x^{15}+109480a\,b^{4}x^{12}+172040a^{2}b^{3}x^{9}+141680a^{3}b^{2}x^{6}+60214a^{4}b\,x^{3}+10472a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{240856x^{23}\left(b\,x^{3}+a\right)^{5}}$	80

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^24,x,method=_RETURNVERBOSE)
```

output
```
((b*x^3+a)^2)^(1/2)/(b*x^3+a)*(-1/23*a^5-1/4*a^4*b*x^3-10/17*a^3*b^2*x^6-5
/7*a^2*b^3*x^9-5/11*a*b^4*x^12-1/8*b^5*x^15)/x^23
```

3.87.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.23

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{24}}\,dx = -\frac{30107\,b^5x^{15}+109480\,ab^4x^{12}+172040\,a^2b^3x^9+141680\,a^3b^2x^6+60214\,a^4bx^3+10472\,a^5}{240856\,x^{23}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^24,x, algorithm="fricas")
```

output
```
-1/240856*(30107*b^5*x^15 + 109480*a*b^4*x^12 + 172040*a^2*b^3*x^9 + 14168
0*a^3*b^2*x^6 + 60214*a^4*b*x^3 + 10472*a^5)/x^23
```

3.87.6 Sympy [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{24}}\,dx = \int \frac{\left((a+bx^3)^2\right)^{\frac{5}{2}}}{x^{24}}\,dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**24,x)
```

output
```
Integral(((a + b*x**3)**2)**(5/2)/x**24, x)
```

3.87.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.23

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{24}}\,dx = -\frac{30107\,b^5x^{15}+109480\,ab^4x^{12}+172040\,a^2b^3x^9+141680\,a^3b^2x^6+60214\,a^4bx^3+10472\,a^5}{240856\,x^{23}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^24,x, algorithm="maxima")
```

output
```
-1/240856*(30107*b^5*x^15 + 109480*a*b^4*x^12 + 172040*a^2*b^3*x^9 + 14168
0*a^3*b^2*x^6 + 60214*a^4*b*x^3 + 10472*a^5)/x^23
```

3.87.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.42

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{24}}\,dx = -\frac{30107\,b^5x^{15}\mathrm{sgn}(bx^3+a) + 109480\,ab^4x^{12}\mathrm{sgn}(bx^3+a) + 172040\,a^2b^3x^9\mathrm{sgn}(bx^3+a) + 141680\,a^3b^2x^6\mathrm{sgn}(b}{240856\,x^{23}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^24,x, algorithm="giac")
```

output
```
-1/240856*(30107*b^5*x^15*sgn(b*x^3 + a) + 109480*a*b^4*x^12*sgn(b*x^3 + a
) + 172040*a^2*b^3*x^9*sgn(b*x^3 + a) + 141680*a^3*b^2*x^6*sgn(b*x^3 + a)
+ 60214*a^4*b*x^3*sgn(b*x^3 + a) + 10472*a^5*sgn(b*x^3 + a))/x^23
```

3.87.9 Mupad [B] (verification not implemented)

Time = 8.34 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.91

$$\begin{aligned}\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{24}}\,dx &= -\frac{a^5\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{23\,x^{23}\,\left(b\,x^3+a\right)} - \frac{b^5\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{8\,x^8\,\left(b\,x^3+a\right)}\\ &\quad - \frac{5\,a\,b^4\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{11\,x^{11}\,\left(b\,x^3+a\right)} - \frac{a^4\,b\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{4\,x^{20}\,\left(b\,x^3+a\right)}\\ &\quad - \frac{5\,a^2\,b^3\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{7\,x^{14}\,\left(b\,x^3+a\right)} - \frac{10\,a^3\,b^2\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{17\,x^{17}\,\left(b\,x^3+a\right)}\end{aligned}$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^24,x)
```

output
```
- (a^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(23*x^23*(a + b*x^3)) - (b^5*(a^
2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(8*x^8*(a + b*x^3)) - (5*a*b^4*(a^2 + b^2*
x^6 + 2*a*b*x^3)^(1/2))/(11*x^11*(a + b*x^3)) - (a^4*b*(a^2 + b^2*x^6 + 2*
a*b*x^3)^(1/2))/(4*x^20*(a + b*x^3)) - (5*a^2*b^3*(a^2 + b^2*x^6 + 2*a*b*x
^3)^(1/2))/(7*x^14*(a + b*x^3)) - (10*a^3*b^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^
(1/2))/(17*x^17*(a + b*x^3))
```

3.87. $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{24}} \, dx$

3.88 $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{25}}\,dx$

3.88.1 Optimal result

Integrand size = 26, antiderivative size = 128

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{25}}\,dx=-\frac{(a+bx^3)^5\sqrt{a^2+2abx^3+b^2x^6}}{24ax^{24}}+\frac{b(a+bx^3)^5\sqrt{a^2+2abx^3+b^2x^6}}{84a^2x^{21}}-\frac{b^2(a+bx^3)^5\sqrt{a^2+2abx^3+b^2x^6}}{504a^3x^{18}}$$

output
```
-1/24*(b*x^3+a)^5*((b*x^3+a)^2)^(1/2)/a/x^24+1/84*b*(b*x^3+a)^5*((b*x^3+a)
^2)^(1/2)/a^2/x^21-1/504*b^2*(b*x^3+a)^5*((b*x^3+a)^2)^(1/2)/a^3/x^18
```

3.88.2 Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.65

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{25}}\,dx=-\frac{\sqrt{(a+bx^3)^2}(21a^5+120a^4bx^3+280a^3b^2x^6+336a^2b^3x^9+210ab^4x^{12}+56b^5x^{15})}{504x^{24}(a+bx^3)}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^25,x]
```

output
```
-1/504*(Sqrt[(a + b*x^3)^2]*(21*a^5 + 120*a^4*b*x^3 + 280*a^3*b^2*x^6 + 33
6*a^2*b^3*x^9 + 210*a*b^4*x^12 + 56*b^5*x^15))/(x^24*(a + b*x^3))
```

3.88.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.80, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1384, 27, 798, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{25}}dx$$

$$\Big\downarrow \ 1384$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b^5\left(bx^3+a\right)^5}{x^{25}}dx}{b^5\left(a+bx^3\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(bx^3+a\right)^5}{x^{25}}dx}{a+bx^3}$$

$$\Big\downarrow \ 798$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(bx^3+a\right)^5}{x^{27}}dx^3}{3\left(a+bx^3\right)}$$

$$\Big\downarrow \ 55$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(-\frac{b\int \frac{\left(bx^3+a\right)^5}{x^{24}}dx^3}{4a}-\frac{\left(a+bx^3\right)^6}{8ax^{24}}\right)}{3\left(a+bx^3\right)}$$

$$\Big\downarrow \ 55$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(-\frac{b\left(-\frac{b\int \frac{\left(bx^3+a\right)^5}{x^{21}}dx^3}{7a}-\frac{\left(a+bx^3\right)^6}{7ax^{21}}\right)}{4a}-\frac{\left(a+bx^3\right)^6}{8ax^{24}}\right)}{3\left(a+bx^3\right)}$$

$$\Big\downarrow \ 48$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(-\frac{b\left(\frac{b\left(a+bx^3\right)^6}{42a^2x^{18}}-\frac{\left(a+bx^3\right)^6}{7ax^{21}}\right)}{4a}-\frac{\left(a+bx^3\right)^6}{8ax^{24}}\right)}{3\left(a+bx^3\right)}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^25,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-1/8*(a + b*x^3)^6/(a*x^24) - (b*(-1/7*(
a + b*x^3)^6/(a*x^21) + (b*(a + b*x^3)^6)/(42*a^2*x^18)))/(4*a)))/(3*(a +
b*x^3))
```

3.88.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

3.88.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 4.14 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.52

method	result	size
pseudoelliptic	$-\frac{\operatorname{csgn}\left(b\,x^{3}+a\right)\left(\frac{8}{3}b^{5}x^{15}+10a\,b^{4}x^{12}+16a^{2}b^{3}x^{9}+\frac{40}{3}a^{3}b^{2}x^{6}+\frac{40}{7}a^{4}b\,x^{3}+a^{5}\right)}{24x^{24}}$	66
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(-\frac{1}{24}a^{5}-\frac{5}{9}a^{3}b^{2}x^{6}-\frac{2}{3}a^{2}b^{3}x^{9}-\frac{5}{12}a\,b^{4}x^{12}-\frac{1}{9}b^{5}x^{15}-\frac{5}{21}a^{4}b\,x^{3}\right)}{\left(b\,x^{3}+a\right)x^{24}}$	79
gosper	$-\frac{\left(56b^{5}x^{15}+210a\,b^{4}x^{12}+336a^{2}b^{3}x^{9}+280a^{3}b^{2}x^{6}+120a^{4}b\,x^{3}+21a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{504x^{24}\left(b\,x^{3}+a\right)^{5}}$	80
default	$-\frac{\left(56b^{5}x^{15}+210a\,b^{4}x^{12}+336a^{2}b^{3}x^{9}+280a^{3}b^{2}x^{6}+120a^{4}b\,x^{3}+21a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{504x^{24}\left(b\,x^{3}+a\right)^{5}}$	80

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^25,x,method=_RETURNVERBOSE)
```

output
```
-1/24*csgn(b*x^3+a)*(8/3*b^5*x^15+10*a*b^4*x^12+16*a^2*b^3*x^9+40/3*a^3*b^
2*x^6+40/7*a^4*b*x^3+a^5)/x^24
```

3.88.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.46

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{25}}\,dx = -\frac{56\,b^5x^{15}+210\,ab^4x^{12}+336\,a^2b^3x^9+280\,a^3b^2x^6+120\,a^4bx^3+21\,a^5}{504\,x^{24}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^25,x, algorithm="fricas")
```

output
```
-1/504*(56*b^5*x^15 + 210*a*b^4*x^12 + 336*a^2*b^3*x^9 + 280*a^3*b^2*x^6 +
 120*a^4*b*x^3 + 21*a^5)/x^24
```

3.88.6 Sympy [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{25}}\,dx=\int \frac{\left((a+bx^3)^2\right)^{\frac{5}{2}}}{x^{25}}\,dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**25,x)
```

output
```
Integral(((a + b*x**3)**2)**(5/2)/x**25, x)
```

3.88.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(89)=178$.

Time = 0.21 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.12

$$\begin{aligned}\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{25}}\,dx&=\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}b^8}{18\,a^8}\\&+\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}b^7}{18\,a^7x^3}-\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{7}{2}}b^6}{18\,a^8x^6}+\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{7}{2}}b^5}{18\,a^7x^9}\\&-\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{7}{2}}b^4}{18\,a^6x^{12}}+\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{7}{2}}b^3}{18\,a^5x^{15}}\\&-\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{7}{2}}b^2}{18\,a^4x^{18}}+\frac{3\,(b^2x^6+2\,abx^3+a^2)^{\frac{7}{2}}b}{56\,a^3x^{21}}-\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{7}{2}}}{24\,a^2x^{24}}\end{aligned}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^25,x, algorithm="maxima")
```

output
```
1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^8/a^8 + 1/18*(b^2*x^6 + 2*a*b*x^3
 + a^2)^(5/2)*b^7/(a^7*x^3) - 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b^6/(
a^8*x^6) + 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b^5/(a^7*x^9) - 1/18*(b^
2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b^4/(a^6*x^12) + 1/18*(b^2*x^6 + 2*a*b*x^3
+ a^2)^(7/2)*b^3/(a^5*x^15) - 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b^2/(
a^4*x^18) + 3/56*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b/(a^3*x^21) - 1/24*(b^
2*x^6 + 2*a*b*x^3 + a^2)^(7/2)/(a^2*x^24)
```

3.88.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.84

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{25}}\,dx = -\frac{56\,b^5x^{15}\mathrm{sgn}(bx^3+a)+210\,ab^4x^{12}\mathrm{sgn}(bx^3+a)+336\,a^2b^3x^9\mathrm{sgn}(bx^3+a)+280\,a^3b^2x^6\mathrm{sgn}(bx^3+a)+120}{504\,x^{24}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^25,x, algorithm="giac")
```

output
```
-1/504*(56*b^5*x^15*sgn(b*x^3 + a) + 210*a*b^4*x^12*sgn(b*x^3 + a) + 336*a
^2*b^3*x^9*sgn(b*x^3 + a) + 280*a^3*b^2*x^6*sgn(b*x^3 + a) + 120*a^4*b*x^3
*sgn(b*x^3 + a) + 21*a^5*sgn(b*x^3 + a))/x^24
```

3.88.9 Mupad [B] (verification not implemented)

Time = 8.35 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.80

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{25}}\,dx = -\frac{a^5\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{24\,x^{24}\,\left(b\,x^3+a\right)} - \frac{b^5\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{9\,x^9\,\left(b\,x^3+a\right)} - \frac{5\,a\,b^4\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{12\,x^{12}\,\left(b\,x^3+a\right)} - \frac{5\,a^4\,b\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{21\,x^{21}\,\left(b\,x^3+a\right)} - \frac{2\,a^2\,b^3\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{3\,x^{15}\,\left(b\,x^3+a\right)} - \frac{5\,a^3\,b^2\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{9\,x^{18}\,\left(b\,x^3+a\right)}$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^25,x)
```

output
```
- (a^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(24*x^24*(a + b*x^3)) - (b^5*(a^
2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(9*x^9*(a + b*x^3)) - (5*a*b^4*(a^2 + b^2*
x^6 + 2*a*b*x^3)^(1/2))/(12*x^12*(a + b*x^3)) - (5*a^4*b*(a^2 + b^2*x^6 +
2*a*b*x^3)^(1/2))/(21*x^21*(a + b*x^3)) - (2*a^2*b^3*(a^2 + b^2*x^6 + 2*a*
b*x^3)^(1/2))/(3*x^15*(a + b*x^3)) - (5*a^3*b^2*(a^2 + b^2*x^6 + 2*a*b*x^3
)^(1/2))/(9*x^18*(a + b*x^3))
```

3.89 $\int \frac{x^4}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

3.89.1 Optimal result

Integrand size = 26, antiderivative size = 240

$$\int \frac{x^4}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{x^2(a+bx^3)}{2b\sqrt{a^2+2abx^3+b^2x^6}} + \frac{a^{2/3}(a+bx^3)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{a^{2/3}(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{a^{2/3}(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

output
```
1/2*x^2*(b*x^3+a)/b/((b*x^3+a)^2)^(1/2)+1/3*a^(2/3)*(b*x^3+a)*ln(a^(1/3)+b
^(1/3)*x)/b^(5/3)/((b*x^3+a)^2)^(1/2)-1/6*a^(2/3)*(b*x^3+a)*ln(a^(2/3)-a^(
1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(5/3)/((b*x^3+a)^2)^(1/2)+1/3*a^(2/3)*(b*x^3
+a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/b^(5/3)*3^(1/2)/((b*
x^3+a)^2)^(1/2)
```

3.89.2 Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.55

$$\int \frac{x^4}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx$$
$$= \frac{(a+bx^3)\left(3b^{2/3}x^2+2\sqrt{3}a^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+\cdots\right.\right.}{6b^{5/3}\sqrt{(a+bx^3)^2}}$$

input
```
Integrate[x^4/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

output
```
((a + b*x^3)*(3*b^(2/3)*x^2 + 2*Sqrt[3]*a^(2/3)*ArcTan[(1 - (2*b^(1/3)*x)/
a^(1/3))/Sqrt[3]] + 2*a^(2/3)*Log[a^(1/3) + b^(1/3)*x] - a^(2/3)*Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(6*b^(5/3)*Sqrt[(a + b*x^3)^2])
```

3.89.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.68, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {1384, 27, 843, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx$$
$$\Big\downarrow \ 1384$$
$$\frac{b(a+bx^3)\int \frac{x^4}{b(bx^3+a)}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$
$$\Big\downarrow \ 27$$
$$\frac{(a+bx^3)\int \frac{x^4}{bx^3+a}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$
$$\Big\downarrow \ 843$$

$$\frac{(a+bx^3)\left(\frac{x^2}{2b}-\frac{a\int\frac{x}{bx^3+a}dx}{b}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\Big\downarrow$ 821

$$\frac{(a+bx^3)\left(\frac{x^2}{2b}-\frac{a\left(\frac{\int\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int\frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{b}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\Big\downarrow$ 16

$$\frac{(a+bx^3)\left(\frac{x^2}{2b}-\frac{a\left(\frac{\int\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\Big\downarrow$ 1142

$$\frac{(a+bx^3)\left(\frac{x^2}{2b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\Big\downarrow$ 25

3.89. $\int\frac{x^4}{\sqrt{a^2+2abx^3+b^2x^6}}dx$

$$\frac{\left(a+bx^3\right)\left(\frac{x^2}{2b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(a+bx^3\right)\left(\frac{x^2}{2b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{1082}$$

$$\frac{\left(a+bx^3\right)\left(\frac{x^2}{2b}-\frac{a\left(\frac{\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{217}$$

3.89. $\int \frac{x^4}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

$$\frac{(a+bx^3)\left(\frac{x^2}{2b}-\frac{a\left(\frac{-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\Big\downarrow$ 1103

$$\frac{(a+bx^3)\left(\frac{x^2}{2b}-\frac{a\left(\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

input
```
Int[x^4/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

3.89. $\int\frac{x^4}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

output
```
((a + b*x^3)*(x^2/(2*b) - (a*(-1/3*Log[a^(1/3) + b^(1/3)*x]/(a^(1/3)*b^(2/
3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) +
Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(1/3)*b^(
1/3))))/b))/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]
```

3.89.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

3.89.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.10 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.32

method	result	size
risch	$\frac{x^{2}\sqrt{\left(b\,x^{3}+a\right)^{2}}}{2\left(b\,x^{3}+a\right)b}-\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,a\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{3}b+a\right)}\frac{\ln\left(x-\_R\right)}{\_R}\right)}{3\left(b\,x^{3}+a\right)b^{2}}$	77
default	$\frac{\left(b\,x^{3}+a\right)\left(3x^{2}b\left(\frac{a}{b}\right)^{\frac{1}{3}}+2\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)\sqrt{3}\,a+2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)a-\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)a\right)}{6\sqrt{\left(b\,x^{3}+a\right)^{2}}\,b^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}}$	113

input
```
int(x^4/((b*x^3+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*((b*x^3+a)^2)^(1/2)/(b*x^3+a)/b-1/3*((b*x^3+a)^2)^(1/2)/(b*x^3+a)/
b^2*a*sum(1/_R*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

3.89.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.51

$$\int \frac{x^4}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{3\,x^2 - 2\,\sqrt{3}\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}} \arctan\left(\frac{2\,\sqrt{3}bx\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}-\sqrt{3}a}{3\,a}\right) - \left(\frac{a^2}{b^2}\right)^{\frac{1}{3}} \log\left(ax^2 - bx\left(\frac{a^2}{b^2}\right)^{\frac{2}{3}} + a\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\right) + 2\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}} \log\left(a\right.}{6\,b}$$

input
```
integrate(x^4/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/6*(3*x^2 - 2*sqrt(3)*(a^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a^2/b^2)
^(1/3) - sqrt(3)*a)/a) - (a^2/b^2)^(1/3)*log(a*x^2 - b*x*(a^2/b^2)^(2/3) +
 a*(a^2/b^2)^(1/3)) + 2*(a^2/b^2)^(1/3)*log(a*x + b*(a^2/b^2)^(2/3)))/b
```

3.89.6 Sympy [F]

$$\int \frac{x^4}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \int \frac{x^4}{\sqrt{(a+bx^3)^2}}\,dx$$

input
```
integrate(x**4/((b*x**3+a)**2)**(1/2),x)
```

output
```
Integral(x**4/sqrt((a + b*x**3)**2), x)
```

3.89.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.45

$$\int \frac{x^4}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{x^2}{2\,b} - \frac{\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{a\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{a\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate(x^4/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*x^2/b - 1/3*sqrt(3)*a*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/
3))/(b^2*(a/b)^(1/3)) - 1/6*a*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*
(a/b)^(1/3)) + 1/3*a*log(x + (a/b)^(1/3))/(b^2*(a/b)^(1/3))
```

3.89.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.61

$$\int \frac{x^4}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{x^2\mathrm{sgn}(bx^3+a)}{2\,b} + \frac{\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)\mathrm{sgn}(bx^3+a)}{3\,b} + \frac{\sqrt{3}(-ab^2)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)\mathrm{sgn}(bx^3+a)}{3\,b^3} - \frac{(-ab^2)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)\mathrm{sgn}(bx^3+a)}{6\,b^3}$$

input
```
integrate(x^4/((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/2*x^2*sgn(b*x^3 + a)/b + 1/3*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))*sgn
(b*x^3 + a)/b + 1/3*sqrt(3)*(-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b
)^(1/3))/(-a/b)^(1/3))*sgn(b*x^3 + a)/b^3 - 1/6*(-a*b^2)^(2/3)*log(x^2 + x
*(-a/b)^(1/3) + (-a/b)^(2/3))*sgn(b*x^3 + a)/b^3
```

3.89.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \int \frac{x^4}{\sqrt{{\left(b\,x^3+a\right)}^2}}\,dx$$

input
```
int(x^4/((a + b*x^3)^2)^(1/2),x)
```

output
```
int(x^4/((a + b*x^3)^2)^(1/2), x)
```

3.90 $\int \frac{x^3}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

3.90.1 Optimal result

Integrand size = 26, antiderivative size = 235

$$\int \frac{x^3}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx=\frac{x(a+bx^3)}{b\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\sqrt[3]{a}(a+bx^3)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\sqrt[3]{a}(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\sqrt[3]{a}(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

output
```
x*(b*x^3+a)/b/((b*x^3+a)^2)^(1/2)-1/3*a^(1/3)*(b*x^3+a)*ln(a^(1/3)+b^(1/3)
*x)/b^(4/3)/((b*x^3+a)^2)^(1/2)+1/6*a^(1/3)*(b*x^3+a)*ln(a^(2/3)-a^(1/3)*b
^(1/3)*x+b^(2/3)*x^2)/b^(4/3)/((b*x^3+a)^2)^(1/2)+1/3*a^(1/3)*(b*x^3+a)*ar
ctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/b^(4/3)*3^(1/2)/((b*x^3+a)
^2)^(1/2)
```

3.90.2 Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.54

$$\int \frac{x^3}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{(a+bx^3)\left(6\sqrt[3]{b}x+2\sqrt{3}\sqrt[3]{a}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}\right.\right.}{6b^{4/3}\sqrt{(a+bx^3)^2}}$$

input
```
Integrate[x^3/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

output
```
((a + b*x^3)*(6*b^(1/3)*x + 2*Sqrt[3]*a^(1/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^
(1/3))/Sqrt[3]] - 2*a^(1/3)*Log[a^(1/3) + b^(1/3)*x] + a^(1/3)*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(6*b^(4/3)*Sqrt[(a + b*x^3)^2])
```

3.90.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.65, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {1384, 27, 843, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx$$

$$\Big\downarrow \; 1384$$

$$\frac{b(a+bx^3)\int \frac{x^3}{b(bx^3+a)}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 27$$

$$\frac{(a+bx^3)\int \frac{x^3}{bx^3+a}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 843$$

$$\frac{(a+bx^3)\left(\frac{x}{b}-\frac{a\int \frac{1}{bx^3+a}dx}{b}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 750$$

$$\frac{(a+bx^3)\left(\frac{x}{b}-\frac{a\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{b}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 16$$

$$\frac{(a+bx^3)\left(\frac{x}{b}-\frac{a\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 1142$$

$$\frac{(a+bx^3)\left(\frac{x}{b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 25$$

3.90. $\int \frac{x^3}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

$$\frac{(a+bx^3)\left(\frac{x}{b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\Big\downarrow$ 27

$$\frac{(a+bx^3)\left(\frac{x}{b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\Big\downarrow$ 1082

$$\frac{(a+bx^3)\left(\frac{x}{b}-\frac{a\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\Big\downarrow$ 217

3.90. $\int \frac{x^3}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

$$\frac{\left(a+bx^3\right)\left(\frac{x}{b}-\frac{a\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\downarrow$ 1103

$$\frac{\left(a+bx^3\right)\left(\frac{x}{b}-\frac{a\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

input
```
Int[x^3/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

3.90. $\int \frac{x^3}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

output
```
((a + b*x^3)*(x/b - (a*(Log[a^(1/3) + b^(1/3)*x]/(3*a^(2/3)*b^(1/3)) + (-(
(Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) - Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(2/3))))/b))/Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6]
```

3.90.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.90. $\int \frac{x^3}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

3.90.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.21 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.31

method	result	size
risch	$\frac{x\sqrt{(b\,x^3+a)^2}}{(b\,x^3+a)b}-\frac{\sqrt{(b\,x^3+a)^2}\,a\left(\sum_{\_R=\mathrm{RootOf}(\_Z^3b+a)}\frac{\ln(x-\_R)}{\_R^2}\right)}{3(b\,x^3+a)b^2}$	74
default	$\frac{(b\,x^3+a)\left(6xb\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)\sqrt{3}\,a-2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)a+\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)a\right)}{6\sqrt{(b\,x^3+a)^2}\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$	110

input
```
int(x^3/((b*x^3+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x*((b*x^3+a)^2)^(1/2)/(b*x^3+a)/b-1/3*((b*x^3+a)^2)^(1/2)/(b*x^3+a)/b^2*a*
sum(1/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

3.90.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.45

$$\int \frac{x^3}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{2\sqrt{3}\left(-\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bx\left(-\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}a}{3a}\right)-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)+2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right.}{6b}$$

input
```
integrate(x^3/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/6*(2*sqrt(3)*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(-a/b)^(2/3) - sqrt(
3)*a)/a) - (-a/b)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3)) + 2*(-a/b
)^(1/3)*log(x - (-a/b)^(1/3)) + 6*x)/b
```

3.90.6 Sympy [F]

$$\int \frac{x^3}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \int \frac{x^3}{\sqrt{(a+bx^3)^2}}\,dx$$

input
```
integrate(x**3/((b*x**3+a)**2)**(1/2),x)
```

output
```
Integral(x**3/sqrt((a + b*x**3)**2), x)
```

3.90.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.45

$$\int \frac{x^3}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{x}{b}-\frac{\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{a\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{a\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(x^3/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
x/b - 1/3*sqrt(3)*a*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b
^2*(a/b)^(2/3)) + 1/6*a*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*(a/b)^
(2/3)) - 1/3*a*log(x + (a/b)^(1/3))/(b^2*(a/b)^(2/3))
```

3.90.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.61

$$\int \frac{x^3}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)\operatorname{sgn}(bx^3+a)}{3\,b} + \frac{x\operatorname{sgn}(bx^3+a)}{b} - \frac{\sqrt{3}(-ab^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)\operatorname{sgn}(bx^3+a)}{3\,b^2} - \frac{(-ab^2)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)\operatorname{sgn}(bx^3+a)}{6\,b^2}$$

input
```
integrate(x^3/((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/3*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))*sgn(b*x^3 + a)/b + x*sgn(b*x^3
 + a)/b - 1/3*sqrt(3)*(-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3
))/(-a/b)^(1/3))*sgn(b*x^3 + a)/b^2 - 1/6*(-a*b^2)^(1/3)*log(x^2 + x*(-a/b
)^(1/3) + (-a/b)^(2/3))*sgn(b*x^3 + a)/b^2
```

3.90.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \int \frac{x^3}{\sqrt{(b\,x^3+a)^2}}\,dx$$

input
```
int(x^3/((a + b*x^3)^2)^(1/2),x)
```

output
```
int(x^3/((a + b*x^3)^2)^(1/2), x)
```

3.91 $\int \frac{x^2}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

3.91.1 Optimal result

Integrand size = 26, antiderivative size = 44

$$\int \frac{x^2}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx=\frac{(a+bx^3)\log(a+bx^3)}{3b\sqrt{a^2+2abx^3+b^2x^6}}$$

output
```
1/3*(b*x^3+a)*ln(b*x^3+a)/b/((b*x^3+a)^2)^(1/2)
```

3.91.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.16

$$\int \frac{x^2}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx=-\frac{2\text{arctanh}\left(\frac{\frac{\sqrt{a^2}}{b}-\frac{\sqrt{a^2+2abx^3+b^2x^6}}{b}}{x^3}\right)}{3b}$$

input
```
Integrate[x^2/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

output
```
(-2*ArcTanh[(Sqrt[a^2]/b - Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/b)/x^3])/(3*b)
```

3.91.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1690, 1079, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx$$
$$\Big\downarrow \text{1690}$$
$$\frac{1}{3}\int \frac{1}{\sqrt{b^2x^6+2abx^3+a^2}}dx^3$$
$$\Big\downarrow \text{1079}$$
$$\frac{b(a+bx^3)\int \frac{1}{b^2x^3+ab}dx^3}{3\sqrt{a^2+2abx^3+b^2x^6}}$$
$$\Big\downarrow \text{16}$$
$$\frac{(a+bx^3)\log(a+bx^3)}{3b\sqrt{a^2+2abx^3+b^2x^6}}$$

input
```
Int[x^2/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

output
```
((a + b*x^3)*Log[a + b*x^3])/(3*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

3.91.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

3.91.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.50

method	result	size
pseudoelliptic	$\frac{\ln(b\,x^3+a)\,\operatorname{csgn}(b\,x^3+a)}{3b}$	22
default	$\frac{(b\,x^3+a)\ln(b\,x^3+a)}{3b\sqrt{(b\,x^3+a)^2}}$	32
risch	$\frac{\sqrt{(b\,x^3+a)^2}\,\ln(b\,x^3+a)}{3(b\,x^3+a)b}$	34

input
```
int(x^2/((b*x^3+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*ln(b*x^3+a)/b*csgn(b*x^3+a)
```

3.91.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.30

$$\int \frac{x^2}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{\log(bx^3+a)}{3\,b}$$

input
```
integrate(x^2/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/3*log(b*x^3 + a)/b
```

3.91.6 Sympy [F]

$$\int \frac{x^2}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx=\int \frac{x^2}{\sqrt{(a+bx^3)^2}}\,dx$$

input
```
integrate(x**2/((b*x**3+a)**2)**(1/2),x)
```

output
```
Integral(x**2/sqrt((a + b*x**3)**2), x)
```

3.91.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.34

$$\int \frac{x^2}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx=\frac{\log\left(x^3+\frac{a}{b}\right)}{3\,b}$$

input
```
integrate(x^2/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/3*log(x^3 + a/b)/b
```

3.91.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.50

$$\int \frac{x^2}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx=\frac{\log\left(|bx^3+a|\right)\operatorname{sgn}(bx^3+a)}{3\,b}$$

input
```
integrate(x^2/((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/3*log(abs(b*x^3 + a))*sgn(b*x^3 + a)/b
```

3.91.9 Mupad [B] (verification not implemented)

Time = 8.43 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.75

$$\int \frac{x^2}{\sqrt{a^2 + 2abx^3 + b^2x^6}}\,dx = \frac{\ln\left(b^2\,x^3 + a\,b\right)\,\mathrm{sign}(2\,b^2\,x^3 + 2\,a\,b)}{3\,\sqrt{b^2}}$$

input
```
int(x^2/((a + b*x^3)^2)^(1/2),x)
```

output
```
(log(a*b + b^2*x^3)*sign(2*a*b + 2*b^2*x^3))/(3*(b^2)^(1/2))
```

3.92 $\int \frac{x}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

3.92.1 Optimal result

Integrand size = 24, antiderivative size = 202

$$\int \frac{x}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = -\frac{(a+bx^3)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

output
```
-1/3*(b*x^3+a)*ln(a^(1/3)+b^(1/3)*x)/a^(1/3)/b^(2/3)/((b*x^3+a)^2)^(1/2)+1
/6*(b*x^3+a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(1/3)/b^(2/3)/((b
*x^3+a)^2)^(1/2)-1/3*(b*x^3+a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^
(1/2))/a^(1/3)/b^(2/3)*3^(1/2)/((b*x^3+a)^2)^(1/2)
```

3.92.2 Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.54

$$\int \frac{x}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{(a+bx^3)\left(-2\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\right)}{6\sqrt[3]{a}b^{2/3}\sqrt{(a+bx^3)^2}}$$

input
```
Integrate[x/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

output
```
((a + b*x^3)*(-2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*L
og[a^(1/3) + b^(1/3)*x] + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))
/(6*a^(1/3)*b^(2/3)*Sqrt[(a + b*x^3)^2])
```

3.92.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.72, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {1384, 27, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a^2+2abx^3+b^2x^6}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b\left(a+bx^3\right)\int \frac{x}{b(bx^3+a)}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(a+bx^3\right)\int \frac{x}{bx^3+a}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{821}$$

$$\frac{(a+bx^3)\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{16}$$

$$\frac{(a+bx^3)\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{1142}$$

$$\frac{(a+bx^3)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\big\downarrow$ 25

$$\frac{(a+bx^3)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\big\downarrow$ 27

$$\frac{(a+bx^3)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\big\downarrow$ 1082

$$\frac{(a+bx^3)\left(\frac{\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\big\downarrow$ 217

3.92. $\int\frac{x}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

$$\frac{(a+bx^3)\left(\frac{-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 1103$$

$$\frac{(a+bx^3)\left(\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

input
```
Int[x/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

output
```
((a + b*x^3)*(-1/3*Log[a^(1/3) + b^(1/3)*x]/(a^(1/3)*b^(2/3)) + (-((Sqrt[3
]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) + Log[a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(1/3)*b^(1/3))))/Sqrt[a^2
 + 2*a*b*x^3 + b^2*x^6]
```

3.92.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

3.92.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.23

method	result	size
risch	$\frac{\sqrt{(b\,x^3+a)^2}\left(\sum_{\_R=\operatorname{RootOf}(\_Z^3 b+a)}\frac{\ln(x-\_R)}{\_R}\right)}{3(b\,x^3+a)b}$	47
default	$-\frac{(b\,x^3+a)\left(2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)+2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)-\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\right)}{6\sqrt{(b\,x^3+a)^2}\,b\left(\frac{a}{b}\right)^{\frac{1}{3}}}$	97

input
```
int(x/((b*x^3+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*((b*x^3+a)^2)^(1/2)/(b*x^3+a)/b*sum(1/_R*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

3.92.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.50

$$\int \frac{x}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx$$
$$=\left[\frac{3\sqrt{\frac{1}{3}}ab\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,b^2x^3-ab+3\sqrt{\frac{1}{3}}\left(abx+2\,(-ab^2)^{\frac{2}{3}}x^2+(-ab^2)^{\frac{1}{3}}a\right)\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}}-3\,(-ab^2)^{\frac{2}{3}}x}{bx^3+a}\right)+(-ab^2)^{\frac{2}{3}}\log\left(b^2x^2\right.}{6\,ab^2}\right.$$

input
```
integrate(x/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/6*(3*sqrt(1/3)*a*b*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt
(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3
)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) + (-a*b^2)^(2/3)*log(b^2*x^2 + (-a
*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 2*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1
/3)))/(a*b^2), 1/6*(6*sqrt(1/3)*a*b*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/
3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + (-a*b^2)^(2/3)*lo
g(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 2*(-a*b^2)^(2/3)*log(b*
x - (-a*b^2)^(1/3)))/(a*b^2)]
```

3.92.6 Sympy [F]

$$\int \frac{x}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx=\int \frac{x}{\sqrt{(a+bx^3)^2}}\,dx$$

input
```
integrate(x/((b*x**3+a)**2)**(1/2),x)
```

output
```
Integral(x/sqrt((a + b*x**3)**2), x)
```

3.92.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.49

$$\int \frac{x}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx=\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate(x/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b*(a/b)^(
1/3)) + 1/6*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(1/3)) - 1/3*l
og(x + (a/b)^(1/3))/(b*(a/b)^(1/3))
```

3.92.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.61

$$\int \frac{x}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)\operatorname{sgn}(bx^3+a)}{3\left(-ab^2\right)^{\frac{1}{3}}} - \frac{\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)\operatorname{sgn}(bx^3+a)}{6\left(-ab^2\right)^{\frac{1}{3}}} - \frac{\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)\operatorname{sgn}(bx^3+a)}{3\,a}$$

input
```
integrate(x/((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))*sgn(b*x^
3 + a)/(-a*b^2)^(1/3) - 1/6*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))*sgn(b
*x^3 + a)/(-a*b^2)^(1/3) - 1/3*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))*sgn
(b*x^3 + a)/a
```

3.92.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \int \frac{x}{\sqrt{\left(b\,x^3+a\right)^2}}\,dx$$

input
```
int(x/((a + b*x^3)^2)^(1/2),x)
```

output
```
int(x/((a + b*x^3)^2)^(1/2), x)
```

3.93 $\int \frac{1}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

3.93.1 Optimal result

Integrand size = 22, antiderivative size = 202

$$\int \frac{1}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx=-\frac{(a+bx^3)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{b}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}\sqrt{a^2+2abx^3+b^2x^6}}$$

output
```
1/3*(b*x^3+a)*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/b^(1/3)/((b*x^3+a)^2)^(1/2)-1/
6*(b*x^3+a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/b^(1/3)/((b*
x^3+a)^2)^(1/2)-1/3*(b*x^3+a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(
1/2))/a^(2/3)/b^(1/3)*3^(1/2)/((b*x^3+a)^2)^(1/2)
```

3.93.2 Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.54

$$\int \frac{1}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx=-\frac{(a+bx^3)\left(2\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\right)}{6a^{2/3}\sqrt[3]{b}\sqrt{(a+bx^3)^2}}$$

input
```
Integrate[1/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

output
```
-1/6*((a + b*x^3)*(2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] -
 2*Log[a^(1/3) + b^(1/3)*x] + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^
2]))/(a^(2/3)*b^(1/3)*Sqrt[(a + b*x^3)^2])
```

3.93.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.71, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {1384, 750, 16, 27, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a^2+2abx^3+b^2x^6}}dx$$

$$\Big\downarrow \ 1384$$

$$\frac{b(a+bx^3)\int \frac{1}{b^2x^3+ab}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \ 750$$

$$\frac{b(a+bx^3)\left(\frac{\int \frac{\sqrt[3]{b}\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)}{b^{4/3}x^2-\sqrt[3]{a}bx+a^{2/3}b^{2/3}}dx}{3a^{2/3}b^{2/3}}+\frac{\int \frac{1}{b^{2/3}x+\sqrt[3]{a}\sqrt[3]{b}}dx}{3a^{2/3}b^{2/3}}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \ 16$$

$$\frac{b(a+bx^3)\left(\frac{\int \frac{\sqrt[3]{b}\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)}{b^{4/3}x^2-\sqrt[3]{a}bx+a^{2/3}b^{2/3}}dx}{3a^{2/3}b^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \ 27$$

$$\frac{b(a+bx^3)\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{4/3}x^2-\sqrt[3]{a}bx+a^{2/3}b^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\Big\downarrow$ 1142

$$\frac{b(a+bx^3)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{4/3}x^2-\sqrt[3]{a}bx+a^{2/3}b^{2/3}}dx-\frac{\int -\frac{b\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{4/3}x^2-\sqrt[3]{a}bx+a^{2/3}b^{2/3}}dx}{2b}}{3a^{2/3}\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\Big\downarrow$ 25

$$\frac{b(a+bx^3)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{4/3}x^2-\sqrt[3]{a}bx+a^{2/3}b^{2/3}}dx+\frac{\int \frac{b\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{4/3}x^2-\sqrt[3]{a}bx+a^{2/3}b^{2/3}}dx}{2b}}{3a^{2/3}\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\Big\downarrow$ 27

$$\frac{b(a+bx^3)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{4/3}x^2-\sqrt[3]{a}bx+a^{2/3}b^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{4/3}x^2-\sqrt[3]{a}bx+a^{2/3}b^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\Big\downarrow$ 1082

$$\frac{b(a+bx^3)\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{4/3}x^2-\sqrt[3]{a}bx+a^{2/3}b^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{b}}{3a^{2/3}\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\Big\downarrow$ 217

3.93. $\int \frac{1}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

$$\frac{b\left(a+bx^3\right)\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{4/3}x^2-\sqrt[3]{a}bx+a^{2/3}b^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b}}{3a^{2/3}\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 1103$$

$$\frac{b\left(a+bx^3\right)\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2b}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b}}{3a^{2/3}\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

input
```
Int[1/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

output
```
(b*(a + b*x^3)*(Log[a^(1/3) + b^(1/3)*x]/(3*a^(2/3)*b^(4/3)) + (-((Sqrt[3]
*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b) - Log[a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2]/(2*b))/(3*a^(2/3)*b^(1/3))))/Sqrt[a^2 + 2*a*b*x^3
+ b^2*x^6]
```

3.93.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

3.93.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.88 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.23

method	result	size
risch	$\frac{\sqrt{(b x^3+a)^2}\left(\sum_{\_R=\mathrm{RootOf}(\_Z^3 b+a)}\frac{\ln(x-\_R)}{\_R^2}\right)}{3(b x^3+a)b}$	47
default	$\frac{(b x^3+a)\left(-2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)+2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)-\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\right)}{6\sqrt{(b x^3+a)^2}\, b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$	97

input
```
int(1/((b*x^3+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*((b*x^3+a)^2)^(1/2)/(b*x^3+a)/b*sum(1/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a
))
```

3.93.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.48

$$\int \frac{1}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx$$

$$= \left[\frac{3\sqrt{\frac{1}{3}}ab\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^3-3\,(a^2b)^{\frac{1}{3}}ax-a^2+3\sqrt{\frac{1}{3}}\left(2\,abx^2+(a^2b)^{\frac{2}{3}}x-(a^2b)^{\frac{1}{3}}a\right)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}}{bx^3+a}\right)-(a^2b)^{\frac{2}{3}}\log\left(abx^2-(a\right.}{6\,a^2b}\right.$$

input
```
integrate(1/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/6*(3*sqrt(1/3)*a*b*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1
/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a
)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a)) - (a^2*b)^(2/3)*log(a*b*x^2 - (a^2*
b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3))
)/(a^2*b), 1/6*(6*sqrt(1/3)*a*b*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*
(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - (a^2*b)^(2
/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*(a^2*b)^(2/3)*log
(a*b*x + (a^2*b)^(2/3)))/(a^2*b)]
```

3.93.6 Sympy [F]

$$\int \frac{1}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \int \frac{1}{\sqrt{(a+bx^3)^2}}\,dx$$

input
```
integrate(1/((b*x**3+a)**2)**(1/2),x)
```

output
```
Integral(1/sqrt((a + b*x**3)**2), x)
```

3.93.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.49

$$\int \frac{1}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(1/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b*(a/b)^(
2/3)) - 1/6*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(2/3)) + 1/3*l
og(x + (a/b)^(1/3))/(b*(a/b)^(2/3))
```

3.93.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.60

$$\int \frac{1}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx =$$

$$-\frac{1}{6}\left(\frac{2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{a} - \frac{2\sqrt{3}(-ab^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab} - \frac{(-ab^2)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)\right.}{ab}\right.$$

$$+a)$$

input
```
integrate(1/((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/6*(2*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a - 2*sqrt(3)*(-a*b^2)^(1/
3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b) - (-a*b^2)^
(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b))*sgn(b*x^3 + a)
```

3.93.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a^2 + 2abx^3 + b^2x^6}}\, dx = \int \frac{1}{\sqrt{(b\,x^3 + a)^2}}\, dx$$

input
```
int(1/((a + b*x^3)^2)^(1/2),x)
```

output
```
int(1/((a + b*x^3)^2)^(1/2), x)
```

3.94 $\int \frac{1}{x\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

3.94.1 Optimal result

Integrand size = 26, antiderivative size = 80

$$\int \frac{1}{x\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{(a+bx^3)\log(x)}{a\sqrt{a^2+2abx^3+b^2x^6}} - \frac{(a+bx^3)\log(a+bx^3)}{3a\sqrt{a^2+2abx^3+b^2x^6}}$$

output
```
(b*x^3+a)*ln(x)/a/((b*x^3+a)^2)^(1/2)-1/3*(b*x^3+a)*ln(b*x^3+a)/a/((b*x^3+
a)^2)^(1/2)
```

3.94.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.68

$$\int \frac{1}{x\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{-2a\log(x^3)+\left(a-\sqrt{a^2}\right)\log\left(\sqrt{a^2}-bx^3-\sqrt{(a+bx^3)^2}\right)+a\log\left(\sqrt{a^2}+bx^3-\sqrt{(a+bx^3)^2}\right)+\sqrt{a^2}}{6a\sqrt{a^2}}$$

input
```
Integrate[1/(x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
```

output
```
(-2*a*Log[x^3] + (a - Sqrt[a^2])*Log[Sqrt[a^2] - b*x^3 - Sqrt[(a + b*x^3)^
2]] + a*Log[Sqrt[a^2] + b*x^3 - Sqrt[(a + b*x^3)^2]] + Sqrt[a^2]*Log[a*(Sq
rt[a^2] + b*x^3 - Sqrt[(a + b*x^3)^2])])/(6*a*Sqrt[a^2])
```

3.94.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.69, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1384, 27, 798, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a^2+2abx^3+b^2x^6}}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{b(a+bx^3)\int \frac{1}{bx(bx^3+a)}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(a+bx^3)\int \frac{1}{x(bx^3+a)}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{ 798}$$

$$\frac{(a+bx^3)\int \frac{1}{x^3(bx^3+a)}dx^3}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{ 47}$$

$$\frac{(a+bx^3)\left(\frac{\int \frac{1}{x^3}dx^3}{a}-\frac{b\int \frac{1}{bx^3+a}dx^3}{a}\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{ 14}$$

$$\frac{(a+bx^3)\left(\frac{\log(x^3)}{a}-\frac{b\int \frac{1}{bx^3+a}dx^3}{a}\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{(a+bx^3)\left(\frac{\log(x^3)}{a}-\frac{\log(a+bx^3)}{a}\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

input
```
Int[1/(x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
```

output
```
((a + b*x^3)*(Log[x^3]/a - Log[a + b*x^3]/a))/(3*Sqrt[a^2 + 2*a*b*x^3 + b^
2*x^6])
```

3.94.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

3.94.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.45 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.39

method	result	size
pseudoelliptic	$-\frac{(\ln(b\,x^3+a)-\ln(b\,x^3))\,\mathrm{csgn}(b\,x^3+a)}{3a}$	31
default	$\frac{(b\,x^3+a)\,(3\ln(x)-\ln(b\,x^3+a))}{3\sqrt{(b\,x^3+a)^2}\,a}$	39
risch	$\frac{\sqrt{(b\,x^3+a)^2}\,\ln(x)}{(b\,x^3+a)a}-\frac{\sqrt{(b\,x^3+a)^2}\,\ln(b\,x^3+a)}{3(b\,x^3+a)a}$	61

input
```
int(1/x/((b*x^3+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(ln(b*x^3+a)-ln(b*x^3))*csgn(b*x^3+a)/a
```

3.94.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.22

$$\int \frac{1}{x\sqrt{a^2+2abx^3+b^2x^6}}\,dx = -\frac{\log\left(bx^3+a\right)-3\,\log\left(x\right)}{3\,a}$$

input
```
integrate(1/x/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*(log(b*x^3 + a) - 3*log(x))/a
```

3.94.6 Sympy [F]

$$\int \frac{1}{x\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \int \frac{1}{x\sqrt{(a+bx^3)^2}}\,dx$$

input
```
integrate(1/x/((b*x**3+a)**2)**(1/2),x)
```

output
```
Integral(1/(x*sqrt((a + b*x**3)**2)), x)
```

3.94.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.54

$$\int \frac{1}{x\sqrt{a^2+2abx^3+b^2x^6}}\,dx = -\frac{(-1)^{2\,abx^3+2\,a^2}\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right)}{3\,a}$$

input
```
integrate(1/x/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/3*(-1)^(2*a*b*x^3 + 2*a^2)*log(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x)))/a
```

3.94.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.40

$$\int \frac{1}{x\sqrt{a^2+2abx^3+b^2x^6}}\,dx = -\frac{1}{3}\left(\frac{\log(|bx^3+a|)}{a}-\frac{3\log(|x|)}{a}\right)\mathrm{sgn}\left(bx^3+a\right)$$

input
```
integrate(1/x/((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/3*(log(abs(b*x^3 + a))/a - 3*log(abs(x))/a)*sgn(b*x^3 + a)
```

3.94.9 Mupad [B] (verification not implemented)

Time = 8.57 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.60

$$\int \frac{1}{x\sqrt{a^2+2abx^3+b^2x^6}}\,dx = -\frac{\ln\left(a\,b+\frac{a^2}{x^3}+\frac{\sqrt{a^2}\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{x^3}\right)}{3\,\sqrt{a^2}}$$

input
```
int(1/(x*((a + b*x^3)^2)^(1/2)),x)
```

output
```
-log(a*b + a^2/x^3 + ((a^2)^(1/2)*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/x^3)/
(3*(a^2)^(1/2))
```

3.95 $\int \frac{1}{x^2\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

3.95.1 Optimal result

Integrand size = 26, antiderivative size = 238

$$\int \frac{1}{x^2\sqrt{a^2+2abx^3+b^2x^6}}\,dx = -\frac{a+bx^3}{ax\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\sqrt[3]{b}(a+bx^3)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\sqrt[3]{b}(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\sqrt[3]{b}(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

output
```
(-b*x^3-a)/a/x/((b*x^3+a)^2)^(1/2)+1/3*b^(1/3)*(b*x^3+a)*ln(a^(1/3)+b^(1/3
)*x)/a^(4/3)/((b*x^3+a)^2)^(1/2)-1/6*b^(1/3)*(b*x^3+a)*ln(a^(2/3)-a^(1/3)*
b^(1/3)*x+b^(2/3)*x^2)/a^(4/3)/((b*x^3+a)^2)^(1/2)+1/3*b^(1/3)*(b*x^3+a)*a
rctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(4/3)*3^(1/2)/((b*x^3+a
)^2)^(1/2)
```

3.95.2 Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.56

$$\int \frac{1}{x^2\sqrt{a^2+2abx^3+b^2x^6}}\,dx = -\frac{(a+bx^3)\left(6\sqrt[3]{a}-2\sqrt{3}\sqrt[3]{b}x\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2\sqrt[3]{b}x\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+\sqrt[3]{b}x\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+\right.\right.}{6a^{4/3}x\sqrt{(a+bx^3)^2}}$$

input
```
Integrate[1/(x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
```

output
```
-1/6*((a + b*x^3)*(6*a^(1/3) - 2*Sqrt[3]*b^(1/3)*x*ArcTan[(1 - (2*b^(1/3)*
x)/a^(1/3))/Sqrt[3]] - 2*b^(1/3)*x*Log[a^(1/3) + b^(1/3)*x] + b^(1/3)*x*Lo
g[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(a^(4/3)*x*Sqrt[(a + b*x^3)
^2])
```

3.95.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.68, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {1384, 27, 847, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{a^2+2abx^3+b^2x^6}}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b(a+bx^3)\int \frac{1}{bx^2(bx^3+a)}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx^3)\int \frac{1}{x^2(bx^3+a)}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{847}$$

$$\frac{(a+bx^3)\left(-\frac{b\int \frac{x}{bx^3+a}dx}{a}-\frac{1}{ax}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\downarrow$ 821

$$\frac{(a+bx^3)\left(-\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{a}-\frac{1}{ax}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\downarrow$ 16

$$\frac{(a+bx^3)\left(-\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\downarrow$ 1142

$$\frac{(a+bx^3)\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\downarrow$ 25

3.95. $\int \frac{1}{x^2\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

$$\frac{(a+bx^3)\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \ 27$$

$$\frac{(a+bx^3)\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \ 1082$$

$$\frac{(a+bx^3)\left(-\frac{b\left(\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \ 217$$

3.95. $\int \frac{1}{x^2\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

$$\frac{\left(a+bx^3\right)\left(-\frac{b\left(\frac{-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\Big\downarrow$ 1103

$$\frac{\left(a+bx^3\right)\left(-\frac{b\left(\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

input
```
Int[1/(x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
```

3.95. $\int \frac{1}{x^2\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

output
```
((a + b*x^3)*(-(1/(a*x)) - (b*(-1/3*Log[a^(1/3) + b^(1/3)*x]/(a^(1/3)*b^(2
/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) +
 Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(1/3)*b^
(1/3))))/a))/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]
```

3.95.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

3.95.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.74 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.39

method	result	size
risch	$-\frac{\sqrt{(b\,x^{3}+a)^{2}}}{(b\,x^{3}+a)ax}+\frac{\sqrt{(b\,x^{3}+a)^{2}}\left(\sum_{\_R=\operatorname{RootOf}\left(a^{4}\_Z^{3}-b\right)}\_R\ln\left(\left(-4\_R^{3}a^{4}+3b\right)x-a^{3}\_R^{2}\right)\right)}{3b\,x^{3}+3a}$	93
default	$-\frac{(b\,x^{3}+a)\left(-2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)x+\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)x-2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)x+6\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6\sqrt{(b\,x^{3}+a)^{2}}\,\left(\frac{a}{b}\right)^{\frac{1}{3}}ax}$	111

input
```
int(1/x^2/((b*x^3+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-((b*x^3+a)^2)^(1/2)/(b*x^3+a)/a/x+1/3*((b*x^3+a)^2)^(1/2)/(b*x^3+a)*sum(_
R*ln((-4*_R^3*a^4+3*b)*x-a^3*_R^2),_R=RootOf(_Z^3*a^4-b))
```

3.95.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.43

$$\int \frac{1}{x^2\sqrt{a^2+2abx^3+b^2x^6}}\,dx =$$
$$-\frac{2\sqrt{3}x\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+x\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^2-ax\left(\frac{b}{a}\right)^{\frac{2}{3}}+a\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)-2x\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx+a\left(\right.\right.}{6\,ax}$$

input
```
integrate(1/x^2/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/6*(2*sqrt(3)*x*(b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(b/a)^(1/3) - 1/3*sqrt(
3)) + x*(b/a)^(1/3)*log(b*x^2 - a*x*(b/a)^(2/3) + a*(b/a)^(1/3)) - 2*x*(b/
a)^(1/3)*log(b*x + a*(b/a)^(2/3)) + 6)/(a*x)
```

3.95.6 Sympy [F]

$$\int \frac{1}{x^2\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \int \frac{1}{x^2\sqrt{(a+bx^3)^2}}\,dx$$

input
```
integrate(1/x**2/((b*x**3+a)**2)**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt((a + b*x**3)**2)), x)
```

3.95.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.45

$$\int \frac{1}{x^2\sqrt{a^2+2abx^3+b^2x^6}}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,a\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{1}{ax}$$

input
```
integrate(1/x^2/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*(a/b)^
(1/3)) - 1/6*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*(a/b)^(1/3)) + 1/3*
log(x + (a/b)^(1/3))/(a*(a/b)^(1/3)) - 1/(a*x)
```

3.95.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.55

$$\int \frac{1}{x^2\sqrt{a^2+2abx^3+b^2x^6}}\,dx$$
$$= \frac{1}{6}\left(\frac{2\,b\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{a^2}+\frac{2\sqrt{3}(-ab^2)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^2b}-\frac{(-ab^2)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right.}{a^2b}\right.$$
$$+a)$$

input
```
integrate(1/x^2/((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/6*(2*b*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/a^2 + 2*sqrt(3)*(-a*b^2)^
(2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b) - (-a*
b^2)^(2/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b) - 6/(a*x))*sgn
(b*x^3 + a)
```

3.95.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \int \frac{1}{x^2\,\sqrt{(b\,x^3+a)^2}}\,dx$$

input
```
int(1/(x^2*((a + b*x^3)^2)^(1/2)),x)
```

output
```
int(1/(x^2*((a + b*x^3)^2)^(1/2)), x)
```

3.96 $\int \frac{1}{x^3\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

3.96.1 Optimal result

Integrand size = 26, antiderivative size = 243

$$\int \frac{1}{x^3\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{-a-bx^3}{2ax^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{b^{2/3}(a+bx^3)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{b^{2/3}(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{b^{2/3}(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

output
```
1/2*(-b*x^3-a)/a/x^2/((b*x^3+a)^2)^(1/2)-1/3*b^(2/3)*(b*x^3+a)*ln(a^(1/3)+
b^(1/3)*x)/a^(5/3)/((b*x^3+a)^2)^(1/2)+1/6*b^(2/3)*(b*x^3+a)*ln(a^(2/3)-a^
(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/((b*x^3+a)^2)^(1/2)+1/3*b^(2/3)*(b*x^
3+a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(5/3)*3^(1/2)/((b
*x^3+a)^2)^(1/2)
```

3.96.2 Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.58

$$\int \frac{1}{x^3\sqrt{a^2+2abx^3+b^2x^6}}\,dx = -\frac{(a+bx^3)\left(3a^{2/3}-2\sqrt{3}b^{2/3}x^2\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2b^{2/3}x^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-b^{2/3}x^2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\right)}{6a^{5/3}x^2\sqrt{(a+bx^3)^2}}$$

input
```
Integrate[1/(x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
```

output
```
-1/6*((a + b*x^3)*(3*a^(2/3) - 2*Sqrt[3]*b^(2/3)*x^2*ArcTan[(1 - (2*b^(1/3
)*x)/a^(1/3))/Sqrt[3]] + 2*b^(2/3)*x^2*Log[a^(1/3) + b^(1/3)*x] - b^(2/3)*
x^2*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(a^(5/3)*x^2*Sqrt[(a
+ b*x^3)^2])
```

3.96.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.65, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {1384, 27, 847, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{a^2+2abx^3+b^2x^6}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b(a+bx^3)\int \frac{1}{bx^3(bx^3+a)}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx^3)\int \frac{1}{x^3(bx^3+a)}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{847}$$

$$\frac{(a+bx^3)\left(-\frac{b\int \frac{1}{bx^3+a}dx}{a}-\frac{1}{2ax^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \ 750$$

$$\frac{(a+bx^3)\left(-\frac{b\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{a}-\frac{1}{2ax^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \ 16$$

$$\frac{(a+bx^3)\left(-\frac{b\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \ 1142$$

$$\frac{(a+bx^3)\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \ 25$$

3.96. $\int \frac{1}{x^3\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

$$\frac{(a+bx^3)\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\Big\downarrow$ 27

$$\frac{(a+bx^3)\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\Big\downarrow$ 1082

$$\frac{(a+bx^3)\left(-\frac{b\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\Big\downarrow$ 217

3.96. $\int \frac{1}{x^3\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

$$\frac{(a+bx^3)\left(-\frac{b\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\downarrow$ 1103

$$\frac{(a+bx^3)\left(-\frac{b\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

input
```
Int[1/(x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
```

3.96. $\int \frac{1}{x^3\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

output
```
((a + b*x^3)*(-1/2*1/(a*x^2) - (b*(Log[a^(1/3) + b^(1/3)*x]/(3*a^(2/3)*b^(
1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3))
- Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(2/3)))
)/a))/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]
```

3.96.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

3.96.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.57 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.39

method	result	size
risch	$-\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}}{2\left(b\,x^{3}+a\right)a\,x^{2}}+\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(\sum_{\_R=\operatorname{RootOf}\left(a^{5}\_Z^{3}+b^{2}\right)}\_R\ln\left(\left(-4\_R^{3}a^{5}-3b^{2}\right)x-a^{2}b\_R\right)\right)}{3b\,x^{3}+3a}$	94
default	$-\frac{\left(b\,x^{3}+a\right)\left(-2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)x^{2}+2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)x^{2}-\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)x^{2}+3\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\sqrt{\left(b\,x^{3}+a\right)^{2}}\,a\,x^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}$	118

input
```
int(1/x^3/((b*x^3+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*((b*x^3+a)^2)^(1/2)/(b*x^3+a)/a/x^2+1/3*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
*sum(_R*ln((-4*_R^3*a^5-3*b^2)*x-a^2*b*_R),_R=RootOf(_Z^3*a^5+b^2))
```

3.96.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.59

$$\int \frac{1}{x^3\sqrt{a^2+2abx^3+b^2x^6}}\,dx$$
$$= \frac{2\sqrt{3}x^2\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ax\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}b}{3b}\right)-x^2\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^2+abx\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}+a^2\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}\right)+2x^2\left(\right.}{6ax^2}$$

input
```
integrate(1/x^3/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/6*(2*sqrt(3)*x^2*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(-b^2/a^2)^(
2/3) - sqrt(3)*b)/b) - x^2*(-b^2/a^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b^2/a^2)
^(1/3) + a^2*(-b^2/a^2)^(2/3)) + 2*x^2*(-b^2/a^2)^(1/3)*log(b*x - a*(-b^2/
a^2)^(1/3)) - 3)/(a*x^2)
```

3.96.6 Sympy [F]

$$\int \frac{1}{x^3\sqrt{a^2+2abx^3+b^2x^6}}\,dx=\int \frac{1}{x^3\sqrt{(a+bx^3)^2}}\,dx$$

input
```
integrate(1/x**3/((b*x**3+a)**2)**(1/2),x)
```

output
```
Integral(1/(x**3*sqrt((a + b*x**3)**2)), x)
```

3.96.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.44

$$\int \frac{1}{x^3\sqrt{a^2+2abx^3+b^2x^6}}\,dx=-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3a\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{1}{2ax^2}$$

input
```
integrate(1/x^3/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*(a/b)^
(2/3)) + 1/6*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*(a/b)^(2/3)) - 1/3*
log(x + (a/b)^(1/3))/(a*(a/b)^(2/3)) - 1/2/(a*x^2)
```

3.96.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.51

$$\int \frac{1}{x^3\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{1}{6}\left(\frac{2\,b\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{a^2} - \frac{2\,\sqrt{3}(-ab^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^2} - \frac{(-ab^2)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right.}{a^2} \right.$$
$$\left. + a\right)$$

input
```
integrate(1/x^3/((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/6*(2*b*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^2 - 2*sqrt(3)*(-a*b^2)^
(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/a^2 - (-a*b^2)
^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^2 - 3/(a*x^2))*sgn(b*x^3
 + a)
```

3.96.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \int \frac{1}{x^3\,\sqrt{{\left(b\,x^3+a\right)}^2}}\,dx$$

input
```
int(1/(x^3*((a + b*x^3)^2)^(1/2)),x)
```

output
```
int(1/(x^3*((a + b*x^3)^2)^(1/2)), x)
```

3.97 $\int \frac{1}{x^4\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

3.97.1 Optimal result

Integrand size = 26, antiderivative size = 125

$$\int \frac{1}{x^4\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{-a-bx^3}{3ax^3\sqrt{a^2+2abx^3+b^2x^6}} - \frac{b(a+bx^3)\log(x)}{a^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{b(a+bx^3)\log\left(a+bx^3\right)}{3a^2\sqrt{a^2+2abx^3+b^2x^6}}$$

output
```
1/3*(-b*x^3-a)/a/x^3/((b*x^3+a)^2)^(1/2)-b*(b*x^3+a)*ln(x)/a^2/((b*x^3+a)^
2)^(1/2)+1/3*b*(b*x^3+a)*ln(b*x^3+a)/a^2/((b*x^3+a)^2)^(1/2)
```

3.97.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.40

$$\int \frac{1}{x^4\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{a^2-\sqrt{a^2}\sqrt{\left(a+bx^3\right)^2}+2abx^3\log\left(x^3\right)+\left(-a+\sqrt{a^2}\right)bx^3\log\left(\sqrt{a^2}-bx^3-\sqrt{\left(a+bx^3\right)^2}\right)-abx^3\log\Big(}{6\left(a^2\right)^{3/2}x^3}$$

input
```
Integrate[1/(x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
```

output
```
(a^2 - Sqrt[a^2]*Sqrt[(a + b*x^3)^2] + 2*a*b*x^3*Log[x^3] + (-a + Sqrt[a^2
])*b*x^3*Log[Sqrt[a^2] - b*x^3 - Sqrt[(a + b*x^3)^2]] - a*b*x^3*Log[Sqrt[a
^2] + b*x^3 - Sqrt[(a + b*x^3)^2]] - Sqrt[a^2]*b*x^3*Log[Sqrt[a^2] + b*x^3
 - Sqrt[(a + b*x^3)^2]])/(6*(a^2)^(3/2)*x^3)
```

3.97.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.52, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1384, 27, 798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{a^2+2abx^3+b^2x^6}}dx$$
$$\Big\downarrow \text{1384}$$
$$\frac{b(a+bx^3)\int \frac{1}{bx^4(bx^3+a)}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$
$$\Big\downarrow \text{27}$$
$$\frac{(a+bx^3)\int \frac{1}{x^4(bx^3+a)}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$
$$\Big\downarrow \text{798}$$
$$\frac{(a+bx^3)\int \frac{1}{x^6(bx^3+a)}dx^3}{3\sqrt{a^2+2abx^3+b^2x^6}}$$
$$\Big\downarrow \text{54}$$
$$\frac{(a+bx^3)\int \left(\frac{b^2}{a^2(bx^3+a)}-\frac{b}{a^2x^3}+\frac{1}{ax^6}\right)dx^3}{3\sqrt{a^2+2abx^3+b^2x^6}}$$
$$\Big\downarrow \text{2009}$$
$$\frac{(a+bx^3)\left(-\frac{b\log(x^3)}{a^2}+\frac{b\log(a+bx^3)}{a^2}-\frac{1}{ax^3}\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

input
```
Int[1/(x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
```

output
```
((a + b*x^3)*(-(1/(a*x^3)) - (b*Log[x^3])/a^2 + (b*Log[a + b*x^3])/a^2))/(
3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

3.97.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.97.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.47 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.35

method	result	size
pseudoelliptic	$-\frac{\left(\ln \left(b \,x^{3}\right) b \,x^{3}-b \ln \left(b \,x^{3}+a \right) x^{3}+a \right) \operatorname{csgn}\left(b \,x^{3}+a \right)}{3 a^{2} x^{3}}$	44
default	$-\frac{\left(b \,x^{3}+a \right) \left(3 b \ln \left(x \right) x^{3}-b \ln \left(b \,x^{3}+a \right) x^{3}+a \right)}{3 \sqrt{\left(b \,x^{3}+a \right)^{2}}\, a^{2} x^{3}}$	51
risch	$-\frac{\sqrt{\left(b \,x^{3}+a \right)^{2}}}{3 \left(b \,x^{3}+a \right) a \,x^{3}}-\frac{\sqrt{\left(b \,x^{3}+a \right)^{2}}\, b \ln \left(x \right)}{\left(b \,x^{3}+a \right) a^{2}}+\frac{\sqrt{\left(b \,x^{3}+a \right)^{2}}\, b \ln \left(-b \,x^{3}-a \right)}{3 \left(b \,x^{3}+a \right) a^{2}}$	95

input
```
int(1/x^4/((b*x^3+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(ln(b*x^3)*b*x^3-b*ln(b*x^3+a)*x^3+a)*csgn(b*x^3+a)/a^2/x^3
```

3.97.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.26

$$\int \frac{1}{x^4\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{bx^3\log\left(bx^3+a\right)-3\,bx^3\log\left(x\right)-a}{3\,a^2x^3}$$

input
```
integrate(1/x^4/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(b*x^3*log(b*x^3 + a) - 3*b*x^3*log(x) - a)/(a^2*x^3)
```

3.97.6 Sympy [F]

$$\int \frac{1}{x^4\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \int \frac{1}{x^4\sqrt{\left(a+bx^3\right)^2}}\,dx$$

input
```
integrate(1/x**4/((b*x**3+a)**2)**(1/2),x)
```

output
```
Integral(1/(x**4*sqrt((a + b*x**3)**2)), x)
```

3.97.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.58

$$\int \frac{1}{x^4\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{(-1)^{2\,abx^3+2\,a^2}\,b\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right)}{3\,a^2}-\frac{\sqrt{b^2x^6+2\,abx^3+a^2}}{3\,a^2x^3}$$

input
```
integrate(1/x^4/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/3*(-1)^(2*a*b*x^3 + 2*a^2)*b*log(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x)))/a^
2 - 1/3*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)/(a^2*x^3)
```

3.97.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.40

$$\int \frac{1}{x^4\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{1}{3}\left(\frac{b\log\left(|bx^3+a|\right)}{a^2}-\frac{3\,b\log\left(|x|\right)}{a^2}+\frac{bx^3-a}{a^2x^3}\right)\operatorname{sgn}\left(bx^3+a\right)$$

input
```
integrate(1/x^4/((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/3*(b*log(abs(b*x^3 + a))/a^2 - 3*b*log(abs(x))/a^2 + (b*x^3 - a)/(a^2*x^
3))*sgn(b*x^3 + a)
```

3.97.9 Mupad [B] (verification not implemented)

Time = 8.52 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.60

$$\int \frac{1}{x^4\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{a\,b\,\mathrm{atanh}\left(\frac{a^2+b\,a\,x^3}{\sqrt{a^2}\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}\right)}{3\,{\left(a^2\right)}^{3/2}}-\frac{\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{3\,a^2\,x^3}$$

input
```
int(1/(x^4*((a + b*x^3)^2)^(1/2)),x)
```

output
```
(a*b*atanh((a^2 + a*b*x^3)/((a^2)^(1/2)*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))
))/(3*(a^2)^(3/2)) - (a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2)/(3*a^2*x^3)
```

3.98 $\int \frac{x^4}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$

3.98.1 Optimal result

Integrand size = 26, antiderivative size = 280

$$\begin{aligned}\int \frac{x^4}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx &= \frac{x^2}{9ab\sqrt{a^2+2abx^3+b^2x^6}} \\ &- \frac{x^2}{6b(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{(a+bx^3)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{4/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} \\ &- \frac{(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{4/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{4/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}\end{aligned}$$

output
```
1/9*x^2/a/b/((b*x^3+a)^2)^(1/2)-1/6*x^2/b/(b*x^3+a)/((b*x^3+a)^2)^(1/2)-1/
27*(b*x^3+a)*ln(a^(1/3)+b^(1/3)*x)/a^(4/3)/b^(5/3)/((b*x^3+a)^2)^(1/2)+1/5
4*(b*x^3+a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(4/3)/b^(5/3)/((b*
x^3+a)^2)^(1/2)-1/27*(b*x^3+a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^
(1/2))/a^(4/3)/b^(5/3)*3^(1/2)/((b*x^3+a)^2)^(1/2)
```

3.98.2 Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.84

$$\int \frac{x^4}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \frac{-3a^{4/3}b^{2/3}x^2+6\sqrt[3]{a}b^{5/3}x^5-2\sqrt{3}(a+bx^3)^2\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2(a+bx^3}{}$$

input

```
Integrate[x^4/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

output

```
(-3*a^(4/3)*b^(2/3)*x^2 + 6*a^(1/3)*b^(5/3)*x^5 - 2*Sqrt[3]*(a + b*x^3)^2*
ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*(a + b*x^3)^2*Log[a^(1/3)
+ b^(1/3)*x] + a^2*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 2*a*b*
x^3*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + b^2*x^6*Log[a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(4/3)*b^(5/3)*(a + b*x^3)*Sqrt[(a
 + b*x^3)^2])
```

3.98.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.71, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {1384, 27, 817, 819, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a^2+2abx^3+b^2x^6)^{3/2}}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{b^3(a+bx^3)\int \frac{x^4}{b^3(bx^3+a)^3}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(a+bx^3)\int \frac{x^4}{(bx^3+a)^3}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{ 817}$$

$$\frac{(a+bx^3)\left(\frac{\int \frac{x}{(bx^3+a)^2}dx}{3b}-\frac{x^2}{6b(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{819}$$

$$\frac{(a+bx^3)\left(\frac{\frac{\int \frac{x}{bx^3+a}dx}{3a}+\frac{x^2}{3a(a+bx^3)}}{3b}-\frac{x^2}{6b(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{821}$$

$$\frac{(a+bx^3)\left(\frac{\frac{\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}}{3a}+\frac{x^2}{3a(a+bx^3)}}{3b}-\frac{x^2}{6b(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{16}$$

$$\frac{(a+bx^3)\left(\frac{\frac{\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a(a+bx^3)}}{3b}-\frac{x^2}{6b(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{1142}$$

3.98. $\int \frac{x^4}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$

$$\frac{(a+bx^3)\left(\frac{\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}}{3b}-\frac{x^2}{6b(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 25$$

$$\frac{(a+bx^3)\left(\frac{\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}}{3b}-\frac{x^2}{6b(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 27$$

$$\frac{(a+bx^3)\left(\frac{\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}}{3b}-\frac{x^2}{6b(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 1082$$

3.98. $\int \frac{x^4}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx$

$$\frac{(a+bx^3)\left(\frac{\frac{\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}}{3b}-\frac{x^2}{6b(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \ 217$$

$$\frac{(a+bx^3)\left(\frac{\frac{\frac{-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}}{3b}-\frac{x^2}{6b(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \ 1103$$

$$\frac{(a+bx^3)\left(\frac{\frac{\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}}{3b}-\frac{x^2}{6b(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

3.98. $\int \frac{x^4}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$

input
```
Int[x^4/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

output
```
((a + b*x^3)*(-1/6*x^2/(b*(a + b*x^3)^2) + (x^2/(3*a*(a + b*x^3)) + (-1/3*
Log[a^(1/3) + b^(1/3)*x]/(a^(1/3)*b^(2/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^
(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(1/3)*b^(1/3)))/(3*a))/(3*b)))/Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6]
```

3.98.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

3.98.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.86 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.32

method	result
risch	$\frac{\sqrt{(b\,x^{3}+a)^{2}}\,\left(\frac{x^{5}}{9a}-\frac{x^{2}}{18b}\right)}{(b\,x^{3}+a)^{3}}+\frac{\sqrt{(b\,x^{3}+a)^{2}}\,\left(\sum_{\_R=\mathrm{RootOf}(\_Z^{3}b+a)}\frac{\ln(x-\_R)}{\_R}\right)}{27(b\,x^{3}+a)b^{2}a}$
default	$-\frac{\left(2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)b^{2}x^{6}+2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)b^{2}x^{6}-\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)b^{2}x^{6}-6\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{2}x^{5}+4\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(-\right.}{3}\right.}{}$

input
```
int(x^4/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^3+a)^2)^(1/2)/(b*x^3+a)^3*(1/9/a*x^5-1/18/b*x^2)+1/27*((b*x^3+a)^2)^
(1/2)/(b*x^3+a)/b^2/a*sum(1/_R*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

3.98.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 512, normalized size of antiderivative = 1.83

$$\int \frac{x^4}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \left[\frac{6\,ab^{3}x^{5}-3\,a^{2}b^{2}x^{2}+3\,\sqrt{\frac{1}{3}}(ab^{3}x^{6}+2\,a^{2}b^{2}x^{3}+a^{3}b)\sqrt{\frac{(-ab^{2})^{\frac{1}{3}}}{a}}\log\left(\frac{2\,b^{2}x^{3}-ab-}{}\right.}{}\right.$$

input
```
integrate(x^4/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/54*(6*a*b^3*x^5 - 3*a^2*b^2*x^2 + 3*sqrt(1/3)*(a*b^3*x^6 + 2*a^2*b^2*x^
3 + a^3*b)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*
x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-
a*b^2)^(2/3)*x)/(b*x^3 + a)) + (b^2*x^6 + 2*a*b*x^3 + a^2)*(-a*b^2)^(2/3)*
log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 2*(b^2*x^6 + 2*a*b*x^
3 + a^2)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^2*b^5*x^6 + 2*a^3*b^
4*x^3 + a^4*b^3), 1/54*(6*a*b^3*x^5 - 3*a^2*b^2*x^2 + 6*sqrt(1/3)*(a*b^3*x
^6 + 2*a^2*b^2*x^3 + a^3*b)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*
x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + (b^2*x^6 + 2*a*b*x^3 + a^
2)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 2*(
b^2*x^6 + 2*a*b*x^3 + a^2)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^2*
b^5*x^6 + 2*a^3*b^4*x^3 + a^4*b^3)]
```

3.98.6 Sympy [F]

$$\int \frac{x^4}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \int \frac{x^4}{\left((a+bx^3)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**4/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

output
```
Integral(x**4/((a + b*x**3)**2)**(3/2), x)
```

3.98.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.53

$$\int \frac{x^4}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \frac{2\,bx^5-ax^2}{18\,(ab^3x^6+2\,a^2b^2x^3+a^3b)} + \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,ab^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,ab^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,ab^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate(x^4/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

output
```
1/18*(2*b*x^5 - a*x^2)/(a*b^3*x^6 + 2*a^2*b^2*x^3 + a^3*b) + 1/27*sqrt(3)*
arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2*(a/b)^(1/3)) +
1/54*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^2*(a/b)^(1/3)) - 1/27*log
(x + (a/b)^(1/3))/(a*b^2*(a/b)^(1/3))
```

3.98.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.66

$$\int \frac{x^4}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = -\frac{\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,(-ab^2)^{\frac{1}{3}}\,ab\mathrm{sgn}\,(bx^3+a)} - \frac{\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^2b\mathrm{sgn}\,(bx^3+a)} - \frac{\sqrt{3}(-ab^2)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^2b^3\mathrm{sgn}\,(bx^3+a)} + \frac{2\,bx^5-ax^2}{18\,(bx^3+a)^2ab\mathrm{sgn}\,(bx^3+a)}$$

input
```
integrate(x^4/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

output
```
-1/54*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(1/3)*a*b*sgn(b*x
^3 + a)) - 1/27*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b*sgn(b*x^3 +
 a)) - 1/27*sqrt(3)*(-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))
/(-a/b)^(1/3))/(a^2*b^3*sgn(b*x^3 + a)) + 1/18*(2*b*x^5 - a*x^2)/((b*x^3 +
 a)^2*a*b*sgn(b*x^3 + a))
```

3.98.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \int \frac{x^4}{(a^2+2\,a\,b\,x^3+b^2\,x^6)^{3/2}}\,dx$$

input
```
int(x^4/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

output
```
int(x^4/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

3.99 $\int \frac{x^3}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx$

3.99.1 Optimal result

Integrand size = 26, antiderivative size = 276

$$\int \frac{x^3}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx=\frac{x}{18ab\sqrt{a^2+2abx^3+b^2x^6}}-\frac{x}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\left(a+bx^3\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{5/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{5/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\left(a+bx^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{5/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

output
```
1/18*x/a/b/((b*x^3+a)^2)^(1/2)-1/6*x/b/(b*x^3+a)/((b*x^3+a)^2)^(1/2)+1/27*
(b*x^3+a)*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)/b^(4/3)/((b*x^3+a)^2)^(1/2)-1/54*(
b*x^3+a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/b^(4/3)/((b*x^3
+a)^2)^(1/2)-1/27*(b*x^3+a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/
2))/a^(5/3)/b^(4/3)*3^(1/2)/((b*x^3+a)^2)^(1/2)
```

3.99.2 Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.85

$$\int \frac{x^3}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = \frac{-6a^{5/3}\sqrt[3]{b}x+3a^{2/3}b^{4/3}x^4-2\sqrt{3}\left(a+bx^3\right)^2\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2\left(a+bx^3\right)}{}$$

input
```
Integrate[x^3/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

output
```
(-6*a^(5/3)*b^(1/3)*x + 3*a^(2/3)*b^(4/3)*x^4 - 2*Sqrt[3]*(a + b*x^3)^2*Ar
cTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*(a + b*x^3)^2*Log[a^(1/3) +
b^(1/3)*x] - a^2*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 2*a*b*x^
3*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - b^2*x^6*Log[a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(5/3)*b^(4/3)*(a + b*x^3)*Sqrt[(a +
 b*x^3)^2])
```

3.99.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.69, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {1384, 27, 817, 749, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}dx$$

$$\Big\downarrow \ 1384$$

$$\frac{b^3\left(a+bx^3\right)\int \frac{x^3}{b^3\left(bx^3+a\right)^3}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \ 27$$

$$\frac{\left(a+bx^3\right)\int \frac{x^3}{\left(bx^3+a\right)^3}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \ 817$$

$$\frac{(a+bx^3)\left(\frac{\int \frac{1}{(bx^3+a)^2}dx}{6b}-\frac{x}{6b(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\Big\downarrow$ 749

$$\frac{(a+bx^3)\left(\frac{\frac{2\int \frac{1}{bx^3+a}dx}{3a}+\frac{x}{3a(a+bx^3)}}{6b}-\frac{x}{6b(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\Big\downarrow$ 750

$$\frac{(a+bx^3)\left(\frac{\frac{2\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{3a}+\frac{x}{3a(a+bx^3)}}{6b}-\frac{x}{6b(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\Big\downarrow$ 16

$$\frac{(a+bx^3)\left(\frac{\frac{2\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a(a+bx^3)}}{6b}-\frac{x}{6b(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\Big\downarrow$ 1142

3.99. $\int \frac{x^3}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$

$$\frac{\left(a+bx^3\right)\left(\frac{\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a\left(a+bx^3\right)}}{6b}-\frac{x}{6b\left(a+bx^3\right)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 25$$

$$\frac{\left(a+bx^3\right)\left(\frac{\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a\left(a+bx^3\right)}}{6b}-\frac{x}{6b\left(a+bx^3\right)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 27$$

3.99. $\int \frac{x^3}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx$

$$\frac{\left(a+bx^3\right)\left(\frac{\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a\left(a+bx^3\right)}}{6b}-\frac{x}{6b\left(a+bx^3\right)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\Big\downarrow$ 1082

$$\frac{\left(a+bx^3\right)\left(\frac{\frac{2\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a\left(a+bx^3\right)}}{6b}-\frac{x}{6b\left(a+bx^3\right)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\Big\downarrow$ 217

3.99. $\int \frac{x^3}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx$

$$\frac{(a+bx^3)\left(\frac{\frac{2\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a\left(a+bx^3\right)}}{6b}-\frac{x}{6b(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\downarrow$ 1103

$$\frac{(a+bx^3)\left(\frac{\frac{2\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a\left(a+bx^3\right)}}{6b}-\frac{x}{6b(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

3.99. $\int\frac{x^3}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$

input
```
Int[x^3/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

output
```
((a + b*x^3)*(-1/6*x/(b*(a + b*x^3)^2) + (x/(3*a*(a + b*x^3)) + (2*(Log[a^
(1/3) + b^(1/3)*x]/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3
)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) - Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/
3)*x^2]/(2*b^(1/3)))/(3*a^(2/3))))/(3*a))/(6*b)))/Sqrt[a^2 + 2*a*b*x^3 + b
^2*x^6]
```

3.99.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

3.99.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.24 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.32

method	result
risch	$\frac{\sqrt{(b\,x^3+a)^2}\left(\frac{x^4}{18a}-\frac{x}{9b}\right)}{(b\,x^3+a)^3}+\frac{\sqrt{(b\,x^3+a)^2}\left(\sum_{\_R=\mathrm{RootOf}(\_Z^3b+a)}\frac{\ln(x-\_R)}{\_R^2}\right)}{27(b\,x^3+a)b^2a}$
default	$\frac{\left(-2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)b^2x^6+2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)b^2x^6-\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)b^2x^6+3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2x^4-4\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(-2\right.}{3(}\right.\right.}{}$

input
```
int(x^3/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^3+a)^2)^(1/2)/(b*x^3+a)^3*(1/18/a*x^4-1/9/b*x)+1/27*((b*x^3+a)^2)^(1
/2)/(b*x^3+a)/b^2/a*sum(1/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

3.99.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 503, normalized size of antiderivative = 1.82

$$\int \frac{x^3}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \left[\frac{3\,a^2b^2x^4-6\,a^3bx+3\,\sqrt{\frac{1}{3}}(ab^3x^6+2\,a^2b^2x^3+a^3b)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^3-3\,(a}{}\right.}{}\right.$$

input
```
integrate(x^3/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/54*(3*a^2*b^2*x^4 - 6*a^3*b*x + 3*sqrt(1/3)*(a*b^3*x^6 + 2*a^2*b^2*x^3
+ a^3*b)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2
 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*
b)^(1/3)/b))/(b*x^3 + a)) - (b^2*x^6 + 2*a*b*x^3 + a^2)*(a^2*b)^(2/3)*log(
a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*(b^2*x^6 + 2*a*b*x^3 + a^
2)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^3*b^4*x^6 + 2*a^4*b^3*x^3
+ a^5*b^2), 1/54*(3*a^2*b^2*x^4 - 6*a^3*b*x + 6*sqrt(1/3)*(a*b^3*x^6 + 2*a
^2*b^2*x^3 + a^3*b)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3
)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - (b^2*x^6 + 2*a*b*x^3 +
 a^2)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*(
b^2*x^6 + 2*a*b*x^3 + a^2)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^3*
b^4*x^6 + 2*a^4*b^3*x^3 + a^5*b^2)]
```

3.99.6 Sympy [F]

$$\int \frac{x^3}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \int \frac{x^3}{\left((a+bx^3)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

output
```
Integral(x**3/((a + b*x**3)**2)**(3/2), x)
```

3.99.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.53

$$\int \frac{x^3}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \frac{bx^4-2\,ax}{18\,(ab^3x^6+2\,a^2b^2x^3+a^3b)} + \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(x^3/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

output
```
1/18*(b*x^4 - 2*a*x)/(a*b^3*x^6 + 2*a^2*b^2*x^3 + a^3*b) + 1/27*sqrt(3)*ar
ctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2*(a/b)^(2/3)) - 1/
54*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^2*(a/b)^(2/3)) + 1/27*log(x
 + (a/b)^(1/3))/(a*b^2*(a/b)^(2/3))
```

3.99.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.64

$$\int \frac{x^3}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\left(-ab^2\right)^{\frac{2}{3}}a\mathrm{sgn}\left(bx^3+a\right)} - \frac{\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(-ab^2\right)^{\frac{2}{3}}a\mathrm{sgn}\left(bx^3+a\right)} - \frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^2b\mathrm{sgn}\left(bx^3+a\right)} + \frac{bx^4-2\,ax}{18\left(bx^3+a\right)^2ab\mathrm{sgn}\left(bx^3+a\right)}$$

input
```
integrate(x^3/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

output
```
-1/27*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b
^2)^(2/3)*a*sgn(b*x^3 + a)) - 1/54*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3)
)/((-a*b^2)^(2/3)*a*sgn(b*x^3 + a)) - 1/27*(-a/b)^(1/3)*log(abs(x - (-a/b)
^(1/3)))/(a^2*b*sgn(b*x^3 + a)) + 1/18*(b*x^4 - 2*a*x)/((b*x^3 + a)^2*a*b*
sgn(b*x^3 + a))
```

3.99.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \int \frac{x^3}{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{3/2}}\,dx$$

input
```
int(x^3/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

output
```
int(x^3/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

3.100 $\int \frac{x^2}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$

3.100.1 Optimal result

Integrand size = 26, antiderivative size = 38

$$\int \frac{x^2}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx=-\frac{1}{6b\,(a+bx^3)\,\sqrt{a^2+2abx^3+b^2x^6}}$$

output
```
-1/6/b/(b*x^3+a)/((b*x^3+a)^2)^(1/2)
```

3.100.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 143 vs. 2(38) = 76.

Time = 0.35 (sec) , antiderivative size = 143, normalized size of antiderivative = 3.76

$$\int \frac{x^2}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = -\frac{x^3\left(2a^4+a^3bx^3-ab^3x^9+a\sqrt{a^2}bx^3\sqrt{(a+bx^3)^2}-\sqrt{a^2}\sqrt{(a+bx^3)^2}(2a^2+b^2x^6)\right)}{6a^4\,(a+bx^3)\left(\sqrt{a^2}bx^3+a\left(\sqrt{a^2}-\sqrt{(a+bx^3)^2}\right)\right)}$$

input
```
Integrate[x^2/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

output
```
-1/6*(x^3*(2*a^4 + a^3*b*x^3 - a*b^3*x^9 + a*Sqrt[a^2]*b*x^3*Sqrt[(a + b*x
^3)^2] - Sqrt[a^2]*Sqrt[(a + b*x^3)^2]*(2*a^2 + b^2*x^6)))/(a^4*(a + b*x^3
)*(Sqrt[a^2]*b*x^3 + a*(Sqrt[a^2] - Sqrt[(a + b*x^3)^2])))
```

3.100.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1690, 1078}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$$

$$\Big\downarrow \text{1690}$$

$$\frac{1}{3}\int \frac{1}{(b^2x^6+2abx^3+a^2)^{3/2}}dx^3$$

$$\Big\downarrow \text{1078}$$

$$-\frac{1}{6b\,(a+bx^3)\,\sqrt{a^2+2abx^3+b^2x^6}}$$

input
```
Int[x^2/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

output
```
-1/6*1/(b*(a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

3.100.3.1 Defintions of rubi rules used

rule 1078
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[2*((a + b*x
+ c*x^2)^(p + 1)/((2*p + 1)*(b + 2*c*x))), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

3.100.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.61

method	result	size
pseudoelliptic	$-\frac{\operatorname{csgn}\left(b\,x^{3}+a\right)}{6\left(b\,x^{3}+a\right)^{2} b}$	23
gosper	$-\frac{b\,x^{3}+a}{6b\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}$	24
default	$-\frac{b\,x^{3}+a}{6b\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}$	24
risch	$-\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}}{6\left(b\,x^{3}+a\right)^{3} b}$	26

input
```
int(x^2/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6/(b*x^3+a)^2/b*csgn(b*x^3+a)
```

3.100.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.68

$$\int \frac{x^2}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = -\frac{1}{6\,(b^3x^6+2\,ab^2x^3+a^2b)}$$

input
```
integrate(x^2/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/6/(b^3*x^6 + 2*a*b^2*x^3 + a^2*b)
```

3.100.6 Sympy [F]

$$\int \frac{x^2}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \int \frac{x^2}{\left((a+bx^3)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

output
```
Integral(x**2/((a + b*x**3)**2)**(3/2), x)
```

3.100.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.42

$$\int \frac{x^2}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = -\frac{1}{6\left(x^3+\frac{a}{b}\right)^2 b^3}$$

input
```
integrate(x^2/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/6/((x^3 + a/b)^2*b^3)
```

3.100.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.63

$$\int \frac{x^2}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = -\frac{1}{6\,(bx^3+a)^2 b\,\mathrm{sgn}\,(bx^3+a)}$$

input
```
integrate(x^2/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

output
```
-1/6/((b*x^3 + a)^2*b*sgn(b*x^3 + a))
```

3.100.9 Mupad [B] (verification not implemented)

Time = 8.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \frac{x^2}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx=-\frac{\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{6\,b\,\left(b\,x^3+a\right)^3}$$

input
```
int(x^2/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

output
```
-(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2)/(6*b*(a + b*x^3)^3)
```

3.101 $\int \frac{x}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx$

3.101.1 Optimal result

Integrand size = 24, antiderivative size = 277

$$\begin{aligned}\int \frac{x}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx &= \frac{2x^2}{9a^2\sqrt{a^2+2abx^3+b^2x^6}} \\ &+ \frac{x^2}{6a\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{2(a+bx^3)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{7/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}} \\ &- \frac{2(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{7/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{7/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}\end{aligned}$$

output
```
2/9*x^2/a^2/((b*x^3+a)^2)^(1/2)+1/6*x^2/a/(b*x^3+a)/((b*x^3+a)^2)^(1/2)-2/
27*(b*x^3+a)*ln(a^(1/3)+b^(1/3)*x)/a^(7/3)/b^(2/3)/((b*x^3+a)^2)^(1/2)+1/2
7*(b*x^3+a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(7/3)/b^(2/3)/((b*
x^3+a)^2)^(1/2)-2/27*(b*x^3+a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^
(1/2))/a^(7/3)/b^(2/3)*3^(1/2)/((b*x^3+a)^2)^(1/2)
```

3.101.2 Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.86

$$\int \frac{x}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \frac{21a^{4/3}b^{2/3}x^2+12\sqrt[3]{a}b^{5/3}x^5-4\sqrt{3}(a+bx^3)^2\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-4(a+bx^3}{}$$

input
```
Integrate[x/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

output
```
(21*a^(4/3)*b^(2/3)*x^2 + 12*a^(1/3)*b^(5/3)*x^5 - 4*Sqrt[3]*(a + b*x^3)^2
*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 4*(a + b*x^3)^2*Log[a^(1/3)
 + b^(1/3)*x] + 2*a^2*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 4*a
*b*x^3*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 2*b^2*x^6*Log[a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(7/3)*b^(2/3)*(a + b*x^3)*Sq
rt[(a + b*x^3)^2])
```

3.101.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.72, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1384, 27, 819, 819, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a^2+2abx^3+b^2x^6)^{3/2}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b^3(a+bx^3)\int \frac{x}{b^3(bx^3+a)^3}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx^3)\int \frac{x}{(bx^3+a)^3}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{819}$$

$$\frac{\left(a+bx^3\right)\left(\frac{2\int \frac{x}{\left(bx^3+a\right)^2}dx}{3a}+\frac{x^2}{6a(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 819$$

$$\frac{\left(a+bx^3\right)\left(\frac{2\left(\frac{\int \frac{x}{bx^3+a}dx}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}\right)}{3a}+\frac{x^2}{6a(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 821$$

$$\frac{\left(a+bx^3\right)\left(\frac{2\left(\frac{\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}\right)}{3a}+\frac{x^2}{6a(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 16$$

$$\frac{\left(a+bx^3\right)\left(\frac{2\left(\frac{\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}\right)}{3a}+\frac{x^2}{6a(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 1142$$

3.101. $\int \frac{x}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$

$$\frac{\left(a+bx^3\right)\left(\frac{2\left(\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}\right)}{3a}+\frac{x^2}{6a\left(a+bx^3\right)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\Big\downarrow$ 25

$$\frac{\left(a+bx^3\right)\left(\frac{2\left(\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}\right)}{3a}+\frac{x^2}{6a\left(a+bx^3\right)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\Big\downarrow$ 27

3.101. $\int \frac{x}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx$

$$\frac{(a+bx^3)\left(\frac{2\left(\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}\right)}{3a}+\frac{x^2}{6a(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 1082$$

$$\frac{(a+bx^3)\left(\frac{2\left(\frac{\frac{\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}\right)}{3a}+\frac{x^2}{6a(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 217$$

3.101. $\int \frac{x}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$

$$\frac{(a+bx^3)\left(\frac{2\left(\frac{\frac{-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}\right)}{3a}+\frac{x^2}{6a(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 1103$$

3.101. $\int \frac{x}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$

$$\frac{(a+bx^3)\left(\frac{2\left(\frac{\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}\right)}{3a}+\frac{x^2}{6a(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

input
```
Int[x/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

output
```
((a + b*x^3)*(x^2/(6*a*(a + b*x^3)^2) + (2*(x^2/(3*a*(a + b*x^3)) + (-1/3*
Log[a^(1/3) + b^(1/3)*x]/(a^(1/3)*b^(2/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^
(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(1/3)*b^(1/3)))/(3*a)))/(3*a)))/Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6]
```

3.101.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.101. $\int \frac{x}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

3.101.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.93 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.32

method	result
risch	$\frac{\sqrt{(b x^3+a)^2}\left(\frac{2b x^5}{9a^2}+\frac{7x^2}{18a}\right)}{(b x^3+a)^3}+\frac{2\sqrt{(b x^3+a)^2}\left(\sum_{\_R=\operatorname{RootOf}(\_Z^3 b+a)}\frac{\ln(x-\_R)}{\_R}\right)}{27(b x^3+a)a^2 b}$
default	$-\frac{\left(4\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)b^2x^6+4\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)b^2x^6-2\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)b^2x^6-12\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2x^5+8\sqrt{3}\arctan\left(\frac{\sqrt{3}\,(\right.\right.}{}$

input
```
int(x/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^3+a)^2)^(1/2)/(b*x^3+a)^3*(2/9*b/a^2*x^5+7/18*x^2/a)+2/27*((b*x^3+a)
^2)^(1/2)/(b*x^3+a)/a^2/b*sum(1/_R*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

3.101.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 514, normalized size of antiderivative = 1.86

$$\int \frac{x}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \left[\frac{12\,ab^3x^5+21\,a^2b^2x^2+6\,\sqrt{\frac{1}{3}}(ab^3x^6+2\,a^2b^2x^3+a^3b)\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,b^2x^3-}{}\right.}{}\right.$$

input
```
integrate(x/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/54*(12*a*b^3*x^5 + 21*a^2*b^2*x^2 + 6*sqrt(1/3)*(a*b^3*x^6 + 2*a^2*b^2*
x^3 + a^3*b)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*
b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*
(-a*b^2)^(2/3)*x)/(b*x^3 + a)) + 2*(b^2*x^6 + 2*a*b*x^3 + a^2)*(-a*b^2)^(2
/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 4*(b^2*x^6 + 2*a*
b*x^3 + a^2)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^3*b^4*x^6 + 2*a^
4*b^3*x^3 + a^5*b^2), 1/54*(12*a*b^3*x^5 + 21*a^2*b^2*x^2 + 12*sqrt(1/3)*(
a*b^3*x^6 + 2*a^2*b^2*x^3 + a^3*b)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3
)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + 2*(b^2*x^6 + 2*a*b
*x^3 + a^2)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/
3)) - 4*(b^2*x^6 + 2*a*b*x^3 + a^2)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3
)))/(a^3*b^4*x^6 + 2*a^4*b^3*x^3 + a^5*b^2)]
```

3.101.6 Sympy [F]

$$\int \frac{x}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \int \frac{x}{\left((a+bx^3)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

output
```
Integral(x/((a + b*x**3)**2)**(3/2), x)
```

3.101.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.53

$$\int \frac{x}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \frac{4\,bx^5+7\,ax^2}{18\,(a^2b^2x^6+2\,a^3bx^3+a^4)} + \frac{2\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^2b\,\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,a^2b\,\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{2\,\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^2b\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate(x/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

output
```
1/18*(4*b*x^5 + 7*a*x^2)/(a^2*b^2*x^6 + 2*a^3*b*x^3 + a^4) + 2/27*sqrt(3)*
arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b*(a/b)^(1/3)) +
1/27*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b*(a/b)^(1/3)) - 2/27*log
(x + (a/b)^(1/3))/(a^2*b*(a/b)^(1/3))
```

3.101.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.64

$$\int \frac{x}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = -\frac{\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(-ab^2\right)^{\frac{1}{3}}a^2\mathrm{sgn}\left(bx^3+a\right)} - \frac{2\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^3\mathrm{sgn}\left(bx^3+a\right)} - \frac{2\sqrt{3}(-ab^2)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^3b^2\mathrm{sgn}\left(bx^3+a\right)} + \frac{4\,bx^5+7\,ax^2}{18\left(bx^3+a\right)^2a^2\mathrm{sgn}\left(bx^3+a\right)}$$

input
```
integrate(x/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

output
```
-1/27*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(1/3)*a^2*sgn(b*x
^3 + a)) - 2/27*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*sgn(b*x^3 + a
)) - 2/27*sqrt(3)*(-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(
-a/b)^(1/3))/(a^3*b^2*sgn(b*x^3 + a)) + 1/18*(4*b*x^5 + 7*a*x^2)/((b*x^3 +
 a)^2*a^2*sgn(b*x^3 + a))
```

3.101.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \int \frac{x}{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{3/2}}\,dx$$

input
```
int(x/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

output
```
int(x/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

3.102 $\int \frac{1}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$

3.102.1 Optimal result

Integrand size = 22, antiderivative size = 286

$$\int \frac{1}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \frac{x(a+bx^3)}{6a\left(a^2+2abx^3+b^2x^6\right)^{3/2}} + \frac{5x(a+bx^3)^2}{18a^2\left(a^2+2abx^3+b^2x^6\right)^{3/2}} - \frac{5(a+bx^3)^3\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}\sqrt[3]{b}\left(a^2+2abx^3+b^2x^6\right)^{3/2}} + \frac{5(a+bx^3)^3\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}\sqrt[3]{b}\left(a^2+2abx^3+b^2x^6\right)^{3/2}} - \frac{5(a+bx^3)^3\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{8/3}\sqrt[3]{b}\left(a^2+2abx^3+b^2x^6\right)^{3/2}}$$

output
```
1/6*x*(b*x^3+a)/a/(b^2*x^6+2*a*b*x^3+a^2)^(3/2)+5/18*x*(b*x^3+a)^2/a^2/(b^
2*x^6+2*a*b*x^3+a^2)^(3/2)+5/27*(b*x^3+a)^3*ln(a^(1/3)+b^(1/3)*x)/a^(8/3)/
b^(1/3)/(b^2*x^6+2*a*b*x^3+a^2)^(3/2)-5/54*(b*x^3+a)^3*ln(a^(2/3)-a^(1/3)*
b^(1/3)*x+b^(2/3)*x^2)/a^(8/3)/b^(1/3)/(b^2*x^6+2*a*b*x^3+a^2)^(3/2)-5/27*
(b*x^3+a)^3*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(8/3)/b^(1
/3)/(b^2*x^6+2*a*b*x^3+a^2)^(3/2)*3^(1/2)
```

3.102.2 Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.82

$$\int \frac{1}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \frac{24a^{5/3}\sqrt[3]{b}x+15a^{2/3}b^{4/3}x^4-10\sqrt{3}(a+bx^3)^2\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+10(a+b\cdots}{\cdots}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(-3/2),x]
```

output
```
(24*a^(5/3)*b^(1/3)*x + 15*a^(2/3)*b^(4/3)*x^4 - 10*Sqrt[3]*(a + b*x^3)^2*
ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 10*(a + b*x^3)^2*Log[a^(1/3)
 + b^(1/3)*x] - 5*a^2*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 10*
a*b*x^3*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 5*b^2*x^6*Log[a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(8/3)*b^(1/3)*(a + b*x^3)*S
qrt[(a + b*x^3)^2])
```

3.102.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.72, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {1384, 749, 749, 750, 16, 27, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b^3(a+bx^3)\int \frac{1}{(b^2x^3+ab)^3}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{749}$$

$$\frac{b^3(a+bx^3)\left(\frac{5\int \frac{1}{(b^2x^3+ab)^2}dx}{6ab}+\frac{x}{6ab^3(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 749$$

$$\frac{b^3\left(a+bx^3\right)\left(\frac{5\left(\frac{2\int \frac{1}{b^2x^3+ab}dx}{3ab}+\frac{x}{3ab^2\left(a+bx^3\right)}\right)}{6ab}+\frac{x}{6ab^3\left(a+bx^3\right)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 750$$

$$\frac{b^3\left(a+bx^3\right)\left(\frac{5\left(\frac{2\left(\frac{\int \frac{\sqrt[3]{b}\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)}{b^{4/3}x^2-\sqrt[3]{a}bx+a^{2/3}b^{2/3}}dx}{3a^{2/3}b^{2/3}}+\frac{\int \frac{1}{b^{2/3}x+\sqrt[3]{a}\sqrt[3]{b}}dx}{3a^{2/3}b^{2/3}}\right)}{3ab}+\frac{x}{3ab^2\left(a+bx^3\right)}\right)}{6ab}+\frac{x}{6ab^3\left(a+bx^3\right)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 16$$

3.102. $\int \frac{1}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx$

$$\frac{b^3\left(a+bx^3\right)\left(\frac{5\left(\frac{2\left(\frac{\int \frac{\sqrt[3]{b}\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)}{b^{4/3}x^2-\sqrt[3]{a}bx+a^{2/3}b^{2/3}}dx}{3a^{2/3}b^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}}\right)}{3ab}+\frac{x}{3ab^2\left(a+bx^3\right)}\right)}{6ab}+\frac{x}{6ab^3\left(a+bx^3\right)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 27$$

$$\frac{b^3\left(a+bx^3\right)\left(\frac{5\left(\frac{2\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{4/3}x^2-\sqrt[3]{a}bx+a^{2/3}b^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}}\right)}{3ab}+\frac{x}{3ab^2\left(a+bx^3\right)}\right)}{6ab}+\frac{x}{6ab^3\left(a+bx^3\right)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 1142$$

3.102. $\int \frac{1}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx$

$$\frac{b^3(a+bx^3)\left(\frac{5\left(\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{4/3}x^2-\sqrt[3]{a}bx+a^{2/3}b^{2/3}}dx-\frac{\int-\frac{b\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{4/3}x^2-\sqrt[3]{a}bx+a^{2/3}b^{2/3}}dx}{2b}}{3a^{2/3}\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}}\right)}{3ab}+\frac{x}{3ab^2\left(a+bx^3\right)}\right)}{6ab}+\frac{x}{6ab^3(a+b\cdots}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\downarrow$ 25

3.102. $\int \frac{1}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$

$$\frac{b^3\left(a+bx^3\right)\left(\frac{5\left(\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{4/3}x^2-\sqrt[3]{a}bx+a^{2/3}b^{2/3}}dx+\frac{\int \frac{b\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{4/3}x^2-\sqrt[3]{a}bx+a^{2/3}b^{2/3}}dx}{2b}}{3a^{2/3}\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}}\right)}{3ab}+\frac{x}{3ab^2\left(a+bx^3\right)}\right)}{6ab}+\frac{x}{6ab^3(a+bx\ldots}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \ 27$$

$$\frac{b^3\left(a+bx^3\right)\left(\frac{5\left(\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{4/3}x^2-\sqrt[3]{a}bx+a^{2/3}b^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{4/3}x^2-\sqrt[3]{a}bx+a^{2/3}b^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}}\right)}{3ab}+\frac{x}{3ab^2\left(a+bx^3\right)}\right)}{6ab}+\frac{x}{6ab^3(a+b\ldots}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

3.102. $\int \frac{1}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx$

$\downarrow$ 1082

$$\frac{b^3\left(a+bx^3\right)\left(\frac{5\left(\frac{2\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{4/3}x^2-\sqrt[3]{a}bx+a^{2/3}b^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{b}}{3a^{2/3}\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}}\right)}{3ab}+\frac{x}{3ab^2\left(a+bx^3\right)}\right)}{6ab}+\frac{x}{6ab^3(a+\right.}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\downarrow$ 217

3.102. $\int \frac{1}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$

$$\frac{b^3\left(a+bx^3\right)\left(\frac{5\left(\frac{2\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{4/3}x^2-\sqrt[3]{a}bx+a^{2/3}b^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b}}{3a^{2/3}\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}}\right)}{3ab}+\frac{x}{3ab^2\left(a+bx^3\right)}\right)}{6ab}+\frac{x}{6ab^3\left(a+bx^3\right)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{1103}$$

3.102. $\int \frac{1}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx$

$$\frac{b^3(a+bx^3)\left(\frac{5\left(\frac{2\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2b}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b}}{3a^{2/3}\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}}\right)}{3ab}+\frac{x}{3ab^2(a+bx^3)}\right)}{6ab}+\frac{x}{6ab^3(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(-3/2),x]
```

output
```
(b^3*(a + b*x^3)*(x/(6*a*b^3*(a + b*x^3)^2) + (5*(x/(3*a*b^2*(a + b*x^3))
+ (2*(Log[a^(1/3) + b^(1/3)*x]/(3*a^(2/3)*b^(4/3)) + (-((Sqrt[3]*ArcTan[(1
 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b) - Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2]/(2*b))/(3*a^(2/3)*b^(1/3))))/(3*a*b)))/(6*a*b)))/Sqrt[a^2 +
2*a*b*x^3 + b^2*x^6]
```

3.102. $\int \frac{1}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$

3.102.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

3.102.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.53 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.31

method	result
risch	$\frac{\sqrt{(b\,x^3+a)^2}\left(\frac{5b\,x^4}{18a^2}+\frac{4x}{9a}\right)}{(b\,x^3+a)^3}+\frac{5\sqrt{(b\,x^3+a)^2}\left(\sum_{\_R=\mathrm{RootOf}(\_Z^3b+a)}\frac{\ln(x-\_R)}{\_R^2}\right)}{27(b\,x^3+a)a^2b}$
default	$\frac{\left(-10\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)b^2x^6+10\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)b^2x^6-5\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)b^2x^6+15\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2x^4-20\sqrt{3}\,\arctan\left(\right.\right.}{}$

input
```
int(1/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^3+a)^2)^(1/2)/(b*x^3+a)^3*(5/18*b/a^2*x^4+4/9*x/a)+5/27*((b*x^3+a)^2
)^(1/2)/(b*x^3+a)/a^2/b*sum(1/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

3.102.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 499, normalized size of antiderivative = 1.74

$$\int \frac{1}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \left[\frac{15\,a^2b^2x^4+24\,a^3bx+15\,\sqrt{\frac{1}{3}}(ab^3x^6+2\,a^2b^2x^3+a^3b)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^3\cdots}{\cdots}\right.}{\cdots}\right.$$

input
```
integrate(1/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/54*(15*a^2*b^2*x^4 + 24*a^3*b*x + 15*sqrt(1/3)*(a*b^3*x^6 + 2*a^2*b^2*x
^3 + a^3*b)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x -
a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a
^2*b)^(1/3)/b))/(b*x^3 + a)) - 5*(b^2*x^6 + 2*a*b*x^3 + a^2)*(a^2*b)^(2/3)
*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 10*(b^2*x^6 + 2*a*b*x^
3 + a^2)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^4*b^3*x^6 + 2*a^5*b^
2*x^3 + a^6*b), 1/54*(15*a^2*b^2*x^4 + 24*a^3*b*x + 30*sqrt(1/3)*(a*b^3*x^
6 + 2*a^2*b^2*x^3 + a^3*b)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*
b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - 5*(b^2*x^6 + 2*
a*b*x^3 + a^2)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)
*a) + 10*(b^2*x^6 + 2*a*b*x^3 + a^2)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/
3)))/(a^4*b^3*x^6 + 2*a^5*b^2*x^3 + a^6*b)]
```

3.102.6 Sympy [F]

$$\int \frac{1}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \int \frac{1}{(a^2+2abx^3+b^2x^6)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

output
```
Integral((a**2 + 2*a*b*x**3 + b**2*x**6)**(-3/2), x)
```

3.102.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.51

$$\int \frac{1}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \frac{5\,bx^4+8\,ax}{18\,(a^2b^2x^6+2\,a^3bx^3+a^4)} + \frac{5\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{5\,\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{5\,\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(1/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

output
```
1/18*(5*b*x^4 + 8*a*x)/(a^2*b^2*x^6 + 2*a^3*b*x^3 + a^4) + 5/27*sqrt(3)*ar
ctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b*(a/b)^(2/3)) - 5/
54*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b*(a/b)^(2/3)) + 5/27*log(x
 + (a/b)^(1/3))/(a^2*b*(a/b)^(2/3))
```

3.102.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.62

$$\int \frac{1}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = -\frac{5\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^3\mathrm{sgn}\left(bx^3+a\right)} + \frac{5\,\sqrt{3}(-ab^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^3b\mathrm{sgn}\left(bx^3+a\right)} + \frac{5\,(-ab^2)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^3b\mathrm{sgn}\left(bx^3+a\right)} + \frac{5\,bx^4+8\,ax}{18\,(bx^3+a)^2a^2\mathrm{sgn}\left(bx^3+a\right)}$$

input
```
integrate(1/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

output
```
-5/27*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*sgn(b*x^3 + a)) + 5/27*
sqrt(3)*(-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3
))/(a^3*b*sgn(b*x^3 + a)) + 5/54*(-a*b^2)^(1/3)*log(x^2 + x*(-a/b)^(1/3) +
 (-a/b)^(2/3))/(a^3*b*sgn(b*x^3 + a)) + 1/18*(5*b*x^4 + 8*a*x)/((b*x^3 + a
)^2*a^2*sgn(b*x^3 + a))
```

3.102.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a^2 + 2abx^3 + b^2x^6)^{3/2}}\, dx = \int \frac{1}{(a^2 + 2\,a\,b\,x^3 + b^2\,x^6)^{3/2}}\, dx$$

input
```
int(1/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

output
```
int(1/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

3.103 $\int \frac{1}{x(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$

3.103.1 Optimal result

Integrand size = 26, antiderivative size = 147

$$\int \frac{1}{x\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = \frac{1}{3a^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{6a(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{(a+bx^3)\log(x)}{a^3\sqrt{a^2+2abx^3+b^2x^6}} - \frac{(a+bx^3)\log(a+bx^3)}{3a^3\sqrt{a^2+2abx^3+b^2x^6}}$$

output

```
1/3/a^2/((b*x^3+a)^2)^(1/2)+1/6/a/(b*x^3+a)/((b*x^3+a)^2)^(1/2)+(b*x^3+a)*
ln(x)/a^3/((b*x^3+a)^2)^(1/2)-1/3*(b*x^3+a)*ln(b*x^3+a)/a^3/((b*x^3+a)^2)^
(1/2)
```

3.103.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 790 vs. 2(147) = 294.

Time = 1.10 (sec) , antiderivative size = 790, normalized size of antiderivative = 5.37

$$\int \frac{1}{x\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = \frac{4a^4bx^3+3a^3b^2x^6-ab^4x^{12}-4(a^2)^{3/2}bx^3\sqrt{(a+bx^3)^2}+a\sqrt{a^2}b^2x^6\sqrt{(a+\cdots}}{}$$

input

```
Integrate[1/(x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]
```

output
```
(4*a^4*b*x^3 + 3*a^3*b^2*x^6 - a*b^4*x^12 - 4*(a^2)^(3/2)*b*x^3*Sqrt[(a +
b*x^3)^2] + a*Sqrt[a^2]*b^2*x^6*Sqrt[(a + b*x^3)^2] - Sqrt[a^2]*b^3*x^9*Sq
rt[(a + b*x^3)^2] + 2*((a^2)^(3/2)*b^2*x^6 + a^4*(Sqrt[a^2] - Sqrt[(a + b*
x^3)^2]) + a^3*b*x^3*(2*Sqrt[a^2] - Sqrt[(a + b*x^3)^2]))*ArcTanh[(b*x^3)/
(Sqrt[a^2] - Sqrt[(a + b*x^3)^2])] - 2*(a^5 + 2*a^4*b*x^3 - (a^2)^(3/2)*b*
x^3*Sqrt[(a + b*x^3)^2] + a^3*(b^2*x^6 - Sqrt[a^2]*Sqrt[(a + b*x^3)^2]))*L
og[x^3] + a^5*Log[Sqrt[a^2] - b*x^3 - Sqrt[(a + b*x^3)^2]] + 2*a^4*b*x^3*L
og[Sqrt[a^2] - b*x^3 - Sqrt[(a + b*x^3)^2]] + a^3*b^2*x^6*Log[Sqrt[a^2] -
b*x^3 - Sqrt[(a + b*x^3)^2]] - a^3*Sqrt[a^2]*Sqrt[(a + b*x^3)^2]*Log[Sqrt[
a^2] - b*x^3 - Sqrt[(a + b*x^3)^2]] - (a^2)^(3/2)*b*x^3*Sqrt[(a + b*x^3)^2
]*Log[Sqrt[a^2] - b*x^3 - Sqrt[(a + b*x^3)^2]] + a^5*Log[Sqrt[a^2] + b*x^3
 - Sqrt[(a + b*x^3)^2]] + 2*a^4*b*x^3*Log[Sqrt[a^2] + b*x^3 - Sqrt[(a + b*
x^3)^2]] + a^3*b^2*x^6*Log[Sqrt[a^2] + b*x^3 - Sqrt[(a + b*x^3)^2]] - a^3*
Sqrt[a^2]*Sqrt[(a + b*x^3)^2]*Log[Sqrt[a^2] + b*x^3 - Sqrt[(a + b*x^3)^2]]
 - (a^2)^(3/2)*b*x^3*Sqrt[(a + b*x^3)^2]*Log[Sqrt[a^2] + b*x^3 - Sqrt[(a +
 b*x^3)^2]])/(3*a^3*Sqrt[a^2]*(a^2 + a*b*x^3 - Sqrt[a^2]*Sqrt[(a + b*x^3)^
2])^2)
```

3.103.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.57, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1384, 27, 798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a^2+2abx^3+b^2x^6\right)^{3/2}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b^3\left(a+bx^3\right)\int \frac{1}{b^3x\left(bx^3+a\right)^3}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(a+bx^3\right)\int \frac{1}{x\left(bx^3+a\right)^3}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{798}$$

$$\frac{\left(a+bx^3\right)\int \frac{1}{x^3\left(bx^3+a\right)^3}dx^3}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{54}$$

$$\frac{(a+bx^3)\int\left(-\frac{b}{a^3(bx^3+a)}-\frac{b}{a^2(bx^3+a)^2}-\frac{b}{a(bx^3+a)^3}+\frac{1}{a^3x^3}\right)dx^3}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx^3)\left(-\frac{\log(a+bx^3)}{a^3}+\frac{\log(x^3)}{a^3}+\frac{1}{a^2(a+bx^3)}+\frac{1}{2a(a+bx^3)^2}\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

input
```
Int[1/(x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]
```

output
```
((a + b*x^3)*(1/(2*a*(a + b*x^3)^2) + 1/(a^2*(a + b*x^3)) + Log[x^3]/a^3 -
 Log[a + b*x^3]/a^3))/(3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

3.103.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.103. $\int \frac{1}{x(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$

3.103.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.48

method	result	si
pseudoelliptic	$\frac{\operatorname{csgn}\left(b\,x^{3}+a\right)\left(-\ln\left(b\,x^{3}+a\right)\left(b\,x^{3}+a\right)^{2}+\ln\left(b\,x^{3}\right)\left(b\,x^{3}+a\right)^{2}+a b\,x^{3}+\frac{3 a^{2}}{2}\right)}{3 \left(b\,x^{3}+a\right)^{2} a^{3}}$	7
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(\frac{b\,x^{3}}{3 a^{2}}+\frac{1}{2 a}\right)}{\left(b\,x^{3}+a\right)^{3}}+\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\ln\left(x\right)}{\left(b\,x^{3}+a\right) a^{3}}-\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\ln\left(b\,x^{3}+a\right)}{3 \left(b\,x^{3}+a\right) a^{3}}$	9
default	$\frac{\left(6 \ln\left(x\right) b^{2} x^{6}-2 \ln\left(b\,x^{3}+a\right) b^{2} x^{6}+12 \ln\left(x\right) a b\,x^{3}-4 \ln\left(b\,x^{3}+a\right) a b\,x^{3}+2 a b\,x^{3}+6 a^{2} \ln\left(x\right)-2 \ln\left(b\,x^{3}+a\right) a^{2}+3 a^{2}\right) \left(b\,x^{3}+a\right)}{6 a^{3} \left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}$	1

input
```
int(1/x/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*csgn(b*x^3+a)*(-ln(b*x^3+a)*(b*x^3+a)^2+ln(b*x^3)*(b*x^3+a)^2+a*b*x^3+
3/2*a^2)/(b*x^3+a)^2/a^3
```

3.103.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.61

$$\int \frac{1}{x\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = \frac{2\,abx^3+3\,a^2-2\left(b^2x^6+2\,abx^3+a^2\right)\log\left(bx^3+a\right)+6\left(b^2x^6+2\,abx^3+a^2\right.}{6\left(a^3b^2x^6+2\,a^4bx^3+a^5\right)}$$

input
```
integrate(1/x/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

output
```
1/6*(2*a*b*x^3 + 3*a^2 - 2*(b^2*x^6 + 2*a*b*x^3 + a^2)*log(b*x^3 + a) + 6*
(b^2*x^6 + 2*a*b*x^3 + a^2)*log(x))/(a^3*b^2*x^6 + 2*a^4*b*x^3 + a^5)
```

3.103.6 Sympy [F]

$$\int \frac{1}{x\,(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \int \frac{1}{x\left((a+bx^3)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

output
```
Integral(1/(x*((a + b*x**3)**2)**(3/2)), x)
```

3.103.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.60

$$\int \frac{1}{x\,(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = -\frac{(-1)^{2\,abx^3+2\,a^2}\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right)}{3\,a^3} + \frac{1}{3\,\sqrt{b^2x^6+2\,abx^3+a^2}a^2} + \frac{1}{6\,\left(x^3+\frac{a}{b}\right)^2ab^2}$$

input
```
integrate(1/x/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/3*(-1)^(2*a*b*x^3 + 2*a^2)*log(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x)))/a^3
 + 1/3/(sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a^2) + 1/6/((x^3 + a/b)^2*a*b^2)
```

3.103.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.59

$$\int \frac{1}{x\,(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = -\frac{\log\left(|bx^3+a|\right)}{3\,a^3\mathrm{sgn}\left(bx^3+a\right)} + \frac{\log\left(|x|\right)}{a^3\mathrm{sgn}\left(bx^3+a\right)} + \frac{3\,b^2x^6+8\,abx^3+6\,a^2}{6\,(bx^3+a)^2a^3\mathrm{sgn}\left(bx^3+a\right)}$$

input
```
integrate(1/x/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

output
```
-1/3*log(abs(b*x^3 + a))/(a^3*sgn(b*x^3 + a)) + log(abs(x))/(a^3*sgn(b*x^3
 + a)) + 1/6*(3*b^2*x^6 + 8*a*b*x^3 + 6*a^2)/((b*x^3 + a)^2*a^3*sgn(b*x^3
+ a))
```

3.103.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx=\int \frac{1}{x\,\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{3/2}} \,dx$$

input
```
int(1/(x*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)),x)
```

output
```
int(1/(x*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)), x)
```

3.104 $\int \frac{1}{x^2\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx$

3.104.1 Optimal result

Integrand size = 26, antiderivative size = 316

$$\begin{aligned}\int \frac{1}{x^2\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx &= \frac{7}{18a^2x\sqrt{a^2+2abx^3+b^2x^6}} \\ &+ \frac{1}{6ax\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{14(a+bx^3)}{9a^3x\sqrt{a^2+2abx^3+b^2x^6}} \\ &+ \frac{14\sqrt[3]{b}(a+bx^3)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{10/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{14\sqrt[3]{b}(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{10/3}\sqrt{a^2+2abx^3+b^2x^6}} \\ &- \frac{7\sqrt[3]{b}(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{10/3}\sqrt{a^2+2abx^3+b^2x^6}}\end{aligned}$$

output
```
7/18/a^2/x/((b*x^3+a)^2)^(1/2)+1/6/a/x/(b*x^3+a)/((b*x^3+a)^2)^(1/2)-14/9*
(b*x^3+a)/a^3/x/((b*x^3+a)^2)^(1/2)+14/27*b^(1/3)*(b*x^3+a)*ln(a^(1/3)+b^(
1/3)*x)/a^(10/3)/((b*x^3+a)^2)^(1/2)-7/27*b^(1/3)*(b*x^3+a)*ln(a^(2/3)-a^(
1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(10/3)/((b*x^3+a)^2)^(1/2)+14/27*b^(1/3)*(b*
x^3+a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(10/3)*3^(1/2)/
((b*x^3+a)^2)^(1/2)
```

3.104.2 Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^2 \left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = \frac{-54a^{7/3}-147a^{4/3}bx^3-84\sqrt[3]{a}b^2x^6+28\sqrt{3}\sqrt[3]{b}x\left(a+bx^3\right)^2\arctan\left(\frac{1-\frac{2\sqrt[3]{\cdots}}{\sqrt[3]{\cdots}}}{\sqrt{3}}\right.\cdots}{\cdots}$$

input
```
Integrate[1/(x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]
```

output
```
(-54*a^(7/3) - 147*a^(4/3)*b*x^3 - 84*a^(1/3)*b^2*x^6 + 28*Sqrt[3]*b^(1/3)
*x*(a + b*x^3)^2*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 28*b^(1/3)*
x*(a + b*x^3)^2*Log[a^(1/3) + b^(1/3)*x] - 14*a^2*b^(1/3)*x*Log[a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 28*a*b^(4/3)*x^4*Log[a^(2/3) - a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2] - 14*b^(7/3)*x^7*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2])/(54*a^(10/3)*x*(a + b*x^3)*Sqrt[(a + b*x^3)^2])
```

3.104.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.68, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1384, 27, 819, 819, 847, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(a^2+2abx^3+b^2x^6\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{b^3\left(a+bx^3\right)\int \frac{1}{b^3x^2\left(bx^3+a\right)^3}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\left(a+bx^3\right)\int \frac{1}{x^2\left(bx^3+a\right)^3}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{ 819}$$

$$\frac{(a+bx^3)\left(\frac{7\int \frac{1}{x^2\left(bx^3+a\right)^2}dx}{6a}+\frac{1}{6ax(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{ 819}$$

$$\frac{(a+bx^3)\left(\frac{7\left(\frac{4\int \frac{1}{x^2\left(bx^3+a\right)}dx}{3a}+\frac{1}{3ax\left(a+bx^3\right)}\right)}{6a}+\frac{1}{6ax(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{ 847}$$

$$\frac{(a+bx^3)\left(\frac{7\left(\frac{4\left(-\frac{b\int \frac{x}{bx^3+a}dx}{a}-\frac{1}{ax}\right)}{3a}+\frac{1}{3ax\left(a+bx^3\right)}\right)}{6a}+\frac{1}{6ax(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{ 821}$$

3.104. $\int \frac{1}{x^2(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$

$$\frac{\left(a+bx^3\right)\left(\frac{7\left(\frac{4\left(-\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{a}-\frac{1}{ax}\right)}{3a}+\frac{1}{3ax\left(a+bx^3\right)}\right)}{6a}+\frac{1}{6ax(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 16$$

3.104. $\int \frac{1}{x^2(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$

$$\frac{\left(a+bx^3\right)\left(\frac{7\left(\frac{4\left(-\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{3a}+\frac{1}{3ax\left(a+bx^3\right)}\right)}{6a}+\frac{1}{6ax(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \ 1142$$

3.104. $\int \frac{1}{x^2(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$

$$\left(a+bx^3\right)\left(\frac{7\left(\frac{4\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{3a}+\frac{1}{3ax\left(a+bx^3\right)}\right)}{6a}+\frac{}{6ax}\right.$$

3.104. $\int\frac{1}{x^2(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$

3.104. $\downarrow$ 25

3.104. $\int \frac{1}{x^2(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$

$$\left(a+bx^3\right)\left(\frac{7\left(\frac{4\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{3a}+\frac{1}{3ax\left(a+bx^3\right)}\right)}{6a}+\frac{}{6ax(}$$

3.104. $\int\frac{1}{x^2\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx$

$$\downarrow 27$$

$$\frac{(a+bx^3)\left(\frac{7\left(\frac{4\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{3a}+\frac{1}{3ax\left(a+bx^3\right)}\right)}{6a}+\frac{}{6ax\ldots}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\downarrow 1082$$

3.104. $\int \frac{1}{x^2(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$

$$(a+bx^3)\left(\frac{7\left(\frac{4\left(-\frac{b\left(\frac{\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{3a}+\frac{1}{3ax\left(a+bx^3\right)}\right)}{6a}+\frac{\cdots}{6\cdots}\right.$$

3.104. $\int\frac{1}{x^2(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$

↓ 217

3.104. $\int \frac{1}{x^2(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$

$$\left(a+bx^3\right)\left(\frac{7\left(\frac{4\left(-\frac{b\left(\frac{-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{3a}+\frac{1}{3ax\left(a+bx^3\right)}\right)}{6a}+\frac{1}{6ax\left(a+bx^3\right)^2}\right.$$

3.104. $\int \frac{1}{x^2\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx$

↓ 1103

3.104. $\int \frac{1}{x^2(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$

$$\left(a+bx^3\right)\left(\frac{7\left(\frac{4\left(-\frac{b\left(\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{3a}+\frac{1}{3ax\left(a+bx^3\right)}\right)}{6a}+\frac{1}{6ax(a+bx^3)^2}\right)$$

3.104. $\displaystyle\int \frac{1}{x^2(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$

input
```
Int[1/(x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]
```

output
```
((a + b*x^3)*(1/(6*a*x*(a + b*x^3)^2) + (7*(1/(3*a*x*(a + b*x^3)) + (4*(-(
1/(a*x)) - (b*(-1/3*Log[a^(1/3) + b^(1/3)*x]/(a^(1/3)*b^(2/3)) + (-((Sqrt[
3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) + Log[a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(1/3)*b^(1/3))))/a))/(3*
a)))/(6*a)))/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]
```

3.104.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

3.104.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.42 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.36

method	result
risch	$\frac{\sqrt{(b\,x^3+a)^2}\left(-\frac{14b^2x^6}{9a^3}-\frac{49b\,x^3}{18a^2}-\frac{1}{a}\right)}{(b\,x^3+a)^3x}+\frac{14\sqrt{(b\,x^3+a)^2}\left(\sum_{\_R=\mathrm{RootOf}\left(a^{10}\_Z^3-b\right)}\_R\ln\left(\left(-4\_R^3a^{10}+3b\right)x-a^7\_R^2\right)\right)}{27(b\,x^3+a)}$
default	$\frac{\left(28\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)b^2x^7+28\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)b^2x^7-14\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)b^2x^7-84\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2x^6+56\sqrt{3}\,\arctan\left(\right.\right.}{}$

input
```
int(1/x^2/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^3+a)^2)^(1/2)/(b*x^3+a)^3*(-14/9*b^2/a^3*x^6-49/18*b/a^2*x^3-1/a)/x+
14/27*((b*x^3+a)^2)^(1/2)/(b*x^3+a)*sum(_R*ln((-4*_R^3*a^10+3*b)*x-a^7*_R^
2),_R=RootOf(_Z^3*a^10-b))
```

3.104.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.64

$$\int \frac{1}{x^2\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx =$$
$$-\frac{84\,b^2x^6+147\,abx^3+28\,\sqrt{3}(b^2x^7+2\,abx^4+a^2x)\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\,\sqrt{3}x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\,\sqrt{3}\right)+14\,(b^2x^7+2\,abx^4+}{54\,(a^3b^2x^7+2\,a^4b}$$

input
```
integrate(1/x^2/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/54*(84*b^2*x^6 + 147*a*b*x^3 + 28*sqrt(3)*(b^2*x^7 + 2*a*b*x^4 + a^2*x)
*(b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(b/a)^(1/3) - 1/3*sqrt(3)) + 14*(b^2*x^7
 + 2*a*b*x^4 + a^2*x)*(b/a)^(1/3)*log(b*x^2 - a*x*(b/a)^(2/3) + a*(b/a)^(1
/3)) - 28*(b^2*x^7 + 2*a*b*x^4 + a^2*x)*(b/a)^(1/3)*log(b*x + a*(b/a)^(2/3
)) + 54*a^2)/(a^3*b^2*x^7 + 2*a^4*b*x^4 + a^5*x)
```

3.104.6 Sympy [F]

$$\int \frac{1}{x^2\,(a^2+2abx^3+b^2x^6)^{3/2}}\,dx=\int \frac{1}{x^2\left((a+bx^3)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**2/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

output
```
Integral(1/(x**2*((a + b*x**3)**2)**(3/2)), x)
```

3.104.7 Maxima [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.47

$$\int \frac{1}{x^2\,(a^2+2abx^3+b^2x^6)^{3/2}}\,dx=-\frac{28\,b^2x^6+49\,abx^3+18\,a^2}{18\,(a^3b^2x^7+2\,a^4bx^4+a^5x)}-\frac{14\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{7\,\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,a^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{14\,\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate(1/x^2/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/18*(28*b^2*x^6 + 49*a*b*x^3 + 18*a^2)/(a^3*b^2*x^7 + 2*a^4*b*x^4 + a^5*
x) - 14/27*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^
3*(a/b)^(1/3)) - 7/27*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*(a/b)^(1
/3)) + 14/27*log(x + (a/b)^(1/3))/(a^3*(a/b)^(1/3))
```

3.104.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.64

$$\int \frac{1}{x^2\,(a^2+2abx^3+b^2x^6)^{3/2}}\,dx=\frac{14\,b\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^4\mathrm{sgn}\left(bx^3+a\right)}+\frac{14\,\sqrt{3}(-ab^2)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^4b\mathrm{sgn}\left(bx^3+a\right)}-\frac{7\,(-ab^2)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,a^4b\mathrm{sgn}\left(bx^3+a\right)}-\frac{10\,b^2x^5+13\,abx^2}{18\,(bx^3+a)^2a^3\mathrm{sgn}\left(bx^3+a\right)}-\frac{1}{a^3x\mathrm{sgn}\left(bx^3+a\right)}$$

input
```
integrate(1/x^2/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

output
```
14/27*b*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a^4*sgn(b*x^3 + a)) + 14/
27*sqrt(3)*(-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(
1/3))/(a^4*b*sgn(b*x^3 + a)) - 7/27*(-a*b^2)^(2/3)*log(x^2 + x*(-a/b)^(1/3
) + (-a/b)^(2/3))/(a^4*b*sgn(b*x^3 + a)) - 1/18*(10*b^2*x^5 + 13*a*b*x^2)/
((b*x^3 + a)^2*a^3*sgn(b*x^3 + a)) - 1/(a^3*x*sgn(b*x^3 + a))
```

3.104.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a^2+2abx^3+b^2x^6)^{3/2}} \, dx = \int \frac{1}{x^2\,{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{3/2}} \,dx$$

input
```
int(1/(x^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)),x)
```

output
```
int(1/(x^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)), x)
```

3.105 $\int \frac{1}{x^3(a^2+2abx^3+b^2x^6)^{3/2}} dx$

3.105.1 Optimal result

Integrand size = 26, antiderivative size = 316

$$\int \frac{1}{x^3(a^2+2abx^3+b^2x^6)^{3/2}} dx = \frac{4}{9a^2x^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{6ax^2(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{10(a+bx^3)}{9a^3x^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{20b^{2/3}(a+bx^3)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{11/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{20b^{2/3}(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{11/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{10b^{2/3}(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{11/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

output
```
4/9/a^2/x^2/((b*x^3+a)^2)^(1/2)+1/6/a/x^2/(b*x^3+a)/((b*x^3+a)^2)^(1/2)-10
/9*(b*x^3+a)/a^3/x^2/((b*x^3+a)^2)^(1/2)-20/27*b^(2/3)*(b*x^3+a)*ln(a^(1/3
)+b^(1/3)*x)/a^(11/3)/((b*x^3+a)^2)^(1/2)+10/27*b^(2/3)*(b*x^3+a)*ln(a^(2/
3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(11/3)/((b*x^3+a)^2)^(1/2)+20/27*b^(2/
3)*(b*x^3+a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(11/3)*3^
(1/2)/((b*x^3+a)^2)^(1/2)
```

3.105.2 Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^3 \left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = \frac{-27a^{8/3}-96a^{5/3}bx^3-60a^{2/3}b^2x^6+40\sqrt{3}b^{2/3}x^2\left(a+bx^3\right)^2\arctan\left(\cdots\right.}{\cdots}$$ [truncated]

input
```
Integrate[1/(x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]
```

output
```
(-27*a^(8/3) - 96*a^(5/3)*b*x^3 - 60*a^(2/3)*b^2*x^6 + 40*Sqrt[3]*b^(2/3)*
x^2*(a + b*x^3)^2*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 40*b^(2/3)
*x^2*(a + b*x^3)^2*Log[a^(1/3) + b^(1/3)*x] + 20*a^2*b^(2/3)*x^2*Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 40*a*b^(5/3)*x^5*Log[a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2] + 20*b^(8/3)*x^8*Log[a^(2/3) - a^(1/3)*b^(1/
3)*x + b^(2/3)*x^2])/(54*a^(11/3)*x^2*(a + b*x^3)*Sqrt[(a + b*x^3)^2])
```

3.105.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.67, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1384, 27, 819, 819, 847, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \left(a^2+2abx^3+b^2x^6\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{b^3\left(a+bx^3\right)\int \frac{1}{b^3x^3\left(bx^3+a\right)^3}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\left(a+bx^3\right)\int \frac{1}{x^3\left(bx^3+a\right)^3}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{ 819}$$

$$\frac{(a+bx^3)\left(\frac{4\int \frac{1}{x^3\left(bx^3+a\right)^2}dx}{3a}+\frac{1}{6ax^2(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\big\downarrow$ 819

$$\frac{(a+bx^3)\left(\frac{4\left(\frac{5\int \frac{1}{x^3\left(bx^3+a\right)}dx}{3a}+\frac{1}{3ax^2\left(a+bx^3\right)}\right)}{3a}+\frac{1}{6ax^2(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\big\downarrow$ 847

$$\frac{(a+bx^3)\left(\frac{4\left(\frac{5\left(-\frac{b\int \frac{1}{bx^3+a}dx}{a}-\frac{1}{2ax^2}\right)}{3a}+\frac{1}{3ax^2\left(a+bx^3\right)}\right)}{3a}+\frac{1}{6ax^2(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\big\downarrow$ 750

3.105. $\int \frac{1}{x^3(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$

$$\frac{\left(a+bx^3\right)\left(\frac{4\left(\frac{5\left(-\frac{b\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{a}-\frac{1}{2ax^2}\right)}{3a}+\frac{1}{3ax^2\left(a+bx^3\right)}\right)}{3a}+\frac{1}{6ax^2(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\downarrow 16$$

3.105. $\int \frac{1}{x^3(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$

$$\frac{\left(a+bx^3\right)\left(\frac{4\left(\frac{5\left(-\frac{b\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{3a}+\frac{1}{3ax^2\left(a+bx^3\right)}\right)}{3a}+\frac{1}{6ax^2(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{1142}$$

3.105. $\int \frac{1}{x^3(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$

$$(a+bx^3)\left(\frac{4\left(\frac{5\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{3a}+\frac{1}{3ax^2\left(a+bx^3\right)}\right)}{3a}+$$

3.105. $\int\frac{1}{x^3(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$

$\downarrow$ 25

3.105. $\int \frac{1}{x^3(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$

$$\left(a+bx^3\right) \left(\frac{4\left(\frac{5\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{3a}+\frac{1}{3ax^2\left(a+bx^3\right)}\right)}{3a}+\frac{}{6\ldots} \right.$$

3.105. $\int \frac{1}{x^3\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx$

$\downarrow$ 27

$$\frac{(a+bx^3)\left(\frac{4\left(\frac{5\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{3a}+\frac{1}{3ax^2(a+bx^3)}\right)}{3a}+\right.}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\downarrow$ 1082

3.105. $\int \frac{1}{x^3(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$

$$(a+bx^3)\left(\frac{4\left(\frac{5\left(-\frac{b\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{3a}+\frac{1}{3ax^2\left(a+bx^3\right)}\right)}{3a}+\right.$$

3.105. $\int \frac{1}{x^3(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$

$\downarrow$ 217

3.105. $\int \frac{1}{x^3(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$

$$(a+bx^3)\left(\frac{4\left(\frac{5\left(-\frac{b\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{3a}+\frac{1}{3ax^2\left(a+bx^3\right)}\right)}{3a}+\frac{1}{6ax^2(a+bx^3}\right.$$

3.105. $\int\frac{1}{x^3\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx$

↓ 1103

3.105. $\int \frac{1}{x^3(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$

$$\left(a+bx^3\right)\left(\frac{4\left(\frac{5\left(-\frac{b\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{3a}+\frac{1}{3ax^2\left(a+bx^3\right)}\right)}{3a}+\frac{1}{6ax^2(a+bx\ldots}\right.$$

3.105. $\int \frac{1}{x^3(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$

input
```
Int[1/(x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]
```

output
```
((a + b*x^3)*(1/(6*a*x^2*(a + b*x^3)^2) + (4*(1/(3*a*x^2*(a + b*x^3)) + (5
*(-1/2*1/(a*x^2) - (b*(Log[a^(1/3) + b^(1/3)*x]/(3*a^(2/3)*b^(1/3)) + (-((
Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) - Log[a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(2/3))))/a))/(3*a))
)/(3*a)))/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]
```

3.105.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

3.105.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.80 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.37

method	result
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(-\frac{10b^{2}x^{6}}{9a^{3}}-\frac{16b\,x^{3}}{9a^{2}}-\frac{1}{2a}\right)}{\left(b\,x^{3}+a\right)^{3}x^{2}}+\frac{20\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(\sum_{\_R=\mathrm{RootOf}\left(a^{11}\_Z^{3}+b^{2}\right)}\_R\ln\left(\left(-4\_R^{3}a^{11}-3b^{2}\right)x-a^{4}b\_R\right)\right)}{27\left(b\,x^{3}+a\right)}$
default	$-\frac{\left(-40\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)b^{2}x^{8}+40\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)b^{2}x^{8}-20\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)b^{2}x^{8}+60\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{2}x^{6}-80\sqrt{3}\,\arctan\right.}{}$

input
```
int(1/x^3/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^3+a)^2)^(1/2)/(b*x^3+a)^3*(-10/9*b^2/a^3*x^6-16/9*b/a^2*x^3-1/2/a)/x
^2+20/27*((b*x^3+a)^2)^(1/2)/(b*x^3+a)*sum(_R*ln((-4*_R^3*a^11-3*b^2)*x-a^
4*b*_R),_R=RootOf(_Z^3*a^11+b^2))
```

3.105.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^3\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx =$$

$$-\frac{60\,b^{2}x^{6}+96\,abx^{3}-40\,\sqrt{3}(b^{2}x^{8}+2\,abx^{5}+a^{2}x^{2})\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}ax\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{2}{3}}-\sqrt{3}b}{3\,b}\right)+20\,(b^{2}x^{8}+2\,abx^{5}}{54\,(a^{3}}$$

input
```
integrate(1/x^3/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/54*(60*b^2*x^6 + 96*a*b*x^3 - 40*sqrt(3)*(b^2*x^8 + 2*a*b*x^5 + a^2*x^2
)*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(-b^2/a^2)^(2/3) - sqrt(3)*b)
/b) + 20*(b^2*x^8 + 2*a*b*x^5 + a^2*x^2)*(-b^2/a^2)^(1/3)*log(b^2*x^2 + a*
b*x*(-b^2/a^2)^(1/3) + a^2*(-b^2/a^2)^(2/3)) - 40*(b^2*x^8 + 2*a*b*x^5 + a
^2*x^2)*(-b^2/a^2)^(1/3)*log(b*x - a*(-b^2/a^2)^(1/3)) + 27*a^2)/(a^3*b^2*
x^8 + 2*a^4*b*x^5 + a^5*x^2)
```

3.105.6 Sympy [F]

$$\int \frac{1}{x^3 (a^2 + 2abx^3 + b^2x^6)^{3/2}}\,dx = \int \frac{1}{x^3 \left((a + bx^3)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**3/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

output
```
Integral(1/(x**3*((a + b*x**3)**2)**(3/2)), x)
```

3.105.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.47

$$\int \frac{1}{x^3 (a^2 + 2abx^3 + b^2x^6)^{3/2}}\,dx = -\frac{20\,b^2x^6 + 32\,abx^3 + 9\,a^2}{18\,(a^3b^2x^8 + 2\,a^4bx^5 + a^5x^2)} - \frac{20\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^3\,\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{10\,\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,a^3\,\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{20\,\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^3\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(1/x^3/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/18*(20*b^2*x^6 + 32*a*b*x^3 + 9*a^2)/(a^3*b^2*x^8 + 2*a^4*b*x^5 + a^5*x
^2) - 20/27*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a
^3*(a/b)^(2/3)) + 10/27*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*(a/b)^
(2/3)) - 20/27*log(x + (a/b)^(1/3))/(a^3*(a/b)^(2/3))
```

3.105.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.58

$$\int \frac{1}{x^3 (a^2 + 2abx^3 + b^2x^6)^{3/2}}\,dx = \frac{20\,b\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^4 \mathrm{sgn}\left(bx^3 + a\right)} - \frac{20\,\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^4 \mathrm{sgn}\left(bx^3 + a\right)} - \frac{10\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,a^4 \mathrm{sgn}\left(bx^3 + a\right)} - \frac{20\,b^2x^6 + 32\,abx^3 + 9\,a^2}{18\left(bx^4 + ax\right)^2 a^3 \mathrm{sgn}\left(bx^3 + a\right)}$$

input
```
integrate(1/x^3/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

output
```
20/27*b*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^4*sgn(b*x^3 + a)) - 20/
27*sqrt(3)*(-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(
1/3))/(a^4*sgn(b*x^3 + a)) - 10/27*(-a*b^2)^(1/3)*log(x^2 + x*(-a/b)^(1/3)
 + (-a/b)^(2/3))/(a^4*sgn(b*x^3 + a)) - 1/18*(20*b^2*x^6 + 32*a*b*x^3 + 9*
a^2)/((b*x^4 + a*x)^2*a^3*sgn(b*x^3 + a))
```

3.105.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a^2 + 2abx^3 + b^2x^6)^{3/2}}\,dx = \int \frac{1}{x^3\left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^{3/2}}\,dx$$

input
```
int(1/(x^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)),x)
```

output
```
int(1/(x^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)), x)
```

3.106 $\int \frac{1}{x^4(a^2+2abx^3+b^2x^6)^{3/2}} dx$

3.106.1 Optimal result

Integrand size = 26, antiderivative size = 188

$$\int \frac{1}{x^4\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx=-\frac{2b}{3a^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{b}{6a^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}-\frac{a+bx^3}{3a^3x^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{3b(a+bx^3)\log(x)}{a^4\sqrt{a^2+2abx^3+b^2x^6}}+\frac{b(a+bx^3)\log\left(a+bx^3\right)}{a^4\sqrt{a^2+2abx^3+b^2x^6}}$$

output
```
-2/3*b/a^3/((b*x^3+a)^2)^(1/2)-1/6*b/a^2/(b*x^3+a)/((b*x^3+a)^2)^(1/2)+1/3
*(-b*x^3-a)/a^3/x^3/((b*x^3+a)^2)^(1/2)-3*b*(b*x^3+a)*ln(x)/a^4/((b*x^3+a)
^2)^(1/2)+b*(b*x^3+a)*ln(b*x^3+a)/a^4/((b*x^3+a)^2)^(1/2)
```

3.106.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 901 vs. 2(188) = 376.

Time = 1.33 (sec) , antiderivative size = 901, normalized size of antiderivative = 4.79

$$\int \frac{1}{x^4\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx=$$

$$-\frac{-2a^6-5a^5bx^3+2a^4b^2x^6+4a^3b^3x^9-ab^5x^{15}+2a^4\sqrt{a^2}\sqrt{\left(a+bx^3\right)^2}+3a^3\sqrt{a^2}bx^3\sqrt{\left(a+bx^3\right)^2}-5(a^2)^{3/2}\cdots}{\cdots}$$

input
```
Integrate[1/(x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]
```

output
```
-1/3*(-2*a^6 - 5*a^5*b*x^3 + 2*a^4*b^2*x^6 + 4*a^3*b^3*x^9 - a*b^5*x^15 +
2*a^4*Sqrt[a^2]*Sqrt[(a + b*x^3)^2] + 3*a^3*Sqrt[a^2]*b*x^3*Sqrt[(a + b*x^
3)^2] - 5*(a^2)^(3/2)*b^2*x^6*Sqrt[(a + b*x^3)^2] + a*Sqrt[a^2]*b^3*x^9*Sq
rt[(a + b*x^3)^2] - Sqrt[a^2]*b^4*x^12*Sqrt[(a + b*x^3)^2] + 6*b*x^3*((a^2
)^(3/2)*b^2*x^6 + a^4*(Sqrt[a^2] - Sqrt[(a + b*x^3)^2]) + a^3*b*x^3*(2*Sqr
t[a^2] - Sqrt[(a + b*x^3)^2]))*ArcTanh[(b*x^3)/(Sqrt[a^2] - Sqrt[(a + b*x^
3)^2])] - 6*b*x^3*(a^5 + 2*a^4*b*x^3 - (a^2)^(3/2)*b*x^3*Sqrt[(a + b*x^3)^
2] + a^3*(b^2*x^6 - Sqrt[a^2]*Sqrt[(a + b*x^3)^2]))*Log[x^3] + 3*a^5*b*x^3
*Log[Sqrt[a^2] - b*x^3 - Sqrt[(a + b*x^3)^2]] + 6*a^4*b^2*x^6*Log[Sqrt[a^2
] - b*x^3 - Sqrt[(a + b*x^3)^2]] + 3*a^3*b^3*x^9*Log[Sqrt[a^2] - b*x^3 - S
qrt[(a + b*x^3)^2]] - 3*a^3*Sqrt[a^2]*b*x^3*Sqrt[(a + b*x^3)^2]*Log[Sqrt[a
^2] - b*x^3 - Sqrt[(a + b*x^3)^2]] - 3*(a^2)^(3/2)*b^2*x^6*Sqrt[(a + b*x^3
)^2]*Log[Sqrt[a^2] - b*x^3 - Sqrt[(a + b*x^3)^2]] + 3*a^5*b*x^3*Log[Sqrt[a
^2] + b*x^3 - Sqrt[(a + b*x^3)^2]] + 6*a^4*b^2*x^6*Log[Sqrt[a^2] + b*x^3 -
 Sqrt[(a + b*x^3)^2]] + 3*a^3*b^3*x^9*Log[Sqrt[a^2] + b*x^3 - Sqrt[(a + b*
x^3)^2]] - 3*a^3*Sqrt[a^2]*b*x^3*Sqrt[(a + b*x^3)^2]*Log[Sqrt[a^2] + b*x^3
 - Sqrt[(a + b*x^3)^2]] - 3*(a^2)^(3/2)*b^2*x^6*Sqrt[(a + b*x^3)^2]*Log[Sq
rt[a^2] + b*x^3 - Sqrt[(a + b*x^3)^2]])/(a^4*Sqrt[a^2]*x^3*(a^2 + a*b*x^3
- Sqrt[a^2]*Sqrt[(a + b*x^3)^2])^2)
```

3.106.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.52, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1384, 27, 798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a^2+2abx^3+b^2x^6)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{b^3 (a+bx^3) \int \frac{1}{b^3 x^4 (bx^3+a)^3}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(a+bx^3) \int \frac{1}{x^4 (bx^3+a)^3}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{798}$$

$$\frac{(a+bx^3)\int \frac{1}{x^6(bx^3+a)^3}dx^3}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{54}$$

$$\frac{(a+bx^3)\int \left(\frac{3b^2}{a^4(bx^3+a)}+\frac{2b^2}{a^3(bx^3+a)^2}+\frac{b^2}{a^2(bx^3+a)^3}-\frac{3b}{a^4x^3}+\frac{1}{a^3x^6}\right)dx^3}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx^3)\left(-\frac{3b\log(x^3)}{a^4}+\frac{3b\log(a+bx^3)}{a^4}-\frac{2b}{a^3(a+bx^3)}-\frac{1}{a^3x^3}-\frac{b}{2a^2(a+bx^3)^2}\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

input
```
Int[1/(x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]
```

output
```
((a + b*x^3)*(-(1/(a^3*x^3)) - b/(2*a^2*(a + b*x^3)^2) - (2*b)/(a^3*(a + b
*x^3)) - (3*b*Log[x^3])/a^4 + (3*b*Log[a + b*x^3])/a^4))/(3*Sqrt[a^2 + 2*a
*b*x^3 + b^2*x^6])
```

3.106.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

3.106. $\int \frac{1}{x^4(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.106.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.49

method	result
pseudoelliptic	$-\frac{\left(-3b\,x^{3}\left(b\,x^{3}+a\right)^{2}\ln\left(b\,x^{3}+a\right)+3b\,x^{3}\left(b\,x^{3}+a\right)^{2}\ln\left(b\,x^{3}\right)+a\left(3b^{2}x^{6}+\frac{9}{2}ab\,x^{3}+a^{2}\right)\right)\operatorname{csgn}\left(b\,x^{3}+a\right)}{3\left(b\,x^{3}+a\right)^{2}a^{4}x^{3}}$
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(-\frac{b^{2}x^{6}}{a^{3}}-\frac{3b\,x^{3}}{2a^{2}}-\frac{1}{3a}\right)}{\left(b\,x^{3}+a\right)^{3}x^{3}}-\frac{3\sqrt{\left(b\,x^{3}+a\right)^{2}}\,b\ln\left(x\right)}{\left(b\,x^{3}+a\right)a^{4}}+\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,b\ln\left(-b\,x^{3}-a\right)}{\left(b\,x^{3}+a\right)a^{4}}$
default	$-\frac{\left(18b^{3}\ln\left(x\right)x^{9}-6\ln\left(b\,x^{3}+a\right)b^{3}x^{9}+36b^{2}a\ln\left(x\right)x^{6}-12\ln\left(b\,x^{3}+a\right)a\,b^{2}x^{6}+6b^{2}x^{6}a+18a^{2}b\ln\left(x\right)x^{3}-6\ln\left(b\,x^{3}+a\right)a^{2}b\,x^{3}+9a^{2}\right.}{6x^{3}a^{4}\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}$

input
```
int(1/x^4/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-3*b*x^3*(b*x^3+a)^2*ln(b*x^3+a)+3*b*x^3*(b*x^3+a)^2*ln(b*x^3)+a*(3*
b^2*x^6+9/2*a*b*x^3+a^2))*csgn(b*x^3+a)/(b*x^3+a)^2/a^4/x^3
```

3.106.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.63

$$\int \frac{1}{x^4 \left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = -\frac{6\,ab^2x^6+9\,a^2bx^3+2\,a^3-6\,\left(b^3x^9+2\,ab^2x^6+a^2bx^3\right)\log\left(bx^3+a\right)+18\,\left(b^3x^9+2\,ab^2x^6+a^2bx^3\right)\log\left(x\right)}{6\,\left(a^4b^2x^9+2\,a^5bx^6+a^6x^3\right)}$$

input
```
integrate(1/x^4/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/6*(6*a*b^2*x^6 + 9*a^2*b*x^3 + 2*a^3 - 6*(b^3*x^9 + 2*a*b^2*x^6 + a^2*b
*x^3)*log(b*x^3 + a) + 18*(b^3*x^9 + 2*a*b^2*x^6 + a^2*b*x^3)*log(x))/(a^4
*b^2*x^9 + 2*a^5*b*x^6 + a^6*x^3)
```

3.106.6 Sympy [F]

$$\int \frac{1}{x^4 (a^2 + 2abx^3 + b^2x^6)^{3/2}}\,dx = \int \frac{1}{x^4 \left((a + bx^3)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**4/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

output
```
Integral(1/(x**4*((a + b*x**3)**2)**(3/2)), x)
```

3.106.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.62

$$\int \frac{1}{x^4 (a^2 + 2abx^3 + b^2x^6)^{3/2}}\,dx = \frac{(-1)^{2\,abx^3+2\,a^2}\, b \log\left(\frac{2\,abx}{|x|} + \frac{2\,a^2}{x^2|x|}\right)}{a^4} - \frac{b}{\sqrt{b^2x^6 + 2\,abx^3 + a^2}a^3} - \frac{1}{6\left(x^3 + \frac{a}{b}\right)^2 a^2 b} - \frac{1}{3\sqrt{b^2x^6 + 2\,abx^3 + a^2}a^2x^3}$$

input
```
integrate(1/x^4/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

output
```
(-1)^(2*a*b*x^3 + 2*a^2)*b*log(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x)))/a^4 -
b/(sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a^3) - 1/6/((x^3 + a/b)^2*a^2*b) - 1/3/
(sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a^2*x^3)
```

3.106.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.64

$$\int \frac{1}{x^4 (a^2 + 2abx^3 + b^2x^6)^{3/2}} \, dx = \frac{b \log (|bx^3 + a|)}{a^4 \text{sgn} (bx^3 + a)} - \frac{3 \, b \log (|x|)}{a^4 \text{sgn} (bx^3 + a)} - \frac{9 \, b^3 x^6 + 22 \, ab^2 x^3 + 14 \, a^2 b}{6 \, (bx^3 + a)^2 a^4 \text{sgn} (bx^3 + a)} + \frac{3 \, bx^3 - a}{3 \, a^4 x^3 \text{sgn} (bx^3 + a)}$$

input
```
integrate(1/x^4/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

output
```
b*log(abs(b*x^3 + a))/(a^4*sgn(b*x^3 + a)) - 3*b*log(abs(x))/(a^4*sgn(b*x^
3 + a)) - 1/6*(9*b^3*x^6 + 22*a*b^2*x^3 + 14*a^2*b)/((b*x^3 + a)^2*a^4*sgn
(b*x^3 + a)) + 1/3*(3*b*x^3 - a)/(a^4*x^3*sgn(b*x^3 + a))
```

3.106.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a^2 + 2abx^3 + b^2x^6)^{3/2}} \, dx = \int \frac{1}{x^4 \, (a^2 + 2 \, a \, b \, x^3 + b^2 \, x^6)^{3/2}} \, dx$$

input
```
int(1/(x^4*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)),x)
```

output
```
int(1/(x^4*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)), x)
```

3.107 $\int \frac{x^6}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$

3.107.1 Optimal result

Integrand size = 26, antiderivative size = 359

$$\begin{aligned}\int \frac{x^6}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx &= \frac{5x}{486a^2b^2\sqrt{a^2+2abx^3+b^2x^6}} \\ &- \frac{x^4}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x}{27b^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}} \\ &+ \frac{x}{162ab^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{5(a+bx^3)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}a^{8/3}b^{7/3}\sqrt{a^2+2abx^3+b^2x^6}} \\ &+ \frac{5(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{8/3}b^{7/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{5(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{1458a^{8/3}b^{7/3}\sqrt{a^2+2abx^3+b^2x^6}}\end{aligned}$$

output
```
5/486*x/a^2/b^2/((b*x^3+a)^2)^(1/2)-1/12*x^4/b/(b*x^3+a)^3/((b*x^3+a)^2)^(
1/2)-1/27*x/b^2/(b*x^3+a)^2/((b*x^3+a)^2)^(1/2)+1/162*x/a/b^2/(b*x^3+a)/((
b*x^3+a)^2)^(1/2)+5/729*(b*x^3+a)*ln(a^(1/3)+b^(1/3)*x)/a^(8/3)/b^(7/3)/((
b*x^3+a)^2)^(1/2)-5/1458*(b*x^3+a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^
2)/a^(8/3)/b^(7/3)/((b*x^3+a)^2)^(1/2)-5/729*(b*x^3+a)*arctan(1/3*(a^(1/3)
-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(8/3)/b^(7/3)*3^(1/2)/((b*x^3+a)^2)^(1/2)
```

3.107.2 Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.61

$$\int \frac{x^6}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx = \frac{\left(a+bx^3\right)\left(243a\sqrt[3]{b}x-351\sqrt[3]{b}x\left(a+bx^3\right)+\frac{18\sqrt[3]{b}x\left(a+bx^3\right)^2}{a}+\frac{30\sqrt[3]{b}x\left(a+bx^3\right)^3}{a^2}+\right.}{29}$$

input
```
Integrate[x^6/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
((a + b*x^3)*(243*a*b^(1/3)*x - 351*b^(1/3)*x*(a + b*x^3) + (18*b^(1/3)*x*
(a + b*x^3)^2)/a + (30*b^(1/3)*x*(a + b*x^3)^3)/a^2 + (20*Sqrt[3]*(a + b*x
^3)^4*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/a^(8/3) + (20*(a
 + b*x^3)^4*Log[a^(1/3) + b^(1/3)*x])/a^(8/3) - (10*(a + b*x^3)^4*Log[a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(8/3)))/(2916*b^(7/3)*((a + b*x^
3)^2)^(5/2))
```

3.107.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.68, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {1384, 27, 817, 817, 749, 749, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b^5\left(a+bx^3\right)\int \frac{x^6}{b^5\left(bx^3+a\right)^5}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(a+bx^3\right)\int \frac{x^6}{\left(bx^3+a\right)^5}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{817}$$

$$\frac{\left(a+bx^3\right)\left(\frac{\int \frac{x^3}{\left(bx^3+a\right)^4}dx}{3b}-\frac{x^4}{12b\left(a+bx^3\right)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{ 817}$$

$$\frac{\left(a+bx^3\right)\left(\frac{\frac{\int \frac{1}{\left(bx^3+a\right)^3}dx}{9b}-\frac{x}{9b\left(a+bx^3\right)^3}}{3b}-\frac{x^4}{12b\left(a+bx^3\right)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{ 749}$$

$$\frac{\left(a+bx^3\right)\left(\frac{\frac{\frac{5\int \frac{1}{\left(bx^3+a\right)^2}dx}{6a}+\frac{x}{6a\left(a+bx^3\right)^2}}{9b}-\frac{x}{9b\left(a+bx^3\right)^3}}{3b}-\frac{x^4}{12b\left(a+bx^3\right)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{ 749}$$

$$\frac{\left(a+bx^3\right)\left(\frac{\frac{\frac{5\left(\frac{2\int \frac{1}{bx^3+a}dx}{3a}+\frac{x}{3a\left(a+bx^3\right)}\right)}{6a}+\frac{x}{6a\left(a+bx^3\right)^2}}{9b}-\frac{x}{9b\left(a+bx^3\right)^3}}{3b}-\frac{x^4}{12b\left(a+bx^3\right)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{ 750}$$

3.107. $\int \frac{x^6}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$

$$\frac{\left(a+bx^3\right)\left(\frac{\frac{5\left(\frac{2\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{3a}+\frac{x}{3a\left(a+bx^3\right)}\right)}{6a}+\frac{x}{6a\left(a+bx^3\right)^2}}{9b}-\frac{x}{9b\left(a+bx^3\right)^3}}{3b}-\frac{x^4}{12b(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\big\downarrow$ 16

$$\frac{\left(a+bx^3\right)\left(\frac{\frac{5\left(\frac{2\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a\left(a+bx^3\right)}\right)}{6a}+\frac{x}{6a\left(a+bx^3\right)^2}}{9b}-\frac{x}{9b\left(a+bx^3\right)^3}}{3b}-\frac{x^4}{12b(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\big\downarrow$ 1142

3.107. $\int \frac{x^6}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

$$\frac{\left(a+bx^3\right)\left(\frac{\frac{5\left(\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a\left(a+bx^3\right)}\right)}{6a}+\frac{x}{6a\left(a+bx^3\right)^2}}{9b}-\frac{}{9b\left(a\ldots\right.}}{3b}\ldots\right.}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\downarrow$ 25

3.107. $\int\frac{x^6}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$

$$\frac{\left(a+bx^3\right)\left(\frac{\frac{5\left(\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a\left(a+bx^3\right)}\right)}{6a}+\frac{x}{6a\left(a+bx^3\right)^2}}{9b}-\frac{\cdots}{9b\left(a+\cdots\right.}}{3b}\cdots\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\big\downarrow$ 27

3.107. $\int \frac{x^6}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

$$\frac{\left(a+bx^3\right)\left(\frac{\frac{5\left(\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a\left(a+bx^3\right)}\right)}{6a}+\frac{x}{6a\left(a+bx^3\right)^2}}{9b}-\frac{\cdots}{9b\left(a\cdots\right.}}{3b}\cdots\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\downarrow$ 1082

3.107. $\int \frac{x^6}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

$$\frac{\left(a+bx^3\right)\left(\frac{\frac{5\left(\frac{2\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a\left(a+bx^3\right)}\right)}{6a}+\frac{x}{6a\left(a+bx^3\right)^2}}{9b}-\frac{\cdots}{9b}}{3b}\cdots\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 217$$

3.107. $\int \frac{x^6}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

$$\frac{\left(a+bx^3\right)\left(\frac{\frac{\frac{5\left(\frac{2\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a\left(a+bx^3\right)}\right)}{6a}+\frac{x}{6a\left(a+bx^3\right)^2}}{9b}-\frac{x}{9b\left(a+bx^3\right)^3}}{3b}-\right.}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\downarrow$ 1103

3.107. $\int \frac{x^6}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

$$\frac{\left(a+bx^3\right)\left(\frac{\frac{\frac{5\left(\frac{2\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a\left(a+bx^3\right)}\right)}{6a}+\frac{x}{6a\left(a+bx^3\right)^2}}{9b}-\frac{x}{9b\left(a+bx^3\right)^3}}{3b}-\right.}{\sqrt{a^2+2abx^3+b^2x^6}}$$

input
```
Int[x^6/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

3.107. $\int \frac{x^6}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

output
```
((a + b*x^3)*(-1/12*x^4/(b*(a + b*x^3)^4) + (-1/9*x/(b*(a + b*x^3)^3) + (x
/(6*a*(a + b*x^3)^2) + (5*(x/(3*a*(a + b*x^3)) + (2*(Log[a^(1/3) + b^(1/3)
*x]/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/S
qrt[3]])/b^(1/3)) - Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1
/3)))/(3*a^(2/3))))/(3*a)))/(6*a))/(9*b))/(3*b)))/Sqrt[a^2 + 2*a*b*x^3 + b
^2*x^6]
```

3.107.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

3.107. $\int \frac{x^6}{(a^2+2abx^3+b^2x^6)^{5/2}} \, dx$

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

3.107.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 5.37 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.29

method	result
risch	$\frac{\sqrt{(b x^3+a)^2}\left(\frac{5b x^{10}}{486a^2}+\frac{x^7}{27a}-\frac{25x^4}{324b}-\frac{5ax}{243b^2}\right)}{(b x^3+a)^5}+\frac{5\sqrt{(b x^3+a)^2}\left(\sum_{\_R=\mathrm{RootOf}(\_Z^3b+a)}\frac{\ln(x-\_R)}{\_R^2}\right)}{729(b x^3+a)a^2b^3}$
default	$-\frac{\left(20\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)b^4x^{12}-20\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)b^4x^{12}+10\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)b^4x^{12}-30\left(\frac{a}{b}\right)^{\frac{2}{3}}b^4x^{10}+80\sqrt{3}\,\mathrm{arct}}{}$

input
```
int(x^6/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^3+a)^2)^(1/2)/(b*x^3+a)^5*(5/486*b/a^2*x^10+1/27/a*x^7-25/324/b*x^4-
5/243*a/b^2*x)+5/729*((b*x^3+a)^2)^(1/2)/(b*x^3+a)/a^2/b^3*sum(1/_R^2*ln(x
-_R),_R=RootOf(_Z^3*b+a))
```

3.107.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 723, normalized size of antiderivative = 2.01

$$\int \frac{x^6}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \left[\frac{30\,a^2b^4x^{10}+108\,a^3b^3x^7-225\,a^4b^2x^4-60\,a^5bx+30\,\sqrt{\tfrac{1}{3}}(ab^5x^{12}+4\,a^2b^4x^9}{}\right.$$

input
```
integrate(x^6/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/2916*(30*a^2*b^4*x^10 + 108*a^3*b^3*x^7 - 225*a^4*b^2*x^4 - 60*a^5*b*x
+ 30*sqrt(1/3)*(a*b^5*x^12 + 4*a^2*b^4*x^9 + 6*a^3*b^3*x^6 + 4*a^4*b^2*x^3
 + a^5*b)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^
2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2
*b)^(1/3)/b))/(b*x^3 + a)) - 10*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 +
4*a^3*b*x^3 + a^4)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(
1/3)*a) + 20*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*
(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^4*b^7*x^12 + 4*a^5*b^6*x^9 +
6*a^6*b^5*x^6 + 4*a^7*b^4*x^3 + a^8*b^3), 1/2916*(30*a^2*b^4*x^10 + 108*a^
3*b^3*x^7 - 225*a^4*b^2*x^4 - 60*a^5*b*x + 60*sqrt(1/3)*(a*b^5*x^12 + 4*a^
2*b^4*x^9 + 6*a^3*b^3*x^6 + 4*a^4*b^2*x^3 + a^5*b)*sqrt((a^2*b)^(1/3)/b)*a
rctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b
)/a^2) - 10*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(
a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 20*(b^4*x^
12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(a^2*b)^(2/3)*log(a*
b*x + (a^2*b)^(2/3)))/(a^4*b^7*x^12 + 4*a^5*b^6*x^9 + 6*a^6*b^5*x^6 + 4*a^
7*b^4*x^3 + a^8*b^3)]
```

3.107.6 Sympy [F]

$$\int \frac{x^6}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \int \frac{x^6}{\left((a+bx^3)^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**6/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

output
```
Integral(x**6/((a + b*x**3)**2)**(5/2), x)
```

3.107.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.54

$$\int \frac{x^6}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \frac{10\,b^3x^{10}+36\,ab^2x^7-75\,a^2bx^4-20\,a^3x}{972\,(a^2b^6x^{12}+4\,a^3b^5x^9+6\,a^4b^4x^6+4\,a^5b^3x^3+a^6b^2)} + \frac{5\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729\,a^2b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{5\,\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{1458\,a^2b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{5\,\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{729\,a^2b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(x^6/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

output
```
1/972*(10*b^3*x^10 + 36*a*b^2*x^7 - 75*a^2*b*x^4 - 20*a^3*x)/(a^2*b^6*x^12
 + 4*a^3*b^5*x^9 + 6*a^4*b^4*x^6 + 4*a^5*b^3*x^3 + a^6*b^2) + 5/729*sqrt(3
)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b^3*(a/b)^(2/3)
) - 5/1458*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b^3*(a/b)^(2/3)) +
5/729*log(x + (a/b)^(1/3))/(a^2*b^3*(a/b)^(2/3))
```

3.107.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.57

$$
\begin{aligned}
\int \frac{x^6}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx =& -\frac{5\,\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{1458\,(-ab^2)^{\frac{2}{3}}\,a^2b\mathrm{sgn}\left(bx^3+a\right)} \\
&-\frac{5\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{729\,a^3b^2\mathrm{sgn}\left(bx^3+a\right)}+\frac{5\sqrt{3}(-ab^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729\,a^3b^3\mathrm{sgn}\left(bx^3+a\right)} \\
&+\frac{10\,b^3x^{10}+36\,ab^2x^7-75\,a^2bx^4-20\,a^3x}{972\,(bx^3+a)^4a^2b^2\mathrm{sgn}\left(bx^3+a\right)}
\end{aligned}
$$

input
```
integrate(x^6/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

output
```
-5/1458*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^2*b*sgn
(b*x^3 + a)) - 5/729*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b^2*sgn(
b*x^3 + a)) + 5/729*sqrt(3)*(-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b
)^(1/3))/(-a/b)^(1/3))/(a^3*b^3*sgn(b*x^3 + a)) + 1/972*(10*b^3*x^10 + 36*
a*b^2*x^7 - 75*a^2*b*x^4 - 20*a^3*x)/((b*x^3 + a)^4*a^2*b^2*sgn(b*x^3 + a)
)
```

3.107.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx=\int \frac{x^6}{{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{5/2}} \,d x$$

input
```
int(x^6/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

output
```
int(x^6/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

3.108 $\int \frac{x^5}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$

3.108.1 Optimal result

Integrand size = 26, antiderivative size = 78

$$\int \frac{x^5}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx=\frac{a}{12b^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{1}{9b^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}$$

output
```
1/12*a/b^2/(b*x^3+a)^3/((b*x^3+a)^2)^(1/2)-1/9/b^2/(b*x^3+a)^2/((b*x^3+a)^
2)^(1/2)
```

3.108.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 232 vs. 2(78) = 156.

Time = 0.50 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.97

$$\int \frac{x^5}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx=-\frac{x^6\left(3\sqrt{a^2}b^6x^{18}+3a^3b^3x^9\sqrt{\left(a+bx^3\right)^2}-3a^2b^4x^{12}\sqrt{\left(a+bx^3\right)^2}+3ab^5x^{15}\sqrt{\left(a+bx^3\right)^2}+a^4b^2x^6\left(\sqrt{a^2}-3\sqrt{\right.\right.}{36a^7\left(a+bx^3\right)^3\left(a^2+abx^3-\sqrt{a^2}\sqrt{(\right.}$$

input
```
Integrate[x^5/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
-1/36*(x^6*(3*Sqrt[a^2]*b^6*x^18 + 3*a^3*b^3*x^9*Sqrt[(a + b*x^3)^2] - 3*a
^2*b^4*x^12*Sqrt[(a + b*x^3)^2] + 3*a*b^5*x^15*Sqrt[(a + b*x^3)^2] + a^4*b
^2*x^6*(Sqrt[a^2] - 3*Sqrt[(a + b*x^3)^2]) + 6*a^6*(Sqrt[a^2] - Sqrt[(a +
b*x^3)^2]) + 2*a^5*b*x^3*(2*Sqrt[a^2] + Sqrt[(a + b*x^3)^2])))/(a^7*(a + b
*x^3)^3*(a^2 + a*b*x^3 - Sqrt[a^2]*Sqrt[(a + b*x^3)^2]))
```

3.108.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1693, 1100, 1078}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a^2+2abx^3+b^2x^6)^{5/2}}dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int \frac{x^3}{(b^2x^6+2abx^3+a^2)^{5/2}}dx^3$$

$$\Big\downarrow \text{1100}$$

$$\frac{1}{3}\left(-\frac{a\int \frac{1}{(b^2x^6+2abx^3+a^2)^{5/2}}dx^3}{b}-\frac{1}{3b^2(a^2+2abx^3+b^2x^6)^{3/2}}\right)$$

$$\Big\downarrow \text{1078}$$

$$\frac{1}{3}\left(\frac{a}{4b^2(a+bx^3)(a^2+2abx^3+b^2x^6)^{3/2}}-\frac{1}{3b^2(a^2+2abx^3+b^2x^6)^{3/2}}\right)$$

input
```
Int[x^5/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
(-1/3*1/(b^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)) + a/(4*b^2*(a + b*x^3)*(a^
2 + 2*a*b*x^3 + b^2*x^6)^(3/2)))/3
```

3.108.3.1 Defintions of rubi rules used

rule 1078
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[2*((a + b*x
+ c*x^2)^(p + 1)/((2*p + 1)*(b + 2*c*x))), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 1100
```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.108.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.40

method	result	size
pseudoelliptic	$-\frac{\left(4 b \,x^{3}+a \right) \operatorname{csgn}\left(b \,x^{3}+a \right)}{36 b^{2} \left(b \,x^{3}+a \right)^{4}}$	31
gosper	$-\frac{\left(b \,x^{3}+a \right) \left(4 b \,x^{3}+a \right)}{36 b^{2} \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{5}{2}}}$	32
default	$-\frac{\left(b \,x^{3}+a \right) \left(4 b \,x^{3}+a \right)}{36 b^{2} \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{5}{2}}}$	32
risch	$\frac{\sqrt{\left(b \,x^{3}+a \right)^{2}}\, \left(-\frac{x^{3}}{9 b}-\frac{a}{36 b^{2}}\right)}{\left(b \,x^{3}+a \right)^{5}}$	37

input
```
int(x^5/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/36*(4*b*x^3+a)*csgn(b*x^3+a)/b^2/(b*x^3+a)^4
```

3.108.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.74

$$\int \frac{x^5}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx = -\frac{4\,bx^3+a}{36\left(b^6x^{12}+4\,ab^5x^9+6\,a^2b^4x^6+4\,a^3b^3x^3+a^4b^2\right)}$$

input
```
integrate(x^5/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/36*(4*b*x^3 + a)/(b^6*x^12 + 4*a*b^5*x^9 + 6*a^2*b^4*x^6 + 4*a^3*b^3*x^
3 + a^4*b^2)
```

3.108.6 Sympy [F]

$$\int \frac{x^5}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx = \int \frac{x^5}{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**5/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

output
```
Integral(x**5/((a + b*x**3)**2)**(5/2), x)
```

3.108.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.55

$$\int \frac{x^5}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx = -\frac{1}{9\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{3}{2}}b^2}+\frac{a}{12\left(x^3+\frac{a}{b}\right)^4b^6}$$

input
```
integrate(x^5/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/9/((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*b^2) + 1/12*a/((x^3 + a/b)^4*b^6)
```

3.108.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.41

$$\int \frac{x^5}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = -\frac{4\,bx^3+a}{36\,(bx^3+a)^4b^2\mathrm{sgn}\,(bx^3+a)}$$

input
```
integrate(x^5/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

output
```
-1/36*(4*b*x^3 + a)/((b*x^3 + a)^4*b^2*sgn(b*x^3 + a))
```

3.108.9 Mupad [B] (verification not implemented)

Time = 8.39 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.54

$$\int \frac{x^5}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = -\frac{(4\,b\,x^3+a)\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{36\,b^2\,(b\,x^3+a)^5}$$

input
```
int(x^5/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

output
```
-((a + 4*b*x^3)*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(36*b^2*(a + b*x^3)^5)
```

3.109 $\int \frac{x^4}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

3.109.1 Optimal result

Integrand size = 26, antiderivative size = 368

$$\begin{aligned}\int \frac{x^4}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx &= \frac{7x^2}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}}\\ &- \frac{x^2}{12b(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{54ab(a+bx^3)^2\sqrt{a^2+2abx^3+b^2x^6}}\\ &+ \frac{7x^2}{324a^2b(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{7(a+bx^3)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}a^{10/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}\\ &- \frac{7(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{10/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{7(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{1458a^{10/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}\end{aligned}$$

output
```
7/243*x^2/a^3/b/((b*x^3+a)^2)^(1/2)-1/12*x^2/b/(b*x^3+a)^3/((b*x^3+a)^2)^(
1/2)+1/54*x^2/a/b/(b*x^3+a)^2/((b*x^3+a)^2)^(1/2)+7/324*x^2/a^2/b/(b*x^3+a
)/((b*x^3+a)^2)^(1/2)-7/729*(b*x^3+a)*ln(a^(1/3)+b^(1/3)*x)/a^(10/3)/b^(5/
3)/((b*x^3+a)^2)^(1/2)+7/1458*(b*x^3+a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/
3)*x^2)/a^(10/3)/b^(5/3)/((b*x^3+a)^2)^(1/2)-7/729*(b*x^3+a)*arctan(1/3*(a
^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(10/3)/b^(5/3)*3^(1/2)/((b*x^3+a)^2
)^(1/2)
```

3.109.2 Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.62

$$\int \frac{x^4}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx = \frac{(a+bx^3)\left(-243a^{10/3}b^{2/3}x^2+54a^{7/3}b^{2/3}x^2(a+bx^3)+63a^{4/3}b^{2/3}x^2(a+bx^3)\right.}{}$$

input
```
Integrate[x^4/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
((a + b*x^3)*(-243*a^(10/3)*b^(2/3)*x^2 + 54*a^(7/3)*b^(2/3)*x^2*(a + b*x^
3) + 63*a^(4/3)*b^(2/3)*x^2*(a + b*x^3)^2 + 84*a^(1/3)*b^(2/3)*x^2*(a + b*
x^3)^3 + 28*Sqrt[3]*(a + b*x^3)^4*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]
*a^(1/3))] - 28*(a + b*x^3)^4*Log[a^(1/3) + b^(1/3)*x] + 14*(a + b*x^3)^4*
Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(2916*a^(10/3)*b^(5/3)*((
a + b*x^3)^2)^(5/2))
```

3.109.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.69, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {1384, 27, 817, 819, 819, 819, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b^5\left(a+bx^3\right)\int \frac{x^4}{b^5\left(bx^3+a\right)^5}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(a+bx^3\right)\int \frac{x^4}{\left(bx^3+a\right)^5}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{817}$$

$$\frac{(a+bx^3)\left(\frac{\int \frac{x}{(bx^3+a)^4}dx}{6b}-\frac{x^2}{12b(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 819$$

$$\frac{(a+bx^3)\left(\frac{\frac{7\int \frac{x}{(bx^3+a)^3}dx}{9a}+\frac{x^2}{9a(a+bx^3)^3}}{6b}-\frac{x^2}{12b(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 819$$

$$\frac{(a+bx^3)\left(\frac{\frac{7\left(\frac{2\int \frac{x}{(bx^3+a)^2}dx}{3a}+\frac{x^2}{6a(a+bx^3)^2}\right)}{9a}+\frac{x^2}{9a(a+bx^3)^3}}{6b}-\frac{x^2}{12b(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 819$$

$$\frac{(a+bx^3)\left(\frac{\frac{7\left(\frac{2\left(\frac{\int \frac{x}{bx^3+a}dx}{3a}+\frac{x^2}{3a(a+bx^3)}\right)}{3a}+\frac{x^2}{6a(a+bx^3)^2}\right)}{9a}+\frac{x^2}{9a(a+bx^3)^3}}{6b}-\frac{x^2}{12b(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 821$$

3.109. $\int \frac{x^4}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

$$\frac{(a+bx^3)\left(\frac{\frac{7\left(\frac{2\left(\frac{\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}\right)}{3a}+\frac{x^2}{6a\left(a+bx^3\right)^2}\right)}{9a}+\frac{x^2}{9a\left(a+bx^3\right)^3}}{6b}-\frac{x^2}{12b(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 16$$

3.109. $\int \frac{x^4}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

$$\frac{(a+bx^3)\left(\frac{\frac{7\left(\frac{2\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}\right)}{3a}+\frac{x^2}{6a\left(a+bx^3\right)^2}\right)}{9a}+\frac{x^2}{9a\left(a+bx^3\right)^3}}{6b}-\frac{x^2}{12b(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 1142$$

3.109. $\int \frac{x^4}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

$$\frac{(a+bx^3)\left(\frac{7\left(\frac{2\left(\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}\right)}{3a}+\frac{x^2}{6a\left(a+bx^3\right)^2}\right)}{9a}+\frac{\cdots}{9a\left(\cdots\right.}}{6b}\cdots\right.}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\downarrow$ 25

3.109. $\int \frac{x^4}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

$$\frac{(a+bx^3)\left(\frac{7\left(\frac{2\left(\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a(a+bx^3)}\right)}{3a}+\frac{x^2}{6a(a+bx^3)^2}\right)}{9a}+\frac{x}{9a(a+\cdots}}{6b}\cdots\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\downarrow$ 27

3.109. $\int \frac{x^4}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

$$\frac{\left(a+bx^3\right)\left(\frac{\frac{7\left(\frac{2\left(\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}\right)}{3a}+\frac{x^2}{6a\left(a+bx^3\right)^2}\right)}{9a}+\frac{\cdots}{9a\left(a \cdots\right.}}{6b}\cdots\right.}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\downarrow$ 1082

3.109. $\int \frac{x^4}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$

$$\frac{\left(a+bx^3\right)\left(\frac{\frac{7\left(\frac{2\left(\frac{\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}\right)}{3a}+\frac{x^2}{6a\left(a+bx^3\right)^2}\right)}{9a}+\frac{\cdots}{9a}}{6b}\cdots\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 217$$

3.109. $\int \frac{x^4}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

$$\frac{\left(a+bx^3\right)\left(\frac{\frac{7\left(\frac{2\left(\frac{\frac{-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}\right)}{3a}+\frac{x^2}{6a\left(a+bx^3\right)^2}\right)}{9a}+\frac{x^2}{9a\left(a+bx^3\right)^3}}{6b}-\right.}{\sqrt{a^2+2abx^3+b^2x^6}}$$

3.109. $\int\frac{x^4}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$

$\downarrow$ 1103

3.109. $\int \frac{x^4}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

$$\frac{\left(a+bx^3\right)\left(\frac{\frac{7\left(\frac{2\left(\frac{\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}\right)}{3a}+\frac{x^2}{6a\left(a+bx^3\right)^2}\right)}{9a}+\frac{x^2}{9a\left(a+bx^3\right)^3}}{6b}-\right.}{\sqrt{a^2+2abx^3+b^2x^6}}$$

3.109. $\int \frac{x^4}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

input
```
Int[x^4/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
((a + b*x^3)*(-1/12*x^2/(b*(a + b*x^3)^4) + (x^2/(9*a*(a + b*x^3)^3) + (7*
(x^2/(6*a*(a + b*x^3)^2) + (2*(x^2/(3*a*(a + b*x^3)) + (-1/3*Log[a^(1/3) +
 b^(1/3)*x]/(a^(1/3)*b^(2/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1
/3))/Sqrt[3]])/b^(1/3)) + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(
2*b^(1/3)))/(3*a^(1/3)*b^(1/3))))/(3*a)))/(3*a)))/(9*a))/(6*b)))/Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6]
```

3.109.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

3.109.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.70 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.30

method	result
risch	$\frac{\sqrt{(b\,x^3+a)^2}\,\left(\frac{7b^2x^{11}}{243a^3}+\frac{35b\,x^8}{324a^2}+\frac{4x^5}{27a}-\frac{7x^2}{486b}\right)}{(b\,x^3+a)^5}+\frac{7\sqrt{(b\,x^3+a)^2}\,\left(\sum_{\_R=\mathrm{RootOf}(\_Z^3b+a)}\frac{\ln(x-\_R)}{\_R}\right)}{729(b\,x^3+a)a^3b^2}$
default	$-\frac{\left(28\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)b^4x^{12}+28\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)b^4x^{12}-14\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)b^4x^{12}-84\left(\frac{a}{b}\right)^{\frac{1}{3}}b^4x^{11}+112\sqrt{3}\,\mathrm{arc}}{}$

input
```
int(x^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^3+a)^2)^(1/2)/(b*x^3+a)^5*(7/243*b^2/a^3*x^11+35/324*b/a^2*x^8+4/27/
a*x^5-7/486/b*x^2)+7/729*((b*x^3+a)^2)^(1/2)/(b*x^3+a)/a^3/b^2*sum(1/_R*ln
(x-_R),_R=RootOf(_Z^3*b+a))
```

3.109.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 734, normalized size of antiderivative = 1.99

$$\int \frac{x^4}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \left[\frac{84\,ab^5x^{11}+315\,a^2b^4x^8+432\,a^3b^3x^5-42\,a^4b^2x^2+42\,\sqrt{\frac{1}{3}}(ab^5x^{12}+4\,a^2b^4x}{}\right.$$

input
```
integrate(x^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/2916*(84*a*b^5*x^11 + 315*a^2*b^4*x^8 + 432*a^3*b^3*x^5 - 42*a^4*b^2*x^
2 + 42*sqrt(1/3)*(a*b^5*x^12 + 4*a^2*b^4*x^9 + 6*a^3*b^3*x^6 + 4*a^4*b^2*x
^3 + a^5*b)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b
*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(
-a*b^2)^(2/3)*x)/(b*x^3 + a)) + 14*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6
 + 4*a^3*b*x^3 + a^4)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-
a*b^2)^(2/3)) - 28*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 +
 a^4)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^4*b^7*x^12 + 4*a^5*b^6*
x^9 + 6*a^6*b^5*x^6 + 4*a^7*b^4*x^3 + a^8*b^3), 1/2916*(84*a*b^5*x^11 + 31
5*a^2*b^4*x^8 + 432*a^3*b^3*x^5 - 42*a^4*b^2*x^2 + 84*sqrt(1/3)*(a*b^5*x^1
2 + 4*a^2*b^4*x^9 + 6*a^3*b^3*x^6 + 4*a^4*b^2*x^3 + a^5*b)*sqrt(-(-a*b^2)^
(1/3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)
/b) + 14*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(-a*
b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 28*(b^4*x^
12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(-a*b^2)^(2/3)*log(b
*x - (-a*b^2)^(1/3)))/(a^4*b^7*x^12 + 4*a^5*b^6*x^9 + 6*a^6*b^5*x^6 + 4*a^
7*b^4*x^3 + a^8*b^3)]
```

3.109.6 Sympy [F]

$$\int \frac{x^4}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \int \frac{x^4}{\left((a+bx^3)^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**4/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

output
```
Integral(x**4/((a + b*x**3)**2)**(5/2), x)
```

3.109.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.53

$$\int \frac{x^4}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \frac{28\,b^3x^{11}+105\,ab^2x^8+144\,a^2bx^5-14\,a^3x^2}{972\,(a^3b^5x^{12}+4\,a^4b^4x^9+6\,a^5b^3x^6+4\,a^6b^2x^3+a^7b)} + \frac{7\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729\,a^3b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{7\,\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{1458\,a^3b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{7\,\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{729\,a^3b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate(x^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

output
```
1/972*(28*b^3*x^11 + 105*a*b^2*x^8 + 144*a^2*b*x^5 - 14*a^3*x^2)/(a^3*b^5*
x^12 + 4*a^4*b^4*x^9 + 6*a^5*b^3*x^6 + 4*a^6*b^2*x^3 + a^7*b) + 7/729*sqrt
(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^3*b^2*(a/b)^(1/
3)) + 7/1458*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*b^2*(a/b)^(1/3))
- 7/729*log(x + (a/b)^(1/3))/(a^3*b^2*(a/b)^(1/3))
```

3.109.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.56

$$\int \frac{x^4}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = -\frac{7\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{1458\,(-ab^2)^{\frac{1}{3}}\,a^3b\,\mathrm{sgn}\,(bx^3+a)} - \frac{7\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{729\,a^4b\,\mathrm{sgn}\,(bx^3+a)} - \frac{7\sqrt{3}(-ab^2)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729\,a^4b^3\,\mathrm{sgn}\,(bx^3+a)} + \frac{28\,b^3x^{11}+105\,ab^2x^8+144\,a^2bx^5-14\,a^3x^2}{972\,(bx^3+a)^4a^3b\,\mathrm{sgn}\,(bx^3+a)}$$

input
```
integrate(x^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

output
```
-7/1458*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(1/3)*a^3*b*sgn
(b*x^3 + a)) - 7/729*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a^4*b*sgn(b*
x^3 + a)) - 7/729*sqrt(3)*(-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^
(1/3))/(-a/b)^(1/3))/(a^4*b^3*sgn(b*x^3 + a)) + 1/972*(28*b^3*x^11 + 105*a
*b^2*x^8 + 144*a^2*b*x^5 - 14*a^3*x^2)/((b*x^3 + a)^4*a^3*b*sgn(b*x^3 + a)
)
```

3.109.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \int \frac{x^4}{{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{5/2}} \,d x$$

input
```
int(x^4/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

output
```
int(x^4/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

3.110 $\int \frac{x^3}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$

3.110.1 Optimal result

Integrand size = 26, antiderivative size = 360

$$\int \frac{x^3}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx=\frac{5x}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}}-\frac{x}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}+\frac{x}{108ab\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{x}{81a^2b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}-\frac{10(a+bx^3)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}a^{11/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{10(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{11/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{5(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{729a^{11/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

output
```
5/243*x/a^3/b/((b*x^3+a)^2)^(1/2)-1/12*x/b/(b*x^3+a)^3/((b*x^3+a)^2)^(1/2)
+1/108*x/a/b/(b*x^3+a)^2/((b*x^3+a)^2)^(1/2)+1/81*x/a^2/b/(b*x^3+a)/((b*x^
3+a)^2)^(1/2)+10/729*(b*x^3+a)*ln(a^(1/3)+b^(1/3)*x)/a^(11/3)/b^(4/3)/((b*
x^3+a)^2)^(1/2)-5/729*(b*x^3+a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/
a^(11/3)/b^(4/3)/((b*x^3+a)^2)^(1/2)-10/729*(b*x^3+a)*arctan(1/3*(a^(1/3)-
2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(11/3)/b^(4/3)*3^(1/2)/((b*x^3+a)^2)^(1/2)
```

3.110.2 Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.61

$$\int \frac{x^3}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \frac{(a+bx^3)\left(-243a^{11/3}\sqrt[3]{b}x+27a^{8/3}\sqrt[3]{b}x(a+bx^3)+36a^{5/3}\sqrt[3]{b}x(a+bx^3)^2+\cdots\right.}{\cdots}$$

input
```
Integrate[x^3/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
((a + b*x^3)*(-243*a^(11/3)*b^(1/3)*x + 27*a^(8/3)*b^(1/3)*x*(a + b*x^3) +
 36*a^(5/3)*b^(1/3)*x*(a + b*x^3)^2 + 60*a^(2/3)*b^(1/3)*x*(a + b*x^3)^3 +
 40*Sqrt[3]*(a + b*x^3)^4*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3)
)] + 40*(a + b*x^3)^4*Log[a^(1/3) + b^(1/3)*x] - 20*(a + b*x^3)^4*Log[a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(2916*a^(11/3)*b^(4/3)*((a + b*x^
3)^2)^(5/2))
```

3.110.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.67, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {1384, 27, 817, 749, 749, 749, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a^2+2abx^3+b^2x^6)^{5/2}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b^5(a+bx^3)\int \frac{x^3}{b^5(bx^3+a)^5}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx^3)\int \frac{x^3}{(bx^3+a)^5}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{817}$$

$$\frac{\left(a+bx^3\right)\left(\frac{\int \frac{1}{\left(bx^3+a\right)^4}dx}{12b}-\frac{x}{12b(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\Big\downarrow$ 749

$$\frac{\left(a+bx^3\right)\left(\frac{\frac{8\int \frac{1}{\left(bx^3+a\right)^3}dx}{9a}+\frac{x}{9a\left(a+bx^3\right)^3}}{12b}-\frac{x}{12b(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\Big\downarrow$ 749

$$\frac{\left(a+bx^3\right)\left(\frac{\frac{8\left(\frac{5\int \frac{1}{\left(bx^3+a\right)^2}dx}{6a}+\frac{x}{6a\left(a+bx^3\right)^2}\right)}{9a}+\frac{x}{9a\left(a+bx^3\right)^3}}{12b}-\frac{x}{12b(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\Big\downarrow$ 749

$$\frac{\left(a+bx^3\right)\left(\frac{\frac{8\left(\frac{5\left(\frac{2\int \frac{1}{bx^3+a}dx}{3a}+\frac{x}{3a\left(a+bx^3\right)}\right)}{6a}+\frac{x}{6a\left(a+bx^3\right)^2}\right)}{9a}+\frac{x}{9a\left(a+bx^3\right)^3}}{12b}-\frac{x}{12b(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\Big\downarrow$ 750

3.110. $\int \frac{x^3}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

$$\frac{(a+bx^3)\left(\frac{\frac{8\left(\frac{5\left(\frac{2\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{3a}+\frac{x}{3a\left(a+bx^3\right)}\right)}{6a}+\frac{x}{6a\left(a+bx^3\right)^2}\right)}{9a}+\frac{x}{9a\left(a+bx^3\right)^3}}{12b}-\frac{x}{12b(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \ 16$$

3.110. $\int \frac{x^3}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

$$\frac{(a+bx^3)\left(\frac{\frac{8\left(\frac{5\left(\frac{2\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a\left(a+bx^3\right)}\right)}{6a}+\frac{x}{6a\left(a+bx^3\right)^2}\right)}{9a}+\frac{x}{9a\left(a+bx^3\right)^3}}{12b}-\frac{x}{12b(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 1142$$

3.110. $\int \frac{x^3}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

$$\left(a+bx^3\right)\left(\frac{\frac{8\left(\frac{5\left(\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a\left(a+bx^3\right)}\right)}{6a}+\frac{x}{6a\left(a+bx^3\right)^2}\right)}{9a}+}{12b}\right.$$

3.110. $\int\frac{x^3}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

$$\downarrow 25$$

3.110. $\int \frac{x^3}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

$$(a+bx^3)\left(\frac{8\left(\frac{5\left(\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a\left(a+bx^3\right)}\right)}{6a}+\frac{x}{6a\left(a+bx^3\right)^2}\right)}{9a}+\frac{\ldots}{9\ldots}}{12b}\right.$$

3.110. $\int\frac{x^3}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

$$\Big\downarrow \; 27$$

$$\frac{(a+bx^3)\left(\frac{8\left(\frac{5\left(\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a\left(a+bx^3\right)}\right)}{6a}+\frac{x}{6a\left(a+bx^3\right)^2}\right)}{9a}+}{12b}\right.}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 1082$$

3.110. $\int \frac{x^3}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

$$\left(a+bx^3\right)\frac{\frac{8\left(\frac{5\left(\frac{2\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a\left(a+bx^3\right)}\right)}{6a}+\frac{x}{6a\left(a+bx^3\right)^2}\right)}{9a}}{12b}$$

3.110. $\displaystyle\int\frac{x^3}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

$$\downarrow 217$$

3.110. $\int \frac{x^3}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

$$(a+bx^3)\left(\frac{\frac{8\left(\frac{5\left(\frac{2\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}\,dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a\left(a+bx^3\right)}\right)}{6a}+\frac{x}{6a\left(a+bx^3\right)^2}\right)}{9a}+\frac{x}{9a\left(a+bx^3\right)^3}}{12b}\right.$$

3.110. $\int\frac{x^3}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

$\downarrow$ 1103

3.110. $\int \frac{x^3}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

$$\left(a+bx^3\right)\left(\frac{8\left(\frac{5\left(\frac{2\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a\left(a+bx^3\right)}\right)}{6a}+\frac{x}{6a\left(a+bx^3\right)^2}\right)}{9a}+\frac{x}{9a\left(a+bx^3\right)}}{12b}\right.$$

3.110. $\int \frac{x^3}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

input
```
Int[x^3/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
((a + b*x^3)*(-1/12*x/(b*(a + b*x^3)^4) + (x/(9*a*(a + b*x^3)^3) + (8*(x/(
6*a*(a + b*x^3)^2) + (5*(x/(3*a*(a + b*x^3)) + (2*(Log[a^(1/3) + b^(1/3)*x
]/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqr
t[3]])/b^(1/3)) - Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3
)))/(3*a^(2/3))))/(3*a)))/(6*a)))/(9*a))/(12*b)))/Sqrt[a^2 + 2*a*b*x^3 + b
^2*x^6]
```

3.110.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

3.110.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.81 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.30

method	result
risch	$\frac{\sqrt{(b\,x^3+a)^2}\left(\frac{5b^2x^{10}}{243a^3}+\frac{2b\,x^7}{27a^2}+\frac{31x^4}{324a}-\frac{10x}{243b}\right)}{(b\,x^3+a)^5}+\frac{10\sqrt{(b\,x^3+a)^2}\left(\sum_{\_R=\text{RootOf}\left(\_Z^3b+a\right)}\frac{\ln\left(x-\_R\right)}{\_R^2}\right)}{729(b\,x^3+a)a^3b^2}$
default	$\frac{\left(-40\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)b^4x^{12}+40\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)b^4x^{12}-20\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)b^4x^{12}+60\left(\frac{a}{b}\right)^{\frac{2}{3}}b^4x^{10}-160\sqrt{3}\,\text{arc}\right.}{}$

input
```
int(x^3/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^3+a)^2)^(1/2)/(b*x^3+a)^5*(5/243*b^2/a^3*x^10+2/27*b/a^2*x^7+31/324/
a*x^4-10/243/b*x)+10/729*((b*x^3+a)^2)^(1/2)/(b*x^3+a)/a^3/b^2*sum(1/_R^2*
ln(x-_R),_R=RootOf(_Z^3*b+a))
```

3.110.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 723, normalized size of antiderivative = 2.01

$$\int \frac{x^3}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \left[\frac{60\,a^2b^4x^{10}+216\,a^3b^3x^7+279\,a^4b^2x^4-120\,a^5bx+60\,\sqrt{\frac{1}{3}}(ab^5x^{12}+4\,a^2b^4x}{}\right.$$

input
```
integrate(x^3/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/2916*(60*a^2*b^4*x^10 + 216*a^3*b^3*x^7 + 279*a^4*b^2*x^4 - 120*a^5*b*x
 + 60*sqrt(1/3)*(a*b^5*x^12 + 4*a^2*b^4*x^9 + 6*a^3*b^3*x^6 + 4*a^4*b^2*x^
3 + a^5*b)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a
^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^
2*b)^(1/3)/b))/(b*x^3 + a)) - 20*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 +
 4*a^3*b*x^3 + a^4)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^
(1/3)*a) + 40*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)
*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^5*b^6*x^12 + 4*a^6*b^5*x^9 +
 6*a^7*b^4*x^6 + 4*a^8*b^3*x^3 + a^9*b^2), 1/2916*(60*a^2*b^4*x^10 + 216*a
^3*b^3*x^7 + 279*a^4*b^2*x^4 - 120*a^5*b*x + 120*sqrt(1/3)*(a*b^5*x^12 + 4
*a^2*b^4*x^9 + 6*a^3*b^3*x^6 + 4*a^4*b^2*x^3 + a^5*b)*sqrt((a^2*b)^(1/3)/b
)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3
)/b)/a^2) - 20*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4
)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 40*(b^4
*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(a^2*b)^(2/3)*log
(a*b*x + (a^2*b)^(2/3)))/(a^5*b^6*x^12 + 4*a^6*b^5*x^9 + 6*a^7*b^4*x^6 + 4
*a^8*b^3*x^3 + a^9*b^2)]
```

3.110.6 Sympy [F]

$$\int \frac{x^3}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx=\int \frac{x^3}{\left((a+bx^3)^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**3/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

output
```
Integral(x**3/((a + b*x**3)**2)**(5/2), x)
```

3.110.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.54

$$\int \frac{x^3}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx=\frac{20\,b^3x^{10}+72\,ab^2x^7+93\,a^2bx^4-40\,a^3x}{972\,(a^3b^5x^{12}+4\,a^4b^4x^9+6\,a^5b^3x^6+4\,a^6b^2x^3+a^7b)}+\frac{10\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729\,a^3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{5\,\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{729\,a^3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{10\,\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{729\,a^3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(x^3/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

output
```
1/972*(20*b^3*x^10 + 72*a*b^2*x^7 + 93*a^2*b*x^4 - 40*a^3*x)/(a^3*b^5*x^12
 + 4*a^4*b^4*x^9 + 6*a^5*b^3*x^6 + 4*a^6*b^2*x^3 + a^7*b) + 10/729*sqrt(3)
*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^3*b^2*(a/b)^(2/3))
 - 5/729*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*b^2*(a/b)^(2/3)) + 10
/729*log(x + (a/b)^(1/3))/(a^3*b^2*(a/b)^(2/3))
```

3.110.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.55

$$\int \frac{x^3}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = -\frac{10\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729\,(-ab^2)^{\frac{2}{3}}\,a^3\mathrm{sgn}\,(bx^3+a)} - \frac{5\,\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{729\,(-ab^2)^{\frac{2}{3}}\,a^3\mathrm{sgn}\,(bx^3+a)} - \frac{10\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{729\,a^4b\mathrm{sgn}\,(bx^3+a)} + \frac{20\,b^3x^{10}+72\,ab^2x^7+93\,a^2bx^4-40\,a^3x}{972\,(bx^3+a)^4a^3b\mathrm{sgn}\,(bx^3+a)}$$

input
```
integrate(x^3/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

output
```
-10/729*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a
*b^2)^(2/3)*a^3*sgn(b*x^3 + a)) - 5/729*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^
(2/3))/((-a*b^2)^(2/3)*a^3*sgn(b*x^3 + a)) - 10/729*(-a/b)^(1/3)*log(abs(x
 - (-a/b)^(1/3)))/(a^4*b*sgn(b*x^3 + a)) + 1/972*(20*b^3*x^10 + 72*a*b^2*x
^7 + 93*a^2*b*x^4 - 40*a^3*x)/((b*x^3 + a)^4*a^3*b*sgn(b*x^3 + a))
```

3.110.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(a^2 + 2abx^3 + b^2x^6)^{5/2}}\,dx = \int \frac{x^3}{(a^2 + 2\,a\,b\,x^3 + b^2\,x^6)^{5/2}}\,dx$$

input
```
int(x^3/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

output
```
int(x^3/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

3.111 $\int \frac{x^2}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$

3.111.1 Optimal result

Integrand size = 26, antiderivative size = 38

$$\int \frac{x^2}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx=-\frac{1}{12b\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{3/2}}$$

output
```
-1/12/b/(b*x^3+a)/(b^2*x^6+2*a*b*x^3+a^2)^(3/2)
```

3.111.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 267 vs. 2(38) = 76.

Time = 0.59 (sec) , antiderivative size = 267, normalized size of antiderivative = 7.03

$$\int \frac{x^2}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx =$$
$$-\frac{x^3\left(-\sqrt{a^2}b^7x^{21}-a^3b^4x^{12}\sqrt{\left(a+bx^3\right)^2}+a^2b^5x^{15}\sqrt{\left(a+bx^3\right)^2}-ab^6x^{18}\sqrt{\left(a+bx^3\right)^2}+4a^7\left(\sqrt{a^2}-\sqrt{(a+\cdots}\right.\right.}{12a^8\left(a+bx^3\right)^3\left(a^2+\cdots\right.}$$

input
```
Integrate[x^2/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
-1/12*(x^3*(-(Sqrt[a^2]*b^7*x^21) - a^3*b^4*x^12*Sqrt[(a + b*x^3)^2] + a^2
*b^5*x^15*Sqrt[(a + b*x^3)^2] - a*b^6*x^18*Sqrt[(a + b*x^3)^2] + 4*a^7*(Sq
rt[a^2] - Sqrt[(a + b*x^3)^2]) + 2*a^5*b^2*x^6*(2*Sqrt[a^2] - Sqrt[(a + b*
x^3)^2]) + 2*a^6*b*x^3*(3*Sqrt[a^2] - Sqrt[(a + b*x^3)^2]) + a^4*b^3*x^9*(
Sqrt[a^2] + Sqrt[(a + b*x^3)^2])))/(a^8*(a + b*x^3)^3*(a^2 + a*b*x^3 - Sqr
t[a^2]*Sqrt[(a + b*x^3)^2]))
```

3.111.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1690, 1078}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a^2+2abx^3+b^2x^6)^{5/2}} \, dx$$

$$\Big\downarrow \text{1690}$$

$$\frac{1}{3}\int \frac{1}{(b^2x^6+2abx^3+a^2)^{5/2}}dx^3$$

$$\Big\downarrow \text{1078}$$

$$-\frac{1}{12b(a+bx^3)(a^2+2abx^3+b^2x^6)^{3/2}}$$

input
```
Int[x^2/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
-1/12*1/(b*(a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2))
```

3.111.3.1 Defintions of rubi rules used

rule 1078
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[2*((a + b*x
+ c*x^2)^(p + 1)/((2*p + 1)*(b + 2*c*x))), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

3.111.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.61

method	result	size
pseudoelliptic	$-\frac{\operatorname{csgn}\left(b\,x^{3}+a\right)}{12\left(b\,x^{3}+a\right)^{4} b}$	23
gosper	$-\frac{b\,x^{3}+a}{12 b \left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}$	24
default	$-\frac{b\,x^{3}+a}{12 b \left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}$	24
risch	$-\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}}{12\left(b\,x^{3}+a\right)^{5} b}$	26

input
```
int(x^2/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12/(b*x^3+a)^4/b*csgn(b*x^3+a)
```

3.111.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.26

$$\int \frac{x^2}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = -\frac{1}{12\,(b^5x^{12}+4\,ab^4x^9+6\,a^2b^3x^6+4\,a^3b^2x^3+a^4b)}$$

input
```
integrate(x^2/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/12/(b^5*x^12 + 4*a*b^4*x^9 + 6*a^2*b^3*x^6 + 4*a^3*b^2*x^3 + a^4*b)
```

3.111.6 Sympy [F]

$$\int \frac{x^2}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \int \frac{x^2}{\left((a+bx^3)^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**2/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

output
```
Integral(x**2/((a + b*x**3)**2)**(5/2), x)
```

3.111.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.42

$$\int \frac{x^2}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = -\frac{1}{12\left(x^3+\frac{a}{b}\right)^4 b^5}$$

input
```
integrate(x^2/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/12/((x^3 + a/b)^4*b^5)
```

3.111.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.63

$$\int \frac{x^2}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = -\frac{1}{12\,(bx^3+a)^4 b\mathrm{sgn}\,(bx^3+a)}$$

input
```
integrate(x^2/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

output
```
-1/12/((b*x^3 + a)^4*b*sgn(b*x^3 + a))
```

3.111.9 Mupad [B] (verification not implemented)

Time = 8.37 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \frac{x^2}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = -\frac{\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{12\,b\,(b\,x^3+a)^5}$$

input
```
int(x^2/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

output
```
-(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2)/(12*b*(a + b*x^3)^5)
```

3.112 $\int \frac{x}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$

3.112.1 Optimal result

Integrand size = 24, antiderivative size = 359

$$\begin{aligned}\int \frac{x}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx &= \frac{35x^2}{243a^4\sqrt{a^2+2abx^3+b^2x^6}}\\ &+ \frac{x^2}{12a\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{5x^2}{54a^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}\\ &+ \frac{35x^2}{324a^3\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{35(a+bx^3)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}a^{13/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}\\ &- \frac{35(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{13/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{35(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{1458a^{13/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}\end{aligned}$$

output
```
35/243*x^2/a^4/((b*x^3+a)^2)^(1/2)+1/12*x^2/a/(b*x^3+a)^3/((b*x^3+a)^2)^(1
/2)+5/54*x^2/a^2/(b*x^3+a)^2/((b*x^3+a)^2)^(1/2)+35/324*x^2/a^3/(b*x^3+a)/
((b*x^3+a)^2)^(1/2)-35/729*(b*x^3+a)*ln(a^(1/3)+b^(1/3)*x)/a^(13/3)/b^(2/3
)/((b*x^3+a)^2)^(1/2)+35/1458*(b*x^3+a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/
3)*x^2)/a^(13/3)/b^(2/3)/((b*x^3+a)^2)^(1/2)-35/729*(b*x^3+a)*arctan(1/3*(
a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(13/3)/b^(2/3)*3^(1/2)/((b*x^3+a)^
2)^(1/2)
```

3.112.2 Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.61

$$\int \frac{x}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \frac{(a+bx^3)\left(243a^{10/3}x^2+270a^{7/3}x^2(a+bx^3)+315a^{4/3}x^2(a+bx^3)^2+420\sqrt[3]{\right.}{}}{}$$

input
```
Integrate[x/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
((a + b*x^3)*(243*a^(10/3)*x^2 + 270*a^(7/3)*x^2*(a + b*x^3) + 315*a^(4/3)
*x^2*(a + b*x^3)^2 + 420*a^(1/3)*x^2*(a + b*x^3)^3 + (140*Sqrt[3]*(a + b*x
^3)^4*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/b^(2/3) - (140*(
a + b*x^3)^4*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) + (70*(a + b*x^3)^4*Log[a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3)))/(2916*a^(13/3)*((a + b*
x^3)^2)^(5/2))
```

3.112.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.71, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {1384, 27, 819, 819, 819, 819, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$$

$$\Big\downarrow \ 1384$$

$$\frac{b^5(a+bx^3)\int \frac{x}{b^5(bx^3+a)^5}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \ 27$$

$$\frac{(a+bx^3)\int \frac{x}{(bx^3+a)^5}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \ 819$$

$$\frac{(a+bx^3)\left(\frac{5\int \frac{x}{(bx^3+a)^4}dx}{6a}+\frac{x^2}{12a(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\Big\downarrow$ 819

$$\frac{(a+bx^3)\left(\frac{5\left(\frac{7\int \frac{x}{(bx^3+a)^3}dx}{9a}+\frac{x^2}{9a(a+bx^3)^3}\right)}{6a}+\frac{x^2}{12a(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\Big\downarrow$ 819

$$\frac{(a+bx^3)\left(\frac{5\left(\frac{7\left(\frac{2\int \frac{x}{(bx^3+a)^2}dx}{3a}+\frac{x^2}{6a(a+bx^3)^2}\right)}{9a}+\frac{x^2}{9a(a+bx^3)^3}\right)}{6a}+\frac{x^2}{12a(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\Big\downarrow$ 819

$$\frac{(a+bx^3)\left(\frac{5\left(\frac{7\left(\frac{2\left(\frac{\int \frac{x}{bx^3+a}dx}{3a}+\frac{x^2}{3a(a+bx^3)}\right)}{3a}+\frac{x^2}{6a(a+bx^3)^2}\right)}{9a}+\frac{x^2}{9a(a+bx^3)^3}\right)}{6a}+\frac{x^2}{12a(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\Big\downarrow$ 821

3.112. $\int \frac{x}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

$$\frac{(a+bx^3)\left(\frac{5\left(\frac{7\left(\frac{2\left(\frac{\frac{\int\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int\frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}\right)}{3a}+\frac{x^2}{6a\left(a+bx^3\right)^2}\right)}{9a}+\frac{x^2}{9a\left(a+bx^3\right)^3}\right)}{6a}+\frac{x^2}{12a(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

3.112. $\int \frac{x}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

$\downarrow$ 16

3.112. $\int \frac{x}{(a^2+2abx^3+b^2x^6)^{5/2}}\, dx$

$$\frac{(a+bx^3)\left(\frac{5\left(\frac{7\left(\frac{2\left(\frac{\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}\,dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}\right)}{3a}+\frac{x^2}{6a\left(a+bx^3\right)^2}\right)}{9a}+\frac{x^2}{9a\left(a+bx^3\right)^3}\right)}{6a}+\frac{x^2}{12a(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

3.112. $\int \frac{x}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

$\downarrow$ 1142

3.112. $\int \frac{x}{(a^2+2abx^3+b^2x^6)^{5/2}}\, dx$

$$\left(a+bx^3\right)\left(\frac{5\left(\frac{7\left(\frac{2\left(\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}\right)}{3a}+\frac{x^2}{6a\left(a+bx^3\right)^2}\right)}{9a}+\cdots}{6a}\cdots\right.$$

3.112. $\int\frac{x}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

$$\downarrow 25$$

3.112. $\int \frac{x}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

$$\left(a+bx^3\right)\left(\frac{5\left(\frac{7\left(\frac{2\left(\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}\right)}{3a}+\frac{x^2}{6a\left(a+bx^3\right)^2}\right)}{9a}+\frac{}{9a}\right.}{6a}\right.$$

3.112. $\int\frac{x}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$

$$\downarrow 27$$

3.112. $\int \frac{x}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

$$\frac{(a+bx^3)\left(\frac{5\left(\frac{7\left(\frac{2\left(\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}\right)}{3a}+\frac{x^2}{6a\left(a+bx^3\right)^2}\right)}{9a}+\cdots}{6a}\cdots\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

3.112. $\int \frac{x}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

↓ 1082

3.112. $\int \frac{x}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

$$(a+bx^3)\left(\frac{5\left(\frac{7\left(\frac{2\left(\frac{\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}\right)}{3a}+\frac{x^2}{6a\left(a+bx^3\right)^2}\right)}{9a}+\cdots}{6a}\right.$$

3.112. $\int\frac{x}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

↓ 217

3.112. $\int \frac{x}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

$$\left(a+bx^3\right)\left(\frac{5\left(\frac{7\left(\frac{2\left(\frac{\frac{-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}\right)}{3a}+\frac{x^2}{6a\left(a+bx^3\right)^2}\right)}{9a}+\frac{x^2}{9a\left(a+bx^3\right)^3}\right)}{6a}\right.$$

3.112. $\int \frac{x}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

$\downarrow$ 1103

3.112. $\int \frac{x}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

$$(a+bx^3)\left(\frac{5\left(\frac{7\left(\frac{2\left(\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}\right)}{3a}+\frac{x^2}{6a\left(a+bx^3\right)^2}\right)}{9a}+\frac{x^2}{9a\left(a+bx^3\right)^3}\right)}{6a}\right.$$

3.112. $\int \frac{x}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

input
```
Int[x/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
((a + b*x^3)*(x^2/(12*a*(a + b*x^3)^4) + (5*(x^2/(9*a*(a + b*x^3)^3) + (7*
(x^2/(6*a*(a + b*x^3)^2) + (2*(x^2/(3*a*(a + b*x^3)) + (-1/3*Log[a^(1/3) +
 b^(1/3)*x]/(a^(1/3)*b^(2/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1
/3))/Sqrt[3]])/b^(1/3)) + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(
2*b^(1/3)))/(3*a^(1/3)*b^(1/3))))/(3*a)))/(3*a)))/(9*a)))/(6*a)))/Sqrt[a^2
+ 2*a*b*x^3 + b^2*x^6]
```

3.112.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

3.112.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.87 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.31

method	result
risch	$\frac{\sqrt{(b\,x^3+a)^2}\,\left(\frac{35 b^3 x^{11}}{243 a^4}+\frac{175 b^2 x^8}{324 a^3}+\frac{20 b\,x^5}{27 a^2}+\frac{104 x^2}{243 a}\right)}{(b\,x^3+a)^5}+\frac{35\sqrt{(b\,x^3+a)^2}\,\left(\sum_{\_R=\mathrm{RootOf}(\_Z^3 b+a)}\frac{\ln(x-\_R)}{\_R}\right)}{729(b\,x^3+a)b\,a^4}$
default	$\frac{\left(-140\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)b^4x^{12}-140\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)b^4x^{12}+70\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)b^4x^{12}+420\left(\frac{a}{b}\right)^{\frac{1}{3}}b^4x^{11}-560\sqrt{3}\right.}{}$

input
```
int(x/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^3+a)^2)^(1/2)/(b*x^3+a)^5*(35/243/a^4*b^3*x^11+175/324*b^2/a^3*x^8+2
0/27*b/a^2*x^5+104/243*x^2/a)+35/729*((b*x^3+a)^2)^(1/2)/(b*x^3+a)/b/a^4*s
um(1/_R*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

3.112.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 734, normalized size of antiderivative = 2.04

$$\int \frac{x}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \left[\frac{420\,ab^5x^{11}+1575\,a^2b^4x^8+2160\,a^3b^3x^5+1248\,a^4b^2x^2+210\,\sqrt{\frac{1}{3}}(ab^5x^{12}+}{}\right.$$

input
```
integrate(x/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/2916*(420*a*b^5*x^11 + 1575*a^2*b^4*x^8 + 2160*a^3*b^3*x^5 + 1248*a^4*b
^2*x^2 + 210*sqrt(1/3)*(a*b^5*x^12 + 4*a^2*b^4*x^9 + 6*a^3*b^3*x^6 + 4*a^4
*b^2*x^3 + a^5*b)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3
)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a)
 - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) + 70*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b
^2*x^6 + 4*a^3*b*x^3 + a^4)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*
x + (-a*b^2)^(2/3)) - 140*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*
b*x^3 + a^4)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^5*b^6*x^12 + 4*a
^6*b^5*x^9 + 6*a^7*b^4*x^6 + 4*a^8*b^3*x^3 + a^9*b^2), 1/2916*(420*a*b^5*x
^11 + 1575*a^2*b^4*x^8 + 2160*a^3*b^3*x^5 + 1248*a^4*b^2*x^2 + 420*sqrt(1/
3)*(a*b^5*x^12 + 4*a^2*b^4*x^9 + 6*a^3*b^3*x^6 + 4*a^4*b^2*x^3 + a^5*b)*sq
rt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*
b^2)^(1/3)/a)/b) + 70*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^
3 + a^4)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3))
 - 140*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(-a*b^
2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^5*b^6*x^12 + 4*a^6*b^5*x^9 + 6*a^7*
b^4*x^6 + 4*a^8*b^3*x^3 + a^9*b^2)]
```

3.112.6 Sympy [F]

$$\int \frac{x}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \int \frac{x}{\left((a+bx^3)^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

output
```
Integral(x/((a + b*x**3)**2)**(5/2), x)
```

3.112.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.53

$$\int \frac{x}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \frac{140\,b^3x^{11}+525\,ab^2x^8+720\,a^2bx^5+416\,a^3x^2}{972\,(a^4b^4x^{12}+4\,a^5b^3x^9+6\,a^6b^2x^6+4\,a^7bx^3+a^8)} + \frac{35\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729\,a^4b\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{35\,\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{1458\,a^4b\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{35\,\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{729\,a^4b\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate(x/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

output
```
1/972*(140*b^3*x^11 + 525*a*b^2*x^8 + 720*a^2*b*x^5 + 416*a^3*x^2)/(a^4*b^
4*x^12 + 4*a^5*b^3*x^9 + 6*a^6*b^2*x^6 + 4*a^7*b*x^3 + a^8) + 35/729*sqrt(
3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^4*b*(a/b)^(1/3))
 + 35/1458*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^4*b*(a/b)^(1/3)) - 35
/729*log(x + (a/b)^(1/3))/(a^4*b*(a/b)^(1/3))
```

3.112.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.55

$$\int \frac{x}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = -\frac{35\,\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{1458\,(-ab^2)^{\frac{1}{3}}\,a^4\mathrm{sgn}\,(bx^3+a)} - \frac{35\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{729\,a^5\mathrm{sgn}\,(bx^3+a)} - \frac{35\,\sqrt{3}(-ab^2)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729\,a^5b^2\mathrm{sgn}\,(bx^3+a)} + \frac{140\,b^3x^{11}+525\,ab^2x^8+720\,a^2bx^5+416\,a^3x^2}{972\,(bx^3+a)^4a^4\mathrm{sgn}\,(bx^3+a)}$$

input
```
integrate(x/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

output
```
-35/1458*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(1/3)*a^4*sgn(
b*x^3 + a)) - 35/729*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a^5*sgn(b*x^
3 + a)) - 35/729*sqrt(3)*(-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(
1/3))/(-a/b)^(1/3))/(a^5*b^2*sgn(b*x^3 + a)) + 1/972*(140*b^3*x^11 + 525*a
*b^2*x^8 + 720*a^2*b*x^5 + 416*a^3*x^2)/((b*x^3 + a)^4*a^4*sgn(b*x^3 + a))
```

3.112.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \int \frac{x}{(a^2+2\,a\,b\,x^3+b^2\,x^6)^{5/2}}\,dx$$

input
```
int(x/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

output
```
int(x/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

3.113 $\int \frac{1}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

3.113.1 Optimal result

Integrand size = 22, antiderivative size = 364

$$\begin{aligned}\int \frac{1}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx &= \frac{x(a+bx^3)}{12a\,(a^2+2abx^3+b^2x^6)^{5/2}} \\ &+ \frac{11x(a+bx^3)^2}{108a^2\,(a^2+2abx^3+b^2x^6)^{5/2}} + \frac{11x(a+bx^3)^3}{81a^3\,(a^2+2abx^3+b^2x^6)^{5/2}} \\ &+ \frac{55x(a+bx^3)^4}{243a^4\,(a^2+2abx^3+b^2x^6)^{5/2}} - \frac{110(a+bx^3)^5\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}a^{14/3}\sqrt[3]{b}\,(a^2+2abx^3+b^2x^6)^{5/2}} \\ &+ \frac{110(a+bx^3)^5\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{14/3}\sqrt[3]{b}\,(a^2+2abx^3+b^2x^6)^{5/2}} - \frac{55(a+bx^3)^5\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{729a^{14/3}\sqrt[3]{b}\,(a^2+2abx^3+b^2x^6)^{5/2}}\end{aligned}$$

output
```
1/12*x*(b*x^3+a)/a/(b^2*x^6+2*a*b*x^3+a^2)^(5/2)+11/108*x*(b*x^3+a)^2/a^2/
(b^2*x^6+2*a*b*x^3+a^2)^(5/2)+11/81*x*(b*x^3+a)^3/a^3/(b^2*x^6+2*a*b*x^3+a
^2)^(5/2)+55/243*x*(b*x^3+a)^4/a^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2)+110/729*(
b*x^3+a)^5*ln(a^(1/3)+b^(1/3)*x)/a^(14/3)/b^(1/3)/(b^2*x^6+2*a*b*x^3+a^2)^
(5/2)-55/729*(b*x^3+a)^5*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(14/3
)/b^(1/3)/(b^2*x^6+2*a*b*x^3+a^2)^(5/2)-110/729*(b*x^3+a)^5*arctan(1/3*(a^
(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(14/3)/b^(1/3)/(b^2*x^6+2*a*b*x^3+a^
2)^(5/2)*3^(1/2)
```

3.113.2 Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.58

$$\int \frac{1}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \frac{(a+bx^3)\left(243a^{11/3}x+297a^{8/3}x(a+bx^3)+396a^{5/3}x(a+bx^3)^2+660a^{2/3}x\right.}{}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(-5/2),x]
```

output
```
((a + b*x^3)*(243*a^(11/3)*x + 297*a^(8/3)*x*(a + b*x^3) + 396*a^(5/3)*x*(
a + b*x^3)^2 + 660*a^(2/3)*x*(a + b*x^3)^3 + (440*Sqrt[3]*(a + b*x^3)^4*Ar
cTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/b^(1/3) + (440*(a + b*x^
3)^4*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) - (220*(a + b*x^3)^4*Log[a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(1/3)))/(2916*a^(14/3)*((a + b*x^3)^2)
^(5/2))
```

3.113.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.74, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {1384, 749, 749, 749, 749, 750, 16, 27, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a^2+2abx^3+b^2x^6)^{5/2}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b^5(a+bx^3)\int \frac{1}{(b^2x^3+ab)^5}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{749}$$

$$\frac{b^5(a+bx^3)\left(\frac{11\int \frac{1}{(b^2x^3+ab)^4}dx}{12ab}+\frac{x}{12ab^5(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\downarrow 749$$

$$\frac{b^5\left(a+bx^3\right)\left(\frac{11\left(\frac{8\int\frac{1}{\left(b^2x^3+ab\right)^3}dx}{9ab}+\frac{x}{9ab^4\left(a+bx^3\right)^3}\right)}{12ab}+\frac{x}{12ab^5\left(a+bx^3\right)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\downarrow 749$$

$$\frac{b^5\left(a+bx^3\right)\left(\frac{11\left(\frac{8\left(\frac{5\int\frac{1}{\left(b^2x^3+ab\right)^2}dx}{6ab}+\frac{x}{6ab^3\left(a+bx^3\right)^2}\right)}{9ab}+\frac{x}{9ab^4\left(a+bx^3\right)^3}\right)}{12ab}+\frac{x}{12ab^5\left(a+bx^3\right)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\downarrow 749$$

$$\frac{b^5\left(a+bx^3\right)\left(\frac{11\left(\frac{8\left(\frac{5\left(\frac{2\int\frac{1}{b^2x^3+ab}dx}{3ab}+\frac{x}{3ab^2\left(a+bx^3\right)}\right)}{6ab}+\frac{x}{6ab^3\left(a+bx^3\right)^2}\right)}{9ab}+\frac{x}{9ab^4\left(a+bx^3\right)^3}\right)}{12ab}+\frac{x}{12ab^5\left(a+bx^3\right)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\downarrow 750$$

3.113. $\int\frac{1}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}dx$

$$b^5(a+bx^3)\left(\frac{11\left(\frac{8\left(\frac{5\left(\frac{2\left(\frac{\int \frac{\sqrt[3]{b}\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)}{b^{4/3}x^2-\sqrt[3]{a}bx+a^{2/3}b^{2/3}}\,dx}{3a^{2/3}b^{2/3}}+\frac{\int \frac{1}{b^{2/3}x+\sqrt[3]{a}\sqrt[3]{b}}\,dx}{3a^{2/3}b^{2/3}}\right)}{3ab}+\frac{x}{3ab^2\left(a+bx^3\right)}\right)}{6ab}+\frac{x}{6ab^3\left(a+bx^3\right)^2}\right)}{9ab}+\frac{x}{9ab^4\left(a+bx^3\right)^3}\right)}{12ab}+\frac{\cdots}{12a\cdots}\right.$$

3.113. $\int \frac{1}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

$$\downarrow 16$$

3.113. $\int \frac{1}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

$$b^5\left(a+bx^3\right)\quad \left(\frac{11\left(\frac{8\left(\frac{5\left(\frac{2\left(\frac{\int \frac{\sqrt[3]{b}\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)}{b^{4/3}x^2-\sqrt[3]{a}bx+a^{2/3}b^{2/3}}\,dx}{3a^{2/3}b^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}}\right)}{3ab}+\frac{x}{3ab^2\left(a+bx^3\right)}\right)}{6ab}+\frac{x}{6ab^3\left(a+bx^3\right)^2}\right)}{9ab}+\frac{x}{9ab^4\left(a+bx^3\right)^3}\right)}{12ab}+\frac{}{12ab^5}\right.$$

3.113. $\int \frac{1}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$

3.113.

$$\downarrow 27$$

3.113. $\int \frac{1}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

$$b^5\left(a+bx^3\right) \quad \frac{11\left(\frac{8\left(\frac{5\left(\frac{2\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{4/3}x^2-\sqrt[3]{a}bx+a^{2/3}b^{2/3}}\,dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}}\right)}{3ab}+\frac{x}{3ab^2\left(a+bx^3\right)}\right)}{6ab}+\frac{x}{6ab^3\left(a+bx^3\right)^2}\right)}{9ab}+\frac{x}{9ab^4\left(a+bx^3\right)^3}\right)}{12ab}+\frac{\ }{12ab^5}$$

3.113. $\int \frac{1}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$

↓ 1142

3.113. $\int \frac{1}{(a^2+2abx^3+b^2x^6)^{5/2}}\, dx$

$$\left(11\,\frac{\left(8\,\frac{\left(5\,\frac{\left(2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{4/3}x^2-\sqrt[3]{a}bx+a^{2/3}b^{2/3}}dx-\frac{\int-\frac{b\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{4/3}x^2-\sqrt[3]{a}bx+a^{2/3}b^{2/3}}dx}{2b}}{3a^{2/3}\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}}\right)\right)}{3ab}+\frac{x}{3ab^2\left(a+bx^3\right)}\right)}{6ab}+\frac{\cdots}{6ab^3(\cdots}\right)}{9ab}\cdots}{12ab}\cdots$$

3.113. $\int \frac{1}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

$b^5\left(a+bx^3\right)$

$$\downarrow 25$$

3.113. $\int \frac{1}{(a^2+2abx^3+b^2x^6)^{5/2}} \, dx$

$$\frac{11\left(\frac{8\left(\frac{5\left(\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{4/3}x^2-\sqrt[3]{a}bx+a^{2/3}b^{2/3}}\,dx+\frac{\int\frac{b\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{4/3}x^2-\sqrt[3]{a}bx+a^{2/3}b^{2/3}}\,dx}{2b}}{3a^{2/3}\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}}\right)}{3ab}+\frac{x}{3ab^2\left(a+bx^3\right)}\right)}{6ab}+\frac{\cdots}{6ab^3\left(a\cdots\right.}\cdots}{9ab}\cdots}{12ab}$$

3.113. $\int \frac{1}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$

$b^5\left(a+bx^3\right)$

3.113. ↓ 27

3.113. $\int \frac{1}{(a^2+2abx^3+b^2x^6)^{5/2}}\, dx$

$$b^5\left(a+bx^3\right)\left(\frac{11\left(\frac{8\left(\frac{5\left(\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{4/3}x^2-\sqrt[3]{a}bx+a^{2/3}b^{2/3}}dx+\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{4/3}x^2-\sqrt[3]{a}bx+a^{2/3}b^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}}\right)}{3ab}+\frac{x}{3ab^2\left(a+bx^3\right)}\right)}{6ab}+\frac{\cdots}{6ab^3(\cdots}\cdots\right)}{9ab}\cdots\right)}{12ab}\cdots\right.$$

3.113. $\int \frac{1}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

$$\downarrow 1082$$

3.113. $\int \frac{1}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

$$\left(\frac{11\left(\frac{8\left(\frac{5\left(\frac{2\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{4/3}x^2-\sqrt[3]{a}bx+a^{2/3}b^{2/3}}dx+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{b}}{3a^{2/3}\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}}\right)}{3ab}+\frac{x}{3ab^2\left(a+bx^3\right)}\right)}{6ab}+\frac{\cdots}{6ab^3\cdots}\cdots\right)}{9ab}\cdots\right)}{\cdots}\cdots\right)$$

3.113. $\displaystyle\int\frac{1}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$

↓ 217

3.113. $\int \frac{1}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

$$\left(11 \left(\frac{8 \left(\frac{5 \left(\frac{2 \left(\frac{\frac{1}{2} \int \frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{b^{4/3}x^2 - \sqrt[3]{a}bx + a^{2/3}b^{2/3}}\, dx - \frac{\sqrt{3}\arctan\left(\frac{1 - \frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b}}{3a^{2/3}\sqrt[3]{b}} + \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}} \right)}{3ab} + \frac{x}{3ab^2\left(a+bx^3\right)} \right)}{6ab} + \frac{x}{6ab^3\left(a+bx^3\right)^2} \right)}{9ab} \right. \right. +$$

3.113. $\int \frac{1}{(a^2+2abx^3+b^2x^6)^{5/2}}\, dx$

3.113. $\displaystyle\int \frac{1}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$

↓ 1103

$$\left(\frac{11\left(\frac{8\left(\frac{5\left(\frac{2\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2b}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b}}{3a^{2/3}\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}}\right)}{3ab}+\frac{x}{3ab^2\left(a+bx^3\right)}\right)}{6ab}+\frac{x}{6ab^3\left(a+bx^3\right)^2}\right)}{9ab}+\right.$$

3.113. $\int \frac{1}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(-5/2),x]
```

output
```
(b^5*(a + b*x^3)*(x/(12*a*b^5*(a + b*x^3)^4) + (11*(x/(9*a*b^4*(a + b*x^3)
^3) + (8*(x/(6*a*b^3*(a + b*x^3)^2) + (5*(x/(3*a*b^2*(a + b*x^3)) + (2*(Lo
g[a^(1/3) + b^(1/3)*x]/(3*a^(2/3)*b^(4/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^
(1/3)*x)/a^(1/3))/Sqrt[3]])/b) - Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)
*x^2]/(2*b))/(3*a^(2/3)*b^(1/3))))/(3*a*b)))/(6*a*b)))/(9*a*b)))/(12*a*b))
)/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]
```

3.113.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

3.113.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.53 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.30

method	result
risch	$\frac{\sqrt{(b\,x^{3}+a)^{2}}\,\left(\frac{55b^{3}x^{10}}{243a^{4}}+\frac{22b^{2}x^{7}}{27a^{3}}+\frac{341b\,x^{4}}{324a^{2}}+\frac{133x}{243a}\right)}{(b\,x^{3}+a)^{5}}+\frac{110\sqrt{(b\,x^{3}+a)^{2}}\left(\sum_{\_R=\mathrm{RootOf}(\_Z^{3}b+a)}\frac{\ln(x-\_R)}{\_R^{2}}\right)}{729(b\,x^{3}+a)b\,a^{4}}$
default	$\frac{\left(-440\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)b^{4}x^{12}+440\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)b^{4}x^{12}-220\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)b^{4}x^{12}+660\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{4}x^{10}-1760\sqrt{}\right.}{}$

input
```
int(1/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^3+a)^2)^(1/2)/(b*x^3+a)^5*(55/243/a^4*b^3*x^10+22/27*b^2/a^3*x^7+341
/324*b/a^2*x^4+133/243*x/a)+110/729*((b*x^3+a)^2)^(1/2)/(b*x^3+a)/b/a^4*su
m(1/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

3.113.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 719, normalized size of antiderivative = 1.98

$$\int \frac{1}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \left[\frac{660\,a^2b^4x^{10}+2376\,a^3b^3x^7+3069\,a^4b^2x^4+1596\,a^5bx+660\,\sqrt{\frac{1}{3}}(ab^5x^{12}+4}{}\right.$$

input
```
integrate(1/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/2916*(660*a^2*b^4*x^10 + 2376*a^3*b^3*x^7 + 3069*a^4*b^2*x^4 + 1596*a^5
*b*x + 660*sqrt(1/3)*(a*b^5*x^12 + 4*a^2*b^4*x^9 + 6*a^3*b^3*x^6 + 4*a^4*b
^2*x^3 + a^5*b)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*
x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt
(-(a^2*b)^(1/3)/b))/(b*x^3 + a)) - 220*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2
*x^6 + 4*a^3*b*x^3 + a^4)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a
^2*b)^(1/3)*a) + 440*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3
 + a^4)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^6*b^5*x^12 + 4*a^7*b^
4*x^9 + 6*a^8*b^3*x^6 + 4*a^9*b^2*x^3 + a^10*b), 1/2916*(660*a^2*b^4*x^10
+ 2376*a^3*b^3*x^7 + 3069*a^4*b^2*x^4 + 1596*a^5*b*x + 1320*sqrt(1/3)*(a*b
^5*x^12 + 4*a^2*b^4*x^9 + 6*a^3*b^3*x^6 + 4*a^4*b^2*x^3 + a^5*b)*sqrt((a^2
*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((
a^2*b)^(1/3)/b)/a^2) - 220*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3
*b*x^3 + a^4)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*
a) + 440*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(a^2
*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^6*b^5*x^12 + 4*a^7*b^4*x^9 + 6*a^
8*b^3*x^6 + 4*a^9*b^2*x^3 + a^10*b)]
```

3.113.6 Sympy [F]

$$\int \frac{1}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \int \frac{1}{(a^2+2abx^3+b^2x^6)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

output
```
Integral((a**2 + 2*a*b*x**3 + b**2*x**6)**(-5/2), x)
```

3.113.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.52

$$\int \frac{1}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \frac{220\,b^3x^{10}+792\,ab^2x^7+1023\,a^2bx^4+532\,a^3x}{972\,(a^4b^4x^{12}+4\,a^5b^3x^9+6\,a^6b^2x^6+4\,a^7bx^3+a^8)} + \frac{110\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729\,a^4b\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{55\,\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{729\,a^4b\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{110\,\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{729\,a^4b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(1/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

output
```
1/972*(220*b^3*x^10 + 792*a*b^2*x^7 + 1023*a^2*b*x^4 + 532*a^3*x)/(a^4*b^4
*x^12 + 4*a^5*b^3*x^9 + 6*a^6*b^2*x^6 + 4*a^7*b*x^3 + a^8) + 110/729*sqrt(
3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^4*b*(a/b)^(2/3))
 - 55/729*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^4*b*(a/b)^(2/3)) + 110
/729*log(x + (a/b)^(1/3))/(a^4*b*(a/b)^(2/3))
```

3.113.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.55

$$\int \frac{1}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = -\frac{110\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{729\,a^5\mathrm{sgn}\left(bx^3+a\right)} + \frac{110\,\sqrt{3}(-ab^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729\,a^5b\mathrm{sgn}\left(bx^3+a\right)} + \frac{55\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{729\,a^5b\mathrm{sgn}\left(bx^3+a\right)} + \frac{220\,b^3x^{10}+792\,ab^2x^7+1023\,a^2bx^4+532\,a^3x}{972\left(bx^3+a\right)^4a^4\mathrm{sgn}\left(bx^3+a\right)}$$

input
```
integrate(1/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

output
```
-110/729*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^5*sgn(b*x^3 + a)) + 11
0/729*sqrt(3)*(-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b
)^(1/3))/(a^5*b*sgn(b*x^3 + a)) + 55/729*(-a*b^2)^(1/3)*log(x^2 + x*(-a/b)
^(1/3) + (-a/b)^(2/3))/(a^5*b*sgn(b*x^3 + a)) + 1/972*(220*b^3*x^10 + 792*
a*b^2*x^7 + 1023*a^2*b*x^4 + 532*a^3*x)/((b*x^3 + a)^4*a^4*sgn(b*x^3 + a))
```

3.113.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \int \frac{1}{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}}\,dx$$

input
```
int(1/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

output
```
int(1/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

3.114 $\int \frac{1}{x\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$

3.114.1 Optimal result

Integrand size = 26, antiderivative size = 223

$$\begin{aligned}\int \frac{1}{x\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx &= \frac{1}{3a^4\sqrt{a^2+2abx^3+b^2x^6}}\\ &+\frac{1}{12a\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{9a^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}\\ &+\frac{1}{6a^3\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\left(a+bx^3\right)\log(x)}{a^5\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\left(a+bx^3\right)\log\left(a+bx^3\right)}{3a^5\sqrt{a^2+2abx^3+b^2x^6}}\end{aligned}$$

output
```
1/3/a^4/((b*x^3+a)^2)^(1/2)+1/12/a/(b*x^3+a)^3/((b*x^3+a)^2)^(1/2)+1/9/a^2
/(b*x^3+a)^2/((b*x^3+a)^2)^(1/2)+1/6/a^3/(b*x^3+a)/((b*x^3+a)^2)^(1/2)+(b*
x^3+a)*ln(x)/a^5/((b*x^3+a)^2)^(1/2)-1/3*(b*x^3+a)*ln(b*x^3+a)/a^5/((b*x^3
+a)^2)^(1/2)
```

3.114.2 Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.43

$$\int \frac{1}{x\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx=\frac{a(25a^3+52a^2bx^3+42ab^2x^6+12b^3x^9)+36(a+bx^3)^4\log(x)-12(a+bx^3}{36a^5\left(a+bx^3\right)^3\sqrt{\left(a+bx^3\right)^2}}$$

input
```
Integrate[1/(x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)),x]
```

output
```
(a*(25*a^3 + 52*a^2*b*x^3 + 42*a*b^2*x^6 + 12*b^3*x^9) + 36*(a + b*x^3)^4*
Log[x] - 12*(a + b*x^3)^4*Log[a + b*x^3])/(36*a^5*(a + b*x^3)^3*Sqrt[(a +
b*x^3)^2])
```

3.114.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.52, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1384, 27, 798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a^2+2abx^3+b^2x^6\right)^{5/2}}dx$$

$$\Big\downarrow \ 1384$$

$$\frac{b^5\left(a+bx^3\right)\int \frac{1}{b^5x\left(bx^3+a\right)^5}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \ 27$$

$$\frac{\left(a+bx^3\right)\int \frac{1}{x\left(bx^3+a\right)^5}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \ 798$$

$$\frac{\left(a+bx^3\right)\int \frac{1}{x^3\left(bx^3+a\right)^5}dx^3}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \ 54$$

$$\frac{\left(a+bx^3\right)\int \left(-\frac{b}{a^5\left(bx^3+a\right)}-\frac{b}{a^4\left(bx^3+a\right)^2}-\frac{b}{a^3\left(bx^3+a\right)^3}-\frac{b}{a^2\left(bx^3+a\right)^4}-\frac{b}{a\left(bx^3+a\right)^5}+\frac{1}{a^5x^3}\right)dx^3}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \ 2009$$

$$\frac{\left(a+bx^3\right)\left(-\frac{\log\left(a+bx^3\right)}{a^5}+\frac{\log\left(x^3\right)}{a^5}+\frac{1}{a^4\left(a+bx^3\right)}+\frac{1}{2a^3\left(a+bx^3\right)^2}+\frac{1}{3a^2\left(a+bx^3\right)^3}+\frac{1}{4a\left(a+bx^3\right)^4}\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

input
```
Int[1/(x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)),x]
```

output
```
((a + b*x^3)*(1/(4*a*(a + b*x^3)^4) + 1/(3*a^2*(a + b*x^3)^3) + 1/(2*a^3*(
a + b*x^3)^2) + 1/(a^4*(a + b*x^3)) + Log[x^3]/a^5 - Log[a + b*x^3]/a^5))/
(3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

3.114.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.114.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.41

method	result
pseudoelliptic	$\frac{\left(-\ln(b\,x^3+a)\,(b\,x^3+a)^4+\ln(b\,x^3)\,(b\,x^3+a)^4+a\,b^3x^9+\frac{7a^2b^2x^6}{2}+\frac{13a^3b\,x^3}{3}+\frac{25a^4}{12}\right)\mathrm{csgn}(b\,x^3+a)}{3(b\,x^3+a)^4a^5}$
risch	$\frac{\sqrt{(b\,x^3+a)^2}\left(\frac{b^3x^9}{3a^4}+\frac{7b^2x^6}{6a^3}+\frac{13b\,x^3}{9a^2}+\frac{25}{36a}\right)}{(b\,x^3+a)^5}+\frac{\sqrt{(b\,x^3+a)^2}\,\ln(x)}{(b\,x^3+a)a^5}-\frac{\sqrt{(b\,x^3+a)^2}\,\ln(b\,x^3+a)}{3(b\,x^3+a)a^5}$
default	$\frac{(36\ln(x)b^4x^{12}-12\ln(b\,x^3+a)b^4x^{12}+144\ln(x)a\,b^3x^9-48\ln(b\,x^3+a)a\,b^3x^9+12a\,b^3x^9+216\ln(x)a^2b^2x^6-72\ln(b\,x^3+a)a^2b^2}{36a^5\left((b\,x^3+a)^2\right.}$

input
```
int(1/x/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(-ln(b*x^3+a)*(b*x^3+a)^4+ln(b*x^3)*(b*x^3+a)^4+a*b^3*x^9+7/2*a^2*b^2*
x^6+13/3*a^3*b*x^3+25/12*a^4)*csgn(b*x^3+a)/(b*x^3+a)^4/a^5
```

3.114.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.80

$$\int \frac{1}{x\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx = \frac{12\,ab^3x^9+42\,a^2b^2x^6+52\,a^3bx^3+25\,a^4-12\,(b^4x^{12}+4\,ab^3x^9+6\,a^2b^2x^6+}{36\,(a^5b^4x^{12}+4\,a^6b^3}$$

input
```
integrate(1/x/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

output
```
1/36*(12*a*b^3*x^9 + 42*a^2*b^2*x^6 + 52*a^3*b*x^3 + 25*a^4 - 12*(b^4*x^12
 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*log(b*x^3 + a) + 36*(b
^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*log(x))/(a^5*b^
4*x^12 + 4*a^6*b^3*x^9 + 6*a^7*b^2*x^6 + 4*a^8*b*x^3 + a^9)
```

3.114.6 Sympy [F]

$$\int \frac{1}{x(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \int \frac{1}{x\left((a+bx^3)^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

output
```
Integral(1/(x*((a + b*x**3)**2)**(5/2)), x)
```

3.114.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.59

$$\int \frac{1}{x(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = -\frac{(-1)^{2\,abx^3+2\,a^2}\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right)}{3\,a^5} + \frac{1}{9\,(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}a^2} + \frac{1}{3\,\sqrt{b^2x^6+2\,abx^3+a^2}a^4} + \frac{1}{6\,\left(x^3+\frac{a}{b}\right)^2a^3b^2} + \frac{1}{12\,\left(x^3+\frac{a}{b}\right)^4ab^4}$$

input
```
integrate(1/x/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*(-1)^(2*a*b*x^3 + 2*a^2)*log(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x)))/a^5
 + 1/9/((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*a^2) + 1/3/(sqrt(b^2*x^6 + 2*a*b
*x^3 + a^2)*a^4) + 1/6/((x^3 + a/b)^2*a^3*b^2) + 1/12/((x^3 + a/b)^4*a*b^4
)
```

3.114.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.49

$$\int \frac{1}{x(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = -\frac{\log\left(|bx^3+a|\right)}{3\,a^5\mathrm{sgn}\left(bx^3+a\right)} + \frac{\log\left(|x|\right)}{a^5\mathrm{sgn}\left(bx^3+a\right)} + \frac{25\,b^4x^{12}+112\,ab^3x^9+192\,a^2b^2x^6+152\,a^3bx^3+50\,a^4}{36\,(bx^3+a)^4a^5\mathrm{sgn}\left(bx^3+a\right)}$$

input
```
integrate(1/x/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

output
```
-1/3*log(abs(b*x^3 + a))/(a^5*sgn(b*x^3 + a)) + log(abs(x))/(a^5*sgn(b*x^3
 + a)) + 1/36*(25*b^4*x^12 + 112*a*b^3*x^9 + 192*a^2*b^2*x^6 + 152*a^3*b*x
^3 + 50*a^4)/((b*x^3 + a)^4*a^5*sgn(b*x^3 + a))
```

3.114.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x(a^2+2abx^3+b^2x^6)^{5/2}}\,dx=\int \frac{1}{x\,(a^2+2\,a\,b\,x^3+b^2\,x^6)^{5/2}}\,dx$$

input
```
int(1/(x*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)),x)
```

output
```
int(1/(x*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)), x)
```

3.115 $\int \frac{1}{x^2(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

3.115.1 Optimal result

Integrand size = 26, antiderivative size = 398

$$\begin{aligned}\int \frac{1}{x^2\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx &= \frac{455}{972a^4x\sqrt{a^2+2abx^3+b^2x^6}} \\ &+ \frac{1}{12ax\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}+\frac{13}{108a^2x\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}} \\ &+ \frac{65}{324a^3x\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}-\frac{455(a+bx^3)}{243a^5x\sqrt{a^2+2abx^3+b^2x^6}} \\ &+ \frac{455\sqrt[3]{b}(a+bx^3)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}a^{16/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{455\sqrt[3]{b}(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{16/3}\sqrt{a^2+2abx^3+b^2x^6}} \\ &- \frac{455\sqrt[3]{b}(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{1458a^{16/3}\sqrt{a^2+2abx^3+b^2x^6}}\end{aligned}$$

output
```
455/972/a^4/x/((b*x^3+a)^2)^(1/2)+1/12/a/x/(b*x^3+a)^3/((b*x^3+a)^2)^(1/2)
+13/108/a^2/x/(b*x^3+a)^2/((b*x^3+a)^2)^(1/2)+65/324/a^3/x/(b*x^3+a)/((b*x
^3+a)^2)^(1/2)-455/243*(b*x^3+a)/a^5/x/((b*x^3+a)^2)^(1/2)+455/729*b^(1/3)
*(b*x^3+a)*ln(a^(1/3)+b^(1/3)*x)/a^(16/3)/((b*x^3+a)^2)^(1/2)-455/1458*b^(
1/3)*(b*x^3+a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(16/3)/((b*x^3+
a)^2)^(1/2)+455/729*b^(1/3)*(b*x^3+a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(
1/3)*3^(1/2))/a^(16/3)*3^(1/2)/((b*x^3+a)^2)^(1/2)
```

3.115.2 Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.61

$$\int \frac{1}{x^2 \left(a^2+2abx^3+b^2x^6\right)^{5/2}} \, dx = \frac{(a+bx^3)\left(-243a^{10/3}bx^2-594a^{7/3}bx^2(a+bx^3)-1179a^{4/3}bx^2(a+bx^3)^2\right.}{}$$

input
```
Integrate[1/(x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)),x]
```

output
```
((a + b*x^3)*(-243*a^(10/3)*b*x^2 - 594*a^(7/3)*b*x^2*(a + b*x^3) - 1179*a
^(4/3)*b*x^2*(a + b*x^3)^2 - 2544*a^(1/3)*b*x^2*(a + b*x^3)^3 - (2916*a^(1
/3)*(a + b*x^3)^4)/x - 1820*Sqrt[3]*b^(1/3)*(a + b*x^3)^4*ArcTan[(-a^(1/3)
 + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))] + 1820*b^(1/3)*(a + b*x^3)^4*Log[a^(1/3
) + b^(1/3)*x] - 910*b^(1/3)*(a + b*x^3)^4*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2]))/(2916*a^(16/3)*((a + b*x^3)^2)^(5/2))
```

3.115.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.68, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.577$, Rules used = {1384, 27, 819, 819, 819, 819, 847, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(a^2+2abx^3+b^2x^6\right)^{5/2}} \, dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b^5\left(a+bx^3\right)\int \frac{1}{b^5x^2\left(bx^3+a\right)^5}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(a+bx^3\right)\int \frac{1}{x^2\left(bx^3+a\right)^5}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{819}$$

$$\frac{(a+bx^3)\left(\frac{13\int\frac{1}{x^2\left(bx^3+a\right)^4}dx}{12a}+\frac{1}{12ax(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 819$$

$$\frac{(a+bx^3)\left(\frac{13\left(\frac{10\int\frac{1}{x^2\left(bx^3+a\right)^3}dx}{9a}+\frac{1}{9ax\left(a+bx^3\right)^3}\right)}{12a}+\frac{1}{12ax(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 819$$

$$\frac{(a+bx^3)\left(\frac{13\left(\frac{10\left(\frac{7\int\frac{1}{x^2\left(bx^3+a\right)^2}dx}{6a}+\frac{1}{6ax\left(a+bx^3\right)^2}\right)}{9a}+\frac{1}{9ax\left(a+bx^3\right)^3}\right)}{12a}+\frac{1}{12ax(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 819$$

3.115. $\int\frac{1}{x^2(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

$$\frac{(a+bx^3)\left(\frac{13\left(\frac{10\left(\frac{7\left(\frac{4\int\frac{1}{x^2\left(bx^3+a\right)}dx}{3a}+\frac{1}{3ax\left(a+bx^3\right)}\right)}{6a}+\frac{1}{6ax\left(a+bx^3\right)^2}\right)}{9a}+\frac{1}{9ax\left(a+bx^3\right)^3}\right)}{12a}+\frac{1}{12ax(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{ 847}$$

3.115. $\int \frac{1}{x^2(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

$$\frac{\left(a+bx^3\right)\left(\frac{13\left(\frac{10\left(\frac{7\left(\frac{4\left(-\frac{b\int\frac{x}{bx^3+a}dx}{a}-\frac{1}{ax}\right)}{3a}+\frac{1}{3ax\left(a+bx^3\right)}\right)}{6a}+\frac{1}{6ax\left(a+bx^3\right)^2}\right)}{9a}+\frac{1}{9ax\left(a+bx^3\right)^3}\right)}{12a}+\frac{1}{12ax(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 821$$

3.115. $\int \frac{1}{x^2(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

$$\left(13\left(\frac{10\left(\frac{7\left(\frac{4\left(-\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{a}-\frac{1}{ax}\right)}{3a}+\frac{1}{3ax\left(a+bx^3\right)}\right)}{6a}+\frac{1}{6ax\left(a+bx^3\right)^2}\right)}{9a}+\frac{1}{9ax\left(a+bx^3\right)^3}\right)$$

3.115. $\int \frac{1}{x^2(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

3.115. $\int \frac{1}{x^2(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

$\downarrow$ 16

$$\left(\left(13\,\frac{\left(10\,\frac{\left(7\,\frac{\left(4\left(-\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}\,dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)\right)}{3a}+\frac{1}{3ax\left(a+bx^3\right)}\right)}{6a}+\frac{1}{6ax\left(a+bx^3\right)^2}\right)}{9a}+\frac{1}{9ax\left(a+bx^3\right)^3}\right.\right.$$

3.115. $\displaystyle\int \frac{1}{x^2(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

↓ 1142

3.115. $\int \frac{1}{x^2(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

$$\left(\left(\left(\frac{10\left(\frac{7\left(4\left(-\frac{b\left(\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)\right)}{3a}+\frac{1}{3ax\left(a+bx^3\right)}\right)}{6a}\right.\right.\right.$$

3.115. $\int \frac{1}{x^2(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

$\downarrow$ 25

3.115. $\int \frac{1}{x^2(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

$$\left(\left(\left(10\,\frac{\left(7\,\frac{\left(4\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)\right)}{3a}+\frac{1}{3ax\left(a+bx^3\right)}\right)}{6a}+\right.\right.\right.$$

3.115. $\int\frac{1}{x^2(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

$\downarrow$ 27

3.115. $\int \frac{1}{x^2(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

$$\left(13\frac{\left(10\frac{\left(7\frac{\left(4\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)\right)}{3a}+\frac{1}{3ax\left(a+bx^3\right)}\right)}{6a}}{9a}\right.$$

3.115. $\int \frac{1}{x^2\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$

$\downarrow$ 1082

3.115. $\int \frac{1}{x^2(a^2+2abx^3+b^2x^6)^{5/2}}\, dx$

$$\left(\left(\left(10\,\frac{\left(7\,\frac{\left(4\left(-\frac{b\left(\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}\,d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}\,dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)\right)}{3a}+\frac{1}{3ax\left(a+bx^3\right)}\right.}{6a}\right.\right.\right.$$

3.115. $\int \frac{1}{x^2(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

↓ 217

3.115. $\int \frac{1}{x^2(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

$$10\left(\frac{7\left(\frac{4\left(-\frac{b\left(\frac{-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{3a}+\frac{1}{3ax\left(a+bx^3\right)}\right)}{6a}+\frac{1}{6ax\left(a+b\ldots\right.}\right.$$

3.115. $\int \frac{1}{x^2(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

↓ 1103

3.115. $\int \frac{1}{x^2(a^2+2abx^3+b^2x^6)^{5/2}}\, dx$

$$\left(\left(\left(10\,\frac{\left(7\,\frac{\left(4\left(-\frac{b\left(\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\,\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)\right)}{3a}+\frac{1}{3ax\left(a+bx^3\right)}\right)}{6a}+\frac{1}{6ax\left(a+bx^3\right.}$$

3.115. $\int \frac{1}{x^2(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

input
```
Int[1/(x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)),x]
```

output
```
((a + b*x^3)*(1/(12*a*x*(a + b*x^3)^4) + (13*(1/(9*a*x*(a + b*x^3)^3) + (1
0*(1/(6*a*x*(a + b*x^3)^2) + (7*(1/(3*a*x*(a + b*x^3)) + (4*(-(1/(a*x)) -
(b*(-1/3*Log[a^(1/3) + b^(1/3)*x]/(a^(1/3)*b^(2/3)) + (-((Sqrt[3]*ArcTan[(
1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) + Log[a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(1/3)*b^(1/3))))/a))/(3*a)))/(6*a))
)/(9*a)))/(12*a)))/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]
```

3.115.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

3.115.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.34 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.34

method	result
risch	$\frac{\sqrt{(b\,x^3+a)^2}\left(-\frac{455b^4x^{12}}{243a^5}-\frac{2275b^3x^9}{324a^4}-\frac{260b^2x^6}{27a^3}-\frac{1352b\,x^3}{243a^2}-\frac{1}{a}\right)}{(b\,x^3+a)^5x}+\frac{455\sqrt{(b\,x^3+a)^2}\left(\sum_{\_R=\mathrm{RootOf}(a^{16}\_Z^3-b)}\_R\ln\left(\left(-4\_R^3a^{16}+\right.\right.\right.}{729(b\,x^3+a)}$
default	$-\frac{\left(-1820\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)b^4x^{13}-1820\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)b^4x^{13}+910\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)b^4x^{13}+5460\left(\frac{a}{b}\right)^{\frac{1}{3}}b^4x^{12}-\right.}{}$

input
```
int(1/x^2/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^3+a)^2)^(1/2)/(b*x^3+a)^5*(-455/243/a^5*b^4*x^12-2275/324*b^3/a^4*x^
9-260/27*b^2/a^3*x^6-1352/243*b/a^2*x^3-1/a)/x+455/729*((b*x^3+a)^2)^(1/2)
/(b*x^3+a)*sum(_R*ln((-4*_R^3*a^16+3*b)*x-a^11*_R^2),_R=RootOf(_Z^3*a^16-b
))
```

3.115.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^2\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx =$$
$$-\frac{5460\,b^4x^{12}+20475\,ab^3x^9+28080\,a^2b^2x^6+16224\,a^3bx^3+2916\,a^4+1820\,\sqrt{3}(b^4x^{13}+4\,ab^3x^{10}+6\,a^2b^2x^7+}{}$$

input
```
integrate(1/x^2/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/2916*(5460*b^4*x^12 + 20475*a*b^3*x^9 + 28080*a^2*b^2*x^6 + 16224*a^3*b
*x^3 + 2916*a^4 + 1820*sqrt(3)*(b^4*x^13 + 4*a*b^3*x^10 + 6*a^2*b^2*x^7 +
4*a^3*b*x^4 + a^4*x)*(b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(b/a)^(1/3) - 1/3*sq
rt(3)) + 910*(b^4*x^13 + 4*a*b^3*x^10 + 6*a^2*b^2*x^7 + 4*a^3*b*x^4 + a^4*
x)*(b/a)^(1/3)*log(b*x^2 - a*x*(b/a)^(2/3) + a*(b/a)^(1/3)) - 1820*(b^4*x^
13 + 4*a*b^3*x^10 + 6*a^2*b^2*x^7 + 4*a^3*b*x^4 + a^4*x)*(b/a)^(1/3)*log(b
*x + a*(b/a)^(2/3)))/(a^5*b^4*x^13 + 4*a^6*b^3*x^10 + 6*a^7*b^2*x^7 + 4*a^
8*b*x^4 + a^9*x)
```

3.115.6 Sympy [F]

$$\int \frac{1}{x^2 (a^2 + 2abx^3 + b^2x^6)^{5/2}} dx = \int \frac{1}{x^2 \left((a + bx^3)^2\right)^{\frac{5}{2}}} dx$$

input
```
integrate(1/x**2/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

output
```
Integral(1/(x**2*((a + b*x**3)**2)**(5/2)), x)
```

3.115.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.48

$$\begin{aligned}\int \frac{1}{x^2 (a^2 + 2abx^3 + b^2x^6)^{5/2}} dx = & -\frac{1820 b^4 x^{12} + 6825 ab^3 x^9 + 9360 a^2 b^2 x^6 + 5408 a^3 b x^3 + 972 a^4}{972 (a^5 b^4 x^{13} + 4 a^6 b^3 x^{10} + 6 a^7 b^2 x^7 + 4 a^8 b x^4 + a^9 x)} \\ & - \frac{455 \sqrt{3} \arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729 a^5 \left(\frac{a}{b}\right)^{\frac{1}{3}}} \\ & - \frac{455 \log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{1458 a^5 \left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{455 \log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{729 a^5 \left(\frac{a}{b}\right)^{\frac{1}{3}}}\end{aligned}$$

input
```
integrate(1/x^2/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/972*(1820*b^4*x^12 + 6825*a*b^3*x^9 + 9360*a^2*b^2*x^6 + 5408*a^3*b*x^3
 + 972*a^4)/(a^5*b^4*x^13 + 4*a^6*b^3*x^10 + 6*a^7*b^2*x^7 + 4*a^8*b*x^4 +
 a^9*x) - 455/729*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/
3))/(a^5*(a/b)^(1/3)) - 455/1458*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a
^5*(a/b)^(1/3)) + 455/729*log(x + (a/b)^(1/3))/(a^5*(a/b)^(1/3))
```

3.115.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.56

$$\int \frac{1}{x^2 (a^2 + 2abx^3 + b^2x^6)^{5/2}}\, dx = \frac{455\, b\left(-\frac{a}{b}\right)^{\frac{2}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{729\, a^6 \mathrm{sgn}\left(bx^3 + a\right)} + \frac{455\, \sqrt{3}(-ab^2)^{\frac{2}{3}} \arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729\, a^6 b \mathrm{sgn}\left(bx^3 + a\right)} - \frac{455\,(-ab^2)^{\frac{2}{3}} \log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{1458\, a^6 b \mathrm{sgn}\left(bx^3 + a\right)} - \frac{1}{a^5 x \mathrm{sgn}\left(bx^3 + a\right)} - \frac{848\, b^4 x^{11} + 2937\, ab^3 x^8 + 3528\, a^2 b^2 x^5 + 1520\, a^3 b x^2}{972\, (bx^3 + a)^4 a^5 \mathrm{sgn}\left(bx^3 + a\right)}$$

input
```
integrate(1/x^2/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

output
```
455/729*b*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a^6*sgn(b*x^3 + a)) + 4
55/729*sqrt(3)*(-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/
b)^(1/3))/(a^6*b*sgn(b*x^3 + a)) - 455/1458*(-a*b^2)^(2/3)*log(x^2 + x*(-a
/b)^(1/3) + (-a/b)^(2/3))/(a^6*b*sgn(b*x^3 + a)) - 1/(a^5*x*sgn(b*x^3 + a)
) - 1/972*(848*b^4*x^11 + 2937*a*b^3*x^8 + 3528*a^2*b^2*x^5 + 1520*a^3*b*x
^2)/((b*x^3 + a)^4*a^5*sgn(b*x^3 + a))
```

3.115.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a^2 + 2abx^3 + b^2x^6)^{5/2}}\, dx = \int \frac{1}{x^2\, (a^2 + 2\,a\,b\,x^3 + b^2\,x^6)^{5/2}}\, dx$$

input
```
int(1/(x^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)),x)
```

output
```
int(1/(x^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)), x)
```

3.116 $\int \frac{1}{x^3(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

3.116.1 Optimal result

Integrand size = 26, antiderivative size = 398

$$\begin{aligned}\int \frac{1}{x^3\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx &= \frac{154}{243a^4x^2\sqrt{a^2+2abx^3+b^2x^6}} \\ &+ \frac{1}{12ax^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}+\frac{7}{54a^2x^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}} \\ &+ \frac{77}{324a^3x^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}-\frac{385(a+bx^3)}{243a^5x^2\sqrt{a^2+2abx^3+b^2x^6}} \\ &+ \frac{770b^{2/3}(a+bx^3)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}a^{17/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{770b^{2/3}(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{17/3}\sqrt{a^2+2abx^3+b^2x^6}} \\ &+ \frac{385b^{2/3}(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{729a^{17/3}\sqrt{a^2+2abx^3+b^2x^6}}\end{aligned}$$

output
```
154/243/a^4/x^2/((b*x^3+a)^2)^(1/2)+1/12/a/x^2/(b*x^3+a)^3/((b*x^3+a)^2)^(
1/2)+7/54/a^2/x^2/(b*x^3+a)^2/((b*x^3+a)^2)^(1/2)+77/324/a^3/x^2/(b*x^3+a)
/((b*x^3+a)^2)^(1/2)-385/243*(b*x^3+a)/a^5/x^2/((b*x^3+a)^2)^(1/2)-770/729
*b^(2/3)*(b*x^3+a)*ln(a^(1/3)+b^(1/3)*x)/a^(17/3)/((b*x^3+a)^2)^(1/2)+385/
729*b^(2/3)*(b*x^3+a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(17/3)/(
(b*x^3+a)^2)^(1/2)+770/729*b^(2/3)*(b*x^3+a)*arctan(1/3*(a^(1/3)-2*b^(1/3)
*x)/a^(1/3)*3^(1/2))/a^(17/3)*3^(1/2)/((b*x^3+a)^2)^(1/2)
```

3.116.2 Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.59

$$\int \frac{1}{x^3 (a^2 + 2abx^3 + b^2x^6)^{5/2}} \, dx = \frac{(a + bx^3) \left(-243a^{11/3}bx - 621a^{8/3}bx(a + bx^3) - 1314a^{5/3}bx(a + bx^3)^2 - \right.}{}$$

input
```
Integrate[1/(x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)),x]
```

output
```
((a + b*x^3)*(-243*a^(11/3)*b*x - 621*a^(8/3)*b*x*(a + b*x^3) - 1314*a^(5/
3)*b*x*(a + b*x^3)^2 - 3162*a^(2/3)*b*x*(a + b*x^3)^3 - (1458*a^(2/3)*(a +
 b*x^3)^4)/x^2 - 3080*Sqrt[3]*b^(2/3)*(a + b*x^3)^4*ArcTan[(-a^(1/3) + 2*b
^(1/3)*x)/(Sqrt[3]*a^(1/3))] - 3080*b^(2/3)*(a + b*x^3)^4*Log[a^(1/3) + b^
(1/3)*x] + 1540*b^(2/3)*(a + b*x^3)^4*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2]))/(2916*a^(17/3)*((a + b*x^3)^2)^(5/2))
```

3.116.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.67, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.577$, Rules used = {1384, 27, 819, 819, 819, 819, 847, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a^2 + 2abx^3 + b^2x^6)^{5/2}} \, dx$$

$$\Big\downarrow \; 1384$$

$$\frac{b^5 (a + bx^3) \int \frac{1}{b^5 x^3 (bx^3+a)^5} dx}{\sqrt{a^2 + 2abx^3 + b^2x^6}}$$

$$\Big\downarrow \; 27$$

$$\frac{(a + bx^3) \int \frac{1}{x^3 (bx^3+a)^5} dx}{\sqrt{a^2 + 2abx^3 + b^2x^6}}$$

$$\Big\downarrow \; 819$$

$$\frac{\left(a+bx^3\right)\left(\frac{7\int \frac{1}{x^3\left(bx^3+a\right)^4}dx}{6a}+\frac{1}{12ax^2(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{ 819}$$

$$\frac{\left(a+bx^3\right)\left(\frac{7\left(\frac{11\int \frac{1}{x^3\left(bx^3+a\right)^3}dx}{9a}+\frac{1}{9ax^2\left(a+bx^3\right)^3}\right)}{6a}+\frac{1}{12ax^2(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{ 819}$$

$$\frac{\left(a+bx^3\right)\left(\frac{7\left(\frac{11\left(\frac{4\int \frac{1}{x^3\left(bx^3+a\right)^2}dx}{3a}+\frac{1}{6ax^2\left(a+bx^3\right)^2}\right)}{9a}+\frac{1}{9ax^2\left(a+bx^3\right)^3}\right)}{6a}+\frac{1}{12ax^2(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{ 819}$$

3.116. $\int \frac{1}{x^3(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

$$\frac{\left(a+bx^3\right)\left(\frac{7\left(\frac{11\left(\frac{4\left(\frac{5\int \frac{1}{x^3\left(bx^3+a\right)}dx}{3a}+\frac{1}{3ax^2\left(a+bx^3\right)}\right)}{3a}+\frac{1}{6ax^2\left(a+bx^3\right)^2}\right)}{9a}+\frac{1}{9ax^2\left(a+bx^3\right)^3}\right)}{6a}+\frac{1}{12ax^2(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \ 847$$

3.116. $\int \frac{1}{x^3(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

$$\frac{(a+bx^3)\left(\frac{7\left(\frac{11\left(\frac{4\left(\frac{5\left(-\frac{b\int\frac{1}{bx^3+a}dx}{a}-\frac{1}{2ax^2}\right)}{3a}+\frac{1}{3ax^2(a+bx^3)}\right)}{3a}+\frac{1}{6ax^2(a+bx^3)^2}\right)}{9a}+\frac{1}{9ax^2(a+bx^3)^3}\right)}{6a}+\frac{1}{12ax^2(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\downarrow$ 750

3.116. $\int \frac{1}{x^3(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

$$\left(7\left(\frac{11\left(\frac{4\left(\frac{5\left(-\frac{b\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{a}-\frac{1}{2ax^2}\right)}{3a}+\frac{1}{3ax^2(a+bx^3)}\right)}{3a}+\frac{1}{6ax^2(a+bx^3)^2}\right)}{9a}+\frac{1}{9ax^2(a+bx^3)^3}\right)\right)$$

3.116. $\int \frac{1}{x^3(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

$$\downarrow 16$$

3.116. $\int \frac{1}{x^3(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

$$\left(\left(\frac{7\left(\frac{11\left(\frac{4\left(5\left(-\frac{b\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)\right)}{3a}+\frac{1}{3ax^2\left(a+bx^3\right)}\right)}{3a}+\frac{1}{6ax^2\left(a+bx^3\right)^2}\right)}{9a}+\frac{1}{9ax^2\left(a+bx^3\right)^3}\right.\right.$$

3.116. $\int \frac{1}{x^3\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$

↓ 1142

3.116. $\int \frac{1}{x^3(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

$$\frac{4\left(\frac{5\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{3a}+\frac{1}{3ax^2\left(a+bx^3\right)}\right)}{3a}$$

3.116. $\int\frac{1}{x^3\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$

$$\Big\downarrow \; 25$$

3.116. $\int \frac{1}{x^3(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

$$\left(\left(\left(11\left(\frac{4\left(\frac{5\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}\right)-\frac{1}{2ax^2}\right)}{3a}+\frac{1}{3ax^2\left(a+bx^3\right)}\right)}{3a}\right.\right.\right.$$

3.116. $\int \frac{1}{x^3\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$

$$\downarrow 27$$

3.116. $\int \frac{1}{x^3(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

$$7\left(\frac{11\left(\frac{4\left(\frac{5\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{3a}+\frac{1}{3ax^2\left(a+bx^3\right)}\cdots}{3a}\cdots\right)}{9a}\cdots\right)$$

3.116. $\int \frac{1}{x^3\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$

3.116. $\int \frac{1}{x^3(a^2+2abx^3+b^2x^6)^{5/2}} \, dx$

$\downarrow$ 1082

$$\left(\left(\left(11\,\frac{4\left(5\left(-\frac{b\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{3a}+\frac{1}{3ax^2\left(a+bx^3\right.}}{3a}\right.\right.\right.$$

3.116. $\int\frac{1}{x^3\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$

$\downarrow$ 217

3.116. $\int \frac{1}{x^3(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

$$11\left(4\left(5\left(-\frac{b\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)\frac{1}{3a}+\frac{1}{3ax^2\left(a+bx^3\right)}\right)\frac{1}{3a}+\frac{1}{6ax^2\left(a+\right.}$$

3.116. $\int \frac{1}{x^3\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$

↓ 1103

3.116. $\int \frac{1}{x^3(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

$$\left(\left(\frac{11\left(\frac{4\left(\frac{5\left(-\frac{b\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{3a}+\frac{1}{3ax^2\left(a+bx^3\right)}\right)}{3a}+\frac{\cdots}{6ax^2\left(a\cdots\right.}\right.\right.\right.$$

3.116. $\int \frac{1}{x^3\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$

input
```
Int[1/(x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)),x]
```

output
```
((a + b*x^3)*(1/(12*a*x^2*(a + b*x^3)^4) + (7*(1/(9*a*x^2*(a + b*x^3)^3) +
 (11*(1/(6*a*x^2*(a + b*x^3)^2) + (4*(1/(3*a*x^2*(a + b*x^3)) + (5*(-1/2*1
/(a*x^2) - (b*(Log[a^(1/3) + b^(1/3)*x]/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*
ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) - Log[a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(2/3))))/a))/(3*a)))/(3*a))
)/(9*a)))/(6*a)))/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]
```

3.116.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

3.116.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.86 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.35

method	result
risch	$\frac{\sqrt{(b x^3+a)^2}\left(-\frac{385 b^4 x^{12}}{243 a^5}-\frac{154 b^3 x^9}{27 a^4}-\frac{2387 b^2 x^6}{324 a^3}-\frac{931 b x^3}{243 a^2}-\frac{1}{2 a}\right)}{(b x^3+a)^5 x^2}+\frac{770\sqrt{(b x^3+a)^2}\left(\sum_{\_R=\mathrm{RootOf}(a^{17}\_Z^3+b^2)}\_R\ln\left(\left(-4\_R^3 a^{17}\right.\right.\right.}{729(b x^3+a)}$
default	$-\frac{\left(-3080\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)b^4x^{14}+3080\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)b^4x^{14}-1540\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)b^4x^{14}+4620\left(\frac{a}{b}\right)^{\frac{2}{3}}b^4x^{12}\right.}{}$

input
```
int(1/x^3/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^3+a)^2)^(1/2)/(b*x^3+a)^5*(-385/243/a^5*b^4*x^12-154/27*b^3/a^4*x^9-
2387/324*b^2/a^3*x^6-931/243*b/a^2*x^3-1/2/a)/x^2+770/729*((b*x^3+a)^2)^(1
/2)/(b*x^3+a)*sum(_R*ln((-4*_R^3*a^17-3*b^2)*x-a^6*b*_R),_R=RootOf(_Z^3*a^
17+b^2))
```

3.116.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 352, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^3 (a^2+2abx^3+b^2x^6)^{5/2}}\,dx =$$

$$-\frac{4620\,b^4x^{12}+16632\,ab^3x^9+21483\,a^2b^2x^6+11172\,a^3bx^3+1458\,a^4-3080\,\sqrt{3}(b^4x^{14}+4\,ab^3x^{11}+6\,a^2b^2x^8}{}$$

input
```
integrate(1/x^3/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/2916*(4620*b^4*x^12 + 16632*a*b^3*x^9 + 21483*a^2*b^2*x^6 + 11172*a^3*b
*x^3 + 1458*a^4 - 3080*sqrt(3)*(b^4*x^14 + 4*a*b^3*x^11 + 6*a^2*b^2*x^8 +
4*a^3*b*x^5 + a^4*x^2)*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(-b^2/a^
2)^(2/3) - sqrt(3)*b)/b) + 1540*(b^4*x^14 + 4*a*b^3*x^11 + 6*a^2*b^2*x^8 +
 4*a^3*b*x^5 + a^4*x^2)*(-b^2/a^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b^2/a^2)^(1
/3) + a^2*(-b^2/a^2)^(2/3)) - 3080*(b^4*x^14 + 4*a*b^3*x^11 + 6*a^2*b^2*x^
8 + 4*a^3*b*x^5 + a^4*x^2)*(-b^2/a^2)^(1/3)*log(b*x - a*(-b^2/a^2)^(1/3)))
/(a^5*b^4*x^14 + 4*a^6*b^3*x^11 + 6*a^7*b^2*x^8 + 4*a^8*b*x^5 + a^9*x^2)
```

3.116.6 Sympy [F]

$$\int \frac{1}{x^3 (a^2 + 2abx^3 + b^2x^6)^{5/2}}\,dx = \int \frac{1}{x^3 \left((a+bx^3)^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x**3/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

output
```
Integral(1/(x**3*((a + b*x**3)**2)**(5/2)), x)
```

3.116.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.49

$$\int \frac{1}{x^3 (a^2 + 2abx^3 + b^2x^6)^{5/2}}\,dx = -\frac{1540\,b^4x^{12} + 5544\,ab^3x^9 + 7161\,a^2b^2x^6 + 3724\,a^3bx^3 + 486\,a^4}{972\,(a^5b^4x^{14} + 4\,a^6b^3x^{11} + 6\,a^7b^2x^8 + 4\,a^8bx^5 + a^9x^2)} - \frac{770\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729\,a^5\,\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{385\,\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{729\,a^5\,\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{770\,\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{729\,a^5\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(1/x^3/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/972*(1540*b^4*x^12 + 5544*a*b^3*x^9 + 7161*a^2*b^2*x^6 + 3724*a^3*b*x^3
 + 486*a^4)/(a^5*b^4*x^14 + 4*a^6*b^3*x^11 + 6*a^7*b^2*x^8 + 4*a^8*b*x^5 +
 a^9*x^2) - 770/729*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(
1/3))/(a^5*(a/b)^(2/3)) + 385/729*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(
a^5*(a/b)^(2/3)) - 770/729*log(x + (a/b)^(1/3))/(a^5*(a/b)^(2/3))
```

3.116.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.54

$$\int \frac{1}{x^3(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \frac{770\,b\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{729\,a^6\mathrm{sgn}\left(bx^3+a\right)} - \frac{770\,\sqrt{3}(-ab^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729\,a^6\mathrm{sgn}\left(bx^3+a\right)} - \frac{385\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{729\,a^6\mathrm{sgn}\left(bx^3+a\right)} - \frac{1}{2\,a^5x^2\mathrm{sgn}\left(bx^3+a\right)} - \frac{1054\,b^4x^{10}+3600\,ab^3x^7+4245\,a^2b^2x^4+1780\,a^3bx}{972\left(bx^3+a\right)^4a^5\mathrm{sgn}\left(bx^3+a\right)}$$

input
```
integrate(1/x^3/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

output
```
770/729*b*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^6*sgn(b*x^3 + a)) - 7
70/729*sqrt(3)*(-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/
b)^(1/3))/(a^6*sgn(b*x^3 + a)) - 385/729*(-a*b^2)^(1/3)*log(x^2 + x*(-a/b)
^(1/3) + (-a/b)^(2/3))/(a^6*sgn(b*x^3 + a)) - 1/2/(a^5*x^2*sgn(b*x^3 + a))
 - 1/972*(1054*b^4*x^10 + 3600*a*b^3*x^7 + 4245*a^2*b^2*x^4 + 1780*a^3*b*x
)/((b*x^3 + a)^4*a^5*sgn(b*x^3 + a))
```

3.116.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \int \frac{1}{x^3\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}}\,dx$$

input
```
int(1/(x^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)),x)
```

output
```
int(1/(x^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)), x)
```

3.117 $\int \frac{1}{x^4\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$

3.117.1 Optimal result

Integrand size = 26, antiderivative size = 269

$$\begin{aligned}\int \frac{1}{x^4\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx &= -\frac{4b}{3a^5\sqrt{a^2+2abx^3+b^2x^6}} \\ &- \frac{b}{12a^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} - \frac{2b}{9a^3\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}} \\ &- \frac{b}{2a^4\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{a+bx^3}{3a^5x^3\sqrt{a^2+2abx^3+b^2x^6}} \\ &- \frac{5b(a+bx^3)\log(x)}{a^6\sqrt{a^2+2abx^3+b^2x^6}} + \frac{5b(a+bx^3)\log\left(a+bx^3\right)}{3a^6\sqrt{a^2+2abx^3+b^2x^6}}\end{aligned}$$

output
```
-4/3*b/a^5/((b*x^3+a)^2)^(1/2)-1/12*b/a^2/(b*x^3+a)^3/((b*x^3+a)^2)^(1/2)-
2/9*b/a^3/(b*x^3+a)^2/((b*x^3+a)^2)^(1/2)-1/2*b/a^4/(b*x^3+a)/((b*x^3+a)^2
)^(1/2)+1/3*(-b*x^3-a)/a^5/x^3/((b*x^3+a)^2)^(1/2)-5*b*(b*x^3+a)*ln(x)/a^6
/((b*x^3+a)^2)^(1/2)+5/3*b*(b*x^3+a)*ln(b*x^3+a)/a^6/((b*x^3+a)^2)^(1/2)
```

3.117.2 Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.44

$$\int \frac{1}{x^4 \left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx = \frac{-a(12a^4+125a^3bx^3+260a^2b^2x^6+210ab^3x^9+60b^4x^{12})-180bx^3(a+bx}{36a^6x^3\left(a+bx^3\right)^3\sqrt{(a+bx^3)^2}}$$

input
```
Integrate[1/(x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)),x]
```

output
```
(-(a*(12*a^4 + 125*a^3*b*x^3 + 260*a^2*b^2*x^6 + 210*a*b^3*x^9 + 60*b^4*x^
12)) - 180*b*x^3*(a + b*x^3)^4*Log[x] + 60*b*x^3*(a + b*x^3)^4*Log[a + b*x
^3])/(36*a^6*x^3*(a + b*x^3)^3*Sqrt[(a + b*x^3)^2])
```

3.117.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.49, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1384, 27, 798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \left(a^2+2abx^3+b^2x^6\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{b^5\left(a+bx^3\right)\int \frac{1}{b^5x^4\left(bx^3+a\right)^5}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\left(a+bx^3\right)\int \frac{1}{x^4\left(bx^3+a\right)^5}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{ 798}$$

$$\frac{\left(a+bx^3\right)\int \frac{1}{x^6\left(bx^3+a\right)^5}dx^3}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{ 54}$$

$$\frac{(a+bx^3)\int\left(\frac{5b^2}{a^6(bx^3+a)}+\frac{4b^2}{a^5(bx^3+a)^2}+\frac{3b^2}{a^4(bx^3+a)^3}+\frac{2b^2}{a^3(bx^3+a)^4}+\frac{b^2}{a^2(bx^3+a)^5}-\frac{5b}{a^6x^3}+\frac{1}{a^5x^6}\right)dx^3}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx^3)\left(-\frac{5b\log(x^3)}{a^6}+\frac{5b\log(a+bx^3)}{a^6}-\frac{4b}{a^5(a+bx^3)}-\frac{1}{a^5x^3}-\frac{3b}{2a^4(a+bx^3)^2}-\frac{2b}{3a^3(a+bx^3)^3}-\frac{b}{4a^2(a+bx^3)^4}\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

input
```
Int[1/(x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)),x]
```

output
```
((a + b*x^3)*(-(1/(a^5*x^3)) - b/(4*a^2*(a + b*x^3)^4) - (2*b)/(3*a^3*(a +
 b*x^3)^3) - (3*b)/(2*a^4*(a + b*x^3)^2) - (4*b)/(a^5*(a + b*x^3)) - (5*b*
Log[x^3])/a^6 + (5*b*Log[a + b*x^3])/a^6))/(3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x
^6])
```

3.117.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.117.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.14 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.42

method	result
pseudoelliptic	$-\frac{\left(-5b\,x^3(b\,x^3+a)^4\ln(b\,x^3+a)+5b\,x^3(b\,x^3+a)^4\ln(b\,x^3)+a(5b^4x^{12}+\frac{35}{2}a\,b^3x^9+\frac{65}{3}a^2b^2x^6+\frac{125}{12}a^3b\,x^3+a^4)\right)\operatorname{csgn}(b\,x^3+a}{3(b\,x^3+a)^4a^6x^3}$
risch	$\frac{\sqrt{(b\,x^3+a)^2}\left(-\frac{1}{3a}-\frac{125b\,x^3}{36a^2}-\frac{65b^2x^6}{9a^3}-\frac{35b^3x^9}{6a^4}-\frac{5b^4x^{12}}{3a^5}\right)}{(b\,x^3+a)^5x^3}-\frac{5\sqrt{(b\,x^3+a)^2}\,b\ln(x)}{(b\,x^3+a)a^6}+\frac{5\sqrt{(b\,x^3+a)^2}\,b\ln(-b\,x^3-a)}{3(b\,x^3+a)a^6}$
default	$\frac{(60\ln(b\,x^3+a)b^5x^{15}-180b^5\ln(x)x^{15}+240\ln(b\,x^3+a)a\,b^4x^{12}-720b^4a\ln(x)x^{12}-60a\,b^4x^{12}+360\ln(b\,x^3+a)a^2b^3x^9-1080a^2}{3}$

input
```
int(1/x^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-5*b*x^3*(b*x^3+a)^4*ln(b*x^3+a)+5*b*x^3*(b*x^3+a)^4*ln(b*x^3)+a*(5*
b^4*x^12+35/2*a*b^3*x^9+65/3*a^2*b^2*x^6+125/12*a^3*b*x^3+a^4))*csgn(b*x^3
+a)/(b*x^3+a)^4/a^6/x^3
```

3.117.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^4(a^2+2abx^3+b^2x^6)^{5/2}}\,dx =$$
$$-\frac{60\,ab^4x^{12}+210\,a^2b^3x^9+260\,a^3b^2x^6+125\,a^4bx^3+12\,a^5-60\,(b^5x^{15}+4\,ab^4x^{12}+6\,a^2b^3x^9+4\,a^3b^2x^6+a^4}{36\,(a^6b^4x^{15}+4\,a^7b^3x^{12}+6\,a^8b^2x^9+4\,a}$$

input
```
integrate(1/x^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/36*(60*a*b^4*x^12 + 210*a^2*b^3*x^9 + 260*a^3*b^2*x^6 + 125*a^4*b*x^3 +
 12*a^5 - 60*(b^5*x^15 + 4*a*b^4*x^12 + 6*a^2*b^3*x^9 + 4*a^3*b^2*x^6 + a^
4*b*x^3)*log(b*x^3 + a) + 180*(b^5*x^15 + 4*a*b^4*x^12 + 6*a^2*b^3*x^9 + 4
*a^3*b^2*x^6 + a^4*b*x^3)*log(x))/(a^6*b^4*x^15 + 4*a^7*b^3*x^12 + 6*a^8*b
^2*x^9 + 4*a^9*b*x^6 + a^10*x^3)
```

3.117.6 Sympy [F]

$$\int \frac{1}{x^4 (a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \int \frac{1}{x^4 \left((a+bx^3)^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x**4/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

output
```
Integral(1/(x**4*((a + b*x**3)**2)**(5/2)), x)
```

3.117.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.61

$$\int \frac{1}{x^4 (a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \frac{5\,(-1)^{2\,abx^3+2\,a^2}\,b\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right)}{3\,a^6} - \frac{5\,b}{9\,(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}a^3} - \frac{5\,b}{3\,\sqrt{b^2x^6+2\,abx^3+a^2}a^5} - \frac{1}{3\,(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}a^2x^3} - \frac{5}{6\,\left(x^3+\frac{a}{b}\right)^2a^4b} - \frac{1}{12\,\left(x^3+\frac{a}{b}\right)^4a^2b^3}$$

input
```
integrate(1/x^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

output
```
5/3*(-1)^(2*a*b*x^3 + 2*a^2)*b*log(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x)))/a^
6 - 5/9*b/((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*a^3) - 5/3*b/(sqrt(b^2*x^6 +
2*a*b*x^3 + a^2)*a^5) - 1/3/((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*a^2*x^3) -
5/6/((x^3 + a/b)^2*a^4*b) - 1/12/((x^3 + a/b)^4*a^2*b^3)
```

3.117.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.53

$$\int \frac{1}{x^4 (a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \frac{5\,b\log\left(|bx^3+a|\right)}{3\,a^6\mathrm{sgn}\left(bx^3+a\right)} - \frac{5\,b\log\left(|x|\right)}{a^6\mathrm{sgn}\left(bx^3+a\right)} + \frac{5\,bx^3-a}{3\,a^6x^3\mathrm{sgn}\left(bx^3+a\right)} - \frac{125\,b^5x^{12}+548\,ab^4x^9+912\,a^2b^3x^6+688\,a^3b^2x^3+202\,a^4b}{36\,(bx^3+a)^4a^6\mathrm{sgn}\left(bx^3+a\right)}$$

input
```
integrate(1/x^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

output
```
5/3*b*log(abs(b*x^3 + a))/(a^6*sgn(b*x^3 + a)) - 5*b*log(abs(x))/(a^6*sgn(
b*x^3 + a)) + 1/3*(5*b*x^3 - a)/(a^6*x^3*sgn(b*x^3 + a)) - 1/36*(125*b^5*x
^12 + 548*a*b^4*x^9 + 912*a^2*b^3*x^6 + 688*a^3*b^2*x^3 + 202*a^4*b)/((b*x
^3 + a)^4*a^6*sgn(b*x^3 + a))
```

3.117.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a^2 + 2abx^3 + b^2x^6)^{5/2}} \, dx = \int \frac{1}{x^4 \left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^{5/2}} \, dx$$

input
```
int(1/(x^4*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)),x)
```

output
```
int(1/(x^4*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)), x)
```

3.118 $\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx$

3.118.1 Optimal result

Integrand size = 28, antiderivative size = 313

$$\begin{aligned}\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx &= \frac{a^5(dx)^{1+m}\sqrt{a^2+2abx^3+b^2x^6}}{d(1+m)\left(a+bx^3\right)} \\ &+ \frac{5a^4b(dx)^{4+m}\sqrt{a^2+2abx^3+b^2x^6}}{d^4(4+m)\left(a+bx^3\right)}+\frac{10a^3b^2(dx)^{7+m}\sqrt{a^2+2abx^3+b^2x^6}}{d^7(7+m)\left(a+bx^3\right)} \\ &+ \frac{10a^2b^3(dx)^{10+m}\sqrt{a^2+2abx^3+b^2x^6}}{d^{10}(10+m)\left(a+bx^3\right)} \\ &+ \frac{5ab^4(dx)^{13+m}\sqrt{a^2+2abx^3+b^2x^6}}{d^{13}(13+m)\left(a+bx^3\right)}+\frac{b^5(dx)^{16+m}\sqrt{a^2+2abx^3+b^2x^6}}{d^{16}(16+m)\left(a+bx^3\right)}\end{aligned}$$

output
```
a^5*(d*x)^(1+m)*((b*x^3+a)^2)^(1/2)/d/(1+m)/(b*x^3+a)+5*a^4*b*(d*x)^(4+m)*
((b*x^3+a)^2)^(1/2)/d^4/(4+m)/(b*x^3+a)+10*a^3*b^2*(d*x)^(7+m)*((b*x^3+a)^
2)^(1/2)/d^7/(7+m)/(b*x^3+a)+10*a^2*b^3*(d*x)^(10+m)*((b*x^3+a)^2)^(1/2)/d
^10/(10+m)/(b*x^3+a)+5*a*b^4*(d*x)^(13+m)*((b*x^3+a)^2)^(1/2)/d^13/(13+m)/
(b*x^3+a)+b^5*(d*x)^(16+m)*((b*x^3+a)^2)^(1/2)/d^16/(16+m)/(b*x^3+a)
```

3.118.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.35

$$\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx = \frac{x(dx)^m \left(\left(a+bx^3\right)^2\right)^{5/2} \left(\frac{a^5}{1+m}+\frac{5a^4bx^3}{4+m}+\frac{10a^3b^2x^6}{7+m}+\frac{10a^2b^3x^9}{10+m}+\frac{5ab^4x^{12}}{13+m}+\frac{b^5x^{15}}{16+m}\right)}{\left(a+bx^3\right)^5}$$

input
```
Integrate[(d*x)^m*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
(x*(d*x)^m*((a + b*x^3)^2)^(5/2)*(a^5/(1 + m) + (5*a^4*b*x^3)/(4 + m) + (1
0*a^3*b^2*x^6)/(7 + m) + (10*a^2*b^3*x^9)/(10 + m) + (5*a*b^4*x^12)/(13 +
m) + (b^5*x^15)/(16 + m)))/(a + b*x^3)^5
```

3.118.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.51, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+2abx^3+b^2x^6\right)^{5/2} (dx)^m \, dx$$

$$\Big\downarrow \ 1384$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6} \int b^5(dx)^m \left(bx^3+a\right)^5 dx}{b^5 \left(a+bx^3\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6} \int (dx)^m \left(bx^3+a\right)^5 dx}{a+bx^3}$$

$$\Big\downarrow \ 802$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6} \int \left(a^5(dx)^m+\frac{5a^4b(dx)^{m+3}}{d^3}+\frac{10a^3b^2(dx)^{m+6}}{d^6}+\frac{10a^2b^3(dx)^{m+9}}{d^9}+\frac{5ab^4(dx)^{m+12}}{d^{12}}+\frac{b^5(dx)^{m+15}}{d^{15}}\right) dx}{a+bx^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(\frac{a^5(dx)^{m+1}}{d(m+1)}+\frac{5a^4b(dx)^{m+4}}{d^4(m+4)}+\frac{10a^3b^2(dx)^{m+7}}{d^7(m+7)}+\frac{10a^2b^3(dx)^{m+10}}{d^{10}(m+10)}+\frac{5ab^4(dx)^{m+13}}{d^{13}(m+13)}+\frac{b^5(dx)^{m+16}}{d^{16}(m+16)}\right)}{a+bx^3}$$

input
```
Int[(d*x)^m*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*((a^5*(d*x)^(1 + m))/(d*(1 + m)) + (5*a^4
*b*(d*x)^(4 + m))/(d^4*(4 + m)) + (10*a^3*b^2*(d*x)^(7 + m))/(d^7*(7 + m))
 + (10*a^2*b^3*(d*x)^(10 + m))/(d^10*(10 + m)) + (5*a*b^4*(d*x)^(13 + m))/
(d^13*(13 + m)) + (b^5*(d*x)^(16 + m))/(d^16*(16 + m))))/(a + b*x^3)
```

3.118.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.118.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.45

method	result
gosper	$\frac{x(b^5m^5x^{15}+35b^5m^4x^{15}+445b^5m^3x^{15}+5a\,b^4m^5x^{12}+2485b^5m^2x^{15}+190a\,b^4m^4x^{12}+5714m\,x^{15}b^5+2555a\,b^4m^3x^{12}+3640b^5x^{15}+10a}{}$
risch	$\frac{\sqrt{(b\,x^3+a)^2}\,(b^5m^5x^{15}+35b^5m^4x^{15}+445b^5m^3x^{15}+5a\,b^4m^5x^{12}+2485b^5m^2x^{15}+190a\,b^4m^4x^{12}+5714m\,x^{15}b^5+2555a\,b^4m^3x^{12}+364}{}$

input
```
int((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
x*(b^5*m^5*x^15+35*b^5*m^4*x^15+445*b^5*m^3*x^15+5*a*b^4*m^5*x^12+2485*b^5
*m^2*x^15+190*a*b^4*m^4*x^12+5714*b^5*m*x^15+2555*a*b^4*m^3*x^12+3640*b^5*
x^15+10*a^2*b^3*m^5*x^9+14810*a*b^4*m^2*x^12+410*a^2*b^3*m^4*x^9+34840*a*b
^4*m*x^12+5950*a^2*b^3*m^3*x^9+22400*a*b^4*x^12+10*a^3*b^2*m^5*x^6+36550*a
^2*b^3*m^2*x^9+440*a^3*b^2*m^4*x^6+89240*a^2*b^3*m*x^9+6970*a^3*b^2*m^3*x^
6+58240*a^2*b^3*x^9+5*a^4*b*m^5*x^3+47260*a^3*b^2*m^2*x^6+235*a^4*b*m^4*x^
3+123920*a^3*b^2*m*x^6+4085*a^4*b*m^3*x^3+83200*a^3*b^2*x^6+a^5*m^5+31685*
a^4*b*m^2*x^3+50*a^5*m^4+100630*a^4*b*m*x^3+955*a^5*m^3+72800*a^4*b*x^3+86
50*a^5*m^2+36824*a^5*m+58240*a^5)*(d*x)^m*((b*x^3+a)^2)^(5/2)/(1+m)/(4+m)/
(7+m)/(10+m)/(13+m)/(16+m)/(b*x^3+a)^5
```

3.118.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.18

$$\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{((b^5m^5+35\,b^5m^4+445\,b^5m^3+2485\,b^5m^2+5714\,b^5m+3640\,b^5)x^{16}+5\,(ab^4m^5+38\,ab^4m^4}{}$$

input
```
integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

output
```
((b^5*m^5 + 35*b^5*m^4 + 445*b^5*m^3 + 2485*b^5*m^2 + 5714*b^5*m + 3640*b^
5)*x^16 + 5*(a*b^4*m^5 + 38*a*b^4*m^4 + 511*a*b^4*m^3 + 2962*a*b^4*m^2 + 6
968*a*b^4*m + 4480*a*b^4)*x^13 + 10*(a^2*b^3*m^5 + 41*a^2*b^3*m^4 + 595*a^
2*b^3*m^3 + 3655*a^2*b^3*m^2 + 8924*a^2*b^3*m + 5824*a^2*b^3)*x^10 + 10*(a
^3*b^2*m^5 + 44*a^3*b^2*m^4 + 697*a^3*b^2*m^3 + 4726*a^3*b^2*m^2 + 12392*a
^3*b^2*m + 8320*a^3*b^2)*x^7 + 5*(a^4*b*m^5 + 47*a^4*b*m^4 + 817*a^4*b*m^3
 + 6337*a^4*b*m^2 + 20126*a^4*b*m + 14560*a^4*b)*x^4 + (a^5*m^5 + 50*a^5*m
^4 + 955*a^5*m^3 + 8650*a^5*m^2 + 36824*a^5*m + 58240*a^5)*x)*(d*x)^m/(m^6
 + 51*m^5 + 1005*m^4 + 9605*m^3 + 45474*m^2 + 95064*m + 58240)
```

3.118.6 Sympy [F]

$$\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^{5/2} \, dx = \int (dx)^m \left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}} \, dx$$

input
```
integrate((d*x)**m*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

output
```
Integral((d*x)**m*((a + b*x**3)**2)**(5/2), x)
```

3.118.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.78

$$\int (dx)^m \left(a^2+2abx^3 +b^2x^6\right)^{5/2} \, dx = \frac{((m^5+35\,m^4+445\,m^3+2485\,m^2+5714\,m+3640)b^5d^mx^{16}+5\,(m^5+38\,m^4+511\,m^3+2$$

input
```
integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

output
```
((m^5 + 35*m^4 + 445*m^3 + 2485*m^2 + 5714*m + 3640)*b^5*d^m*x^16 + 5*(m^5
 + 38*m^4 + 511*m^3 + 2962*m^2 + 6968*m + 4480)*a*b^4*d^m*x^13 + 10*(m^5 +
 41*m^4 + 595*m^3 + 3655*m^2 + 8924*m + 5824)*a^2*b^3*d^m*x^10 + 10*(m^5 +
 44*m^4 + 697*m^3 + 4726*m^2 + 12392*m + 8320)*a^3*b^2*d^m*x^7 + 5*(m^5 +
47*m^4 + 817*m^3 + 6337*m^2 + 20126*m + 14560)*a^4*b*d^m*x^4 + (m^5 + 50*m
^4 + 955*m^3 + 8650*m^2 + 36824*m + 58240)*a^5*d^m*x)*x^m/(m^6 + 51*m^5 +
1005*m^4 + 9605*m^3 + 45474*m^2 + 95064*m + 58240)
```

3.118.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 900 vs. $2(247) = 494$.

Time = 0.36 (sec) , antiderivative size = 900, normalized size of antiderivative = 2.88

$$\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx = \text{Too large to display}$$

input
```
integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

output
```
((d*x)^m*b^5*m^5*x^16*sgn(b*x^3 + a) + 35*(d*x)^m*b^5*m^4*x^16*sgn(b*x^3 +
 a) + 445*(d*x)^m*b^5*m^3*x^16*sgn(b*x^3 + a) + 5*(d*x)^m*a*b^4*m^5*x^13*s
gn(b*x^3 + a) + 2485*(d*x)^m*b^5*m^2*x^16*sgn(b*x^3 + a) + 190*(d*x)^m*a*b
^4*m^4*x^13*sgn(b*x^3 + a) + 5714*(d*x)^m*b^5*m*x^16*sgn(b*x^3 + a) + 2555
*(d*x)^m*a*b^4*m^3*x^13*sgn(b*x^3 + a) + 3640*(d*x)^m*b^5*x^16*sgn(b*x^3 +
 a) + 10*(d*x)^m*a^2*b^3*m^5*x^10*sgn(b*x^3 + a) + 14810*(d*x)^m*a*b^4*m^2
*x^13*sgn(b*x^3 + a) + 410*(d*x)^m*a^2*b^3*m^4*x^10*sgn(b*x^3 + a) + 34840
*(d*x)^m*a*b^4*m*x^13*sgn(b*x^3 + a) + 5950*(d*x)^m*a^2*b^3*m^3*x^10*sgn(b
*x^3 + a) + 22400*(d*x)^m*a*b^4*x^13*sgn(b*x^3 + a) + 10*(d*x)^m*a^3*b^2*m
^5*x^7*sgn(b*x^3 + a) + 36550*(d*x)^m*a^2*b^3*m^2*x^10*sgn(b*x^3 + a) + 44
0*(d*x)^m*a^3*b^2*m^4*x^7*sgn(b*x^3 + a) + 89240*(d*x)^m*a^2*b^3*m*x^10*sg
n(b*x^3 + a) + 6970*(d*x)^m*a^3*b^2*m^3*x^7*sgn(b*x^3 + a) + 58240*(d*x)^m
*a^2*b^3*x^10*sgn(b*x^3 + a) + 5*(d*x)^m*a^4*b*m^5*x^4*sgn(b*x^3 + a) + 47
260*(d*x)^m*a^3*b^2*m^2*x^7*sgn(b*x^3 + a) + 235*(d*x)^m*a^4*b*m^4*x^4*sgn
(b*x^3 + a) + 123920*(d*x)^m*a^3*b^2*m*x^7*sgn(b*x^3 + a) + 4085*(d*x)^m*a
^4*b*m^3*x^4*sgn(b*x^3 + a) + 83200*(d*x)^m*a^3*b^2*x^7*sgn(b*x^3 + a) + (
d*x)^m*a^5*m^5*x*sgn(b*x^3 + a) + 31685*(d*x)^m*a^4*b*m^2*x^4*sgn(b*x^3 +
a) + 50*(d*x)^m*a^5*m^4*x*sgn(b*x^3 + a) + 100630*(d*x)^m*a^4*b*m*x^4*sgn(
b*x^3 + a) + 955*(d*x)^m*a^5*m^3*x*sgn(b*x^3 + a) + 72800*(d*x)^m*a^4*b*x^
4*sgn(b*x^3 + a) + 8650*(d*x)^m*a^5*m^2*x*sgn(b*x^3 + a) + 36824*(d*x)^...
```

3.118.9 Mupad [F(-1)]

Timed out.

$$\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx = \int (d\,x)^m\,\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2} dx$$

input
```
int((d*x)^m*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

output
```
int((d*x)^m*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

3.119 $\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx$

3.119.1 Optimal result

Integrand size = 28, antiderivative size = 205

$$\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{a^3(dx)^{1+m}\sqrt{a^2+2abx^3+b^2x^6}}{d(1+m)\left(a+bx^3\right)}+\frac{3a^2b(dx)^{4+m}\sqrt{a^2+2abx^3+b^2x^6}}{d^4(4+m)\left(a+bx^3\right)}+\frac{3ab^2(dx)^{7+m}\sqrt{a^2+2abx^3+b^2x^6}}{d^7(7+m)\left(a+bx^3\right)}+\frac{b^3(dx)^{10+m}\sqrt{a^2+2abx^3+b^2x^6}}{d^{10}(10+m)\left(a+bx^3\right)}$$

output
```
a^3*(d*x)^(1+m)*((b*x^3+a)^2)^(1/2)/d/(1+m)/(b*x^3+a)+3*a^2*b*(d*x)^(4+m)*
((b*x^3+a)^2)^(1/2)/d^4/(4+m)/(b*x^3+a)+3*a*b^2*(d*x)^(7+m)*((b*x^3+a)^2)^
(1/2)/d^7/(7+m)/(b*x^3+a)+b^3*(d*x)^(10+m)*((b*x^3+a)^2)^(1/2)/d^10/(10+m)
/(b*x^3+a)
```

3.119.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.64

$$\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{x(dx)^m\sqrt{\left(a+bx^3\right)^2}\left(a^3\left(280+138m+21m^2+m^3\right)+3a^2b\left(70+87m+18m^2+m^3\right)x^3+3a\right.}{(1+m)(4+m)(7+m)(10+m)\,(}$$

input
```
Integrate[(d*x)^m*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

output
```
(x*(d*x)^m*Sqrt[(a + b*x^3)^2]*(a^3*(280 + 138*m + 21*m^2 + m^3) + 3*a^2*b
*(70 + 87*m + 18*m^2 + m^3)*x^3 + 3*a*b^2*(40 + 54*m + 15*m^2 + m^3)*x^6 +
 b^3*(28 + 39*m + 12*m^2 + m^3)*x^9))/((1 + m)*(4 + m)*(7 + m)*(10 + m)*(a
 + b*x^3))
```

3.119.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.55, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+2abx^3+b^2x^6\right)^{3/2} (dx)^m \, dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6} \int b^3 (dx)^m \left(bx^3+a\right)^3 dx}{b^3 \left(a+bx^3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6} \int (dx)^m \left(bx^3+a\right)^3 dx}{a+bx^3}$$

$$\Big\downarrow \text{802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6} \int \left(a^3 (dx)^m+\frac{3a^2b(dx)^{m+3}}{d^3}+\frac{3ab^2(dx)^{m+6}}{d^6}+\frac{b^3(dx)^{m+9}}{d^9}\right) dx}{a+bx^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6} \left(\frac{a^3(dx)^{m+1}}{d(m+1)}+\frac{3a^2b(dx)^{m+4}}{d^4(m+4)}+\frac{3ab^2(dx)^{m+7}}{d^7(m+7)}+\frac{b^3(dx)^{m+10}}{d^{10}(m+10)}\right)}{a+bx^3}$$

input
```
Int[(d*x)^m*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*((a^3*(d*x)^(1 + m))/(d*(1 + m)) + (3*a^2
*b*(d*x)^(4 + m))/(d^4*(4 + m)) + (3*a*b^2*(d*x)^(7 + m))/(d^7*(7 + m)) +
(b^3*(d*x)^(10 + m))/(d^10*(10 + m))))/(a + b*x^3)
```

3.119.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.119.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.97

method	result
gosper	$\frac{x\left(b^3m^3x^9+12b^3m^2x^9+39m\,x^9b^3+3a\,b^2m^3x^6+28b^3x^9+45a\,b^2m^2x^6+162m\,x^6b^2a+3a^2b\,m^3x^3+120b^2x^6a+54a^2b\,m^2x^3+261m\,x^3a^{2}\right.}{(10+m)(7+m)(4+m)(1+m)(b\,x^3+a)^3}$
risch	$\frac{\sqrt{(b\,x^3+a)^2}\,\left(b^3m^3x^9+12b^3m^2x^9+39m\,x^9b^3+3a\,b^2m^3x^6+28b^3x^9+45a\,b^2m^2x^6+162m\,x^6b^2a+3a^2b\,m^3x^3+120b^2x^6a+54a^2b\,m^2x^3\right.}{(b\,x^3+a)(10+m)(7+m)(4+m)(1+m)}$

input
```
int((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
x*(b^3*m^3*x^9+12*b^3*m^2*x^9+39*b^3*m*x^9+3*a*b^2*m^3*x^6+28*b^3*x^9+45*a
*b^2*m^2*x^6+162*a*b^2*m*x^6+3*a^2*b*m^3*x^3+120*a*b^2*x^6+54*a^2*b*m^2*x^
3+261*a^2*b*m*x^3+a^3*m^3+210*a^2*b*x^3+21*a^3*m^2+138*a^3*m+280*a^3)*(d*x
)^m*((b*x^3+a)^2)^(3/2)/(10+m)/(7+m)/(4+m)/(1+m)/(b*x^3+a)^3
```

3.119.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.78

$$\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx = \frac{((b^3m^3+12\,b^3m^2+39\,b^3m+28\,b^3)x^{10}+3\,(ab^2m^3+15\,ab^2m^2+54\,ab^2m+40\,ab^2)x^7+3\,(a}{m^4+22\,m^3+159\,m}$$

input
```
integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

output
```
((b^3*m^3 + 12*b^3*m^2 + 39*b^3*m + 28*b^3)*x^10 + 3*(a*b^2*m^3 + 15*a*b^2
*m^2 + 54*a*b^2*m + 40*a*b^2)*x^7 + 3*(a^2*b*m^3 + 18*a^2*b*m^2 + 87*a^2*b
*m + 70*a^2*b)*x^4 + (a^3*m^3 + 21*a^3*m^2 + 138*a^3*m + 280*a^3)*x)*(d*x)
^m/(m^4 + 22*m^3 + 159*m^2 + 418*m + 280)
```

3.119.6 Sympy [F]

$$\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx = \int (dx)^m \left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}} dx$$

input
```
integrate((d*x)**m*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

output
```
Integral((d*x)**m*((a + b*x**3)**2)**(3/2), x)
```

3.119.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.58

$$\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx = \frac{((m^3+12\,m^2+39\,m+28)b^3d^mx^{10}+3\,(m^3+15\,m^2+54\,m+40)ab^2d^mx^7+3\,(m^3+18\,m^2}{m^4+22\,m^3+159\,m^2+418\,m+}$$

input
```
integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

output
```
((m^3 + 12*m^2 + 39*m + 28)*b^3*d^m*x^10 + 3*(m^3 + 15*m^2 + 54*m + 40)*a*
b^2*d^m*x^7 + 3*(m^3 + 18*m^2 + 87*m + 70)*a^2*b*d^m*x^4 + (m^3 + 21*m^2 +
 138*m + 280)*a^3*d^m*x)*x^m/(m^4 + 22*m^3 + 159*m^2 + 418*m + 280)
```

3.119.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 384 vs. $2(161) = 322$.

Time = 0.31 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.87

$$\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx = \frac{(dx)^m\,b^3m^3x^{10}\mathrm{sgn}(bx^3+a)+12\,(dx)^m\,b^3m^2x^{10}\mathrm{sgn}(bx^3+a)+39\,(dx)^m\,b^3mx^{10}\mathrm{sgn}(bx^3+a)}{}$$

input
```
integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

output
```
((d*x)^m*b^3*m^3*x^10*sgn(b*x^3 + a) + 12*(d*x)^m*b^3*m^2*x^10*sgn(b*x^3 +
 a) + 39*(d*x)^m*b^3*m*x^10*sgn(b*x^3 + a) + 3*(d*x)^m*a*b^2*m^3*x^7*sgn(b
*x^3 + a) + 28*(d*x)^m*b^3*x^10*sgn(b*x^3 + a) + 45*(d*x)^m*a*b^2*m^2*x^7*
sgn(b*x^3 + a) + 162*(d*x)^m*a*b^2*m*x^7*sgn(b*x^3 + a) + 3*(d*x)^m*a^2*b*
m^3*x^4*sgn(b*x^3 + a) + 120*(d*x)^m*a*b^2*x^7*sgn(b*x^3 + a) + 54*(d*x)^m
*a^2*b*m^2*x^4*sgn(b*x^3 + a) + 261*(d*x)^m*a^2*b*m*x^4*sgn(b*x^3 + a) + (
d*x)^m*a^3*m^3*x*sgn(b*x^3 + a) + 210*(d*x)^m*a^2*b*x^4*sgn(b*x^3 + a) + 2
1*(d*x)^m*a^3*m^2*x*sgn(b*x^3 + a) + 138*(d*x)^m*a^3*m*x*sgn(b*x^3 + a) +
280*(d*x)^m*a^3*x*sgn(b*x^3 + a))/(m^4 + 22*m^3 + 159*m^2 + 418*m + 280)
```

3.119.9 Mupad [F(-1)]

Timed out.

$$\int (dx)^m \left(a^2 + 2abx^3 + b^2x^6\right)^{3/2} dx = \int (d\,x)^m \left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^{3/2} dx$$

input
```
int((d*x)^m*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

output
```
int((d*x)^m*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

3.120 $\int (dx)^m \sqrt{a^2+2abx^3+b^2x^6}\,dx$

3.120.1 Optimal result

Integrand size = 28, antiderivative size = 97

$$\int (dx)^m \sqrt{a^2+2abx^3+b^2x^6}\,dx = \frac{a(dx)^{1+m}\sqrt{a^2+2abx^3+b^2x^6}}{d(1+m)(a+bx^3)}+\frac{b(dx)^{4+m}\sqrt{a^2+2abx^3+b^2x^6}}{d^4(4+m)(a+bx^3)}$$

output
```
a*(d*x)^(1+m)*((b*x^3+a)^2)^(1/2)/d/(1+m)/(b*x^3+a)+b*(d*x)^(4+m)*((b*x^3+
a)^2)^(1/2)/d^4/(4+m)/(b*x^3+a)
```

3.120.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.55

$$\int (dx)^m \sqrt{a^2+2abx^3+b^2x^6}\,dx = \frac{x(dx)^m\sqrt{(a+bx^3)^2}(a(4+m)+b(1+m)x^3)}{(1+m)(4+m)(a+bx^3)}$$

input
```
Integrate[(d*x)^m*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

output
```
(x*(d*x)^m*Sqrt[(a + b*x^3)^2]*(a*(4 + m) + b*(1 + m)*x^3))/((1 + m)*(4 +
m)*(a + b*x^3))
```

3.120.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.69, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a^2+2abx^3+b^2x^6}(dx)^m\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int b(dx)^m\left(bx^3+a\right)dx}{b\left(a+bx^3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int (dx)^m\left(bx^3+a\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(a(dx)^m+\frac{b(dx)^{m+3}}{d^3}\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(\frac{a(dx)^{m+1}}{d(m+1)}+\frac{b(dx)^{m+4}}{d^4(m+4)}\right)}{a+bx^3}$$

input
```
Int[(d*x)^m*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*((a*(d*x)^(1 + m))/(d*(1 + m)) + (b*(d*x)
^(4 + m))/(d^4*(4 + m))))/(a + b*x^3)
```

3.120.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.120.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.58

method	result	size
gosper	$\frac{x\left(bm\,x^{3}+b\,x^{3}+am+4a\right)\left(dx\right)^{m}\sqrt{\left(b\,x^{3}+a\right)^{2}}}{\left(4+m\right)\left(1+m\right)\left(b\,x^{3}+a\right)}$	56
risch	$\frac{x\left(bm\,x^{3}+b\,x^{3}+am+4a\right)\left(dx\right)^{m}\sqrt{\left(b\,x^{3}+a\right)^{2}}}{\left(4+m\right)\left(1+m\right)\left(b\,x^{3}+a\right)}$	56

input
```
int((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x*(b*m*x^3+b*x^3+a*m+4*a)*(d*x)^m*((b*x^3+a)^2)^(1/2)/(4+m)/(1+m)/(b*x^3+a
)
```

3.120.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.36

$$\int (dx)^m \sqrt{a^2 + 2abx^3 + b^2x^6}\, dx = \frac{((bm + b)x^4 + (am + 4\,a)x)(dx)^m}{m^2 + 5\,m + 4}$$

input
```
integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(1/2),x, algorithm="fricas")
```

output
```
((b*m + b)*x^4 + (a*m + 4*a)*x)*(d*x)^m/(m^2 + 5*m + 4)
```

3.120.6 Sympy [F]

$$\int (dx)^m \sqrt{a^2 + 2abx^3 + b^2x^6}\, dx = \int (dx)^m \sqrt{(a + bx^3)^2}\, dx$$

input
```
integrate((d*x)**m*(b**2*x**6+2*a*b*x**3+a**2)**(1/2),x)
```

output
```
Integral((d*x)**m*sqrt((a + b*x**3)**2), x)
```

3.120.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.36

$$\int (dx)^m \sqrt{a^2 + 2abx^3 + b^2x^6}\, dx = \frac{(bd^m(m + 1)x^4 + ad^m(m + 4)x)x^m}{m^2 + 5\,m + 4}$$

input
```
integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(1/2),x, algorithm="maxima")
```

output
```
(b*d^m*(m + 1)*x^4 + a*d^m*(m + 4)*x)*x^m/(m^2 + 5*m + 4)
```

3.120.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.86

$$\int (dx)^m\sqrt{a^2+2abx^3+b^2x^6}\,dx$$
$$=\frac{(dx)^m\,bmx^4\mathrm{sgn}(bx^3+a)+(dx)^m\,bx^4\mathrm{sgn}(bx^3+a)+(dx)^m\,amx\mathrm{sgn}(bx^3+a)+4\,(dx)^m\,ax\mathrm{sgn}(bx^3+a)}{m^2+5\,m+4}$$

input
```
integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(1/2),x, algorithm="giac")
```

output
```
((d*x)^m*b*m*x^4*sgn(b*x^3 + a) + (d*x)^m*b*x^4*sgn(b*x^3 + a) + (d*x)^m*a
*m*x*sgn(b*x^3 + a) + 4*(d*x)^m*a*x*sgn(b*x^3 + a))/(m^2 + 5*m + 4)
```

3.120.9 Mupad [F(-1)]

Timed out.

$$\int (dx)^m\sqrt{a^2+2abx^3+b^2x^6}\,dx=\int (d\,x)^m\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}\,dx$$

input
```
int((d*x)^m*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2),x)
```

output
```
int((d*x)^m*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2), x)
```

3.121 $\int \frac{(dx)^m}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

3.121.1 Optimal result

Integrand size = 28, antiderivative size = 73

$$\int \frac{(dx)^m}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx=\frac{(dx)^{1+m}(a+bx^3)\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)}{ad(1+m)\sqrt{a^2+2abx^3+b^2x^6}}$$

output
```
(d*x)^(1+m)*(b*x^3+a)*hypergeom([1, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)/a/d/(
1+m)/((b*x^3+a)^2)^(1/2)
```

3.121.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.85

$$\int \frac{(dx)^m}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx=\frac{x(dx)^m(a+bx^3)\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{3},1+\frac{1+m}{3},-\frac{bx^3}{a}\right)}{a(1+m)\sqrt{(a+bx^3)^2}}$$

input
```
Integrate[(d*x)^m/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

output
```
(x*(d*x)^m*(a + b*x^3)*Hypergeometric2F1[1, (1 + m)/3, 1 + (1 + m)/3, -((b
*x^3)/a)])/(a*(1 + m)*Sqrt[(a + b*x^3)^2])
```

3.121.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {1384, 27, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{\sqrt{a^2+2abx^3+b^2x^6}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b(a+bx^3)\int \frac{(dx)^m}{b(bx^3+a)}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx^3)\int \frac{(dx)^m}{bx^3+a}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{888}$$

$$\frac{(a+bx^3)(dx)^{m+1}\text{Hypergeometric2F1}\left(1,\frac{m+1}{3},\frac{m+4}{3},-\frac{bx^3}{a}\right)}{ad(m+1)\sqrt{a^2+2abx^3+b^2x^6}}$$

input
```
Int[(d*x)^m/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

output
```
((d*x)^(1 + m)*(a + b*x^3)*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, -((b
*x^3)/a)])/(a*d*(1 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

3.121.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

3.121.4 Maple [F]

$$\int \frac{(dx)^m}{\sqrt{b^2x^6 + 2ab\,x^3 + a^2}}dx$$

input
```
int((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(1/2),x)
```

output
```
int((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(1/2),x)
```

3.121.5 Fricas [F]

$$\int \frac{(dx)^m}{\sqrt{a^2 + 2abx^3 + b^2x^6}}\,dx = \int \frac{(dx)^m}{\sqrt{b^2x^6 + 2\,abx^3 + a^2}}\,dx$$

input
```
integrate((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(1/2),x, algorithm="fricas")
```

output
```
integral((d*x)^m/sqrt(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

3.121.6 Sympy [F]

$$\int \frac{(dx)^m}{\sqrt{a^2 + 2abx^3 + b^2x^6}}\,dx = \int \frac{(dx)^m}{\sqrt{(a + bx^3)^2}}\,dx$$

input
```
integrate((d*x)**m/(b**2*x**6+2*a*b*x**3+a**2)**(1/2),x)
```

output
```
Integral((d*x)**m/sqrt((a + b*x**3)**2), x)
```

3.121.7 Maxima [F]

$$\int \frac{(dx)^m}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx=\int \frac{(dx)^m}{\sqrt{b^2x^6+2\,abx^3+a^2}}\,dx$$

input
```
integrate((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x)^m/sqrt(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

3.121.8 Giac [F]

$$\int \frac{(dx)^m}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx=\int \frac{(dx)^m}{\sqrt{b^2x^6+2\,abx^3+a^2}}\,dx$$

input
```
integrate((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x)^m/sqrt(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

3.121.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx=\int \frac{(d\,x)^m}{\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}\,dx$$

input
```
int((d*x)^m/(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2),x)
```

output
```
int((d*x)^m/(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2), x)
```

3.122 $\int \frac{(dx)^m}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$

3.122.1 Optimal result

Integrand size = 28, antiderivative size = 73

$$\int \frac{(dx)^m}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx=\frac{(dx)^{1+m}\,(a+bx^3)\,\text{Hypergeometric2F1}\left(3,\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)}{a^3d(1+m)\sqrt{a^2+2abx^3+b^2x^6}}$$

output
```
(d*x)^(1+m)*(b*x^3+a)*hypergeom([3, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)/a^3/d
/(1+m)/((b*x^3+a)^2)^(1/2)
```

3.122.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.82

$$\int \frac{(dx)^m}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx=\frac{x(dx)^m\,(a+bx^3)\,\text{Hypergeometric2F1}\left(3,\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)}{a^3(1+m)\sqrt{(a+bx^3)^2}}$$

input
```
Integrate[(d*x)^m/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

output
```
(x*(d*x)^m*(a + b*x^3)*Hypergeometric2F1[3, (1 + m)/3, (4 + m)/3, -((b*x^3
)/a)])/(a^3*(1 + m)*Sqrt[(a + b*x^3)^2])
```

3.122.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {1384, 27, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{(a^2+2abx^3+b^2x^6)^{3/2}}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{b^3(a+bx^3)\int \frac{(dx)^m}{b^3(bx^3+a)^3}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(a+bx^3)\int \frac{(dx)^m}{(bx^3+a)^3}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{(a+bx^3)(dx)^{m+1}\text{Hypergeometric2F1}\left(3,\frac{m+1}{3},\frac{m+4}{3},-\frac{bx^3}{a}\right)}{a^3d(m+1)\sqrt{a^2+2abx^3+b^2x^6}}$$

input
```
Int[(d*x)^m/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

output
```
((d*x)^(1 + m)*(a + b*x^3)*Hypergeometric2F1[3, (1 + m)/3, (4 + m)/3, -((b
*x^3)/a)])/(a^3*d*(1 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

3.122.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

3.122.4 Maple [F]

$$\int \frac{(dx)^m}{(b^2x^6 + 2ab\,x^3 + a^2)^{\frac{3}{2}}}dx$$

input
```
int((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

output
```
int((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

3.122.5 Fricas [F]

$$\int \frac{(dx)^m}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \int \frac{(dx)^m}{(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*(d*x)^m/(b^4*x^12 + 4*a*b^3*x^9 +
 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4), x)
```

3.122.6 Sympy [F]

$$\int \frac{(dx)^m}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \int \frac{(dx)^m}{\left((a+bx^3)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)**m/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

output
```
Integral((d*x)**m/((a + b*x**3)**2)**(3/2), x)
```

3.122.7 Maxima [F]

$$\int \frac{(dx)^m}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \int \frac{(dx)^m}{(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x)^m/(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2), x)
```

3.122.8 Giac [F]

$$\int \frac{(dx)^m}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \int \frac{(dx)^m}{(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x)^m/(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2), x)
```

3.122.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \int \frac{(d\,x)^m}{(a^2+2\,a\,b\,x^3+b^2\,x^6)^{3/2}}\,dx$$

input
```
int((d*x)^m/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

output
```
int((d*x)^m/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

3.123 $\int \frac{(dx)^m}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

3.123.1 Optimal result

Integrand size = 28, antiderivative size = 73

$$\int \frac{(dx)^m}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \frac{(dx)^{1+m}(a+bx^3)\,\text{Hypergeometric2F1}\left(5,\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)}{a^5d(1+m)\sqrt{a^2+2abx^3+b^2x^6}}$$

output
```
(d*x)^(1+m)*(b*x^3+a)*hypergeom([5, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)/a^5/d
/(1+m)/((b*x^3+a)^2)^(1/2)
```

3.123.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.82

$$\int \frac{(dx)^m}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \frac{x(dx)^m(a+bx^3)\,\text{Hypergeometric2F1}\left(5,\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)}{a^5(1+m)\sqrt{(a+bx^3)^2}}$$

input
```
Integrate[(d*x)^m/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
(x*(d*x)^m*(a + b*x^3)*Hypergeometric2F1[5, (1 + m)/3, (4 + m)/3, -((b*x^3
)/a)])/(a^5*(1 + m)*Sqrt[(a + b*x^3)^2])
```

3.123.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {1384, 27, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{(a^2+2abx^3+b^2x^6)^{5/2}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b^5(a+bx^3)\int \frac{(dx)^m}{b^5(bx^3+a)^5}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx^3)\int \frac{(dx)^m}{(bx^3+a)^5}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{888}$$

$$\frac{(a+bx^3)(dx)^{m+1}\text{Hypergeometric2F1}\left(5,\frac{m+1}{3},\frac{m+4}{3},-\frac{bx^3}{a}\right)}{a^5d(m+1)\sqrt{a^2+2abx^3+b^2x^6}}$$

input
```
Int[(d*x)^m/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
((d*x)^(1 + m)*(a + b*x^3)*Hypergeometric2F1[5, (1 + m)/3, (4 + m)/3, -((b
*x^3)/a)])/(a^5*d*(1 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

3.123.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

3.123.4 Maple [F]

$$\int \frac{(dx)^m}{(b^2x^6 + 2ab\,x^3 + a^2)^{\frac{5}{2}}}dx$$

input
```
int((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

output
```
int((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

3.123.5 Fricas [F]

$$\int \frac{(dx)^m}{(a^2 + 2abx^3 + b^2x^6)^{5/2}}\,dx = \int \frac{(dx)^m}{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*(d*x)^m/(b^6*x^18 + 6*a*b^5*x^15
+ 15*a^2*b^4*x^12 + 20*a^3*b^3*x^9 + 15*a^4*b^2*x^6 + 6*a^5*b*x^3 + a^6),
x)
```

3.123.6 Sympy [F]

$$\int \frac{(dx)^m}{(a^2 + 2abx^3 + b^2x^6)^{5/2}}\,dx = \int \frac{(dx)^m}{((a + bx^3)^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x)**m/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

output
```
Integral((d*x)**m/((a + b*x**3)**2)**(5/2), x)
```

3.123.7 Maxima [F]

$$\int \frac{(dx)^m}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx=\int \frac{(dx)^m}{(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*x)^m/(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2), x)
```

3.123.8 Giac [F]

$$\int \frac{(dx)^m}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx=\int \frac{(dx)^m}{(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

output
```
integrate((d*x)^m/(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2), x)
```

3.123.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx=\int \frac{(d\,x)^m}{(a^2+2\,a\,b\,x^3+b^2\,x^6)^{5/2}}\,dx$$

input
```
int((d*x)^m/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

output
```
int((d*x)^m/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

3.124 $\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^p \, dx$

3.124.1 Optimal result

Integrand size = 26, antiderivative size = 77

$$\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^p \, dx = \frac{(dx)^{1+m} \left(1+\frac{bx^3}{a}\right)^{-2p} \left(a^2+2abx^3+b^2x^6\right)^p \text{Hypergeometric2F1}\left(\frac{1+m}{3},-2p,\frac{4+m}{3},-\frac{bx^3}{a}\right)}{d(1+m)}$$

output
```
(d*x)^(1+m)*(b^2*x^6+2*a*b*x^3+a^2)^p*hypergeom([-2*p, 1/3+1/3*m],[4/3+1/3
*m],-b*x^3/a)/d/(1+m)/((1+b*x^3/a)^(2*p))
```

3.124.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.86

$$\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^p \, dx = \frac{x(dx)^m \left(\left(a+bx^3\right)^2\right)^p \left(1+\frac{bx^3}{a}\right)^{-2p} \text{Hypergeometric2F1}\left(\frac{1+m}{3},-2p,1+\frac{1+m}{3},-\frac{bx^3}{a}\right)}{1+m}$$

input
```
Integrate[(d*x)^m*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

output
```
(x*(d*x)^m*((a + b*x^3)^2)^p*Hypergeometric2F1[(1 + m)/3, -2*p, 1 + (1 + m
)/3, -((b*x^3)/a)])/((1 + m)*(1 + (b*x^3)/a)^(2*p))
```

3.124.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1385, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^p \, dx$$

$$\Big\downarrow \text{1385}$$

$$\left(\frac{bx^3}{a}+1\right)^{-2p} \left(a^2+2abx^3+b^2x^6\right)^p \int (dx)^m \left(\frac{bx^3}{a}+1\right)^{2p} dx$$

$$\Big\downarrow \text{888}$$

$$\frac{(dx)^{m+1} \left(\frac{bx^3}{a}+1\right)^{-2p} \left(a^2+2abx^3+b^2x^6\right)^p \text{Hypergeometric2F1}\left(\frac{m+1}{3},-2p,\frac{m+4}{3},-\frac{bx^3}{a}\right)}{d(m+1)}$$

input
```
Int[(d*x)^m*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

output
```
((d*x)^(1 + m)*(a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[(1 + m)/3,
-2*p, (4 + m)/3, -((b*x^3)/a)])/(d*(1 + m)*(1 + (b*x^3)/a)^(2*p))
```

3.124.3.1 Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

3.124.4 Maple [F]

$$\int (dx)^m \left(b^2x^6 + 2ab\,x^3 + a^2\right)^p dx$$

input
```
int((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

output
```
int((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

3.124.5 Fricas [F]

$$\int (dx)^m \left(a^2 + 2abx^3 + b^2x^6\right)^p dx = \int \left(b^2x^6 + 2\,abx^3 + a^2\right)^p (dx)^m \, dx$$

input
```
integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="fricas")
```

output
```
integral((b^2*x^6 + 2*a*b*x^3 + a^2)^p*(d*x)^m, x)
```

3.124.6 Sympy [F]

$$\int (dx)^m \left(a^2 + 2abx^3 + b^2x^6\right)^p dx = \int (dx)^m \left(\left(a + bx^3\right)^2\right)^p dx$$

input
```
integrate((d*x)**m*(b**2*x**6+2*a*b*x**3+a**2)**p,x)
```

output
```
Integral((d*x)**m*((a + b*x**3)**2)**p, x)
```

3.124.7 Maxima [F]

$$\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^p \, dx = \int \left(b^2x^6+2\,abx^3+a^2\right)^p (dx)^m \, dx$$

input
```
integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="maxima")
```

output
```
integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*(d*x)^m, x)
```

3.124.8 Giac [F]

$$\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^p \, dx = \int \left(b^2x^6+2\,abx^3+a^2\right)^p (dx)^m \, dx$$

input
```
integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="giac")
```

output
```
integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*(d*x)^m, x)
```

3.124.9 Mupad [F(-1)]

Timed out.

$$\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^p \, dx = \int (d\,x)^m \left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^p \, dx$$

input
```
int((d*x)^m*(a^2 + b^2*x^6 + 2*a*b*x^3)^p,x)
```

output
```
int((d*x)^m*(a^2 + b^2*x^6 + 2*a*b*x^3)^p, x)
```

3.125 $\int x^{11}(a^2+2abx^3+b^2x^6)^p\,dx$

3.125.1 Optimal result

Integrand size = 24, antiderivative size = 172

$$\int x^{11}\left(a^2+2abx^3+b^2x^6\right)^p\,dx=-\frac{a^3(a+bx^3)\left(a^2+2abx^3+b^2x^6\right)^p}{3b^4(1+2p)}+\frac{a^2(a+bx^3)^2\left(a^2+2abx^3+b^2x^6\right)^p}{2b^4(1+p)}-\frac{a(a+bx^3)^3\left(a^2+2abx^3+b^2x^6\right)^p}{b^4(3+2p)}+\frac{(a+bx^3)^4\left(a^2+2abx^3+b^2x^6\right)^p}{6b^4(2+p)}$$

output
```
-1/3*a^3*(b*x^3+a)*(b^2*x^6+2*a*b*x^3+a^2)^p/b^4/(1+2*p)+1/2*a^2*(b*x^3+a)
^2*(b^2*x^6+2*a*b*x^3+a^2)^p/b^4/(p+1)-a*(b*x^3+a)^3*(b^2*x^6+2*a*b*x^3+a^
2)^p/b^4/(3+2*p)+1/6*(b*x^3+a)^4*(b^2*x^6+2*a*b*x^3+a^2)^p/b^4/(2+p)
```

3.125.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.64

$$\int x^{11}\left(a^2+2abx^3+b^2x^6\right)^p\,dx=\frac{(a+bx^3)\left((a+bx^3)^2\right)^p\left(-3a^3+3a^2b(1+2p)x^3-3ab^2\left(1+3p+2p^2\right)x^6+b^3\left(3+11p+12p^2+4p^3\right)x^9\right)}{6b^4(1+p)(2+p)(1+2p)(3+2p)}$$

input
```
Integrate[x^11*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

output
```
((a + b*x^3)*((a + b*x^3)^2)^p*(-3*a^3 + 3*a^2*b*(1 + 2*p)*x^3 - 3*a*b^2*(
1 + 3*p + 2*p^2)*x^6 + b^3*(3 + 11*p + 12*p^2 + 4*p^3)*x^9))/(6*b^4*(1 + p
)*(2 + p)*(1 + 2*p)*(3 + 2*p))
```

3.125.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1385, 798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{11}\left(a^2+2abx^3+b^2x^6\right)^p\,dx$$

$$\Big\downarrow \text{1385}$$

$$\left(\frac{bx^3}{a}+1\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\int x^{11}\left(\frac{bx^3}{a}+1\right)^{2p}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\left(\frac{bx^3}{a}+1\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\int x^{9}\left(\frac{bx^3}{a}+1\right)^{2p}dx^3$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{3}\left(\frac{bx^3}{a}+1\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\int \left(-\frac{a^3\left(\frac{bx^3}{a}+1\right)^{2p}}{b^3}+\frac{3a^3\left(\frac{bx^3}{a}+1\right)^{2p+1}}{b^3}-\frac{3a^3\left(\frac{bx^3}{a}+1\right)^{2p+2}}{b^3}+\frac{a^3\left(\frac{bx^3}{a}\right.}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{bx^3}{a}+1\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\left(\frac{3a^4\left(\frac{bx^3}{a}+1\right)^{2(p+1)}}{2b^4(p+1)}+\frac{a^4\left(\frac{bx^3}{a}+1\right)^{2(p+2)}}{2b^4(p+2)}-\frac{a^4\left(\frac{bx^3}{a}+1\right)^{2p+1}}{b^4(2p+1)}-\frac{3a^4\left(\frac{bx^3}{a}\right.}{b^4(2p}\right.$$

input
```
Int[x^11*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

output
```
((a^2 + 2*a*b*x^3 + b^2*x^6)^p*((3*a^4*(1 + (b*x^3)/a)^(2*(1 + p)))/(2*b^4
*(1 + p)) + (a^4*(1 + (b*x^3)/a)^(2*(2 + p)))/(2*b^4*(2 + p)) - (a^4*(1 +
(b*x^3)/a)^(1 + 2*p))/(b^4*(1 + 2*p)) - (3*a^4*(1 + (b*x^3)/a)^(3 + 2*p))/
(b^4*(3 + 2*p))))/(3*(1 + (b*x^3)/a)^(2*p))
```

3.125.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.125.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.87

method	result
gosper	$-\frac{\left(b^{2} x^{6}+2 a b \,x^{3}+a^{2}\right)^{p} \left(-4 b^{3} p^{3} x^{9}-12 b^{3} p^{2} x^{9}-11 b^{3} p \,x^{9}-3 b^{3} x^{9}+6 a \,b^{2} p^{2} x^{6}+9 a \,b^{2} p \,x^{6}+3 b^{2} x^{6} a -6 a^{2} b p \,x^{3}-3 a^{2} b \,x^{3}+3 a^{3}\right) (b \,x^{3}}{6 b^{4} \left(4 p^{4}+20 p^{3}+35 p^{2}+25 p +6\right)}$
risch	$-\frac{\left(-4 b^{4} p^{3} x^{12}-12 b^{4} p^{2} x^{12}-11 b^{4} p \,x^{12}-4 a \,b^{3} p^{3} x^{9}-3 b^{4} x^{12}-6 a \,b^{3} p^{2} x^{9}-2 a p \,x^{9} b^{3}+6 a^{2} b^{2} p^{2} x^{6}+3 a^{2} p \,x^{6} b^{2}-6 a^{3} p \,x^{3} b +3 a^{4}\right) ((b}{6 (3+2 p) (2+p) (1+p) (1+2 p) b^{4}}$
parallelrisch	$4 x^{12} \left(b^{2} x^{6}+2 a b \,x^{3}+a^{2}\right)^{p} b^{4} p^{3}+12 x^{12} \left(b^{2} x^{6}+2 a b \,x^{3}+a^{2}\right)^{p} b^{4} p^{2}+11 x^{12} \left(b^{2} x^{6}+2 a b \,x^{3}+a^{2}\right)^{p} b^{4} p +3 x^{12} \left(b^{2} x^{6}+2 a b \,x^{3}+a^{2}\right)^{p} b^{4}+4$

input
```
int(x^11*(b^2*x^6+2*a*b*x^3+a^2)^p,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(b^2*x^6+2*a*b*x^3+a^2)^p*(-4*b^3*p^3*x^9-12*b^3*p^2*x^9-11*b^3*p*x^9
-3*b^3*x^9+6*a*b^2*p^2*x^6+9*a*b^2*p*x^6+3*a*b^2*x^6-6*a^2*b*p*x^3-3*a^2*b
*x^3+3*a^3)*(b*x^3+a)/b^4/(4*p^4+20*p^3+35*p^2+25*p+6)
```

3.125.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.95

$$\int x^{11}\left(a^2+2abx^3+b^2x^6\right)^p\,dx$$
$$=\frac{((4\,b^4p^3+12\,b^4p^2+11\,b^4p+3\,b^4)x^{12}+2\,(2\,ab^3p^3+3\,ab^3p^2+ab^3p)x^9+6\,a^3bpx^3-3\,(2\,a^2b^2p^2+a^2b^2p)x^6}{6\,(4\,b^4p^4+20\,b^4p^3+35\,b^4p^2+25\,b^4p+6\,b^4)}$$

input
```
integrate(x^11*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="fricas")
```

output
```
1/6*((4*b^4*p^3 + 12*b^4*p^2 + 11*b^4*p + 3*b^4)*x^12 + 2*(2*a*b^3*p^3 + 3
*a*b^3*p^2 + a*b^3*p)*x^9 + 6*a^3*b*p*x^3 - 3*(2*a^2*b^2*p^2 + a^2*b^2*p)*
x^6 - 3*a^4)*(b^2*x^6 + 2*a*b*x^3 + a^2)^p/(4*b^4*p^4 + 20*b^4*p^3 + 35*b^
4*p^2 + 25*b^4*p + 6*b^4)
```

3.125.6 Sympy [F]

$$\int x^{11}\left(a^2+2abx^3+b^2x^6\right)^p\,dx=\text{Too large to display}$$

input
```
integrate(x**11*(b**2*x**6+2*a*b*x**3+a**2)**p,x)
```

output
```
Piecewise((x**12*(a**2)**p/12, Eq(b, 0)), (6*a**3*log(x - (-a/b)**(1/3))/(
18*a**3*b**4 + 54*a**2*b**5*x**3 + 54*a*b**6*x**6 + 18*b**7*x**9) + 6*a**3
*log(4*x**2 + 4*x*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(18*a**3*b**4 + 54*a**2
*b**5*x**3 + 54*a*b**6*x**6 + 18*b**7*x**9) - 12*a**3*log(2)/(18*a**3*b**4
 + 54*a**2*b**5*x**3 + 54*a*b**6*x**6 + 18*b**7*x**9) + 11*a**3/(18*a**3*b
**4 + 54*a**2*b**5*x**3 + 54*a*b**6*x**6 + 18*b**7*x**9) + 18*a**2*b*x**3*
log(x - (-a/b)**(1/3))/(18*a**3*b**4 + 54*a**2*b**5*x**3 + 54*a*b**6*x**6
+ 18*b**7*x**9) + 18*a**2*b*x**3*log(4*x**2 + 4*x*(-a/b)**(1/3) + 4*(-a/b)
**(2/3))/(18*a**3*b**4 + 54*a**2*b**5*x**3 + 54*a*b**6*x**6 + 18*b**7*x**9
) - 36*a**2*b*x**3*log(2)/(18*a**3*b**4 + 54*a**2*b**5*x**3 + 54*a*b**6*x*
*6 + 18*b**7*x**9) + 27*a**2*b*x**3/(18*a**3*b**4 + 54*a**2*b**5*x**3 + 54
*a*b**6*x**6 + 18*b**7*x**9) + 18*a*b**2*x**6*log(x - (-a/b)**(1/3))/(18*a
**3*b**4 + 54*a**2*b**5*x**3 + 54*a*b**6*x**6 + 18*b**7*x**9) + 18*a*b**2*
x**6*log(4*x**2 + 4*x*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(18*a**3*b**4 + 54*
a**2*b**5*x**3 + 54*a*b**6*x**6 + 18*b**7*x**9) - 36*a*b**2*x**6*log(2)/(1
8*a**3*b**4 + 54*a**2*b**5*x**3 + 54*a*b**6*x**6 + 18*b**7*x**9) + 18*a*b*
*2*x**6/(18*a**3*b**4 + 54*a**2*b**5*x**3 + 54*a*b**6*x**6 + 18*b**7*x**9)
 + 6*b**3*x**9*log(x - (-a/b)**(1/3))/(18*a**3*b**4 + 54*a**2*b**5*x**3 +
54*a*b**6*x**6 + 18*b**7*x**9) + 6*b**3*x**9*log(4*x**2 + 4*x*(-a/b)**(1/3
) + 4*(-a/b)**(2/3))/(18*a**3*b**4 + 54*a**2*b**5*x**3 + 54*a*b**6*x**6...
```

3.125.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.67

$$\int x^{11}(a^2 + 2abx^3 + b^2x^6)^p\,dx$$
$$= \frac{((4\,p^3 + 12\,p^2 + 11\,p + 3)b^4x^{12} + 2\,(2\,p^3 + 3\,p^2 + p)ab^3x^9 - 3\,(2\,p^2 + p)a^2b^2x^6 + 6\,a^3bpx^3 - 3\,a^4)(bx^3 + a)}{6\,(4\,p^4 + 20\,p^3 + 35\,p^2 + 25\,p + 6)b^4}$$

input
```
integrate(x^11*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="maxima")
```

output
```
1/6*((4*p^3 + 12*p^2 + 11*p + 3)*b^4*x^12 + 2*(2*p^3 + 3*p^2 + p)*a*b^3*x^
9 - 3*(2*p^2 + p)*a^2*b^2*x^6 + 6*a^3*b*p*x^3 - 3*a^4)*(b*x^3 + a)^(2*p)/(
(4*p^4 + 20*p^3 + 35*p^2 + 25*p + 6)*b^4)
```

3.125.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 375 vs. $2(166) = 332$.

Time = 0.31 (sec) , antiderivative size = 375, normalized size of antiderivative = 2.18

$$\int x^{11}\left(a^2+2abx^3+b^2x^6\right)^p\,dx$$
$$= \frac{4\left(b^2x^6+2\,abx^3+a^2\right)^p b^4p^3x^{12}+12\left(b^2x^6+2\,abx^3+a^2\right)^p b^4p^2x^{12}+11\left(b^2x^6+2\,abx^3+a^2\right)^p b^4px^{12}+4\,(b^2}{}$$

input
```
integrate(x^11*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="giac")
```

output
```
1/6*(4*(b^2*x^6 + 2*a*b*x^3 + a^2)^p*b^4*p^3*x^12 + 12*(b^2*x^6 + 2*a*b*x^
3 + a^2)^p*b^4*p^2*x^12 + 11*(b^2*x^6 + 2*a*b*x^3 + a^2)^p*b^4*p*x^12 + 4*
(b^2*x^6 + 2*a*b*x^3 + a^2)^p*a*b^3*p^3*x^9 + 3*(b^2*x^6 + 2*a*b*x^3 + a^2
)^p*b^4*x^12 + 6*(b^2*x^6 + 2*a*b*x^3 + a^2)^p*a*b^3*p^2*x^9 + 2*(b^2*x^6
+ 2*a*b*x^3 + a^2)^p*a*b^3*p*x^9 - 6*(b^2*x^6 + 2*a*b*x^3 + a^2)^p*a^2*b^2
*p^2*x^6 - 3*(b^2*x^6 + 2*a*b*x^3 + a^2)^p*a^2*b^2*p*x^6 + 6*(b^2*x^6 + 2*
a*b*x^3 + a^2)^p*a^3*b*p*x^3 - 3*(b^2*x^6 + 2*a*b*x^3 + a^2)^p*a^4)/(4*b^4
*p^4 + 20*b^4*p^3 + 35*b^4*p^2 + 25*b^4*p + 6*b^4)
```

3.125.9 Mupad [B] (verification not implemented)

Time = 8.39 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.20

$$\int x^{11}\left(a^2+2abx^3+b^2x^6\right)^p\,dx = \left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^p\left(\frac{x^{12}\,\left(4\,p^3+12\,p^2+11\,p+3\right)}{6\,\left(4\,p^4+20\,p^3+35\,p^2+25\,p+6\right)} - \frac{a^4}{2\,b^4\,\left(4\,p^4+20\,p^3+35\,p^2+25\,p+6\right)} + \frac{a^3\,p\,x^3}{b^3\,\left(4\,p^4+20\,p^3+35\,p^2+25\,p+6\right)} + \frac{a\,p\,x^9\,\left(2\,p^2+3\,p+1\right)}{3\,b\,\left(4\,p^4+20\,p^3+35\,p^2+25\,p+6\right)} - \frac{a^2\,p\,x^6\,\left(2\,p+1\right)}{2\,b^2\,\left(4\,p^4+20\,p^3+35\,p^2+25\,p+6\right)}\right)$$

input
```
int(x^11*(a^2 + b^2*x^6 + 2*a*b*x^3)^p,x)
```

output
```
(a^2 + b^2*x^6 + 2*a*b*x^3)^p*((x^12*(11*p + 12*p^2 + 4*p^3 + 3))/(6*(25*p
 + 35*p^2 + 20*p^3 + 4*p^4 + 6)) - a^4/(2*b^4*(25*p + 35*p^2 + 20*p^3 + 4*
p^4 + 6)) + (a^3*p*x^3)/(b^3*(25*p + 35*p^2 + 20*p^3 + 4*p^4 + 6)) + (a*p*
x^9*(3*p + 2*p^2 + 1))/(3*b*(25*p + 35*p^2 + 20*p^3 + 4*p^4 + 6)) - (a^2*p
*x^6*(2*p + 1))/(2*b^2*(25*p + 35*p^2 + 20*p^3 + 4*p^4 + 6)))
```

3.125. $\int x^{11}(a^2 + 2abx^3 + b^2x^6)^p \, dx$

3.126 $\int x^8(a^2+2abx^3+b^2x^6)^p\,dx$

3.126.1 Optimal result

Integrand size = 24, antiderivative size = 130

$$\int x^8(a^2+2abx^3+b^2x^6)^p\,dx=\frac{a^2(a+bx^3)\,(a^2+2abx^3+b^2x^6)^p}{3b^3(1+2p)}-\frac{a(a+bx^3)^2\,(a^2+2abx^3+b^2x^6)^p}{3b^3(1+p)}+\frac{(a+bx^3)^3\,(a^2+2abx^3+b^2x^6)^p}{3b^3(3+2p)}$$

output
```
1/3*a^2*(b*x^3+a)*(b^2*x^6+2*a*b*x^3+a^2)^p/b^3/(1+2*p)-1/3*a*(b*x^3+a)^2*
(b^2*x^6+2*a*b*x^3+a^2)^p/b^3/(p+1)+1/3*(b*x^3+a)^3*(b^2*x^6+2*a*b*x^3+a^2
)^p/b^3/(3+2*p)
```

3.126.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.59

$$\int x^8(a^2+2abx^3+b^2x^6)^p\,dx=\frac{(a+bx^3)\left((a+bx^3)^2\right)^p(a^2-ab(1+2p)x^3+b^2(1+3p+2p^2)\,x^6)}{3b^3(1+p)(1+2p)(3+2p)}$$

input
```
Integrate[x^8*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

output
```
((a + b*x^3)*((a + b*x^3)^2)^p*(a^2 - a*b*(1 + 2*p)*x^3 + b^2*(1 + 3*p + 2
*p^2)*x^6))/(3*b^3*(1 + p)*(1 + 2*p)*(3 + 2*p))
```

3.126.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1385, 798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8 \left(a^2+2abx^3+b^2x^6\right)^p \, dx$$

$$\Big\downarrow \text{1385}$$

$$\left(\frac{bx^3}{a}+1\right)^{-2p} \left(a^2+2abx^3+b^2x^6\right)^p \int x^8 \left(\frac{bx^3}{a}+1\right)^{2p} dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\left(\frac{bx^3}{a}+1\right)^{-2p} \left(a^2+2abx^3+b^2x^6\right)^p \int x^6 \left(\frac{bx^3}{a}+1\right)^{2p} dx^3$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{3}\left(\frac{bx^3}{a}+1\right)^{-2p} \left(a^2+2abx^3+b^2x^6\right)^p \int \left(\frac{a^2\left(\frac{bx^3}{a}+1\right)^{2p}}{b^2}-\frac{2a^2\left(\frac{bx^3}{a}+1\right)^{2p+1}}{b^2}+\frac{a^2\left(\frac{bx^3}{a}+1\right)^{2p+2}}{b^2}\right) dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{bx^3}{a}+1\right)^{-2p} \left(a^2+2abx^3+b^2x^6\right)^p \left(-\frac{a^3\left(\frac{bx^3}{a}+1\right)^{2(p+1)}}{b^3(p+1)}+\frac{a^3\left(\frac{bx^3}{a}+1\right)^{2p+1}}{b^3(2p+1)}+\frac{a^3\left(\frac{bx^3}{a}+1\right)^{2p+3}}{b^3(2p+3)}\right)$$

input
```
Int[x^8*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

output
```
((a^2 + 2*a*b*x^3 + b^2*x^6)^p*(-((a^3*(1 + (b*x^3)/a)^(2*(1 + p)))/(b^3*(
1 + p))) + (a^3*(1 + (b*x^3)/a)^(1 + 2*p))/(b^3*(1 + 2*p)) + (a^3*(1 + (b*
x^3)/a)^(3 + 2*p))/(b^3*(3 + 2*p))))/(3*(1 + (b*x^3)/a)^(2*p))
```

3.126.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.126.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.74

method	result
gosper	$\frac{\left(b\,x^{3}+a\right)\left(2b^{2}p^{2}x^{6}+3b^{2}p\,x^{6}+b^{2}x^{6}-2abp\,x^{3}-ab\,x^{3}+a^{2}\right)\left(b^{2}x^{6}+2ab\,x^{3}+a^{2}\right)^{p}}{3b^{3}\left(4p^{3}+12p^{2}+11p+3\right)}$
risch	$\frac{\left(2b^{3}p^{2}x^{9}+3b^{3}p\,x^{9}+b^{3}x^{9}+2a\,b^{2}p^{2}x^{6}+a\,b^{2}p\,x^{6}-2a^{2}bp\,x^{3}+a^{3}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{p}}{3(1+p)(3+2p)(1+2p)b^{3}}$
parallelrisch	$\frac{2x^{9}\left(b^{2}x^{6}+2ab\,x^{3}+a^{2}\right)^{p}a\,b^{3}p^{2}+3x^{9}\left(b^{2}x^{6}+2ab\,x^{3}+a^{2}\right)^{p}a\,b^{3}p+x^{9}\left(b^{2}x^{6}+2ab\,x^{3}+a^{2}\right)^{p}a\,b^{3}+2x^{6}\left(b^{2}x^{6}+2ab\,x^{3}+a^{2}\right)^{p}a^{2}b^{2}p^{2}+x}{3(3+2p)(1+p)(1+2p)a\,b^{3}}$

input
```
int(x^8*(b^2*x^6+2*a*b*x^3+a^2)^p,x,method=_RETURNVERBOSE)
```

output
```
1/3*(b*x^3+a)*(2*b^2*p^2*x^6+3*b^2*p*x^6+b^2*x^6-2*a*b*p*x^3-a*b*x^3+a^2)*
(b^2*x^6+2*a*b*x^3+a^2)^p/b^3/(4*p^3+12*p^2+11*p+3)
```

3.126.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.83

$$\int x^8 \left(a^2+2abx^3+b^2x^6\right)^p \, dx = \frac{((2\,b^3p^2+3\,b^3p+b^3)x^9-2\,a^2bpx^3+(2\,ab^2p^2+ab^2p)x^6+a^3)(b^2x^6+2\,abx^3+a^2)^p}{3\,(4\,b^3p^3+12\,b^3p^2+11\,b^3p+3\,b^3)}$$

input
```
integrate(x^8*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="fricas")
```

output
```
1/3*((2*b^3*p^2 + 3*b^3*p + b^3)*x^9 - 2*a^2*b*p*x^3 + (2*a*b^2*p^2 + a*b^
2*p)*x^6 + a^3)*(b^2*x^6 + 2*a*b*x^3 + a^2)^p/(4*b^3*p^3 + 12*b^3*p^2 + 11
*b^3*p + 3*b^3)
```

3.126.6 Sympy [F]

$$\int x^8 \left(a^2+2abx^3+b^2x^6\right)^p \, dx = \begin{cases} \frac{x^9 (a^2)^p}{9} \\ \int \frac{x^8}{\left((a+bx^3)^2\right)^{\frac{3}{2}}}\, dx \\ -\frac{2a^2\log\left(x-\sqrt[3]{-\frac{a}{b}}\right)}{3ab^3+3b^4x^3} - \frac{2a^2\log\left(4x^2+4x\sqrt[3]{-\frac{a}{b}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3ab^3+3b^4x^3} - \frac{2a^2}{3ab^3+3b^4x^3} + \frac{4a^2\log(2)}{3ab^3+3b^4x^3} - \frac{2abx^3\log\left(x-\sqrt[3]{-\frac{a}{b}}\right)}{3ab^3+3b^4x^3} - \frac{2abx^3 \mathrm{l}}{} \\ \int \frac{x^8}{\sqrt{(a+bx^3)^2}}\, dx \\ \frac{a^3(a^2+2abx^3+b^2x^6)^p}{12b^3p^3+36b^3p^2+33b^3p+9b^3} - \frac{2a^2bpx^3(a^2+2abx^3+b^2x^6)^p}{12b^3p^3+36b^3p^2+33b^3p+9b^3} + \frac{2ab^2p^2x^6(a^2+2abx^3+b^2x^6)^p}{12b^3p^3+36b^3p^2+33b^3p+9b^3} + \frac{ab^2px^6(a^2+2abx^3+b^2x^6)^p}{12b^3p^3+36b^3p^2+33b^3p+9b^3} + \frac{2b^3p^2x^9(}{12b^3p^3+} \end{cases}$$

input
```
integrate(x**8*(b**2*x**6+2*a*b*x**3+a**2)**p,x)
```

output
```
Piecewise((x**9*(a**2)**p/9, Eq(b, 0)), (Integral(x**8/((a + b*x**3)**2)**
(3/2), x), Eq(p, -3/2)), (-2*a**2*log(x - (-a/b)**(1/3))/(3*a*b**3 + 3*b**
4*x**3) - 2*a**2*log(4*x**2 + 4*x*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(3*a*b*
*3 + 3*b**4*x**3) - 2*a**2/(3*a*b**3 + 3*b**4*x**3) + 4*a**2*log(2)/(3*a*b
**3 + 3*b**4*x**3) - 2*a*b*x**3*log(x - (-a/b)**(1/3))/(3*a*b**3 + 3*b**4*
x**3) - 2*a*b*x**3*log(4*x**2 + 4*x*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(3*a*
b**3 + 3*b**4*x**3) + 4*a*b*x**3*log(2)/(3*a*b**3 + 3*b**4*x**3) + b**2*x*
*6/(3*a*b**3 + 3*b**4*x**3), Eq(p, -1)), (Integral(x**8/sqrt((a + b*x**3)*
*2), x), Eq(p, -1/2)), (a**3*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(12*b**3*p
**3 + 36*b**3*p**2 + 33*b**3*p + 9*b**3) - 2*a**2*b*p*x**3*(a**2 + 2*a*b*x
**3 + b**2*x**6)**p/(12*b**3*p**3 + 36*b**3*p**2 + 33*b**3*p + 9*b**3) + 2
*a*b**2*p**2*x**6*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(12*b**3*p**3 + 36*b*
*3*p**2 + 33*b**3*p + 9*b**3) + a*b**2*p*x**6*(a**2 + 2*a*b*x**3 + b**2*x*
*6)**p/(12*b**3*p**3 + 36*b**3*p**2 + 33*b**3*p + 9*b**3) + 2*b**3*p**2*x*
*9*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(12*b**3*p**3 + 36*b**3*p**2 + 33*b*
*3*p + 9*b**3) + 3*b**3*p*x**9*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(12*b**3
*p**3 + 36*b**3*p**2 + 33*b**3*p + 9*b**3) + b**3*x**9*(a**2 + 2*a*b*x**3
+ b**2*x**6)**p/(12*b**3*p**3 + 36*b**3*p**2 + 33*b**3*p + 9*b**3), True))
```

3.126.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.61

$$\int x^8 \left(a^2 + 2abx^3 + b^2x^6\right)^p \, dx = \frac{((2\,p^2 + 3\,p + 1)b^3x^9 + (2\,p^2 + p)ab^2x^6 - 2\,a^2bpx^3 + a^3)(bx^3 + a)^{2\,p}}{3\,(4\,p^3 + 12\,p^2 + 11\,p + 3)b^3}$$

input
```
integrate(x^8*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="maxima")
```

output
```
1/3*((2*p^2 + 3*p + 1)*b^3*x^9 + (2*p^2 + p)*a*b^2*x^6 - 2*a^2*b*p*x^3 + a
^3)*(b*x^3 + a)^(2*p)/((4*p^3 + 12*p^2 + 11*p + 3)*b^3)
```

3.126.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.81

$$\int x^8 \left(a^2+2abx^3+b^2x^6\right)^p \, dx$$
$$= \frac{2\,(b^2x^6+2\,abx^3+a^2)^p b^3p^2x^9+3\,(b^2x^6+2\,abx^3+a^2)^p b^3px^9+(b^2x^6+2\,abx^3+a^2)^p b^3x^9+2\,(b^2x^6+2\,ab}{3\,(4\,b^3p^3+12\,b^3p}$$

input
```
integrate(x^8*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="giac")
```

output
```
1/3*(2*(b^2*x^6 + 2*a*b*x^3 + a^2)^p*b^3*p^2*x^9 + 3*(b^2*x^6 + 2*a*b*x^3
+ a^2)^p*b^3*p*x^9 + (b^2*x^6 + 2*a*b*x^3 + a^2)^p*b^3*x^9 + 2*(b^2*x^6 +
2*a*b*x^3 + a^2)^p*a*b^2*p^2*x^6 + (b^2*x^6 + 2*a*b*x^3 + a^2)^p*a*b^2*p*x
^6 - 2*(b^2*x^6 + 2*a*b*x^3 + a^2)^p*a^2*b*p*x^3 + (b^2*x^6 + 2*a*b*x^3 +
a^2)^p*a^3)/(4*b^3*p^3 + 12*b^3*p^2 + 11*b^3*p + 3*b^3)
```

3.126.9 Mupad [B] (verification not implemented)

Time = 8.36 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.05

$$\int x^8 \left(a^2+2abx^3+b^2x^6\right)^p \, dx = \left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^p \left(\frac{x^9\,\left(\frac{2\,p^2}{3}+p+\frac{1}{3}\right)}{4\,p^3+12\,p^2+11\,p+3} + \frac{a^3}{3\,b^3\,\left(4\,p^3+12\,p^2+11\,p+3\right)} - \frac{2\,a^2\,p\,x^3}{3\,b^2\,\left(4\,p^3+12\,p^2+11\,p+3\right)} + \frac{a\,p\,x^6\,\left(2\,p+1\right)}{3\,b\,\left(4\,p^3+12\,p^2+11\,p+3\right)}\right)$$

input
```
int(x^8*(a^2 + b^2*x^6 + 2*a*b*x^3)^p,x)
```

output
```
(a^2 + b^2*x^6 + 2*a*b*x^3)^p*((x^9*(p + (2*p^2)/3 + 1/3))/(11*p + 12*p^2
+ 4*p^3 + 3) + a^3/(3*b^3*(11*p + 12*p^2 + 4*p^3 + 3)) - (2*a^2*p*x^3)/(3*
b^2*(11*p + 12*p^2 + 4*p^3 + 3)) + (a*p*x^6*(2*p + 1))/(3*b*(11*p + 12*p^2
 + 4*p^3 + 3)))
```

3.127 $\int x^5(a^2+2abx^3+b^2x^6)^p\,dx$

3.127.1 Optimal result

Integrand size = 24, antiderivative size = 84

$$\int x^5\left(a^2+2abx^3+b^2x^6\right)^p\,dx=-\frac{a(a+bx^3)\left(a^2+2abx^3+b^2x^6\right)^p}{3b^2(1+2p)}+\frac{\left(a+bx^3\right)^2\left(a^2+2abx^3+b^2x^6\right)^p}{6b^2(1+p)}$$

output
```
-1/3*a*(b*x^3+a)*(b^2*x^6+2*a*b*x^3+a^2)^p/b^2/(1+2*p)+1/6*(b*x^3+a)^2*(b^
2*x^6+2*a*b*x^3+a^2)^p/b^2/(p+1)
```

3.127.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.61

$$\int x^5\left(a^2+2abx^3+b^2x^6\right)^p\,dx=\frac{(a+bx^3)\left((a+bx^3)^2\right)^p(-a+b(1+2p)x^3)}{6b^2(1+p)(1+2p)}$$

input
```
Integrate[x^5*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

output
```
((a + b*x^3)*((a + b*x^3)^2)^p*(-a + b*(1 + 2*p)*x^3))/(6*b^2*(1 + p)*(1 +
 2*p))
```

3.127.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1693, 1100, 1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(a^2+2 a b x^3+b^2 x^6\right)^p \, dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3} \int x^3 \left(b^2 x^6+2 a b x^3+a^2\right)^p dx^3$$

$$\Big\downarrow \text{1100}$$

$$\frac{1}{3} \left(\frac{\left(a^2+2 a b x^3+b^2 x^6\right)^{p+1}}{2 b^2 (p+1)}-\frac{a \int \left(b^2 x^6+2 a b x^3+a^2\right)^p dx^3}{b}\right)$$

$$\Big\downarrow \text{1079}$$

$$\frac{1}{3} \left(\frac{\left(a^2+2 a b x^3+b^2 x^6\right)^{p+1}}{2 b^2 (p+1)}-\frac{a \left(a b+b^2 x^3\right)^{-2 p} \left(a^2+2 a b x^3+b^2 x^6\right)^p \int \left(b^2 x^3+a b\right)^{2 p} dx^3}{b}\right)$$

$$\Big\downarrow \text{17}$$

$$\frac{1}{3} \left(\frac{\left(a^2+2 a b x^3+b^2 x^6\right)^{p+1}}{2 b^2 (p+1)}-\frac{a \left(a b+b^2 x^3\right) \left(a^2+2 a b x^3+b^2 x^6\right)^p}{b^3 (2 p+1)}\right)$$

input
```
Int[x^5*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

output
```
(-((a*(a*b + b^2*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(b^3*(1 + 2*p))) + (a
^2 + 2*a*b*x^3 + b^2*x^6)^(1 + p)/(2*b^2*(1 + p)))/3
```

3.127.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1100
```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.127.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.69

method	result	size
risch	$-\frac{\left(-2 b^{2} p \,x^{6}-b^{2} x^{6}-2 a b p \,x^{3}+a^{2}\right) \left(\left(b \,x^{3}+a \right)^{2}\right)^{p}}{6 b^{2} \left(1+p \right) \left(1+2 p \right)}$	58
gosper	$-\frac{\left(b^{2} x^{6}+2 a b \,x^{3}+a^{2}\right)^{p} \left(-2 x^{3} p b -b \,x^{3}+a \right) \left(b \,x^{3}+a \right)}{6 b^{2} \left(2 p^{2}+3 p +1\right)}$	60
norman	$\frac{x^{6} {\mathrm e}^{p \ln \left(b^{2} x^{6}+2 a b \,x^{3}+a^{2}\right)}}{6 p +6}-\frac{a^{2} {\mathrm e}^{p \ln \left(b^{2} x^{6}+2 a b \,x^{3}+a^{2}\right)}}{6 b^{2} \left(2 p^{2}+3 p +1\right)}+\frac{p a \,x^{3} {\mathrm e}^{p \ln \left(b^{2} x^{6}+2 a b \,x^{3}+a^{2}\right)}}{3 b \left(2 p^{2}+3 p +1\right)}$	120
parallelrisch	$\frac{2 x^{6} \left(b^{2} x^{6}+2 a b \,x^{3}+a^{2}\right)^{p} b^{2} p +x^{6} \left(b^{2} x^{6}+2 a b \,x^{3}+a^{2}\right)^{p} b^{2}+2 x^{3} \left(b^{2} x^{6}+2 a b \,x^{3}+a^{2}\right)^{p} a b p -\left(b^{2} x^{6}+2 a b \,x^{3}+a^{2}\right)^{p} a^{2}}{6 b^{2} \left(2 p^{2}+3 p +1\right)}$	128

input
```
int(x^5*(b^2*x^6+2*a*b*x^3+a^2)^p,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(-2*b^2*p*x^6-b^2*x^6-2*a*b*p*x^3+a^2)/b^2/(1+p)/(1+2*p)*((b*x^3+a)^2
)^p
```

3.127.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.83

$$\int x^5 (a^2 + 2abx^3 + b^2x^6)^p \, dx = \frac{((2\,b^2p + b^2)x^6 + 2\,abpx^3 - a^2)(b^2x^6 + 2\,abx^3 + a^2)^p}{6\,(2\,b^2p^2 + 3\,b^2p + b^2)}$$

input
```
integrate(x^5*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="fricas")
```

output
```
1/6*((2*b^2*p + b^2)*x^6 + 2*a*b*p*x^3 - a^2)*(b^2*x^6 + 2*a*b*x^3 + a^2)^
p/(2*b^2*p^2 + 3*b^2*p + b^2)
```

3.127.6 Sympy [F]

$$\int x^5 (a^2 + 2abx^3 + b^2x^6)^p \, dx$$
$$= \begin{cases} \frac{x^6 (a^2)^p}{6} \\ \frac{a \log\left(x - \sqrt[3]{-\frac{a}{b}}\right)}{3ab^2+3b^3x^3} + \frac{a \log\left(4x^2+4x\sqrt[3]{-\frac{a}{b}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3ab^2+3b^3x^3} - \frac{2a\log(2)}{3ab^2+3b^3x^3} + \frac{a}{3ab^2+3b^3x^3} + \frac{bx^3 \log\left(x - \sqrt[3]{-\frac{a}{b}}\right)}{3ab^2+3b^3x^3} + \frac{bx^3 \log\left(4x^2+4x \cdots\right.}{3ab^2 \cdots} \\ \int \frac{x^5}{\sqrt{(a+bx^3)^2}}\, dx \\ -\frac{a^2(a^2+2abx^3+b^2x^6)^p}{12b^2p^2+18b^2p+6b^2} + \frac{2abpx^3(a^2+2abx^3+b^2x^6)^p}{12b^2p^2+18b^2p+6b^2} + \frac{2b^2px^6(a^2+2abx^3+b^2x^6)^p}{12b^2p^2+18b^2p+6b^2} + \frac{b^2x^6(a^2+2abx^3+b^2x^6)^p}{12b^2p^2+18b^2p+6b^2} \end{cases}$$

input
```
integrate(x**5*(b**2*x**6+2*a*b*x**3+a**2)**p,x)
```

output
```
Piecewise((x**6*(a**2)**p/6, Eq(b, 0)), (a*log(x - (-a/b)**(1/3))/(3*a*b**
2 + 3*b**3*x**3) + a*log(4*x**2 + 4*x*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(3*
a*b**2 + 3*b**3*x**3) - 2*a*log(2)/(3*a*b**2 + 3*b**3*x**3) + a/(3*a*b**2
+ 3*b**3*x**3) + b*x**3*log(x - (-a/b)**(1/3))/(3*a*b**2 + 3*b**3*x**3) +
b*x**3*log(4*x**2 + 4*x*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(3*a*b**2 + 3*b**
3*x**3) - 2*b*x**3*log(2)/(3*a*b**2 + 3*b**3*x**3), Eq(p, -1)), (Integral(
x**5/sqrt((a + b*x**3)**2), x), Eq(p, -1/2)), (-a**2*(a**2 + 2*a*b*x**3 +
b**2*x**6)**p/(12*b**2*p**2 + 18*b**2*p + 6*b**2) + 2*a*b*p*x**3*(a**2 + 2
*a*b*x**3 + b**2*x**6)**p/(12*b**2*p**2 + 18*b**2*p + 6*b**2) + 2*b**2*p*x
**6*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(12*b**2*p**2 + 18*b**2*p + 6*b**2)
 + b**2*x**6*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(12*b**2*p**2 + 18*b**2*p
+ 6*b**2), True))
```

3.127.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.64

$$\int x^5 (a^2 + 2abx^3 + b^2x^6)^p \, dx = \frac{(b^2(2\,p+1)x^6 + 2\,abpx^3 - a^2)(bx^3 + a)^{2\,p}}{6\,(2\,p^2 + 3\,p + 1)b^2}$$

input
```
integrate(x^5*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="maxima")
```

output
```
1/6*(b^2*(2*p + 1)*x^6 + 2*a*b*p*x^3 - a^2)*(b*x^3 + a)^(2*p)/((2*p^2 + 3*
p + 1)*b^2)
```

3.127.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.57

$$\int x^5 (a^2 + 2abx^3 + b^2x^6)^p \, dx$$
$$= \frac{2\,(b^2x^6 + 2\,abx^3 + a^2)^p b^2px^6 + (b^2x^6 + 2\,abx^3 + a^2)^p b^2x^6 + 2\,(b^2x^6 + 2\,abx^3 + a^2)^p abpx^3 - (b^2x^6 + 2\,abx^3}{6\,(2\,b^2p^2 + 3\,b^2p + b^2)}$$

input
```
integrate(x^5*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="giac")
```

output
```
1/6*(2*(b^2*x^6 + 2*a*b*x^3 + a^2)^p*b^2*p*x^6 + (b^2*x^6 + 2*a*b*x^3 + a^
2)^p*b^2*x^6 + 2*(b^2*x^6 + 2*a*b*x^3 + a^2)^p*a*b*p*x^3 - (b^2*x^6 + 2*a*
b*x^3 + a^2)^p*a^2)/(2*b^2*p^2 + 3*b^2*p + b^2)
```

3.127.9 Mupad [B] (verification not implemented)

Time = 8.31 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.01

$$\int x^5 (a^2 + 2abx^3 + b^2x^6)^p \, dx = (a^2 + 2\,a\,b\,x^3 + b^2\,x^6)^p \left(\frac{x^6\,(2\,p+1)}{6\,(2\,p^2 + 3\,p + 1)} - \frac{a^2}{6\,b^2\,(2\,p^2 + 3\,p + 1)} + \frac{a\,p\,x^3}{3\,b\,(2\,p^2 + 3\,p + 1)}\right)$$

input
```
int(x^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^p,x)
```

output
```
(a^2 + b^2*x^6 + 2*a*b*x^3)^p*((x^6*(2*p + 1))/(6*(3*p + 2*p^2 + 1)) - a^2
/(6*b^2*(3*p + 2*p^2 + 1)) + (a*p*x^3)/(3*b*(3*p + 2*p^2 + 1)))
```

3.127. $\int x^5(a^2 + 2abx^3 + b^2x^6)^p \, dx$

3.128 $\int x^4(a^2+2abx^3+b^2x^6)^p\,dx$

3.128.1 Optimal result

Integrand size = 24, antiderivative size = 60

$$\int x^4(a^2+2abx^3+b^2x^6)^p\,dx = \frac{1}{5}x^5\left(1+\frac{bx^3}{a}\right)^{-2p}(a^2+2abx^3+b^2x^6)^p\,\text{Hypergeometric2F1}\left(\frac{5}{3},-2p,\frac{8}{3},-\frac{bx^3}{a}\right)$$

output
```
1/5*x^5*(b^2*x^6+2*a*b*x^3+a^2)^p*hypergeom([5/3, -2*p],[8/3],-b*x^3/a)/((
1+b*x^3/a)^(2*p))
```

3.128.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.85

$$\int x^4(a^2+2abx^3+b^2x^6)^p\,dx = \frac{1}{5}x^5\left((a+bx^3)^2\right)^p\left(1+\frac{bx^3}{a}\right)^{-2p}\text{Hypergeometric2F1}\left(\frac{5}{3},-2p,\frac{8}{3},-\frac{bx^3}{a}\right)$$

input
```
Integrate[x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

output
```
(x^5*((a + b*x^3)^2)^p*Hypergeometric2F1[5/3, -2*p, 8/3, -((b*x^3)/a)])/(5
*(1 + (b*x^3)/a)^(2*p))
```

3.128.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1385, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\left(a^2+2abx^3+b^2x^6\right)^p\,dx$$

$$\Big\downarrow \text{1385}$$

$$\left(\frac{bx^3}{a}+1\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\int x^4\left(\frac{bx^3}{a}+1\right)^{2p}dx$$

$$\Big\downarrow \text{888}$$

$$\frac{1}{5}x^5\left(\frac{bx^3}{a}+1\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\text{Hypergeometric2F1}\left(\frac{5}{3},-2p,\frac{8}{3},-\frac{bx^3}{a}\right)$$

input
```
Int[x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

output
```
(x^5*(a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[5/3, -2*p, 8/3, -((b*
x^3)/a)])/(5*(1 + (b*x^3)/a)^(2*p))
```

3.128.3.1 Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

3.128.4 Maple [F]

$$\int x^4 \left(b^2 x^6 + 2ab\, x^3 + a^2\right)^p dx$$

input
```
int(x^4*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

output
```
int(x^4*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

3.128.5 Fricas [F]

$$\int x^4 \left(a^2 + 2abx^3 + b^2 x^6\right)^p dx = \int \left(b^2 x^6 + 2\, abx^3 + a^2\right)^p x^4\, dx$$

input
```
integrate(x^4*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="fricas")
```

output
```
integral((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x^4, x)
```

3.128.6 Sympy [F]

$$\int x^4 \left(a^2 + 2abx^3 + b^2 x^6\right)^p dx = \int x^4 \left(\left(a + bx^3\right)^2\right)^p dx$$

input
```
integrate(x**4*(b**2*x**6+2*a*b*x**3+a**2)**p,x)
```

output
```
Integral(x**4*((a + b*x**3)**2)**p, x)
```

3.128.7 Maxima [F]

$$\int x^4\left(a^2+2abx^3+b^2x^6\right)^p\,dx=\int \left(b^2x^6+2\,abx^3+a^2\right)^p x^4\,dx$$

input
```
integrate(x^4*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="maxima")
```

output
```
integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x^4, x)
```

3.128.8 Giac [F]

$$\int x^4\left(a^2+2abx^3+b^2x^6\right)^p\,dx=\int \left(b^2x^6+2\,abx^3+a^2\right)^p x^4\,dx$$

input
```
integrate(x^4*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="giac")
```

output
```
integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x^4, x)
```

3.128.9 Mupad [F(-1)]

Timed out.

$$\int x^4\left(a^2+2abx^3+b^2x^6\right)^p\,dx=\int x^4\,\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^p\,dx$$

input
```
int(x^4*(a^2 + b^2*x^6 + 2*a*b*x^3)^p,x)
```

output
```
int(x^4*(a^2 + b^2*x^6 + 2*a*b*x^3)^p, x)
```

3.129 $\int x^3(a^2+2abx^3+b^2x^6)^p\,dx$

3.129.1 Optimal result

Integrand size = 24, antiderivative size = 60

$$\int x^3(a^2+2abx^3+b^2x^6)^p\,dx=\frac{1}{4}x^4\left(1+\frac{bx^3}{a}\right)^{-2p}(a^2+2abx^3+b^2x^6)^p\text{Hypergeometric2F1}\left(\frac{4}{3},-2p,\frac{7}{3},-\frac{bx^3}{a}\right)$$

output
```
1/4*x^4*(b^2*x^6+2*a*b*x^3+a^2)^p*hypergeom([4/3, -2*p],[7/3],-b*x^3/a)/((
1+b*x^3/a)^(2*p))
```

3.129.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.85

$$\int x^3(a^2+2abx^3+b^2x^6)^p\,dx=\frac{1}{4}x^4\left((a+bx^3)^2\right)^p\left(1+\frac{bx^3}{a}\right)^{-2p}\text{Hypergeometric2F1}\left(\frac{4}{3},-2p,\frac{7}{3},-\frac{bx^3}{a}\right)$$

input
```
Integrate[x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

output
```
(x^4*((a + b*x^3)^2)^p*Hypergeometric2F1[4/3, -2*p, 7/3, -((b*x^3)/a)])/(4
*(1 + (b*x^3)/a)^(2*p))
```

3.129.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1385, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a^2+2abx^3+b^2x^6\right)^p \, dx$$

$$\Big\downarrow \text{ 1385}$$

$$\left(\frac{bx^3}{a}+1\right)^{-2p} \left(a^2+2abx^3+b^2x^6\right)^p \int x^3 \left(\frac{bx^3}{a}+1\right)^{2p} dx$$

$$\Big\downarrow \text{ 888}$$

$$\frac{1}{4}x^4\left(\frac{bx^3}{a}+1\right)^{-2p} \left(a^2+2abx^3+b^2x^6\right)^p \text{Hypergeometric2F1}\left(\frac{4}{3},-2p,\frac{7}{3},-\frac{bx^3}{a}\right)$$

input
```
Int[x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

output
```
(x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[4/3, -2*p, 7/3, -((b*
x^3)/a)])/(4*(1 + (b*x^3)/a)^(2*p))
```

3.129.3.1 Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

3.129.4 Maple [F]

$$\int x^3 \left(b^2 x^6 + 2ab\, x^3 + a^2\right)^p dx$$

input
```
int(x^3*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

output
```
int(x^3*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

3.129.5 Fricas [F]

$$\int x^3 \left(a^2 + 2abx^3 + b^2 x^6\right)^p \, dx = \int \left(b^2 x^6 + 2\, abx^3 + a^2\right)^p x^3 \, dx$$

input
```
integrate(x^3*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="fricas")
```

output
```
integral((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x^3, x)
```

3.129.6 Sympy [F]

$$\int x^3 \left(a^2 + 2abx^3 + b^2 x^6\right)^p \, dx = \int x^3 \left(\left(a + bx^3\right)^2\right)^p \, dx$$

input
```
integrate(x**3*(b**2*x**6+2*a*b*x**3+a**2)**p,x)
```

output
```
Integral(x**3*((a + b*x**3)**2)**p, x)
```

3.129.7 Maxima [F]

$$\int x^3 \left(a^2+2abx^3+b^2x^6\right)^p\,dx=\int \left(b^2x^6+2\,abx^3+a^2\right)^p x^3\,dx$$

input
```
integrate(x^3*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="maxima")
```

output
```
integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x^3, x)
```

3.129.8 Giac [F]

$$\int x^3 \left(a^2+2abx^3+b^2x^6\right)^p\,dx=\int \left(b^2x^6+2\,abx^3+a^2\right)^p x^3\,dx$$

input
```
integrate(x^3*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="giac")
```

output
```
integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x^3, x)
```

3.129.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \left(a^2+2abx^3+b^2x^6\right)^p\,dx=\int x^3\,\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^p\,dx$$

input
```
int(x^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^p,x)
```

output
```
int(x^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^p, x)
```

3.130 $\int x^2(a^2+2abx^3+b^2x^6)^p\,dx$

3.130.1 Optimal result

Integrand size = 24, antiderivative size = 41

$$\int x^2\left(a^2+2abx^3+b^2x^6\right)^p\,dx=\frac{(a+bx^3)\left(a^2+2abx^3+b^2x^6\right)^p}{3b(1+2p)}$$

output
```
1/3*(b*x^3+a)*(b^2*x^6+2*a*b*x^3+a^2)^p/b/(1+2*p)
```

3.130.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

$$\int x^2\left(a^2+2abx^3+b^2x^6\right)^p\,dx=\frac{(a+bx^3)\left((a+bx^3)^2\right)^p}{3b(1+2p)}$$

input
```
Integrate[x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

output
```
((a + b*x^3)*((a + b*x^3)^2)^p)/(3*b*(1 + 2*p))
```

3.130.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1690, 1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a^2+2 a b x^3+b^2 x^6\right)^p \, dx$$

$$\Big\downarrow \ 1690$$

$$\frac{1}{3} \int \left(b^2 x^6+2 a b x^3+a^2\right)^p dx^3$$

$$\Big\downarrow \ 1079$$

$$\frac{1}{3} \left(a b+b^2 x^3\right)^{-2 p} \left(a^2+2 a b x^3+b^2 x^6\right)^p \int \left(b^2 x^3+a b\right)^{2 p} dx^3$$

$$\Big\downarrow \ 17$$

$$\frac{\left(a b+b^2 x^3\right) \left(a^2+2 a b x^3+b^2 x^6\right)^p}{3 b^2 (2 p+1)}$$

input
```
Int[x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

output
```
((a*b + b^2*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(3*b^2*(1 + 2*p))
```

3.130.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

3.130.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

method	result	size
risch	$\frac{(b\,x^3+a)\left(\left(b\,x^3+a\right)^2\right)^p}{3b(1+2p)}$	31
gosper	$\frac{(b\,x^3+a)\,(b^2x^6+2ab\,x^3+a^2)^p}{3b(1+2p)}$	40
parallelrisch	$\frac{x^3\,(b^2x^6+2ab\,x^3+a^2)^p ab+(b^2x^6+2ab\,x^3+a^2)^p a^2}{3a(1+2p)b}$	67
norman	$\frac{x^3 \mathrm{e}^{p\ln\left(b^2x^6+2ab\,x^3+a^2\right)}}{6p+3}+\frac{a\,\mathrm{e}^{p\ln\left(b^2x^6+2ab\,x^3+a^2\right)}}{3b(1+2p)}$	71

input
```
int(x^2*(b^2*x^6+2*a*b*x^3+a^2)^p,x,method=_RETURNVERBOSE)
```

output
```
1/3*(b*x^3+a)/b/(1+2*p)*((b*x^3+a)^2)^p
```

3.130.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

$$\int x^2\left(a^2+2abx^3+b^2x^6\right)^p\,dx=\frac{(bx^3+a)(b^2x^6+2\,abx^3+a^2)^p}{3\,(2\,bp+b)}$$

input
```
integrate(x^2*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="fricas")
```

output
```
1/3*(b*x^3 + a)*(b^2*x^6 + 2*a*b*x^3 + a^2)^p/(2*b*p + b)
```

3.130.6 Sympy [F]

$$\int x^2(a^2+2abx^3+b^2x^6)^p\,dx = \begin{cases} \frac{x^3}{3\sqrt{a^2}} & \text{for } b=0 \wedge p=-\frac{1}{2} \\ \frac{x^3(a^2)^p}{3} & \text{for } b=0 \\ \int \frac{x^2}{\sqrt{(a+bx^3)^2}}\,dx & \text{for } p=-\frac{1}{2} \\ \frac{a(a^2+2abx^3+b^2x^6)^p}{6bp+3b} + \frac{bx^3(a^2+2abx^3+b^2x^6)^p}{6bp+3b} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*(b**2*x**6+2*a*b*x**3+a**2)**p,x)
```

output
```
Piecewise((x**3/(3*sqrt(a**2)), Eq(b, 0) & Eq(p, -1/2)), (x**3*(a**2)**p/3
, Eq(b, 0)), (Integral(x**2/sqrt((a + b*x**3)**2), x), Eq(p, -1/2)), (a*(a
**2 + 2*a*b*x**3 + b**2*x**6)**p/(6*b*p + 3*b) + b*x**3*(a**2 + 2*a*b*x**3
 + b**2*x**6)**p/(6*b*p + 3*b), True))
```

3.130.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.73

$$\int x^2(a^2+2abx^3+b^2x^6)^p\,dx = \frac{(bx^3+a)(bx^3+a)^{2p}}{3b(2p+1)}$$

input
```
integrate(x^2*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="maxima")
```

output
```
1/3*(b*x^3 + a)*(b*x^3 + a)^(2*p)/(b*(2*p + 1))
```

3.130.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.41

$$\int x^2(a^2+2abx^3+b^2x^6)^p\,dx = \frac{(b^2x^6+2abx^3+a^2)^p bx^3 + (b^2x^6+2abx^3+a^2)^p a}{3(2bp+b)}$$

input
```
integrate(x^2*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="giac")
```

output
```
1/3*((b^2*x^6 + 2*a*b*x^3 + a^2)^p*b*x^3 + (b^2*x^6 + 2*a*b*x^3 + a^2)^p*a
)/(2*b*p + b)
```

3.130.9 Mupad [B] (verification not implemented)

Time = 8.36 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12

$$\int x^2 (a^2 + 2abx^3 + b^2x^6)^p \, dx = \left(\frac{x^3}{3\,(2\,p+1)} + \frac{a}{3\,b\,(2\,p+1)}\right)\,(a^2 + 2\,a\,b\,x^3 + b^2\,x^6)^p$$

input
```
int(x^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^p,x)
```

output
```
(x^3/(3*(2*p + 1)) + a/(3*b*(2*p + 1)))*(a^2 + b^2*x^6 + 2*a*b*x^3)^p
```

3.131 $\int x(a^2+2abx^3+b^2x^6)^p\,dx$

3.131.1 Optimal result

Integrand size = 22, antiderivative size = 58

$$\int x\left(a^2+2abx^3+b^2x^6\right)^p\,dx = \frac{x^2(a+bx^3)\left(a^2+2abx^3+b^2x^6\right)^p \text{Hypergeometric2F1}\left(1,\frac{5}{3}+2p,\frac{5}{3},-\frac{bx^3}{a}\right)}{2a}$$

output
```
1/2*x^2*(b*x^3+a)*(b^2*x^6+2*a*b*x^3+a^2)^p*hypergeom([1, 5/3+2*p],[5/3],-
b*x^3/a)/a
```

3.131.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.88

$$\int x\left(a^2+2abx^3+b^2x^6\right)^p\,dx = \frac{1}{2}x^2\left(\left(a+bx^3\right)^2\right)^p\left(1+\frac{bx^3}{a}\right)^{-2p}\text{Hypergeometric2F1}\left(\frac{2}{3},-2p,\frac{5}{3},-\frac{bx^3}{a}\right)$$

input
```
Integrate[x*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

output
```
(x^2*((a + b*x^3)^2)^p*Hypergeometric2F1[2/3, -2*p, 5/3, -((b*x^3)/a)])/(2
*(1 + (b*x^3)/a)^(2*p))
```

3.131.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.03, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1385, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a^2+2abx^3+b^2x^6\right)^p\,dx$$

$$\Big\downarrow \text{1385}$$

$$\left(\frac{bx^3}{a}+1\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\int x\left(\frac{bx^3}{a}+1\right)^{2p}dx$$

$$\Big\downarrow \text{888}$$

$$\frac{1}{2}x^2\left(\frac{bx^3}{a}+1\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\text{Hypergeometric2F1}\left(\frac{2}{3},-2p,\frac{5}{3},-\frac{bx^3}{a}\right)$$

input
```
Int[x*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

output
```
(x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[2/3, -2*p, 5/3, -((b*
x^3)/a)])/(2*(1 + (b*x^3)/a)^(2*p))
```

3.131.3.1 Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

3.131.4 Maple [F]

$$\int x\left(b^2x^6+2ab\,x^3+a^2\right)^p dx$$

input
```
int(x*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

output
```
int(x*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

3.131.5 Fricas [F]

$$\int x\left(a^2+2abx^3+b^2x^6\right)^p\,dx=\int\left(b^2x^6+2\,abx^3+a^2\right)^px\,dx$$

input
```
integrate(x*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="fricas")
```

output
```
integral((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x, x)
```

3.131.6 Sympy [F]

$$\int x\left(a^2+2abx^3+b^2x^6\right)^p\,dx=\int x\left(\left(a+bx^3\right)^2\right)^p\,dx$$

input
```
integrate(x*(b**2*x**6+2*a*b*x**3+a**2)**p,x)
```

output
```
Integral(x*((a + b*x**3)**2)**p, x)
```

3.131.7 Maxima [F]

$$\int x(a^2 + 2abx^3 + b^2x^6)^p \, dx = \int (b^2x^6 + 2\,abx^3 + a^2)^p x \, dx$$

input
```
integrate(x*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="maxima")
```

output
```
integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x, x)
```

3.131.8 Giac [F]

$$\int x(a^2 + 2abx^3 + b^2x^6)^p \, dx = \int (b^2x^6 + 2\,abx^3 + a^2)^p x \, dx$$

input
```
integrate(x*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="giac")
```

output
```
integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x, x)
```

3.131.9 Mupad [F(-1)]

Timed out.

$$\int x(a^2 + 2abx^3 + b^2x^6)^p \, dx = \int x\,(a^2 + 2\,a\,b\,x^3 + b^2\,x^6)^p \, dx$$

input
```
int(x*(a^2 + b^2*x^6 + 2*a*b*x^3)^p,x)
```

output
```
int(x*(a^2 + b^2*x^6 + 2*a*b*x^3)^p, x)
```

3.132 $\int (a^2+2abx^3+b^2x^6)^p \, dx$

3.132.1 Optimal result

Integrand size = 20, antiderivative size = 53

$$\int (a^2+2abx^3+b^2x^6)^p \, dx = \frac{x(a+bx^3)(a^2+2abx^3+b^2x^6)^p \text{Hypergeometric2F1}\left(1,\frac{4}{3}+2p,\frac{4}{3},-\frac{bx^3}{a}\right)}{a}$$

output
```
x*(b*x^3+a)*(b^2*x^6+2*a*b*x^3+a^2)^p*hypergeom([1, 4/3+2*p],[4/3],-b*x^3/
a)/a
```

3.132.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.16 (sec) , antiderivative size = 211, normalized size of antiderivative = 3.98

$$\int (a^2+2abx^3+b^2x^6)^p \, dx = \frac{4^{-p}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}\right)^{-2p}\left(\frac{i\left(1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3i+\sqrt{3}}\right)^{-2p}\left((a+bx^3)^2\right)^p \text{AppellF1}\left(1+2p,-2p,\right.}{\sqrt[3]{b}(1+2p)}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

output
```
(((-1)^(2/3)*a^(1/3) + b^(1/3)*x)*((a + b*x^3)^2)^p*AppellF1[1 + 2*p, -2*p
, -2*p, 2*(1 + p), -(((-1)^(2/3)*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/((1 + (
-1)^(1/3))*a^(1/3))), (I + Sqrt[3] - ((2*I)*b^(1/3)*x)/a^(1/3))/(3*I + Sqr
t[3])])/(4^p*b^(1/3)*(1 + 2*p)*((a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1
)^(1/3))*a^(1/3)))^(2*p)*((I*(1 + (b^(1/3)*x)/a^(1/3)))/(3*I + Sqrt[3]))^(
2*p))
```

3.132.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.04, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1385, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+2abx^3+b^2x^6\right)^p\,dx$$

$$\Big\downarrow \text{1385}$$

$$\left(\frac{bx^3}{a}+1\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\int \left(\frac{bx^3}{a}+1\right)^{2p}dx$$

$$\Big\downarrow \text{778}$$

$$x\left(\frac{bx^3}{a}+1\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\text{Hypergeometric2F1}\left(\frac{1}{3},-2p,\frac{4}{3},-\frac{bx^3}{a}\right)$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

output
```
(x*(a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[1/3, -2*p, 4/3, -((b*x^
3)/a)])/(1 + (b*x^3)/a)^(2*p)
```

3.132.3.1 Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

3.132.4 Maple [F]

$$\int \left(b^2x^6 + 2ab\,x^3 + a^2\right)^p dx$$

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

output
```
int((b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

3.132.5 Fricas [F]

$$\int \left(a^2 + 2abx^3 + b^2x^6\right)^p\,dx = \int \left(b^2x^6 + 2\,abx^3 + a^2\right)^p dx$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="fricas")
```

output
```
integral((b^2*x^6 + 2*a*b*x^3 + a^2)^p, x)
```

3.132.6 Sympy [F]

$$\int \left(a^2+2abx^3+b^2x^6\right)^p\,dx=\int \left(a^2+2abx^3+b^2x^6\right)^p\,dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**p,x)
```

output
```
Integral((a**2 + 2*a*b*x**3 + b**2*x**6)**p, x)
```

3.132.7 Maxima [F]

$$\int \left(a^2+2abx^3+b^2x^6\right)^p\,dx=\int \left(b^2x^6+2\,abx^3+a^2\right)^p\,dx$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="maxima")
```

output
```
integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p, x)
```

3.132.8 Giac [F]

$$\int \left(a^2+2abx^3+b^2x^6\right)^p\,dx=\int \left(b^2x^6+2\,abx^3+a^2\right)^p\,dx$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="giac")
```

output
```
integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p, x)
```

3.132.9 Mupad [F(-1)]

Timed out.

$$\int \left(a^2 + 2abx^3 + b^2x^6\right)^p \, dx = \int \left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^p \, dx$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^p,x)
```

output
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^p, x)
```

3.133 $\int \frac{(a^2+2abx^3+b^2x^6)^p}{x}\,dx$

3.133.1 Optimal result

Integrand size = 24, antiderivative size = 63

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x}\,dx = -\frac{(a+bx^3)(a^2+2abx^3+b^2x^6)^p \,\text{Hypergeometric2F1}\left(1,1+2p,2(1+p),1+\frac{bx^3}{a}\right)}{3a(1+2p)}$$

output
```
-1/3*(b*x^3+a)*(b^2*x^6+2*a*b*x^3+a^2)^p*hypergeom([1, 1+2*p],[2+2*p],1+b*
x^3/a)/a/(1+2*p)
```

3.133.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.86

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x}\,dx = -\frac{(a+bx^3)\left((a+bx^3)^2\right)^p \text{Hypergeometric2F1}\left(1,1+2p,2+2p,1+\frac{bx^3}{a}\right)}{3a(1+2p)}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^p/x,x]
```

output
```
-1/3*((a + b*x^3)*((a + b*x^3)^2)^p*Hypergeometric2F1[1, 1 + 2*p, 2 + 2*p,
 1 + (b*x^3)/a])/(a*(1 + 2*p))
```

3.133.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1385, 798, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^p}{x}dx$$

$$\Big\downarrow \text{1385}$$

$$\left(\frac{bx^3}{a}+1\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\int \frac{\left(\frac{bx^3}{a}+1\right)^{2p}}{x}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\left(\frac{bx^3}{a}+1\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\int \frac{\left(\frac{bx^3}{a}+1\right)^{2p}}{x^3}dx^3$$

$$\Big\downarrow \text{75}$$

$$-\frac{\left(\frac{bx^3}{a}+1\right)\left(a^2+2abx^3+b^2x^6\right)^p \text{Hypergeometric2F1}\left(1,2p+1,2(p+1),\frac{bx^3}{a}+1\right)}{3(2p+1)}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^p/x,x]
```

output
```
-1/3*((1 + (b*x^3)/a)*(a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[1, 1
 + 2*p, 2*(1 + p), 1 + (b*x^3)/a])/(1 + 2*p)
```

3.133.3.1 Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

3.133.4 Maple [F]

$$\int \frac{(b^2x^6 + 2ab\,x^3 + a^2)^p}{x} dx$$

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^p/x,x)
```

output
```
int((b^2*x^6+2*a*b*x^3+a^2)^p/x,x)
```

3.133.5 Fricas [F]

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^p}{x}\, dx = \int \frac{(b^2x^6 + 2\,abx^3 + a^2)^p}{x}\, dx$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x,x, algorithm="fricas")
```

output
```
integral((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x, x)
```

3.133.6 Sympy [F]

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^p}{x} \, dx = \int \frac{\left((a + bx^3)^2\right)^p}{x} \, dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**p/x,x)
```

output
```
Integral(((a + b*x**3)**2)**p/x, x)
```

3.133.7 Maxima [F]

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^p}{x} \, dx = \int \frac{(b^2x^6 + 2\,abx^3 + a^2)^p}{x} \, dx$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x,x, algorithm="maxima")
```

output
```
integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x, x)
```

3.133.8 Giac [F]

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^p}{x} \, dx = \int \frac{(b^2x^6 + 2\,abx^3 + a^2)^p}{x} \, dx$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x,x, algorithm="giac")
```

output
```
integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x, x)
```

3.133.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^p}{x} \, dx = \int \frac{(a^2 + 2\,a\,b\,x^3 + b^2\,x^6)^p}{x} \, dx$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^p/x,x)
```

output
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^p/x, x)
```

3.134 $\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^2}\,dx$

3.134.1 Optimal result

Integrand size = 24, antiderivative size = 58

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^2}\,dx = -\frac{\left(1+\frac{bx^3}{a}\right)^{-2p}(a^2+2abx^3+b^2x^6)^p \operatorname{Hypergeometric2F1}\left(-\frac{1}{3},-2p,\frac{2}{3},-\frac{bx^3}{a}\right)}{x}$$

output
```
-(b^2*x^6+2*a*b*x^3+a^2)^p*hypergeom([-1/3, -2*p],[2/3],-b*x^3/a)/x/((1+b*
x^3/a)^(2*p))
```

3.134.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.84

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^2}\,dx = -\frac{\left((a+bx^3)^2\right)^p\left(1+\frac{bx^3}{a}\right)^{-2p} \operatorname{Hypergeometric2F1}\left(-\frac{1}{3},-2p,\frac{2}{3},-\frac{bx^3}{a}\right)}{x}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^p/x^2,x]
```

output
```
-((((a + b*x^3)^2)^p*Hypergeometric2F1[-1/3, -2*p, 2/3, -((b*x^3)/a)])/(x*
(1 + (b*x^3)/a)^(2*p)))
```

3.134.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1385, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^2} dx$$

$$\Big\downarrow \text{1385}$$

$$\left(\frac{bx^3}{a}+1\right)^{-2p} (a^2+2abx^3+b^2x^6)^p \int \frac{\left(\frac{bx^3}{a}+1\right)^{2p}}{x^2} dx$$

$$\Big\downarrow \text{888}$$

$$-\frac{\left(\frac{bx^3}{a}+1\right)^{-2p} (a^2+2abx^3+b^2x^6)^p \text{Hypergeometric2F1}\left(-\frac{1}{3},-2p,\frac{2}{3},-\frac{bx^3}{a}\right)}{x}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^p/x^2,x]
```

output
```
-(((a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[-1/3, -2*p, 2/3, -((b*x
^3)/a)])/(x*(1 + (b*x^3)/a)^(2*p)))
```

3.134.3.1 Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

3.134.4 Maple [F]

$$\int \frac{(b^2x^6 + 2ab\,x^3 + a^2)^p}{x^2} dx$$

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^p/x^2,x)
```

output
```
int((b^2*x^6+2*a*b*x^3+a^2)^p/x^2,x)
```

3.134.5 Fricas [F]

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^p}{x^2} dx = \int \frac{(b^2x^6 + 2\,abx^3 + a^2)^p}{x^2} dx$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x^2,x, algorithm="fricas")
```

output
```
integral((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^2, x)
```

3.134.6 Sympy [F]

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^p}{x^2} dx = \int \frac{\left((a + bx^3)^2\right)^p}{x^2} dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**p/x**2,x)
```

output
```
Integral(((a + b*x**3)**2)**p/x**2, x)
```

3.134.7 Maxima [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^2}\,dx = \int \frac{(b^2x^6+2\,abx^3+a^2)^p}{x^2}\,dx$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x^2,x, algorithm="maxima")
```

output
```
integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^2, x)
```

3.134.8 Giac [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^2}\,dx = \int \frac{(b^2x^6+2\,abx^3+a^2)^p}{x^2}\,dx$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x^2,x, algorithm="giac")
```

output
```
integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^2, x)
```

3.134.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^2}\,dx = \int \frac{(a^2+2\,a\,b\,x^3+b^2\,x^6)^p}{x^2}\,dx$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^p/x^2,x)
```

output
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^p/x^2, x)
```

3.135 $\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^3}\,dx$

3.135.1 Optimal result

Integrand size = 24, antiderivative size = 60

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^3}\,dx = -\frac{\left(1+\frac{bx^3}{a}\right)^{-2p}(a^2+2abx^3+b^2x^6)^p \operatorname{Hypergeometric2F1}\left(-\frac{2}{3},-2p,\frac{1}{3},-\frac{bx^3}{a}\right)}{2x^2}$$

output
```
-1/2*(b^2*x^6+2*a*b*x^3+a^2)^p*hypergeom([-2/3, -2*p],[1/3],-b*x^3/a)/x^2/
((1+b*x^3/a)^(2*p))
```

3.135.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.85

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^3}\,dx = -\frac{\left((a+bx^3)^2\right)^p\left(1+\frac{bx^3}{a}\right)^{-2p} \operatorname{Hypergeometric2F1}\left(-\frac{2}{3},-2p,\frac{1}{3},-\frac{bx^3}{a}\right)}{2x^2}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^p/x^3,x]
```

output
```
-1/2*(((a + b*x^3)^2)^p*Hypergeometric2F1[-2/3, -2*p, 1/3, -((b*x^3)/a)])/
(x^2*(1 + (b*x^3)/a)^(2*p))
```

3.135.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1385, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^3}\,dx$$

$$\Big\downarrow \text{1385}$$

$$\left(\frac{bx^3}{a}+1\right)^{-2p}(a^2+2abx^3+b^2x^6)^p\int \frac{\left(\frac{bx^3}{a}+1\right)^{2p}}{x^3}dx$$

$$\Big\downarrow \text{888}$$

$$-\frac{\left(\frac{bx^3}{a}+1\right)^{-2p}(a^2+2abx^3+b^2x^6)^p\,\text{Hypergeometric2F1}\left(-\frac{2}{3},-2p,\frac{1}{3},-\frac{bx^3}{a}\right)}{2x^2}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^p/x^3,x]
```

output
```
-1/2*((a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[-2/3, -2*p, 1/3, -((
b*x^3)/a)])/(x^2*(1 + (b*x^3)/a)^(2*p))
```

3.135.3.1 Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

3.135.4 Maple [F]

$$\int \frac{(b^2x^6 + 2ab\,x^3 + a^2)^p}{x^3}dx$$

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^p/x^3,x)
```

output
```
int((b^2*x^6+2*a*b*x^3+a^2)^p/x^3,x)
```

3.135.5 Fricas [F]

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^p}{x^3}\,dx = \int \frac{(b^2x^6 + 2\,abx^3 + a^2)^p}{x^3}\,dx$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x^3,x, algorithm="fricas")
```

output
```
integral((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^3, x)
```

3.135.6 Sympy [F]

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^p}{x^3}\,dx = \int \frac{\left((a + bx^3)^2\right)^p}{x^3}\,dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**p/x**3,x)
```

output
```
Integral(((a + b*x**3)**2)**p/x**3, x)
```

3.135.7 Maxima [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^3}\,dx=\int \frac{(b^2x^6+2\,abx^3+a^2)^p}{x^3}\,dx$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x^3,x, algorithm="maxima")
```

output
```
integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^3, x)
```

3.135.8 Giac [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^3}\,dx=\int \frac{(b^2x^6+2\,abx^3+a^2)^p}{x^3}\,dx$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x^3,x, algorithm="giac")
```

output
```
integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^3, x)
```

3.135.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^3}\,dx=\int \frac{(a^2+2\,a\,b\,x^3+b^2\,x^6)^p}{x^3}\,dx$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^p/x^3,x)
```

output
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^p/x^3, x)
```

3.136 $\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^4} dx$

3.136.1 Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^4} dx = \frac{b(a+bx^3)(a^2+2abx^3+b^2x^6)^p \text{Hypergeometric2F1}\left(2,1+2p,2(1+p),1+\frac{bx^3}{a}\right)}{3a^2(1+2p)}$$

output
```
1/3*b*(b*x^3+a)*(b^2*x^6+2*a*b*x^3+a^2)^p*hypergeom([2, 1+2*p],[2+2*p],1+b
*x^3/a)/a^2/(1+2*p)
```

3.136.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.86

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^4} dx = \frac{b(a+bx^3)\left((a+bx^3)^2\right)^p \text{Hypergeometric2F1}\left(2,1+2p,2+2p,1+\frac{bx^3}{a}\right)}{3a^2(1+2p)}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^p/x^4,x]
```

output
```
(b*(a + b*x^3)*((a + b*x^3)^2)^p*Hypergeometric2F1[2, 1 + 2*p, 2 + 2*p, 1
+ (b*x^3)/a])/(3*a^2*(1 + 2*p))
```

3.136.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1385, 798, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^4}\,dx$$

$$\Big\downarrow \ 1385$$

$$\left(\frac{bx^3}{a}+1\right)^{-2p}(a^2+2abx^3+b^2x^6)^p\int \frac{\left(\frac{bx^3}{a}+1\right)^{2p}}{x^4}dx$$

$$\Big\downarrow \ 798$$

$$\frac{1}{3}\left(\frac{bx^3}{a}+1\right)^{-2p}(a^2+2abx^3+b^2x^6)^p\int \frac{\left(\frac{bx^3}{a}+1\right)^{2p}}{x^6}dx^3$$

$$\Big\downarrow \ 75$$

$$\frac{b\left(\frac{bx^3}{a}+1\right)(a^2+2abx^3+b^2x^6)^p\,\text{Hypergeometric2F1}\left(2,2p+1,2(p+1),\frac{bx^3}{a}+1\right)}{3a(2p+1)}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^p/x^4,x]
```

output
```
(b*(1 + (b*x^3)/a)*(a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[2, 1 +
2*p, 2*(1 + p), 1 + (b*x^3)/a])/(3*a*(1 + 2*p))
```

3.136.3.1 Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

3.136.4 Maple [F]

$$\int \frac{(b^2x^6 + 2ab\,x^3 + a^2)^p}{x^4}dx$$

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^p/x^4,x)
```

output
```
int((b^2*x^6+2*a*b*x^3+a^2)^p/x^4,x)
```

3.136.5 Fricas [F]

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^p}{x^4}\,dx = \int \frac{(b^2x^6 + 2\,abx^3 + a^2)^p}{x^4}\,dx$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x^4,x, algorithm="fricas")
```

output
```
integral((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^4, x)
```

3.136.6 Sympy [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^4}\,dx = \int \frac{\left((a+bx^3)^2\right)^p}{x^4}\,dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**p/x**4,x)
```

output
```
Integral(((a + b*x**3)**2)**p/x**4, x)
```

3.136.7 Maxima [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^4}\,dx = \int \frac{(b^2x^6+2\,abx^3+a^2)^p}{x^4}\,dx$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x^4,x, algorithm="maxima")
```

output
```
integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^4, x)
```

3.136.8 Giac [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^4}\,dx = \int \frac{(b^2x^6+2\,abx^3+a^2)^p}{x^4}\,dx$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x^4,x, algorithm="giac")
```

output
```
integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^4, x)
```

3.136.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^p}{x^4}\,dx = \int \frac{(a^2 + 2\,a\,b\,x^3 + b^2\,x^6)^p}{x^4}\,dx$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^p/x^4,x)
```

output
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^p/x^4, x)
```

3.137 $\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^5} dx$

3.137.1 Optimal result

Integrand size = 24, antiderivative size = 60

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^5} dx = -\frac{\left(1+\frac{bx^3}{a}\right)^{-2p} (a^2+2abx^3+b^2x^6)^p \text{Hypergeometric2F1}\left(-\frac{4}{3},-2p,-\frac{1}{3},-\frac{bx^3}{a}\right)}{4x^4}$$

output
```
-1/4*(b^2*x^6+2*a*b*x^3+a^2)^p*hypergeom([-4/3, -2*p],[-1/3],-b*x^3/a)/x^4
/((1+b*x^3/a)^(2*p))
```

3.137.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.85

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^5} dx = -\frac{\left((a+bx^3)^2\right)^p \left(1+\frac{bx^3}{a}\right)^{-2p} \text{Hypergeometric2F1}\left(-\frac{4}{3},-2p,-\frac{1}{3},-\frac{bx^3}{a}\right)}{4x^4}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^p/x^5,x]
```

output
```
-1/4*(((a + b*x^3)^2)^p*Hypergeometric2F1[-4/3, -2*p, -1/3, -((b*x^3)/a)])
/(x^4*(1 + (b*x^3)/a)^(2*p))
```

3.137.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1385, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^5}\,dx$$

$$\Big\downarrow \text{1385}$$

$$\left(\frac{bx^3}{a}+1\right)^{-2p}(a^2+2abx^3+b^2x^6)^p\int \frac{\left(\frac{bx^3}{a}+1\right)^{2p}}{x^5}dx$$

$$\Big\downarrow \text{888}$$

$$-\frac{\left(\frac{bx^3}{a}+1\right)^{-2p}(a^2+2abx^3+b^2x^6)^p\,\text{Hypergeometric2F1}\left(-\frac{4}{3},-2p,-\frac{1}{3},-\frac{bx^3}{a}\right)}{4x^4}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^p/x^5,x]
```

output
```
-1/4*((a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[-4/3, -2*p, -1/3, -(
(b*x^3)/a)])/(x^4*(1 + (b*x^3)/a)^(2*p))
```

3.137.3.1 Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

3.137.4 Maple [F]

$$\int \frac{(b^2x^6 + 2ab\,x^3 + a^2)^p}{x^5}dx$$

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^p/x^5,x)
```

output
```
int((b^2*x^6+2*a*b*x^3+a^2)^p/x^5,x)
```

3.137.5 Fricas [F]

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^p}{x^5}\,dx = \int \frac{(b^2x^6 + 2\,abx^3 + a^2)^p}{x^5}\,dx$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x^5,x, algorithm="fricas")
```

output
```
integral((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^5, x)
```

3.137.6 Sympy [F]

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^p}{x^5}\,dx = \int \frac{\left((a + bx^3)^2\right)^p}{x^5}\,dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**p/x**5,x)
```

output
```
Integral(((a + b*x**3)**2)**p/x**5, x)
```

3.137.7 Maxima [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^5}\,dx=\int \frac{(b^2x^6+2\,abx^3+a^2)^p}{x^5}\,dx$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x^5,x, algorithm="maxima")
```

output
```
integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^5, x)
```

3.137.8 Giac [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^5}\,dx=\int \frac{(b^2x^6+2\,abx^3+a^2)^p}{x^5}\,dx$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x^5,x, algorithm="giac")
```

output
```
integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^5, x)
```

3.137.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^5}\,dx=\int \frac{(a^2+2\,a\,b\,x^3+b^2\,x^6)^p}{x^5}\,dx$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^p/x^5,x)
```

output
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^p/x^5, x)
```

3.138 $\int \frac{x^8}{a+bx^3+cx^6} \, dx$

3.138.1 Optimal result

Integrand size = 18, antiderivative size = 81

$$\int \frac{x^8}{a+bx^3+cx^6} \, dx = \frac{x^3}{3c} - \frac{(b^2-2ac)\,\text{arctanh}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c^2\sqrt{b^2-4ac}} - \frac{b\log\left(a+bx^3+cx^6\right)}{6c^2}$$

output
```
1/3*x^3/c-1/6*b*ln(c*x^6+b*x^3+a)/c^2-1/3*(-2*a*c+b^2)*arctanh((2*c*x^3+b)
/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(1/2)
```

3.138.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.96

$$\int \frac{x^8}{a+bx^3+cx^6} \, dx = \frac{2cx^3 + \frac{2(b^2-2ac)\arctan\left(\frac{b+2cx^3}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}} - b\log\left(a+bx^3+cx^6\right)}{6c^2}$$

input
```
Integrate[x^8/(a + b*x^3 + c*x^6),x]
```

output
```
(2*c*x^3 + (2*(b^2 - 2*a*c)*ArcTan[(b + 2*c*x^3)/Sqrt[-b^2 + 4*a*c]])/Sqrt
[-b^2 + 4*a*c] - b*Log[a + b*x^3 + c*x^6])/(6*c^2)
```

3.138.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1693, 1143, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{a+bx^3+cx^6}\,dx$$

$$\Big\downarrow \text{ 1693}$$

$$\frac{1}{3}\int \frac{x^6}{cx^6+bx^3+a}dx^3$$

$$\Big\downarrow \text{ 1143}$$

$$\frac{1}{3}\int \left(\frac{1}{c}-\frac{bx^3+a}{c\,(cx^6+bx^3+a)}\right)dx^3$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}\left(-\frac{(b^2-2ac)\,\text{arctanh}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx^3+cx^6\right)}{2c^2}+\frac{x^3}{c}\right)$$

input
```
Int[x^8/(a + b*x^3 + c*x^6),x]
```

output
```
(x^3/c - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(c^2*Sqr
t[b^2 - 4*a*c]) - (b*Log[a + b*x^3 + c*x^6])/(2*c^2))/3
```

3.138.3.1 Defintions of rubi rules used

rule 1143
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x)^m/(a + b*x + c*x^2), x], x] /; FreeQ[{a,
b, c, d, e}, x] && IGtQ[m, 1]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.138.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.02

method	result
default	$\frac{x^3}{3c}+\frac{-\frac{b\ln\left(c\,x^6+b\,x^3+a\right)}{2c}+\frac{2\left(-a+\frac{b^2}{2c}\right)\arctan\left(\frac{2c\,x^3+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{3c}$
risch	$\frac{x^3}{3c}-\frac{2\ln\left(\left(\left(-8a^2c^2+6a\,b^2c-b^4+\sqrt{-(4ac-b^2)(2ac-b^2)^2}\,b\right)x^3+2\sqrt{-(4ac-b^2)(2ac-b^2)^2}\,a\right)ab}{3c(4ac-b^2)}+\frac{\ln\left(\left(-8a^2c^2+6a\,b^2c-b^4+\sqrt{-}\right.\right.}{}$

input
```
int(x^8/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3/c+1/3/c*(-1/2*b/c*ln(c*x^6+b*x^3+a)+2*(-a+1/2/c*b^2)/(4*a*c-b^2)^(
1/2)*arctan((2*c*x^3+b)/(4*a*c-b^2)^(1/2)))
```

3.138.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 254, normalized size of antiderivative = 3.14

$$\int\frac{x^8}{a+bx^3+cx^6}\,dx$$
$$=\left[\frac{2\,(b^2c-4\,ac^2)x^3-(b^2-2\,ac)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^6+2\,bcx^3+b^2-2\,ac+(2\,cx^3+b)\sqrt{b^2-4\,ac}}{cx^6+bx^3+a}\right)-(b^3-4\,abc)\log\left(cx^6\right.}{6\,(b^2c^2-4\,ac^3)}\right.$$

input
```
integrate(x^8/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

output
```
[1/6*(2*(b^2*c - 4*a*c^2)*x^3 - (b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2
*x^6 + 2*b*c*x^3 + b^2 - 2*a*c + (2*c*x^3 + b)*sqrt(b^2 - 4*a*c))/(c*x^6 +
 b*x^3 + a)) - (b^3 - 4*a*b*c)*log(c*x^6 + b*x^3 + a))/(b^2*c^2 - 4*a*c^3)
, 1/6*(2*(b^2*c - 4*a*c^2)*x^3 - 2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*arctan
(-(2*c*x^3 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^3 - 4*a*b*c)*log(c*
x^6 + b*x^3 + a))/(b^2*c^2 - 4*a*c^3)]
```

3.138.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. 2(75) = 150.

Time = 1.72 (sec) , antiderivative size = 316, normalized size of antiderivative = 3.90

$$\int \frac{x^8}{a+bx^3+cx^6}\,dx = \left(-\frac{b}{6c^2} - \frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{6c^2\cdot(4ac-b^2)}\right)\log\left(x^3 + \frac{-ab-12ac^2\left(-\frac{b}{6c^2}-\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{6c^2\cdot(4ac-b^2)}\right)+3b^2c\left(-\frac{b}{6c^2}-\frac{\sqrt{-4ac+b^2}\cdot(}{6c^2\cdot(4ac}\right.}{2ac-b^2}\right. + \left(-\frac{b}{6c^2} + \frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{6c^2\cdot(4ac-b^2)}\right)\log\left(x^3 + \frac{-ab-12ac^2\left(-\frac{b}{6c^2}+\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{6c^2\cdot(4ac-b^2)}\right)+3b^2c\left(-\frac{b}{6c^2}+\frac{\sqrt{-4ac+b^2}\cdot(}{6c^2\cdot(4ac}\right.}{2ac-b^2}\right. + \frac{x^3}{3c}$$

input
```
integrate(x**8/(c*x**6+b*x**3+a),x)
```

output
```
(-b/(6*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(6*c**2*(4*a*c - b**2)))
*log(x**3 + (-a*b - 12*a*c**2*(-b/(6*c**2) - sqrt(-4*a*c + b**2)*(2*a*c -
b**2)/(6*c**2*(4*a*c - b**2))) + 3*b**2*c*(-b/(6*c**2) - sqrt(-4*a*c + b**
2)*(2*a*c - b**2)/(6*c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + (-b/(6*c**2)
 + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(6*c**2*(4*a*c - b**2)))*log(x**3 +
(-a*b - 12*a*c**2*(-b/(6*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(6*c**
2*(4*a*c - b**2))) + 3*b**2*c*(-b/(6*c**2) + sqrt(-4*a*c + b**2)*(2*a*c -
b**2)/(6*c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + x**3/(3*c)
```

3.138.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^8}{a+bx^3+cx^6}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^8/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.138.8 Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.93

$$\int \frac{x^8}{a+bx^3+cx^6}\,dx = \frac{x^3}{3\,c} - \frac{b\log\left(cx^6+bx^3+a\right)}{6\,c^2} + \frac{\left(b^2-2\,ac\right)\arctan\left(\frac{2\,cx^3+b}{\sqrt{-b^2+4\,ac}}\right)}{3\,\sqrt{-b^2+4\,ac}c^2}$$

input
```
integrate(x^8/(c*x^6+b*x^3+a),x, algorithm="giac")
```

output
```
1/3*x^3/c - 1/6*b*log(c*x^6 + b*x^3 + a)/c^2 + 1/3*(b^2 - 2*a*c)*arctan((2
*c*x^3 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

3.138.9 Mupad [B] (verification not implemented)

Time = 8.89 (sec) , antiderivative size = 1758, normalized size of antiderivative = 21.70

$$\int \frac{x^8}{a+bx^3+cx^6}\,dx = \text{Too large to display}$$

input
```
int(x^8/(a + b*x^3 + c*x^6),x)
```

output
```
x^3/(3*c) + (log(a + b*x^3 + c*x^6)*(3*b^3 - 12*a*b*c))/(2*(36*a*c^3 - 9*b
^2*c^2)) + (atan((4*c^3*x^3*(4*a*c - b^2)^(3/2)*((b*((b^5 + a^2*b*c^2 - 2*
a*b^3*c)/c^3 + ((3*b^3 - 12*a*b*c)*((6*a^2*c^4 + 12*b^4*c^2 - 18*a*b^2*c^3
)/c^3 + ((3*b^3 - 12*a*b*c)*((45*b^3*c^4 - 36*a*b*c^5)/c^3 + (27*b^2*c^3*(
3*b^3 - 12*a*b*c))/(36*a*c^3 - 9*b^2*c^2)))/(2*(36*a*c^3 - 9*b^2*c^2))))/(
2*(36*a*c^3 - 9*b^2*c^2)) - ((((2*a*c - b^2)*((45*b^3*c^4 - 36*a*b*c^5)/c^
3 + (27*b^2*c^3*(3*b^3 - 12*a*b*c))/(36*a*c^3 - 9*b^2*c^2)))/(6*c^2*(4*a*c
 - b^2)^(1/2)) + (9*b^2*c*(3*b^3 - 12*a*b*c)*(2*a*c - b^2))/(2*(4*a*c - b^
2)^(1/2)*(36*a*c^3 - 9*b^2*c^2)))*(2*a*c - b^2))/(6*c^2*(4*a*c - b^2)^(1/2
)) - (3*b^2*(3*b^3 - 12*a*b*c)*(2*a*c - b^2)^2)/(4*c*(4*a*c - b^2)*(36*a*c
^3 - 9*b^2*c^2))))/(4*a^2*c) + ((2*a*c - b^2)*(((3*b^3 - 12*a*b*c)*(((2*a*
c - b^2)*((45*b^3*c^4 - 36*a*b*c^5)/c^3 + (27*b^2*c^3*(3*b^3 - 12*a*b*c))/
(36*a*c^3 - 9*b^2*c^2)))/(6*c^2*(4*a*c - b^2)^(1/2)) + (9*b^2*c*(3*b^3 - 1
2*a*b*c)*(2*a*c - b^2))/(2*(4*a*c - b^2)^(1/2)*(36*a*c^3 - 9*b^2*c^2))))/(
2*(36*a*c^3 - 9*b^2*c^2)) - (b^2*(2*a*c - b^2)^3)/(4*c^3*(4*a*c - b^2)^(3/
2)) + (((6*a^2*c^4 + 12*b^4*c^2 - 18*a*b^2*c^3)/c^3 + ((3*b^3 - 12*a*b*c)*
((45*b^3*c^4 - 36*a*b*c^5)/c^3 + (27*b^2*c^3*(3*b^3 - 12*a*b*c))/(36*a*c^3
 - 9*b^2*c^2)))/(2*(36*a*c^3 - 9*b^2*c^2)))*(2*a*c - b^2))/(6*c^2*(4*a*c -
 b^2)^(1/2))))/(4*a^2*c*(4*a*c - b^2)^(1/2))))/(b^6 - 8*a^3*c^3 + 12*a^2*b
^2*c^2 - 6*a*b^4*c) - (c^2*(2*a*c - b^2)*(4*a*c - b^2)*(((3*b^3 - 12*a*...
```

3.138. $\int \frac{x^8}{a+bx^3+cx^6} dx$

3.139 $\int \frac{x^5}{a+bx^3+cx^6} dx$

3.139.1 Optimal result

Integrand size = 18, antiderivative size = 63

$$\int \frac{x^5}{a+bx^3+cx^6} dx = \frac{b\text{arctanh}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c\sqrt{b^2-4ac}} + \frac{\log(a+bx^3+cx^6)}{6c}$$

output
```
1/6*ln(c*x^6+b*x^3+a)/c+1/3*b*arctanh((2*c*x^3+b)/(-4*a*c+b^2)^(1/2))/c/(-
4*a*c+b^2)^(1/2)
```

3.139.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.98

$$\int \frac{x^5}{a+bx^3+cx^6} dx = \frac{-\frac{2b\arctan\left(\frac{b+2cx^3}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}} + \log(a+bx^3+cx^6)}{6c}$$

input
```
Integrate[x^5/(a + b*x^3 + c*x^6),x]
```

output
```
((-2*b*ArcTan[(b + 2*c*x^3)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + Log[
a + b*x^3 + c*x^6])/(6*c)
```

3.139.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1693, 1142, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{a+bx^3+cx^6} \, dx$$

$$\Big\downarrow \text{ 1693}$$

$$\frac{1}{3} \int \frac{x^3}{cx^6+bx^3+a}dx^3$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{1}{3} \left(\frac{\int \frac{2cx^3+b}{cx^6+bx^3+a}dx^3}{2c}-\frac{b \int \frac{1}{cx^6+bx^3+a}dx^3}{2c}\right)$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{1}{3} \left(\frac{b \int \frac{1}{-x^6+b^2-4ac}d\left(2cx^3+b\right)}{c}+\frac{\int \frac{2cx^3+b}{cx^6+bx^3+a}dx^3}{2c}\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{3} \left(\frac{\int \frac{2cx^3+b}{cx^6+bx^3+a}dx^3}{2c}+\frac{b \text{arctanh}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}\right)$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{1}{3} \left(\frac{b \text{arctanh}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^3+cx^6\right)}{2c}\right)$$

input
```
Int[x^5/(a + b*x^3 + c*x^6),x]
```

output
```
((b*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 - 4*a*c]) + Log[
a + b*x^3 + c*x^6]/(2*c))/3
```

3.139.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.139.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

method	result
default	$\frac{\ln\left(c\,x^{6}+b\,x^{3}+a\right)}{6c}-\frac{b\arctan\left(\frac{2c\,x^{3}+b}{\sqrt{4ac-b^{2}}}\right)}{3c\sqrt{4ac-b^{2}}}$
risch	$\frac{2\ln\left(\left(-4abc+b^{3}+\sqrt{-b^{2}\left(4ac-b^{2}\right)}\,b\right)x^{3}+2\sqrt{-b^{2}\left(4ac-b^{2}\right)}\,a\right)a}{3\left(4ac-b^{2}\right)}-\frac{\ln\left(\left(-4abc+b^{3}+\sqrt{-b^{2}\left(4ac-b^{2}\right)}\,b\right)x^{3}+2\sqrt{-b^{2}\left(4ac-b^{2}\right)}\,a\right)b^{2}}{6c\left(4ac-b^{2}\right)}+$

input
```
int(x^5/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/6*ln(c*x^6+b*x^3+a)/c-1/3*b/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x^3+b)/(4*a*
c-b^2)^(1/2))
```

3.139.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 197, normalized size of antiderivative = 3.13

$$\int \frac{x^5}{a+bx^3+cx^6}\,dx$$
$$= \left[\frac{\sqrt{b^2-4\,ac}b\log\left(\frac{2\,c^2x^6+2\,bcx^3+b^2-2\,ac+(2\,cx^3+b)\sqrt{b^2-4\,ac}}{cx^6+bx^3+a}\right)+(b^2-4\,ac)\log\left(cx^6+bx^3+a\right)}{6\,(b^2c-4\,ac^2)},\ \frac{2\,\sqrt{-b^2+4\,ac}b\,\mathrm{ar}}{}\right.$$

input
```
integrate(x^5/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

output
```
[1/6*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^6 + 2*b*c*x^3 + b^2 - 2*a*c + (2*c*
x^3 + b)*sqrt(b^2 - 4*a*c))/(c*x^6 + b*x^3 + a)) + (b^2 - 4*a*c)*log(c*x^6
 + b*x^3 + a))/(b^2*c - 4*a*c^2), 1/6*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-(2*c
*x^3 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^2 - 4*a*c)*log(c*x^6 + b*
x^3 + a))/(b^2*c - 4*a*c^2)]
```

3.139.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. 2(54) = 108.

Time = 0.84 (sec) , antiderivative size = 223, normalized size of antiderivative = 3.54

$$\int \frac{x^5}{a+bx^3+cx^6}\,dx = \left(-\frac{b\sqrt{-4ac+b^2}}{6c\,(4ac-b^2)}+\frac{1}{6c}\right)\log\left(x^3+\frac{-12ac\left(-\frac{b\sqrt{-4ac+b^2}}{6c(4ac-b^2)}+\frac{1}{6c}\right)+2a+3b^2\left(-\frac{b\sqrt{-4ac+b^2}}{6c(4ac-b^2)}+\frac{1}{6c}\right)}{b}\right) + \left(\frac{b\sqrt{-4ac+b^2}}{6c\,(4ac-b^2)}+\frac{1}{6c}\right)\log\left(x^3+\frac{-12ac\left(\frac{b\sqrt{-4ac+b^2}}{6c(4ac-b^2)}+\frac{1}{6c}\right)+2a+3b^2\left(\frac{b\sqrt{-4ac+b^2}}{6c(4ac-b^2)}+\frac{1}{6c}\right)}{b}\right)$$

input
```
integrate(x**5/(c*x**6+b*x**3+a),x)
```

output
```
(-b*sqrt(-4*a*c + b**2)/(6*c*(4*a*c - b**2)) + 1/(6*c))*log(x**3 + (-12*a*
c*(-b*sqrt(-4*a*c + b**2)/(6*c*(4*a*c - b**2)) + 1/(6*c)) + 2*a + 3*b**2*(
-b*sqrt(-4*a*c + b**2)/(6*c*(4*a*c - b**2)) + 1/(6*c)))/b) + (b*sqrt(-4*a*
c + b**2)/(6*c*(4*a*c - b**2)) + 1/(6*c))*log(x**3 + (-12*a*c*(b*sqrt(-4*a
*c + b**2)/(6*c*(4*a*c - b**2)) + 1/(6*c)) + 2*a + 3*b**2*(b*sqrt(-4*a*c +
 b**2)/(6*c*(4*a*c - b**2)) + 1/(6*c)))/b)
```

3.139.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{a+bx^3+cx^6}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.139.8 Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.94

$$\int \frac{x^5}{a+bx^3+cx^6}\,dx = -\frac{b\arctan\left(\frac{2\,cx^3+b}{\sqrt{-b^2+4\,ac}}\right)}{3\,\sqrt{-b^2+4\,ac}c} + \frac{\log\left(cx^6+bx^3+a\right)}{6\,c}$$

input
```
integrate(x^5/(c*x^6+b*x^3+a),x, algorithm="giac")
```

output
```
-1/3*b*arctan((2*c*x^3 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c) + 1
/6*log(c*x^6 + b*x^3 + a)/c
```

3.139.9 Mupad [B] (verification not implemented)

Time = 8.64 (sec) , antiderivative size = 1199, normalized size of antiderivative = 19.03

$$\int \frac{x^5}{a+bx^3+cx^6}\,dx = \text{Too large to display}$$

input
```
int(x^5/(a + b*x^3 + c*x^6),x)
```

output
```
(log(a + b*x^3 + c*x^6)*(12*a*c - 3*b^2))/(2*(36*a*c^2 - 9*b^2*c)) + (b*at
an((4*x^3*((b*(b^2 - ((12*b^2*c - ((45*b^2*c^2 - (27*b^2*c^3*(12*a*c - 3*b
^2))/(36*a*c^2 - 9*b^2*c))*(12*a*c - 3*b^2))/(2*(36*a*c^2 - 9*b^2*c)))*(12
*a*c - 3*b^2))/(2*(36*a*c^2 - 9*b^2*c)) - (b*((b*(45*b^2*c^2 - (27*b^2*c^3
*(12*a*c - 3*b^2))/(36*a*c^2 - 9*b^2*c)))/(6*c*(4*a*c - b^2)^(1/2)) - (9*b
^3*c^2*(12*a*c - 3*b^2))/(2*(36*a*c^2 - 9*b^2*c)*(4*a*c - b^2)^(1/2))))/(6
*c*(4*a*c - b^2)^(1/2)) + (3*b^4*c*(12*a*c - 3*b^2))/(4*(36*a*c^2 - 9*b^2*
c)*(4*a*c - b^2))))/(4*a^2*c) + ((2*a*c - b^2)*(b^5/(4*(4*a*c - b^2)^(3/2)
) + ((12*a*c - 3*b^2)*((b*(45*b^2*c^2 - (27*b^2*c^3*(12*a*c - 3*b^2))/(36*
a*c^2 - 9*b^2*c)))/(6*c*(4*a*c - b^2)^(1/2)) - (9*b^3*c^2*(12*a*c - 3*b^2)
)/(2*(36*a*c^2 - 9*b^2*c)*(4*a*c - b^2)^(1/2))))/(2*(36*a*c^2 - 9*b^2*c))
- (b*(12*b^2*c - ((45*b^2*c^2 - (27*b^2*c^3*(12*a*c - 3*b^2))/(36*a*c^2 -
9*b^2*c))*(12*a*c - 3*b^2))/(2*(36*a*c^2 - 9*b^2*c))))/(6*c*(4*a*c - b^2)^
(1/2))))/(4*a^2*c*(4*a*c - b^2)^(1/2)))*(4*a*c - b^2)^(3/2))/b^3 + ((4*a*c
 - b^2)^(3/2)*(a*b + ((((72*a*b*c^2 - (54*a*b*c^3*(12*a*c - 3*b^2))/(36*a*
c^2 - 9*b^2*c))*(12*a*c - 3*b^2))/(2*(36*a*c^2 - 9*b^2*c)) - 15*a*b*c)*(12
*a*c - 3*b^2))/(2*(36*a*c^2 - 9*b^2*c)) - (b*((b*(72*a*b*c^2 - (54*a*b*c^3
*(12*a*c - 3*b^2))/(36*a*c^2 - 9*b^2*c)))/(6*c*(4*a*c - b^2)^(1/2)) - (9*a
*b^2*c^2*(12*a*c - 3*b^2))/((36*a*c^2 - 9*b^2*c)*(4*a*c - b^2)^(1/2))))/(6
*c*(4*a*c - b^2)^(1/2)) + (3*a*b^3*c*(12*a*c - 3*b^2))/(2*(36*a*c^2 - 9...
```

3.140 $\int \frac{x^2}{a+bx^3+cx^6}\,dx$

3.140.1 Optimal result

Integrand size = 18, antiderivative size = 38

$$\int \frac{x^2}{a+bx^3+cx^6}\,dx = -\frac{2\mathrm{arctanh}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3\sqrt{b^2-4ac}}$$

output
```
-2/3*arctanh((2*c*x^3+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)
```

3.140.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{a+bx^3+cx^6}\,dx = \frac{2\arctan\left(\frac{b+2cx^3}{\sqrt{-b^2+4ac}}\right)}{3\sqrt{-b^2+4ac}}$$

input
```
Integrate[x^2/(a + b*x^3 + c*x^6),x]
```

output
```
(2*ArcTan[(b + 2*c*x^3)/Sqrt[-b^2 + 4*a*c]])/(3*Sqrt[-b^2 + 4*a*c])
```

3.140.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1690, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{a+bx^3+cx^6}\,dx$$

$$\Big\downarrow \text{1690}$$

$$\frac{1}{3}\int \frac{1}{cx^6+bx^3+a}dx^3$$

$$\Big\downarrow \text{1083}$$

$$-\frac{2}{3}\int \frac{1}{-x^6+b^2-4ac}d\left(2cx^3+b\right)$$

$$\Big\downarrow \text{219}$$

$$-\frac{2\text{arctanh}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3\sqrt{b^2-4ac}}$$

input
```
Int[x^2/(a + b*x^3 + c*x^6),x]
```

output
```
(-2*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(3*Sqrt[b^2 - 4*a*c])
```

3.140.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

3.140.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{2\arctan\left(\frac{2c\,x^{3}+b}{\sqrt{4ac-b^{2}}}\right)}{3\sqrt{4ac-b^{2}}}$	37
risch	$-\frac{\ln\left(\left(-b+\sqrt{-4ac+b^{2}}\right)x^{3}-2a\right)}{3\sqrt{-4ac+b^{2}}}+\frac{\ln\left(\left(b+\sqrt{-4ac+b^{2}}\right)x^{3}+2a\right)}{3\sqrt{-4ac+b^{2}}}$	70

input
```
int(x^2/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
2/3/(4*a*c-b^2)^(1/2)*arctan((2*c*x^3+b)/(4*a*c-b^2)^(1/2))
```

3.140.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 129, normalized size of antiderivative = 3.39

$$\int \frac{x^2}{a+bx^3+cx^6}\,dx = \left[\frac{\log\left(\frac{2\,c^2x^6+2\,bcx^3+b^2-2\,ac-(2\,cx^3+b)\sqrt{b^2-4\,ac}}{cx^6+bx^3+a}\right)}{3\,\sqrt{b^2-4\,ac}},\ -\frac{2\,\sqrt{-b^2+4\,ac}\arctan\left(-\frac{(2\,cx^3+b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)}{3\,(b^2-4\,ac)}\right]$$

input
```
integrate(x^2/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

output
```
[1/3*log((2*c^2*x^6 + 2*b*c*x^3 + b^2 - 2*a*c - (2*c*x^3 + b)*sqrt(b^2 - 4
*a*c))/(c*x^6 + b*x^3 + a))/sqrt(b^2 - 4*a*c), -2/3*sqrt(-b^2 + 4*a*c)*arc
tan(-(2*c*x^3 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c))/(b^2 - 4*a*c)]
```

3.140.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(37) = 74$.

Time = 0.38 (sec) , antiderivative size = 131, normalized size of antiderivative = 3.45

$$\int \frac{x^2}{a+bx^3+cx^6}\,dx = -\frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(x^3+\frac{-4ac\sqrt{-\frac{1}{4ac-b^2}}+b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)}{3} + \frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(x^3+\frac{4ac\sqrt{-\frac{1}{4ac-b^2}}-b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)}{3}$$

input
```
integrate(x**2/(c*x**6+b*x**3+a),x)
```

output
```
-sqrt(-1/(4*a*c - b**2))*log(x**3 + (-4*a*c*sqrt(-1/(4*a*c - b**2)) + b**2
*sqrt(-1/(4*a*c - b**2)) + b)/(2*c))/3 + sqrt(-1/(4*a*c - b**2))*log(x**3
+ (4*a*c*sqrt(-1/(4*a*c - b**2)) - b**2*sqrt(-1/(4*a*c - b**2)) + b)/(2*c)
)/3
```

3.140.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{a+bx^3+cx^6}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.140.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

$$\int \frac{x^2}{a+bx^3+cx^6}\,dx = \frac{2\arctan\left(\frac{2cx^3+b}{\sqrt{-b^2+4ac}}\right)}{3\sqrt{-b^2+4ac}}$$

input
```
integrate(x^2/(c*x^6+b*x^3+a),x, algorithm="giac")
```

output
```
2/3*arctan((2*c*x^3 + b)/sqrt(-b^2 + 4*a*c))/sqrt(-b^2 + 4*a*c)
```

3.140.9 Mupad [B] (verification not implemented)

Time = 8.25 (sec) , antiderivative size = 174, normalized size of antiderivative = 4.58

$$\int \frac{x^2}{a+bx^3+cx^6}\,dx = -\frac{2\,\mathrm{atan}\left(\frac{\frac{x^3\,{\left(4\,a\,c-b^2\right)}^4}{2}+a\,b\,{\left(4\,a\,c-b^2\right)}^3+a\,b^3\,{\left(4\,a\,c-b^2\right)}^2+b^2\,x^3\,{\left(4\,a\,c-b^2\right)}^3+\frac{b^4\,x^3\,{\left(4\,a\,c-b^2\right)}^2}{2}}{b^2\,\left(32\,a^3\,c^2\,\sqrt{4\,a\,c-b^2}-4\,a^2\,b^2\,c\,\sqrt{4\,a\,c-b^2}\right)-64\,a^4\,c^3\,\sqrt{4\,a\,c-b^2}}\right)}{3\,\sqrt{4\,a\,c-b^2}}$$

input
```
int(x^2/(a + b*x^3 + c*x^6),x)
```

output
```
-(2*atan(((x^3*(4*a*c - b^2)^4)/2 + a*b*(4*a*c - b^2)^3 + a*b^3*(4*a*c - b
^2)^2 + b^2*x^3*(4*a*c - b^2)^3 + (b^4*x^3*(4*a*c - b^2)^2)/2)/(b^2*(32*a^
3*c^2*(4*a*c - b^2)^(1/2) - 4*a^2*b^2*c*(4*a*c - b^2)^(1/2)) - 64*a^4*c^3*
(4*a*c - b^2)^(1/2))))/(3*(4*a*c - b^2)^(1/2))
```

3.141 $\int \frac{1}{x(a+bx^3+cx^6)}\,dx$

3.141.1 Optimal result

Integrand size = 18, antiderivative size = 69

$$\int \frac{1}{x\left(a+bx^3+cx^6\right)}\,dx=\frac{b\operatorname{arctanh}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a\sqrt{b^2-4ac}}+\frac{\log(x)}{a}-\frac{\log\left(a+bx^3+cx^6\right)}{6a}$$

output
```
ln(x)/a-1/6*ln(c*x^6+b*x^3+a)/a+1/3*b*arctanh((2*c*x^3+b)/(-4*a*c+b^2)^(1/
2))/a/(-4*a*c+b^2)^(1/2)
```

3.141.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int \frac{1}{x\left(a+bx^3+cx^6\right)}\,dx=\frac{\log(x)}{a}-\frac{\operatorname{RootSum}\left[a+b\#1^3+c\#1^6\&,\frac{b\log(x-\#1)+c\log(x-\#1)\#1^3}{b+2c\#1^3}\&\right]}{3a}$$

input
```
Integrate[1/(x*(a + b*x^3 + c*x^6)),x]
```

output
```
Log[x]/a - RootSum[a + b*#1^3 + c*#1^6 & , (b*Log[x - #1] + c*Log[x - #1]*
#1^3)/(b + 2*c*#1^3) & ]/(3*a)
```

3.141.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1693, 1144, 25, 1142, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+bx^3+cx^6\right)}dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int \frac{1}{x^3\left(cx^6+bx^3+a\right)}dx^3$$

$$\Big\downarrow \text{1144}$$

$$\frac{1}{3}\left(\frac{\int -\frac{cx^3+b}{cx^6+bx^3+a}dx^3}{a}+\frac{\log\left(x^3\right)}{a}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\left(\frac{\log\left(x^3\right)}{a}-\frac{\int \frac{cx^3+b}{cx^6+bx^3+a}dx^3}{a}\right)$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{3}\left(\frac{\log\left(x^3\right)}{a}-\frac{\frac{1}{2}b\int \frac{1}{cx^6+bx^3+a}dx^3+\frac{1}{2}\int \frac{2cx^3+b}{cx^6+bx^3+a}dx^3}{a}\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{3}\left(\frac{\log\left(x^3\right)}{a}-\frac{\frac{1}{2}\int \frac{2cx^3+b}{cx^6+bx^3+a}dx^3-b\int \frac{1}{-x^6+b^2-4ac}d\left(2cx^3+b\right)}{a}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{\log\left(x^3\right)}{a}-\frac{\frac{1}{2}\int \frac{2cx^3+b}{cx^6+bx^3+a}dx^3-\frac{b\,\text{arctanh}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}}{a}\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{3}\left(\frac{\log\left(x^3\right)}{a}-\frac{\frac{1}{2}\log\left(a+bx^3+cx^6\right)-\frac{b\operatorname{arctanh}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}}{a}\right)$$

input
```
Int[1/(x*(a + b*x^3 + c*x^6)),x]
```

output
```
(Log[x^3]/a - (-((b*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4
*a*c]) + Log[a + b*x^3 + c*x^6]/2)/a)/3
```

3.141.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1144
```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Simp[e*(Log[RemoveContent[d + e*x, x]]/(c*d^2 - b*d*e + a*e^2)), x] + S
imp[1/(c*d^2 - b*d*e + a*e^2)   Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2),
x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.141.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{\ln(x)}{a}-\frac{\frac{\ln\left(c\,x^{6}+b\,x^{3}+a\right)}{2}+\frac{b\arctan\left(\frac{2c\,x^{3}+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{3a}$	65
risch	$\frac{\ln(x)}{a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4c\,a^{2}-b^{2}a\right)\_Z^{2}+\left(4ac-b^{2}\right)\_Z+c\right)}\_R\ln\left(\left(\left(-14ac+4b^{2}\right)\_R-7c\right)x^{3}+ab\_R-3b\right)\right)}{3}$	76

input
```
int(1/x/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
ln(x)/a-1/3/a*(1/2*ln(c*x^6+b*x^3+a)+b/(4*a*c-b^2)^(1/2)*arctan((2*c*x^3+b
)/(4*a*c-b^2)^(1/2)))
```

3.141.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 223, normalized size of antiderivative = 3.23

$$\int \frac{1}{x\left(a+bx^3+cx^6\right)}\,dx$$
$$=\left[\frac{\sqrt{b^2-4\,ac}b\log\left(\frac{2\,c^2x^6+2\,bcx^3+b^2-2\,ac+\left(2\,cx^3+b\right)\sqrt{b^2-4\,ac}}{cx^6+bx^3+a}\right)-\left(b^2-4\,ac\right)\log\left(cx^6+bx^3+a\right)+6\left(b^2-4\,ac\right)\log}{6\left(ab^2-4\,a^2c\right)}\right.$$

input
```
integrate(1/x/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

output
```
[1/6*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^6 + 2*b*c*x^3 + b^2 - 2*a*c + (2*c*
x^3 + b)*sqrt(b^2 - 4*a*c))/(c*x^6 + b*x^3 + a)) - (b^2 - 4*a*c)*log(c*x^6
 + b*x^3 + a) + 6*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c), 1/6*(2*sqrt(-b^
2 + 4*a*c)*b*arctan(-(2*c*x^3 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^
2 - 4*a*c)*log(c*x^6 + b*x^3 + a) + 6*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2
*c)]
```

3.141.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(60) = 120$.

Time = 16.91 (sec) , antiderivative size = 253, normalized size of antiderivative = 3.67

$$\begin{aligned}
&\int \frac{1}{x\left(a+bx^3+cx^6\right)}\,dx \\
&= \left(-\frac{b\sqrt{-4ac+b^2}}{6a\left(4ac-b^2\right)} - \frac{1}{6a}\right)\log\left(x^3+\frac{-12a^2c\left(-\frac{b\sqrt{-4ac+b^2}}{6a(4ac-b^2)}-\frac{1}{6a}\right)+3ab^2\left(-\frac{b\sqrt{-4ac+b^2}}{6a(4ac-b^2)}-\frac{1}{6a}\right)-2ac+b^2}{bc}\right) \\
&+ \left(\frac{b\sqrt{-4ac+b^2}}{6a\left(4ac-b^2\right)} - \frac{1}{6a}\right)\log\left(x^3+\frac{-12a^2c\left(\frac{b\sqrt{-4ac+b^2}}{6a(4ac-b^2)}-\frac{1}{6a}\right)+3ab^2\left(\frac{b\sqrt{-4ac+b^2}}{6a(4ac-b^2)}-\frac{1}{6a}\right)-2ac+b^2}{bc}\right) \\
&+ \frac{\log(x)}{a}
\end{aligned}$$

input
```
integrate(1/x/(c*x**6+b*x**3+a),x)
```

output
```
(-b*sqrt(-4*a*c + b**2)/(6*a*(4*a*c - b**2)) - 1/(6*a))*log(x**3 + (-12*a*
*2*c*(-b*sqrt(-4*a*c + b**2)/(6*a*(4*a*c - b**2)) - 1/(6*a)) + 3*a*b**2*(-
b*sqrt(-4*a*c + b**2)/(6*a*(4*a*c - b**2)) - 1/(6*a)) - 2*a*c + b**2)/(b*c
)) + (b*sqrt(-4*a*c + b**2)/(6*a*(4*a*c - b**2)) - 1/(6*a))*log(x**3 + (-1
2*a**2*c*(b*sqrt(-4*a*c + b**2)/(6*a*(4*a*c - b**2)) - 1/(6*a)) + 3*a*b**2
*(b*sqrt(-4*a*c + b**2)/(6*a*(4*a*c - b**2)) - 1/(6*a)) - 2*a*c + b**2)/(b
*c)) + log(x)/a
```

3.141.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(a+bx^3+cx^6\right)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.141.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int \frac{1}{x\left(a+bx^3+cx^6\right)}\,dx = -\frac{b\arctan\left(\frac{2\,cx^3+b}{\sqrt{-b^2+4\,ac}}\right)}{3\,\sqrt{-b^2+4\,ac}a} - \frac{\log\left(cx^6+bx^3+a\right)}{6\,a} + \frac{\log\left(|x|\right)}{a}$$

input
```
integrate(1/x/(c*x^6+b*x^3+a),x, algorithm="giac")
```

output
```
-1/3*b*arctan((2*c*x^3 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a) - 1
/6*log(c*x^6 + b*x^3 + a)/a + log(abs(x))/a
```

3.141.9 Mupad [B] (verification not implemented)

Time = 8.77 (sec) , antiderivative size = 1362, normalized size of antiderivative = 19.74

$$\int \frac{1}{x\left(a+bx^3+cx^6\right)}\,dx = \text{Too large to display}$$

input
```
int(1/(x*(a + b*x^3 + c*x^6)),x)
```

output
```
log(x)/a + (log(a + b*x^3 + c*x^6)*(12*a*c - 3*b^2))/(2*(9*a*b^2 - 36*a^2*
c)) - (b*atan((3*(4*a*c - b^2)^2*(4*b^4 + 7*a^2*c^2 - 15*a*b^2*c)*((b^3*(2
7*b^3*c^3 - (27*a*b^3*c^3*(12*a*c - 3*b^2))/(2*(9*a*b^2 - 36*a^2*c))))/(21
6*a^3*(4*a*c - b^2)^(3/2)) + (9*b^4*c^3*(12*a*c - 3*b^2)^3)/(16*(9*a*b^2 -
 36*a^2*c)^3*(4*a*c - b^2)^(1/2)) - (3*b^6*c^3*(12*a*c - 3*b^2))/(16*a^2*(
9*a*b^2 - 36*a^2*c)*(4*a*c - b^2)^(3/2)) - (b*(12*a*c - 3*b^2)^2*(27*b^3*c
^3 - (27*a*b^3*c^3*(12*a*c - 3*b^2))/(2*(9*a*b^2 - 36*a^2*c))))/(8*a*(9*a*
b^2 - 36*a^2*c)^2*(4*a*c - b^2)^(1/2))))/(b^3*c^6*(49*a*c - 12*b^2)) - (3*
(4*a*c - b^2)^(3/2)*(4*b^5 + 29*a^2*b*c^2 - 23*a*b^3*c)*(((12*a*c - 3*b^2)
^3*(27*b^3*c^3 - (27*a*b^3*c^3*(12*a*c - 3*b^2))/(2*(9*a*b^2 - 36*a^2*c)))
)/(8*(9*a*b^2 - 36*a^2*c)^3) - (b^7*c^3)/(48*a^3*(4*a*c - b^2)^2) - (b^2*(
12*a*c - 3*b^2)*(27*b^3*c^3 - (27*a*b^3*c^3*(12*a*c - 3*b^2))/(2*(9*a*b^2
- 36*a^2*c))))/(24*a^2*(9*a*b^2 - 36*a^2*c)*(4*a*c - b^2)) + (9*b^5*c^3*(1
2*a*c - 3*b^2)^2)/(16*a*(9*a*b^2 - 36*a^2*c)^2*(4*a*c - b^2))))/(b^3*c^6*(
49*a*c - 12*b^2)) + (48*a^4*x^3*(((4*b^4 + 7*a^2*c^2 - 15*a*b^2*c)*((b^3*(
63*b^2*c^4 - ((108*b^4*c^3 - 378*a*b^2*c^4)*(12*a*c - 3*b^2))/(2*(9*a*b^2
- 36*a^2*c))))/(216*a^3*(4*a*c - b^2)^(3/2)) + (b*(108*b^4*c^3 - 378*a*b^2
*c^4)*(12*a*c - 3*b^2)^3)/(48*a*(9*a*b^2 - 36*a^2*c)^3*(4*a*c - b^2)^(1/2)
) - (b^3*(108*b^4*c^3 - 378*a*b^2*c^4)*(12*a*c - 3*b^2))/(144*a^3*(9*a*b^2
 - 36*a^2*c)*(4*a*c - b^2)^(3/2)) - (b*(63*b^2*c^4 - ((108*b^4*c^3 - 37...
```

3.141. $\int \frac{1}{x(a+bx^3+cx^6)}\,dx$

3.142 $\int \frac{1}{x^4(a+bx^3+cx^6)}\,dx$

3.142.1 Optimal result

Integrand size = 18, antiderivative size = 89

$$\int \frac{1}{x^4(a+bx^3+cx^6)}\,dx = -\frac{1}{3ax^3} - \frac{(b^2-2ac)\operatorname{arctanh}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a^2\sqrt{b^2-4ac}} - \frac{b\log(x)}{a^2} + \frac{b\log(a+bx^3+cx^6)}{6a^2}$$

output
```
-1/3/a/x^3-b*ln(x)/a^2+1/6*b*ln(c*x^6+b*x^3+a)/a^2-1/3*(-2*a*c+b^2)*arctan
h((2*c*x^3+b)/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(1/2)
```

3.142.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^4(a+bx^3+cx^6)}\,dx = -\frac{1}{3ax^3} - \frac{b\log(x)}{a^2} + \frac{\operatorname{RootSum}\left[a+b\#1^3+c\#1^6\&,\frac{b^2\log(x-\#1)-ac\log(x-\#1)+bc\log(x-\#1)\#1^3}{b+2c\#1^3}\&\right]}{3a^2}$$

input
```
Integrate[1/(x^4*(a + b*x^3 + c*x^6)),x]
```

output
```
-1/3*1/(a*x^3) - (b*Log[x])/a^2 + RootSum[a + b*#1^3 + c*#1^6 & , (b^2*Log
[x - #1] - a*c*Log[x - #1] + b*c*Log[x - #1]*#1^3)/(b + 2*c*#1^3) & ]/(3*a
^2)
```

3.142.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1693, 1145, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a+bx^3+cx^6)}dx$$

$$\Big\downarrow \text{ 1693}$$

$$\frac{1}{3}\int \frac{1}{x^6 (cx^6+bx^3+a)}dx^3$$

$$\Big\downarrow \text{ 1145}$$

$$\frac{1}{3}\left(\frac{\int -\frac{cx^3+b}{x^3(cx^6+bx^3+a)}dx^3}{a}-\frac{1}{ax^3}\right)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{3}\left(-\frac{\int \frac{cx^3+b}{x^3(cx^6+bx^3+a)}dx^3}{a}-\frac{1}{ax^3}\right)$$

$$\Big\downarrow \text{ 1200}$$

$$\frac{1}{3}\left(-\frac{\int \left(\frac{b}{ax^3}+\frac{-bcx^3-b^2+ac}{a(cx^6+bx^3+a)}\right)dx^3}{a}-\frac{1}{ax^3}\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}\left(-\frac{\frac{(b^2-2ac)\text{arctanh}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}-\frac{b\log(a+bx^3+cx^6)}{2a}+\frac{b\log(x^3)}{a}}{a}-\frac{1}{ax^3}\right)$$

input
```
Int[1/(x^4*(a + b*x^3 + c*x^6)),x]
```

output
```
(-(1/(a*x^3)) - (((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/
(a*Sqrt[b^2 - 4*a*c]) + (b*Log[x^3])/a - (b*Log[a + b*x^3 + c*x^6])/(2*a))
/a)/3
```

3.142.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1145
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp
[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[m, -1]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.142.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.96

method	result
default	$-\frac{1}{3a\,x^{3}}-\frac{b\ln\left(x\right)}{a^{2}}-\frac{-\frac{b\ln\left(c\,x^{6}+b\,x^{3}+a\right)}{2}+\frac{2\left(ac-\frac{b^{2}}{2}\right)\arctan\left(\frac{2c\,x^{3}+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{3a^{2}}$
risch	$-\frac{1}{3a\,x^{3}}-\frac{b\ln\left(x\right)}{a^{2}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4a^{3}c-a^{2}b^{2}\right)\_Z^{2}+\left(-4abc+b^{3}\right)\_Z+c^{2}\right)}\_R\ln\left(\left(\left(-14a^{3}c+4a^{2}b^{2}\right)\_R^{2}+6\_Rabc-3c^{2}\right)x^{3}+\right.\right.}{3}$

input
```
int(1/x^4/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/3/a/x^3-b*ln(x)/a^2-1/3/a^2*(-1/2*b*ln(c*x^6+b*x^3+a)+2*(a*c-1/2*b^2)/(
4*a*c-b^2)^(1/2)*arctan((2*c*x^3+b)/(4*a*c-b^2)^(1/2)))
```

3.142.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 293, normalized size of antiderivative = 3.29

$$\int \frac{1}{x^4\left(a+bx^3+cx^6\right)}\,dx$$
$$=\left[-\frac{(b^2-2\,ac)\sqrt{b^2-4\,ac}x^3\log\left(\frac{2\,c^2x^6+2\,bcx^3+b^2-2\,ac+(2\,cx^3+b)\sqrt{b^2-4\,ac}}{cx^6+bx^3+a}\right)-(b^3-4\,abc)x^3\log\left(cx^6+bx^3+a\right)+}{6\left(a^2b^2-4\,a^3c\right)x^3}\right.$$
$$-\frac{2\,(b^2-2\,ac)\sqrt{-b^2+4\,ac}x^3\arctan\left(-\frac{(2\,cx^3+b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)-(b^3-4\,abc)x^3\log\left(cx^6+bx^3+a\right)+6\,(b^3-}{6\left(a^2b^2-4\,a^3c\right)x^3}$$

input
```
integrate(1/x^4/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

output
```
[-1/6*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*x^3*log((2*c^2*x^6 + 2*b*c*x^3 + b^
2 - 2*a*c + (2*c*x^3 + b)*sqrt(b^2 - 4*a*c))/(c*x^6 + b*x^3 + a)) - (b^3 -
 4*a*b*c)*x^3*log(c*x^6 + b*x^3 + a) + 6*(b^3 - 4*a*b*c)*x^3*log(x) + 2*a*
b^2 - 8*a^2*c)/((a^2*b^2 - 4*a^3*c)*x^3), -1/6*(2*(b^2 - 2*a*c)*sqrt(-b^2
+ 4*a*c)*x^3*arctan(-(2*c*x^3 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^
3 - 4*a*b*c)*x^3*log(c*x^6 + b*x^3 + a) + 6*(b^3 - 4*a*b*c)*x^3*log(x) + 2
*a*b^2 - 8*a^2*c)/((a^2*b^2 - 4*a^3*c)*x^3)]
```

3.142.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a + bx^3 + cx^6)} \, dx = \text{Timed out}$$

input
```
integrate(1/x**4/(c*x**6+b*x**3+a),x)
```

output
```
Timed out
```

3.142.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^4 (a + bx^3 + cx^6)} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x^4/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.142.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^4 (a + bx^3 + cx^6)} \, dx = \frac{b \log (cx^6 + bx^3 + a)}{6 a^2} - \frac{b \log (|x|)}{a^2} + \frac{(b^2 - 2 ac) \arctan \left(\frac{2 cx^3 + b}{\sqrt{-b^2 + 4 ac}}\right)}{3 \sqrt{-b^2 + 4 ac} a^2} + \frac{bx^3 - a}{3 a^2 x^3}$$

input
```
integrate(1/x^4/(c*x^6+b*x^3+a),x, algorithm="giac")
```

output
```
1/6*b*log(c*x^6 + b*x^3 + a)/a^2 - b*log(abs(x))/a^2 + 1/3*(b^2 - 2*a*c)*a
rctan((2*c*x^3 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^2) + 1/3*(b*
x^3 - a)/(a^2*x^3)
```

3.142.9 Mupad [B] (verification not implemented)

Time = 8.78 (sec) , antiderivative size = 4281, normalized size of antiderivative = 48.10

$$\int \frac{1}{x^4 (a + bx^3 + cx^6)} \, dx = \text{Too large to display}$$

input
```
int(1/(x^4*(a + b*x^3 + c*x^6)),x)
```

output
```
(atan((48*a^8*x^3*(((4*b^5 + 9*a^2*b*c^2 - 16*a*b^3*c)*(((3*b^3 - 12*a*b*c
)*(((3*b^3 - 12*a*b*c)*(((2*a*c - b^2)*((252*a^4*b*c^5 - 18*a^3*b^3*c^4)/a
^4 - ((3*b^3 - 12*a*b*c)*(108*a^4*b^4*c^3 - 378*a^5*b^2*c^4))/(2*a^4*(36*a
^3*c - 9*a^2*b^2))))/(6*a^2*(4*a*c - b^2)^(1/2)) - ((3*b^3 - 12*a*b*c)*(2*
a*c - b^2)*(108*a^4*b^4*c^3 - 378*a^5*b^2*c^4))/(12*a^6*(4*a*c - b^2)^(1/2
)*(36*a^3*c - 9*a^2*b^2))))/(2*(36*a^3*c - 9*a^2*b^2)) + (((42*a^3*c^6 + 3
3*a^2*b^2*c^5)/a^4 + ((3*b^3 - 12*a*b*c)*((252*a^4*b*c^5 - 18*a^3*b^3*c^4)
/a^4 - ((3*b^3 - 12*a*b*c)*(108*a^4*b^4*c^3 - 378*a^5*b^2*c^4))/(2*a^4*(36
*a^3*c - 9*a^2*b^2))))/(2*(36*a^3*c - 9*a^2*b^2)))*(2*a*c - b^2))/(6*a^2*(
4*a*c - b^2)^(1/2))))/(2*(36*a^3*c - 9*a^2*b^2)) - ((((((2*a*c - b^2)*((25
2*a^4*b*c^5 - 18*a^3*b^3*c^4)/a^4 - ((3*b^3 - 12*a*b*c)*(108*a^4*b^4*c^3 -
 378*a^5*b^2*c^4))/(2*a^4*(36*a^3*c - 9*a^2*b^2))))/(6*a^2*(4*a*c - b^2)^(
1/2)) - ((3*b^3 - 12*a*b*c)*(2*a*c - b^2)*(108*a^4*b^4*c^3 - 378*a^5*b^2*c
^4))/(12*a^6*(4*a*c - b^2)^(1/2)*(36*a^3*c - 9*a^2*b^2)))*(2*a*c - b^2))/(
6*a^2*(4*a*c - b^2)^(1/2)) - ((3*b^3 - 12*a*b*c)*(2*a*c - b^2)^2*(108*a^4*
b^4*c^3 - 378*a^5*b^2*c^4))/(72*a^8*(4*a*c - b^2)*(36*a^3*c - 9*a^2*b^2)))
*(2*a*c - b^2))/(6*a^2*(4*a*c - b^2)^(1/2)) + ((2*a*c - b^2)*(((3*b^3 - 12
*a*b*c)*((42*a^3*c^6 + 33*a^2*b^2*c^5)/a^4 + ((3*b^3 - 12*a*b*c)*((252*a^4
*b*c^5 - 18*a^3*b^3*c^4)/a^4 - ((3*b^3 - 12*a*b*c)*(108*a^4*b^4*c^3 - 378*
a^5*b^2*c^4))/(2*a^4*(36*a^3*c - 9*a^2*b^2))))/(2*(36*a^3*c - 9*a^2*b^2...
```

3.143 $\int \frac{x^7}{a+bx^3+cx^6}\,dx$

3.143.1 Optimal result

Integrand size = 18, antiderivative size = 636

$$\int \frac{x^7}{a+bx^3+cx^6}\,dx$$

$$= \frac{x^2}{2c} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}$$

$$+ \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}c^{5/3}\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

$$+ \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}$$

$$+ \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{5/3}\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

$$- \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}$$

$$- \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}c^{5/3}\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

output
```
1/2*x^2/c+1/6*ln(2^(1/3)*c^(1/3)*x+(b-(-4*a*c+b^2)^(1/2))^(1/3))*(b+(2*a*c
-b^2)/(-4*a*c+b^2)^(1/2))*2^(1/3)/c^(5/3)/(b-(-4*a*c+b^2)^(1/2))^(1/3)-1/1
2*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b-(-4*a*c+b^2)^(1/2))^(1/3)+(b
-(-4*a*c+b^2)^(1/2))^(2/3))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))*2^(1/3)/c^(
5/3)/(b-(-4*a*c+b^2)^(1/2))^(1/3)+1/6*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b
-(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))*2^
(1/3)/c^(5/3)*3^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/3)+1/6*ln(2^(1/3)*c^(1/3)*
x+(b+(-4*a*c+b^2)^(1/2))^(1/3))*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*2^(1/3
)/c^(5/3)/(b+(-4*a*c+b^2)^(1/2))^(1/3)-1/12*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)
*c^(1/3)*x*(b+(-4*a*c+b^2)^(1/2))^(1/3)+(b+(-4*a*c+b^2)^(1/2))^(2/3))*(b+(
-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*2^(1/3)/c^(5/3)/(b+(-4*a*c+b^2)^(1/2))^(1/
3)+1/6*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b+(-4*a*c+b^2)^(1/2))^(1/3))*3^(
1/2))*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*2^(1/3)/c^(5/3)*3^(1/2)/(b+(-4*a
*c+b^2)^(1/2))^(1/3)
```

3.143.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.11

$$\int \frac{x^7}{a+bx^3+cx^6}\,dx = \frac{3x^2-2\text{RootSum}\left[a+b\#1^3+c\#1^6\&,\frac{a\log(x-\#1)+b\log(x-\#1)\#1^3}{b\#1+2c\#1^4}\&\right]}{6c}$$

input
```
Integrate[x^7/(a + b*x^3 + c*x^6),x]
```

output
```
(3*x^2 - 2*RootSum[a + b*#1^3 + c*#1^6 & , (a*Log[x - #1] + b*Log[x - #1]*
#1^3)/(b*#1 + 2*c*#1^4) & ])/(6*c)
```

3.143.3 Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 546, normalized size of antiderivative = 0.86, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {1703, 27, 1834, 27, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{a+bx^3+cx^6}\,dx$$

$\downarrow$ 1703

$$\frac{x^2}{2c}-\frac{\int \frac{2x(bx^3+a)}{cx^6+bx^3+a}dx}{2c}$$

$\downarrow$ 27

$$\frac{x^2}{2c}-\frac{\int \frac{x(bx^3+a)}{cx^6+bx^3+a}dx}{c}$$

$\downarrow$ 1834

$$\frac{x^2}{2c}-\frac{\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{2x}{2cx^3+b-\sqrt{b^2-4ac}}dx+\frac{1}{2}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\int \frac{2x}{2cx^3+b+\sqrt{b^2-4ac}}dx}{c}$$

$\downarrow$ 27

$$\frac{x^2}{2c}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{x}{2cx^3+b-\sqrt{b^2-4ac}}dx+\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\int \frac{x}{2cx^3+b+\sqrt{b^2-4ac}}dx}{c}$$

$\downarrow$ 821

$$\frac{x^2}{2c}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{\int \frac{\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b-\sqrt{b^2-4ac}}}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\int \frac{1}{\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b-\sqrt{b^2-4ac}}}dx}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)+\left(\frac{b^2-\cdots}{\sqrt{b^2\cdots}}\right.}{c}$$

$\downarrow$ 16

$$\frac{x^2}{2c}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{\int \frac{\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b-\sqrt{b^2-4ac}}}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\,2^{2/3}c^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)+\left(\frac{b\cdots}{\sqrt{\cdots}}\right.}{c}$$

$\downarrow$ 1142

3.143. $\int \frac{x^7}{a+bx^3+cx^6}\,dx$

$$\frac{x^2}{2c}-$$

$$\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{\frac{3}{2}\sqrt[3]{b-\sqrt{b^2-4ac}}\int\frac{1}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx+\frac{\int-\frac{\sqrt[3]{2}\sqrt[3]{c}\left(\sqrt[3]{b-\ldots}\right.}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b\ldots}}}{\ldots}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\right.$$

$\downarrow$ 25

$$\frac{x^2}{2c}-$$

$$\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{\frac{3}{2}\sqrt[3]{b-\sqrt{b^2-4ac}}\int\frac{1}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{\int\frac{\sqrt[3]{2}\sqrt[3]{c}\left(\sqrt[3]{b-\ldots}\right.}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\ldots}}}{2\ldots}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\right.$$

$\downarrow$ 27

$$\frac{x^2}{2c}-$$

$$\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{\frac{3}{2}\sqrt[3]{b-\sqrt{b^2-4ac}}\int\frac{1}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{1}{2}\int\frac{\sqrt[3]{b-\sqrt{\ldots}}}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b\ldots}}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\right.$$

$\downarrow$ 1082

3.143. $\int\frac{x^7}{a+bx^3+cx^6}\,dx$

$$\frac{x^2}{2c}-\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)}{\sqrt[3]{2}\sqrt[3]{c}}-\frac{1}{2}\int \frac{\sqrt[3]{b-\sqrt{b^2-4ac}}-2\sqrt[3]{2}\sqrt[3]{c}x}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\right.}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\right.$$

$\downarrow$ 217

$$\frac{x^2}{2c}-\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{-\frac{1}{2}\int \frac{\sqrt[3]{b-\sqrt{b^2-4ac}}-2\sqrt[3]{2}\sqrt[3]{c}x}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{c}}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\log\left(\sqrt[3]{\cdots}\right.}{3\ 2\cdots}\right.$$

$\downarrow$ 1103

3.143. $\int \frac{x^7}{a+bx^3+cx^6}\,dx$

$$\frac{x^2}{2c} - \left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{\frac{\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{2\sqrt[3]{2}\sqrt[3]{c}} - \frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{c}}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}} - \frac{\log\left(\sqrt[3]{\cdots}\right.}{3\ 2^{\cdots}}\right.$$

input
```
Int[x^7/(a + b*x^3 + c*x^6),x]
```

output
```
x^2/(2*c) - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*(-1/3*Log[(b - Sqrt[b^2
 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x]/(2^(2/3)*c^(2/3)*(b - Sqrt[b^2 - 4*a
*c])^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2
- 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*c^(1/3))) + Log[(b - Sqrt[b^2 - 4*a*c]
)^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3
)*x^2]/(2*2^(1/3)*c^(1/3)))/(3*2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/
3))) + (b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*(-1/3*Log[(b + Sqrt[b^2 - 4*a
*c])^(1/3) + 2^(1/3)*c^(1/3)*x]/(2^(2/3)*c^(2/3)*(b + Sqrt[b^2 - 4*a*c])^(
1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*
c])^(1/3))/Sqrt[3]])/(2^(1/3)*c^(1/3))) + Log[(b + Sqrt[b^2 - 4*a*c])^(2/3
) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2]
/(2*2^(1/3)*c^(1/3)))/(3*2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3))))/
c
```

3.143.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1703
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(2*n - 1)*(d*x)^(m - 2*n + 1)*((a + b*x^n + c*x^(2*n))^(
p + 1)/(c*(m + 2*n*p + 1))), x] - Simp[d^(2*n)/(c*(m + 2*n*p + 1))   Int[(d
*x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x
^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && N
eQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0
] && IntegerQ[p]
```

3.143. $\int \frac{x^7}{a+bx^3+cx^6}\,dx$

rule 1834
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 +
 (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2
 - (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ
[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n
, 0]
```

3.143.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.50 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.10

method	result	size
default	$\frac{x^2}{2c} - \frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^6+\_Z^3b+a\right)} \frac{\left(\_R^4b+\_Ra\right)\ln\left(x-\_R\right)}{2\_R^5c+b\_R^2}}{3c}$	61
risch	$\frac{x^2}{2c} + \frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^6+\_Z^3b+a\right)} \frac{\left(-\_R^4b-\_Ra\right)\ln\left(x-\_R\right)}{2\_R^5c+b\_R^2}}{3c}$	63

input
```
int(x^7/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2/c-1/3/c*sum((_R^4*b+_R*a)/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^
6*c+_Z^3*b+a))
```

3.143.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3402 vs. 2(498) = 996.

Time = 0.49 (sec) , antiderivative size = 3402, normalized size of antiderivative = 5.35

$$\int \frac{x^7}{a+bx^3+cx^6}\,dx = \text{Too large to display}$$

input
```
integrate(x^7/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

output
```
1/6*((1/2)^(1/3)*(sqrt(-3)*c - c)*((b^4 - 3*a*b^2*c + a^2*c^2 + (b^2*c^5 -
 4*a*c^6)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a
^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(
b^2*c^5 - 4*a*c^6))^(1/3)*log(-(1/2)^(2/3)*(b^10 - 12*a*b^8*c + 52*a^2*b^6
*c^2 - 95*a^3*b^4*c^3 + 60*a^4*b^2*c^4 + sqrt(-3)*(b^10 - 12*a*b^8*c + 52*
a^2*b^6*c^2 - 95*a^3*b^4*c^3 + 60*a^4*b^2*c^4) - (b^8*c^5 - 13*a*b^6*c^6 +
 60*a^2*b^4*c^7 - 112*a^3*b^2*c^8 + 64*a^4*c^9 + sqrt(-3)*(b^8*c^5 - 13*a*
b^6*c^6 + 60*a^2*b^4*c^7 - 112*a^3*b^2*c^8 + 64*a^4*c^9))*sqrt((b^10 - 10*
a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12
*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*((b^4 - 3*a*b^2*c + a^2*c^2
 + (b^2*c^5 - 4*a*c^6)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b
^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*
a^3*c^13)))/(b^2*c^5 - 4*a*c^6))^(2/3) + 4*(a^3*b^5 - 5*a^4*b^3*c + 5*a^5*
b*c^2)*x) - (1/2)^(1/3)*(sqrt(-3)*c + c)*((b^4 - 3*a*b^2*c + a^2*c^2 + (b^
2*c^5 - 4*a*c^6)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3
 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^
13)))/(b^2*c^5 - 4*a*c^6))^(1/3)*log(-(1/2)^(2/3)*(b^10 - 12*a*b^8*c + 52*
a^2*b^6*c^2 - 95*a^3*b^4*c^3 + 60*a^4*b^2*c^4 - sqrt(-3)*(b^10 - 12*a*b^8*
c + 52*a^2*b^6*c^2 - 95*a^3*b^4*c^3 + 60*a^4*b^2*c^4) - (b^8*c^5 - 13*a*b^
6*c^6 + 60*a^2*b^4*c^7 - 112*a^3*b^2*c^8 + 64*a^4*c^9 - sqrt(-3)*(b^8*c...
```

3.143.6 Sympy [A] (verification not implemented)

Time = 147.58 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.44

$$\int \frac{x^7}{a+bx^3+cx^6}\,dx$$
$$=\text{RootSum}\left(t^6\cdot\left(46656a^3c^8-34992a^2b^2c^7+8748ab^4c^6-729b^6c^5\right)+t^3\cdot\left(432a^4c^4-1512a^3b^2c^3+1107a^2b\right.\right.$$
$$+\frac{x^2}{2c}$$

input
```
integrate(x**7/(c*x**6+b*x**3+a),x)
```

output
```
RootSum(_t**6*(46656*a**3*c**8 - 34992*a**2*b**2*c**7 + 8748*a*b**4*c**6 -
 729*b**6*c**5) + _t**3*(432*a**4*c**4 - 1512*a**3*b**2*c**3 + 1107*a**2*b
**4*c**2 - 297*a*b**6*c + 27*b**8) + a**5, Lambda(_t, _t*log(x + (-15552*_
t**5*a**4*c**9 + 27216*_t**5*a**3*b**2*c**8 - 14580*_t**5*a**2*b**4*c**7 +
 3159*_t**5*a*b**6*c**6 - 243*_t**5*b**8*c**5 - 72*_t**2*a**5*c**5 + 594*_
t**2*a**4*b**2*c**4 - 864*_t**2*a**3*b**4*c**3 + 468*_t**2*a**2*b**6*c**2
- 108*_t**2*a*b**8*c + 9*_t**2*b**10)/(5*a**5*b*c**2 - 5*a**4*b**3*c + a**
3*b**5)))) + x**2/(2*c)
```

3.143.7 Maxima [F]

$$\int \frac{x^7}{a+bx^3+cx^6}\,dx=\int \frac{x^7}{cx^6+bx^3+a}\,dx$$

input
```
integrate(x^7/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

output
```
1/2*x^2/c - integrate((b*x^4 + a*x)/(c*x^6 + b*x^3 + a), x)/c
```

3.143.8 Giac [F]

$$\int \frac{x^7}{a+bx^3+cx^6}\,dx=\int \frac{x^7}{cx^6+bx^3+a}\,dx$$

input
```
integrate(x^7/(c*x^6+b*x^3+a),x, algorithm="giac")
```

output
```
integrate(x^7/(c*x^6 + b*x^3 + a), x)
```

3.143.9 Mupad [B] (verification not implemented)

Time = 14.73 (sec) , antiderivative size = 4069, normalized size of antiderivative = 6.40

$$\int \frac{x^7}{a+bx^3+cx^6}\,dx = \text{Too large to display}$$

input
```
int(x^7/(a + b*x^3 + c*x^6),x)
```

output
```
log((2^(1/3)*((2^(2/3)*(27*a^2*c*x*(b^4 + 8*a^2*c^2 - 6*a*b^2*c) + (27*2^(
1/3)*a*b*c^3*(4*a*c - b^2)^2*(-(b^8 + 16*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^
(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^2*(-(4*a*
c - b^2)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)
^3))^(2/3))/2)*(-(b^8 + 16*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2
*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1
/2) - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(1/3))/6
- (9*a*b*(b^6 - 12*a^3*c^3 + 19*a^2*b^2*c^2 - 8*a*b^4*c))/c^2)*(-(b^8 + 16
*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3
- 11*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c -
 b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(2/3))/18 + (a^4*x*(a*c - b^2))/c^2
)*(-(b^8 + 16*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56
*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^3
*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3*c^8 - b^6*c^5 + 12*a*b^4*c^6 - 48
*a^2*b^2*c^7)))^(1/3) + log((2^(1/3)*((2^(2/3)*(27*a^2*c*x*(b^4 + 8*a^2*c^
2 - 6*a*b^2*c) + (27*2^(1/3)*a*b*c^3*(4*a*c - b^2)^2*(-(b^8 + 16*a^4*c^4 -
 b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6
*c - 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^3*c*(-(4*a*c - b^2)^3)^(
1/2))/(c^5*(4*a*c - b^2)^3))^(2/3))/2)*(-(b^8 + 16*a^4*c^4 - b^5*(-(4*a*c
- b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c - 5*a^2*...
```

3.144 $\int \frac{x^6}{a+bx^3+cx^6} \, dx$

3.144.1 Optimal result

Integrand size = 18, antiderivative size = 631

$$
\begin{aligned}
\int \frac{x^6}{a+bx^3+cx^6}\,dx
&= \frac{x}{c}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} \\
&+\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}} \\
&-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} \\
&-\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}} \\
&+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} \\
&+\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}
\end{aligned}
$$

output
```
x/c-1/6*ln(2^(1/3)*c^(1/3)*x+(b-(-4*a*c+b^2)^(1/2))^(1/3))*(b+(2*a*c-b^2)/
(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)/(b-(-4*a*c+b^2)^(1/2))^(2/3)+1/12*ln(2
^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b-(-4*a*c+b^2)^(1/2))^(1/3)+(b-(-4*a
*c+b^2)^(1/2))^(2/3))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)/(
b-(-4*a*c+b^2)^(1/2))^(2/3)+1/6*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b-(-4*a
*c+b^2)^(1/2))^(1/3))*3^(1/2))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))*2^(2/3)/
c^(4/3)*3^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(2/3)-1/6*ln(2^(1/3)*c^(1/3)*x+(b+(
-4*a*c+b^2)^(1/2))^(1/3))*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4
/3)/(b+(-4*a*c+b^2)^(1/2))^(2/3)+1/12*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/
3)*x*(b+(-4*a*c+b^2)^(1/2))^(1/3)+(b+(-4*a*c+b^2)^(1/2))^(2/3))*(b+(-2*a*c
+b^2)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)/(b+(-4*a*c+b^2)^(1/2))^(2/3)+1/6
*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b+(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*
(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)*3^(1/2)/(b+(-4*a*c+b^2
)^(1/2))^(2/3)
```

3.144.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.11

$$\int \frac{x^6}{a+bx^3+cx^6}\,dx=\frac{x}{c}-\frac{\text{RootSum}\left[a+b\#1^3+c\#1^6\&,\frac{a\log(x-\#1)+b\log(x-\#1)\#1^3}{b\#1^2+2c\#1^5}\&\right]}{3c}$$

input
```
Integrate[x^6/(a + b*x^3 + c*x^6),x]
```

output
```
x/c - RootSum[a + b*#1^3 + c*#1^6 & , (a*Log[x - #1] + b*Log[x - #1]*#1^3)
/(b*#1^2 + 2*c*#1^5) & ]/(3*c)
```

3.144.3 Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 521, normalized size of antiderivative = 0.83, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.611, Rules used = {1703, 1752, 750, 16, 27, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{a+bx^3+cx^6}\,dx$$
$$\Big\downarrow \text{ 1703}$$
$$\frac{x}{c}-\frac{\int \frac{bx^3+a}{cx^6+bx^3+a}dx}{c}$$
$$\Big\downarrow \text{ 1752}$$
$$\frac{x}{c}-\frac{\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^3+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\int \frac{1}{cx^3+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{c}$$
$$\Big\downarrow \text{ 750}$$
$$\frac{x}{c}-\frac{\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{2^{2/3}\int \frac{2\left(2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}x\right)}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\int \frac{1}{\sqrt[3]{c}x+\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}}dx}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right)+\cdots}{\cdots}$$
$$\Big\downarrow \text{ 16}$$
$$\frac{x}{c}-\frac{\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{2^{2/3}\int \frac{2\left(2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}x\right)}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\cdots\right.}{3\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\cdots\right.}{\cdots}$$
$$\Big\downarrow \text{ 27}$$
$$\frac{x}{c}-\frac{\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{2\ 2^{2/3}\int \frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}x}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\cdots\right.}{3\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\cdots\right.}{\cdots}$$

3.144. $\int \frac{x^6}{a+bx^3+cx^6}\,dx$

$\big\downarrow$ 1142

$$\frac{x}{c}-$$

$$\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{2\,2^{2/3}\left(\frac{3\sqrt[3]{b-\sqrt{b^2-4ac}}\int\frac{1}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{2\sqrt[3]{2}}-\frac{\int-\frac{\sqrt[3]{c}\ \cdots}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{\cdots}}}{\cdots}\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\cdots\right.$$

$\big\downarrow$ 25

$$\frac{x}{c}-$$

$$\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{2\,2^{2/3}\left(\frac{3\sqrt[3]{b-\sqrt{b^2-4ac}}\int\frac{1}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{2\sqrt[3]{2}}+\frac{\int\frac{\sqrt[3]{c}\left(\cdots\right.}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{\cdots}}}{\cdots}\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\cdots\right.$$

$\big\downarrow$ 27

3.144. $\int\frac{x^6}{a+bx^3+cx^6}\,dx$

$$\frac{\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{x}{c}-\frac{2\,2^{2/3}\left(\frac{3\sqrt[3]{b-\sqrt{b^2-4ac}}\int\frac{1}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{2\sqrt[3]{2}}+\frac{1}{4}\int\frac{\cdots}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{\cdots}}\cdots\right)}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\cdots\right)}{\cdots}$$

$\downarrow$ 1082

$$\frac{\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{x}{c}-\frac{2\,2^{2/3}\left(\frac{1}{4}\int\frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-4\sqrt[3]{c}x}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)^2-3}d\left(\cdots\right)}{2\sqrt[3]{c}}\cdots\right)}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\cdots\right)}{\cdots}$$

$\downarrow$ 217

3.144. $\int \frac{x^6}{a+bx^3+cx^6}\,dx$

$$\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{x}{c}-\frac{2\ 2^{2/3}\left(\frac{1}{4}\int\frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-4\sqrt[3]{c}x}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2\sqrt[3]{c}}\right)}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right)$$

$\big\downarrow$ 1103

$$\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{x}{c}-\frac{2\ 2^{2/3}\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2\sqrt[3]{c}}-\frac{\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{4\sqrt[3]{c}}\right)}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right)$$

3.144. $\int\frac{x^6}{a+bx^3+cx^6}\,dx$

input
```
Int[x^6/(a + b*x^3 + c*x^6),x]
```

output
```
x/c - (((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*((2^(2/3)*Log[(b - Sqrt[b^2
- 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(
2/3)) + (2*2^(2/3)*(-1/2*(Sqrt[3]*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - S
qrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/c^(1/3) - Log[(b - Sqrt[b^2 - 4*a*c])^(
2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x
^2]/(4*c^(1/3))))/(3*(b - Sqrt[b^2 - 4*a*c])^(2/3))))/2 + ((b + (b^2 - 2*a
*c)/Sqrt[b^2 - 4*a*c])*((2^(2/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/
3)*c^(1/3)*x])/(3*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)) + (2*2^(2/3)*(-1/
2*(Sqrt[3]*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3)
)/Sqrt[3]])/c^(1/3) - Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*
(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2]/(4*c^(1/3))))/(3*(b
 + Sqrt[b^2 - 4*a*c])^(2/3))))/2)/c
```

3.144.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1703
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(2*n - 1)*(d*x)^(m - 2*n + 1)*((a + b*x^n + c*x^(2*n))^(
p + 1)/(c*(m + 2*n*p + 1))), x] - Simp[d^(2*n)/(c*(m + 2*n*p + 1))   Int[(d
*x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x
^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && N
eQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0
] && IntegerQ[p]
```

rule 1752
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

3.144. $\int \frac{x^6}{a+bx^3+cx^6} \, dx$

3.144.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.09

method	result	size
default	$\frac{x}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{6}+\_Z^{3}b+a\right)}\frac{\left(-\_R^{3}b-a\right)\ln\left(x-\_R\right)}{2\_R^{5}c+b\_R^{2}}}{3c}$	59
risch	$\frac{x}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{6}+\_Z^{3}b+a\right)}\frac{\left(-\_R^{3}b-a\right)\ln\left(x-\_R\right)}{2\_R^{5}c+b\_R^{2}}}{3c}$	59

input
```
int(x^6/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
x/c+1/3/c*sum((-_R^3*b-a)/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^6*c+_Z^3
*b+a))
```

3.144.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2882 vs. 2(495) = 990.

Time = 0.41 (sec) , antiderivative size = 2882, normalized size of antiderivative = 4.57

$$\int \frac{x^6}{a+bx^3+cx^6}\,dx = \text{Too large to display}$$

input
```
integrate(x^6/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

output
```
-1/6*((1/2)^(1/3)*(sqrt(-3)*c + c)*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*
sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*
c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))
^(1/3)*log(4*(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*x - (1/2)^(1/3)*(b^6 - 8*a*
b^4*c + 18*a^2*b^2*c^2 - 8*a^3*c^3 + sqrt(-3)*(b^6 - 8*a*b^4*c + 18*a^2*b^
2*c^2 - 8*a^3*c^3) - (b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6 + sqrt(-3)*(b^5
*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6))*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2
 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 -
 64*a^3*c^11)))*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6
*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9
+ 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)) - (1/2)^(1/
3)*(sqrt(-3)*c - c)*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a
*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*
c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)*log(4*(a
*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*x - (1/2)^(1/3)*(b^6 - 8*a*b^4*c + 18*a^2*
b^2*c^2 - 8*a^3*c^3 - sqrt(-3)*(b^6 - 8*a*b^4*c + 18*a^2*b^2*c^2 - 8*a^3*c
^3) - (b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6 - sqrt(-3)*(b^5*c^4 - 8*a*b^3*
c^5 + 16*a^2*b*c^6))*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c
^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))
*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*...
```

3.144.6 Sympy [A] (verification not implemented)

Time = 53.13 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.31

$$\int \frac{x^6}{a+bx^3+cx^6}\,dx$$

$$=\operatorname{RootSum}\left(t^6\cdot\left(46656a^3c^7-34992a^2b^2c^6+8748ab^4c^5-729b^6c^4\right)+t^3\cdot\left(864a^3bc^3-864a^2b^3c^2+270ab^5c\right.\right.$$
$$+\frac{x}{c}$$

input
```
integrate(x**6/(c*x**6+b*x**3+a),x)
```

output
```
RootSum(_t**6*(46656*a**3*c**7 - 34992*a**2*b**2*c**6 + 8748*a*b**4*c**5 -
 729*b**6*c**4) + _t**3*(864*a**3*b*c**3 - 864*a**2*b**3*c**2 + 270*a*b**5
*c - 27*b**7) + a**4, Lambda(_t, _t*log(x + (1296*_t**4*a**2*b*c**6 - 648*
_t**4*a*b**3*c**5 + 81*_t**4*b**5*c**4 - 12*_t*a**3*c**3 + 39*_t*a**2*b**2
*c**2 - 21*_t*a*b**4*c + 3*_t*b**6)/(2*a**3*c**2 - 4*a**2*b**2*c + a*b**4)
))) + x/c
```

3.144.7 Maxima [F]

$$\int \frac{x^6}{a+bx^3+cx^6}\,dx=\int \frac{x^6}{cx^6+bx^3+a}\,dx$$

input
```
integrate(x^6/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

output
```
x/c - integrate((b*x^3 + a)/(c*x^6 + b*x^3 + a), x)/c
```

3.144.8 Giac [F]

$$\int \frac{x^6}{a+bx^3+cx^6}\,dx=\int \frac{x^6}{cx^6+bx^3+a}\,dx$$

input
```
integrate(x^6/(c*x^6+b*x^3+a),x, algorithm="giac")
```

output
```
integrate(x^6/(c*x^6 + b*x^3 + a), x)
```

3.144.9 Mupad [B] (verification not implemented)

Time = 1.91 (sec) , antiderivative size = 2280, normalized size of antiderivative = 3.61

$$\int \frac{x^6}{a+bx^3+cx^6}\,dx=\text{Too large to display}$$

input
```
int(x^6/(a + b*x^3 + c*x^6),x)
```

output
```
log((3*a^2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c - (3*2^(2/3)*a*(-(b^4*(-(4*a
*c - b^2)^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*
a*c - b^2)^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(c^
4*(4*a*c - b^2)^3))^(1/3)*(b^4 + 2*a^2*c^2 - 4*a*b^2*c)*(b*(-(4*a*c - b^2)
^3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(4*c*(4*a*c - b^2)))*(-(b^4*(-(
4*a*c - b^2)^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-
(4*a*c - b^2)^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/
(54*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6)))^(1/3) + x/c +
 log((3*a^2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c + (3*2^(2/3)*a*((b^7 + b^4*
(-(4*a*c - b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*
a*c - b^2)^3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(c^
4*(4*a*c - b^2)^3))^(1/3)*(b^4 + 2*a^2*c^2 - 4*a*b^2*c)*(b*(-(4*a*c - b^2)
^3)^(1/2) - b^4 - 16*a^2*c^2 + 8*a*b^2*c))/(4*c*(4*a*c - b^2)))*((b^7 + b^
4*(-(4*a*c - b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(
4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(
54*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6)))^(1/3) + log((3
*a^2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c + (3*2^(2/3)*a*(3^(1/2)*1i - 1)*((
b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*a^2
*c^2*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^
(1/2))/(c^4*(4*a*c - b^2)^3))^(1/3)*(b^4 + 2*a^2*c^2 - 4*a*b^2*c)*(b*(-...
```

3.144. $\int \frac{x^6}{a+bx^3+cx^6}\,dx$

3.145 $\int \frac{x^4}{a+bx^3+cx^6} \, dx$

3.145.1 Optimal result

Integrand size = 18, antiderivative size = 558

$$\int \frac{x^4}{a+bx^3+cx^6}\,dx$$

$$= \frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}c^{2/3}\sqrt{b^2-4ac}} - \frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}c^{2/3}\sqrt{b^2-4ac}}$$

$$+ \frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{2/3}\sqrt{b^2-4ac}} - \frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{2/3}\sqrt{b^2-4ac}}$$

$$- \frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}c^{2/3}\sqrt{b^2-4ac}}$$

$$+ \frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}c^{2/3}\sqrt{b^2-4ac}}$$

output
```
1/6*ln(2^(1/3)*c^(1/3)*x+(b-(-4*a*c+b^2)^(1/2))^(1/3))*(b-(-4*a*c+b^2)^(1/
2))^(2/3)*2^(1/3)/c^(2/3)/(-4*a*c+b^2)^(1/2)-1/12*ln(2^(2/3)*c^(2/3)*x^2-2
^(1/3)*c^(1/3)*x*(b-(-4*a*c+b^2)^(1/2))^(1/3)+(b-(-4*a*c+b^2)^(1/2))^(2/3)
)*(b-(-4*a*c+b^2)^(1/2))^(2/3)*2^(1/3)/c^(2/3)/(-4*a*c+b^2)^(1/2)+1/6*arct
an(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b-(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*(b-(-
4*a*c+b^2)^(1/2))^(2/3)*2^(1/3)/c^(2/3)*3^(1/2)/(-4*a*c+b^2)^(1/2)-1/6*ln(
2^(1/3)*c^(1/3)*x+(b+(-4*a*c+b^2)^(1/2))^(1/3))*(b+(-4*a*c+b^2)^(1/2))^(2/
3)*2^(1/3)/c^(2/3)/(-4*a*c+b^2)^(1/2)+1/12*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*
c^(1/3)*x*(b+(-4*a*c+b^2)^(1/2))^(1/3)+(b+(-4*a*c+b^2)^(1/2))^(2/3))*(b+(-
4*a*c+b^2)^(1/2))^(2/3)*2^(1/3)/c^(2/3)/(-4*a*c+b^2)^(1/2)-1/6*arctan(1/3*
(1-2*2^(1/3)*c^(1/3)*x/(b+(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*(b+(-4*a*c+b
^2)^(1/2))^(2/3)*2^(1/3)/c^(2/3)*3^(1/2)/(-4*a*c+b^2)^(1/2)
```

3.145.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.08

$$\int \frac{x^4}{a+bx^3+cx^6}\,dx=\frac{1}{3}\text{RootSum}\left[a+b\#1^3+c\#1^6\&,\frac{\log(x-\#1)\#1^2}{b+2c\#1^3}\&\right]$$

input
```
Integrate[x^4/(a + b*x^3 + c*x^6),x]
```

output
```
RootSum[a + b*#1^3 + c*#1^6 & , (Log[x - #1]*#1^2)/(b + 2*c*#1^3) & ]/3
```

3.145.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 516, normalized size of antiderivative = 0.92, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {1710, 27, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{a+bx^3+cx^6}\,dx$$

$$\Big\downarrow \text{1710}$$

$$\frac{1}{2}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{2x}{2cx^3+b-\sqrt{b^2-4ac}}dx+\frac{1}{2}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\int \frac{2x}{2cx^3+b+\sqrt{b^2-4ac}}dx$$

$$\Big\downarrow \text{27}$$

$$\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{x}{2cx^3+b-\sqrt{b^2-4ac}}dx+\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\int \frac{x}{2cx^3+b+\sqrt{b^2-4ac}}dx$$

$$\Big\downarrow \text{821}$$

$$\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(\frac{\int \frac{\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b-\sqrt{b^2-4ac}}}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\int \frac{1}{\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b-\sqrt{b^2-4ac}}}dx}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)$$
$$\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{\int \frac{\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b+\sqrt{b^2-4ac}}}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{\sqrt{b^2-4ac}+b}}-\frac{\int \frac{1}{\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b+\sqrt{b^2-4ac}}}dx}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{\sqrt{b^2-4ac}+b}}\right)$$

$$\Big\downarrow \text{16}$$

$$\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(\frac{\int \frac{\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b-\sqrt{b^2-4ac}}}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}\right.$$
$$\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{\int \frac{\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b+\sqrt{b^2-4ac}}}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{\sqrt{b^2-4ac}+b}}-\frac{\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{2/3}\sqrt[3]{\sqrt{b^2-4ac}+b}}\right.$$

$$\Big\downarrow \text{1142}$$

3.145. $\int \frac{x^4}{a+bx^3+cx^6}\,dx$

$$\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(\frac{\frac{3}{2}\sqrt[3]{b-\sqrt{b^2-4ac}}\int\frac{1}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx+\frac{\int-\frac{\sqrt[3]{2}\sqrt[3]{}}{2^{2/3}c^{2/3}x^2-\sqrt[3]{}}}{}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\right.$$

$$\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{\frac{3}{2}\sqrt[3]{\sqrt{b^2-4ac}+b}\int\frac{1}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx+\frac{\int-\frac{\sqrt[3]{2}\sqrt[3]{}}{2^{2/3}c^{2/3}x^2-\sqrt[3]{}}}{}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{\sqrt{b^2-4ac}+b}}\right.$$

$$\downarrow 25$$

3.145. $\int \frac{x^4}{a+bx^3+cx^6}\,dx$

$$\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(\frac{\frac{3}{2}\sqrt[3]{b-\sqrt{b^2-4ac}}\int \frac{1}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{\int \frac{\sqrt[3]{2}\sqrt[3]{c}}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}}}{}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\right.$$

$$\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{\frac{3}{2}\sqrt[3]{\sqrt{b^2-4ac}+b}\int \frac{1}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{\int \frac{\sqrt[3]{2}\sqrt[3]{c}}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}}}{}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{\sqrt{b^2-4ac}+b}}\right.$$

$$\Big\downarrow 27$$

$$\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(\frac{\frac{3}{2}\sqrt[3]{b-\sqrt{b^2-4ac}}\int \frac{1}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{1}{2}\int \frac{}{2^{2/3}c^{2/3}x^2-\sqrt[3]{}}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\right.$$

$$\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{\frac{3}{2}\sqrt[3]{\sqrt{b^2-4ac}+b}\int \frac{1}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{1}{2}\int \frac{}{2^{2/3}c^{2/3}x^2-\sqrt[3]{}}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{\sqrt{b^2-4ac}+b}}\right.$$

$$\Big\downarrow 1082$$

3.145. $\int \frac{x^4}{a+bx^3+cx^6}\,dx$

$$\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)^2-3}\,d\left(1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)}{\sqrt[3]{2}\sqrt[3]{c}}-\frac{1}{2}\int \frac{\sqrt[3]{b-\sqrt{b^2-4ac}}-2\cdots}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4\cdots}}}\cdots}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\cdots\right.$$

$$\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}\right)^2-3}\,d\left(1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}\right)}{\sqrt[3]{2}\sqrt[3]{c}}-\frac{1}{2}\int \frac{\sqrt[3]{b+\sqrt{b^2-4ac}}-2\cdots}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4\cdots}}}\cdots}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{\sqrt{b^2-4ac}+b}}\cdots\right.$$

$$\downarrow \; 217$$

3.145. $\int \frac{x^4}{a+bx^3+cx^6}\,dx$

$$\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(\frac{-\frac{1}{2}\int \frac{\sqrt[3]{b-\sqrt{b^2-4ac}}-2\sqrt[3]{2}\sqrt[3]{c}x}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right.}{\sqrt[3]{2}\sqrt[3]{c}}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\right.$$

$$\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{-\frac{1}{2}\int \frac{\sqrt[3]{b+\sqrt{b^2-4ac}}-2\sqrt[3]{2}\sqrt[3]{c}x}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right.}{\sqrt[3]{2}\sqrt[3]{c}}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{\sqrt{b^2-4ac}+b}}\right.$$

$\downarrow$ 1103

3.145. $\int \frac{x^4}{a+bx^3+cx^6}\,dx$

$$\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(\frac{\frac{\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{2\sqrt[3]{2}\sqrt[3]{c}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{c}}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\right.$$

$$\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{\frac{\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{2\sqrt[3]{2}\sqrt[3]{c}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{c}}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{\sqrt{b^2-4ac}+b}}\right.$$

input
```
Int[x^4/(a + b*x^3 + c*x^6),x]
```

output
```
(1 - b/Sqrt[b^2 - 4*a*c])*(-1/3*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3
)*c^(1/3)*x]/(2^(2/3)*c^(2/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)) + (-((Sqrt[3]
*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]]
)/(2^(1/3)*c^(1/3))) + Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)
*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2]/(2*2^(1/3)*c^(1/3)
))/(3*2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3))) + (1 + b/Sqrt[b^2 -
4*a*c])*(-1/3*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x]/(2^(2
/3)*c^(2/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*2^
(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*c^(1/3)
)) + Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4
*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2]/(2*2^(1/3)*c^(1/3)))/(3*2^(1/3)*c^(1
/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)))
```

3.145. $\int \frac{x^4}{a+bx^3+cx^6}\,dx$

3.145.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1710
```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^n/2)*(b/q + 1)   Int[(d*x)^(m
- n)/(b/2 + q/2 + c*x^n), x], x] - Simp[(d^n/2)*(b/q - 1)   Int[(d*x)^(m -
n)/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

3.145.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.08

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^6+\_Z^3b+a\right)}\frac{\_R^4\ln\left(x-\_R\right)}{2\_R^5c+b\_R^2}\right)}{3}$	43
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^6+\_Z^3b+a\right)}\frac{\_R^4\ln\left(x-\_R\right)}{2\_R^5c+b\_R^2}\right)}{3}$	43

input
```
int(x^4/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/3*sum(_R^4/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^6*c+_Z^3*b+a))
```

3.145.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2314 vs. 2(421) = 842.

Time = 0.31 (sec) , antiderivative size = 2314, normalized size of antiderivative = 4.15

$$\int \frac{x^4}{a+bx^3+cx^6}\,dx = \text{Too large to display}$$

input
```
integrate(x^4/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

output
```
1/6*(1/2)^(1/3)*(sqrt(-3) - 1)*(-((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*
c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) + b
)/(b^2*c^2 - 4*a*c^3))^(1/3)*log((1/2)^(2/3)*(b^5 - 6*a*b^3*c + 8*a^2*b*c^
2 + sqrt(-3)*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2) - (b^6*c^2 - 12*a*b^4*c^3 + 4
8*a^2*b^2*c^4 - 64*a^3*c^5 + sqrt(-3)*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2
*c^4 - 64*a^3*c^5))*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4
*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)))*(-((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4
*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^
7)) + b)/(b^2*c^2 - 4*a*c^3))^(2/3) - 4*(a*b^2 - 2*a^2*c)*x) - 1/6*(1/2)^(
1/3)*(sqrt(-3) + 1)*(-((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c
^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) + b)/(b^2*c^2
- 4*a*c^3))^(1/3)*log((1/2)^(2/3)*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2 - sqrt(-3
)*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2) - (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c
^4 - 64*a^3*c^5 - sqrt(-3)*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a
^3*c^5))*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a
^2*b^2*c^6 - 64*a^3*c^7)))*(-((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c +
4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) + b)/(b
^2*c^2 - 4*a*c^3))^(2/3) - 4*(a*b^2 - 2*a^2*c)*x) + 1/6*(1/2)^(1/3)*(sqrt(
-3) - 1)*(((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4
 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) - b)/(b^2*c^2 - 4*a*c^3...
```

3.145.6 Sympy [A] (verification not implemented)

Time = 1.45 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.31

$$\int \frac{x^4}{a+bx^3+cx^6}\,dx$$
$$=\operatorname{RootSum}\left(t^6\cdot\left(46656a^3c^5-34992a^2b^2c^4+8748ab^4c^3-729b^6c^2\right)+t^3\left(-432a^2bc^2+216ab^3c-27b^5\right)+a\right.$$

input
```
integrate(x**4/(c*x**6+b*x**3+a),x)
```

output
```
RootSum(_t**6*(46656*a**3*c**5 - 34992*a**2*b**2*c**4 + 8748*a*b**4*c**3 -
 729*b**6*c**2) + _t**3*(-432*a**2*b*c**2 + 216*a*b**3*c - 27*b**5) + a**2
, Lambda(_t, _t*log(x + (15552*_t**5*a**3*c**5 - 11664*_t**5*a**2*b**2*c**
4 + 2916*_t**5*a*b**4*c**3 - 243*_t**5*b**6*c**2 - 108*_t**2*a**2*b*c**2 +
 63*_t**2*a*b**3*c - 9*_t**2*b**5)/(2*a**2*c - a*b**2))))
```

3.145.7 Maxima [F]

$$\int \frac{x^4}{a+bx^3+cx^6}\,dx = \int \frac{x^4}{cx^6+bx^3+a}\,dx$$

input
```
integrate(x^4/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

output
```
integrate(x^4/(c*x^6 + b*x^3 + a), x)
```

3.145.8 Giac [F]

$$\int \frac{x^4}{a+bx^3+cx^6}\,dx = \int \frac{x^4}{cx^6+bx^3+a}\,dx$$

input
```
integrate(x^4/(c*x^6+b*x^3+a),x, algorithm="giac")
```

output
```
integrate(x^4/(c*x^6 + b*x^3 + a), x)
```

3.145.9 Mupad [B] (verification not implemented)

Time = 12.28 (sec) , antiderivative size = 2695, normalized size of antiderivative = 4.83

$$\int \frac{x^4}{a+bx^3+cx^6}\,dx = \text{Too large to display}$$

input
```
int(x^4/(a + b*x^3 + c*x^6),x)
```

output
```
log((2^(1/3)*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3
*c - 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(c^2*(4*a*c - b^2)^3))^(2/3)*(36*a^3*
c^3 - (2^(2/3)*(54*a^2*c^3*x*(4*a*c - b^2) - (27*2^(1/3)*a*b*c^3*(4*a*c -
b^2)^2*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c - 2
*a*c*(-(4*a*c - b^2)^3)^(1/2))/(c^2*(4*a*c - b^2)^3))^(2/3))/2)*((b^5 + b^
2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c - 2*a*c*(-(4*a*c - b
^2)^3)^(1/2))/(c^2*(4*a*c - b^2)^3))^(1/3))/6 - 45*a^2*b^2*c^2 + 9*a*b^4*c
))/18 + a^2*b*c*x)*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8
*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3*c^5 - b^6*c^2 + 12*
a*b^4*c^3 - 48*a^2*b^2*c^4)))^(1/3) + log((2^(1/3)*((b^5 - b^2*(-(4*a*c -
b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^(1/2))
/(c^2*(4*a*c - b^2)^3))^(2/3)*(36*a^3*c^3 - (2^(2/3)*(54*a^2*c^3*x*(4*a*c
- b^2) - (27*2^(1/3)*a*b*c^3*(4*a*c - b^2)^2*((b^5 - b^2*(-(4*a*c - b^2)^3
)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(c^2*
(4*a*c - b^2)^3))^(2/3))/2)*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*
b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(c^2*(4*a*c - b^2)^3))
^(1/3))/6 - 45*a^2*b^2*c^2 + 9*a*b^4*c))/18 + a^2*b*c*x)*((b^5 - b^2*(-(4*
a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^
(1/2))/(54*(64*a^3*c^5 - b^6*c^2 + 12*a*b^4*c^3 - 48*a^2*b^2*c^4)))^(1/3)
- log((2^(1/3)*(3^(1/2)*1i - 1)*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + ...
```

3.145. $\int \frac{x^4}{a+bx^3+cx^6}\,dx$

3.146 $\int \frac{x^3}{a+bx^3+cx^6}\,dx$

3.146.1 Optimal result

Integrand size = 18, antiderivative size = 558

$$\int \frac{x^3}{a+bx^3+cx^6}\,dx$$
$$= \frac{\sqrt[3]{b-\sqrt{b^2-4ac}}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\sqrt[3]{c}\sqrt{b^2-4ac}}$$
$$- \frac{\sqrt[3]{b+\sqrt{b^2-4ac}}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\sqrt[3]{c}\sqrt{b^2-4ac}}$$
$$- \frac{\sqrt[3]{b-\sqrt{b^2-4ac}}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt{b^2-4ac}}$$
$$+ \frac{\sqrt[3]{b+\sqrt{b^2-4ac}}\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt{b^2-4ac}}$$
$$+ \frac{\sqrt[3]{b-\sqrt{b^2-4ac}}\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\sqrt[3]{c}\sqrt{b^2-4ac}}$$
$$- \frac{\sqrt[3]{b+\sqrt{b^2-4ac}}\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\sqrt[3]{c}\sqrt{b^2-4ac}}$$

output
```
-1/6*ln(2^(1/3)*c^(1/3)*x+(b-(-4*a*c+b^2)^(1/2))^(1/3))*(b-(-4*a*c+b^2)^(1
/2))^(1/3)*2^(2/3)/c^(1/3)/(-4*a*c+b^2)^(1/2)+1/12*ln(2^(2/3)*c^(2/3)*x^2-
2^(1/3)*c^(1/3)*x*(b-(-4*a*c+b^2)^(1/2))^(1/3)+(b-(-4*a*c+b^2)^(1/2))^(2/3
))*(b-(-4*a*c+b^2)^(1/2))^(1/3)*2^(2/3)/c^(1/3)/(-4*a*c+b^2)^(1/2)+1/6*arc
tan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b-(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*(b-(
-4*a*c+b^2)^(1/2))^(1/3)*2^(2/3)/c^(1/3)*3^(1/2)/(-4*a*c+b^2)^(1/2)+1/6*ln
(2^(1/3)*c^(1/3)*x+(b+(-4*a*c+b^2)^(1/2))^(1/3))*(b+(-4*a*c+b^2)^(1/2))^(1
/3)*2^(2/3)/c^(1/3)/(-4*a*c+b^2)^(1/2)-1/12*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)
*c^(1/3)*x*(b+(-4*a*c+b^2)^(1/2))^(1/3)+(b+(-4*a*c+b^2)^(1/2))^(2/3))*(b+(
-4*a*c+b^2)^(1/2))^(1/3)*2^(2/3)/c^(1/3)/(-4*a*c+b^2)^(1/2)-1/6*arctan(1/3
*(1-2*2^(1/3)*c^(1/3)*x/(b+(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*(b+(-4*a*c+
b^2)^(1/2))^(1/3)*2^(2/3)/c^(1/3)*3^(1/2)/(-4*a*c+b^2)^(1/2)
```

3.146.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.08

$$\int \frac{x^3}{a+bx^3+cx^6}\,dx = \frac{1}{3}\text{RootSum}\left[a+b\#1^3+c\#1^6\&,\frac{\log(x-\#1)\#1}{b+2c\#1^3}\&\right]$$

input
```
Integrate[x^3/(a + b*x^3 + c*x^6),x]
```

output
```
RootSum[a + b*#1^3 + c*#1^6 & , (Log[x - #1]*#1)/(b + 2*c*#1^3) & ]/3
```

3.146.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 496, normalized size of antiderivative = 0.89, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {1710, 750, 16, 27, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{a+bx^3+cx^6}\,dx$$

$$\Big\downarrow \text{ 1710}$$

$$\frac{1}{2}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^3+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+$$
$$\frac{1}{2}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\int \frac{1}{cx^3+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx$$

$\downarrow$ 750

$$\frac{1}{2}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(\frac{2^{2/3}\int \frac{2\left(2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}x\right)}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\int \frac{1}{\sqrt[3]{c}x+\frac{\sqrt[3]{b-\sqrt{b^2-}}}{\sqrt[3]{2}}}}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right.$$
$$\frac{1}{2}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{2^{2/3}\int \frac{2\left(2^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}-\sqrt[3]{c}x\right)}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{2^{2/3}\int \frac{1}{\sqrt[3]{c}x+\frac{\sqrt[3]{b+\sqrt{b^2-}}}{\sqrt[3]{2}}}}{3\left(\sqrt{b^2-4ac}+b\right)^{2/3}}\right.$$

$\downarrow$ 16

$$\frac{1}{2}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(\frac{2^{2/3}\int \frac{2\left(2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}x\right)}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}\right.}{3\sqrt[3]{c}\left(b-\sqrt{b^2-4a}\right.}\right.$$
$$\frac{1}{2}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{2^{2/3}\int \frac{2\left(2^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}-\sqrt[3]{c}x\right)}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{2^{2/3}\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}\right.}{3\sqrt[3]{c}\left(\sqrt{b^2-4ac}+\right.}\right.$$

$\downarrow$ 27

3.146. $\int \frac{x^3}{a+bx^3+cx^6}\,dx$

$$\frac{1}{2}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(\frac{2\ 2^{2/3}\int \frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}x}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4a}\right.}{3\sqrt[3]{c}\left(b-\sqrt{b^2-\right.}\right.$$

$$\frac{1}{2}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{2\ 2^{2/3}\int \frac{2^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}-\sqrt[3]{c}x}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{2^{2/3}\log\left(\sqrt[3]{\sqrt{b^2-4ac}+\right.}{3\sqrt[3]{c}\left(\sqrt{b^2-4ac}\right.}\right.$$

$\downarrow$ 1142

$$\frac{1}{2}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(\frac{2\ 2^{2/3}\left(\frac{3\sqrt[3]{b-\sqrt{b^2-4ac}}\int\frac{1}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{2\sqrt[3]{2}}-\frac{\int -}{2c^{2/3}}\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right.$$

$$\frac{1}{2}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{2\ 2^{2/3}\left(\frac{3\sqrt[3]{\sqrt{b^2-4ac}+b}\int\frac{1}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx}{2\sqrt[3]{2}}-\frac{\int -}{2c^{2/3}}\right.}{3\left(\sqrt{b^2-4ac}+b\right)^{2/3}}\right.$$

3.146. $\int \frac{x^3}{a+bx^3+cx^6}\,dx$

$$\downarrow 25$$

$$\frac{1}{2}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(\frac{2\,2^{2/3}\left(\frac{3\sqrt[3]{b-\sqrt{b^2-4ac}}\int\frac{1}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{2\sqrt[3]{2}}+\frac{\int}{2c^{2/3}x^2}\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right.$$

$$\frac{1}{2}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{2\,2^{2/3}\left(\frac{3\sqrt[3]{\sqrt{b^2-4ac}+b}\int\frac{1}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx}{2\sqrt[3]{2}}+\frac{\int}{2c^{2/3}x^2}\right.}{3\left(\sqrt{b^2-4ac}+b\right)^{2/3}}\right.$$

$$\downarrow 27$$

3.146. $\int \frac{x^3}{a+bx^3+cx^6}\,dx$

$$\frac{1}{2}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(\frac{2\,2^{2/3}\left(\frac{3\sqrt[3]{b-\sqrt{b^2-4ac}}\int\frac{1}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{2\sqrt[3]{2}}+\frac{1}{4}\int\frac{}{2c^{2/3}}\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right.$$

$$\frac{1}{2}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{2\,2^{2/3}\left(\frac{3\sqrt[3]{\sqrt{b^2-4ac}+b}\int\frac{1}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx}{2\sqrt[3]{2}}+\frac{1}{4}\int\frac{}{2c^{2/3}}\right.}{3\left(\sqrt{b^2-4ac}+b\right)^{2/3}}\right.$$

↓ 1082

3.146. $\int\frac{x^3}{a+bx^3+cx^6}\,dx$

$$\frac{1}{2}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(\frac{2\,2^{2/3}\left(\frac{1}{4}\int\frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-4\sqrt[3]{c}x}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-}}}\right.}}{}\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right.$$

$$\frac{1}{2}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{2\,2^{2/3}\left(\frac{1}{4}\int\frac{2^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}-4\sqrt[3]{c}x}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-}}}\right.}}{}\right.}{3\left(\sqrt{b^2-4ac}+b\right)^{2/3}}\right.$$

$\downarrow$ 217

3.146. $\int \frac{x^3}{a+bx^3+cx^6}\,dx$

$$\frac{1}{2}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\frac{2\,2^{2/3}\left(\frac{1}{4}\int\frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-4\sqrt[3]{c}x}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\cdots}{\sqrt[3]{b-\sqrt{b\cdots}}}}{\sqrt{3}}\right)}{2\sqrt[3]{c}}\cdots\right)}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}\cdots}$$

$$\frac{1}{2}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\frac{2\,2^{2/3}\left(\frac{1}{4}\int\frac{2^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}-4\sqrt[3]{c}x}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\cdots}{\sqrt[3]{\sqrt{b^2-\cdots}}}}{\sqrt{3}}\right)}{2\sqrt[3]{c}}\cdots\right)}{3\left(\sqrt{b^2-4ac}+b\right)^{2/3}\cdots}$$

$\downarrow$ 1103

3.146. $\int\frac{x^3}{a+bx^3+cx^6}\,dx$

$$\frac{1}{2}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(\frac{2\,2^{2/3}\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2\sqrt[3]{c}}-\frac{\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}\right.}{4\sqrt[3]{c}}\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right.$$

$$\frac{1}{2}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{2\,2^{2/3}\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{2\sqrt[3]{c}}-\frac{\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}\right.}{4\sqrt[3]{c}}\right.}{3\left(\sqrt{b^2-4ac}+b\right)^{2/3}}\right.$$

input
```
Int[x^3/(a + b*x^3 + c*x^6),x]
```

3.146. $\int \frac{x^3}{a+bx^3+cx^6}\,dx$

output
```
((1 - b/Sqrt[b^2 - 4*a*c])*((2^(2/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2
^(1/3)*c^(1/3)*x])/(3*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + (2*2^(2/3)*
(-1/2*(Sqrt[3]*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(
1/3))/Sqrt[3]])/c^(1/3) - Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1
/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2]/(4*c^(1/3))))/(
3*(b - Sqrt[b^2 - 4*a*c])^(2/3))))/2 + ((1 + b/Sqrt[b^2 - 4*a*c])*((2^(2/3
)*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*c^(1/3)*(b +
Sqrt[b^2 - 4*a*c])^(2/3)) + (2*2^(2/3)*(-1/2*(Sqrt[3]*ArcTan[(1 - (2*2^(1/
3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/c^(1/3) - Log[(b +
Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x
 + 2^(2/3)*c^(2/3)*x^2]/(4*c^(1/3))))/(3*(b + Sqrt[b^2 - 4*a*c])^(2/3))))/
2
```

3.146.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1710
```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^n/2)*(b/q + 1)   Int[(d*x)^(m
- n)/(b/2 + q/2 + c*x^n), x], x] - Simp[(d^n/2)*(b/q - 1)   Int[(d*x)^(m -
n)/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

3.146.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.08

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^6+\_Z^3b+a\right)}\frac{\_R^3\ln\left(x-\_R\right)}{2\_R^5c+b\_R^2}\right)}{3}$	43
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^6+\_Z^3b+a\right)}\frac{\_R^3\ln\left(x-\_R\right)}{2\_R^5c+b\_R^2}\right)}{3}$	43

input
```
int(x^3/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/3*sum(_R^3/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^6*c+_Z^3*b+a))
```

3.146.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1542 vs. $2(421) = 842$.

Time = 0.29 (sec) , antiderivative size = 1542, normalized size of antiderivative = 2.76

$$\int \frac{x^3}{a + bx^3 + cx^6}\, dx = \text{Too large to display}$$

input
```
integrate(x^3/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

output
```
-1/6*(1/2)^(1/3)*(sqrt(-3) + 1)*(((b^2*c - 4*a*c^2)*sqrt(b^2/(b^6*c^2 - 12
*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) + 1)/(b^2*c - 4*a*c^2))^(1/3)*l
og(-(1/2)^(1/3)*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3 + sqrt(-3)*(b^4*c - 8*a*
b^2*c^2 + 16*a^2*c^3))*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 -
 64*a^3*c^5))*(((b^2*c - 4*a*c^2)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^
2*b^2*c^4 - 64*a^3*c^5)) + 1)/(b^2*c - 4*a*c^2))^(1/3) + 2*b*x) + 1/6*(1/2
)^(1/3)*(sqrt(-3) - 1)*(((b^2*c - 4*a*c^2)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^
3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) + 1)/(b^2*c - 4*a*c^2))^(1/3)*log(-(1/2)
^(1/3)*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3 - sqrt(-3)*(b^4*c - 8*a*b^2*c^2 +
 16*a^2*c^3))*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c
^5))*(((b^2*c - 4*a*c^2)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4
 - 64*a^3*c^5)) + 1)/(b^2*c - 4*a*c^2))^(1/3) + 2*b*x) - 1/6*(1/2)^(1/3)*(
sqrt(-3) + 1)*(-((b^2*c - 4*a*c^2)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a
^2*b^2*c^4 - 64*a^3*c^5)) - 1)/(b^2*c - 4*a*c^2))^(1/3)*log((1/2)^(1/3)*(b
^4*c - 8*a*b^2*c^2 + 16*a^2*c^3 + sqrt(-3)*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c
^3))*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*(-((
b^2*c - 4*a*c^2)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^
3*c^5)) - 1)/(b^2*c - 4*a*c^2))^(1/3) + 2*b*x) + 1/6*(1/2)^(1/3)*(sqrt(-3)
 - 1)*(-((b^2*c - 4*a*c^2)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c
^4 - 64*a^3*c^5)) - 1)/(b^2*c - 4*a*c^2))^(1/3)*log((1/2)^(1/3)*(b^4*c ...
```

3.146.6 Sympy [A] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.22

$$\int \frac{x^3}{a + bx^3 + cx^6}\, dx$$
$$= \operatorname{RootSum}\left(t^6 \cdot \left(46656a^3c^4 - 34992a^2b^2c^3 + 8748ab^4c^2 - 729b^6c\right) + t^3 \cdot \left(432a^2c^2 - 216ab^2c + 27b^4\right) + a, \left(\right.\right.$$

input
```
integrate(x**3/(c*x**6+b*x**3+a),x)
```

output
```
RootSum(_t**6*(46656*a**3*c**4 - 34992*a**2*b**2*c**3 + 8748*a*b**4*c**2 -
 729*b**6*c) + _t**3*(432*a**2*c**2 - 216*a*b**2*c + 27*b**4) + a, Lambda(
_t, _t*log(x + (2592*_t**4*a**2*c**3 - 1296*_t**4*a*b**2*c**2 + 162*_t**4*
b**4*c + 12*_t*a*c - 3*_t*b**2)/b)))
```

3.146.7 Maxima [F]

$$\int \frac{x^3}{a+bx^3+cx^6}\,dx = \int \frac{x^3}{cx^6+bx^3+a}\,dx$$

input
```
integrate(x^3/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

output
```
integrate(x^3/(c*x^6 + b*x^3 + a), x)
```

3.146.8 Giac [F]

$$\int \frac{x^3}{a+bx^3+cx^6}\,dx = \int \frac{x^3}{cx^6+bx^3+a}\,dx$$

input
```
integrate(x^3/(c*x^6+b*x^3+a),x, algorithm="giac")
```

output
```
integrate(x^3/(c*x^6 + b*x^3 + a), x)
```

3.146.9 Mupad [B] (verification not implemented)

Time = 12.04 (sec) , antiderivative size = 2129, normalized size of antiderivative = 3.82

$$\int \frac{x^3}{a+bx^3+cx^6}\,dx = \text{Too large to display}$$

input
```
int(x^3/(a + b*x^3 + c*x^6),x)
```

output
```
log((2^(2/3)*(-(b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c)
/(c*(4*a*c - b^2)^3))^(1/3)*(9*a*b^3*c^2 - 36*a^2*b*c^3 + (9*2^(1/3)*a*c^3
*(4*a*c - b^2)^2*(x - (2^(2/3)*b*(-(b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*
a^2*c^2 - 8*a*b^2*c)/(c*(4*a*c - b^2)^3))^(1/3))/2)*(-(b*(-(4*a*c - b^2)^3
)^(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c)/(c*(4*a*c - b^2)^3))^(2/3))/2))/6
+ 3*a*c^2*x*(2*a*c - b^2))*((b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*a^2*c^2
 - 8*a*b^2*c)/(54*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)))^(
1/3) + log((2^(2/3)*(9*a*b^3*c^2 - 36*a^2*b*c^3 + (9*2^(1/3)*a*c^3*(x - (2
^(2/3)*b*((b*(-(4*a*c - b^2)^3)^(1/2) - b^4 - 16*a^2*c^2 + 8*a*b^2*c)/(c*(
4*a*c - b^2)^3))^(1/3))/2)*(4*a*c - b^2)^2*((b*(-(4*a*c - b^2)^3)^(1/2) -
b^4 - 16*a^2*c^2 + 8*a*b^2*c)/(c*(4*a*c - b^2)^3))^(2/3))/2)*((b*(-(4*a*c
- b^2)^3)^(1/2) - b^4 - 16*a^2*c^2 + 8*a*b^2*c)/(c*(4*a*c - b^2)^3))^(1/3)
)/6 + 3*a*c^2*x*(2*a*c - b^2))*(-(b*(-(4*a*c - b^2)^3)^(1/2) - b^4 - 16*a^
2*c^2 + 8*a*b^2*c)/(54*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3
)))^(1/3) + log((2^(2/3)*(3^(1/2)*1i - 1)*(36*a^2*b*c^3 - 9*a*b^3*c^2 + (2
^(1/3)*(3^(1/2)*1i + 1)*(81*a*c^3*x*(4*a*c - b^2)^2 - (81*2^(2/3)*a*b*c^3*
(3^(1/2)*1i - 1)*(4*a*c - b^2)^2*(-(b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*
a^2*c^2 - 8*a*b^2*c)/(c*(4*a*c - b^2)^3))^(1/3))/4)*(-(b*(-(4*a*c - b^2)^3
)^(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c)/(c*(4*a*c - b^2)^3))^(2/3))/36)*(-
(b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c)/(c*(4*a*c -...
```

3.146. $\int \frac{x^3}{a+bx^3+cx^6}\,dx$

3.147 $\int \frac{x}{a+bx^3+cx^6}\,dx$

3.147.1 Optimal result

Integrand size = 16, antiderivative size = 558

$$\int \frac{x}{a+bx^3+cx^6}\,dx = -\frac{\sqrt[3]{2}\sqrt[3]{c}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt{b^2-4ac}\sqrt[3]{b-\sqrt{b^2-4ac}}}+\frac{\sqrt[3]{2}\sqrt[3]{c}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt{b^2-4ac}\sqrt[3]{b+\sqrt{b^2-4ac}}}$$
$$-\frac{\sqrt[3]{2}\sqrt[3]{c}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt{b^2-4ac}\sqrt[3]{b-\sqrt{b^2-4ac}}}+\frac{\sqrt[3]{2}\sqrt[3]{c}\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt{b^2-4ac}\sqrt[3]{b+\sqrt{b^2-4ac}}}$$
$$+\frac{\sqrt[3]{c}\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{3\,2^{2/3}\sqrt{b^2-4ac}\sqrt[3]{b-\sqrt{b^2-4ac}}}$$
$$-\frac{\sqrt[3]{c}\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{3\,2^{2/3}\sqrt{b^2-4ac}\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

output
```
-1/3*2^(1/3)*c^(1/3)*ln(2^(1/3)*c^(1/3)*x+(b-(-4*a*c+b^2)^(1/2))^(1/3))/(b
-(-4*a*c+b^2)^(1/2))^(1/3)/(-4*a*c+b^2)^(1/2)+1/6*c^(1/3)*ln(2^(2/3)*c^(2/
3)*x^2-2^(1/3)*c^(1/3)*x*(b-(-4*a*c+b^2)^(1/2))^(1/3)+(b-(-4*a*c+b^2)^(1/2
))^(2/3))*2^(1/3)/(b-(-4*a*c+b^2)^(1/2))^(1/3)/(-4*a*c+b^2)^(1/2)-1/3*2^(1
/3)*c^(1/3)*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b-(-4*a*c+b^2)^(1/2))^(1/3)
)*3^(1/2))*3^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/3)/(-4*a*c+b^2)^(1/2)+1/3*2^(
1/3)*c^(1/3)*ln(2^(1/3)*c^(1/3)*x+(b+(-4*a*c+b^2)^(1/2))^(1/3))/(-4*a*c+b^
2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/3)-1/6*c^(1/3)*ln(2^(2/3)*c^(2/3)*x^2-2
^(1/3)*c^(1/3)*x*(b+(-4*a*c+b^2)^(1/2))^(1/3)+(b+(-4*a*c+b^2)^(1/2))^(2/3)
)*2^(1/3)/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/3)+1/3*2^(1/3)*c^(1
/3)*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b+(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2
))*3^(1/2)/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/3)
```

3.147.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.08

$$\int \frac{x}{a+bx^3+cx^6}\,dx=\frac{1}{3}\text{RootSum}\left[a+b\#1^3+c\#1^6\&,\frac{\log(x-\#1)}{b\#1+2c\#1^4}\&\right]$$

input
```
Integrate[x/(a + b*x^3 + c*x^6),x]
```

output
```
RootSum[a + b*#1^3 + c*#1^6 & , Log[x - #1]/(b*#1 + 2*c*#1^4) & ]/3
```

3.147.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 511, normalized size of antiderivative = 0.92, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {1711, 27, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{a+bx^3+cx^6}\,dx$$

$$\Big\downarrow \text{1711}$$

$$\frac{c\int \frac{2x}{2cx^3+b-\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}-\frac{c\int \frac{2x}{2cx^3+b+\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}$$

$\Big\downarrow$ 27

$$\frac{2c\int \frac{x}{2cx^3+b-\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}-\frac{2c\int \frac{x}{2cx^3+b+\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}$$

$\Big\downarrow$ 821

$$\frac{2c\left(\frac{\int \frac{\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b-\sqrt{b^2-4ac}}}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\int \frac{1}{\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b-\sqrt{b^2-4ac}}}dx}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}}-\frac{2c\left(\frac{\int \frac{\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b+\sqrt{b^2-4ac}}}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{\sqrt{b^2-4ac}+b}}-\frac{\int \frac{1}{\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b+\sqrt{b^2-4ac}}}dx}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}}$$

$\Big\downarrow$ 16

$$\frac{2c\left(\frac{\int \frac{\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b-\sqrt{b^2-4ac}}}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}}-\frac{2c\left(\frac{\int \frac{\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b+\sqrt{b^2-4ac}}}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{\sqrt{b^2-4ac}+b}}-\frac{\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{2/3}\sqrt[3]{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}}$$

$\Big\downarrow$ 1142

3.147. $\int \frac{x}{a+bx^3+cx^6}\,dx$

$$\frac{2c\left(\frac{\frac{3}{2}\sqrt[3]{b-\sqrt{b^2-4ac}}\int\frac{1}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx+\frac{\int-\frac{\sqrt[3]{2}\sqrt[3]{c}\left(\sqrt[3]{b-\sqrt{b^2-4ac}}-\cdots\right.}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}\cdots}}{2\sqrt[3]{2}\sqrt[3]{c}}\cdots}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\right.\cdots}{\sqrt{b^2-4ac}}\cdots$$

$$\frac{2c\left(\frac{\frac{3}{2}\sqrt[3]{\sqrt{b^2-4ac}+b}\int\frac{1}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx+\frac{\int-\frac{\sqrt[3]{2}\sqrt[3]{c}\left(\sqrt[3]{b+\sqrt{b^2-4ac}}-\cdots\right.}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}\cdots}}{2\sqrt[3]{2}\sqrt[3]{c}}\cdots}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{\sqrt{b^2-4ac}+b}}\right.\cdots}{\sqrt{b^2-4ac}}\cdots$$

$$\Big\downarrow 25$$

3.147. $\int \frac{x}{a+bx^3+cx^6}\,dx$

$$\frac{2c\left(\frac{\frac{3}{2}\sqrt[3]{b-\sqrt{b^2-4ac}}\int \frac{1}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}\,x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{\int \frac{\sqrt[3]{2}\sqrt[3]{c}\left(\sqrt[3]{b-\sqrt{b^2-4ac}}-2\ldots\right.}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}\,x\ldots}}{2\sqrt[3]{2}\sqrt[3]{c}}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\ldots\right.}{\sqrt{b^2-4ac}}$$

$$\frac{2c\left(\frac{\frac{3}{2}\sqrt[3]{\sqrt{b^2-4ac}+b}\int \frac{1}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}\,x+\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{\int \frac{\sqrt[3]{2}\sqrt[3]{c}\left(\sqrt[3]{b+\sqrt{b^2-4ac}}-2\ldots\right.}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}\,x\ldots}}{2\sqrt[3]{2}\sqrt[3]{c}}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{\sqrt{b^2-4ac}+b}}\ldots\right.}{\sqrt{b^2-4ac}}$$

$\downarrow$ 27

$$\frac{2c\left(\frac{\frac{3}{2}\sqrt[3]{b-\sqrt{b^2-4ac}}\int \frac{1}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}\,x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{1}{2}\int \frac{\sqrt[3]{b-\sqrt{b^2-4ac}}-2\sqrt[3]{\ldots}}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4a\ldots}}}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\ldots\right.}{\sqrt{b^2-4ac}}$$

$$\frac{2c\left(\frac{\frac{3}{2}\sqrt[3]{\sqrt{b^2-4ac}+b}\int \frac{1}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}\,x+\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{1}{2}\int \frac{\sqrt[3]{b+\sqrt{b^2-4ac}}-2\sqrt[3]{\ldots}}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4a\ldots}}}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{\sqrt{b^2-4ac}+b}}\ldots\right.}{\sqrt{b^2-4ac}}$$

$\downarrow$ 1082

3.147. $\int \frac{x}{a+bx^3+cx^6}\,dx$

$$\frac{2c\left(\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)}{\sqrt[3]{2}\sqrt[3]{c}}-\frac{1}{2}\int \frac{\sqrt[3]{b-\sqrt{b^2-4ac}}-2\sqrt[3]{2}\sqrt[3]{c}x}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}}$$

$$\frac{2c\left(\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}\right)}{\sqrt[3]{2}\sqrt[3]{c}}-\frac{1}{2}\int \frac{\sqrt[3]{b+\sqrt{b^2-4ac}}-2\sqrt[3]{2}\sqrt[3]{c}x}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \; 217$$

3.147. $\int \frac{x}{a+bx^3+cx^6}\,dx$

$$\frac{2c\left(\frac{-\frac{1}{2}\int \frac{\sqrt[3]{b-\sqrt{b^2-4ac}}-2\sqrt[3]{2}\sqrt[3]{c}x}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{c}}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\log\left(\sqrt[3]{b-\sqrt{b^2-4}\right.}{3\ 2^{2/3}c^{2/3}\sqrt[3]{b-}}\right.}{\sqrt{b^2-4ac}}$$

$$\frac{2c\left(\frac{-\frac{1}{2}\int \frac{\sqrt[3]{b+\sqrt{b^2-4ac}}-2\sqrt[3]{2}\sqrt[3]{c}x}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{c}}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{\sqrt{b^2-4ac}+b}}-\frac{\log\left(\sqrt[3]{\sqrt{b^2-4ac}-\right.}{3\ 2^{2/3}c^{2/3}\sqrt[3]{\sqrt{b^2}}}\right.}{\sqrt{b^2-4ac}}$$

$$\downarrow$$ 1103

3.147. $\int \frac{x}{a+bx^3+cx^6}\,dx$

$$\frac{2c\left(\frac{\frac{\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{2\sqrt[3]{2}\sqrt[3]{c}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{c}}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\log\left(\sqrt[3]{b-\sqrt{b^2-4}}\right.}{3\ 2^{2/3}c^{2/3}\sqrt[3]{b-}}\right.}{\sqrt{b^2-4ac}}$$

$$\frac{2c\left(\frac{\frac{\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{2\sqrt[3]{2}\sqrt[3]{c}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{c}}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{\sqrt{b^2-4ac}+b}}-\frac{\log\left(\sqrt[3]{\sqrt{b^2-4ac}+}\right.}{3\ 2^{2/3}c^{2/3}\sqrt[3]{\sqrt{b^2}}}\right.}{\sqrt{b^2-4ac}}$$

input
```
Int[x/(a + b*x^3 + c*x^6),x]
```

output
```
(2*c*(-1/3*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x]/(2^(2/3)
*c^(2/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*2^(1/
3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*c^(1/3)))
+ Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*
c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2]/(2*2^(1/3)*c^(1/3)))/(3*2^(1/3)*c^(1/3)
*(b - Sqrt[b^2 - 4*a*c])^(1/3))))/Sqrt[b^2 - 4*a*c] - (2*c*(-1/3*Log[(b +
Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x]/(2^(2/3)*c^(2/3)*(b + Sqrt[b
^2 - 4*a*c])^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + S
qrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*c^(1/3))) + Log[(b + Sqrt[b^2
- 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3
)*c^(2/3)*x^2]/(2*2^(1/3)*c^(1/3)))/(3*2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a
*c])^(1/3))))/Sqrt[b^2 - 4*a*c]
```

3.147. $\int \frac{x}{a+bx^3+cx^6}\,dx$

3.147.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1711
```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[c/q   Int[(d*x)^m/(b/2 - q/2 + c
*x^n), x], x] - Simp[c/q   Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; Free
Q[{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

3.147.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.07

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^6+\_Z^3b+a\right)}\frac{\_R\ln\left(x-\_R\right)}{2\_R^5c+b\_R^2}\right)}{3}$	41
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^6+\_Z^3b+a\right)}\frac{\_R\ln\left(x-\_R\right)}{2\_R^5c+b\_R^2}\right)}{3}$	41

input
```
int(x/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/3*sum(_R/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^6*c+_Z^3*b+a))
```

3.147.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1798 vs. 2(421) = 842.

Time = 0.29 (sec) , antiderivative size = 1798, normalized size of antiderivative = 3.22

$$\int \frac{x}{a+bx^3+cx^6}\,dx = \text{Too large to display}$$

input
```
integrate(x/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

output
```
1/6*(1/2)^(1/3)*(sqrt(-3) - 1)*(-((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12
*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)) + 1)/(a*b^2 - 4*a^2*c))^(1/3)*l
og(4*b*c*x - (1/2)^(2/3)*(b^4 - 4*a*b^2*c + sqrt(-3)*(b^4 - 4*a*b^2*c) - (
a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3 + sqrt(-3)*(a*b^6 - 12*
a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c
 + 48*a^4*b^2*c^2 - 64*a^5*c^3)))*(-((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 -
 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)) + 1)/(a*b^2 - 4*a^2*c))^(2/3
)) - 1/6*(1/2)^(1/3)*(sqrt(-3) + 1)*(-((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6
 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)) + 1)/(a*b^2 - 4*a^2*c))^(1
/3)*log(4*b*c*x - (1/2)^(2/3)*(b^4 - 4*a*b^2*c - sqrt(-3)*(b^4 - 4*a*b^2*c
) - (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3 - sqrt(-3)*(a*b^6
- 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))*sqrt(b^2/(a^2*b^6 - 12*a^3*
b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)))*(-((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2*
b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)) + 1)/(a*b^2 - 4*a^2*c))
^(2/3)) + 1/6*(1/2)^(1/3)*(sqrt(-3) - 1)*(((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2
*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)) - 1)/(a*b^2 - 4*a^2*c)
)^(1/3)*log(4*b*c*x - (1/2)^(2/3)*(b^4 - 4*a*b^2*c + sqrt(-3)*(b^4 - 4*a*b
^2*c) + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3 + sqrt(-3)*(a*
b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))*sqrt(b^2/(a^2*b^6 - 12*
a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)))*(((a*b^2 - 4*a^2*c)*sqrt(b^2...
```

3.147.6 Sympy [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.28

$$\int \frac{x}{a+bx^3+cx^6}\,dx$$
$$= \text{RootSum}\left(t^6\cdot\left(46656a^4c^3-34992a^3b^2c^2+8748a^2b^4c-729ab^6\right)+t^3\left(-432a^2c^2+216ab^2c-27b^4\right)+c,\ \left(\right.\right.$$

input
```
integrate(x/(c*x**6+b*x**3+a),x)
```

output
```
RootSum(_t**6*(46656*a**4*c**3 - 34992*a**3*b**2*c**2 + 8748*a**2*b**4*c -
 729*a*b**6) + _t**3*(-432*a**2*c**2 + 216*a*b**2*c - 27*b**4) + c, Lambda
(_t, _t*log(x + (-15552*_t**5*a**4*c**3 + 11664*_t**5*a**3*b**2*c**2 - 291
6*_t**5*a**2*b**4*c + 243*_t**5*a*b**6 + 72*_t**2*a**2*c**2 - 54*_t**2*a*b
**2*c + 9*_t**2*b**4)/(b*c))))
```

3.147.7 Maxima [F]

$$\int \frac{x}{a+bx^3+cx^6}\,dx=\int \frac{x}{cx^6+bx^3+a}\,dx$$

input
```
integrate(x/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

output
```
integrate(x/(c*x^6 + b*x^3 + a), x)
```

3.147.8 Giac [F]

$$\int \frac{x}{a+bx^3+cx^6}\,dx=\int \frac{x}{cx^6+bx^3+a}\,dx$$

input
```
integrate(x/(c*x^6+b*x^3+a),x, algorithm="giac")
```

output
```
integrate(x/(c*x^6 + b*x^3 + a), x)
```

3.147.9 Mupad [B] (verification not implemented)

Time = 10.79 (sec) , antiderivative size = 1543, normalized size of antiderivative = 2.77

$$\int \frac{x}{a+bx^3+cx^6}\,dx=\text{Too large to display}$$

input
```
int(x/(a + b*x^3 + c*x^6),x)
```

output
```
log(c^4*x - ((27*c^3*x*(b^4 + 8*a^2*c^2 - 6*a*b^2*c) + (27*2^(1/3)*a*b*c^3
*(4*a*c - b^2)^2*((b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2
*c)/(a*(4*a*c - b^2)^3))^(2/3))/2)*(b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*
a^2*c^2 - 8*a*b^2*c))/(54*a*(4*a*c - b^2)^3))*(-(b*(-(4*a*c - b^2)^3)^(1/2
) + b^4 + 16*a^2*c^2 - 8*a*b^2*c)/(54*(a*b^6 - 64*a^4*c^3 - 12*a^2*b^4*c +
 48*a^3*b^2*c^2)))^(1/3) + log(c^4*x + ((27*c^3*x*(b^4 + 8*a^2*c^2 - 6*a*b
^2*c) + (27*2^(1/3)*a*b*c^3*(4*a*c - b^2)^2*(-(b*(-(4*a*c - b^2)^3)^(1/2)
- b^4 - 16*a^2*c^2 + 8*a*b^2*c)/(a*(4*a*c - b^2)^3))^(2/3))/2)*(b*(-(4*a*c
 - b^2)^3)^(1/2) - b^4 - 16*a^2*c^2 + 8*a*b^2*c))/(54*a*(4*a*c - b^2)^3))*
((b*(-(4*a*c - b^2)^3)^(1/2) - b^4 - 16*a^2*c^2 + 8*a*b^2*c)/(54*(a*b^6 -
64*a^4*c^3 - 12*a^2*b^4*c + 48*a^3*b^2*c^2)))^(1/3) - log(c^4*x - ((27*c^3
*x*(b^4 + 8*a^2*c^2 - 6*a*b^2*c) + (27*2^(1/3)*a*b*c^3*(3^(1/2)*1i - 1)*(4
*a*c - b^2)^2*((b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c)
/(a*(4*a*c - b^2)^3))^(2/3))/4)*(b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*a^2
*c^2 - 8*a*b^2*c))/(54*a*(4*a*c - b^2)^3))*((3^(1/2)*1i)/2 + 1/2)*(-(b*(-(
4*a*c - b^2)^3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c)/(54*(a*b^6 - 64*a^4*
c^3 - 12*a^2*b^4*c + 48*a^3*b^2*c^2)))^(1/3) + log(c^4*x - ((27*c^3*x*(b^4
 + 8*a^2*c^2 - 6*a*b^2*c) - (27*2^(1/3)*a*b*c^3*(3^(1/2)*1i + 1)*(4*a*c -
b^2)^2*((b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c)/(a*(4*
a*c - b^2)^3))^(2/3))/4)*(b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*a^2*c^2...
```

3.147. $\int \frac{x}{a+bx^3+cx^6}\,dx$

3.148 $\int \frac{1}{a+bx^3+cx^6}\,dx$

3.148.1 Optimal result

Integrand size = 14, antiderivative size = 558

$$\int \frac{1}{a+bx^3+cx^6}\,dx = -\frac{2^{2/3}c^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}c^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$
$$+\frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$
$$-\frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$
$$-\frac{c^{2/3}\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{3\sqrt[3]{2}\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$
$$+\frac{c^{2/3}\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{3\sqrt[3]{2}\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

output
```
1/3*2^(2/3)*c^(2/3)*ln(2^(1/3)*c^(1/3)*x+(b-(-4*a*c+b^2)^(1/2))^(1/3))/(b-
(-4*a*c+b^2)^(1/2))^(2/3)/(-4*a*c+b^2)^(1/2)-1/6*c^(2/3)*ln(2^(2/3)*c^(2/3
)*x^2-2^(1/3)*c^(1/3)*x*(b-(-4*a*c+b^2)^(1/2))^(1/3)+(b-(-4*a*c+b^2)^(1/2)
)^(2/3))*2^(2/3)/(b-(-4*a*c+b^2)^(1/2))^(2/3)/(-4*a*c+b^2)^(1/2)-1/3*2^(2/
3)*c^(2/3)*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b-(-4*a*c+b^2)^(1/2))^(1/3))
*3^(1/2))*3^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(2/3)/(-4*a*c+b^2)^(1/2)-1/3*2^(2
/3)*c^(2/3)*ln(2^(1/3)*c^(1/3)*x+(b+(-4*a*c+b^2)^(1/2))^(1/3))/(-4*a*c+b^2
)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(2/3)+1/6*c^(2/3)*ln(2^(2/3)*c^(2/3)*x^2-2^
(1/3)*c^(1/3)*x*(b+(-4*a*c+b^2)^(1/2))^(1/3)+(b+(-4*a*c+b^2)^(1/2))^(2/3))
*2^(2/3)/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(2/3)+1/3*2^(2/3)*c^(2/
3)*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b+(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2)
)*3^(1/2)/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(2/3)
```

3.148.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.08

$$\int \frac{1}{a+bx^3+cx^6}\,dx=\frac{1}{3}\text{RootSum}\left[a+b\#1^3+c\#1^6\&,\frac{\log(x-\#1)}{b\#1^2+2c\#1^5}\&\right]$$

input
```
Integrate[(a + b*x^3 + c*x^6)^(-1),x]
```

output
```
RootSum[a + b*#1^3 + c*#1^6 & , Log[x - #1]/(b*#1^2 + 2*c*#1^5) & ]/3
```

3.148.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 484, normalized size of antiderivative = 0.87, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {1685, 750, 16, 27, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+bx^3+cx^6}dx$$

$$\Big\downarrow \text{1685}$$

$$\frac{c\int \frac{1}{cx^3+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx}{\sqrt{b^2-4ac}}-\frac{c\int \frac{1}{cx^3+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{\sqrt{b^2-4ac}}$$

$\downarrow$ 750

$$\frac{c\left(\frac{2^{2/3}\int \frac{2\left(2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}x\right)}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\int \frac{1}{\sqrt[3]{c}x+\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}}dx}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right)}{\sqrt{b^2-4ac}}-\frac{c\left(\frac{2^{2/3}\int \frac{2\left(2^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}-\sqrt[3]{c}x\right)}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{2^{2/3}\int \frac{1}{\sqrt[3]{c}x+\frac{\sqrt[3]{b+\sqrt{b^2-4ac}}}{\sqrt[3]{2}}}dx}{3\left(\sqrt{b^2-4ac}+b\right)^{2/3}}\right)}{\sqrt{b^2-4ac}}$$

$\downarrow$ 16

$$\frac{c\left(\frac{2^{2/3}\int \frac{2\left(2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}x\right)}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right)}{\sqrt{b^2-4ac}}-\frac{c\left(\frac{2^{2/3}\int \frac{2\left(2^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}-\sqrt[3]{c}x\right)}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{2^{2/3}\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{c}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}\right)}{\sqrt{b^2-4ac}}$$

$\downarrow$ 27

3.148. $\int \frac{1}{a+bx^3+cx^6}\,dx$

$$\frac{c\left(\frac{2\ 2^{2/3}\int \frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}x}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right)}{\sqrt{b^2-4ac}}-$$

$$\frac{c\left(\frac{2\ 2^{2/3}\int \frac{2^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}-\sqrt[3]{c}x}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{2^{2/3}\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{c}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}\right)}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \ 1142$$

3.148. $\int \frac{1}{a+bx^3+cx^6}\,dx$

$$\frac{c\left(\frac{2\,2^{2/3}\left(\frac{3\sqrt[3]{b-\sqrt{b^2-4ac}}\int\frac{1}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{2\sqrt[3]{2}}-\frac{\int-\frac{\sqrt[3]{c}\left(2^{2/3}\sqrt[3]{b-\sqrt{b^2}}\right.}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-}}}}{4\sqrt[3]{c}}\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right.}{\sqrt{b^2-4ac}}$$

$$\frac{c\left(\frac{2\,2^{2/3}\left(\frac{3\sqrt[3]{\sqrt{b^2-4ac}+b}\int\frac{1}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx}{2\sqrt[3]{2}}-\frac{\int-\frac{\sqrt[3]{c}\left(2^{2/3}\sqrt[3]{b+\sqrt{b^2}}\right.}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-}}}}{4\sqrt[3]{c}}\right.}{3\left(\sqrt{b^2-4ac}+b\right)^{2/3}}\right.}{\sqrt{b^2-4ac}}$$

$\downarrow$ 25

3.148. $\int \frac{1}{a+bx^3+cx^6}\,dx$

$$c\left(\frac{2\,2^{2/3}\left(\frac{3\sqrt[3]{b-\sqrt{b^2-4ac}}\int\frac{1}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{2\sqrt[3]{2}}+\frac{\int\frac{\sqrt[3]{c}\left(2^{2/3}\sqrt[3]{b-\sqrt{b^2-\cdots}}\right.}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4\cdots}}}}{4\sqrt[3]{c}}\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right.$$

$$\overline{\qquad\qquad\sqrt{b^2-4ac}\qquad\qquad}$$

$$c\left(\frac{2\,2^{2/3}\left(\frac{3\sqrt[3]{\sqrt{b^2-4ac}+b}\int\frac{1}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx}{2\sqrt[3]{2}}+\frac{\int\frac{\sqrt[3]{c}\left(2^{2/3}\sqrt[3]{b+\sqrt{b^2-\cdots}}\right.}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4\cdots}}}}{4\sqrt[3]{c}}\right.}{3\left(\sqrt{b^2-4ac}+b\right)^{2/3}}\right.$$

$$\overline{\qquad\qquad\sqrt{b^2-4ac}\qquad\qquad}$$

$$\Big\downarrow \; 27$$

3.148. $\int \frac{1}{a+bx^3+cx^6}\,dx$

$$\frac{c\left(\frac{2\ 2^{2/3}\left(\frac{3\sqrt[3]{b-\sqrt{b^2-4ac}}\int \frac{1}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{2\sqrt[3]{2}}+\frac{1}{4}\int \frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-\cdots}}}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-\cdots}}}\cdots}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\cdots\right.}{\sqrt{b^2-4ac}}$$

$$\frac{c\left(\frac{2\ 2^{2/3}\left(\frac{3\sqrt[3]{\sqrt{b^2-4ac}+b}\int \frac{1}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx}{2\sqrt[3]{2}}+\frac{1}{4}\int \frac{2^{2/3}\sqrt[3]{b+\sqrt{b^2-\cdots}}}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-\cdots}}}\cdots}{3\left(\sqrt{b^2-4ac}+b\right)^{2/3}}\cdots\right.}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \; 1082$$

3.148. $\int \frac{1}{a+bx^3+cx^6}\,dx$

$$c\left(\frac{2\,2^{2/3}\left(\frac{1}{4}\int \frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-4\sqrt[3]{c}x}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-}}}\right.}{2\sqrt[3]{c}}\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right.$$

$$\overline{\sqrt{b^2-4ac}}$$

$$c\left(\frac{2\,2^{2/3}\left(\frac{1}{4}\int \frac{2^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}-4\sqrt[3]{c}x}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-}}}\right.}{2\sqrt[3]{c}}\right.}{3\left(\sqrt{b^2-4ac}+b\right)^{2/3}}\right.$$

$$\overline{\sqrt{b^2-4ac}}$$

$\downarrow$ 217

3.148. $\int \frac{1}{a+bx^3+cx^6}\,dx$

$$\frac{c\left(\frac{2\,2^{2/3}\left(\frac{1}{4}\int\frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-4\sqrt[3]{c}x}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2\sqrt[3]{c}}\right)}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\log\left(\sqrt[3]{b\cdots}\right.}{3\sqrt[3]{\cdots}}\right.}{\sqrt{b^2-4ac}}$$

$$\frac{c\left(\frac{2\,2^{2/3}\left(\frac{1}{4}\int\frac{2^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}-4\sqrt[3]{c}x}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{2\sqrt[3]{c}}\right)}{3\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{2^{2/3}\log\left(\sqrt[3]{\sqrt{\cdots}}\right.}{3\sqrt[3]{\cdots}}\right.}{\sqrt{b^2-4ac}}$$

$\downarrow$ 1103

3.148. $\int \frac{1}{a+bx^3+cx^6}\,dx$

$$c\left(\frac{2\,2^{2/3}\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2\sqrt[3]{c}}-\frac{\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{4\sqrt[3]{c}}\right)}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\log\left(\sqrt[3]{\vphantom{x}}\right.}{3}\right.$$

$$\overline{\sqrt{b^2-4ac}}$$

$$c\left(\frac{2\,2^{2/3}\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{2\sqrt[3]{c}}-\frac{\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{4\sqrt[3]{c}}\right)}{3\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{2^{2/3}\log\left(\sqrt[3]{\vphantom{x}}\right.}{3}\right.$$

$$\overline{\sqrt{b^2-4ac}}$$

input
```
Int[(a + b*x^3 + c*x^6)^(-1),x]
```

3.148. $\int \frac{1}{a+bx^3+cx^6}\,dx$

output
```
(c*((2^(2/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*c^
(1/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + (2*2^(2/3)*(-1/2*(Sqrt[3]*ArcTan[(1
 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/c^(1/3)
- Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*
c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2]/(4*c^(1/3))))/(3*(b - Sqrt[b^2 - 4*a*c]
)^(2/3))))/Sqrt[b^2 - 4*a*c] - (c*((2^(2/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(1
/3) + 2^(1/3)*c^(1/3)*x])/(3*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)) + (2*2
^(2/3)*(-1/2*(Sqrt[3]*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*
a*c])^(1/3))/Sqrt[3]])/c^(1/3) - Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/
3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2]/(4*c^(1/
3))))/(3*(b + Sqrt[b^2 - 4*a*c])^(2/3))))/Sqrt[b^2 - 4*a*c]
```

3.148.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1685
```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^n), x], x] - Simp[
c/q   Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2,
2*n] && NeQ[b^2 - 4*a*c, 0]
```

3.148.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.07

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^6+\_Z^3b+a\right)}\frac{\ln\left(x-\_R\right)}{2\_R^5c+b\_R^2}\right)}{3}$	40
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^6+\_Z^3b+a\right)}\frac{\ln\left(x-\_R\right)}{2\_R^5c+b\_R^2}\right)}{3}$	40

input
```
int(1/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/3*sum(1/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^6*c+_Z^3*b+a))
```

3.148.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2206 vs. $2(421) = 842$.

Time = 0.33 (sec) , antiderivative size = 2206, normalized size of antiderivative = 3.95

$$\int \frac{1}{a + bx^3 + cx^6} \, dx = \text{Too large to display}$$

input
```
integrate(1/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

output
```
-1/6*(1/2)^(1/3)*(sqrt(-3) + 1)*(((a^2*b^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b^2*
c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) + b
)/(a^2*b^2 - 4*a^3*c))^(1/3)*log(-4*(b^2*c - 2*a*c^2)*x - (1/2)^(1/3)*(b^4
 - 6*a*b^2*c + 8*a^2*c^2 + sqrt(-3)*(b^4 - 6*a*b^2*c + 8*a^2*c^2) - (a^2*b
^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 + sqrt(-3)*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4
*b*c^2))*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a
^6*b^2*c^2 - 64*a^7*c^3)))*(((a^2*b^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b^2*c + 4
*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) + b)/(a^
2*b^2 - 4*a^3*c))^(1/3)) + 1/6*(1/2)^(1/3)*(sqrt(-3) - 1)*(((a^2*b^2 - 4*a
^3*c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*
b^2*c^2 - 64*a^7*c^3)) + b)/(a^2*b^2 - 4*a^3*c))^(1/3)*log(-4*(b^2*c - 2*a
*c^2)*x - (1/2)^(1/3)*(b^4 - 6*a*b^2*c + 8*a^2*c^2 - sqrt(-3)*(b^4 - 6*a*b
^2*c + 8*a^2*c^2) - (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 - sqrt(-3)*(a^2*
b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2))*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4
*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)))*(((a^2*b^2 - 4*a^3*c)
*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c
^2 - 64*a^7*c^3)) + b)/(a^2*b^2 - 4*a^3*c))^(1/3)) - 1/6*(1/2)^(1/3)*(sqrt
(-3) + 1)*(-((a^2*b^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b
^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) - b)/(a^2*b^2 - 4*a^3*c)
)^(1/3)*log(-4*(b^2*c - 2*a*c^2)*x - (1/2)^(1/3)*(b^4 - 6*a*b^2*c + 8*a...
```

3.148.6 Sympy [A] (verification not implemented)

Time = 3.64 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.28

$$\int \frac{1}{a + bx^3 + cx^6} \, dx$$
$$= \text{RootSum}\left(t^6 \cdot \left(46656a^5c^3 - 34992a^4b^2c^2 + 8748a^3b^4c - 729a^2b^6\right) + t^3 \cdot \left(432a^2bc^2 - 216ab^3c + 27b^5\right) + c\right.$$

input
```
integrate(1/(c*x**6+b*x**3+a),x)
```

output
```
RootSum(_t**6*(46656*a**5*c**3 - 34992*a**4*b**2*c**2 + 8748*a**3*b**4*c -
 729*a**2*b**6) + _t**3*(432*a**2*b*c**2 - 216*a*b**3*c + 27*b**5) + c**2,
 Lambda(_t, _t*log(x + (-1296*_t**4*a**4*b*c**2 + 648*_t**4*a**3*b**3*c -
81*_t**4*a**2*b**5 + 12*_t*a**2*c**2 - 15*_t*a*b**2*c + 3*_t*b**4)/(2*a*c*
*2 - b**2*c))))
```

3.148.7 Maxima [F]

$$\int \frac{1}{a+bx^3+cx^6}\,dx=\int \frac{1}{cx^6+bx^3+a}\,dx$$

input
```
integrate(1/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

output
```
integrate(1/(c*x^6 + b*x^3 + a), x)
```

3.148.8 Giac [F]

$$\int \frac{1}{a+bx^3+cx^6}\,dx=\int \frac{1}{cx^6+bx^3+a}\,dx$$

input
```
integrate(1/(c*x^6+b*x^3+a),x, algorithm="giac")
```

output
```
integrate(1/(c*x^6 + b*x^3 + a), x)
```

3.148.9 Mupad [B] (verification not implemented)

Time = 12.31 (sec) , antiderivative size = 2597, normalized size of antiderivative = 4.65

$$\int \frac{1}{a+bx^3+cx^6}\,dx=\text{Too large to display}$$

input
```
int(1/(a + b*x^3 + c*x^6),x)
```

output
```
log(6*c^5*x + (2^(2/3)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^
2 - 8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(a^2*(4*a*c - b^2)^3))^(1/
3)*(36*a*c^5 - 9*b^2*c^4 + (9*2^(1/3)*b*c^3*(x + (2^(2/3)*a*(-(b^5 + b^2*(
-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)
^3)^(1/2))/(a^2*(4*a*c - b^2)^3))^(1/3))/2)*(4*a*c - b^2)^2*(-(b^5 + b^2*(
-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)
^3)^(1/2))/(a^2*(4*a*c - b^2)^3))^(2/3))/2))/6)*((b^5 + b^2*(-(4*a*c - b^2
)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(5
4*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)))^(1/3) + log(6*c
^5*x + (2^(2/3)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a
*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(a^2*(4*a*c - b^2)^3))^(1/3)*(36*
a*c^5 - 9*b^2*c^4 + (9*2^(1/3)*b*c^3*(x + (2^(2/3)*a*(-(b^5 - b^2*(-(4*a*c
 - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^(1/
2))/(a^2*(4*a*c - b^2)^3))^(1/3))/2)*(4*a*c - b^2)^2*(-(b^5 - b^2*(-(4*a*c
 - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^(1/
2))/(a^2*(4*a*c - b^2)^3))^(2/3))/2))/6)*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1
/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(a^2*
b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)))^(1/3) + log(6*c^5*x -
(2^(2/3)*(3^(1/2)*1i - 1)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b
*c^2 - 8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(a^2*(4*a*c - b^2)^3...
```

3.148. $\int \frac{1}{a+bx^3+cx^6}\,dx$

3.149 $\int \frac{1}{x^2(a+bx^3+cx^6)}\,dx$

3.149.1 Optimal result

Integrand size = 18, antiderivative size = 610

$$\int \frac{1}{x^2\left(a+bx^3+cx^6\right)}\,dx$$

$$= -\frac{1}{ax} + \frac{\sqrt[3]{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}a\sqrt[3]{b-\sqrt{b^2-4ac}}}$$

$$+ \frac{\sqrt[3]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}a\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

$$+ \frac{\sqrt[3]{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}}$$

$$+ \frac{\sqrt[3]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}a\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

$$- \frac{\sqrt[3]{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}}$$

$$- \frac{\sqrt[3]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}a\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

output
```
-1/a/x+1/6*c^(1/3)*ln(2^(1/3)*c^(1/3)*x+(b-(-4*a*c+b^2)^(1/2))^(1/3))*(1+b
/(-4*a*c+b^2)^(1/2))*2^(1/3)/a/(b-(-4*a*c+b^2)^(1/2))^(1/3)-1/12*c^(1/3)*l
n(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b-(-4*a*c+b^2)^(1/2))^(1/3)+(b-(-
4*a*c+b^2)^(1/2))^(2/3))*(1+b/(-4*a*c+b^2)^(1/2))*2^(1/3)/a/(b-(-4*a*c+b^2
)^(1/2))^(1/3)+1/6*c^(1/3)*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b-(-4*a*c+b^
2)^(1/2))^(1/3))*3^(1/2))*(1+b/(-4*a*c+b^2)^(1/2))*2^(1/3)/a*3^(1/2)/(b-(-
4*a*c+b^2)^(1/2))^(1/3)+1/6*c^(1/3)*ln(2^(1/3)*c^(1/3)*x+(b+(-4*a*c+b^2)^(
1/2))^(1/3))*(1-b/(-4*a*c+b^2)^(1/2))*2^(1/3)/a/(b+(-4*a*c+b^2)^(1/2))^(1/
3)-1/12*c^(1/3)*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b+(-4*a*c+b^2)^(
1/2))^(1/3)+(b+(-4*a*c+b^2)^(1/2))^(2/3))*(1-b/(-4*a*c+b^2)^(1/2))*2^(1/3)
/a/(b+(-4*a*c+b^2)^(1/2))^(1/3)+1/6*c^(1/3)*arctan(1/3*(1-2*2^(1/3)*c^(1/3
)*x/(b+(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*(1-b/(-4*a*c+b^2)^(1/2))*2^(1/3
)/a*3^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/3)
```

3.149.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.12

$$\int \frac{1}{x^2(a+bx^3+cx^6)}\,dx = -\frac{1}{ax} - \frac{\text{RootSum}\left[a+b\#1^3+c\#1^6\&,\frac{b\log(x-\#1)+c\log(x-\#1)\#1^3}{b\#1+2c\#1^4}\&\right]}{3a}$$

input
```
Integrate[1/(x^2*(a + b*x^3 + c*x^6)),x]
```

output
```
-(1/(a*x)) - RootSum[a + b*#1^3 + c*#1^6 & , (b*Log[x - #1] + c*Log[x - #1
]*#1^3)/(b*#1 + 2*c*#1^4) & ]/(3*a)
```

3.149.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 532, normalized size of antiderivative = 0.87, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {1704, 25, 1834, 27, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(a+bx^3+cx^6\right)}dx$$

$$\Big\downarrow \; 1704$$

$$\frac{\int -\frac{x\left(cx^3+b\right)}{cx^6+bx^3+a}dx}{a}-\frac{1}{ax}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int \frac{x\left(cx^3+b\right)}{cx^6+bx^3+a}dx}{a}-\frac{1}{ax}$$

$$\Big\downarrow \; 1834$$

$$-\frac{\frac{1}{2}c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\int \frac{2x}{2cx^3+b-\sqrt{b^2-4ac}}dx+\frac{1}{2}c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{2x}{2cx^3+b+\sqrt{b^2-4ac}}dx}{a}-\frac{1}{ax}$$

$$\Big\downarrow \; 27$$

$$-\frac{c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\int \frac{x}{2cx^3+b-\sqrt{b^2-4ac}}dx+c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{x}{2cx^3+b+\sqrt{b^2-4ac}}dx}{a}-\frac{1}{ax}$$

$$\Big\downarrow \; 821$$

$$-\frac{c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{\int \frac{\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b-\sqrt{b^2-4ac}}}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\int \frac{1}{\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b-\sqrt{b^2-4ac}}}dx}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)+c\Big(\cdots}{a}-\frac{1}{ax}$$

$$\Big\downarrow \; 16$$

$$-\frac{c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{\int \frac{\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b-\sqrt{b^2-4ac}}}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)+\cdots}{a}-\frac{1}{ax}$$

$$\Big\downarrow \; 1142$$

3.149. $\int \frac{1}{x^2(a+bx^3+cx^6)}dx$

$$-\frac{c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{\frac{3}{2}\sqrt[3]{b-\sqrt{b^2-4ac}}\int\frac{1}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx+\int-\frac{\sqrt[3]{2}\sqrt[3]{c}\left(\sqrt[3]{b}\right.}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{}}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\right.}{}$$

$$\frac{1}{ax}$$

$\downarrow$ 25

$$-\frac{c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{\frac{3}{2}\sqrt[3]{b-\sqrt{b^2-4ac}}\int\frac{1}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx-\int\frac{\sqrt[3]{2}\sqrt[3]{c}\left(\sqrt[3]{b}\right.}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{}}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\right.}{}$$

$$\frac{1}{ax}$$

$\downarrow$ 27

$$-\frac{c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{\frac{3}{2}\sqrt[3]{b-\sqrt{b^2-4ac}}\int\frac{1}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{1}{2}\int\frac{\sqrt[3]{b-}}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\right.}{}$$

$$\frac{1}{ax}$$

$\downarrow$ 1082

3.149. $\int \frac{1}{x^2(a+bx^3+cx^6)}\,dx$

$$-\frac{c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)}{\sqrt[3]{2}\sqrt[3]{c}}-\frac{1}{2}\int \frac{\sqrt[3]{b-\sqrt{b^2-4ac}}-2\sqrt[3]{2}\sqrt[3]{\cdots}}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\cdots}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\cdots\right.}{\cdots}$$

$$\frac{1}{ax}$$

$\downarrow$ 217

$$-\frac{c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{-\frac{1}{2}\int \frac{\sqrt[3]{b-\sqrt{b^2-4ac}}-2\sqrt[3]{2}\sqrt[3]{c}x}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{c}}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\log\cdots}{\cdots}\right.}{\cdots}$$

$$\frac{1}{ax}$$

$\downarrow$ 1103

3.149. $\int \frac{1}{x^2\left(a+bx^3+cx^6\right)}\,dx$

$$-\frac{c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{\frac{\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{2\sqrt[3]{2}\sqrt[3]{c}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{c}}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\log\left(\cdots\right.}{\cdots}\right.}{\cdots}$$

$$\frac{1}{ax}$$

input
```
Int[1/(x^2*(a + b*x^3 + c*x^6)),x]
```

output
```
-(1/(a*x)) - (c*(1 + b/Sqrt[b^2 - 4*a*c])*(-1/3*Log[(b - Sqrt[b^2 - 4*a*c]
)^(1/3) + 2^(1/3)*c^(1/3)*x]/(2^(2/3)*c^(2/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3
)) + (-((Sqrt[3]*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])
^(1/3))/Sqrt[3]])/(2^(1/3)*c^(1/3))) + Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) -
 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2]/(2
*2^(1/3)*c^(1/3)))/(3*2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3))) + c*
(1 - b/Sqrt[b^2 - 4*a*c])*(-1/3*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3
)*c^(1/3)*x]/(2^(2/3)*c^(2/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)) + (-((Sqrt[3]
*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]]
)/(2^(1/3)*c^(1/3))) + Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)
*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2]/(2*2^(1/3)*c^(1/3)
))/(3*2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3))))/a
```

3.149.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1704
```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Simp[1/(a*d^n*(m + 1))   Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1)
+ c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{
a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] &&
LtQ[m, -1] && IntegerQ[p]
```

3.149. $\int \frac{1}{x^2(a+bx^3+cx^6)}\,dx$

rule 1834
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 +
 (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2
 - (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ
[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n
, 0]
```

3.149.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.10

method	result
default	$-\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{6}+\_Z^{3}b+a\right)}\frac{\left(\_R^{4}c+\_Rb\right)\ln\left(x-\_R\right)}{2\_R^{5}c+b\_R^{2}}}{3a}-\frac{1}{ax}$
risch	$-\frac{1}{ax}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(64c^{3}a^{7}-48b^{2}c^{2}a^{6}+12b^{4}c\,a^{5}-b^{6}a^{4}\right)\_Z^{6}+\left(-32b\,c^{3}a^{3}+32b^{3}c^{2}a^{2}-10b^{5}ca+b^{7}\right)\_Z^{3}+c^{4}\right)}\_R\ln\left(\left(\left(224c^{3}a^{7}-176\right.\right.\right.$

input
```
int(1/x^2/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/3/a*sum((_R^4*c+_R*b)/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^6*c+_Z^3*
b+a))-1/a/x
```

3.149.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3225 vs. 2(471) = 942.

Time = 0.39 (sec) , antiderivative size = 3225, normalized size of antiderivative = 5.29

$$\int \frac{1}{x^2\left(a+bx^3+cx^6\right)}\,dx=\text{Too large to display}$$

input
```
integrate(1/x^2/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

output
```
1/6*(2*(1/2)^(1/3)*a*x*((b^3 - 2*a*b*c + (a^4*b^2 - 4*a^5*c)*sqrt((b^8 - 8
*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*
b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(1/3)*log((1
/2)^(2/3)*(b^9 - 11*a*b^7*c + 42*a^2*b^5*c^2 - 62*a^3*b^3*c^3 + 24*a^4*b*c
^4 - (a^4*b^8 - 13*a^5*b^6*c + 60*a^6*b^4*c^2 - 112*a^7*b^2*c^3 + 64*a^8*c
^4)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(
a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)))*((b^3 - 2*a*b*c
+ (a^4*b^2 - 4*a^5*c)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*
c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3))
)/(a^4*b^2 - 4*a^5*c))^(2/3) + 2*(b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*c^5)*x) +
2*(1/2)^(1/3)*a*x*((b^3 - 2*a*b*c - (a^4*b^2 - 4*a^5*c)*sqrt((b^8 - 8*a*b^
6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c
 + 48*a^10*b^2*c^2 - 64*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(1/3)*log((1/2)^(
2/3)*(b^9 - 11*a*b^7*c + 42*a^2*b^5*c^2 - 62*a^3*b^3*c^3 + 24*a^4*b*c^4 +
(a^4*b^8 - 13*a^5*b^6*c + 60*a^6*b^4*c^2 - 112*a^7*b^2*c^3 + 64*a^8*c^4)*s
qrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b
^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)))*((b^3 - 2*a*b*c - (a^
4*b^2 - 4*a^5*c)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 +
 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)))/(a^
4*b^2 - 4*a^5*c))^(2/3) + 2*(b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*c^5)*x) + (1...
```

3.149.6 Sympy [A] (verification not implemented)

Time = 2.22 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.41

$$\int \frac{1}{x^2\left(a+bx^3+cx^6\right)}\,dx$$
$$= \text{RootSum}\left(t^6\cdot\left(46656a^7c^3-34992a^6b^2c^2+8748a^5b^4c-729a^4b^6\right)+t^3\left(-864a^3bc^3+864a^2b^3c^2-270ab^5c\right.\right.$$
$$-\frac{1}{ax}$$

input
```
integrate(1/x**2/(c*x**6+b*x**3+a),x)
```

output
```
RootSum(_t**6*(46656*a**7*c**3 - 34992*a**6*b**2*c**2 + 8748*a**5*b**4*c -
 729*a**4*b**6) + _t**3*(-864*a**3*b*c**3 + 864*a**2*b**3*c**2 - 270*a*b**
5*c + 27*b**7) + c**4, Lambda(_t, _t*log(x + (-15552*_t**5*a**8*c**4 + 272
16*_t**5*a**7*b**2*c**3 - 14580*_t**5*a**6*b**4*c**2 + 3159*_t**5*a**5*b**
6*c - 243*_t**5*a**4*b**8 + 252*_t**2*a**4*b*c**4 - 567*_t**2*a**3*b**3*c*
*3 + 378*_t**2*a**2*b**5*c**2 - 99*_t**2*a*b**7*c + 9*_t**2*b**9)/(2*a**2*
c**5 - 4*a*b**2*c**4 + b**4*c**3)))) - 1/(a*x)
```

3.149.7 Maxima [F]

$$\int \frac{1}{x^2\left(a+bx^3+cx^6\right)}\,dx=\int \frac{1}{(cx^6+bx^3+a)x^2}\,dx$$

input
```
integrate(1/x^2/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

output
```
-integrate((c*x^4 + b*x)/(c*x^6 + b*x^3 + a), x)/a - 1/(a*x)
```

3.149.8 Giac [F]

$$\int \frac{1}{x^2\left(a+bx^3+cx^6\right)}\,dx=\int \frac{1}{(cx^6+bx^3+a)x^2}\,dx$$

input
```
integrate(1/x^2/(c*x^6+b*x^3+a),x, algorithm="giac")
```

output
```
integrate(1/((c*x^6 + b*x^3 + a)*x^2), x)
```

3.149.9 Mupad [B] (verification not implemented)

Time = 11.56 (sec) , antiderivative size = 2978, normalized size of antiderivative = 4.88

$$\int \frac{1}{x^2\left(a+bx^3+cx^6\right)}\,dx=\text{Too large to display}$$

input
```
int(1/(x^2*(a + b*x^3 + c*x^6)),x)
```

output
```
log(36*a^9*c^6 + 9*a^7*b^4*c^4 - 45*a^8*b^2*c^5 - (2^(2/3)*(27*a^7*c^3*x*(
b^6 - 8*a^3*c^3 + 18*a^2*b^2*c^2 - 8*a*b^4*c) + (27*2^(1/3)*a^10*b*c^3*(4*
a*c - b^2)^2*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2
*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(
4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(2/3))/2)*(-(b^7 + b^4*(-(4*
a*c - b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c -
 b^2)^3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*
a*c - b^2)^3))^(1/3))/6)*((b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 32*a^3*b*c
^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c - 4*
a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(a^4*b^6 - 64*a^7*c^3 - 12*a^5*b^4*c
 + 48*a^6*b^2*c^2)))^(1/3) + log(36*a^9*c^6 + 9*a^7*b^4*c^4 - 45*a^8*b^2*c
^5 - (2^(2/3)*(27*a^7*c^3*x*(b^6 - 8*a^3*c^3 + 18*a^2*b^2*c^2 - 8*a*b^4*c)
 + (27*2^(1/3)*a^10*b*c^3*(4*a*c - b^2)^2*((b^4*(-(4*a*c - b^2)^3)^(1/2) -
 b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2)
+ 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^
(2/3))/2)*((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3
*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*
c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(1/3))/6)*(-(b^4*(-(4*a*c - b^2)
^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2
)^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(a^4*...
```

3.149. $\int \frac{1}{x^2(a+bx^3+cx^6)}\,dx$

3.150 $\int \frac{1}{x^3(a+bx^3+cx^6)}\,dx$

3.150.1 Optimal result

Integrand size = 18, antiderivative size = 612

$$\int \frac{1}{x^3 (a+bx^3+cx^6)}\,dx$$

$$= -\frac{1}{2ax^2}+\frac{c^{2/3}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$
$$+\frac{c^{2/3}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}a\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$
$$-\frac{c^{2/3}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$
$$-\frac{c^{2/3}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}a\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$
$$+\frac{c^{2/3}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$
$$+\frac{c^{2/3}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}a\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

output
```
-1/2/a/x^2-1/6*c^(2/3)*ln(2^(1/3)*c^(1/3)*x+(b-(-4*a*c+b^2)^(1/2))^(1/3))*
(1+b/(-4*a*c+b^2)^(1/2))*2^(2/3)/a/(b-(-4*a*c+b^2)^(1/2))^(2/3)+1/12*c^(2/
3)*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b-(-4*a*c+b^2)^(1/2))^(1/3)+(
b-(-4*a*c+b^2)^(1/2))^(2/3))*(1+b/(-4*a*c+b^2)^(1/2))*2^(2/3)/a/(b-(-4*a*c
+b^2)^(1/2))^(2/3)+1/6*c^(2/3)*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b-(-4*a*
c+b^2)^(1/2))^(1/3))*3^(1/2))*(1+b/(-4*a*c+b^2)^(1/2))*2^(2/3)/a*3^(1/2)/(
b-(-4*a*c+b^2)^(1/2))^(2/3)-1/6*c^(2/3)*ln(2^(1/3)*c^(1/3)*x+(b+(-4*a*c+b^
2)^(1/2))^(1/3))*(1-b/(-4*a*c+b^2)^(1/2))*2^(2/3)/a/(b+(-4*a*c+b^2)^(1/2))
^(2/3)+1/12*c^(2/3)*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b+(-4*a*c+b^
2)^(1/2))^(1/3)+(b+(-4*a*c+b^2)^(1/2))^(2/3))*(1-b/(-4*a*c+b^2)^(1/2))*2^(
2/3)/a/(b+(-4*a*c+b^2)^(1/2))^(2/3)+1/6*c^(2/3)*arctan(1/3*(1-2*2^(1/3)*c^
(1/3)*x/(b+(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*(1-b/(-4*a*c+b^2)^(1/2))*2^
(2/3)/a*3^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(2/3)
```

3.150.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.12

$$\int \frac{1}{x^3 (a + bx^3 + cx^6)} \, dx = -\frac{1}{2ax^2} - \frac{\text{RootSum}\left[a + b\#1^3 + c\#1^6\&, \frac{b\log(x-\#1)+c\log(x-\#1)\#1^3}{b\#1^2+2c\#1^5}\&\right]}{3a}$$

input
```
Integrate[1/(x^3*(a + b*x^3 + c*x^6)),x]
```

output
```
-1/2*1/(a*x^2) - RootSum[a + b*#1^3 + c*#1^6 & , (b*Log[x - #1] + c*Log[x
- #1]*#1^3)/(b*#1^2 + 2*c*#1^5) & ]/(3*a)
```

3.150.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 514, normalized size of antiderivative = 0.84, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {1704, 27, 1752, 750, 16, 27, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\left(a+bx^3+cx^6\right)}\,dx$$

$\Big\downarrow$ 1704

$$\frac{\int -\frac{2(cx^3+b)}{cx^6+bx^3+a}dx}{2a}-\frac{1}{2ax^2}$$

$\Big\downarrow$ 27

$$-\frac{\int \frac{cx^3+b}{cx^6+bx^3+a}dx}{a}-\frac{1}{2ax^2}$$

$\Big\downarrow$ 1752

$$-\frac{\frac{1}{2}c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\int \frac{1}{cx^3+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^3+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{a}-\frac{1}{2ax^2}$$

$\Big\downarrow$ 750

$$-\frac{\frac{1}{2}c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{2^{2/3}\int \frac{2\left(2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}x\right)}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\int \frac{1}{\sqrt[3]{c}x+\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}}dx}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right.}{}$$

$$\frac{1}{2ax^2}$$

$\Big\downarrow$ 16

$$-\frac{\frac{1}{2}c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{2^{2/3}\int \frac{2\left(2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}x\right)}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{\cdots}\right.}{3\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right.}{}$$

$$\frac{1}{2ax^2}$$

$\Big\downarrow$ 27

3.150. $\int \frac{1}{x^3(a+bx^3+cx^6)}\,dx$

$$-\frac{\frac{1}{2}c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{2\ 2^{2/3}\int \frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}x}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\cdots\right.}{3\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\cdots\right.}{\cdots}$$

$$\frac{1}{2ax^2}$$

$\downarrow$ 1142

$$-\frac{\frac{1}{2}c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{2\ 2^{2/3}\left(\frac{3\sqrt[3]{b-\sqrt{b^2-4ac}}\int \frac{1}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{2\sqrt[3]{2}}-\frac{\int -\cdots}{2c^{2/3}x^2-2^{2/3}\cdots}\cdots\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\cdots\right.}{\cdots}$$

$$\frac{1}{2ax^2}$$

$\downarrow$ 25

3.150. $\int \frac{1}{x^3(a+bx^3+cx^6)}\,dx$

$$-\frac{\frac{1}{2}c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{2\ 2^{2/3}\left(\frac{3\sqrt[3]{b-\sqrt{b^2-4ac}}\int\frac{1}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{2\sqrt[3]{2}}+\frac{\int}{2c^{2/3}x^2-2^{2/3}\cdots}\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right.}{\cdots}$$

$$\frac{1}{2ax^2}$$

$\downarrow$ 27

$$-\frac{\frac{1}{2}c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{2\ 2^{2/3}\left(\frac{3\sqrt[3]{b-\sqrt{b^2-4ac}}\int\frac{1}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{2\sqrt[3]{2}}+\frac{1}{4}\int\frac{}{2c^{2/3}x^2-2^{2}\cdots}\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right.}{\cdots}$$

$$\frac{1}{2ax^2}$$

$\downarrow$ 1082

3.150. $\int \frac{1}{x^3(a+bx^3+cx^6)}\,dx$

$$-\frac{\frac{1}{2}c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{2\;2^{2/3}\left(\frac{1}{4}\int\frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-4\sqrt[3]{c}x}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)^2-\cdots}\cdots}{2\sqrt[3]{c}\cdots}\cdots\right)}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}\cdots}\cdots\right)}{\cdots}$$

$$\frac{1}{2ax^2}$$

$\downarrow$ 217

$$-\frac{\frac{1}{2}c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{2\;2^{2/3}\left(\frac{1}{4}\int\frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-4\sqrt[3]{c}x}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}\cdots}{\sqrt{3}}\cdots\right)\cdots}{2\sqrt[3]{c}}\cdots\right)}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}\cdots}\cdots\right)}{\cdots}$$

$$\frac{1}{2ax^2}$$

3.150. $\int\frac{1}{x^3\left(a+bx^3+cx^6\right)}\,dx$

$$\Big\downarrow \text{1103}$$

$$-\frac{\frac{1}{2}c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{2\,2^{2/3}\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2\sqrt[3]{c}}-\frac{\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}\right.}{4\sqrt[3]{c}}\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right.}{\frac{1}{2ax^2}}$$

input
```
Int[1/(x^3*(a + b*x^3 + c*x^6)),x]
```

output
```
-1/2*1/(a*x^2) - ((c*(1 + b/Sqrt[b^2 - 4*a*c])*((2^(2/3)*Log[(b - Sqrt[b^2
 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^
(2/3)) + (2*2^(2/3)*(-1/2*(Sqrt[3]*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b -
Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/c^(1/3) - Log[(b - Sqrt[b^2 - 4*a*c])^
(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*
x^2]/(4*c^(1/3))))/(3*(b - Sqrt[b^2 - 4*a*c])^(2/3))))/2 + (c*(1 - b/Sqrt[
b^2 - 4*a*c])*((2^(2/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3
)*x])/(3*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)) + (2*2^(2/3)*(-1/2*(Sqrt[3
]*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]
])/c^(1/3) - Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt
[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2]/(4*c^(1/3))))/(3*(b + Sqrt[b
^2 - 4*a*c])^(2/3))))/2)/a
```

3.150. $\int \frac{1}{x^3(a+bx^3+cx^6)}\,dx$

3.150.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1704
```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Simp[1/(a*d^n*(m + 1))   Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1)
+ c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{
a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] &&
LtQ[m, -1] && IntegerQ[p]
```

rule 1752
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

3.150.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.10

method	result
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^6+\_Z^3b+a\right)}\frac{\left(-c\_R^3-b\right)\ln\left(x-\_R\right)}{2\_R^5c+b\_R^2}}{3a}-\frac{1}{2a\,x^2}$
risch	$-\frac{1}{2a\,x^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(64c^3a^8-48a^7b^2c^2+12a^6b^4c-a^5b^6\right)\_Z^6+\left(-16a^4c^4+56a^3b^2c^3-41b^4c^2a^2+11a\,b^6c-b^8\right)\_Z^3+c^5\right)}\_R\ln\left(\left(\left(22\right.\right.\right.$

input
```
int(1/x^3/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/3/a*sum((-_R^3*c-b)/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^6*c+_Z^3*b+a
))-1/2/a/x^2
```

3.150.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3225 vs. $2(471) = 942$.

Time = 0.42 (sec) , antiderivative size = 3225, normalized size of antiderivative = 5.27

$$\int \frac{1}{x^3 (a + bx^3 + cx^6)} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^3/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

output
```
1/6*(2*(1/2)^(1/3)*a*x^2*(-(b^4 - 3*a*b^2*c + a^2*c^2 + (a^5*b^2 - 4*a^6*c
)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c
^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^2
- 4*a^6*c))^(1/3)*log(2*(b^5*c^2 - 5*a*b^3*c^3 + 5*a^2*b*c^4)*x + (1/2)^(1
/3)*(b^8 - 9*a*b^6*c + 25*a^2*b^4*c^2 - 20*a^3*b^2*c^3 - (a^5*b^6 - 10*a^6
*b^4*c + 32*a^7*b^2*c^2 - 32*a^8*c^3)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6
*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^1
2*b^2*c^2 - 64*a^13*c^3)))*(-(b^4 - 3*a*b^2*c + a^2*c^2 + (a^5*b^2 - 4*a^6
*c)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2
*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^
2 - 4*a^6*c))^(1/3)) + 2*(1/2)^(1/3)*a*x^2*(-(b^4 - 3*a*b^2*c + a^2*c^2 -
(a^5*b^2 - 4*a^6*c)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*
c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^1
3*c^3)))/(a^5*b^2 - 4*a^6*c))^(1/3)*log(2*(b^5*c^2 - 5*a*b^3*c^3 + 5*a^2*b
*c^4)*x + (1/2)^(1/3)*(b^8 - 9*a*b^6*c + 25*a^2*b^4*c^2 - 20*a^3*b^2*c^3 +
 (a^5*b^6 - 10*a^6*b^4*c + 32*a^7*b^2*c^2 - 32*a^8*c^3)*sqrt((b^10 - 10*a*
b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a
^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*(-(b^4 - 3*a*b^2*c + a^2*c^2
- (a^5*b^2 - 4*a^6*c)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^
4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 6...
```

3.150.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a + bx^3 + cx^6)} \, dx = \text{Timed out}$$

input
```
integrate(1/x**3/(c*x**6+b*x**3+a),x)
```

output
```
Timed out
```

3.150.7 Maxima [F]

$$\int \frac{1}{x^3 (a + bx^3 + cx^6)} \, dx = \int \frac{1}{(cx^6 + bx^3 + a)x^3} \, dx$$

input
```
integrate(1/x^3/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

output
```
-integrate((c*x^3 + b)/(c*x^6 + b*x^3 + a), x)/a - 1/2/(a*x^2)
```

3.150.8 Giac [F]

$$\int \frac{1}{x^3 (a + bx^3 + cx^6)} \, dx = \int \frac{1}{(cx^6 + bx^3 + a)x^3} \, dx$$

input
```
integrate(1/x^3/(c*x^6+b*x^3+a),x, algorithm="giac")
```

output
```
integrate(1/((c*x^6 + b*x^3 + a)*x^3), x)
```

3.150.9 Mupad [B] (verification not implemented)

Time = 13.75 (sec) , antiderivative size = 4063, normalized size of antiderivative = 6.64

$$\int \frac{1}{x^3 (a + bx^3 + cx^6)} \, dx = \text{Too large to display}$$

input
```
int(1/(x^3*(a + b*x^3 + c*x^6)),x)
```

output
```
log((2^(2/3)*((b^8 + 16*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^
4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2)
 - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))^(1/3)*(72*a^
8*b*c^6 + (2^(1/3)*(81*a^8*c^3*x*(a*c - b^2)*(4*a*c - b^2)^2 + (81*2^(2/3)
*a^10*b*c^3*(4*a*c - b^2)^2*((b^8 + 16*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1
/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c
- b^2)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3
))^(1/3))/2)*((b^8 + 16*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^
4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2)
 - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))^(2/3))/18 +
9*a^6*b^5*c^4 - 54*a^7*b^3*c^5))/6 - 3*a^6*c^6*x*(2*a*c - b^2))*(-(b^8 + 1
6*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3
 - 11*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c
- b^2)^3)^(1/2))/(54*(a^5*b^6 - 64*a^8*c^3 - 12*a^6*b^4*c + 48*a^7*b^2*c^2
)))^(1/3) + log((2^(2/3)*((b^8 + 16*a^4*c^4 - b^5*(-(4*a*c - b^2)^3)^(1/2)
 + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c - 5*a^2*b*c^2*(-(4*a*c - b
^2)^3)^(1/2) + 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))^
(1/3)*(72*a^8*b*c^6 + (2^(1/3)*(81*a^8*c^3*x*(a*c - b^2)*(4*a*c - b^2)^2 +
 (81*2^(2/3)*a^10*b*c^3*(4*a*c - b^2)^2*((b^8 + 16*a^4*c^4 - b^5*(-(4*a*c
- b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c - 5*a^2*...
```

3.150. $\int \frac{1}{x^3(a+bx^3+cx^6)}\,dx$

3.151 $\int \frac{x^{11}}{3+4x^3+x^6}\,dx$

3.151.1 Optimal result

Integrand size = 16, antiderivative size = 35

$$\int \frac{x^{11}}{3+4x^3+x^6}\,dx=-\frac{4x^3}{3}+\frac{x^6}{6}-\frac{1}{6}\log\left(1+x^3\right)+\frac{9}{2}\log\left(3+x^3\right)$$

output
```
-4/3*x^3+1/6*x^6-1/6*ln(x^3+1)+9/2*ln(x^3+3)
```

3.151.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{x^{11}}{3+4x^3+x^6}\,dx=-\frac{4x^3}{3}+\frac{x^6}{6}-\frac{1}{6}\log\left(1+x^3\right)+\frac{9}{2}\log\left(3+x^3\right)$$

input
```
Integrate[x^11/(3 + 4*x^3 + x^6),x]
```

output
```
(-4*x^3)/3 + x^6/6 - Log[1 + x^3]/6 + (9*Log[3 + x^3])/2
```

3.151.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1693, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{x^6+4x^3+3}dx$$
$$\Big\downarrow \text{1693}$$
$$\frac{1}{3}\int \frac{x^9}{x^6+4x^3+3}dx^3$$
$$\Big\downarrow \text{1141}$$
$$\frac{1}{3}\int \left(x^3-\frac{1}{2(x^3+1)}+\frac{27}{2(x^3+3)}-4\right)dx^3$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{3}\left(\frac{x^6}{2}-4x^3-\frac{1}{2}\log\left(x^3+1\right)+\frac{27}{2}\log\left(x^3+3\right)\right)$$

input
```
Int[x^11/(3 + 4*x^3 + x^6),x]
```

output
```
(-4*x^3 + x^6/2 - Log[1 + x^3]/2 + (27*Log[3 + x^3])/2)/3
```

3.151.3.1 Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.151.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

method	result	size
default	$-\frac{4x^3}{3}+\frac{x^6}{6}-\frac{\ln(x^3+1)}{6}+\frac{9\ln(x^3+3)}{2}$	28
risch	$\frac{x^6}{6}-\frac{4x^3}{3}+\frac{8}{3}-\frac{\ln(x^3+1)}{6}+\frac{9\ln(x^3+3)}{2}$	29
norman	$-\frac{4x^3}{3}+\frac{x^6}{6}-\frac{\ln(x+1)}{6}+\frac{9\ln(x^3+3)}{2}-\frac{\ln(x^2-x+1)}{6}$	37
parallelrisch	$-\frac{4x^3}{3}+\frac{x^6}{6}-\frac{\ln(x+1)}{6}+\frac{9\ln(x^3+3)}{2}-\frac{\ln(x^2-x+1)}{6}$	37

input
```
int(x^11/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
```

output
```
-4/3*x^3+1/6*x^6-1/6*ln(x^3+1)+9/2*ln(x^3+3)
```

3.151.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{x^{11}}{3+4x^3+x^6}\,dx=\frac{1}{6}\,x^6-\frac{4}{3}\,x^3+\frac{9}{2}\,\log\left(x^3+3\right)-\frac{1}{6}\,\log\left(x^3+1\right)$$

input
```
integrate(x^11/(x^6+4*x^3+3),x, algorithm="fricas")
```

output
```
1/6*x^6 - 4/3*x^3 + 9/2*log(x^3 + 3) - 1/6*log(x^3 + 1)
```

3.151.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \frac{x^{11}}{3+4x^3+x^6}\,dx = \frac{x^6}{6} - \frac{4x^3}{3} - \frac{\log(x^3+1)}{6} + \frac{9\log(x^3+3)}{2}$$

input
```
integrate(x**11/(x**6+4*x**3+3),x)
```

output
```
x**6/6 - 4*x**3/3 - log(x**3 + 1)/6 + 9*log(x**3 + 3)/2
```

3.151.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{x^{11}}{3+4x^3+x^6}\,dx = \frac{1}{6}x^6 - \frac{4}{3}x^3 + \frac{9}{2}\log(x^3+3) - \frac{1}{6}\log(x^3+1)$$

input
```
integrate(x^11/(x^6+4*x^3+3),x, algorithm="maxima")
```

output
```
1/6*x^6 - 4/3*x^3 + 9/2*log(x^3 + 3) - 1/6*log(x^3 + 1)
```

3.151.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \frac{x^{11}}{3+4x^3+x^6}\,dx = \frac{1}{6}x^6 - \frac{4}{3}x^3 + \frac{9}{2}\log(|x^3+3|) - \frac{1}{6}\log(|x^3+1|)$$

input
```
integrate(x^11/(x^6+4*x^3+3),x, algorithm="giac")
```

output
```
1/6*x^6 - 4/3*x^3 + 9/2*log(abs(x^3 + 3)) - 1/6*log(abs(x^3 + 1))
```

3.151.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{x^{11}}{3+4x^3+x^6}\,dx = \frac{9\,\ln\left(x^3+3\right)}{2} - \frac{\ln\left(x^3+1\right)}{6} - \frac{4\,x^3}{3} + \frac{x^6}{6}$$

input
```
int(x^11/(4*x^3 + x^6 + 3),x)
```

output
```
(9*log(x^3 + 3))/2 - log(x^3 + 1)/6 - (4*x^3)/3 + x^6/6
```

3.152 $\int \frac{x^8}{3+4x^3+x^6} \, dx$

3.152.1 Optimal result

Integrand size = 16, antiderivative size = 28

$$\int \frac{x^8}{3+4x^3+x^6} \, dx = \frac{x^3}{3} + \frac{1}{6} \log \left(1+x^3\right) - \frac{3}{2} \log \left(3+x^3\right)$$

output
```
1/3*x^3+1/6*ln(x^3+1)-3/2*ln(x^3+3)
```

3.152.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^8}{3+4x^3+x^6} \, dx = \frac{x^3}{3} + \frac{1}{6} \log \left(1+x^3\right) - \frac{3}{2} \log \left(3+x^3\right)$$

input
```
Integrate[x^8/(3 + 4*x^3 + x^6),x]
```

output
```
x^3/3 + Log[1 + x^3]/6 - (3*Log[3 + x^3])/2
```

3.152.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1693, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{x^6+4x^3+3} \, dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int \frac{x^6}{x^6+4x^3+3}dx^3$$

$$\Big\downarrow \text{1141}$$

$$\frac{1}{3}\int \left(-\frac{9}{2\left(x^3+3\right)}+1+\frac{1}{2\left(x^3+1\right)}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(x^3+\frac{1}{2}\log\left(x^3+1\right)-\frac{9}{2}\log\left(x^3+3\right)\right)$$

input
```
Int[x^8/(3 + 4*x^3 + x^6),x]
```

output
```
(x^3 + Log[1 + x^3]/2 - (9*Log[3 + x^3])/2)/3
```

3.152.3.1 Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.152.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{x^3}{3}+\frac{\ln(x^3+1)}{6}-\frac{3\ln(x^3+3)}{2}$	23
risch	$\frac{x^3}{3}+\frac{\ln(x^3+1)}{6}-\frac{3\ln(x^3+3)}{2}$	23
norman	$\frac{x^3}{3}+\frac{\ln(x+1)}{6}-\frac{3\ln(x^3+3)}{2}+\frac{\ln(x^2-x+1)}{6}$	32
parallelrisch	$\frac{x^3}{3}+\frac{\ln(x+1)}{6}-\frac{3\ln(x^3+3)}{2}+\frac{\ln(x^2-x+1)}{6}$	32

input
```
int(x^8/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3+1/6*ln(x^3+1)-3/2*ln(x^3+3)
```

3.152.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \frac{x^8}{3+4x^3+x^6}\,dx=\frac{1}{3}\,x^3-\frac{3}{2}\,\log\left(x^3+3\right)+\frac{1}{6}\,\log\left(x^3+1\right)$$

input
```
integrate(x^8/(x^6+4*x^3+3),x, algorithm="fricas")
```

output
```
1/3*x^3 - 3/2*log(x^3 + 3) + 1/6*log(x^3 + 1)
```

3.152.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \frac{x^8}{3+4x^3+x^6}\,dx = \frac{x^3}{3} + \frac{\log\left(x^3+1\right)}{6} - \frac{3\log\left(x^3+3\right)}{2}$$

input
```
integrate(x**8/(x**6+4*x**3+3),x)
```

output
```
x**3/3 + log(x**3 + 1)/6 - 3*log(x**3 + 3)/2
```

3.152.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \frac{x^8}{3+4x^3+x^6}\,dx = \frac{1}{3}\,x^3 - \frac{3}{2}\,\log\left(x^3+3\right) + \frac{1}{6}\,\log\left(x^3+1\right)$$

input
```
integrate(x^8/(x^6+4*x^3+3),x, algorithm="maxima")
```

output
```
1/3*x^3 - 3/2*log(x^3 + 3) + 1/6*log(x^3 + 1)
```

3.152.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{x^8}{3+4x^3+x^6}\,dx = \frac{1}{3}\,x^3 - \frac{3}{2}\,\log\left(\left|x^3+3\right|\right) + \frac{1}{6}\,\log\left(\left|x^3+1\right|\right)$$

input
```
integrate(x^8/(x^6+4*x^3+3),x, algorithm="giac")
```

output
```
1/3*x^3 - 3/2*log(abs(x^3 + 3)) + 1/6*log(abs(x^3 + 1))
```

3.152.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \frac{x^8}{3+4x^3+x^6}\,dx = \frac{\ln(x^3+1)}{6} - \frac{3\,\ln(x^3+3)}{2} + \frac{x^3}{3}$$

input
```
int(x^8/(4*x^3 + x^6 + 3),x)
```

output
```
log(x^3 + 1)/6 - (3*log(x^3 + 3))/2 + x^3/3
```

3.153 $\int \frac{x^5}{3+4x^3+x^6}\,dx$

3.153.1 Optimal result

Integrand size = 16, antiderivative size = 21

$$\int \frac{x^5}{3+4x^3+x^6}\,dx=-\frac{1}{6}\log\left(1+x^3\right)+\frac{1}{2}\log\left(3+x^3\right)$$

output
```
-1/6*ln(x^3+1)+1/2*ln(x^3+3)
```

3.153.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{3+4x^3+x^6}\,dx=-\frac{1}{6}\log\left(1+x^3\right)+\frac{1}{2}\log\left(3+x^3\right)$$

input
```
Integrate[x^5/(3 + 4*x^3 + x^6),x]
```

output
```
-1/6*Log[1 + x^3] + Log[3 + x^3]/2
```

3.153.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1693, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{x^6+4x^3+3} \, dx$$

$$\Big\downarrow \text{ 1693}$$

$$\frac{1}{3}\int \frac{x^3}{x^6+4x^3+3}dx^3$$

$$\Big\downarrow \text{ 1141}$$

$$\frac{1}{3}\int \left(\frac{3}{2(x^3+3)}-\frac{1}{2(x^3+1)}\right)dx^3$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}\left(\frac{3}{2}\log\left(x^3+3\right)-\frac{1}{2}\log\left(x^3+1\right)\right)$$

input
```
Int[x^5/(3 + 4*x^3 + x^6),x]
```

output
```
(-1/2*Log[1 + x^3] + (3*Log[3 + x^3])/2)/3
```

3.153.3.1 Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.153.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
default	$-\frac{\ln(x^3+1)}{6}+\frac{\ln(x^3+3)}{2}$	18
risch	$-\frac{\ln(x^3+1)}{6}+\frac{\ln(x^3+3)}{2}$	18
norman	$-\frac{\ln(x+1)}{6}+\frac{\ln(x^3+3)}{2}-\frac{\ln(x^2-x+1)}{6}$	27
parallelrisch	$-\frac{\ln(x+1)}{6}+\frac{\ln(x^3+3)}{2}-\frac{\ln(x^2-x+1)}{6}$	27

input
```
int(x^5/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
```

output
```
-1/6*ln(x^3+1)+1/2*ln(x^3+3)
```

3.153.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{x^5}{3+4x^3+x^6}\,dx=\frac{1}{2}\log\left(x^3+3\right)-\frac{1}{6}\log\left(x^3+1\right)$$

input
```
integrate(x^5/(x^6+4*x^3+3),x, algorithm="fricas")
```

output
```
1/2*log(x^3 + 3) - 1/6*log(x^3 + 1)
```

3.153.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{x^5}{3+4x^3+x^6}\,dx = -\frac{\log\left(x^3+1\right)}{6}+\frac{\log\left(x^3+3\right)}{2}$$

input
```
integrate(x**5/(x**6+4*x**3+3),x)
```

output
```
-log(x**3 + 1)/6 + log(x**3 + 3)/2
```

3.153.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{x^5}{3+4x^3+x^6}\,dx = \frac{1}{2}\log\left(x^3+3\right)-\frac{1}{6}\log\left(x^3+1\right)$$

input
```
integrate(x^5/(x^6+4*x^3+3),x, algorithm="maxima")
```

output
```
1/2*log(x^3 + 3) - 1/6*log(x^3 + 1)
```

3.153.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{x^5}{3+4x^3+x^6}\,dx = \frac{1}{2}\log\left(\left|x^3+3\right|\right)-\frac{1}{6}\log\left(\left|x^3+1\right|\right)$$

input
```
integrate(x^5/(x^6+4*x^3+3),x, algorithm="giac")
```

output
```
1/2*log(abs(x^3 + 3)) - 1/6*log(abs(x^3 + 1))
```

3.153.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{x^5}{3+4x^3+x^6}\,dx = \frac{\ln(x^3+3)}{2} - \frac{\ln(x^3+1)}{6}$$

input
```
int(x^5/(4*x^3 + x^6 + 3),x)
```

output
```
log(x^3 + 3)/2 - log(x^3 + 1)/6
```

3.154 $\int \frac{x^2}{3+4x^3+x^6}\,dx$

3.154.1 Optimal result

Integrand size = 16, antiderivative size = 10

$$\int \frac{x^2}{3+4x^3+x^6}\,dx = -\frac{1}{3}\text{arctanh}\left(2+x^3\right)$$

output
```
-1/3*arctanh(x^3+2)
```

3.154.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 21 vs. 2(10) = 20.

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 2.10

$$\int \frac{x^2}{3+4x^3+x^6}\,dx = \frac{1}{6}\log\left(1+x^3\right)-\frac{1}{6}\log\left(3+x^3\right)$$

input
```
Integrate[x^2/(3 + 4*x^3 + x^6),x]
```

output
```
Log[1 + x^3]/6 - Log[3 + x^3]/6
```

3.154.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 25 vs. 2(10) = 20.

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.50, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1690, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{x^6+4x^3+3}\,dx$$

$$\Big\downarrow \text{1690}$$

$$\frac{1}{3}\int \frac{1}{x^6+4x^3+3}dx^3$$

$$\Big\downarrow \text{1081}$$

$$\frac{1}{3}\int \left(\frac{1}{2(x^3+1)}-\frac{1}{2(x^3+3)}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{1}{2}\log\left(x^3+1\right)-\frac{1}{2}\log\left(x^3+3\right)\right)$$

input
```
Int[x^2/(3 + 4*x^3 + x^6),x]
```

output
```
(Log[1 + x^3]/2 - Log[3 + x^3]/2)/3
```

3.154.3.1 Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.154.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(8) = 16.

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.80

method	result	size
default	$-\frac{\ln\left(x^{3}+3\right)}{6}+\frac{\ln\left(x^{3}+1\right)}{6}$	18
risch	$-\frac{\ln\left(x^{3}+3\right)}{6}+\frac{\ln\left(x^{3}+1\right)}{6}$	18
norman	$\frac{\ln\left(x+1\right)}{6}-\frac{\ln\left(x^{3}+3\right)}{6}+\frac{\ln\left(x^{2}-x+1\right)}{6}$	27
parallelrisch	$\frac{\ln\left(x+1\right)}{6}-\frac{\ln\left(x^{3}+3\right)}{6}+\frac{\ln\left(x^{2}-x+1\right)}{6}$	27

input
```
int(x^2/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
```

output
```
-1/6*ln(x^3+3)+1/6*ln(x^3+1)
```

3.154.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(8) = 16.

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.70

$$\int \frac{x^2}{3+4x^3+x^6}\,dx=-\frac{1}{6}\,\log\left(x^3+3\right)+\frac{1}{6}\,\log\left(x^3+1\right)$$

input
```
integrate(x^2/(x^6+4*x^3+3),x, algorithm="fricas")
```

output
```
-1/6*log(x^3 + 3) + 1/6*log(x^3 + 1)
```

3.154.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.50

$$\int \frac{x^2}{3+4x^3+x^6}\,dx = \frac{\log(x^3+1)}{6} - \frac{\log(x^3+3)}{6}$$

input
```
integrate(x**2/(x**6+4*x**3+3),x)
```

output
```
log(x**3 + 1)/6 - log(x**3 + 3)/6
```

3.154.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(8) = 16.

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.70

$$\int \frac{x^2}{3+4x^3+x^6}\,dx = -\frac{1}{6}\log\left(x^3+3\right) + \frac{1}{6}\log\left(x^3+1\right)$$

input
```
integrate(x^2/(x^6+4*x^3+3),x, algorithm="maxima")
```

output
```
-1/6*log(x^3 + 3) + 1/6*log(x^3 + 1)
```

3.154.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(8) = 16.

Time = 0.30 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.90

$$\int \frac{x^2}{3+4x^3+x^6}\,dx = -\frac{1}{6}\log\left(\left|x^3+3\right|\right) + \frac{1}{6}\log\left(\left|x^3+1\right|\right)$$

input
```
integrate(x^2/(x^6+4*x^3+3),x, algorithm="giac")
```

output
```
-1/6*log(abs(x^3 + 3)) + 1/6*log(abs(x^3 + 1))
```

3.154.9 Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

$$\int \frac{x^2}{3+4x^3+x^6}\,dx = \frac{\operatorname{atanh}\left(\frac{9}{2\,(8\,x^3+6)}+\frac{5}{4}\right)}{3}$$

input
```
int(x^2/(4*x^3 + x^6 + 3),x)
```

output
```
atanh(9/(2*(8*x^3 + 6)) + 5/4)/3
```

3.155 $\int \frac{1}{x(3+4x^3+x^6)} dx$

3.155.1 Optimal result

Integrand size = 16, antiderivative size = 27

$$\int \frac{1}{x(3+4x^3+x^6)} dx = \frac{\log(x)}{3} - \frac{1}{6}\log\left(1+x^3\right) + \frac{1}{18}\log\left(3+x^3\right)$$

output
```
1/3*ln(x)-1/6*ln(x^3+1)+1/18*ln(x^3+3)
```

3.155.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(3+4x^3+x^6)} dx = \frac{\log(x)}{3} - \frac{1}{6}\log\left(1+x^3\right) + \frac{1}{18}\log\left(3+x^3\right)$$

input
```
Integrate[1/(x*(3 + 4*x^3 + x^6)),x]
```

output
```
Log[x]/3 - Log[1 + x^3]/6 + Log[3 + x^3]/18
```

3.155.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.22, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1693, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(x^6+4x^3+3)}dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int \frac{1}{x^3(x^6+4x^3+3)}dx^3$$

$$\Big\downarrow \text{1141}$$

$$\frac{1}{3}\int \left(\frac{1}{6(x^3+3)}+\frac{1}{3x^3}-\frac{1}{2(x^3+1)}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{\log(x^3)}{3}-\frac{1}{2}\log(x^3+1)+\frac{1}{6}\log(x^3+3)\right)$$

input
```
Int[1/(x*(3 + 4*x^3 + x^6)),x]
```

output
```
(Log[x^3]/3 - Log[1 + x^3]/2 + Log[3 + x^3]/6)/3
```

3.155.3.1 Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.155.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

method	result	size
risch	$\frac{\ln(x)}{3}-\frac{\ln(x^3+1)}{6}+\frac{\ln(x^3+3)}{18}$	22
default	$\frac{\ln(x)}{3}-\frac{\ln(x+1)}{6}+\frac{\ln(x^3+3)}{18}-\frac{\ln(x^2-x+1)}{6}$	31
norman	$\frac{\ln(x)}{3}-\frac{\ln(x+1)}{6}+\frac{\ln(x^3+3)}{18}-\frac{\ln(x^2-x+1)}{6}$	31
parallelrisch	$\frac{\ln(x)}{3}-\frac{\ln(x+1)}{6}+\frac{\ln(x^3+3)}{18}-\frac{\ln(x^2-x+1)}{6}$	31

input
```
int(1/x/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
```

output
```
1/3*ln(x)-1/6*ln(x^3+1)+1/18*ln(x^3+3)
```

3.155.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{1}{x(3+4x^3+x^6)}\,dx = \frac{1}{18}\log\left(x^3+3\right)-\frac{1}{6}\log\left(x^3+1\right)+\frac{1}{3}\log(x)$$

input
```
integrate(1/x/(x^6+4*x^3+3),x, algorithm="fricas")
```

output
```
1/18*log(x^3 + 3) - 1/6*log(x^3 + 1) + 1/3*log(x)
```

3.155.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \frac{1}{x\left(3+4x^3+x^6\right)}\,dx = \frac{\log\left(x\right)}{3} - \frac{\log\left(x^3+1\right)}{6} + \frac{\log\left(x^3+3\right)}{18}$$

input
```
integrate(1/x/(x**6+4*x**3+3),x)
```

output
```
log(x)/3 - log(x**3 + 1)/6 + log(x**3 + 3)/18
```

3.155.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{1}{x\left(3+4x^3+x^6\right)}\,dx = \frac{1}{18}\log\left(x^3+3\right) - \frac{1}{6}\log\left(x^3+1\right) + \frac{1}{9}\log\left(x^3\right)$$

input
```
integrate(1/x/(x^6+4*x^3+3),x, algorithm="maxima")
```

output
```
1/18*log(x^3 + 3) - 1/6*log(x^3 + 1) + 1/9*log(x^3)
```

3.155.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{1}{x\left(3+4x^3+x^6\right)}\,dx = \frac{1}{18}\log\left(\left|x^3+3\right|\right) - \frac{1}{6}\log\left(\left|x^3+1\right|\right) + \frac{1}{3}\log\left(\left|x\right|\right)$$

input
```
integrate(1/x/(x^6+4*x^3+3),x, algorithm="giac")
```

output
```
1/18*log(abs(x^3 + 3)) - 1/6*log(abs(x^3 + 1)) + 1/3*log(abs(x))
```

3.155.9 Mupad [B] (verification not implemented)

Time = 8.39 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{1}{x\left(3+4x^3+x^6\right)}\,dx = \frac{\ln\left(x^3+3\right)}{18} - \frac{\ln\left(x^3+1\right)}{6} + \frac{\ln\left(x\right)}{3}$$

input
```
int(1/(x*(4*x^3 + x^6 + 3)),x)
```

output
```
log(x^3 + 3)/18 - log(x^3 + 1)/6 + log(x)/3
```

3.156 $\int \frac{1}{x^4(3+4x^3+x^6)}\,dx$

3.156.1 Optimal result

Integrand size = 16, antiderivative size = 34

$$\int \frac{1}{x^4(3+4x^3+x^6)}\,dx = -\frac{1}{9x^3}-\frac{4\log(x)}{9}+\frac{1}{6}\log\left(1+x^3\right)-\frac{1}{54}\log\left(3+x^3\right)$$

output
```
-1/9/x^3-4/9*ln(x)+1/6*ln(x^3+1)-1/54*ln(x^3+3)
```

3.156.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4(3+4x^3+x^6)}\,dx = -\frac{1}{9x^3}-\frac{4\log(x)}{9}+\frac{1}{6}\log\left(1+x^3\right)-\frac{1}{54}\log\left(3+x^3\right)$$

input
```
Integrate[1/(x^4*(3 + 4*x^3 + x^6)),x]
```

output
```
-1/9*1/x^3 - (4*Log[x])/9 + Log[1 + x^3]/6 - Log[3 + x^3]/54
```

3.156.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.18, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1693, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (x^6+4x^3+3)} \, dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3} \int \frac{1}{x^6 (x^6+4x^3+3)}dx^3$$

$$\Big\downarrow \text{1141}$$

$$\frac{1}{3} \int \left(-\frac{1}{18 (x^3+3)}-\frac{4}{9x^3}+\frac{1}{3x^6}+\frac{1}{2 (x^3+1)}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3} \left(-\frac{1}{3x^3}-\frac{4}{9} \log \left(x^3\right)+\frac{1}{2} \log \left(x^3+1\right)-\frac{1}{18} \log \left(x^3+3\right)\right)$$

input
```
Int[1/(x^4*(3 + 4*x^3 + x^6)),x]
```

output
```
(-1/3*1/x^3 - (4*Log[x^3])/9 + Log[1 + x^3]/2 - Log[3 + x^3]/18)/3
```

3.156.3.1 Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.156.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

method	result	size
risch	$-\frac{1}{9x^3}-\frac{4\ln(x)}{9}+\frac{\ln(x^3+1)}{6}-\frac{\ln(x^3+3)}{54}$	27
default	$-\frac{1}{9x^3}-\frac{4\ln(x)}{9}+\frac{\ln(x+1)}{6}-\frac{\ln(x^3+3)}{54}+\frac{\ln(x^2-x+1)}{6}$	36
norman	$-\frac{1}{9x^3}-\frac{4\ln(x)}{9}+\frac{\ln(x+1)}{6}-\frac{\ln(x^3+3)}{54}+\frac{\ln(x^2-x+1)}{6}$	36
parallelrisch	$-\frac{24\ln(x)x^3-9\ln(x+1)x^3+\ln(x^3+3)x^3-9\ln(x^2-x+1)x^3+6}{54x^3}$	48

input
```
int(1/x^4/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
```

output
```
-1/9/x^3-4/9*ln(x)+1/6*ln(x^3+1)-1/54*ln(x^3+3)
```

3.156.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^4(3+4x^3+x^6)}\,dx=-\frac{x^3\log(x^3+3)-9\,x^3\log(x^3+1)+24\,x^3\log(x)+6}{54\,x^3}$$

input
```
integrate(1/x^4/(x^6+4*x^3+3),x, algorithm="fricas")
```

output
```
-1/54*(x^3*log(x^3 + 3) - 9*x^3*log(x^3 + 1) + 24*x^3*log(x) + 6)/x^3
```

3.156.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^4 (3 + 4x^3 + x^6)} dx = -\frac{4 \log (x)}{9} + \frac{\log (x^3 + 1)}{6} - \frac{\log (x^3 + 3)}{54} - \frac{1}{9x^3}$$

input
```
integrate(1/x**4/(x**6+4*x**3+3),x)
```

output
```
-4*log(x)/9 + log(x**3 + 1)/6 - log(x**3 + 3)/54 - 1/(9*x**3)
```

3.156.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^4 (3 + 4x^3 + x^6)} dx = -\frac{1}{9 x^3} - \frac{1}{54} \log \left(x^3 + 3\right) + \frac{1}{6} \log \left(x^3 + 1\right) - \frac{4}{27} \log \left(x^3\right)$$

input
```
integrate(1/x^4/(x^6+4*x^3+3),x, algorithm="maxima")
```

output
```
-1/9/x^3 - 1/54*log(x^3 + 3) + 1/6*log(x^3 + 1) - 4/27*log(x^3)
```

3.156.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^4 (3 + 4x^3 + x^6)} dx = \frac{4 x^3 - 3}{27 x^3} - \frac{1}{54} \log \left(\left|x^3 + 3\right|\right) + \frac{1}{6} \log \left(\left|x^3 + 1\right|\right) - \frac{4}{9} \log \left(\left|x\right|\right)$$

input
```
integrate(1/x^4/(x^6+4*x^3+3),x, algorithm="giac")
```

output
```
1/27*(4*x^3 - 3)/x^3 - 1/54*log(abs(x^3 + 3)) + 1/6*log(abs(x^3 + 1)) - 4/
9*log(abs(x))
```

3.156.9 Mupad [B] (verification not implemented)

Time = 8.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^4 (3+4x^3+x^6)} \, dx = \frac{\ln (x^3+1)}{6} - \frac{\ln (x^3+3)}{54} - \frac{4 \ln (x)}{9} - \frac{1}{9 \, x^3}$$

input
```
int(1/(x^4*(4*x^3 + x^6 + 3)),x)
```

output
```
log(x^3 + 1)/6 - log(x^3 + 3)/54 - (4*log(x))/9 - 1/(9*x^3)
```

3.157 $\int \frac{1}{x^7(3+4x^3+x^6)}\,dx$

3.157.1 Optimal result

Integrand size = 16, antiderivative size = 41

$$\int \frac{1}{x^7(3+4x^3+x^6)}\,dx = -\frac{1}{18x^6}+\frac{4}{27x^3}+\frac{13\log(x)}{27}-\frac{1}{6}\log\left(1+x^3\right)+\frac{1}{162}\log\left(3+x^3\right)$$

output
```
-1/18/x^6+4/27/x^3+13/27*ln(x)-1/6*ln(x^3+1)+1/162*ln(x^3+3)
```

3.157.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^7(3+4x^3+x^6)}\,dx = -\frac{1}{18x^6}+\frac{4}{27x^3}+\frac{13\log(x)}{27}-\frac{1}{6}\log\left(1+x^3\right)+\frac{1}{162}\log\left(3+x^3\right)$$

input
```
Integrate[1/(x^7*(3 + 4*x^3 + x^6)),x]
```

output
```
-1/18*1/x^6 + 4/(27*x^3) + (13*Log[x])/27 - Log[1 + x^3]/6 + Log[3 + x^3]/
162
```

3.157.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1693, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7 (x^6+4x^3+3)} \, dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int \frac{1}{x^9 (x^6+4x^3+3)}dx^3$$

$$\Big\downarrow \text{1141}$$

$$\frac{1}{3}\int \left(\frac{1}{54 (x^3+3)}+\frac{13}{27x^3}-\frac{4}{9x^6}+\frac{1}{3x^9}-\frac{1}{2 (x^3+1)}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{1}{6x^6}+\frac{4}{9x^3}+\frac{13\log\left(x^3\right)}{27}-\frac{1}{2}\log\left(x^3+1\right)+\frac{1}{54}\log\left(x^3+3\right)\right)$$

input
```
Int[1/(x^7*(3 + 4*x^3 + x^6)),x]
```

output
```
(-1/6*1/x^6 + 4/(9*x^3) + (13*Log[x^3])/27 - Log[1 + x^3]/2 + Log[3 + x^3]
/54)/3
```

3.157.3.1 Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.157.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

method	result	size
risch	$\frac{-\frac{1}{18}+\frac{4x^3}{27}}{x^6}+\frac{13\ln(x)}{27}-\frac{\ln(x^3+1)}{6}+\frac{\ln(x^3+3)}{162}$	33
default	$-\frac{1}{18x^6}+\frac{4}{27x^3}+\frac{13\ln(x)}{27}-\frac{\ln(x+1)}{6}+\frac{\ln(x^3+3)}{162}-\frac{\ln(x^2-x+1)}{6}$	41
norman	$\frac{-\frac{1}{18}+\frac{4x^3}{27}}{x^6}+\frac{13\ln(x)}{27}-\frac{\ln(x+1)}{6}+\frac{\ln(x^3+3)}{162}-\frac{\ln(x^2-x+1)}{6}$	42
parallelrisch	$\frac{78\ln(x)x^6-27\ln(x+1)x^6+\ln(x^3+3)x^6-27\ln(x^2-x+1)x^6-9+24x^3}{162x^6}$	53

input
```
int(1/x^7/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
```

output
```
(-1/18+4/27*x^3)/x^6+13/27*ln(x)-1/6*ln(x^3+1)+1/162*ln(x^3+3)
```

3.157.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98

$$\int \frac{1}{x^7(3+4x^3+x^6)}\,dx=\frac{x^6\log(x^3+3)-27\,x^6\log(x^3+1)+78\,x^6\log(x)+24\,x^3-9}{162\,x^6}$$

input
```
integrate(1/x^7/(x^6+4*x^3+3),x, algorithm="fricas")
```

output
```
1/162*(x^6*log(x^3 + 3) - 27*x^6*log(x^3 + 1) + 78*x^6*log(x) + 24*x^3 - 9
)/x^6
```

3.157.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^7(3+4x^3+x^6)}\,dx = \frac{13\log(x)}{27} - \frac{\log(x^3+1)}{6} + \frac{\log(x^3+3)}{162} + \frac{8x^3-3}{54x^6}$$

input
```
integrate(1/x**7/(x**6+4*x**3+3),x)
```

output
```
13*log(x)/27 - log(x**3 + 1)/6 + log(x**3 + 3)/162 + (8*x**3 - 3)/(54*x**6
)
```

3.157.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^7(3+4x^3+x^6)}\,dx = \frac{8\,x^3-3}{54\,x^6} + \frac{1}{162}\log\left(x^3+3\right) - \frac{1}{6}\log\left(x^3+1\right) + \frac{13}{81}\log\left(x^3\right)$$

input
```
integrate(1/x^7/(x^6+4*x^3+3),x, algorithm="maxima")
```

output
```
1/54*(8*x^3 - 3)/x^6 + 1/162*log(x^3 + 3) - 1/6*log(x^3 + 1) + 13/81*log(x
^3)
```

3.157.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^7(3+4x^3+x^6)}\,dx = -\frac{13\,x^6-8\,x^3+3}{54\,x^6} + \frac{1}{162}\log\left(|x^3+3|\right) - \frac{1}{6}\log\left(|x^3+1|\right) + \frac{13}{27}\log\left(|x|\right)$$

input
```
integrate(1/x^7/(x^6+4*x^3+3),x, algorithm="giac")
```

output
```
-1/54*(13*x^6 - 8*x^3 + 3)/x^6 + 1/162*log(abs(x^3 + 3)) - 1/6*log(abs(x^3
 + 1)) + 13/27*log(abs(x))
```

3.157.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^7 (3+4x^3+x^6)} \, dx = \frac{\ln (x^3+3)}{162} - \frac{\ln (x^3+1)}{6} + \frac{13 \ln (x)}{27} + \frac{\frac{4x^3}{27} - \frac{1}{18}}{x^6}$$

input
```
int(1/(x^7*(4*x^3 + x^6 + 3)),x)
```

output
```
log(x^3 + 3)/162 - log(x^3 + 1)/6 + (13*log(x))/27 + ((4*x^3)/27 - 1/18)/x
^6
```

3.158 $\int \frac{x^{10}}{3+4x^3+x^6}\,dx$

3.158.1 Optimal result

Integrand size = 16, antiderivative size = 124

$$\int \frac{x^{10}}{3+4x^3+x^6}\,dx=-2x^2+\frac{x^5}{5}+\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{9}{2}\sqrt[6]{3}\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)+\frac{1}{6}\log(1+x)-\frac{3}{2}3^{2/3}\log\left(\sqrt[3]{3}+x\right)-\frac{1}{12}\log\left(1-x+x^2\right)+\frac{3}{4}3^{2/3}\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)$$

output

```
-2*x^2+1/5*x^5-9/2*3^(1/6)*arctan(1/3*(3^(1/3)-2*x)*3^(1/6))+1/6*ln(1+x)-3
/2*3^(2/3)*ln(3^(1/3)+x)-1/12*ln(x^2-x+1)+3/4*3^(2/3)*ln(3^(2/3)-3^(1/3)*x
+x^2)+1/6*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

3.158.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.95

$$\begin{aligned}&\int \frac{x^{10}}{3+4x^3+x^6}\,dx\\&=\frac{1}{60}\left(-120x^2+12x^5-270\sqrt[6]{3}\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)-10\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)+10\log(1+x)\right.\\&\left.-90\ 3^{2/3}\log\left(3+3^{2/3}x\right)-5\log\left(1-x+x^2\right)+45\ 3^{2/3}\log\left(3-3^{2/3}x+\sqrt[3]{3}x^2\right)\right)\end{aligned}$$

input
```
Integrate[x^10/(3 + 4*x^3 + x^6),x]
```

output
```
(-120*x^2 + 12*x^5 - 270*3^(1/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] - 10*Sqrt
[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 10*Log[1 + x] - 90*3^(2/3)*Log[3 + 3^(2/3
)*x] - 5*Log[1 - x + x^2] + 45*3^(2/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/6
0
```

3.158.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.14, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {1703, 27, 1826, 27, 1834, 821, 16, 1142, 25, 1082, 217, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{10}}{x^6+4x^3+3}dx$$

$$\Big\downarrow \text{1703}$$

$$\frac{x^5}{5}-\frac{1}{5}\int \frac{5x^4\left(4x^3+3\right)}{x^6+4x^3+3}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{x^5}{5}-\int \frac{x^4\left(4x^3+3\right)}{x^6+4x^3+3}dx$$

$$\Big\downarrow \text{1826}$$

$$\frac{1}{2}\int \frac{2x\left(13x^3+12\right)}{x^6+4x^3+3}dx+\frac{x^5}{5}-2x^2$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x\left(13x^3+12\right)}{x^6+4x^3+3}dx+\frac{x^5}{5}-2x^2$$

$$\Big\downarrow \text{1834}$$

$$-\frac{1}{2}\int \frac{x}{x^3+1}dx+\frac{27}{2}\int \frac{x}{x^3+3}dx+\frac{x^5}{5}-2x^2$$

$$\Big\downarrow \text{821}$$

$$\frac{1}{2}\left(\frac{1}{3}\int \frac{1}{x+1}dx-\frac{1}{3}\int \frac{x+1}{x^2-x+1}dx\right)+\frac{27}{2}\left(\frac{\int \frac{x+\sqrt[3]{3}}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\sqrt[3]{3}}-\frac{\int \frac{1}{x+\sqrt[3]{3}}dx}{3\sqrt[3]{3}}\right)+\frac{x^5}{5}-2x^2$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}\left(\frac{1}{3}\log(x+1)-\frac{1}{3}\int \frac{x+1}{x^2-x+1}dx\right)+\frac{27}{2}\left(\frac{\int \frac{x+\sqrt[3]{3}}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\sqrt[3]{3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}\right)+\frac{x^5}{5}-2x^2$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(-\frac{3}{2}\int \frac{1}{x^2-x+1}dx-\frac{1}{2}\int -\frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{3}\log(x+1)\right)+$$
$$\frac{27}{2}\left(\frac{\frac{3}{2}\sqrt[3]{3}\int \frac{1}{x^2-\sqrt[3]{3}x+3^{2/3}}dx+\frac{1}{2}\int -\frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\sqrt[3]{3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}\right)+\frac{x^5}{5}-2x^2$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx-\frac{3}{2}\int \frac{1}{x^2-x+1}dx\right)+\frac{1}{3}\log(x+1)\right)+$$
$$\frac{27}{2}\left(\frac{\frac{3}{2}\sqrt[3]{3}\int \frac{1}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\sqrt[3]{3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}\right)+\frac{x^5}{5}-2x^2$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx-\frac{3}{2}\int \frac{1}{x^2-x+1}dx\right)+\frac{1}{3}\log(x+1)\right)+$$
$$\frac{27}{2}\left(\frac{3\int \frac{1}{-\left(1-\frac{2x}{\sqrt[3]{3}}\right)^2-3}d\left(1-\frac{2x}{\sqrt[3]{3}}\right)-\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\sqrt[3]{3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}\right)+\frac{x^5}{5}-2x^2$$

$$\Big\downarrow \text{217}$$

$$\frac{27}{2}\left(\frac{-\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\sqrt[3]{3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}\right)+$$
$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx-\frac{3}{2}\int \frac{1}{x^2-x+1}dx\right)+\frac{1}{3}\log(x+1)\right)+\frac{x^5}{5}-2x^2$$

3.158. $\int \frac{x^{10}}{3+4x^3+x^6}\,dx$

$$\Big\downarrow \text{1083}$$

$$\frac{27}{2}\left(\frac{-\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\sqrt[3]{3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}\right)+$$
$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx+3\int \frac{1}{-(2x-1)^2-3}d(2x-1)\right)+\frac{1}{3}\log(x+1)\right)+\frac{x^5}{5}-2x^2$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx-\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)\right)+\frac{1}{3}\log(x+1)\right)+$$
$$\frac{27}{2}\left(\frac{-\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\sqrt[3]{3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}\right)+\frac{x^5}{5}-2x^2$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(-\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)-\frac{1}{2}\log\left(x^2-x+1\right)\right)+\frac{1}{3}\log(x+1)\right)+$$
$$\frac{27}{2}\left(\frac{\frac{1}{2}\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\sqrt[3]{3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}\right)+\frac{x^5}{5}-2x^2$$

input
```
Int[x^10/(3 + 4*x^3 + x^6),x]
```

output
```
-2*x^2 + x^5/5 + (Log[1 + x]/3 + (-(Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]]) -
Log[1 - x + x^2]/2)/3)/2 + (27*(-1/3*Log[3^(1/3) + x]/3^(1/3) + (-(Sqrt[3]
*ArcTan[(1 - (2*x)/3^(1/3))/Sqrt[3]]) + Log[3^(2/3) - 3^(1/3)*x + x^2]/2)/
(3*3^(1/3))))/2
```

3.158. $\int \frac{x^{10}}{3+4x^3+x^6}\,dx$

3.158.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + 
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.158. $\int \frac{x^{10}}{3+4x^3+x^6}\,dx$

rule 1703
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(2*n - 1)*(d*x)^(m - 2*n + 1)*((a + b*x^n + c*x^(2*n))^(
p + 1)/(c*(m + 2*n*p + 1))), x] - Simp[d^(2*n)/(c*(m + 2*n*p + 1))   Int[(d
*x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x
^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && N
eQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0
] && IntegerQ[p]
```

rule 1826
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[e*f^(n - 1)*(f*x)^(m - n + 1)*((a
+ b*x^n + c*x^(2*n))^(p + 1)/(c*(m + n*(2*p + 1) + 1))), x] - Simp[f^n/(c*(
m + n*(2*p + 1) + 1))   Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*
e*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x]
, x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Intege
rQ[p]
```

rule 1834
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 +
 (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2
 - (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ
[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n
, 0]
```

3.158.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.55

method	result
risch	$\frac{x^5}{5} - 2x^2 - \frac{\ln(4x^2-4x+4)}{12} - \frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6} + \frac{3\left(\sum_{\_R=\text{RootOf}(\_Z^3+9)} \_R\ln\left(\_R^2+3x\right)\right)}{2} + \frac{\ln(x+1)}{6}$
default	$\frac{x^5}{5} - 2x^2 + \frac{\ln(x+1)}{6} - \frac{3\,3^{\frac{2}{3}}\ln\left(3^{\frac{1}{3}}+x\right)}{2} + \frac{3\,3^{\frac{2}{3}}\ln\left(3^{\frac{2}{3}}-3^{\frac{1}{3}}x+x^2\right)}{4} + \frac{9\,3^{\frac{1}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,3^{\frac{2}{3}}x}{3}-1\right)}{3}\right)}{2} - \frac{\ln(x^2-x+1)}{12} - \cdots$

3.158. $\int \frac{x^{10}}{3+4x^3+x^6}\,dx$

input
```
int(x^10/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
```

output
```
1/5*x^5-2*x^2-1/12*ln(4*x^2-4*x+4)-1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
+3/2*sum(_R*ln(_R^2+3*x),_R=RootOf(_Z^3+9))+1/6*ln(x+1)
```

3.158.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.82

$$\int \frac{x^{10}}{3+4x^3+x^6}\,dx = \frac{1}{5}\,x^5 - 2\,x^2 + \frac{3}{2}\,\sqrt{3}(-9)^{\frac{1}{3}}\arctan\left(\frac{1}{9}\,\sqrt{3}\Big(2\,(-9)^{\frac{1}{3}}\,x+3\Big)\right) - \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) - \frac{3}{4}\,(-9)^{\frac{1}{3}}\log\left(3\,x^2-(-9)^{\frac{2}{3}}\,x-3\,(-9)^{\frac{1}{3}}\right) + \frac{3}{2}\,(-9)^{\frac{1}{3}}\log\left(3\,x+(-9)^{\frac{2}{3}}\right) - \frac{1}{12}\,\log\left(x^2-x+1\right) + \frac{1}{6}\,\log\left(x+1\right)$$

input
```
integrate(x^10/(x^6+4*x^3+3),x, algorithm="fricas")
```

output
```
1/5*x^5 - 2*x^2 + 3/2*sqrt(3)*(-9)^(1/3)*arctan(1/9*sqrt(3)*(2*(-9)^(1/3)*
x + 3)) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 3/4*(-9)^(1/3)*log(3
*x^2 - (-9)^(2/3)*x - 3*(-9)^(1/3)) + 3/2*(-9)^(1/3)*log(3*x + (-9)^(2/3))
 - 1/12*log(x^2 - x + 1) + 1/6*log(x + 1)
```

3.158.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.16

$$\int \frac{x^{10}}{3+4x^3+x^6}\,dx = \frac{x^5}{5} - 2x^2 + \frac{\log(x+1)}{6} + \left(-\frac{1}{12} - \frac{\sqrt{3}i}{12}\right)\log\left(x + \frac{3872\left(-\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)^5}{3281} + \frac{3188648\left(-\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)^2}{88587}\right) + \left(-\frac{1}{12} + \frac{\sqrt{3}i}{12}\right)\log\left(x + \frac{3188648\left(-\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)^2}{88587} + \frac{3872\left(-\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)^5}{3281}\right) + \operatorname{RootSum}\left(8t^3+243, \left(t \mapsto t\log\left(\frac{3872t^5}{3281} + \frac{3188648t^2}{88587} + x\right)\right)\right)$$

input
```
integrate(x**10/(x**6+4*x**3+3),x)
```

output
```
x**5/5 - 2*x**2 + log(x + 1)/6 + (-1/12 - sqrt(3)*I/12)*log(x + 3872*(-1/1
2 - sqrt(3)*I/12)**5/3281 + 3188648*(-1/12 - sqrt(3)*I/12)**2/88587) + (-1
/12 + sqrt(3)*I/12)*log(x + 3188648*(-1/12 + sqrt(3)*I/12)**2/88587 + 3872
*(-1/12 + sqrt(3)*I/12)**5/3281) + RootSum(8*_t**3 + 243, Lambda(_t, _t*lo
g(3872*_t**5/3281 + 3188648*_t**2/88587 + x)))
```

3.158.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.76

$$\int \frac{x^{10}}{3+4x^3+x^6}\,dx = \frac{1}{5}x^5 - 2x^2 + \frac{3}{4}\cdot 3^{\frac{2}{3}}\log\left(x^2 - 3^{\frac{1}{3}}x + 3^{\frac{2}{3}}\right) - \frac{3}{2}\cdot 3^{\frac{2}{3}}\log\left(x + 3^{\frac{1}{3}}\right) - \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{9}{2}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x - 3^{\frac{1}{3}}\right)\right) - \frac{1}{12}\log\left(x^2-x+1\right) + \frac{1}{6}\log(x+1)$$

input
```
integrate(x^10/(x^6+4*x^3+3),x, algorithm="maxima")
```

output
```
1/5*x^5 - 2*x^2 + 3/4*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 3/2*3^(2/3)
*log(x + 3^(1/3)) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 9/2*3^(1/6
)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) - 1/12*log(x^2 - x + 1) + 1/6*log(x
+ 1)
```

3.158.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.77

$$\int \frac{x^{10}}{3+4x^3+x^6}\,dx = \frac{1}{5}\,x^5 - 2\,x^2 + \frac{3}{4}\cdot 3^{\frac{2}{3}}\log\left(x^2 - 3^{\frac{1}{3}}x + 3^{\frac{2}{3}}\right) - \frac{3}{2}\cdot 3^{\frac{2}{3}}\log\left(\left|x + 3^{\frac{1}{3}}\right|\right) - \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) + \frac{9}{2}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2\,x - 3^{\frac{1}{3}}\right)\right) - \frac{1}{12}\,\log\left(x^2 - x + 1\right) + \frac{1}{6}\,\log\left(|x+1|\right)$$

input
```
integrate(x^10/(x^6+4*x^3+3),x, algorithm="giac")
```

output
```
1/5*x^5 - 2*x^2 + 3/4*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 3/2*3^(2/3)
*log(abs(x + 3^(1/3))) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 9/2*3
^(1/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) - 1/12*log(x^2 - x + 1) + 1/6*l
og(abs(x + 1))
```

3.158.9 Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00

$$\int \frac{x^{10}}{3+4x^3+x^6}\,dx = \frac{\ln(x+1)}{6} - \frac{3\,3^{2/3}\,\ln\left(x+3^{1/3}\right)}{2} + \ln\left(x - \frac{1}{2} - \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{12} + \frac{\sqrt{3}\,1\mathrm{i}}{12}\right) - \ln\left(x - \frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{12} + \frac{\sqrt{3}\,1\mathrm{i}}{12}\right) - 2\,x^2 + \frac{x^5}{5} - \frac{3\,(-1)^{1/3}\,\ln\left(x - \frac{(-1)^{1/3}\,3^{1/3}}{2} - \frac{(-1)^{1/6}\,3^{5/6}}{2} + \frac{3^{1/3}}{2}\right)\left(3^{2/3} + 3^{1/6}\,3\mathrm{i}\right)}{4} + \frac{3\,(-1)^{1/3}\,3^{2/3}\,\ln\left(x + (-1)^{2/3}\,3^{1/3}\right)}{2}$$

input
```
int(x^10/(4*x^3 + x^6 + 3),x)
```

output
```
log(x + 1)/6 - (3*3^(2/3)*log(x + 3^(1/3)))/2 + log(x - (3^(1/2)*1i)/2 - 1
/2)*((3^(1/2)*1i)/12 - 1/12) - log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)
/12 + 1/12) - 2*x^2 + x^5/5 - (3*(-1)^(1/3)*log(x - ((-1)^(1/3)*3^(1/3))/2
 - ((-1)^(1/6)*3^(5/6))/2 + 3^(1/3)/2)*(3^(2/3) + 3^(1/6)*3i))/4 + (3*(-1)
^(1/3)*3^(2/3)*log(x + (-1)^(2/3)*3^(1/3)))/2
```

3.158. $\int \frac{x^{10}}{3+4x^3+x^6}\,dx$

3.159 $\int \frac{x^9}{3+4x^3+x^6}\,dx$

3.159.1 Optimal result

Integrand size = 16, antiderivative size = 122

$$\int \frac{x^9}{3+4x^3+x^6}\,dx = -4x+\frac{x^4}{4}+\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{3}{2}3^{5/6}\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)-\frac{1}{6}\log(1+x)+\frac{3}{2}\sqrt[3]{3}\log\left(\sqrt[3]{3}+x\right)+\frac{1}{12}\log\left(1-x+x^2\right)-\frac{3}{4}\sqrt[3]{3}\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)$$

output
```
-4*x+1/4*x^4-3/2*3^(5/6)*arctan(1/3*(3^(1/3)-2*x)*3^(1/6))-1/6*ln(1+x)+3/2
*3^(1/3)*ln(3^(1/3)+x)+1/12*ln(x^2-x+1)-3/4*3^(1/3)*ln(3^(2/3)-3^(1/3)*x+x
^2)+1/6*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

3.159.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.93

$$\int \frac{x^9}{3+4x^3+x^6}\,dx = \frac{1}{12}\Bigg(-48x+3x^4-18\,3^{5/6}\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)-2\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)-2\log(1+x)+18\sqrt[3]{3}\log\left(3+3^{2/3}x\right)+\log\left(1-x+$$

input
```
Integrate[x^9/(3 + 4*x^3 + x^6),x]
```

output
```
(-48*x + 3*x^4 - 18*3^(5/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] - 2*Sqrt[3]*Ar
cTan[(-1 + 2*x)/Sqrt[3]] - 2*Log[1 + x] + 18*3^(1/3)*Log[3 + 3^(2/3)*x] +
Log[1 - x + x^2] - 9*3^(1/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/12
```

3.159.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.14, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.812$, Rules used = {1703, 27, 1826, 1752, 750, 16, 1142, 25, 1082, 217, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{x^6+4x^3+3}\,dx$$

$$\Big\downarrow \text{1703}$$

$$\frac{x^4}{4}-\frac{1}{4}\int \frac{4x^3\left(4x^3+3\right)}{x^6+4x^3+3}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{x^4}{4}-\int \frac{x^3\left(4x^3+3\right)}{x^6+4x^3+3}dx$$

$$\Big\downarrow \text{1826}$$

$$\int \frac{13x^3+12}{x^6+4x^3+3}dx+\frac{x^4}{4}-4x$$

$$\Big\downarrow \text{1752}$$

$$-\frac{1}{2}\int \frac{1}{x^3+1}dx+\frac{27}{2}\int \frac{1}{x^3+3}dx+\frac{x^4}{4}-4x$$

$$\Big\downarrow \text{750}$$

$$\frac{1}{2}\left(-\frac{1}{3}\int \frac{2-x}{x^2-x+1}dx-\frac{1}{3}\int \frac{1}{x+1}dx\right)+\frac{27}{2}\left(\frac{\int \frac{2\sqrt[3]{3}-x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\ 3^{2/3}}+\frac{\int \frac{1}{x+\sqrt[3]{3}}dx}{3\ 3^{2/3}}\right)+\frac{x^4}{4}-4x$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}\left(-\frac{1}{3}\int \frac{2-x}{x^2-x+1}dx-\frac{1}{3}\log(x+1)\right)+\frac{27}{2}\left(\frac{\int \frac{2\sqrt[3]{3}-x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\ 3^{2/3}}+\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)+\frac{x^4}{4}-4x$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{1}{2}\int -\frac{1-2x}{x^2-x+1}dx-\frac{3}{2}\int \frac{1}{x^2-x+1}dx\right)-\frac{1}{3}\log(x+1)\right)+\frac{27}{2}\left(\frac{\frac{3}{2}\sqrt[3]{3}\int \frac{1}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\frac{1}{2}\int -\frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\ 3^{2/3}}+\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)+\frac{x^4}{4}-4x$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(-\frac{3}{2}\int \frac{1}{x^2-x+1}dx-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx\right)-\frac{1}{3}\log(x+1)\right)+\frac{27}{2}\left(\frac{\frac{3}{2}\sqrt[3]{3}\int \frac{1}{x^2-\sqrt[3]{3}x+3^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\ 3^{2/3}}+\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)+\frac{x^4}{4}-4x$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(-\frac{3}{2}\int \frac{1}{x^2-x+1}dx-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx\right)-\frac{1}{3}\log(x+1)\right)+\frac{27}{2}\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx+3\int \frac{1}{-\left(1-\frac{2x}{\sqrt[3]{3}}\right)^2-3}d\left(1-\frac{2x}{\sqrt[3]{3}}\right)}{3\ 3^{2/3}}+\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)+\frac{x^4}{4}-4x$$

$$\Big\downarrow \text{217}$$

$$\frac{27}{2}\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\ 3^{2/3}}+\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)+\frac{1}{2}\left(\frac{1}{3}\left(-\frac{3}{2}\int \frac{1}{x^2-x+1}dx-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx\right)-\frac{1}{3}\log(x+1)\right)+\frac{x^4}{4}-4x$$

$$\Big\downarrow \text{1083}$$

$$\frac{27}{2}\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\ 3^{2/3}}+\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)+\frac{1}{2}\left(\frac{1}{3}\left(3\int \frac{1}{-(2x-1)^2-3}d(2x-1)-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx\right)-\frac{1}{3}\log(x+1)\right)+\frac{x^4}{4}-4x$$

3.159. $\int \frac{x^9}{3+4x^3+x^6}\,dx$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx-\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)\right)-\frac{1}{3}\log(x+1)\right)+\frac{27}{2}\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\ 3^{2/3}}+\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)+\frac{x^4}{4}-4x$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{1}{2}\log\left(x^2-x+1\right)-\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)\right)-\frac{1}{3}\log(x+1)\right)+\frac{27}{2}\left(\frac{-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)-\frac{1}{2}\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{3\ 3^{2/3}}+\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)+\frac{x^4}{4}-4x$$

input
```
Int[x^9/(3 + 4*x^3 + x^6),x]
```

output
```
-4*x + x^4/4 + (-1/3*Log[1 + x] + (-(Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]]) +
 Log[1 - x + x^2]/2)/3)/2 + (27*(Log[3^(1/3) + x]/(3*3^(2/3)) + (-(Sqrt[3]
*ArcTan[(1 - (2*x)/3^(1/3))/Sqrt[3]]) - Log[3^(2/3) - 3^(1/3)*x + x^2]/2)/
(3*3^(2/3))))/2
```

3.159.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1703
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(2*n - 1)*(d*x)^(m - 2*n + 1)*((a + b*x^n + c*x^(2*n))^(
p + 1)/(c*(m + 2*n*p + 1))), x] - Simp[d^(2*n)/(c*(m + 2*n*p + 1))   Int[(d
*x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x
^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && N
eQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0
] && IntegerQ[p]
```

3.159. $\int \frac{x^9}{3+4x^3+x^6} \, dx$

rule 1752
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

rule 1826
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[e*f^(n - 1)*(f*x)^(m - n + 1)*((a
+ b*x^n + c*x^(2*n))^(p + 1)/(c*(m + n*(2*p + 1) + 1))), x] - Simp[f^n/(c*(
m + n*(2*p + 1) + 1))   Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*
e*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x]
, x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Intege
rQ[p]
```

3.159.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.51

method	result
risch	$\frac{x^4}{4}-4x+\frac{\ln(4x^2-4x+4)}{12}-\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}-\frac{\ln(x+1)}{6}+\frac{3\left(\sum_{\_R=\operatorname{RootOf}(\_Z^3-3)}\_R\ln(x+\_R)\right)}{2}$
default	$\frac{x^4}{4}-4x-\frac{\ln(x+1)}{6}+\frac{3\,3^{\frac{1}{3}}\ln\left(3^{\frac{1}{3}}+x\right)}{2}-\frac{3\,3^{\frac{1}{3}}\ln\left(3^{\frac{2}{3}}-3^{\frac{1}{3}}x+x^2\right)}{4}+\frac{3\,3^{\frac{5}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,3^{\frac{2}{3}}x}{3}-1\right)}{3}\right)}{2}+\frac{\ln(x^2-x+1)}{12}-\sqrt{3}$

input
```
int(x^9/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4-4*x+1/12*ln(4*x^2-4*x+4)-1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))-1
/6*ln(x+1)+3/2*sum(_R*ln(x+_R),_R=RootOf(_Z^3-3))
```

3.159.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.74

$$\int \frac{x^9}{3+4x^3+x^6}\,dx = \frac{1}{4}\,x^4 + \frac{3}{2}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{2}{3}\cdot 3^{\frac{1}{6}}x - \frac{1}{3}\sqrt{3}\right) - \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right) - \frac{3}{4}\cdot 3^{\frac{1}{3}}\log\left(x^2 - 3^{\frac{1}{3}}x + 3^{\frac{2}{3}}\right) + \frac{3}{2}\cdot 3^{\frac{1}{3}}\log\left(x + 3^{\frac{1}{3}}\right) - 4\,x + \frac{1}{12}\log\left(x^2 - x + 1\right) - \frac{1}{6}\log\left(x+1\right)$$

input
```
integrate(x^9/(x^6+4*x^3+3),x, algorithm="fricas")
```

output
```
1/4*x^4 + 3/2*3^(5/6)*arctan(2/3*3^(1/6)*x - 1/3*sqrt(3)) - 1/6*sqrt(3)*ar
ctan(1/3*sqrt(3)*(2*x - 1)) - 3/4*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) +
 3/2*3^(1/3)*log(x + 3^(1/3)) - 4*x + 1/12*log(x^2 - x + 1) - 1/6*log(x +
1)
```

3.159.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.06

$$\int \frac{x^9}{3+4x^3+x^6}\,dx = \frac{x^4}{4} - 4x - \frac{\log(x+1)}{6} + \left(\frac{1}{12} + \frac{\sqrt{3}i}{12}\right)\log\left(x - \frac{9841}{19692} - \frac{9841\sqrt{3}i}{19692} + \frac{360\left(\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)^4}{547}\right) + \left(\frac{1}{12} - \frac{\sqrt{3}i}{12}\right)\log\left(x - \frac{9841}{19692} + \frac{360\left(\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)^4}{547} + \frac{9841\sqrt{3}i}{19692}\right) + \operatorname{RootSum}\left(8t^3 - 81, \left(t \mapsto t\log\left(\frac{360t^4}{547} - \frac{9841t}{1641} + x\right)\right)\right)$$

input
```
integrate(x**9/(x**6+4*x**3+3),x)
```

output
```
x**4/4 - 4*x - log(x + 1)/6 + (1/12 + sqrt(3)*I/12)*log(x - 9841/19692 - 9
841*sqrt(3)*I/19692 + 360*(1/12 + sqrt(3)*I/12)**4/547) + (1/12 - sqrt(3)*
I/12)*log(x - 9841/19692 + 360*(1/12 - sqrt(3)*I/12)**4/547 + 9841*sqrt(3)
*I/19692) + RootSum(8*_t**3 - 81, Lambda(_t, _t*log(360*_t**4/547 - 9841*_
t/1641 + x)))
```

3.159.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.75

$$\int \frac{x^9}{3+4x^3+x^6}\,dx = \frac{1}{4}\,x^4 + \frac{3}{2}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2\,x - 3^{\frac{1}{3}}\right)\right) - \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) - \frac{3}{4}\cdot 3^{\frac{1}{3}}\log\left(x^2 - 3^{\frac{1}{3}}x + 3^{\frac{2}{3}}\right) + \frac{3}{2}\cdot 3^{\frac{1}{3}}\log\left(x + 3^{\frac{1}{3}}\right) - 4\,x + \frac{1}{12}\,\log\left(x^2 - x + 1\right) - \frac{1}{6}\,\log\left(x+1\right)$$

input
```
integrate(x^9/(x^6+4*x^3+3),x, algorithm="maxima")
```

output
```
1/4*x^4 + 3/2*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) - 1/6*sqrt(3)*ar
ctan(1/3*sqrt(3)*(2*x - 1)) - 3/4*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) +
 3/2*3^(1/3)*log(x + 3^(1/3)) - 4*x + 1/12*log(x^2 - x + 1) - 1/6*log(x +
1)
```

3.159.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.77

$$\int \frac{x^9}{3+4x^3+x^6}\,dx = \frac{1}{4}\,x^4 + \frac{3}{2}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2\,x - 3^{\frac{1}{3}}\right)\right) - \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) - \frac{3}{4}\cdot 3^{\frac{1}{3}}\log\left(x^2 - 3^{\frac{1}{3}}x + 3^{\frac{2}{3}}\right) + \frac{3}{2}\cdot 3^{\frac{1}{3}}\log\left(\left|x + 3^{\frac{1}{3}}\right|\right) - 4\,x + \frac{1}{12}\,\log\left(x^2 - x + 1\right) - \frac{1}{6}\,\log\left(|x+1|\right)$$

input
```
integrate(x^9/(x^6+4*x^3+3),x, algorithm="giac")
```

output
```
1/4*x^4 + 3/2*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) - 1/6*sqrt(3)*ar
ctan(1/3*sqrt(3)*(2*x - 1)) - 3/4*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) +
 3/2*3^(1/3)*log(abs(x + 3^(1/3))) - 4*x + 1/12*log(x^2 - x + 1) - 1/6*log
(abs(x + 1))
```

3.159.9 Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.98

$$\begin{aligned}\int \frac{x^9}{3+4x^3+x^6}\,dx &= \frac{3\,3^{1/3}\,\ln\left(x+3^{1/3}\right)}{2}-\frac{\ln\left(x+1\right)}{6}-4\,x \\ &\quad +\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)+\frac{x^4}{4} \\ &\quad -\ln\left(x-\frac{3^{1/3}}{2}-\frac{3^{5/6}\,1\mathrm{i}}{2}\right)\left(\frac{3\,3^{1/3}}{4}+\frac{3^{5/6}\,3\mathrm{i}}{4}\right)+3^{1/3}\,\ln\left(x-\frac{3^{1/3}}{2}+\frac{3^{5/6}\,1\mathrm{i}}{2}\right)\left(-\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\end{aligned}$$

input
```
int(x^9/(4*x^3 + x^6 + 3),x)
```

output
```
(3*3^(1/3)*log(x + 3^(1/3)))/2 - log(x + 1)/6 - 4*x + log(x - (3^(1/2)*1i)
/2 - 1/2)*((3^(1/2)*1i)/12 + 1/12) - log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/
2)*1i)/12 - 1/12) + x^4/4 - log(x - 3^(1/3)/2 - (3^(5/6)*1i)/2)*((3*3^(1/3
))/4 + (3^(5/6)*3i)/4) + 3^(1/3)*log(x - 3^(1/3)/2 + (3^(5/6)*1i)/2)*((3^(
1/2)*3i)/4 - 3/4)
```

3.160 $\int \frac{x^7}{3+4x^3+x^6}\,dx$

3.160.1 Optimal result

Integrand size = 16, antiderivative size = 119

$$\int \frac{x^7}{3+4x^3+x^6}\,dx=\frac{x^2}{2}-\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{3}{2}\sqrt[6]{3}\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)-\frac{1}{6}\log(1+x)+\frac{1}{2}3^{2/3}\log\left(\sqrt[3]{3}+x\right)+\frac{1}{12}\log\left(1-x+x^2\right)-\frac{1}{4}3^{2/3}\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)$$

output
```
1/2*x^2+3/2*3^(1/6)*arctan(1/3*(3^(1/3)-2*x)*3^(1/6))-1/6*ln(1+x)+1/2*3^(2
/3)*ln(3^(1/3)+x)+1/12*ln(x^2-x+1)-1/4*3^(2/3)*ln(3^(2/3)-3^(1/3)*x+x^2)-1
/6*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

3.160.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.93

$$\int \frac{x^7}{3+4x^3+x^6}\,dx=\frac{1}{12}\left(6x^2+18\sqrt[6]{3}\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)+2\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)-2\log(1+x)+6\,3^{2/3}\log\left(3+3^{2/3}x\right)+\log\left(1-x+x^2\right)-3\,3^{2/3}\log\left(3-3^{2/3}x+\sqrt[3]{3}x^2\right)\right)$$

input
```
Integrate[x^7/(3 + 4*x^3 + x^6),x]
```

output
```
(6*x^2 + 18*3^(1/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] + 2*Sqrt[3]*ArcTan[(-1
 + 2*x)/Sqrt[3]] - 2*Log[1 + x] + 6*3^(2/3)*Log[3 + 3^(2/3)*x] + Log[1 - x
 + x^2] - 3*3^(2/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/12
```

3.160.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.13, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {1703, 27, 1834, 821, 16, 1142, 25, 1082, 217, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{x^6+4x^3+3}dx$$

$$\Big\downarrow \text{1703}$$

$$\frac{x^2}{2}-\frac{1}{2}\int \frac{2x(4x^3+3)}{x^6+4x^3+3}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{x^2}{2}-\int \frac{x(4x^3+3)}{x^6+4x^3+3}dx$$

$$\Big\downarrow \text{1834}$$

$$\frac{1}{2}\int \frac{x}{x^3+1}dx-\frac{9}{2}\int \frac{x}{x^3+3}dx+\frac{x^2}{2}$$

$$\Big\downarrow \text{821}$$

$$\frac{1}{2}\left(\frac{1}{3}\int \frac{x+1}{x^2-x+1}dx-\frac{1}{3}\int \frac{1}{x+1}dx\right)-\frac{9}{2}\left(\frac{\int \frac{x+\sqrt[3]{3}}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\sqrt[3]{3}}-\frac{\int \frac{1}{x+\sqrt[3]{3}}dx}{3\sqrt[3]{3}}\right)+\frac{x^2}{2}$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}\left(\frac{1}{3}\int \frac{x+1}{x^2-x+1}dx-\frac{1}{3}\log(x+1)\right)-\frac{9}{2}\left(\frac{\int \frac{x+\sqrt[3]{3}}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\sqrt[3]{3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}\right)+\frac{x^2}{2}$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int\frac{1}{x^2-x+1}dx+\frac{1}{2}\int-\frac{1-2x}{x^2-x+1}dx\right)-\frac{1}{3}\log(x+1)\right)-\frac{9}{2}\left(\frac{\frac{3}{2}\sqrt[3]{3}\int\frac{1}{x^2-\sqrt[3]{3}x+3^{2/3}}dx+\frac{1}{2}\int-\frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\sqrt[3]{3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}\right)+\frac{x^2}{2}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int\frac{1}{x^2-x+1}dx-\frac{1}{2}\int\frac{1-2x}{x^2-x+1}dx\right)-\frac{1}{3}\log(x+1)\right)-\frac{9}{2}\left(\frac{\frac{3}{2}\sqrt[3]{3}\int\frac{1}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\frac{1}{2}\int\frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\sqrt[3]{3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}\right)+\frac{x^2}{2}$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int\frac{1}{x^2-x+1}dx-\frac{1}{2}\int\frac{1-2x}{x^2-x+1}dx\right)-\frac{1}{3}\log(x+1)\right)-\frac{9}{2}\left(\frac{3\int\frac{1}{-\left(1-\frac{2x}{\sqrt[3]{3}}\right)^2-3}d\left(1-\frac{2x}{\sqrt[3]{3}}\right)-\frac{1}{2}\int\frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\sqrt[3]{3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}\right)+\frac{x^2}{2}$$

$$\Big\downarrow \text{217}$$

$$-\frac{9}{2}\left(\frac{-\frac{1}{2}\int\frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\sqrt[3]{3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}\right)+\frac{1}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int\frac{1}{x^2-x+1}dx-\frac{1}{2}\int\frac{1-2x}{x^2-x+1}dx\right)-\frac{1}{3}\log(x+1)\right)+\frac{x^2}{2}$$

$$\Big\downarrow \text{1083}$$

$$-\frac{9}{2}\left(\frac{-\frac{1}{2}\int\frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\sqrt[3]{3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}\right)+\frac{1}{2}\left(\frac{1}{3}\left(-\frac{1}{2}\int\frac{1-2x}{x^2-x+1}dx-3\int\frac{1}{-(2x-1)^2-3}d(2x-1)\right)-\frac{1}{3}\log(x+1)\right)+\frac{x^2}{2}$$

3.160. $\int\frac{x^7}{3+4x^3+x^6}\,dx$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)-\frac{1}{2}\int\frac{1-2x}{x^2-x+1}dx\right)-\frac{1}{3}\log(x+1)\right)-\frac{9}{2}\left(\frac{-\frac{1}{2}\int\frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\sqrt[3]{3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}\right)+\frac{x^2}{2}$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)+\frac{1}{2}\log\left(x^2-x+1\right)\right)-\frac{1}{3}\log(x+1)\right)-\frac{9}{2}\left(\frac{\frac{1}{2}\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\sqrt[3]{3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}\right)+\frac{x^2}{2}$$

input
```
Int[x^7/(3 + 4*x^3 + x^6),x]
```

output
```
x^2/2 + (-1/3*Log[1 + x] + (Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + Log[1 - x
 + x^2]/2)/3)/2 - (9*(-1/3*Log[3^(1/3) + x]/3^(1/3) + (-(Sqrt[3]*ArcTan[(1
 - (2*x)/3^(1/3))/Sqrt[3]]) + Log[3^(2/3) - 3^(1/3)*x + x^2]/2)/(3*3^(1/3)
)))/2
```

3.160.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1703
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(2*n - 1)*(d*x)^(m - 2*n + 1)*((a + b*x^n + c*x^(2*n))^(
p + 1)/(c*(m + 2*n*p + 1))), x] - Simp[d^(2*n)/(c*(m + 2*n*p + 1))   Int[(d
*x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x
^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && N
eQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0
] && IntegerQ[p]
```

3.160. $\int \frac{x^7}{3+4x^3+x^6} \, dx$

rule 1834
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 +
 (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2
 - (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ
[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n
, 0]
```

3.160.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.53

method	result
risch	$\frac{x^2}{2}+\frac{\ln\left(4x^2-4x+4\right)}{12}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^3-9\right)}\_R\ln\left(\_R^2+3x\right)\right)}{2}-\frac{\ln(x+1)}{6}$
default	$\frac{x^2}{2}-\frac{\ln(x+1)}{6}+\frac{3^{\frac{2}{3}}\ln\left(3^{\frac{1}{3}}+x\right)}{2}-\frac{3^{\frac{2}{3}}\ln\left(3^{\frac{2}{3}}-3^{\frac{1}{3}}x+x^2\right)}{4}-\frac{3\,3^{\frac{1}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,3^{\frac{2}{3}}x}{3}-1\right)}{3}\right)}{2}+\frac{\ln\left(x^2-x+1\right)}{12}+\frac{\sqrt{3}\,\arctan\left(\right.}{6}$

input
```
int(x^7/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2+1/12*ln(4*x^2-4*x+4)+1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))+1/2*s
um(_R*ln(_R^2+3*x),_R=RootOf(_Z^3-9))-1/6*ln(x+1)
```

3.160.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.83

$$\begin{aligned}\int \frac{x^7}{3+4x^3+x^6}\,dx &= \frac{1}{2}\,x^2-\frac{1}{2}\cdot 9^{\frac{1}{3}}\sqrt{3}\arctan\left(\frac{2}{9}\cdot 9^{\frac{1}{3}}\sqrt{3}x-\frac{1}{3}\sqrt{3}\right)\\ &\quad+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right)-\frac{1}{4}\cdot 9^{\frac{1}{3}}\log\left(3\,x^2-9^{\frac{2}{3}}x+3\cdot 9^{\frac{1}{3}}\right)\\ &\quad+\frac{1}{2}\cdot 9^{\frac{1}{3}}\log\left(3\,x+9^{\frac{2}{3}}\right)+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{1}{6}\log\left(x+1\right)\end{aligned}$$

input
```
integrate(x^7/(x^6+4*x^3+3),x, algorithm="fricas")
```

output
```
1/2*x^2 - 1/2*9^(1/3)*sqrt(3)*arctan(2/9*9^(1/3)*sqrt(3)*x - 1/3*sqrt(3))
+ 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/4*9^(1/3)*log(3*x^2 - 9^(2
/3)*x + 3*9^(1/3)) + 1/2*9^(1/3)*log(3*x + 9^(2/3)) + 1/12*log(x^2 - x + 1
) - 1/6*log(x + 1)
```

3.160.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.13

$$\int \frac{x^7}{3+4x^3+x^6}\,dx = \frac{x^2}{2} - \frac{\log(x+1)}{6} + \left(\frac{1}{12} - \frac{\sqrt{3}i}{12}\right)\log\left(x + \frac{6562\left(\frac{1}{12} - \frac{\sqrt{3}i}{12}\right)^2}{183} - \frac{1872\left(\frac{1}{12} - \frac{\sqrt{3}i}{12}\right)^5}{61}\right) + \left(\frac{1}{12} + \frac{\sqrt{3}i}{12}\right)\log\left(x - \frac{1872\left(\frac{1}{12} + \frac{\sqrt{3}i}{12}\right)^5}{61} + \frac{6562\left(\frac{1}{12} + \frac{\sqrt{3}i}{12}\right)^2}{183}\right) + \operatorname{RootSum}\left(8t^3 - 9, \left(t \mapsto t\log\left(-\frac{1872t^5}{61} + \frac{6562t^2}{183} + x\right)\right)\right)$$

input
```
integrate(x**7/(x**6+4*x**3+3),x)
```

output
```
x**2/2 - log(x + 1)/6 + (1/12 - sqrt(3)*I/12)*log(x + 6562*(1/12 - sqrt(3)
*I/12)**2/183 - 1872*(1/12 - sqrt(3)*I/12)**5/61) + (1/12 + sqrt(3)*I/12)*
log(x - 1872*(1/12 + sqrt(3)*I/12)**5/61 + 6562*(1/12 + sqrt(3)*I/12)**2/1
83) + RootSum(8*_t**3 - 9, Lambda(_t, _t*log(-1872*_t**5/61 + 6562*_t**2/1
83 + x)))
```

3.160.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.75

$$\int \frac{x^7}{3+4x^3+x^6}\,dx = \frac{1}{2}\,x^2 - \frac{1}{4}\cdot 3^{\frac{2}{3}}\log\left(x^2 - 3^{\frac{1}{3}}x + 3^{\frac{2}{3}}\right) + \frac{1}{2}\cdot 3^{\frac{2}{3}}\log\left(x + 3^{\frac{1}{3}}\right) + \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) - \frac{3}{2}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2\,x - 3^{\frac{1}{3}}\right)\right) + \frac{1}{12}\,\log\left(x^2-x+1\right) - \frac{1}{6}\,\log\left(x+1\right)$$

input
```
integrate(x^7/(x^6+4*x^3+3),x, algorithm="maxima")
```

output
```
1/2*x^2 - 1/4*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/2*3^(2/3)*log(x +
 3^(1/3)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 3/2*3^(1/6)*arctan
(1/3*3^(1/6)*(2*x - 3^(1/3))) + 1/12*log(x^2 - x + 1) - 1/6*log(x + 1)
```

3.160.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.76

$$\int \frac{x^7}{3+4x^3+x^6}\,dx = \frac{1}{2}\,x^2 - \frac{1}{4}\cdot 3^{\frac{2}{3}}\log\left(x^2 - 3^{\frac{1}{3}}x + 3^{\frac{2}{3}}\right) + \frac{1}{2}\cdot 3^{\frac{2}{3}}\log\left(\left|x + 3^{\frac{1}{3}}\right|\right) + \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) - \frac{3}{2}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2\,x - 3^{\frac{1}{3}}\right)\right) + \frac{1}{12}\,\log\left(x^2-x+1\right) - \frac{1}{6}\,\log\left(|x+1|\right)$$

input
```
integrate(x^7/(x^6+4*x^3+3),x, algorithm="giac")
```

output
```
1/2*x^2 - 1/4*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/2*3^(2/3)*log(abs
(x + 3^(1/3))) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 3/2*3^(1/6)*a
rctan(1/3*3^(1/6)*(2*x - 3^(1/3))) + 1/12*log(x^2 - x + 1) - 1/6*log(abs(x
 + 1))
```

3.160.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.99

$$\int \frac{x^7}{3+4x^3+x^6}\,dx = \frac{3^{2/3}\,\ln\left(x+3^{1/3}\right)}{2} - \frac{\ln(x+1)}{6} - \ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) + \ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) + \frac{x^2}{2} - \ln\left(x-\frac{3^{1/3}}{2}-\frac{3^{5/6}\,1\mathrm{i}}{2}\right)\left(\frac{3^{2/3}}{4}-\frac{3^{1/6}\,3\mathrm{i}}{4}\right) - \ln\left(x-\frac{3^{1/3}}{2}+\frac{3^{5/6}\,1\mathrm{i}}{2}\right)\left(\frac{3^{2/3}}{4}+\frac{3^{1/6}\,3\mathrm{i}}{4}\right)$$

input
```
int(x^7/(4*x^3 + x^6 + 3),x)
```

output
```
(3^(2/3)*log(x + 3^(1/3)))/2 - log(x + 1)/6 - log(x - (3^(1/2)*1i)/2 - 1/2
)*((3^(1/2)*1i)/12 - 1/12) + log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/1
2 + 1/12) + x^2/2 - log(x - 3^(1/3)/2 - (3^(5/6)*1i)/2)*(3^(2/3)/4 - (3^(1
/6)*3i)/4) - log(x - 3^(1/3)/2 + (3^(5/6)*1i)/2)*(3^(2/3)/4 + (3^(1/6)*3i)
/4)
```

3.161 $\int \frac{x^6}{3+4x^3+x^6} \, dx$

3.161.1 Optimal result

Integrand size = 16, antiderivative size = 113

$$\int \frac{x^6}{3+4x^3+x^6} \, dx = x-\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{1}{2}3^{5/6}\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)+\frac{1}{6}\log(1+x)-\frac{1}{2}\sqrt[3]{3}\log\left(\sqrt[3]{3}+x\right)-\frac{1}{12}\log\left(1-x+x^2\right)+\frac{1}{4}\sqrt[3]{3}\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)$$

output
```
x+1/2*3^(5/6)*arctan(1/3*(3^(1/3)-2*x)*3^(1/6))+1/6*ln(1+x)-1/2*3^(1/3)*ln
(3^(1/3)+x)-1/12*ln(x^2-x+1)+1/4*3^(1/3)*ln(3^(2/3)-3^(1/3)*x+x^2)-1/6*arc
tan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

3.161.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.98

$$\int \frac{x^6}{3+4x^3+x^6} \, dx = \frac{1}{12}\left(12x+6\,3^{5/6}\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)+2\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)+2\log(1+x)-6\sqrt[3]{3}\log\left(3+3^{2/3}x\right)-\log\left(1-x+x^2\right)\right.$$

input
```
Integrate[x^6/(3 + 4*x^3 + x^6),x]
```

output
```
(12*x + 6*3^(5/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] + 2*Sqrt[3]*ArcTan[(-1 +
 2*x)/Sqrt[3]] + 2*Log[1 + x] - 6*3^(1/3)*Log[3 + 3^(2/3)*x] - Log[1 - x +
 x^2] + 3*3^(1/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/12
```

3.161.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.14, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.688, Rules used = {1703, 1752, 750, 16, 1142, 25, 1082, 217, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{x^6+4x^3+3}dx$$

$$\Big\downarrow \text{1703}$$

$$x-\int \frac{4x^3+3}{x^6+4x^3+3}dx$$

$$\Big\downarrow \text{1752}$$

$$\frac{1}{2}\int \frac{1}{x^3+1}dx-\frac{9}{2}\int \frac{1}{x^3+3}dx+x$$

$$\Big\downarrow \text{750}$$

$$\frac{1}{2}\left(\frac{1}{3}\int \frac{2-x}{x^2-x+1}dx+\frac{1}{3}\int \frac{1}{x+1}dx\right)-\frac{9}{2}\left(\frac{\int \frac{2\sqrt[3]{3}-x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\ 3^{2/3}}+\frac{\int \frac{1}{x+\sqrt[3]{3}}dx}{3\ 3^{2/3}}\right)+x$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}\left(\frac{1}{3}\int \frac{2-x}{x^2-x+1}dx+\frac{1}{3}\log(x+1)\right)-\frac{9}{2}\left(\frac{\int \frac{2\sqrt[3]{3}-x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\ 3^{2/3}}+\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)+x$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^2-x+1}dx-\frac{1}{2}\int -\frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{3}\log(x+1)\right)-\frac{9}{2}\left(\frac{\frac{3}{2}\sqrt[3]{3}\int \frac{1}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\frac{1}{2}\int -\frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\ 3^{2/3}}+\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)+x$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^2-x+1}dx+\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{3}\log(x+1)\right)-\frac{9}{2}\left(\frac{\frac{3}{2}\sqrt[3]{3}\int \frac{1}{x^2-\sqrt[3]{3}x+3^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\ 3^{2/3}}+\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)+x$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^2-x+1}dx+\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{3}\log(x+1)\right)-\frac{9}{2}\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx+3\int \frac{1}{-\left(1-\frac{2x}{\sqrt[3]{3}}\right)^2-3}d\left(1-\frac{2x}{\sqrt[3]{3}}\right)}{3\ 3^{2/3}}+\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)+x$$

$$\Big\downarrow \text{217}$$

$$-\frac{9}{2}\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\ 3^{2/3}}+\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)+\frac{1}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^2-x+1}dx+\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{3}\log(x+1)\right)+x$$

$$\Big\downarrow \text{1083}$$

$$-\frac{9}{2}\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\ 3^{2/3}}+\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)+\frac{1}{2}\left(\frac{1}{3}\left(\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx-3\int \frac{1}{-(2x-1)^2-3}d(2x-1)\right)+\frac{1}{3}\log(x+1)\right)+x$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx+\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)\right)+\frac{1}{3}\log(x+1)\right)-\frac{9}{2}\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\ 3^{2/3}}+\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)+x$$

3.161. $\int \frac{x^6}{3+4x^3+x^6}\,dx$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)-\frac{1}{2}\log\left(x^2-x+1\right)\right)+\frac{1}{3}\log(x+1)\right)-\frac{9}{2}\left(\frac{-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)-\frac{1}{2}\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{3\ 3^{2/3}}+\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)+x$$

input
```
Int[x^6/(3 + 4*x^3 + x^6),x]
```

output
```
x + (Log[1 + x]/3 + (Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - Log[1 - x + x^2]
/2)/3)/2 - (9*(Log[3^(1/3) + x]/(3*3^(2/3)) + (-(Sqrt[3]*ArcTan[(1 - (2*x)
/3^(1/3))/Sqrt[3]]) - Log[3^(2/3) - 3^(1/3)*x + x^2]/2)/(3*3^(2/3))))/2
```

3.161.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.161. $\int \frac{x^6}{3+4x^3+x^6} \, dx$

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1703
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(2*n - 1)*(d*x)^(m - 2*n + 1)*((a + b*x^n + c*x^(2*n))^(
p + 1)/(c*(m + 2*n*p + 1))), x] - Simp[d^(2*n)/(c*(m + 2*n*p + 1))   Int[(d
*x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x
^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && N
eQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0
] && IntegerQ[p]
```

rule 1752
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

3.161. $\int \frac{x^6}{3+4x^3+x^6} \, dx$

3.161.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.47

method	result
risch	$x+\frac{\ln(x+1)}{6}-\frac{\ln(x^2-x+1)}{12}+\frac{\sqrt{3}\arctan\left(\frac{2\left(x-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{6}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^3+3\right)}\_R\ln\left(x-\_R\right)\right)}{2}$
default	$x+\frac{\ln(x+1)}{6}-\frac{3^{\frac{1}{3}}\ln\left(3^{\frac{1}{3}}+x\right)}{2}+\frac{3^{\frac{1}{3}}\ln\left(3^{\frac{2}{3}}-3^{\frac{1}{3}}x+x^2\right)}{4}-\frac{3^{\frac{5}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,3^{\frac{2}{3}}x}{3}-1\right)}{3}\right)}{2}-\frac{\ln(x^2-x+1)}{12}+\frac{\sqrt{3}\arctan\left(\frac{(2x-$

input
```
int(x^6/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
```

output
```
x+1/6*ln(x+1)-1/12*ln(x^2-x+1)+1/6*3^(1/2)*arctan(2/3*(x-1/2)*3^(1/2))+1/2
*sum(_R*ln(x-_R),_R=RootOf(_Z^3+3))
```

3.161.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.78

$$\int \frac{x^6}{3+4x^3+x^6}\,dx = \frac{1}{2}\sqrt{3}(-3)^{\frac{1}{3}}\arctan\left(\frac{1}{9}\sqrt{3}\left(2\,(-3)^{\frac{2}{3}}\,x-3\right)\right) + \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right) - \frac{1}{4}\,(-3)^{\frac{1}{3}}\log\left(x^2+(-3)^{\frac{1}{3}}\,x+(-3)^{\frac{2}{3}}\right) + \frac{1}{2}\,(-3)^{\frac{1}{3}}\log\left(x-(-3)^{\frac{1}{3}}\right) + x - \frac{1}{12}\log\left(x^2-x+1\right) + \frac{1}{6}\log\left(x+1\right)$$

input
```
integrate(x^6/(x^6+4*x^3+3),x, algorithm="fricas")
```

output
```
1/2*sqrt(3)*(-3)^(1/3)*arctan(1/9*sqrt(3)*(2*(-3)^(2/3)*x - 3)) + 1/6*sqrt
(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/4*(-3)^(1/3)*log(x^2 + (-3)^(1/3)*x
+ (-3)^(2/3)) + 1/2*(-3)^(1/3)*log(x - (-3)^(1/3)) + x - 1/12*log(x^2 - x
+ 1) + 1/6*log(x + 1)
```

3.161.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.12

$$\int \frac{x^6}{3+4x^3+x^6}\,dx = x+\frac{\log(x+1)}{6} + \left(-\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)\log\left(x-\frac{121}{246}-\frac{121\sqrt{3}i}{246}+\frac{864\left(-\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)^4}{41}\right) + \left(-\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)\log\left(x-\frac{121}{246}+\frac{864\left(-\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)^4}{41}+\frac{121\sqrt{3}i}{246}\right) + \operatorname{RootSum}\left(8t^3+3,\left(t\mapsto t\log\left(\frac{864t^4}{41}+\frac{242t}{41}+x\right)\right)\right)$$

input
```
integrate(x**6/(x**6+4*x**3+3),x)
```

output
```
x + log(x + 1)/6 + (-1/12 - sqrt(3)*I/12)*log(x - 121/246 - 121*sqrt(3)*I/
246 + 864*(-1/12 - sqrt(3)*I/12)**4/41) + (-1/12 + sqrt(3)*I/12)*log(x - 1
21/246 + 864*(-1/12 + sqrt(3)*I/12)**4/41 + 121*sqrt(3)*I/246) + RootSum(8
*_t**3 + 3, Lambda(_t, _t*log(864*_t**4/41 + 242*_t/41 + x)))
```

3.161.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.75

$$\int \frac{x^6}{3+4x^3+x^6}\,dx = -\frac{1}{2}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{4}\cdot 3^{\frac{1}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)-\frac{1}{2}\cdot 3^{\frac{1}{3}}\log\left(x+3^{\frac{1}{3}}\right) + x-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{1}{6}\log(x+1)$$

input
```
integrate(x^6/(x^6+4*x^3+3),x, algorithm="maxima")
```

output
```
-1/2*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) + 1/6*sqrt(3)*arctan(1/3*
sqrt(3)*(2*x - 1)) + 1/4*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 1/2*3^(1
/3)*log(x + 3^(1/3)) + x - 1/12*log(x^2 - x + 1) + 1/6*log(x + 1)
```

3.161.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.77

$$\int \frac{x^6}{3+4x^3+x^6}\,dx = -\frac{1}{2}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2\,x-3^{\frac{1}{3}}\right)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right) + \frac{1}{4}\cdot 3^{\frac{1}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)-\frac{1}{2}\cdot 3^{\frac{1}{3}}\log\left(\left|x+3^{\frac{1}{3}}\right|\right) + x-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{1}{6}\log\left(|x+1|\right)$$

input
```
integrate(x^6/(x^6+4*x^3+3),x, algorithm="giac")
```

output
```
-1/2*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) + 1/6*sqrt(3)*arctan(1/3*
sqrt(3)*(2*x - 1)) + 1/4*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 1/2*3^(1
/3)*log(abs(x + 3^(1/3))) + x - 1/12*log(x^2 - x + 1) + 1/6*log(abs(x + 1)
)
```

3.161.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.92

$$\int \frac{x^6}{3+4x^3+x^6}\,dx = x+\frac{\ln(x+1)}{6}-\frac{3^{1/3}\,\ln\left(x+3^{1/3}\right)}{2} - \ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)+\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) + \ln\left(x-\frac{3^{1/3}}{2}+\frac{3^{5/6}\,1\mathrm{i}}{2}\right)\left(\frac{3^{1/3}}{4}-\frac{3^{5/6}\,1\mathrm{i}}{4}\right)+\frac{(-1)^{1/3}\,3^{1/3}\,\ln\left(x-(-1)^{1/3}\,3^{1/3}\right)}{2}$$

input
```
int(x^6/(4*x^3 + x^6 + 3),x)
```

output
```
x + log(x + 1)/6 - (3^(1/3)*log(x + 3^(1/3)))/2 - log(x - (3^(1/2)*1i)/2 -
 1/2)*((3^(1/2)*1i)/12 + 1/12) + log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1
i)/12 - 1/12) + log(x - 3^(1/3)/2 + (3^(5/6)*1i)/2)*(3^(1/3)/4 - (3^(5/6)*
1i)/4) + ((-1)^(1/3)*3^(1/3)*log(x - (-1)^(1/3)*3^(1/3)))/2
```

3.162 $\int \frac{x^4}{3+4x^3+x^6}\,dx$

3.162.1 Optimal result

Integrand size = 16, antiderivative size = 112

$$\int \frac{x^4}{3+4x^3+x^6}\,dx=\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{2}\sqrt[6]{3}\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)+\frac{1}{6}\log(1+x)-\frac{\log\left(\sqrt[3]{3}+x\right)}{2\sqrt[3]{3}}-\frac{1}{12}\log\left(1-x+x^2\right)+\frac{\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)}{4\sqrt[3]{3}}$$

output
```
-1/2*3^(1/6)*arctan(1/3*(3^(1/3)-2*x)*3^(1/6))+1/6*ln(1+x)-1/6*3^(2/3)*ln(
3^(1/3)+x)-1/12*ln(x^2-x+1)+1/12*3^(2/3)*ln(3^(2/3)-3^(1/3)*x+x^2)+1/6*arc
tan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

3.162.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.96

$$\int \frac{x^4}{3+4x^3+x^6}\,dx=\frac{1}{12}\left(-6\sqrt[6]{3}\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)-2\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)+2\log(1+x)-2\,3^{2/3}\log\left(3+3^{2/3}x\right)-\log\left(1-x+x^2\right)+3^{2/3}\log\left(3-3^{2/3}x+\sqrt[3]{3}x^2\right)\right)$$

input
```
Integrate[x^4/(3 + 4*x^3 + x^6),x]
```

output
```
(-6*3^(1/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/
Sqrt[3]] + 2*Log[1 + x] - 2*3^(2/3)*Log[3 + 3^(2/3)*x] - Log[1 - x + x^2]
+ 3^(2/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/12
```

3.162.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.15, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {1710, 821, 16, 1142, 25, 1082, 217, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{x^6+4x^3+3}\,dx$$

$$\Big\downarrow \text{1710}$$

$$\frac{3}{2}\int \frac{x}{x^3+3}dx-\frac{1}{2}\int \frac{x}{x^3+1}dx$$

$$\Big\downarrow \text{821}$$

$$\frac{1}{2}\left(\frac{1}{3}\int \frac{1}{x+1}dx-\frac{1}{3}\int \frac{x+1}{x^2-x+1}dx\right)+\frac{3}{2}\left(\frac{\int \frac{x+\sqrt[3]{3}}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\sqrt[3]{3}}-\frac{\int \frac{1}{x+\sqrt[3]{3}}dx}{3\sqrt[3]{3}}\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}\left(\frac{1}{3}\log(x+1)-\frac{1}{3}\int \frac{x+1}{x^2-x+1}dx\right)+\frac{3}{2}\left(\frac{\int \frac{x+\sqrt[3]{3}}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\sqrt[3]{3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}\right)$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(-\frac{3}{2}\int \frac{1}{x^2-x+1}dx-\frac{1}{2}\int -\frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{3}\log(x+1)\right)+\frac{3}{2}\left(\frac{\frac{3}{2}\sqrt[3]{3}\int \frac{1}{x^2-\sqrt[3]{3}x+3^{2/3}}dx+\frac{1}{2}\int -\frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\sqrt[3]{3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx-\frac{3}{2}\int \frac{1}{x^2-x+1}dx\right)+\frac{1}{3}\log(x+1)\right)+\frac{3}{2}\left(\frac{\frac{3}{2}\sqrt[3]{3}\int \frac{1}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\sqrt[3]{3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}\right)$$

$\Big\downarrow$ 1082

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx-\frac{3}{2}\int \frac{1}{x^2-x+1}dx\right)+\frac{1}{3}\log(x+1)\right)+\frac{3}{2}\left(\frac{3\int \frac{1}{-\left(1-\frac{2x}{\sqrt[3]{3}}\right)^2-3}d\left(1-\frac{2x}{\sqrt[3]{3}}\right)-\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\sqrt[3]{3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}\right)$$

$\Big\downarrow$ 217

$$\frac{3}{2}\left(\frac{-\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\sqrt[3]{3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}\right)+\frac{1}{2}\left(\frac{1}{3}\left(\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx-\frac{3}{2}\int \frac{1}{x^2-x+1}dx\right)+\frac{1}{3}\log(x+1)\right)$$

$\Big\downarrow$ 1083

$$\frac{3}{2}\left(\frac{-\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\sqrt[3]{3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}\right)+\frac{1}{2}\left(\frac{1}{3}\left(\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx+3\int \frac{1}{-(2x-1)^2-3}d(2x-1)\right)+\frac{1}{3}\log(x+1)\right)$$

$\Big\downarrow$ 217

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx-\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)\right)+\frac{1}{3}\log(x+1)\right)+\frac{3}{2}\left(\frac{-\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\sqrt[3]{3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}\right)$$

$\Big\downarrow$ 1103

3.162. $\int \frac{x^4}{3+4x^3+x^6}\,dx$

$$\frac{1}{2}\left(\frac{1}{3}\left(-\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)-\frac{1}{2}\log\left(x^2-x+1\right)\right)+\frac{1}{3}\log(x+1)\right)+\frac{3}{2}\left(\frac{\frac{1}{2}\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\sqrt[3]{3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}\right)$$

input
```
Int[x^4/(3 + 4*x^3 + x^6),x]
```

output
```
(Log[1 + x]/3 + (-(Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]]) - Log[1 - x + x^2]/
2)/3)/2 + (3*(-1/3*Log[3^(1/3) + x]/3^(1/3) + (-(Sqrt[3]*ArcTan[(1 - (2*x)
/3^(1/3))/Sqrt[3]]) + Log[3^(2/3) - 3^(1/3)*x + x^2]/2)/(3*3^(1/3))))/2
```

3.162.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1710
```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^n/2)*(b/q + 1)   Int[(d*x)^(m
- n)/(b/2 + q/2 + c*x^n), x], x] - Simp[(d^n/2)*(b/q - 1)   Int[(d*x)^(m -
n)/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

3.162.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.54

method	result
risch	$-\frac{\ln\left(4x^{2}-4x+4\right)}{12}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}+\frac{\ln(x+1)}{6}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(3\_Z^{3}+1\right)}\_R\ln\left(3\_R^{2}+x\right)\right)}{2}$
default	$\frac{\ln(x+1)}{6}-\frac{3^{\frac{2}{3}}\ln\left(3^{\frac{1}{3}}+x\right)}{6}+\frac{3^{\frac{2}{3}}\ln\left(3^{\frac{2}{3}}-3^{\frac{1}{3}}x+x^{2}\right)}{12}+\frac{3^{\frac{1}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,3^{\frac{2}{3}}x}{3}-1\right)}{3}\right)}{2}-\frac{\ln\left(x^{2}-x+1\right)}{12}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right.}{6}$

input
```
int(x^4/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
```

output
```
-1/12*ln(4*x^2-4*x+4)-1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))+1/6*ln(x+1)+
1/2*sum(_R*ln(3*_R^2+x),_R=RootOf(3*_Z^3+1))
```

3.162.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.95

$$\int \frac{x^4}{3+4x^3+x^6}\,dx = -\frac{1}{12}\cdot 3^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-3^{\frac{1}{3}}(-1)^{\frac{2}{3}}x+x^2-3^{\frac{2}{3}}(-1)^{\frac{1}{3}}\right)+\frac{1}{6}\cdot 3^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(3^{\frac{1}{3}}(-1)^{\frac{2}{3}}+x\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{2}\cdot 3^{\frac{1}{6}}(-1)^{\frac{1}{3}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2(-1)^{\frac{1}{3}}x+3^{\frac{1}{3}}\right)\right)-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{1}{6}\log\left(x+1\right)$$

input
```
integrate(x^4/(x^6+4*x^3+3),x, algorithm="fricas")
```

output
```
-1/12*3^(2/3)*(-1)^(1/3)*log(-3^(1/3)*(-1)^(2/3)*x + x^2 - 3^(2/3)*(-1)^(1
/3)) + 1/6*3^(2/3)*(-1)^(1/3)*log(3^(1/3)*(-1)^(2/3) + x) - 1/6*sqrt(3)*ar
ctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*3^(1/6)*(-1)^(1/3)*arctan(1/3*3^(1/6)*(2
*(-1)^(1/3)*x + 3^(1/3))) - 1/12*log(x^2 - x + 1) + 1/6*log(x + 1)
```

3.162.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.20

$$\int \frac{x^4}{3+4x^3+x^6}\,dx = \frac{\log(x+1)}{6}+\left(-\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)\log\left(x+\frac{2592\left(-\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)^5}{5}+\frac{168\left(-\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)^2}{5}\right)+\left(-\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)\log\left(x+\frac{168\left(-\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)^2}{5}+\frac{2592\left(-\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)^5}{5}\right)+\operatorname{RootSum}\left(24t^3+1,\left(t\mapsto t\log\left(\frac{2592t^5}{5}+\frac{168t^2}{5}+x\right)\right)\right)$$

input
```
integrate(x**4/(x**6+4*x**3+3),x)
```

output
```
log(x + 1)/6 + (-1/12 - sqrt(3)*I/12)*log(x + 2592*(-1/12 - sqrt(3)*I/12)*
*5/5 + 168*(-1/12 - sqrt(3)*I/12)**2/5) + (-1/12 + sqrt(3)*I/12)*log(x + 1
68*(-1/12 + sqrt(3)*I/12)**2/5 + 2592*(-1/12 + sqrt(3)*I/12)**5/5) + RootS
um(24*_t**3 + 1, Lambda(_t, _t*log(2592*_t**5/5 + 168*_t**2/5 + x)))
```

3.162.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.75

$$\int \frac{x^4}{3+4x^3+x^6}\,dx = \frac{1}{12}\cdot 3^{\frac{2}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)-\frac{1}{6}\cdot 3^{\frac{2}{3}}\log\left(x+3^{\frac{1}{3}}\right) -\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{2}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right)\right) -\frac{1}{12}\log\left(x^2-x+1\right)+\frac{1}{6}\log\left(x+1\right)$$

input
```
integrate(x^4/(x^6+4*x^3+3),x, algorithm="maxima")
```

output
```
1/12*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 1/6*3^(2/3)*log(x + 3^(1/3))
 - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*3^(1/6)*arctan(1/3*3^(1
/6)*(2*x - 3^(1/3))) - 1/12*log(x^2 - x + 1) + 1/6*log(x + 1)
```

3.162.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.77

$$\int \frac{x^4}{3+4x^3+x^6}\,dx = \frac{1}{12}\cdot 3^{\frac{2}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)-\frac{1}{6}\cdot 3^{\frac{2}{3}}\log\left(\left|x+3^{\frac{1}{3}}\right|\right) -\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{2}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right)\right) -\frac{1}{12}\log\left(x^2-x+1\right)+\frac{1}{6}\log\left(|x+1|\right)$$

input
```
integrate(x^4/(x^6+4*x^3+3),x, algorithm="giac")
```

output
```
1/12*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 1/6*3^(2/3)*log(abs(x + 3^(1
/3))) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*3^(1/6)*arctan(1/3
*3^(1/6)*(2*x - 3^(1/3))) - 1/12*log(x^2 - x + 1) + 1/6*log(abs(x + 1))
```

3.162.9 Mupad [B] (verification not implemented)

Time = 8.35 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.02

$$\int \frac{x^4}{3+4x^3+x^6}\,dx = \frac{\ln(x+1)}{6} - \frac{3^{2/3}\ln\left(x+3^{1/3}\right)}{6} + \ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) - \ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) - \frac{(-1)^{1/3}\ln\left(x-\frac{(-1)^{1/3}\,3^{1/3}}{2}-\frac{(-1)^{1/6}\,3^{5/6}}{2}+\frac{3^{1/3}}{2}\right)\left(3^{2/3}+3^{1/6}\,3\mathrm{i}\right)}{12} + \frac{(-1)^{1/3}\,3^{2/3}\ln\left(x+(-1)^{2/3}\,3^{1/3}\right)}{6}$$

input
```
int(x^4/(4*x^3 + x^6 + 3),x)
```

output
```
log(x + 1)/6 - (3^(2/3)*log(x + 3^(1/3)))/6 + log(x - (3^(1/2)*1i)/2 - 1/2
)*((3^(1/2)*1i)/12 - 1/12) - log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/1
2 + 1/12) - ((-1)^(1/3)*log(x - ((-1)^(1/3)*3^(1/3))/2 - ((-1)^(1/6)*3^(5/
6))/2 + 3^(1/3)/2)*(3^(2/3) + 3^(1/6)*3i))/12 + ((-1)^(1/3)*3^(2/3)*log(x
+ (-1)^(2/3)*3^(1/3)))/6
```

3.163 $\int \frac{x^3}{3+4x^3+x^6} \, dx$

3.163.1 Optimal result

Integrand size = 16, antiderivative size = 112

$$\int \frac{x^3}{3+4x^3+x^6} \, dx = \frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{2\sqrt[6]{3}} - \frac{1}{6}\log(1+x) + \frac{\log\left(\sqrt[3]{3}+x\right)}{2\ 3^{2/3}} + \frac{1}{12}\log\left(1-x+x^2\right) - \frac{\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)}{4\ 3^{2/3}}$$

output
```
-1/6*3^(5/6)*arctan(1/3*(3^(1/3)-2*x)*3^(1/6))-1/6*ln(1+x)+1/6*3^(1/3)*ln(
3^(1/3)+x)+1/12*ln(x^2-x+1)-1/12*3^(1/3)*ln(3^(2/3)-3^(1/3)*x+x^2)+1/6*arc
tan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

3.163.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.95

$$\int \frac{x^3}{3+4x^3+x^6} \, dx$$
$$= \frac{1}{12}\left(-23^{5/6}\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right) - 2\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right) - 2\log(1+x) + 2\sqrt[3]{3}\log\left(3+3^{2/3}x\right) + \log\left(1-x+\right.\right.$$

input
```
Integrate[x^3/(3 + 4*x^3 + x^6),x]
```

output
```
(-2*3^(5/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/
Sqrt[3]] - 2*Log[1 + x] + 2*3^(1/3)*Log[3 + 3^(2/3)*x] + Log[1 - x + x^2]
- 3^(1/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/12
```

3.163.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.15, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {1710, 750, 16, 1142, 25, 1082, 217, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{x^6+4x^3+3}\,dx$$

$$\Big\downarrow \; 1710$$

$$\frac{3}{2}\int \frac{1}{x^3+3}dx-\frac{1}{2}\int \frac{1}{x^3+1}dx$$

$$\Big\downarrow \; 750$$

$$\frac{1}{2}\left(-\frac{1}{3}\int \frac{2-x}{x^2-x+1}dx-\frac{1}{3}\int \frac{1}{x+1}dx\right)+\frac{3}{2}\left(\frac{\int \frac{2\sqrt[3]{3}-x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\ 3^{2/3}}+\frac{\int \frac{1}{x+\sqrt[3]{3}}dx}{3\ 3^{2/3}}\right)$$

$$\Big\downarrow \; 16$$

$$\frac{1}{2}\left(-\frac{1}{3}\int \frac{2-x}{x^2-x+1}dx-\frac{1}{3}\log(x+1)\right)+\frac{3}{2}\left(\frac{\int \frac{2\sqrt[3]{3}-x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\ 3^{2/3}}+\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)$$

$$\Big\downarrow \; 1142$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{1}{2}\int -\frac{1-2x}{x^2-x+1}dx-\frac{3}{2}\int \frac{1}{x^2-x+1}dx\right)-\frac{1}{3}\log(x+1)\right)+\frac{3}{2}\left(\frac{\frac{3}{2}\sqrt[3]{3}\int \frac{1}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\frac{1}{2}\int -\frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\ 3^{2/3}}+\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)$$

$$\Big\downarrow \; 25$$

$$\frac{1}{2}\left(\frac{1}{3}\left(-\frac{3}{2}\int\frac{1}{x^2-x+1}dx-\frac{1}{2}\int\frac{1-2x}{x^2-x+1}dx\right)-\frac{1}{3}\log(x+1)\right)+\frac{3}{2}\left(\frac{\frac{3}{2}\sqrt[3]{3}\int\frac{1}{x^2-\sqrt[3]{3}x+3^{2/3}}dx+\frac{1}{2}\int\frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\ 3^{2/3}}+\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)$$

$$\Big\downarrow \ 1082$$

$$\frac{1}{2}\left(\frac{1}{3}\left(-\frac{3}{2}\int\frac{1}{x^2-x+1}dx-\frac{1}{2}\int\frac{1-2x}{x^2-x+1}dx\right)-\frac{1}{3}\log(x+1)\right)+\frac{3}{2}\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx+3\int\frac{1}{-\left(1-\frac{2x}{\sqrt[3]{3}}\right)^2-3}d\left(1-\frac{2x}{\sqrt[3]{3}}\right)}{3\ 3^{2/3}}+\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)$$

$$\Big\downarrow \ 217$$

$$\frac{3}{2}\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\ 3^{2/3}}+\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)+\frac{1}{2}\left(\frac{1}{3}\left(-\frac{3}{2}\int\frac{1}{x^2-x+1}dx-\frac{1}{2}\int\frac{1-2x}{x^2-x+1}dx\right)-\frac{1}{3}\log(x+1)\right)$$

$$\Big\downarrow \ 1083$$

$$\frac{3}{2}\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\ 3^{2/3}}+\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)+\frac{1}{2}\left(\frac{1}{3}\left(3\int\frac{1}{-(2x-1)^2-3}d(2x-1)-\frac{1}{2}\int\frac{1-2x}{x^2-x+1}dx\right)-\frac{1}{3}\log(x+1)\right)$$

$$\Big\downarrow \ 217$$

$$\frac{1}{2}\left(\frac{1}{3}\left(-\frac{1}{2}\int\frac{1-2x}{x^2-x+1}dx-\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)\right)-\frac{1}{3}\log(x+1)\right)+\frac{3}{2}\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\ 3^{2/3}}+\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)$$

$$\Big\downarrow \ 1103$$

3.163. $\int \frac{x^3}{3+4x^3+x^6}\,dx$

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{1}{2}\log\left(x^2-x+1\right)-\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)\right)-\frac{1}{3}\log(x+1)\right)+\frac{3}{2}\left(\frac{-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)-\frac{1}{2}\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{3\ 3^{2/3}}+\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)$$

input
```
Int[x^3/(3 + 4*x^3 + x^6),x]
```

output
```
(-1/3*Log[1 + x] + (-(Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]]) + Log[1 - x + x^
2]/2)/3)/2 + (3*(Log[3^(1/3) + x]/(3*3^(2/3)) + (-(Sqrt[3]*ArcTan[(1 - (2*
x)/3^(1/3))/Sqrt[3]]) - Log[3^(2/3) - 3^(1/3)*x + x^2]/2)/(3*3^(2/3))))/2
```

3.163.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1710
```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^n/2)*(b/q + 1)   Int[(d*x)^(m
- n)/(b/2 + q/2 + c*x^n), x], x] - Simp[(d^n/2)*(b/q - 1)   Int[(d*x)^(m -
n)/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

3.163.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.48

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(9\_Z^{3}-1\right)}\_R\ln\left(x+3\_R\right)\right)}{2}-\frac{\ln\left(x+1\right)}{6}+\frac{\ln\left(x^{2}-x+1\right)}{12}-\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{6}$
default	$-\frac{\ln\left(x+1\right)}{6}+\frac{3^{\frac{1}{3}}\ln\left(3^{\frac{1}{3}}+x\right)}{6}-\frac{3^{\frac{1}{3}}\ln\left(3^{\frac{2}{3}}-3^{\frac{1}{3}}x+x^{2}\right)}{12}+\frac{3^{\frac{5}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,3^{\frac{2}{3}}x}{3}-1\right)}{3}\right)}{6}+\frac{\ln\left(x^{2}-x+1\right)}{12}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{}}{3}\right.}{6}$

input
```
int(x^3/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
```

output
```
1/2*sum(_R*ln(x+3*_R),_R=RootOf(9*_Z^3-1))-1/6*ln(x+1)+1/12*ln(x^2-x+1)-1/
6*3^(1/2)*arctan(2/3*(x-1/2)*3^(1/2))
```

3.163.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.91

$$\int \frac{x^3}{3+4x^3+x^6}\,dx = \frac{1}{6}\cdot 9^{\frac{1}{6}}\sqrt{3}\arctan\left(\frac{1}{27}\cdot 9^{\frac{1}{6}}\left(2\cdot 9^{\frac{2}{3}}\sqrt{3}x-3\cdot 9^{\frac{1}{3}}\sqrt{3}\right)\right)-\frac{1}{36}\cdot 9^{\frac{2}{3}}\log\left(3\,x^2-9^{\frac{2}{3}}x+3\cdot 9^{\frac{1}{3}}\right)+\frac{1}{18}\cdot 9^{\frac{2}{3}}\log\left(3\,x+9^{\frac{2}{3}}\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right)+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{1}{6}\log\left(x+1\right)$$

input
```
integrate(x^3/(x^6+4*x^3+3),x, algorithm="fricas")
```

output
```
1/6*9^(1/6)*sqrt(3)*arctan(1/27*9^(1/6)*(2*9^(2/3)*sqrt(3)*x - 3*9^(1/3)*s
qrt(3))) - 1/36*9^(2/3)*log(3*x^2 - 9^(2/3)*x + 3*9^(1/3)) + 1/18*9^(2/3)*
log(3*x + 9^(2/3)) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/12*log(
x^2 - x + 1) - 1/6*log(x + 1)
```

3.163.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.98

$$\int \frac{x^3}{3+4x^3+x^6}\,dx = -\frac{\log(x+1)}{6}+\left(\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)\log\left(x-\frac{1}{4}+648\left(\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)^4+\frac{\sqrt{3}i}{4}\right)+\left(\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)\log\left(x-\frac{1}{4}+648\left(\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)^4-\frac{\sqrt{3}i}{4}\right)+\operatorname{RootSum}\left(72t^3-1,\left(t\mapsto t\log\left(648t^4-3t+x\right)\right)\right)$$

input
```
integrate(x**3/(x**6+4*x**3+3),x)
```

output
```
-log(x + 1)/6 + (1/12 - sqrt(3)*I/12)*log(x - 1/4 + 648*(1/12 - sqrt(3)*I/
12)**4 + sqrt(3)*I/4) + (1/12 + sqrt(3)*I/12)*log(x - 1/4 + 648*(1/12 + sq
rt(3)*I/12)**4 - sqrt(3)*I/4) + RootSum(72*_t**3 - 1, Lambda(_t, _t*log(64
8*_t**4 - 3*_t + x)))
```

3.163.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.75

$$\int \frac{x^3}{3+4x^3+x^6}\,dx = \frac{1}{6}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2\,x-3^{\frac{1}{3}}\right)\right)-\frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) - \frac{1}{12}\cdot 3^{\frac{1}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 3^{\frac{1}{3}}\log\left(x+3^{\frac{1}{3}}\right) + \frac{1}{12}\,\log\left(x^2-x+1\right)-\frac{1}{6}\,\log\left(x+1\right)$$

input
```
integrate(x^3/(x^6+4*x^3+3),x, algorithm="maxima")
```

output
```
1/6*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) - 1/6*sqrt(3)*arctan(1/3*s
qrt(3)*(2*x - 1)) - 1/12*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/6*3^(1
/3)*log(x + 3^(1/3)) + 1/12*log(x^2 - x + 1) - 1/6*log(x + 1)
```

3.163.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.77

$$\int \frac{x^3}{3+4x^3+x^6}\,dx = \frac{1}{6}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2\,x-3^{\frac{1}{3}}\right)\right)-\frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) - \frac{1}{12}\cdot 3^{\frac{1}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 3^{\frac{1}{3}}\log\left(\left|x+3^{\frac{1}{3}}\right|\right) + \frac{1}{12}\,\log\left(x^2-x+1\right)-\frac{1}{6}\,\log\left(|x+1|\right)$$

input
```
integrate(x^3/(x^6+4*x^3+3),x, algorithm="giac")
```

output
```
1/6*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) - 1/6*sqrt(3)*arctan(1/3*s
qrt(3)*(2*x - 1)) - 1/12*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/6*3^(1
/3)*log(abs(x + 3^(1/3))) + 1/12*log(x^2 - x + 1) - 1/6*log(abs(x + 1))
```

3.163.9 Mupad [B] (verification not implemented)

Time = 8.37 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.01

$$\int \frac{x^3}{3+4x^3+x^6}\,dx$$
$$= \frac{3^{1/3}\ln\left(x+3^{1/3}\right)}{6} - \frac{\ln(x+1)}{6}$$
$$+ \ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) - \ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)$$
$$- \ln\left(x-\frac{3^{1/3}}{2}-\frac{3^{5/6}\,1\mathrm{i}}{2}\right)\left(\frac{3^{1/3}}{12}+\frac{3^{5/6}\,1\mathrm{i}}{12}\right) - \ln\left(x-\frac{3^{1/3}}{2}+\frac{3^{5/6}\,1\mathrm{i}}{2}\right)\left(\frac{3^{1/3}}{12}-\frac{3^{5/6}\,1\mathrm{i}}{12}\right)$$

input
```
int(x^3/(4*x^3 + x^6 + 3),x)
```

output
```
(3^(1/3)*log(x + 3^(1/3)))/6 - log(x + 1)/6 + log(x - (3^(1/2)*1i)/2 - 1/2
)*((3^(1/2)*1i)/12 + 1/12) - log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/1
2 - 1/12) - log(x - 3^(1/3)/2 - (3^(5/6)*1i)/2)*(3^(1/3)/12 + (3^(5/6)*1i)
/12) - log(x - 3^(1/3)/2 + (3^(5/6)*1i)/2)*(3^(1/3)/12 - (3^(5/6)*1i)/12)
```

3.164 $\int \frac{x}{3+4x^3+x^6}\,dx$

3.164.1 Optimal result

Integrand size = 14, antiderivative size = 112

$$\int \frac{x}{3+4x^3+x^6}\,dx = -\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{2\ 3^{5/6}}-\frac{1}{6}\log(1+x)+\frac{\log\left(\sqrt[3]{3}+x\right)}{6\sqrt[3]{3}}+\frac{1}{12}\log\left(1-x+x^2\right)-\frac{\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)}{12\sqrt[3]{3}}$$

output
```
1/6*3^(1/6)*arctan(1/3*(3^(1/3)-2*x)*3^(1/6))-1/6*ln(1+x)+1/18*3^(2/3)*ln(
3^(1/3)+x)+1/12*ln(x^2-x+1)-1/36*3^(2/3)*ln(3^(2/3)-3^(1/3)*x+x^2)-1/6*arc
tan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

3.164.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.96

$$\int \frac{x}{3+4x^3+x^6}\,dx = \frac{1}{36}\left(6\sqrt[6]{3}\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)+6\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)-6\log(1+x)+2\ 3^{2/3}\log\left(3+3^{2/3}x\right)+3\log\left(1-x+x^2\right)-3^{2/3}\log\left(3-3^{2/3}x+\sqrt[3]{3}x^2\right)\right)$$

input
```
Integrate[x/(3 + 4*x^3 + x^6),x]
```

output
```
(6*3^(1/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] + 6*Sqrt[3]*ArcTan[(-1 + 2*x)/S
qrt[3]] - 6*Log[1 + x] + 2*3^(2/3)*Log[3 + 3^(2/3)*x] + 3*Log[1 - x + x^2]
 - 3^(2/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/36
```

3.164.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.14, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {1711, 821, 16, 1142, 25, 1082, 217, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{x^6+4x^3+3}\,dx$$

$$\Big\downarrow \text{1711}$$

$$\frac{1}{2}\int \frac{x}{x^3+1}dx-\frac{1}{2}\int \frac{x}{x^3+3}dx$$

$$\Big\downarrow \text{821}$$

$$\frac{1}{2}\left(\frac{1}{3}\int \frac{x+1}{x^2-x+1}dx-\frac{1}{3}\int \frac{1}{x+1}dx\right)+\frac{1}{2}\left(\frac{\int \frac{1}{x+\sqrt[3]{3}}dx}{3\sqrt[3]{3}}-\frac{\int \frac{x+\sqrt[3]{3}}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\sqrt[3]{3}}\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}\left(\frac{1}{3}\int \frac{x+1}{x^2-x+1}dx-\frac{1}{3}\log(x+1)\right)+\frac{1}{2}\left(\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}-\frac{\int \frac{x+\sqrt[3]{3}}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\sqrt[3]{3}}\right)$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^2-x+1}dx+\frac{1}{2}\int -\frac{1-2x}{x^2-x+1}dx\right)-\frac{1}{3}\log(x+1)\right)+\frac{1}{2}\left(\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}-\frac{\frac{3}{2}\sqrt[3]{3}\int \frac{1}{x^2-\sqrt[3]{3}x+3^{2/3}}dx+\frac{1}{2}\int -\frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\sqrt[3]{3}}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^2-x+1}dx-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx\right)-\frac{1}{3}\log(x+1)\right)+\frac{1}{2}\left(\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}-\frac{\frac{3}{2}\sqrt[3]{3}\int \frac{1}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\sqrt[3]{3}}\right)$$

$\downarrow$ 1082

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^2-x+1}dx-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx\right)-\frac{1}{3}\log(x+1)\right)+\frac{1}{2}\left(\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}-\frac{3\int \frac{1}{-\left(1-\frac{2x}{\sqrt[3]{3}}\right)^2-3}d\left(1-\frac{2x}{\sqrt[3]{3}}\right)-\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\sqrt[3]{3}}\right)$$

$\downarrow$ 217

$$\frac{1}{2}\left(\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}-\frac{-\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\sqrt[3]{3}}\right)+\frac{1}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^2-x+1}dx-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx\right)-\frac{1}{3}\log(x+1)\right)$$

$\downarrow$ 1083

$$\frac{1}{2}\left(\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}-\frac{-\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\sqrt[3]{3}}\right)+\frac{1}{2}\left(\frac{1}{3}\left(-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx-3\int \frac{1}{-(2x-1)^2-3}d(2x-1)\right)-\frac{1}{3}\log(x+1)\right)$$

$\downarrow$ 217

$$\frac{1}{2}\left(\frac{1}{3}\left(\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx\right)-\frac{1}{3}\log(x+1)\right)+\frac{1}{2}\left(\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}-\frac{-\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\sqrt[3]{3}}\right)$$

$\downarrow$ 1103

3.164. $\int \frac{x}{3+4x^3+x^6}\,dx$

$$\frac{1}{2}\left(\frac{1}{3}\left(\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)+\frac{1}{2}\log\left(x^2-x+1\right)\right)-\frac{1}{3}\log(x+1)\right)+\frac{1}{2}\left(\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}-\frac{\frac{1}{2}\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\sqrt[3]{3}}\right)$$

input
```
Int[x/(3 + 4*x^3 + x^6),x]
```

output
```
(-1/3*Log[1 + x] + (Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + Log[1 - x + x^2]/
2)/3)/2 + (Log[3^(1/3) + x]/(3*3^(1/3)) - (-(Sqrt[3]*ArcTan[(1 - (2*x)/3^(
1/3))/Sqrt[3]]) + Log[3^(2/3) - 3^(1/3)*x + x^2]/2)/(3*3^(1/3)))/2
```

3.164.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1711
```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[c/q   Int[(d*x)^m/(b/2 - q/2 + c
*x^n), x], x] - Simp[c/q   Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; Free
Q[{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

3.164.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.50

method	result
risch	$-\frac{\ln(x+1)}{6}+\frac{\ln(x^2-x+1)}{12}+\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{6}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(3\_Z^3-1\right)}\_R\ln\left(3\_R^2+x\right)\right)}{6}$
default	$-\frac{\ln(x+1)}{6}+\frac{3^{\frac{2}{3}}\ln\left(3^{\frac{1}{3}}+x\right)}{18}-\frac{3^{\frac{2}{3}}\ln\left(3^{\frac{2}{3}}-3^{\frac{1}{3}}x+x^2\right)}{36}-\frac{3^{\frac{1}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,3^{\frac{2}{3}}x}{3}-1\right)}{3}\right)}{6}+\frac{\ln(x^2-x+1)}{12}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)}{3}\right.}{6}$

input
```
int(x/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
```

output
```
-1/6*ln(x+1)+1/12*ln(x^2-x+1)+1/6*3^(1/2)*arctan(2/3*(x-1/2)*3^(1/2))+1/6*
sum(_R*ln(3*_R^2+x),_R=RootOf(3*_Z^3-1))
```

3.164.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.75

$$\int \frac{x}{3+4x^3+x^6}\,dx = -\frac{1}{36}\cdot 3^{\frac{2}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)+\frac{1}{18}\cdot 3^{\frac{2}{3}}\log\left(x+3^{\frac{1}{3}}\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{6}\cdot 3^{\frac{1}{6}}\arctan\left(-\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right)\right)+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{1}{6}\log(x+1)$$

input
```
integrate(x/(x^6+4*x^3+3),x, algorithm="fricas")
```

output
```
-1/36*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/18*3^(2/3)*log(x + 3^(1/3
)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/6*3^(1/6)*arctan(-1/3*3
^(1/6)*(2*x - 3^(1/3))) + 1/12*log(x^2 - x + 1) - 1/6*log(x + 1)
```

3.164.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.21 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.06

$$\int \frac{x}{3+4x^3+x^6}\,dx = -\frac{\log(x+1)}{6}+\left(\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)\log\left(x+90\left(\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)^2+11664\left(\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)^5\right)+\left(\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)\log\left(x+11664\left(\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)^5+90\left(\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)^2\right)+\operatorname{RootSum}\left(648t^3-1,\left(t\mapsto t\log\left(11664t^5+90t^2+x\right)\right)\right)$$

input
```
integrate(x/(x**6+4*x**3+3),x)
```

output
```
-log(x + 1)/6 + (1/12 - sqrt(3)*I/12)*log(x + 90*(1/12 - sqrt(3)*I/12)**2
+ 11664*(1/12 - sqrt(3)*I/12)**5) + (1/12 + sqrt(3)*I/12)*log(x + 11664*(1
/12 + sqrt(3)*I/12)**5 + 90*(1/12 + sqrt(3)*I/12)**2) + RootSum(648*_t**3
- 1, Lambda(_t, _t*log(11664*_t**5 + 90*_t**2 + x)))
```

3.164.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.75

$$\int \frac{x}{3+4x^3+x^6}\,dx = -\frac{1}{36}\cdot 3^{\frac{2}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)+\frac{1}{18}\cdot 3^{\frac{2}{3}}\log\left(x+3^{\frac{1}{3}}\right) + \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{6}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right)\right) + \frac{1}{12}\log\left(x^2-x+1\right)-\frac{1}{6}\log\left(x+1\right)$$

input
```
integrate(x/(x^6+4*x^3+3),x, algorithm="maxima")
```

output
```
-1/36*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/18*3^(2/3)*log(x + 3^(1/3
)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/6*3^(1/6)*arctan(1/3*3^
(1/6)*(2*x - 3^(1/3))) + 1/12*log(x^2 - x + 1) - 1/6*log(x + 1)
```

3.164.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.77

$$\int \frac{x}{3+4x^3+x^6}\,dx = -\frac{1}{36}\cdot 3^{\frac{2}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)+\frac{1}{18}\cdot 3^{\frac{2}{3}}\log\left(\left|x+3^{\frac{1}{3}}\right|\right) + \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{6}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right)\right) + \frac{1}{12}\log\left(x^2-x+1\right)-\frac{1}{6}\log\left(|x+1|\right)$$

input
```
integrate(x/(x^6+4*x^3+3),x, algorithm="giac")
```

output
```
-1/36*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/18*3^(2/3)*log(abs(x + 3^
(1/3))) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/6*3^(1/6)*arctan(1
/3*3^(1/6)*(2*x - 3^(1/3))) + 1/12*log(x^2 - x + 1) - 1/6*log(abs(x + 1))
```

3.164.9 Mupad [B] (verification not implemented)

Time = 8.48 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.01

$$\int \frac{x}{3+4x^3+x^6}\,dx = \frac{3^{2/3}\ln\left(x+3^{1/3}\right)}{18}-\frac{\ln(x+1)}{6} - \ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)+\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) - \ln\left(x-\frac{3^{1/3}}{2}-\frac{3^{5/6}\,1\mathrm{i}}{2}\right)\left(\frac{3^{2/3}}{36}-\frac{3^{1/6}\,1\mathrm{i}}{12}\right)-\ln\left(x-\frac{3^{1/3}}{2}+\frac{3^{5/6}\,1\mathrm{i}}{2}\right)\left(\frac{3^{2/3}}{36}+\frac{3^{1/6}\,1\mathrm{i}}{12}\right)$$

input
```
int(x/(4*x^3 + x^6 + 3),x)
```

output
```
(3^(2/3)*log(x + 3^(1/3)))/18 - log(x + 1)/6 - log(x - (3^(1/2)*1i)/2 - 1/
2)*((3^(1/2)*1i)/12 - 1/12) + log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/
12 + 1/12) - log(x - 3^(1/3)/2 - (3^(5/6)*1i)/2)*(3^(2/3)/36 - (3^(1/6)*1i
)/12) - log(x - 3^(1/3)/2 + (3^(5/6)*1i)/2)*(3^(2/3)/36 + (3^(1/6)*1i)/12)
```

3.165 $\int \frac{1}{3+4x^3+x^6}\,dx$

3.165.1 Optimal result

Integrand size = 12, antiderivative size = 112

$$\int \frac{1}{3+4x^3+x^6}\,dx=-\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{6\sqrt[6]{3}}+\frac{1}{6}\log(1+x)-\frac{\log\left(\sqrt[3]{3}+x\right)}{6\ 3^{2/3}}-\frac{1}{12}\log\left(1-x+x^2\right)+\frac{\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)}{12\ 3^{2/3}}$$

output
```
1/18*3^(5/6)*arctan(1/3*(3^(1/3)-2*x)*3^(1/6))+1/6*ln(1+x)-1/18*3^(1/3)*ln
(3^(1/3)+x)-1/12*ln(x^2-x+1)+1/36*3^(1/3)*ln(3^(2/3)-3^(1/3)*x+x^2)-1/6*ar
ctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

3.165.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.96

$$\int \frac{1}{3+4x^3+x^6}\,dx$$
$$=\frac{1}{36}\left(2\,3^{5/6}\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)+6\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)+6\log(1+x)-2\sqrt[3]{3}\log\left(3+3^{2/3}x\right)-3\log\left(1-x+\right.\right.$$

input
```
Integrate[(3 + 4*x^3 + x^6)^(-1),x]
```

output
```
(2*3^(5/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] + 6*Sqrt[3]*ArcTan[(-1 + 2*x)/S
qrt[3]] + 6*Log[1 + x] - 2*3^(1/3)*Log[3 + 3^(2/3)*x] - 3*Log[1 - x + x^2]
 + 3^(1/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/36
```

3.165.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.14, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {1685, 750, 16, 1142, 25, 1082, 217, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6+4x^3+3}\,dx$$

$$\Big\downarrow \text{1685}$$

$$\frac{1}{2}\int \frac{1}{x^3+1}dx-\frac{1}{2}\int \frac{1}{x^3+3}dx$$

$$\Big\downarrow \text{750}$$

$$\frac{1}{2}\left(\frac{1}{3}\int \frac{2-x}{x^2-x+1}dx+\frac{1}{3}\int \frac{1}{x+1}dx\right)+\frac{1}{2}\left(-\frac{\int \frac{2\sqrt[3]{3}-x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\ 3^{2/3}}-\frac{\int \frac{1}{x+\sqrt[3]{3}}dx}{3\ 3^{2/3}}\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}\left(\frac{1}{3}\int \frac{2-x}{x^2-x+1}dx+\frac{1}{3}\log(x+1)\right)+\frac{1}{2}\left(-\frac{\int \frac{2\sqrt[3]{3}-x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\ 3^{2/3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^2-x+1}dx-\frac{1}{2}\int -\frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{3}\log(x+1)\right)+\frac{1}{2}\left(-\frac{\frac{3}{2}\sqrt[3]{3}\int \frac{1}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\frac{1}{2}\int -\frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\ 3^{2/3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^2-x+1}dx+\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{3}\log(x+1)\right)+\frac{1}{2}\left(-\frac{\frac{3}{2}\sqrt[3]{3}\int \frac{1}{x^2-\sqrt[3]{3}x+3^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\ 3^{2/3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)$$

$$\downarrow 1082$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^2-x+1}dx+\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{3}\log(x+1)\right)+\frac{1}{2}\left(-\frac{\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx+3\int \frac{1}{-\left(1-\frac{2x}{\sqrt[3]{3}}\right)^2-3}d\left(1-\frac{2x}{\sqrt[3]{3}}\right)}{3\ 3^{2/3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)$$

$$\downarrow 217$$

$$\frac{1}{2}\left(-\frac{\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\ 3^{2/3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)+\frac{1}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^2-x+1}dx+\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{3}\log(x+1)\right)$$

$$\downarrow 1083$$

$$\frac{1}{2}\left(-\frac{\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\ 3^{2/3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)+\frac{1}{2}\left(\frac{1}{3}\left(\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx-3\int \frac{1}{-(2x-1)^2-3}d(2x-1)\right)+\frac{1}{3}\log(x+1)\right)$$

$$\downarrow 217$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx+\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)\right)+\frac{1}{3}\log(x+1)\right)+\frac{1}{2}\left(-\frac{\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\ 3^{2/3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)$$

$$\downarrow 1103$$

3.165. $\int \frac{1}{3+4x^3+x^6}\,dx$

$$\frac{1}{2}\left(\frac{1}{3}\left(\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)-\frac{1}{2}\log\left(x^2-x+1\right)\right)+\frac{1}{3}\log(x+1)\right)+\frac{1}{2}\left(-\frac{-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)-\frac{1}{2}\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{3\ 3^{2/3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)$$

input
```
Int[(3 + 4*x^3 + x^6)^(-1),x]
```

output
```
(Log[1 + x]/3 + (Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - Log[1 - x + x^2]/2)/
3)/2 + (-1/3*Log[3^(1/3) + x]/3^(2/3) - (-(Sqrt[3]*ArcTan[(1 - (2*x)/3^(1/
3))/Sqrt[3]]) - Log[3^(2/3) - 3^(1/3)*x + x^2]/2)/(3*3^(2/3)))/2
```

3.165.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1685
```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^n), x], x] - Simp[
c/q   Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2,
2*n] && NeQ[b^2 - 4*a*c, 0]
```

3.165.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.52

method	result
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(9\_Z^{3}+1\right)}\_R\ln\left(x-3\_R\right)\right)}{6}-\frac{\ln\left(4x^{2}-4x+4\right)}{12}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}+\frac{\ln(x+1)}{6}$
default	$\frac{\ln(x+1)}{6}-\frac{3^{\frac{1}{3}}\ln\left(3^{\frac{1}{3}}+x\right)}{18}+\frac{3^{\frac{1}{3}}\ln\left(3^{\frac{2}{3}}-3^{\frac{1}{3}}x+x^{2}\right)}{36}-\frac{3^{\frac{5}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,3^{\frac{2}{3}}x}{3}-1\right)}{3}\right)}{18}-\frac{\ln\left(x^{2}-x+1\right)}{12}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}$

input
```
int(1/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
```

output
```
1/6*sum(_R*ln(x-3*_R),_R=RootOf(9*_Z^3+1))-1/12*ln(4*x^2-4*x+4)+1/6*3^(1/2
)*arctan(1/3*(2*x-1)*3^(1/2))+1/6*ln(x+1)
```

3.165.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.11

$$\int \frac{1}{3+4x^3+x^6}\,dx = \frac{1}{18}\cdot 9^{\frac{1}{6}}\sqrt{3}(-1)^{\frac{1}{3}}\arctan\left(\frac{1}{27}\cdot 9^{\frac{1}{6}}\left(2\cdot 9^{\frac{2}{3}}\sqrt{3}(-1)^{\frac{2}{3}}\,x-3\cdot 9^{\frac{1}{3}}\sqrt{3}\right)\right) - \frac{1}{108}\cdot 9^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(9^{\frac{2}{3}}(-1)^{\frac{1}{3}}\,x+3\,x^2+3\cdot 9^{\frac{1}{3}}(-1)^{\frac{2}{3}}\right)+\frac{1}{54}\cdot 9^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-9^{\frac{2}{3}}(-1)^{\frac{1}{3}}+3\,x\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right) - \frac{1}{12}\log\left(x^2-x+1\right)+\frac{1}{6}\log\left(x+1\right)$$

input
```
integrate(1/(x^6+4*x^3+3),x, algorithm="fricas")
```

output
```
1/18*9^(1/6)*sqrt(3)*(-1)^(1/3)*arctan(1/27*9^(1/6)*(2*9^(2/3)*sqrt(3)*(-1
)^(2/3)*x - 3*9^(1/3)*sqrt(3))) - 1/108*9^(2/3)*(-1)^(1/3)*log(9^(2/3)*(-1
)^(1/3)*x + 3*x^2 + 3*9^(1/3)*(-1)^(2/3)) + 1/54*9^(2/3)*(-1)^(1/3)*log(-9
^(2/3)*(-1)^(1/3) + 3*x) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/1
2*log(x^2 - x + 1) + 1/6*log(x + 1)
```

3.165.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.15 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.11

$$\int \frac{1}{3+4x^3+x^6}\,dx = \frac{\log\left(x+1\right)}{6} + \left(-\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)\log\left(x+\frac{13}{10}-\frac{13\sqrt{3}i}{10}+\frac{23328\left(-\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)^4}{5}\right) + \left(-\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)\log\left(x+\frac{13}{10}+\frac{23328\left(-\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)^4}{5}+\frac{13\sqrt{3}i}{10}\right) + \operatorname{RootSum}\left(1944t^3+1,\left(t\mapsto t\log\left(\frac{23328t^4}{5}-\frac{78t}{5}+x\right)\right)\right)$$

input
```
integrate(1/(x**6+4*x**3+3),x)
```

output
```
log(x + 1)/6 + (-1/12 + sqrt(3)*I/12)*log(x + 13/10 - 13*sqrt(3)*I/10 + 23
328*(-1/12 + sqrt(3)*I/12)**4/5) + (-1/12 - sqrt(3)*I/12)*log(x + 13/10 +
23328*(-1/12 - sqrt(3)*I/12)**4/5 + 13*sqrt(3)*I/10) + RootSum(1944*_t**3
+ 1, Lambda(_t, _t*log(23328*_t**4/5 - 78*_t/5 + x)))
```

3.165.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.75

$$\int \frac{1}{3+4x^3+x^6}\,dx = -\frac{1}{18}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{36}\cdot 3^{\frac{1}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)-\frac{1}{18}\cdot 3^{\frac{1}{3}}\log\left(x+3^{\frac{1}{3}}\right) - \frac{1}{12}\log\left(x^2-x+1\right)+\frac{1}{6}\log\left(x+1\right)$$

input
```
integrate(1/(x^6+4*x^3+3),x, algorithm="maxima")
```

output
```
-1/18*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) + 1/6*sqrt(3)*arctan(1/3
*sqrt(3)*(2*x - 1)) + 1/36*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 1/18*3
^(1/3)*log(x + 3^(1/3)) - 1/12*log(x^2 - x + 1) + 1/6*log(x + 1)
```

3.165.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.77

$$\int \frac{1}{3+4x^3+x^6}\,dx = -\frac{1}{18}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{36}\cdot 3^{\frac{1}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)-\frac{1}{18}\cdot 3^{\frac{1}{3}}\log\left(\left|x+3^{\frac{1}{3}}\right|\right) - \frac{1}{12}\log\left(x^2-x+1\right)+\frac{1}{6}\log\left(|x+1|\right)$$

input
```
integrate(1/(x^6+4*x^3+3),x, algorithm="giac")
```

output
```
-1/18*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) + 1/6*sqrt(3)*arctan(1/3
*sqrt(3)*(2*x - 1)) + 1/36*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 1/18*3
^(1/3)*log(abs(x + 3^(1/3))) - 1/12*log(x^2 - x + 1) + 1/6*log(abs(x + 1))
```

3.165.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.98

$$\int \frac{1}{3+4x^3+x^6}\,dx = \frac{\ln(x+1)}{6} - \frac{3^{1/3}\ln\left(x+3^{1/3}\right)}{18} - \ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) + \ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) + \frac{(-1)^{1/3}\,3^{1/3}\ln\left(x-(-1)^{1/3}\,3^{1/3}\right)}{18} - \frac{(-1)^{1/3}\ln\left(x+\frac{(-1)^{1/3}\,3^{1/3}}{2}+\frac{(-1)^{1/3}\,3^{5/6}\,1\mathrm{i}}{2}\right)\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}$$

input
```
int(1/(4*x^3 + x^6 + 3),x)
```

output
```
log(x + 1)/6 - (3^(1/3)*log(x + 3^(1/3)))/18 - log(x - (3^(1/2)*1i)/2 - 1/
2)*((3^(1/2)*1i)/12 + 1/12) + log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/
12 - 1/12) + ((-1)^(1/3)*3^(1/3)*log(x - (-1)^(1/3)*3^(1/3)))/18 - ((-1)^(
1/3)*log(x + ((-1)^(1/3)*3^(1/3))/2 + ((-1)^(1/3)*3^(5/6)*1i)/2)*(3^(1/3)
+ 3^(5/6)*1i))/36
```

3.166 $\int \frac{1}{x^2(3+4x^3+x^6)}\,dx$

3.166.1 Optimal result

Integrand size = 16, antiderivative size = 119

$$\int \frac{1}{x^2(3+4x^3+x^6)}\,dx = -\frac{1}{3x} + \frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{6\ 3^{5/6}} + \frac{1}{6}\log(1+x) - \frac{\log\left(\sqrt[3]{3}+x\right)}{18\sqrt[3]{3}} - \frac{1}{12}\log\left(1-x+x^2\right) + \frac{\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)}{36\sqrt[3]{3}}$$

output
```
-1/3/x-1/18*3^(1/6)*arctan(1/3*(3^(1/3)-2*x)*3^(1/6))+1/6*ln(1+x)-1/54*3^(
2/3)*ln(3^(1/3)+x)-1/12*ln(x^2-x+1)+1/108*3^(2/3)*ln(3^(2/3)-3^(1/3)*x+x^2
)+1/6*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

3.166.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.99

$$\int \frac{1}{x^2(3+4x^3+x^6)}\,dx = -\frac{36+6\sqrt[6]{3}x\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)+18\sqrt{3}x\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)-18x\log(1+x)+2\ 3^{2/3}x\log\left(3+3^{2/3}x\right)+9x}{108x}$$

input
```
Integrate[1/(x^2*(3 + 4*x^3 + x^6)),x]
```

output
```
-1/108*(36 + 6*3^(1/6)*x*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] + 18*Sqrt[3]*x*Ar
cTan[(-1 + 2*x)/Sqrt[3]] - 18*x*Log[1 + x] + 2*3^(2/3)*x*Log[3 + 3^(2/3)*x
] + 9*x*Log[1 - x + x^2] - 3^(2/3)*x*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/x
```

3.166.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.18, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {1704, 25, 1834, 821, 16, 1142, 25, 1082, 217, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (x^6+4x^3+3)} \, dx$$

$$\Big\downarrow \; 1704$$

$$\frac{1}{3}\int -\frac{x(x^3+4)}{x^6+4x^3+3}dx-\frac{1}{3x}$$

$$\Big\downarrow \; 25$$

$$-\frac{1}{3}\int \frac{x(x^3+4)}{x^6+4x^3+3}dx-\frac{1}{3x}$$

$$\Big\downarrow \; 1834$$

$$\frac{1}{3}\left(\frac{1}{2}\int \frac{x}{x^3+3}dx-\frac{3}{2}\int \frac{x}{x^3+1}dx\right)-\frac{1}{3x}$$

$$\Big\downarrow \; 821$$

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{\int \frac{x+\sqrt[3]{3}}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\sqrt[3]{3}}-\frac{\int \frac{1}{x+\sqrt[3]{3}}dx}{3\sqrt[3]{3}}\right)-\frac{3}{2}\left(\frac{1}{3}\int \frac{x+1}{x^2-x+1}dx-\frac{1}{3}\int \frac{1}{x+1}dx\right)\right)-\frac{1}{3x}$$

$$\Big\downarrow \; 16$$

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{\int \frac{x+\sqrt[3]{3}}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\sqrt[3]{3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}\right)-\frac{3}{2}\left(\frac{1}{3}\int \frac{x+1}{x^2-x+1}dx-\frac{1}{3}\log(x+1)\right)\right)-\frac{1}{3x}$$

$$\Big\downarrow \; 1142$$

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{\frac{3}{2}\sqrt[3]{3}\int\frac{1}{x^2-\sqrt[3]{3}x+3^{2/3}}dx+\frac{1}{2}\int-\frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\sqrt[3]{3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}\right)-\frac{3}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int\frac{1}{x^2-x+1}dx+\frac{1}{2}\int-\frac{}{x}\right.\right.\right.$$

$$\frac{1}{3x}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{\frac{3}{2}\sqrt[3]{3}\int\frac{1}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\frac{1}{2}\int\frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\sqrt[3]{3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}\right)-\frac{3}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int\frac{1}{x^2-x+1}dx-\frac{1}{2}\int\frac{1-}{x^2-}\right.\right.\right.$$

$$\frac{1}{3x}$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{3\int\frac{1}{-\left(1-\frac{2x}{\sqrt[3]{3}}\right)^2-3}d\left(1-\frac{2x}{\sqrt[3]{3}}\right)-\frac{1}{2}\int\frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\sqrt[3]{3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}\right)-\frac{3}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int\frac{1}{x^2-x+1}dx-\frac{1}{2}\right.\right.\right.$$

$$\frac{1}{3x}$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{-\frac{1}{2}\int\frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\sqrt[3]{3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}\right)-\frac{3}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int\frac{1}{x^2-x+1}dx-\frac{1}{2}\int\frac{1-}{x^2-}\right.\right.\right.$$

$$\frac{1}{3x}$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{-\frac{1}{2}\int\frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\sqrt[3]{3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}\right)-\frac{3}{2}\left(\frac{1}{3}\left(-\frac{1}{2}\int\frac{1-2x}{x^2-x+1}dx-3\int\frac{}{-(2}\right.\right.\right.$$

$$\frac{1}{3x}$$

3.166. $\int\frac{1}{x^2(3+4x^3+x^6)}\,dx$

$$\Big\downarrow \ 217$$

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{-\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\sqrt[3]{3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}\right)-\frac{3}{2}\left(\frac{1}{3}\left(\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)-\frac{1}{2}\int \frac{1}{x^2}\right.\right.\right.$$

$$\frac{1}{3x}$$

$$\Big\downarrow \ 1103$$

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{\frac{1}{2}\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\sqrt[3]{3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}\right)-\frac{3}{2}\left(\frac{1}{3}\left(\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)+\frac{1}{2}\log\right.\right.\right.$$

$$\frac{1}{3x}$$

input
```
Int[1/(x^2*(3 + 4*x^3 + x^6)),x]
```

output
```
-1/3*1/x + ((-3*(-1/3*Log[1 + x] + (Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + L
og[1 - x + x^2]/2)/3))/2 + (-1/3*Log[3^(1/3) + x]/3^(1/3) + (-(Sqrt[3]*Arc
Tan[(1 - (2*x)/3^(1/3))/Sqrt[3]]) + Log[3^(2/3) - 3^(1/3)*x + x^2]/2)/(3*3
^(1/3)))/2)/3
```

3.166.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1704
```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Simp[1/(a*d^n*(m + 1))   Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1)
+ c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{
a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] &&
LtQ[m, -1] && IntegerQ[p]
```

rule 1834
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 +
 (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2
 - (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ
[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n
, 0]
```

3.166. $\int \frac{1}{x^2(3+4x^3+x^6)}\,dx$

3.166.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.51

method	result
risch	$-\frac{1}{3x}+\frac{\ln(x+1)}{6}-\frac{\ln(x^2-x+1)}{12}-\frac{\sqrt{3}\arctan\left(\frac{2\left(x-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{6}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(3\_Z^3+1\right)}\_R\ln\left(3\_R^2+x\right)\right)}{18}$
default	$\frac{\ln(x+1)}{6}-\frac{3^{\frac{2}{3}}\ln\left(3^{\frac{1}{3}}+x\right)}{54}+\frac{3^{\frac{2}{3}}\ln\left(3^{\frac{2}{3}}-3^{\frac{1}{3}}x+x^2\right)}{108}+\frac{3^{\frac{1}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,3^{\frac{2}{3}}x}{3}-1\right)}{3}\right)}{18}-\frac{\ln(x^2-x+1)}{12}-\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}$

input
```
int(1/x^2/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
```

output
```
-1/3/x+1/6*ln(x+1)-1/12*ln(x^2-x+1)-1/6*3^(1/2)*arctan(2/3*(x-1/2)*3^(1/2)
)+1/18*sum(_R*ln(3*_R^2+x),_R=RootOf(3*_Z^3+1))
```

3.166.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.98

$$\int \frac{1}{x^2(3+4x^3+x^6)}\,dx = -\frac{3^{\frac{2}{3}}(-1)^{\frac{1}{3}}x\log\left(-3^{\frac{1}{3}}(-1)^{\frac{2}{3}}x+x^2-3^{\frac{2}{3}}(-1)^{\frac{1}{3}}\right)-2\cdot 3^{\frac{2}{3}}(-1)^{\frac{1}{3}}x\log\left(3^{\frac{1}{3}}(-1)^{\frac{2}{3}}+x\right)+18\sqrt{3}x\arctan\left(\frac{1}{3}\right.}{}$$

input
```
integrate(1/x^2/(x^6+4*x^3+3),x, algorithm="fricas")
```

output
```
-1/108*(3^(2/3)*(-1)^(1/3)*x*log(-3^(1/3)*(-1)^(2/3)*x + x^2 - 3^(2/3)*(-1
)^(1/3)) - 2*3^(2/3)*(-1)^(1/3)*x*log(3^(1/3)*(-1)^(2/3) + x) + 18*sqrt(3)
*x*arctan(1/3*sqrt(3)*(2*x - 1)) - 6*3^(1/6)*(-1)^(1/3)*x*arctan(1/3*3^(1/
6)*(2*(-1)^(1/3)*x + 3^(1/3))) + 9*x*log(x^2 - x + 1) - 18*x*log(x + 1) +
36)/x
```

3.166.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.17 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.17

$$\int \frac{1}{x^2(3+4x^3+x^6)}\,dx = \frac{\log(x+1)}{6} + \left(-\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)\log\left(x - \frac{8188128\left(-\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)^5}{41} + \frac{39384\left(-\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)^2}{41}\right) + \left(-\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)\log\left(x + \frac{39384\left(-\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)^2}{41} - \frac{8188128\left(-\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)^5}{41}\right) + \operatorname{RootSum}\left(17496t^3+1, \left(t \mapsto t\log\left(-\frac{8188128t^5}{41}+\frac{39384t^2}{41}+x\right)\right)\right) - \frac{1}{3x}$$

input
```
integrate(1/x**2/(x**6+4*x**3+3),x)
```

output
```
log(x + 1)/6 + (-1/12 - sqrt(3)*I/12)*log(x - 8188128*(-1/12 - sqrt(3)*I/1
2)**5/41 + 39384*(-1/12 - sqrt(3)*I/12)**2/41) + (-1/12 + sqrt(3)*I/12)*lo
g(x + 39384*(-1/12 + sqrt(3)*I/12)**2/41 - 8188128*(-1/12 + sqrt(3)*I/12)*
*5/41) + RootSum(17496*_t**3 + 1, Lambda(_t, _t*log(-8188128*_t**5/41 + 39
384*_t**2/41 + x))) - 1/(3*x)
```

3.166.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.75

$$\int \frac{1}{x^2(3+4x^3+x^6)}\,dx = \frac{1}{108}\cdot 3^{\frac{2}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right) - \frac{1}{54}\cdot 3^{\frac{2}{3}}\log\left(x+3^{\frac{1}{3}}\right) - \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{18}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right)\right) - \frac{1}{3x} - \frac{1}{12}\log\left(x^2-x+1\right) + \frac{1}{6}\log(x+1)$$

input
```
integrate(1/x^2/(x^6+4*x^3+3),x, algorithm="maxima")
```

output
```
1/108*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 1/54*3^(2/3)*log(x + 3^(1/3
)) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/18*3^(1/6)*arctan(1/3*3
^(1/6)*(2*x - 3^(1/3))) - 1/3/x - 1/12*log(x^2 - x + 1) + 1/6*log(x + 1)
```

3.166.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^2(3+4x^3+x^6)}\,dx = \frac{1}{108}\cdot 3^{\frac{2}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)-\frac{1}{54}\cdot 3^{\frac{2}{3}}\log\left(\left|x+3^{\frac{1}{3}}\right|\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{18}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right)\right)-\frac{1}{3x}-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{1}{6}\log\left(|x+1|\right)$$

input
```
integrate(1/x^2/(x^6+4*x^3+3),x, algorithm="giac")
```

output
```
1/108*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 1/54*3^(2/3)*log(abs(x + 3^
(1/3))) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/18*3^(1/6)*arctan(
1/3*3^(1/6)*(2*x - 3^(1/3))) - 1/3/x - 1/12*log(x^2 - x + 1) + 1/6*log(abs
(x + 1))
```

3.166.9 Mupad [B] (verification not implemented)

Time = 8.39 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2(3+4x^3+x^6)}\,dx = \frac{\ln(x+1)}{6}-\frac{3^{2/3}\ln\left(x+3^{1/3}\right)}{54}+\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\,1i}{12}\right)-\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\,1i}{12}\right)-\frac{1}{3x}-\frac{(-1)^{1/3}\ln\left(x-\frac{(-1)^{1/3}3^{1/3}}{2}-\frac{(-1)^{1/6}3^{5/6}}{2}+\frac{3^{1/3}}{2}\right)\left(3^{2/3}+3^{1/6}\,3i\right)}{108}+\frac{(-1)^{1/3}3^{2/3}\ln\left(x+(-1)^{2/3}3^{1/3}\right)}{54}$$

input
```
int(1/(x^2*(4*x^3 + x^6 + 3)),x)
```

output
```
log(x + 1)/6 - (3^(2/3)*log(x + 3^(1/3)))/54 + log(x - (3^(1/2)*1i)/2 - 1/
2)*((3^(1/2)*1i)/12 - 1/12) - log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/
12 + 1/12) - 1/(3*x) - ((-1)^(1/3)*log(x - ((-1)^(1/3)*3^(1/3))/2 - ((-1)^
(1/6)*3^(5/6))/2 + 3^(1/3)/2)*(3^(2/3) + 3^(1/6)*3i))/108 + ((-1)^(1/3)*3^
(2/3)*log(x + (-1)^(2/3)*3^(1/3)))/54
```

3.166. $\int \frac{1}{x^2(3+4x^3+x^6)}\,dx$

3.167 $\int \frac{1}{x^3(3+4x^3+x^6)}\,dx$

3.167.1 Optimal result

Integrand size = 16, antiderivative size = 119

$$\int \frac{1}{x^3(3+4x^3+x^6)}\,dx = -\frac{1}{6x^2}+\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{18\sqrt[6]{3}}-\frac{1}{6}\log(1+x)+\frac{\log\left(\sqrt[3]{3}+x\right)}{18\ 3^{2/3}}+\frac{1}{12}\log\left(1-x+x^2\right)-\frac{\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)}{36\ 3^{2/3}}$$

output
```
-1/6/x^2-1/54*3^(5/6)*arctan(1/3*(3^(1/3)-2*x)*3^(1/6))-1/6*ln(1+x)+1/54*3
^(1/3)*ln(3^(1/3)+x)+1/12*ln(x^2-x+1)-1/108*3^(1/3)*ln(3^(2/3)-3^(1/3)*x+x
^2)+1/6*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

3.167.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^3(3+4x^3+x^6)}\,dx = \frac{1}{108}\left(-\frac{18}{x^2}-2\ 3^{5/6}\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)-18\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)-18\log(1+x)+2\sqrt[3]{3}\log\left(3+3^{2/3}x\right)+9\log\left(1-x-\right.\right.$$

input
```
Integrate[1/(x^3*(3 + 4*x^3 + x^6)),x]
```

output
```
(-18/x^2 - 2*3^(5/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] - 18*Sqrt[3]*ArcTan[(
-1 + 2*x)/Sqrt[3]] - 18*Log[1 + x] + 2*3^(1/3)*Log[3 + 3^(2/3)*x] + 9*Log[
1 - x + x^2] - 3^(1/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/108
```

3.167.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.18, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {1704, 27, 1752, 750, 16, 1142, 25, 1082, 217, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (x^6+4x^3+3)}dx$$

$$\Big\downarrow \text{1704}$$

$$\frac{1}{6}\int -\frac{2(x^3+4)}{x^6+4x^3+3}dx-\frac{1}{6x^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{3}\int \frac{x^3+4}{x^6+4x^3+3}dx-\frac{1}{6x^2}$$

$$\Big\downarrow \text{1752}$$

$$\frac{1}{3}\left(\frac{1}{2}\int \frac{1}{x^3+3}dx-\frac{3}{2}\int \frac{1}{x^3+1}dx\right)-\frac{1}{6x^2}$$

$$\Big\downarrow \text{750}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{\int \frac{2\sqrt[3]{3}-x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\ 3^{2/3}}+\frac{\int \frac{1}{x+\sqrt[3]{3}}dx}{3\ 3^{2/3}}\right)-\frac{3}{2}\left(\frac{1}{3}\int \frac{2-x}{x^2-x+1}dx+\frac{1}{3}\int \frac{1}{x+1}dx\right)\right)-\frac{1}{6x^2}$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{\int \frac{2\sqrt[3]{3}-x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\ 3^{2/3}}+\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)-\frac{3}{2}\left(\frac{1}{3}\int \frac{2-x}{x^2-x+1}dx+\frac{1}{3}\log(x+1)\right)\right)-\frac{1}{6x^2}$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{\frac{3}{2}\sqrt[3]{3}\int\frac{1}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\frac{1}{2}\int-\frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\ 3^{2/3}}+\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)-\frac{3}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int\frac{1}{x^2-x+1}dx-\frac{1}{2}\int-\frac{}{x}\right.\right.\right.$$

$$\frac{1}{6x^2}$$

$\downarrow$ 25

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{\frac{3}{2}\sqrt[3]{3}\int\frac{1}{x^2-\sqrt[3]{3}x+3^{2/3}}dx+\frac{1}{2}\int\frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\ 3^{2/3}}+\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)-\frac{3}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int\frac{1}{x^2-x+1}dx+\frac{1}{2}\int\frac{1-}{x^2-}\right.\right.\right.$$

$$\frac{1}{6x^2}$$

$\downarrow$ 1082

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx+3\int\frac{1}{-\left(1-\frac{2x}{\sqrt[3]{3}}\right)^2-3}d\left(1-\frac{2x}{\sqrt[3]{3}}\right)}{3\ 3^{2/3}}+\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)-\frac{3}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int\frac{1}{x^2-x+1}dx+\frac{1}{2}\right.\right.\right.$$

$$\frac{1}{6x^2}$$

$\downarrow$ 217

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\ 3^{2/3}}+\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)-\frac{3}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int\frac{1}{x^2-x+1}dx+\frac{1}{2}\int\frac{1-2}{x^2-x}\right.\right.\right.$$

$$\frac{1}{6x^2}$$

$\downarrow$ 1083

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\ 3^{2/3}}+\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)-\frac{3}{2}\left(\frac{1}{3}\left(\frac{1}{2}\int\frac{1-2x}{x^2-x+1}dx-3\int\frac{}{-(2x-}\right.\right.\right.$$

$$\frac{1}{6x^2}$$

3.167. $\int\frac{1}{x^3(3+4x^3+x^6)}\,dx$

$\Big\downarrow$ 217

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\ 3^{2/3}}+\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)-\frac{3}{2}\left(\frac{1}{3}\left(\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx+\sqrt{3}\arctan\left(\right.\right.\right.\right.$$

$$\frac{1}{6x^2}$$

$\Big\downarrow$ 1103

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)-\frac{1}{2}\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{3\ 3^{2/3}}+\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)-\frac{3}{2}\left(\frac{1}{3}\left(\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)-\frac{1}{2}\right.\right.\right.$$

$$\frac{1}{6x^2}$$

input
```
Int[1/(x^3*(3 + 4*x^3 + x^6)),x]
```

output
```
-1/6*1/x^2 + ((-3*(Log[1 + x]/3 + (Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - Lo
g[1 - x + x^2]/2)/3))/2 + (Log[3^(1/3) + x]/(3*3^(2/3)) + (-(Sqrt[3]*ArcTa
n[(1 - (2*x)/3^(1/3))/Sqrt[3]]) - Log[3^(2/3) - 3^(1/3)*x + x^2]/2)/(3*3^(
2/3)))/2)/3
```

3.167.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1704
```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Simp[1/(a*d^n*(m + 1))   Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1)
+ c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{
a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] &&
LtQ[m, -1] && IntegerQ[p]
```

3.167. $\int \frac{1}{x^3(3+4x^3+x^6)}\,dx$

rule 1752
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

3.167.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.50

method	result
risch	$-\frac{1}{6x^2}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(9\_Z^3-1\right)}\_R\ln\left(x+3\_R\right)\right)}{18}+\frac{\ln(x^2-x+1)}{12}-\frac{\sqrt{3}\arctan\left(\frac{2\left(x-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{6}-\frac{\ln(x+1)}{6}$
default	$-\frac{1}{6x^2}-\frac{\ln(x+1)}{6}+\frac{3^{\frac{1}{3}}\ln\left(3^{\frac{1}{3}}+x\right)}{54}-\frac{3^{\frac{1}{3}}\ln\left(3^{\frac{2}{3}}-3^{\frac{1}{3}}x+x^2\right)}{108}+\frac{3^{\frac{5}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,3^{\frac{2}{3}}x}{3}-1\right)}{3}\right)}{54}+\frac{\ln(x^2-x+1)}{12}-\frac{\sqrt{3}\arctan}{}$

input
```
int(1/x^3/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
```

output
```
-1/6/x^2+1/18*sum(_R*ln(x+3*_R),_R=RootOf(9*_Z^3-1))+1/12*ln(x^2-x+1)-1/6*
3^(1/2)*arctan(2/3*(x-1/2)*3^(1/2))-1/6*ln(x+1)
```

3.167.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^3(3+4x^3+x^6)}\,dx$$
$$=\frac{6\cdot 9^{\frac{1}{6}}\sqrt{3}x^2\arctan\left(\frac{1}{27}\cdot 9^{\frac{1}{6}}\left(2\cdot 9^{\frac{2}{3}}\sqrt{3}x-3\cdot 9^{\frac{1}{3}}\sqrt{3}\right)\right)-9^{\frac{2}{3}}x^2\log\left(3\,x^2-9^{\frac{2}{3}}x+3\cdot 9^{\frac{1}{3}}\right)+2\cdot 9^{\frac{2}{3}}x^2\log\left(3\,x\right.}{324\,x^2}$$

input
```
integrate(1/x^3/(x^6+4*x^3+3),x, algorithm="fricas")
```

output
```
1/324*(6*9^(1/6)*sqrt(3)*x^2*arctan(1/27*9^(1/6)*(2*9^(2/3)*sqrt(3)*x - 3*
9^(1/3)*sqrt(3))) - 9^(2/3)*x^2*log(3*x^2 - 9^(2/3)*x + 3*9^(1/3)) + 2*9^(
2/3)*x^2*log(3*x + 9^(2/3)) - 54*sqrt(3)*x^2*arctan(1/3*sqrt(3)*(2*x - 1))
 + 27*x^2*log(x^2 - x + 1) - 54*x^2*log(x + 1) - 54)/x^2
```

3.167.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.04 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.08

$$\begin{aligned}\int \frac{1}{x^3(3+4x^3+x^6)}\,dx &= -\frac{\log(x+1)}{6} + \left(\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)\log\left(x+\frac{1093}{244}-\frac{1093\sqrt{3}i}{244}+\frac{787320\left(\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)^4}{61}\right)\\ &\quad + \left(\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)\log\left(x+\frac{1093}{244}+\frac{787320\left(\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)^4}{61}+\frac{1093\sqrt{3}i}{244}\right)\\ &\quad + \operatorname{RootSum}\left(52488t^3-1,\left(t\mapsto t\log\left(\frac{787320t^4}{61}+\frac{3279t}{61}+x\right)\right)\right)-\frac{1}{6x^2}\end{aligned}$$

input
```
integrate(1/x**3/(x**6+4*x**3+3),x)
```

output
```
-log(x + 1)/6 + (1/12 - sqrt(3)*I/12)*log(x + 1093/244 - 1093*sqrt(3)*I/24
4 + 787320*(1/12 - sqrt(3)*I/12)**4/61) + (1/12 + sqrt(3)*I/12)*log(x + 10
93/244 + 787320*(1/12 + sqrt(3)*I/12)**4/61 + 1093*sqrt(3)*I/244) + RootSu
m(52488*_t**3 - 1, Lambda(_t, _t*log(787320*_t**4/61 + 3279*_t/61 + x))) -
 1/(6*x**2)
```

3.167.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.75

$$\int \frac{1}{x^3(3+4x^3+x^6)}\,dx = \frac{1}{54}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2\,x-3^{\frac{1}{3}}\right)\right)-\frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) - \frac{1}{108}\cdot 3^{\frac{1}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)+\frac{1}{54}\cdot 3^{\frac{1}{3}}\log\left(x+3^{\frac{1}{3}}\right) - \frac{1}{6\,x^2}+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{1}{6}\log\left(x+1\right)$$

input
```
integrate(1/x^3/(x^6+4*x^3+3),x, algorithm="maxima")
```

output
```
1/54*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) - 1/6*sqrt(3)*arctan(1/3*
sqrt(3)*(2*x - 1)) - 1/108*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/54*3
^(1/3)*log(x + 3^(1/3)) - 1/6/x^2 + 1/12*log(x^2 - x + 1) - 1/6*log(x + 1)
```

3.167.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^3(3+4x^3+x^6)}\,dx = \frac{1}{54}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2\,x-3^{\frac{1}{3}}\right)\right)-\frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) - \frac{1}{108}\cdot 3^{\frac{1}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)+\frac{1}{54}\cdot 3^{\frac{1}{3}}\log\left(\left|x+3^{\frac{1}{3}}\right|\right) - \frac{1}{6\,x^2}+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{1}{6}\log\left(|x+1|\right)$$

input
```
integrate(1/x^3/(x^6+4*x^3+3),x, algorithm="giac")
```

output
```
1/54*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) - 1/6*sqrt(3)*arctan(1/3*
sqrt(3)*(2*x - 1)) - 1/108*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/54*3
^(1/3)*log(abs(x + 3^(1/3))) - 1/6/x^2 + 1/12*log(x^2 - x + 1) - 1/6*log(a
bs(x + 1))
```

3.167.9 Mupad [B] (verification not implemented)

Time = 8.46 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.99

$$\int \frac{1}{x^3 (3+4x^3+x^6)} dx = \frac{3^{1/3} \ln \left(x+3^{1/3}\right)}{54} - \frac{\ln (x+1)}{6} + \ln \left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) - \ln \left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) - \frac{1}{6\,x^2} - \ln \left(x-\frac{3^{1/3}}{2}-\frac{3^{5/6}\,1\mathrm{i}}{2}\right)\left(\frac{3^{1/3}}{108}+\frac{3^{5/6}\,1\mathrm{i}}{108}\right) - \ln \left(x-\frac{3^{1/3}}{2}+\frac{3^{5/6}\,1\mathrm{i}}{2}\right)\left(\frac{3^{1/3}}{108}-\frac{3^{5/6}\,1\mathrm{i}}{108}\right)$$

input
```
int(1/(x^3*(4*x^3 + x^6 + 3)),x)
```

output
```
(3^(1/3)*log(x + 3^(1/3)))/54 - log(x + 1)/6 + log(x - (3^(1/2)*1i)/2 - 1/
2)*((3^(1/2)*1i)/12 + 1/12) - log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/
12 - 1/12) - 1/(6*x^2) - log(x - 3^(1/3)/2 - (3^(5/6)*1i)/2)*(3^(1/3)/108
+ (3^(5/6)*1i)/108) - log(x - 3^(1/3)/2 + (3^(5/6)*1i)/2)*(3^(1/3)/108 - (
3^(5/6)*1i)/108)
```

3.168 $\int \frac{1}{x^5(3+4x^3+x^6)}\,dx$

3.168.1 Optimal result

Integrand size = 16, antiderivative size = 126

$$\int \frac{1}{x^5\left(3+4x^3+x^6\right)}\,dx=-\frac{1}{12x^4}+\frac{4}{9x}-\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{18\ 3^{5/6}}-\frac{1}{6}\log(1+x)+\frac{\log\left(\sqrt[3]{3}+x\right)}{54\sqrt[3]{3}}+\frac{1}{12}\log\left(1-x+x^2\right)-\frac{\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)}{108\sqrt[3]{3}}$$

output
```
-1/12/x^4+4/9/x+1/54*3^(1/6)*arctan(1/3*(3^(1/3)-2*x)*3^(1/6))-1/6*ln(1+x)
+1/162*3^(2/3)*ln(3^(1/3)+x)+1/12*ln(x^2-x+1)-1/324*3^(2/3)*ln(3^(2/3)-3^(
1/3)*x+x^2)-1/6*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

3.168.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^5\left(3+4x^3+x^6\right)}\,dx=\frac{1}{324}\left(-\frac{27}{x^4}+\frac{144}{x}+6\sqrt[6]{3}\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)+54\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)-54\log(1+x)+2\ 3^{2/3}\log\left(3+3^{2/3}x\right)+27\log\left(1-x+x^2\right)-3^{2/3}\log\left(3-3^{2/3}x+\sqrt[3]{3}x^2\right)\right)$$

input
```
Integrate[1/(x^5*(3 + 4*x^3 + x^6)),x]
```

output
```
(-27/x^4 + 144/x + 6*3^(1/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] + 54*Sqrt[3]*
ArcTan[(-1 + 2*x)/Sqrt[3]] - 54*Log[1 + x] + 2*3^(2/3)*Log[3 + 3^(2/3)*x]
+ 27*Log[1 - x + x^2] - 3^(2/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/324
```

3.168.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.21, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.812$, Rules used = {1704, 27, 1828, 1834, 821, 16, 1142, 25, 1082, 217, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 \left(x^6+4 x^3+3\right)} \, dx$$

$$\Big\downarrow \text{1704}$$

$$\frac{1}{12} \int -\frac{4 \left(x^3+4\right)}{x^2 \left(x^6+4 x^3+3\right)}dx-\frac{1}{12 x^4}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{3} \int \frac{x^3+4}{x^2 \left(x^6+4 x^3+3\right)}dx-\frac{1}{12 x^4}$$

$$\Big\downarrow \text{1828}$$

$$\frac{1}{3} \left(\frac{1}{3} \int \frac{x \left(4 x^3+13\right)}{x^6+4 x^3+3}dx+\frac{4}{3 x}\right)-\frac{1}{12 x^4}$$

$$\Big\downarrow \text{1834}$$

$$\frac{1}{3} \left(\frac{1}{3} \left(\frac{9}{2} \int \frac{x}{x^3+1}dx-\frac{1}{2} \int \frac{x}{x^3+3}dx\right)+\frac{4}{3 x}\right)-\frac{1}{12 x^4}$$

$$\Big\downarrow \text{821}$$

$$\frac{1}{3} \left(\frac{1}{3} \left(\frac{9}{2} \left(\frac{1}{3} \int \frac{x+1}{x^2-x+1}dx-\frac{1}{3} \int \frac{1}{x+1}dx\right)+\frac{1}{2} \left(\frac{\int \frac{1}{x+\sqrt[3]{3}}dx}{3 \sqrt[3]{3}}-\frac{\int \frac{x+\sqrt[3]{3}}{x^2-\sqrt[3]{3} x+3^{2/3}}dx}{3 \sqrt[3]{3}}\right)\right)+\frac{4}{3 x}\right)-\frac{1}{12 x^4}$$

$\Big\downarrow$ 16

$$\frac{1}{3}\left(\frac{1}{3}\left(\frac{9}{2}\left(\frac{1}{3}\int\frac{x+1}{x^2-x+1}dx-\frac{1}{3}\log(x+1)\right)+\frac{1}{2}\left(\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}-\frac{\int\frac{x+\sqrt[3]{3}}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\sqrt[3]{3}}\right)\right)+\frac{4}{3x}\right)-\frac{1}{12x^4}$$

$\Big\downarrow$ 1142

$$\frac{1}{3}\left(\frac{1}{3}\left(\frac{9}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int\frac{1}{x^2-x+1}dx+\frac{1}{2}\int-\frac{1-2x}{x^2-x+1}dx\right)-\frac{1}{3}\log(x+1)\right)+\frac{1}{2}\left(\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}-\frac{\frac{3}{2}\sqrt[3]{3}\int\frac{1}{x^2-\sqrt[3]{3}x}\right.\right.\right.$$

$$\frac{1}{12x^4}$$

$\Big\downarrow$ 25

$$\frac{1}{3}\left(\frac{1}{3}\left(\frac{9}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int\frac{1}{x^2-x+1}dx-\frac{1}{2}\int\frac{1-2x}{x^2-x+1}dx\right)-\frac{1}{3}\log(x+1)\right)+\frac{1}{2}\left(\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}-\frac{\frac{3}{2}\sqrt[3]{3}\int\frac{1}{x^2-\sqrt[3]{3}x+}\right.\right.\right.$$

$$\frac{1}{12x^4}$$

$\Big\downarrow$ 1082

$$\frac{1}{3}\left(\frac{1}{3}\left(\frac{9}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int\frac{1}{x^2-x+1}dx-\frac{1}{2}\int\frac{1-2x}{x^2-x+1}dx\right)-\frac{1}{3}\log(x+1)\right)+\frac{1}{2}\left(\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}-\frac{3\int\frac{1}{-\left(1-\frac{2x}{\sqrt[3]{3}}\right)^2-}}{}\right.\right.\right.$$

$$\frac{1}{12x^4}$$

$\Big\downarrow$ 217

$$\frac{1}{3}\left(\frac{1}{3}\left(\frac{1}{2}\left(\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}-\frac{-\frac{1}{2}\int\frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\sqrt[3]{3}}\right)+\frac{9}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int\frac{1}{x^2-x+1}dx-\frac{1}{2}\int\frac{}{x}\right.\right.\right.\right.$$

$$\frac{1}{12x^4}$$

3.168. $\int\frac{1}{x^5(3+4x^3+x^6)}\,dx$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{3}\left(\frac{1}{3}\left(\frac{1}{2}\left(\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}-\frac{-\frac{1}{2}\int\frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\sqrt[3]{3}}\right)+\frac{9}{2}\left(\frac{1}{3}\left(-\frac{1}{2}\int\frac{1-2x}{x^2-x+1}dx-3\int\right.\right.\right.\right.$$

$$\frac{1}{12x^4}$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{3}\left(\frac{1}{3}\left(\frac{9}{2}\left(\frac{1}{3}\left(\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)-\frac{1}{2}\int\frac{1-2x}{x^2-x+1}dx\right)-\frac{1}{3}\log(x+1)\right)+\frac{1}{2}\left(\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}-\frac{-\frac{1}{2}\int\frac{\sqrt[3]{3}-2}{x^2-\sqrt[3]{3}x-}}{}\right.\right.\right.$$

$$\frac{1}{12x^4}$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{3}\left(\frac{1}{3}\left(\frac{9}{2}\left(\frac{1}{3}\left(\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)+\frac{1}{2}\log\left(x^2-x+1\right)\right)-\frac{1}{3}\log(x+1)\right)+\frac{1}{2}\left(\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}-\frac{\frac{1}{2}\log\left(x^2-\sqrt[3]{3}\right.}{}\right.\right.\right.$$

$$\frac{1}{12x^4}$$

input
```
Int[1/(x^5*(3 + 4*x^3 + x^6)),x]
```

output
```
-1/12*1/x^4 + (4/(3*x) + ((9*(-1/3*Log[1 + x] + (Sqrt[3]*ArcTan[(-1 + 2*x)
/Sqrt[3]] + Log[1 - x + x^2]/2)/3))/2 + (Log[3^(1/3) + x]/(3*3^(1/3)) - (-
(Sqrt[3]*ArcTan[(1 - (2*x)/3^(1/3))/Sqrt[3]]) + Log[3^(2/3) - 3^(1/3)*x +
x^2]/2)/(3*3^(1/3)))/2)/3)/3
```

3.168. $\int \frac{1}{x^5(3+4x^3+x^6)}\,dx$

3.168.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1704
```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Simp[1/(a*d^n*(m + 1))   Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1)
+ c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{
a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] &&
LtQ[m, -1] && IntegerQ[p]
```

rule 1828
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Simp[1/(a*f^n*(m + 1))   Int[(f*x)^(m +
 n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) -
c*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x
] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Int
egerQ[p]
```

rule 1834
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 +
 (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2
 - (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ
[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n
, 0]
```

3.168.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.53

method	result
risch	$\frac{\frac{4x^3}{9}-\frac{1}{12}}{x^4}-\frac{\ln(x+1)}{6}+\frac{\ln(x^2-x+1)}{12}+\frac{\sqrt{3}\arctan\left(\frac{2\left(x-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{6}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(3\_Z^3-1\right)}\_R\ln\left(3\_R^2+x\right)\right)}{54}$
default	$-\frac{1}{12x^4}+\frac{4}{9x}-\frac{\ln(x+1)}{6}+\frac{3^{\frac{2}{3}}\ln\left(3^{\frac{1}{3}}+x\right)}{162}-\frac{3^{\frac{2}{3}}\ln\left(3^{\frac{2}{3}}-3^{\frac{1}{3}}x+x^2\right)}{324}-\frac{3^{\frac{1}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,3^{\frac{2}{3}}x}{3}-1\right)}{3}\right)}{54}+\frac{\ln(x^2-x+1)}{12}+\sqrt{3}$

input
```
int(1/x^5/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
```

output
```
(4/9*x^3-1/12)/x^4-1/6*ln(x+1)+1/12*ln(x^2-x+1)+1/6*3^(1/2)*arctan(2/3*(x-
1/2)*3^(1/2))+1/54*sum(_R*ln(3*_R^2+x),_R=RootOf(3*_Z^3-1))
```

3.168.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^5(3+4x^3+x^6)}\,dx = -\frac{3^{\frac{2}{3}}x^4\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)-2\cdot 3^{\frac{2}{3}}x^4\log\left(x+3^{\frac{1}{3}}\right)-54\sqrt{3}x^4\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-6\cdot 3^{\frac{1}{6}}x^4\arctan}{324\,x^4}$$

input
```
integrate(1/x^5/(x^6+4*x^3+3),x, algorithm="fricas")
```

output
```
-1/324*(3^(2/3)*x^4*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 2*3^(2/3)*x^4*log(x +
 3^(1/3)) - 54*sqrt(3)*x^4*arctan(1/3*sqrt(3)*(2*x - 1)) - 6*3^(1/6)*x^4*a
rctan(-1/3*3^(1/6)*(2*x - 3^(1/3))) - 27*x^4*log(x^2 - x + 1) + 54*x^4*log
(x + 1) - 144*x^3 + 27)/x^4
```

3.168.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.20 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.12

$$\begin{aligned}\int \frac{1}{x^5(3+4x^3+x^6)}\,dx &= -\frac{\log(x+1)}{6} \\ &+ \left(\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)\log\left(x+\frac{4782978\left(\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)^2}{547}+\frac{1028869776\left(\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)^5}{547}\right) \\ &+ \left(\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)\log\left(x+\frac{1028869776\left(\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)^5}{547}+\frac{4782978\left(\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)^2}{547}\right) \\ &+ \operatorname{RootSum}\left(472392t^3-1,\left(t\mapsto t\log\left(\frac{1028869776t^5}{547}+\frac{4782978t^2}{547}+x\right)\right)\right) \\ &+ \frac{16x^3-3}{36x^4}\end{aligned}$$

input
```
integrate(1/x**5/(x**6+4*x**3+3),x)
```

output
```
-log(x + 1)/6 + (1/12 - sqrt(3)*I/12)*log(x + 4782978*(1/12 - sqrt(3)*I/12
)**2/547 + 1028869776*(1/12 - sqrt(3)*I/12)**5/547) + (1/12 + sqrt(3)*I/12
)*log(x + 1028869776*(1/12 + sqrt(3)*I/12)**5/547 + 4782978*(1/12 + sqrt(3
)*I/12)**2/547) + RootSum(472392*_t**3 - 1, Lambda(_t, _t*log(1028869776*_
t**5/547 + 4782978*_t**2/547 + x))) + (16*x**3 - 3)/(36*x**4)
```

3.168.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^5(3+4x^3+x^6)}\,dx = -\frac{1}{324}\cdot 3^{\frac{2}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)+\frac{1}{162}\cdot 3^{\frac{2}{3}}\log\left(x+3^{\frac{1}{3}}\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{54}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right)\right)+\frac{16x^3-3}{36x^4}+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{1}{6}\log\left(x+1\right)$$

input
```
integrate(1/x^5/(x^6+4*x^3+3),x, algorithm="maxima")
```

output
```
-1/324*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/162*3^(2/3)*log(x + 3^(1
/3)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/54*3^(1/6)*arctan(1/3
*3^(1/6)*(2*x - 3^(1/3))) + 1/36*(16*x^3 - 3)/x^4 + 1/12*log(x^2 - x + 1)
- 1/6*log(x + 1)
```

3.168.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^5(3+4x^3+x^6)}\,dx = -\frac{1}{324}\cdot 3^{\frac{2}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)+\frac{1}{162}\cdot 3^{\frac{2}{3}}\log\left(\left|x+3^{\frac{1}{3}}\right|\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{54}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right)\right)+\frac{16x^3-3}{36x^4}+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{1}{6}\log\left(|x+1|\right)$$

input
```
integrate(1/x^5/(x^6+4*x^3+3),x, algorithm="giac")
```

output
```
-1/324*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/162*3^(2/3)*log(abs(x +
3^(1/3))) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/54*3^(1/6)*arcta
n(1/3*3^(1/6)*(2*x - 3^(1/3))) + 1/36*(16*x^3 - 3)/x^4 + 1/12*log(x^2 - x
+ 1) - 1/6*log(abs(x + 1))
```

3.168.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.98

$$\int \frac{1}{x^5 (3 + 4x^3 + x^6)} \, dx = \frac{3^{2/3} \ln \left(x + 3^{1/3}\right)}{162} - \frac{\ln (x+1)}{6} - \ln \left(x - \frac{1}{2} - \frac{\sqrt{3}\,1\mathrm{i}}{2}\right) \left(-\frac{1}{12} + \frac{\sqrt{3}\,1\mathrm{i}}{12}\right) + \ln \left(x - \frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{2}\right) \left(\frac{1}{12} + \frac{\sqrt{3}\,1\mathrm{i}}{12}\right) + \frac{\frac{4x^3}{9} - \frac{1}{12}}{x^4} - \ln \left(x - \frac{3^{1/3}}{2} - \frac{3^{5/6}\,1\mathrm{i}}{2}\right) \left(\frac{3^{2/3}}{324} - \frac{3^{1/6}\,1\mathrm{i}}{108}\right) - \ln \left(x - \frac{3^{1/3}}{2} + \frac{3^{5/6}\,1\mathrm{i}}{2}\right) \left(\frac{3^{2/3}}{324} + \frac{3^{1/6}\,1\mathrm{i}}{108}\right)$$

input
```
int(1/(x^5*(4*x^3 + x^6 + 3)),x)
```

output
```
(3^(2/3)*log(x + 3^(1/3)))/162 - log(x + 1)/6 - log(x - (3^(1/2)*1i)/2 - 1
/2)*((3^(1/2)*1i)/12 - 1/12) + log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)
/12 + 1/12) + ((4*x^3)/9 - 1/12)/x^4 - log(x - 3^(1/3)/2 - (3^(5/6)*1i)/2)
*(3^(2/3)/324 - (3^(1/6)*1i)/108) - log(x - 3^(1/3)/2 + (3^(5/6)*1i)/2)*(3
^(2/3)/324 + (3^(1/6)*1i)/108)
```

3.169 $\int \frac{1}{x^6(3+4x^3+x^6)}\,dx$

3.169.1 Optimal result

Integrand size = 16, antiderivative size = 126

$$\int \frac{1}{x^6\left(3+4x^3+x^6\right)}\,dx = -\frac{1}{15x^5}+\frac{2}{9x^2}-\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{54\sqrt[6]{3}}+\frac{1}{6}\log(1+x)-\frac{\log\left(\sqrt[3]{3}+x\right)}{54\ 3^{2/3}}-\frac{1}{12}\log\left(1-x+x^2\right)+\frac{\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)}{108\ 3^{2/3}}$$

output
```
-1/15/x^5+2/9/x^2+1/162*3^(5/6)*arctan(1/3*(3^(1/3)-2*x)*3^(1/6))+1/6*ln(1
+x)-1/162*3^(1/3)*ln(3^(1/3)+x)-1/12*ln(x^2-x+1)+1/324*3^(1/3)*ln(3^(2/3)-
3^(1/3)*x+x^2)-1/6*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

3.169.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^6\left(3+4x^3+x^6\right)}\,dx = \frac{-\frac{108}{x^5}+\frac{360}{x^2}+10\ 3^{5/6}\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)+270\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)+270\log(1+x)-10\sqrt[3]{3}\log\left(3+3^{2/3}x\right)}{1620}$$

input
```
Integrate[1/(x^6*(3 + 4*x^3 + x^6)),x]
```

output
```
(-108/x^5 + 360/x^2 + 10*3^(5/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] + 270*Sqr
t[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 270*Log[1 + x] - 10*3^(1/3)*Log[3 + 3^(2
/3)*x] - 135*Log[1 - x + x^2] + 5*3^(1/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2]
)/1620
```

3.169.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.21, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {1704, 27, 1828, 27, 1752, 750, 16, 1142, 25, 1082, 217, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6\left(x^6+4x^3+3\right)}dx$$

$$\Big\downarrow \text{ 1704}$$

$$\frac{1}{15}\int -\frac{5\left(x^3+4\right)}{x^3\left(x^6+4x^3+3\right)}dx-\frac{1}{15x^5}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{1}{3}\int \frac{x^3+4}{x^3\left(x^6+4x^3+3\right)}dx-\frac{1}{15x^5}$$

$$\Big\downarrow \text{ 1828}$$

$$\frac{1}{3}\left(\frac{1}{6}\int \frac{2\left(4x^3+13\right)}{x^6+4x^3+3}dx+\frac{2}{3x^2}\right)-\frac{1}{15x^5}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{3}\left(\frac{1}{3}\int \frac{4x^3+13}{x^6+4x^3+3}dx+\frac{2}{3x^2}\right)-\frac{1}{15x^5}$$

$$\Big\downarrow \text{ 1752}$$

$$\frac{1}{3}\left(\frac{1}{3}\left(\frac{9}{2}\int \frac{1}{x^3+1}dx-\frac{1}{2}\int \frac{1}{x^3+3}dx\right)+\frac{2}{3x^2}\right)-\frac{1}{15x^5}$$

$$\Big\downarrow \text{ 750}$$

$$\frac{1}{3}\left(\frac{1}{3}\left(\frac{9}{2}\left(\frac{1}{3}\int \frac{2-x}{x^2-x+1}dx+\frac{1}{3}\int \frac{1}{x+1}dx\right)+\frac{1}{2}\left(-\frac{\int \frac{2\sqrt[3]{3}-x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\ 3^{2/3}}-\frac{\int \frac{1}{x+\sqrt[3]{3}}dx}{3\ 3^{2/3}}\right)\right)+\frac{2}{3x^2}\right)-\frac{1}{15x^5}$$

$\downarrow$ 16

$$\frac{1}{3}\left(\frac{1}{3}\left(\frac{9}{2}\left(\frac{1}{3}\int \frac{2-x}{x^2-x+1}dx+\frac{1}{3}\log(x+1)\right)+\frac{1}{2}\left(-\frac{\int \frac{2\sqrt[3]{3}-x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\ 3^{2/3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)\right)+\frac{2}{3x^2}\right)-\frac{1}{15x^5}$$

$\downarrow$ 1142

$$\frac{1}{3}\left(\frac{1}{3}\left(\frac{9}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^2-x+1}dx-\frac{1}{2}\int -\frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{3}\log(x+1)\right)+\frac{1}{2}\left(-\frac{\frac{3}{2}\sqrt[3]{3}\int \frac{1}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\frac{1}{2}\int -}{3\ 3^{2/3}}\right.\right.\right.$$

$$\frac{1}{15x^5}$$

$\downarrow$ 25

$$\frac{1}{3}\left(\frac{1}{3}\left(\frac{9}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^2-x+1}dx+\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{3}\log(x+1)\right)+\frac{1}{2}\left(-\frac{\frac{3}{2}\sqrt[3]{3}\int \frac{1}{x^2-\sqrt[3]{3}x+3^{2/3}}dx+\frac{1}{2}\int \frac{}{x^2}}{3\ 3^{2/3}}\right.\right.\right.$$

$$\frac{1}{15x^5}$$

$\downarrow$ 1082

$$\frac{1}{3}\left(\frac{1}{3}\left(\frac{9}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^2-x+1}dx+\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{3}\log(x+1)\right)+\frac{1}{2}\left(-\frac{\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx+3\int \frac{}{-\left(1-\right.}}{3\ 3^{2/3}}\right.\right.\right.$$

$$\frac{1}{15x^5}$$

$\downarrow$ 217

3.169. $\int \frac{1}{x^6(3+4x^3+x^6)}\,dx$

$$\frac{1}{3}\left(\frac{1}{3}\left(\frac{1}{2}\left(-\frac{\frac{1}{2}\int\frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\ 3^{2/3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)+\frac{9}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int\frac{1}{x^2-x+1}dx+\frac{1}{2}\int\frac{}{x}\right.\right.\right.\right.$$

$$\frac{1}{15x^5}$$

$$\downarrow 1083$$

$$\frac{1}{3}\left(\frac{1}{3}\left(\frac{1}{2}\left(-\frac{\frac{1}{2}\int\frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\ 3^{2/3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)+\frac{9}{2}\left(\frac{1}{3}\left(\frac{1}{2}\int\frac{1-2x}{x^2-x+1}dx-3\int\frac{}{-}\right.\right.\right.\right.$$

$$\frac{1}{15x^5}$$

$$\downarrow 217$$

$$\frac{1}{3}\left(\frac{1}{3}\left(\frac{9}{2}\left(\frac{1}{3}\left(\frac{1}{2}\int\frac{1-2x}{x^2-x+1}dx+\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)\right)+\frac{1}{3}\log(x+1)\right)+\frac{1}{2}\left(-\frac{\frac{1}{2}\int\frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\sqrt{3}\,\mathrm{arct}}{3\ 3^{2/3}}\right.\right.\right.$$

$$\frac{1}{15x^5}$$

$$\downarrow 1103$$

$$\frac{1}{3}\left(\frac{1}{3}\left(\frac{9}{2}\left(\frac{1}{3}\left(\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)-\frac{1}{2}\log\left(x^2-x+1\right)\right)+\frac{1}{3}\log(x+1)\right)+\frac{1}{2}\left(-\frac{-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)-\frac{1}{2}\log}{3\ 3^{2/3}}\right.\right.\right.$$

$$\frac{1}{15x^5}$$

input
```
Int[1/(x^6*(3 + 4*x^3 + x^6)),x]
```

3.169. $\int\frac{1}{x^6(3+4x^3+x^6)}\,dx$

output
```
-1/15*1/x^5 + (2/(3*x^2) + ((9*(Log[1 + x]/3 + (Sqrt[3]*ArcTan[(-1 + 2*x)/
Sqrt[3]] - Log[1 - x + x^2]/2)/3))/2 + (-1/3*Log[3^(1/3) + x]/3^(2/3) - (-
(Sqrt[3]*ArcTan[(1 - (2*x)/3^(1/3))/Sqrt[3]]) - Log[3^(2/3) - 3^(1/3)*x +
x^2]/2)/(3*3^(2/3)))/2)/3)/3
```

3.169.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1704
```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Simp[1/(a*d^n*(m + 1))   Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1)
+ c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{
a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] &&
LtQ[m, -1] && IntegerQ[p]
```

rule 1752
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

rule 1828
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Simp[1/(a*f^n*(m + 1))   Int[(f*x)^(m +
 n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) -
c*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x
] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Int
egerQ[p]
```

3.169. $\int \frac{1}{x^6(3+4x^3+x^6)}\,dx$

3.169.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.52

method	result
risch	$\frac{\frac{2x^3}{9}-\frac{1}{15}}{x^5}+\frac{\ln(x+1)}{6}+\frac{\left(\sum_{\_R=\mathrm{RootOf}(9\_Z^3+1)}\_R\ln\left(x-3\_R\right)\right)}{54}-\frac{\ln(x^2-x+1)}{12}+\frac{\sqrt{3}\arctan\left(\frac{2\left(x-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{6}$
default	$-\frac{1}{15x^5}+\frac{2}{9x^2}+\frac{\ln(x+1)}{6}-\frac{3^{\frac{1}{3}}\ln\left(3^{\frac{1}{3}}+x\right)}{162}+\frac{3^{\frac{1}{3}}\ln\left(3^{\frac{2}{3}}-3^{\frac{1}{3}}x+x^2\right)}{324}-\frac{3^{\frac{5}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,3^{\frac{2}{3}}x}{3}-1\right)}{3}\right)}{162}-\frac{\ln(x^2-x+1)}{12}+$

input
```
int(1/x^6/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
```

output
```
(2/9*x^3-1/15)/x^5+1/6*ln(x+1)+1/54*sum(_R*ln(x-3*_R),_R=RootOf(9*_Z^3+1))
-1/12*ln(x^2-x+1)+1/6*3^(1/2)*arctan(2/3*(x-1/2)*3^(1/2))
```

3.169.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.21

$$\int \frac{1}{x^6\left(3+4x^3+x^6\right)}\,dx$$
$$=\frac{30\cdot 9^{\frac{1}{6}}\sqrt{3}(-1)^{\frac{1}{3}}\,x^5\arctan\left(\frac{1}{27}\cdot 9^{\frac{1}{6}}\left(2\cdot 9^{\frac{2}{3}}\sqrt{3}(-1)^{\frac{2}{3}}\,x-3\cdot 9^{\frac{1}{3}}\sqrt{3}\right)\right)-5\cdot 9^{\frac{2}{3}}(-1)^{\frac{1}{3}}\,x^5\log\left(9^{\frac{2}{3}}(-1)^{\frac{1}{3}}\,x+3\,x\right.}{}$$

input
```
integrate(1/x^6/(x^6+4*x^3+3),x, algorithm="fricas")
```

output
```
1/4860*(30*9^(1/6)*sqrt(3)*(-1)^(1/3)*x^5*arctan(1/27*9^(1/6)*(2*9^(2/3)*s
qrt(3)*(-1)^(2/3)*x - 3*9^(1/3)*sqrt(3))) - 5*9^(2/3)*(-1)^(1/3)*x^5*log(9
^(2/3)*(-1)^(1/3)*x + 3*x^2 + 3*9^(1/3)*(-1)^(2/3)) + 10*9^(2/3)*(-1)^(1/3
)*x^5*log(-9^(2/3)*(-1)^(1/3) + 3*x) + 810*sqrt(3)*x^5*arctan(1/3*sqrt(3)*
(2*x - 1)) - 405*x^5*log(x^2 - x + 1) + 810*x^5*log(x + 1) + 1080*x^3 - 32
4)/x^5
```

3.169.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.04 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^6(3+4x^3+x^6)}\,dx = \frac{\log(x+1)}{6} + \left(-\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)\log\left(x+\frac{88573}{6562}-\frac{88573\sqrt{3}i}{6562}+\frac{119042784\left(-\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)^4}{3281}\right) + \left(-\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)\log\left(x+\frac{88573}{6562}+\frac{119042784\left(-\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)^4}{3281}+\frac{88573\sqrt{3}i}{6562}\right) + \operatorname{RootSum}\left(1417176t^3+1,\left(t\mapsto t\log\left(\frac{119042784t^4}{3281}-\frac{531438t}{3281}+x\right)\right)\right)+\frac{10x^3-3}{45x^5}$$

input
```
integrate(1/x**6/(x**6+4*x**3+3),x)
```

output
```
log(x + 1)/6 + (-1/12 + sqrt(3)*I/12)*log(x + 88573/6562 - 88573*sqrt(3)*I
/6562 + 119042784*(-1/12 + sqrt(3)*I/12)**4/3281) + (-1/12 - sqrt(3)*I/12)
*log(x + 88573/6562 + 119042784*(-1/12 - sqrt(3)*I/12)**4/3281 + 88573*sqr
t(3)*I/6562) + RootSum(1417176*_t**3 + 1, Lambda(_t, _t*log(119042784*_t**
4/3281 - 531438*_t/3281 + x))) + (10*x**3 - 3)/(45*x**5)
```

3.169.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^6(3+4x^3+x^6)}\,dx = -\frac{1}{162}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right)\right) + \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{324}\cdot 3^{\frac{1}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right) - \frac{1}{162}\cdot 3^{\frac{1}{3}}\log\left(x+3^{\frac{1}{3}}\right) + \frac{10x^3-3}{45x^5} - \frac{1}{12}\log\left(x^2-x+1\right) + \frac{1}{6}\log(x+1)$$

input
```
integrate(1/x^6/(x^6+4*x^3+3),x, algorithm="maxima")
```

output
```
-1/162*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) + 1/6*sqrt(3)*arctan(1/
3*sqrt(3)*(2*x - 1)) + 1/324*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 1/16
2*3^(1/3)*log(x + 3^(1/3)) + 1/45*(10*x^3 - 3)/x^5 - 1/12*log(x^2 - x + 1)
 + 1/6*log(x + 1)
```

3.169.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^6(3+4x^3+x^6)}\,dx = -\frac{1}{162}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2\,x-3^{\frac{1}{3}}\right)\right) + \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right) + \frac{1}{324}\cdot 3^{\frac{1}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right) - \frac{1}{162}\cdot 3^{\frac{1}{3}}\log\left(\left|x+3^{\frac{1}{3}}\right|\right) + \frac{10\,x^3-3}{45\,x^5} - \frac{1}{12}\log\left(x^2-x+1\right) + \frac{1}{6}\log\left(|x+1|\right)$$

input
```
integrate(1/x^6/(x^6+4*x^3+3),x, algorithm="giac")
```

output
```
-1/162*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) + 1/6*sqrt(3)*arctan(1/
3*sqrt(3)*(2*x - 1)) + 1/324*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 1/16
2*3^(1/3)*log(abs(x + 3^(1/3))) + 1/45*(10*x^3 - 3)/x^5 - 1/12*log(x^2 - x
 + 1) + 1/6*log(abs(x + 1))
```

3.169.9 Mupad [B] (verification not implemented)

Time = 8.50 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^6(3+4x^3+x^6)}\,dx = \frac{\ln(x+1)}{6} - \frac{3^{1/3}\ln\left(x+3^{1/3}\right)}{162} - \ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) + \ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) + \frac{\frac{2\,x^3}{9}-\frac{1}{15}}{x^5} + \frac{(-1)^{1/3}\,3^{1/3}\ln\left(x-(-1)^{1/3}\,3^{1/3}\right)}{162} - \frac{(-1)^{1/3}\ln\left(x+\frac{(-1)^{1/3}\,3^{1/3}}{2}+\frac{(-1)^{1/3}\,3^{5/6}\,1\mathrm{i}}{2}\right)\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{324}$$

input
```
int(1/(x^6*(4*x^3 + x^6 + 3)),x)
```

output
```
log(x + 1)/6 - (3^(1/3)*log(x + 3^(1/3)))/162 - log(x - (3^(1/2)*1i)/2 - 1
/2)*((3^(1/2)*1i)/12 + 1/12) + log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)
/12 - 1/12) + ((2*x^3)/9 - 1/15)/x^5 + ((-1)^(1/3)*3^(1/3)*log(x - (-1)^(1
/3)*3^(1/3)))/162 - ((-1)^(1/3)*log(x + ((-1)^(1/3)*3^(1/3))/2 + ((-1)^(1/
3)*3^(5/6)*1i)/2)*(3^(1/3) + 3^(5/6)*1i))/324
```

3.169. $\int \frac{1}{x^6(3+4x^3+x^6)}\,dx$

3.170 $\int \frac{x^6}{1-x^3+x^6} \, dx$

3.170.1 Optimal result

Integrand size = 16, antiderivative size = 412

$$\int \frac{x^6}{1-x^3+x^6}\,dx = x + \frac{(i-\sqrt{3})\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} - \frac{(i+\sqrt{3})\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} + \frac{(3-i\sqrt{3})\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} + \frac{(3+i\sqrt{3})\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} - \frac{(3-i\sqrt{3})\log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} - \frac{(3+i\sqrt{3})\log\left(\left(1+i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

output
```
x+1/6*arctan(1/3*(1+2*2^(1/3)*x/(1-I*3^(1/2))^(1/3))*3^(1/2))*(I-3^(1/2))*
2^(2/3)/(1-I*3^(1/2))^(2/3)+1/18*ln(-2^(1/3)*x+(1-I*3^(1/2))^(1/3))*(3-I*3
^(1/2))*2^(2/3)/(1-I*3^(1/2))^(2/3)-1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1-I*3^(
1/2))^(1/3)+(1-I*3^(1/2))^(2/3))*(3-I*3^(1/2))*2^(2/3)/(1-I*3^(1/2))^(2/3)
+1/18*ln(-2^(1/3)*x+(1+I*3^(1/2))^(1/3))*(3+I*3^(1/2))*2^(2/3)/(1+I*3^(1/2
))^(2/3)-1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1+I*3^(1/2))^(1/3)+(1+I*3^(1/2))^(
2/3))*(3+I*3^(1/2))*2^(2/3)/(1+I*3^(1/2))^(2/3)-1/6*arctan(1/3*(1+2*2^(1/3
)*x/(1+I*3^(1/2))^(1/3))*3^(1/2))*(3^(1/2)+I)*2^(2/3)/(1+I*3^(1/2))^(2/3)
```

3.170.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.14

$$\int \frac{x^6}{1-x^3+x^6}\,dx = x+\frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&,\frac{-\log(x-\#1)+\log(x-\#1)\#1^3}{-\#1^2+2\#1^5}\&\right]$$

input
```
Integrate[x^6/(1 - x^3 + x^6),x]
```

output
```
x + RootSum[1 - #1^3 + #1^6 & , (-Log[x - #1] + Log[x - #1]*#1^3)/(-#1^2 +
 2*#1^5) & ]/3
```

3.170.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 350, normalized size of antiderivative = 0.85, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {1703, 1752, 750, 16, 25, 1142, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{x^6-x^3+1}dx$$

$$\Big\downarrow \text{1703}$$

$$x-\int \frac{1-x^3}{x^6-x^3+1}dx$$

$$\Big\downarrow \text{1752}$$

$$\frac{1}{6}\left(3+i\sqrt{3}\right)\int \frac{1}{x^3+\frac{1}{2}\left(-1-i\sqrt{3}\right)}dx+\frac{1}{6}\left(3-i\sqrt{3}\right)\int \frac{1}{x^3+\frac{1}{2}\left(-1+i\sqrt{3}\right)}dx+x$$

$$\Big\downarrow \text{750}$$

$$\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\int -\frac{x+2^{2/3}\sqrt[3]{1-i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}+\frac{\int \frac{1}{x-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right)+$$

$$\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\int -\frac{x+2^{2/3}\sqrt[3]{1+i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}+\frac{\int \frac{1}{x-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right)+x$$

$$\downarrow 16$$

$$\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\int -\frac{x+2^{2/3}\sqrt[3]{1-i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}+\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right)+$$

$$\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\int -\frac{x+2^{2/3}\sqrt[3]{1+i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}+\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right)+x$$

$$\downarrow 25$$

3.170. $\int \frac{x^6}{1-x^3+x^6}\,dx$

$$\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\int\frac{x+2^{2/3}\sqrt[3]{1-i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right)+$$

$$\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\int\frac{x+2^{2/3}\sqrt[3]{1+i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right)+x$$

$$\Big\downarrow \; 1142$$

$$\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{3}{2}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}\int\frac{1}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx+\frac{1}{2}\int\frac{\cdots}{x^2+\sqrt[3]{\cdots}}\cdots}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\cdots\right.$$

$$\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{3}{2}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}\int\frac{1}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx+\frac{1}{2}\int\frac{\cdots}{x^2+\sqrt[3]{\cdots}}\cdots}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\cdots\right.$$

$$x$$

$$\Big\downarrow \; 1082$$

3.170. $\int \frac{x^6}{1-x^3+x^6}\,dx$

$$\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{1}{2}\int\frac{2x+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx-3\int\frac{1}{-\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}+\right.}}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right.$$

$$\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{1}{2}\int\frac{2x+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx-3\int\frac{1}{-\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}+\right.}}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right.$$

$$x$$

$$\downarrow 217$$

3.170. $\int \frac{x^6}{1-x^3+x^6}\,dx$

$$\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{1}{2}\int\frac{2x+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx+\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-\dots\right.}}}{\sqrt{3}}\right.}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right.$$

$$\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{1}{2}\int\frac{2x+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx+\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+\dots\right.}}}{\sqrt{3}}\right.}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right.$$

x

$\downarrow$ 1103

$$\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)+\frac{1}{2}\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x\dots\right.}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right.$$

$$\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)+\frac{1}{2}\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x\dots\right.}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right.$$

x

3.170. $\int\frac{x^6}{1-x^3+x^6}\,dx$

input
```
Int[x^6/(1 - x^3 + x^6),x]
```

output
```
x + ((3 - I*Sqrt[3])*(Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x]/(3*((1 - I*Sq
rt[3])/2)^(2/3)) - (Sqrt[3]*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/S
qrt[3]] + Log[(1 - I*Sqrt[3])^(2/3) + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3
)*x^2]/2)/(3*((1 - I*Sqrt[3])/2)^(2/3))))/6 + ((3 + I*Sqrt[3])*(Log[(1 + I
*Sqrt[3])^(1/3) - 2^(1/3)*x]/(3*((1 + I*Sqrt[3])/2)^(2/3)) - (Sqrt[3]*ArcT
an[(1 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]] + Log[(1 + I*Sqrt[3])^(2
/3) + (2*(1 + I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2]/2)/(3*((1 + I*Sqrt[3])/2)
^(2/3))))/6
```

3.170.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1703
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(2*n - 1)*(d*x)^(m - 2*n + 1)*((a + b*x^n + c*x^(2*n))^(
p + 1)/(c*(m + 2*n*p + 1))), x] - Simp[d^(2*n)/(c*(m + 2*n*p + 1))   Int[(d
*x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x
^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && N
eQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0
] && IntegerQ[p]
```

rule 1752
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

3.170.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.11

method	result	size
default	$x+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)}\frac{\left(\_R^3-1\right)\ln\left(x-\_R\right)}{2\_R^5-\_R^2}\right)}{3}$	44
risch	$x+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)}\frac{\left(\_R^3-1\right)\ln\left(x-\_R\right)}{2\_R^5-\_R^2}\right)}{3}$	44

input
```
int(x^6/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
x+1/3*sum((_R^3-1)/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1))
```

3.170.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.73

$$\int \frac{x^6}{1-x^3+x^6}\,dx = \frac{1}{108}\cdot 18^{\frac{2}{3}}\left(i\sqrt{3}+3\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\log\left(18^{\frac{2}{3}}\left(\sqrt{3}(i\sqrt{-3}-i)-3\sqrt{-3}+3\right)\left(i\sqrt{3}+3\right)^{\frac{1}{3}}+72\,x\right)-\frac{1}{108}\cdot 18^{\frac{2}{3}}\left(i\sqrt{3}+3\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\log\left(18^{\frac{2}{3}}\left(\sqrt{3}(-i\sqrt{-3}-i)+3\sqrt{-3}+3\right)\left(i\sqrt{3}+3\right)^{\frac{1}{3}}+72\,x\right)-\frac{1}{108}\cdot 18^{\frac{2}{3}}\left(-i\sqrt{3}+3\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\log\left(18^{\frac{2}{3}}\left(\sqrt{3}(i\sqrt{-3}+i)+3\sqrt{-3}+3\right)\left(-i\sqrt{3}+3\right)^{\frac{1}{3}}+72\,x\right)+\frac{1}{108}\cdot 18^{\frac{2}{3}}\left(-i\sqrt{3}+3\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\log\left(18^{\frac{2}{3}}\left(\sqrt{3}(-i\sqrt{-3}+i)-3\sqrt{-3}+3\right)\left(-i\sqrt{3}+3\right)^{\frac{1}{3}}+72\,x\right)+\frac{1}{54}\cdot 18^{\frac{2}{3}}\left(i\sqrt{3}+3\right)^{\frac{1}{3}}\log\left(18^{\frac{2}{3}}\left(i\sqrt{3}+3\right)^{\frac{1}{3}}\left(i\sqrt{3}-3\right)+36\,x\right)+\frac{1}{54}\cdot 18^{\frac{2}{3}}\left(-i\sqrt{3}+3\right)^{\frac{1}{3}}\log\left(18^{\frac{2}{3}}\left(-i\sqrt{3}+3\right)^{\frac{1}{3}}\left(-i\sqrt{3}-3\right)+36\,x\right)+x$$

input
```
integrate(x^6/(x^6-x^3+1),x, algorithm="fricas")
```

output
```
1/108*18^(2/3)*(I*sqrt(3) + 3)^(1/3)*(sqrt(-3) - 1)*log(18^(2/3)*(sqrt(3)*
(I*sqrt(-3) - I) - 3*sqrt(-3) + 3)*(I*sqrt(3) + 3)^(1/3) + 72*x) - 1/108*1
8^(2/3)*(I*sqrt(3) + 3)^(1/3)*(sqrt(-3) + 1)*log(18^(2/3)*(sqrt(3)*(-I*sqr
t(-3) - I) + 3*sqrt(-3) + 3)*(I*sqrt(3) + 3)^(1/3) + 72*x) - 1/108*18^(2/3
)*(-I*sqrt(3) + 3)^(1/3)*(sqrt(-3) + 1)*log(18^(2/3)*(sqrt(3)*(I*sqrt(-3)
+ I) + 3*sqrt(-3) + 3)*(-I*sqrt(3) + 3)^(1/3) + 72*x) + 1/108*18^(2/3)*(-I
*sqrt(3) + 3)^(1/3)*(sqrt(-3) - 1)*log(18^(2/3)*(sqrt(3)*(-I*sqrt(-3) + I)
 - 3*sqrt(-3) + 3)*(-I*sqrt(3) + 3)^(1/3) + 72*x) + 1/54*18^(2/3)*(I*sqrt(
3) + 3)^(1/3)*log(18^(2/3)*(I*sqrt(3) + 3)^(1/3)*(I*sqrt(3) - 3) + 36*x) +
 1/54*18^(2/3)*(-I*sqrt(3) + 3)^(1/3)*log(18^(2/3)*(-I*sqrt(3) + 3)^(1/3)*
(-I*sqrt(3) - 3) + 36*x) + x
```

3.170.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.06

$$\int \frac{x^6}{1-x^3+x^6}\,dx = x + \operatorname{RootSum}\left(19683t^6 - 243t^3 + 1, \left(t \mapsto t\log\left(729t^4 - 9t + x\right)\right)\right)$$

input
```
integrate(x**6/(x**6-x**3+1),x)
```

output
```
x + RootSum(19683*_t**6 - 243*_t**3 + 1, Lambda(_t, _t*log(729*_t**4 - 9*_
t + x)))
```

3.170.7 Maxima [F]

$$\int \frac{x^6}{1-x^3+x^6}\,dx = \int \frac{x^6}{x^6-x^3+1}\,dx$$

input
```
integrate(x^6/(x^6-x^3+1),x, algorithm="maxima")
```

output
```
x + integrate((x^3 - 1)/(x^6 - x^3 + 1), x)
```

3.170.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 641 vs. 2(268) = 536.

Time = 0.33 (sec) , antiderivative size = 641, normalized size of antiderivative = 1.56

$$\int \frac{x^6}{1-x^3+x^6}\,dx = \text{Too large to display}$$

input
```
integrate(x^6/(x^6-x^3+1),x, algorithm="giac")
```

output
```
-1/9*(sqrt(3)*cos(4/9*pi)^4 - 6*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^2 + sqrt
(3)*sin(4/9*pi)^4 + 4*cos(4/9*pi)^3*sin(4/9*pi) - 4*cos(4/9*pi)*sin(4/9*pi
)^3 + 2*sqrt(3)*cos(4/9*pi) + 2*sin(4/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*
cos(4/9*pi) + 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(4/9*pi))) - 1/9*(sqrt(3)*cos
(2/9*pi)^4 - 6*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^2 + sqrt(3)*sin(2/9*pi)^4
 + 4*cos(2/9*pi)^3*sin(2/9*pi) - 4*cos(2/9*pi)*sin(2/9*pi)^3 + 2*sqrt(3)*c
os(2/9*pi) + 2*sin(2/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(2/9*pi) + 2*x
)/((1/2*I*sqrt(3) + 1/2)*sin(2/9*pi))) - 1/9*(sqrt(3)*cos(1/9*pi)^4 - 6*sq
rt(3)*cos(1/9*pi)^2*sin(1/9*pi)^2 + sqrt(3)*sin(1/9*pi)^4 - 4*cos(1/9*pi)^
3*sin(1/9*pi) + 4*cos(1/9*pi)*sin(1/9*pi)^3 - 2*sqrt(3)*cos(1/9*pi) + 2*si
n(1/9*pi))*arctan(-1/2*((-I*sqrt(3) - 1)*cos(1/9*pi) - 2*x)/((1/2*I*sqrt(3
) + 1/2)*sin(1/9*pi))) - 1/18*(4*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi) - 4*sqr
t(3)*cos(4/9*pi)*sin(4/9*pi)^3 - cos(4/9*pi)^4 + 6*cos(4/9*pi)^2*sin(4/9*p
i)^2 - sin(4/9*pi)^4 + 2*sqrt(3)*sin(4/9*pi) - 2*cos(4/9*pi))*log((-I*sqrt
(3)*cos(4/9*pi) - cos(4/9*pi))*x + x^2 + 1) - 1/18*(4*sqrt(3)*cos(2/9*pi)^
3*sin(2/9*pi) - 4*sqrt(3)*cos(2/9*pi)*sin(2/9*pi)^3 - cos(2/9*pi)^4 + 6*co
s(2/9*pi)^2*sin(2/9*pi)^2 - sin(2/9*pi)^4 + 2*sqrt(3)*sin(2/9*pi) - 2*cos(
2/9*pi))*log((-I*sqrt(3)*cos(2/9*pi) - cos(2/9*pi))*x + x^2 + 1) + 1/18*(4
*sqrt(3)*cos(1/9*pi)^3*sin(1/9*pi) - 4*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^3 +
 cos(1/9*pi)^4 - 6*cos(1/9*pi)^2*sin(1/9*pi)^2 + sin(1/9*pi)^4 - 2*sqrt...
```

3.170. $\int \frac{x^6}{1-x^3+x^6}\,dx$

3.170.9 Mupad [B] (verification not implemented)

Time = 8.73 (sec) , antiderivative size = 320, normalized size of antiderivative = 0.78

$$\int \frac{x^6}{1-x^3+x^6}\,dx = x + \frac{\ln\left(x+\frac{\left(-\frac{27}{2}+\frac{\sqrt{3}\,9\mathrm{i}}{2}\right)\left(36+\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{54}\right)\left(36+\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18} + \frac{\ln\left(x-\frac{\left(\frac{27}{2}+\frac{\sqrt{3}\,9\mathrm{i}}{2}\right)\left(36-\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{54}\right)\left(36-\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18} - \frac{2^{2/3}\ln\left(x-\frac{2^{2/3}\left(3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)\left(\frac{3\left(-3+\sqrt{3}\,1\mathrm{i}\right)\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)^3}{16}-27\right)}{108}\right)\left(3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36} - \frac{2^{2/3}\ln\left(x+\frac{2^{2/3}\left(3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)\left(\frac{3\left(3+\sqrt{3}\,1\mathrm{i}\right)\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)^3}{16}+27\right)}{108}\right)\left(3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36} - \frac{2^{2/3}\ln\left(x+\frac{2^{2/3}\,3^{5/6}\left(3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}1\mathrm{i}}{6}\right)\left(3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36} - \frac{2^{2/3}\ln\left(x-\frac{2^{2/3}\,3^{5/6}\left(3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}1\mathrm{i}}{6}\right)\left(3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}$$

input

```
int(x^6/(x^6 - x^3 + 1),x)
```

output

```
x + (log(x + (((3^(1/2)*9i)/2 - 27/2)*(3^(1/2)*12i + 36)^(1/3))/54)*(3^(1/
2)*12i + 36)^(1/3))/18 + (log(x - (((3^(1/2)*9i)/2 + 27/2)*(36 - 3^(1/2)*1
2i)^(1/3))/54)*(36 - 3^(1/2)*12i)^(1/3))/18 - (2^(2/3)*log(x - (2^(2/3)*(3
 - 3^(1/2)*1i)^(1/3)*(3^(1/3) - 3^(5/6)*1i)*((3*(3^(1/2)*1i - 3)*(3^(1/3)
- 3^(5/6)*1i)^3)/16 - 27))/108)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) - 3^(5/6)*
1i))/36 - (2^(2/3)*log(x + (2^(2/3)*(3^(1/2)*1i + 3)^(1/3)*(3^(1/3) + 3^(5
/6)*1i)*((3*(3^(1/2)*1i + 3)*(3^(1/3) + 3^(5/6)*1i)^3)/16 + 27))/108)*(3^(
1/2)*1i + 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*log(x + (2^(2/3)*
3^(5/6)*(3 - 3^(1/2)*1i)^(1/3)*1i)/6)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) + 3^
(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(2/3)*3^(5/6)*(3^(1/2)*1i + 3)^(1/3)*1
i)/6)*(3^(1/2)*1i + 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36
```

3.171 $\int \frac{x^5}{1-x^3+x^6} \, dx$

3.171.1 Optimal result

Integrand size = 16, antiderivative size = 39

$$\int \frac{x^5}{1-x^3+x^6} \, dx = -\frac{\arctan\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{1}{6}\log\left(1-x^3+x^6\right)$$

output
```
1/6*ln(x^6-x^3+1)-1/9*arctan(1/3*(-2*x^3+1)*3^(1/2))*3^(1/2)
```

3.171.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{1-x^3+x^6} \, dx = \frac{\arctan\left(\frac{-1+2x^3}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{1}{6}\log\left(1-x^3+x^6\right)$$

input
```
Integrate[x^5/(1 - x^3 + x^6),x]
```

output
```
ArcTan[(-1 + 2*x^3)/Sqrt[3]]/(3*Sqrt[3]) + Log[1 - x^3 + x^6]/6
```

3.171.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1693, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{x^6-x^3+1}dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int \frac{x^3}{x^6-x^3+1}dx^3$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{3}\left(\frac{1}{2}\int \frac{1}{x^6-x^3+1}dx^3+\frac{1}{2}\int -\frac{1-2x^3}{x^6-x^3+1}dx^3\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\left(\frac{1}{2}\int \frac{1}{x^6-x^3+1}dx^3-\frac{1}{2}\int \frac{1-2x^3}{x^6-x^3+1}dx^3\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{3}\left(-\int \frac{1}{-x^6-3}d(2x^3-1)-\frac{1}{2}\int \frac{1-2x^3}{x^6-x^3+1}dx^3\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{3}\left(\frac{\arctan\left(\frac{2x^3-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}\int \frac{1-2x^3}{x^6-x^3+1}dx^3\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{3}\left(\frac{\arctan\left(\frac{2x^3-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(x^6-x^3+1\right)\right)$$

input
```
Int[x^5/(1 - x^3 + x^6),x]
```

output
```
(ArcTan[(-1 + 2*x^3)/Sqrt[3]]/Sqrt[3] + Log[1 - x^3 + x^6]/2)/3
```

3.171.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.171.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{\ln\left(x^{6}-x^{3}+1\right)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{\left(2x^{3}-1\right)\sqrt{3}}{3}\right)}{9}$	33
risch	$\frac{\ln\left(4x^{6}-4x^{3}+4\right)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{\left(2x^{3}-1\right)\sqrt{3}}{3}\right)}{9}$	35

input
```
int(x^5/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/6*ln(x^6-x^3+1)+1/9*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))
```

3.171.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{x^5}{1-x^3+x^6}\,dx = \frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x^3-1)\right)+\frac{1}{6}\log\left(x^6-x^3+1\right)$$

input
```
integrate(x^5/(x^6-x^3+1),x, algorithm="fricas")
```

output
```
1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 1/6*log(x^6 - x^3 + 1)
```

3.171.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{x^5}{1-x^3+x^6}\,dx = \frac{\log\left(x^6-x^3+1\right)}{6}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^3}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

input
```
integrate(x**5/(x**6-x**3+1),x)
```

output
```
log(x**6 - x**3 + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3)/9
```

3.171.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{x^5}{1-x^3+x^6}\,dx = \frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x^3-1)\right)+\frac{1}{6}\log\left(x^6-x^3+1\right)$$

input
```
integrate(x^5/(x^6-x^3+1),x, algorithm="maxima")
```

output
```
1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 1/6*log(x^6 - x^3 + 1)
```

3.171.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{x^5}{1-x^3+x^6}\,dx = \frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^3-1)\right)+\frac{1}{6}\log\left(x^6-x^3+1\right)$$

input
```
integrate(x^5/(x^6-x^3+1),x, algorithm="giac")
```

output
```
1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 1/6*log(x^6 - x^3 + 1)
```

3.171.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

$$\int \frac{x^5}{1-x^3+x^6}\,dx = \frac{\ln\left(x^6-x^3+1\right)}{6}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}\,x^3}{3}\right)}{9}$$

input
```
int(x^5/(x^6 - x^3 + 1),x)
```

output
```
log(x^6 - x^3 + 1)/6 - (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/3))/9
```

3.172 $\int \frac{x^4}{1-x^3+x^6}\,dx$

3.172.1 Optimal result

Integrand size = 16, antiderivative size = 411

$$\int \frac{x^4}{1-x^3+x^6}\,dx = \frac{(i+\sqrt{3})\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} - \frac{(i-\sqrt{3})\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}} + \frac{(3+i\sqrt{3})\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} + \frac{(3-i\sqrt{3})\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}} - \frac{(3+i\sqrt{3})\log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} - \frac{(3-i\sqrt{3})\log\left(\left(1+i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

output
```
-1/6*arctan(1/3*(1+2*2^(1/3)*x/(1+I*3^(1/2))^(1/3))*3^(1/2))*(I-3^(1/2))*2
^(1/3)/(1+I*3^(1/2))^(1/3)+1/18*ln(-2^(1/3)*x+(1+I*3^(1/2))^(1/3))*(3-I*3^
(1/2))*2^(1/3)/(1+I*3^(1/2))^(1/3)-1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1+I*3^(1
/2))^(1/3)+(1+I*3^(1/2))^(2/3))*(3-I*3^(1/2))*2^(1/3)/(1+I*3^(1/2))^(1/3)+
1/18*ln(-2^(1/3)*x+(1-I*3^(1/2))^(1/3))*(3+I*3^(1/2))*2^(1/3)/(1-I*3^(1/2)
)^(1/3)-1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1-I*3^(1/2))^(1/3)+(1-I*3^(1/2))^(2
/3))*(3+I*3^(1/2))*2^(1/3)/(1-I*3^(1/2))^(1/3)+1/6*arctan(1/3*(1+2*2^(1/3)
*x/(1-I*3^(1/2))^(1/3))*3^(1/2))*(3^(1/2)+I)*2^(1/3)/(1-I*3^(1/2))^(1/3)
```

3.172.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.10

$$\int \frac{x^4}{1-x^3+x^6}\,dx=\frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&,\frac{\log(x-\#1)\#1^2}{-1+2\#1^3}\&\right]$$

input
```
Integrate[x^4/(1 - x^3 + x^6),x]
```

output
```
RootSum[1 - #1^3 + #1^6 & , (Log[x - #1]*#1^2)/(-1 + 2*#1^3) & ]/3
```

3.172.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 367, normalized size of antiderivative = 0.89, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1710, 27, 821, 16, 1142, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{x^6-x^3+1}dx$$

$$\Big\downarrow \text{1710}$$

$$\frac{1}{6}\left(3+i\sqrt{3}\right)\int -\frac{2x}{-2x^3-i\sqrt{3}+1}dx+\frac{1}{6}\left(3-i\sqrt{3}\right)\int -\frac{2x}{-2x^3+i\sqrt{3}+1}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{3}\left(3+i\sqrt{3}\right)\int \frac{x}{-2x^3-i\sqrt{3}+1}dx-\frac{1}{3}\left(3-i\sqrt{3}\right)\int \frac{x}{-2x^3+i\sqrt{3}+1}dx$$

$$\Big\downarrow \text{821}$$

$$-\frac{1}{3}\left(3+i\sqrt{3}\right)\left(\frac{\int \frac{1}{\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x}dx}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\frac{\int \frac{\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}\right)-\frac{1}{3}\left(3-i\sqrt{3}\right)\left(\frac{\int \frac{1}{\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x}dx}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\frac{\int \frac{\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}\right)$$

$$\Big\downarrow \text{16}$$

$$-\frac{1}{3}\left(3+i\sqrt{3}\right)\left(-\frac{\int \frac{\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}\right)-\frac{1}{3}\left(3-i\sqrt{3}\right)\left(-\frac{\int \frac{\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}\right)$$

$$\Big\downarrow \text{1142}$$

3.172. $\int \frac{x^4}{1-x^3+x^6}\,dx$

$$-\frac{1}{3}\left(3+i\sqrt{3}\right)\left(-\frac{\frac{3}{2}\sqrt[3]{1-i\sqrt{3}}\int\frac{1}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx-\frac{\int\frac{2\,2^{2/3}x+\sqrt[3]{2\left(1-i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\right.$$

$$\frac{1}{3}\left(3-i\sqrt{3}\right)\left(-\frac{\frac{3}{2}\sqrt[3]{1+i\sqrt{3}}\int\frac{1}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx-\frac{\int\frac{2\,2^{2/3}x+\sqrt[3]{2\left(1+i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\frac{\text{lo}}{}\right.$$

↓ 1082

3.172. $\int \frac{x^4}{1-x^3+x^6}\,dx$

$$-\frac{1}{3}\left(3+i\sqrt{3}\right)\left(-\frac{-\frac{\int \frac{2\,2^{2/3}x+\sqrt[3]{2\left(1-i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}-\frac{3\int \frac{1}{-\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}+1\right)^2-3}d\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}+1\right)}{\sqrt[3]{2}}}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\cdots\right.$$

$$\frac{1}{3}\left(3-i\sqrt{3}\right)\left(-\frac{-\frac{\int \frac{2\,2^{2/3}x+\sqrt[3]{2\left(1+i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}-\frac{3\int \frac{1}{-\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}+1\right)^2-3}d\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}+1\right)}{\sqrt[3]{2}}}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\cdots\right.$$

$\downarrow$ 217

3.172. $\int \frac{x^4}{1-x^3+x^6}\,dx$

$$-\frac{1}{3}\left(3+i\sqrt{3}\right)\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\int\frac{2\,2^{2/3}x+\sqrt[3]{2\left(1-i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-\cdots}\right.}{3\,2^{2/3}\sqrt[3]{1-i\sqrt{3}}\cdots}\right.$$

$$\frac{1}{3}\left(3-i\sqrt{3}\right)\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\int\frac{2\,2^{2/3}x+\sqrt[3]{2\left(1+i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\cdots}\right.}{3\,2^{2/3}\sqrt[3]{1+i\sqrt{3}}}\right.$$

↓ 1103

3.172. $\int\frac{x^4}{1-x^3+x^6}\,dx$

$$-\frac{1}{3}\left(3+i\sqrt{3}\right)\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1\cdots}\right.}{3\ 2^{2/3}\sqrt[3]{1-\cdots}}\right.$$

$$\frac{1}{3}\left(3-i\sqrt{3}\right)\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+\cdots}\right.}{3\ 2^{2/3}\sqrt[3]{1+i\cdots}}\right.$$

input
```
Int[x^4/(1 - x^3 + x^6),x]
```

output
```
-1/3*((3 + I*Sqrt[3])*(-1/3*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x]/(2^(2/3
)*(1 - I*Sqrt[3])^(1/3)) - ((Sqrt[3]*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)
^(1/3))/Sqrt[3]])/2^(1/3) - Log[(1 - I*Sqrt[3])^(2/3) + (2*(1 - I*Sqrt[3])
)^(1/3)*x + 2^(2/3)*x^2]/(2*2^(1/3)))/(3*(2*(1 - I*Sqrt[3]))^(1/3)))) - ((
3 - I*Sqrt[3])*(-1/3*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x]/(2^(2/3)*(1 +
I*Sqrt[3])^(1/3)) - ((Sqrt[3]*ArcTan[(1 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))
/Sqrt[3]])/2^(1/3) - Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3]))^(1/3)
*x + 2^(2/3)*x^2]/(2*2^(1/3)))/(3*(2*(1 + I*Sqrt[3]))^(1/3))))/3
```

3.172. $\int \frac{x^4}{1-x^3+x^6}\,dx$

3.172.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1710
```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^n/2)*(b/q + 1)   Int[(d*x)^(m
- n)/(b/2 + q/2 + c*x^n), x], x] - Simp[(d^n/2)*(b/q - 1)   Int[(d*x)^(m -
n)/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

3.172. $\int \frac{x^4}{1-x^3+x^6} \, dx$

3.172.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.10

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)}\frac{\_R^{4}\ln\left(x-\_R\right)}{2\_R^{5}-\_R^{2}}\right)}{3}$	40
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)}\frac{\_R^{4}\ln\left(x-\_R\right)}{2\_R^{5}-\_R^{2}}\right)}{3}$	40

input
```
int(x^4/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/3*sum(_R^4/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1))
```

3.172.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 295, normalized size of antiderivative = 0.72

$$\int \frac{x^4}{1-x^3+x^6}\,dx = -\frac{1}{108}\cdot 18^{\frac{2}{3}}\left(i\sqrt{3}-3\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\log\left(18^{\frac{1}{3}}\left(\sqrt{3}(i\sqrt{-3}-i)+\sqrt{-3}-1\right)\left(i\sqrt{3}-3\right)^{\frac{2}{3}}+24x\right)+\frac{1}{108}\cdot 18^{\frac{2}{3}}\left(i\sqrt{3}-3\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\log\left(18^{\frac{1}{3}}\left(\sqrt{3}(-i\sqrt{-3}-i)-\sqrt{-3}-1\right)\left(i\sqrt{3}-3\right)^{\frac{2}{3}}+24x\right)+\frac{1}{108}\cdot 18^{\frac{2}{3}}\left(-i\sqrt{3}-3\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\log\left(18^{\frac{1}{3}}\left(\sqrt{3}(i\sqrt{-3}+i)-\sqrt{-3}-1\right)\left(-i\sqrt{3}-3\right)^{\frac{2}{3}}+24x\right)-\frac{1}{108}\cdot 18^{\frac{2}{3}}\left(-i\sqrt{3}-3\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\log\left(18^{\frac{1}{3}}\left(\sqrt{3}(-i\sqrt{-3}+i)+\sqrt{-3}-1\right)\left(-i\sqrt{3}-3\right)^{\frac{2}{3}}+24x\right)+\frac{1}{54}\cdot 18^{\frac{2}{3}}\left(i\sqrt{3}-3\right)^{\frac{1}{3}}\log\left(18^{\frac{1}{3}}\left(i\sqrt{3}+1\right)\left(i\sqrt{3}-3\right)^{\frac{2}{3}}+12x\right)+\frac{1}{54}\cdot 18^{\frac{2}{3}}\left(-i\sqrt{3}-3\right)^{\frac{1}{3}}\log\left(18^{\frac{1}{3}}\left(-i\sqrt{3}+1\right)\left(-i\sqrt{3}-3\right)^{\frac{2}{3}}+12x\right)$$

input
```
integrate(x^4/(x^6-x^3+1),x, algorithm="fricas")
```

output
```
-1/108*18^(2/3)*(I*sqrt(3) - 3)^(1/3)*(sqrt(-3) + 1)*log(18^(1/3)*(sqrt(3)
*(I*sqrt(-3) - I) + sqrt(-3) - 1)*(I*sqrt(3) - 3)^(2/3) + 24*x) + 1/108*18
^(2/3)*(I*sqrt(3) - 3)^(1/3)*(sqrt(-3) - 1)*log(18^(1/3)*(sqrt(3)*(-I*sqrt
(-3) - I) - sqrt(-3) - 1)*(I*sqrt(3) - 3)^(2/3) + 24*x) + 1/108*18^(2/3)*(
-I*sqrt(3) - 3)^(1/3)*(sqrt(-3) - 1)*log(18^(1/3)*(sqrt(3)*(I*sqrt(-3) + I
) - sqrt(-3) - 1)*(-I*sqrt(3) - 3)^(2/3) + 24*x) - 1/108*18^(2/3)*(-I*sqrt
(3) - 3)^(1/3)*(sqrt(-3) + 1)*log(18^(1/3)*(sqrt(3)*(-I*sqrt(-3) + I) + sq
rt(-3) - 1)*(-I*sqrt(3) - 3)^(2/3) + 24*x) + 1/54*18^(2/3)*(I*sqrt(3) - 3)
^(1/3)*log(18^(1/3)*(I*sqrt(3) + 1)*(I*sqrt(3) - 3)^(2/3) + 12*x) + 1/54*1
8^(2/3)*(-I*sqrt(3) - 3)^(1/3)*log(18^(1/3)*(-I*sqrt(3) + 1)*(-I*sqrt(3) -
 3)^(2/3) + 12*x)
```

3.172.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.06

$$\int \frac{x^4}{1-x^3+x^6}\,dx = \text{RootSum}\left(19683t^6+243t^3+1,\left(t\mapsto t\log\left(6561t^5+54t^2+x\right)\right)\right)$$

input
```
integrate(x**4/(x**6-x**3+1),x)
```

output
```
RootSum(19683*_t**6 + 243*_t**3 + 1, Lambda(_t, _t*log(6561*_t**5 + 54*_t*
*2 + x)))
```

3.172.7 Maxima [F]

$$\int \frac{x^4}{1-x^3+x^6}\,dx = \int \frac{x^4}{x^6-x^3+1}\,dx$$

input
```
integrate(x^4/(x^6-x^3+1),x, algorithm="maxima")
```

output
```
integrate(x^4/(x^6 - x^3 + 1), x)
```

3.172.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 827 vs. 2(267) = 534.

Time = 0.32 (sec) , antiderivative size = 827, normalized size of antiderivative = 2.01

$$\int \frac{x^4}{1-x^3+x^6}\,dx = \text{Too large to display}$$

input
```
integrate(x^4/(x^6-x^3+1),x, algorithm="giac")
```

output
```
-1/9*(2*sqrt(3)*cos(4/9*pi)^5 - 20*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi)^2 + 1
0*sqrt(3)*cos(4/9*pi)*sin(4/9*pi)^4 - 10*cos(4/9*pi)^4*sin(4/9*pi) + 20*co
s(4/9*pi)^2*sin(4/9*pi)^3 - 2*sin(4/9*pi)^5 + sqrt(3)*cos(4/9*pi)^2 - sqrt
(3)*sin(4/9*pi)^2 - 2*cos(4/9*pi)*sin(4/9*pi))*arctan(1/2*((-I*sqrt(3) - 1
)*cos(4/9*pi) + 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(4/9*pi))) - 1/9*(2*sqrt(3)
*cos(2/9*pi)^5 - 20*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi)^2 + 10*sqrt(3)*cos(2
/9*pi)*sin(2/9*pi)^4 - 10*cos(2/9*pi)^4*sin(2/9*pi) + 20*cos(2/9*pi)^2*sin
(2/9*pi)^3 - 2*sin(2/9*pi)^5 + sqrt(3)*cos(2/9*pi)^2 - sqrt(3)*sin(2/9*pi)
^2 - 2*cos(2/9*pi)*sin(2/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(2/9*pi) +
 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(2/9*pi))) + 1/9*(2*sqrt(3)*cos(1/9*pi)^5
- 20*sqrt(3)*cos(1/9*pi)^3*sin(1/9*pi)^2 + 10*sqrt(3)*cos(1/9*pi)*sin(1/9*
pi)^4 + 10*cos(1/9*pi)^4*sin(1/9*pi) - 20*cos(1/9*pi)^2*sin(1/9*pi)^3 + 2*
sin(1/9*pi)^5 - sqrt(3)*cos(1/9*pi)^2 + sqrt(3)*sin(1/9*pi)^2 - 2*cos(1/9*
pi)*sin(1/9*pi))*arctan(-1/2*((-I*sqrt(3) - 1)*cos(1/9*pi) - 2*x)/((1/2*I*
sqrt(3) + 1/2)*sin(1/9*pi))) - 1/18*(10*sqrt(3)*cos(4/9*pi)^4*sin(4/9*pi)
- 20*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^3 + 2*sqrt(3)*sin(4/9*pi)^5 + 2*cos
(4/9*pi)^5 - 20*cos(4/9*pi)^3*sin(4/9*pi)^2 + 10*cos(4/9*pi)*sin(4/9*pi)^4
 + 2*sqrt(3)*cos(4/9*pi)*sin(4/9*pi) + cos(4/9*pi)^2 - sin(4/9*pi)^2)*log(
(-I*sqrt(3)*cos(4/9*pi) - cos(4/9*pi))*x + x^2 + 1) - 1/18*(10*sqrt(3)*cos
(2/9*pi)^4*sin(2/9*pi) - 20*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^3 + 2*sqr...
```

3.172. $\int \frac{x^4}{1-x^3+x^6} \, dx$

3.172.9 Mupad [B] (verification not implemented)

Time = 8.61 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.74

$$\int \frac{x^4}{1-x^3+x^6}\,dx = \frac{\ln\left(x+\left(162\,x+\frac{27\left(-36+\sqrt{3}\,12\mathrm{i}\right)^{2/3}}{4}\right)\left(-\frac{1}{162}+\frac{\sqrt{3}\,1\mathrm{i}}{486}\right)\right)\left(-36+\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18} + \frac{\ln\left(x-\left(162\,x+\frac{27\left(-36-\sqrt{3}\,12\mathrm{i}\right)^{2/3}}{4}\right)\left(\frac{1}{162}+\frac{\sqrt{3}\,1\mathrm{i}}{486}\right)\right)\left(-36-\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18} - \frac{2^{2/3}\ln\left(x+\frac{2^{1/3}\,3^{2/3}\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{2/3}}{12}+\frac{2^{1/3}\,3^{1/6}\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{2/3}1\mathrm{i}}{4}\right)\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36} - \frac{2^{2/3}\ln\left(x+\frac{2^{1/3}\,3^{2/3}\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{2/3}}{12}-\frac{2^{1/3}\,3^{1/6}\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{2/3}1\mathrm{i}}{4}\right)\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36} - \frac{2^{2/3}\ln\left(x-\frac{2^{1/3}\,3^{2/3}\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{2/3}}{6}\right)\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36} - \frac{2^{2/3}\ln\left(x-\frac{2^{1/3}\,3^{2/3}\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{2/3}}{6}\right)\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}$$

input
```
int(x^4/(x^6 - x^3 + 1),x)
```

output
```
(log(x + (162*x + (27*(3^(1/2)*12i - 36)^(2/3))/4)*((3^(1/2)*1i)/486 - 1/1
62))*(3^(1/2)*12i - 36)^(1/3))/18 + (log(x - (162*x + (27*(- 3^(1/2)*12i -
 36)^(2/3))/4)*((3^(1/2)*1i)/486 + 1/162))*(- 3^(1/2)*12i - 36)^(1/3))/18
- (2^(2/3)*log(x + (2^(1/3)*3^(2/3)*(- 3^(1/2)*1i - 3)^(2/3))/12 + (2^(1/3
)*3^(1/6)*(- 3^(1/2)*1i - 3)^(2/3)*1i)/4)*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1/3
) + 3^(5/6)*1i))/36 - (2^(2/3)*log(x + (2^(1/3)*3^(2/3)*(3^(1/2)*1i - 3)^(
2/3))/12 - (2^(1/3)*3^(1/6)*(3^(1/2)*1i - 3)^(2/3)*1i)/4)*(3^(1/2)*1i - 3)
^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(1/3)*3^(2/3)*(- 3
^(1/2)*1i - 3)^(2/3))/6)*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/
36 - (2^(2/3)*log(x - (2^(1/3)*3^(2/3)*(3^(1/2)*1i - 3)^(2/3))/6)*(3^(1/2)
*1i - 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36
```

3.173 $\int \frac{x^3}{1-x^3+x^6} \, dx$

3.173.1 Optimal result

Integrand size = 16, antiderivative size = 411

$$\int \frac{x^3}{1-x^3+x^6}\,dx = -\frac{\left(i+\sqrt{3}\right)\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} + \frac{\left(i-\sqrt{3}\right)\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} + \frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} + \frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} - \frac{\left(3+i\sqrt{3}\right)\log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} - \frac{\left(3-i\sqrt{3}\right)\log\left(\left(1+i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

output
```
1/6*arctan(1/3*(1+2*2^(1/3)*x/(1+I*3^(1/2))^(1/3))*3^(1/2))*(I-3^(1/2))*2^
(2/3)/(1+I*3^(1/2))^(2/3)+1/18*ln(-2^(1/3)*x+(1+I*3^(1/2))^(1/3))*(3-I*3^(
1/2))*2^(2/3)/(1+I*3^(1/2))^(2/3)-1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1+I*3^(1/
2))^(1/3)+(1+I*3^(1/2))^(2/3))*(3-I*3^(1/2))*2^(2/3)/(1+I*3^(1/2))^(2/3)+1
/18*ln(-2^(1/3)*x+(1-I*3^(1/2))^(1/3))*(3+I*3^(1/2))*2^(2/3)/(1-I*3^(1/2))
^(2/3)-1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1-I*3^(1/2))^(1/3)+(1-I*3^(1/2))^(2/
3))*(3+I*3^(1/2))*2^(2/3)/(1-I*3^(1/2))^(2/3)-1/6*arctan(1/3*(1+2*2^(1/3)*
x/(1-I*3^(1/2))^(1/3))*3^(1/2))*(3^(1/2)+I)*2^(2/3)/(1-I*3^(1/2))^(2/3)
```

3.173.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.09

$$\int \frac{x^3}{1-x^3+x^6}\,dx = \frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&,\frac{\log(x-\#1)\#1}{-1+2\#1^3}\&\right]$$

input
```
Integrate[x^3/(1 - x^3 + x^6),x]
```

output
```
RootSum[1 - #1^3 + #1^6 & , (Log[x - #1]*#1)/(-1 + 2*#1^3) & ]/3
```

3.173.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 349, normalized size of antiderivative = 0.85, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1710, 750, 16, 25, 1142, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{x^6-x^3+1}dx$$

$$\Big\downarrow \; 1710$$

$$\frac{1}{6}\left(3-i\sqrt{3}\right)\int \frac{1}{x^3+\frac{1}{2}\left(-1-i\sqrt{3}\right)}dx+\frac{1}{6}\left(3+i\sqrt{3}\right)\int \frac{1}{x^3+\frac{1}{2}\left(-1+i\sqrt{3}\right)}dx$$

$$\Big\downarrow \; 750$$

$$\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\int -\frac{x+2^{2/3}\sqrt[3]{1-i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}+\frac{\int \frac{1}{x-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right)+$$
$$\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\int -\frac{x+2^{2/3}\sqrt[3]{1+i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}+\frac{\int \frac{1}{x-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right)$$

$$\downarrow 16$$

$$\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\int -\frac{x+2^{2/3}\sqrt[3]{1-i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}+\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right)+$$
$$\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\int -\frac{x+2^{2/3}\sqrt[3]{1+i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}+\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right)$$

$$\downarrow 25$$

3.173. $\int \frac{x^3}{1-x^3+x^6}\,dx$

$$\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\int\frac{x+2^{2/3}\sqrt[3]{1-i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right)+$$

$$\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\int\frac{x+2^{2/3}\sqrt[3]{1+i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right)$$

$\downarrow$ 1142

$$\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{3}{2}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}\int\frac{1}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx+\frac{1}{2}\int\frac{\ }{x^2+\sqrt[3]{\ }}}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right.$$

$$\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{3}{2}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}\int\frac{1}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx+\frac{1}{2}\int\frac{\ }{x^2+\sqrt[3]{\ }}}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right.$$

$\downarrow$ 1082

3.173. $\int\frac{x^3}{1-x^3+x^6}\,dx$

$$\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{1}{2}\int\frac{2x+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx-3\int\frac{1}{-\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}+\right.}}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right.$$

$$\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{1}{2}\int\frac{2x+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx-3\int\frac{1}{-\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}+\right.}}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right.$$

$\downarrow$ 217

3.173. $\int \frac{x^3}{1-x^3+x^6}\,dx$

$$\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{1}{2}\int\frac{2x+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx+\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-\cdots\right.}}}{\sqrt{3}}\right.\cdots}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right.\cdots$$

$$\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{1}{2}\int\frac{2x+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx+\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+\cdots\right.}}}{\sqrt{3}}\right.\cdots}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right.\cdots$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)+\frac{1}{2}\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x\cdots\right.}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right.\cdots$$

$$\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)+\frac{1}{2}\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x\cdots\right.}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right.\cdots$$

input
```
Int[x^3/(1 - x^3 + x^6),x]
```

3.173. $\int \frac{x^3}{1-x^3+x^6}\,dx$

output
```
((3 + I*Sqrt[3])*(Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x]/(3*((1 - I*Sqrt[3
])/2)^(2/3)) - (Sqrt[3]*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[
3]] + Log[(1 - I*Sqrt[3])^(2/3) + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^
2]/2)/(3*((1 - I*Sqrt[3])/2)^(2/3))))/6 + ((3 - I*Sqrt[3])*(Log[(1 + I*Sqr
t[3])^(1/3) - 2^(1/3)*x]/(3*((1 + I*Sqrt[3])/2)^(2/3)) - (Sqrt[3]*ArcTan[(
1 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]] + Log[(1 + I*Sqrt[3])^(2/3)
+ (2*(1 + I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2]/2)/(3*((1 + I*Sqrt[3])/2)^(2/
3))))/6
```

3.173.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

3.173. $\int \frac{x^3}{1-x^3+x^6}\,dx$

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1710
```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^n/2)*(b/q + 1)   Int[(d*x)^(m
- n)/(b/2 + q/2 + c*x^n), x], x] - Simp[(d^n/2)*(b/q - 1)   Int[(d*x)^(m -
n)/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

3.173.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.10

method	result	size
default	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)}\frac{\_R^{3}\ln\left(x-\_R\right)}{2\_R^{5}-\_R^{2}}\right)}{3}$	40
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)}\frac{\_R^{3}\ln\left(x-\_R\right)}{2\_R^{5}-\_R^{2}}\right)}{3}$	40

input
```
int(x^3/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/3*sum(_R^3/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1))
```

3.173.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.64

$$\int \frac{x^3}{1-x^3+x^6}\,dx = -\frac{1}{108}\cdot 18^{\frac{2}{3}}\left(i\sqrt{3}-3\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\log\left(18^{\frac{2}{3}}\sqrt{3}\left(i\sqrt{3}-3\right)^{\frac{1}{3}}\left(i\sqrt{-3}+i\right)+36\,x\right)+\frac{1}{108}\cdot 18^{\frac{2}{3}}\left(-i\sqrt{3}-3\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\log\left(18^{\frac{2}{3}}\sqrt{3}\left(-i\sqrt{3}-3\right)^{\frac{1}{3}}\left(i\sqrt{-3}-i\right)+36\,x\right)+\frac{1}{108}\cdot 18^{\frac{2}{3}}\left(i\sqrt{3}-3\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\log\left(18^{\frac{2}{3}}\sqrt{3}\left(i\sqrt{3}-3\right)^{\frac{1}{3}}\left(-i\sqrt{-3}+i\right)+36\,x\right)-\frac{1}{108}\cdot 18^{\frac{2}{3}}\left(-i\sqrt{3}-3\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\log\left(18^{\frac{2}{3}}\sqrt{3}\left(-i\sqrt{3}-3\right)^{\frac{1}{3}}\left(-i\sqrt{-3}-i\right)+36\,x\right)+\frac{1}{54}\cdot 18^{\frac{2}{3}}\left(i\sqrt{3}-3\right)^{\frac{1}{3}}\log\left(-i\cdot 18^{\frac{2}{3}}\sqrt{3}\left(i\sqrt{3}-3\right)^{\frac{1}{3}}+18\,x\right)+\frac{1}{54}\cdot 18^{\frac{2}{3}}\left(-i\sqrt{3}-3\right)^{\frac{1}{3}}\log\left(i\cdot 18^{\frac{2}{3}}\sqrt{3}\left(-i\sqrt{3}-3\right)^{\frac{1}{3}}+18\,x\right)$$

input
```
integrate(x^3/(x^6-x^3+1),x, algorithm="fricas")
```

output
```
-1/108*18^(2/3)*(I*sqrt(3) - 3)^(1/3)*(sqrt(-3) + 1)*log(18^(2/3)*sqrt(3)*
(I*sqrt(3) - 3)^(1/3)*(I*sqrt(-3) + I) + 36*x) + 1/108*18^(2/3)*(-I*sqrt(3
) - 3)^(1/3)*(sqrt(-3) - 1)*log(18^(2/3)*sqrt(3)*(-I*sqrt(3) - 3)^(1/3)*(I
*sqrt(-3) - I) + 36*x) + 1/108*18^(2/3)*(I*sqrt(3) - 3)^(1/3)*(sqrt(-3) -
1)*log(18^(2/3)*sqrt(3)*(I*sqrt(3) - 3)^(1/3)*(-I*sqrt(-3) + I) + 36*x) -
1/108*18^(2/3)*(-I*sqrt(3) - 3)^(1/3)*(sqrt(-3) + 1)*log(18^(2/3)*sqrt(3)*
(-I*sqrt(3) - 3)^(1/3)*(-I*sqrt(-3) - I) + 36*x) + 1/54*18^(2/3)*(I*sqrt(3
) - 3)^(1/3)*log(-I*18^(2/3)*sqrt(3)*(I*sqrt(3) - 3)^(1/3) + 18*x) + 1/54*
18^(2/3)*(-I*sqrt(3) - 3)^(1/3)*log(I*18^(2/3)*sqrt(3)*(-I*sqrt(3) - 3)^(1
/3) + 18*x)
```

3.173.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.06

$$\int \frac{x^3}{1-x^3+x^6} \, dx = \text{RootSum}\left(19683t^6+243t^3+1,\left(t\mapsto t\log\left(-1458t^4-9t+x\right)\right)\right)$$

input
```
integrate(x**3/(x**6-x**3+1),x)
```

output
```
RootSum(19683*_t**6 + 243*_t**3 + 1, Lambda(_t, _t*log(-1458*_t**4 - 9*_t
+ x)))
```

3.173.7 Maxima [F]

$$\int \frac{x^3}{1-x^3+x^6} \, dx = \int \frac{x^3}{x^6-x^3+1} \, dx$$

input
```
integrate(x^3/(x^6-x^3+1),x, algorithm="maxima")
```

output
```
integrate(x^3/(x^6 - x^3 + 1), x)
```

3.173.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 640 vs. 2(267) = 534.

Time = 0.33 (sec) , antiderivative size = 640, normalized size of antiderivative = 1.56

$$\int \frac{x^3}{1-x^3+x^6} \, dx = \text{Too large to display}$$

input
```
integrate(x^3/(x^6-x^3+1),x, algorithm="giac")
```

output
```
-1/9*(2*sqrt(3)*cos(4/9*pi)^4 - 12*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^2 + 2
*sqrt(3)*sin(4/9*pi)^4 + 8*cos(4/9*pi)^3*sin(4/9*pi) - 8*cos(4/9*pi)*sin(4
/9*pi)^3 + sqrt(3)*cos(4/9*pi) + sin(4/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)
*cos(4/9*pi) + 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(4/9*pi))) - 1/9*(2*sqrt(3)*
cos(2/9*pi)^4 - 12*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^2 + 2*sqrt(3)*sin(2/9
*pi)^4 + 8*cos(2/9*pi)^3*sin(2/9*pi) - 8*cos(2/9*pi)*sin(2/9*pi)^3 + sqrt(
3)*cos(2/9*pi) + sin(2/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(2/9*pi) + 2
*x)/((1/2*I*sqrt(3) + 1/2)*sin(2/9*pi))) - 1/9*(2*sqrt(3)*cos(1/9*pi)^4 -
12*sqrt(3)*cos(1/9*pi)^2*sin(1/9*pi)^2 + 2*sqrt(3)*sin(1/9*pi)^4 - 8*cos(1
/9*pi)^3*sin(1/9*pi) + 8*cos(1/9*pi)*sin(1/9*pi)^3 - sqrt(3)*cos(1/9*pi) +
 sin(1/9*pi))*arctan(-1/2*((-I*sqrt(3) - 1)*cos(1/9*pi) - 2*x)/((1/2*I*sqr
t(3) + 1/2)*sin(1/9*pi))) - 1/18*(8*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi) - 8*
sqrt(3)*cos(4/9*pi)*sin(4/9*pi)^3 - 2*cos(4/9*pi)^4 + 12*cos(4/9*pi)^2*sin
(4/9*pi)^2 - 2*sin(4/9*pi)^4 + sqrt(3)*sin(4/9*pi) - cos(4/9*pi))*log((-I*
sqrt(3)*cos(4/9*pi) - cos(4/9*pi))*x + x^2 + 1) - 1/18*(8*sqrt(3)*cos(2/9*
pi)^3*sin(2/9*pi) - 8*sqrt(3)*cos(2/9*pi)*sin(2/9*pi)^3 - 2*cos(2/9*pi)^4
+ 12*cos(2/9*pi)^2*sin(2/9*pi)^2 - 2*sin(2/9*pi)^4 + sqrt(3)*sin(2/9*pi) -
 cos(2/9*pi))*log((-I*sqrt(3)*cos(2/9*pi) - cos(2/9*pi))*x + x^2 + 1) + 1/
18*(8*sqrt(3)*cos(1/9*pi)^3*sin(1/9*pi) - 8*sqrt(3)*cos(1/9*pi)*sin(1/9*pi
)^3 + 2*cos(1/9*pi)^4 - 12*cos(1/9*pi)^2*sin(1/9*pi)^2 + 2*sin(1/9*pi)^...
```

3.173. $\int \frac{x^3}{1-x^3+x^6}\,dx$

3.173.9 Mupad [B] (verification not implemented)

Time = 8.68 (sec) , antiderivative size = 327, normalized size of antiderivative = 0.80

$$\int \frac{x^3}{1-x^3+x^6}\,dx = \frac{\ln\left(x+\frac{2^{2/3}\,3^{5/6}\,\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{6}\right)\,\left(-36-\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18}$$
$$+\frac{\ln\left(x-\frac{2^{2/3}\,3^{5/6}\,\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{6}\right)\,\left(-36+\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18}$$
$$-\frac{2^{2/3}\,\ln\left(x+\frac{2^{2/3}\,3^{1/3}\,\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}}{2}+\frac{2^{2/3}\,3^{1/3}\,\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{4/3}}{12}\right)\,\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}$$
$$-\frac{2^{2/3}\,\ln\left(x+\frac{2^{2/3}\,3^{1/3}\,\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}}{2}+\frac{2^{2/3}\,3^{1/3}\,\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{4/3}}{12}\right)\,\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}$$
$$-\frac{2^{2/3}\,\ln\left(x-\frac{2^{2/3}\,3^{1/3}\,\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}}{4}-\frac{2^{2/3}\,3^{5/6}\,\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{12}\right)\,\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}$$
$$-\frac{2^{2/3}\,\ln\left(x-\frac{2^{2/3}\,3^{1/3}\,\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}}{4}+\frac{2^{2/3}\,3^{5/6}\,\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{12}\right)\,\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}$$

input
```
int(x^3/(x^6 - x^3 + 1),x)
```

output
```
(log(x + (2^(2/3)*3^(5/6)*(- 3^(1/2)*1i - 3)^(1/3)*1i)/6)*(- 3^(1/2)*12i -
 36)^(1/3))/18 + (log(x - (2^(2/3)*3^(5/6)*(3^(1/2)*1i - 3)^(1/3)*1i)/6)*(
3^(1/2)*12i - 36)^(1/3))/18 - (2^(2/3)*log(x + (2^(2/3)*3^(1/3)*(- 3^(1/2)
*1i - 3)^(1/3))/2 + (2^(2/3)*3^(1/3)*(- 3^(1/2)*1i - 3)^(4/3))/12)*(- 3^(1
/2)*1i - 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*log(x + (2^(2/3)*3
^(1/3)*(3^(1/2)*1i - 3)^(1/3))/2 + (2^(2/3)*3^(1/3)*(3^(1/2)*1i - 3)^(4/3)
)/12)*(3^(1/2)*1i - 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x -
 (2^(2/3)*3^(1/3)*(- 3^(1/2)*1i - 3)^(1/3))/4 - (2^(2/3)*3^(5/6)*(- 3^(1/2
)*1i - 3)^(1/3)*1i)/12)*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/3
6 - (2^(2/3)*log(x - (2^(2/3)*3^(1/3)*(3^(1/2)*1i - 3)^(1/3))/4 + (2^(2/3)
*3^(5/6)*(3^(1/2)*1i - 3)^(1/3)*1i)/12)*(3^(1/2)*1i - 3)^(1/3)*(3^(1/3) +
3^(5/6)*1i))/36
```

3.174 $\int \frac{x^2}{1-x^3+x^6} dx$

3.174.1 Optimal result

Integrand size = 16, antiderivative size = 23

$$\int \frac{x^2}{1-x^3+x^6} dx = -\frac{2\arctan\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}$$

output
```
-2/9*arctan(1/3*(-2*x^3+1)*3^(1/2))*3^(1/2)
```

3.174.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{1-x^3+x^6} dx = \frac{2\arctan\left(\frac{-1+2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}$$

input
```
Integrate[x^2/(1 - x^3 + x^6),x]
```

output
```
(2*ArcTan[(-1 + 2*x^3)/Sqrt[3]])/(3*Sqrt[3])
```

3.174.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1690, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{x^6-x^3+1}dx$$
$$\Big\downarrow \text{1690}$$
$$\frac{1}{3}\int \frac{1}{x^6-x^3+1}dx^3$$
$$\Big\downarrow \text{1083}$$
$$-\frac{2}{3}\int \frac{1}{-x^6-3}d(2x^3-1)$$
$$\Big\downarrow \text{217}$$
$$\frac{2\arctan\left(\frac{2x^3-1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

input
```
Int[x^2/(1 - x^3 + x^6),x]
```

output
```
(2*ArcTan[(-1 + 2*x^3)/Sqrt[3]])/(3*Sqrt[3])
```

3.174.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

3.174.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{2\sqrt{3}\,\arctan\left(\frac{\left(2x^3-1\right)\sqrt{3}}{3}\right)}{9}$	19
risch	$\frac{2\sqrt{3}\,\arctan\left(\frac{\left(2x^3-1\right)\sqrt{3}}{3}\right)}{9}$	19

input
```
int(x^2/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
2/9*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))
```

3.174.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{x^2}{1-x^3+x^6}\,dx = \frac{2}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^3-1)\right)$$

input
```
integrate(x^2/(x^6-x^3+1),x, algorithm="fricas")
```

output
```
2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1))
```

3.174.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x^2}{1-x^3+x^6}\,dx = \frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^3}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

input
```
integrate(x**2/(x**6-x**3+1),x)
```

output
```
2*sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3)/9
```

3.174.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{x^2}{1-x^3+x^6}\,dx = \frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^3-1)\right)$$

input
```
integrate(x^2/(x^6-x^3+1),x, algorithm="maxima")
```

output
```
2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1))
```

3.174.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{x^2}{1-x^3+x^6}\,dx = \frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^3-1)\right)$$

input
```
integrate(x^2/(x^6-x^3+1),x, algorithm="giac")
```

output
```
2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1))
```

3.174.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{x^2}{1-x^3+x^6}\,dx = -\frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}\,x^3}{3}\right)}{9}$$

input
```
int(x^2/(x^6 - x^3 + 1),x)
```

output
```
-(2*3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/3))/9
```

3.175 $\int \frac{x}{1-x^3+x^6}\,dx$

3.175.1 Optimal result

Integrand size = 14, antiderivative size = 375

$$\int \frac{x}{1-x^3+x^6}\,dx = \frac{i\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}} - \frac{i\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}} + \frac{i\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}} - \frac{i\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}} - \frac{i\log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{3\,2^{2/3}\sqrt{3}\sqrt[3]{1-i\sqrt{3}}} + \frac{i\log\left(\left(1+i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{3\,2^{2/3}\sqrt{3}\sqrt[3]{1+i\sqrt{3}}}$$

output
```
1/3*I*2^(1/3)*arctan(1/3*(1+2*2^(1/3)*x/(1-I*3^(1/2))^(1/3))*3^(1/2))/(1-I
*3^(1/2))^(1/3)-1/3*I*2^(1/3)*arctan(1/3*(1+2*2^(1/3)*x/(1+I*3^(1/2))^(1/3
))*3^(1/2))/(1+I*3^(1/2))^(1/3)+1/9*I*2^(1/3)*ln(-2^(1/3)*x+(1-I*3^(1/2))^
(1/3))/(1-I*3^(1/2))^(1/3)*3^(1/2)-1/18*I*ln(2^(2/3)*x^2+2^(1/3)*x*(1-I*3^
(1/2))^(1/3)+(1-I*3^(1/2))^(2/3))*2^(1/3)/(1-I*3^(1/2))^(1/3)*3^(1/2)-1/9*
I*2^(1/3)*ln(-2^(1/3)*x+(1+I*3^(1/2))^(1/3))/(1+I*3^(1/2))^(1/3)*3^(1/2)+1
/18*I*ln(2^(2/3)*x^2+2^(1/3)*x*(1+I*3^(1/2))^(1/3)+(1+I*3^(1/2))^(2/3))*2^
(1/3)/(1+I*3^(1/2))^(1/3)*3^(1/2)
```

3.175.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.11

$$\int \frac{x}{1-x^3+x^6} \, dx = \frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&,\frac{\log(x-\#1)}{-\#1+2\#1^4}\&\right]$$

input
```
Integrate[x/(1 - x^3 + x^6),x]
```

output
```
RootSum[1 - #1^3 + #1^6 & , Log[x - #1]/(-#1 + 2*#1^4) & ]/3
```

3.175.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 355, normalized size of antiderivative = 0.95, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {1711, 27, 821, 16, 1142, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{x^6-x^3+1} \, dx$$

$$\Big\downarrow \text{1711}$$

$$\frac{i\int -\frac{2x}{-2x^3-i\sqrt{3}+1}dx}{\sqrt{3}}-\frac{i\int -\frac{2x}{-2x^3+i\sqrt{3}+1}dx}{\sqrt{3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2i\int \frac{x}{-2x^3+i\sqrt{3}+1}dx}{\sqrt{3}}-\frac{2i\int \frac{x}{-2x^3-i\sqrt{3}+1}dx}{\sqrt{3}}$$

$$\Big\downarrow \; 821$$

$$\frac{2i\left(\frac{\int \frac{1}{\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x}dx}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\frac{\int \frac{\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}\right)}{\sqrt{3}}-\frac{2i\left(\frac{\int \frac{1}{\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x}dx}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\frac{\int \frac{\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}\right)}{\sqrt{3}}$$

$$\Big\downarrow \; 16$$

$$\frac{2i\left(-\frac{\int \frac{\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}\right)}{\sqrt{3}}-\frac{2i\left(-\frac{\int \frac{\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}\right)}{\sqrt{3}}$$

$$\Big\downarrow \; 1142$$

3.175. $\int \frac{x}{1-x^3+x^6}\,dx$

$$\frac{2i\left(-\frac{\frac{3}{2}\sqrt[3]{1+i\sqrt{3}}\int\frac{1}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx-\frac{\int\frac{2\ 2^{2/3}x+\sqrt[3]{2\left(1+i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+\ldots}\right.}{3\ 2^{2/3}\sqrt[3]{1+i\ldots}}\right.}{\sqrt{3}}$$

$$\frac{2i\left(-\frac{\frac{3}{2}\sqrt[3]{1-i\sqrt{3}}\int\frac{1}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx-\frac{\int\frac{2\ 2^{2/3}x+\sqrt[3]{2\left(1-i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-\ldots}\right.}{3\ 2^{2/3}\sqrt[3]{1-i\ldots}}\right.}{\sqrt{3}}$$

$\downarrow$ 1082

3.175. $\int \frac{x}{1-x^3+x^6}\,dx$

$$\frac{2i\left(-\frac{-\frac{\int \frac{2\,2^{2/3}x+\sqrt[3]{2\left(1+i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}-\frac{3\int \frac{1}{-\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}+1\right)^2-3}d\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}+1\right)}{\sqrt[3]{2}}}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1\ldots}\right.}{3\,2^{2/3}\sqrt[3]{1+\ldots}}\right.}{\sqrt{3}}$$

$$\frac{2i\left(-\frac{-\frac{\int \frac{2\,2^{2/3}x+\sqrt[3]{2\left(1-i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}-\frac{3\int \frac{1}{-\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}+1\right)^2-3}d\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}+1\right)}{\sqrt[3]{2}}}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1\ldots}\right.}{3\,2^{2/3}\sqrt[3]{1-\ldots}}\right.}{\sqrt{3}}$$

$$\Big\downarrow \; 217$$

3.175. $\int \frac{x}{1-x^3+x^6}\,dx$

$$\frac{2i\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\int\frac{2\ 2^{2/3}x+\sqrt[3]{2\left(1+i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}\right)}{\sqrt{3}}-\frac{2i\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\int\frac{2\ 2^{2/3}x+\sqrt[3]{2\left(1-i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}\right)}{\sqrt{3}}$$

$\downarrow$ 1103

3.175. $\int \frac{x}{1-x^3+x^6}\,dx$

$$\frac{2i\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}\right)}{\sqrt{3}}-\frac{2i\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}\right)}{\sqrt{3}}$$

input
```
Int[x/(1 - x^3 + x^6),x]
```

output
```
((-2*I)*(-1/3*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x]/(2^(2/3)*(1 - I*Sqrt[
3])^(1/3)) - ((Sqrt[3]*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3
]])/2^(1/3) - Log[(1 - I*Sqrt[3])^(2/3) + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^
(2/3)*x^2]/(2*2^(1/3)))/(3*(2*(1 - I*Sqrt[3]))^(1/3))))/Sqrt[3] + ((2*I)*(
-1/3*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x]/(2^(2/3)*(1 + I*Sqrt[3])^(1/3)
) - ((Sqrt[3]*ArcTan[(1 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/2^(1/
3) - Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2
]/(2*2^(1/3)))/(3*(2*(1 + I*Sqrt[3]))^(1/3))))/Sqrt[3]
```

3.175. $\int \frac{x}{1-x^3+x^6}\,dx$

3.175.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1711
```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[c/q   Int[(d*x)^m/(b/2 - q/2 + c
*x^n), x], x] - Simp[c/q   Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; Free
Q[{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

3.175. $\int \frac{x}{1-x^3+x^6} \, dx$

3.175.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.10

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)}\frac{\_R\ln\left(x-\_R\right)}{2\_R^{5}-\_R^{2}}\right)}{3}$	38
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)}\frac{\_R\ln\left(x-\_R\right)}{2\_R^{5}-\_R^{2}}\right)}{3}$	38

input
```
int(x/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/3*sum(_R/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1))
```

3.175.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 295, normalized size of antiderivative = 0.79

$$\int \frac{x}{1-x^3+x^6}\,dx = -\frac{1}{108}\cdot 18^{\frac{2}{3}}\left(i\sqrt{3}+3\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\log\left(18^{\frac{1}{3}}\left(\sqrt{3}\left(i\sqrt{-3}-i\right)+\sqrt{-3}-1\right)\left(i\sqrt{3}+3\right)^{\frac{2}{3}}+24\,x\right)+\frac{1}{108}\cdot 18^{\frac{2}{3}}\left(i\sqrt{3}+3\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\log\left(18^{\frac{1}{3}}\left(\sqrt{3}\left(-i\sqrt{-3}-i\right)-\sqrt{-3}-1\right)\left(i\sqrt{3}+3\right)^{\frac{2}{3}}+24\,x\right)+\frac{1}{108}\cdot 18^{\frac{2}{3}}\left(-i\sqrt{3}+3\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\log\left(18^{\frac{1}{3}}\left(\sqrt{3}\left(i\sqrt{-3}+i\right)-\sqrt{-3}-1\right)\left(-i\sqrt{3}+3\right)^{\frac{2}{3}}+24\,x\right)-\frac{1}{108}\cdot 18^{\frac{2}{3}}\left(-i\sqrt{3}+3\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\log\left(18^{\frac{1}{3}}\left(\sqrt{3}\left(-i\sqrt{-3}+i\right)+\sqrt{-3}-1\right)\left(-i\sqrt{3}+3\right)^{\frac{2}{3}}+24\,x\right)+\frac{1}{54}\cdot 18^{\frac{2}{3}}\left(i\sqrt{3}+3\right)^{\frac{1}{3}}\log\left(18^{\frac{1}{3}}\left(i\sqrt{3}+3\right)^{\frac{2}{3}}\left(i\sqrt{3}+1\right)+12\,x\right)+\frac{1}{54}\cdot 18^{\frac{2}{3}}\left(-i\sqrt{3}+3\right)^{\frac{1}{3}}\log\left(18^{\frac{1}{3}}\left(-i\sqrt{3}+3\right)^{\frac{2}{3}}\left(-i\sqrt{3}+1\right)+12\,x\right)$$

input

```
integrate(x/(x^6-x^3+1),x, algorithm="fricas")
```

output

```
-1/108*18^(2/3)*(I*sqrt(3) + 3)^(1/3)*(sqrt(-3) + 1)*log(18^(1/3)*(sqrt(3)
*(I*sqrt(-3) - I) + sqrt(-3) - 1)*(I*sqrt(3) + 3)^(2/3) + 24*x) + 1/108*18
^(2/3)*(I*sqrt(3) + 3)^(1/3)*(sqrt(-3) - 1)*log(18^(1/3)*(sqrt(3)*(-I*sqrt
(-3) - I) - sqrt(-3) - 1)*(I*sqrt(3) + 3)^(2/3) + 24*x) + 1/108*18^(2/3)*(
-I*sqrt(3) + 3)^(1/3)*(sqrt(-3) - 1)*log(18^(1/3)*(sqrt(3)*(I*sqrt(-3) + I
) - sqrt(-3) - 1)*(-I*sqrt(3) + 3)^(2/3) + 24*x) - 1/108*18^(2/3)*(-I*sqrt
(3) + 3)^(1/3)*(sqrt(-3) + 1)*log(18^(1/3)*(sqrt(3)*(-I*sqrt(-3) + I) + sq
rt(-3) - 1)*(-I*sqrt(3) + 3)^(2/3) + 24*x) + 1/54*18^(2/3)*(I*sqrt(3) + 3)
^(1/3)*log(18^(1/3)*(I*sqrt(3) + 3)^(2/3)*(I*sqrt(3) + 1) + 12*x) + 1/54*1
8^(2/3)*(-I*sqrt(3) + 3)^(1/3)*log(18^(1/3)*(-I*sqrt(3) + 3)^(2/3)*(-I*sqr
t(3) + 1) + 12*x)
```

3.175.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.07

$$\int \frac{x}{1-x^3+x^6}\,dx = \text{RootSum}\left(19683t^6-243t^3+1,\left(t\mapsto t\log\left(6561t^5-27t^2+x\right)\right)\right)$$

input
```
integrate(x/(x**6-x**3+1),x)
```

output
```
RootSum(19683*_t**6 - 243*_t**3 + 1, Lambda(_t, _t*log(6561*_t**5 - 27*_t*
*2 + x)))
```

3.175.7 Maxima [F]

$$\int \frac{x}{1-x^3+x^6}\,dx = \int \frac{x}{x^6-x^3+1}\,dx$$

input
```
integrate(x/(x^6-x^3+1),x, algorithm="maxima")
```

output
```
integrate(x/(x^6 - x^3 + 1), x)
```

3.175.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 815 vs. 2(241) = 482.

Time = 0.31 (sec) , antiderivative size = 815, normalized size of antiderivative = 2.17

$$\int \frac{x}{1-x^3+x^6}\,dx = \text{Too large to display}$$

input
```
integrate(x/(x^6-x^3+1),x, algorithm="giac")
```

output
```
-1/9*(sqrt(3)*cos(4/9*pi)^5 - 10*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi)^2 + 5*s
qrt(3)*cos(4/9*pi)*sin(4/9*pi)^4 - 5*cos(4/9*pi)^4*sin(4/9*pi) + 10*cos(4/
9*pi)^2*sin(4/9*pi)^3 - sin(4/9*pi)^5 - sqrt(3)*cos(4/9*pi)^2 + sqrt(3)*si
n(4/9*pi)^2 + 2*cos(4/9*pi)*sin(4/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(
4/9*pi) + 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(4/9*pi))) - 1/9*(sqrt(3)*cos(2/9
*pi)^5 - 10*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi)^2 + 5*sqrt(3)*cos(2/9*pi)*si
n(2/9*pi)^4 - 5*cos(2/9*pi)^4*sin(2/9*pi) + 10*cos(2/9*pi)^2*sin(2/9*pi)^3
 - sin(2/9*pi)^5 - sqrt(3)*cos(2/9*pi)^2 + sqrt(3)*sin(2/9*pi)^2 + 2*cos(2
/9*pi)*sin(2/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(2/9*pi) + 2*x)/((1/2*
I*sqrt(3) + 1/2)*sin(2/9*pi))) + 1/9*(sqrt(3)*cos(1/9*pi)^5 - 10*sqrt(3)*c
os(1/9*pi)^3*sin(1/9*pi)^2 + 5*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^4 + 5*cos(1
/9*pi)^4*sin(1/9*pi) - 10*cos(1/9*pi)^2*sin(1/9*pi)^3 + sin(1/9*pi)^5 + sq
rt(3)*cos(1/9*pi)^2 - sqrt(3)*sin(1/9*pi)^2 + 2*cos(1/9*pi)*sin(1/9*pi))*a
rctan(-1/2*((-I*sqrt(3) - 1)*cos(1/9*pi) - 2*x)/((1/2*I*sqrt(3) + 1/2)*sin
(1/9*pi))) - 1/18*(5*sqrt(3)*cos(4/9*pi)^4*sin(4/9*pi) - 10*sqrt(3)*cos(4/
9*pi)^2*sin(4/9*pi)^3 + sqrt(3)*sin(4/9*pi)^5 + cos(4/9*pi)^5 - 10*cos(4/9
*pi)^3*sin(4/9*pi)^2 + 5*cos(4/9*pi)*sin(4/9*pi)^4 - 2*sqrt(3)*cos(4/9*pi)
*sin(4/9*pi) - cos(4/9*pi)^2 + sin(4/9*pi)^2)*log((-I*sqrt(3)*cos(4/9*pi)
- cos(4/9*pi))*x + x^2 + 1) - 1/18*(5*sqrt(3)*cos(2/9*pi)^4*sin(2/9*pi) -
10*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^3 + sqrt(3)*sin(2/9*pi)^5 + cos(2/...
```

3.175. $\int \frac{x}{1-x^3+x^6}\,dx$

3.175.9 Mupad [B] (verification not implemented)

Time = 8.52 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.81

$$\int \frac{x}{1-x^3+x^6}\,dx$$
$$= \frac{\ln\left(x+\left(81\,x-\frac{27\,{\left(36-\sqrt{3}\,12{}\mathrm{i}\right)}^{2/3}}{4}\right)\,\left(-\frac{1}{162}+\frac{\sqrt{3}\,1{}\mathrm{i}}{486}\right)\right)\,{\left(36-\sqrt{3}\,12{}\mathrm{i}\right)}^{1/3}}{18}$$
$$+\frac{\ln\left(x-\left(81\,x-\frac{27\,{\left(36+\sqrt{3}\,12{}\mathrm{i}\right)}^{2/3}}{4}\right)\,\left(\frac{1}{162}+\frac{\sqrt{3}\,1{}\mathrm{i}}{486}\right)\right)\,{\left(36+\sqrt{3}\,12{}\mathrm{i}\right)}^{1/3}}{18}$$
$$-\frac{2^{2/3}\,\ln\left(x+\frac{2^{1/3}\,3^{2/3}\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{2/3}}{12}+\frac{2^{1/3}\,3^{1/6}\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{2/3}\,1{}\mathrm{i}}{4}\right)\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}+3^{5/6}\,1{}\mathrm{i}\right)}{36}$$
$$-\frac{2^{2/3}\,\ln\left(x+\frac{2^{1/3}\,3^{2/3}\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{2/3}}{12}-\frac{2^{1/3}\,3^{1/6}\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{2/3}\,1{}\mathrm{i}}{4}\right)\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}-3^{5/6}\,1{}\mathrm{i}\right)}{36}$$
$$-\frac{2^{2/3}\,\ln\left(x-\frac{2^{1/3}\,3^{2/3}\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{2/3}}{6}\right)\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}-3^{5/6}\,1{}\mathrm{i}\right)}{36}$$
$$-\frac{2^{2/3}\,\ln\left(x-\frac{2^{1/3}\,3^{2/3}\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{2/3}}{6}\right)\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}+3^{5/6}\,1{}\mathrm{i}\right)}{36}$$

input
```
int(x/(x^6 - x^3 + 1),x)
```

output
```
(log(x + (81*x - (27*(36 - 3^(1/2)*12i)^(2/3))/4)*((3^(1/2)*1i)/486 - 1/16
2))*(36 - 3^(1/2)*12i)^(1/3))/18 + (log(x - (81*x - (27*(3^(1/2)*12i + 36)
^(2/3))/4)*((3^(1/2)*1i)/486 + 1/162))*(3^(1/2)*12i + 36)^(1/3))/18 - (2^(
2/3)*log(x + (2^(1/3)*3^(2/3)*(3 - 3^(1/2)*1i)^(2/3))/12 + (2^(1/3)*3^(1/6
)*(3 - 3^(1/2)*1i)^(2/3)*1i)/4)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) + 3^(5/6)*
1i))/36 - (2^(2/3)*log(x + (2^(1/3)*3^(2/3)*(3^(1/2)*1i + 3)^(2/3))/12 - (
2^(1/3)*3^(1/6)*(3^(1/2)*1i + 3)^(2/3)*1i)/4)*(3^(1/2)*1i + 3)^(1/3)*(3^(1
/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(1/3)*3^(2/3)*(3 - 3^(1/2)*1i)
^(2/3))/6)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*lo
g(x - (2^(1/3)*3^(2/3)*(3^(1/2)*1i + 3)^(2/3))/6)*(3^(1/2)*1i + 3)^(1/3)*(
3^(1/3) + 3^(5/6)*1i))/36
```

3.176 $\int \frac{1}{1-x^3+x^6}\,dx$

3.176.1 Optimal result

Integrand size = 12, antiderivative size = 186

$$\int \frac{1}{1-x^3+x^6}\,dx = -\frac{1}{3}(-1)^{13/18}\arctan\left(\frac{1+2\sqrt[9]{-1}x}{\sqrt{3}}\right) + \frac{1}{3}(-1)^{5/18}\arctan\left(\frac{1-2(-1)^{8/9}x}{\sqrt{3}}\right) - \frac{(-1)^{5/18}\left(\log(2)+3\log\left(\sqrt[9]{-1}-x\right)\right)}{9\sqrt{3}} + \frac{(-1)^{13/18}\log\left(-\sqrt[3]{2}((-1)\right.}{3\sqrt{3}}$$

output
```
-1/3*(-1)^(13/18)*arctan(1/3*(1+2*(-1)^(1/9)*x)*3^(1/2))+1/3*(-1)^(5/18)*a
rctan(1/3*(1-2*(-1)^(8/9)*x)*3^(1/2))-1/27*(-1)^(5/18)*(ln(2)+3*ln((-1)^(1
/9)-x))*3^(1/2)+1/9*(-1)^(13/18)*ln(-2^(1/3)*((-1)^(8/9)+x))*3^(1/2)-1/18*
(-1)^(13/18)*ln(-2^(2/3)*((-1)^(7/9)+((-1)^(8/9)-x)*x))*3^(1/2)+1/18*(-1)^
(5/18)*ln(2^(2/3)*((-1)^(2/9)+x*((-1)^(1/9)+x)))*3^(1/2)
```

3.176.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.23

$$\int \frac{1}{1-x^3+x^6}\,dx = \frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&,\frac{\log(x-\#1)}{-\#1^2+2\#1^5}\&\right]$$

input
```
Integrate[(1 - x^3 + x^6)^(-1),x]
```

output
```
RootSum[1 - #1^3 + #1^6 & , Log[x - #1]/(-#1^2 + 2*#1^5) & ]/3
```

3.176.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.81, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {1685, 750, 16, 25, 1142, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6-x^3+1} \, dx$$

$$\Big\downarrow \text{1685}$$

$$\frac{i\int \frac{1}{x^3+\frac{1}{2}\left(-1+i\sqrt{3}\right)}dx}{\sqrt{3}}-\frac{i\int \frac{1}{x^3+\frac{1}{2}\left(-1-i\sqrt{3}\right)}dx}{\sqrt{3}}$$

$$\Big\downarrow \text{750}$$

$$\frac{i\left(\frac{\int -\frac{x+2^{2/3}\sqrt[3]{1-i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}}\left(1-i\sqrt{3}\right)x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}+\frac{\int \frac{1}{x-\sqrt[3]{\frac{1}{2}}\left(1-i\sqrt{3}\right)}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right)}{\sqrt{3}}-\frac{i\left(\frac{\int -\frac{x+2^{2/3}\sqrt[3]{1+i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}}\left(1+i\sqrt{3}\right)x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}+\frac{\int \frac{1}{x-\sqrt[3]{\frac{1}{2}}\left(1+i\sqrt{3}\right)}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right)}{\sqrt{3}}$$

$$\Big\downarrow \text{16}$$

$$\frac{i\left(\frac{\int -\frac{x+2^{2/3}\sqrt[3]{1-i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}+\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right)}{\sqrt{3}}-\frac{i\left(\frac{\int -\frac{x+2^{2/3}\sqrt[3]{1+i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}+\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right)}{\sqrt{3}}$$

$$\Big\downarrow \ 25$$

$$\frac{i\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\int \frac{x+2^{2/3}\sqrt[3]{1-i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right)}{\sqrt{3}}-\frac{i\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\int \frac{x+2^{2/3}\sqrt[3]{1+i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right)}{\sqrt{3}}$$

$$\Big\downarrow \ 1142$$

3.176. $\int \frac{1}{1-x^3+x^6}\,dx$

$$\frac{i\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{3}{2}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}\int\frac{1}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx+\frac{1}{2}\int\frac{2x+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1\right.\right.}}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right.}{\sqrt{3}}$$

$$\frac{i\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{3}{2}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}\int\frac{1}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx+\frac{1}{2}\int\frac{2x+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1\right.\right.}}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right.}{\sqrt{3}}$$

$\downarrow$ 1082

3.176. $\int \frac{1}{1-x^3+x^6}\,dx$

$$\frac{i\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{1}{2}\int\frac{2x+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx-3\int\frac{1}{-\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}+1\right)^2-3}d\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{}\right.}}\right.}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right.}{\sqrt{3}}$$

$$\frac{i\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{1}{2}\int\frac{2x+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx-3\int\frac{1}{-\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}+1\right)^2-3}d\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{}\right.}}\right.}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right.}{\sqrt{3}}$$

$\downarrow$ 217

3.176. $\int \frac{1}{1-x^3+x^6}\,dx$

$$\frac{i\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{1}{2}\int\frac{2x+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx+\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right)}{\sqrt{3}}-\frac{i\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{1}{2}\int\frac{2x+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx+\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right)}{\sqrt{3}}$$

$$\Big\downarrow \; 1103$$

$$\frac{i\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)+\frac{1}{2}\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right)}{\sqrt{3}}-\frac{i\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)+\frac{1}{2}\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right)}{\sqrt{3}}$$

3.176. $\int \frac{1}{1-x^3+x^6}\,dx$

input
```
Int[(1 - x^3 + x^6)^(-1),x]
```

output
```
(I*(Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x]/(3*((1 - I*Sqrt[3])/2)^(2/3)) -
 (Sqrt[3]*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3]] + Log[(1 -
 I*Sqrt[3])^(2/3) + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2]/2)/(3*((1 -
 I*Sqrt[3])/2)^(2/3))))/Sqrt[3] - (I*(Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*
x]/(3*((1 + I*Sqrt[3])/2)^(2/3)) - (Sqrt[3]*ArcTan[(1 + (2*x)/((1 + I*Sqrt
[3])/2)^(1/3))/Sqrt[3]] + Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3]))^
(1/3)*x + 2^(2/3)*x^2]/2)/(3*((1 + I*Sqrt[3])/2)^(2/3))))/Sqrt[3]
```

3.176.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1685
```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^n), x], x] - Simp[
c/q   Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2,
2*n] && NeQ[b^2 - 4*a*c, 0]
```

3.176.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.20

method	result	size
default	$\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^6-\_Z^3+1\right)} \frac{\ln\left(x-\_R\right)}{2\_R^5-\_R^2}}{3}$	37
risch	$\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^6-\_Z^3+1\right)} \frac{\ln\left(x-\_R\right)}{2\_R^5-\_R^2}}{3}$	37

input
```
int(1/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/3*sum(1/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1))
```

3.176.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.53

$$\int \frac{1}{1-x^3+x^6}\,dx = \frac{1}{108}\cdot 18^{\frac{2}{3}}\left(i\sqrt{3}+3\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\log\left(18^{\frac{2}{3}}\left(\sqrt{3}(i\sqrt{-3}-i)+3\sqrt{-3}-3\right)\left(i\sqrt{3}+3\right)^{\frac{1}{3}}+72\,x\right)-\frac{1}{108}\cdot 18^{\frac{2}{3}}\left(i\sqrt{3}+3\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\log\left(18^{\frac{2}{3}}\left(\sqrt{3}(-i\sqrt{-3}-i)-3\sqrt{-3}-3\right)\left(i\sqrt{3}+3\right)^{\frac{1}{3}}+72\,x\right)-\frac{1}{108}\cdot 18^{\frac{2}{3}}\left(-i\sqrt{3}+3\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\log\left(18^{\frac{2}{3}}\left(\sqrt{3}(i\sqrt{-3}+i)-3\sqrt{-3}-3\right)\left(-i\sqrt{3}+3\right)^{\frac{1}{3}}+72\,x\right)+\frac{1}{108}\cdot 18^{\frac{2}{3}}\left(-i\sqrt{3}+3\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\log\left(18^{\frac{2}{3}}\left(\sqrt{3}(-i\sqrt{-3}+i)+3\sqrt{-3}-3\right)\left(-i\sqrt{3}+3\right)^{\frac{1}{3}}+72\,x\right)+\frac{1}{54}\cdot 18^{\frac{2}{3}}\left(i\sqrt{3}+3\right)^{\frac{1}{3}}\log\left(18^{\frac{2}{3}}\left(i\sqrt{3}+3\right)^{\frac{4}{3}}+36\,x\right)+\frac{1}{54}\cdot 18^{\frac{2}{3}}\left(-i\sqrt{3}+3\right)^{\frac{1}{3}}\log\left(18^{\frac{2}{3}}\left(-i\sqrt{3}+3\right)^{\frac{4}{3}}+36\,x\right)$$

input
```
integrate(1/(x^6-x^3+1),x, algorithm="fricas")
```

output
```
1/108*18^(2/3)*(I*sqrt(3) + 3)^(1/3)*(sqrt(-3) - 1)*log(18^(2/3)*(sqrt(3)*
(I*sqrt(-3) - I) + 3*sqrt(-3) - 3)*(I*sqrt(3) + 3)^(1/3) + 72*x) - 1/108*1
8^(2/3)*(I*sqrt(3) + 3)^(1/3)*(sqrt(-3) + 1)*log(18^(2/3)*(sqrt(3)*(-I*sqr
t(-3) - I) - 3*sqrt(-3) - 3)*(I*sqrt(3) + 3)^(1/3) + 72*x) - 1/108*18^(2/3
)*(-I*sqrt(3) + 3)^(1/3)*(sqrt(-3) + 1)*log(18^(2/3)*(sqrt(3)*(I*sqrt(-3)
+ I) - 3*sqrt(-3) - 3)*(-I*sqrt(3) + 3)^(1/3) + 72*x) + 1/108*18^(2/3)*(-I
*sqrt(3) + 3)^(1/3)*(sqrt(-3) - 1)*log(18^(2/3)*(sqrt(3)*(-I*sqrt(-3) + I)
 + 3*sqrt(-3) - 3)*(-I*sqrt(3) + 3)^(1/3) + 72*x) + 1/54*18^(2/3)*(I*sqrt(
3) + 3)^(1/3)*log(18^(2/3)*(I*sqrt(3) + 3)^(4/3) + 36*x) + 1/54*18^(2/3)*(
-I*sqrt(3) + 3)^(1/3)*log(18^(2/3)*(-I*sqrt(3) + 3)^(4/3) + 36*x)
```

3.176.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.11

$$\int \frac{1}{1-x^3+x^6}\,dx = \operatorname{RootSum}\left(19683t^6-243t^3+1,\left(t\mapsto t\log\left(729t^4+x\right)\right)\right)$$

input
```
integrate(1/(x**6-x**3+1),x)
```

output
```
RootSum(19683*_t**6 - 243*_t**3 + 1, Lambda(_t, _t*log(729*_t**4 + x)))
```

3.176.7 Maxima [F]

$$\int \frac{1}{1-x^3+x^6}\,dx = \int \frac{1}{x^6-x^3+1}\,dx$$

input
```
integrate(1/(x^6-x^3+1),x, algorithm="maxima")
```

output
```
integrate(1/(x^6 - x^3 + 1), x)
```

3.176.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 632, normalized size of antiderivative = 3.40

$$\int \frac{1}{1-x^3+x^6}\,dx = \text{Too large to display}$$

input
```
integrate(1/(x^6-x^3+1),x, algorithm="giac")
```

output
```
-1/9*(sqrt(3)*cos(4/9*pi)^4 - 6*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^2 + sqrt
(3)*sin(4/9*pi)^4 + 4*cos(4/9*pi)^3*sin(4/9*pi) - 4*cos(4/9*pi)*sin(4/9*pi
)^3 - sqrt(3)*cos(4/9*pi) - sin(4/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(
4/9*pi) + 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(4/9*pi))) - 1/9*(sqrt(3)*cos(2/9
*pi)^4 - 6*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^2 + sqrt(3)*sin(2/9*pi)^4 + 4
*cos(2/9*pi)^3*sin(2/9*pi) - 4*cos(2/9*pi)*sin(2/9*pi)^3 - sqrt(3)*cos(2/9
*pi) - sin(2/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(2/9*pi) + 2*x)/((1/2*
I*sqrt(3) + 1/2)*sin(2/9*pi))) - 1/9*(sqrt(3)*cos(1/9*pi)^4 - 6*sqrt(3)*co
s(1/9*pi)^2*sin(1/9*pi)^2 + sqrt(3)*sin(1/9*pi)^4 - 4*cos(1/9*pi)^3*sin(1/
9*pi) + 4*cos(1/9*pi)*sin(1/9*pi)^3 + sqrt(3)*cos(1/9*pi) - sin(1/9*pi))*a
rctan(-1/2*((-I*sqrt(3) - 1)*cos(1/9*pi) - 2*x)/((1/2*I*sqrt(3) + 1/2)*sin
(1/9*pi))) - 1/18*(4*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi) - 4*sqrt(3)*cos(4/9
*pi)*sin(4/9*pi)^3 - cos(4/9*pi)^4 + 6*cos(4/9*pi)^2*sin(4/9*pi)^2 - sin(4
/9*pi)^4 - sqrt(3)*sin(4/9*pi) + cos(4/9*pi))*log((-I*sqrt(3)*cos(4/9*pi)
- cos(4/9*pi))*x + x^2 + 1) - 1/18*(4*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi) -
4*sqrt(3)*cos(2/9*pi)*sin(2/9*pi)^3 - cos(2/9*pi)^4 + 6*cos(2/9*pi)^2*sin(
2/9*pi)^2 - sin(2/9*pi)^4 - sqrt(3)*sin(2/9*pi) + cos(2/9*pi))*log((-I*sqr
t(3)*cos(2/9*pi) - cos(2/9*pi))*x + x^2 + 1) + 1/18*(4*sqrt(3)*cos(1/9*pi)
^3*sin(1/9*pi) - 4*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^3 + cos(1/9*pi)^4 - 6*c
os(1/9*pi)^2*sin(1/9*pi)^2 + sin(1/9*pi)^4 + sqrt(3)*sin(1/9*pi) + cos(...
```

3.176. $\int \frac{1}{1-x^3+x^6} \, dx$

3.176.9 Mupad [B] (verification not implemented)

Time = 8.51 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.76

$$\int \frac{1}{1-x^3+x^6}\,dx = \frac{\ln\left(x+\frac{2^{2/3}\,3^{1/3}\,{\left(3-\sqrt{3}\,1\mathrm{i}\right)}^{1/3}}{4}-\frac{2^{2/3}\,3^{5/6}\,{\left(3-\sqrt{3}\,1\mathrm{i}\right)}^{1/3}\,1\mathrm{i}}{12}\right)\,{\left(36-\sqrt{3}\,12\mathrm{i}\right)}^{1/3}}{18} + \frac{\ln\left(x+\frac{2^{2/3}\,3^{1/3}\,{\left(3+\sqrt{3}\,1\mathrm{i}\right)}^{1/3}}{4}+\frac{2^{2/3}\,3^{5/6}\,{\left(3+\sqrt{3}\,1\mathrm{i}\right)}^{1/3}\,1\mathrm{i}}{12}\right)\,{\left(36+\sqrt{3}\,12\mathrm{i}\right)}^{1/3}}{18} - \frac{2^{2/3}\,\ln\left(x-\frac{2^{2/3}\,3^{1/3}\,{\left(3-\sqrt{3}\,1\mathrm{i}\right)}^{1/3}}{2}+\frac{2^{2/3}\,3^{1/3}\,{\left(3-\sqrt{3}\,1\mathrm{i}\right)}^{4/3}}{12}\right)\,{\left(3-\sqrt{3}\,1\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36} - \frac{2^{2/3}\,\ln\left(x-\frac{2^{2/3}\,3^{1/3}\,{\left(3+\sqrt{3}\,1\mathrm{i}\right)}^{1/3}}{2}+\frac{2^{2/3}\,3^{1/3}\,{\left(3+\sqrt{3}\,1\mathrm{i}\right)}^{4/3}}{12}\right)\,{\left(3+\sqrt{3}\,1\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36} - \frac{2^{2/3}\,\ln\left(x+\frac{2^{2/3}\,3^{5/6}\,{\left(3-\sqrt{3}\,1\mathrm{i}\right)}^{1/3}\,1\mathrm{i}}{6}\right)\,{\left(3-\sqrt{3}\,1\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36} - \frac{2^{2/3}\,\ln\left(x-\frac{2^{2/3}\,3^{5/6}\,{\left(3+\sqrt{3}\,1\mathrm{i}\right)}^{1/3}\,1\mathrm{i}}{6}\right)\,{\left(3+\sqrt{3}\,1\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}$$

input
```
int(1/(x^6 - x^3 + 1),x)
```

output
```
(log(x + (2^(2/3)*3^(1/3)*(3 - 3^(1/2)*1i)^(1/3))/4 - (2^(2/3)*3^(5/6)*(3
- 3^(1/2)*1i)^(1/3)*1i)/12)*(36 - 3^(1/2)*12i)^(1/3))/18 + (log(x + (2^(2/
3)*3^(1/3)*(3^(1/2)*1i + 3)^(1/3))/4 + (2^(2/3)*3^(5/6)*(3^(1/2)*1i + 3)^(
1/3)*1i)/12)*(3^(1/2)*12i + 36)^(1/3))/18 - (2^(2/3)*log(x - (2^(2/3)*3^(1
/3)*(3 - 3^(1/2)*1i)^(1/3))/2 + (2^(2/3)*3^(1/3)*(3 - 3^(1/2)*1i)^(4/3))/1
2)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2
^(2/3)*3^(1/3)*(3^(1/2)*1i + 3)^(1/3))/2 + (2^(2/3)*3^(1/3)*(3^(1/2)*1i +
3)^(4/3))/12)*(3^(1/2)*1i + 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)
*log(x + (2^(2/3)*3^(5/6)*(3 - 3^(1/2)*1i)^(1/3)*1i)/6)*(3 - 3^(1/2)*1i)^(
1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(2/3)*3^(5/6)*(3^(1/
2)*1i + 3)^(1/3)*1i)/6)*(3^(1/2)*1i + 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36
```

3.177 $\int \frac{1}{x(1-x^3+x^6)}\,dx$

3.177.1 Optimal result

Integrand size = 16, antiderivative size = 41

$$\int \frac{1}{x(1-x^3+x^6)}\,dx = -\frac{\arctan\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}} + \log(x) - \frac{1}{6}\log\left(1-x^3+x^6\right)$$

output
```
ln(x)-1/6*ln(x^6-x^3+1)-1/9*arctan(1/3*(-2*x^3+1)*3^(1/2))*3^(1/2)
```

3.177.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.34

$$\int \frac{1}{x(1-x^3+x^6)}\,dx = \log(x) - \frac{1}{3}\text{RootSum}\left[1-\#1^3 + \#1^6\&, \frac{-\log(x-\#1)+\log(x-\#1)\#1^3}{-1+2\#1^3}\&\right]$$

input
```
Integrate[1/(x*(1 - x^3 + x^6)),x]
```

output
```
Log[x] - RootSum[1 - #1^3 + #1^6 & , (-Log[x - #1] + Log[x - #1]*#1^3)/(-1
 + 2*#1^3) & ]/3
```

3.177.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1693, 1144, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(x^6-x^3+1)}dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int \frac{1}{x^3(x^6-x^3+1)}dx^3$$

$$\Big\downarrow \text{1144}$$

$$\frac{1}{3}\left(\int \frac{1-x^3}{x^6-x^3+1}dx^3+\log(x^3)\right)$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{3}\left(\frac{1}{2}\int \frac{1}{x^6-x^3+1}dx^3-\frac{1}{2}\int -\frac{1-2x^3}{x^6-x^3+1}dx^3+\log(x^3)\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\left(\frac{1}{2}\int \frac{1}{x^6-x^3+1}dx^3+\frac{1}{2}\int \frac{1-2x^3}{x^6-x^3+1}dx^3+\log(x^3)\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{3}\left(-\int \frac{1}{-x^6-3}d(2x^3-1)+\frac{1}{2}\int \frac{1-2x^3}{x^6-x^3+1}dx^3+\log(x^3)\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{3}\left(\frac{1}{2}\int \frac{1-2x^3}{x^6-x^3+1}dx^3+\frac{\arctan\left(\frac{2x^3-1}{\sqrt{3}}\right)}{\sqrt{3}}+\log(x^3)\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{3}\left(\frac{\arctan\left(\frac{2x^3-1}{\sqrt{3}}\right)}{\sqrt{3}}+\log(x^3)-\frac{1}{2}\log(x^6-x^3+1)\right)$$

input
```
Int[1/(x*(1 - x^3 + x^6)),x]
```

output
```
(ArcTan[(-1 + 2*x^3)/Sqrt[3]]/Sqrt[3] + Log[x^3] - Log[1 - x^3 + x^6]/2)/3
```

3.177.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1144
```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Simp[e*(Log[RemoveContent[d + e*x, x]]/(c*d^2 - b*d*e + a*e^2)), x] + S
imp[1/(c*d^2 - b*d*e + a*e^2)   Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2),
x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.177.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

method	result	size
risch	$\ln(x)-\frac{\ln(x^6-x^3+1)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x^3-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{9}$	33
default	$-\frac{\ln(x^6-x^3+1)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{\left(2x^3-1\right)\sqrt{3}}{3}\right)}{9}+\ln(x)$	35

input
```
int(1/x/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
ln(x)-1/6*ln(x^6-x^3+1)+1/9*3^(1/2)*arctan(2/3*(x^3-1/2)*3^(1/2))
```

3.177.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{1}{x(1-x^3+x^6)}\,dx = \frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^3-1)\right)-\frac{1}{6}\log\left(x^6-x^3+1\right)+\log(x)$$

input
```
integrate(1/x/(x^6-x^3+1),x, algorithm="fricas")
```

output
```
1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + log
(x)
```

3.177.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(1-x^3+x^6)}\,dx = \log(x)-\frac{\log(x^6-x^3+1)}{6}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^3}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

input
```
integrate(1/x/(x**6-x**3+1),x)
```

output
```
log(x) - log(x**6 - x**3 + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/
3)/9
```

3.177.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

$$\int \frac{1}{x(1-x^3+x^6)}\,dx = \frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^3-1)\right) - \frac{1}{6}\log\left(x^6-x^3+1\right) + \frac{1}{3}\log\left(x^3\right)$$

input
```
integrate(1/x/(x^6-x^3+1),x, algorithm="maxima")
```

output
```
1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + 1/3
*log(x^3)
```

3.177.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \frac{1}{x(1-x^3+x^6)}\,dx = \frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^3-1)\right) - \frac{1}{6}\log\left(x^6-x^3+1\right) + \log\left(|x|\right)$$

input
```
integrate(1/x/(x^6-x^3+1),x, algorithm="giac")
```

output
```
1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + log
(abs(x))
```

3.177.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int \frac{1}{x(1-x^3+x^6)}\,dx = \ln(x) - \frac{\ln(x^6-x^3+1)}{6} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3} - \frac{2\sqrt{3}\,x^3}{3}\right)}{9}$$

input
```
int(1/(x*(x^6 - x^3 + 1)),x)
```

output
```
log(x) - log(x^6 - x^3 + 1)/6 - (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/
3))/9
```

3.178 $\int \frac{1}{x^2(1-x^3+x^6)}\,dx$

3.178.1 Optimal result

Integrand size = 16, antiderivative size = 416

$$\int \frac{1}{x^2(1-x^3+x^6)}\,dx = -\frac{1}{x} + \frac{\left(i-\sqrt{3}\right)\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} - \frac{\left(i+\sqrt{3}\right)\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}} - \frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} - \frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}} + \frac{\left(3-i\sqrt{3}\right)\log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} + \frac{\left(3+i\sqrt{3}\right)\log\left(\left(1+i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

output
```
-1/x+1/6*arctan(1/3*(1+2*2^(1/3)*x/(1-I*3^(1/2))^(1/3))*3^(1/2))*(I-3^(1/2
))*2^(1/3)/(1-I*3^(1/2))^(1/3)-1/18*ln(-2^(1/3)*x+(1-I*3^(1/2))^(1/3))*(3-
I*3^(1/2))*2^(1/3)/(1-I*3^(1/2))^(1/3)+1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1-I*
3^(1/2))^(1/3)+(1-I*3^(1/2))^(2/3))*(3-I*3^(1/2))*2^(1/3)/(1-I*3^(1/2))^(1
/3)-1/18*ln(-2^(1/3)*x+(1+I*3^(1/2))^(1/3))*(3+I*3^(1/2))*2^(1/3)/(1+I*3^(
1/2))^(1/3)+1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1+I*3^(1/2))^(1/3)+(1+I*3^(1/2)
)^(2/3))*(3+I*3^(1/2))*2^(1/3)/(1+I*3^(1/2))^(1/3)-1/6*arctan(1/3*(1+2*2^(
1/3)*x/(1+I*3^(1/2))^(1/3))*3^(1/2))*(3^(1/2)+I)*2^(1/3)/(1+I*3^(1/2))^(1/
3)
```

3.178.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.15

$$\int \frac{1}{x^2 (1-x^3+x^6)} \, dx = -\frac{1}{x}-\frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&,\frac{-\log(x-\#1)+\log(x-\#1)\#1^3}{-\#1+2\#1^4}\&\right]$$

input
```
Integrate[1/(x^2*(1 - x^3 + x^6)),x]
```

output
```
-x^(-1) - RootSum[1 - #1^3 + #1^6 & , (-Log[x - #1] + Log[x - #1]*#1^3)/(-
#1 + 2*#1^4) & ]/3
```

3.178.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 372, normalized size of antiderivative = 0.89, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {1704, 1834, 27, 821, 16, 1142, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (x^6-x^3+1)} \, dx$$

$$\Big\downarrow \text{1704}$$

$$\int \frac{x(1-x^3)}{x^6-x^3+1}dx-\frac{1}{x}$$

$$\downarrow 1834$$

$$-\frac{1}{6}\left(3-i\sqrt{3}\right)\int -\frac{2x}{-2x^3-i\sqrt{3}+1}dx-\frac{1}{6}\left(3+i\sqrt{3}\right)\int -\frac{2x}{-2x^3+i\sqrt{3}+1}dx-\frac{1}{x}$$

$$\downarrow 27$$

$$\frac{1}{3}\left(3-i\sqrt{3}\right)\int \frac{x}{-2x^3-i\sqrt{3}+1}dx+\frac{1}{3}\left(3+i\sqrt{3}\right)\int \frac{x}{-2x^3+i\sqrt{3}+1}dx-\frac{1}{x}$$

$$\downarrow 821$$

$$\frac{1}{3}\left(3-i\sqrt{3}\right)\left(\frac{\int \frac{1}{\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x}dx}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\frac{\int \frac{\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}\right)+\frac{1}{3}\left(3+i\sqrt{3}\right)\left(\frac{\int \frac{1}{\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x}dx}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\frac{\int \frac{\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}\right)-\frac{1}{x}$$

$$\downarrow 16$$

$$\frac{1}{3}\left(3-i\sqrt{3}\right)\left(-\frac{\int \frac{\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}\right)+\frac{1}{3}\left(3+i\sqrt{3}\right)\left(-\frac{\int \frac{\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}\right)-\frac{1}{x}$$

$$\downarrow 1142$$

3.178. $\int \frac{1}{x^2(1-x^3+x^6)}\,dx$

$$\frac{1}{3}\left(3-i\sqrt{3}\right)\left(-\frac{\frac{3}{2}\sqrt[3]{1-i\sqrt{3}}\int\frac{1}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx-\frac{\int\frac{2\,2^{2/3}x+\sqrt[3]{2\left(1-i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\text{lo}\ldots\right.$$

$$\frac{1}{3}\left(3+i\sqrt{3}\right)\left(-\frac{\frac{3}{2}\sqrt[3]{1+i\sqrt{3}}\int\frac{1}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx-\frac{\int\frac{2\,2^{2/3}x+\sqrt[3]{2\left(1+i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\text{lo}\ldots\right.$$

$$\frac{1}{x}$$

$$\downarrow\ 1082$$

3.178. $\int \frac{1}{x^2(1-x^3+x^6)}\,dx$

$$\frac{1}{3}\left(3-i\sqrt{3}\right)\left(-\frac{-\frac{\int \frac{2\ 2^{2/3}x+\sqrt[3]{2\left(1-i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}-\frac{3\int \frac{1}{-\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}+1\right)^2-3}d\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}+1\right)}{\sqrt[3]{2}}}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\right.$$

$$\frac{1}{3}\left(3+i\sqrt{3}\right)\left(-\frac{-\frac{\int \frac{2\ 2^{2/3}x+\sqrt[3]{2\left(1+i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}-\frac{3\int \frac{1}{-\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}+1\right)^2-3}d\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}+1\right)}{\sqrt[3]{2}}}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\right.$$

$$\frac{1}{x}$$

$$\Big\downarrow \ 217$$

3.178. $\int \frac{1}{x^2(1-x^3+x^6)}\,dx$

$$\frac{1}{3}\left(3-i\sqrt{3}\right)\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\int\frac{2\,2^{2/3}x+\sqrt[3]{2\left(1-i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{}}\right.}{3\,2^{2/3}\sqrt[3]{1-i\sqrt{3}}}\right.$$

$$\frac{1}{3}\left(3+i\sqrt{3}\right)\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\int\frac{2\,2^{2/3}x+\sqrt[3]{2\left(1+i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{}}\right.}{3\,2^{2/3}\sqrt[3]{1+i\sqrt{3}}}\right.$$

$$\frac{1}{x}$$

$$\Big\downarrow \; 1103$$

3.178. $\int \frac{1}{x^2(1-x^3+x^6)}\,dx$

$$\frac{1}{3}\left(3-i\sqrt{3}\right)\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-\cdots}\right.}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{\cdots}}}\right.$$

$$\frac{1}{3}\left(3+i\sqrt{3}\right)\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+\cdots}\right.}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{\cdots}}}\right.$$

$$\frac{1}{x}$$

input
```
Int[1/(x^2*(1 - x^3 + x^6)),x]
```

output
```
-x^(-1) + ((3 - I*Sqrt[3])*(-1/3*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x]/(2
^(2/3)*(1 - I*Sqrt[3])^(1/3)) - ((Sqrt[3]*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3
])/2)^(1/3))/Sqrt[3]])/2^(1/3) - Log[(1 - I*Sqrt[3])^(2/3) + (2*(1 - I*Sqr
t[3]))^(1/3)*x + 2^(2/3)*x^2]/(2*2^(1/3)))/(3*(2*(1 - I*Sqrt[3]))^(1/3))))
/3 + ((3 + I*Sqrt[3])*(-1/3*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x]/(2^(2/3
)*(1 + I*Sqrt[3])^(1/3)) - ((Sqrt[3]*ArcTan[(1 + (2*x)/((1 + I*Sqrt[3])/2)
^(1/3))/Sqrt[3]])/2^(1/3) - Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3])
)^(1/3)*x + 2^(2/3)*x^2]/(2*2^(1/3)))/(3*(2*(1 + I*Sqrt[3]))^(1/3))))/3
```

3.178. $\int \frac{1}{x^2(1-x^3+x^6)}\,dx$

3.178.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1704
```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Simp[1/(a*d^n*(m + 1))   Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1)
+ c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{
a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] &&
LtQ[m, -1] && IntegerQ[p]
```

rule 1834
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 +
 (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2
 - (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ
[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n
, 0]
```

3.178.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.08

method	result	size
risch	$-\frac{1}{x}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(27\_Z^{6}+9\_Z^{3}+1\right)}\_R\ln\left(-3\_R^{2}+x\right)\right)}{3}$	35
default	$-\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)}\frac{\left(\_R^{4}-\_R\right)\ln\left(x-\_R\right)}{2\_R^{5}-\_R^{2}}\right)}{3}-\frac{1}{x}$	50

input
```
int(1/x^2/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-1/x+1/3*sum(_R*ln(-3*_R^2+x),_R=RootOf(27*_Z^6+9*_Z^3+1))
```

3.178.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.61

$$\int \frac{1}{x^2(1-x^3+x^6)}\,dx$$
$$= \frac{18^{\frac{2}{3}}\left(\sqrt{-3}x-x\right)\left(i\sqrt{3}-3\right)^{\frac{1}{3}}\log\left(18^{\frac{1}{3}}\left(i\sqrt{3}-3\right)^{\frac{2}{3}}\left(\sqrt{-3}+1\right)+12\,x\right)+18^{\frac{2}{3}}\left(\sqrt{-3}x-x\right)\left(-i\sqrt{3}-3\right)^{\frac{1}{3}}\log}{}$$

input
```
integrate(1/x^2/(x^6-x^3+1),x, algorithm="fricas")
```

output
```
1/108*(18^(2/3)*(sqrt(-3)*x - x)*(I*sqrt(3) - 3)^(1/3)*log(18^(1/3)*(I*sqr
t(3) - 3)^(2/3)*(sqrt(-3) + 1) + 12*x) + 18^(2/3)*(sqrt(-3)*x - x)*(-I*sqr
t(3) - 3)^(1/3)*log(18^(1/3)*(-I*sqrt(3) - 3)^(2/3)*(sqrt(-3) + 1) + 12*x)
 - 18^(2/3)*(sqrt(-3)*x + x)*(I*sqrt(3) - 3)^(1/3)*log(-18^(1/3)*(I*sqrt(3
) - 3)^(2/3)*(sqrt(-3) - 1) + 12*x) - 18^(2/3)*(sqrt(-3)*x + x)*(-I*sqrt(3
) - 3)^(1/3)*log(-18^(1/3)*(-I*sqrt(3) - 3)^(2/3)*(sqrt(-3) - 1) + 12*x) +
 2*18^(2/3)*x*(I*sqrt(3) - 3)^(1/3)*log(6*x - 18^(1/3)*(I*sqrt(3) - 3)^(2/
3)) + 2*18^(2/3)*x*(-I*sqrt(3) - 3)^(1/3)*log(6*x - 18^(1/3)*(-I*sqrt(3) -
 3)^(2/3)) - 108)/x
```

3.178.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.06

$$\int \frac{1}{x^2 (1 - x^3 + x^6)} \, dx = \operatorname{RootSum}\left(19683t^6 + 243t^3 + 1, \left(t \mapsto t \log\left(-27t^2 + x\right)\right)\right) - \frac{1}{x}$$

input
```
integrate(1/x**2/(x**6-x**3+1),x)
```

output
```
RootSum(19683*_t**6 + 243*_t**3 + 1, Lambda(_t, _t*log(-27*_t**2 + x))) -
1/x
```

3.178.7 Maxima [F]

$$\int \frac{1}{x^2 (1 - x^3 + x^6)} \, dx = \int \frac{1}{(x^6 - x^3 + 1)x^2} \, dx$$

input
```
integrate(1/x^2/(x^6-x^3+1),x, algorithm="maxima")
```

output
```
-1/x - integrate((x^4 - x)/(x^6 - x^3 + 1), x)
```

3.178.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 829 vs. $2(272) = 544$.

Time = 0.31 (sec) , antiderivative size = 829, normalized size of antiderivative = 1.99

$$\int \frac{1}{x^2 (1 - x^3 + x^6)} dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(x^6-x^3+1),x, algorithm="giac")
```

output
```
1/9*(sqrt(3)*cos(4/9*pi)^5 - 10*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi)^2 + 5*sq
rt(3)*cos(4/9*pi)*sin(4/9*pi)^4 - 5*cos(4/9*pi)^4*sin(4/9*pi) + 10*cos(4/9
*pi)^2*sin(4/9*pi)^3 - sin(4/9*pi)^5 + 2*sqrt(3)*cos(4/9*pi)^2 - 2*sqrt(3)
*sin(4/9*pi)^2 - 4*cos(4/9*pi)*sin(4/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*c
os(4/9*pi) + 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(4/9*pi))) + 1/9*(sqrt(3)*cos(
2/9*pi)^5 - 10*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi)^2 + 5*sqrt(3)*cos(2/9*pi)
*sin(2/9*pi)^4 - 5*cos(2/9*pi)^4*sin(2/9*pi) + 10*cos(2/9*pi)^2*sin(2/9*pi
)^3 - sin(2/9*pi)^5 + 2*sqrt(3)*cos(2/9*pi)^2 - 2*sqrt(3)*sin(2/9*pi)^2 -
4*cos(2/9*pi)*sin(2/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(2/9*pi) + 2*x)
/((1/2*I*sqrt(3) + 1/2)*sin(2/9*pi))) - 1/9*(sqrt(3)*cos(1/9*pi)^5 - 10*sq
rt(3)*cos(1/9*pi)^3*sin(1/9*pi)^2 + 5*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^4 +
5*cos(1/9*pi)^4*sin(1/9*pi) - 10*cos(1/9*pi)^2*sin(1/9*pi)^3 + sin(1/9*pi)
^5 - 2*sqrt(3)*cos(1/9*pi)^2 + 2*sqrt(3)*sin(1/9*pi)^2 - 4*cos(1/9*pi)*sin
(1/9*pi))*arctan(-1/2*((-I*sqrt(3) - 1)*cos(1/9*pi) - 2*x)/((1/2*I*sqrt(3)
 + 1/2)*sin(1/9*pi))) + 1/18*(5*sqrt(3)*cos(4/9*pi)^4*sin(4/9*pi) - 10*sqr
t(3)*cos(4/9*pi)^2*sin(4/9*pi)^3 + sqrt(3)*sin(4/9*pi)^5 + cos(4/9*pi)^5 -
 10*cos(4/9*pi)^3*sin(4/9*pi)^2 + 5*cos(4/9*pi)*sin(4/9*pi)^4 + 4*sqrt(3)*
cos(4/9*pi)*sin(4/9*pi) + 2*cos(4/9*pi)^2 - 2*sin(4/9*pi)^2)*log((-I*sqrt(
3)*cos(4/9*pi) - cos(4/9*pi))*x + x^2 + 1) + 1/18*(5*sqrt(3)*cos(2/9*pi)^4
*sin(2/9*pi) - 10*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^3 + sqrt(3)*sin(2/9...
```

3.178.9 Mupad [B] (verification not implemented)

Time = 8.55 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.69

$$\int \frac{1}{x^2 (1 - x^3 + x^6)} \, dx$$
$$= \frac{\ln\left(x - \frac{2^{1/3}\,3^{2/3}\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{2/3}}{6}\right)\left(-36+\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18}$$
$$-\frac{1}{x} + \frac{\ln\left(x - \frac{\left(-36-\sqrt{3}\,12\mathrm{i}\right)^{2/3}}{12}\right)\left(-36-\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18}$$
$$-\frac{2^{2/3}\ln\left(x - \frac{2^{1/3}\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{2/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)^2}{24}\right)\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}$$
$$-\frac{2^{2/3}\ln\left(x - \frac{2^{1/3}\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{2/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)^2}{24}\right)\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}$$
$$-\frac{2^{2/3}\ln\left(x - \frac{2^{1/3}\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{2/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)^2}{24}\right)\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}$$
$$-\frac{2^{2/3}\ln\left(x - \frac{2^{1/3}\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{2/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)^2}{24}\right)\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}$$

input
```
int(1/(x^2*(x^6 - x^3 + 1)),x)
```

output
```
(log(x - (2^(1/3)*3^(2/3)*(3^(1/2)*1i - 3)^(2/3))/6)*(3^(1/2)*12i - 36)^(1
/3))/18 - 1/x + (log(x - (- 3^(1/2)*12i - 36)^(2/3)/12)*(- 3^(1/2)*12i - 3
6)^(1/3))/18 - (2^(2/3)*log(x - (2^(1/3)*(- 3^(1/2)*1i - 3)^(2/3)*(3^(1/3)
 - 3^(5/6)*1i)^2)/24)*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36
- (2^(2/3)*log(x - (2^(1/3)*(- 3^(1/2)*1i - 3)^(2/3)*(3^(1/3) + 3^(5/6)*1i
)^2)/24)*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*lo
g(x - (2^(1/3)*(3^(1/2)*1i - 3)^(2/3)*(3^(1/3) - 3^(5/6)*1i)^2)/24)*(3^(1/
2)*1i - 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(1/3)*(3
^(1/2)*1i - 3)^(2/3)*(3^(1/3) + 3^(5/6)*1i)^2)/24)*(3^(1/2)*1i - 3)^(1/3)*
(3^(1/3) + 3^(5/6)*1i))/36
```

3.179 $\int \frac{1}{x^3(1-x^3+x^6)}\,dx$

3.179.1 Optimal result

Integrand size = 16, antiderivative size = 418

$$\int \frac{1}{x^3(1-x^3+x^6)}\,dx = -\frac{1}{2x^2} - \frac{\left(i-\sqrt{3}\right)\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} + \frac{\left(i+\sqrt{3}\right)\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} - \frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} - \frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} + \frac{\left(3-i\sqrt{3}\right)\log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} + \frac{\left(3+i\sqrt{3}\right)\log\left(\left(1+i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

output
```
-1/2/x^2-1/6*arctan(1/3*(1+2*2^(1/3)*x/(1-I*3^(1/2))^(1/3))*3^(1/2))*(I-3^
(1/2))*2^(2/3)/(1-I*3^(1/2))^(2/3)-1/18*ln(-2^(1/3)*x+(1-I*3^(1/2))^(1/3))
*(3-I*3^(1/2))*2^(2/3)/(1-I*3^(1/2))^(2/3)+1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(
1-I*3^(1/2))^(1/3)+(1-I*3^(1/2))^(2/3))*(3-I*3^(1/2))*2^(2/3)/(1-I*3^(1/2)
)^(2/3)-1/18*ln(-2^(1/3)*x+(1+I*3^(1/2))^(1/3))*(3+I*3^(1/2))*2^(2/3)/(1+I
*3^(1/2))^(2/3)+1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1+I*3^(1/2))^(1/3)+(1+I*3^(
1/2))^(2/3))*(3+I*3^(1/2))*2^(2/3)/(1+I*3^(1/2))^(2/3)+1/6*arctan(1/3*(1+2
*2^(1/3)*x/(1+I*3^(1/2))^(1/3))*3^(1/2))*(3^(1/2)+I)*2^(2/3)/(1+I*3^(1/2))
^(2/3)
```

3.179.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.16

$$\int \frac{1}{x^3(1-x^3+x^6)}\,dx = -\frac{1}{2x^2} - \frac{1}{3}\text{RootSum}\Big[1-\#1^3 + \#1^6\&, \frac{-\log(x-\#1)+\log(x-\#1)\#1^3}{-\#1^2+2\#1^5}\&\Big]$$

input
```
Integrate[1/(x^3*(1 - x^3 + x^6)),x]
```

output
```
-1/2*1/x^2 - RootSum[1 - #1^3 + #1^6 & , (-Log[x - #1] + Log[x - #1]*#1^3)
/(-#1^2 + 2*#1^5) & ]/3
```

3.179.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 356, normalized size of antiderivative = 0.85, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {1704, 27, 1752, 750, 16, 25, 1142, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3(x^6-x^3+1)}\,dx$$

$$\Big\downarrow \; 1704$$

$$\frac{1}{2}\int \frac{2(1-x^3)}{x^6-x^3+1}dx - \frac{1}{2x^2}$$

$$\Big\downarrow \; 27$$

$$\int \frac{1-x^3}{x^6-x^3+1}dx - \frac{1}{2x^2}$$

$$\Big\downarrow \; 1752$$

$$-\frac{1}{6}\left(3+i\sqrt{3}\right)\int \frac{1}{x^3+\frac{1}{2}\left(-1-i\sqrt{3}\right)}dx - \frac{1}{6}\left(3-i\sqrt{3}\right)\int \frac{1}{x^3+\frac{1}{2}\left(-1+i\sqrt{3}\right)}dx - \frac{1}{2x^2}$$

$$\Big\downarrow \; 750$$

$$-\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\int -\frac{x+2^{2/3}\sqrt[3]{1-i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}+\frac{\int \frac{1}{x-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right)-\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\int -\frac{x+2^{2/3}\sqrt[3]{1+i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}+\frac{\int \frac{1}{x-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right)-\frac{1}{2x^2}$$

$$\Big\downarrow \textbf{16}$$

$$-\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\int -\frac{x+2^{2/3}\sqrt[3]{1-i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}+\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right)-\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\int -\frac{x+2^{2/3}\sqrt[3]{1+i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}+\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right)-\frac{1}{2x^2}$$

$$\Big\downarrow \textbf{25}$$

3.179. $\int \frac{1}{x^3(1-x^3+x^6)}\,dx$

$$-\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\int\frac{x+2^{2/3}\sqrt[3]{1-i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right)-$$

$$\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\int\frac{x+2^{2/3}\sqrt[3]{1+i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right)-\frac{1}{2x^2}$$

$\downarrow$ 1142

$$-\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{3}{2}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}\int\frac{1}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx+\frac{1}{2}\int\frac{}{x^2-}}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right.$$

$$\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{3}{2}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}\int\frac{1}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx+\frac{1}{2}\int\frac{}{x^2+}}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right.$$

$$\frac{1}{2x^2}$$

$\downarrow$ 1082

3.179. $\int \frac{1}{x^3(1-x^3+x^6)}\,dx$

$$-\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{1}{2}\int\frac{2x+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx-3\int\frac{1}{-\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}\cdots\right.}\cdots}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\cdots\right.$$

$$\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{1}{2}\int\frac{2x+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx-3\int\frac{1}{-\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}+\cdots\right.}\cdots}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\cdots\right.$$

$$\frac{1}{2x^2}$$

$$\Big\downarrow \; 217$$

3.179. $\int \frac{1}{x^3(1-x^3+x^6)}\,dx$

$$-\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{1}{2}\int\frac{2x+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx+\sqrt{3}\arctan\left(\frac{1+\frac{\cdots}{\sqrt[3]{\frac{1}{2}\left(\cdots\right)}}}{\sqrt{\cdots}}\right.}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right.$$

$$\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{1}{2}\int\frac{2x+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx+\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+\cdots\right)}}}{\sqrt{3}}\right.}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right.$$

$$\frac{1}{2x^2}$$

$$\Big\downarrow \ 1103$$

$$-\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)+\frac{1}{2}\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}\cdots\right.}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right.$$

$$\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)+\frac{1}{2}\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x\cdots\right.}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right.$$

$$\frac{1}{2x^2}$$

3.179. $\int \frac{1}{x^3(1-x^3+x^6)}\,dx$

input
```
Int[1/(x^3*(1 - x^3 + x^6)),x]
```

output
```
-1/2*1/x^2 - ((3 - I*Sqrt[3])*(Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x]/(3*(
(1 - I*Sqrt[3])/2)^(2/3)) - (Sqrt[3]*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)
^(1/3))/Sqrt[3]] + Log[(1 - I*Sqrt[3])^(2/3) + (2*(1 - I*Sqrt[3]))^(1/3)*x
 + 2^(2/3)*x^2]/2)/(3*((1 - I*Sqrt[3])/2)^(2/3))))/6 - ((3 + I*Sqrt[3])*(L
og[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x]/(3*((1 + I*Sqrt[3])/2)^(2/3)) - (Sqr
t[3]*ArcTan[(1 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]] + Log[(1 + I*Sq
rt[3])^(2/3) + (2*(1 + I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2]/2)/(3*((1 + I*Sq
rt[3])/2)^(2/3))))/6
```

3.179.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1704
```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Simp[1/(a*d^n*(m + 1))   Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1)
+ c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{
a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] &&
LtQ[m, -1] && IntegerQ[p]
```

rule 1752
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

3.179.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.09

method	result	size
risch	$-\frac{1}{2x^2}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(27\_Z^6+9\_Z^3+1\right)}\_R\ln\left(9\_R^4+3\_R+x\right)\right)}{3}$	38
default	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^6-\_Z^3+1\right)}\frac{\left(-\_R^3+1\right)\ln\left(x-\_R\right)}{2\_R^5-\_R^2}\right)}{3}-\frac{1}{2x^2}$	50

input
```
int(1/x^3/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2/x^2+1/3*sum(_R*ln(9*_R^4+3*_R+x),_R=RootOf(27*_Z^6+9*_Z^3+1))
```

3.179.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 337, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^3 (1 - x^3 + x^6)} \, dx = \frac{2 \cdot 18^{\frac{2}{3}} x^2 \left(i \sqrt{3} - 3\right)^{\frac{1}{3}} \log \left(18^{\frac{2}{3}} \left(i \sqrt{3} + 3\right) \left(i \sqrt{3} - 3\right)^{\frac{1}{3}} + 36 \, x\right) + 2 \cdot 18^{\frac{2}{3}} x^2 \left(-i \sqrt{3} - 3\right)^{\frac{1}{3}} \log \left(18^{\frac{2}{3}} \left(-i \sqrt{3} + \right.\right.}{}$$

input
```
integrate(1/x^3/(x^6-x^3+1),x, algorithm="fricas")
```

output
```
1/108*(2*18^(2/3)*x^2*(I*sqrt(3) - 3)^(1/3)*log(18^(2/3)*(I*sqrt(3) + 3)*(
I*sqrt(3) - 3)^(1/3) + 36*x) + 2*18^(2/3)*x^2*(-I*sqrt(3) - 3)^(1/3)*log(1
8^(2/3)*(-I*sqrt(3) + 3)*(-I*sqrt(3) - 3)^(1/3) + 36*x) + 18^(2/3)*(sqrt(-
3)*x^2 - x^2)*(I*sqrt(3) - 3)^(1/3)*log(18^(2/3)*(sqrt(3)*(I*sqrt(-3) - I)
 + 3*sqrt(-3) - 3)*(I*sqrt(3) - 3)^(1/3) + 72*x) - 18^(2/3)*(sqrt(-3)*x^2
+ x^2)*(I*sqrt(3) - 3)^(1/3)*log(18^(2/3)*(sqrt(3)*(-I*sqrt(-3) - I) - 3*s
qrt(-3) - 3)*(I*sqrt(3) - 3)^(1/3) + 72*x) - 18^(2/3)*(sqrt(-3)*x^2 + x^2)
*(-I*sqrt(3) - 3)^(1/3)*log(18^(2/3)*(sqrt(3)*(I*sqrt(-3) + I) - 3*sqrt(-3
) - 3)*(-I*sqrt(3) - 3)^(1/3) + 72*x) + 18^(2/3)*(sqrt(-3)*x^2 - x^2)*(-I*
sqrt(3) - 3)^(1/3)*log(18^(2/3)*(sqrt(3)*(-I*sqrt(-3) + I) + 3*sqrt(-3) -
3)*(-I*sqrt(3) - 3)^(1/3) + 72*x) - 54)/x^2
```

3.179.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.07

$$\int \frac{1}{x^3 (1 - x^3 + x^6)} \, dx = \operatorname{RootSum}\left(19683t^6 + 243t^3 + 1, \left(t \mapsto t \log \left(729t^4 + 9t + x\right)\right)\right) - \frac{1}{2x^2}$$

input
```
integrate(1/x**3/(x**6-x**3+1),x)
```

output
```
RootSum(19683*_t**6 + 243*_t**3 + 1, Lambda(_t, _t*log(729*_t**4 + 9*_t +
x))) - 1/(2*x**2)
```

3.179.7 Maxima [F]

$$\int \frac{1}{x^3\left(1-x^3+x^6\right)}\,dx = \int \frac{1}{(x^6-x^3+1)x^3}\,dx$$

input
```
integrate(1/x^3/(x^6-x^3+1),x, algorithm="maxima")
```

output
```
-1/2/x^2 - integrate((x^3 - 1)/(x^6 - x^3 + 1), x)
```

3.179.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 645 vs. 2(272) = 544.

Time = 0.31 (sec) , antiderivative size = 645, normalized size of antiderivative = 1.54

$$\int \frac{1}{x^3\left(1-x^3+x^6\right)}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^3/(x^6-x^3+1),x, algorithm="giac")
```

output
```
1/9*(sqrt(3)*cos(4/9*pi)^4 - 6*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^2 + sqrt(
3)*sin(4/9*pi)^4 + 4*cos(4/9*pi)^3*sin(4/9*pi) - 4*cos(4/9*pi)*sin(4/9*pi)
^3 + 2*sqrt(3)*cos(4/9*pi) + 2*sin(4/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*c
os(4/9*pi) + 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(4/9*pi))) + 1/9*(sqrt(3)*cos(
2/9*pi)^4 - 6*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^2 + sqrt(3)*sin(2/9*pi)^4
+ 4*cos(2/9*pi)^3*sin(2/9*pi) - 4*cos(2/9*pi)*sin(2/9*pi)^3 + 2*sqrt(3)*co
s(2/9*pi) + 2*sin(2/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(2/9*pi) + 2*x)
/((1/2*I*sqrt(3) + 1/2)*sin(2/9*pi))) + 1/9*(sqrt(3)*cos(1/9*pi)^4 - 6*sqr
t(3)*cos(1/9*pi)^2*sin(1/9*pi)^2 + sqrt(3)*sin(1/9*pi)^4 - 4*cos(1/9*pi)^3
*sin(1/9*pi) + 4*cos(1/9*pi)*sin(1/9*pi)^3 - 2*sqrt(3)*cos(1/9*pi) + 2*sin
(1/9*pi))*arctan(-1/2*((-I*sqrt(3) - 1)*cos(1/9*pi) - 2*x)/((1/2*I*sqrt(3)
 + 1/2)*sin(1/9*pi))) + 1/18*(4*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi) - 4*sqrt
(3)*cos(4/9*pi)*sin(4/9*pi)^3 - cos(4/9*pi)^4 + 6*cos(4/9*pi)^2*sin(4/9*pi
)^2 - sin(4/9*pi)^4 + 2*sqrt(3)*sin(4/9*pi) - 2*cos(4/9*pi))*log((-I*sqrt(
3)*cos(4/9*pi) - cos(4/9*pi))*x + x^2 + 1) + 1/18*(4*sqrt(3)*cos(2/9*pi)^3
*sin(2/9*pi) - 4*sqrt(3)*cos(2/9*pi)*sin(2/9*pi)^3 - cos(2/9*pi)^4 + 6*cos
(2/9*pi)^2*sin(2/9*pi)^2 - sin(2/9*pi)^4 + 2*sqrt(3)*sin(2/9*pi) - 2*cos(2
/9*pi))*log((-I*sqrt(3)*cos(2/9*pi) - cos(2/9*pi))*x + x^2 + 1) - 1/18*(4*
sqrt(3)*cos(1/9*pi)^3*sin(1/9*pi) - 4*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^3 +
cos(1/9*pi)^4 - 6*cos(1/9*pi)^2*sin(1/9*pi)^2 + sin(1/9*pi)^4 - 2*sqrt(...
```

3.179. $\int \frac{1}{x^3(1-x^3+x^6)}\,dx$

3.179.9 Mupad [B] (verification not implemented)

Time = 8.81 (sec) , antiderivative size = 324, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^3(1-x^3+x^6)}\,dx = \frac{\ln\left(x-\frac{\left(-\frac{27}{2}+\frac{\sqrt{3}\,9\mathrm{i}}{2}\right)\left(-36-\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{54}\right)\left(-36-\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18} + \frac{\ln\left(x+\frac{\left(\frac{27}{2}+\frac{\sqrt{3}\,9\mathrm{i}}{2}\right)\left(-36+\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{54}\right)\left(-36+\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18} - \frac{1}{2x^2} - \frac{2^{2/3}\ln\left(x-\frac{2^{2/3}\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)\left(\frac{3\left(3+\sqrt{3}\,1\mathrm{i}\right)\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)^3}{16}+27\right)}{108}\right)\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36} - \frac{2^{2/3}\ln\left(x+\frac{2^{2/3}\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)\left(\frac{3\left(-3+\sqrt{3}\,1\mathrm{i}\right)\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)^3}{16}-27\right)}{108}\right)\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36} - \frac{2^{2/3}\ln\left(x+\frac{2^{2/3}\,3^{5/6}\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}1\mathrm{i}}{6}\right)\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36} - \frac{2^{2/3}\ln\left(x-\frac{2^{2/3}\,3^{5/6}\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}1\mathrm{i}}{6}\right)\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}$$

input
```
int(1/(x^3*(x^6 - x^3 + 1)),x)
```

output
```
(log(x - (((3^(1/2)*9i)/2 - 27/2)*(- 3^(1/2)*12i - 36)^(1/3))/54)*(- 3^(1/
2)*12i - 36)^(1/3))/18 + (log(x + (((3^(1/2)*9i)/2 + 27/2)*(3^(1/2)*12i -
36)^(1/3))/54)*(3^(1/2)*12i - 36)^(1/3))/18 - 1/(2*x^2) - (2^(2/3)*log(x -
 (2^(2/3)*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i)*((3*(3^(1/2)*1i
+ 3)*(3^(1/3) + 3^(5/6)*1i)^3)/16 + 27))/108)*(- 3^(1/2)*1i - 3)^(1/3)*(3^
(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*log(x + (2^(2/3)*(3^(1/2)*1i - 3)^(1/3)
*(3^(1/3) - 3^(5/6)*1i)*((3*(3^(1/2)*1i - 3)*(3^(1/3) - 3^(5/6)*1i)^3)/16
- 27))/108)*(3^(1/2)*1i - 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*l
og(x + (2^(2/3)*3^(5/6)*(- 3^(1/2)*1i - 3)^(1/3)*1i)/6)*(- 3^(1/2)*1i - 3)
^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(2/3)*3^(5/6)*(3^(
1/2)*1i - 3)^(1/3)*1i)/6)*(3^(1/2)*1i - 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/3
6
```

3.180 $\int \frac{1}{x^4(1-x^3+x^6)} dx$

3.180.1 Optimal result

Integrand size = 16, antiderivative size = 48

$$\int \frac{1}{x^4(1-x^3+x^6)} dx = -\frac{1}{3x^3} + \frac{\arctan\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}} + \log(x) - \frac{1}{6}\log\left(1-x^3+x^6\right)$$

output
```
-1/3/x^3+ln(x)-1/6*ln(x^6-x^3+1)+1/9*arctan(1/3*(-2*x^3+1)*3^(1/2))*3^(1/2
)
```

3.180.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^4(1-x^3+x^6)} dx = -\frac{1}{3x^3} + \log(x) - \frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&, \frac{\log(x-\#1)\#1^3}{-1+2\#1^3}\&\right]$$

input
```
Integrate[1/(x^4*(1 - x^3 + x^6)),x]
```

output
```
-1/3*1/x^3 + Log[x] - RootSum[1 - #1^3 + #1^6 & , (Log[x - #1]*#1^3)/(-1 +
 2*#1^3) & ]/3
```

3.180.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1693, 1145, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (x^6-x^3+1)}dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int \frac{1}{x^6 (x^6-x^3+1)}dx^3$$

$$\Big\downarrow \text{1145}$$

$$\frac{1}{3}\left(\int \frac{1-x^3}{x^3 (x^6-x^3+1)}dx^3-\frac{1}{x^3}\right)$$

$$\Big\downarrow \text{1200}$$

$$\frac{1}{3}\left(\int \left(\frac{1}{x^3}-\frac{x^3}{x^6-x^3+1}\right)dx^3-\frac{1}{x^3}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{\arctan\left(\frac{1-2x^3}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{x^3}+\log\left(x^3\right)-\frac{1}{2}\log\left(x^6-x^3+1\right)\right)$$

input
```
Int[1/(x^4*(1 - x^3 + x^6)),x]
```

output
```
(-x^(-3) + ArcTan[(1 - 2*x^3)/Sqrt[3]]/Sqrt[3] + Log[x^3] - Log[1 - x^3 +
x^6]/2)/3
```

3.180.3.1 Defintions of rubi rules used

rule 1145
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp
[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[m, -1]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.180.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.79

method	result	size
risch	$-\frac{1}{3x^3}+\ln(x)-\frac{\ln(x^6-x^3+1)}{6}-\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x^3-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{9}$	38
default	$-\frac{\ln(x^6-x^3+1)}{6}-\frac{\sqrt{3}\,\arctan\left(\frac{\left(2x^3-1\right)\sqrt{3}}{3}\right)}{9}-\frac{1}{3x^3}+\ln(x)$	40

input
```
int(1/x^4/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-1/3/x^3+ln(x)-1/6*ln(x^6-x^3+1)-1/9*3^(1/2)*arctan(2/3*(x^3-1/2)*3^(1/2))
```

3.180.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^4(1-x^3+x^6)}\,dx = -\frac{2\sqrt{3}x^3\arctan\left(\frac{1}{3}\sqrt{3}(2x^3-1)\right)+3x^3\log\left(x^6-x^3+1\right)-18x^3\log\left(x\right)+6}{18x^3}$$

input
```
integrate(1/x^4/(x^6-x^3+1),x, algorithm="fricas")
```

output
```
-1/18*(2*sqrt(3)*x^3*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 3*x^3*log(x^6 - x^3
 + 1) - 18*x^3*log(x) + 6)/x^3
```

3.180.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4(1-x^3+x^6)}\,dx = \log\left(x\right)-\frac{\log\left(x^6-x^3+1\right)}{6}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^3}{3}-\frac{\sqrt{3}}{3}\right)}{9}-\frac{1}{3x^3}$$

input
```
integrate(1/x**4/(x**6-x**3+1),x)
```

output
```
log(x) - log(x**6 - x**3 + 1)/6 - sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/
3)/9 - 1/(3*x**3)
```

3.180.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^4(1-x^3+x^6)}\,dx = -\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^3-1)\right)-\frac{1}{3x^3}-\frac{1}{6}\log\left(x^6-x^3+1\right)+\frac{1}{3}\log\left(x^3\right)$$

input
```
integrate(1/x^4/(x^6-x^3+1),x, algorithm="maxima")
```

output
```
-1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/3/x^3 - 1/6*log(x^6 - x^3
 + 1) + 1/3*log(x^3)
```

3.180.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^4(1-x^3+x^6)}\,dx = -\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^3-1)\right) - \frac{x^3+1}{3x^3} - \frac{1}{6}\log\left(x^6-x^3+1\right) + \log(|x|)$$

input
```
integrate(1/x^4/(x^6-x^3+1),x, algorithm="giac")
```

output
```
-1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/3*(x^3 + 1)/x^3 - 1/6*log
(x^6 - x^3 + 1) + log(abs(x))
```

3.180.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^4(1-x^3+x^6)}\,dx = \ln(x) - \frac{\ln(x^6-x^3+1)}{6} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}\,x^3}{3}\right)}{9} - \frac{1}{3x^3}$$

input
```
int(1/(x^4*(x^6 - x^3 + 1)),x)
```

output
```
log(x) - log(x^6 - x^3 + 1)/6 + (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/
3))/9 - 1/(3*x^3)
```

3.181 $\int \frac{1}{x^5(1-x^3+x^6)}\,dx$

3.181.1 Optimal result

Integrand size = 16, antiderivative size = 423

$$\int \frac{1}{x^5(1-x^3+x^6)}\,dx = -\frac{1}{4x^4}-\frac{1}{x}-\frac{\left(i+\sqrt{3}\right)\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{\left(i-\sqrt{3}\right)\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}-\frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}+\frac{\left(3+i\sqrt{3}\right)\log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{\left(3-i\sqrt{3}\right)\log\left(\left(1+i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

output
```
-1/4/x^4-1/x+1/6*arctan(1/3*(1+2*2^(1/3)*x/(1+I*3^(1/2))^(1/3))*3^(1/2))*(
I-3^(1/2))*2^(1/3)/(1+I*3^(1/2))^(1/3)-1/18*ln(-2^(1/3)*x+(1+I*3^(1/2))^(1
/3))*(3-I*3^(1/2))*2^(1/3)/(1+I*3^(1/2))^(1/3)+1/36*ln(2^(2/3)*x^2+2^(1/3)
*x*(1+I*3^(1/2))^(1/3)+(1+I*3^(1/2))^(2/3))*(3-I*3^(1/2))*2^(1/3)/(1+I*3^(
1/2))^(1/3)-1/18*ln(-2^(1/3)*x+(1-I*3^(1/2))^(1/3))*(3+I*3^(1/2))*2^(1/3)/
(1-I*3^(1/2))^(1/3)+1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1-I*3^(1/2))^(1/3)+(1-I
*3^(1/2))^(2/3))*(3+I*3^(1/2))*2^(1/3)/(1-I*3^(1/2))^(1/3)-1/6*arctan(1/3*
(1+2*2^(1/3)*x/(1-I*3^(1/2))^(1/3))*3^(1/2))*(3^(1/2)+I)*2^(1/3)/(1-I*3^(1
/2))^(1/3)
```

3.181.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.13

$$\int \frac{1}{x^5 (1-x^3+x^6)} \, dx = -\frac{1}{4x^4} - \frac{1}{x} - \frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&, \frac{\log(x-\#1)\#1^2}{-1+2\#1^3}\&\right]$$

input
```
Integrate[1/(x^5*(1 - x^3 + x^6)),x]
```

output
```
-1/4*1/x^4 - x^(-1) - RootSum[1 - #1^3 + #1^6 & , (Log[x - #1]*#1^2)/(-1 +
 2*#1^3) & ]/3
```

3.181.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 379, normalized size of antiderivative = 0.90, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.688$, Rules used = {1704, 27, 1828, 1710, 27, 821, 16, 1142, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 (x^6-x^3+1)} \, dx$$

$$\Big\downarrow \ 1704$$

$$\frac{1}{4}\int \frac{4(1-x^3)}{x^2 (x^6-x^3+1)}dx - \frac{1}{4x^4}$$

$$\Big\downarrow 27$$

$$\int \frac{1-x^3}{x^2\left(x^6-x^3+1\right)}dx-\frac{1}{4x^4}$$

$$\Big\downarrow 1828$$

$$-\int \frac{x^4}{x^6-x^3+1}dx-\frac{1}{4x^4}-\frac{1}{x}$$

$$\Big\downarrow 1710$$

$$-\frac{1}{6}\left(3+i\sqrt{3}\right)\int -\frac{2x}{-2x^3-i\sqrt{3}+1}dx-\frac{1}{6}\left(3-i\sqrt{3}\right)\int -\frac{2x}{-2x^3+i\sqrt{3}+1}dx-\frac{1}{4x^4}-\frac{1}{x}$$

$$\Big\downarrow 27$$

$$\frac{1}{3}\left(3+i\sqrt{3}\right)\int \frac{x}{-2x^3-i\sqrt{3}+1}dx+\frac{1}{3}\left(3-i\sqrt{3}\right)\int \frac{x}{-2x^3+i\sqrt{3}+1}dx-\frac{1}{4x^4}-\frac{1}{x}$$

$$\Big\downarrow 821$$

$$\frac{1}{3}\left(3+i\sqrt{3}\right)\left(\frac{\int \frac{1}{\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x}dx}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\frac{\int \frac{\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}\right)+$$

$$\frac{1}{3}\left(3-i\sqrt{3}\right)\left(\frac{\int \frac{1}{\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x}dx}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\frac{\int \frac{\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}\right)-\frac{1}{4x^4}-\frac{1}{x}$$

$$\Big\downarrow 16$$

3.181. $\int \frac{1}{x^5(1-x^3+x^6)}\,dx$

$$\frac{1}{3}\left(3+i\sqrt{3}\right)\left(-\frac{\int \frac{\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}\right)+$$

$$\frac{1}{3}\left(3-i\sqrt{3}\right)\left(-\frac{\int \frac{\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}\right)-\frac{1}{4x^4}-\frac{1}{x}$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{3}\left(3+i\sqrt{3}\right)\left(-\frac{\frac{3}{2}\sqrt[3]{1-i\sqrt{3}}\int \frac{1}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx-\frac{\int \frac{2\ 2^{2/3}x+\sqrt[3]{2\left(1-i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\frac{\text{lo}}{}\right.$$

$$\frac{1}{3}\left(3-i\sqrt{3}\right)\left(-\frac{\frac{3}{2}\sqrt[3]{1+i\sqrt{3}}\int \frac{1}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx-\frac{\int \frac{2\ 2^{2/3}x+\sqrt[3]{2\left(1+i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\frac{\text{lo}}{}\right.$$

$$\frac{1}{4x^4}-\frac{1}{x}$$

$$\Big\downarrow \text{1082}$$

3.181. $\int \frac{1}{x^5(1-x^3+x^6)}\,dx$

$$\frac{1}{3}\left(3+i\sqrt{3}\right)\left(-\frac{-\frac{\int \frac{2\, 2^{2/3}x+\sqrt[3]{2\left(1-i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}-\frac{3\int \frac{1}{-\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}+1\right)^2-3}d\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}+1\right)}{\sqrt[3]{2}}}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\cdots\right.$$

$$\frac{1}{3}\left(3-i\sqrt{3}\right)\left(-\frac{-\frac{\int \frac{2\, 2^{2/3}x+\sqrt[3]{2\left(1+i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}-\frac{3\int \frac{1}{-\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}+1\right)^2-3}d\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}+1\right)}{\sqrt[3]{2}}}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\cdots\right.$$

$$\frac{1}{4x^4}-\frac{1}{x}$$

$$\Big\downarrow \; 217$$

3.181. $\int \frac{1}{x^5(1-x^3+x^6)}\,dx$

$$\frac{1}{3}\left(3+i\sqrt{3}\right)\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\int\frac{2\ 2^{2/3}x+\sqrt[3]{2\left(1-i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{}}\right.}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}\right.$$

$$\frac{1}{3}\left(3-i\sqrt{3}\right)\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\int\frac{2\ 2^{2/3}x+\sqrt[3]{2\left(1+i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{}}\right.}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}\right.$$

$$\frac{1}{4x^4}-\frac{1}{x}$$

$$\Big\downarrow \ 1103$$

3.181. $\int \frac{1}{x^5(1-x^3+x^6)}\,dx$

$$\frac{1}{3}\left(3+i\sqrt{3}\right)\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-\cdots}\right.}{3\ 2^{2/3}\sqrt[3]{1-i\cdots}}\right.$$

$$\frac{1}{3}\left(3-i\sqrt{3}\right)\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+\cdots}\right.}{3\ 2^{2/3}\sqrt[3]{1+i\cdots}}\right.$$

$$\frac{1}{4x^4}-\frac{1}{x}$$

input
```
Int[1/(x^5*(1 - x^3 + x^6)),x]
```

output
```
-1/4*1/x^4 - x^(-1) + ((3 + I*Sqrt[3])*(-1/3*Log[(1 - I*Sqrt[3])^(1/3) - 2
^(1/3)*x]/(2^(2/3)*(1 - I*Sqrt[3])^(1/3)) - ((Sqrt[3]*ArcTan[(1 + (2*x)/((
1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/2^(1/3) - Log[(1 - I*Sqrt[3])^(2/3) + (
2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2]/(2*2^(1/3)))/(3*(2*(1 - I*Sqrt[3
]))^(1/3))))/3 + ((3 - I*Sqrt[3])*(-1/3*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3
)*x]/(2^(2/3)*(1 + I*Sqrt[3])^(1/3)) - ((Sqrt[3]*ArcTan[(1 + (2*x)/((1 + I
*Sqrt[3])/2)^(1/3))/Sqrt[3]])/2^(1/3) - Log[(1 + I*Sqrt[3])^(2/3) + (2*(1
+ I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2]/(2*2^(1/3)))/(3*(2*(1 + I*Sqrt[3]))^(
1/3))))/3
```

3.181. $\int \frac{1}{x^5(1-x^3+x^6)}\,dx$

3.181.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1704
```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Simp[1/(a*d^n*(m + 1))   Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1)
+ c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{
a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] &&
LtQ[m, -1] && IntegerQ[p]
```

3.181. $\int \frac{1}{x^5(1-x^3+x^6)}\,dx$

rule 1710
```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^n/2)*(b/q + 1)   Int[(d*x)^(m
- n)/(b/2 + q/2 + c*x^n), x], x] - Simp[(d^n/2)*(b/q - 1)   Int[(d*x)^(m -
n)/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

rule 1828
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Simp[1/(a*f^n*(m + 1))   Int[(f*x)^(m +
 n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) -
c*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x
] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Int
egerQ[p]
```

3.181.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.11

method	result	size
risch	$\frac{-x^3-\frac{1}{4}}{x^4}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(27\_Z^6-9\_Z^3+1\right)}\_R\ln\left(-27\_R^5+6\_R^2+x\right)\right)}{3}$	46
default	$-\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^6-\_Z^3+1\right)}\frac{\_R^4\ln\left(x-\_R\right)}{2\_R^5-\_R^2}\right)}{3}-\frac{1}{4x^4}-\frac{1}{x}$	51

input
```
int(1/x^5/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
(-x^3-1/4)/x^4+1/3*sum(_R*ln(-27*_R^5+6*_R^2+x),_R=RootOf(27*_Z^6-9*_Z^3+1
))
```

3.181.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 338, normalized size of antiderivative = 0.80

$$\int \frac{1}{x^5\,(1-x^3+x^6)}\,dx$$
$$= \frac{2\cdot 18^{\frac{2}{3}}x^4\left(-i\sqrt{3}+3\right)^{\frac{1}{3}}\log\left(18^{\frac{1}{3}}\left(i\sqrt{3}+1\right)\left(-i\sqrt{3}+3\right)^{\frac{2}{3}}+12\,x\right)+2\cdot 18^{\frac{2}{3}}x^4\left(i\sqrt{3}+3\right)^{\frac{1}{3}}\log\left(18^{\frac{1}{3}}\left(i\sqrt{3}+\right.\right.}{}$$

input
```
integrate(1/x^5/(x^6-x^3+1),x, algorithm="fricas")
```

output
```
1/108*(2*18^(2/3)*x^4*(-I*sqrt(3) + 3)^(1/3)*log(18^(1/3)*(I*sqrt(3) + 1)*
(-I*sqrt(3) + 3)^(2/3) + 12*x) + 2*18^(2/3)*x^4*(I*sqrt(3) + 3)^(1/3)*log(
18^(1/3)*(I*sqrt(3) + 3)^(2/3)*(-I*sqrt(3) + 1) + 12*x) - 108*x^3 + 18^(2/
3)*(sqrt(-3)*x^4 - x^4)*(I*sqrt(3) + 3)^(1/3)*log(18^(1/3)*(sqrt(3)*(I*sqr
t(-3) + I) - sqrt(-3) - 1)*(I*sqrt(3) + 3)^(2/3) + 24*x) - 18^(2/3)*(sqrt(
-3)*x^4 + x^4)*(I*sqrt(3) + 3)^(1/3)*log(18^(1/3)*(sqrt(3)*(-I*sqrt(-3) +
I) + sqrt(-3) - 1)*(I*sqrt(3) + 3)^(2/3) + 24*x) - 18^(2/3)*(sqrt(-3)*x^4
+ x^4)*(-I*sqrt(3) + 3)^(1/3)*log(18^(1/3)*(sqrt(3)*(I*sqrt(-3) - I) + sqr
t(-3) - 1)*(-I*sqrt(3) + 3)^(2/3) + 24*x) + 18^(2/3)*(sqrt(-3)*x^4 - x^4)*
(-I*sqrt(3) + 3)^(1/3)*log(18^(1/3)*(sqrt(3)*(-I*sqrt(-3) - I) - sqrt(-3)
- 1)*(-I*sqrt(3) + 3)^(2/3) + 24*x) - 27)/x^4
```

3.181.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.09

$$\int \frac{1}{x^5\,(1-x^3+x^6)}\,dx = \operatorname{RootSum}\left(19683t^6-243t^3+1,\left(t\mapsto t\log\left(-6561t^5+54t^2+x\right)\right)\right) + \frac{-4x^3-1}{4x^4}$$

input
```
integrate(1/x**5/(x**6-x**3+1),x)
```

output
```
RootSum(19683*_t**6 - 243*_t**3 + 1, Lambda(_t, _t*log(-6561*_t**5 + 54*_t
**2 + x))) + (-4*x**3 - 1)/(4*x**4)
```

3.181.7 Maxima [F]

$$\int \frac{1}{x^5 (1 - x^3 + x^6)} \, dx = \int \frac{1}{(x^6 - x^3 + 1)x^5} \, dx$$

input
```
integrate(1/x^5/(x^6-x^3+1),x, algorithm="maxima")
```

output
```
-1/4*(4*x^3 + 1)/x^4 - integrate(x^4/(x^6 - x^3 + 1), x)
```

3.181.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 839 vs. 2(277) = 554.

Time = 0.29 (sec) , antiderivative size = 839, normalized size of antiderivative = 1.98

$$\int \frac{1}{x^5 (1 - x^3 + x^6)} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^5/(x^6-x^3+1),x, algorithm="giac")
```

output
```
1/9*(2*sqrt(3)*cos(4/9*pi)^5 - 20*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi)^2 + 10
*sqrt(3)*cos(4/9*pi)*sin(4/9*pi)^4 - 10*cos(4/9*pi)^4*sin(4/9*pi) + 20*cos
(4/9*pi)^2*sin(4/9*pi)^3 - 2*sin(4/9*pi)^5 + sqrt(3)*cos(4/9*pi)^2 - sqrt(
3)*sin(4/9*pi)^2 - 2*cos(4/9*pi)*sin(4/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)
*cos(4/9*pi) + 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(4/9*pi))) + 1/9*(2*sqrt(3)*
cos(2/9*pi)^5 - 20*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi)^2 + 10*sqrt(3)*cos(2/
9*pi)*sin(2/9*pi)^4 - 10*cos(2/9*pi)^4*sin(2/9*pi) + 20*cos(2/9*pi)^2*sin(
2/9*pi)^3 - 2*sin(2/9*pi)^5 + sqrt(3)*cos(2/9*pi)^2 - sqrt(3)*sin(2/9*pi)^
2 - 2*cos(2/9*pi)*sin(2/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(2/9*pi) +
2*x)/((1/2*I*sqrt(3) + 1/2)*sin(2/9*pi))) - 1/9*(2*sqrt(3)*cos(1/9*pi)^5 -
 20*sqrt(3)*cos(1/9*pi)^3*sin(1/9*pi)^2 + 10*sqrt(3)*cos(1/9*pi)*sin(1/9*p
i)^4 + 10*cos(1/9*pi)^4*sin(1/9*pi) - 20*cos(1/9*pi)^2*sin(1/9*pi)^3 + 2*s
in(1/9*pi)^5 - sqrt(3)*cos(1/9*pi)^2 + sqrt(3)*sin(1/9*pi)^2 - 2*cos(1/9*p
i)*sin(1/9*pi))*arctan(-1/2*((-I*sqrt(3) - 1)*cos(1/9*pi) - 2*x)/((1/2*I*s
qrt(3) + 1/2)*sin(1/9*pi))) + 1/18*(10*sqrt(3)*cos(4/9*pi)^4*sin(4/9*pi) -
 20*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^3 + 2*sqrt(3)*sin(4/9*pi)^5 + 2*cos(
4/9*pi)^5 - 20*cos(4/9*pi)^3*sin(4/9*pi)^2 + 10*cos(4/9*pi)*sin(4/9*pi)^4
+ 2*sqrt(3)*cos(4/9*pi)*sin(4/9*pi) + cos(4/9*pi)^2 - sin(4/9*pi)^2)*log((
-I*sqrt(3)*cos(4/9*pi) - cos(4/9*pi))*x + x^2 + 1) + 1/18*(10*sqrt(3)*cos(
2/9*pi)^4*sin(2/9*pi) - 20*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^3 + 2*sqrt...
```

3.181. $\int \frac{1}{x^5(1-x^3+x^6)}\,dx$

3.181.9 Mupad [B] (verification not implemented)

Time = 8.72 (sec) , antiderivative size = 318, normalized size of antiderivative = 0.75

$$\int \frac{1}{x^5 (1 - x^3 + x^6)} dx$$
$$= \frac{\ln\left(-x + \left(162\,x + \frac{27\left(36+\sqrt{3}\,12\mathrm{i}\right)^{2/3}}{4}\right)\left(\frac{1}{162} + \frac{\sqrt{3}\,1\mathrm{i}}{486}\right)\right)\left(36 + \sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18}$$
$$+ \frac{\ln\left(-x - \left(162\,x + \frac{27\left(36-\sqrt{3}\,12\mathrm{i}\right)^{2/3}}{4}\right)\left(-\frac{1}{162} + \frac{\sqrt{3}\,1\mathrm{i}}{486}\right)\right)\left(36 - \sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18} - \frac{x^3 + \frac{1}{4}}{x^4}$$
$$- \frac{2^{2/3}\ln\left(x + \frac{2^{1/3}\,3^{2/3}\left(3-\sqrt{3}\,1\mathrm{i}\right)^{2/3}}{12} - \frac{2^{1/3}\,3^{1/6}\left(3-\sqrt{3}\,1\mathrm{i}\right)^{2/3}1\mathrm{i}}{4}\right)\left(3 - \sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3} - 3^{5/6}\,1\mathrm{i}\right)}{36}$$
$$- \frac{2^{2/3}\ln\left(x + \frac{2^{1/3}\,3^{2/3}\left(3+\sqrt{3}\,1\mathrm{i}\right)^{2/3}}{12} + \frac{2^{1/3}\,3^{1/6}\left(3+\sqrt{3}\,1\mathrm{i}\right)^{2/3}1\mathrm{i}}{4}\right)\left(3 + \sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3} + 3^{5/6}\,1\mathrm{i}\right)}{36}$$
$$- \frac{2^{2/3}\ln\left(x - \frac{2^{1/3}\,3^{2/3}\left(3-\sqrt{3}\,1\mathrm{i}\right)^{2/3}}{6}\right)\left(3 - \sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3} + 3^{5/6}\,1\mathrm{i}\right)}{36}$$
$$- \frac{2^{2/3}\ln\left(x - \frac{2^{1/3}\,3^{2/3}\left(3+\sqrt{3}\,1\mathrm{i}\right)^{2/3}}{6}\right)\left(3 + \sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3} - 3^{5/6}\,1\mathrm{i}\right)}{36}$$

input
```
int(1/(x^5*(x^6 - x^3 + 1)),x)
```

output
```
(log((162*x + (27*(3^(1/2)*12i + 36)^(2/3))/4)*((3^(1/2)*1i)/486 + 1/162)
- x)*(3^(1/2)*12i + 36)^(1/3))/18 + (log(- x - (162*x + (27*(36 - 3^(1/2)*
12i)^(2/3))/4)*((3^(1/2)*1i)/486 - 1/162))*(36 - 3^(1/2)*12i)^(1/3))/18 -
(x^3 + 1/4)/x^4 - (2^(2/3)*log(x + (2^(1/3)*3^(2/3)*(3 - 3^(1/2)*1i)^(2/3)
)/12 - (2^(1/3)*3^(1/6)*(3 - 3^(1/2)*1i)^(2/3)*1i)/4)*(3 - 3^(1/2)*1i)^(1/
3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x + (2^(1/3)*3^(2/3)*(3^(1/2)
*1i + 3)^(2/3))/12 + (2^(1/3)*3^(1/6)*(3^(1/2)*1i + 3)^(2/3)*1i)/4)*(3^(1/
2)*1i + 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(1/3)*3^
(2/3)*(3 - 3^(1/2)*1i)^(2/3))/6)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) + 3^(5/6)
*1i))/36 - (2^(2/3)*log(x - (2^(1/3)*3^(2/3)*(3^(1/2)*1i + 3)^(2/3))/6)*(3
^(1/2)*1i + 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36
```

3.182 $\int \frac{1}{2+x^3+x^6}\,dx$

3.182.1 Optimal result

Integrand size = 10, antiderivative size = 381

$$\int \frac{1}{2+x^3+x^6}\,dx = \frac{i\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}}}{\sqrt{3}}\right)}{\sqrt{21}\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}} - \frac{i\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}}}{\sqrt{3}}\right)}{\sqrt{21}\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}} - \frac{i\log\left(\sqrt[3]{1-i\sqrt{7}}+\sqrt[3]{2}x\right)}{3\sqrt{7}\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}} + \frac{i\log\left(\sqrt[3]{1+i\sqrt{7}}+\sqrt[3]{2}x\right)}{3\sqrt{7}\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}} + \frac{i\log\left(\left(1-i\sqrt{7}\right)^{2/3}-\sqrt[3]{2\left(1-i\sqrt{7}\right)}x+2^{2/3}x^2\right)}{3\sqrt[3]{2}\sqrt{7}\left(1-i\sqrt{7}\right)^{2/3}} - \frac{i\log\left(\left(1+i\sqrt{7}\right)^{2/3}-\sqrt[3]{2\left(1+i\sqrt{7}\right)}x+2^{2/3}x^2\right)}{3\sqrt[3]{2}\sqrt{7}\left(1+i\sqrt{7}\right)^{2/3}}$$

output
```
-1/21*I*2^(2/3)*ln(2^(1/3)*x+(1-I*7^(1/2))^(1/3))/(1-I*7^(1/2))^(2/3)*7^(1
/2)+1/42*I*ln(2^(2/3)*x^2-2^(1/3)*x*(1-I*7^(1/2))^(1/3)+(1-I*7^(1/2))^(2/3
))*2^(2/3)/(1-I*7^(1/2))^(2/3)*7^(1/2)+1/21*I*2^(2/3)*ln(2^(1/3)*x+(1+I*7^
(1/2))^(1/3))/(1+I*7^(1/2))^(2/3)*7^(1/2)-1/42*I*ln(2^(2/3)*x^2-2^(1/3)*x*
(1+I*7^(1/2))^(1/3)+(1+I*7^(1/2))^(2/3))*2^(2/3)/(1+I*7^(1/2))^(2/3)*7^(1/
2)+1/21*I*2^(2/3)*arctan(1/3*(1-2*2^(1/3)*x/(1-I*7^(1/2))^(1/3))*3^(1/2))/
(1-I*7^(1/2))^(2/3)*21^(1/2)-1/21*I*2^(2/3)*arctan(1/3*(1-2*2^(1/3)*x/(1+I
*7^(1/2))^(1/3))*3^(1/2))/(1+I*7^(1/2))^(2/3)*21^(1/2)
```

3.182.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.10

$$\int \frac{1}{2+x^3+x^6}\,dx=\frac{1}{3}\text{RootSum}\left[2+\#1^3+\#1^6\&,\frac{\log(x-\#1)}{\#1^2+2\#1^5}\&\right]$$

input
```
Integrate[(2 + x^3 + x^6)^(-1),x]
```

output
```
RootSum[2 + #1^3 + #1^6 & , Log[x - #1]/(#1^2 + 2*#1^5) & ]/3
```

3.182.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 339, normalized size of antiderivative = 0.89, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {1685, 750, 16, 1142, 25, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6+x^3+2}dx$$

$$\Big\downarrow \text{1685}$$

$$\frac{i\int \frac{1}{x^3+\frac{1}{2}\left(1+i\sqrt{7}\right)}dx}{\sqrt{7}}-\frac{i\int \frac{1}{x^3+\frac{1}{2}\left(1-i\sqrt{7}\right)}dx}{\sqrt{7}}$$

$$\Big\downarrow \text{750}$$

$$\frac{i\left(\frac{\int \frac{2^{2/3}\sqrt[3]{1+i\sqrt{7}}-x}{x^2-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}+\frac{\int \frac{1}{x+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}\right)}{\sqrt{7}}-\frac{i\left(\frac{\int \frac{2^{2/3}\sqrt[3]{1-i\sqrt{7}}-x}{x^2-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}+\frac{\int \frac{1}{x+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}\right)}{\sqrt{7}}$$

$$\Big\downarrow \text{16}$$

$$\frac{i\left(\frac{\int \frac{2^{2/3}\sqrt[3]{1+i\sqrt{7}}-x}{x^2-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}+\frac{\log\left(\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{7}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}\right)}{\sqrt{7}}-\frac{i\left(\frac{\int \frac{2^{2/3}\sqrt[3]{1-i\sqrt{7}}-x}{x^2-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}+\frac{\log\left(\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{7}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}\right)}{\sqrt{7}}$$

$$\Big\downarrow \text{1142}$$

3.182. $\int \frac{1}{2+x^3+x^6}\,dx$

$$\frac{i\left(\frac{\frac{3}{2}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}\int \frac{1}{x^2-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}dx-\frac{1}{2}\int -\frac{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}-2x}{x^2-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}+\frac{\log\left(\sqrt[3]{2}x+\sqrt[3]{\cdots}\right.}{3\left(\frac{1}{2}\left(1+i\cdots\right.\right.}\right.}{\sqrt{7}}$$

$$\frac{i\left(\frac{\frac{3}{2}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}\int \frac{1}{x^2-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}dx-\frac{1}{2}\int -\frac{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}-2x}{x^2-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}+\frac{\log\left(\sqrt[3]{2}x+\sqrt[3]{\cdots}\right.}{3\left(\frac{1}{2}\left(1-i\cdots\right.\right.}\right.}{\sqrt{7}}$$

$\downarrow$ 25

$$\frac{i\left(\frac{\frac{3}{2}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}\int \frac{1}{x^2-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}-2x}{x^2-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}+\frac{\log\left(\sqrt[3]{2}x+\sqrt[3]{1\cdots}\right.}{3\left(\frac{1}{2}\left(1+i\sqrt{7}\cdots\right.\right.}\right.}{\sqrt{7}}$$

$$\frac{i\left(\frac{\frac{3}{2}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}\int \frac{1}{x^2-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}-2x}{x^2-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}+\frac{\log\left(\sqrt[3]{2}x+\sqrt[3]{1\cdots}\right.}{3\left(\frac{1}{2}\left(1-i\sqrt{7}\cdots\right.\right.}\right.}{\sqrt{7}}$$

$\downarrow$ 1082

3.182. $\int \frac{1}{2+x^3+x^6}\,dx$

$$\frac{i\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}-2x}{x^2-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}dx+3\int \frac{1}{-\left(1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}}\right)^2-3}d\left(1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}+\frac{\log\left(\sqrt[3]{2}x+\sqrt[3]{1+\right.}{3\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2}}\right.}{\sqrt{7}}$$

$$\frac{i\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}-2x}{x^2-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}dx+3\int \frac{1}{-\left(1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}}\right)^2-3}d\left(1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}+\frac{\log\left(\sqrt[3]{2}x+\sqrt[3]{1-\right.}{3\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2}}\right.}{\sqrt{7}}$$

$$\Big\downarrow \; 217$$

3.182. $\int \frac{1}{2+x^3+x^6}\,dx$

$$\frac{i\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}-2x}{x^2-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}}}{\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}+\frac{\log\left(\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{7}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}\right)}{\sqrt{7}}-\frac{i\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}-2x}{x^2-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}}}{\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}+\frac{\log\left(\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{7}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}\right)}{\sqrt{7}}$$

$$\Big\downarrow \ 1103$$

$$\frac{i\left(\frac{-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}}}{\sqrt{3}}\right)-\frac{1}{2}\log\left(2^{2/3}x^2-\sqrt[3]{2\left(1+i\sqrt{7}\right)}x+\left(1+i\sqrt{7}\right)^{2/3}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}+\frac{\log\left(\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{7}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}\right)}{\sqrt{7}}-\frac{i\left(\frac{-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}}}{\sqrt{3}}\right)-\frac{1}{2}\log\left(2^{2/3}x^2-\sqrt[3]{2\left(1-i\sqrt{7}\right)}x+\left(1-i\sqrt{7}\right)^{2/3}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}+\frac{\log\left(\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{7}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}\right)}{\sqrt{7}}$$

3.182. $\int \frac{1}{2+x^3+x^6}\,dx$

input
```
Int[(2 + x^3 + x^6)^(-1),x]
```

output
```
((-I)*(Log[(1 - I*Sqrt[7])^(1/3) + 2^(1/3)*x]/(3*((1 - I*Sqrt[7])/2)^(2/3)
) + (-(Sqrt[3]*ArcTan[(1 - (2*x)/((1 - I*Sqrt[7])/2)^(1/3))/Sqrt[3]]) - Lo
g[(1 - I*Sqrt[7])^(2/3) - (2*(1 - I*Sqrt[7]))^(1/3)*x + 2^(2/3)*x^2]/2)/(3
*((1 - I*Sqrt[7])/2)^(2/3))))/Sqrt[7] + (I*(Log[(1 + I*Sqrt[7])^(1/3) + 2^
(1/3)*x]/(3*((1 + I*Sqrt[7])/2)^(2/3)) + (-(Sqrt[3]*ArcTan[(1 - (2*x)/((1
+ I*Sqrt[7])/2)^(1/3))/Sqrt[3]]) - Log[(1 + I*Sqrt[7])^(2/3) - (2*(1 + I*S
qrt[7]))^(1/3)*x + 2^(2/3)*x^2]/2)/(3*((1 + I*Sqrt[7])/2)^(2/3))))/Sqrt[7]
```

3.182.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1685
```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^n), x], x] - Simp[
c/q   Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2,
2*n] && NeQ[b^2 - 4*a*c, 0]
```

3.182.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.09

method	result	size
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^6+\_Z^3+2\right)}\frac{\ln\left(x-\_R\right)}{2\_R^5+\_R^2}}{3}$	33
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^6+\_Z^3+2\right)}\frac{\ln\left(x-\_R\right)}{2\_R^5+\_R^2}}{3}$	33

input
```
int(1/(x^6+x^3+2),x,method=_RETURNVERBOSE)
```

output
```
1/3*sum(1/(2*_R^5+_R^2)*ln(x-_R),_R=RootOf(_Z^6+_Z^3+2))
```

3.182.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.78

$$\int \frac{1}{2+x^3+x^6}\,dx = -\frac{1}{588} \cdot 49^{\frac{2}{3}}\left(3i\sqrt{7}-7\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\log\left(49^{\frac{2}{3}}\left(\sqrt{7}(i\sqrt{-3}+i)-7\sqrt{-3}-7\right)\left(3i\sqrt{7}-7\right)^{\frac{1}{3}}+392\,x\right)+\frac{1}{588} \cdot 49^{\frac{2}{3}}\left(3i\sqrt{7}-7\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\log\left(49^{\frac{2}{3}}\left(\sqrt{7}(-i\sqrt{-3}+i)+7\sqrt{-3}-7\right)\left(3i\sqrt{7}-7\right)^{\frac{1}{3}}+392\,x\right)+\frac{1}{588} \cdot 49^{\frac{2}{3}}\left(-3i\sqrt{7}-7\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\log\left(49^{\frac{2}{3}}\left(\sqrt{7}(i\sqrt{-3}-i)+7\sqrt{-3}-7\right)\left(-3i\sqrt{7}-7\right)^{\frac{1}{3}}+392\,x\right)-\frac{1}{588} \cdot 49^{\frac{2}{3}}\left(-3i\sqrt{7}-7\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\log\left(49^{\frac{2}{3}}\left(\sqrt{7}(-i\sqrt{-3}-i)-7\sqrt{-3}-7\right)\left(-3i\sqrt{7}-7\right)^{\frac{1}{3}}+392\,x\right)+\frac{1}{294} \cdot 49^{\frac{2}{3}}\left(3i\sqrt{7}-7\right)^{\frac{1}{3}}\log\left(49^{\frac{2}{3}}\left(3i\sqrt{7}-7\right)^{\frac{1}{3}}\left(-i\sqrt{7}+7\right)+196\,x\right)+\frac{1}{294} \cdot 49^{\frac{2}{3}}\left(-3i\sqrt{7}-7\right)^{\frac{1}{3}}\log\left(49^{\frac{2}{3}}\left(i\sqrt{7}+7\right)\left(-3i\sqrt{7}-7\right)^{\frac{1}{3}}+196\,x\right)$$

input
```
integrate(1/(x^6+x^3+2),x, algorithm="fricas")
```

output
```
-1/588*49^(2/3)*(3*I*sqrt(7) - 7)^(1/3)*(sqrt(-3) + 1)*log(49^(2/3)*(sqrt(
7)*(I*sqrt(-3) + I) - 7*sqrt(-3) - 7)*(3*I*sqrt(7) - 7)^(1/3) + 392*x) + 1
/588*49^(2/3)*(3*I*sqrt(7) - 7)^(1/3)*(sqrt(-3) - 1)*log(49^(2/3)*(sqrt(7)
*(-I*sqrt(-3) + I) + 7*sqrt(-3) - 7)*(3*I*sqrt(7) - 7)^(1/3) + 392*x) + 1/
588*49^(2/3)*(-3*I*sqrt(7) - 7)^(1/3)*(sqrt(-3) - 1)*log(49^(2/3)*(sqrt(7)
*(I*sqrt(-3) - I) + 7*sqrt(-3) - 7)*(-3*I*sqrt(7) - 7)^(1/3) + 392*x) - 1/
588*49^(2/3)*(-3*I*sqrt(7) - 7)^(1/3)*(sqrt(-3) + 1)*log(49^(2/3)*(sqrt(7)
*(-I*sqrt(-3) - I) - 7*sqrt(-3) - 7)*(-3*I*sqrt(7) - 7)^(1/3) + 392*x) + 1
/294*49^(2/3)*(3*I*sqrt(7) - 7)^(1/3)*log(49^(2/3)*(3*I*sqrt(7) - 7)^(1/3)
*(-I*sqrt(7) + 7) + 196*x) + 1/294*49^(2/3)*(-3*I*sqrt(7) - 7)^(1/3)*log(4
9^(2/3)*(I*sqrt(7) + 7)*(-3*I*sqrt(7) - 7)^(1/3) + 196*x)
```

3.182.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.06

$$\int \frac{1}{2+x^3+x^6}\,dx = \text{RootSum}\left(1000188t^6+1323t^3+1,\left(t\mapsto t\log\left(-5292t^4+7t+x\right)\right)\right)$$

input
```
integrate(1/(x**6+x**3+2),x)
```

output
```
RootSum(1000188*_t**6 + 1323*_t**3 + 1, Lambda(_t, _t*log(-5292*_t**4 + 7*
_t + x)))
```

3.182.7 Maxima [F]

$$\int \frac{1}{2+x^3+x^6}\,dx = \int \frac{1}{x^6+x^3+2}\,dx$$

input
```
integrate(1/(x^6+x^3+2),x, algorithm="maxima")
```

output
```
integrate(1/(x^6 + x^3 + 2), x)
```

3.182.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{2+x^3+x^6}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(x^6+x^3+2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Invalid _EXT in replace_ext Error: Bad Argument Valuein
tegrate(1/(sageVARx^6+sageVARx^3+2),sageVARx)
```

3.182.9 Mupad [B] (verification not implemented)

Time = 9.36 (sec) , antiderivative size = 513, normalized size of antiderivative = 1.35

$$
\begin{aligned}
\int \frac{1}{2+x^3+x^6}\,dx &= \frac{\ln\left(x+\frac{7^{1/3}\left(-7-\sqrt{7}\,3i\right)^{1/3}}{4}+\frac{7^{5/6}\left(-7-\sqrt{7}\,3i\right)^{1/3}\,1i}{28}\right)\left(-49-\sqrt{7}\,21i\right)^{1/3}}{42} \\
&+ \frac{\ln\left(x+\frac{7^{1/3}\left(-7+\sqrt{7}\,3i\right)^{1/3}}{4}-\frac{7^{5/6}\left(-7+\sqrt{7}\,3i\right)^{1/3}\,1i}{28}\right)\left(-49+\sqrt{7}\,21i\right)^{1/3}}{42} \\
&+ \frac{7^{1/3}\ln\left(6\,x+\frac{7^{1/3}\left(-1+\sqrt{3}\,1i\right)\left(-7-\sqrt{7}\,3i\right)^{1/3}\left(\frac{7^{2/3}\left(-1+\sqrt{3}\,1i\right)^2\left(-7-\sqrt{7}\,3i\right)^{2/3}\left(3969\,x+\frac{567\,7^{1/3}\left(-1+\sqrt{3}\,1i\right)\left(-7-\sqrt{7}\,3i\right)^{1/3}}{2}\right)}{7056}+63\right)}{84}\right)}{84} \\
&+ \frac{7^{1/3}\ln\left(6\,x+\frac{7^{1/3}\left(-1+\sqrt{3}\,1i\right)\left(-7+\sqrt{7}\,3i\right)^{1/3}\left(\frac{7^{2/3}\left(-1+\sqrt{3}\,1i\right)^2\left(-7+\sqrt{7}\,3i\right)^{2/3}\left(3969\,x+\frac{567\,7^{1/3}\left(-1+\sqrt{3}\,1i\right)\left(-7+\sqrt{7}\,3i\right)^{1/3}}{2}\right)}{7056}+63\right)}{84}\right)}{84} \\
&- \frac{7^{1/3}\ln\left(6\,x-\frac{7^{1/3}\left(1+\sqrt{3}\,1i\right)\left(-7-\sqrt{7}\,3i\right)^{1/3}\left(\frac{7^{2/3}\left(1+\sqrt{3}\,1i\right)^2\left(-7-\sqrt{7}\,3i\right)^{2/3}\left(3969\,x-\frac{567\,7^{1/3}\left(1+\sqrt{3}\,1i\right)\left(-7-\sqrt{7}\,3i\right)^{1/3}}{2}\right)}{7056}+63\right)}{84}\right)}{84} \\
&- \frac{7^{1/3}\ln\left(6\,x-\frac{7^{1/3}\left(1+\sqrt{3}\,1i\right)\left(-7+\sqrt{7}\,3i\right)^{1/3}\left(\frac{7^{2/3}\left(1+\sqrt{3}\,1i\right)^2\left(-7+\sqrt{7}\,3i\right)^{2/3}\left(3969\,x-\frac{567\,7^{1/3}\left(1+\sqrt{3}\,1i\right)\left(-7+\sqrt{7}\,3i\right)^{1/3}}{2}\right)}{7056}+63\right)}{84}\right)}{84}
\end{aligned}
$$

input
```
int(1/(x^3 + x^6 + 2),x)
```

output
```
(log(x + (7^(1/3)*(- 7^(1/2)*3i - 7)^(1/3))/4 + (7^(5/6)*(- 7^(1/2)*3i - 7
)^(1/3)*1i)/28)*(- 7^(1/2)*21i - 49)^(1/3))/42 + (log(x + (7^(1/3)*(7^(1/2
)*3i - 7)^(1/3))/4 - (7^(5/6)*(7^(1/2)*3i - 7)^(1/3)*1i)/28)*(7^(1/2)*21i
- 49)^(1/3))/42 + (7^(1/3)*log(6*x + (7^(1/3)*(3^(1/2)*1i - 1)*(- 7^(1/2)*
3i - 7)^(1/3)*((7^(2/3)*(3^(1/2)*1i - 1)^2*(- 7^(1/2)*3i - 7)^(2/3)*(3969*
x + (567*7^(1/3)*(3^(1/2)*1i - 1)*(- 7^(1/2)*3i - 7)^(1/3))/2))/7056 + 63)
)/84)*(3^(1/2)*1i - 1)*(- 7^(1/2)*3i - 7)^(1/3))/84 + (7^(1/3)*log(6*x + (
7^(1/3)*(3^(1/2)*1i - 1)*(7^(1/2)*3i - 7)^(1/3)*((7^(2/3)*(3^(1/2)*1i - 1)
^2*(7^(1/2)*3i - 7)^(2/3)*(3969*x + (567*7^(1/3)*(3^(1/2)*1i - 1)*(7^(1/2)
*3i - 7)^(1/3))/2))/7056 + 63))/84)*(3^(1/2)*1i - 1)*(7^(1/2)*3i - 7)^(1/3
))/84 - (7^(1/3)*log(6*x - (7^(1/3)*(3^(1/2)*1i + 1)*(- 7^(1/2)*3i - 7)^(1
/3)*((7^(2/3)*(3^(1/2)*1i + 1)^2*(- 7^(1/2)*3i - 7)^(2/3)*(3969*x - (567*7
^(1/3)*(3^(1/2)*1i + 1)*(- 7^(1/2)*3i - 7)^(1/3))/2))/7056 + 63))/84)*(3^(
1/2)*1i + 1)*(- 7^(1/2)*3i - 7)^(1/3))/84 - (7^(1/3)*log(6*x - (7^(1/3)*(3
^(1/2)*1i + 1)*(7^(1/2)*3i - 7)^(1/3)*((7^(2/3)*(3^(1/2)*1i + 1)^2*(7^(1/2
)*3i - 7)^(2/3)*(3969*x - (567*7^(1/3)*(3^(1/2)*1i + 1)*(7^(1/2)*3i - 7)^(
1/3))/2))/7056 + 63))/84)*(3^(1/2)*1i + 1)*(7^(1/2)*3i - 7)^(1/3))/84
```

3.182. $\int \frac{1}{2+x^3+x^6}\,dx$

3.183 $\int \frac{x^2}{2+x^3+x^6} dx$

3.183.1 Optimal result

Integrand size = 14, antiderivative size = 23

$$\int \frac{x^2}{2+x^3+x^6} \, dx = \frac{2\arctan\left(\frac{1+2x^3}{\sqrt{7}}\right)}{3\sqrt{7}}$$

output
```
2/21*arctan(1/7*(2*x^3+1)*7^(1/2))*7^(1/2)
```

3.183.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{2+x^3+x^6} \, dx = \frac{2\arctan\left(\frac{1+2x^3}{\sqrt{7}}\right)}{3\sqrt{7}}$$

input
```
Integrate[x^2/(2 + x^3 + x^6),x]
```

output
```
(2*ArcTan[(1 + 2*x^3)/Sqrt[7]])/(3*Sqrt[7])
```

3.183.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1690, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{x^6+x^3+2}\,dx$$

$$\Big\downarrow \; 1690$$

$$\frac{1}{3}\int \frac{1}{x^6+x^3+2}dx^3$$

$$\Big\downarrow \; 1083$$

$$-\frac{2}{3}\int \frac{1}{-x^6-7}d(2x^3+1)$$

$$\Big\downarrow \; 217$$

$$\frac{2\arctan\left(\frac{2x^3+1}{\sqrt{7}}\right)}{3\sqrt{7}}$$

input
```
Int[x^2/(2 + x^3 + x^6),x]
```

output
```
(2*ArcTan[(1 + 2*x^3)/Sqrt[7]])/(3*Sqrt[7])
```

3.183.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

3.183.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{2\arctan\left(\frac{(2x^3+1)\sqrt{7}}{7}\right)\sqrt{7}}{21}$	19
risch	$\frac{2\arctan\left(\frac{(2x^3+1)\sqrt{7}}{7}\right)\sqrt{7}}{21}$	19

input
```
int(x^2/(x^6+x^3+2),x,method=_RETURNVERBOSE)
```

output
```
2/21*arctan(1/7*(2*x^3+1)*7^(1/2))*7^(1/2)
```

3.183.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{x^2}{2+x^3+x^6}\,dx = \frac{2}{21}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2x^3+1)\right)$$

input
```
integrate(x^2/(x^6+x^3+2),x, algorithm="fricas")
```

output
```
2/21*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^3 + 1))
```

3.183.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x^2}{2+x^3+x^6}\,dx=\frac{2\sqrt{7}\,\mathrm{atan}\left(\frac{2\sqrt{7}x^3}{7}+\frac{\sqrt{7}}{7}\right)}{21}$$

input
```
integrate(x**2/(x**6+x**3+2),x)
```

output
```
2*sqrt(7)*atan(2*sqrt(7)*x**3/7 + sqrt(7)/7)/21
```

3.183.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{x^2}{2+x^3+x^6}\,dx=\frac{2}{21}\,\sqrt{7}\arctan\left(\frac{1}{7}\,\sqrt{7}(2\,x^3+1)\right)$$

input
```
integrate(x^2/(x^6+x^3+2),x, algorithm="maxima")
```

output
```
2/21*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^3 + 1))
```

3.183.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{x^2}{2+x^3+x^6}\,dx=\frac{2}{21}\,\sqrt{7}\arctan\left(\frac{1}{7}\,\sqrt{7}(2\,x^3+1)\right)$$

input
```
integrate(x^2/(x^6+x^3+2),x, algorithm="giac")
```

output
```
2/21*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^3 + 1))
```

3.183.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{x^2}{2+x^3+x^6}\,dx = \frac{2\sqrt{7}\,\mathrm{atan}\left(\frac{2\sqrt{7}\,x^3}{7}+\frac{\sqrt{7}}{7}\right)}{21}$$

input
```
int(x^2/(x^3 + x^6 + 2),x)
```

output
```
(2*7^(1/2)*atan(7^(1/2)/7 + (2*7^(1/2)*x^3)/7))/21
```

3.184 $\int \frac{x^3}{2+x^3+x^6} \, dx$

3.184.1 Optimal result

Integrand size = 14, antiderivative size = 399

$$\int \frac{x^3}{2+x^3+x^6}\,dx = -\frac{i\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}}}{\sqrt{3}}\right)}{\sqrt{21}} + \frac{i\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}}}{\sqrt{3}}\right)}{\sqrt{21}} + \frac{(7+i\sqrt{7})\log\left(\sqrt[3]{1-i\sqrt{7}}+\sqrt[3]{2}x\right)}{21\sqrt[3]{2}\left(1-i\sqrt{7}\right)^{2/3}} + \frac{(7-i\sqrt{7})\log\left(\sqrt[3]{1+i\sqrt{7}}+\sqrt[3]{2}x\right)}{21\sqrt[3]{2}\left(1+i\sqrt{7}\right)^{2/3}} - \frac{(7+i\sqrt{7})\log\left(\left(1-i\sqrt{7}\right)^{2/3}-\sqrt[3]{2\left(1-i\sqrt{7}\right)}x+2^{2/3}x^2\right)}{42\sqrt[3]{2}\left(1-i\sqrt{7}\right)^{2/3}} - \frac{(7-i\sqrt{7})\log\left(\left(1+i\sqrt{7}\right)^{2/3}-\sqrt[3]{2\left(1+i\sqrt{7}\right)}x+2^{2/3}x^2\right)}{42\sqrt[3]{2}\left(1+i\sqrt{7}\right)^{2/3}}$$

output
```
1/42*ln(2^(1/3)*x+(1+I*7^(1/2))^(1/3))*(7-I*7^(1/2))*2^(2/3)/(1+I*7^(1/2))
^(2/3)-1/84*ln(2^(2/3)*x^2-2^(1/3)*x*(1+I*7^(1/2))^(1/3)+(1+I*7^(1/2))^(2/
3))*(7-I*7^(1/2))*2^(2/3)/(1+I*7^(1/2))^(2/3)+1/42*ln(2^(1/3)*x+(1-I*7^(1/
2))^(1/3))*(7+I*7^(1/2))*2^(2/3)/(1-I*7^(1/2))^(2/3)-1/84*ln(2^(2/3)*x^2-2
^(1/3)*x*(1-I*7^(1/2))^(1/3)+(1-I*7^(1/2))^(2/3))*(7+I*7^(1/2))*2^(2/3)/(1
-I*7^(1/2))^(2/3)-1/42*I*arctan(1/3*(1-2*2^(1/3)*x/(1-I*7^(1/2))^(1/3))*3^
(1/2))*(1-I*7^(1/2))^(1/3)*2^(2/3)*21^(1/2)+1/42*I*arctan(1/3*(1-2*2^(1/3)
*x/(1+I*7^(1/2))^(1/3))*3^(1/2))*(1+I*7^(1/2))^(1/3)*2^(2/3)*21^(1/2)
```

3.184.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.09

$$\int \frac{x^3}{2+x^3+x^6}\,dx=\frac{1}{3}\text{RootSum}\left[2+\#1^3+\#1^6\&,\frac{\log(x-\#1)\#1}{1+2\#1^3}\&\right]$$

input
```
Integrate[x^3/(2 + x^3 + x^6),x]
```

output
```
RootSum[2 + #1^3 + #1^6 & , (Log[x - #1]*#1)/(1 + 2*#1^3) & ]/3
```

3.184.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 351, normalized size of antiderivative = 0.88, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {1710, 750, 16, 1142, 25, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{x^6+x^3+2}dx$$

$$\Big\downarrow \text{1710}$$

$$\frac{1}{14}\left(7+i\sqrt{7}\right)\int \frac{1}{x^3+\frac{1}{2}\left(1-i\sqrt{7}\right)}dx+\frac{1}{14}\left(7-i\sqrt{7}\right)\int \frac{1}{x^3+\frac{1}{2}\left(1+i\sqrt{7}\right)}dx$$

$$\Big\downarrow \text{750}$$

$$\frac{1}{14}\left(7+i\sqrt{7}\right)\left(\frac{\int \frac{2^{2/3}\sqrt[3]{1-i\sqrt{7}}-x}{x^2-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}+\frac{\int \frac{1}{x+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}\right)+\frac{1}{14}\left(7-i\sqrt{7}\right)\left(\frac{\int \frac{2^{2/3}\sqrt[3]{1+i\sqrt{7}}-x}{x^2-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}+\frac{\int \frac{1}{x+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}\right)$$

$$\Big\downarrow \; 16$$

$$\frac{1}{14}\left(7+i\sqrt{7}\right)\left(\frac{\int \frac{2^{2/3}\sqrt[3]{1-i\sqrt{7}}-x}{x^2-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}+\frac{\log\left(\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{7}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}\right)+\frac{1}{14}\left(7-i\sqrt{7}\right)\left(\frac{\int \frac{2^{2/3}\sqrt[3]{1+i\sqrt{7}}-x}{x^2-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}+\frac{\log\left(\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{7}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}\right)$$

$$\Big\downarrow \; 1142$$

3.184. $\int \frac{x^3}{2+x^3+x^6}\,dx$

$$\frac{1}{14}\left(7+i\sqrt{7}\right)\left(\frac{\frac{3}{2}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}\int\frac{1}{x^2-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}dx-\frac{1}{2}\int-\frac{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}-2x}{x^2-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right.}}{3\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}\right.$$

$$\frac{1}{14}\left(7-i\sqrt{7}\right)\left(\frac{\frac{3}{2}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}\int\frac{1}{x^2-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}dx-\frac{1}{2}\int-\frac{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}-2x}{x^2-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right.}}{3\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}\right.$$

$$\Big\downarrow \; 25$$

$$\frac{1}{14}\left(7+i\sqrt{7}\right)\left(\frac{\frac{3}{2}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}\int\frac{1}{x^2-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}dx+\frac{1}{2}\int\frac{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}-2x}{x^2-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2}}}{3\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}\right.$$

$$\frac{1}{14}\left(7-i\sqrt{7}\right)\left(\frac{\frac{3}{2}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}\int\frac{1}{x^2-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}dx+\frac{1}{2}\int\frac{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}-2x}{x^2-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2}}}{3\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}\right.$$

$$\Big\downarrow \; 1082$$

3.184. $\int \frac{x^3}{2+x^3+x^6}\,dx$

$$\frac{1}{14}\left(7+i\sqrt{7}\right)\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}-2x}{x^2-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}dx+3\int \frac{1}{-\left(1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}}\right)^2-3}d\left(1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}}\right.}{3\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}\right.$$

$$\frac{1}{14}\left(7-i\sqrt{7}\right)\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}-2x}{x^2-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}dx+3\int \frac{1}{-\left(1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}}\right)^2-3}d\left(1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}}\right.}{3\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}\right.$$

$$\Big\downarrow \; 217$$

3.184. $\int \frac{x^3}{2+x^3+x^6}\,dx$

$$\frac{1}{14}\left(7+i\sqrt{7}\right)\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}-2x}{x^2-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}}}{\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}+\frac{\log\left(\sqrt[3]{2}x+\sqrt[3]{1}\right.}{3\left(\frac{1}{2}\left(1-i\sqrt{}\right.\right.}\right.$$

$$\frac{1}{14}\left(7-i\sqrt{7}\right)\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}-2x}{x^2-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}}}{\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}+\frac{\log\left(\sqrt[3]{2}x+\sqrt[3]{1}\right.}{3\left(\frac{1}{2}\left(1+i\sqrt{}\right.\right.}\right.$$

$\downarrow$ 1103

$$\frac{1}{14}\left(7+i\sqrt{7}\right)\left(\frac{-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}}}{\sqrt{3}}\right)-\frac{1}{2}\log\left(2^{2/3}x^2-\sqrt[3]{2\left(1-i\sqrt{7}\right)}x+\left(1-i\sqrt{7}\right)^{2/3}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}+\frac{\log\left(\right.}{3\left(\right.}\right.$$

$$\frac{1}{14}\left(7-i\sqrt{7}\right)\left(\frac{-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}}}{\sqrt{3}}\right)-\frac{1}{2}\log\left(2^{2/3}x^2-\sqrt[3]{2\left(1+i\sqrt{7}\right)}x+\left(1+i\sqrt{7}\right)^{2/3}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}+\frac{\log\left(\right.}{3\left(\right.}\right.$$

input
```
Int[x^3/(2 + x^3 + x^6),x]
```

3.184. $\int \frac{x^3}{2+x^3+x^6}\,dx$

output
```
((7 + I*Sqrt[7])*(Log[(1 - I*Sqrt[7])^(1/3) + 2^(1/3)*x]/(3*((1 - I*Sqrt[7
])/2)^(2/3)) + (-(Sqrt[3]*ArcTan[(1 - (2*x)/((1 - I*Sqrt[7])/2)^(1/3))/Sqr
t[3]]) - Log[(1 - I*Sqrt[7])^(2/3) - (2*(1 - I*Sqrt[7]))^(1/3)*x + 2^(2/3)
*x^2]/2)/(3*((1 - I*Sqrt[7])/2)^(2/3))))/14 + ((7 - I*Sqrt[7])*(Log[(1 + I
*Sqrt[7])^(1/3) + 2^(1/3)*x]/(3*((1 + I*Sqrt[7])/2)^(2/3)) + (-(Sqrt[3]*Ar
cTan[(1 - (2*x)/((1 + I*Sqrt[7])/2)^(1/3))/Sqrt[3]]) - Log[(1 + I*Sqrt[7])
^(2/3) - (2*(1 + I*Sqrt[7]))^(1/3)*x + 2^(2/3)*x^2]/2)/(3*((1 + I*Sqrt[7])
/2)^(2/3))))/14
```

3.184.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1710
```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^n/2)*(b/q + 1)   Int[(d*x)^(m
- n)/(b/2 + q/2 + c*x^n), x], x] - Simp[(d^n/2)*(b/q - 1)   Int[(d*x)^(m -
n)/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

3.184.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.09

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}+\_Z^{3}+2\right)}\frac{\_R^{3}\ln\left(x-\_R\right)}{2\_R^{5}+\_R^{2}}\right)}{3}$	36
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}+\_Z^{3}+2\right)}\frac{\_R^{3}\ln\left(x-\_R\right)}{2\_R^{5}+\_R^{2}}\right)}{3}$	36

input
```
int(x^3/(x^6+x^3+2),x,method=_RETURNVERBOSE)
```

output
```
1/3*sum(_R^3/(2*_R^5+_R^2)*ln(x-_R),_R=RootOf(_Z^6+_Z^3+2))
```

3.184.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.65

$$\int \frac{x^3}{2+x^3+x^6}\,dx = -\frac{1}{588} \cdot 98^{\frac{2}{3}}\left(-i\sqrt{7}-7\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\log\left(98^{\frac{2}{3}}\sqrt{7}\left(-i\sqrt{7}-7\right)^{\frac{1}{3}}\left(i\sqrt{-3}+i\right)+196\,x\right)+\frac{1}{588} \cdot 98^{\frac{2}{3}}\left(i\sqrt{7}-7\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\log\left(98^{\frac{2}{3}}\sqrt{7}\left(i\sqrt{7}-7\right)^{\frac{1}{3}}\left(i\sqrt{-3}-i\right)+196\,x\right)+\frac{1}{588} \cdot 98^{\frac{2}{3}}\left(-i\sqrt{7}-7\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\log\left(98^{\frac{2}{3}}\sqrt{7}\left(-i\sqrt{7}-7\right)^{\frac{1}{3}}\left(-i\sqrt{-3}+i\right)+196\,x\right)-\frac{1}{588} \cdot 98^{\frac{2}{3}}\left(i\sqrt{7}-7\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\log\left(98^{\frac{2}{3}}\sqrt{7}\left(i\sqrt{7}-7\right)^{\frac{1}{3}}\left(-i\sqrt{-3}-i\right)+196\,x\right)+\frac{1}{294} \cdot 98^{\frac{2}{3}}\left(i\sqrt{7}-7\right)^{\frac{1}{3}}\log\left(i \cdot 98^{\frac{2}{3}}\sqrt{7}\left(i\sqrt{7}-7\right)^{\frac{1}{3}}+98\,x\right)+\frac{1}{294} \cdot 98^{\frac{2}{3}}\left(-i\sqrt{7}-7\right)^{\frac{1}{3}}\log\left(-i \cdot 98^{\frac{2}{3}}\sqrt{7}\left(-i\sqrt{7}-7\right)^{\frac{1}{3}}+98\,x\right)$$

input
```
integrate(x^3/(x^6+x^3+2),x, algorithm="fricas")
```

output
```
-1/588*98^(2/3)*(-I*sqrt(7) - 7)^(1/3)*(sqrt(-3) + 1)*log(98^(2/3)*sqrt(7)
*(-I*sqrt(7) - 7)^(1/3)*(I*sqrt(-3) + I) + 196*x) + 1/588*98^(2/3)*(I*sqrt
(7) - 7)^(1/3)*(sqrt(-3) - 1)*log(98^(2/3)*sqrt(7)*(I*sqrt(7) - 7)^(1/3)*(
I*sqrt(-3) - I) + 196*x) + 1/588*98^(2/3)*(-I*sqrt(7) - 7)^(1/3)*(sqrt(-3)
 - 1)*log(98^(2/3)*sqrt(7)*(-I*sqrt(7) - 7)^(1/3)*(-I*sqrt(-3) + I) + 196*
x) - 1/588*98^(2/3)*(I*sqrt(7) - 7)^(1/3)*(sqrt(-3) + 1)*log(98^(2/3)*sqrt
(7)*(I*sqrt(7) - 7)^(1/3)*(-I*sqrt(-3) - I) + 196*x) + 1/294*98^(2/3)*(I*s
qrt(7) - 7)^(1/3)*log(I*98^(2/3)*sqrt(7)*(I*sqrt(7) - 7)^(1/3) + 98*x) + 1
/294*98^(2/3)*(-I*sqrt(7) - 7)^(1/3)*log(-I*98^(2/3)*sqrt(7)*(-I*sqrt(7) -
 7)^(1/3) + 98*x)
```

3.184.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.06

$$\int \frac{x^3}{2+x^3+x^6} \, dx = \text{RootSum}\left(250047t^6+1323t^3+2, \left(t \mapsto t\log\left(7938t^4+21t+x\right)\right)\right)$$

input
```
integrate(x**3/(x**6+x**3+2),x)
```

output
```
RootSum(250047*_t**6 + 1323*_t**3 + 2, Lambda(_t, _t*log(7938*_t**4 + 21*_
t + x)))
```

3.184.7 Maxima [F]

$$\int \frac{x^3}{2+x^3+x^6} \, dx = \int \frac{x^3}{x^6+x^3+2} \, dx$$

input
```
integrate(x^3/(x^6+x^3+2),x, algorithm="maxima")
```

output
```
integrate(x^3/(x^6 + x^3 + 2), x)
```

3.184.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{2+x^3+x^6} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(x^6+x^3+2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Invalid _EXT in replace_ext Error:
Bad Argument ValueDone
```

3.184.9 Mupad [B] (verification not implemented)

Time = 9.29 (sec) , antiderivative size = 351, normalized size of antiderivative = 0.88

$$\int \frac{x^3}{2+x^3+x^6}\,dx = \frac{\ln\left(x-\frac{2^{2/3}\,7^{5/6}\,\left(-7-\sqrt{7}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{14}\right)\,\left(-196-\sqrt{7}\,28\mathrm{i}\right)^{1/3}}{42} + \frac{2^{2/3}\,7^{1/3}\,\ln\left(x+\frac{2^{2/3}\,7^{5/6}\,\left(-7+\sqrt{7}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{14}\right)\,\left(-7+\sqrt{7}\,1\mathrm{i}\right)^{1/3}}{42} - \frac{2^{2/3}\,7^{1/3}\,\ln\left(x+\frac{2^{2/3}\,7^{5/6}\,\left(-7-\sqrt{7}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{28}-\frac{2^{2/3}\,\sqrt{3}\,7^{5/6}\,\left(-7-\sqrt{7}\,1\mathrm{i}\right)^{1/3}}{28}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)\,\left(-7-\sqrt{7}\,1\mathrm{i}\right)^{1/3}}{84} + \frac{2^{2/3}\,7^{1/3}\,\ln\left(x+\frac{2^{2/3}\,7^{5/6}\,\left(-7-\sqrt{7}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{28}+\frac{2^{2/3}\,\sqrt{3}\,7^{5/6}\,\left(-7-\sqrt{7}\,1\mathrm{i}\right)^{1/3}}{28}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)\,\left(-7-\sqrt{7}\,1\mathrm{i}\right)^{1/3}}{84} + \frac{2^{2/3}\,7^{1/3}\,\ln\left(x-\frac{2^{2/3}\,7^{5/6}\,\left(-7+\sqrt{7}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{28}-\frac{2^{2/3}\,\sqrt{3}\,7^{5/6}\,\left(-7+\sqrt{7}\,1\mathrm{i}\right)^{1/3}}{28}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)\,\left(-7+\sqrt{7}\,1\mathrm{i}\right)^{1/3}}{84} - \frac{2^{2/3}\,7^{1/3}\,\ln\left(x-\frac{2^{2/3}\,7^{5/6}\,\left(-7+\sqrt{7}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{28}+\frac{2^{2/3}\,\sqrt{3}\,7^{5/6}\,\left(-7+\sqrt{7}\,1\mathrm{i}\right)^{1/3}}{28}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)\,\left(-7+\sqrt{7}\,1\mathrm{i}\right)^{1/3}}{84}$$

input
```
int(x^3/(x^3 + x^6 + 2),x)
```

output
```
(log(x - (2^(2/3)*7^(5/6)*(- 7^(1/2)*1i - 7)^(1/3)*1i)/14)*(- 7^(1/2)*28i
- 196)^(1/3))/42 + (2^(2/3)*7^(1/3)*log(x + (2^(2/3)*7^(5/6)*(7^(1/2)*1i -
 7)^(1/3)*1i)/14)*(7^(1/2)*1i - 7)^(1/3))/42 - (2^(2/3)*7^(1/3)*log(x + (2
^(2/3)*7^(5/6)*(- 7^(1/2)*1i - 7)^(1/3)*1i)/28 - (2^(2/3)*3^(1/2)*7^(5/6)*
(- 7^(1/2)*1i - 7)^(1/3))/28)*(3^(1/2)*1i + 1)*(- 7^(1/2)*1i - 7)^(1/3))/8
4 + (2^(2/3)*7^(1/3)*log(x + (2^(2/3)*7^(5/6)*(- 7^(1/2)*1i - 7)^(1/3)*1i)
/28 + (2^(2/3)*3^(1/2)*7^(5/6)*(- 7^(1/2)*1i - 7)^(1/3))/28)*(3^(1/2)*1i -
 1)*(- 7^(1/2)*1i - 7)^(1/3))/84 + (2^(2/3)*7^(1/3)*log(x - (2^(2/3)*7^(5/
6)*(7^(1/2)*1i - 7)^(1/3)*1i)/28 - (2^(2/3)*3^(1/2)*7^(5/6)*(7^(1/2)*1i -
7)^(1/3))/28)*(3^(1/2)*1i - 1)*(7^(1/2)*1i - 7)^(1/3))/84 - (2^(2/3)*7^(1/
3)*log(x - (2^(2/3)*7^(5/6)*(7^(1/2)*1i - 7)^(1/3)*1i)/28 + (2^(2/3)*3^(1/
2)*7^(5/6)*(7^(1/2)*1i - 7)^(1/3))/28)*(3^(1/2)*1i + 1)*(7^(1/2)*1i - 7)^(
1/3))/84
```

3.185 $\int x^{14}\sqrt{a+bx^3+cx^6}\,dx$

3.185.1 Optimal result

Integrand size = 20, antiderivative size = 231

$$\begin{aligned}\int x^{14}\sqrt{a+bx^3+cx^6}\,dx = &\frac{(21b^4-56ab^2c+16a^2c^2)\,(b+2cx^3)\,\sqrt{a+bx^3+cx^6}}{1536c^5}\\&-\frac{bx^6(a+bx^3+cx^6)^{3/2}}{20c^2}+\frac{x^9(a+bx^3+cx^6)^{3/2}}{18c}\\&-\frac{(7b(15b^2-28ac)-6c(21b^2-20ac)\,x^3)\,(a+bx^3+cx^6)^{3/2}}{2880c^4}\\&-\frac{(b^2-4ac)\,(21b^4-56ab^2c+16a^2c^2)\,\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3072c^{11/2}}\end{aligned}$$

output
```
-1/20*b*x^6*(c*x^6+b*x^3+a)^(3/2)/c^2+1/18*x^9*(c*x^6+b*x^3+a)^(3/2)/c-1/2
880*(7*b*(-28*a*c+15*b^2)-6*c*(-20*a*c+21*b^2)*x^3)*(c*x^6+b*x^3+a)^(3/2)/
c^4-1/3072*(-4*a*c+b^2)*(16*a^2*c^2-56*a*b^2*c+21*b^4)*arctanh(1/2*(2*c*x^
3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))/c^(11/2)+1/1536*(16*a^2*c^2-56*a*b^2*c
+21*b^4)*(2*c*x^3+b)*(c*x^6+b*x^3+a)^(1/2)/c^5
```

3.185.2 Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.89

$$\int x^{14}\sqrt{a+bx^3+cx^6}\,dx$$
$$= \frac{2\sqrt{c}\sqrt{a+bx^3+cx^6}(315b^5-210b^4cx^3+16b^2c^2x^3(56a-9cx^6)+168b^3c(-10a+cx^6)+16bc^2(113a^2-34}{}$$

input
```
Integrate[x^14*Sqrt[a + b*x^3 + c*x^6],x]
```

output
```
(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]*(315*b^5 - 210*b^4*c*x^3 + 16*b^2*c^2*x
^3*(56*a - 9*c*x^6) + 168*b^3*c*(-10*a + c*x^6) + 16*b*c^2*(113*a^2 - 34*a
*c*x^6 + 8*c^2*x^12) + 160*c^3*x^3*(-3*a^2 + 2*a*c*x^6 + 8*c^2*x^12)) + 15
*(21*b^6 - 140*a*b^4*c + 240*a^2*b^2*c^2 - 64*a^3*c^3)*Log[b + 2*c*x^3 - 2
*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]])/(46080*c^(11/2))
```

3.185.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {1693, 1166, 27, 1236, 27, 1225, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{14}\sqrt{a+bx^3+cx^6}\,dx$$
$$\Big\downarrow \text{1693}$$
$$\frac{1}{3}\int x^{12}\sqrt{cx^6+bx^3+a}dx^3$$
$$\Big\downarrow \text{1166}$$
$$\frac{1}{3}\left(\frac{\int -\frac{3}{2}x^6\left(3bx^3+2a\right)\sqrt{cx^6+bx^3+a}dx^3}{6c}+\frac{x^9\left(a+bx^3+cx^6\right)^{3/2}}{6c}\right)$$
$$\Big\downarrow \text{27}$$
$$\frac{1}{3}\left(\frac{x^9\left(a+bx^3+cx^6\right)^{3/2}}{6c}-\frac{\int x^6\left(3bx^3+2a\right)\sqrt{cx^6+bx^3+a}dx^3}{4c}\right)$$

$$\Big\downarrow \text{1236}$$

$$\frac{1}{3}\left(\frac{x^9\left(a+bx^3+cx^6\right)^{3/2}}{6c}-\frac{\frac{\int -\frac{1}{2}x^3\left(\left(21b^2-20ac\right)x^3+12ab\right)\sqrt{cx^6+bx^3+a}dx^3}{5c}+\frac{3bx^6\left(a+bx^3+cx^6\right)^{3/2}}{5c}}{4c}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{x^9\left(a+bx^3+cx^6\right)^{3/2}}{6c}-\frac{\frac{3bx^6\left(a+bx^3+cx^6\right)^{3/2}}{5c}-\frac{\int x^3\left(\left(21b^2-20ac\right)x^3+12ab\right)\sqrt{cx^6+bx^3+a}dx^3}{10c}}{4c}\right)$$

$$\Big\downarrow \text{1225}$$

$$\frac{1}{3}\left(\frac{x^9\left(a+bx^3+cx^6\right)^{3/2}}{6c}-\frac{\frac{3bx^6\left(a+bx^3+cx^6\right)^{3/2}}{5c}-\frac{\frac{5\left(16a^2c^2-56ab^2c+21b^4\right)\int \sqrt{cx^6+bx^3+a}dx^3}{16c^2}-\frac{\left(7b\left(15b^2-28ac\right)-6cx^3\left(21b^2-20ac\right)\right)\left(a+b\cdots\right.}{24c^2}}{10c}}{4c}\cdots\right.$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{3}\left(\frac{x^9\left(a+bx^3+cx^6\right)^{3/2}}{6c}-\frac{\frac{3bx^6\left(a+bx^3+cx^6\right)^{3/2}}{5c}-\frac{\frac{5\left(16a^2c^2-56ab^2c+21b^4\right)\left(\frac{\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{4c}-\frac{\left(b^2-4ac\right)\int \frac{1}{\sqrt{cx^6+bx^3+a}}dx^3}{8c}\right)}{16c^2}-\frac{\left(\cdots\right.}{}}{10c}}{4c}\cdots\right.$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{3}\left(\frac{x^9\left(a+bx^3+cx^6\right)^{3/2}}{6c}-\frac{\frac{3bx^6\left(a+bx^3+cx^6\right)^{3/2}}{5c}-\frac{\frac{5\left(16a^2c^2-56ab^2c+21b^4\right)\left(\frac{\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{4c}-\frac{\left(b^2-4ac\right)\int \frac{1}{4c-x^6}d\frac{2cx^3+b}{\sqrt{cx^6+bx^3+a}}}{4c}\right)}{16c^2}\cdots}{10c}}{4c}\cdots\right.$$

$$\Big\downarrow \text{219}$$

3.185. $\int x^{14}\sqrt{a+bx^3+cx^6}\,dx$

$$\frac{1}{3}\left(\frac{x^9\left(a+bx^3+cx^6\right)^{3/2}}{6c}-\frac{\frac{3bx^6\left(a+bx^3+cx^6\right)^{3/2}}{5c}-\frac{\frac{5\left(16a^2c^2-56ab^2c+21b^4\right)\left(\frac{\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{4c}-\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+}}\right.}{8c^{3/2}}\right.}{16c^2}}{10c}}{4c}\right.$$

input
```
Int[x^14*Sqrt[a + b*x^3 + c*x^6],x]
```

output
```
((x^9*(a + b*x^3 + c*x^6)^(3/2))/(6*c) - ((3*b*x^6*(a + b*x^3 + c*x^6)^(3/
2))/(5*c) - (-1/24*((7*b*(15*b^2 - 28*a*c) - 6*c*(21*b^2 - 20*a*c)*x^3)*(a
 + b*x^3 + c*x^6)^(3/2))/c^2 + (5*(21*b^4 - 56*a*b^2*c + 16*a^2*c^2)*(((b
+ 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(4*c) - ((b^2 - 4*a*c)*ArcTanh[(b + 2*
c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(8*c^(3/2))))/(16*c^2))/(10*c
))/(4*c))/3
```

3.185.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.185.4 Maple [F]

$$\int x^{14}\sqrt{c\,x^{6}+b\,x^{3}+a}dx$$

input
```
int(x^14*(c*x^6+b*x^3+a)^(1/2),x)
```

output
```
int(x^14*(c*x^6+b*x^3+a)^(1/2),x)
```

3.185.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 451, normalized size of antiderivative = 1.95

$$\int x^{14}\sqrt{a+bx^3+cx^6}\,dx$$
$$=\left[-\frac{15\,(21\,b^6-140\,ab^4c+240\,a^2b^2c^2-64\,a^3c^3)\sqrt{c}\log\left(-8\,c^2x^6-8\,bcx^3-b^2-4\,\sqrt{cx^6+bx^3+a}(2\,cx^3+\right.}{}\right.$$

input
```
integrate(x^14*(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/92160*(15*(21*b^6 - 140*a*b^4*c + 240*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(c
)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 +
b)*sqrt(c) - 4*a*c) - 4*(1280*c^6*x^15 + 128*b*c^5*x^12 - 16*(9*b^2*c^4 -
20*a*c^5)*x^9 + 8*(21*b^3*c^3 - 68*a*b*c^4)*x^6 + 315*b^5*c - 1680*a*b^3*c
^2 + 1808*a^2*b*c^3 - 2*(105*b^4*c^2 - 448*a*b^2*c^3 + 240*a^2*c^4)*x^3)*s
qrt(c*x^6 + b*x^3 + a))/c^6, 1/46080*(15*(21*b^6 - 140*a*b^4*c + 240*a^2*b
^2*c^2 - 64*a^3*c^3)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3
+ b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*(1280*c^6*x^15 + 128*b*c^5*x^
12 - 16*(9*b^2*c^4 - 20*a*c^5)*x^9 + 8*(21*b^3*c^3 - 68*a*b*c^4)*x^6 + 315
*b^5*c - 1680*a*b^3*c^2 + 1808*a^2*b*c^3 - 2*(105*b^4*c^2 - 448*a*b^2*c^3
+ 240*a^2*c^4)*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^6]
```

3.185.6 Sympy [F]

$$\int x^{14}\sqrt{a+bx^3+cx^6}\,dx=\int x^{14}\sqrt{a+bx^3+cx^6}\,dx$$

input
```
integrate(x**14*(c*x**6+b*x**3+a)**(1/2),x)
```

output
```
Integral(x**14*sqrt(a + b*x**3 + c*x**6), x)
```

3.185.7 Maxima [F(-2)]

Exception generated.

$$\int x^{14}\sqrt{a+bx^3+cx^6}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^14*(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.185.8 Giac [F]

$$\int x^{14}\sqrt{a+bx^3+cx^6}\,dx = \int \sqrt{cx^6+bx^3+a}x^{14}\,dx$$

input
```
integrate(x^14*(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^6 + b*x^3 + a)*x^14, x)
```

3.185.9 Mupad [B] (verification not implemented)

Time = 9.33 (sec) , antiderivative size = 543, normalized size of antiderivative = 2.35

$$\int x^{14}\sqrt{a+bx^3+cx^6}\,dx = \frac{x^9\left(c\,x^6+b\,x^3+a\right)^{3/2}}{18\,c} - \frac{b\left(\frac{x^6\left(c\,x^6+b\,x^3+a\right)^{3/2}}{5\,c} + \frac{7\,b\left(\frac{a\left(\left(\frac{b}{4\,c}+\frac{x^3}{2}\right)\sqrt{c\,x^6+b\,x^3+a}+\frac{\ln\left(\sqrt{c\,x^6+b\,x^3+a}+\frac{c\,x^3+\frac{b}{2}}{\sqrt{c}}\right)\left(a\,c-\frac{b^2}{4}\right)}{2\,c^{3/2}}\right)}{4\,c} - \frac{x^3\left(c\,x^6+b\,x^3+a\right)^{3/2}}{4\,c} + \frac{5\,b\left(\frac{\left(8\,c\left(c\,x^6 \cdots\right.\right.}{\cdots}\right.}{\cdots}\right.}{10\,c}\cdots}{\cdots} + \frac{a\left(\frac{a\left(\left(\frac{b}{4\,c}+\frac{x^3}{2}\right)\sqrt{c\,x^6+b\,x^3+a}+\frac{\ln\left(\sqrt{c\,x^6+b\,x^3+a}+\frac{c\,x^3+\frac{b}{2}}{\sqrt{c}}\right)\left(a\,c-\frac{b^2}{4}\right)}{2\,c^{3/2}}\right)}{4\,c} - \frac{x^3\left(c\,x^6+b\,x^3+a\right)^{3/2}}{4\,c} + \frac{5\,b\left(\frac{\left(8\,c\left(c\,x^6+a\right)-3\,b^2+2\,b\,c\,x^3\right)\sqrt{c \cdots}}{24\,c^2}\cdots\right.}{\cdots}\right.}{6\,c}$$

input
```
int(x^14*(a + b*x^3 + c*x^6)^(1/2),x)
```

output
```
(x^9*(a + b*x^3 + c*x^6)^(3/2))/(18*c) - (b*((x^6*(a + b*x^3 + c*x^6)^(3/2
))/(5*c) + (7*b*((a*((b/(4*c) + x^3/2)*(a + b*x^3 + c*x^6)^(1/2) + (log((a
 + b*x^3 + c*x^6)^(1/2) + (b/2 + c*x^3)/c^(1/2))*(a*c - b^2/4))/(2*c^(3/2)
)))/(4*c) - (x^3*(a + b*x^3 + c*x^6)^(3/2))/(4*c) + (5*b*(((8*c*(a + c*x^6
) - 3*b^2 + 2*b*c*x^3)*(a + b*x^3 + c*x^6)^(1/2))/(24*c^2) + (log(2*(a + b
*x^3 + c*x^6)^(1/2) + (b + 2*c*x^3)/c^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2))
))/(8*c)))/(10*c) - (2*a*(((8*c*(a + c*x^6) - 3*b^2 + 2*b*c*x^3)*(a + b*x^
3 + c*x^6)^(1/2))/(24*c^2) + (log(2*(a + b*x^3 + c*x^6)^(1/2) + (b + 2*c*x
^3)/c^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2))))/(5*c)))/(4*c) + (a*((a*((b/(4
*c) + x^3/2)*(a + b*x^3 + c*x^6)^(1/2) + (log((a + b*x^3 + c*x^6)^(1/2) +
(b/2 + c*x^3)/c^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c) - (x^3*(a + b*x^
3 + c*x^6)^(3/2))/(4*c) + (5*b*(((8*c*(a + c*x^6) - 3*b^2 + 2*b*c*x^3)*(a
+ b*x^3 + c*x^6)^(1/2))/(24*c^2) + (log(2*(a + b*x^3 + c*x^6)^(1/2) + (b +
 2*c*x^3)/c^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2))))/(8*c)))/(6*c)
```

3.186 $\int x^{11}\sqrt{a+bx^3+cx^6}\,dx$

3.186.1 Optimal result

Integrand size = 20, antiderivative size = 171

$$\int x^{11}\sqrt{a+bx^3+cx^6}\,dx = -\frac{b(7b^2-12ac)\,(b+2cx^3)\sqrt{a+bx^3+cx^6}}{384c^4}+\frac{x^6(a+bx^3+cx^6)^{3/2}}{15c}+\frac{(35b^2-32ac-42bcx^3)\,(a+bx^3+cx^6)^{3/2}}{720c^3}+\frac{b(7b^2-12ac)\,(b^2-4ac)\,\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{768c^{9/2}}$$

output
```
1/15*x^6*(c*x^6+b*x^3+a)^(3/2)/c+1/720*(-42*b*c*x^3-32*a*c+35*b^2)*(c*x^6+
b*x^3+a)^(3/2)/c^3+1/768*b*(-12*a*c+7*b^2)*(-4*a*c+b^2)*arctanh(1/2*(2*c*x
^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))/c^(9/2)-1/384*b*(-12*a*c+7*b^2)*(2*c*
x^3+b)*(c*x^6+b*x^3+a)^(1/2)/c^4
```

3.186.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.97

$$\int x^{11}\sqrt{a+bx^3+cx^6}\,dx = \frac{\sqrt{a+bx^3+cx^6}(-105b^4+70b^3cx^3+4b^2c(115a-14cx^6)+8bc^2x^3(-29a+6cx^6)+128c^2(-2a^2+acx^6+}{5760c^4} - \frac{(7b^5-40ab^3c+48a^2bc^2)\log\left(c^4\left(b+2cx^3-2\sqrt{c}\sqrt{a+bx^3+cx^6}\right)\right)}{768c^{9/2}}$$

input
```
Integrate[x^11*Sqrt[a + b*x^3 + c*x^6],x]
```

output
```
(Sqrt[a + b*x^3 + c*x^6]*(-105*b^4 + 70*b^3*c*x^3 + 4*b^2*c*(115*a - 14*c*
x^6) + 8*b*c^2*x^3*(-29*a + 6*c*x^6) + 128*c^2*(-2*a^2 + a*c*x^6 + 3*c^2*x
^12)))/(5760*c^4) - ((7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*Log[c^4*(b + 2*c*
x^3 - 2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(768*c^(9/2))
```

3.186.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1693, 1166, 27, 1225, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{11}\sqrt{a+bx^3+cx^6}\,dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int x^9\sqrt{cx^6+bx^3+a}dx^3$$

$$\Big\downarrow \text{1166}$$

$$\frac{1}{3}\left(\frac{\int -\frac{1}{2}x^3\left(7bx^3+4a\right)\sqrt{cx^6+bx^3+a}dx^3}{5c}+\frac{x^6\left(a+bx^3+cx^6\right)^{3/2}}{5c}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{x^6\left(a+bx^3+cx^6\right)^{3/2}}{5c}-\frac{\int x^3\left(7bx^3+4a\right)\sqrt{cx^6+bx^3+a}dx^3}{10c}\right)$$

$$\Big\downarrow \text{1225}$$

$$\frac{1}{3}\left(\frac{x^6\left(a+bx^3+cx^6\right)^{3/2}}{5c}-\frac{\frac{5b\left(7b^2-12ac\right)\int\sqrt{cx^6+bx^3+a}dx^3}{16c^2}-\frac{\left(-32ac+35b^2-42bcx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{24c^2}}{10c}\right)$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{3}\left(\frac{x^6\left(a+bx^3+cx^6\right)^{3/2}}{5c}-\frac{\frac{5b(7b^2-12ac)\left(\frac{\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{4c}-\frac{\left(b^2-4ac\right)\int\frac{1}{\sqrt{cx^6+bx^3+a}}dx^3}{8c}\right)}{16c^2}-\frac{\left(-32ac+35b^2-42bcx^3\right)\left(a+bx^3+c\right.}{24c^2}}{10c}\right.$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{3}\left(\frac{x^6\left(a+bx^3+cx^6\right)^{3/2}}{5c}-\frac{\frac{5b(7b^2-12ac)\left(\frac{\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{4c}-\frac{\left(b^2-4ac\right)\int\frac{1}{4c-x^6}d\frac{2cx^3+b}{\sqrt{cx^6+bx^3+a}}}{4c}\right)}{16c^2}-\frac{\left(-32ac+35b^2-42bcx^3\right)\left(a+bx\right.}{24c^2}}{10c}\right.$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{x^6\left(a+bx^3+cx^6\right)^{3/2}}{5c}-\frac{\frac{5b(7b^2-12ac)\left(\frac{\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{4c}-\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{8c^{3/2}}\right)}{16c^2}-\frac{\left(-32ac+35b^2-42bcx^3\right.}{24c^2}}{10c}\right.$$

input
```
Int[x^11*Sqrt[a + b*x^3 + c*x^6],x]
```

output
```
((x^6*(a + b*x^3 + c*x^6)^(3/2))/(5*c) - (-1/24*((35*b^2 - 32*a*c - 42*b*c
*x^3)*(a + b*x^3 + c*x^6)^(3/2))/c^2 + (5*b*(7*b^2 - 12*a*c)*(((b + 2*c*x^
3)*Sqrt[a + b*x^3 + c*x^6])/(4*c) - ((b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/(
2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]])])/(8*c^(3/2))))/(16*c^2))/(10*c))/3
```

3.186. $\int x^{11}\sqrt{a+bx^3+cx^6}\,dx$

3.186.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.186.4 Maple [F]

$$\int x^{11}\sqrt{c\,x^6+b\,x^3+a}dx$$

input
```
int(x^11*(c*x^6+b*x^3+a)^(1/2),x)
```

output
```
int(x^11*(c*x^6+b*x^3+a)^(1/2),x)
```

3.186.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 367, normalized size of antiderivative = 2.15

$$\int x^{11}\sqrt{a+bx^3+cx^6}\,dx$$
$$= \left[\frac{15\,(7\,b^5-40\,ab^3c+48\,a^2bc^2)\sqrt{c}\log\left(-8\,c^2x^6-8\,bcx^3-b^2-4\,\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{c}-4\,ac\right)+ \right.$$
$$-\frac{15\,(7\,b^5-40\,ab^3c+48\,a^2bc^2)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{-c}}{2\,(c^2x^6+bcx^3+ac)}\right)-2\,(384\,c^5x^{12}+48\,bc^4x^9-8\,(7\,b^2c^3}{11520\,c^5}$$

input
```
integrate(x^11*(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/23040*(15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*sqrt(c)*log(-8*c^2*x^6 -
8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c)
 + 4*(384*c^5*x^12 + 48*b*c^4*x^9 - 8*(7*b^2*c^3 - 16*a*c^4)*x^6 - 105*b^4
*c + 460*a*b^2*c^2 - 256*a^2*c^3 + 2*(35*b^3*c^2 - 116*a*b*c^3)*x^3)*sqrt(
c*x^6 + b*x^3 + a))/c^5, -1/11520*(15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*
sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^
6 + b*c*x^3 + a*c)) - 2*(384*c^5*x^12 + 48*b*c^4*x^9 - 8*(7*b^2*c^3 - 16*a
*c^4)*x^6 - 105*b^4*c + 460*a*b^2*c^2 - 256*a^2*c^3 + 2*(35*b^3*c^2 - 116*
a*b*c^3)*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^5]
```

3.186.6 Sympy [F]

$$\int x^{11}\sqrt{a+bx^3+cx^6}\,dx=\int x^{11}\sqrt{a+bx^3+cx^6}\,dx$$

input
```
integrate(x**11*(c*x**6+b*x**3+a)**(1/2),x)
```

output
```
Integral(x**11*sqrt(a + b*x**3 + c*x**6), x)
```

3.186.7 Maxima [F(-2)]

Exception generated.

$$\int x^{11}\sqrt{a+bx^3+cx^6}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^11*(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.186.8 Giac [F]

$$\int x^{11}\sqrt{a+bx^3+cx^6}\,dx=\int\sqrt{cx^6+bx^3+a}\,x^{11}\,dx$$

input
```
integrate(x^11*(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^6 + b*x^3 + a)*x^11, x)
```

3.186.9 Mupad [B] (verification not implemented)

Time = 8.69 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.84

$$\int x^{11}\sqrt{a+bx^3+cx^6}\,dx=\frac{x^6\,{\left(c\,x^6+b\,x^3+a\right)}^{3/2}}{15\,c}$$

$$+\frac{7\,b\left(\frac{a\left(\left(\frac{b}{4\,c}+\frac{x^3}{2}\right)\sqrt{c\,x^6+b\,x^3+a}+\frac{\ln\left(\sqrt{c\,x^6+b\,x^3+a}+\frac{c\,x^3+\frac{b}{2}}{\sqrt{c}}\right)\left(a\,c-\frac{b^2}{4}\right)}{2\,c^{3/2}}\right)}{4\,c}-\frac{x^3\,{\left(c\,x^6+b\,x^3+a\right)}^{3/2}}{4\,c}+\frac{5\,b\left(\frac{\left(8\,c\,\left(c\,x^6+a\right)-3\,b^2+2\,b\,c\,x^3\right)\sqrt{\dots}}{24\,c^2}\dots\right.}{\dots}\right)}{30\,c}$$

$$-\frac{2\,a\left(\frac{\left(8\,c\,\left(c\,x^6+a\right)-3\,b^2+2\,b\,c\,x^3\right)\sqrt{c\,x^6+b\,x^3+a}}{24\,c^2}+\frac{\ln\left(2\,\sqrt{c\,x^6+b\,x^3+a}+\frac{2\,c\,x^3+b}{\sqrt{c}}\right)\left(b^3-4\,a\,b\,c\right)}{16\,c^{5/2}}\right)}{15\,c}$$

input
```
int(x^11*(a + b*x^3 + c*x^6)^(1/2),x)
```

output
```
(x^6*(a + b*x^3 + c*x^6)^(3/2))/(15*c) + (7*b*((a*((b/(4*c) + x^3/2)*(a +
b*x^3 + c*x^6)^(1/2) + (log((a + b*x^3 + c*x^6)^(1/2) + (b/2 + c*x^3)/c^(1
/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c) - (x^3*(a + b*x^3 + c*x^6)^(3/2))/
(4*c) + (5*b*(((8*c*(a + c*x^6) - 3*b^2 + 2*b*c*x^3)*(a + b*x^3 + c*x^6)^(
1/2))/(24*c^2) + (log(2*(a + b*x^3 + c*x^6)^(1/2) + (b + 2*c*x^3)/c^(1/2))
*(b^3 - 4*a*b*c))/(16*c^(5/2))))/(8*c)))/(30*c) - (2*a*(((8*c*(a + c*x^6)
- 3*b^2 + 2*b*c*x^3)*(a + b*x^3 + c*x^6)^(1/2))/(24*c^2) + (log(2*(a + b*x
^3 + c*x^6)^(1/2) + (b + 2*c*x^3)/c^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2))))
/(15*c)
```

3.187 $\int x^8\sqrt{a+bx^3+cx^6}\,dx$

3.187.1 Optimal result

Integrand size = 20, antiderivative size = 153

$$\int x^8\sqrt{a+bx^3+cx^6}\,dx=\frac{(5b^2-4ac)\,(b+2cx^3)\,\sqrt{a+bx^3+cx^6}}{192c^3}-\frac{5b(a+bx^3+cx^6)^{3/2}}{72c^2}+\frac{x^3(a+bx^3+cx^6)^{3/2}}{12c}-\frac{(b^2-4ac)\,(5b^2-4ac)\,\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{384c^{7/2}}$$

output
```
-5/72*b*(c*x^6+b*x^3+a)^(3/2)/c^2+1/12*x^3*(c*x^6+b*x^3+a)^(3/2)/c-1/384*(
-4*a*c+b^2)*(-4*a*c+5*b^2)*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)
^(1/2))/c^(7/2)+1/192*(-4*a*c+5*b^2)*(2*c*x^3+b)*(c*x^6+b*x^3+a)^(1/2)/c^3
```

3.187.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.86

$$\int x^8\sqrt{a+bx^3+cx^6}\,dx=\frac{\sqrt{a+bx^3+cx^6}(15b^3-52abc-10b^2cx^3+24ac^2x^3+8bc^2x^6+48c^3x^9)}{576c^3}+\frac{(5b^4-24ab^2c+16a^2c^2)\log\left(b+2cx^3-2\sqrt{c}\sqrt{a+bx^3+cx^6}\right)}{384c^{7/2}}$$

input
```
Integrate[x^8*Sqrt[a + b*x^3 + c*x^6],x]
```

output
```
(Sqrt[a + b*x^3 + c*x^6]*(15*b^3 - 52*a*b*c - 10*b^2*c*x^3 + 24*a*c^2*x^3
+ 8*b*c^2*x^6 + 48*c^3*x^9))/(576*c^3) + ((5*b^4 - 24*a*b^2*c + 16*a^2*c^2
)*Log[b + 2*c*x^3 - 2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]])/(384*c^(7/2))
```

3.187.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1693, 1166, 27, 1160, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8 \sqrt{a+bx^3+cx^6}\,dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int x^6\sqrt{cx^6+bx^3+a}dx^3$$

$$\Big\downarrow \text{1166}$$

$$\frac{1}{3}\left(\frac{\int -\frac{1}{2}\left(5bx^3+2a\right)\sqrt{cx^6+bx^3+a}dx^3}{4c}+\frac{x^3\left(a+bx^3+cx^6\right)^{3/2}}{4c}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{x^3\left(a+bx^3+cx^6\right)^{3/2}}{4c}-\frac{\int \left(5bx^3+2a\right)\sqrt{cx^6+bx^3+a}dx^3}{8c}\right)$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{3}\left(\frac{x^3\left(a+bx^3+cx^6\right)^{3/2}}{4c}-\frac{\frac{5b\left(a+bx^3+cx^6\right)^{3/2}}{3c}-\frac{\left(5b^2-4ac\right)\int \sqrt{cx^6+bx^3+a}dx^3}{2c}}{8c}\right)$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{3}\left(\frac{x^3\left(a+bx^3+cx^6\right)^{3/2}}{4c}-\frac{\frac{5b\left(a+bx^3+cx^6\right)^{3/2}}{3c}-\frac{(5b^2-4ac)\left(\frac{\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{4c}-\frac{\left(b^2-4ac\right)\int\frac{1}{\sqrt{cx^6+bx^3+a}}dx^3}{8c}\right)}{2c}}{8c}\right)$$

$$\Big\downarrow \text{ 1092}$$

$$\frac{1}{3}\left(\frac{x^3\left(a+bx^3+cx^6\right)^{3/2}}{4c}-\frac{\frac{5b\left(a+bx^3+cx^6\right)^{3/2}}{3c}-\frac{(5b^2-4ac)\left(\frac{\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{4c}-\frac{\left(b^2-4ac\right)\int\frac{1}{4c-x^6}d\frac{2cx^3+b}{\sqrt{cx^6+bx^3+a}}}{4c}\right)}{2c}}{8c}\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{3}\left(\frac{x^3\left(a+bx^3+cx^6\right)^{3/2}}{4c}-\frac{\frac{5b\left(a+bx^3+cx^6\right)^{3/2}}{3c}-\frac{(5b^2-4ac)\left(\frac{\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{4c}-\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{8c^{3/2}}\right)}{2c}}{8c}\right)$$

input
```
Int[x^8*Sqrt[a + b*x^3 + c*x^6],x]
```

output
```
((x^3*(a + b*x^3 + c*x^6)^(3/2))/(4*c) - ((5*b*(a + b*x^3 + c*x^6)^(3/2))/
(3*c) - ((5*b^2 - 4*a*c)*(((b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(4*c) -
((b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])
/(8*c^(3/2))))/(2*c))/(8*c))/3
```

3.187. $\int x^8\sqrt{a+bx^3+cx^6}\,dx$

3.187.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.187.4 Maple [F]

$$\int x^8\sqrt{c\,x^6+b\,x^3+a}dx$$

input
```
int(x^8*(c*x^6+b*x^3+a)^(1/2),x)
```

output
```
int(x^8*(c*x^6+b*x^3+a)^(1/2),x)
```

3.187.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.98

$$\int x^8\sqrt{a+bx^3+cx^6}\,dx$$
$$= \left[\frac{3\,(5\,b^4-24\,ab^2c+16\,a^2c^2)\sqrt{c}\log\left(-8\,c^2x^6-8\,bcx^3-b^2+4\,\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{c}-4\,ac\right)+4\,(}{2304\,c^4}\right.$$

input
```
integrate(x^8*(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/2304*(3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*sqrt(c)*log(-8*c^2*x^6 - 8*b*
c*x^3 - b^2 + 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 4
*(48*c^4*x^9 + 8*b*c^3*x^6 + 15*b^3*c - 52*a*b*c^2 - 2*(5*b^2*c^2 - 12*a*c
^3)*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^4, 1/1152*(3*(5*b^4 - 24*a*b^2*c + 16*
a^2*c^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c
)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*(48*c^4*x^9 + 8*b*c^3*x^6 + 15*b^3*c - 52
*a*b*c^2 - 2*(5*b^2*c^2 - 12*a*c^3)*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^4]
```

3.187.6 Sympy [F]

$$\int x^8\sqrt{a+bx^3+cx^6}\,dx=\int x^8\sqrt{a+bx^3+cx^6}\,dx$$

input
```
integrate(x**8*(c*x**6+b*x**3+a)**(1/2),x)
```

output
```
Integral(x**8*sqrt(a + b*x**3 + c*x**6), x)
```

3.187.7 Maxima [F(-2)]

Exception generated.

$$\int x^8\sqrt{a+bx^3+cx^6}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^8*(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.187.8 Giac [F]

$$\int x^8\sqrt{a+bx^3+cx^6}\,dx = \int \sqrt{cx^6+bx^3+a}\,x^8\,dx$$

input
```
integrate(x^8*(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^6 + b*x^3 + a)*x^8, x)
```

3.187.9 Mupad [B] (verification not implemented)

Time = 8.36 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.26

$$\begin{aligned}&\int x^8\sqrt{a+bx^3+cx^6}\,dx\\&=\frac{x^3\,\left(c\,x^6+b\,x^3+a\right)^{3/2}}{12\,c}\\&-\frac{a\,\left(\left(\frac{b}{4\,c}+\frac{x^3}{2}\right)\,\sqrt{c\,x^6+b\,x^3+a}+\frac{\ln\left(\sqrt{c\,x^6+b\,x^3+a}+\frac{c\,x^3+\frac{b}{2}}{\sqrt{c}}\right)\,\left(a\,c-\frac{b^2}{4}\right)}{2\,c^{3/2}}\right)}{12\,c}\\&-\frac{5\,b\,\left(\frac{\left(8\,c\,\left(c\,x^6+a\right)-3\,b^2+2\,b\,c\,x^3\right)\,\sqrt{c\,x^6+b\,x^3+a}}{24\,c^2}+\frac{\ln\left(2\,\sqrt{c\,x^6+b\,x^3+a}+\frac{2\,c\,x^3+b}{\sqrt{c}}\right)\,\left(b^3-4\,a\,b\,c\right)}{16\,c^{5/2}}\right)}{24\,c}\end{aligned}$$

input
```
int(x^8*(a + b*x^3 + c*x^6)^(1/2),x)
```

output
```
(x^3*(a + b*x^3 + c*x^6)^(3/2))/(12*c) - (a*((b/(4*c) + x^3/2)*(a + b*x^3
+ c*x^6)^(1/2) + (log((a + b*x^3 + c*x^6)^(1/2) + (b/2 + c*x^3)/c^(1/2))*(
a*c - b^2/4))/(2*c^(3/2))))/(12*c) - (5*b*(((8*c*(a + c*x^6) - 3*b^2 + 2*b
*c*x^3)*(a + b*x^3 + c*x^6)^(1/2))/(24*c^2) + (log(2*(a + b*x^3 + c*x^6)^(
1/2) + (b + 2*c*x^3)/c^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2))))/(24*c)
```

3.187. $\int x^8\sqrt{a+bx^3+cx^6}\,dx$

3.188 $\int x^5\sqrt{a+bx^3+cx^6}\,dx$

3.188.1 Optimal result

Integrand size = 20, antiderivative size = 108

$$\int x^5\sqrt{a+bx^3+cx^6}\,dx=-\frac{b(b+2cx^3)\sqrt{a+bx^3+cx^6}}{24c^2}+\frac{(a+bx^3+cx^6)^{3/2}}{9c}+\frac{b(b^2-4ac)\,\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{48c^{5/2}}$$

output
```
1/9*(c*x^6+b*x^3+a)^(3/2)/c+1/48*b*(-4*a*c+b^2)*arctanh(1/2*(2*c*x^3+b)/c^
(1/2)/(c*x^6+b*x^3+a)^(1/2))/c^(5/2)-1/24*b*(2*c*x^3+b)*(c*x^6+b*x^3+a)^(1
/2)/c^2
```

3.188.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.94

$$\int x^5\sqrt{a+bx^3+cx^6}\,dx=\frac{\sqrt{a+bx^3+cx^6}(-3b^2+2bcx^3+8c(a+cx^6))}{72c^2}-\frac{(b^3-4abc)\log\left(c^2\left(b+2cx^3-2\sqrt{c}\sqrt{a+bx^3+cx^6}\right)\right)}{48c^{5/2}}$$

input
```
Integrate[x^5*Sqrt[a + b*x^3 + c*x^6],x]
```

output
```
(Sqrt[a + b*x^3 + c*x^6]*(-3*b^2 + 2*b*c*x^3 + 8*c*(a + c*x^6)))/(72*c^2)
- ((b^3 - 4*a*b*c)*Log[c^2*(b + 2*c*x^3 - 2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6
])])/(48*c^(5/2))
```

3.188.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1693, 1160, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\sqrt{a+bx^3+cx^6}\,dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int x^3\sqrt{cx^6+bx^3+a}dx^3$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{3}\left(\frac{\left(a+bx^3+cx^6\right)^{3/2}}{3c}-\frac{b\int \sqrt{cx^6+bx^3+a}dx^3}{2c}\right)$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{3}\left(\frac{\left(a+bx^3+cx^6\right)^{3/2}}{3c}-\frac{b\left(\frac{(b+2cx^3)\sqrt{a+bx^3+cx^6}}{4c}-\frac{(b^2-4ac)\int \frac{1}{\sqrt{cx^6+bx^3+a}}dx^3}{8c}\right)}{2c}\right)$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{3}\left(\frac{\left(a+bx^3+cx^6\right)^{3/2}}{3c}-\frac{b\left(\frac{(b+2cx^3)\sqrt{a+bx^3+cx^6}}{4c}-\frac{(b^2-4ac)\int \frac{1}{4c-x^6}d\frac{2cx^3+b}{\sqrt{cx^6+bx^3+a}}}{4c}\right)}{2c}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{\left(a+bx^3+cx^6\right)^{3/2}}{3c}-\frac{b\left(\frac{(b+2cx^3)\sqrt{a+bx^3+cx^6}}{4c}-\frac{(b^2-4ac)\operatorname{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{8c^{3/2}}\right)}{2c}\right)$$

input
```
Int[x^5*Sqrt[a + b*x^3 + c*x^6],x]
```

output
```
((a + b*x^3 + c*x^6)^(3/2)/(3*c) - (b*(((b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x
^6])/(4*c) - ((b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^
3 + c*x^6])])/(8*c^(3/2))))/(2*c))/3
```

3.188.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.188.4 Maple [F]

$$\int x^5\sqrt{c\,x^6+b\,x^3+a}dx$$

input
```
int(x^5*(c*x^6+b*x^3+a)^(1/2),x)
```

output
```
int(x^5*(c*x^6+b*x^3+a)^(1/2),x)
```

3.188.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.19

$$\int x^5\sqrt{a+bx^3+cx^6}\,dx$$
$$= \left[-\frac{3\left(b^3-4\,abc\right)\sqrt{c}\log\left(-8\,c^2x^6-8\,bcx^3-b^2+4\,\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{c}-4\,ac\right)-4\left(8\,c^3x^6+2\,bc\right.}{288\,c^3}\right.$$
$$-\frac{3\left(b^3-4\,abc\right)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{-c}}{2\,(c^2x^6+bcx^3+ac)}\right)-2\left(8\,c^3x^6+2\,bc^2x^3-3\,b^2c+8\,ac^2\right)\sqrt{cx^6+bx^3+a}}{144\,c^3}$$

input
```
integrate(x^5*(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/288*(3*(b^3 - 4*a*b*c)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 + 4*sq
rt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) - 4*(8*c^3*x^6 + 2*b*
c^2*x^3 - 3*b^2*c + 8*a*c^2)*sqrt(c*x^6 + b*x^3 + a))/c^3, -1/144*(3*(b^3
- 4*a*b*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(
-c)/(c^2*x^6 + b*c*x^3 + a*c)) - 2*(8*c^3*x^6 + 2*b*c^2*x^3 - 3*b^2*c + 8*
a*c^2)*sqrt(c*x^6 + b*x^3 + a))/c^3]
```

3.188.6 Sympy [F]

$$\int x^5\sqrt{a+bx^3+cx^6}\,dx=\int x^5\sqrt{a+bx^3+cx^6}\,dx$$

input
```
integrate(x**5*(c*x**6+b*x**3+a)**(1/2),x)
```

output
```
Integral(x**5*sqrt(a + b*x**3 + c*x**6), x)
```

3.188.7 Maxima [F(-2)]

Exception generated.

$$\int x^5\sqrt{a+bx^3+cx^6}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^5*(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.188.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.89

$$\int x^5\sqrt{a+bx^3+cx^6}\,dx=\frac{1}{72}\sqrt{cx^6+bx^3+a}\left(2\left(4x^3+\frac{b}{c}\right)x^3-\frac{3b^2-8ac}{c^2}\right)-\frac{(b^3-4abc)\log\left(\left|2\left(\sqrt{c}x^3-\sqrt{cx^6+bx^3+a}\right)\sqrt{c}+b\right|\right)}{48c^{\frac{5}{2}}}$$

input
```
integrate(x^5*(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
1/72*sqrt(c*x^6 + b*x^3 + a)*(2*(4*x^3 + b/c)*x^3 - (3*b^2 - 8*a*c)/c^2) -
 1/48*(b^3 - 4*a*b*c)*log(abs(2*(sqrt(c)*x^3 - sqrt(c*x^6 + b*x^3 + a))*sq
rt(c) + b))/c^(5/2)
```

3.188.9 Mupad [B] (verification not implemented)

Time = 8.39 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.81

$$\int x^5\sqrt{a+bx^3+cx^6}\,dx=\frac{\left(8\,c\,\left(c\,x^6+a\right)-3\,b^2+2\,b\,c\,x^3\right)\,\sqrt{c\,x^6+b\,x^3+a}}{72\,c^2}+\frac{\ln\left(2\,\sqrt{c\,x^6+b\,x^3+a}+\frac{2\,c\,x^3+b}{\sqrt{c}}\right)\,\left(b^3-4\,a\,b\,c\right)}{48\,c^{5/2}}$$

input
```
int(x^5*(a + b*x^3 + c*x^6)^(1/2),x)
```

output
```
((8*c*(a + c*x^6) - 3*b^2 + 2*b*c*x^3)*(a + b*x^3 + c*x^6)^(1/2))/(72*c^2)
 + (log(2*(a + b*x^3 + c*x^6)^(1/2) + (b + 2*c*x^3)/c^(1/2))*(b^3 - 4*a*b*
c))/(48*c^(5/2))
```

3.189 $\int x^2\sqrt{a+bx^3+cx^6}\,dx$

3.189.1 Optimal result

Integrand size = 20, antiderivative size = 83

$$\int x^2\sqrt{a+bx^3+cx^6}\,dx=\frac{(b+2cx^3)\sqrt{a+bx^3+cx^6}}{12c}-\frac{(b^2-4ac)\,\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{24c^{3/2}}$$

output
```
-1/24*(-4*a*c+b^2)*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))/
c^(3/2)+1/12*(2*c*x^3+b)*(c*x^6+b*x^3+a)^(1/2)/c
```

3.189.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.05

$$\begin{aligned}&\int x^2\sqrt{a+bx^3+cx^6}\,dx\\&=\frac{(b+2cx^3)\sqrt{a+bx^3+cx^6}}{12c}+\frac{(-b^2+4ac)\,\text{arctanh}\left(\frac{\sqrt{c}x^3}{-\sqrt{a}+\sqrt{a+bx^3+cx^6}}\right)}{12c^{3/2}}\end{aligned}$$

input
```
Integrate[x^2*Sqrt[a + b*x^3 + c*x^6],x]
```

output
```
((b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(12*c) + ((-b^2 + 4*a*c)*ArcTanh[(
Sqrt[c]*x^3)/(-Sqrt[a] + Sqrt[a + b*x^3 + c*x^6])])/(12*c^(3/2))
```

3.189.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1690, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sqrt{a+bx^3+cx^6} \, dx$$

$$\Big\downarrow \text{1690}$$

$$\frac{1}{3} \int \sqrt{cx^6+bx^3+a}dx^3$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{3} \left(\frac{(b+2cx^3)\sqrt{a+bx^3+cx^6}}{4c} - \frac{(b^2-4ac)\int \frac{1}{\sqrt{cx^6+bx^3+a}}dx^3}{8c}\right)$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{3} \left(\frac{(b+2cx^3)\sqrt{a+bx^3+cx^6}}{4c} - \frac{(b^2-4ac)\int \frac{1}{4c-x^6}d\frac{2cx^3+b}{\sqrt{cx^6+bx^3+a}}}{4c}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3} \left(\frac{(b+2cx^3)\sqrt{a+bx^3+cx^6}}{4c} - \frac{(b^2-4ac)\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{8c^{3/2}}\right)$$

input
```
Int[x^2*Sqrt[a + b*x^3 + c*x^6],x]
```

output
```
(((b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(4*c) - ((b^2 - 4*a*c)*ArcTanh[(b
 + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(8*c^(3/2)))/3
```

3.189.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

3.189.4 Maple [F]

$$\int x^{2}\sqrt{c\,x^{6}+b\,x^{3}+a}dx$$

input
```
int(x^2*(c*x^6+b*x^3+a)^(1/2),x)
```

output
```
int(x^2*(c*x^6+b*x^3+a)^(1/2),x)
```

3.189.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 197, normalized size of antiderivative = 2.37

$$\int x^2\sqrt{a+bx^3+cx^6}\,dx$$

$$= \left[-\frac{(b^2-4\,ac)\sqrt{c}\log\left(-8\,c^2x^6-8\,bcx^3-b^2-4\,\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{c}-4\,ac\right)-4\,\sqrt{cx^6+bx^3+a}}{48\,c^2}\right.$$

input
```
integrate(x^2*(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/48*((b^2 - 4*a*c)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*
x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) - 4*sqrt(c*x^6 + b*x^3 + a
)*(2*c^2*x^3 + b*c))/c^2, 1/24*((b^2 - 4*a*c)*sqrt(-c)*arctan(1/2*sqrt(c*x
^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*sqrt
(c*x^6 + b*x^3 + a)*(2*c^2*x^3 + b*c))/c^2]
```

3.189.6 Sympy [F]

$$\int x^2\sqrt{a+bx^3+cx^6}\,dx=\int x^2\sqrt{a+bx^3+cx^6}\,dx$$

input
```
integrate(x**2*(c*x**6+b*x**3+a)**(1/2),x)
```

output
```
Integral(x**2*sqrt(a + b*x**3 + c*x**6), x)
```

3.189.7 Maxima [F(-2)]

Exception generated.

$$\int x^2\sqrt{a+bx^3+cx^6}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^2*(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.189.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.89

$$\int x^2\sqrt{a+bx^3+cx^6}\,dx = \frac{1}{12}\sqrt{cx^6+bx^3+a}\left(2x^3+\frac{b}{c}\right) + \frac{(b^2-4ac)\log\left(\left|2\left(\sqrt{c}x^3-\sqrt{cx^6+bx^3+a}\right)\sqrt{c}+b\right|\right)}{24c^{\frac{3}{2}}}$$

input
```
integrate(x^2*(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
1/12*sqrt(c*x^6 + b*x^3 + a)*(2*x^3 + b/c) + 1/24*(b^2 - 4*a*c)*log(abs(2*
(sqrt(c)*x^3 - sqrt(c*x^6 + b*x^3 + a))*sqrt(c) + b))/c^(3/2)
```

3.189.9 Mupad [B] (verification not implemented)

Time = 8.49 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.87

$$\int x^2\sqrt{a+bx^3+cx^6}\,dx = \frac{\left(\frac{b}{4c}+\frac{x^3}{2}\right)\sqrt{c\,x^6+b\,x^3+a}}{3} + \frac{\ln\left(\sqrt{c\,x^6+b\,x^3+a}+\frac{c\,x^3+\frac{b}{2}}{\sqrt{c}}\right)\left(a\,c-\frac{b^2}{4}\right)}{6\,c^{3/2}}$$

input
```
int(x^2*(a + b*x^3 + c*x^6)^(1/2),x)
```

output
```
((b/(4*c) + x^3/2)*(a + b*x^3 + c*x^6)^(1/2))/3 + (log((a + b*x^3 + c*x^6)
^(1/2) + (b/2 + c*x^3)/c^(1/2))*(a*c - b^2/4))/(6*c^(3/2))
```

3.190 $\int \frac{\sqrt{a+bx^3+cx^6}}{x}\,dx$

3.190.1 Optimal result

Integrand size = 20, antiderivative size = 109

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x}\,dx = \frac{1}{3}\sqrt{a+bx^3+cx^6} - \frac{1}{3}\sqrt{a}\,\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right) + \frac{b\,\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{6\sqrt{c}}$$

output
```
-1/3*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2))*a^(1/2)+1/6*b*
arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))/c^(1/2)+1/3*(c*x^6+
b*x^3+a)^(1/2)
```

3.190.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x}\,dx = \frac{1}{6}\left(2\sqrt{a+bx^3+cx^6} + 4\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{c}x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right) - \frac{b\log\left(b+2cx^3-2\sqrt{c}\sqrt{a+bx^3+cx^6}\right)}{\sqrt{c}}\right)$$

input
```
Integrate[Sqrt[a + b*x^3 + c*x^6]/x,x]
```

output
```
(2*Sqrt[a + b*x^3 + c*x^6] + 4*Sqrt[a]*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b*x
^3 + c*x^6])/Sqrt[a]] - (b*Log[b + 2*c*x^3 - 2*Sqrt[c]*Sqrt[a + b*x^3 + c*
x^6]])/Sqrt[c])/6
```

3.190.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1693, 1162, 25, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x} \, dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3} \int \frac{\sqrt{cx^6+bx^3+a}}{x^3}dx^3$$

$$\Big\downarrow \text{1162}$$

$$\frac{1}{3} \left(\sqrt{a+bx^3+cx^6}-\frac{1}{2} \int -\frac{bx^3+2a}{x^3\sqrt{cx^6+bx^3+a}}dx^3\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3} \left(\frac{1}{2} \int \frac{bx^3+2a}{x^3\sqrt{cx^6+bx^3+a}}dx^3+\sqrt{a+bx^3+cx^6}\right)$$

$$\Big\downarrow \text{1269}$$

$$\frac{1}{3} \left(\frac{1}{2} \left(b\int \frac{1}{\sqrt{cx^6+bx^3+a}}dx^3+2a\int \frac{1}{x^3\sqrt{cx^6+bx^3+a}}dx^3\right)+\sqrt{a+bx^3+cx^6}\right)$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{3} \left(\frac{1}{2} \left(2b\int \frac{1}{4c-x^6}d\frac{2cx^3+b}{\sqrt{cx^6+bx^3+a}}+2a\int \frac{1}{x^3\sqrt{cx^6+bx^3+a}}dx^3\right)+\sqrt{a+bx^3+cx^6}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3} \left(\frac{1}{2} \left(2a\int \frac{1}{x^3\sqrt{cx^6+bx^3+a}}dx^3+\frac{b\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{\sqrt{c}}\right)+\sqrt{a+bx^3+cx^6}\right)$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{b\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{\sqrt{c}}-4a\int\frac{1}{4a-x^6}d\frac{bx^3+2a}{\sqrt{cx^6+bx^3+a}}\right)+\sqrt{a+bx^3+cx^6}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{b\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{\sqrt{c}}-2\sqrt{a}\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)\right)+\sqrt{a+bx^3+cx^6}\right)$$

input
```
Int[Sqrt[a + b*x^3 + c*x^6]/x,x]
```

output
```
(Sqrt[a + b*x^3 + c*x^6] + (-2*Sqrt[a]*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sq
rt[a + b*x^3 + c*x^6])] + (b*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x
^3 + c*x^6])])/Sqrt[c])/2)/3
```

3.190.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1162
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p/(e*(m + 2*p + 1))   Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d -
b*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&
!ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.190.4 Maple [F]

$$\int \frac{\sqrt{c\,x^{6}+b\,x^{3}+a}}{x}dx$$

input
```
int((c*x^6+b*x^3+a)^(1/2)/x,x)
```

output
```
int((c*x^6+b*x^3+a)^(1/2)/x,x)
```

3.190.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 566, normalized size of antiderivative = 5.19

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x}\,dx$$
$$= \left[\frac{b\sqrt{c}\log\left(-8\,c^2x^6-8\,bcx^3-b^2-4\,\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{c}-4\,ac\right)+2\,\sqrt{a}c\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3}{\quad}\right.}{12\,c}\right.$$
$$-\frac{b\sqrt{-c}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{-c}}{2\,(c^2x^6+bcx^3+ac)}\right)-\sqrt{a}c\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3-4\,\sqrt{cx^6+bx^3+a}(bx^3+2\,a)\sqrt{a}+8\,a^2}{x^6}\right)-2\,\sqrt{c}}{6\,c}$$

input
```
integrate((c*x^6+b*x^3+a)^(1/2)/x,x, algorithm="fricas")
```

output
```
[1/12*(b*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 +
 a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 2*sqrt(a)*c*log(-((b^2 + 4*a*c)*x^6 +
 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6)
 + 4*sqrt(c*x^6 + b*x^3 + a)*c)/c, -1/6*(b*sqrt(-c)*arctan(1/2*sqrt(c*x^6
+ b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) - sqrt(a)*c
*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 +
2*a)*sqrt(a) + 8*a^2)/x^6) - 2*sqrt(c*x^6 + b*x^3 + a)*c)/c, 1/12*(4*sqrt(
-a)*c*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 +
 a*b*x^3 + a^2)) + b*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x
^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 4*sqrt(c*x^6 + b*x^3 + a)
*c)/c, 1/6*(2*sqrt(-a)*c*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*
sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) - b*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b
*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*sqrt(c*x^6
 + b*x^3 + a)*c)/c]
```

3.190.6 Sympy [F]

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x}\,dx = \int \frac{\sqrt{a+bx^3+cx^6}}{x}\,dx$$

input
```
integrate((c*x**6+b*x**3+a)**(1/2)/x,x)
```

output
```
Integral(sqrt(a + b*x**3 + c*x**6)/x, x)
```

3.190.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^6+b*x^3+a)^(1/2)/x,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.190.8 Giac [F]

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x}\,dx=\int \frac{\sqrt{cx^6+bx^3+a}}{x}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^(1/2)/x,x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^6 + b*x^3 + a)/x, x)
```

3.190.9 Mupad [B] (verification not implemented)

Time = 8.37 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x}\,dx=\frac{\sqrt{c\,x^6+b\,x^3+a}}{3}-\frac{\sqrt{a}\,\ln\left(\frac{b}{2}+\frac{a}{x^3}+\frac{\sqrt{a}\,\sqrt{c\,x^6+b\,x^3+a}}{x^3}\right)}{3}+\frac{b\,\ln\left(\sqrt{c\,x^6+b\,x^3+a}+\frac{c\,x^3+\frac{b}{2}}{\sqrt{c}}\right)}{6\,\sqrt{c}}$$

input
```
int((a + b*x^3 + c*x^6)^(1/2)/x,x)
```

output
```
(a + b*x^3 + c*x^6)^(1/2)/3 - (a^(1/2)*log(b/2 + a/x^3 + (a^(1/2)*(a + b*x
^3 + c*x^6)^(1/2))/x^3))/3 + (b*log((a + b*x^3 + c*x^6)^(1/2) + (b/2 + c*x
^3)/c^(1/2)))/(6*c^(1/2))
```

3.191 $\int \frac{\sqrt{a+bx^3+cx^6}}{x^4}\,dx$

3.191.1 Optimal result

Integrand size = 20, antiderivative size = 112

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^4}\,dx=-\frac{\sqrt{a+bx^3+cx^6}}{3x^3}-\frac{b\,\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{6\sqrt{a}}+\frac{1}{3}\sqrt{c}\,\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)$$

output
```
-1/6*b*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2))/a^(1/2)+1/3*
arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))*c^(1/2)-1/3*(c*x^6+
b*x^3+a)^(1/2)/x^3
```

3.191.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^4}\,dx=\frac{1}{3}\left(-\frac{\sqrt{a+bx^3+cx^6}}{x^3}+\frac{b\,\text{arctanh}\left(\frac{\sqrt{c}x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{\sqrt{a}}-\sqrt{c}\log\left(b+2cx^3-2\sqrt{c}\sqrt{a+bx^3+cx^6}\right)\right)$$

input
```
Integrate[Sqrt[a + b*x^3 + c*x^6]/x^4,x]
```

output
```
(-(Sqrt[a + b*x^3 + c*x^6]/x^3) + (b*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3
 + c*x^6])/Sqrt[a]])/Sqrt[a] - Sqrt[c]*Log[b + 2*c*x^3 - 2*Sqrt[c]*Sqrt[a
+ b*x^3 + c*x^6]])/3
```

3.191.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1693, 1161, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^4}dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int \frac{\sqrt{cx^6+bx^3+a}}{x^6}dx^3$$

$$\Big\downarrow \text{1161}$$

$$\frac{1}{3}\left(\frac{1}{2}\int \frac{2cx^3+b}{x^3\sqrt{cx^6+bx^3+a}}dx^3-\frac{\sqrt{a+bx^3+cx^6}}{x^3}\right)$$

$$\Big\downarrow \text{1269}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(2c\int \frac{1}{\sqrt{cx^6+bx^3+a}}dx^3+b\int \frac{1}{x^3\sqrt{cx^6+bx^3+a}}dx^3\right)-\frac{\sqrt{a+bx^3+cx^6}}{x^3}\right)$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(4c\int \frac{1}{4c-x^6}d\frac{2cx^3+b}{\sqrt{cx^6+bx^3+a}}+b\int \frac{1}{x^3\sqrt{cx^6+bx^3+a}}dx^3\right)-\frac{\sqrt{a+bx^3+cx^6}}{x^3}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(b\int \frac{1}{x^3\sqrt{cx^6+bx^3+a}}dx^3+2\sqrt{c}\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)\right)-\frac{\sqrt{a+bx^3+cx^6}}{x^3}\right)$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(2\sqrt{c}\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)-2b\int \frac{1}{4a-x^6}d\frac{bx^3+2a}{\sqrt{cx^6+bx^3+a}}\right)-\frac{\sqrt{a+bx^3+cx^6}}{x^3}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(2\sqrt{c}\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)-\frac{b\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{\sqrt{a}}\right)-\frac{\sqrt{a+bx^3+cx^6}}{x^3}\right)$$

input
```
Int[Sqrt[a + b*x^3 + c*x^6]/x^4,x]
```

output
```
(-(Sqrt[a + b*x^3 + c*x^6]/x^3) + (-((b*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*S
qrt[a + b*x^3 + c*x^6])])/Sqrt[a]) + 2*Sqrt[c]*ArcTanh[(b + 2*c*x^3)/(2*Sq
rt[c]*Sqrt[a + b*x^3 + c*x^6])])/2)/3
```

3.191.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1161
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Si
mp[p/(e*(m + 1))   Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||
 LtQ[m, -1]) && NeQ[m, -1] && !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b,
 c, d, e, m, p, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.191.4 Maple [F]

$$\int \frac{\sqrt{c\,x^{6}+b\,x^{3}+a}}{x^{4}}dx$$

input
```
int((c*x^6+b*x^3+a)^(1/2)/x^4,x)
```

output
```
int((c*x^6+b*x^3+a)^(1/2)/x^4,x)
```

3.191.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 601, normalized size of antiderivative = 5.37

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^4}\,dx = \left[\frac{2\,a\sqrt{c}x^3\log\left(-8\,c^2x^6-8\,bcx^3-b^2-4\,\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{c}-4\,ac\right)+\sqrt{a}bx^3\log\left(-\frac{(b^2+4\,ac)x^6+\cdots}{\cdots}\right.}{12\,ax^3}\right.$$
$$-\frac{4\,a\sqrt{-c}x^3\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{-c}}{2\,(c^2x^6+bcx^3+ac)}\right)-\sqrt{a}bx^3\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3-4\,\sqrt{cx^6+bx^3+a}(bx^3+2\,a)\sqrt{a}+8\,a^2}{x^6}\right)}{12\,ax^3}$$

input
```
integrate((c*x^6+b*x^3+a)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
[1/12*(2*a*sqrt(c)*x^3*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b
*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + sqrt(a)*b*x^3*log(-((b^2 + 4*a*
c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a
^2)/x^6) - 4*sqrt(c*x^6 + b*x^3 + a)*a)/(a*x^3), -1/12*(4*a*sqrt(-c)*x^3*a
rctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^
3 + a*c)) - sqrt(a)*b*x^3*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x
^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) + 4*sqrt(c*x^6 + b*x^3
 + a)*a)/(a*x^3), 1/6*(sqrt(-a)*b*x^3*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(
b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + a*sqrt(c)*x^3*log(-8*c^
2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c)
- 4*a*c) - 2*sqrt(c*x^6 + b*x^3 + a)*a)/(a*x^3), 1/6*(sqrt(-a)*b*x^3*arcta
n(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 +
a^2)) - 2*a*sqrt(-c)*x^3*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*
sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) - 2*sqrt(c*x^6 + b*x^3 + a)*a)/(a*x^3)
]
```

3.191.6 Sympy [F]

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^4}\,dx = \int \frac{\sqrt{a+bx^3+cx^6}}{x^4}\,dx$$

input
```
integrate((c*x**6+b*x**3+a)**(1/2)/x**4,x)
```

output
```
Integral(sqrt(a + b*x**3 + c*x**6)/x**4, x)
```

3.191.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^6+b*x^3+a)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.191.8 Giac [F]

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^4}\,dx = \int \frac{\sqrt{cx^6+bx^3+a}}{x^4}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^(1/2)/x^4,x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^6 + b*x^3 + a)/x^4, x)
```

3.191.9 Mupad [B] (verification not implemented)

Time = 8.57 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^4}\,dx = \frac{\sqrt{c}\,\ln\left(\sqrt{c\,x^6+b\,x^3+a}+\frac{c\,x^3+\frac{b}{2}}{\sqrt{c}}\right)}{3} - \frac{\sqrt{c\,x^6+b\,x^3+a}}{3\,x^3} - \frac{b\,\ln\left(\frac{b}{2}+\frac{a}{x^3}+\frac{\sqrt{a}\,\sqrt{c\,x^6+b\,x^3+a}}{x^3}\right)}{6\,\sqrt{a}}$$

input
```
int((a + b*x^3 + c*x^6)^(1/2)/x^4,x)
```

output
```
(c^(1/2)*log((a + b*x^3 + c*x^6)^(1/2) + (b/2 + c*x^3)/c^(1/2)))/3 - (a +
b*x^3 + c*x^6)^(1/2)/(3*x^3) - (b*log(b/2 + a/x^3 + (a^(1/2)*(a + b*x^3 +
c*x^6)^(1/2))/x^3))/(6*a^(1/2))
```

3.192 $\int \frac{\sqrt{a+bx^3+cx^6}}{x^7}\,dx$

3.192.1 Optimal result

Integrand size = 20, antiderivative size = 88

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^7}\,dx = -\frac{(2a+bx^3)\sqrt{a+bx^3+cx^6}}{12ax^6} + \frac{(b^2-4ac)\,\mathrm{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{24a^{3/2}}$$

output
```
1/24*(-4*a*c+b^2)*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2))/a
^(3/2)-1/12*(b*x^3+2*a)*(c*x^6+b*x^3+a)^(1/2)/a/x^6
```

3.192.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^7}\,dx = \frac{(-2a-bx^3)\sqrt{a+bx^3+cx^6}}{12ax^6} + \frac{(-b^2+4ac)\,\mathrm{arctanh}\left(\frac{\sqrt{c}x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{12a^{3/2}}$$

input
```
Integrate[Sqrt[a + b*x^3 + c*x^6]/x^7,x]
```

output
```
((-2*a - b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(12*a*x^6) + ((-b^2 + 4*a*c)*ArcT
anh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3 + c*x^6])/Sqrt[a]])/(12*a^(3/2))
```

3.192.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1693, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^7} \, dx$$

$$\Big\downarrow \text{ 1693}$$

$$\frac{1}{3}\int \frac{\sqrt{cx^6+bx^3+a}}{x^9}dx^3$$

$$\Big\downarrow \text{ 1152}$$

$$\frac{1}{3}\left(-\frac{(b^2-4ac)\int \frac{1}{x^3\sqrt{cx^6+bx^3+a}}dx^3}{8a}-\frac{(2a+bx^3)\sqrt{a+bx^3+cx^6}}{4ax^6}\right)$$

$$\Big\downarrow \text{ 1154}$$

$$\frac{1}{3}\left(\frac{(b^2-4ac)\int \frac{1}{4a-x^6}d\frac{bx^3+2a}{\sqrt{cx^6+bx^3+a}}}{4a}-\frac{(2a+bx^3)\sqrt{a+bx^3+cx^6}}{4ax^6}\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{3}\left(\frac{(b^2-4ac)\,\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{8a^{3/2}}-\frac{(2a+bx^3)\sqrt{a+bx^3+cx^6}}{4ax^6}\right)$$

input
```
Int[Sqrt[a + b*x^3 + c*x^6]/x^7,x]
```

output
```
(-1/4*((2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(a*x^6) + ((b^2 - 4*a*c)*Arc
Tanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(8*a^(3/2)))/3
```

3.192.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.192.4 Maple [F]

$$\int \frac{\sqrt{c\,x^{6}+b\,x^{3}+a}}{x^{7}}dx$$

input
```
int((c*x^6+b*x^3+a)^(1/2)/x^7,x)
```

output
```
int((c*x^6+b*x^3+a)^(1/2)/x^7,x)
```

3.192.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.44

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^7}\,dx$$
$$= \left[-\frac{(b^2-4\,ac)\sqrt{a}x^6\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3-4\,\sqrt{cx^6+bx^3+a}(bx^3+2\,a)\sqrt{a}+8\,a^2}{x^6}\right)+4\,\sqrt{cx^6+bx^3+a}(abx^3+2\,a^2)}{48\,a^2x^6}, -\frac{(b^2-4\,ac)\sqrt{-a}x^6\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(bx^3+2\,a)\sqrt{-a}}{2\,(acx^6+abx^3+a^2)}\right)+2\,\sqrt{cx^6+bx^3+a}(abx^3+2\,a^2)}{24\,a^2x^6}\right]$$

input
```
integrate((c*x^6+b*x^3+a)^(1/2)/x^7,x, algorithm="fricas")
```

output
```
[-1/48*((b^2 - 4*a*c)*sqrt(a)*x^6*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*
sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) + 4*sqrt(c*x^6
 + b*x^3 + a)*(a*b*x^3 + 2*a^2))/(a^2*x^6), -1/24*((b^2 - 4*a*c)*sqrt(-a)*
x^6*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a
*b*x^3 + a^2)) + 2*sqrt(c*x^6 + b*x^3 + a)*(a*b*x^3 + 2*a^2))/(a^2*x^6)]
```

3.192.6 Sympy [F]

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^7}\,dx = \int \frac{\sqrt{a+bx^3+cx^6}}{x^7}\,dx$$

input
```
integrate((c*x**6+b*x**3+a)**(1/2)/x**7,x)
```

output
```
Integral(sqrt(a + b*x**3 + c*x**6)/x**7, x)
```

3.192.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^7}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^6+b*x^3+a)^(1/2)/x^7,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.192.8 Giac [F]

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^7}\,dx = \int \frac{\sqrt{cx^6+bx^3+a}}{x^7}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^(1/2)/x^7,x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^6 + b*x^3 + a)/x^7, x)
```

3.192.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^7}\,dx = \int \frac{\sqrt{c\,x^6+b\,x^3+a}}{x^7}\,dx$$

input
```
int((a + b*x^3 + c*x^6)^(1/2)/x^7,x)
```

output
```
int((a + b*x^3 + c*x^6)^(1/2)/x^7, x)
```

3.193 $\int \frac{\sqrt{a+bx^3+cx^6}}{x^{10}}\,dx$

3.193.1 Optimal result

Integrand size = 20, antiderivative size = 116

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{10}}\,dx = \frac{b(2a+bx^3)\sqrt{a+bx^3+cx^6}}{24a^2x^6} - \frac{(a+bx^3+cx^6)^{3/2}}{9ax^9} - \frac{b(b^2-4ac)\,\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{48a^{5/2}}$$

output
```
-1/9*(c*x^6+b*x^3+a)^(3/2)/a/x^9-1/48*b*(-4*a*c+b^2)*arctanh(1/2*(b*x^3+2*
a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2))/a^(5/2)+1/24*b*(b*x^3+2*a)*(c*x^6+b*x^3+
a)^(1/2)/a^2/x^6
```

3.193.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{10}}\,dx = \frac{\sqrt{a+bx^3+cx^6}(-8a^2-2abx^3+3b^2x^6-8acx^6)}{72a^2x^9} + \frac{(b^3-4abc)\,\text{arctanh}\left(\frac{\sqrt{c}x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{24a^{5/2}}$$

input
```
Integrate[Sqrt[a + b*x^3 + c*x^6]/x^10,x]
```

output
```
(Sqrt[a + b*x^3 + c*x^6]*(-8*a^2 - 2*a*b*x^3 + 3*b^2*x^6 - 8*a*c*x^6))/(72
*a^2*x^9) + ((b^3 - 4*a*b*c)*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3 + c*x^6
])/Sqrt[a]])/(24*a^(5/2))
```

3.193.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1693, 1157, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{10}}dx$$

$$\Big\downarrow \; 1693$$

$$\frac{1}{3}\int \frac{\sqrt{cx^6+bx^3+a}}{x^{12}}dx^3$$

$$\Big\downarrow \; 1157$$

$$\frac{1}{3}\left(-\frac{b\int \frac{\sqrt{cx^6+bx^3+a}}{x^9}dx^3}{2a}-\frac{\left(a+bx^3+cx^6\right)^{3/2}}{3ax^9}\right)$$

$$\Big\downarrow \; 1152$$

$$\frac{1}{3}\left(-\frac{b\left(-\frac{\left(b^2-4ac\right)\int \frac{1}{x^3\sqrt{cx^6+bx^3+a}}dx^3}{8a}-\frac{\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{4ax^6}\right)}{2a}-\frac{\left(a+bx^3+cx^6\right)^{3/2}}{3ax^9}\right)$$

$$\Big\downarrow \; 1154$$

$$\frac{1}{3}\left(-\frac{b\left(\frac{\left(b^2-4ac\right)\int \frac{1}{4a-x^6}d\frac{bx^3+2a}{\sqrt{cx^6+bx^3+a}}}{4a}-\frac{\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{4ax^6}\right)}{2a}-\frac{\left(a+bx^3+cx^6\right)^{3/2}}{3ax^9}\right)$$

$$\Big\downarrow \; 219$$

$$\frac{1}{3}\left(-\frac{b\left(\frac{(b^2-4ac)\operatorname{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{8a^{3/2}}-\frac{(2a+bx^3)\sqrt{a+bx^3+cx^6}}{4ax^6}\right)}{2a}-\frac{(a+bx^3+cx^6)^{3/2}}{3ax^9}\right)$$

input
```
Int[Sqrt[a + b*x^3 + c*x^6]/x^10,x]
```

output
```
(-1/3*(a + b*x^3 + c*x^6)^(3/2)/(a*x^9) - (b*(-1/4*((2*a + b*x^3)*Sqrt[a +
 b*x^3 + c*x^6])/(a*x^6) + ((b^2 - 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]
*Sqrt[a + b*x^3 + c*x^6])])/(8*a^(3/2))))/(2*a))/3
```

3.193.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1157
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Simp[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2))
  Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && EqQ[m + 2*p + 3, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.193.4 Maple [F]

$$\int \frac{\sqrt{c\,x^{6}+b\,x^{3}+a}}{x^{10}}dx$$

input
```
int((c*x^6+b*x^3+a)^(1/2)/x^10,x)
```

output
```
int((c*x^6+b*x^3+a)^(1/2)/x^10,x)
```

3.193.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 259, normalized size of antiderivative = 2.23

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{10}}\,dx$$
$$= \left[-\frac{3\,(b^3-4\,abc)\sqrt{a}x^9\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3+4\,\sqrt{cx^6+bx^3+a}\,(bx^3+2\,a)\,\sqrt{a}+8\,a^2}{x^6}\right)-4\,((3\,ab^2-8\,a^2c)x^6-2\,a^2bx^3}{288\,a^3x^9}\right.$$

input
```
integrate((c*x^6+b*x^3+a)^(1/2)/x^10,x, algorithm="fricas")
```

output
```
[-1/288*(3*(b^3 - 4*a*b*c)*sqrt(a)*x^9*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3
 + 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) - 4*((3*a
*b^2 - 8*a^2*c)*x^6 - 2*a^2*b*x^3 - 8*a^3)*sqrt(c*x^6 + b*x^3 + a))/(a^3*x
^9), 1/144*(3*(b^3 - 4*a*b*c)*sqrt(-a)*x^9*arctan(1/2*sqrt(c*x^6 + b*x^3 +
 a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((3*a*b^2 - 8*a^
2*c)*x^6 - 2*a^2*b*x^3 - 8*a^3)*sqrt(c*x^6 + b*x^3 + a))/(a^3*x^9)]
```

3.193.6 Sympy [F]

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{10}}\,dx = \int \frac{\sqrt{a+bx^3+cx^6}}{x^{10}}\,dx$$

input
```
integrate((c*x**6+b*x**3+a)**(1/2)/x**10,x)
```

output
```
Integral(sqrt(a + b*x**3 + c*x**6)/x**10, x)
```

3.193.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{10}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^6+b*x^3+a)^(1/2)/x^10,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.193.8 Giac [F]

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{10}}\,dx = \int \frac{\sqrt{cx^6+bx^3+a}}{x^{10}}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^(1/2)/x^10,x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^6 + b*x^3 + a)/x^10, x)
```

3.193.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{10}}\,dx = \int \frac{\sqrt{c\,x^6+b\,x^3+a}}{x^{10}}\,dx$$

input
```
int((a + b*x^3 + c*x^6)^(1/2)/x^10,x)
```

output
```
int((a + b*x^3 + c*x^6)^(1/2)/x^10, x)
```

3.194 $\int \frac{\sqrt{a+bx^3+cx^6}}{x^{13}}\,dx$

3.194.1 Optimal result

Integrand size = 20, antiderivative size = 161

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{13}}\,dx = -\frac{(5b^2-4ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{192a^3x^6} - \frac{(a+bx^3+cx^6)^{3/2}}{12ax^{12}} + \frac{5b(a+bx^3+cx^6)^{3/2}}{72a^2x^9} + \frac{(b^2-4ac)(5b^2-4ac)\,\mathrm{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{384a^{7/2}}$$

output
```
-1/12*(c*x^6+b*x^3+a)^(3/2)/a/x^12+5/72*b*(c*x^6+b*x^3+a)^(3/2)/a^2/x^9+1/
384*(-4*a*c+b^2)*(-4*a*c+5*b^2)*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x
^3+a)^(1/2))/a^(7/2)-1/192*(-4*a*c+5*b^2)*(b*x^3+2*a)*(c*x^6+b*x^3+a)^(1/2
)/a^3/x^6
```

3.194.2 Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{13}}\,dx = \frac{\sqrt{a+bx^3+cx^6}(-48a^3-8a^2bx^3+10ab^2x^6-24a^2cx^6-15b^3x^9+52abcx^9)}{576a^3x^{12}} + \frac{(-5b^4+24ab^2c-16a^2c^2)\,\mathrm{arctanh}\left(\frac{\sqrt{c}x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{192a^{7/2}}$$

input
```
Integrate[Sqrt[a + b*x^3 + c*x^6]/x^13,x]
```

output
```
(Sqrt[a + b*x^3 + c*x^6]*(-48*a^3 - 8*a^2*b*x^3 + 10*a*b^2*x^6 - 24*a^2*c*
x^6 - 15*b^3*x^9 + 52*a*b*c*x^9))/(576*a^3*x^12) + ((-5*b^4 + 24*a*b^2*c -
 16*a^2*c^2)*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3 + c*x^6])/Sqrt[a]])/(19
2*a^(7/2))
```

3.194.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1693, 1167, 27, 1228, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{13}} \, dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3} \int \frac{\sqrt{cx^6+bx^3+a}}{x^{15}}dx^3$$

$$\Big\downarrow \text{1167}$$

$$\frac{1}{3} \left(-\frac{\int \frac{(2cx^3+5b)\sqrt{cx^6+bx^3+a}}{2x^{12}}dx^3}{4a}-\frac{(a+bx^3+cx^6)^{3/2}}{4ax^{12}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3} \left(-\frac{\int \frac{(2cx^3+5b)\sqrt{cx^6+bx^3+a}}{x^{12}}dx^3}{8a}-\frac{(a+bx^3+cx^6)^{3/2}}{4ax^{12}}\right)$$

$$\Big\downarrow \text{1228}$$

$$\frac{1}{3} \left(-\frac{-\frac{(5b^2-4ac)\int \frac{\sqrt{cx^6+bx^3+a}}{x^9}dx^3}{2a}-\frac{5b(a+bx^3+cx^6)^{3/2}}{3ax^9}}{8a}-\frac{(a+bx^3+cx^6)^{3/2}}{4ax^{12}}\right)$$

$$\Big\downarrow \text{1152}$$

$$\frac{1}{3}\left(-\frac{-\frac{(5b^2-4ac)\left(-\frac{(b^2-4ac)\int\frac{1}{x^3\sqrt{cx^6+bx^3+a}}dx^3}{8a}-\frac{(2a+bx^3)\sqrt{a+bx^3+cx^6}}{4ax^6}\right)}{2a}-\frac{5b(a+bx^3+cx^6)^{3/2}}{3ax^9}}{8a}-\frac{(a+bx^3+cx^6)^{3/2}}{4ax^{12}}\right)$$

$$\Big\downarrow \; 1154$$

$$\frac{1}{3}\left(-\frac{-\frac{(5b^2-4ac)\left(\frac{(b^2-4ac)\int\frac{1}{4a-x^6}d\frac{bx^3+2a}{\sqrt{cx^6+bx^3+a}}}{4a}-\frac{(2a+bx^3)\sqrt{a+bx^3+cx^6}}{4ax^6}\right)}{2a}-\frac{5b(a+bx^3+cx^6)^{3/2}}{3ax^9}}{8a}-\frac{(a+bx^3+cx^6)^{3/2}}{4ax^{12}}\right)$$

$$\Big\downarrow \; 219$$

$$\frac{1}{3}\left(-\frac{-\frac{(5b^2-4ac)\left(\frac{(b^2-4ac)\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{8a^{3/2}}-\frac{(2a+bx^3)\sqrt{a+bx^3+cx^6}}{4ax^6}\right)}{2a}-\frac{5b(a+bx^3+cx^6)^{3/2}}{3ax^9}}{8a}-\frac{(a+bx^3+cx^6)^{3/2}}{4ax^{12}}\right)$$

input
```
Int[Sqrt[a + b*x^3 + c*x^6]/x^13,x]
```

output
```
(-1/4*(a + b*x^3 + c*x^6)^(3/2)/(a*x^12) - ((-5*b*(a + b*x^3 + c*x^6)^(3/2
))/(3*a*x^9) - ((5*b^2 - 4*a*c)*(-1/4*((2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^
6])/(a*x^6) + ((b^2 - 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x
^3 + c*x^6])])/(8*a^(3/2))))/(2*a))/(8*a))/3
```

3.194. $\int \frac{\sqrt{a+bx^3+cx^6}}{x^{13}}\,dx$

3.194.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1167
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 - b*d*e + a*e^2))   Int[
(d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
 x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[m
, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumSimp
lerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

3.194. $\int \frac{\sqrt{a+bx^3+cx^6}}{x^{13}}\,dx$

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.194.4 Maple [F]

$$\int \frac{\sqrt{c\,x^{6}+b\,x^{3}+a}}{x^{13}}dx$$

input
```
int((c*x^6+b*x^3+a)^(1/2)/x^13,x)
```

output
```
int((c*x^6+b*x^3+a)^(1/2)/x^13,x)
```

3.194.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 325, normalized size of antiderivative = 2.02

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{13}}\,dx = \left[\frac{3\,(5\,b^4-24\,ab^2c+16\,a^2c^2)\sqrt{a}x^{12}\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3+4\,\sqrt{cx^6+bx^3+a}(bx^3+2\,a)\sqrt{a}+8\,a^2}{x^6}\right)-4\,((15\,ab^3-52\,a^2\cdots}{2304\,a^4x^{12}}\right.$$
$$-\frac{3\,(5\,b^4-24\,ab^2c+16\,a^2c^2)\sqrt{-a}x^{12}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(bx^3+2\,a)\sqrt{-a}}{2\,(acx^6+abx^3+a^2)}\right)+2\,((15\,ab^3-52\,a^2bc)x^9+8\,a^3bx^3\cdots}{1152\,a^4x^{12}}$$

input
```
integrate((c*x^6+b*x^3+a)^(1/2)/x^13,x, algorithm="fricas")
```

output
```
[1/2304*(3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*sqrt(a)*x^12*log(-((b^2 + 4*a
*c)*x^6 + 8*a*b*x^3 + 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*
a^2)/x^6) - 4*((15*a*b^3 - 52*a^2*b*c)*x^9 + 8*a^3*b*x^3 - 2*(5*a^2*b^2 -
12*a^3*c)*x^6 + 48*a^4)*sqrt(c*x^6 + b*x^3 + a))/(a^4*x^12), -1/1152*(3*(5
*b^4 - 24*a*b^2*c + 16*a^2*c^2)*sqrt(-a)*x^12*arctan(1/2*sqrt(c*x^6 + b*x^
3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((15*a*b^3 -
52*a^2*b*c)*x^9 + 8*a^3*b*x^3 - 2*(5*a^2*b^2 - 12*a^3*c)*x^6 + 48*a^4)*sqr
t(c*x^6 + b*x^3 + a))/(a^4*x^12)]
```

3.194.6 Sympy [F]

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{13}}\,dx=\int \frac{\sqrt{a+bx^3+cx^6}}{x^{13}}\,dx$$

input
```
integrate((c*x**6+b*x**3+a)**(1/2)/x**13,x)
```

output
```
Integral(sqrt(a + b*x**3 + c*x**6)/x**13, x)
```

3.194.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{13}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((c*x^6+b*x^3+a)^(1/2)/x^13,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.194.8 Giac [F]

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{13}}\,dx=\int \frac{\sqrt{cx^6+bx^3+a}}{x^{13}}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^(1/2)/x^13,x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^6 + b*x^3 + a)/x^13, x)
```

3.194.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{13}}\,dx = \int \frac{\sqrt{c\,x^6+b\,x^3+a}}{x^{13}}\,dx$$

input
```
int((a + b*x^3 + c*x^6)^(1/2)/x^13,x)
```

output
```
int((a + b*x^3 + c*x^6)^(1/2)/x^13, x)
```

3.195 $\int \frac{\sqrt{a+bx^3+cx^6}}{x^{16}}\,dx$

3.195.1 Optimal result

Integrand size = 20, antiderivative size = 199

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{16}}\,dx = \frac{b(7b^2-12ac)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{384a^4x^6} - \frac{\left(a+bx^3+cx^6\right)^{3/2}}{15ax^{15}} + \frac{7b\left(a+bx^3+cx^6\right)^{3/2}}{120a^2x^{12}} - \frac{\left(35b^2-32ac\right)\left(a+bx^3+cx^6\right)^{3/2}}{720a^3x^9} - \frac{b(7b^2-12ac)\left(b^2-4ac\right)\operatorname{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{768a^{9/2}}$$

output
```
-1/15*(c*x^6+b*x^3+a)^(3/2)/a/x^15+7/120*b*(c*x^6+b*x^3+a)^(3/2)/a^2/x^12-
1/720*(-32*a*c+35*b^2)*(c*x^6+b*x^3+a)^(3/2)/a^3/x^9-1/768*b*(-12*a*c+7*b^
2)*(-4*a*c+b^2)*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2))/a^(
9/2)+1/384*b*(-12*a*c+7*b^2)*(b*x^3+2*a)*(c*x^6+b*x^3+a)^(1/2)/a^4/x^6
```

3.195.2 Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{16}}\,dx = \frac{\sqrt{a+bx^3+cx^6}\left(-384a^4-48a^3bx^3+56a^2b^2x^6-128a^3cx^6-70ab^3x^9+232a^2bcx^9+105b^4x^{12}-460ab^2c\right.}{5760a^4x^{15}} + \frac{\left(7b^5-40ab^3c+48a^2bc^2\right)\operatorname{arctanh}\left(\frac{\sqrt{c}x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{384a^{9/2}}$$

input
```
Integrate[Sqrt[a + b*x^3 + c*x^6]/x^16,x]
```

output
```
(Sqrt[a + b*x^3 + c*x^6]*(-384*a^4 - 48*a^3*b*x^3 + 56*a^2*b^2*x^6 - 128*a
^3*c*x^6 - 70*a*b^3*x^9 + 232*a^2*b*c*x^9 + 105*b^4*x^12 - 460*a*b^2*c*x^1
2 + 256*a^2*c^2*x^12))/(5760*a^4*x^15) + ((7*b^5 - 40*a*b^3*c + 48*a^2*b*c
^2)*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3 + c*x^6])/Sqrt[a]])/(384*a^(9/2)
)
```

3.195.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {1693, 1167, 27, 1237, 27, 1228, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{16}}\,dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int \frac{\sqrt{cx^6+bx^3+a}}{x^{18}}dx^3$$

$$\Big\downarrow \text{1167}$$

$$\frac{1}{3}\left(-\frac{\int \frac{(4cx^3+7b)\sqrt{cx^6+bx^3+a}}{2x^{15}}dx^3}{5a}-\frac{\left(a+bx^3+cx^6\right)^{3/2}}{5ax^{15}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(-\frac{\int \frac{(4cx^3+7b)\sqrt{cx^6+bx^3+a}}{x^{15}}dx^3}{10a}-\frac{\left(a+bx^3+cx^6\right)^{3/2}}{5ax^{15}}\right)$$

$$\Big\downarrow \text{1237}$$

$$\frac{1}{3}\left(-\frac{-\frac{\int \frac{\left(14bcx^3+35b^2-32ac\right)\sqrt{cx^6+bx^3+a}}{2x^{12}}dx^3}{4a}-\frac{7b\left(a+bx^3+cx^6\right)^{3/2}}{4ax^{12}}}{10a}-\frac{\left(a+bx^3+cx^6\right)^{3/2}}{5ax^{15}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(-\frac{-\frac{\int \frac{\left(14bcx^3+35b^2-32ac\right)\sqrt{cx^6+bx^3+a}}{x^{12}}dx^3}{8a}-\frac{7b\left(a+bx^3+cx^6\right)^{3/2}}{4ax^{12}}}{10a}-\frac{\left(a+bx^3+cx^6\right)^{3/2}}{5ax^{15}}\right)$$

$\big\downarrow$ 1228

$$\frac{1}{3}\left(-\frac{-\frac{-\frac{5b\left(7b^2-12ac\right)\int \frac{\sqrt{cx^6+bx^3+a}}{x^9}dx^3}{2a}-\frac{\left(35b^2-32ac\right)\left(a+bx^3+cx^6\right)^{3/2}}{3ax^9}}{8a}-\frac{7b\left(a+bx^3+cx^6\right)^{3/2}}{4ax^{12}}}{10a}-\frac{\left(a+bx^3+cx^6\right)^{3/2}}{5ax^{15}}\right)$$

$\big\downarrow$ 1152

$$\frac{1}{3}\left(-\frac{-\frac{-\frac{5b\left(7b^2-12ac\right)\left(-\frac{\left(b^2-4ac\right)\int \frac{1}{x^3\sqrt{cx^6+bx^3+a}}dx^3}{8a}-\frac{\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{4ax^6}\right)}{2a}-\frac{\left(35b^2-32ac\right)\left(a+bx^3+cx^6\right)^{3/2}}{3ax^9}}{8a}-\frac{7b\left(a+bx^3+cx^6\right)^{3/2}}{4ax^{12}}}{10a}-\right.$$

$\big\downarrow$ 1154

$$\frac{1}{3}\left(-\frac{-\frac{-\frac{5b\left(7b^2-12ac\right)\left(\frac{\left(b^2-4ac\right)\int \frac{1}{4a-x^6}d\frac{bx^3+2a}{\sqrt{cx^6+bx^3+a}}}{4a}-\frac{\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{4ax^6}\right)}{2a}-\frac{\left(35b^2-32ac\right)\left(a+bx^3+cx^6\right)^{3/2}}{3ax^9}}{8a}-\frac{7b\left(a+bx^3+cx^6\right)^{3/2}}{4ax^{12}}}{10a}-\right.$$

$\big\downarrow$ 219

$$\frac{1}{3}\left(-\frac{-\frac{-\frac{5b\left(7b^2-12ac\right)\left(\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{8a^{3/2}}-\frac{\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{4ax^6}\right)}{2a}-\frac{\left(35b^2-32ac\right)\left(a+bx^3+cx^6\right)^{3/2}}{3ax^9}}{8a}-\frac{7b\left(a+bx^3+cx^6\right)^{3/}}{4ax^{12}}}{10a}\right.$$

input
```
Int[Sqrt[a + b*x^3 + c*x^6]/x^16,x]
```

3.195. $\int \frac{\sqrt{a+bx^3+cx^6}}{x^{16}}\,dx$

output
```
(-1/5*(a + b*x^3 + c*x^6)^(3/2)/(a*x^15) - ((-7*b*(a + b*x^3 + c*x^6)^(3/2
))/(4*a*x^12) - (-1/3*((35*b^2 - 32*a*c)*(a + b*x^3 + c*x^6)^(3/2))/(a*x^9
) - (5*b*(7*b^2 - 12*a*c)*(-1/4*((2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(a
*x^6) + ((b^2 - 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c
*x^6])])/(8*a^(3/2))))/(2*a))/(8*a))/(10*a))/3
```

3.195.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1167
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 - b*d*e + a*e^2))   Int[
(d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
 x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[m
, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumSimp
lerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

3.195. $\int \frac{\sqrt{a+bx^3+cx^6}}{x^{16}}\,dx$

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.195.4 Maple [F]

$$\int \frac{\sqrt{c\,x^{6}+b\,x^{3}+a}}{x^{16}}dx$$

input
```
int((c*x^6+b*x^3+a)^(1/2)/x^16,x)
```

output
```
int((c*x^6+b*x^3+a)^(1/2)/x^16,x)
```

3.195.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.95

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{16}}\,dx$$
$$= \left[\frac{15\left(7\,b^5-40\,ab^3c+48\,a^2bc^2\right)\sqrt{a}x^{15}\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3-4\,\sqrt{cx^6+bx^3+a}(bx^3+2\,a)\sqrt{a}+8\,a^2}{x^6}\right)+4\left((105\,ab^4-4\right.}{23}\right.$$

input
```
integrate((c*x^6+b*x^3+a)^(1/2)/x^16,x, algorithm="fricas")
```

output
```
[1/23040*(15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*sqrt(a)*x^15*log(-((b^2 +
 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a)
+ 8*a^2)/x^6) + 4*((105*a*b^4 - 460*a^2*b^2*c + 256*a^3*c^2)*x^12 - 2*(35*
a^2*b^3 - 116*a^3*b*c)*x^9 - 48*a^4*b*x^3 + 8*(7*a^3*b^2 - 16*a^4*c)*x^6 -
 384*a^5)*sqrt(c*x^6 + b*x^3 + a))/(a^5*x^15), 1/11520*(15*(7*b^5 - 40*a*b
^3*c + 48*a^2*b*c^2)*sqrt(-a)*x^15*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x
^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((105*a*b^4 - 460*a^2*b^
2*c + 256*a^3*c^2)*x^12 - 2*(35*a^2*b^3 - 116*a^3*b*c)*x^9 - 48*a^4*b*x^3
+ 8*(7*a^3*b^2 - 16*a^4*c)*x^6 - 384*a^5)*sqrt(c*x^6 + b*x^3 + a))/(a^5*x^
15)]
```

3.195.6 Sympy [F]

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{16}}\,dx=\int \frac{\sqrt{a+bx^3+cx^6}}{x^{16}}\,dx$$

input
```
integrate((c*x**6+b*x**3+a)**(1/2)/x**16,x)
```

output
```
Integral(sqrt(a + b*x**3 + c*x**6)/x**16, x)
```

3.195.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{16}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((c*x^6+b*x^3+a)^(1/2)/x^16,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.195.8 Giac [F]

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{16}}\,dx=\int \frac{\sqrt{cx^6+bx^3+a}}{x^{16}}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^(1/2)/x^16,x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^6 + b*x^3 + a)/x^16, x)
```

3.195.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{16}}\,dx=\int \frac{\sqrt{c\,x^6+b\,x^3+a}}{x^{16}}\,dx$$

input
```
int((a + b*x^3 + c*x^6)^(1/2)/x^16,x)
```

output
```
int((a + b*x^3 + c*x^6)^(1/2)/x^16, x)
```

3.196 $\int x^3\sqrt{a+bx^3+cx^6}\,dx$

3.196.1 Optimal result

Integrand size = 20, antiderivative size = 140

$$\int x^3\sqrt{a+bx^3+cx^6}\,dx = \frac{x^4\sqrt{a+bx^3+cx^6}\,\text{AppellF1}\left(\frac{4}{3},-\frac{1}{2},-\frac{1}{2},\frac{7}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

output
```
1/4*x^4*AppellF1(4/3,-1/2,-1/2,7/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^
3/(b+(-4*a*c+b^2)^(1/2)))*(c*x^6+b*x^3+a)^(1/2)/(1+2*c*x^3/(b-(-4*a*c+b^2)
^(1/2)))^(1/2)/(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.196.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 358 vs. 2(140) = 280.

Time = 8.91 (sec) , antiderivative size = 358, normalized size of antiderivative = 2.56

$$\int x^3\sqrt{a+bx^3+cx^6}\,dx = \frac{x\left(8(3b+8cx^3)(a+bx^3+cx^6)-24ab\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\right.\right.}{448c\sqrt{}}$$

input
```
Integrate[x^3*Sqrt[a + b*x^3 + c*x^6],x]
```

output
```
(x*(8*(3*b + 8*c*x^3)*(a + b*x^3 + c*x^6) - 24*a*b*Sqrt[(b - Sqrt[b^2 - 4*
a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c
*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3)/(b
+ Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + 3*(-5*b^2 + 16
*a*c)*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*
Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[4
/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqr
t[b^2 - 4*a*c])]))/(448*c*Sqrt[a + b*x^3 + c*x^6])
```

3.196.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{a+bx^3+cx^6}\,dx$$

$$\Big\downarrow \text{1721}$$

$$\frac{\sqrt{a+bx^3+cx^6}\int x^3\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{b+\sqrt{b^2-4ac}}+1}dx}{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^4\sqrt{a+bx^3+cx^6}\,\text{AppellF1}\left(\frac{4}{3},-\frac{1}{2},-\frac{1}{2},\frac{7}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[x^3*Sqrt[a + b*x^3 + c*x^6],x]
```

output
```
(x^4*Sqrt[a + b*x^3 + c*x^6]*AppellF1[4/3, -1/2, -1/2, 7/3, (-2*c*x^3)/(b
- Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(4*Sqrt[1 + (2*
c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])
])
```

3.196.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

3.196.4 Maple [F]

$$\int x^3\sqrt{c\,x^6+b\,x^3+a}dx$$

input
```
int(x^3*(c*x^6+b*x^3+a)^(1/2),x)
```

output
```
int(x^3*(c*x^6+b*x^3+a)^(1/2),x)
```

3.196.5 Fricas [F]

$$\int x^3\sqrt{a+bx^3+cx^6}\,dx=\int \sqrt{cx^6+bx^3+a}x^3\,dx$$

input
```
integrate(x^3*(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^6 + b*x^3 + a)*x^3, x)
```

3.196.6 Sympy [F]

$$\int x^3\sqrt{a+bx^3+cx^6}\,dx=\int x^3\sqrt{a+bx^3+cx^6}\,dx$$

input
```
integrate(x**3*(c*x**6+b*x**3+a)**(1/2),x)
```

output
```
Integral(x**3*sqrt(a + b*x**3 + c*x**6), x)
```

3.196.7 Maxima [F]

$$\int x^3\sqrt{a+bx^3+cx^6}\,dx=\int \sqrt{cx^6+bx^3+a}x^3\,dx$$

input
```
integrate(x^3*(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^6 + b*x^3 + a)*x^3, x)
```

3.196.8 Giac [F]

$$\int x^3\sqrt{a+bx^3+cx^6}\,dx=\int \sqrt{cx^6+bx^3+a}x^3\,dx$$

input
```
integrate(x^3*(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^6 + b*x^3 + a)*x^3, x)
```

3.196.9 Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{a+bx^3+cx^6}\,dx=\int x^3\,\sqrt{c\,x^6+b\,x^3+a}\,dx$$

input
```
int(x^3*(a + b*x^3 + c*x^6)^(1/2),x)
```

output
```
int(x^3*(a + b*x^3 + c*x^6)^(1/2), x)
```

3.197 $\int x\sqrt{a+bx^3+cx^6}\,dx$

3.197.1 Optimal result

Integrand size = 18, antiderivative size = 140

$$\int x\sqrt{a+bx^3+cx^6}\,dx = \frac{x^2\sqrt{a+bx^3+cx^6}\,\text{AppellF1}\left(\frac{2}{3},-\frac{1}{2},-\frac{1}{2},\frac{5}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

output
```
1/2*x^2*AppellF1(2/3,-1/2,-1/2,5/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^
3/(b+(-4*a*c+b^2)^(1/2)))*(c*x^6+b*x^3+a)^(1/2)/(1+2*c*x^3/(b-(-4*a*c+b^2)
^(1/2)))^(1/2)/(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.197.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 337 vs. 2(140) = 280.

Time = 10.03 (sec) , antiderivative size = 337, normalized size of antiderivative = 2.41

$$\int x\sqrt{a+bx^3+cx^6}\,dx = \frac{x^2\left(10(a+bx^3+cx^6)+15a\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{5}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right.\right.}{50\sqrt{a+bx^3+cx}}$$

input
```
Integrate[x*Sqrt[a + b*x^3 + c*x^6],x]
```

output
```
(x^2*(10*(a + b*x^3 + c*x^6) + 15*a*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)
/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt
[b^2 - 4*a*c])]*AppellF1[2/3, 1/2, 1/2, 5/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*
a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + 3*b*x^3*Sqrt[(b - Sqrt[b^2 -
4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2
*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/3, 1/2, 1/2, 8/3, (-2*c*x^3)/(
b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]))/(50*Sqrt[a +
 b*x^3 + c*x^6])
```

3.197.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{a+bx^3+cx^6}\,dx$$

$$\Big\downarrow \text{1721}$$

$$\frac{\sqrt{a+bx^3+cx^6}\int x\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{b+\sqrt{b^2-4ac}}+1}dx}{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^2\sqrt{a+bx^3+cx^6}\,\text{AppellF1}\left(\frac{2}{3},-\frac{1}{2},-\frac{1}{2},\frac{5}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[x*Sqrt[a + b*x^3 + c*x^6],x]
```

output
```
(x^2*Sqrt[a + b*x^3 + c*x^6]*AppellF1[2/3, -1/2, -1/2, 5/3, (-2*c*x^3)/(b
- Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(2*Sqrt[1 + (2*
c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])
])
```

3.197.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

3.197.4 Maple [F]

$$\int x\sqrt{c\,x^{6}+b\,x^{3}+a}dx$$

input
```
int(x*(c*x^6+b*x^3+a)^(1/2),x)
```

output
```
int(x*(c*x^6+b*x^3+a)^(1/2),x)
```

3.197.5 Fricas [F]

$$\int x\sqrt{a+bx^3+cx^6}\,dx=\int\sqrt{cx^6+bx^3+a}x\,dx$$

input
```
integrate(x*(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^6 + b*x^3 + a)*x, x)
```

3.197.6 Sympy [F]

$$\int x\sqrt{a+bx^3+cx^6}\,dx=\int x\sqrt{a+bx^3+cx^6}\,dx$$

input
```
integrate(x*(c*x**6+b*x**3+a)**(1/2),x)
```

output
```
Integral(x*sqrt(a + b*x**3 + c*x**6), x)
```

3.197.7 Maxima [F]

$$\int x\sqrt{a+bx^3+cx^6}\,dx=\int \sqrt{cx^6+bx^3+a}x\,dx$$

input
```
integrate(x*(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^6 + b*x^3 + a)*x, x)
```

3.197.8 Giac [F]

$$\int x\sqrt{a+bx^3+cx^6}\,dx=\int \sqrt{cx^6+bx^3+a}x\,dx$$

input
```
integrate(x*(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^6 + b*x^3 + a)*x, x)
```

3.197.9 Mupad [F(-1)]

Timed out.

$$\int x\sqrt{a+bx^3+cx^6}\,dx=\int x\,\sqrt{c\,x^6+b\,x^3+a}\,dx$$

input
```
int(x*(a + b*x^3 + c*x^6)^(1/2),x)
```

output
```
int(x*(a + b*x^3 + c*x^6)^(1/2), x)
```

3.198 $\int \sqrt{a+bx^3+cx^6}\,dx$

3.198.1 Optimal result

Integrand size = 16, antiderivative size = 135

$$\int \sqrt{a+bx^3+cx^6}\,dx=\frac{x\sqrt{a+bx^3+cx^6}\operatorname{AppellF1}\left(\frac{1}{3},-\frac{1}{2},-\frac{1}{2},\frac{4}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

output
```
x*AppellF1(1/3,-1/2,-1/2,4/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/(b+(
-4*a*c+b^2)^(1/2)))*(c*x^6+b*x^3+a)^(1/2)/(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)
))^(1/2)/(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.198.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 335 vs. 2(135) = 270.

Time = 10.24 (sec) , antiderivative size = 335, normalized size of antiderivative = 2.48

$$\int \sqrt{a+bx^3+cx^6}\,dx$$
$$=\frac{x\left(8(a+bx^3+cx^6)+24a\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)\right.}{32\sqrt{a+bx^3+cx^6}}$$

input
```
Integrate[Sqrt[a + b*x^3 + c*x^6],x]
```

output
```
(x*(8*(a + b*x^3 + c*x^6) + 24*a*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b
 - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^
2 - 4*a*c])]*AppellF1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c
]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + 3*b*x^3*Sqrt[(b - Sqrt[b^2 - 4*a
*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*
x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b +
 Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]))/(32*Sqrt[a + b*
x^3 + c*x^6])
```

3.198.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1686, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^3+cx^6}\,dx$$

$$\Big\downarrow \text{1686}$$

$$\frac{\sqrt{a+bx^3+cx^6}\int \sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{b+\sqrt{b^2-4ac}}+1}dx}{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

$$\Big\downarrow \text{936}$$

$$\frac{x\sqrt{a+bx^3+cx^6}\,\text{AppellF1}\left(\frac{1}{3},-\frac{1}{2},-\frac{1}{2},\frac{4}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[Sqrt[a + b*x^3 + c*x^6],x]
```

output
```
(x*Sqrt[a + b*x^3 + c*x^6]*AppellF1[1/3, -1/2, -1/2, 4/3, (-2*c*x^3)/(b -
Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt[1 + (2*c*x^
3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])
```

3.198.3.1 Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1686
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^
IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*c*(x^n/(b + Rt[b^2
- 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4*a*c, 2])))^FracPar
t[p]))   Int[(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2*c*(x^n/(b - S
qrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n]
&& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

3.198.4 Maple [F]

$$\int \sqrt{c\,x^{6}+b\,x^{3}+a}dx$$

input
```
int((c*x^6+b*x^3+a)^(1/2),x)
```

output
```
int((c*x^6+b*x^3+a)^(1/2),x)
```

3.198.5 Fricas [F]

$$\int \sqrt{a+bx^3+cx^6}\,dx=\int \sqrt{cx^6+bx^3+a}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^6 + b*x^3 + a), x)
```

3.198.6 Sympy [F]

$$\int \sqrt{a+bx^3+cx^6}\,dx = \int \sqrt{a+bx^3+cx^6}\,dx$$

input
```
integrate((c*x**6+b*x**3+a)**(1/2),x)
```

output
```
Integral(sqrt(a + b*x**3 + c*x**6), x)
```

3.198.7 Maxima [F]

$$\int \sqrt{a+bx^3+cx^6}\,dx = \int \sqrt{cx^6+bx^3+a}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^6 + b*x^3 + a), x)
```

3.198.8 Giac [F]

$$\int \sqrt{a+bx^3+cx^6}\,dx = \int \sqrt{cx^6+bx^3+a}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^6 + b*x^3 + a), x)
```

3.198.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx^3+cx^6}\,dx=\int \sqrt{c\,x^6+b\,x^3+a}\,dx$$

input
```
int((a + b*x^3 + c*x^6)^(1/2),x)
```

output
```
int((a + b*x^3 + c*x^6)^(1/2), x)
```

3.199 $\int \frac{\sqrt{a+bx^3+cx^6}}{x^2} dx$

3.199.1 Optimal result

Integrand size = 20, antiderivative size = 138

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^2}\,dx = -\frac{\sqrt{a+bx^3+cx^6}\,\text{AppellF1}\left(-\frac{1}{3},-\frac{1}{2},-\frac{1}{2},\frac{2}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

output
```
-AppellF1(-1/3,-1/2,-1/2,2/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/(b+(
-4*a*c+b^2)^(1/2)))*(c*x^6+b*x^3+a)^(1/2)/x/(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/
2)))^(1/2)/(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.199.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 340 vs. 2(138) = 276.

Time = 10.26 (sec) , antiderivative size = 340, normalized size of antiderivative = 2.46

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^2}\,dx = \frac{-20(a+bx^3+cx^6)+15bx^3\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{5}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right.}{20x\sqrt{a+bx^3+c}}$$

input
```
Integrate[Sqrt[a + b*x^3 + c*x^6]/x^2,x]
```

output
```
(-20*(a + b*x^3 + c*x^6) + 15*b*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)
/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt
[b^2 - 4*a*c])]*AppellF1[2/3, 1/2, 1/2, 5/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*
a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + 12*c*x^6*Sqrt[(b - Sqrt[b^2 -
 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] +
2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/3, 1/2, 1/2, 8/3, (-2*c*x^3)/
(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(20*x*Sqrt[a
 + b*x^3 + c*x^6])
```

3.199.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^2} \, dx$$

$$\Big\downarrow \text{ 1721}$$

$$\frac{\sqrt{a+bx^3+cx^6}\int \frac{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{b+\sqrt{b^2-4ac}}+1}}{x^2}dx}{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

$$\Big\downarrow \text{ 1012}$$

$$-\frac{\sqrt{a+bx^3+cx^6}\,\text{AppellF1}\left(-\frac{1}{3},-\frac{1}{2},-\frac{1}{2},\frac{2}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[Sqrt[a + b*x^3 + c*x^6]/x^2,x]
```

output
```
-((Sqrt[a + b*x^3 + c*x^6]*AppellF1[-1/3, -1/2, -1/2, 2/3, (-2*c*x^3)/(b -
 Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(x*Sqrt[1 + (2*c
*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]
))
```

3.199.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

3.199.4 Maple [F]

$$\int \frac{\sqrt{c\,x^{6}+b\,x^{3}+a}}{x^{2}}dx$$

input
```
int((c*x^6+b*x^3+a)^(1/2)/x^2,x)
```

output
```
int((c*x^6+b*x^3+a)^(1/2)/x^2,x)
```

3.199.5 Fricas [F]

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^2}\,dx = \int \frac{\sqrt{cx^6+bx^3+a}}{x^2}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^6 + b*x^3 + a)/x^2, x)
```

3.199.6 Sympy [F]

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^2}\,dx = \int \frac{\sqrt{a+bx^3+cx^6}}{x^2}\,dx$$

input
```
integrate((c*x**6+b*x**3+a)**(1/2)/x**2,x)
```

output
```
Integral(sqrt(a + b*x**3 + c*x**6)/x**2, x)
```

3.199.7 Maxima [F]

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^2}\,dx = \int \frac{\sqrt{cx^6+bx^3+a}}{x^2}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^6 + b*x^3 + a)/x^2, x)
```

3.199.8 Giac [F]

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^2}\,dx = \int \frac{\sqrt{cx^6+bx^3+a}}{x^2}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^(1/2)/x^2,x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^6 + b*x^3 + a)/x^2, x)
```

3.199.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^2}\,dx=\int \frac{\sqrt{c\,x^6+b\,x^3+a}}{x^2}\,dx$$

input
```
int((a + b*x^3 + c*x^6)^(1/2)/x^2,x)
```

output
```
int((a + b*x^3 + c*x^6)^(1/2)/x^2, x)
```

3.200 $\int \frac{\sqrt{a+bx^3+cx^6}}{x^3}\,dx$

3.200.1 Optimal result

Integrand size = 20, antiderivative size = 140

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^3}\,dx = -\frac{\sqrt{a+bx^3+cx^6}\,\text{AppellF1}\left(-\frac{2}{3},-\frac{1}{2},-\frac{1}{2},\frac{1}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

output
```
-1/2*AppellF1(-2/3,-1/2,-1/2,1/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/
(b+(-4*a*c+b^2)^(1/2)))*(c*x^6+b*x^3+a)^(1/2)/x^2/(1+2*c*x^3/(b-(-4*a*c+b^
2)^(1/2)))^(1/2)/(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.200.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 340 vs. 2(140) = 280.

Time = 10.21 (sec) , antiderivative size = 340, normalized size of antiderivative = 2.43

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^3}\,dx = \frac{-4(a+bx^3+cx^6)+6bx^3\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)}{8x^2\sqrt{a+bx^3+cx^6}}$$

input
```
Integrate[Sqrt[a + b*x^3 + c*x^6]/x^3,x]
```

output
```
(-4*(a + b*x^3 + c*x^6) + 6*b*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(
b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b
^2 - 4*a*c])]*AppellF1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*
c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + 3*c*x^6*Sqrt[(b - Sqrt[b^2 - 4*
a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c
*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b
+ Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(8*x^2*Sqrt[a +
 b*x^3 + c*x^6])
```

3.200.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^3} \, dx$$

$$\Big\downarrow \text{ 1721}$$

$$\frac{\sqrt{a+bx^3+cx^6} \int \frac{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{b+\sqrt{b^2-4ac}}+1}}{x^3}dx}{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

$$\Big\downarrow \text{ 1012}$$

$$-\frac{\sqrt{a+bx^3+cx^6}\,\text{AppellF1}\left(-\frac{2}{3},-\frac{1}{2},-\frac{1}{2},\frac{1}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[Sqrt[a + b*x^3 + c*x^6]/x^3,x]
```

output
```
-1/2*(Sqrt[a + b*x^3 + c*x^6]*AppellF1[-2/3, -1/2, -1/2, 1/3, (-2*c*x^3)/(
b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(x^2*Sqrt[1 +
 (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a
*c])])
```

3.200.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

3.200.4 Maple [F]

$$\int \frac{\sqrt{c\,x^{6}+b\,x^{3}+a}}{x^{3}}dx$$

input
```
int((c*x^6+b*x^3+a)^(1/2)/x^3,x)
```

output
```
int((c*x^6+b*x^3+a)^(1/2)/x^3,x)
```

3.200.5 Fricas [F]

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^3}\,dx=\int \frac{\sqrt{cx^6+bx^3+a}}{x^3}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^6 + b*x^3 + a)/x^3, x)
```

3.200.6 Sympy [F]

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^3}\,dx = \int \frac{\sqrt{a+bx^3+cx^6}}{x^3}\,dx$$

input
```
integrate((c*x**6+b*x**3+a)**(1/2)/x**3,x)
```

output
```
Integral(sqrt(a + b*x**3 + c*x**6)/x**3, x)
```

3.200.7 Maxima [F]

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^3}\,dx = \int \frac{\sqrt{cx^6+bx^3+a}}{x^3}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^6 + b*x^3 + a)/x^3, x)
```

3.200.8 Giac [F]

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^3}\,dx = \int \frac{\sqrt{cx^6+bx^3+a}}{x^3}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^(1/2)/x^3,x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^6 + b*x^3 + a)/x^3, x)
```

3.200.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^3}\,dx=\int \frac{\sqrt{c\,x^6+b\,x^3+a}}{x^3}\,dx$$

input
```
int((a + b*x^3 + c*x^6)^(1/2)/x^3,x)
```

output
```
int((a + b*x^3 + c*x^6)^(1/2)/x^3, x)
```

3.201 $\int x^{14}(a+bx^3+cx^6)^{3/2}\,dx$

3.201.1 Optimal result

Integrand size = 20, antiderivative size = 293

$$\int x^{14}(a+bx^3+cx^6)^{3/2}\,dx = -\frac{(b^2-4ac)(33b^4-72ab^2c+16a^2c^2)(b+2cx^3)\sqrt{a+bx^3+cx^6}}{16384c^6} + \frac{(33b^4-72ab^2c+16a^2c^2)(b+2cx^3)(a+bx^3+cx^6)^{3/2}}{6144c^5} - \frac{11bx^6(a+bx^3+cx^6)^{5/2}}{336c^2} + \frac{x^9(a+bx^3+cx^6)^{5/2}}{24c} - \frac{(3b(77b^2-124ac)-10c(33b^2-28ac)x^3)(a+bx^3+cx^6)^{5/2}}{13440c^4} + \frac{(b^2-4ac)^2(33b^4-72ab^2c+16a^2c^2)\operatorname{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{32768c^{13/2}}$$

output
```
1/6144*(16*a^2*c^2-72*a*b^2*c+33*b^4)*(2*c*x^3+b)*(c*x^6+b*x^3+a)^(3/2)/c^
5-11/336*b*x^6*(c*x^6+b*x^3+a)^(5/2)/c^2+1/24*x^9*(c*x^6+b*x^3+a)^(5/2)/c-
1/13440*(3*b*(-124*a*c+77*b^2)-10*c*(-28*a*c+33*b^2)*x^3)*(c*x^6+b*x^3+a)^
(5/2)/c^4+1/32768*(-4*a*c+b^2)^2*(16*a^2*c^2-72*a*b^2*c+33*b^4)*arctanh(1/
2*(2*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))/c^(13/2)-1/16384*(-4*a*c+b^2)
*(16*a^2*c^2-72*a*b^2*c+33*b^4)*(2*c*x^3+b)*(c*x^6+b*x^3+a)^(1/2)/c^6
```

3.201.2 Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.99

$$\int x^{14}(a+bx^3+cx^6)^{3/2}\,dx = \frac{2\sqrt{c}\sqrt{a+bx^3+cx^6}(-3465b^7+2310b^6cx^3+84b^5c(365a-22cx^6)+24b^4c^2x^3(-749a+66cx^6}{}$$

input
```
Integrate[x^14*(a + b*x^3 + c*x^6)^(3/2),x]
```

output
```
(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]*(-3465*b^7 + 2310*b^6*c*x^3 + 84*b^5*c*
(365*a - 22*c*x^6) + 24*b^4*c^2*x^3*(-749*a + 66*c*x^6) + 32*b^2*c^3*x^3*(
1181*a^2 - 284*a*c*x^6 + 40*c^2*x^12) - 16*b^3*c^2*(5103*a^2 - 780*a*c*x^6
 + 88*c^2*x^12) + 4480*c^4*x^3*(-3*a^3 + 2*a^2*c*x^6 + 24*a*c^2*x^12 + 16*
c^3*x^18) + 64*b*c^3*(919*a^3 - 302*a^2*c*x^6 + 104*a*c^2*x^12 + 1360*c^3*
x^18)) - 105*(b^2 - 4*a*c)^2*(33*b^4 - 72*a*b^2*c + 16*a^2*c^2)*Log[b + 2*
c*x^3 - 2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]])/(3440640*c^(13/2))
```

3.201.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.97, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1693, 1166, 27, 1236, 27, 1225, 1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{14}(a+bx^3+cx^6)^{3/2}\,dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int x^{12}(cx^6+bx^3+a)^{3/2}\,dx^3$$

$$\Big\downarrow \text{1166}$$

$$\frac{1}{3}\left(\frac{\int -\frac{1}{2}x^6(11bx^3+6a)(cx^6+bx^3+a)^{3/2}\,dx^3}{8c}+\frac{x^9(a+bx^3+cx^6)^{5/2}}{8c}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{x^9\left(a+bx^3+cx^6\right)^{5/2}}{8c}-\frac{\int x^6\left(11bx^3+6a\right)\left(cx^6+bx^3+a\right)^{3/2}dx^3}{16c}\right)$$

$$\Big\downarrow \; 1236$$

$$\frac{1}{3}\left(\frac{x^9\left(a+bx^3+cx^6\right)^{5/2}}{8c}-\frac{\frac{\int -\frac{1}{2}x^3\left(3\left(33b^2-28ac\right)x^3+44ab\right)\left(cx^6+bx^3+a\right)^{3/2}dx^3}{7c}+\frac{11bx^6\left(a+bx^3+cx^6\right)^{5/2}}{7c}}{16c}\right)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3}\left(\frac{x^9\left(a+bx^3+cx^6\right)^{5/2}}{8c}-\frac{\frac{11bx^6\left(a+bx^3+cx^6\right)^{5/2}}{7c}-\frac{\int x^3\left(3\left(33b^2-28ac\right)x^3+44ab\right)\left(cx^6+bx^3+a\right)^{3/2}dx^3}{14c}}{16c}\right)$$

$$\Big\downarrow \; 1225$$

$$\frac{1}{3}\left(\frac{x^9\left(a+bx^3+cx^6\right)^{5/2}}{8c}-\frac{\frac{11bx^6\left(a+bx^3+cx^6\right)^{5/2}}{7c}-\frac{\frac{7\left(16a^2c^2-72ab^2c+33b^4\right)\int\left(cx^6+bx^3+a\right)^{3/2}dx^3}{8c^2}-\frac{\left(3b\left(77b^2-124ac\right)-10cx^3\left(33b^2-28ac\right)\cdots\right.}{20c^2}}{14c}}{16c}\right.$$

$$\Big\downarrow \; 1087$$

$$\frac{1}{3}\left(\frac{x^9\left(a+bx^3+cx^6\right)^{5/2}}{8c}-\frac{\frac{11bx^6\left(a+bx^3+cx^6\right)^{5/2}}{7c}-\frac{\frac{7\left(16a^2c^2-72ab^2c+33b^4\right)\left(\frac{\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\int\sqrt{cx^6+bx^3+a}\,dx^3}{16c}\right)}{8c^2}\cdots}{14c}}{16c}\right.$$

$$\Big\downarrow \; 1087$$

$$\frac{1}{3}\left(\frac{x^9\left(a+bx^3+cx^6\right)^{5/2}}{8c}-\frac{\frac{11bx^6\left(a+bx^3+cx^6\right)^{5/2}}{7c}-\frac{\frac{7\left(16a^2c^2-72ab^2c+33b^4\right)\left(\frac{\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b+2cx^3\right)\sqrt{a+bx^3\cdots}}{4c}\cdots\right.}{\cdots}\right.}{8c^2}\cdots}{16c}\right.$$

$$\Big\downarrow \; 1092$$

3.201. $\int x^{14}\left(a+bx^3+cx^6\right)^{3/2}\,dx$

$$\frac{1}{3}\left(\frac{x^9\left(a+bx^3+cx^6\right)^{5/2}}{8c}-\frac{\frac{11bx^6\left(a+bx^3+cx^6\right)^{5/2}}{7c}-\frac{\frac{7\left(16a^2c^2-72ab^2c+33b^4\right)\left(\frac{\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b+2cx^3\right)\sqrt{a+bx^3}}{4c}\cdots\right.}{\cdots}\right.}{8c^2}}{\cdots}}{16c}\cdots\right.$$

$\downarrow$ 219

$$\frac{1}{3}\left(\frac{x^9\left(a+bx^3+cx^6\right)^{5/2}}{8c}-\frac{\frac{11bx^6\left(a+bx^3+cx^6\right)^{5/2}}{7c}-\frac{\frac{7\left(16a^2c^2-72ab^2c+33b^4\right)\left(\frac{\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b+2cx^3\right)\sqrt{a+bx^3}}{4c}\cdots\right.}{\cdots}\right.}{8c^2}}{\cdots}}{16c}\cdots\right.$$

input
```
Int[x^14*(a + b*x^3 + c*x^6)^(3/2),x]
```

output
```
((x^9*(a + b*x^3 + c*x^6)^(5/2))/(8*c) - ((11*b*x^6*(a + b*x^3 + c*x^6)^(5
/2))/(7*c) - (-1/20*((3*b*(77*b^2 - 124*a*c) - 10*c*(33*b^2 - 28*a*c)*x^3)
*(a + b*x^3 + c*x^6)^(5/2))/c^2 + (7*(33*b^4 - 72*a*b^2*c + 16*a^2*c^2)*((
(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(8*c) - (3*(b^2 - 4*a*c)*(((b + 2
*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(4*c) - ((b^2 - 4*a*c)*ArcTanh[(b + 2*c*x
^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(8*c^(3/2))))/(16*c)))/(8*c^2))/
(14*c))/(16*c))/3
```

3.201. $\int x^{14}\left(a+bx^3+cx^6\right)^{3/2}\,dx$

3.201.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

3.201. $\int x^{14}(a+bx^3+cx^6)^{3/2}\,dx$

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.201.4 Maple [F]

$$\int x^{14}\left(c\,x^{6}+b\,x^{3}+a\right)^{\frac{3}{2}}dx$$

input
```
int(x^14*(c*x^6+b*x^3+a)^(3/2),x)
```

output
```
int(x^14*(c*x^6+b*x^3+a)^(3/2),x)
```

3.201.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 641, normalized size of antiderivative = 2.19

$$\int x^{14}(a+bx^3+cx^6)^{3/2}\,dx = \left[\frac{105\,(33\,b^8-336\,ab^6c+1120\,a^2b^4c^2-1280\,a^3b^2c^3+256\,a^4c^4)\sqrt{c}\log\left(-8\,c^2x^6-8\,bcx^3-b^2\right.}{}\right.$$

$$-\frac{105\,(33\,b^8-336\,ab^6c+1120\,a^2b^4c^2-1280\,a^3b^2c^3+256\,a^4c^4)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{-c}}{2\,(c^2x^6+bcx^3+ac)}\right)-2\,(71}{}$$

input
```
integrate(x^14*(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/6881280*(105*(33*b^8 - 336*a*b^6*c + 1120*a^2*b^4*c^2 - 1280*a^3*b^2*c^
3 + 256*a^4*c^4)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 +
 b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 4*(71680*c^8*x^21 + 87040*b*c
^7*x^18 + 1280*(b^2*c^6 + 84*a*c^7)*x^15 - 128*(11*b^3*c^5 - 52*a*b*c^6)*x
^12 + 16*(99*b^4*c^4 - 568*a*b^2*c^5 + 560*a^2*c^6)*x^9 - 3465*b^7*c + 306
60*a*b^5*c^2 - 81648*a^2*b^3*c^3 + 58816*a^3*b*c^4 - 8*(231*b^5*c^3 - 1560
*a*b^3*c^4 + 2416*a^2*b*c^5)*x^6 + 2*(1155*b^6*c^2 - 8988*a*b^4*c^3 + 1889
6*a^2*b^2*c^4 - 6720*a^3*c^5)*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^7, -1/344064
0*(105*(33*b^8 - 336*a*b^6*c + 1120*a^2*b^4*c^2 - 1280*a^3*b^2*c^3 + 256*a
^4*c^4)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)
/(c^2*x^6 + b*c*x^3 + a*c)) - 2*(71680*c^8*x^21 + 87040*b*c^7*x^18 + 1280*
(b^2*c^6 + 84*a*c^7)*x^15 - 128*(11*b^3*c^5 - 52*a*b*c^6)*x^12 + 16*(99*b^
4*c^4 - 568*a*b^2*c^5 + 560*a^2*c^6)*x^9 - 3465*b^7*c + 30660*a*b^5*c^2 -
81648*a^2*b^3*c^3 + 58816*a^3*b*c^4 - 8*(231*b^5*c^3 - 1560*a*b^3*c^4 + 24
16*a^2*b*c^5)*x^6 + 2*(1155*b^6*c^2 - 8988*a*b^4*c^3 + 18896*a^2*b^2*c^4 -
 6720*a^3*c^5)*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^7]
```

3.201.6 Sympy [F]

$$\int x^{14}(a+bx^3+cx^6)^{3/2}\,dx=\int x^{14}(a+bx^3+cx^6)^{\frac{3}{2}}\,dx$$

input
```
integrate(x**14*(c*x**6+b*x**3+a)**(3/2),x)
```

output
```
Integral(x**14*(a + b*x**3 + c*x**6)**(3/2), x)
```

3.201.7 Maxima [F(-2)]

Exception generated.

$$\int x^{14}(a+bx^3+cx^6)^{3/2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^14*(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.201.8 Giac [F]

$$\int x^{14}(a+bx^3+cx^6)^{3/2}\,dx=\int (cx^6+bx^3+a)^{\frac{3}{2}}x^{14}\,dx$$

input
```
integrate(x^14*(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^(3/2)*x^14, x)
```

3.201.9 Mupad [F(-1)]

Timed out.

$$\int x^{14}(a+bx^3+cx^6)^{3/2}\,dx=\int x^{14}\left(c\,x^6+b\,x^3+a\right)^{3/2}\,dx$$

input
```
int(x^14*(a + b*x^3 + c*x^6)^(3/2),x)
```

output
```
int(x^14*(a + b*x^3 + c*x^6)^(3/2), x)
```

3.202 $\int x^{11}(a+bx^3+cx^6)^{3/2}\,dx$

3.202.1 Optimal result

Integrand size = 20, antiderivative size = 223

$$\int x^{11}\left(a+bx^3+cx^6\right)^{3/2}\,dx=\frac{b(b^2-4ac)\,(3b^2-4ac)\,(b+2cx^3)\,\sqrt{a+bx^3+cx^6}}{1024c^5}-\frac{b(3b^2-4ac)\,(b+2cx^3)\,\left(a+bx^3+cx^6\right)^{3/2}}{384c^4}+\frac{x^6\left(a+bx^3+cx^6\right)^{5/2}}{21c}+\frac{(21b^2-16ac-30bcx^3)\,\left(a+bx^3+cx^6\right)^{5/2}}{840c^3}-\frac{b(b^2-4ac)^2\,(3b^2-4ac)\,\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{2048c^{11/2}}$$

output
```
-1/384*b*(-4*a*c+3*b^2)*(2*c*x^3+b)*(c*x^6+b*x^3+a)^(3/2)/c^4+1/21*x^6*(c*
x^6+b*x^3+a)^(5/2)/c+1/840*(-30*b*c*x^3-16*a*c+21*b^2)*(c*x^6+b*x^3+a)^(5/
2)/c^3-1/2048*b*(-4*a*c+b^2)^2*(-4*a*c+3*b^2)*arctanh(1/2*(2*c*x^3+b)/c^(1
/2)/(c*x^6+b*x^3+a)^(1/2))/c^(11/2)+1/1024*b*(-4*a*c+b^2)*(-4*a*c+3*b^2)*(
2*c*x^3+b)*(c*x^6+b*x^3+a)^(1/2)/c^5
```

3.202.2 Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.99

$$\int x^{11}\left(a+bx^3+cx^6\right)^{3/2}\,dx=\frac{\sqrt{a+bx^3+cx^6}\left(315b^6-210b^5cx^3+16b^3c^2x^3(91a-9cx^6)+168b^4c(-15a+cx^6)+1024c^3(a\cdots\right.}{1\cdots}+\frac{b(b^2-4ac)^2\left(3b^2-4ac\right)\log\left(b+2cx^3-2\sqrt{c}\sqrt{a+bx^3+cx^6}\right)}{2048c^{11/2}}$$

input
```
Integrate[x^11*(a + b*x^3 + c*x^6)^(3/2),x]
```

output
```
(Sqrt[a + b*x^3 + c*x^6]*(315*b^6 - 210*b^5*c*x^3 + 16*b^3*c^2*x^3*(91*a -
 9*c*x^6) + 168*b^4*c*(-15*a + c*x^6) + 1024*c^3*(a + c*x^6)^2*(-2*a + 5*c
*x^6) + 16*b^2*c^2*(343*a^2 - 62*a*c*x^6 + 8*c^2*x^12) + 32*b*c^3*x^3*(-73
*a^2 + 22*a*c*x^6 + 200*c^2*x^12)))/(107520*c^5) + (b*(b^2 - 4*a*c)^2*(3*b
^2 - 4*a*c)*Log[b + 2*c*x^3 - 2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]])/(2048*c^
(11/2))
```

3.202.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1693, 1166, 27, 1225, 1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{11}\left(a+bx^3+cx^6\right)^{3/2}dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int x^9\left(cx^6+bx^3+a\right)^{3/2}dx^3$$

$$\Big\downarrow \text{1166}$$

$$\frac{1}{3}\left(\frac{\int -\frac{1}{2}x^3\left(9bx^3+4a\right)\left(cx^6+bx^3+a\right)^{3/2}dx^3}{7c}+\frac{x^6\left(a+bx^3+cx^6\right)^{5/2}}{7c}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{x^6\left(a+bx^3+cx^6\right)^{5/2}}{7c}-\frac{\int x^3\left(9bx^3+4a\right)\left(cx^6+bx^3+a\right)^{3/2}dx^3}{14c}\right)$$

$\downarrow$ 1225

$$\frac{1}{3}\left(\frac{x^6\left(a+bx^3+cx^6\right)^{5/2}}{7c}-\frac{\frac{7b(3b^2-4ac)\int\left(cx^6+bx^3+a\right)^{3/2}dx^3}{8c^2}-\frac{\left(-16ac+21b^2-30bcx^3\right)\left(a+bx^3+cx^6\right)^{5/2}}{20c^2}}{14c}\right)$$

$\downarrow$ 1087

$$\frac{1}{3}\left(\frac{x^6\left(a+bx^3+cx^6\right)^{5/2}}{7c}-\frac{\frac{7b(3b^2-4ac)\left(\frac{\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\int\sqrt{cx^6+bx^3+a}dx^3}{16c}\right)}{8c^2}-\frac{\left(-16ac+21b^2-30bcx^3\right)\left(a+bx^3\right.}{20c^2}}{14c}\right.$$

$\downarrow$ 1087

$$\frac{1}{3}\left(\frac{x^6\left(a+bx^3+cx^6\right)^{5/2}}{7c}-\frac{\frac{7b(3b^2-4ac)\left(\frac{\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{4c}-\frac{\left(b^2-4ac\right)\int\frac{1}{\sqrt{cx^6+bx^3+a}}dx^3}{8c}\right)}{16c}\right.}{8c^2}}{14c}\right.$$

$\downarrow$ 1092

$$\frac{1}{3}\left(\frac{x^6\left(a+bx^3+cx^6\right)^{5/2}}{7c}-\frac{\frac{7b(3b^2-4ac)\left(\frac{\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{4c}-\frac{\left(b^2-4ac\right)\int\frac{1}{4c-x^6}d\frac{2cx^3+b}{\sqrt{cx^6+bx^3+}}}{4c}\right.}{16c}\right.}{8c^2}}{14c}\right.$$

$\downarrow$ 219

3.202. $\int x^{11}\left(a+bx^3+cx^6\right)^{3/2}\,dx$

$$\frac{1}{3}\left(\frac{x^6\left(a+bx^3+cx^6\right)^{5/2}}{7c}-\frac{\frac{7b(3b^2-4ac)\left(\frac{\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{4c}-\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{8c^{3/2}}\right)}{16c}\right)}{8c^2}}{14c}\right)$$

input
```
Int[x^11*(a + b*x^3 + c*x^6)^(3/2),x]
```

output
```
((x^6*(a + b*x^3 + c*x^6)^(5/2))/(7*c) - (-1/20*((21*b^2 - 16*a*c - 30*b*c
*x^3)*(a + b*x^3 + c*x^6)^(5/2))/c^2 + (7*b*(3*b^2 - 4*a*c)*(((b + 2*c*x^3
)*(a + b*x^3 + c*x^6)^(3/2))/(8*c) - (3*(b^2 - 4*a*c)*(((b + 2*c*x^3)*Sqrt
[a + b*x^3 + c*x^6])/(4*c) - ((b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[
c]*Sqrt[a + b*x^3 + c*x^6])])/(8*c^(3/2))))/(16*c)))/(8*c^2))/(14*c))/3
```

3.202.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.202.4 Maple [F]

$$\int x^{11} \left(c\,x^{6}+b\,x^{3}+a\right)^{\frac{3}{2}}d x$$

input
```
int(x^11*(c*x^6+b*x^3+a)^(3/2),x)
```

output
```
int(x^11*(c*x^6+b*x^3+a)^(3/2),x)
```

3.202.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 535, normalized size of antiderivative = 2.40

$$\int x^{11}(a+bx^3+cx^6)^{3/2}\,dx = \left[-\frac{105\,(3\,b^7-28\,ab^5c+80\,a^2b^3c^2-64\,a^3bc^3)\sqrt{c}\log\left(-8\,c^2x^6-8\,bcx^3-b^2-4\,\sqrt{cx^6+bx^3+\right.}{}\right.$$

input
```
integrate(x^11*(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/430080*(105*(3*b^7 - 28*a*b^5*c + 80*a^2*b^3*c^2 - 64*a^3*b*c^3)*sqrt(
c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 +
 b)*sqrt(c) - 4*a*c) - 4*(5120*c^7*x^18 + 6400*b*c^6*x^15 + 128*(b^2*c^5 +
 64*a*c^6)*x^12 - 16*(9*b^3*c^4 - 44*a*b*c^5)*x^9 + 315*b^6*c - 2520*a*b^4
*c^2 + 5488*a^2*b^2*c^3 - 2048*a^3*c^4 + 8*(21*b^4*c^3 - 124*a*b^2*c^4 + 1
28*a^2*c^5)*x^6 - 2*(105*b^5*c^2 - 728*a*b^3*c^3 + 1168*a^2*b*c^4)*x^3)*sq
rt(c*x^6 + b*x^3 + a))/c^6, 1/215040*(105*(3*b^7 - 28*a*b^5*c + 80*a^2*b^3
*c^2 - 64*a^3*b*c^3)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3
+ b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*(5120*c^7*x^18 + 6400*b*c^6*x
^15 + 128*(b^2*c^5 + 64*a*c^6)*x^12 - 16*(9*b^3*c^4 - 44*a*b*c^5)*x^9 + 31
5*b^6*c - 2520*a*b^4*c^2 + 5488*a^2*b^2*c^3 - 2048*a^3*c^4 + 8*(21*b^4*c^3
 - 124*a*b^2*c^4 + 128*a^2*c^5)*x^6 - 2*(105*b^5*c^2 - 728*a*b^3*c^3 + 116
8*a^2*b*c^4)*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^6]
```

3.202.6 Sympy [F]

$$\int x^{11}(a+bx^3+cx^6)^{3/2}\,dx = \int x^{11}(a+bx^3+cx^6)^{\frac{3}{2}}\,dx$$

input
```
integrate(x**11*(c*x**6+b*x**3+a)**(3/2),x)
```

output
```
Integral(x**11*(a + b*x**3 + c*x**6)**(3/2), x)
```

3.202.7 Maxima [F(-2)]

Exception generated.

$$\int x^{11}(a+bx^3+cx^6)^{3/2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^11*(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.202.8 Giac [F]

$$\int x^{11}(a+bx^3+cx^6)^{3/2}\,dx = \int (cx^6+bx^3+a)^{\frac{3}{2}}x^{11}\,dx$$

input
```
integrate(x^11*(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^(3/2)*x^11, x)
```

3.202.9 Mupad [F(-1)]

Timed out.

$$\int x^{11}(a+bx^3+cx^6)^{3/2}\,dx = \int x^{11}\,(c\,x^6+b\,x^3+a)^{3/2}\,dx$$

input
```
int(x^11*(a + b*x^3 + c*x^6)^(3/2),x)
```

output
```
int(x^11*(a + b*x^3 + c*x^6)^(3/2), x)
```

3.203 $\int x^8(a+bx^3+cx^6)^{3/2}\,dx$

3.203.1 Optimal result

Integrand size = 20, antiderivative size = 204

$$\int x^8(a+bx^3+cx^6)^{3/2}\,dx = -\frac{(b^2-4ac)(7b^2-4ac)(b+2cx^3)\sqrt{a+bx^3+cx^6}}{1536c^4} + \frac{(7b^2-4ac)(b+2cx^3)(a+bx^3+cx^6)^{3/2}}{576c^3} - \frac{7b(a+bx^3+cx^6)^{5/2}}{180c^2} + \frac{x^3(a+bx^3+cx^6)^{5/2}}{18c} + \frac{(b^2-4ac)^2(7b^2-4ac)\operatorname{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3072c^{9/2}}$$

output
```
1/576*(-4*a*c+7*b^2)*(2*c*x^3+b)*(c*x^6+b*x^3+a)^(3/2)/c^3-7/180*b*(c*x^6+
b*x^3+a)^(5/2)/c^2+1/18*x^3*(c*x^6+b*x^3+a)^(5/2)/c+1/3072*(-4*a*c+b^2)^2*
(-4*a*c+7*b^2)*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))/c^(9
/2)-1/1536*(-4*a*c+b^2)*(-4*a*c+7*b^2)*(2*c*x^3+b)*(c*x^6+b*x^3+a)^(1/2)/c
^4
```

3.203.2 Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.95

$$\int x^8(a+bx^3+cx^6)^{3/2}\,dx = \frac{2\sqrt{c}\sqrt{a+bx^3+cx^6}(-105b^5+70b^4cx^3+8b^3c(95a-7cx^6)+48b^2c^2x^3(-9a+cx^6)+160c^3x^3}{}$$

input
```
Integrate[x^8*(a + b*x^3 + c*x^6)^(3/2),x]
```

output
```
(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]*(-105*b^5 + 70*b^4*c*x^3 + 8*b^3*c*(95*
a - 7*c*x^6) + 48*b^2*c^2*x^3*(-9*a + c*x^6) + 160*c^3*x^3*(3*a^2 + 14*a*c
*x^6 + 8*c^2*x^12) + 16*b*c^2*(-81*a^2 + 18*a*c*x^6 + 104*c^2*x^12)) - 15*
(b^2 - 4*a*c)^2*(7*b^2 - 4*a*c)*Log[b + 2*c*x^3 - 2*Sqrt[c]*Sqrt[a + b*x^3
 + c*x^6]])/(46080*c^(9/2))
```

3.203.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1693, 1166, 27, 1160, 1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8 \left(a+bx^3+cx^6\right)^{3/2} \, dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int x^6 \left(cx^6+bx^3+a\right)^{3/2}dx^3$$

$$\Big\downarrow \text{1166}$$

$$\frac{1}{3}\left(\frac{\int -\frac{1}{2}\left(7bx^3+2a\right)\left(cx^6+bx^3+a\right)^{3/2}dx^3}{6c}+\frac{x^3\left(a+bx^3+cx^6\right)^{5/2}}{6c}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{x^3\left(a+bx^3+cx^6\right)^{5/2}}{6c}-\frac{\int \left(7bx^3+2a\right)\left(cx^6+bx^3+a\right)^{3/2}dx^3}{12c}\right)$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{3}\left(\frac{x^3\left(a+bx^3+cx^6\right)^{5/2}}{6c}-\frac{\frac{7b\left(a+bx^3+cx^6\right)^{5/2}}{5c}-\frac{\left(7b^2-4ac\right)\int \left(cx^6+bx^3+a\right)^{3/2}dx^3}{2c}}{12c}\right)$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{3}\left(\frac{x^3\left(a+bx^3+cx^6\right)^{5/2}}{6c}-\frac{\frac{7b\left(a+bx^3+cx^6\right)^{5/2}}{5c}-\frac{(7b^2-4ac)\left(\frac{\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\int\sqrt{cx^6+bx^3+a}dx^3}{16c}\right)}{2c}}{12c}\right)$$

$\downarrow$ 1087

$$\frac{1}{3}\left(\frac{x^3\left(a+bx^3+cx^6\right)^{5/2}}{6c}-\frac{\frac{7b\left(a+bx^3+cx^6\right)^{5/2}}{5c}-\frac{(7b^2-4ac)\left(\frac{\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{4c}-\frac{\left(b^2-4ac\right)}{}\right.}{16c}\right.}{2c}}{12c}\right.$$

$\downarrow$ 1092

$$\frac{1}{3}\left(\frac{x^3\left(a+bx^3+cx^6\right)^{5/2}}{6c}-\frac{\frac{7b\left(a+bx^3+cx^6\right)^{5/2}}{5c}-\frac{(7b^2-4ac)\left(\frac{\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{4c}-\frac{\left(b^2-4ac\right)}{}\right.}{16c}\right.}{2c}}{12c}\right.$$

$\downarrow$ 219

3.203. $\int x^8\left(a+bx^3+cx^6\right)^{3/2}\,dx$

$$\frac{1}{3}\left(\frac{x^3\left(a+bx^3+cx^6\right)^{5/2}}{6c}-\frac{\frac{7b\left(a+bx^3+cx^6\right)^{5/2}}{5c}-\frac{(7b^2-4ac)\left(\frac{\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{4c}-\frac{\left(b^2-4ac\right)\cdots}{\cdots}\right)}{16c}\right)}{2c}}{12c}\right.$$

input
```
Int[x^8*(a + b*x^3 + c*x^6)^(3/2),x]
```

output
```
((x^3*(a + b*x^3 + c*x^6)^(5/2))/(6*c) - ((7*b*(a + b*x^3 + c*x^6)^(5/2))/
(5*c) - ((7*b^2 - 4*a*c)*(((b + 2*c*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(8*c)
- (3*(b^2 - 4*a*c)*(((b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(4*c) - ((b^2
- 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(8*c^
(3/2))))/(16*c)))/(2*c))/(12*c))/3
```

3.203.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.203.4 Maple [F]

$$\int x^{8} \left(c\,x^{6}+b\,x^{3}+a\right)^{\frac{3}{2}}d x$$

input
```
int(x^8*(c*x^6+b*x^3+a)^(3/2),x)
```

output
```
int(x^8*(c*x^6+b*x^3+a)^(3/2),x)
```

3.203.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 451, normalized size of antiderivative = 2.21

$$\int x^8(a+bx^3+cx^6)^{3/2}\,dx = \left[-\frac{15\,(7\,b^6-60\,ab^4c+144\,a^2b^2c^2-64\,a^3c^3)\sqrt{c}\log\left(-8\,c^2x^6-8\,bcx^3-b^2+4\,\sqrt{cx^6+bx^3+}\right.}{}\right.$$

$$-\frac{15\,(7\,b^6-60\,ab^4c+144\,a^2b^2c^2-64\,a^3c^3)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{-c}}{2\,(c^2x^6+bcx^3+ac)}\right)-2\,(1280\,c^6x^{15}+1664\,bc^5x}{}$$

input
```
integrate(x^8*(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/92160*(15*(7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(c)*
log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 + 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)
*sqrt(c) - 4*a*c) - 4*(1280*c^6*x^15 + 1664*b*c^5*x^12 + 16*(3*b^2*c^4 + 1
40*a*c^5)*x^9 - 8*(7*b^3*c^3 - 36*a*b*c^4)*x^6 - 105*b^5*c + 760*a*b^3*c^2
 - 1296*a^2*b*c^3 + 2*(35*b^4*c^2 - 216*a*b^2*c^3 + 240*a^2*c^4)*x^3)*sqrt
(c*x^6 + b*x^3 + a))/c^5, -1/46080*(15*(7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c
^2 - 64*a^3*c^3)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)
*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) - 2*(1280*c^6*x^15 + 1664*b*c^5*x^12
+ 16*(3*b^2*c^4 + 140*a*c^5)*x^9 - 8*(7*b^3*c^3 - 36*a*b*c^4)*x^6 - 105*b^
5*c + 760*a*b^3*c^2 - 1296*a^2*b*c^3 + 2*(35*b^4*c^2 - 216*a*b^2*c^3 + 240
*a^2*c^4)*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^5]
```

3.203.6 Sympy [F]

$$\int x^8(a+bx^3+cx^6)^{3/2}\,dx = \int x^8(a+bx^3+cx^6)^{\frac{3}{2}}\,dx$$

input
```
integrate(x**8*(c*x**6+b*x**3+a)**(3/2),x)
```

output
```
Integral(x**8*(a + b*x**3 + c*x**6)**(3/2), x)
```

3.203.7 Maxima [F(-2)]

Exception generated.

$$\int x^8 \left(a + bx^3 + cx^6\right)^{3/2} dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^8*(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.203.8 Giac [F]

$$\int x^8 \left(a + bx^3 + cx^6\right)^{3/2} dx = \int \left(cx^6 + bx^3 + a\right)^{\frac{3}{2}} x^8 \, dx$$

input
```
integrate(x^8*(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^(3/2)*x^8, x)
```

3.203.9 Mupad [F(-1)]

Timed out.

$$\int x^8 \left(a + bx^3 + cx^6\right)^{3/2} dx = \int x^8 \left(c\,x^6 + b\,x^3 + a\right)^{3/2} dx$$

input
```
int(x^8*(a + b*x^3 + c*x^6)^(3/2),x)
```

output
```
int(x^8*(a + b*x^3 + c*x^6)^(3/2), x)
```

3.204 $\int x^5(a+bx^3+cx^6)^{3/2}\,dx$

3.204.1 Optimal result

Integrand size = 20, antiderivative size = 150

$$\int x^5\left(a+bx^3+cx^6\right)^{3/2}\,dx=\frac{b(b^2-4ac)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{128c^3}-\frac{b(b+2cx^3)\left(a+bx^3+cx^6\right)^{3/2}}{48c^2}+\frac{\left(a+bx^3+cx^6\right)^{5/2}}{15c}-\frac{b(b^2-4ac)^2\operatorname{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{256c^{7/2}}$$

output
```
-1/48*b*(2*c*x^3+b)*(c*x^6+b*x^3+a)^(3/2)/c^2+1/15*(c*x^6+b*x^3+a)^(5/2)/c
-1/256*b*(-4*a*c+b^2)^2*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1
/2))/c^(7/2)+1/128*b*(-4*a*c+b^2)*(2*c*x^3+b)*(c*x^6+b*x^3+a)^(1/2)/c^3
```

3.204.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.93

$$\int x^5\left(a+bx^3+cx^6\right)^{3/2}\,dx=\frac{\sqrt{a+bx^3+cx^6}\left(15b^4-10b^3cx^3+128c^2(a+cx^6)^2+4b^2c(-25a+2cx^6)+8bc^2x^3(7a+22cx\right.}{1920c^3}+\frac{b(b^2-4ac)^2\log\left(b+2cx^3-2\sqrt{c}\sqrt{a+bx^3+cx^6}\right)}{256c^{7/2}}$$

input
```
Integrate[x^5*(a + b*x^3 + c*x^6)^(3/2),x]
```

output
```
(Sqrt[a + b*x^3 + c*x^6]*(15*b^4 - 10*b^3*c*x^3 + 128*c^2*(a + c*x^6)^2 +
4*b^2*c*(-25*a + 2*c*x^6) + 8*b*c^2*x^3*(7*a + 22*c*x^6)))/(1920*c^3) + (b
*(b^2 - 4*a*c)^2*Log[b + 2*c*x^3 - 2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]])/(25
6*c^(7/2))
```

3.204.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1693, 1160, 1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(a+bx^3+cx^6\right)^{3/2} \, dx$$

$$\Big\downarrow \; 1693$$

$$\frac{1}{3} \int x^3 \left(cx^6+bx^3+a\right)^{3/2} dx^3$$

$$\Big\downarrow \; 1160$$

$$\frac{1}{3} \left(\frac{\left(a+bx^3+cx^6\right)^{5/2}}{5c}-\frac{b \int \left(cx^6+bx^3+a\right)^{3/2} dx^3}{2c}\right)$$

$$\Big\downarrow \; 1087$$

$$\frac{1}{3} \left(\frac{\left(a+bx^3+cx^6\right)^{5/2}}{5c}-\frac{b \left(\frac{\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\int \sqrt{cx^6+bx^3+a}dx^3}{16c}\right)}{2c}\right)$$

$$\Big\downarrow \; 1087$$

$$\frac{1}{3}\left(\frac{\left(a+bx^3+cx^6\right)^{5/2}}{5c}-\frac{b\left(\frac{\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{4c}-\frac{\left(b^2-4ac\right)\int\frac{1}{\sqrt{cx^6+bx^3+a}}dx^3}{8c}\right)}{16c}\right)}{2c}\right)$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{3}\left(\frac{\left(a+bx^3+cx^6\right)^{5/2}}{5c}-\frac{b\left(\frac{\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{4c}-\frac{\left(b^2-4ac\right)\int\frac{1}{4c-x^6}d\frac{2cx^3+b}{\sqrt{cx^6+bx^3+a}}}{4c}\right)}{16c}\right)}{2c}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{\left(a+bx^3+cx^6\right)^{5/2}}{5c}-\frac{b\left(\frac{\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{4c}-\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{8c^{3/2}}\right)}{16c}\right)}{2c}\right)$$

input
```
Int[x^5*(a + b*x^3 + c*x^6)^(3/2),x]
```

output
```
((a + b*x^3 + c*x^6)^(5/2)/(5*c) - (b*(((b + 2*c*x^3)*(a + b*x^3 + c*x^6)^
(3/2))/(8*c) - (3*(b^2 - 4*a*c)*(((b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(
4*c) - ((b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*
x^6])])/(8*c^(3/2))))/(16*c)))/(2*c))/3
```

3.204. $\int x^5(a+bx^3+cx^6)^{3/2}\,dx$

3.204.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.204.4 Maple [F]

$$\int x^{5} \left(c\,x^{6}+b\,x^{3}+a\right)^{\frac{3}{2}}d x$$

input
```
int(x^5*(c*x^6+b*x^3+a)^(3/2),x)
```

output
```
int(x^5*(c*x^6+b*x^3+a)^(3/2),x)
```

3.204.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 361, normalized size of antiderivative = 2.41

$$\int x^5(a+bx^3+cx^6)^{3/2}\,dx = \left[\frac{15\,(b^5-8\,ab^3c+16\,a^2bc^2)\sqrt{c}\log\left(-8\,c^2x^6-8\,bcx^3-b^2+4\,\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{c}-\right.}{}\right.$$

input
```
integrate(x^5*(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/7680*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*sqrt(c)*log(-8*c^2*x^6 - 8*b*
c*x^3 - b^2 + 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 4
*(128*c^5*x^12 + 176*b*c^4*x^9 + 8*(b^2*c^3 + 32*a*c^4)*x^6 + 15*b^4*c - 1
00*a*b^2*c^2 + 128*a^2*c^3 - 2*(5*b^3*c^2 - 28*a*b*c^3)*x^3)*sqrt(c*x^6 +
b*x^3 + a))/c^4, 1/3840*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*sqrt(-c)*arct
an(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 +
 a*c)) + 2*(128*c^5*x^12 + 176*b*c^4*x^9 + 8*(b^2*c^3 + 32*a*c^4)*x^6 + 15
*b^4*c - 100*a*b^2*c^2 + 128*a^2*c^3 - 2*(5*b^3*c^2 - 28*a*b*c^3)*x^3)*sqr
t(c*x^6 + b*x^3 + a))/c^4]
```

3.204.6 Sympy [F]

$$\int x^5(a+bx^3+cx^6)^{3/2}\,dx = \int x^5(a+bx^3+cx^6)^{\frac{3}{2}}\,dx$$

input
```
integrate(x**5*(c*x**6+b*x**3+a)**(3/2),x)
```

output
```
Integral(x**5*(a + b*x**3 + c*x**6)**(3/2), x)
```

3.204.7 Maxima [F(-2)]

Exception generated.

$$\int x^5 \left(a + bx^3 + cx^6\right)^{3/2} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5*(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.204.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.13

$$\int x^5 \left(a + bx^3 + cx^6\right)^{3/2} \, dx = \frac{1}{1920} \sqrt{cx^6 + bx^3 + a} \left(2 \left(4 \left(2 \left(8\, cx^3 + 11\, b\right) x^3 + \frac{b^2 c^3 + 32\, ac^4}{c^4}\right) x^3 - \frac{5\, b^3 c^2 - 28\, abc^3}{c^4}\right) x^3 + \right.$$
$$+ \frac{\left(b^5 - 8\, ab^3 c + 16\, a^2 bc^2\right) \log\left(\left|2 \left(\sqrt{c} x^3 - \sqrt{cx^6 + bx^3 + a}\right) \sqrt{c} + b\right|\right)}{256\, c^{\frac{7}{2}}}$$

input
```
integrate(x^5*(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
1/1920*sqrt(c*x^6 + b*x^3 + a)*(2*(4*(2*(8*c*x^3 + 11*b)*x^3 + (b^2*c^3 +
32*a*c^4)/c^4)*x^3 - (5*b^3*c^2 - 28*a*b*c^3)/c^4)*x^3 + (15*b^4*c - 100*a
*b^2*c^2 + 128*a^2*c^3)/c^4) + 1/256*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*log(
abs(2*(sqrt(c)*x^3 - sqrt(c*x^6 + b*x^3 + a))*sqrt(c) + b))/c^(7/2)
```

3.204.9 Mupad [B] (verification not implemented)

Time = 8.57 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.49

$$\int x^5 (a + bx^3 + cx^6)^{3/2} \, dx = \frac{(c\,x^6 + b\,x^3 + a)^{5/2}}{15\,c} - \frac{b\left(\frac{3\,a\left(\ln\left(\sqrt{c\,x^6+b\,x^3+a}+\frac{c\,x^3+\frac{b}{2}}{\sqrt{c}}\right)\left(\frac{a}{2\sqrt{c}}-\frac{b^2}{8\,c^{3/2}}\right)+\frac{\left(2\,c\,x^3+b\right)\sqrt{c\,x^6+b\,x^3+a}}{4\,c}\right)}{4}+\frac{x^3\left(c\,x^6+b\,x^3+a\right)^{3/2}}{4}-\frac{3\,b^2\left(\ln\left(\sqrt{c\,x^6+b\,x^3+a}+\frac{c\,x^3+}{\sqrt{c}}\right.\right.}{}\right.}{6\,c}$$

input
```
int(x^5*(a + b*x^3 + c*x^6)^(3/2),x)
```

output
```
(a + b*x^3 + c*x^6)^(5/2)/(15*c) - (b*((3*a*(log((a + b*x^3 + c*x^6)^(1/2)
 + (b/2 + c*x^3)/c^(1/2))*(a/(2*c^(1/2)) - b^2/(8*c^(3/2))) + ((b + 2*c*x^
3)*(a + b*x^3 + c*x^6)^(1/2))/(4*c)))/4 + (x^3*(a + b*x^3 + c*x^6)^(3/2))/
4 - (3*b^2*(log((a + b*x^3 + c*x^6)^(1/2) + (b/2 + c*x^3)/c^(1/2))*(a/(2*c
^(1/2)) - b^2/(8*c^(3/2))) + ((b + 2*c*x^3)*(a + b*x^3 + c*x^6)^(1/2))/(4*
c)))/(16*c) + (b*(a + b*x^3 + c*x^6)^(3/2))/(8*c)))/(6*c)
```

3.205 $\int x^2(a+bx^3+cx^6)^{3/2}\,dx$

3.205.1 Optimal result

Integrand size = 20, antiderivative size = 124

$$\int x^2(a+bx^3+cx^6)^{3/2}\,dx=-\frac{(b^2-4ac)(b+2cx^3)\sqrt{a+bx^3+cx^6}}{64c^2}+\frac{(b+2cx^3)(a+bx^3+cx^6)^{3/2}}{24c}+\frac{(b^2-4ac)^2\operatorname{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{128c^{5/2}}$$

output
```
1/24*(2*c*x^3+b)*(c*x^6+b*x^3+a)^(3/2)/c+1/128*(-4*a*c+b^2)^2*arctanh(1/2*
(2*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))/c^(5/2)-1/64*(-4*a*c+b^2)*(2*c*
x^3+b)*(c*x^6+b*x^3+a)^(1/2)/c^2
```

3.205.2 Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.92

$$\int x^2(a+bx^3+cx^6)^{3/2}\,dx=\frac{(b+2cx^3)\sqrt{a+bx^3+cx^6}(-3b^2+20ac+8bcx^3+8c^2x^6)}{192c^2}+\frac{(-b^2+4ac)^2\operatorname{arctanh}\left(\frac{\sqrt{c}x^3}{-\sqrt{a}+\sqrt{a+bx^3+cx^6}}\right)}{64c^{5/2}}$$

input
```
Integrate[x^2*(a + b*x^3 + c*x^6)^(3/2),x]
```

output
```
((b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6]*(-3*b^2 + 20*a*c + 8*b*c*x^3 + 8*c^
2*x^6))/(192*c^2) + ((-b^2 + 4*a*c)^2*ArcTanh[(Sqrt[c]*x^3)/(-Sqrt[a] + Sq
rt[a + b*x^3 + c*x^6])])/(64*c^(5/2))
```

3.205.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1690, 1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+bx^3+cx^6\right)^{3/2} dx$$

$$\Big\downarrow \text{1690}$$

$$\frac{1}{3}\int \left(cx^6+bx^3+a\right)^{3/2}dx^3$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{3}\left(\frac{\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\int \sqrt{cx^6+bx^3+a}dx^3}{16c}\right)$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{3}\left(\frac{\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{4c}-\frac{\left(b^2-4ac\right)\int \frac{1}{\sqrt{cx^6+bx^3+a}}dx^3}{8c}\right)}{16c}\right)$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{3}\left(\frac{\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{4c}-\frac{\left(b^2-4ac\right)\int \frac{1}{4c-x^6}d\frac{2cx^3+b}{\sqrt{cx^6+bx^3+a}}}{4c}\right)}{16c}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{(b+2cx^3)\,(a+bx^3+cx^6)^{3/2}}{8c}-\frac{3(b^2-4ac)\left(\frac{(b+2cx^3)\sqrt{a+bx^3+cx^6}}{4c}-\frac{(b^2-4ac)\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{8c^{3/2}}\right)}{16c}\right)$$

input
```
Int[x^2*(a + b*x^3 + c*x^6)^(3/2),x]
```

output
```
(((b + 2*c*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(8*c) - (3*(b^2 - 4*a*c)*(((b +
 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(4*c) - ((b^2 - 4*a*c)*ArcTanh[(b + 2*c
*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(8*c^(3/2))))/(16*c))/3
```

3.205.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

3.205.4 Maple [F]

$$\int x^2 \left(c\,x^6+b\,x^3+a\right)^{\frac{3}{2}}dx$$

input
```
int(x^2*(c*x^6+b*x^3+a)^(3/2),x)
```

output
```
int(x^2*(c*x^6+b*x^3+a)^(3/2),x)
```

3.205.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 297, normalized size of antiderivative = 2.40

$$\int x^2(a+bx^3+cx^6)^{3/2}\,dx = \left[\frac{3\,(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{c}\log\left(-8\,c^2x^6-8\,bcx^3-b^2-4\,\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{c}-4\right.}{768\,c^3}\right.$$
$$-\frac{3\,(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{-c}}{2\,(c^2x^6+bcx^3+ac)}\right)-2\,(16\,c^4x^9+24\,bc^3x^6-3\,b^3c+20\,abc^2+2}{384\,c^3}$$

input
```
integrate(x^2*(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/768*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^
3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 4*(16
*c^4*x^9 + 24*b*c^3*x^6 - 3*b^3*c + 20*a*b*c^2 + 2*(b^2*c^2 + 20*a*c^3)*x^
3)*sqrt(c*x^6 + b*x^3 + a))/c^3, -1/384*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*
sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^
6 + b*c*x^3 + a*c)) - 2*(16*c^4*x^9 + 24*b*c^3*x^6 - 3*b^3*c + 20*a*b*c^2
+ 2*(b^2*c^2 + 20*a*c^3)*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^3]
```

3.205.6 Sympy [F]

$$\int x^2 \left(a+bx^3+cx^6\right)^{3/2}\,dx=\int x^2 \left(a+bx^3+cx^6\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x**2*(c*x**6+b*x**3+a)**(3/2),x)
```

output
```
Integral(x**2*(a + b*x**3 + c*x**6)**(3/2), x)
```

3.205.7 Maxima [F(-2)]

Exception generated.

$$\int x^2 \left(a+bx^3+cx^6\right)^{3/2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^2*(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.205.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.07

$$\int x^2 \left(a+bx^3+cx^6\right)^{3/2}\,dx=\frac{1}{192}\sqrt{cx^6+bx^3+a}\left(2\left(4\left(2\,cx^3+3\,b\right)x^3+\frac{b^2c^2+20\,ac^3}{c^3}\right)x^3-\frac{3\,b^3c-20\,abc^2}{c^3}\right)-\frac{\left(b^4-8\,ab^2c+16\,a^2c^2\right)\log\left(\left|2\left(\sqrt{c}x^3-\sqrt{cx^6+bx^3+a}\right)\sqrt{c}+b\right|\right)}{128\,c^{\frac{5}{2}}}$$

input
```
integrate(x^2*(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
1/192*sqrt(c*x^6 + b*x^3 + a)*(2*(4*(2*c*x^3 + 3*b)*x^3 + (b^2*c^2 + 20*a*
c^3)/c^3)*x^3 - (3*b^3*c - 20*a*b*c^2)/c^3) - 1/128*(b^4 - 8*a*b^2*c + 16*
a^2*c^2)*log(abs(2*(sqrt(c)*x^3 - sqrt(c*x^6 + b*x^3 + a))*sqrt(c) + b))/c
^(5/2)
```

3.205.9 Mupad [B] (verification not implemented)

Time = 8.53 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.93

$$\int x^2 \left(a+bx^3+cx^6\right)^{3/2} dx = \frac{\left(c\,x^3+\frac{b}{2}\right)\,\left(c\,x^6+b\,x^3+a\right)^{3/2}}{12\,c} + \frac{\left(3\,a\,c-\frac{3\,b^2}{4}\right)\,\left(\left(\frac{b}{4\,c}+\frac{x^3}{2}\right)\,\sqrt{c\,x^6+b\,x^3+a}+\frac{\ln\left(\sqrt{c\,x^6+b\,x^3+a}+\frac{c\,x^3+\frac{b}{2}}{\sqrt{c}}\right)\,\left(a\,c-\frac{b^2}{4}\right)}{2\,c^{3/2}}\right)}{12\,c}$$

input
```
int(x^2*(a + b*x^3 + c*x^6)^(3/2),x)
```

output
```
((b/2 + c*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(12*c) + ((3*a*c - (3*b^2)/4)*((
b/(4*c) + x^3/2)*(a + b*x^3 + c*x^6)^(1/2) + (log((a + b*x^3 + c*x^6)^(1/2
) + (b/2 + c*x^3)/c^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(12*c)
```

3.206 $\int \frac{(a+bx^3+cx^6)^{3/2}}{x}\,dx$

3.206.1 Optimal result

Integrand size = 20, antiderivative size = 155

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x}\,dx = \frac{(b^2+8ac+2bcx^3)\sqrt{a+bx^3+cx^6}}{24c}+\frac{1}{9}(a+bx^3+cx^6)^{3/2} - \frac{1}{3}a^{3/2}\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)-\frac{b(b^2-12ac)\,\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{48c^{3/2}}$$

output
```
1/9*(c*x^6+b*x^3+a)^(3/2)-1/3*a^(3/2)*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x
^6+b*x^3+a)^(1/2))-1/48*b*(-12*a*c+b^2)*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c
*x^6+b*x^3+a)^(1/2))/c^(3/2)+1/24*(2*b*c*x^3+8*a*c+b^2)*(c*x^6+b*x^3+a)^(1
/2)/c
```

3.206.2 Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x}\,dx = \frac{1}{144}\left(\frac{2\sqrt{a+bx^3+cx^6}(3b^2+14bcx^3+8c(4a+cx^6))}{c} - \frac{3(b^3-12abc)\,\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{c^{3/2}}+96a^{3/2}\text{arctanh}\left(\frac{\sqrt{c}x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)\right)$$

input
```
Integrate[(a + b*x^3 + c*x^6)^(3/2)/x,x]
```

output
```
((2*Sqrt[a + b*x^3 + c*x^6]*(3*b^2 + 14*b*c*x^3 + 8*c*(4*a + c*x^6)))/c -
(3*(b^3 - 12*a*b*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^
6])])/c^(3/2) + 96*a^(3/2)*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3 + c*x^6])
/Sqrt[a]])/144
```

3.206.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.09, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1693, 1162, 25, 1231, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3+cx^6\right)^{3/2}}{x}dx$$

$$\Big\downarrow \ 1693$$

$$\frac{1}{3}\int \frac{\left(cx^6+bx^3+a\right)^{3/2}}{x^3}dx^3$$

$$\Big\downarrow \ 1162$$

$$\frac{1}{3}\left(\frac{1}{3}\left(a+bx^3+cx^6\right)^{3/2}-\frac{1}{2}\int -\frac{\left(bx^3+2a\right)\sqrt{cx^6+bx^3+a}}{x^3}dx^3\right)$$

$$\Big\downarrow \ 25$$

$$\frac{1}{3}\left(\frac{1}{2}\int \frac{\left(bx^3+2a\right)\sqrt{cx^6+bx^3+a}}{x^3}dx^3+\frac{1}{3}\left(a+bx^3+cx^6\right)^{3/2}\right)$$

$$\Big\downarrow \ 1231$$

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{\left(8ac+b^2+2bcx^3\right)\sqrt{a+bx^3+cx^6}}{4c}-\frac{\int -\frac{16a^2c-b\left(b^2-12ac\right)x^3}{2x^3\sqrt{cx^6+bx^3+a}}dx^3}{4c}\right)+\frac{1}{3}\left(a+bx^3+cx^6\right)^{3/2}\right)$$

$$\Big\downarrow \ 27$$

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{\int \frac{16a^2c-b\left(b^2-12ac\right)x^3}{x^3\sqrt{cx^6+bx^3+a}}dx^3}{8c}+\frac{\sqrt{a+bx^3+cx^6}\left(8ac+b^2+2bcx^3\right)}{4c}\right)+\frac{1}{3}\left(a+bx^3+cx^6\right)^{3/2}\right)$$

$$\Big\downarrow \text{1269}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{16a^2c\int \frac{1}{x^3\sqrt{cx^6+bx^3+a}}dx^3-b(b^2-12ac)\int \frac{1}{\sqrt{cx^6+bx^3+a}}dx^3}{8c}+\frac{\sqrt{a+bx^3+cx^6}(8ac+b^2+2bcx^3)}{4c}\right)+\frac{1}{3}(a+$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{16a^2c\int \frac{1}{x^3\sqrt{cx^6+bx^3+a}}dx^3-2b(b^2-12ac)\int \frac{1}{4c-x^6}d\frac{2cx^3+b}{\sqrt{cx^6+bx^3+a}}}{8c}+\frac{\sqrt{a+bx^3+cx^6}(8ac+b^2+2bcx^3)}{4c}\right)+\frac{1}{3}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{16a^2c\int \frac{1}{x^3\sqrt{cx^6+bx^3+a}}dx^3-\frac{b(b^2-12ac)\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{\sqrt{c}}}{8c}+\frac{\sqrt{a+bx^3+cx^6}(8ac+b^2+2bcx^3)}{4c}\right)+\frac{1}{3}$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{-32a^2c\int \frac{1}{4a-x^6}d\frac{bx^3+2a}{\sqrt{cx^6+bx^3+a}}-\frac{b(b^2-12ac)\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{\sqrt{c}}}{8c}+\frac{\sqrt{a+bx^3+cx^6}(8ac+b^2+2bcx^3)}{4c}\right)-$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{-16a^{3/2}c\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)-\frac{b(b^2-12ac)\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{\sqrt{c}}}{8c}+\frac{\sqrt{a+bx^3+cx^6}(8ac+b^2+2bcx^3)}{4c}\right.\right.$$

input
```
Int[(a + b*x^3 + c*x^6)^(3/2)/x,x]
```

output
```
((a + b*x^3 + c*x^6)^(3/2)/3 + (((b^2 + 8*a*c + 2*b*c*x^3)*Sqrt[a + b*x^3
+ c*x^6])/(4*c) + (-16*a^(3/2)*c*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a +
 b*x^3 + c*x^6])] - (b*(b^2 - 12*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqr
t[a + b*x^3 + c*x^6])])/Sqrt[c])/(8*c))/2)/3
```

3.206. $\int \frac{(a+bx^3+cx^6)^{3/2}}{x}\,dx$

3.206.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1162
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p/(e*(m + 2*p + 1))   Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d -
b*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&
!ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.206.4 Maple [F]

$$\int \frac{\left(c\,x^{6}+b\,x^{3}+a\right)^{\frac{3}{2}}}{x}dx$$

input
```
int((c*x^6+b*x^3+a)^(3/2)/x,x)
```

output
```
int((c*x^6+b*x^3+a)^(3/2)/x,x)
```

3.206.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 727, normalized size of antiderivative = 4.69

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x}\,dx = \left[\frac{48\,a^{\frac{3}{2}}c^2\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3-4\,\sqrt{cx^6+bx^3+a}\,(bx^3+2\,a)\sqrt{a}+8\,a^2}{x^6}\right)-3\,(b^3-12\,abc)\sqrt{c}}{}\right.$$

input
```
integrate((c*x^6+b*x^3+a)^(3/2)/x,x, algorithm="fricas")
```

output
```
[1/288*(48*a^(3/2)*c^2*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6
+ b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) - 3*(b^3 - 12*a*b*c)*sqrt
(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3
+ b)*sqrt(c) - 4*a*c) + 4*(8*c^3*x^6 + 14*b*c^2*x^3 + 3*b^2*c + 32*a*c^2)*
sqrt(c*x^6 + b*x^3 + a))/c^2, 1/144*(24*a^(3/2)*c^2*log(-((b^2 + 4*a*c)*x^
6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x
^6) + 3*(b^3 - 12*a*b*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*
x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*(8*c^3*x^6 + 14*b*c^2*x^3
 + 3*b^2*c + 32*a*c^2)*sqrt(c*x^6 + b*x^3 + a))/c^2, 1/288*(96*sqrt(-a)*a*
c^2*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a
*b*x^3 + a^2)) - 3*(b^3 - 12*a*b*c)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b
^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 4*(8*c^3*x
^6 + 14*b*c^2*x^3 + 3*b^2*c + 32*a*c^2)*sqrt(c*x^6 + b*x^3 + a))/c^2, 1/14
4*(48*sqrt(-a)*a*c^2*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt
(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 3*(b^3 - 12*a*b*c)*sqrt(-c)*arctan(1/2*s
qrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) +
 2*(8*c^3*x^6 + 14*b*c^2*x^3 + 3*b^2*c + 32*a*c^2)*sqrt(c*x^6 + b*x^3 + a)
)/c^2]
```

3.206.6 Sympy [F]

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x}\,dx = \int \frac{(a+bx^3+cx^6)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((c*x**6+b*x**3+a)**(3/2)/x,x)
```

output
```
Integral((a + b*x**3 + c*x**6)**(3/2)/x, x)
```

3.206.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^6+b*x^3+a)^(3/2)/x,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.206.8 Giac [F]

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x}\,dx = \int \frac{(cx^6+bx^3+a)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^(3/2)/x,x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^(3/2)/x, x)
```

3.206.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x}\,dx = \int \frac{(c\,x^6+b\,x^3+a)^{3/2}}{x}\,dx$$

input
```
int((a + b*x^3 + c*x^6)^(3/2)/x,x)
```

output
```
int((a + b*x^3 + c*x^6)^(3/2)/x, x)
```

3.207 $\int \frac{(a+bx^3+cx^6)^{3/2}}{x^4}\,dx$

3.207.1 Optimal result

Integrand size = 20, antiderivative size = 150

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^4}\,dx = \frac{1}{4}(3b+2cx^3)\sqrt{a+bx^3+cx^6} - \frac{(a+bx^3+cx^6)^{3/2}}{3x^3} - \frac{1}{2}\sqrt{a}b\,\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right) + \frac{(b^2+4ac)\,\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{8\sqrt{c}}$$

output
```
-1/3*(c*x^6+b*x^3+a)^(3/2)/x^3-1/2*b*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^
6+b*x^3+a)^(1/2))*a^(1/2)+1/8*(4*a*c+b^2)*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/
(c*x^6+b*x^3+a)^(1/2))/c^(1/2)+1/4*(2*c*x^3+3*b)*(c*x^6+b*x^3+a)^(1/2)
```

3.207.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^4}\,dx = \frac{\sqrt{a+bx^3+cx^6}(-4a+5bx^3+2cx^6)}{12x^3} + \sqrt{a}b\,\text{arctanh}\left(\frac{\sqrt{c}x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right) - \frac{(b^2+4ac)\log\left(b+2cx^3-2\sqrt{c}\sqrt{a+bx^3+cx^6}\right)}{8\sqrt{c}}$$

input
```
Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^4,x]
```

output
```
(Sqrt[a + b*x^3 + c*x^6]*(-4*a + 5*b*x^3 + 2*c*x^6))/(12*x^3) + Sqrt[a]*b*
ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3 + c*x^6])/Sqrt[a]] - ((b^2 + 4*a*c)*
Log[b + 2*c*x^3 - 2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]])/(8*Sqrt[c])
```

3.207.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1693, 1161, 1231, 25, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^4}dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int \frac{\left(cx^6+bx^3+a\right)^{3/2}}{x^6}dx^3$$

$$\Big\downarrow \text{1161}$$

$$\frac{1}{3}\left(\frac{3}{2}\int \frac{\left(2cx^3+b\right)\sqrt{cx^6+bx^3+a}}{x^3}dx^3-\frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^3}\right)$$

$$\Big\downarrow \text{1231}$$

$$\frac{1}{3}\left(\frac{3}{2}\left(\frac{1}{2}\left(3b+2cx^3\right)\sqrt{a+bx^3+cx^6}-\frac{\int -\frac{c\left(\left(b^2+4ac\right)x^3+4ab\right)}{x^3\sqrt{cx^6+bx^3+a}}dx^3}{4c}\right)-\frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^3}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\left(\frac{3}{2}\left(\frac{\int \frac{c\left(\left(b^2+4ac\right)x^3+4ab\right)}{x^3\sqrt{cx^6+bx^3+a}}dx^3}{4c}+\frac{1}{2}\left(3b+2cx^3\right)\sqrt{a+bx^3+cx^6}\right)-\frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^3}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{3}{2}\left(\frac{1}{4}\int \frac{\left(b^2+4ac\right)x^3+4ab}{x^3\sqrt{cx^6+bx^3+a}}dx^3+\frac{1}{2}\left(3b+2cx^3\right)\sqrt{a+bx^3+cx^6}\right)-\frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^3}\right)$$

$$\Big\downarrow \text{1269}$$

$$\frac{1}{3}\left(\frac{3}{2}\left(\frac{1}{4}\left(\left(4ac+b^2\right)\int\frac{1}{\sqrt{cx^6+bx^3+a}}dx^3+4ab\int\frac{1}{x^3\sqrt{cx^6+bx^3+a}}dx^3\right)+\frac{1}{2}\left(3b+2cx^3\right)\sqrt{a+bx^3+cx^6}\right)-\right.$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{3}\left(\frac{3}{2}\left(\frac{1}{4}\left(2\left(4ac+b^2\right)\int\frac{1}{4c-x^6}d\frac{2cx^3+b}{\sqrt{cx^6+bx^3+a}}+4ab\int\frac{1}{x^3\sqrt{cx^6+bx^3+a}}dx^3\right)+\frac{1}{2}\left(3b+2cx^3\right)\sqrt{a+bx^3+}\right.\right.$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{3}{2}\left(\frac{1}{4}\left(4ab\int\frac{1}{x^3\sqrt{cx^6+bx^3+a}}dx^3+\frac{\left(4ac+b^2\right)\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{\sqrt{c}}\right)+\frac{1}{2}\left(3b+2cx^3\right)\sqrt{a+bx^3+cx^6}\right.\right.$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{3}\left(\frac{3}{2}\left(\frac{1}{4}\left(\frac{\left(4ac+b^2\right)\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{\sqrt{c}}-8ab\int\frac{1}{4a-x^6}d\frac{bx^3+2a}{\sqrt{cx^6+bx^3+a}}\right)+\frac{1}{2}\left(3b+2cx^3\right)\sqrt{a+bx^3+}\right.\right.$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{3}{2}\left(\frac{1}{4}\left(\frac{\left(4ac+b^2\right)\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{\sqrt{c}}-4\sqrt{a}b\,\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)\right)+\frac{1}{2}\left(3b+2cx^3\right)\sqrt{a+b}\right.\right.$$

input
```
Int[(a + b*x^3 + c*x^6)^(3/2)/x^4,x]
```

output
```
(-((a + b*x^3 + c*x^6)^(3/2)/x^3) + (3*(((3*b + 2*c*x^3)*Sqrt[a + b*x^3 +
c*x^6])/2 + (-4*Sqrt[a]*b*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3
+ c*x^6])] + ((b^2 + 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^
3 + c*x^6])])/Sqrt[c])/4))/2)/3
```

3.207. $\int\frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^4}\,dx$

3.207.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1161
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Si
mp[p/(e*(m + 1))   Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||
 LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b,
 c, d, e, m, p, x]
```

3.207. $\int \frac{(a+bx^3+cx^6)^{3/2}}{x^4} dx$

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.207.4 Maple [F]

$$\int \frac{(c\,x^{6}+b\,x^{3}+a)^{\frac{3}{2}}}{x^{4}}dx$$

input
```
int((c*x^6+b*x^3+a)^(3/2)/x^4,x)
```

output
```
int((c*x^6+b*x^3+a)^(3/2)/x^4,x)
```

3.207.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 713, normalized size of antiderivative = 4.75

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^4}\,dx = \left[\frac{12\sqrt{a}bcx^3\log\left(-\frac{(b^2+4ac)x^6+8abx^3-4\sqrt{cx^6+bx^3+a}(bx^3+2a)\sqrt{a}+8a^2}{x^6}\right)+3(b^2+4ac)\sqrt{c}}{}\right.$$

input
```
integrate((c*x^6+b*x^3+a)^(3/2)/x^4,x, algorithm="fricas")
```

output
```
[1/48*(12*sqrt(a)*b*c*x^3*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x
^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) + 3*(b^2 + 4*a*c)*sqrt
(c)*x^3*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*
x^3 + b)*sqrt(c) - 4*a*c) + 4*(2*c^2*x^6 + 5*b*c*x^3 - 4*a*c)*sqrt(c*x^6 +
 b*x^3 + a))/(c*x^3), 1/24*(6*sqrt(a)*b*c*x^3*log(-((b^2 + 4*a*c)*x^6 + 8*
a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) -
3*(b^2 + 4*a*c)*sqrt(-c)*x^3*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 +
 b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*(2*c^2*x^6 + 5*b*c*x^3 - 4*a*c
)*sqrt(c*x^6 + b*x^3 + a))/(c*x^3), 1/48*(24*sqrt(-a)*b*c*x^3*arctan(1/2*s
qrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) +
 3*(b^2 + 4*a*c)*sqrt(c)*x^3*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x
^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 4*(2*c^2*x^6 + 5*b*c*x^3
- 4*a*c)*sqrt(c*x^6 + b*x^3 + a))/(c*x^3), 1/24*(12*sqrt(-a)*b*c*x^3*arcta
n(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 +
a^2)) - 3*(b^2 + 4*a*c)*sqrt(-c)*x^3*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2
*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*(2*c^2*x^6 + 5*b*c*x^3
 - 4*a*c)*sqrt(c*x^6 + b*x^3 + a))/(c*x^3)]
```

3.207.6 Sympy [F]

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^4}\,dx = \int \frac{(a+bx^3+cx^6)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((c*x**6+b*x**3+a)**(3/2)/x**4,x)
```

output
```
Integral((a + b*x**3 + c*x**6)**(3/2)/x**4, x)
```

3.207.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^6+b*x^3+a)^(3/2)/x^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.207.8 Giac [F]

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^4}\,dx = \int \frac{(cx^6+bx^3+a)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^(3/2)/x^4,x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^(3/2)/x^4, x)
```

3.207.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^4}\,dx = \int \frac{(c\,x^6+b\,x^3+a)^{3/2}}{x^4}\,dx$$

input
```
int((a + b*x^3 + c*x^6)^(3/2)/x^4,x)
```

output
```
int((a + b*x^3 + c*x^6)^(3/2)/x^4, x)
```

3.208 $\int \frac{(a+bx^3+cx^6)^{3/2}}{x^7}\,dx$

3.208.1 Optimal result

Integrand size = 20, antiderivative size = 151

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^7}\,dx = -\frac{(b-2cx^3)\sqrt{a+bx^3+cx^6}}{4x^3}-\frac{(a+bx^3+cx^6)^{3/2}}{6x^6} -\frac{(b^2+4ac)\,\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{8\sqrt{a}}+\frac{1}{2}b\sqrt{c}\,\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)$$

output
```
-1/6*(c*x^6+b*x^3+a)^(3/2)/x^6-1/8*(4*a*c+b^2)*arctanh(1/2*(b*x^3+2*a)/a^(
1/2)/(c*x^6+b*x^3+a)^(1/2))/a^(1/2)+1/2*b*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/
(c*x^6+b*x^3+a)^(1/2))*c^(1/2)-1/4*(-2*c*x^3+b)*(c*x^6+b*x^3+a)^(1/2)/x^3
```

3.208.2 Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^7}\,dx = \frac{1}{12}\left(\frac{\sqrt{a+bx^3+cx^6}(-2a-5bx^3+4cx^6)}{x^6} +\frac{3(b^2+4ac)\,\text{arctanh}\left(\frac{\sqrt{c}x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{\sqrt{a}}-6b\sqrt{c}\log\left(b+2cx^3-2\sqrt{c}\sqrt{a+bx^3+cx^6}\right)\right)$$

input
```
Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^7,x]
```

output
```
((Sqrt[a + b*x^3 + c*x^6]*(-2*a - 5*b*x^3 + 4*c*x^6))/x^6 + (3*(b^2 + 4*a*
c)*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3 + c*x^6])/Sqrt[a]])/Sqrt[a] - 6*b
*Sqrt[c]*Log[b + 2*c*x^3 - 2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]])/12
```

3.208.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {1693, 1161, 1230, 25, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^7}dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int \frac{\left(cx^6+bx^3+a\right)^{3/2}}{x^9}dx^3$$

$$\Big\downarrow \text{1161}$$

$$\frac{1}{3}\left(\frac{3}{4}\int \frac{\left(2cx^3+b\right)\sqrt{cx^6+bx^3+a}}{x^6}dx^3-\frac{\left(a+bx^3+cx^6\right)^{3/2}}{2x^6}\right)$$

$$\Big\downarrow \text{1230}$$

$$\frac{1}{3}\left(\frac{3}{4}\left(-\frac{1}{2}\int -\frac{4bcx^3+b^2+4ac}{x^3\sqrt{cx^6+bx^3+a}}dx^3-\frac{\left(b-2cx^3\right)\sqrt{a+bx^3+cx^6}}{x^3}\right)-\frac{\left(a+bx^3+cx^6\right)^{3/2}}{2x^6}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{4bcx^3+b^2+4ac}{x^3\sqrt{cx^6+bx^3+a}}dx^3-\frac{\left(b-2cx^3\right)\sqrt{a+bx^3+cx^6}}{x^3}\right)-\frac{\left(a+bx^3+cx^6\right)^{3/2}}{2x^6}\right)$$

$$\Big\downarrow \text{1269}$$

$$\frac{1}{3}\left(\frac{3}{4}\left(\frac{1}{2}\left(\left(4ac+b^2\right)\int \frac{1}{x^3\sqrt{cx^6+bx^3+a}}dx^3+4bc\int \frac{1}{\sqrt{cx^6+bx^3+a}}dx^3\right)-\frac{\left(b-2cx^3\right)\sqrt{a+bx^3+cx^6}}{x^3}\right)-$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{3}\left(\frac{3}{4}\left(\frac{1}{2}\left(\left(4ac+b^2\right)\int\frac{1}{x^3\sqrt{cx^6+bx^3+a}}dx^3+8bc\int\frac{1}{4c-x^6}d\frac{2cx^3+b}{\sqrt{cx^6+bx^3+a}}\right)-\frac{\left(b-2cx^3\right)\sqrt{a+bx^3+cx^6}}{x^3}\right.\right.$$

$\downarrow$ 219

$$\frac{1}{3}\left(\frac{3}{4}\left(\frac{1}{2}\left(\left(4ac+b^2\right)\int\frac{1}{x^3\sqrt{cx^6+bx^3+a}}dx^3+4b\sqrt{c}\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)\right)-\frac{\left(b-2cx^3\right)\sqrt{a+bx^3+}}{x^3}\right.\right.$$

$\downarrow$ 1154

$$\frac{1}{3}\left(\frac{3}{4}\left(\frac{1}{2}\left(4b\sqrt{c}\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)-2\left(4ac+b^2\right)\int\frac{1}{4a-x^6}d\frac{bx^3+2a}{\sqrt{cx^6+bx^3+a}}\right)-\frac{\left(b-2cx^3\right)\sqrt{a+b}}{x^3}\right.\right.$$

$\downarrow$ 219

$$\frac{1}{3}\left(\frac{3}{4}\left(\frac{1}{2}\left(4b\sqrt{c}\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)-\frac{\left(4ac+b^2\right)\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{\sqrt{a}}\right)-\frac{\left(b-2cx^3\right)\sqrt{a+bx^3}}{x^3}\right.\right.$$

input
```
Int[(a + b*x^3 + c*x^6)^(3/2)/x^7,x]
```

output
```
(-1/2*(a + b*x^3 + c*x^6)^(3/2)/x^6 + (3*(-(((b - 2*c*x^3)*Sqrt[a + b*x^3
+ c*x^6])/x^3) + (-(((b^2 + 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a
 + b*x^3 + c*x^6])])/Sqrt[a]) + 4*b*Sqrt[c]*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[
c]*Sqrt[a + b*x^3 + c*x^6])])/2))/4)/3
```

3.208.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1161
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Si
mp[p/(e*(m + 1))   Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||
 LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b,
 c, d, e, m, p, x]
```

rule 1230
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Simp[p/(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a
+ b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m
+ 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -
1] || EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ
[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.208. $\int \frac{(a+bx^3+cx^6)^{3/2}}{x^7}\,dx$

3.208.4 Maple [F]

$$\int \frac{(c\,x^6+b\,x^3+a)^{\frac{3}{2}}}{x^7}dx$$

input
```
int((c*x^6+b*x^3+a)^(3/2)/x^7,x)
```

output
```
int((c*x^6+b*x^3+a)^(3/2)/x^7,x)
```

3.208.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 713, normalized size of antiderivative = 4.72

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^7}\,dx = \Bigg[\frac{12\,ab\sqrt{c}x^6\log\left(-8\,c^2x^6-8\,bcx^3-b^2-4\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{c}-4\,ac\right)}{}$$

$$-\frac{24\,ab\sqrt{-c}x^6\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{-c}}{2\,(c^2x^6+bcx^3+ac)}\right)-3\,(b^2+4\,ac)\sqrt{a}x^6\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3-4\sqrt{cx^6+bx^3+a}(bx^3+2}{x^6}\right.}{48\,ax^6}$$

$$-\frac{12\,ab\sqrt{-c}x^6\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{-c}}{2\,(c^2x^6+bcx^3+ac)}\right)-3\,(b^2+4\,ac)\sqrt{-a}x^6\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(bx^3+2\,a)\sqrt{-a}}{2\,(acx^6+abx^3+a^2)}\right)-2\,(4}{24\,ax^6}$$

input
```
integrate((c*x^6+b*x^3+a)^(3/2)/x^7,x, algorithm="fricas")
```

output
```
[1/48*(12*a*b*sqrt(c)*x^6*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6
+ b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 3*(b^2 + 4*a*c)*sqrt(a)*x^6*
log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2
*a)*sqrt(a) + 8*a^2)/x^6) + 4*(4*a*c*x^6 - 5*a*b*x^3 - 2*a^2)*sqrt(c*x^6 +
 b*x^3 + a))/(a*x^6), -1/48*(24*a*b*sqrt(-c)*x^6*arctan(1/2*sqrt(c*x^6 + b
*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) - 3*(b^2 + 4*a
*c)*sqrt(a)*x^6*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3
 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) - 4*(4*a*c*x^6 - 5*a*b*x^3 - 2*a
^2)*sqrt(c*x^6 + b*x^3 + a))/(a*x^6), 1/24*(6*a*b*sqrt(c)*x^6*log(-8*c^2*x
^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4
*a*c) + 3*(b^2 + 4*a*c)*sqrt(-a)*x^6*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b
*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*(4*a*c*x^6 - 5*a*b*x^3
 - 2*a^2)*sqrt(c*x^6 + b*x^3 + a))/(a*x^6), -1/24*(12*a*b*sqrt(-c)*x^6*arc
tan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3
+ a*c)) - 3*(b^2 + 4*a*c)*sqrt(-a)*x^6*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*
(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) - 2*(4*a*c*x^6 - 5*a*b*x
^3 - 2*a^2)*sqrt(c*x^6 + b*x^3 + a))/(a*x^6)]
```

3.208.6 Sympy [F]

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^7}\,dx = \int \frac{(a+bx^3+cx^6)^{\frac{3}{2}}}{x^7}\,dx$$

input
```
integrate((c*x**6+b*x**3+a)**(3/2)/x**7,x)
```

output
```
Integral((a + b*x**3 + c*x**6)**(3/2)/x**7, x)
```

3.208.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^7}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^6+b*x^3+a)^(3/2)/x^7,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.208.8 Giac [F]

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^7}\,dx = \int \frac{(cx^6+bx^3+a)^{\frac{3}{2}}}{x^7}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^(3/2)/x^7,x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^(3/2)/x^7, x)
```

3.208.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^7}\,dx = \int \frac{(c\,x^6+b\,x^3+a)^{3/2}}{x^7}\,dx$$

input
```
int((a + b*x^3 + c*x^6)^(3/2)/x^7,x)
```

output
```
int((a + b*x^3 + c*x^6)^(3/2)/x^7, x)
```

3.209 $\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{10}}\,dx$

3.209.1 Optimal result

Integrand size = 20, antiderivative size = 163

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{10}}\,dx = -\frac{(2ab+(b^2+8ac)\,x^3)\sqrt{a+bx^3+cx^6}}{24ax^6} - \frac{(a+bx^3+cx^6)^{3/2}}{9x^9} + \frac{b(b^2-12ac)\,\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{48a^{3/2}} + \frac{1}{3}c^{3/2}\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)$$

output
```
-1/9*(c*x^6+b*x^3+a)^(3/2)/x^9+1/48*b*(-12*a*c+b^2)*arctanh(1/2*(b*x^3+2*a
)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2))/a^(3/2)+1/3*c^(3/2)*arctanh(1/2*(2*c*x^3+
b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))-1/24*(2*a*b+(8*a*c+b^2)*x^3)*(c*x^6+b*x^
3+a)^(1/2)/a/x^6
```

3.209.2 Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{10}}\,dx = \frac{\sqrt{a+bx^3+cx^6}(-8a^2-14abx^3-3b^2x^6-32acx^6)}{72ax^9} + \frac{(b^3-12abc)\,\text{arctanh}\left(\frac{-\sqrt{c}x^3+\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{24a^{3/2}} - \frac{1}{3}c^{3/2}\log\left(b+2cx^3-2\sqrt{c}\sqrt{a+bx^3+cx^6}\right)$$

input
```
Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^10,x]
```

output
```
(Sqrt[a + b*x^3 + c*x^6]*(-8*a^2 - 14*a*b*x^3 - 3*b^2*x^6 - 32*a*c*x^6))/(
72*a*x^9) + ((b^3 - 12*a*b*c)*ArcTanh[(-(Sqrt[c]*x^3) + Sqrt[a + b*x^3 + c
*x^6])/Sqrt[a]])/(24*a^(3/2)) - (c^(3/2)*Log[b + 2*c*x^3 - 2*Sqrt[c]*Sqrt[
a + b*x^3 + c*x^6]])/3
```

3.209.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {1693, 1161, 1229, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{10}}dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int \frac{(cx^6+bx^3+a)^{3/2}}{x^{12}}dx^3$$

$$\Big\downarrow \text{1161}$$

$$\frac{1}{3}\left(\frac{1}{2}\int \frac{(2cx^3+b)\sqrt{cx^6+bx^3+a}}{x^9}dx^3-\frac{(a+bx^3+cx^6)^{3/2}}{3x^9}\right)$$

$$\Big\downarrow \text{1229}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(-\frac{\int \frac{b(b^2-12ac)-16ac^2x^3}{2x^3\sqrt{cx^6+bx^3+a}}dx^3}{4a}-\frac{\sqrt{a+bx^3+cx^6}(x^3(8ac+b^2)+2ab)}{4ax^6}\right)-\frac{(a+bx^3+cx^6)^{3/2}}{3x^9}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(-\frac{\int \frac{b(b^2-12ac)-16ac^2x^3}{x^3\sqrt{cx^6+bx^3+a}}dx^3}{8a}-\frac{\sqrt{a+bx^3+cx^6}(x^3(8ac+b^2)+2ab)}{4ax^6}\right)-\frac{(a+bx^3+cx^6)^{3/2}}{3x^9}\right)$$

$$\Big\downarrow \text{1269}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(-\frac{b(b^2-12ac)\int \frac{1}{x^3\sqrt{cx^6+bx^3+a}}dx^3-16ac^2\int \frac{1}{\sqrt{cx^6+bx^3+a}}dx^3}{8a}-\frac{\sqrt{a+bx^3+cx^6}(x^3(8ac+b^2)+2ab)}{4ax^6}\right)-\frac{(}{}$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(-\frac{b(b^2-12ac)\int \frac{1}{x^3\sqrt{cx^6+bx^3+a}}dx^3-32ac^2\int \frac{1}{4c-x^6}d\frac{2cx^3+b}{\sqrt{cx^6+bx^3+a}}}{8a}-\frac{\sqrt{a+bx^3+cx^6}\left(x^3(8ac+b^2)+2ab\right)}{4ax^6}\right)\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(-\frac{b(b^2-12ac)\int \frac{1}{x^3\sqrt{cx^6+bx^3+a}}dx^3-16ac^{3/2}\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{8a}-\frac{\sqrt{a+bx^3+cx^6}\left(x^3(8ac+b^2)+2\right.}{4ax^6}\right.\right.$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(-\frac{-2b(b^2-12ac)\int \frac{1}{4a-x^6}d\frac{bx^3+2a}{\sqrt{cx^6+bx^3+a}}-16ac^{3/2}\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{8a}-\frac{\sqrt{a+bx^3+cx^6}\left(x^3(8ac+b^2\right.}{4ax^6}\right.\right.$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(-\frac{-\frac{b(b^2-12ac)\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{\sqrt{a}}-16ac^{3/2}\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{8a}-\frac{\sqrt{a+bx^3+cx^6}\left(x^3(8ac+b^2)\right.}{4ax^6}\right.\right.$$

input
```
Int[(a + b*x^3 + c*x^6)^(3/2)/x^10,x]
```

output
```
(-1/3*(a + b*x^3 + c*x^6)^(3/2)/x^9 + (-1/4*((2*a*b + (b^2 + 8*a*c)*x^3)*S
qrt[a + b*x^3 + c*x^6])/(a*x^6) - (-((b*(b^2 - 12*a*c)*ArcTanh[(2*a + b*x^
3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/Sqrt[a]) - 16*a*c^(3/2)*ArcTanh[(
b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(8*a))/2)/3
```

3.209. $\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{10}}\,dx$

3.209.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1161
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Si
mp[p/(e*(m + 1))   Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||
 LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b,
 c, d, e, m, p, x]
```

rule 1229
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Simp[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m +
p + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c
*(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(
m - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3,
0]
```

3.209. $\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{10}}\,dx$

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.209.4 Maple [F]

$$\int \frac{\left(c\,x^{6}+b\,x^{3}+a\right)^{\frac{3}{2}}}{x^{10}}dx$$

input
```
int((c*x^6+b*x^3+a)^(3/2)/x^10,x)
```

output
```
int((c*x^6+b*x^3+a)^(3/2)/x^10,x)
```

3.209.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 771, normalized size of antiderivative = 4.73

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{10}}\,dx = \left[\frac{48\,a^2c^{\frac{3}{2}}x^9\log\left(-8\,c^2x^6-8\,bcx^3-b^2-4\,\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{c}-4\,ac\right)\cdots}{\cdots}\right.$$

$$-\frac{96\,a^2\sqrt{-c}cx^9\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{-c}}{2\,(c^2x^6+bcx^3+ac)}\right)+3\,(b^3-12\,abc)\sqrt{a}x^9\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3-4\,\sqrt{cx^6+bx^3+a}(bx^3\cdots}{x^6}\cdots\right.}{288\,a^2x^9}$$

$$-\frac{48\,a^2\sqrt{-c}cx^9\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{-c}}{2\,(c^2x^6+bcx^3+ac)}\right)+3\,(b^3-12\,abc)\sqrt{-a}x^9\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(bx^3+2\,a)\sqrt{-a}}{2\,(acx^6+abx^3+a^2)}\right)+2\cdots}{144\,a^2x^9}$$

input
```
integrate((c*x^6+b*x^3+a)^(3/2)/x^10,x, algorithm="fricas")
```

output
```
[1/288*(48*a^2*c^(3/2)*x^9*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6
 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) - 3*(b^3 - 12*a*b*c)*sqrt(a)*
x^9*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3
 + 2*a)*sqrt(a) + 8*a^2)/x^6) - 4*((3*a*b^2 + 32*a^2*c)*x^6 + 14*a^2*b*x^3
 + 8*a^3)*sqrt(c*x^6 + b*x^3 + a))/(a^2*x^9), -1/288*(96*a^2*sqrt(-c)*c*x^
9*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c
*x^3 + a*c)) + 3*(b^3 - 12*a*b*c)*sqrt(a)*x^9*log(-((b^2 + 4*a*c)*x^6 + 8*
a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) +
4*((3*a*b^2 + 32*a^2*c)*x^6 + 14*a^2*b*x^3 + 8*a^3)*sqrt(c*x^6 + b*x^3 + a
))/(a^2*x^9), 1/144*(24*a^2*c^(3/2)*x^9*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 -
 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) - 3*(b^3 - 12*a*
b*c)*sqrt(-a)*x^9*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a
)/(a*c*x^6 + a*b*x^3 + a^2)) - 2*((3*a*b^2 + 32*a^2*c)*x^6 + 14*a^2*b*x^3
+ 8*a^3)*sqrt(c*x^6 + b*x^3 + a))/(a^2*x^9), -1/144*(48*a^2*sqrt(-c)*c*x^9
*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*
x^3 + a*c)) + 3*(b^3 - 12*a*b*c)*sqrt(-a)*x^9*arctan(1/2*sqrt(c*x^6 + b*x^
3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((3*a*b^2 + 3
2*a^2*c)*x^6 + 14*a^2*b*x^3 + 8*a^3)*sqrt(c*x^6 + b*x^3 + a))/(a^2*x^9)]
```

3.209.6 Sympy [F]

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{10}}\,dx = \int \frac{(a+bx^3+cx^6)^{\frac{3}{2}}}{x^{10}}\,dx$$

input
```
integrate((c*x**6+b*x**3+a)**(3/2)/x**10,x)
```

output
```
Integral((a + b*x**3 + c*x**6)**(3/2)/x**10, x)
```

3.209.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{10}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^6+b*x^3+a)^(3/2)/x^10,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.209.8 Giac [F]

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{10}}\,dx = \int \frac{(cx^6+bx^3+a)^{\frac{3}{2}}}{x^{10}}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^(3/2)/x^10,x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^(3/2)/x^10, x)
```

3.209.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{10}}\,dx = \int \frac{(c\,x^6+b\,x^3+a)^{3/2}}{x^{10}}\,dx$$

input
```
int((a + b*x^3 + c*x^6)^(3/2)/x^10,x)
```

output
```
int((a + b*x^3 + c*x^6)^(3/2)/x^10, x)
```

3.210 $\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{13}}\,dx$

3.210.1 Optimal result

Integrand size = 20, antiderivative size = 133

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{13}}\,dx = \frac{(b^2-4ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{64a^2x^6} - \frac{(2a+bx^3)(a+bx^3+cx^6)^{3/2}}{24ax^{12}} - \frac{(b^2-4ac)^2\operatorname{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{128a^{5/2}}$$

output
```
-1/24*(b*x^3+2*a)*(c*x^6+b*x^3+a)^(3/2)/a/x^12-1/128*(-4*a*c+b^2)^2*arctan
h(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2))/a^(5/2)+1/64*(-4*a*c+b^2)
*(b*x^3+2*a)*(c*x^6+b*x^3+a)^(1/2)/a^2/x^6
```

3.210.2 Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{13}}\,dx = -\frac{(2a+bx^3)\sqrt{a+bx^3+cx^6}(8a^2+8abx^3-3b^2x^6+20acx^6)}{192a^2x^{12}} + \frac{(b^2-4ac)^2\operatorname{arctanh}\left(\frac{\sqrt{c}x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{64a^{5/2}}$$

input
```
Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^13,x]
```

output
```
-1/192*((2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6]*(8*a^2 + 8*a*b*x^3 - 3*b^2*x
^6 + 20*a*c*x^6))/(a^2*x^12) + ((b^2 - 4*a*c)^2*ArcTanh[(Sqrt[c]*x^3 - Sqr
t[a + b*x^3 + c*x^6])/Sqrt[a]])/(64*a^(5/2))
```

3.210.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1693, 1152, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^{13}}dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int \frac{\left(cx^6+bx^3+a\right)^{3/2}}{x^{15}}dx^3$$

$$\Big\downarrow \text{1152}$$

$$\frac{1}{3}\left(-\frac{3\left(b^2-4ac\right)\int \frac{\sqrt{cx^6+bx^3+a}}{x^9}dx^3}{16a}-\frac{\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{8ax^{12}}\right)$$

$$\Big\downarrow \text{1152}$$

$$\frac{1}{3}\left(-\frac{3\left(b^2-4ac\right)\left(-\frac{\left(b^2-4ac\right)\int \frac{1}{x^3\sqrt{cx^6+bx^3+a}}dx^3}{8a}-\frac{\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{4ax^6}\right)}{16a}-\frac{\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{8ax^{12}}\right)$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{3}\left(-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b^2-4ac\right)\int \frac{1}{4a-x^6}d\frac{bx^3+2a}{\sqrt{cx^6+bx^3+a}}}{4a}-\frac{\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{4ax^6}\right)}{16a}-\frac{\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{8ax^{12}}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(-\frac{3(b^2-4ac)\left(\frac{(b^2-4ac)\operatorname{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{8a^{3/2}}-\frac{(2a+bx^3)\sqrt{a+bx^3+cx^6}}{4ax^6}\right)}{16a}-\frac{(2a+bx^3)\left(a+bx^3+cx^6\right)^{3/2}}{8ax^{12}}\right)$$

input
```
Int[(a + b*x^3 + c*x^6)^(3/2)/x^13,x]
```

output
```
(-1/8*((2*a + b*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(a*x^12) - (3*(b^2 - 4*a*c
)*(-1/4*((2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(a*x^6) + ((b^2 - 4*a*c)*A
rcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(8*a^(3/2))))/(
16*a))/3
```

3.210.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.210.4 Maple [F]

$$\int \frac{(c\,x^{6}+b\,x^{3}+a)^{\frac{3}{2}}}{x^{13}}dx$$

input
```
int((c*x^6+b*x^3+a)^(3/2)/x^13,x)
```

output
```
int((c*x^6+b*x^3+a)^(3/2)/x^13,x)
```

3.210.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 319, normalized size of antiderivative = 2.40

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{13}}\,dx = \left[\frac{3\,(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{a}x^{12}\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3-4\,\sqrt{cx^6+bx^3+a}(bx^3+2\,a)\sqrt{a}+8\,a}{x^6}\right.}{76}\right.$$

input
```
integrate((c*x^6+b*x^3+a)^(3/2)/x^13,x, algorithm="fricas")
```

output
```
[1/768*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(a)*x^12*log(-((b^2 + 4*a*c)*
x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)
/x^6) + 4*((3*a*b^3 - 20*a^2*b*c)*x^9 - 24*a^3*b*x^3 - 2*(a^2*b^2 + 20*a^3
*c)*x^6 - 16*a^4)*sqrt(c*x^6 + b*x^3 + a))/(a^3*x^12), 1/384*(3*(b^4 - 8*a
*b^2*c + 16*a^2*c^2)*sqrt(-a)*x^12*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x
^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((3*a*b^3 - 20*a^2*b*c)*
x^9 - 24*a^3*b*x^3 - 2*(a^2*b^2 + 20*a^3*c)*x^6 - 16*a^4)*sqrt(c*x^6 + b*x
^3 + a))/(a^3*x^12)]
```

3.210.6 Sympy [F]

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{13}}\,dx = \int \frac{(a+bx^3+cx^6)^{\frac{3}{2}}}{x^{13}}\,dx$$

input
```
integrate((c*x**6+b*x**3+a)**(3/2)/x**13,x)
```

output
```
Integral((a + b*x**3 + c*x**6)**(3/2)/x**13, x)
```

3.210.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{13}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^6+b*x^3+a)^(3/2)/x^13,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.210.8 Giac [F]

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{13}}\,dx = \int \frac{(cx^6+bx^3+a)^{\frac{3}{2}}}{x^{13}}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^(3/2)/x^13,x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^(3/2)/x^13, x)
```

3.210.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{13}}\,dx = \int \frac{(c\,x^6+b\,x^3+a)^{3/2}}{x^{13}}\,dx$$

input
```
int((a + b*x^3 + c*x^6)^(3/2)/x^13,x)
```

output
```
int((a + b*x^3 + c*x^6)^(3/2)/x^13, x)
```

3.211 $\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{16}}\,dx$

3.211.1 Optimal result

Integrand size = 20, antiderivative size = 162

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{16}}\,dx = -\frac{b(b^2-4ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{128a^3x^6} + \frac{b(2a+bx^3)(a+bx^3+cx^6)^{3/2}}{48a^2x^{12}} - \frac{(a+bx^3+cx^6)^{5/2}}{15ax^{15}} + \frac{b(b^2-4ac)^2\operatorname{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{256a^{7/2}}$$

output
```
1/48*b*(b*x^3+2*a)*(c*x^6+b*x^3+a)^(3/2)/a^2/x^12-1/15*(c*x^6+b*x^3+a)^(5/
2)/a/x^15+1/256*b*(-4*a*c+b^2)^2*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*
x^3+a)^(1/2))/a^(7/2)-1/128*b*(-4*a*c+b^2)*(b*x^3+2*a)*(c*x^6+b*x^3+a)^(1/
2)/a^3/x^6
```

3.211.2 Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{16}}\,dx = \frac{-\frac{\sqrt{a}\sqrt{a+bx^3+cx^6}\left(128a^4+15b^4x^{12}-10ab^2x^9\left(b+10cx^3\right)+16a^3\left(11bx^3+16cx^6\right)+8a^2x^6\left(b^2+7bcx^3+16c^2x^6\right)\right)}{x^{15}}}{1920a^{7/2}}$$

input
```
Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^16,x]
```

output
```
(-((Sqrt[a]*Sqrt[a + b*x^3 + c*x^6]*(128*a^4 + 15*b^4*x^12 - 10*a*b^2*x^9*
(b + 10*c*x^3) + 16*a^3*(11*b*x^3 + 16*c*x^6) + 8*a^2*x^6*(b^2 + 7*b*c*x^3
 + 16*c^2*x^6)))/x^15) - 15*b*(b^2 - 4*a*c)^2*ArcTanh[(Sqrt[c]*x^3 - Sqrt[
a + b*x^3 + c*x^6])/Sqrt[a]])/(1920*a^(7/2))
```

3.211.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1693, 1157, 1152, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^{16}}dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int \frac{\left(cx^6+bx^3+a\right)^{3/2}}{x^{18}}dx^3$$

$$\Big\downarrow \text{1157}$$

$$\frac{1}{3}\left(-\frac{b\int \frac{\left(cx^6+bx^3+a\right)^{3/2}}{x^{15}}dx^3}{2a}-\frac{\left(a+bx^3+cx^6\right)^{5/2}}{5ax^{15}}\right)$$

$$\Big\downarrow \text{1152}$$

$$\frac{1}{3}\left(-\frac{b\left(-\frac{3\left(b^2-4ac\right)\int \frac{\sqrt{cx^6+bx^3+a}}{x^9}dx^3}{16a}-\frac{\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{8ax^{12}}\right)}{2a}-\frac{\left(a+bx^3+cx^6\right)^{5/2}}{5ax^{15}}\right)$$

$$\Big\downarrow \text{1152}$$

$$\frac{1}{3}\left(-\frac{b\left(-\frac{3(b^2-4ac)\left(-\frac{(b^2-4ac)\int \frac{1}{x^3\sqrt{cx^6+bx^3+a}}dx^3}{8a}-\frac{(2a+bx^3)\sqrt{a+bx^3+cx^6}}{4ax^6}\right)}{16a}-\frac{(2a+bx^3)(a+bx^3+cx^6)^{3/2}}{8ax^{12}}\right)}{2a}-\frac{(a+bx^3+cx^6)^5}{5ax^{15}}\right.$$

$\downarrow$ 1154

$$\frac{1}{3}\left(-\frac{b\left(-\frac{3(b^2-4ac)\left(\frac{(b^2-4ac)\int \frac{1}{4a-x^6}d\frac{bx^3+2a}{\sqrt{cx^6+bx^3+a}}}{4a}-\frac{(2a+bx^3)\sqrt{a+bx^3+cx^6}}{4ax^6}\right)}{16a}-\frac{(2a+bx^3)(a+bx^3+cx^6)^{3/2}}{8ax^{12}}\right)}{2a}-\frac{(a+bx^3+cx^6)^5}{5ax^{15}}\right.$$

$\downarrow$ 219

$$\frac{1}{3}\left(-\frac{b\left(-\frac{3(b^2-4ac)\left(\frac{(b^2-4ac)\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{8a^{3/2}}-\frac{(2a+bx^3)\sqrt{a+bx^3+cx^6}}{4ax^6}\right)}{16a}-\frac{(2a+bx^3)(a+bx^3+cx^6)^{3/2}}{8ax^{12}}\right)}{2a}-\frac{(a+bx^3+}{5ax^1}\right.$$

input
```
Int[(a + b*x^3 + c*x^6)^(3/2)/x^16,x]
```

output
```
(-1/5*(a + b*x^3 + c*x^6)^(5/2)/(a*x^15) - (b*(-1/8*((2*a + b*x^3)*(a + b*
x^3 + c*x^6)^(3/2))/(a*x^12) - (3*(b^2 - 4*a*c)*(-1/4*((2*a + b*x^3)*Sqrt[
a + b*x^3 + c*x^6])/(a*x^6) + ((b^2 - 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt
[a]*Sqrt[a + b*x^3 + c*x^6])])/(8*a^(3/2))))/(16*a)))/(2*a))/3
```

3.211. $\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{16}}\,dx$

3.211.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1157
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Simp[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2))
  Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && EqQ[m + 2*p + 3, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.211.4 Maple [F]

$$\int \frac{(c\,x^{6}+b\,x^{3}+a)^{\frac{3}{2}}}{x^{16}}dx$$

input
```
int((c*x^6+b*x^3+a)^(3/2)/x^16,x)
```

output
```
int((c*x^6+b*x^3+a)^(3/2)/x^16,x)
```

3.211.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 383, normalized size of antiderivative = 2.36

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{16}}\,dx = \left[\frac{15\,(b^5-8\,ab^3c+16\,a^2bc^2)\sqrt{a}x^{15}\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3+4\,\sqrt{cx^6+bx^3+a}(bx^3+2\,a)\sqrt{a}+}{x^6}\right.}{}\right.$$

$$-\frac{15\,(b^5-8\,ab^3c+16\,a^2bc^2)\sqrt{-a}x^{15}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(bx^3+2\,a)\sqrt{-a}}{2\,(acx^6+abx^3+a^2)}\right)+2\,((15\,ab^4-100\,a^2b^2c+128\,a^3c^2)x^{12}}{3840\,a^4x^{15}}$$

input
```
integrate((c*x^6+b*x^3+a)^(3/2)/x^16,x, algorithm="fricas")
```

output
```
[1/7680*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*sqrt(a)*x^15*log(-((b^2 + 4*a
*c)*x^6 + 8*a*b*x^3 + 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*
a^2)/x^6) - 4*((15*a*b^4 - 100*a^2*b^2*c + 128*a^3*c^2)*x^12 - 2*(5*a^2*b^
3 - 28*a^3*b*c)*x^9 + 176*a^4*b*x^3 + 8*(a^3*b^2 + 32*a^4*c)*x^6 + 128*a^5
)*sqrt(c*x^6 + b*x^3 + a))/(a^4*x^15), -1/3840*(15*(b^5 - 8*a*b^3*c + 16*a
^2*b*c^2)*sqrt(-a)*x^15*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*s
qrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((15*a*b^4 - 100*a^2*b^2*c + 128*a^
3*c^2)*x^12 - 2*(5*a^2*b^3 - 28*a^3*b*c)*x^9 + 176*a^4*b*x^3 + 8*(a^3*b^2
+ 32*a^4*c)*x^6 + 128*a^5)*sqrt(c*x^6 + b*x^3 + a))/(a^4*x^15)]
```

3.211.6 Sympy [F]

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{16}}\,dx = \int \frac{(a+bx^3+cx^6)^{\frac{3}{2}}}{x^{16}}\,dx$$

input
```
integrate((c*x**6+b*x**3+a)**(3/2)/x**16,x)
```

output
```
Integral((a + b*x**3 + c*x**6)**(3/2)/x**16, x)
```

3.211.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{16}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^6+b*x^3+a)^(3/2)/x^16,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.211.8 Giac [F]

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{16}}\,dx = \int \frac{(cx^6+bx^3+a)^{\frac{3}{2}}}{x^{16}}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^(3/2)/x^16,x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^(3/2)/x^16, x)
```

3.211.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{16}}\,dx = \int \frac{(c\,x^6+b\,x^3+a)^{3/2}}{x^{16}}\,dx$$

input
```
int((a + b*x^3 + c*x^6)^(3/2)/x^16,x)
```

output
```
int((a + b*x^3 + c*x^6)^(3/2)/x^16, x)
```

3.212 $\int \frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^{19}}\,dx$

3.212.1 Optimal result

Integrand size = 20, antiderivative size = 216

$$\begin{aligned}\int \frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^{19}}\,dx &= \frac{(b^2-4ac)\,(7b^2-4ac)\,(2a+bx^3)\sqrt{a+bx^3+cx^6}}{1536a^4x^6}\\ &-\frac{(7b^2-4ac)\,(2a+bx^3)\,(a+bx^3+cx^6)^{3/2}}{576a^3x^{12}}-\frac{(a+bx^3+cx^6)^{5/2}}{18ax^{18}}\\ &+\frac{7b(a+bx^3+cx^6)^{5/2}}{180a^2x^{15}}-\frac{(b^2-4ac)^2\,(7b^2-4ac)\,\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{3072a^{9/2}}\end{aligned}$$

output
```
-1/576*(-4*a*c+7*b^2)*(b*x^3+2*a)*(c*x^6+b*x^3+a)^(3/2)/a^3/x^12-1/18*(c*x
^6+b*x^3+a)^(5/2)/a/x^18+7/180*b*(c*x^6+b*x^3+a)^(5/2)/a^2/x^15-1/3072*(-4
*a*c+b^2)^2*(-4*a*c+7*b^2)*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)
^(1/2))/a^(9/2)+1/1536*(-4*a*c+b^2)*(-4*a*c+7*b^2)*(b*x^3+2*a)*(c*x^6+b*x^
3+a)^(1/2)/a^4/x^6
```

3.212.2 Mathematica [A] (verified)

Time = 1.33 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.93

$$\int \frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^{19}}\,dx = \frac{-\frac{\sqrt{a}\sqrt{a+bx^3+cx^6}\left(1280a^5-105b^5x^{15}+10ab^3x^{12}(7b+76cx^3)+64a^4(26bx^3+35cx^6)+48a^3x^6(b^2+6bcx^3+10c^2\right.}{x^{18}}}{230}$$

input
```
Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^19,x]
```

output
```
(-((Sqrt[a]*Sqrt[a + b*x^3 + c*x^6]*(1280*a^5 - 105*b^5*x^15 + 10*a*b^3*x^
12*(7*b + 76*c*x^3) + 64*a^4*(26*b*x^3 + 35*c*x^6) + 48*a^3*x^6*(b^2 + 6*b
*c*x^3 + 10*c^2*x^6) - 8*a^2*b*x^9*(7*b^2 + 54*b*c*x^3 + 162*c^2*x^6)))/x^
18) + 15*(b^2 - 4*a*c)^2*(7*b^2 - 4*a*c)*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b
*x^3 + c*x^6])/Sqrt[a]])/(23040*a^(9/2))
```

3.212.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1693, 1167, 27, 1228, 1152, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^{19}}dx$$

$$\Big\downarrow \ 1693$$

$$\frac{1}{3}\int \frac{\left(cx^6+bx^3+a\right)^{3/2}}{x^{21}}dx^3$$

$$\Big\downarrow \ 1167$$

$$\frac{1}{3}\left(-\frac{\int \frac{(2cx^3+7b)\left(cx^6+bx^3+a\right)^{3/2}}{2x^{18}}dx^3}{6a}-\frac{\left(a+bx^3+cx^6\right)^{5/2}}{6ax^{18}}\right)$$

$$\Big\downarrow \ 27$$

$$\frac{1}{3}\left(-\frac{\int \frac{(2cx^3+7b)\left(cx^6+bx^3+a\right)^{3/2}}{x^{18}}dx^3}{12a}-\frac{\left(a+bx^3+cx^6\right)^{5/2}}{6ax^{18}}\right)$$

$$\Big\downarrow \ 1228$$

$$\frac{1}{3}\left(-\frac{-\frac{(7b^2-4ac)\int \frac{\left(cx^6+bx^3+a\right)^{3/2}}{x^{15}}dx^3}{2a}-\frac{7b\left(a+bx^3+cx^6\right)^{5/2}}{5ax^{15}}}{12a}-\frac{\left(a+bx^3+cx^6\right)^{5/2}}{6ax^{18}}\right)$$

$$\Big\downarrow \ 1152$$

$$\frac{1}{3}\left(-\frac{-\frac{(7b^2-4ac)\left(-\frac{3\left(b^2-4ac\right)\int\frac{\sqrt{cx^6+bx^3+a}}{x^9}dx^3}{16a}-\frac{\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{8ax^{12}}\right)}{2a}-\frac{7b\left(a+bx^3+cx^6\right)^{5/2}}{5ax^{15}}}{12a}-\frac{\left(a+bx^3+cx^6\right)^{5/2}}{6ax^{18}}\right)$$

$\downarrow$ 1152

$$\frac{1}{3}\left(-\frac{-\frac{(7b^2-4ac)\left(-\frac{3\left(b^2-4ac\right)\left(-\frac{\left(b^2-4ac\right)\int\frac{1}{x^3\sqrt{cx^6+bx^3+a}}dx^3}{8a}-\frac{\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{4ax^6}\right)}{16a}-\frac{\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{8ax^{12}}\right)}{2a}-\frac{7b\left(a+bx^3+cx^6\right)^{5}}{5ax^{15}}}{12a}\right.$$

$\downarrow$ 1154

$$\frac{1}{3}\left(-\frac{-\frac{(7b^2-4ac)\left(-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b^2-4ac\right)\int\frac{1}{4a-x^6}d\frac{bx^3+2a}{\sqrt{cx^6+bx^3+a}}}{4a}-\frac{\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{4ax^6}\right)}{16a}-\frac{\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{8ax^{12}}\right)}{2a}-\frac{7b\left(a+bx^3+cx^6\right)}{5ax^{15}}}{12a}\right.$$

$\downarrow$ 219

3.212. $\int\frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^{19}}\,dx$

$$\frac{1}{3}\left(-\frac{-\frac{(7b^2-4ac)\left(-\frac{3(b^2-4ac)\left(\frac{(b^2-4ac)\operatorname{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{8a^{3/2}}-\frac{(2a+bx^3)\sqrt{a+bx^3+cx^6}}{4ax^6}\right)}{16a}-\frac{(2a+bx^3)(a+bx^3+cx^6)^{3/2}}{8ax^{12}}\right)}{2a}-\frac{7b(a+bx^3}{5a}}{12a}\right.$$

input
```
Int[(a + b*x^3 + c*x^6)^(3/2)/x^19,x]
```

output
```
(-1/6*(a + b*x^3 + c*x^6)^(5/2)/(a*x^18) - ((-7*b*(a + b*x^3 + c*x^6)^(5/2
))/(5*a*x^15) - ((7*b^2 - 4*a*c)*(-1/8*((2*a + b*x^3)*(a + b*x^3 + c*x^6)^
(3/2))/(a*x^12) - (3*(b^2 - 4*a*c)*(-1/4*((2*a + b*x^3)*Sqrt[a + b*x^3 + c
*x^6])/(a*x^6) + ((b^2 - 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a +
b*x^3 + c*x^6])])/(8*a^(3/2))))/(16*a)))/(2*a))/(12*a))/3
```

3.212.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1167
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 - b*d*e + a*e^2))   Int[
(d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
 x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[m
, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumSimp
lerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.212.4 Maple [F]

$$\int \frac{(c\,x^{6}+b\,x^{3}+a)^{\frac{3}{2}}}{x^{19}}dx$$

input
```
int((c*x^6+b*x^3+a)^(3/2)/x^19,x)
```

output
```
int((c*x^6+b*x^3+a)^(3/2)/x^19,x)
```

3.212.5 Fricas [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 473, normalized size of antiderivative = 2.19

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{19}}\,dx = \left[-\frac{15\,(7\,b^6-60\,ab^4c+144\,a^2b^2c^2-64\,a^3c^3)\sqrt{a}x^{18}\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3+4\,\sqrt{cx^6-}}{x^6}\right.}{}\right.$$

input
```
integrate((c*x^6+b*x^3+a)^(3/2)/x^19,x, algorithm="fricas")
```

output
```
[-1/92160*(15*(7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(a)*
x^18*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 + 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^
3 + 2*a)*sqrt(a) + 8*a^2)/x^6) - 4*((105*a*b^5 - 760*a^2*b^3*c + 1296*a^3*
b*c^2)*x^15 - 2*(35*a^2*b^4 - 216*a^3*b^2*c + 240*a^4*c^2)*x^12 + 8*(7*a^3
*b^3 - 36*a^4*b*c)*x^9 - 1664*a^5*b*x^3 - 16*(3*a^4*b^2 + 140*a^5*c)*x^6 -
 1280*a^6)*sqrt(c*x^6 + b*x^3 + a))/(a^5*x^18), 1/46080*(15*(7*b^6 - 60*a*
b^4*c + 144*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(-a)*x^18*arctan(1/2*sqrt(c*x^6
+ b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((105*a
*b^5 - 760*a^2*b^3*c + 1296*a^3*b*c^2)*x^15 - 2*(35*a^2*b^4 - 216*a^3*b^2*
c + 240*a^4*c^2)*x^12 + 8*(7*a^3*b^3 - 36*a^4*b*c)*x^9 - 1664*a^5*b*x^3 -
16*(3*a^4*b^2 + 140*a^5*c)*x^6 - 1280*a^6)*sqrt(c*x^6 + b*x^3 + a))/(a^5*x
^18)]
```

3.212.6 Sympy [F]

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{19}}\,dx = \int \frac{(a+bx^3+cx^6)^{\frac{3}{2}}}{x^{19}}\,dx$$

input
```
integrate((c*x**6+b*x**3+a)**(3/2)/x**19,x)
```

output
```
Integral((a + b*x**3 + c*x**6)**(3/2)/x**19, x)
```

3.212.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{19}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^6+b*x^3+a)^(3/2)/x^19,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.212.8 Giac [F]

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{19}}\,dx = \int \frac{(cx^6+bx^3+a)^{\frac{3}{2}}}{x^{19}}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^(3/2)/x^19,x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^(3/2)/x^19, x)
```

3.212.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{19}}\,dx = \int \frac{(c\,x^6+b\,x^3+a)^{3/2}}{x^{19}}\,dx$$

input
```
int((a + b*x^3 + c*x^6)^(3/2)/x^19,x)
```

output
```
int((a + b*x^3 + c*x^6)^(3/2)/x^19, x)
```

3.213 $\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{22}}\,dx$

3.213.1 Optimal result

Integrand size = 20, antiderivative size = 255

$$\begin{aligned}\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{22}}\,dx =& -\frac{b(b^2-4ac)\,(3b^2-4ac)\,(2a+bx^3)\sqrt{a+bx^3+cx^6}}{1024a^5x^6}\\&+\frac{b(3b^2-4ac)\,(2a+bx^3)\,(a+bx^3+cx^6)^{3/2}}{384a^4x^{12}}-\frac{(a+bx^3+cx^6)^{5/2}}{21ax^{21}}+\frac{b(a+bx^3+cx^6)^{5/2}}{28a^2x^{18}}\\&-\frac{(21b^2-16ac)\,(a+bx^3+cx^6)^{5/2}}{840a^3x^{15}}+\frac{b(b^2-4ac)^2\,(3b^2-4ac)\,\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{2048a^{11/2}}\end{aligned}$$

output
```
1/384*b*(-4*a*c+3*b^2)*(b*x^3+2*a)*(c*x^6+b*x^3+a)^(3/2)/a^4/x^12-1/21*(c*
x^6+b*x^3+a)^(5/2)/a/x^21+1/28*b*(c*x^6+b*x^3+a)^(5/2)/a^2/x^18-1/840*(-16
*a*c+21*b^2)*(c*x^6+b*x^3+a)^(5/2)/a^3/x^15+1/2048*b*(-4*a*c+b^2)^2*(-4*a*
c+3*b^2)*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2))/a^(11/2)-1
/1024*b*(-4*a*c+b^2)*(-4*a*c+3*b^2)*(b*x^3+2*a)*(c*x^6+b*x^3+a)^(1/2)/a^5/
x^6
```

3.213.2 Mathematica [A] (verified)

Time = 1.53 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{22}}\,dx = \frac{-\frac{\sqrt{a}\sqrt{a+bx^3+cx^6}\left(5120a^6+315b^6x^{18}-210ab^4x^{15}\left(b+12cx^3\right)+256a^5\left(25bx^3+32cx^6\right)+64a^4x^6\left(2b^2+11bcx^3+1\right.}{x^{21}}}{}$$

input
```
Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^22,x]
```

output
```
(-((Sqrt[a]*Sqrt[a + b*x^3 + c*x^6]*(5120*a^6 + 315*b^6*x^18 - 210*a*b^4*x
^15*(b + 12*c*x^3) + 256*a^5*(25*b*x^3 + 32*c*x^6) + 64*a^4*x^6*(2*b^2 + 1
1*b*c*x^3 + 16*c^2*x^6) + 56*a^2*b^2*x^12*(3*b^2 + 26*b*c*x^3 + 98*c^2*x^6
) - 16*a^3*x^9*(9*b^3 + 62*b^2*c*x^3 + 146*b*c^2*x^6 + 128*c^3*x^9)))/x^21
) - 105*b*(b^2 - 4*a*c)^2*(3*b^2 - 4*a*c)*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a +
b*x^3 + c*x^6])/Sqrt[a]])/(107520*a^(11/2))
```

3.213.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1693, 1167, 27, 1237, 27, 1228, 1152, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{22}}\,dx$$

$$\Big\downarrow \ 1693$$

$$\frac{1}{3}\int \frac{(cx^6+bx^3+a)^{3/2}}{x^{24}}dx^3$$

$$\Big\downarrow \ 1167$$

$$\frac{1}{3}\left(-\frac{\int \frac{(4cx^3+9b)(cx^6+bx^3+a)^{3/2}}{2x^{21}}dx^3}{7a}-\frac{(a+bx^3+cx^6)^{5/2}}{7ax^{21}}\right)$$

$$\Big\downarrow \ 27$$

$$\frac{1}{3}\left(-\frac{\int \frac{(4cx^3+9b)(cx^6+bx^3+a)^{3/2}}{x^{21}}dx^3}{14a}-\frac{(a+bx^3+cx^6)^{5/2}}{7ax^{21}}\right)$$

$$\Big\downarrow \; 1237$$

$$\frac{1}{3}\left(-\frac{-\frac{\int \frac{3(6bcx^3+21b^2-16ac)(cx^6+bx^3+a)^{3/2}}{2x^{18}}dx^3}{6a}-\frac{3b(a+bx^3+cx^6)^{5/2}}{2ax^{18}}}{14a}-\frac{(a+bx^3+cx^6)^{5/2}}{7ax^{21}}\right)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3}\left(-\frac{-\frac{\int \frac{(6bcx^3+21b^2-16ac)(cx^6+bx^3+a)^{3/2}}{x^{18}}dx^3}{4a}-\frac{3b(a+bx^3+cx^6)^{5/2}}{2ax^{18}}}{14a}-\frac{(a+bx^3+cx^6)^{5/2}}{7ax^{21}}\right)$$

$$\Big\downarrow \; 1228$$

$$\frac{1}{3}\left(-\frac{-\frac{-\frac{7b(3b^2-4ac)\int \frac{(cx^6+bx^3+a)^{3/2}}{x^{15}}dx^3}{2a}-\frac{(21b^2-16ac)(a+bx^3+cx^6)^{5/2}}{5ax^{15}}}{4a}-\frac{3b(a+bx^3+cx^6)^{5/2}}{2ax^{18}}}{14a}-\frac{(a+bx^3+cx^6)^{5/2}}{7ax^{21}}\right)$$

$$\Big\downarrow \; 1152$$

$$\frac{1}{3}\left(-\frac{-\frac{-\frac{7b(3b^2-4ac)\left(-\frac{3(b^2-4ac)\int \frac{\sqrt{cx^6+bx^3+a}}{x^9}dx^3}{16a}-\frac{(2a+bx^3)(a+bx^3+cx^6)^{3/2}}{8ax^{12}}\right)}{2a}-\frac{(21b^2-16ac)(a+bx^3+cx^6)^{5/2}}{5ax^{15}}}{4a}-\frac{3b(a+bx^3+cx^6)^{5/2}}{2ax^{18}}}{14a}-\cdots\right.$$

$$\Big\downarrow \; 1152$$

$$\frac{1}{3}\left(-\frac{-\frac{-\frac{7b(3b^2-4ac)\left(-\frac{3(b^2-4ac)\left(-\frac{(b^2-4ac)\int \frac{1}{x^3\sqrt{cx^6+bx^3+a}}dx^3}{8a}-\frac{(2a+bx^3)\sqrt{a+bx^3+cx^6}}{4ax^6}\right)}{16a}-\frac{(2a+bx^3)(a+bx^3+cx^6)^{3/2}}{8ax^{12}}\right)}{2a}-\frac{(21b^2-16ac)(a+b\cdots}{5ax^{15}}}{4a}}{14a}\right.$$

3.213. $\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{22}}\,dx$

$\Big\downarrow$ 1154

$$\frac{1}{3}\left(-\frac{-\frac{-\frac{7b\left(3b^2-4ac\right)\left(-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b^2-4ac\right)\int \frac{1}{4a-x^6}d\frac{bx^3+2a}{\sqrt{cx^6+bx^3+a}}}{4a}-\frac{\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{4ax^6}\right)}{16a}-\frac{\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{8ax^{12}}\right)}{2a}-\frac{\left(21b^2-16ac\right)\left(a+\cdots\right.}{5ax^{15}\cdots}}{4a}}{14a}\cdots\right.$$

$\Big\downarrow$ 219

$$\frac{1}{3}\left(-\frac{-\frac{-\frac{7b\left(3b^2-4ac\right)\left(-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b^2-4ac\right)\operatorname{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{8a^{3/2}}-\frac{\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{4ax^6}\right)}{16a}-\frac{\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{8ax^{12}}\right)}{2a}-\frac{\left(21b^2-16\cdots\right.}{\cdots}}{4a}}{14a}\cdots\right.$$

input
```
Int[(a + b*x^3 + c*x^6)^(3/2)/x^22,x]
```

output
```
(-1/7*(a + b*x^3 + c*x^6)^(5/2)/(a*x^21) - ((-3*b*(a + b*x^3 + c*x^6)^(5/2
))/(2*a*x^18) - (-1/5*((21*b^2 - 16*a*c)*(a + b*x^3 + c*x^6)^(5/2))/(a*x^1
5) - (7*b*(3*b^2 - 4*a*c)*(-1/8*((2*a + b*x^3)*(a + b*x^3 + c*x^6)^(3/2))/
(a*x^12) - (3*(b^2 - 4*a*c)*(-1/4*((2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/
(a*x^6) + ((b^2 - 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 +
 c*x^6])])/(8*a^(3/2))))/(16*a)))/(2*a))/(4*a))/(14*a))/3
```

3.213. $\int \frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^{22}}\,dx$

3.213.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1167
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 - b*d*e + a*e^2))   Int[
(d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
 x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[m
, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumSimp
lerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

3.213. $\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{22}}\,dx$

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.213.4 Maple [F]

$$\int \frac{(c\,x^{6}+b\,x^{3}+a)^{\frac{3}{2}}}{x^{22}}dx$$

input
```
int((c*x^6+b*x^3+a)^(3/2)/x^22,x)
```

output
```
int((c*x^6+b*x^3+a)^(3/2)/x^22,x)
```

3.213.5 Fricas [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 557, normalized size of antiderivative = 2.18

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{22}}\,dx = \left[-\frac{105\,(3\,b^7-28\,ab^5c+80\,a^2b^3c^2-64\,a^3bc^3)\sqrt{a}x^{21}\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3-4\,\sqrt{cx^6}}{x^6}\right.}{}\right.$$

$$-\frac{105\,(3\,b^7-28\,ab^5c+80\,a^2b^3c^2-64\,a^3bc^3)\sqrt{-a}x^{21}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(bx^3+2\,a)\sqrt{-a}}{2\,(acx^6+abx^3+a^2)}\right)+2\,((315\,ab^6-2520\,a^2}{}$$

input
```
integrate((c*x^6+b*x^3+a)^(3/2)/x^22,x, algorithm="fricas")
```

output
```
[-1/430080*(105*(3*b^7 - 28*a*b^5*c + 80*a^2*b^3*c^2 - 64*a^3*b*c^3)*sqrt(
a)*x^21*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b
*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) + 4*((315*a*b^6 - 2520*a^2*b^4*c + 5488*
a^3*b^2*c^2 - 2048*a^4*c^3)*x^18 - 2*(105*a^2*b^5 - 728*a^3*b^3*c + 1168*a
^4*b*c^2)*x^15 + 8*(21*a^3*b^4 - 124*a^4*b^2*c + 128*a^5*c^2)*x^12 + 6400*
a^6*b*x^3 - 16*(9*a^4*b^3 - 44*a^5*b*c)*x^9 + 5120*a^7 + 128*(a^5*b^2 + 64
*a^6*c)*x^6)*sqrt(c*x^6 + b*x^3 + a))/(a^6*x^21), -1/215040*(105*(3*b^7 -
28*a*b^5*c + 80*a^2*b^3*c^2 - 64*a^3*b*c^3)*sqrt(-a)*x^21*arctan(1/2*sqrt(
c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*(
(315*a*b^6 - 2520*a^2*b^4*c + 5488*a^3*b^2*c^2 - 2048*a^4*c^3)*x^18 - 2*(1
05*a^2*b^5 - 728*a^3*b^3*c + 1168*a^4*b*c^2)*x^15 + 8*(21*a^3*b^4 - 124*a^
4*b^2*c + 128*a^5*c^2)*x^12 + 6400*a^6*b*x^3 - 16*(9*a^4*b^3 - 44*a^5*b*c)
*x^9 + 5120*a^7 + 128*(a^5*b^2 + 64*a^6*c)*x^6)*sqrt(c*x^6 + b*x^3 + a))/(
a^6*x^21)]
```

3.213.6 Sympy [F]

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{22}}\,dx = \int \frac{(a+bx^3+cx^6)^{\frac{3}{2}}}{x^{22}}\,dx$$

input
```
integrate((c*x**6+b*x**3+a)**(3/2)/x**22,x)
```

output
```
Integral((a + b*x**3 + c*x**6)**(3/2)/x**22, x)
```

3.213.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{22}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^6+b*x^3+a)^(3/2)/x^22,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.213.8 Giac [F]

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{22}}\,dx = \int \frac{(cx^6+bx^3+a)^{\frac{3}{2}}}{x^{22}}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^(3/2)/x^22,x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^(3/2)/x^22, x)
```

3.213.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{22}}\,dx = \int \frac{(c\,x^6+b\,x^3+a)^{3/2}}{x^{22}}\,dx$$

input
```
int((a + b*x^3 + c*x^6)^(3/2)/x^22,x)
```

output
```
int((a + b*x^3 + c*x^6)^(3/2)/x^22, x)
```

3.214 $\int x^3(a+bx^3+cx^6)^{3/2}\,dx$

3.214.1 Optimal result

Integrand size = 20, antiderivative size = 141

$$\int x^3\left(a+bx^3+cx^6\right)^{3/2}\,dx=\frac{ax^4\sqrt{a+bx^3+cx^6}\,\text{AppellF1}\left(\frac{4}{3},-\frac{3}{2},-\frac{3}{2},\frac{7}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

output
```
1/4*a*x^4*AppellF1(4/3,-3/2,-3/2,7/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*
x^3/(b+(-4*a*c+b^2)^(1/2)))*(c*x^6+b*x^3+a)^(1/2)/(1+2*c*x^3/(b-(-4*a*c+b^
2)^(1/2)))^(1/2)/(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.214.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 453 vs. 2(141) = 282.

Time = 10.58 (sec) , antiderivative size = 453, normalized size of antiderivative = 3.21

$$\int x^3\left(a+bx^3+cx^6\right)^{3/2}\,dx=\frac{x\Big(8(-297b^4x^3-81b^3cx^6+3464b^2c^2x^9+5488bc^3x^{12}+2240c^4x^{15}+4a^2c(459b+1280cx^3)+}{}$$

input
```
Integrate[x^3*(a + b*x^3 + c*x^6)^(3/2),x]
```

output
```
(x*(8*(-297*b^4*x^3 - 81*b^3*c*x^6 + 3464*b^2*c^2*x^9 + 5488*b*c^3*x^12 +
2240*c^4*x^15 + 4*a^2*c*(459*b + 1280*c*x^3) + a*(-297*b^3 + 2052*b^2*c*x^
3 + 10204*b*c^2*x^6 + 7360*c^3*x^9)) + 216*a*b*(11*b^2 - 68*a*c)*Sqrt[(b -
 Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2
- 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/3, 1/2, 1/2, 4/3,
(-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] +
27*(55*b^4 - 404*a*b^2*c + 640*a^2*c^2)*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] +
2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(
b + Sqrt[b^2 - 4*a*c])]*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt[
b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]))/(232960*c^2*Sqrt[a +
b*x^3 + c*x^6])
```

3.214.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+bx^3+cx^6\right)^{3/2} dx$$

$$\Big\downarrow \text{ 1721}$$

$$\frac{a\sqrt{a+bx^3+cx^6}\int x^3\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{3/2}\left(\frac{2cx^3}{b+\sqrt{b^2-4ac}}+1\right)^{3/2}dx}{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

$$\Big\downarrow \text{ 1012}$$

$$\frac{ax^4\sqrt{a+bx^3+cx^6}\operatorname{AppellF1}\left(\frac{4}{3},-\frac{3}{2},-\frac{3}{2},\frac{7}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[x^3*(a + b*x^3 + c*x^6)^(3/2),x]
```

output
```
(a*x^4*Sqrt[a + b*x^3 + c*x^6]*AppellF1[4/3, -3/2, -3/2, 7/3, (-2*c*x^3)/(
b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(4*Sqrt[1 + (
2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c
])])
```

3.214.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

3.214.4 Maple [F]

$$\int x^{3} \left(c\,x^{6}+b\,x^{3}+a \right)^{\frac{3}{2}}d x$$

input
```
int(x^3*(c*x^6+b*x^3+a)^(3/2),x)
```

output
```
int(x^3*(c*x^6+b*x^3+a)^(3/2),x)
```

3.214.5 Fricas [F]

$$\int x^3 \left(a+bx^3+cx^6\right)^{3/2} \, dx = \int \left(cx^6+bx^3+a\right)^{\frac{3}{2}} x^3 \, dx$$

input
```
integrate(x^3*(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
integral((c*x^9 + b*x^6 + a*x^3)*sqrt(c*x^6 + b*x^3 + a), x)
```

3.214.6 Sympy [F]

$$\int x^3 \left(a+bx^3+cx^6\right)^{3/2} \, dx = \int x^3 \left(a+bx^3+cx^6\right)^{\frac{3}{2}} \, dx$$

input
```
integrate(x**3*(c*x**6+b*x**3+a)**(3/2),x)
```

output
```
Integral(x**3*(a + b*x**3 + c*x**6)**(3/2), x)
```

3.214.7 Maxima [F]

$$\int x^3 \left(a+bx^3+cx^6\right)^{3/2} \, dx = \int \left(cx^6+bx^3+a\right)^{\frac{3}{2}} x^3 \, dx$$

input
```
integrate(x^3*(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)^(3/2)*x^3, x)
```

3.214.8 Giac [F]

$$\int x^3 \left(a+bx^3+cx^6\right)^{3/2} \, dx = \int \left(cx^6+bx^3+a\right)^{\frac{3}{2}} x^3 \, dx$$

input
```
integrate(x^3*(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^(3/2)*x^3, x)
```

3.214.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \left(a+bx^3+cx^6\right)^{3/2} \, dx = \int x^3 \left(c\,x^6+b\,x^3+a\right)^{3/2} \, dx$$

input
```
int(x^3*(a + b*x^3 + c*x^6)^(3/2),x)
```

output
```
int(x^3*(a + b*x^3 + c*x^6)^(3/2), x)
```

3.215 $\int x(a+bx^3+cx^6)^{3/2}\,dx$

3.215.1 Optimal result

Integrand size = 18, antiderivative size = 141

$$\int x\left(a+bx^3+cx^6\right)^{3/2}\,dx=\frac{ax^2\sqrt{a+bx^3+cx^6}\,\text{AppellF1}\left(\frac{2}{3},-\frac{3}{2},-\frac{3}{2},\frac{5}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

output
```
1/2*a*x^2*AppellF1(2/3,-3/2,-3/2,5/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*
x^3/(b+(-4*a*c+b^2)^(1/2)))*(c*x^6+b*x^3+a)^(1/2)/(1+2*c*x^3/(b-(-4*a*c+b^
2)^(1/2)))^(1/2)/(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.215.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 410 vs. 2(141) = 282.

Time = 10.50 (sec) , antiderivative size = 410, normalized size of antiderivative = 2.91

$$\int x\left(a+bx^3+cx^6\right)^{3/2}\,dx=\frac{x^2\left(10(27ab^2+448a^2c+27b^3x^3+698abcx^3+277b^2cx^6+608ac^2x^6+410bc^2x^9+160c^3x^{12})\right.}{}$$

input
```
Integrate[x*(a + b*x^3 + c*x^6)^(3/2),x]
```

output
```
(x^2*(10*(27*a*b^2 + 448*a^2*c + 27*b^3*x^3 + 698*a*b*c*x^3 + 277*b^2*c*x^
6 + 608*a*c^2*x^6 + 410*b*c^2*x^9 + 160*c^3*x^12) - 270*a*(b^2 - 16*a*c)*S
qrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + S
qrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[2/3, 1/2, 1/
2, 5/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a
*c])] - 27*b*(7*b^2 - 52*a*c)*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(
b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b
^2 - 4*a*c])]*AppellF1[5/3, 1/2, 1/2, 8/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*
c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]))/(17600*c*Sqrt[a + b*x^3 + c*x^6
])
```

3.215.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+bx^3+cx^6\right)^{3/2}dx$$

$$\Big\downarrow \text{ 1721}$$

$$\frac{a\sqrt{a+bx^3+cx^6}\int x\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{3/2}\left(\frac{2cx^3}{b+\sqrt{b^2-4ac}}+1\right)^{3/2}dx}{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

$$\Big\downarrow \text{ 1012}$$

$$\frac{ax^2\sqrt{a+bx^3+cx^6}\operatorname{AppellF1}\left(\frac{2}{3},-\frac{3}{2},-\frac{3}{2},\frac{5}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[x*(a + b*x^3 + c*x^6)^(3/2),x]
```

output
```
(a*x^2*Sqrt[a + b*x^3 + c*x^6]*AppellF1[2/3, -3/2, -3/2, 5/3, (-2*c*x^3)/(
b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(2*Sqrt[1 + (
2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c
])])
```

3.215.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

3.215.4 Maple [F]

$$\int x\left(c\,x^{6}+b\,x^{3}+a\right)^{\frac{3}{2}}dx$$

input
```
int(x*(c*x^6+b*x^3+a)^(3/2),x)
```

output
```
int(x*(c*x^6+b*x^3+a)^(3/2),x)
```

3.215.5 Fricas [F]

$$\int x\left(a+bx^3+cx^6\right)^{3/2}\,dx=\int \left(cx^6+bx^3+a\right)^{\frac{3}{2}}x\,dx$$

input
```
integrate(x*(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
integral((c*x^7 + b*x^4 + a*x)*sqrt(c*x^6 + b*x^3 + a), x)
```

3.215.6 Sympy [F]

$$\int x\left(a+bx^3+cx^6\right)^{3/2}\,dx=\int x\left(a+bx^3+cx^6\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x*(c*x**6+b*x**3+a)**(3/2),x)
```

output
```
Integral(x*(a + b*x**3 + c*x**6)**(3/2), x)
```

3.215.7 Maxima [F]

$$\int x\left(a+bx^3+cx^6\right)^{3/2}\,dx=\int \left(cx^6+bx^3+a\right)^{\frac{3}{2}}x\,dx$$

input
```
integrate(x*(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)^(3/2)*x, x)
```

3.215.8 Giac [F]

$$\int x(a+bx^3+cx^6)^{3/2}\,dx=\int (cx^6+bx^3+a)^{\frac{3}{2}}x\,dx$$

input
```
integrate(x*(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^(3/2)*x, x)
```

3.215.9 Mupad [F(-1)]

Timed out.

$$\int x(a+bx^3+cx^6)^{3/2}\,dx=\int x\,(c\,x^6+b\,x^3+a)^{3/2}\,dx$$

input
```
int(x*(a + b*x^3 + c*x^6)^(3/2),x)
```

output
```
int(x*(a + b*x^3 + c*x^6)^(3/2), x)
```

3.216 $\int (a+bx^3+cx^6)^{3/2}\,dx$

3.216.1 Optimal result

Integrand size = 16, antiderivative size = 136

$$\int (a+bx^3+cx^6)^{3/2}\,dx=\frac{ax\sqrt{a+bx^3+cx^6}\,\text{AppellF1}\left(\frac{1}{3},-\frac{3}{2},-\frac{3}{2},\frac{4}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

output
```
a*x*AppellF1(1/3,-3/2,-3/2,4/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/(b
+(-4*a*c+b^2)^(1/2)))*(c*x^6+b*x^3+a)^(1/2)/(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/
2)))^(1/2)/(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.216.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 408 vs. 2(136) = 272.

Time = 10.48 (sec) , antiderivative size = 408, normalized size of antiderivative = 3.00

$$\int (a+bx^3+cx^6)^{3/2}\,dx=\frac{x\Big(8(27ab^2+364a^2c+27b^3x^3+548abcx^3+211b^2cx^6+476ac^2x^6+296bc^2x^9+112c^3x^{12})-}{}$$

input
```
Integrate[(a + b*x^3 + c*x^6)^(3/2),x]
```

output
```
(x*(8*(27*a*b^2 + 364*a^2*c + 27*b^3*x^3 + 548*a*b*c*x^3 + 211*b^2*c*x^6 +
 476*a*c^2*x^6 + 296*b*c^2*x^9 + 112*c^3*x^12) - 216*a*(b^2 - 28*a*c)*Sqrt
[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt
[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/3, 1/2, 1/2,
4/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c]
)] - 27*b*(5*b^2 - 44*a*c)*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b -
 Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2
- 4*a*c])]*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])
, (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]))/(8960*c*Sqrt[a + b*x^3 + c*x^6])
```

3.216.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1686, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3+cx^6\right)^{3/2} dx$$

$$\Big\downarrow \ 1686$$

$$\frac{a\sqrt{a+bx^3+cx^6}\int \left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{3/2}\left(\frac{2cx^3}{b+\sqrt{b^2-4ac}}+1\right)^{3/2}dx}{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

$$\Big\downarrow \ 936$$

$$\frac{ax\sqrt{a+bx^3+cx^6}\operatorname{AppellF1}\left(\frac{1}{3},-\frac{3}{2},-\frac{3}{2},\frac{4}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[(a + b*x^3 + c*x^6)^(3/2),x]
```

output
```
(a*x*Sqrt[a + b*x^3 + c*x^6]*AppellF1[1/3, -3/2, -3/2, 4/3, (-2*c*x^3)/(b
- Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt[1 + (2*c*
x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])
```

3.216.3.1 Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1686
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^
IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*c*(x^n/(b + Rt[b^2
- 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4*a*c, 2])))^FracPar
t[p]))   Int[(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2*c*(x^n/(b - S
qrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n]
&& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

3.216.4 Maple [F]

$$\int \left(c\,x^{6}+b\,x^{3}+a\right)^{\frac{3}{2}}d x$$

input
```
int((c*x^6+b*x^3+a)^(3/2),x)
```

output
```
int((c*x^6+b*x^3+a)^(3/2),x)
```

3.216.5 Fricas [F]

$$\int \left(a+bx^3+cx^6\right)^{3/2}\,dx=\int \left(cx^6+bx^3+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
integral((c*x^6 + b*x^3 + a)^(3/2), x)
```

3.216.6 Sympy [F]

$$\int \left(a+bx^3+cx^6\right)^{3/2}\,dx=\int \left(a+bx^3+cx^6\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((c*x**6+b*x**3+a)**(3/2),x)
```

output
```
Integral((a + b*x**3 + c*x**6)**(3/2), x)
```

3.216.7 Maxima [F]

$$\int \left(a+bx^3+cx^6\right)^{3/2}\,dx=\int \left(cx^6+bx^3+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)^(3/2), x)
```

3.216.8 Giac [F]

$$\int \left(a+bx^3+cx^6\right)^{3/2}\,dx=\int \left(cx^6+bx^3+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^(3/2), x)
```

3.216.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^3+cx^6\right)^{3/2}\,dx=\int \left(c\,x^6+b\,x^3+a\right)^{3/2}\,dx$$

input
```
int((a + b*x^3 + c*x^6)^(3/2),x)
```

output
```
int((a + b*x^3 + c*x^6)^(3/2), x)
```

3.217 $\int \frac{(a+bx^3+cx^6)^{3/2}}{x^2} dx$

3.217.1 Optimal result

Integrand size = 20, antiderivative size = 139

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^2} dx = -\frac{a\sqrt{a+bx^3+cx^6}\,\text{AppellF1}\left(-\frac{1}{3},-\frac{3}{2},-\frac{3}{2},\frac{2}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

output
```
-a*AppellF1(-1/3,-3/2,-3/2,2/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/(b
+(-4*a*c+b^2)^(1/2)))*(c*x^6+b*x^3+a)^(1/2)/x/(1+2*c*x^3/(b-(-4*a*c+b^2)^(
1/2)))^(1/2)/(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.217.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 379 vs. 2(139) = 278.

Time = 10.35 (sec) , antiderivative size = 379, normalized size of antiderivative = 2.73

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^2} dx = \frac{10(-80a^2-61abx^3+19b^2x^6-70acx^6+29bcx^9+10c^2x^{12})+810abx^3\sqrt{\frac{b-\sqrt{b^2-4ac}}{b-\sqrt{b^2}}}}{}$$

input
```
Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^2,x]
```

output
```
(10*(-80*a^2 - 61*a*b*x^3 + 19*b^2*x^6 - 70*a*c*x^6 + 29*b*c*x^9 + 10*c^2*
x^12) + 810*a*b*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 -
 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*
AppellF1[2/3, 1/2, 1/2, 5/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)
/(-b + Sqrt[b^2 - 4*a*c])] + 27*(b^2 + 20*a*c)*x^6*Sqrt[(b - Sqrt[b^2 - 4*
a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c
*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/3, 1/2, 1/2, 8/3, (-2*c*x^3)/(b
+ Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(800*x*Sqrt[a +
 b*x^3 + c*x^6])
```

3.217.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^2}\,dx$$

$$\Big\downarrow \; 1721$$

$$\frac{a\sqrt{a+bx^3+cx^6}\int \frac{\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{3/2}\left(\frac{2cx^3}{b+\sqrt{b^2-4ac}}+1\right)^{3/2}}{x^2}dx}{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

$$\Big\downarrow \; 1012$$

$$-\frac{a\sqrt{a+bx^3+cx^6}\,\text{AppellF1}\left(-\frac{1}{3},-\frac{3}{2},-\frac{3}{2},\frac{2}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[(a + b*x^3 + c*x^6)^(3/2)/x^2,x]
```

output
```
-((a*Sqrt[a + b*x^3 + c*x^6]*AppellF1[-1/3, -3/2, -3/2, 2/3, (-2*c*x^3)/(b
 - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(x*Sqrt[1 + (2
*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]
)]))
```

3.217.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

3.217.4 Maple [F]

$$\int \frac{\left(c\,x^{6}+b\,x^{3}+a\right)^{\frac{3}{2}}}{x^{2}}d x$$

input
```
int((c*x^6+b*x^3+a)^(3/2)/x^2,x)
```

output
```
int((c*x^6+b*x^3+a)^(3/2)/x^2,x)
```

3.217.5 Fricas [F]

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^2}\,dx = \int \frac{(cx^6+bx^3+a)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^(3/2)/x^2,x, algorithm="fricas")
```

output
```
integral((c*x^6 + b*x^3 + a)^(3/2)/x^2, x)
```

3.217.6 Sympy [F]

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^2}\,dx = \int \frac{(a+bx^3+cx^6)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((c*x**6+b*x**3+a)**(3/2)/x**2,x)
```

output
```
Integral((a + b*x**3 + c*x**6)**(3/2)/x**2, x)
```

3.217.7 Maxima [F]

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^2}\,dx = \int \frac{(cx^6+bx^3+a)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^(3/2)/x^2,x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)^(3/2)/x^2, x)
```

3.217.8 Giac [F]

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^2}\,dx = \int \frac{(cx^6+bx^3+a)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^(3/2)/x^2,x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^(3/2)/x^2, x)
```

3.217.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^2}\,dx = \int \frac{(c\,x^6+b\,x^3+a)^{3/2}}{x^2}\,dx$$

input
```
int((a + b*x^3 + c*x^6)^(3/2)/x^2,x)
```

output
```
int((a + b*x^3 + c*x^6)^(3/2)/x^2, x)
```

3.218 $\int \frac{(a+bx^3+cx^6)^{3/2}}{x^3} \, dx$

3.218.1 Optimal result

Integrand size = 20, antiderivative size = 141

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^3} \, dx = -\frac{a\sqrt{a+bx^3+cx^6}\,\text{AppellF1}\left(-\frac{2}{3},-\frac{3}{2},-\frac{3}{2},\frac{1}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

output
```
-1/2*a*AppellF1(-2/3,-3/2,-3/2,1/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^
3/(b+(-4*a*c+b^2)^(1/2)))*(c*x^6+b*x^3+a)^(1/2)/x^2/(1+2*c*x^3/(b-(-4*a*c+
b^2)^(1/2)))^(1/2)/(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.218.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 379 vs. 2(141) = 282.

Time = 10.35 (sec) , antiderivative size = 379, normalized size of antiderivative = 2.69

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^3} \, dx = \frac{8(-28a^2-11abx^3+17b^2x^6-20acx^6+25bcx^9+8c^2x^{12})+648abx^3\sqrt{\frac{b-\sqrt{b^2-4ac}}{b-\sqrt{b^2-}}}}{}$$

input
```
Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^3,x]
```

output
```
(8*(-28*a^2 - 11*a*b*x^3 + 17*b^2*x^6 - 20*a*c*x^6 + 25*b*c*x^9 + 8*c^2*x^
12) + 648*a*b*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4
*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*Ap
pellF1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(
-b + Sqrt[b^2 - 4*a*c])] + 27*(b^2 + 8*a*c)*x^6*Sqrt[(b - Sqrt[b^2 - 4*a*c
] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^
3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b + S
qrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(448*x^2*Sqrt[a +
b*x^3 + c*x^6])
```

3.218.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^3}dx$$

$$\Big\downarrow \text{ 1721}$$

$$\frac{a\sqrt{a+bx^3+cx^6}\int \frac{\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{3/2}\left(\frac{2cx^3}{b+\sqrt{b^2-4ac}}+1\right)^{3/2}}{x^3}dx}{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

$$\Big\downarrow \text{ 1012}$$

$$-\frac{a\sqrt{a+bx^3+cx^6}\operatorname{AppellF1}\left(-\frac{2}{3},-\frac{3}{2},-\frac{3}{2},\frac{1}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[(a + b*x^3 + c*x^6)^(3/2)/x^3,x]
```

output
```
-1/2*(a*Sqrt[a + b*x^3 + c*x^6]*AppellF1[-2/3, -3/2, -3/2, 1/3, (-2*c*x^3)
/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(x^2*Sqrt[1
 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4
*a*c])])
```

3.218. $\int \frac{(a+bx^3+cx^6)^{3/2}}{x^3}dx$

3.218.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

3.218.4 Maple [F]

$$\int \frac{(c\,x^{6}+b\,x^{3}+a)^{\frac{3}{2}}}{x^{3}}dx$$

input
```
int((c*x^6+b*x^3+a)^(3/2)/x^3,x)
```

output
```
int((c*x^6+b*x^3+a)^(3/2)/x^3,x)
```

3.218.5 Fricas [F]

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^3}\,dx = \int \frac{(cx^6+bx^3+a)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^(3/2)/x^3,x, algorithm="fricas")
```

output
```
integral((c*x^6 + b*x^3 + a)^(3/2)/x^3, x)
```

3.218.6 Sympy [F]

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^3}\,dx = \int \frac{(a+bx^3+cx^6)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((c*x**6+b*x**3+a)**(3/2)/x**3,x)
```

output
```
Integral((a + b*x**3 + c*x**6)**(3/2)/x**3, x)
```

3.218.7 Maxima [F]

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^3}\,dx = \int \frac{(cx^6+bx^3+a)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^(3/2)/x^3,x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)^(3/2)/x^3, x)
```

3.218.8 Giac [F]

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^3}\,dx = \int \frac{(cx^6+bx^3+a)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^(3/2)/x^3,x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^(3/2)/x^3, x)
```

3.218.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^3}\,dx = \int \frac{(c\,x^6+b\,x^3+a)^{3/2}}{x^3}\,dx$$

input
```
int((a + b*x^3 + c*x^6)^(3/2)/x^3,x)
```

output
```
int((a + b*x^3 + c*x^6)^(3/2)/x^3, x)
```

3.219 $\int \frac{x^{14}}{\sqrt{a+bx^3+cx^6}}\,dx$

3.219.1 Optimal result

Integrand size = 20, antiderivative size = 171

$$\int \frac{x^{14}}{\sqrt{a+bx^3+cx^6}}\,dx = -\frac{7bx^6\sqrt{a+bx^3+cx^6}}{72c^2}+\frac{x^9\sqrt{a+bx^3+cx^6}}{12c} - \frac{(5b(21b^2-44ac)-2c(35b^2-36ac)\,x^3)\sqrt{a+bx^3+cx^6}}{576c^4} + \frac{(35b^4-120ab^2c+48a^2c^2)\,\mathrm{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{384c^{9/2}}$$

output
```
1/384*(48*a^2*c^2-120*a*b^2*c+35*b^4)*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x
^6+b*x^3+a)^(1/2))/c^(9/2)-7/72*b*x^6*(c*x^6+b*x^3+a)^(1/2)/c^2+1/12*x^9*(
c*x^6+b*x^3+a)^(1/2)/c-1/576*(5*b*(-44*a*c+21*b^2)-2*c*(-36*a*c+35*b^2)*x^
3)*(c*x^6+b*x^3+a)^(1/2)/c^4
```

3.219.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.81

$$\int \frac{x^{14}}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{\sqrt{a+bx^3+cx^6}(-105b^3+220abc+70b^2cx^3-72ac^2x^3-56bc^2x^6+48c^3x^9)}{576c^4} + \frac{(-35b^4+120ab^2c-48a^2c^2)\log\left(bc^4+2c^5x^3-2c^{9/2}\sqrt{a+bx^3+cx^6}\right)}{384c^{9/2}}$$

input
```
Integrate[x^14/Sqrt[a + b*x^3 + c*x^6],x]
```

output
```
(Sqrt[a + b*x^3 + c*x^6]*(-105*b^3 + 220*a*b*c + 70*b^2*c*x^3 - 72*a*c^2*x
^3 - 56*b*c^2*x^6 + 48*c^3*x^9))/(576*c^4) + ((-35*b^4 + 120*a*b^2*c - 48*
a^2*c^2)*Log[b*c^4 + 2*c^5*x^3 - 2*c^(9/2)*Sqrt[a + b*x^3 + c*x^6]])/(384*
c^(9/2))
```

3.219.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1693, 1166, 27, 1236, 27, 1225, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{14}}{\sqrt{a+bx^3+cx^6}}dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int \frac{x^{12}}{\sqrt{cx^6+bx^3+a}}dx^3$$

$$\Big\downarrow \text{1166}$$

$$\frac{1}{3}\left(\frac{\int -\frac{x^6(7bx^3+6a)}{2\sqrt{cx^6+bx^3+a}}dx^3}{4c}+\frac{x^9\sqrt{a+bx^3+cx^6}}{4c}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{x^9\sqrt{a+bx^3+cx^6}}{4c}-\frac{\int \frac{x^6(7bx^3+6a)}{\sqrt{cx^6+bx^3+a}}dx^3}{8c}\right)$$

$$\Big\downarrow \text{1236}$$

$$\frac{1}{3}\left(\frac{x^9\sqrt{a+bx^3+cx^6}}{4c}-\frac{\frac{\int -\frac{x^3\left(\left(35b^2-36ac\right)x^3+28ab\right)}{2\sqrt{cx^6+bx^3+a}}dx^3}{3c}+\frac{7bx^6\sqrt{a+bx^3+cx^6}}{3c}}{8c}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{x^9\sqrt{a+bx^3+cx^6}}{4c}-\frac{\frac{7bx^6\sqrt{a+bx^3+cx^6}}{3c}-\frac{\int \frac{x^3\left(\left(35b^2-36ac\right)x^3+28ab\right)}{\sqrt{cx^6+bx^3+a}}dx^3}{6c}}{8c}\right)$$

$$\Big\downarrow \text{ 1225}$$

$$\frac{1}{3}\left(\frac{x^9\sqrt{a+bx^3+cx^6}}{4c}-\frac{\frac{7bx^6\sqrt{a+bx^3+cx^6}}{3c}-\frac{\frac{3\left(48a^2c^2-120ab^2c+35b^4\right)\int \frac{1}{\sqrt{cx^6+bx^3+a}}dx^3}{8c^2}-\frac{\left(5b\left(21b^2-44ac\right)-2cx^3\left(35b^2-36ac\right)\right)\sqrt{a+bx^3+c}}{4c^2}}{6c}}{8c}\right.$$

$$\Big\downarrow \text{ 1092}$$

$$\frac{1}{3}\left(\frac{x^9\sqrt{a+bx^3+cx^6}}{4c}-\frac{\frac{7bx^6\sqrt{a+bx^3+cx^6}}{3c}-\frac{\frac{3\left(48a^2c^2-120ab^2c+35b^4\right)\int \frac{1}{4c-x^6}d\frac{2cx^3+b}{\sqrt{cx^6+bx^3+a}}}{4c^2}-\frac{\left(5b\left(21b^2-44ac\right)-2cx^3\left(35b^2-36ac\right)\right)\sqrt{a+bx}}{4c^2}}{6c}}{8c}\right.$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{3}\left(\frac{x^9\sqrt{a+bx^3+cx^6}}{4c}-\frac{\frac{7bx^6\sqrt{a+bx^3+cx^6}}{3c}-\frac{\frac{3\left(48a^2c^2-120ab^2c+35b^4\right)\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{8c^{5/2}}-\frac{\left(5b\left(21b^2-44ac\right)-2cx^3\left(35b^2-36ac\right)\right.}{4c^2}}{6c}}{8c}\right.$$

input
```
Int[x^14/Sqrt[a + b*x^3 + c*x^6],x]
```

output
```
((x^9*Sqrt[a + b*x^3 + c*x^6])/(4*c) - ((7*b*x^6*Sqrt[a + b*x^3 + c*x^6])/
(3*c) - (-1/4*((5*b*(21*b^2 - 44*a*c) - 2*c*(35*b^2 - 36*a*c)*x^3)*Sqrt[a
+ b*x^3 + c*x^6])/c^2 + (3*(35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*ArcTanh[(b
+ 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(8*c^(5/2)))/(6*c))/(8*c)
)/3
```

3.219. $\int \frac{x^{14}}{\sqrt{a+bx^3+cx^6}}\,dx$

3.219.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

3.219. $\int \frac{x^{14}}{\sqrt{a+bx^3+cx^6}}\,dx$

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.219.4 Maple [F]

$$\int \frac{x^{14}}{\sqrt{c\,x^{6}+b\,x^{3}+a}}dx$$

input
```
int(x^14/(c*x^6+b*x^3+a)^(1/2),x)
```

output
```
int(x^14/(c*x^6+b*x^3+a)^(1/2),x)
```

3.219.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.77

$$\int \frac{x^{14}}{\sqrt{a+bx^3+cx^6}}\,dx = \left[\frac{3\,(35\,b^4-120\,ab^2c+48\,a^2c^2)\sqrt{c}\log\left(-8\,c^2x^6-8\,bcx^3-b^2-4\,\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{c}-4\,ac\right)+\cdots}{2304\,c^5}\right.$$
$$-\frac{3\,(35\,b^4-120\,ab^2c+48\,a^2c^2)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\,(2\,cx^3+b)\sqrt{-c}}{2\,(c^2x^6+bcx^3+ac)}\right)-2\,(48\,c^4x^9-56\,bc^3x^6-105\,b^3c+\cdots}{1152\,c^5}$$

input
```
integrate(x^14/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/2304*(3*(35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*sqrt(c)*log(-8*c^2*x^6 - 8*
b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) +
 4*(48*c^4*x^9 - 56*b*c^3*x^6 - 105*b^3*c + 220*a*b*c^2 + 2*(35*b^2*c^2 -
36*a*c^3)*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^5, -1/1152*(3*(35*b^4 - 120*a*b^
2*c + 48*a^2*c^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b
)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) - 2*(48*c^4*x^9 - 56*b*c^3*x^6 - 105
*b^3*c + 220*a*b*c^2 + 2*(35*b^2*c^2 - 36*a*c^3)*x^3)*sqrt(c*x^6 + b*x^3 +
 a))/c^5]
```

3.219.6 Sympy [F]

$$\int \frac{x^{14}}{\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{x^{14}}{\sqrt{a+bx^3+cx^6}}\,dx$$

input
```
integrate(x**14/(c*x**6+b*x**3+a)**(1/2),x)
```

output
```
Integral(x**14/sqrt(a + b*x**3 + c*x**6), x)
```

3.219.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{14}}{\sqrt{a+bx^3+cx^6}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^14/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.219.8 Giac [F]

$$\int \frac{x^{14}}{\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{x^{14}}{\sqrt{cx^6+bx^3+a}}\,dx$$

input
```
integrate(x^14/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^14/sqrt(c*x^6 + b*x^3 + a), x)
```

3.219.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{14}}{\sqrt{a+bx^3+cx^6}}\,dx=\int \frac{x^{14}}{\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

input
```
int(x^14/(a + b*x^3 + c*x^6)^(1/2),x)
```

output
```
int(x^14/(a + b*x^3 + c*x^6)^(1/2), x)
```

3.220 $\int \frac{x^{11}}{\sqrt{a+bx^3+cx^6}}\,dx$

3.220.1 Optimal result

Integrand size = 20, antiderivative size = 121

$$\int \frac{x^{11}}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{x^6\sqrt{a+bx^3+cx^6}}{9c} + \frac{(15b^2-16ac-10bcx^3)\sqrt{a+bx^3+cx^6}}{72c^3} - \frac{b(5b^2-12ac)\operatorname{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{48c^{7/2}}$$

output
```
-1/48*b*(-12*a*c+5*b^2)*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1
/2))/c^(7/2)+1/9*x^6*(c*x^6+b*x^3+a)^(1/2)/c+1/72*(-10*b*c*x^3-16*a*c+15*b
^2)*(c*x^6+b*x^3+a)^(1/2)/c^3
```

3.220.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.83

$$\int \frac{x^{11}}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{\sqrt{a+bx^3+cx^6}(15b^2-16ac-10bcx^3+8c^2x^6)}{72c^3} + \frac{(5b^3-12abc)\log\left(b+2cx^3-2\sqrt{c}\sqrt{a+bx^3+cx^6}\right)}{48c^{7/2}}$$

input
```
Integrate[x^11/Sqrt[a + b*x^3 + c*x^6],x]
```

output
```
(Sqrt[a + b*x^3 + c*x^6]*(15*b^2 - 16*a*c - 10*b*c*x^3 + 8*c^2*x^6))/(72*c
^3) + ((5*b^3 - 12*a*b*c)*Log[b + 2*c*x^3 - 2*Sqrt[c]*Sqrt[a + b*x^3 + c*x
^6]])/(48*c^(7/2))
```

3.220.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1693, 1166, 27, 1225, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{\sqrt{a+bx^3+cx^6}}dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int \frac{x^9}{\sqrt{cx^6+bx^3+a}}dx^3$$

$$\Big\downarrow \text{1166}$$

$$\frac{1}{3}\left(\frac{\int -\frac{x^3(5bx^3+4a)}{2\sqrt{cx^6+bx^3+a}}dx^3}{3c}+\frac{x^6\sqrt{a+bx^3+cx^6}}{3c}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{x^6\sqrt{a+bx^3+cx^6}}{3c}-\frac{\int \frac{x^3(5bx^3+4a)}{\sqrt{cx^6+bx^3+a}}dx^3}{6c}\right)$$

$$\Big\downarrow \text{1225}$$

$$\frac{1}{3}\left(\frac{x^6\sqrt{a+bx^3+cx^6}}{3c}-\frac{\frac{3b(5b^2-12ac)\int \frac{1}{\sqrt{cx^6+bx^3+a}}dx^3}{8c^2}-\frac{(-16ac+15b^2-10bcx^3)\sqrt{a+bx^3+cx^6}}{4c^2}}{6c}\right)$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{3}\left(\frac{x^6\sqrt{a+bx^3+cx^6}}{3c}-\frac{\frac{3b(5b^2-12ac)\int \frac{1}{4c-x^6}d\frac{2cx^3+b}{\sqrt{cx^6+bx^3+a}}}{4c^2}-\frac{(-16ac+15b^2-10bcx^3)\sqrt{a+bx^3+cx^6}}{4c^2}}{6c}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{x^6\sqrt{a+bx^3+cx^6}}{3c}-\frac{\frac{3b(5b^2-12ac)\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{8c^{5/2}}-\frac{(-16ac+15b^2-10bcx^3)\sqrt{a+bx^3+cx^6}}{4c^2}}{6c}\right)$$

input
```
Int[x^11/Sqrt[a + b*x^3 + c*x^6],x]
```

output
```
((x^6*Sqrt[a + b*x^3 + c*x^6])/(3*c) - (-1/4*((15*b^2 - 16*a*c - 10*b*c*x^
3)*Sqrt[a + b*x^3 + c*x^6])/c^2 + (3*b*(5*b^2 - 12*a*c)*ArcTanh[(b + 2*c*x
^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(8*c^(5/2)))/(6*c))/3
```

3.220.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.220.4 Maple [F]

$$\int \frac{x^{11}}{\sqrt{c\,x^{6}+b\,x^{3}+a}}dx$$

input
```
int(x^11/(c*x^6+b*x^3+a)^(1/2),x)
```

output
```
int(x^11/(c*x^6+b*x^3+a)^(1/2),x)
```

3.220.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.99

$$\int \frac{x^{11}}{\sqrt{a+bx^3+cx^6}}\,dx = \left[-\frac{3\,(5\,b^3-12\,abc)\sqrt{c}\log\left(-8\,c^2x^6-8\,bcx^3-b^2-4\,\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{c}-4\,ac\right)-4\,(8\,c^3x^6-}{288\,c^4}\right.$$

input
```
integrate(x^11/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/288*(3*(5*b^3 - 12*a*b*c)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4
*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) - 4*(8*c^3*x^6 - 1
0*b*c^2*x^3 + 15*b^2*c - 16*a*c^2)*sqrt(c*x^6 + b*x^3 + a))/c^4, 1/144*(3*
(5*b^3 - 12*a*b*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 +
b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*(8*c^3*x^6 - 10*b*c^2*x^3 + 15*
b^2*c - 16*a*c^2)*sqrt(c*x^6 + b*x^3 + a))/c^4]
```

3.220.6 Sympy [F]

$$\int \frac{x^{11}}{\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{x^{11}}{\sqrt{a+bx^3+cx^6}}\,dx$$

input
```
integrate(x**11/(c*x**6+b*x**3+a)**(1/2),x)
```

output
```
Integral(x**11/sqrt(a + b*x**3 + c*x**6), x)
```

3.220.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{11}}{\sqrt{a+bx^3+cx^6}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^11/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.220.8 Giac [F]

$$\int \frac{x^{11}}{\sqrt{a+bx^3+cx^6}}\,dx=\int \frac{x^{11}}{\sqrt{cx^6+bx^3+a}}\,dx$$

input
```
integrate(x^11/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^11/sqrt(c*x^6 + b*x^3 + a), x)
```

3.220.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{11}}{\sqrt{a+bx^3+cx^6}}\,dx=\int \frac{x^{11}}{\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

input
```
int(x^11/(a + b*x^3 + c*x^6)^(1/2),x)
```

output
```
int(x^11/(a + b*x^3 + c*x^6)^(1/2), x)
```

3.221 $\int \frac{x^8}{\sqrt{a+bx^3+cx^6}}\,dx$

3.221.1 Optimal result

Integrand size = 20, antiderivative size = 104

$$\int \frac{x^8}{\sqrt{a+bx^3+cx^6}}\,dx = -\frac{b\sqrt{a+bx^3+cx^6}}{4c^2}+\frac{x^3\sqrt{a+bx^3+cx^6}}{6c}+\frac{(3b^2-4ac)\,\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{24c^{5/2}}$$

output
```
1/24*(-4*a*c+3*b^2)*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))
/c^(5/2)-1/4*b*(c*x^6+b*x^3+a)^(1/2)/c^2+1/6*x^3*(c*x^6+b*x^3+a)^(1/2)/c
```

3.221.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.88

$$\int \frac{x^8}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{(-3b+2cx^3)\sqrt{a+bx^3+cx^6}}{12c^2}+\frac{(-3b^2+4ac)\log\left(bc^2+2c^3x^3-2c^{5/2}\sqrt{a+bx^3+cx^6}\right)}{24c^{5/2}}$$

input
```
Integrate[x^8/Sqrt[a + b*x^3 + c*x^6],x]
```

output
```
((-3*b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(12*c^2) + ((-3*b^2 + 4*a*c)*Lo
g[b*c^2 + 2*c^3*x^3 - 2*c^(5/2)*Sqrt[a + b*x^3 + c*x^6]])/(24*c^(5/2))
```

3.221.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1693, 1166, 27, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{\sqrt{a+bx^3+cx^6}}dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int \frac{x^6}{\sqrt{cx^6+bx^3+a}}dx^3$$

$$\Big\downarrow \text{1166}$$

$$\frac{1}{3}\left(\frac{\int -\frac{3bx^3+2a}{2\sqrt{cx^6+bx^3+a}}dx^3}{2c}+\frac{x^3\sqrt{a+bx^3+cx^6}}{2c}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{x^3\sqrt{a+bx^3+cx^6}}{2c}-\frac{\int \frac{3bx^3+2a}{\sqrt{cx^6+bx^3+a}}dx^3}{4c}\right)$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{3}\left(\frac{x^3\sqrt{a+bx^3+cx^6}}{2c}-\frac{\frac{3b\sqrt{a+bx^3+cx^6}}{c}-\frac{(3b^2-4ac)\int \frac{1}{\sqrt{cx^6+bx^3+a}}dx^3}{2c}}{4c}\right)$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{3}\left(\frac{x^3\sqrt{a+bx^3+cx^6}}{2c}-\frac{\frac{3b\sqrt{a+bx^3+cx^6}}{c}-\frac{(3b^2-4ac)\int \frac{1}{4c-x^6}d\frac{2cx^3+b}{\sqrt{cx^6+bx^3+a}}}{c}}{4c}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{x^3\sqrt{a+bx^3+cx^6}}{2c}-\frac{\frac{3b\sqrt{a+bx^3+cx^6}}{c}-\frac{(3b^2-4ac)\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{2c^{3/2}}}{4c}\right)$$

input
```
Int[x^8/Sqrt[a + b*x^3 + c*x^6],x]
```

3.221. $\int \frac{x^8}{\sqrt{a+bx^3+cx^6}}dx$

output
```
((x^3*Sqrt[a + b*x^3 + c*x^6])/(2*c) - ((3*b*Sqrt[a + b*x^3 + c*x^6])/c -
((3*b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])
])/(2*c^(3/2)))/(4*c))/3
```

3.221.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.221. $\int \frac{x^8}{\sqrt{a+bx^3+cx^6}}\,dx$

3.221.4 Maple [F]

$$\int \frac{x^8}{\sqrt{c\,x^6+b\,x^3+a}}dx$$

input
```
int(x^8/(c*x^6+b*x^3+a)^(1/2),x)
```

output
```
int(x^8/(c*x^6+b*x^3+a)^(1/2),x)
```

3.221.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.95

$$\int \frac{x^8}{\sqrt{a+bx^3+cx^6}}\,dx = \left[-\frac{(3\,b^2-4\,ac)\sqrt{c}\log\left(-8\,c^2x^6-8\,bcx^3-b^2+4\,\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{c}-4\,ac\right)-4\,\sqrt{cx^6+bx^3+}}{48\,c^3}\right.$$
$$\left.-\frac{(3\,b^2-4\,ac)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{-c}}{2\,(c^2x^6+bcx^3+ac)}\right)-2\,\sqrt{cx^6+bx^3+a}(2\,c^2x^3-3\,bc)}{24\,c^3}\right]$$

input
```
integrate(x^8/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/48*((3*b^2 - 4*a*c)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 + 4*sqrt(
c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) - 4*sqrt(c*x^6 + b*x^3 +
 a)*(2*c^2*x^3 - 3*b*c))/c^3, -1/24*((3*b^2 - 4*a*c)*sqrt(-c)*arctan(1/2*s
qrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) -
 2*sqrt(c*x^6 + b*x^3 + a)*(2*c^2*x^3 - 3*b*c))/c^3]
```

3.221.6 Sympy [F]

$$\int \frac{x^8}{\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{x^8}{\sqrt{a+bx^3+cx^6}}\,dx$$

input
```
integrate(x**8/(c*x**6+b*x**3+a)**(1/2),x)
```

output
```
Integral(x**8/sqrt(a + b*x**3 + c*x**6), x)
```

3.221.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^8}{\sqrt{a+bx^3+cx^6}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^8/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.221.8 Giac [F]

$$\int \frac{x^8}{\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{x^8}{\sqrt{cx^6+bx^3+a}}\,dx$$

input
```
integrate(x^8/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^8/sqrt(c*x^6 + b*x^3 + a), x)
```

3.221.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^8}{\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{x^8}{\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

input
```
int(x^8/(a + b*x^3 + c*x^6)^(1/2),x)
```

output
```
int(x^8/(a + b*x^3 + c*x^6)^(1/2), x)
```

3.222 $\int \frac{x^5}{\sqrt{a+bx^3+cx^6}}\,dx$

3.222.1 Optimal result

Integrand size = 20, antiderivative size = 68

$$\int \frac{x^5}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{\sqrt{a+bx^3+cx^6}}{3c} - \frac{b\,\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{6c^{3/2}}$$

output
```
-1/6*b*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))/c^(3/2)+1/3*
(c*x^6+b*x^3+a)^(1/2)/c
```

3.222.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{\sqrt{a+bx^3+cx^6}}{3c} - \frac{b\,\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{6c^{3/2}}$$

input
```
Integrate[x^5/Sqrt[a + b*x^3 + c*x^6],x]
```

output
```
Sqrt[a + b*x^3 + c*x^6]/(3*c) - (b*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a
 + b*x^3 + c*x^6])])/(6*c^(3/2))
```

3.222.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1693, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\sqrt{a+bx^3+cx^6}}\,dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int \frac{x^3}{\sqrt{cx^6+bx^3+a}}dx^3$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{3}\left(\frac{\sqrt{a+bx^3+cx^6}}{c}-\frac{b\int \frac{1}{\sqrt{cx^6+bx^3+a}}dx^3}{2c}\right)$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{3}\left(\frac{\sqrt{a+bx^3+cx^6}}{c}-\frac{b\int \frac{1}{4c-x^6}d\frac{2cx^3+b}{\sqrt{cx^6+bx^3+a}}}{c}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{\sqrt{a+bx^3+cx^6}}{c}-\frac{b\,\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{2c^{3/2}}\right)$$

input
```
Int[x^5/Sqrt[a + b*x^3 + c*x^6],x]
```

output
```
(Sqrt[a + b*x^3 + c*x^6]/c - (b*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a +
b*x^3 + c*x^6])])/(2*c^(3/2)))/3
```

3.222.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.222.4 Maple [F]

$$\int \frac{x^5}{\sqrt{c\,x^6+b\,x^3+a}}dx$$

input
```
int(x^5/(c*x^6+b*x^3+a)^(1/2),x)
```

output
```
int(x^5/(c*x^6+b*x^3+a)^(1/2),x)
```

3.222.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.37

$$\int \frac{x^5}{\sqrt{a+bx^3+cx^6}}\,dx$$
$$= \left[\frac{b\sqrt{c}\log\left(-8\,c^2x^6 - 8\,bcx^3 - b^2 + 4\,\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{c} - 4\,ac\right) + 4\,\sqrt{cx^6+bx^3+a}c}{12\,c^2}, \frac{b\sqrt{-c}\,\mathrm{ar}}{}\right.$$

input
```
integrate(x^5/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/12*(b*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 + 4*sqrt(c*x^6 + b*x^3 +
 a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 4*sqrt(c*x^6 + b*x^3 + a)*c)/c^2, 1/6
*(b*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^
2*x^6 + b*c*x^3 + a*c)) + 2*sqrt(c*x^6 + b*x^3 + a)*c)/c^2]
```

3.222.6 Sympy [F]

$$\int \frac{x^5}{\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{x^5}{\sqrt{a+bx^3+cx^6}}\,dx$$

input
```
integrate(x**5/(c*x**6+b*x**3+a)**(1/2),x)
```

output
```
Integral(x**5/sqrt(a + b*x**3 + c*x**6), x)
```

3.222.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{\sqrt{a+bx^3+cx^6}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.222.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.87

$$\int \frac{x^5}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{b\log\left(\left|2\left(\sqrt{c}x^3-\sqrt{cx^6+bx^3+a}\right)\sqrt{c}+b\right|\right)}{6\,c^{\frac{3}{2}}}+\frac{\sqrt{cx^6+bx^3+a}}{3\,c}$$

input
```
integrate(x^5/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
1/6*b*log(abs(2*(sqrt(c)*x^3 - sqrt(c*x^6 + b*x^3 + a))*sqrt(c) + b))/c^(3
/2) + 1/3*sqrt(c*x^6 + b*x^3 + a)/c
```

3.222.9 Mupad [B] (verification not implemented)

Time = 8.53 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.81

$$\int \frac{x^5}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{\sqrt{c\,x^6+b\,x^3+a}}{3\,c}-\frac{b\,\ln\left(\sqrt{c\,x^6+b\,x^3+a}+\frac{c\,x^3+\frac{b}{2}}{\sqrt{c}}\right)}{6\,c^{3/2}}$$

input
```
int(x^5/(a + b*x^3 + c*x^6)^(1/2),x)
```

output
```
(a + b*x^3 + c*x^6)^(1/2)/(3*c) - (b*log((a + b*x^3 + c*x^6)^(1/2) + (b/2
+ c*x^3)/c^(1/2)))/(6*c^(3/2))
```

3.223 $\int \frac{x^2}{\sqrt{a+bx^3+cx^6}}\,dx$

3.223.1 Optimal result

Integrand size = 20, antiderivative size = 43

$$\int \frac{x^2}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{\operatorname{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3\sqrt{c}}$$

output
```
1/3*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))/c^(1/2)
```

3.223.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

$$\int \frac{x^2}{\sqrt{a+bx^3+cx^6}}\,dx = -\frac{\log\left(b+2cx^3-2\sqrt{c}\sqrt{a+bx^3+cx^6}\right)}{3\sqrt{c}}$$

input
```
Integrate[x^2/Sqrt[a + b*x^3 + c*x^6],x]
```

output
```
-1/3*Log[b + 2*c*x^3 - 2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]]/Sqrt[c]
```

3.223.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1690, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a+bx^3+cx^6}}\,dx$$
$$\Big\downarrow \text{1690}$$
$$\frac{1}{3}\int \frac{1}{\sqrt{cx^6+bx^3+a}}dx^3$$
$$\Big\downarrow \text{1092}$$
$$\frac{2}{3}\int \frac{1}{4c-x^6}d\frac{2cx^3+b}{\sqrt{cx^6+bx^3+a}}$$
$$\Big\downarrow \text{219}$$
$$\frac{\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3\sqrt{c}}$$

input
```
Int[x^2/Sqrt[a + b*x^3 + c*x^6],x]
```

output
```
ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])]/(3*Sqrt[c])
```

3.223.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

3.223.4 Maple [F]

$$\int \frac{x^2}{\sqrt{c\,x^6+b\,x^3+a}}dx$$

input
```
int(x^2/(c*x^6+b*x^3+a)^(1/2),x)
```

output
```
int(x^2/(c*x^6+b*x^3+a)^(1/2),x)
```

3.223.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.74

$$\int \frac{x^2}{\sqrt{a+bx^3+cx^6}}\,dx = \left[\frac{\log\left(-8\,c^2x^6-8\,bcx^3-b^2-4\,\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{c}-4\,ac\right)}{6\,\sqrt{c}},\right. \left.-\frac{\sqrt{-c}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{-c}}{2\,(c^2x^6+bcx^3+ac)}\right)}{3\,c}\right]$$

input
```
integrate(x^2/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/6*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3
 + b)*sqrt(c) - 4*a*c)/sqrt(c), -1/3*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^
3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c))/c]
```

3.223.6 Sympy [F]

$$\int \frac{x^2}{\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{x^2}{\sqrt{a+bx^3+cx^6}}\,dx$$

input
```
integrate(x**2/(c*x**6+b*x**3+a)**(1/2),x)
```

output
```
Integral(x**2/sqrt(a + b*x**3 + c*x**6), x)
```

3.223.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{\sqrt{a+bx^3+cx^6}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.223.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(33) = 66$.

Time = 0.32 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.72

$$\int \frac{x^2}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{1}{12}\sqrt{cx^6+bx^3+a}\left(2\,x^3+\frac{b}{c}\right) + \frac{(b^2-4\,ac)\log\left(\left|2\left(\sqrt{c}x^3-\sqrt{cx^6+bx^3+a}\right)\sqrt{c}+b\right|\right)}{24\,c^{\frac{3}{2}}}$$

input
```
integrate(x^2/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
1/12*sqrt(c*x^6 + b*x^3 + a)*(2*x^3 + b/c) + 1/24*(b^2 - 4*a*c)*log(abs(2*
(sqrt(c)*x^3 - sqrt(c*x^6 + b*x^3 + a))*sqrt(c) + b))/c^(3/2)
```

3.223.9 Mupad [B] (verification not implemented)

Time = 8.62 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \frac{x^2}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{\ln\left(\sqrt{c\,x^6+b\,x^3+a}+\frac{c\,x^3+\frac{b}{2}}{\sqrt{c}}\right)}{3\,\sqrt{c}}$$

input
```
int(x^2/(a + b*x^3 + c*x^6)^(1/2),x)
```

output
```
log((a + b*x^3 + c*x^6)^(1/2) + (b/2 + c*x^3)/c^(1/2))/(3*c^(1/2))
```

3.224 $\int \frac{1}{x\sqrt{a+bx^3+cx^6}}\,dx$

3.224.1 Optimal result

Integrand size = 20, antiderivative size = 44

$$\int \frac{1}{x\sqrt{a+bx^3+cx^6}}\,dx=-\frac{\operatorname{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{3\sqrt{a}}$$

output
```
-1/3*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2))/a^(1/2)
```

3.224.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{a+bx^3+cx^6}}\,dx=\frac{2\operatorname{arctanh}\left(\frac{\sqrt{c}x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

input
```
Integrate[1/(x*Sqrt[a + b*x^3 + c*x^6]),x]
```

output
```
(2*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3 + c*x^6])/Sqrt[a]])/(3*Sqrt[a])
```

3.224.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1693, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a+bx^3+cx^6}}dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int \frac{1}{x^3\sqrt{cx^6+bx^3+a}}dx^3$$

$$\Big\downarrow \text{1154}$$

$$-\frac{2}{3}\int \frac{1}{4a-x^6}d\frac{bx^3+2a}{\sqrt{cx^6+bx^3+a}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{3\sqrt{a}}$$

input
```
Int[1/(x*Sqrt[a + b*x^3 + c*x^6]),x]
```

output
```
-1/3*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])]/Sqrt[a]
```

3.224.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.224.4 Maple [F]

$$\int \frac{1}{x\sqrt{c\,x^{6}+b\,x^{3}+a}}dx$$

input
```
int(1/x/(c*x^6+b*x^3+a)^(1/2),x)
```

output
```
int(1/x/(c*x^6+b*x^3+a)^(1/2),x)
```

3.224.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.82

$$\int \frac{1}{x\sqrt{a+bx^3+cx^6}}\,dx = \left[\frac{\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3-4\,\sqrt{cx^6+bx^3+a}(bx^3+2\,a)\sqrt{a}+8\,a^2}{x^6}\right)}{6\,\sqrt{a}}, \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(bx^3+2\,a)\sqrt{-a}}{2\,(acx^6+abx^3+a^2)}\right)}{3\,a}\right]$$

input
```
integrate(1/x/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/6*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^
3 + 2*a)*sqrt(a) + 8*a^2)/x^6)/sqrt(a), 1/3*sqrt(-a)*arctan(1/2*sqrt(c*x^6
 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2))/a]
```

3.224.6 Sympy [F]

$$\int \frac{1}{x\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{1}{x\sqrt{a+bx^3+cx^6}}\,dx$$

input
```
integrate(1/x/(c*x**6+b*x**3+a)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(a + b*x**3 + c*x**6)), x)
```

3.224.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\sqrt{a+bx^3+cx^6}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.224.8 Giac [F]

$$\int \frac{1}{x\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{1}{\sqrt{cx^6+bx^3+ax}}\,dx$$

input
```
integrate(1/x/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x), x)
```

3.224.9 Mupad [B] (verification not implemented)

Time = 8.48 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int \frac{1}{x\sqrt{a+bx^3+cx^6}}\,dx = -\frac{\ln\left(\frac{b}{2}+\frac{a}{x^3}+\frac{\sqrt{a}\,\sqrt{c\,x^6+b\,x^3+a}}{x^3}\right)}{3\,\sqrt{a}}$$

input
```
int(1/(x*(a + b*x^3 + c*x^6)^(1/2)),x)
```

output
```
-log(b/2 + a/x^3 + (a^(1/2)*(a + b*x^3 + c*x^6)^(1/2))/x^3)/(3*a^(1/2))
```

3.225 $\int \frac{1}{x^4\sqrt{a+bx^3+cx^6}}\,dx$

3.225.1 Optimal result

Integrand size = 20, antiderivative size = 72

$$\int \frac{1}{x^4\sqrt{a+bx^3+cx^6}}\,dx=-\frac{\sqrt{a+bx^3+cx^6}}{3ax^3}+\frac{b\,\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{6a^{3/2}}$$

output
```
1/6*b*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2))/a^(3/2)-1/3*(
c*x^6+b*x^3+a)^(1/2)/a/x^3
```

3.225.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4\sqrt{a+bx^3+cx^6}}\,dx=-\frac{\sqrt{a+bx^3+cx^6}}{3ax^3}-\frac{b\,\text{arctanh}\left(\frac{\sqrt{c}x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{3a^{3/2}}$$

input
```
Integrate[1/(x^4*Sqrt[a + b*x^3 + c*x^6]),x]
```

output
```
-1/3*Sqrt[a + b*x^3 + c*x^6]/(a*x^3) - (b*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a +
b*x^3 + c*x^6])/Sqrt[a]])/(3*a^(3/2))
```

3.225.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1693, 1157, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{a+bx^3+cx^6}}dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int \frac{1}{x^6\sqrt{cx^6+bx^3+a}}dx^3$$

$$\Big\downarrow \text{1157}$$

$$\frac{1}{3}\left(-\frac{b\int \frac{1}{x^3\sqrt{cx^6+bx^3+a}}dx^3}{2a}-\frac{\sqrt{a+bx^3+cx^6}}{ax^3}\right)$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{3}\left(\frac{b\int \frac{1}{4a-x^6}d\frac{bx^3+2a}{\sqrt{cx^6+bx^3+a}}}{a}-\frac{\sqrt{a+bx^3+cx^6}}{ax^3}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{b\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{2a^{3/2}}-\frac{\sqrt{a+bx^3+cx^6}}{ax^3}\right)$$

input
```
Int[1/(x^4*Sqrt[a + b*x^3 + c*x^6]),x]
```

output
```
(-(Sqrt[a + b*x^3 + c*x^6]/(a*x^3)) + (b*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*
Sqrt[a + b*x^3 + c*x^6])])/(2*a^(3/2)))/3
```

3.225.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1157
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Simp[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2))
  Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && EqQ[m + 2*p + 3, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.225.4 Maple [F]

$$\int \frac{1}{x^{4}\sqrt{c\,x^{6}+b\,x^{3}+a}}dx$$

input
```
int(1/x^4/(c*x^6+b*x^3+a)^(1/2),x)
```

output
```
int(1/x^4/(c*x^6+b*x^3+a)^(1/2),x)
```

3.225.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 179, normalized size of antiderivative = 2.49

$$\int \frac{1}{x^4\sqrt{a+bx^3+cx^6}}\,dx = \left[\frac{\sqrt{a}bx^3\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3+4\,\sqrt{cx^6+bx^3+a}(bx^3+2\,a)\sqrt{a}+8\,a^2}{x^6}\right)-4\,\sqrt{cx^6+bx^3+a}a}{12\,a^2x^3}, -\frac{\sqrt{-a}bx^3\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(bx^3+2\,a)\sqrt{-a}}{2\,(acx^6+abx^3+a^2)}\right)+2\,\sqrt{cx^6+bx^3+a}a}{6\,a^2x^3}\right]$$

input
```
integrate(1/x^4/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/12*(sqrt(a)*b*x^3*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 + 4*sqrt(c*x^6 +
b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) - 4*sqrt(c*x^6 + b*x^3 + a)
*a)/(a^2*x^3), -1/6*(sqrt(-a)*b*x^3*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*
x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*sqrt(c*x^6 + b*x^3 + a)
*a)/(a^2*x^3)]
```

3.225.6 Sympy [F]

$$\int \frac{1}{x^4\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{1}{x^4\sqrt{a+bx^3+cx^6}}\,dx$$

input
```
integrate(1/x**4/(c*x**6+b*x**3+a)**(1/2),x)
```

output
```
Integral(1/(x**4*sqrt(a + b*x**3 + c*x**6)), x)
```

3.225.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^4\sqrt{a+bx^3+cx^6}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x^4/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.225.8 Giac [F]

$$\int \frac{1}{x^4\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{1}{\sqrt{cx^6+bx^3+a}x^4}\,dx$$

input
```
integrate(1/x^4/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^4), x)
```

3.225.9 Mupad [B] (verification not implemented)

Time = 8.51 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^4\sqrt{a+bx^3+cx^6}}\,dx = \frac{b\,\mathrm{atanh}\left(\frac{\frac{b\,x^3}{2}+a}{\sqrt{a}\,\sqrt{c\,x^6+b\,x^3+a}}\right)}{6\,a^{3/2}} - \frac{\sqrt{c\,x^6+b\,x^3+a}}{3\,a\,x^3}$$

input
```
int(1/(x^4*(a + b*x^3 + c*x^6)^(1/2)),x)
```

output
```
(b*atanh((a + (b*x^3)/2)/(a^(1/2)*(a + b*x^3 + c*x^6)^(1/2))))/(6*a^(3/2))
 - (a + b*x^3 + c*x^6)^(1/2)/(3*a*x^3)
```

3.226 $\int \frac{1}{x^7\sqrt{a+bx^3+cx^6}}\,dx$

3.226.1 Optimal result

Integrand size = 20, antiderivative size = 108

$$\int \frac{1}{x^7\sqrt{a+bx^3+cx^6}}\,dx = -\frac{\sqrt{a+bx^3+cx^6}}{6ax^6}+\frac{b\sqrt{a+bx^3+cx^6}}{4a^2x^3} - \frac{(3b^2-4ac)\operatorname{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{24a^{5/2}}$$

output
```
-1/24*(-4*a*c+3*b^2)*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2)
)/a^(5/2)-1/6*(c*x^6+b*x^3+a)^(1/2)/a/x^6+1/4*b*(c*x^6+b*x^3+a)^(1/2)/a^2/
x^3
```

3.226.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^7\sqrt{a+bx^3+cx^6}}\,dx = \frac{(-2a+3bx^3)\sqrt{a+bx^3+cx^6}}{12a^2x^6} + \frac{(3b^2-4ac)\operatorname{arctanh}\left(\frac{\sqrt{c}x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{12a^{5/2}}$$

input
```
Integrate[1/(x^7*Sqrt[a + b*x^3 + c*x^6]),x]
```

output
```
((-2*a + 3*b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(12*a^2*x^6) + ((3*b^2 - 4*a*c)
*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3 + c*x^6])/Sqrt[a]])/(12*a^(5/2))
```

3.226.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1693, 1167, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7\sqrt{a+bx^3+cx^6}}dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int \frac{1}{x^9\sqrt{cx^6+bx^3+a}}dx^3$$

$$\Big\downarrow \text{1167}$$

$$\frac{1}{3}\left(-\frac{\int \frac{2cx^3+3b}{2x^6\sqrt{cx^6+bx^3+a}}dx^3}{2a}-\frac{\sqrt{a+bx^3+cx^6}}{2ax^6}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(-\frac{\int \frac{2cx^3+3b}{x^6\sqrt{cx^6+bx^3+a}}dx^3}{4a}-\frac{\sqrt{a+bx^3+cx^6}}{2ax^6}\right)$$

$$\Big\downarrow \text{1228}$$

$$\frac{1}{3}\left(-\frac{-\frac{(3b^2-4ac)\int \frac{1}{x^3\sqrt{cx^6+bx^3+a}}dx^3}{2a}-\frac{3b\sqrt{a+bx^3+cx^6}}{ax^3}}{4a}-\frac{\sqrt{a+bx^3+cx^6}}{2ax^6}\right)$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{3}\left(-\frac{\frac{(3b^2-4ac)\int \frac{1}{4a-x^6}d\frac{bx^3+2a}{\sqrt{cx^6+bx^3+a}}}{a}-\frac{3b\sqrt{a+bx^3+cx^6}}{ax^3}}{4a}-\frac{\sqrt{a+bx^3+cx^6}}{2ax^6}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(-\frac{\frac{(3b^2-4ac)\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{2a^{3/2}}-\frac{3b\sqrt{a+bx^3+cx^6}}{ax^3}}{4a}-\frac{\sqrt{a+bx^3+cx^6}}{2ax^6}\right)$$

input
```
Int[1/(x^7*Sqrt[a + b*x^3 + c*x^6]),x]
```

output
```
(-1/2*Sqrt[a + b*x^3 + c*x^6]/(a*x^6) - ((-3*b*Sqrt[a + b*x^3 + c*x^6])/(a
*x^3) + ((3*b^2 - 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 +
 c*x^6])])/(2*a^(3/2)))/(4*a))/3
```

3.226.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1167
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 - b*d*e + a*e^2))   Int[
(d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
 x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[m
, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumSimp
lerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.226.4 Maple [F]

$$\int \frac{1}{x^7\sqrt{c\,x^6+b\,x^3+a}}dx$$

input
```
int(1/x^7/(c*x^6+b*x^3+a)^(1/2),x)
```

output
```
int(1/x^7/(c*x^6+b*x^3+a)^(1/2),x)
```

3.226.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.05

$$\int \frac{1}{x^7\sqrt{a+bx^3+cx^6}}\,dx = \left[-\frac{(3\,b^2-4\,ac)\sqrt{a}x^6\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3+4\,\sqrt{cx^6+bx^3+a}(bx^3+2\,a)\sqrt{a}+8\,a^2}{x^6}\right)-4\,\sqrt{cx^6+bx^3+a}(3\,abx^3-2\,a^2)}{48\,a^3x^6}\right.$$

input
```
integrate(1/x^7/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/48*((3*b^2 - 4*a*c)*sqrt(a)*x^6*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 +
4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) - 4*sqrt(c*x
^6 + b*x^3 + a)*(3*a*b*x^3 - 2*a^2))/(a^3*x^6), 1/24*((3*b^2 - 4*a*c)*sqrt
(-a)*x^6*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^
6 + a*b*x^3 + a^2)) + 2*sqrt(c*x^6 + b*x^3 + a)*(3*a*b*x^3 - 2*a^2))/(a^3*
x^6)]
```

3.226.6 Sympy [F]

$$\int \frac{1}{x^7\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{1}{x^7\sqrt{a+bx^3+cx^6}}\,dx$$

input
```
integrate(1/x**7/(c*x**6+b*x**3+a)**(1/2),x)
```

output
```
Integral(1/(x**7*sqrt(a + b*x**3 + c*x**6)), x)
```

3.226.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^7\sqrt{a+bx^3+cx^6}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x^7/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.226.8 Giac [F]

$$\int \frac{1}{x^7\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{1}{\sqrt{cx^6+bx^3+a}\,x^7}\,dx$$

input
```
integrate(1/x^7/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^7), x)
```

3.226.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^7\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{1}{x^7\,\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

input
```
int(1/(x^7*(a + b*x^3 + c*x^6)^(1/2)),x)
```

output
```
int(1/(x^7*(a + b*x^3 + c*x^6)^(1/2)), x)
```

3.227 $\int \frac{1}{x^{10}\sqrt{a+bx^3+cx^6}}\,dx$

3.227.1 Optimal result

Integrand size = 20, antiderivative size = 145

$$\int \frac{1}{x^{10}\sqrt{a+bx^3+cx^6}}\,dx = -\frac{\sqrt{a+bx^3+cx^6}}{9ax^9}+\frac{5b\sqrt{a+bx^3+cx^6}}{36a^2x^6} - \frac{(15b^2-16ac)\sqrt{a+bx^3+cx^6}}{72a^3x^3} + \frac{b(5b^2-12ac)\,\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{48a^{7/2}}$$

output
```
1/48*b*(-12*a*c+5*b^2)*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/
2))/a^(7/2)-1/9*(c*x^6+b*x^3+a)^(1/2)/a/x^9+5/36*b*(c*x^6+b*x^3+a)^(1/2)/a
^2/x^6-1/72*(-16*a*c+15*b^2)*(c*x^6+b*x^3+a)^(1/2)/a^3/x^3
```

3.227.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^{10}\sqrt{a+bx^3+cx^6}}\,dx = \frac{\sqrt{a+bx^3+cx^6}(-8a^2+10abx^3-15b^2x^6+16acx^6)}{72a^3x^9} + \frac{(-5b^3+12abc)\,\text{arctanh}\left(\frac{\sqrt{c}x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{24a^{7/2}}$$

input
```
Integrate[1/(x^10*Sqrt[a + b*x^3 + c*x^6]),x]
```

output
```
(Sqrt[a + b*x^3 + c*x^6]*(-8*a^2 + 10*a*b*x^3 - 15*b^2*x^6 + 16*a*c*x^6))/
(72*a^3*x^9) + ((-5*b^3 + 12*a*b*c)*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3
+ c*x^6])/Sqrt[a]])/(24*a^(7/2))
```

3.227.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1693, 1167, 27, 1237, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{10}\sqrt{a+bx^3+cx^6}}dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int \frac{1}{x^{12}\sqrt{cx^6+bx^3+a}}dx^3$$

$$\Big\downarrow \text{1167}$$

$$\frac{1}{3}\left(-\frac{\int \frac{4cx^3+5b}{2x^9\sqrt{cx^6+bx^3+a}}dx^3}{3a}-\frac{\sqrt{a+bx^3+cx^6}}{3ax^9}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(-\frac{\int \frac{4cx^3+5b}{x^9\sqrt{cx^6+bx^3+a}}dx^3}{6a}-\frac{\sqrt{a+bx^3+cx^6}}{3ax^9}\right)$$

$$\Big\downarrow \text{1237}$$

$$\frac{1}{3}\left(-\frac{-\frac{\int \frac{10bcx^3+15b^2-16ac}{2x^6\sqrt{cx^6+bx^3+a}}dx^3}{2a}-\frac{5b\sqrt{a+bx^3+cx^6}}{2ax^6}}{6a}-\frac{\sqrt{a+bx^3+cx^6}}{3ax^9}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(-\frac{-\frac{\int \frac{10bcx^3+15b^2-16ac}{x^6\sqrt{cx^6+bx^3+a}}dx^3}{4a}-\frac{5b\sqrt{a+bx^3+cx^6}}{2ax^6}}{6a}-\frac{\sqrt{a+bx^3+cx^6}}{3ax^9}\right)$$

$\Big\downarrow$ 1228

$$\frac{1}{3}\left(-\frac{-\frac{-\frac{3b(5b^2-12ac)\int \frac{1}{x^3\sqrt{cx^6+bx^3+a}}dx^3}{2a}-\frac{(15b^2-16ac)\sqrt{a+bx^3+cx^6}}{ax^3}}{4a}-\frac{5b\sqrt{a+bx^3+cx^6}}{2ax^6}}{6a}-\frac{\sqrt{a+bx^3+cx^6}}{3ax^9}\right)$$

$\Big\downarrow$ 1154

$$\frac{1}{3}\left(-\frac{-\frac{\frac{3b(5b^2-12ac)\int \frac{1}{4a-x^6}d\frac{bx^3+2a}{\sqrt{cx^6+bx^3+a}}}{a}-\frac{(15b^2-16ac)\sqrt{a+bx^3+cx^6}}{ax^3}}{4a}-\frac{5b\sqrt{a+bx^3+cx^6}}{2ax^6}}{6a}-\frac{\sqrt{a+bx^3+cx^6}}{3ax^9}\right)$$

$\Big\downarrow$ 219

$$\frac{1}{3}\left(-\frac{-\frac{\frac{3b(5b^2-12ac)\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{2a^{3/2}}-\frac{(15b^2-16ac)\sqrt{a+bx^3+cx^6}}{ax^3}}{4a}-\frac{5b\sqrt{a+bx^3+cx^6}}{2ax^6}}{6a}-\frac{\sqrt{a+bx^3+cx^6}}{3ax^9}\right)$$

input
```
Int[1/(x^10*Sqrt[a + b*x^3 + c*x^6]),x]
```

output
```
(-1/3*Sqrt[a + b*x^3 + c*x^6]/(a*x^9) - ((-5*b*Sqrt[a + b*x^3 + c*x^6])/(2
*a*x^6) - (-(((15*b^2 - 16*a*c)*Sqrt[a + b*x^3 + c*x^6])/(a*x^3)) + (3*b*(
5*b^2 - 12*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])]
)/(2*a^(3/2)))/(4*a))/(6*a))/3
```

3.227.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1167
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 - b*d*e + a*e^2))   Int[
(d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
 x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[m
, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumSimp
lerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.227. $\int \frac{1}{x^{10}\sqrt{a+bx^3+cx^6}}\,dx$

3.227.4 Maple [F]

$$\int \frac{1}{x^{10}\sqrt{c\,x^6+b\,x^3+a}}dx$$

input
```
int(1/x^10/(c*x^6+b*x^3+a)^(1/2),x)
```

output
```
int(1/x^10/(c*x^6+b*x^3+a)^(1/2),x)
```

3.227.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.81

$$\int \frac{1}{x^{10}\sqrt{a+bx^3+cx^6}}\,dx = \left[-\frac{3\,(5\,b^3-12\,abc)\sqrt{a}x^9\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3-4\,\sqrt{cx^6+bx^3+a}(bx^3+2\,a)\sqrt{a}+8\,a^2}{x^6}\right)+4\,((15\,ab^2-16\,a^2c)x^6-10}{288\,a^4x^9}\right.$$
$$-\frac{3\,(5\,b^3-12\,abc)\sqrt{-a}x^9\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(bx^3+2\,a)\sqrt{-a}}{2\,(acx^6+abx^3+a^2)}\right)+2\,((15\,ab^2-16\,a^2c)x^6-10\,a^2bx^3+8\,a^3)\sqrt{cx}}{144\,a^4x^9}$$

input
```
integrate(1/x^10/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/288*(3*(5*b^3 - 12*a*b*c)*sqrt(a)*x^9*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*
x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) + 4*((
15*a*b^2 - 16*a^2*c)*x^6 - 10*a^2*b*x^3 + 8*a^3)*sqrt(c*x^6 + b*x^3 + a))/
(a^4*x^9), -1/144*(3*(5*b^3 - 12*a*b*c)*sqrt(-a)*x^9*arctan(1/2*sqrt(c*x^6
 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((15*a
*b^2 - 16*a^2*c)*x^6 - 10*a^2*b*x^3 + 8*a^3)*sqrt(c*x^6 + b*x^3 + a))/(a^4
*x^9)]
```

3.227.6 Sympy [F]

$$\int \frac{1}{x^{10}\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{1}{x^{10}\sqrt{a+bx^3+cx^6}}\,dx$$

input
```
integrate(1/x**10/(c*x**6+b*x**3+a)**(1/2),x)
```

output
```
Integral(1/(x**10*sqrt(a + b*x**3 + c*x**6)), x)
```

3.227.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^{10}\sqrt{a+bx^3+cx^6}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x^10/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.227.8 Giac [F]

$$\int \frac{1}{x^{10}\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{1}{\sqrt{cx^6+bx^3+a}\,x^{10}}\,dx$$

input
```
integrate(1/x^10/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^10), x)
```

3.227.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{10}\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{1}{x^{10}\,\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

input
```
int(1/(x^10*(a + b*x^3 + c*x^6)^(1/2)),x)
```

output
```
int(1/(x^10*(a + b*x^3 + c*x^6)^(1/2)), x)
```

3.228 $\int \frac{1}{x^{13}\sqrt{a+bx^3+cx^6}}\,dx$

3.228.1 Optimal result

Integrand size = 20, antiderivative size = 192

$$\int \frac{1}{x^{13}\sqrt{a+bx^3+cx^6}}\,dx = -\frac{\sqrt{a+bx^3+cx^6}}{12ax^{12}}+\frac{7b\sqrt{a+bx^3+cx^6}}{72a^2x^9} - \frac{(35b^2-36ac)\sqrt{a+bx^3+cx^6}}{288a^3x^6} + \frac{5b(21b^2-44ac)\sqrt{a+bx^3+cx^6}}{576a^4x^3} - \frac{(35b^4-120ab^2c+48a^2c^2)\operatorname{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{384a^{9/2}}$$

output
```
-1/384*(48*a^2*c^2-120*a*b^2*c+35*b^4)*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*
x^6+b*x^3+a)^(1/2))/a^(9/2)-1/12*(c*x^6+b*x^3+a)^(1/2)/a/x^12+7/72*b*(c*x^
6+b*x^3+a)^(1/2)/a^2/x^9-1/288*(-36*a*c+35*b^2)*(c*x^6+b*x^3+a)^(1/2)/a^3/
x^6+5/576*b*(-44*a*c+21*b^2)*(c*x^6+b*x^3+a)^(1/2)/a^4/x^3
```

3.228.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.73

$$\int \frac{1}{x^{13}\sqrt{a+bx^3+cx^6}}\,dx = \frac{\sqrt{a+bx^3+cx^6}(-48a^3+56a^2bx^3-70ab^2x^6+72a^2cx^6+105b^3x^9-220abcx^9)}{576a^4x^{12}} + \frac{(35b^4-120ab^2c+48a^2c^2)\operatorname{arctanh}\left(\frac{\sqrt{c}x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{192a^{9/2}}$$

input
```
Integrate[1/(x^13*Sqrt[a + b*x^3 + c*x^6]),x]
```

output
```
(Sqrt[a + b*x^3 + c*x^6]*(-48*a^3 + 56*a^2*b*x^3 - 70*a*b^2*x^6 + 72*a^2*c
*x^6 + 105*b^3*x^9 - 220*a*b*c*x^9))/(576*a^4*x^12) + ((35*b^4 - 120*a*b^2
*c + 48*a^2*c^2)*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3 + c*x^6])/Sqrt[a]])
/(192*a^(9/2))
```

3.228.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.14, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1693, 1167, 27, 1237, 27, 1237, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{13}\sqrt{a+bx^3+cx^6}}\,dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int \frac{1}{x^{15}\sqrt{cx^6+bx^3+a}}dx^3$$

$$\Big\downarrow \text{1167}$$

$$\frac{1}{3}\left(-\frac{\int \frac{6cx^3+7b}{2x^{12}\sqrt{cx^6+bx^3+a}}dx^3}{4a}-\frac{\sqrt{a+bx^3+cx^6}}{4ax^{12}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(-\frac{\int \frac{6cx^3+7b}{x^{12}\sqrt{cx^6+bx^3+a}}dx^3}{8a}-\frac{\sqrt{a+bx^3+cx^6}}{4ax^{12}}\right)$$

$$\Big\downarrow \; 1237$$

$$\frac{1}{3}\left(-\frac{-\frac{\int \frac{28bcx^3+35b^2-36ac}{2x^9\sqrt{cx^6+bx^3+a}}dx^3}{3a}-\frac{7b\sqrt{a+bx^3+cx^6}}{3ax^9}}{8a}-\frac{\sqrt{a+bx^3+cx^6}}{4ax^{12}}\right)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3}\left(-\frac{-\frac{\int \frac{28bcx^3+35b^2-36ac}{x^9\sqrt{cx^6+bx^3+a}}dx^3}{6a}-\frac{7b\sqrt{a+bx^3+cx^6}}{3ax^9}}{8a}-\frac{\sqrt{a+bx^3+cx^6}}{4ax^{12}}\right)$$

$$\Big\downarrow \; 1237$$

$$\frac{1}{3}\left(-\frac{-\frac{-\frac{\int \frac{2c\left(35b^2-36ac\right)x^3+5b\left(21b^2-44ac\right)}{2x^6\sqrt{cx^6+bx^3+a}}dx^3}{2a}-\frac{\left(35b^2-36ac\right)\sqrt{a+bx^3+cx^6}}{2ax^6}}{6a}-\frac{7b\sqrt{a+bx^3+cx^6}}{3ax^9}}{8a}-\frac{\sqrt{a+bx^3+cx^6}}{4ax^{12}}\right)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3}\left(-\frac{-\frac{-\frac{\int \frac{2c\left(35b^2-36ac\right)x^3+5b\left(21b^2-44ac\right)}{x^6\sqrt{cx^6+bx^3+a}}dx^3}{4a}-\frac{\left(35b^2-36ac\right)\sqrt{a+bx^3+cx^6}}{2ax^6}}{6a}-\frac{7b\sqrt{a+bx^3+cx^6}}{3ax^9}}{8a}-\frac{\sqrt{a+bx^3+cx^6}}{4ax^{12}}\right)$$

$$\Big\downarrow \; 1228$$

$$\frac{1}{3}\left(-\frac{-\frac{-\frac{-\frac{3\left(48a^2c^2-120ab^2c+35b^4\right)\int \frac{1}{x^3\sqrt{cx^6+bx^3+a}}dx^3}{2a}-\frac{5b\left(21b^2-44ac\right)\sqrt{a+bx^3+cx^6}}{ax^3}}{4a}-\frac{\left(35b^2-36ac\right)\sqrt{a+bx^3+cx^6}}{2ax^6}}{6a}-\frac{7b\sqrt{a+bx^3+cx^6}}{3ax^9}}{8a}-\sqrt{a \ldots}\right.$$

$$\Big\downarrow \; 1154$$

$$\frac{1}{3}\left(-\frac{-\frac{-\frac{\frac{3\left(48a^2c^2-120ab^2c+35b^4\right)\int \frac{1}{4a-x^6}d\frac{bx^3+2a}{\sqrt{cx^6+bx^3+a}}}{a}-\frac{5b\left(21b^2-44ac\right)\sqrt{a+bx^3+cx^6}}{ax^3}}{4a}-\frac{\left(35b^2-36ac\right)\sqrt{a+bx^3+cx^6}}{2ax^6}}{6a}-\frac{7b\sqrt{a+bx^3+cx^6}}{3ax^9}}{8a}-\sqrt{a \ldots}\right.$$

3.228. $\int \frac{1}{x^{13}\sqrt{a+bx^3+cx^6}}\,dx$

$$\Big\downarrow \; 219$$

$$\frac{1}{3}\left(-\frac{-\frac{-\frac{\frac{3(48a^2c^2-120ab^2c+35b^4)\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{2a^{3/2}}-\frac{5b(21b^2-44ac)\sqrt{a+bx^3+cx^6}}{ax^3}}{4a}-\frac{(35b^2-36ac)\sqrt{a+bx^3+cx^6}}{2ax^6}}{6a}-\frac{7b\sqrt{a+bx^3+cx^6}}{3ax^9}}{8a} - \right.$$

input
```
Int[1/(x^13*Sqrt[a + b*x^3 + c*x^6]),x]
```

output
```
(-1/4*Sqrt[a + b*x^3 + c*x^6]/(a*x^12) - ((-7*b*Sqrt[a + b*x^3 + c*x^6])/(
3*a*x^9) - (-1/2*((35*b^2 - 36*a*c)*Sqrt[a + b*x^3 + c*x^6])/(a*x^6) - ((-
5*b*(21*b^2 - 44*a*c)*Sqrt[a + b*x^3 + c*x^6])/(a*x^3) + (3*(35*b^4 - 120*
a*b^2*c + 48*a^2*c^2)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*
x^6])])/(2*a^(3/2)))/(4*a))/(6*a))/(8*a))/3
```

3.228.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1167
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 - b*d*e + a*e^2))   Int[
(d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
 x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[m
, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumSimp
lerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.228.4 Maple [F]

$$\int \frac{1}{x^{13}\sqrt{c\,x^{6}+b\,x^{3}+a}}dx$$

input
```
int(1/x^13/(c*x^6+b*x^3+a)^(1/2),x)
```

output
```
int(1/x^13/(c*x^6+b*x^3+a)^(1/2),x)
```

3.228.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.70

$$\int \frac{1}{x^{13}\sqrt{a+bx^3+cx^6}}\,dx$$
$$= \left[\frac{3\,(35\,b^4-120\,ab^2c+48\,a^2c^2)\sqrt{a}x^{12}\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3-4\,\sqrt{cx^6+bx^3+a}(bx^3+2\,a)\sqrt{a}+8\,a^2}{x^6}\right)+4\,(5\,(21\,ab^3-4}{2304\,a^5x^{12}}\right.$$

input
```
integrate(1/x^13/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/2304*(3*(35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*sqrt(a)*x^12*log(-((b^2 + 4
*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) +
8*a^2)/x^6) + 4*(5*(21*a*b^3 - 44*a^2*b*c)*x^9 + 56*a^3*b*x^3 - 2*(35*a^2*
b^2 - 36*a^3*c)*x^6 - 48*a^4)*sqrt(c*x^6 + b*x^3 + a))/(a^5*x^12), 1/1152*
(3*(35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*sqrt(-a)*x^12*arctan(1/2*sqrt(c*x^6
 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*(5*(21
*a*b^3 - 44*a^2*b*c)*x^9 + 56*a^3*b*x^3 - 2*(35*a^2*b^2 - 36*a^3*c)*x^6 -
48*a^4)*sqrt(c*x^6 + b*x^3 + a))/(a^5*x^12)]
```

3.228.6 Sympy [F]

$$\int \frac{1}{x^{13}\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{1}{x^{13}\sqrt{a+bx^3+cx^6}}\,dx$$

input
```
integrate(1/x**13/(c*x**6+b*x**3+a)**(1/2),x)
```

output
```
Integral(1/(x**13*sqrt(a + b*x**3 + c*x**6)), x)
```

3.228.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^{13}\sqrt{a+bx^3+cx^6}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x^13/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.228.8 Giac [F]

$$\int \frac{1}{x^{13}\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{1}{\sqrt{cx^6+bx^3+a}x^{13}}\,dx$$

input
```
integrate(1/x^13/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^13), x)
```

3.228.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{13}\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{1}{x^{13}\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

input
```
int(1/(x^13*(a + b*x^3 + c*x^6)^(1/2)),x)
```

output
```
int(1/(x^13*(a + b*x^3 + c*x^6)^(1/2)), x)
```

3.229 $\int \frac{x^3}{\sqrt{a+bx^3+cx^6}}\,dx$

3.229.1 Optimal result

Integrand size = 20, antiderivative size = 140

$$\int \frac{x^3}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{x^4\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},\frac{1}{2},\frac{7}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{a+bx^3+cx^6}}$$

output
```
1/4*x^4*AppellF1(4/3,1/2,1/2,7/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/
(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^
3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(c*x^6+b*x^3+a)^(1/2)
```

3.229.2 Mathematica [A] (verified)

Time = 10.07 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.20

$$\int \frac{x^3}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{x^4\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},\frac{1}{2},\frac{7}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)}{4\sqrt{a+bx^3+cx^6}}$$

input
```
Integrate[x^3/Sqrt[a + b*x^3 + c*x^6],x]
```

output
```
(x^4*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[
(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[4/3, 1
/2, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2
 - 4*a*c])])/(4*Sqrt[a + b*x^3 + c*x^6])
```

3.229.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{a+bx^3+cx^6}}dx$$

$$\Big\downarrow \text{1721}$$

$$\frac{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\int \frac{x^3}{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{b+\sqrt{b^2-4ac}}+1}}dx}{\sqrt{a+bx^3+cx^6}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^4\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\operatorname{AppellF1}\left(\frac{4}{3},\frac{1}{2},\frac{1}{2},\frac{7}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{a+bx^3+cx^6}}$$

input
```
Int[x^3/Sqrt[a + b*x^3 + c*x^6],x]
```

output
```
(x^4*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + S
qrt[b^2 - 4*a*c])]*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b - Sqrt[b^2 -
 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(4*Sqrt[a + b*x^3 + c*x^6])
```

3.229.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

3.229.4 Maple [F]

$$\int \frac{x^3}{\sqrt{c\,x^6+b\,x^3+a}}dx$$

input
```
int(x^3/(c*x^6+b*x^3+a)^(1/2),x)
```

output
```
int(x^3/(c*x^6+b*x^3+a)^(1/2),x)
```

3.229.5 Fricas [F]

$$\int \frac{x^3}{\sqrt{a+bx^3+cx^6}}\,dx=\int \frac{x^3}{\sqrt{cx^6+bx^3+a}}\,dx$$

input
```
integrate(x^3/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^3/sqrt(c*x^6 + b*x^3 + a), x)
```

3.229.6 Sympy [F]

$$\int \frac{x^3}{\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{x^3}{\sqrt{a+bx^3+cx^6}}\,dx$$

input
```
integrate(x**3/(c*x**6+b*x**3+a)**(1/2),x)
```

output
```
Integral(x**3/sqrt(a + b*x**3 + c*x**6), x)
```

3.229.7 Maxima [F]

$$\int \frac{x^3}{\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{x^3}{\sqrt{cx^6+bx^3+a}}\,dx$$

input
```
integrate(x^3/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3/sqrt(c*x^6 + b*x^3 + a), x)
```

3.229.8 Giac [F]

$$\int \frac{x^3}{\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{x^3}{\sqrt{cx^6+bx^3+a}}\,dx$$

input
```
integrate(x^3/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^3/sqrt(c*x^6 + b*x^3 + a), x)
```

3.229.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{a+bx^3+cx^6}}\,dx=\int \frac{x^3}{\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

input
```
int(x^3/(a + b*x^3 + c*x^6)^(1/2),x)
```

output
```
int(x^3/(a + b*x^3 + c*x^6)^(1/2), x)
```

3.230 $\int \frac{x}{\sqrt{a+bx^3+cx^6}}\,dx$

3.230.1 Optimal result

Integrand size = 18, antiderivative size = 140

$$\int \frac{x}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{x^2\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{5}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{a+bx^3+cx^6}}$$

output
```
1/2*x^2*AppellF1(2/3,1/2,1/2,5/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/
(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^
3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(c*x^6+b*x^3+a)^(1/2)
```

3.230.2 Mathematica [A] (verified)

Time = 10.06 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.20

$$\int \frac{x}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{x^2\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{5}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)}{2\sqrt{a+bx^3+cx^6}}$$

input
```
Integrate[x/Sqrt[a + b*x^3 + c*x^6],x]
```

output
```
(x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[
(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[2/3, 1
/2, 1/2, 5/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2
 - 4*a*c])])/(2*Sqrt[a + b*x^3 + c*x^6])
```

3.230.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a+bx^3+cx^6}}dx$$

$$\Big\downarrow \text{ 1721}$$

$$\frac{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\int \frac{x}{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{b+\sqrt{b^2-4ac}}+1}}dx}{\sqrt{a+bx^3+cx^6}}$$

$$\Big\downarrow \text{ 1012}$$

$$\frac{x^2\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{5}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{a+bx^3+cx^6}}$$

input
```
Int[x/Sqrt[a + b*x^3 + c*x^6],x]
```

output
```
(x^2*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + S
qrt[b^2 - 4*a*c])]*AppellF1[2/3, 1/2, 1/2, 5/3, (-2*c*x^3)/(b - Sqrt[b^2 -
 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(2*Sqrt[a + b*x^3 + c*x^6])
```

3.230.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

3.230.4 Maple [F]

$$\int \frac{x}{\sqrt{c\,x^{6}+b\,x^{3}+a}}dx$$

input
```
int(x/(c*x^6+b*x^3+a)^(1/2),x)
```

output
```
int(x/(c*x^6+b*x^3+a)^(1/2),x)
```

3.230.5 Fricas [F]

$$\int \frac{x}{\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{x}{\sqrt{cx^6+bx^3+a}}\,dx$$

input
```
integrate(x/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(x/sqrt(c*x^6 + b*x^3 + a), x)
```

3.230.6 Sympy [F]

$$\int \frac{x}{\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{x}{\sqrt{a+bx^3+cx^6}}\,dx$$

input
```
integrate(x/(c*x**6+b*x**3+a)**(1/2),x)
```

output
```
Integral(x/sqrt(a + b*x**3 + c*x**6), x)
```

3.230.7 Maxima [F]

$$\int \frac{x}{\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{x}{\sqrt{cx^6+bx^3+a}}\,dx$$

input
```
integrate(x/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/sqrt(c*x^6 + b*x^3 + a), x)
```

3.230.8 Giac [F]

$$\int \frac{x}{\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{x}{\sqrt{cx^6+bx^3+a}}\,dx$$

input
```
integrate(x/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/sqrt(c*x^6 + b*x^3 + a), x)
```

3.230.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{x}{\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

input
```
int(x/(a + b*x^3 + c*x^6)^(1/2),x)
```

output
```
int(x/(a + b*x^3 + c*x^6)^(1/2), x)
```

3.231 $\int \frac{1}{\sqrt{a+bx^3+cx^6}}\,dx$

3.231.1 Optimal result

Integrand size = 16, antiderivative size = 135

$$\int \frac{1}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{x\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{a+bx^3+cx^6}}$$

output
```
x*AppellF1(1/3,1/2,1/2,4/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/(b+(-4
*a*c+b^2)^(1/2)))*(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^3/(b+(
-4*a*c+b^2)^(1/2)))^(1/2)/(c*x^6+b*x^3+a)^(1/2)
```

3.231.2 Mathematica [A] (verified)

Time = 10.05 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.21

$$\int \frac{1}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{x\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)}{\sqrt{a+bx^3+cx^6}}$$

input
```
Integrate[1/Sqrt[a + b*x^3 + c*x^6],x]
```

output
```
(x*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b
 + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/3, 1/2
, 1/2, 4/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 -
 4*a*c])])/Sqrt[a + b*x^3 + c*x^6]
```

3.231.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1686, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^3+cx^6}}dx$$

$$\Big\downarrow \text{1686}$$

$$\frac{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\int \frac{1}{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{b+\sqrt{b^2-4ac}}+1}}dx}{\sqrt{a+bx^3+cx^6}}$$

$$\Big\downarrow \text{936}$$

$$\frac{x\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\operatorname{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{a+bx^3+cx^6}}$$

input
```
Int[1/Sqrt[a + b*x^3 + c*x^6],x]
```

output
```
(x*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqr
t[b^2 - 4*a*c])]*AppellF1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4
*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/Sqrt[a + b*x^3 + c*x^6]
```

3.231.3.1 Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1686
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^
IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*c*(x^n/(b + Rt[b^2
- 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4*a*c, 2])))^FracPar
t[p]))   Int[(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2*c*(x^n/(b - S
qrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n]
&& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

3.231.4 Maple [F]

$$\int \frac{1}{\sqrt{c\,x^{6}+b\,x^{3}+a}}dx$$

input
```
int(1/(c*x^6+b*x^3+a)^(1/2),x)
```

output
```
int(1/(c*x^6+b*x^3+a)^(1/2),x)
```

3.231.5 Fricas [F]

$$\int \frac{1}{\sqrt{a+bx^3+cx^6}}\,dx=\int \frac{1}{\sqrt{cx^6+bx^3+a}}\,dx$$

input
```
integrate(1/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(1/sqrt(c*x^6 + b*x^3 + a), x)
```

3.231.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{1}{\sqrt{a+bx^3+cx^6}}\,dx$$

input
```
integrate(1/(c*x**6+b*x**3+a)**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*x**3 + c*x**6), x)
```

3.231.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{1}{\sqrt{cx^6+bx^3+a}}\,dx$$

input
```
integrate(1/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(c*x^6 + b*x^3 + a), x)
```

3.231.8 Giac [F]

$$\int \frac{1}{\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{1}{\sqrt{cx^6+bx^3+a}}\,dx$$

input
```
integrate(1/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(c*x^6 + b*x^3 + a), x)
```

3.231.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^3+cx^6}}\,dx=\int \frac{1}{\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

input
```
int(1/(a + b*x^3 + c*x^6)^(1/2),x)
```

output
```
int(1/(a + b*x^3 + c*x^6)^(1/2), x)
```

3.232 $\int \frac{1}{x^2\sqrt{a+bx^3+cx^6}}\,dx$

3.232.1 Optimal result

Integrand size = 20, antiderivative size = 138

$$\int \frac{1}{x^2\sqrt{a+bx^3+cx^6}}\,dx = -\frac{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(-\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{2}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{a+bx^3+cx^6}}$$

output
```
-AppellF1(-1/3,1/2,1/2,2/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/(b+(-4
*a*c+b^2)^(1/2)))*(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^3/(b+(
-4*a*c+b^2)^(1/2)))^(1/2)/x/(c*x^6+b*x^3+a)^(1/2)
```

3.232.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 343 vs. 2(138) = 276.

Time = 10.23 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.49

$$\int \frac{1}{x^2\sqrt{a+bx^3+cx^6}}\,dx = \frac{-20(a+bx^3+cx^6)+5bx^3\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{5}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)}{20ax\sqrt{a+bx^3+cx}}$$

input
```
Integrate[1/(x^2*Sqrt[a + b*x^3 + c*x^6]),x]
```

output
```
(-20*(a + b*x^3 + c*x^6) + 5*b*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/
(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[
b^2 - 4*a*c])]*AppellF1[2/3, 1/2, 1/2, 5/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a
*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + 8*c*x^6*Sqrt[(b - Sqrt[b^2 - 4
*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*
c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/3, 1/2, 1/2, 8/3, (-2*c*x^3)/(b
 + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(20*a*x*Sqrt[a
 + b*x^3 + c*x^6])
```

3.232.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{a+bx^3+cx^6}}dx$$

$$\Big\downarrow \text{1721}$$

$$\frac{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\int \frac{1}{x^2\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{b+\sqrt{b^2-4ac}}+1}}dx}{\sqrt{a+bx^3+cx^6}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(-\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{2}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{a+bx^3+cx^6}}$$

input
```
Int[1/(x^2*Sqrt[a + b*x^3 + c*x^6]),x]
```

output
```
-((Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqr
t[b^2 - 4*a*c])]*AppellF1[-1/3, 1/2, 1/2, 2/3, (-2*c*x^3)/(b - Sqrt[b^2 -
4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(x*Sqrt[a + b*x^3 + c*x^6]))
```

3.232.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

3.232.4 Maple [F]

$$\int \frac{1}{x^2\sqrt{c\,x^6+b\,x^3+a}}dx$$

input
```
int(1/x^2/(c*x^6+b*x^3+a)^(1/2),x)
```

output
```
int(1/x^2/(c*x^6+b*x^3+a)^(1/2),x)
```

3.232.5 Fricas [F]

$$\int \frac{1}{x^2\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{1}{\sqrt{cx^6+bx^3+ax^2}}\,dx$$

input
```
integrate(1/x^2/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^6 + b*x^3 + a)/(c*x^8 + b*x^5 + a*x^2), x)
```

3.232.6 Sympy [F]

$$\int \frac{1}{x^2\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{1}{x^2\sqrt{a+bx^3+cx^6}}\,dx$$

input
```
integrate(1/x**2/(c*x**6+b*x**3+a)**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(a + b*x**3 + c*x**6)), x)
```

3.232.7 Maxima [F]

$$\int \frac{1}{x^2\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{1}{\sqrt{cx^6+bx^3+a}x^2}\,dx$$

input
```
integrate(1/x^2/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^2), x)
```

3.232.8 Giac [F]

$$\int \frac{1}{x^2\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{1}{\sqrt{cx^6+bx^3+a}x^2}\,dx$$

input
```
integrate(1/x^2/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^2), x)
```

3.232.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{a+bx^3+cx^6}}\,dx=\int \frac{1}{x^2\,\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

input
```
int(1/(x^2*(a + b*x^3 + c*x^6)^(1/2)),x)
```

output
```
int(1/(x^2*(a + b*x^3 + c*x^6)^(1/2)), x)
```

3.233 $\int \frac{1}{x^3\sqrt{a+bx^3+cx^6}}\,dx$

3.233.1 Optimal result

Integrand size = 20, antiderivative size = 140

$$\int \frac{1}{x^3\sqrt{a+bx^3+cx^6}}\,dx = -\frac{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{1}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{a+bx^3+cx^6}}$$

output
```
-1/2*AppellF1(-2/3,1/2,1/2,1/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/(b
+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^3/
(b+(-4*a*c+b^2)^(1/2)))^(1/2)/x^2/(c*x^6+b*x^3+a)^(1/2)
```

3.233.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 342 vs. 2(140) = 280.

Time = 10.20 (sec) , antiderivative size = 342, normalized size of antiderivative = 2.44

$$\int \frac{1}{x^3\sqrt{a+bx^3+cx^6}}\,dx = \frac{-4(a+bx^3+cx^6)-2bx^3\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)}{8ax^2\sqrt{a+bx^3+cx^6}}$$

input
```
Integrate[1/(x^3*Sqrt[a + b*x^3 + c*x^6]),x]
```

output
```
(-4*(a + b*x^3 + c*x^6) - 2*b*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(
b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b
^2 - 4*a*c])]*AppellF1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*
c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + c*x^6*Sqrt[(b - Sqrt[b^2 - 4*a*
c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x
^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b +
Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(8*a*x^2*Sqrt[a +
 b*x^3 + c*x^6])
```

3.233.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{a+bx^3+cx^6}}dx$$

$$\Big\downarrow \text{1721}$$

$$\frac{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\int \frac{1}{x^3\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{b+\sqrt{b^2-4ac}}+1}}dx}{\sqrt{a+bx^3+cx^6}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{1}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{a+bx^3+cx^6}}$$

input
```
Int[1/(x^3*Sqrt[a + b*x^3 + c*x^6]),x]
```

output
```
-1/2*(Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b +
Sqrt[b^2 - 4*a*c])]*AppellF1[-2/3, 1/2, 1/2, 1/3, (-2*c*x^3)/(b - Sqrt[b^2
 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(x^2*Sqrt[a + b*x^3 + c*x
^6])
```

3.233.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

3.233.4 Maple [F]

$$\int \frac{1}{x^{3} \sqrt{c\,x^{6}+b\,x^{3}+a}}dx$$

input
```
int(1/x^3/(c*x^6+b*x^3+a)^(1/2),x)
```

output
```
int(1/x^3/(c*x^6+b*x^3+a)^(1/2),x)
```

3.233.5 Fricas [F]

$$\int \frac{1}{x^3\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{1}{\sqrt{cx^6+bx^3+a}x^3}\,dx$$

input
```
integrate(1/x^3/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^6 + b*x^3 + a)/(c*x^9 + b*x^6 + a*x^3), x)
```

3.233.6 Sympy [F]

$$\int \frac{1}{x^3\sqrt{a+bx^3+cx^6}}\,dx=\int \frac{1}{x^3\sqrt{a+bx^3+cx^6}}\,dx$$

input
```
integrate(1/x**3/(c*x**6+b*x**3+a)**(1/2),x)
```

output
```
Integral(1/(x**3*sqrt(a + b*x**3 + c*x**6)), x)
```

3.233.7 Maxima [F]

$$\int \frac{1}{x^3\sqrt{a+bx^3+cx^6}}\,dx=\int \frac{1}{\sqrt{cx^6+bx^3+a}x^3}\,dx$$

input
```
integrate(1/x^3/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^3), x)
```

3.233.8 Giac [F]

$$\int \frac{1}{x^3\sqrt{a+bx^3+cx^6}}\,dx=\int \frac{1}{\sqrt{cx^6+bx^3+a}x^3}\,dx$$

input
```
integrate(1/x^3/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^3), x)
```

3.233.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{a+bx^3+cx^6}}\,dx=\int \frac{1}{x^3\,\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

input
```
int(1/(x^3*(a + b*x^3 + c*x^6)^(1/2)),x)
```

output
```
int(1/(x^3*(a + b*x^3 + c*x^6)^(1/2)), x)
```

3.234 $\int \frac{x^{14}}{(a+bx^3+cx^6)^{3/2}}\,dx$

3.234.1 Optimal result

Integrand size = 20, antiderivative size = 195

$$\int \frac{x^{14}}{(a+bx^3+cx^6)^{3/2}}\,dx = \frac{2x^9(2a+bx^3)}{3(b^2-4ac)\sqrt{a+bx^3+cx^6}} - \frac{2bx^6\sqrt{a+bx^3+cx^6}}{3c(b^2-4ac)} - \frac{(b(15b^2-52ac)-2c(5b^2-12ac)x^3)\sqrt{a+bx^3+cx^6}}{12c^3(b^2-4ac)} + \frac{(5b^2-4ac)\operatorname{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{8c^{7/2}}$$

output
```
1/8*(-4*a*c+5*b^2)*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))/
c^(7/2)+2/3*x^9*(b*x^3+2*a)/(-4*a*c+b^2)/(c*x^6+b*x^3+a)^(1/2)-2/3*b*x^6*(
c*x^6+b*x^3+a)^(1/2)/c/(-4*a*c+b^2)-1/12*(b*(-52*a*c+15*b^2)-2*c*(-12*a*c+
5*b^2)*x^3)*(c*x^6+b*x^3+a)^(1/2)/c^3/(-4*a*c+b^2)
```

3.234.2 Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.87

$$\int \frac{x^{14}}{(a+bx^3+cx^6)^{3/2}}\,dx = \frac{4a^2c(-13b+6cx^3)+b^2x^3(15b^2+5bcx^3-2c^2x^6)+a(15b^3-62b^2cx^3-20bc^2x^6}{12c^3(-b^2+4ac)\sqrt{a+bx^3+cx^6}} + \frac{(-5b^2+4ac)\log\left(c^3\left(b+2cx^3-2\sqrt{c}\sqrt{a+bx^3+cx^6}\right)\right)}{8c^{7/2}}$$

input
```
Integrate[x^14/(a + b*x^3 + c*x^6)^(3/2),x]
```

output
```
(4*a^2*c*(-13*b + 6*c*x^3) + b^2*x^3*(15*b^2 + 5*b*c*x^3 - 2*c^2*x^6) + a*
(15*b^3 - 62*b^2*c*x^3 - 20*b*c^2*x^6 + 8*c^3*x^9))/(12*c^3*(-b^2 + 4*a*c)
*Sqrt[a + b*x^3 + c*x^6]) + ((-5*b^2 + 4*a*c)*Log[c^3*(b + 2*c*x^3 - 2*Sqr
t[c]*Sqrt[a + b*x^3 + c*x^6])])/(8*c^(7/2))
```

3.234.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1693, 1164, 27, 1236, 27, 1225, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{14}}{(a+bx^3+cx^6)^{3/2}}\,dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int \frac{x^{12}}{(cx^6+bx^3+a)^{3/2}}dx^3$$

$$\Big\downarrow \text{1164}$$

$$\frac{1}{3}\left(\frac{2x^9(2a+bx^3)}{(b^2-4ac)\sqrt{a+bx^3+cx^6}}-\frac{2\int \frac{3x^6(bx^3+2a)}{\sqrt{cx^6+bx^3+a}}dx^3}{b^2-4ac}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{2x^9(2a+bx^3)}{(b^2-4ac)\sqrt{a+bx^3+cx^6}}-\frac{6\int \frac{x^6(bx^3+2a)}{\sqrt{cx^6+bx^3+a}}dx^3}{b^2-4ac}\right)$$

$$\Big\downarrow \text{1236}$$

$$\frac{1}{3}\left(\frac{2x^9(2a+bx^3)}{(b^2-4ac)\sqrt{a+bx^3+cx^6}}-\frac{6\left(\frac{\int -\frac{x^3\left(\left(5b^2-12ac\right)x^3+4ab\right)}{2\sqrt{cx^6+bx^3+a}}dx^3}{3c}+\frac{bx^6\sqrt{a+bx^3+cx^6}}{3c}\right)}{b^2-4ac}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{2x^9(2a+bx^3)}{(b^2-4ac)\sqrt{a+bx^3+cx^6}}-\frac{6\left(\frac{bx^6\sqrt{a+bx^3+cx^6}}{3c}-\frac{\int \frac{x^3\left(\left(5b^2-12ac\right)x^3+4ab\right)}{\sqrt{cx^6+bx^3+a}}dx^3}{6c}\right)}{b^2-4ac}\right)$$

$\Big\downarrow$ 1225

$$\frac{1}{3}\left(\frac{2x^9(2a+bx^3)}{(b^2-4ac)\sqrt{a+bx^3+cx^6}}-\frac{6\left(\frac{bx^6\sqrt{a+bx^3+cx^6}}{3c}-\frac{\frac{3\left(b^2-4ac\right)\left(5b^2-4ac\right)\int \frac{1}{\sqrt{cx^6+bx^3+a}}dx^3}{8c^2}-\frac{\left(b\left(15b^2-52ac\right)-2cx^3\left(5b^2-12ac\right)\right)\sqrt{a}}{4c^2}}{6c}\right.}{b^2-4ac}\right.$$

$\Big\downarrow$ 1092

$$\frac{1}{3}\left(\frac{2x^9(2a+bx^3)}{(b^2-4ac)\sqrt{a+bx^3+cx^6}}-\frac{6\left(\frac{bx^6\sqrt{a+bx^3+cx^6}}{3c}-\frac{\frac{3\left(b^2-4ac\right)\left(5b^2-4ac\right)\int \frac{1}{4c-x^6}d\frac{2cx^3+b}{\sqrt{cx^6+bx^3+a}}}{4c^2}-\frac{\left(b\left(15b^2-52ac\right)-2cx^3\left(5b^2-12ac\right)\right.}{4c^2}}{6c}\right.}{b^2-4ac}\right.$$

$\Big\downarrow$ 219

$$\frac{1}{3}\left(\frac{2x^9(2a+bx^3)}{(b^2-4ac)\sqrt{a+bx^3+cx^6}}-\frac{6\left(\frac{bx^6\sqrt{a+bx^3+cx^6}}{3c}-\frac{\frac{3\left(b^2-4ac\right)\left(5b^2-4ac\right)\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{8c^{5/2}}-\frac{\left(b\left(15b^2-52ac\right)-2cx^3\left(5b^2\right.\right.}{4c^2}}{6c}\right.}{b^2-4ac}\right.$$

input
```
Int[x^14/(a + b*x^3 + c*x^6)^(3/2),x]
```

output
```
((2*x^9*(2*a + b*x^3))/((b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6]) - (6*((b*x^
6*Sqrt[a + b*x^3 + c*x^6])/(3*c) - (-1/4*((b*(15*b^2 - 52*a*c) - 2*c*(5*b^
2 - 12*a*c)*x^3)*Sqrt[a + b*x^3 + c*x^6])/c^2 + (3*(b^2 - 4*a*c)*(5*b^2 -
4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(8*c^(5
/2)))/(6*c)))/(b^2 - 4*a*c))/3
```

3.234. $\int \frac{x^{14}}{(a+bx^3+cx^6)^{3/2}}\,dx$

3.234.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1164
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

3.234. $\int \frac{x^{14}}{(a+bx^3+cx^6)^{3/2}}\,dx$

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.234.4 Maple [F]

$$\int \frac{x^{14}}{\left(c\,x^{6}+b\,x^{3}+a\right)^{\frac{3}{2}}}dx$$

input
```
int(x^14/(c*x^6+b*x^3+a)^(3/2),x)
```

output
```
int(x^14/(c*x^6+b*x^3+a)^(3/2),x)
```

3.234.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 591, normalized size of antiderivative = 3.03

$$\int \frac{x^{14}}{(a+bx^3+cx^6)^{3/2}}\,dx = \left[-\frac{3\left((5\,b^4c-24\,ab^2c^2+16\,a^2c^3)x^6+5\,ab^4-24\,a^2b^2c+16\,a^3c^2+(5\,b^5-24\,ab^3\right.}{}\right.$$

$$-\frac{3\left((5\,b^4c-24\,ab^2c^2+16\,a^2c^3)x^6+5\,ab^4-24\,a^2b^2c+16\,a^3c^2+(5\,b^5-24\,ab^3c+16\,a^2bc^2)x^3\right)\sqrt{-c}\arctan\left(\right.}{24\left(ab^2c^4-\right.}$$

input
```
integrate(x^14/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/48*(3*((5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*x^6 + 5*a*b^4 - 24*a^2*b^
2*c + 16*a^3*c^2 + (5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)*x^3)*sqrt(c)*log(-8
*c^2*x^6 - 8*b*c*x^3 - b^2 + 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(
c) - 4*a*c) - 4*(2*(b^2*c^3 - 4*a*c^4)*x^9 - 5*(b^3*c^2 - 4*a*b*c^3)*x^6 -
 15*a*b^3*c + 52*a^2*b*c^2 - (15*b^4*c - 62*a*b^2*c^2 + 24*a^2*c^3)*x^3)*s
qrt(c*x^6 + b*x^3 + a))/(a*b^2*c^4 - 4*a^2*c^5 + (b^2*c^5 - 4*a*c^6)*x^6 +
 (b^3*c^4 - 4*a*b*c^5)*x^3), -1/24*(3*((5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^
3)*x^6 + 5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2 + (5*b^5 - 24*a*b^3*c + 16*a^
2*b*c^2)*x^3)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sq
rt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) - 2*(2*(b^2*c^3 - 4*a*c^4)*x^9 - 5*(b^3*
c^2 - 4*a*b*c^3)*x^6 - 15*a*b^3*c + 52*a^2*b*c^2 - (15*b^4*c - 62*a*b^2*c^
2 + 24*a^2*c^3)*x^3)*sqrt(c*x^6 + b*x^3 + a))/(a*b^2*c^4 - 4*a^2*c^5 + (b^
2*c^5 - 4*a*c^6)*x^6 + (b^3*c^4 - 4*a*b*c^5)*x^3)]
```

3.234.6 Sympy [F]

$$\int \frac{x^{14}}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{x^{14}}{(a+bx^3+cx^6)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**14/(c*x**6+b*x**3+a)**(3/2),x)
```

output
```
Integral(x**14/(a + b*x**3 + c*x**6)**(3/2), x)
```

3.234.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{14}}{(a+bx^3+cx^6)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^14/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.234.8 Giac [F]

$$\int \frac{x^{14}}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{x^{14}}{(cx^6+bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^14/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^14/(c*x^6 + b*x^3 + a)^(3/2), x)
```

3.234.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{14}}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{x^{14}}{(c\,x^6+b\,x^3+a)^{3/2}}\,dx$$

input
```
int(x^14/(a + b*x^3 + c*x^6)^(3/2),x)
```

output
```
int(x^14/(a + b*x^3 + c*x^6)^(3/2), x)
```

3.235 $\int \frac{x^{11}}{(a+bx^3+cx^6)^{3/2}}\,dx$

3.235.1 Optimal result

Integrand size = 20, antiderivative size = 137

$$\int \frac{x^{11}}{(a+bx^3+cx^6)^{3/2}}\,dx = \frac{2x^6(2a+bx^3)}{3(b^2-4ac)\sqrt{a+bx^3+cx^6}} + \frac{(3b^2-8ac-2bcx^3)\sqrt{a+bx^3+cx^6}}{3c^2(b^2-4ac)} - \frac{b\operatorname{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{2c^{5/2}}$$

output
```
-1/2*b*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))/c^(5/2)+2/3*
x^6*(b*x^3+2*a)/(-4*a*c+b^2)/(c*x^6+b*x^3+a)^(1/2)+1/3*(-2*b*c*x^3-8*a*c+3
*b^2)*(c*x^6+b*x^3+a)^(1/2)/c^2/(-4*a*c+b^2)
```

3.235.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.96

$$\int \frac{x^{11}}{(a+bx^3+cx^6)^{3/2}}\,dx = \frac{-3ab^2+8a^2c-3b^3x^3+10abcx^3-b^2cx^6+4ac^2x^6}{3c^2(-b^2+4ac)\sqrt{a+bx^3+cx^6}} + \frac{b\log\left(bc^2+2c^3x^3-2c^{5/2}\sqrt{a+bx^3+cx^6}\right)}{2c^{5/2}}$$

input
```
Integrate[x^11/(a + b*x^3 + c*x^6)^(3/2),x]
```

output
```
(-3*a*b^2 + 8*a^2*c - 3*b^3*x^3 + 10*a*b*c*x^3 - b^2*c*x^6 + 4*a*c^2*x^6)/
(3*c^2*(-b^2 + 4*a*c)*Sqrt[a + b*x^3 + c*x^6]) + (b*Log[b*c^2 + 2*c^3*x^3
- 2*c^(5/2)*Sqrt[a + b*x^3 + c*x^6]])/(2*c^(5/2))
```

3.235.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1693, 1164, 27, 1225, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{(a+bx^3+cx^6)^{3/2}}dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int \frac{x^9}{(cx^6+bx^3+a)^{3/2}}dx^3$$

$$\Big\downarrow \text{1164}$$

$$\frac{1}{3}\left(\frac{2x^6(2a+bx^3)}{(b^2-4ac)\sqrt{a+bx^3+cx^6}}-\frac{2\int \frac{2x^3(bx^3+2a)}{\sqrt{cx^6+bx^3+a}}dx^3}{b^2-4ac}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{2x^6(2a+bx^3)}{(b^2-4ac)\sqrt{a+bx^3+cx^6}}-\frac{4\int \frac{x^3(bx^3+2a)}{\sqrt{cx^6+bx^3+a}}dx^3}{b^2-4ac}\right)$$

$$\Big\downarrow \text{1225}$$

$$\frac{1}{3}\left(\frac{2x^6(2a+bx^3)}{(b^2-4ac)\sqrt{a+bx^3+cx^6}}-\frac{4\left(\frac{3b(b^2-4ac)\int \frac{1}{\sqrt{cx^6+bx^3+a}}dx^3}{8c^2}-\frac{(-8ac+3b^2-2bcx^3)\sqrt{a+bx^3+cx^6}}{4c^2}\right)}{b^2-4ac}\right)$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{3}\left(\frac{2x^6(2a+bx^3)}{(b^2-4ac)\sqrt{a+bx^3+cx^6}}-\frac{4\left(\frac{3b(b^2-4ac)\int \frac{1}{4c-x^6}d\frac{2cx^3+b}{\sqrt{cx^6+bx^3+a}}}{4c^2}-\frac{(-8ac+3b^2-2bcx^3)\sqrt{a+bx^3+cx^6}}{4c^2}\right)}{b^2-4ac}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{2x^6(2a+bx^3)}{(b^2-4ac)\sqrt{a+bx^3+cx^6}}-\frac{4\left(\frac{3b(b^2-4ac)\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{8c^{5/2}}-\frac{(-8ac+3b^2-2bcx^3)\sqrt{a+bx^3+cx^6}}{4c^2}\right)}{b^2-4ac}\right)$$

input
```
Int[x^11/(a + b*x^3 + c*x^6)^(3/2),x]
```

output
```
((2*x^6*(2*a + b*x^3))/((b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6]) - (4*(-1/4*
((3*b^2 - 8*a*c - 2*b*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/c^2 + (3*b*(b^2 - 4*
a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(8*c^(5/2
))))/(b^2 - 4*a*c))/3
```

3.235.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1164
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

3.235. $\int \frac{x^{11}}{(a+bx^3+cx^6)^{3/2}}\,dx$

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.235.4 Maple [F]

$$\int \frac{x^{11}}{\left(c\,x^{6}+b\,x^{3}+a\right)^{\frac{3}{2}}}dx$$

input
```
int(x^11/(c*x^6+b*x^3+a)^(3/2),x)
```

output
```
int(x^11/(c*x^6+b*x^3+a)^(3/2),x)
```

3.235.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 459, normalized size of antiderivative = 3.35

$$\int \frac{x^{11}}{\left(a+bx^3+cx^6\right)^{3/2}}\,dx = \left[\frac{3\left((b^3c-4\,abc^2)x^6+ab^3-4\,a^2bc+(b^4-4\,ab^2c)x^3\right)\sqrt{c}\log\left(-8\,c^2x^6-8\,bcx^3-\right.}{12\left((b^2c\right.}\right.$$

input
```
integrate(x^11/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/12*(3*((b^3*c - 4*a*b*c^2)*x^6 + a*b^3 - 4*a^2*b*c + (b^4 - 4*a*b^2*c)*
x^3)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 + 4*sqrt(c*x^6 + b*x^3 + a)*
(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 4*((b^2*c^2 - 4*a*c^3)*x^6 + 3*a*b^2*c -
8*a^2*c^2 + (3*b^3*c - 10*a*b*c^2)*x^3)*sqrt(c*x^6 + b*x^3 + a))/((b^2*c^4
 - 4*a*c^5)*x^6 + a*b^2*c^3 - 4*a^2*c^4 + (b^3*c^3 - 4*a*b*c^4)*x^3), 1/6*
(3*((b^3*c - 4*a*b*c^2)*x^6 + a*b^3 - 4*a^2*b*c + (b^4 - 4*a*b^2*c)*x^3)*s
qrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6
 + b*c*x^3 + a*c)) + 2*((b^2*c^2 - 4*a*c^3)*x^6 + 3*a*b^2*c - 8*a^2*c^2 +
(3*b^3*c - 10*a*b*c^2)*x^3)*sqrt(c*x^6 + b*x^3 + a))/((b^2*c^4 - 4*a*c^5)*
x^6 + a*b^2*c^3 - 4*a^2*c^4 + (b^3*c^3 - 4*a*b*c^4)*x^3)]
```

3.235.6 Sympy [F]

$$\int \frac{x^{11}}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{x^{11}}{(a+bx^3+cx^6)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**11/(c*x**6+b*x**3+a)**(3/2),x)
```

output
```
Integral(x**11/(a + b*x**3 + c*x**6)**(3/2), x)
```

3.235.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{11}}{(a+bx^3+cx^6)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^11/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.235.8 Giac [F]

$$\int \frac{x^{11}}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{x^{11}}{(cx^6+bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^11/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^11/(c*x^6 + b*x^3 + a)^(3/2), x)
```

3.235.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{11}}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{x^{11}}{(c\,x^6+b\,x^3+a)^{3/2}}\,dx$$

input
```
int(x^11/(a + b*x^3 + c*x^6)^(3/2),x)
```

output
```
int(x^11/(a + b*x^3 + c*x^6)^(3/2), x)
```

3.236 $\int \frac{x^8}{(a+bx^3+cx^6)^{3/2}}\,dx$

3.236.1 Optimal result

Integrand size = 20, antiderivative size = 120

$$\int \frac{x^8}{(a+bx^3+cx^6)^{3/2}}\,dx = \frac{2x^3(2a+bx^3)}{3(b^2-4ac)\sqrt{a+bx^3+cx^6}} - \frac{2b\sqrt{a+bx^3+cx^6}}{3c(b^2-4ac)} + \frac{\operatorname{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3c^{3/2}}$$

output
```
1/3*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))/c^(3/2)+2/3*x^3
*(b*x^3+2*a)/(-4*a*c+b^2)/(c*x^6+b*x^3+a)^(1/2)-2/3*b*(c*x^6+b*x^3+a)^(1/2
)/c/(-4*a*c+b^2)
```

3.236.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.79

$$\int \frac{x^8}{(a+bx^3+cx^6)^{3/2}}\,dx = \frac{-\frac{2\sqrt{c}(b^2x^3+a(b-2cx^3))}{(b^2-4ac)\sqrt{a+bx^3+cx^6}} + \operatorname{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3c^{3/2}}$$

input
```
Integrate[x^8/(a + b*x^3 + c*x^6)^(3/2),x]
```

output
```
((-2*Sqrt[c]*(b^2*x^3 + a*(b - 2*c*x^3)))/((b^2 - 4*a*c)*Sqrt[a + b*x^3 +
c*x^6]) + ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(3*c
^(3/2))
```

3.236.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1693, 1164, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(a+bx^3+cx^6)^{3/2}}\,dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int \frac{x^6}{(cx^6+bx^3+a)^{3/2}}dx^3$$

$$\Big\downarrow \text{1164}$$

$$\frac{1}{3}\left(\frac{2x^3(2a+bx^3)}{(b^2-4ac)\sqrt{a+bx^3+cx^6}}-\frac{2\int \frac{bx^3+2a}{\sqrt{cx^6+bx^3+a}}dx^3}{b^2-4ac}\right)$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{3}\left(\frac{2x^3(2a+bx^3)}{(b^2-4ac)\sqrt{a+bx^3+cx^6}}-\frac{2\left(\frac{b\sqrt{a+bx^3+cx^6}}{c}-\frac{(b^2-4ac)\int \frac{1}{\sqrt{cx^6+bx^3+a}}dx^3}{2c}\right)}{b^2-4ac}\right)$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{3}\left(\frac{2x^3(2a+bx^3)}{(b^2-4ac)\sqrt{a+bx^3+cx^6}}-\frac{2\left(\frac{b\sqrt{a+bx^3+cx^6}}{c}-\frac{(b^2-4ac)\int \frac{1}{4c-x^6}d\frac{2cx^3+b}{\sqrt{cx^6+bx^3+a}}}{c}\right)}{b^2-4ac}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{2x^3(2a+bx^3)}{(b^2-4ac)\sqrt{a+bx^3+cx^6}}-\frac{2\left(\frac{b\sqrt{a+bx^3+cx^6}}{c}-\frac{(b^2-4ac)\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{2c^{3/2}}\right)}{b^2-4ac}\right)$$

input
```
Int[x^8/(a + b*x^3 + c*x^6)^(3/2),x]
```

output
```
((2*x^3*(2*a + b*x^3))/((b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6]) - (2*((b*Sq
rt[a + b*x^3 + c*x^6])/c - ((b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]
*Sqrt[a + b*x^3 + c*x^6])])/(2*c^(3/2))))/(b^2 - 4*a*c))/3
```

3.236.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1164
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.236. $\int \frac{x^8}{(a+bx^3+cx^6)^{3/2}}\, dx$

3.236.4 Maple [F]

$$\int \frac{x^8}{(c\,x^6+b\,x^3+a)^{\frac{3}{2}}}dx$$

input
```
int(x^8/(c*x^6+b*x^3+a)^(3/2),x)
```

output
```
int(x^8/(c*x^6+b*x^3+a)^(3/2),x)
```

3.236.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 387, normalized size of antiderivative = 3.22

$$\int \frac{x^8}{(a+bx^3+cx^6)^{3/2}}\,dx = \left[\frac{((b^2c-4\,ac^2)x^6+(b^3-4\,abc)x^3+ab^2-4\,a^2c)\sqrt{c}\log\left(-8\,c^2x^6-8\,bcx^3-b^2-\right.}{6\,((b^2c^3-4\,ac^4)x^6+ab^2c^2}\right.$$

$$-\frac{((b^2c-4\,ac^2)x^6+(b^3-4\,abc)x^3+ab^2-4\,a^2c)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{-c}}{2\,(c^2x^6+bcx^3+ac)}\right)+2\,\sqrt{cx^6+bx^3+a}(}{3\,((b^2c^3-4\,ac^4)x^6+ab^2c^2-4\,a^2c^3+(b^3c^2-4\,abc^3)x^3)}$$

input
```
integrate(x^8/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/6*(((b^2*c - 4*a*c^2)*x^6 + (b^3 - 4*a*b*c)*x^3 + a*b^2 - 4*a^2*c)*sqrt
(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3
+ b)*sqrt(c) - 4*a*c) - 4*sqrt(c*x^6 + b*x^3 + a)*((b^2*c - 2*a*c^2)*x^3 +
 a*b*c))/((b^2*c^3 - 4*a*c^4)*x^6 + a*b^2*c^2 - 4*a^2*c^3 + (b^3*c^2 - 4*a
*b*c^3)*x^3), -1/3*(((b^2*c - 4*a*c^2)*x^6 + (b^3 - 4*a*b*c)*x^3 + a*b^2 -
 4*a^2*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-
c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*sqrt(c*x^6 + b*x^3 + a)*((b^2*c - 2*a*c^
2)*x^3 + a*b*c))/((b^2*c^3 - 4*a*c^4)*x^6 + a*b^2*c^2 - 4*a^2*c^3 + (b^3*c
^2 - 4*a*b*c^3)*x^3)]
```

3.236.6 Sympy [F]

$$\int \frac{x^8}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{x^8}{(a+bx^3+cx^6)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**8/(c*x**6+b*x**3+a)**(3/2),x)
```

output
```
Integral(x**8/(a + b*x**3 + c*x**6)**(3/2), x)
```

3.236.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^8}{(a+bx^3+cx^6)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^8/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.236.8 Giac [F]

$$\int \frac{x^8}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{x^8}{(cx^6+bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^8/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^8/(c*x^6 + b*x^3 + a)^(3/2), x)
```

3.236.9 Mupad [B] (verification not implemented)

Time = 8.61 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.70

$$\int \frac{x^8}{(a+bx^3+cx^6)^{3/2}}\,dx = \frac{\ln\left(\sqrt{c\,x^6+b\,x^3+a}+\frac{c\,x^3+\frac{b}{2}}{\sqrt{c}}\right)}{3\,c^{3/2}} + \frac{\frac{a\,b}{2}-x^3\,\left(a\,c-\frac{b^2}{2}\right)}{3\,c\,\left(a\,c-\frac{b^2}{4}\right)\,\sqrt{c\,x^6+b\,x^3+a}}$$

input
```
int(x^8/(a + b*x^3 + c*x^6)^(3/2),x)
```

output
```
log((a + b*x^3 + c*x^6)^(1/2) + (b/2 + c*x^3)/c^(1/2))/(3*c^(3/2)) + ((a*b
)/2 - x^3*(a*c - b^2/2))/(3*c*(a*c - b^2/4)*(a + b*x^3 + c*x^6)^(1/2))
```

3.237 $\int \frac{x^5}{(a+bx^3+cx^6)^{3/2}}\,dx$

3.237.1 Optimal result

Integrand size = 20, antiderivative size = 39

$$\int \frac{x^5}{(a+bx^3+cx^6)^{3/2}}\,dx = \frac{2(2a+bx^3)}{3\,(b^2-4ac)\,\sqrt{a+bx^3+cx^6}}$$

output
```
2/3*(b*x^3+2*a)/(-4*a*c+b^2)/(c*x^6+b*x^3+a)^(1/2)
```

3.237.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{(a+bx^3+cx^6)^{3/2}}\,dx = \frac{2(2a+bx^3)}{3\,(b^2-4ac)\,\sqrt{a+bx^3+cx^6}}$$

input
```
Integrate[x^5/(a + b*x^3 + c*x^6)^(3/2),x]
```

output
```
(2*(2*a + b*x^3))/(3*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6])
```

3.237.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1693, 1158}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a+bx^3+cx^6)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 1693}$$

$$\frac{1}{3}\int \frac{x^3}{(cx^6+bx^3+a)^{3/2}}dx^3$$

$$\Big\downarrow \text{ 1158}$$

$$\frac{2(2a+bx^3)}{3(b^2-4ac)\sqrt{a+bx^3+cx^6}}$$

input
```
Int[x^5/(a + b*x^3 + c*x^6)^(3/2),x]
```

output
```
(2*(2*a + b*x^3))/(3*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6])
```

3.237.3.1 Defintions of rubi rules used

rule 1158
```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.237.4 Maple [A] (verified)

Time = 4.66 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.97

method	result	size
gosper	$-\frac{2(b\,x^{3}+2a)}{3\sqrt{c\,x^{6}+b\,x^{3}+a}\,(4ac-b^{2})}$	38
trager	$-\frac{2(b\,x^{3}+2a)}{3\sqrt{c\,x^{6}+b\,x^{3}+a}\,(4ac-b^{2})}$	38

input
```
int(x^5/(c*x^6+b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/(c*x^6+b*x^3+a)^(1/2)*(b*x^3+2*a)/(4*a*c-b^2)
```

3.237.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.74

$$\int \frac{x^5}{(a+bx^3+cx^6)^{3/2}}\,dx = \frac{2\sqrt{cx^6+bx^3+a}(bx^3+2\,a)}{3\,((b^2c-4\,ac^2)x^6+(b^3-4\,abc)x^3+ab^2-4\,a^2c)}$$

input
```
integrate(x^5/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)/((b^2*c - 4*a*c^2)*x^6 + (b^3 -
4*a*b*c)*x^3 + a*b^2 - 4*a^2*c)
```

3.237.6 Sympy [F]

$$\int \frac{x^5}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{x^5}{(a+bx^3+cx^6)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**5/(c*x**6+b*x**3+a)**(3/2),x)
```

output
```
Integral(x**5/(a + b*x**3 + c*x**6)**(3/2), x)
```

3.237.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{(a+bx^3+cx^6)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.237.8 Giac [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.15

$$\int \frac{x^5}{(a+bx^3+cx^6)^{3/2}}\,dx = \frac{2\left(\frac{bx^3}{b^2-4\,ac}+\frac{2\,a}{b^2-4\,ac}\right)}{3\,\sqrt{cx^6+bx^3+a}}$$

input
```
integrate(x^5/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
2/3*(b*x^3/(b^2 - 4*a*c) + 2*a/(b^2 - 4*a*c))/sqrt(c*x^6 + b*x^3 + a)
```

3.237.9 Mupad [B] (verification not implemented)

Time = 8.65 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.97

$$\int \frac{x^5}{(a+bx^3+cx^6)^{3/2}}\,dx = -\frac{2\,b\,x^3+4\,a}{(12\,a\,c-3\,b^2)\,\sqrt{c\,x^6+b\,x^3+a}}$$

input
```
int(x^5/(a + b*x^3 + c*x^6)^(3/2),x)
```

output
```
-(4*a + 2*b*x^3)/((12*a*c - 3*b^2)*(a + b*x^3 + c*x^6)^(1/2))
```

3.238 $\int \frac{x^2}{(a+bx^3+cx^6)^{3/2}}\,dx$

3.238.1 Optimal result

Integrand size = 20, antiderivative size = 38

$$\int \frac{x^2}{(a+bx^3+cx^6)^{3/2}}\,dx = -\frac{2(b+2cx^3)}{3\,(b^2-4ac)\,\sqrt{a+bx^3+cx^6}}$$

output
```
-2/3*(2*c*x^3+b)/(-4*a*c+b^2)/(c*x^6+b*x^3+a)^(1/2)
```

3.238.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(a+bx^3+cx^6)^{3/2}}\,dx = -\frac{2(b+2cx^3)}{3\,(b^2-4ac)\,\sqrt{a+bx^3+cx^6}}$$

input
```
Integrate[x^2/(a + b*x^3 + c*x^6)^(3/2),x]
```

output
```
(-2*(b + 2*c*x^3))/(3*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6])
```

3.238.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1690, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^3+cx^6)^{3/2}}\,dx$$

$$\Big\downarrow \text{1690}$$

$$\frac{1}{3}\int \frac{1}{(cx^6+bx^3+a)^{3/2}}dx^3$$

$$\Big\downarrow \text{1088}$$

$$-\frac{2(b+2cx^3)}{3(b^2-4ac)\sqrt{a+bx^3+cx^6}}$$

input
```
Int[x^2/(a + b*x^3 + c*x^6)^(3/2),x]
```

output
```
(-2*(b + 2*c*x^3))/(3*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6])
```

3.238.3.1 Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

3.238.4 Maple [A] (verified)

Time = 2.47 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

method	result	size
gosper	$\frac{\frac{4c\,x^{3}}{3}+\frac{2b}{3}}{\sqrt{c\,x^{6}+b\,x^{3}+a}\,(4ac-b^{2})}$	37
trager	$\frac{\frac{4c\,x^{3}}{3}+\frac{2b}{3}}{\sqrt{c\,x^{6}+b\,x^{3}+a}\,(4ac-b^{2})}$	37

input
```
int(x^2/(c*x^6+b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/(c*x^6+b*x^3+a)^(1/2)*(2*c*x^3+b)/(4*a*c-b^2)
```

3.238.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.76

$$\int \frac{x^2}{(a+bx^3+cx^6)^{3/2}}\,dx = -\frac{2\sqrt{cx^6+bx^3+a}(2\,cx^3+b)}{3\,((b^2c-4\,ac^2)x^6+(b^3-4\,abc)x^3+ab^2-4\,a^2c)}$$

input
```
integrate(x^2/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
-2/3*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)/((b^2*c - 4*a*c^2)*x^6 + (b^3 -
 4*a*b*c)*x^3 + a*b^2 - 4*a^2*c)
```

3.238.6 Sympy [F]

$$\int \frac{x^2}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{x^2}{(a+bx^3+cx^6)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2/(c*x**6+b*x**3+a)**(3/2),x)
```

output
```
Integral(x**2/(a + b*x**3 + c*x**6)**(3/2), x)
```

3.238.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(a+bx^3+cx^6)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.238.8 Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.18

$$\int \frac{x^2}{(a+bx^3+cx^6)^{3/2}}\,dx = -\frac{2\left(\frac{2\,cx^3}{b^2-4\,ac}+\frac{b}{b^2-4\,ac}\right)}{3\,\sqrt{cx^6+bx^3+a}}$$

input
```
integrate(x^2/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
-2/3*(2*c*x^3/(b^2 - 4*a*c) + b/(b^2 - 4*a*c))/sqrt(c*x^6 + b*x^3 + a)
```

3.238.9 Mupad [B] (verification not implemented)

Time = 8.68 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int \frac{x^2}{(a+bx^3+cx^6)^{3/2}}\,dx = \frac{4\,c\,x^3+2\,b}{(12\,a\,c-3\,b^2)\,\sqrt{c\,x^6+b\,x^3+a}}$$

input
```
int(x^2/(a + b*x^3 + c*x^6)^(3/2),x)
```

output
```
(2*b + 4*c*x^3)/((12*a*c - 3*b^2)*(a + b*x^3 + c*x^6)^(1/2))
```

3.239 $\int \frac{1}{x(a+bx^3+cx^6)^{3/2}} \, dx$

3.239.1 Optimal result

Integrand size = 20, antiderivative size = 92

$$\int \frac{1}{x(a+bx^3+cx^6)^{3/2}} \, dx = \frac{2(b^2-2ac+bcx^3)}{3a(b^2-4ac)\sqrt{a+bx^3+cx^6}} - \frac{\operatorname{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{3a^{3/2}}$$

output
```
-1/3*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2))/a^(3/2)+2/3*(b
*c*x^3-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^6+b*x^3+a)^(1/2)
```

3.239.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.99

$$\int \frac{1}{x(a+bx^3+cx^6)^{3/2}} \, dx = \frac{2\left(\frac{\sqrt{a}(b^2-2ac+bcx^3)}{(b^2-4ac)\sqrt{a+bx^3+cx^6}} + \operatorname{arctanh}\left(\frac{\sqrt{c}x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)\right)}{3a^{3/2}}$$

input
```
Integrate[1/(x*(a + b*x^3 + c*x^6)^(3/2)),x]
```

output
```
(2*((Sqrt[a]*(b^2 - 2*a*c + b*c*x^3))/((b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^
6]) + ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3 + c*x^6])/Sqrt[a]]))/(3*a^(3/2
))
```

3.239.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1693, 1165, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(a+bx^3+cx^6)^{3/2}}\,dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int \frac{1}{x^3(cx^6+bx^3+a)^{3/2}}dx^3$$

$$\Big\downarrow \text{1165}$$

$$\frac{1}{3}\left(\frac{2(-2ac+b^2+bcx^3)}{a(b^2-4ac)\sqrt{a+bx^3+cx^6}}-\frac{2\int -\frac{b^2-4ac}{2x^3\sqrt{cx^6+bx^3+a}}dx^3}{a(b^2-4ac)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{\int \frac{1}{x^3\sqrt{cx^6+bx^3+a}}dx^3}{a}+\frac{2(-2ac+b^2+bcx^3)}{a(b^2-4ac)\sqrt{a+bx^3+cx^6}}\right)$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{3}\left(\frac{2(-2ac+b^2+bcx^3)}{a(b^2-4ac)\sqrt{a+bx^3+cx^6}}-\frac{2\int \frac{1}{4a-x^6}d\frac{bx^3+2a}{\sqrt{cx^6+bx^3+a}}}{a}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{2(-2ac+b^2+bcx^3)}{a(b^2-4ac)\sqrt{a+bx^3+cx^6}}-\frac{\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{a^{3/2}}\right)$$

input
```
Int[1/(x*(a + b*x^3 + c*x^6)^(3/2)),x]
```

output
```
((2*(b^2 - 2*a*c + b*c*x^3))/(a*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6]) - A
rcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])]/a^(3/2))/3
```

3.239.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.239.4 Maple [F]

$$\int \frac{1}{x\left(c\,x^{6}+b\,x^{3}+a\right)^{\frac{3}{2}}}dx$$

input
```
int(1/x/(c*x^6+b*x^3+a)^(3/2),x)
```

output
```
int(1/x/(c*x^6+b*x^3+a)^(3/2),x)
```

3.239.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(78) = 156$.

Time = 0.30 (sec) , antiderivative size = 389, normalized size of antiderivative = 4.23

$$\int \frac{1}{x\left(a+bx^3+cx^6\right)^{3/2}}\,dx = \left[\frac{((b^2c-4\,ac^2)x^6+(b^3-4\,abc)x^3+ab^2-4\,a^2c)\sqrt{a}\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3-4\,\sqrt{}}{}\right.}{6\left((a^2b^2c-4\,a^3c^2)x^6+a^3b^2-4\,a\right.}\right.$$

input
```
integrate(1/x/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/6*(((b^2*c - 4*a*c^2)*x^6 + (b^3 - 4*a*b*c)*x^3 + a*b^2 - 4*a^2*c)*sqrt
(a)*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3
 + 2*a)*sqrt(a) + 8*a^2)/x^6) + 4*sqrt(c*x^6 + b*x^3 + a)*(a*b*c*x^3 + a*b
^2 - 2*a^2*c))/((a^2*b^2*c - 4*a^3*c^2)*x^6 + a^3*b^2 - 4*a^4*c + (a^2*b^3
 - 4*a^3*b*c)*x^3), 1/3*(((b^2*c - 4*a*c^2)*x^6 + (b^3 - 4*a*b*c)*x^3 + a*
b^2 - 4*a^2*c)*sqrt(-a)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*s
qrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*sqrt(c*x^6 + b*x^3 + a)*(a*b*c*x^3
+ a*b^2 - 2*a^2*c))/((a^2*b^2*c - 4*a^3*c^2)*x^6 + a^3*b^2 - 4*a^4*c + (a^
2*b^3 - 4*a^3*b*c)*x^3)]
```

3.239.6 Sympy [F]

$$\int \frac{1}{x\left(a+bx^3+cx^6\right)^{3/2}}\,dx = \int \frac{1}{x\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/(c*x**6+b*x**3+a)**(3/2),x)
```

output
```
Integral(1/(x*(a + b*x**3 + c*x**6)**(3/2)), x)
```

3.239.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x(a+bx^3+cx^6)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.239.8 Giac [F]

$$\int \frac{1}{x(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{1}{(cx^6+bx^3+a)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(1/x/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x), x)
```

3.239.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{1}{x\,(c\,x^6+b\,x^3+a)^{3/2}}\,dx$$

input
```
int(1/(x*(a + b*x^3 + c*x^6)^(3/2)),x)
```

output
```
int(1/(x*(a + b*x^3 + c*x^6)^(3/2)), x)
```

3.240 $\int \frac{1}{x^4(a+bx^3+cx^6)^{3/2}}\,dx$

3.240.1 Optimal result

Integrand size = 20, antiderivative size = 142

$$\int \frac{1}{x^4\left(a+bx^3+cx^6\right)^{3/2}}\,dx=\frac{2(b^2-2ac+bcx^3)}{3a\left(b^2-4ac\right)x^3\sqrt{a+bx^3+cx^6}}-\frac{\left(3b^2-8ac\right)\sqrt{a+bx^3+cx^6}}{3a^2\left(b^2-4ac\right)x^3}+\frac{b\,\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{2a^{5/2}}$$

output
```
1/2*b*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2))/a^(5/2)+2/3*(
b*c*x^3-2*a*c+b^2)/a/(-4*a*c+b^2)/x^3/(c*x^6+b*x^3+a)^(1/2)-1/3*(-8*a*c+3*
b^2)*(c*x^6+b*x^3+a)^(1/2)/a^2/(-4*a*c+b^2)/x^3
```

3.240.2 Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^4\left(a+bx^3+cx^6\right)^{3/2}}\,dx=\frac{-4a^2c+3b^2x^3(b+cx^3)+a(b^2-10bcx^3-8c^2x^6)}{3a^2\left(-b^2+4ac\right)x^3\sqrt{a+bx^3+cx^6}}-\frac{b\,\text{arctanh}\left(\frac{\sqrt{c}x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{a^{5/2}}$$

input
```
Integrate[1/(x^4*(a + b*x^3 + c*x^6)^(3/2)),x]
```

output
```
(-4*a^2*c + 3*b^2*x^3*(b + c*x^3) + a*(b^2 - 10*b*c*x^3 - 8*c^2*x^6))/(3*a
^2*(-b^2 + 4*a*c)*x^3*Sqrt[a + b*x^3 + c*x^6]) - (b*ArcTanh[(Sqrt[c]*x^3 -
 Sqrt[a + b*x^3 + c*x^6])/Sqrt[a]])/a^(5/2)
```

3.240.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1693, 1165, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \left(a+bx^3+cx^6\right)^{3/2}} \, dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3} \int \frac{1}{x^6 \left(cx^6+bx^3+a\right)^{3/2}}dx^3$$

$$\Big\downarrow \text{1165}$$

$$\frac{1}{3} \left(\frac{2\left(-2ac+b^2+bcx^3\right)}{ax^3 \left(b^2-4ac\right) \sqrt{a+bx^3+cx^6}}-\frac{2 \int -\frac{2bcx^3+3b^2-8ac}{2x^6\sqrt{cx^6+bx^3+a}}dx^3}{a \left(b^2-4ac\right)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3} \left(\frac{\int \frac{2bcx^3+3b^2-8ac}{x^6\sqrt{cx^6+bx^3+a}}dx^3}{a \left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx^3\right)}{ax^3 \left(b^2-4ac\right) \sqrt{a+bx^3+cx^6}}\right)$$

$$\Big\downarrow \text{1228}$$

$$\frac{1}{3} \left(\frac{-\frac{3b\left(b^2-4ac\right) \int \frac{1}{x^3\sqrt{cx^6+bx^3+a}}dx^3}{2a}-\frac{\left(3b^2-8ac\right)\sqrt{a+bx^3+cx^6}}{ax^3}}{a \left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx^3\right)}{ax^3 \left(b^2-4ac\right) \sqrt{a+bx^3+cx^6}}\right)$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{3} \left(\frac{\frac{3b\left(b^2-4ac\right) \int \frac{1}{4a-x^6}d\frac{bx^3+2a}{\sqrt{cx^6+bx^3+a}}}{a}-\frac{\left(3b^2-8ac\right)\sqrt{a+bx^3+cx^6}}{ax^3}}{a \left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx^3\right)}{ax^3 \left(b^2-4ac\right) \sqrt{a+bx^3+cx^6}}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{\frac{3b(b^2-4ac)\operatorname{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{2a^{3/2}}-\frac{(3b^2-8ac)\sqrt{a+bx^3+cx^6}}{ax^3}}{a(b^2-4ac)}+\frac{2(-2ac+b^2+bcx^3)}{ax^3(b^2-4ac)\sqrt{a+bx^3+cx^6}}\right)$$

input
```
Int[1/(x^4*(a + b*x^3 + c*x^6)^(3/2)),x]
```

output
```
((2*(b^2 - 2*a*c + b*c*x^3))/(a*(b^2 - 4*a*c)*x^3*Sqrt[a + b*x^3 + c*x^6])
 + (-(((3*b^2 - 8*a*c)*Sqrt[a + b*x^3 + c*x^6])/(a*x^3)) + (3*b*(b^2 - 4*a
*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(2*a^(3/2)
))/(a*(b^2 - 4*a*c)))/3
```

3.240.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.240.4 Maple [F]

$$\int \frac{1}{x^{4} \left(c\,x^{6}+b\,x^{3}+a\right)^{\frac{3}{2}}}d x$$

input
```
int(1/x^4/(c*x^6+b*x^3+a)^(3/2),x)
```

output
```
int(1/x^4/(c*x^6+b*x^3+a)^(3/2),x)
```

3.240.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 485, normalized size of antiderivative = 3.42

$$\int \frac{1}{x^4 \left(a+bx^3+cx^6\right)^{3/2}}\,dx = \left[\frac{3\left((b^3c-4\,abc^2)x^9+(b^4-4\,ab^2c)x^6+(ab^3-4\,a^2bc)x^3\right)\sqrt{a}\log\left(-\frac{(b^2+4\,ac)x^6 \cdots}{\cdots}\right) \cdots}{12\left((a^3b^2c-\cdots\right)\cdots}\right.$$
$$-\frac{3\left((b^3c-4\,abc^2)x^9+(b^4-4\,ab^2c)x^6+(ab^3-4\,a^2bc)x^3\right)\sqrt{-a}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\,(bx^3+2\,a)\sqrt{-a}}{2\,(acx^6+abx^3+a^2)}\right)+2\left((3\,ab^2c \cdots\right.}{6\left((a^3b^2c-4\,a^4c^2)x^9+(a^3b^3-4\,a^4bc)x^6+(a^4b^2-\cdots\right)\cdots}$$

input
```
integrate(1/x^4/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/12*(3*((b^3*c - 4*a*b*c^2)*x^9 + (b^4 - 4*a*b^2*c)*x^6 + (a*b^3 - 4*a^2
*b*c)*x^3)*sqrt(a)*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 + 4*sqrt(c*x^6 + b*
x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) - 4*((3*a*b^2*c - 8*a^2*c^2)*
x^6 + a^2*b^2 - 4*a^3*c + (3*a*b^3 - 10*a^2*b*c)*x^3)*sqrt(c*x^6 + b*x^3 +
 a))/((a^3*b^2*c - 4*a^4*c^2)*x^9 + (a^3*b^3 - 4*a^4*b*c)*x^6 + (a^4*b^2 -
 4*a^5*c)*x^3), -1/6*(3*((b^3*c - 4*a*b*c^2)*x^9 + (b^4 - 4*a*b^2*c)*x^6 +
 (a*b^3 - 4*a^2*b*c)*x^3)*sqrt(-a)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x
^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((3*a*b^2*c - 8*a^2*c^2)
*x^6 + a^2*b^2 - 4*a^3*c + (3*a*b^3 - 10*a^2*b*c)*x^3)*sqrt(c*x^6 + b*x^3
+ a))/((a^3*b^2*c - 4*a^4*c^2)*x^9 + (a^3*b^3 - 4*a^4*b*c)*x^6 + (a^4*b^2
- 4*a^5*c)*x^3)]
```

3.240.6 Sympy [F]

$$\int \frac{1}{x^4 (a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{1}{x^4 (a+bx^3+cx^6)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**4/(c*x**6+b*x**3+a)**(3/2),x)
```

output
```
Integral(1/(x**4*(a + b*x**3 + c*x**6)**(3/2)), x)
```

3.240.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^4 (a+bx^3+cx^6)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x^4/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.240.8 Giac [F]

$$\int \frac{1}{x^4 (a + bx^3 + cx^6)^{3/2}} \, dx = \int \frac{1}{(cx^6 + bx^3 + a)^{\frac{3}{2}} x^4} \, dx$$

input
```
integrate(1/x^4/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x^4), x)
```

3.240.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a + bx^3 + cx^6)^{3/2}} \, dx = \int \frac{1}{x^4 (c\,x^6 + b\,x^3 + a)^{3/2}} \, dx$$

input
```
int(1/(x^4*(a + b*x^3 + c*x^6)^(3/2)),x)
```

output
```
int(1/(x^4*(a + b*x^3 + c*x^6)^(3/2)), x)
```

3.241 $\int \frac{1}{x^7\left(a+bx^3+cx^6\right)^{3/2}}\,dx$

3.241.1 Optimal result

Integrand size = 20, antiderivative size = 198

$$\begin{aligned}\int \frac{1}{x^7\left(a+bx^3+cx^6\right)^{3/2}}\,dx &= \frac{2(b^2-2ac+bcx^3)}{3a\left(b^2-4ac\right)x^6\sqrt{a+bx^3+cx^6}}\\ &\quad -\frac{\left(5b^2-12ac\right)\sqrt{a+bx^3+cx^6}}{6a^2\left(b^2-4ac\right)x^6}+\frac{b(15b^2-52ac)\sqrt{a+bx^3+cx^6}}{12a^3\left(b^2-4ac\right)x^3}\\ &\quad -\frac{\left(5b^2-4ac\right)\operatorname{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{8a^{7/2}}\end{aligned}$$

output
```
-1/8*(-4*a*c+5*b^2)*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2))
/a^(7/2)+2/3*(b*c*x^3-2*a*c+b^2)/a/(-4*a*c+b^2)/x^6/(c*x^6+b*x^3+a)^(1/2)-
1/6*(-12*a*c+5*b^2)*(c*x^6+b*x^3+a)^(1/2)/a^2/(-4*a*c+b^2)/x^6+1/12*b*(-52
*a*c+15*b^2)*(c*x^6+b*x^3+a)^(1/2)/a^3/(-4*a*c+b^2)/x^3
```

3.241.2 Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.84

$$\begin{aligned}\int \frac{1}{x^7\left(a+bx^3+cx^6\right)^{3/2}}\,dx &= \frac{-8a^3c-15b^3x^6(b+cx^3)+2a^2(b^2+10bcx^3-12c^2x^6)+abx^3(-5b^2+62bcx^3}{12a^3\left(-b^2+4ac\right)x^6\sqrt{a+bx^3+cx^6}}\\ &\quad +\frac{\left(5b^2-4ac\right)\operatorname{arctanh}\left(\frac{\sqrt{c}x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{4a^{7/2}}\end{aligned}$$

input
```
Integrate[1/(x^7*(a + b*x^3 + c*x^6)^(3/2)),x]
```

output
```
(-8*a^3*c - 15*b^3*x^6*(b + c*x^3) + 2*a^2*(b^2 + 10*b*c*x^3 - 12*c^2*x^6)
 + a*b*x^3*(-5*b^2 + 62*b*c*x^3 + 52*c^2*x^6))/(12*a^3*(-b^2 + 4*a*c)*x^6*
Sqrt[a + b*x^3 + c*x^6]) + ((5*b^2 - 4*a*c)*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a
+ b*x^3 + c*x^6])/Sqrt[a]])/(4*a^(7/2))
```

3.241.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1693, 1165, 27, 1237, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7 \left(a+bx^3+cx^6\right)^{3/2}} \, dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3} \int \frac{1}{x^9 \left(cx^6+bx^3+a\right)^{3/2}}dx^3$$

$$\Big\downarrow \text{1165}$$

$$\frac{1}{3} \left(\frac{2\left(-2ac+b^2+bcx^3\right)}{ax^6 \left(b^2-4ac\right) \sqrt{a+bx^3+cx^6}}-\frac{2 \int -\frac{4bcx^3+5b^2-12ac}{2x^9 \sqrt{cx^6+bx^3+a}}dx^3}{a \left(b^2-4ac\right)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3} \left(\frac{\int \frac{4bcx^3+5b^2-12ac}{x^9 \sqrt{cx^6+bx^3+a}}dx^3}{a \left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx^3\right)}{ax^6 \left(b^2-4ac\right) \sqrt{a+bx^3+cx^6}}\right)$$

$$\Big\downarrow \text{1237}$$

$$\frac{1}{3} \left(\frac{-\frac{\int \frac{2c\left(5b^2-12ac\right)x^3+b\left(15b^2-52ac\right)}{2x^6 \sqrt{cx^6+bx^3+a}}dx^3}{2a}-\frac{\left(5b^2-12ac\right)\sqrt{a+bx^3+cx^6}}{2ax^6}}{a \left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx^3\right)}{ax^6 \left(b^2-4ac\right) \sqrt{a+bx^3+cx^6}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{-\frac{\int \frac{2c\left(5b^2-12ac\right)x^3+b\left(15b^2-52ac\right)}{x^6\sqrt{cx^6+bx^3+a}}dx^3}{4a}-\frac{(5b^2-12ac)\sqrt{a+bx^3+cx^6}}{2ax^6}}{a\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx^3\right)}{ax^6\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}\right)$$

$$\Big\downarrow \; 1228$$

$$\frac{1}{3}\left(\frac{-\frac{-\frac{3\left(b^2-4ac\right)\left(5b^2-4ac\right)\int\frac{1}{x^3\sqrt{cx^6+bx^3+a}}dx^3}{2a}-\frac{b\left(15b^2-52ac\right)\sqrt{a+bx^3+cx^6}}{ax^3}}{4a}-\frac{(5b^2-12ac)\sqrt{a+bx^3+cx^6}}{2ax^6}}{a\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx^3\right)}{ax^6\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}\right)$$

$$\Big\downarrow \; 1154$$

$$\frac{1}{3}\left(\frac{-\frac{-\frac{3\left(b^2-4ac\right)\left(5b^2-4ac\right)\int\frac{1}{4a-x^6}d\frac{bx^3+2a}{\sqrt{cx^6+bx^3+a}}}{a}-\frac{b\left(15b^2-52ac\right)\sqrt{a+bx^3+cx^6}}{ax^3}}{4a}-\frac{(5b^2-12ac)\sqrt{a+bx^3+cx^6}}{2ax^6}}{a\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx^3\right)}{ax^6\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}\right)$$

$$\Big\downarrow \; 219$$

$$\frac{1}{3}\left(\frac{-\frac{-\frac{3\left(b^2-4ac\right)\left(5b^2-4ac\right)\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{2a^{3/2}}-\frac{b\left(15b^2-52ac\right)\sqrt{a+bx^3+cx^6}}{ax^3}}{4a}-\frac{(5b^2-12ac)\sqrt{a+bx^3+cx^6}}{2ax^6}}{a\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx^3\right)}{ax^6\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}\right)$$

input
```
Int[1/(x^7*(a + b*x^3 + c*x^6)^(3/2)),x]
```

output
```
((2*(b^2 - 2*a*c + b*c*x^3))/(a*(b^2 - 4*a*c)*x^6*Sqrt[a + b*x^3 + c*x^6])
 + (-1/2*((5*b^2 - 12*a*c)*Sqrt[a + b*x^3 + c*x^6])/(a*x^6) - (-((b*(15*b^
2 - 52*a*c)*Sqrt[a + b*x^3 + c*x^6])/(a*x^3)) + (3*(b^2 - 4*a*c)*(5*b^2 -
4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(2*a^(3
/2)))/(4*a))/(a*(b^2 - 4*a*c)))/3
```

3.241. $\int \frac{1}{x^7\left(a+bx^3+cx^6\right)^{3/2}}\,dx$

3.241.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

3.241. $\int \frac{1}{x^7(a+bx^3+cx^6)^{3/2}}\,dx$

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.241.4 Maple [F]

$$\int \frac{1}{x^{7} \left(c\,x^{6}+b\,x^{3}+a \right)^{\frac{3}{2}}}d x$$

input
```
int(1/x^7/(c*x^6+b*x^3+a)^(3/2),x)
```

output
```
int(1/x^7/(c*x^6+b*x^3+a)^(3/2),x)
```

3.241.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 615, normalized size of antiderivative = 3.11

$$\int \frac{1}{x^7 \left(a+bx^3+cx^6\right)^{3/2}}\,dx = \left[-\frac{3\,((5\,b^4c-24\,ab^2c^2+16\,a^2c^3)x^{12}+(5\,b^5-24\,ab^3c+16\,a^2bc^2)x^9+(5\,ab^4\right.$$

input
```
integrate(1/x^7/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/48*(3*((5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*x^12 + (5*b^5 - 24*a*b^3*
c + 16*a^2*b*c^2)*x^9 + (5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2)*x^6)*sqrt(a)
*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 + 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 +
2*a)*sqrt(a) + 8*a^2)/x^6) - 4*((15*a*b^3*c - 52*a^2*b*c^2)*x^9 + (15*a*b^
4 - 62*a^2*b^2*c + 24*a^3*c^2)*x^6 - 2*a^3*b^2 + 8*a^4*c + 5*(a^2*b^3 - 4*
a^3*b*c)*x^3)*sqrt(c*x^6 + b*x^3 + a))/((a^4*b^2*c - 4*a^5*c^2)*x^12 + (a^
4*b^3 - 4*a^5*b*c)*x^9 + (a^5*b^2 - 4*a^6*c)*x^6), 1/24*(3*((5*b^4*c - 24*
a*b^2*c^2 + 16*a^2*c^3)*x^12 + (5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)*x^9 + (
5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2)*x^6)*sqrt(-a)*arctan(1/2*sqrt(c*x^6 +
 b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((15*a*b
^3*c - 52*a^2*b*c^2)*x^9 + (15*a*b^4 - 62*a^2*b^2*c + 24*a^3*c^2)*x^6 - 2*
a^3*b^2 + 8*a^4*c + 5*(a^2*b^3 - 4*a^3*b*c)*x^3)*sqrt(c*x^6 + b*x^3 + a))/
((a^4*b^2*c - 4*a^5*c^2)*x^12 + (a^4*b^3 - 4*a^5*b*c)*x^9 + (a^5*b^2 - 4*a
^6*c)*x^6)]
```

3.241.6 Sympy [F]

$$\int \frac{1}{x^7 (a + bx^3 + cx^6)^{3/2}} \, dx = \int \frac{1}{x^7 (a + bx^3 + cx^6)^{\frac{3}{2}}} \, dx$$

input
```
integrate(1/x**7/(c*x**6+b*x**3+a)**(3/2),x)
```

output
```
Integral(1/(x**7*(a + b*x**3 + c*x**6)**(3/2)), x)
```

3.241.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^7 (a + bx^3 + cx^6)^{3/2}} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x^7/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.241.8 Giac [F]

$$\int \frac{1}{x^7 (a + bx^3 + cx^6)^{3/2}} \, dx = \int \frac{1}{(cx^6 + bx^3 + a)^{\frac{3}{2}} x^7} \, dx$$

input
```
integrate(1/x^7/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x^7), x)
```

3.241.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^7 \left(a+bx^3+cx^6\right)^{3/2}}\,dx=\int \frac{1}{x^7\,{\left(c\,x^6+b\,x^3+a\right)}^{3/2}} \,dx$$

input
```
int(1/(x^7*(a + b*x^3 + c*x^6)^(3/2)),x)
```

output
```
int(1/(x^7*(a + b*x^3 + c*x^6)^(3/2)), x)
```

3.242 $\int \frac{1}{x^{10}\left(a+bx^3+cx^6\right)^{3/2}}\,dx$

3.242.1 Optimal result

Integrand size = 20, antiderivative size = 256

$$\int \frac{1}{x^{10}\left(a+bx^3+cx^6\right)^{3/2}}\,dx=\frac{2(b^2-2ac+bcx^3)}{3a\left(b^2-4ac\right)x^9\sqrt{a+bx^3+cx^6}}-\frac{\left(7b^2-16ac\right)\sqrt{a+bx^3+cx^6}}{9a^2\left(b^2-4ac\right)x^9}+\frac{b(35b^2-116ac)\sqrt{a+bx^3+cx^6}}{36a^3\left(b^2-4ac\right)x^6}-\frac{\left(105b^4-460ab^2c+256a^2c^2\right)\sqrt{a+bx^3+cx^6}}{72a^4\left(b^2-4ac\right)x^3}+\frac{5b(7b^2-12ac)\,\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{48a^{9/2}}$$

output
```
5/48*b*(-12*a*c+7*b^2)*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/
2))/a^(9/2)+2/3*(b*c*x^3-2*a*c+b^2)/a/(-4*a*c+b^2)/x^9/(c*x^6+b*x^3+a)^(1/
2)-1/9*(-16*a*c+7*b^2)*(c*x^6+b*x^3+a)^(1/2)/a^2/(-4*a*c+b^2)/x^9+1/36*b*(
-116*a*c+35*b^2)*(c*x^6+b*x^3+a)^(1/2)/a^3/(-4*a*c+b^2)/x^6-1/72*(256*a^2*
c^2-460*a*b^2*c+105*b^4)*(c*x^6+b*x^3+a)^(1/2)/a^4/(-4*a*c+b^2)/x^3
```

3.242.2 Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^{10} \left(a+bx^3+cx^6\right)^{3/2}} \, dx = \frac{-32a^4c+105b^4x^9(b+cx^3)+5ab^2x^6(7b^2-106bcx^3-92c^2x^6)+8a^3(b^2+7b}{72a^4\left(-b^2+4ac\right)x^9\sqrt{a+}} + \frac{5b(-7b^2+12ac)\,\text{arctanh}\left(\frac{\sqrt{c}x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{24a^{9/2}}$$

input
```
Integrate[1/(x^10*(a + b*x^3 + c*x^6)^(3/2)),x]
```

output
```
(-32*a^4*c + 105*b^4*x^9*(b + c*x^3) + 5*a*b^2*x^6*(7*b^2 - 106*b*c*x^3 -
92*c^2*x^6) + 8*a^3*(b^2 + 7*b*c*x^3 + 16*c^2*x^6) + 2*a^2*x^3*(-7*b^3 - 8
6*b^2*c*x^3 + 244*b*c^2*x^6 + 128*c^3*x^9))/(72*a^4*(-b^2 + 4*a*c)*x^9*Sqr
t[a + b*x^3 + c*x^6]) + (5*b*(-7*b^2 + 12*a*c)*ArcTanh[(Sqrt[c]*x^3 - Sqrt
[a + b*x^3 + c*x^6])/Sqrt[a]])/(24*a^(9/2))
```

3.242.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1693, 1165, 27, 1237, 27, 1237, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{10} \left(a+bx^3+cx^6\right)^{3/2}} \, dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3} \int \frac{1}{x^{12} \left(cx^6+bx^3+a\right)^{3/2}} dx^3$$

$$\Big\downarrow \text{1165}$$

$$\frac{1}{3} \left(\frac{2\left(-2ac+b^2+bcx^3\right)}{ax^9\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}} - \frac{2\int -\frac{6bcx^3+7b^2-16ac}{2x^{12}\sqrt{cx^6+bx^3+a}}dx^3}{a\left(b^2-4ac\right)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{\int \frac{6bcx^3+7b^2-16ac}{x^{12}\sqrt{cx^6+bx^3+a}}dx^3}{a\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx^3\right)}{ax^9\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}\right)$$

$$\Big\downarrow \text{ 1237}$$

$$\frac{1}{3}\left(\frac{-\frac{\int -\frac{4c\left(7b^2-16ac\right)x^3+b\left(35b^2-116ac\right)}{2x^9\sqrt{cx^6+bx^3+a}}dx^3}{3a}-\frac{\left(7b^2-16ac\right)\sqrt{a+bx^3+cx^6}}{3ax^9}}{a\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx^3\right)}{ax^9\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{3}\left(\frac{-\frac{\int \frac{4c\left(7b^2-16ac\right)x^3+b\left(35b^2-116ac\right)}{x^9\sqrt{cx^6+bx^3+a}}dx^3}{6a}-\frac{\left(7b^2-16ac\right)\sqrt{a+bx^3+cx^6}}{3ax^9}}{a\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx^3\right)}{ax^9\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}\right)$$

$$\Big\downarrow \text{ 1237}$$

$$\frac{1}{3}\left(\frac{-\frac{-\frac{\int \frac{105b^4-460acb^2+2c\left(35b^2-116ac\right)x^3b+256a^2c^2}{2x^6\sqrt{cx^6+bx^3+a}}dx^3}{2a}-\frac{b\left(35b^2-116ac\right)\sqrt{a+bx^3+cx^6}}{2ax^6}}{6a}-\frac{\left(7b^2-16ac\right)\sqrt{a+bx^3+cx^6}}{3ax^9}}{a\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2 \cdots\right.}{ax^9\left(b^2-4ac\right)\sqrt{a \cdots}}\right.$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{3}\left(\frac{-\frac{-\frac{\int \frac{105b^4-460acb^2+2c\left(35b^2-116ac\right)x^3b+256a^2c^2}{x^6\sqrt{cx^6+bx^3+a}}dx^3}{4a}-\frac{b\left(35b^2-116ac\right)\sqrt{a+bx^3+cx^6}}{2ax^6}}{6a}-\frac{\left(7b^2-16ac\right)\sqrt{a+bx^3+cx^6}}{3ax^9}}{a\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2 \cdots\right.}{ax^9\left(b^2-4ac\right)\sqrt{a \cdots}}\right.$$

$$\Big\downarrow \text{ 1228}$$

$$\frac{1}{3}\left(\frac{-\frac{-\frac{-\frac{15b\left(7b^2-12ac\right)\left(b^2-4ac\right)\int \frac{1}{x^3\sqrt{cx^6+bx^3+a}}dx^3}{2a}-\frac{\left(256a^2c^2-460ab^2c+105b^4\right)\sqrt{a+bx^3+cx^6}}{ax^3}}{4a}-\frac{b\left(35b^2-116ac\right)\sqrt{a+bx^3+cx^6}}{2ax^6}}{6a}-\frac{\left(7b^2-16ac\right)\sqrt{a+ \cdots}}{3ax^9}}{a\left(b^2-4ac\right)}\right.$$

$$\Big\downarrow \text{ 1154}$$

3.242. $\int \frac{1}{x^{10}\left(a+bx^3+cx^6\right)^{3/2}}\,dx$

$$\frac{1}{3}\left(\frac{-\frac{-\frac{-\frac{15b\left(7b^2-12ac\right)\left(b^2-4ac\right)\int \frac{1}{4a-x^6}d\frac{bx^3+2a}{\sqrt{cx^6+bx^3+a}}}{a}-\frac{\left(256a^2c^2-460ab^2c+105b^4\right)\sqrt{a+bx^3+cx^6}}{ax^3}}{4a}-\frac{b\left(35b^2-116ac\right)\sqrt{a+bx^3+cx^6}}{2ax^6}}{6a}-\frac{\left(7b^2-16ac\right)\sqrt{a-}}{3ax^9}}{a\left(b^2-4ac\right)}\right.$$

$\downarrow$ 219

$$\frac{1}{3}\left(\frac{-\frac{-\frac{-\frac{15b\left(7b^2-12ac\right)\left(b^2-4ac\right)\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{2a^{3/2}}-\frac{\left(256a^2c^2-460ab^2c+105b^4\right)\sqrt{a+bx^3+cx^6}}{ax^3}}{4a}-\frac{b\left(35b^2-116ac\right)\sqrt{a+bx^3+cx^6}}{2ax^6}}{6a}-\frac{\left(7b^2-16a\right.}{}}{a\left(b^2-4ac\right)}\right.$$

input
```
Int[1/(x^10*(a + b*x^3 + c*x^6)^(3/2)),x]
```

output
```
((2*(b^2 - 2*a*c + b*c*x^3))/(a*(b^2 - 4*a*c)*x^9*Sqrt[a + b*x^3 + c*x^6])
 + (-1/3*((7*b^2 - 16*a*c)*Sqrt[a + b*x^3 + c*x^6])/(a*x^9) - (-1/2*(b*(35
*b^2 - 116*a*c)*Sqrt[a + b*x^3 + c*x^6])/(a*x^6) - (-(((105*b^4 - 460*a*b^
2*c + 256*a^2*c^2)*Sqrt[a + b*x^3 + c*x^6])/(a*x^3)) + (15*b*(7*b^2 - 12*a
*c)*(b^2 - 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6]
)])/(2*a^(3/2)))/(4*a))/(6*a))/(a*(b^2 - 4*a*c)))/3
```

3.242.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.242.4 Maple [F]

$$\int \frac{1}{x^{10} \left(c\,x^{6}+b\,x^{3}+a\right)^{\frac{3}{2}}}dx$$

input
```
int(1/x^10/(c*x^6+b*x^3+a)^(3/2),x)
```

output
```
int(1/x^10/(c*x^6+b*x^3+a)^(3/2),x)
```

3.242.5 Fricas [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 705, normalized size of antiderivative = 2.75

$$\int \frac{1}{x^{10}\left(a+bx^3+cx^6\right)^{3/2}}\,dx = \left[-\frac{15\left((7\,b^5c-40\,ab^3c^2+48\,a^2bc^3)x^{15}+(7\,b^6-40\,ab^4c+48\,a^2b^2c^2)x^{12}+(7\right.}{}\right.$$

$$-\frac{15\left((7\,b^5c-40\,ab^3c^2+48\,a^2bc^3)x^{15}+(7\,b^6-40\,ab^4c+48\,a^2b^2c^2)x^{12}+(7\,ab^5-40\,a^2b^3c+48\,a^3bc^2)x^9\right)\sqrt{-}}{}$$

input
```
integrate(1/x^10/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/288*(15*((7*b^5*c - 40*a*b^3*c^2 + 48*a^2*b*c^3)*x^15 + (7*b^6 - 40*a*
b^4*c + 48*a^2*b^2*c^2)*x^12 + (7*a*b^5 - 40*a^2*b^3*c + 48*a^3*b*c^2)*x^9
)*sqrt(a)*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*
(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) + 4*((105*a*b^4*c - 460*a^2*b^2*c^2 +
256*a^3*c^3)*x^12 + (105*a*b^5 - 530*a^2*b^3*c + 488*a^3*b*c^2)*x^9 + (35*
a^2*b^4 - 172*a^3*b^2*c + 128*a^4*c^2)*x^6 + 8*a^4*b^2 - 32*a^5*c - 14*(a^
3*b^3 - 4*a^4*b*c)*x^3)*sqrt(c*x^6 + b*x^3 + a))/((a^5*b^2*c - 4*a^6*c^2)*
x^15 + (a^5*b^3 - 4*a^6*b*c)*x^12 + (a^6*b^2 - 4*a^7*c)*x^9), -1/144*(15*(
(7*b^5*c - 40*a*b^3*c^2 + 48*a^2*b*c^3)*x^15 + (7*b^6 - 40*a*b^4*c + 48*a^
2*b^2*c^2)*x^12 + (7*a*b^5 - 40*a^2*b^3*c + 48*a^3*b*c^2)*x^9)*sqrt(-a)*ar
ctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3
 + a^2)) + 2*((105*a*b^4*c - 460*a^2*b^2*c^2 + 256*a^3*c^3)*x^12 + (105*a*
b^5 - 530*a^2*b^3*c + 488*a^3*b*c^2)*x^9 + (35*a^2*b^4 - 172*a^3*b^2*c + 1
28*a^4*c^2)*x^6 + 8*a^4*b^2 - 32*a^5*c - 14*(a^3*b^3 - 4*a^4*b*c)*x^3)*sqr
t(c*x^6 + b*x^3 + a))/((a^5*b^2*c - 4*a^6*c^2)*x^15 + (a^5*b^3 - 4*a^6*b*c
)*x^12 + (a^6*b^2 - 4*a^7*c)*x^9)]
```

3.242.6 Sympy [F]

$$\int \frac{1}{x^{10}\left(a+bx^3+cx^6\right)^{3/2}}\,dx = \int \frac{1}{x^{10}\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**10/(c*x**6+b*x**3+a)**(3/2),x)
```

output
```
Integral(1/(x**10*(a + b*x**3 + c*x**6)**(3/2)), x)
```

3.242.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^{10}\left(a+bx^3+cx^6\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x^10/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.242.8 Giac [F]

$$\int \frac{1}{x^{10}\left(a+bx^3+cx^6\right)^{3/2}}\,dx = \int \frac{1}{\left(cx^6+bx^3+a\right)^{\frac{3}{2}}x^{10}}\,dx$$

input
```
integrate(1/x^10/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x^10), x)
```

3.242.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{10}\left(a+bx^3+cx^6\right)^{3/2}}\,dx = \int \frac{1}{x^{10}\left(c\,x^6+b\,x^3+a\right)^{3/2}}\,dx$$

input
```
int(1/(x^10*(a + b*x^3 + c*x^6)^(3/2)),x)
```

output
```
int(1/(x^10*(a + b*x^3 + c*x^6)^(3/2)), x)
```

3.243 $\int \frac{x^3}{(a+bx^3+cx^6)^{3/2}} \, dx$

3.243.1 Optimal result

Integrand size = 20, antiderivative size = 143

$$\int \frac{x^3}{(a+bx^3+cx^6)^{3/2}} \, dx = \frac{x^4\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(\frac{4}{3},\frac{3}{2},\frac{3}{2},\frac{7}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4a\sqrt{a+bx^3+cx^6}}$$

output
```
1/4*x^4*AppellF1(4/3,3/2,3/2,7/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/
(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^
3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/(c*x^6+b*x^3+a)^(1/2)
```

3.243.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 340 vs. 2(143) = 286.

Time = 10.31 (sec) , antiderivative size = 340, normalized size of antiderivative = 2.38

$$\int \frac{x^3}{(a+bx^3+cx^6)^{3/2}} \, dx = \frac{x\left(-2(b+2cx^3)+2b\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\frac{\cdots}{b+\cdots}\right.\right.\cdots}{3\cdots}$$

input
```
Integrate[x^3/(a + b*x^3 + c*x^6)^(3/2),x]
```

output
```
(x*(-2*(b + 2*c*x^3) + 2*b*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqr
t[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*
a*c])]*AppellF1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2
*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + c*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*
c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b
+ Sqrt[b^2 - 4*a*c])]*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt[b^
2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]))/(3*(b^2 - 4*a*c)*Sqrt[a
 + b*x^3 + c*x^6])
```

3.243.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx^3+cx^6)^{3/2}}\,dx$$

$$\Big\downarrow \text{1721}$$

$$\frac{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\int \frac{x^3}{\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{3/2}\left(\frac{2cx^3}{b+\sqrt{b^2-4ac}}+1\right)^{3/2}}dx}{a\sqrt{a+bx^3+cx^6}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^4\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{4}{3},\frac{3}{2},\frac{3}{2},\frac{7}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4a\sqrt{a+bx^3+cx^6}}$$

input
```
Int[x^3/(a + b*x^3 + c*x^6)^(3/2),x]
```

output
```
(x^4*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + S
qrt[b^2 - 4*a*c])]*AppellF1[4/3, 3/2, 3/2, 7/3, (-2*c*x^3)/(b - Sqrt[b^2 -
 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(4*a*Sqrt[a + b*x^3 + c*x^6
])
```

3.243.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

3.243.4 Maple [F]

$$\int \frac{x^3}{(c\,x^6+b\,x^3+a)^{\frac{3}{2}}}dx$$

input
```
int(x^3/(c*x^6+b*x^3+a)^(3/2),x)
```

output
```
int(x^3/(c*x^6+b*x^3+a)^(3/2),x)
```

3.243.5 Fricas [F]

$$\int \frac{x^3}{(a+bx^3+cx^6)^{3/2}}\,dx=\int \frac{x^3}{(cx^6+bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^6 + b*x^3 + a)*x^3/(c^2*x^12 + 2*b*c*x^9 + (b^2 + 2*a*c)
*x^6 + 2*a*b*x^3 + a^2), x)
```

3.243.6 Sympy [F]

$$\int \frac{x^3}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{x^3}{(a+bx^3+cx^6)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3/(c*x**6+b*x**3+a)**(3/2),x)
```

output
```
Integral(x**3/(a + b*x**3 + c*x**6)**(3/2), x)
```

3.243.7 Maxima [F]

$$\int \frac{x^3}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{x^3}{(cx^6+bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^3/(c*x^6 + b*x^3 + a)^(3/2), x)
```

3.243.8 Giac [F]

$$\int \frac{x^3}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{x^3}{(cx^6+bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^3/(c*x^6 + b*x^3 + a)^(3/2), x)
```

3.243.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\left(a+bx^3+cx^6\right)^{3/2}}\,dx = \int \frac{x^3}{\left(c\,x^6+b\,x^3+a\right)^{3/2}}\,dx$$

input
```
int(x^3/(a + b*x^3 + c*x^6)^(3/2),x)
```

output
```
int(x^3/(a + b*x^3 + c*x^6)^(3/2), x)
```

3.244 $\int \frac{x}{\left(a+bx^3+cx^6\right)^{3/2}}\,dx$

3.244.1 Optimal result

Integrand size = 18, antiderivative size = 143

$$\int \frac{x}{\left(a+bx^3+cx^6\right)^{3/2}}\,dx=\frac{x^2\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{2}{3},\frac{3}{2},\frac{3}{2},\frac{5}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2a\sqrt{a+bx^3+cx^6}}$$

output
```
1/2*x^2*AppellF1(2/3,3/2,3/2,5/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/
(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^
3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/(c*x^6+b*x^3+a)^(1/2)
```

3.244.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 362 vs. 2(143) = 286.

Time = 10.32 (sec) , antiderivative size = 362, normalized size of antiderivative = 2.53

$$\int \frac{x}{\left(a+bx^3+cx^6\right)^{3/2}}\,dx=\frac{x^2\left(-20(b^2-2ac+bcx^3)+5(b^2+4ac)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}\,\text{Appell}\right.}{}$$

input
```
Integrate[x/(a + b*x^3 + c*x^6)^(3/2),x]
```

output
```
(x^2*(-20*(b^2 - 2*a*c + b*c*x^3) + 5*(b^2 + 4*a*c)*Sqrt[(b - Sqrt[b^2 - 4
*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*
c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[2/3, 1/2, 1/2, 5/3, (-2*c*x^3)/(b
 + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + 8*b*c*x^3*Sqr
t[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqr
t[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/3, 1/2, 1/2,
 8/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c
])]))/(30*a*(-b^2 + 4*a*c)*Sqrt[a + b*x^3 + c*x^6])
```

3.244.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(a+bx^3+cx^6\right)^{3/2}}dx$$

$$\Big\downarrow \text{1721}$$

$$\frac{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\int \frac{x}{\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{3/2}\left(\frac{2cx^3}{b+\sqrt{b^2-4ac}}+1\right)^{3/2}}dx}{a\sqrt{a+bx^3+cx^6}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^2\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{2}{3},\frac{3}{2},\frac{3}{2},\frac{5}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2a\sqrt{a+bx^3+cx^6}}$$

input
```
Int[x/(a + b*x^3 + c*x^6)^(3/2),x]
```

output
```
(x^2*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + S
qrt[b^2 - 4*a*c])]*AppellF1[2/3, 3/2, 3/2, 5/3, (-2*c*x^3)/(b - Sqrt[b^2 -
 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(2*a*Sqrt[a + b*x^3 + c*x^6
])
```

3.244.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

3.244.4 Maple [F]

$$\int \frac{x}{\left(c\,x^{6}+b\,x^{3}+a\right)^{\frac{3}{2}}}dx$$

input
```
int(x/(c*x^6+b*x^3+a)^(3/2),x)
```

output
```
int(x/(c*x^6+b*x^3+a)^(3/2),x)
```

3.244.5 Fricas [F]

$$\int \frac{x}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{x}{(cx^6+bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^6 + b*x^3 + a)*x/(c^2*x^12 + 2*b*c*x^9 + (b^2 + 2*a*c)*x
^6 + 2*a*b*x^3 + a^2), x)
```

3.244.6 Sympy [F]

$$\int \frac{x}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{x}{(a+bx^3+cx^6)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(c*x**6+b*x**3+a)**(3/2),x)
```

output
```
Integral(x/(a + b*x**3 + c*x**6)**(3/2), x)
```

3.244.7 Maxima [F]

$$\int \frac{x}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{x}{(cx^6+bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x/(c*x^6 + b*x^3 + a)^(3/2), x)
```

3.244.8 Giac [F]

$$\int \frac{x}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{x}{(cx^6+bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(x/(c*x^6 + b*x^3 + a)^(3/2), x)
```

3.244.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{x}{(c\,x^6+b\,x^3+a)^{3/2}}\,dx$$

input
```
int(x/(a + b*x^3 + c*x^6)^(3/2),x)
```

output
```
int(x/(a + b*x^3 + c*x^6)^(3/2), x)
```

3.245 $\int \frac{1}{(a+bx^3+cx^6)^{3/2}}\,dx$

3.245.1 Optimal result

Integrand size = 16, antiderivative size = 138

$$\int \frac{1}{(a+bx^3+cx^6)^{3/2}}\,dx = \frac{x\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{1}{3},\frac{3}{2},\frac{3}{2},\frac{4}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{a\sqrt{a+bx^3+cx^6}}$$

output
```
x*AppellF1(1/3,3/2,3/2,4/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/(b+(-4
*a*c+b^2)^(1/2)))*(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^3/(b+(
-4*a*c+b^2)^(1/2)))^(1/2)/a/(c*x^6+b*x^3+a)^(1/2)
```

3.245.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 359 vs. 2(138) = 276.

Time = 10.34 (sec) , antiderivative size = 359, normalized size of antiderivative = 2.60

$$\int \frac{1}{(a+bx^3+cx^6)^{3/2}}\,dx = \frac{x\left(-4(b^2-2ac+bcx^3)-2(b^2-8ac)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\right.}{}$$

input
```
Integrate[(a + b*x^3 + c*x^6)^(-3/2),x]
```

output
```
(x*(-4*(b^2 - 2*a*c + b*c*x^3) - 2*(b^2 - 8*a*c)*Sqrt[(b - Sqrt[b^2 - 4*a*
c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x
^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3)/(b +
Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + b*c*x^3*Sqrt[(b
- Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2
 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[4/3, 1/2, 1/2, 7/3,
 (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]))
/(6*a*(-b^2 + 4*a*c)*Sqrt[a + b*x^3 + c*x^6])
```

3.245.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1686, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^3+cx^6)^{3/2}} \, dx$$

$$\big\downarrow \text{1686}$$

$$\frac{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\int \frac{1}{\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{3/2}\left(\frac{2cx^3}{b+\sqrt{b^2-4ac}}+1\right)^{3/2}}dx}{a\sqrt{a+bx^3+cx^6}}$$

$$\big\downarrow \text{936}$$

$$\frac{x\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{1}{3},\frac{3}{2},\frac{3}{2},\frac{4}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{a\sqrt{a+bx^3+cx^6}}$$

input
```
Int[(a + b*x^3 + c*x^6)^(-3/2),x]
```

output
```
(x*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqr
t[b^2 - 4*a*c])]*AppellF1[1/3, 3/2, 3/2, 4/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4
*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(a*Sqrt[a + b*x^3 + c*x^6])
```

3.245.3.1 Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1686
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^
IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*c*(x^n/(b + Rt[b^2
- 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4*a*c, 2])))^FracPar
t[p]))   Int[(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2*c*(x^n/(b - S
qrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n]
&& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

3.245.4 Maple [F]

$$\int \frac{1}{(c\,x^6 + b\,x^3 + a)^{\frac{3}{2}}}dx$$

input
```
int(1/(c*x^6+b*x^3+a)^(3/2),x)
```

output
```
int(1/(c*x^6+b*x^3+a)^(3/2),x)
```

3.245.5 Fricas [F]

$$\int \frac{1}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{1}{(cx^6+bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^6 + b*x^3 + a)/(c^2*x^12 + 2*b*c*x^9 + (b^2 + 2*a*c)*x^6
 + 2*a*b*x^3 + a^2), x)
```

3.245.6 Sympy [F]

$$\int \frac{1}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{1}{(a+bx^3+cx^6)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x**6+b*x**3+a)**(3/2),x)
```

output
```
Integral((a + b*x**3 + c*x**6)**(-3/2), x)
```

3.245.7 Maxima [F]

$$\int \frac{1}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{1}{(cx^6+bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)^(-3/2), x)
```

3.245.8 Giac [F]

$$\int \frac{1}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{1}{(cx^6+bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^(-3/2), x)
```

3.245.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{1}{(c\,x^6+b\,x^3+a)^{3/2}}\,dx$$

input
```
int(1/(a + b*x^3 + c*x^6)^(3/2),x)
```

output
```
int(1/(a + b*x^3 + c*x^6)^(3/2), x)
```

3.246 $\int \frac{1}{x^2(a+bx^3+cx^6)^{3/2}}\,dx$

3.246.1 Optimal result

Integrand size = 20, antiderivative size = 141

$$\int \frac{1}{x^2(a+bx^3+cx^6)^{3/2}}\,dx = -\frac{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(-\frac{1}{3},\frac{3}{2},\frac{3}{2},\frac{2}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{ax\sqrt{a+bx^3+cx^6}}$$

output
```
-AppellF1(-1/3,3/2,3/2,2/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/(b+(-4
*a*c+b^2)^(1/2)))*(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^3/(b+(
-4*a*c+b^2)^(1/2)))^(1/2)/a/x/(c*x^6+b*x^3+a)^(1/2)
```

3.246.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 407 vs. 2(141) = 282.

Time = 10.52 (sec) , antiderivative size = 407, normalized size of antiderivative = 2.89

$$\int \frac{1}{x^2(a+bx^3+cx^6)^{3/2}}\,dx = -\frac{5b(-5b^2+12ac)\,x^3\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{5}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)-4(60\cdots}{}$$

input
```
Integrate[1/(x^2*(a + b*x^3 + c*x^6)^(3/2)),x]
```

output
```
-1/60*(5*b*(-5*b^2 + 12*a*c)*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b
 - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^
2 - 4*a*c])]*AppellF1[2/3, 1/2, 1/2, 5/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c
]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] - 4*(60*a^2*c - 25*b^2*x^3*(b + c*
x^3) + 5*a*(-3*b^2 + 18*b*c*x^3 + 16*c^2*x^6) + 2*c*(5*b^2 - 16*a*c)*x^6*S
qrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + S
qrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/3, 1/2, 1/
2, 8/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a
*c])]))/(a^2*(b^2 - 4*a*c)*x*Sqrt[a + b*x^3 + c*x^6])
```

3.246.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(a+b x^3+c x^6\right)^{3/2}} \, dx$$

$$\Big\downarrow \text{1721}$$

$$\frac{\sqrt{\frac{2 c x^3}{b-\sqrt{b^2-4 a c}}+1} \sqrt{\frac{2 c x^3}{\sqrt{b^2-4 a c}+b}+1} \int \frac{1}{x^2 \left(\frac{2 c x^3}{b-\sqrt{b^2-4 a c}}+1\right)^{3/2} \left(\frac{2 c x^3}{b+\sqrt{b^2-4 a c}}+1\right)^{3/2}}dx}{a \sqrt{a+b x^3+c x^6}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt{\frac{2 c x^3}{b-\sqrt{b^2-4 a c}}+1} \sqrt{\frac{2 c x^3}{\sqrt{b^2-4 a c}+b}+1} \operatorname{AppellF1}\left(-\frac{1}{3},\frac{3}{2},\frac{3}{2},\frac{2}{3},-\frac{2 c x^3}{b-\sqrt{b^2-4 a c}},-\frac{2 c x^3}{b+\sqrt{b^2-4 a c}}\right)}{a x \sqrt{a+b x^3+c x^6}}$$

input
```
Int[1/(x^2*(a + b*x^3 + c*x^6)^(3/2)),x]
```

output
```
-((Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqr
t[b^2 - 4*a*c])]*AppellF1[-1/3, 3/2, 3/2, 2/3, (-2*c*x^3)/(b - Sqrt[b^2 -
4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(a*x*Sqrt[a + b*x^3 + c*x^6]
))
```

3.246.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

3.246.4 Maple [F]

$$\int \frac{1}{x^{2} \left(c\,x^{6}+b\,x^{3}+a\right)^{\frac{3}{2}}}d x$$

input
```
int(1/x^2/(c*x^6+b*x^3+a)^(3/2),x)
```

output
```
int(1/x^2/(c*x^6+b*x^3+a)^(3/2),x)
```

3.246.5 Fricas [F]

$$\int \frac{1}{x^2 \left(a+bx^3+cx^6\right)^{3/2}}\, dx = \int \frac{1}{{\left(c x^{6} + b x^{3} + a\right)}^{\frac{3}{2}} x^{2}}\, dx$$

input
```
integrate(1/x^2/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^6 + b*x^3 + a)/(c^2*x^14 + 2*b*c*x^11 + (b^2 + 2*a*c)*x^
8 + 2*a*b*x^5 + a^2*x^2), x)
```

3.246.6 Sympy [F]

$$\int \frac{1}{x^2 (a + bx^3 + cx^6)^{3/2}} \, dx = \int \frac{1}{x^2 (a + bx^3 + cx^6)^{\frac{3}{2}}} \, dx$$

input
```
integrate(1/x**2/(c*x**6+b*x**3+a)**(3/2),x)
```

output
```
Integral(1/(x**2*(a + b*x**3 + c*x**6)**(3/2)), x)
```

3.246.7 Maxima [F]

$$\int \frac{1}{x^2 (a + bx^3 + cx^6)^{3/2}} \, dx = \int \frac{1}{(cx^6 + bx^3 + a)^{\frac{3}{2}} x^2} \, dx$$

input
```
integrate(1/x^2/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x^2), x)
```

3.246.8 Giac [F]

$$\int \frac{1}{x^2 (a + bx^3 + cx^6)^{3/2}} \, dx = \int \frac{1}{(cx^6 + bx^3 + a)^{\frac{3}{2}} x^2} \, dx$$

input
```
integrate(1/x^2/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x^2), x)
```

3.246.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a + bx^3 + cx^6)^{3/2}} \, dx = \int \frac{1}{x^2 (c\,x^6 + b\,x^3 + a)^{3/2}} \, dx$$

input
```
int(1/(x^2*(a + b*x^3 + c*x^6)^(3/2)),x)
```

output
```
int(1/(x^2*(a + b*x^3 + c*x^6)^(3/2)), x)
```

3.247 $\int \frac{1}{x^3(a+bx^3+cx^6)^{3/2}}\,dx$

3.247.1 Optimal result

Integrand size = 20, antiderivative size = 143

$$\int \frac{1}{x^3\left(a+bx^3+cx^6\right)^{3/2}}\,dx = -\frac{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,\mathrm{AppellF1}\left(-\frac{2}{3},\frac{3}{2},\frac{3}{2},\frac{1}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2ax^2\sqrt{a+bx^3+cx^6}}$$

output
```
-1/2*AppellF1(-2/3,3/2,3/2,1/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/(b
+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^3/
(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/x^2/(c*x^6+b*x^3+a)^(1/2)
```

3.247.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 405 vs. $2(143) = 286$.

Time = 10.44 (sec) , antiderivative size = 405, normalized size of antiderivative = 2.83

$$\int \frac{1}{x^3\left(a+bx^3+cx^6\right)^{3/2}}\,dx = \frac{-48a^2c+28b^2x^3(b+cx^3)+4a(3b^2-24bcx^3-20c^2x^6)+2b(7b^2-36ac)\,x^3\sqrt{\ }}{}$$

input
```
Integrate[1/(x^3*(a + b*x^3 + c*x^6)^(3/2)),x]
```

output
```
(-48*a^2*c + 28*b^2*x^3*(b + c*x^3) + 4*a*(3*b^2 - 24*b*c*x^3 - 20*c^2*x^6
) + 2*b*(7*b^2 - 36*a*c)*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - S
qrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 -
4*a*c])]*AppellF1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]),
(2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + c*(-7*b^2 + 20*a*c)*x^6*Sqrt[(b - Sq
rt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4
*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[4/3, 1/2, 1/2, 7/3, (-2
*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(24*
a^2*(-b^2 + 4*a*c)*x^2*Sqrt[a + b*x^3 + c*x^6])
```

3.247.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \left(a+bx^3+cx^6\right)^{3/2}} \, dx$$

$$\Big\downarrow \text{1721}$$

$$\frac{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\int \frac{1}{x^3\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{3/2}\left(\frac{2cx^3}{b+\sqrt{b^2-4ac}}+1\right)^{3/2}}dx}{a\sqrt{a+bx^3+cx^6}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(-\frac{2}{3},\frac{3}{2},\frac{3}{2},\frac{1}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2ax^2\sqrt{a+bx^3+cx^6}}$$

input
```
Int[1/(x^3*(a + b*x^3 + c*x^6)^(3/2)),x]
```

output
```
-1/2*(Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b +
Sqrt[b^2 - 4*a*c])]*AppellF1[-2/3, 3/2, 3/2, 1/3, (-2*c*x^3)/(b - Sqrt[b^2
 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(a*x^2*Sqrt[a + b*x^3 + c
*x^6])
```

3.247.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

3.247.4 Maple [F]

$$\int \frac{1}{x^3\,(c\,x^6+b\,x^3+a)^{\frac{3}{2}}}dx$$

input
```
int(1/x^3/(c*x^6+b*x^3+a)^(3/2),x)
```

output
```
int(1/x^3/(c*x^6+b*x^3+a)^(3/2),x)
```

3.247.5 Fricas [F]

$$\int \frac{1}{x^3\,(a+bx^3+cx^6)^{3/2}}\,dx=\int \frac{1}{(cx^6+bx^3+a)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate(1/x^3/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^6 + b*x^3 + a)/(c^2*x^15 + 2*b*c*x^12 + (b^2 + 2*a*c)*x^
9 + 2*a*b*x^6 + a^2*x^3), x)
```

3.247.6 Sympy [F]

$$\int \frac{1}{x^3 (a + bx^3 + cx^6)^{3/2}} \, dx = \int \frac{1}{x^3 (a + bx^3 + cx^6)^{\frac{3}{2}}} \, dx$$

input
```
integrate(1/x**3/(c*x**6+b*x**3+a)**(3/2),x)
```

output
```
Integral(1/(x**3*(a + b*x**3 + c*x**6)**(3/2)), x)
```

3.247.7 Maxima [F]

$$\int \frac{1}{x^3 (a + bx^3 + cx^6)^{3/2}} \, dx = \int \frac{1}{(cx^6 + bx^3 + a)^{\frac{3}{2}} x^3} \, dx$$

input
```
integrate(1/x^3/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x^3), x)
```

3.247.8 Giac [F]

$$\int \frac{1}{x^3 (a + bx^3 + cx^6)^{3/2}} \, dx = \int \frac{1}{(cx^6 + bx^3 + a)^{\frac{3}{2}} x^3} \, dx$$

input
```
integrate(1/x^3/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x^3), x)
```

3.247.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\left(a+bx^3+cx^6\right)^{3/2}}\,dx=\int \frac{1}{x^3\,{\left(c\,x^6+b\,x^3+a\right)}^{3/2}} \,d x$$

input
```
int(1/(x^3*(a + b*x^3 + c*x^6)^(3/2)),x)
```

output
```
int(1/(x^3*(a + b*x^3 + c*x^6)^(3/2)), x)
```

3.248 $\int (dx)^m (a+bx^3+cx^6)^2 \, dx$

3.248.1 Optimal result

Integrand size = 20, antiderivative size = 101

$$\int (dx)^m \left(a+bx^3+cx^6\right)^2 \, dx = \frac{a^2(dx)^{1+m}}{d(1+m)}+\frac{2ab(dx)^{4+m}}{d^4(4+m)}+\frac{\left(b^2+2ac\right)(dx)^{7+m}}{d^7(7+m)} + \frac{2bc(dx)^{10+m}}{d^{10}(10+m)}+\frac{c^2(dx)^{13+m}}{d^{13}(13+m)}$$

output
```
a^2*(d*x)^(1+m)/d/(1+m)+2*a*b*(d*x)^(4+m)/d^4/(4+m)+(2*a*c+b^2)*(d*x)^(7+m
)/d^7/(7+m)+2*b*c*(d*x)^(10+m)/d^10/(10+m)+c^2*(d*x)^(13+m)/d^13/(13+m)
```

3.248.2 Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.69

$$\int (dx)^m \left(a+bx^3+cx^6\right)^2 \, dx = x(dx)^m \left(\frac{a^2}{1+m}+\frac{2abx^3}{4+m}+\frac{\left(b^2+2ac\right)x^6}{7+m}+\frac{2bcx^9}{10+m}+\frac{c^2x^{12}}{13+m}\right)$$

input
```
Integrate[(d*x)^m*(a + b*x^3 + c*x^6)^2,x]
```

output
```
x*(d*x)^m*(a^2/(1 + m) + (2*a*b*x^3)/(4 + m) + ((b^2 + 2*a*c)*x^6)/(7 + m)
 + (2*b*c*x^9)/(10 + m) + (c^2*x^12)/(13 + m))
```

3.248.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1691, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a+bx^3+cx^6\right)^2 dx$$

$$\Big\downarrow \text{1691}$$

$$\int \left(a^2(dx)^m+\frac{\left(2ac+b^2\right)(dx)^{m+6}}{d^6}+\frac{2ab(dx)^{m+3}}{d^3}+\frac{2bc(dx)^{m+9}}{d^9}+\frac{c^2(dx)^{m+12}}{d^{12}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2(dx)^{m+1}}{d(m+1)}+\frac{\left(2ac+b^2\right)(dx)^{m+7}}{d^7(m+7)}+\frac{2ab(dx)^{m+4}}{d^4(m+4)}+\frac{2bc(dx)^{m+10}}{d^{10}(m+10)}+\frac{c^2(dx)^{m+13}}{d^{13}(m+13)}$$

input
```
Int[(d*x)^m*(a + b*x^3 + c*x^6)^2,x]
```

output
```
(a^2*(d*x)^(1 + m))/(d*(1 + m)) + (2*a*b*(d*x)^(4 + m))/(d^4*(4 + m)) + ((
b^2 + 2*a*c)*(d*x)^(7 + m))/(d^7*(7 + m)) + (2*b*c*(d*x)^(10 + m))/(d^10*(
10 + m)) + (c^2*(d*x)^(13 + m))/(d^13*(13 + m))
```

3.248.3.1 Defintions of rubi rules used

rule 1691
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^n + c*x^(2*n))^p, x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && IGtQ[p, 0] &&  !IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.248.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 300 vs. $2(101) = 202$.

Time = 0.19 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.98

method	result
gosper	$\frac{x(c^2m^4x^{12}+22c^2m^3x^{12}+159c^2m^2x^{12}+2bc\,m^4x^9+418m\,x^{12}c^2+50bc\,m^3x^9+280c^2x^{12}+390bc\,m^2x^9+2ac\,m^4x^6+b^2m^4x^6+10}{}$
risch	$\frac{x(c^2m^4x^{12}+22c^2m^3x^{12}+159c^2m^2x^{12}+2bc\,m^4x^9+418m\,x^{12}c^2+50bc\,m^3x^9+280c^2x^{12}+390bc\,m^2x^9+2ac\,m^4x^6+b^2m^4x^6+10}{}$
parallelrisch	$\frac{2x^{10}(dx)^m bc\,m^4+50x^{10}(dx)^m bc\,m^3+390x^{10}(dx)^m bc\,m^2+520x^7(dx)^m b^2+3640x(dx)^m a^2+1070x^{10}(dx)^m bcm+2x^7(dx)^m ac}{}$

input
```
int((d*x)^m*(c*x^6+b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
x*(c^2*m^4*x^12+22*c^2*m^3*x^12+159*c^2*m^2*x^12+2*b*c*m^4*x^9+418*c^2*m*x
^12+50*b*c*m^3*x^9+280*c^2*x^12+390*b*c*m^2*x^9+2*a*c*m^4*x^6+b^2*m^4*x^6+
1070*b*c*m*x^9+56*a*c*m^3*x^6+28*b^2*m^3*x^6+728*b*c*x^9+498*a*c*m^2*x^6+2
49*b^2*m^2*x^6+2*a*b*m^4*x^3+1484*a*c*m*x^6+742*b^2*m*x^6+62*a*b*m^3*x^3+1
040*a*c*x^6+520*b^2*x^6+642*a*b*m^2*x^3+a^2*m^4+2402*a*b*m*x^3+34*a^2*m^3+
1820*a*b*x^3+411*a^2*m^2+2074*a^2*m+3640*a^2)*(d*x)^m/(13+m)/(10+m)/(7+m)/
(4+m)/(1+m)
```

3.248.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(101) = 202$.

Time = 0.26 (sec) , antiderivative size = 241, normalized size of antiderivative = 2.39

$$\int (dx)^m \left(a+bx^3+cx^6\right)^2\,dx$$
$$= \frac{((c^2m^4+22\,c^2m^3+159\,c^2m^2+418\,c^2m+280\,c^2)x^{13}+2\,(bcm^4+25\,bcm^3+195\,bcm^2+535\,bcm+364\,b}{}$$

input
```
integrate((d*x)^m*(c*x^6+b*x^3+a)^2,x, algorithm="fricas")
```

output
```
((c^2*m^4 + 22*c^2*m^3 + 159*c^2*m^2 + 418*c^2*m + 280*c^2)*x^13 + 2*(b*c*
m^4 + 25*b*c*m^3 + 195*b*c*m^2 + 535*b*c*m + 364*b*c)*x^10 + ((b^2 + 2*a*c
)*m^4 + 28*(b^2 + 2*a*c)*m^3 + 249*(b^2 + 2*a*c)*m^2 + 520*b^2 + 1040*a*c
+ 742*(b^2 + 2*a*c)*m)*x^7 + 2*(a*b*m^4 + 31*a*b*m^3 + 321*a*b*m^2 + 1201*
a*b*m + 910*a*b)*x^4 + (a^2*m^4 + 34*a^2*m^3 + 411*a^2*m^2 + 2074*a^2*m +
3640*a^2)*x)*(d*x)^m/(m^5 + 35*m^4 + 445*m^3 + 2485*m^2 + 5714*m + 3640)
```

3.248.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1459 vs. 2(90) = 180.

Time = 0.82 (sec) , antiderivative size = 1459, normalized size of antiderivative = 14.45

$$\int (dx)^m \left(a+bx^3+cx^6\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate((d*x)**m*(c*x**6+b*x**3+a)**2,x)
```

output
```
Piecewise(((-a**2/(12*x**12) - 2*a*b/(9*x**9) - a*c/(3*x**6) - b**2/(6*x**
6) - 2*b*c/(3*x**3) + c**2*log(x))/d**13, Eq(m, -13)), ((-a**2/(9*x**9) -
a*b/(3*x**6) - 2*a*c/(3*x**3) - b**2/(3*x**3) + 2*b*c*log(x) + c**2*x**3/3
)/d**10, Eq(m, -10)), ((-a**2/(6*x**6) - 2*a*b/(3*x**3) + 2*a*c*log(x) + b
**2*log(x) + 2*b*c*x**3/3 + c**2*x**6/6)/d**7, Eq(m, -7)), ((-a**2/(3*x**3
) + 2*a*b*log(x) + 2*a*c*x**3/3 + b**2*x**3/3 + b*c*x**6/3 + c**2*x**9/9)/
d**4, Eq(m, -4)), ((a**2*log(x) + 2*a*b*x**3/3 + a*c*x**6/3 + b**2*x**6/6
+ 2*b*c*x**9/9 + c**2*x**12/12)/d, Eq(m, -1)), (a**2*m**4*x*(d*x)**m/(m**5
 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 34*a**2*m**3*x*(d*x)*
*m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 411*a**2*m**2
*x*(d*x)**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 2074
*a**2*m*x*(d*x)**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640)
 + 3640*a**2*x*(d*x)**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m +
3640) + 2*a*b*m**4*x**4*(d*x)**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 +
5714*m + 3640) + 62*a*b*m**3*x**4*(d*x)**m/(m**5 + 35*m**4 + 445*m**3 + 24
85*m**2 + 5714*m + 3640) + 642*a*b*m**2*x**4*(d*x)**m/(m**5 + 35*m**4 + 44
5*m**3 + 2485*m**2 + 5714*m + 3640) + 2402*a*b*m*x**4*(d*x)**m/(m**5 + 35*
m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 1820*a*b*x**4*(d*x)**m/(m**
5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 2*a*c*m**4*x**7*(d*x
)**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 56*a*c*m...
```

3.248.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.09

$$\int (dx)^m \left(a+bx^3+cx^6\right)^2 \, dx = \frac{c^2 d^m x^{13} x^m}{m+13} + \frac{2\, bcd^m x^{10} x^m}{m+10} + \frac{b^2 d^m x^7 x^m}{m+7} + \frac{2\, acd^m x^7 x^m}{m+7} + \frac{2\, abd^m x^4 x^m}{m+4} + \frac{(dx)^{m+1}\, a^2}{d(m+1)}$$

input
```
integrate((d*x)^m*(c*x^6+b*x^3+a)^2,x, algorithm="maxima")
```

output
```
c^2*d^m*x^13*x^m/(m + 13) + 2*b*c*d^m*x^10*x^m/(m + 10) + b^2*d^m*x^7*x^m/
(m + 7) + 2*a*c*d^m*x^7*x^m/(m + 7) + 2*a*b*d^m*x^4*x^m/(m + 4) + (d*x)^(m
 + 1)*a^2/(d*(m + 1))
```

3.248.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 449 vs. $2(101) = 202$.

Time = 0.33 (sec) , antiderivative size = 449, normalized size of antiderivative = 4.45

$$\int (dx)^m \left(a+bx^3+cx^6\right)^2 dx$$
$$= \frac{(dx)^m c^2m^4x^{13} + 22 (dx)^m c^2m^3x^{13} + 159 (dx)^m c^2m^2x^{13} + 2 (dx)^m bcm^4x^{10} + 418 (dx)^m c^2mx^{13} + 50 (dx}{}$$

input
```
integrate((d*x)^m*(c*x^6+b*x^3+a)^2,x, algorithm="giac")
```

output
```
((d*x)^m*c^2*m^4*x^13 + 22*(d*x)^m*c^2*m^3*x^13 + 159*(d*x)^m*c^2*m^2*x^13
 + 2*(d*x)^m*b*c*m^4*x^10 + 418*(d*x)^m*c^2*m*x^13 + 50*(d*x)^m*b*c*m^3*x^
10 + 280*(d*x)^m*c^2*x^13 + 390*(d*x)^m*b*c*m^2*x^10 + (d*x)^m*b^2*m^4*x^7
 + 2*(d*x)^m*a*c*m^4*x^7 + 1070*(d*x)^m*b*c*m*x^10 + 28*(d*x)^m*b^2*m^3*x^
7 + 56*(d*x)^m*a*c*m^3*x^7 + 728*(d*x)^m*b*c*x^10 + 249*(d*x)^m*b^2*m^2*x^
7 + 498*(d*x)^m*a*c*m^2*x^7 + 2*(d*x)^m*a*b*m^4*x^4 + 742*(d*x)^m*b^2*m*x^
7 + 1484*(d*x)^m*a*c*m*x^7 + 62*(d*x)^m*a*b*m^3*x^4 + 520*(d*x)^m*b^2*x^7
+ 1040*(d*x)^m*a*c*x^7 + 642*(d*x)^m*a*b*m^2*x^4 + (d*x)^m*a^2*m^4*x + 240
2*(d*x)^m*a*b*m*x^4 + 34*(d*x)^m*a^2*m^3*x + 1820*(d*x)^m*a*b*x^4 + 411*(d
*x)^m*a^2*m^2*x + 2074*(d*x)^m*a^2*m*x + 3640*(d*x)^m*a^2*x)/(m^5 + 35*m^4
 + 445*m^3 + 2485*m^2 + 5714*m + 3640)
```

3.248.9 Mupad [B] (verification not implemented)

Time = 8.49 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.57

$$\int (dx)^m \left(a+bx^3+cx^6\right)^2 \, dx = (d\,x)^m \left(\frac{c^2\,x^{13}\,(m^4+22\,m^3+159\,m^2+418\,m+280)}{m^5+35\,m^4+445\,m^3+2485\,m^2+5714\,m+3640} + \frac{x^7\,(b^2+2\,a\,c)\,\left(m^4+28\,m^3+249\,m^2+742\,m+520\right)}{m^5+35\,m^4+445\,m^3+2485\,m^2+5714\,m+3640} + \frac{a^2\,x\,(m^4+34\,m^3+411\,m^2+2074\,m+3640)}{m^5+35\,m^4+445\,m^3+2485\,m^2+5714\,m+3640} + \frac{2\,a\,b\,x^4\,(m^4+31\,m^3+321\,m^2+1201\,m+910)}{m^5+35\,m^4+445\,m^3+2485\,m^2+5714\,m+3640} + \frac{2\,b\,c\,x^{10}\,(m^4+25\,m^3+195\,m^2+535\,m+364)}{m^5+35\,m^4+445\,m^3+2485\,m^2+5714\,m+3640}\right)$$

input
```
int((d*x)^m*(a + b*x^3 + c*x^6)^2,x)
```

output
```
(d*x)^m*((c^2*x^13*(418*m + 159*m^2 + 22*m^3 + m^4 + 280))/(5714*m + 2485*
m^2 + 445*m^3 + 35*m^4 + m^5 + 3640) + (x^7*(2*a*c + b^2)*(742*m + 249*m^2
 + 28*m^3 + m^4 + 520))/(5714*m + 2485*m^2 + 445*m^3 + 35*m^4 + m^5 + 3640
) + (a^2*x*(2074*m + 411*m^2 + 34*m^3 + m^4 + 3640))/(5714*m + 2485*m^2 +
445*m^3 + 35*m^4 + m^5 + 3640) + (2*a*b*x^4*(1201*m + 321*m^2 + 31*m^3 + m
^4 + 910))/(5714*m + 2485*m^2 + 445*m^3 + 35*m^4 + m^5 + 3640) + (2*b*c*x^
10*(535*m + 195*m^2 + 25*m^3 + m^4 + 364))/(5714*m + 2485*m^2 + 445*m^3 +
35*m^4 + m^5 + 3640))
```

3.249 $\int (dx)^m (a+bx^3+cx^6)\,dx$

3.249.1 Optimal result

Integrand size = 18, antiderivative size = 52

$$\int (dx)^m (a+bx^3+cx^6)\,dx = \frac{a(dx)^{1+m}}{d(1+m)}+\frac{b(dx)^{4+m}}{d^4(4+m)}+\frac{c(dx)^{7+m}}{d^7(7+m)}$$

output
```
a*(d*x)^(1+m)/d/(1+m)+b*(d*x)^(4+m)/d^4/(4+m)+c*(d*x)^(7+m)/d^7/(7+m)
```

3.249.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.67

$$\int (dx)^m (a+bx^3+cx^6)\,dx = x(dx)^m\left(\frac{a}{1+m}+\frac{bx^3}{4+m}+\frac{cx^6}{7+m}\right)$$

input
```
Integrate[(d*x)^m*(a + b*x^3 + c*x^6),x]
```

output
```
x*(d*x)^m*(a/(1 + m) + (b*x^3)/(4 + m) + (c*x^6)/(7 + m))
```

3.249.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1691, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a+bx^3+cx^6\right) \, dx$$

$$\Big\downarrow \text{ 1691}$$

$$\int \left(a(dx)^m+\frac{b(dx)^{m+3}}{d^3}+\frac{c(dx)^{m+6}}{d^6}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a(dx)^{m+1}}{d(m+1)}+\frac{b(dx)^{m+4}}{d^4(m+4)}+\frac{c(dx)^{m+7}}{d^7(m+7)}$$

input
```
Int[(d*x)^m*(a + b*x^3 + c*x^6),x]
```

output
```
(a*(d*x)^(1 + m))/(d*(1 + m)) + (b*(d*x)^(4 + m))/(d^4*(4 + m)) + (c*(d*x)
^(7 + m))/(d^7*(7 + m))
```

3.249.3.1 Defintions of rubi rules used

rule 1691
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^n + c*x^(2*n))^p, x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && IGtQ[p, 0] &&  !IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.249.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.98

method	result
norman	$\frac{ax\,e^{m\ln(dx)}}{1+m}+\frac{b\,x^{4}e^{m\ln(dx)}}{4+m}+\frac{c\,x^{7}e^{m\ln(dx)}}{7+m}$
gosper	$\frac{x(c\,m^{2}x^{6}+5cm\,x^{6}+4c\,x^{6}+b\,m^{2}x^{3}+8bm\,x^{3}+7b\,x^{3}+a\,m^{2}+11am+28a)\,(dx)^{m}}{(7+m)(4+m)(1+m)}$
risch	$\frac{x(c\,m^{2}x^{6}+5cm\,x^{6}+4c\,x^{6}+b\,m^{2}x^{3}+8bm\,x^{3}+7b\,x^{3}+a\,m^{2}+11am+28a)\,(dx)^{m}}{(7+m)(4+m)(1+m)}$
parallelrisch	$\frac{x^{7}(dx)^{m}c\,m^{2}+5x^{7}(dx)^{m}cm+4x^{7}(dx)^{m}c+x^{4}(dx)^{m}b\,m^{2}+8x^{4}(dx)^{m}bm+7x^{4}(dx)^{m}b+x(dx)^{m}a\,m^{2}+11x(dx)^{m}am+28x(dx)^{m}}{(7+m)(4+m)(1+m)}$

input
```
int((d*x)^m*(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
a/(1+m)*x*exp(m*ln(d*x))+b/(4+m)*x^4*exp(m*ln(d*x))+c/(7+m)*x^7*exp(m*ln(d
*x))
```

3.249.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.37

$$\int (dx)^m\left(a+bx^3+cx^6\right)\,dx = \frac{((cm^2+5\,cm+4\,c)x^7+(bm^2+8\,bm+7\,b)x^4+(am^2+11\,am+28\,a)x)(dx)^m}{m^3+12\,m^2+39\,m+28}$$

input
```
integrate((d*x)^m*(c*x^6+b*x^3+a),x, algorithm="fricas")
```

output
```
((c*m^2 + 5*c*m + 4*c)*x^7 + (b*m^2 + 8*b*m + 7*b)*x^4 + (a*m^2 + 11*a*m +
 28*a)*x)*(d*x)^m/(m^3 + 12*m^2 + 39*m + 28)
```

3.249.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(42) = 84$.

Time = 0.37 (sec) , antiderivative size = 299, normalized size of antiderivative = 5.75

$$\int (dx)^m \left(a + bx^3 + cx^6\right)\, dx$$
$$= \begin{cases} \frac{-\frac{a}{6x^6} - \frac{b}{3x^3} + c\log(x)}{d^7} \\ \frac{-\frac{a}{3x^3} + b\log(x) + \frac{cx^3}{3}}{d^4} \\ \frac{a\log(x) + \frac{bx^3}{3} + \frac{cx^6}{6}}{d} \\ \frac{am^2x(dx)^m}{m^3+12m^2+39m+28} + \frac{11amx(dx)^m}{m^3+12m^2+39m+28} + \frac{28ax(dx)^m}{m^3+12m^2+39m+28} + \frac{bm^2x^4(dx)^m}{m^3+12m^2+39m+28} + \frac{8bmx^4(dx)^m}{m^3+12m^2+39m+28} + \frac{7bx^4(dx}{m^3+12m^2+3} \end{cases}$$

input
```
integrate((d*x)**m*(c*x**6+b*x**3+a),x)
```

output
```
Piecewise(((-a/(6*x**6) - b/(3*x**3) + c*log(x))/d**7, Eq(m, -7)), ((-a/(3
*x**3) + b*log(x) + c*x**3/3)/d**4, Eq(m, -4)), ((a*log(x) + b*x**3/3 + c*
x**6/6)/d, Eq(m, -1)), (a*m**2*x*(d*x)**m/(m**3 + 12*m**2 + 39*m + 28) + 1
1*a*m*x*(d*x)**m/(m**3 + 12*m**2 + 39*m + 28) + 28*a*x*(d*x)**m/(m**3 + 12
*m**2 + 39*m + 28) + b*m**2*x**4*(d*x)**m/(m**3 + 12*m**2 + 39*m + 28) + 8
*b*m*x**4*(d*x)**m/(m**3 + 12*m**2 + 39*m + 28) + 7*b*x**4*(d*x)**m/(m**3
+ 12*m**2 + 39*m + 28) + c*m**2*x**7*(d*x)**m/(m**3 + 12*m**2 + 39*m + 28)
 + 5*c*m*x**7*(d*x)**m/(m**3 + 12*m**2 + 39*m + 28) + 4*c*x**7*(d*x)**m/(m
**3 + 12*m**2 + 39*m + 28), True))
```

3.249.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96

$$\int (dx)^m \left(a + bx^3 + cx^6\right)\, dx = \frac{cd^m x^7 x^m}{m+7} + \frac{bd^m x^4 x^m}{m+4} + \frac{(dx)^{m+1} a}{d(m+1)}$$

input
```
integrate((d*x)^m*(c*x^6+b*x^3+a),x, algorithm="maxima")
```

output
```
c*d^m*x^7*x^m/(m + 7) + b*d^m*x^4*x^m/(m + 4) + (d*x)^(m + 1)*a/(d*(m + 1)
)
```

3.249.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(52) = 104.

Time = 0.34 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.29

$$\int (dx)^m \left(a+bx^3+cx^6\right) \, dx$$
$$= \frac{(dx)^m\, cm^2x^7 + 5\,(dx)^m\, cmx^7 + 4\,(dx)^m\, cx^7 + (dx)^m\, bm^2x^4 + 8\,(dx)^m\, bmx^4 + 7\,(dx)^m\, bx^4 + (dx)^m\, am^2x}{m^3 + 12\,m^2 + 39\,m + 28}$$

input
```
integrate((d*x)^m*(c*x^6+b*x^3+a),x, algorithm="giac")
```

output
```
((d*x)^m*c*m^2*x^7 + 5*(d*x)^m*c*m*x^7 + 4*(d*x)^m*c*x^7 + (d*x)^m*b*m^2*x
^4 + 8*(d*x)^m*b*m*x^4 + 7*(d*x)^m*b*x^4 + (d*x)^m*a*m^2*x + 11*(d*x)^m*a*
m*x + 28*(d*x)^m*a*x)/(m^3 + 12*m^2 + 39*m + 28)
```

3.249.9 Mupad [B] (verification not implemented)

Time = 8.35 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.71

$$\int (dx)^m \left(a+bx^3+cx^6\right) \, dx = (d\,x)^m \left(\frac{b\,x^4\,(m^2+8\,m+7)}{m^3+12\,m^2+39\,m+28} + \frac{c\,x^7\,(m^2+5\,m+4)}{m^3+12\,m^2+39\,m+28} + \frac{a\,x\,(m^2+11\,m+28)}{m^3+12\,m^2+39\,m+28}\right)$$

input
```
int((d*x)^m*(a + b*x^3 + c*x^6),x)
```

output
```
(d*x)^m*((b*x^4*(8*m + m^2 + 7))/(39*m + 12*m^2 + m^3 + 28) + (c*x^7*(5*m
+ m^2 + 4))/(39*m + 12*m^2 + m^3 + 28) + (a*x*(11*m + m^2 + 28))/(39*m + 1
2*m^2 + m^3 + 28))
```

3.250 $\int \frac{(dx)^m}{a+bx^3+cx^6}\,dx$

3.250.1 Optimal result

Integrand size = 20, antiderivative size = 173

$$\int \frac{(dx)^m}{a+bx^3+cx^6}\,dx = \frac{2c(dx)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{3},\frac{4+m}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)d(1+m)} - \frac{2c(dx)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{3},\frac{4+m}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)d(1+m)}$$

output
```
2*c*(d*x)^(1+m)*hypergeom([1, 1/3+1/3*m],[4/3+1/3*m],-2*c*x^3/(b-(-4*a*c+b
^2)^(1/2)))/d/(1+m)/(b-(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)-2*c*(d*x)^(1
+m)*hypergeom([1, 1/3+1/3*m],[4/3+1/3*m],-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/
d/(1+m)/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))
```

3.250.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.25 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.49

$$\int \frac{(dx)^m}{a+bx^3+cx^6}\,dx = \frac{(dx)^m\text{RootSum}\left[a+b\#1^3+c\#1^6\&,\frac{\text{Hypergeometric2F1}\left(-m,-m,1-m,-\frac{\#1}{x-\#1}\right)\left(\frac{x}{x-\#1}\right)^{-m}}{b\#1^2+2c\#1^5}\&\right]}{3m}$$

input
```
Integrate[(d*x)^m/(a + b*x^3 + c*x^6),x]
```

output
```
((d*x)^m*RootSum[a + b*#1^3 + c*#1^6 & , Hypergeometric2F1[-m, -m, 1 - m,
-(#1/(x - #1))]/((x/(x - #1))^m*(b*#1^2 + 2*c*#1^5)) & ])/(3*m)
```

3.250.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1711, 27, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{a+bx^3+cx^6} \, dx$$
$$\Big\downarrow \text{ 1711}$$
$$\frac{c\int \frac{2(dx)^m}{2cx^3+b-\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}-\frac{c\int \frac{2(dx)^m}{2cx^3+b+\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}$$
$$\Big\downarrow \text{ 27}$$
$$\frac{2c\int \frac{(dx)^m}{2cx^3+b-\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}-\frac{2c\int \frac{(dx)^m}{2cx^3+b+\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}$$
$$\Big\downarrow \text{ 888}$$
$$\frac{2c(dx)^{m+1}\text{Hypergeometric2F1}\left(1,\frac{m+1}{3},\frac{m+4}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c(dx)^{m+1}\text{Hypergeometric2F1}\left(1,\frac{m+1}{3},\frac{m+4}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}$$

input
```
Int[(d*x)^m/(a + b*x^3 + c*x^6),x]
```

output
```
(2*c*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, (-2*c*x^3)/(
b - Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])*d*(1 +
 m)) - (2*c*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, (-2*c
*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])
*d*(1 + m))
```

3.250.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1711
```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[c/q   Int[(d*x)^m/(b/2 - q/2 + c
*x^n), x], x] - Simp[c/q   Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; Free
Q[{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

3.250.4 Maple [F]

$$\int \frac{(dx)^m}{c\,x^6 + b\,x^3 + a}dx$$

input
```
int((d*x)^m/(c*x^6+b*x^3+a),x)
```

output
```
int((d*x)^m/(c*x^6+b*x^3+a),x)
```

3.250.5 Fricas [F]

$$\int \frac{(dx)^m}{a+bx^3+cx^6}\,dx = \int \frac{(dx)^m}{cx^6+bx^3+a}\,dx$$

input
```
integrate((d*x)^m/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

output
```
integral((d*x)^m/(c*x^6 + b*x^3 + a), x)
```

3.250.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{a+bx^3+cx^6}\,dx = \text{Timed out}$$

input
```
integrate((d*x)**m/(c*x**6+b*x**3+a),x)
```

output
```
Timed out
```

3.250.7 Maxima [F]

$$\int \frac{(dx)^m}{a+bx^3+cx^6}\,dx = \int \frac{(dx)^m}{cx^6+bx^3+a}\,dx$$

input
```
integrate((d*x)^m/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

output
```
integrate((d*x)^m/(c*x^6 + b*x^3 + a), x)
```

3.250.8 Giac [F]

$$\int \frac{(dx)^m}{a+bx^3+cx^6}\,dx = \int \frac{(dx)^m}{cx^6+bx^3+a}\,dx$$

input
```
integrate((d*x)^m/(c*x^6+b*x^3+a),x, algorithm="giac")
```

output
```
integrate((d*x)^m/(c*x^6 + b*x^3 + a), x)
```

3.250.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{a+bx^3+cx^6}\,dx = \int \frac{(d\,x)^m}{c\,x^6+b\,x^3+a}\,dx$$

input
```
int((d*x)^m/(a + b*x^3 + c*x^6),x)
```

output
```
int((d*x)^m/(a + b*x^3 + c*x^6), x)
```

3.251 $\int \frac{(dx)^m}{(a+bx^3+cx^6)^2}\,dx$

3.251.1 Optimal result

Integrand size = 20, antiderivative size = 315

$$\int \frac{(dx)^m}{(a+bx^3+cx^6)^2}\,dx = \frac{(dx)^{1+m}(b^2-2ac+bcx^3)}{3a(b^2-4ac)\,d(a+bx^3+cx^6)} + \frac{c\left(b^2(2-m)+b\sqrt{b^2-4ac}(2-m)-4ac(5-m)\right)(dx)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{3},\frac{4+m}{3},-\frac{2cx^3}{b-\sqrt{b^2-4}}\right.}{3a(b^2-4ac)^{3/2}\left(b-\sqrt{b^2-4ac}\right)d(1+m)} - \frac{c\left(b^2(2-m)-b\sqrt{b^2-4ac}(2-m)-4ac(5-m)\right)(dx)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{3},\frac{4+m}{3},-\frac{2cx^3}{b+\sqrt{b^2-4}}\right.}{3a(b^2-4ac)^{3/2}\left(b+\sqrt{b^2-4ac}\right)d(1+m)}$$

output
```
1/3*(d*x)^(1+m)*(b*c*x^3-2*a*c+b^2)/a/(-4*a*c+b^2)/d/(c*x^6+b*x^3+a)-1/3*c
*(d*x)^(1+m)*hypergeom([1, 1/3+1/3*m],[4/3+1/3*m],-2*c*x^3/(b+(-4*a*c+b^2)
^(1/2)))*(b^2*(2-m)-4*a*c*(5-m)-b*(2-m)*(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)
^(3/2)/d/(1+m)/(b+(-4*a*c+b^2)^(1/2))+1/3*c*(d*x)^(1+m)*hypergeom([1, 1/3+
1/3*m],[4/3+1/3*m],-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))*(b^2*(2-m)-4*a*c*(5-m)
+b*(2-m)*(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(3/2)/d/(1+m)/(b-(-4*a*c+b^2)^
(1/2))
```

3.251.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.38 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.25

$$\int \frac{(dx)^m}{(a+bx^3+cx^6)^2}\,dx = \frac{x(dx)^m \operatorname{AppellF1}\left(\frac{1+m}{3},2,2,\frac{4+m}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)}{a^2(1+m)}$$

input
```
Integrate[(d*x)^m/(a + b*x^3 + c*x^6)^2,x]
```

output
```
(x*(d*x)^m*AppellF1[(1 + m)/3, 2, 2, (4 + m)/3, (-2*c*x^3)/(b + Sqrt[b^2 -
 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(a^2*(1 + m))
```

3.251.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1702, 25, 1834, 27, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{(a+bx^3+cx^6)^2}dx$$

$$\Big\downarrow \text{1702}$$

$$\frac{(dx)^{m+1}\left(-2ac+b^2+bcx^3\right)}{3ad\left(b^2-4ac\right)\left(a+bx^3+cx^6\right)}-\frac{\int -\frac{(dx)^m\left(bc(2-m)x^3+b^2(2-m)-2ac(5-m)\right)}{cx^6+bx^3+a}dx}{3a\left(b^2-4ac\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(dx)^m\left(bc(2-m)x^3+b^2(2-m)-2ac(5-m)\right)}{cx^6+bx^3+a}dx}{3a\left(b^2-4ac\right)}+\frac{(dx)^{m+1}\left(-2ac+b^2+bcx^3\right)}{3ad\left(b^2-4ac\right)\left(a+bx^3+cx^6\right)}$$

$$\Big\downarrow \text{1834}$$

$$\frac{\frac{c\left(b(2-m)\sqrt{b^2-4ac}-4ac(5-m)+b^2(2-m)\right)\int \frac{2(dx)^m}{2cx^3+b-\sqrt{b^2-4ac}}dx}{2\sqrt{b^2-4ac}}-\frac{c\left(-b(2-m)\sqrt{b^2-4ac}-4ac(5-m)+b^2(2-m)\right)\int \frac{2(dx)^m}{2cx^3+b+\sqrt{b^2-4ac}}dx}{2\sqrt{b^2-4ac}}}{3a\left(b^2-4ac\right)}+$$
$$\frac{(dx)^{m+1}\left(-2ac+b^2+bcx^3\right)}{3ad\left(b^2-4ac\right)\left(a+bx^3+cx^6\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{c\left(b(2-m)\sqrt{b^2-4ac}-4ac(5-m)+b^2(2-m)\right)\int \frac{(dx)^m}{2cx^3+b-\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}-\frac{c\left(-b(2-m)\sqrt{b^2-4ac}-4ac(5-m)+b^2(2-m)\right)\int \frac{(dx)^m}{2cx^3+b+\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}}{3a\left(b^2-4ac\right)}+\frac{(dx)^{m+1}\left(-2ac+b^2+bcx^3\right)}{3ad\left(b^2-4ac\right)\left(a+bx^3+cx^6\right)}$$

$$\Big\downarrow \text{888}$$

$$\frac{\frac{c(dx)^{m+1}\left(b(2-m)\sqrt{b^2-4ac}-4ac(5-m)+b^2(2-m)\right)\text{Hypergeometric2F1}\left(1,\frac{m+1}{3},\frac{m+4}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{c(dx)^{m+1}\left(-b(2-m)\sqrt{b^2-4ac}-4ac(5\right.}{d}}{3a\left(b^2-4ac\right)}$$

$$\frac{(dx)^{m+1}\left(-2ac+b^2+bcx^3\right)}{3ad\left(b^2-4ac\right)\left(a+bx^3+cx^6\right)}$$

input
```
Int[(d*x)^m/(a + b*x^3 + c*x^6)^2,x]
```

output
```
((d*x)^(1 + m)*(b^2 - 2*a*c + b*c*x^3))/(3*a*(b^2 - 4*a*c)*d*(a + b*x^3 +
c*x^6)) + ((c*(b^2*(2 - m) + b*Sqrt[b^2 - 4*a*c]*(2 - m) - 4*a*c*(5 - m))*
(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, (-2*c*x^3)/(b - S
qrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])*d*(1 + m))
- (c*(b^2*(2 - m) - b*Sqrt[b^2 - 4*a*c]*(2 - m) - 4*a*c*(5 - m))*(d*x)^(1
+ m)*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, (-2*c*x^3)/(b + Sqrt[b^2 -
 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*d*(1 + m)))/(3*a*(b^
2 - 4*a*c))
```

3.251.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

3.251. $\int \frac{(dx)^m}{(a+bx^3+cx^6)^2}\,dx$

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1702
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(-(d*x)^(m + 1))*(b^2 - 2*a*c + b*c*x^n)*((a + b*x^n + c*x
^(2*n))^(p + 1)/(a*d*n*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(a*n*(p + 1)*(b
^2 - 4*a*c))   Int[(d*x)^m*(a + b*x^n + c*x^(2*n))^(p + 1)*Simp[b^2*(m + n*
(p + 1) + 1) - 2*a*c*(m + 2*n*(p + 1) + 1) + b*c*(m + n*(2*p + 3) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c
, 0] && IGtQ[n, 0] && ILtQ[p, -1]
```

rule 1834
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 +
 (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2
 - (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ
[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n
, 0]
```

3.251.4 Maple [F]

$$\int \frac{(dx)^m}{(c\,x^6+b\,x^3+a)^2}dx$$

input
```
int((d*x)^m/(c*x^6+b*x^3+a)^2,x)
```

output
```
int((d*x)^m/(c*x^6+b*x^3+a)^2,x)
```

3.251.5 Fricas [F]

$$\int \frac{(dx)^m}{(a+bx^3+cx^6)^2}\,dx = \int \frac{(dx)^m}{(cx^6+bx^3+a)^2}\,dx$$

input
```
integrate((d*x)^m/(c*x^6+b*x^3+a)^2,x, algorithm="fricas")
```

output
```
integral((d*x)^m/(c^2*x^12 + 2*b*c*x^9 + (b^2 + 2*a*c)*x^6 + 2*a*b*x^3 + a
^2), x)
```

3.251.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{(a+bx^3+cx^6)^2}\,dx=\text{Timed out}$$

input
```
integrate((d*x)**m/(c*x**6+b*x**3+a)**2,x)
```

output
```
Timed out
```

3.251.7 Maxima [F]

$$\int \frac{(dx)^m}{(a+bx^3+cx^6)^2}\,dx=\int \frac{(dx)^m}{(cx^6+bx^3+a)^2}\,dx$$

input
```
integrate((d*x)^m/(c*x^6+b*x^3+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x)^m/(c*x^6 + b*x^3 + a)^2, x)
```

3.251.8 Giac [F]

$$\int \frac{(dx)^m}{(a+bx^3+cx^6)^2}\,dx=\int \frac{(dx)^m}{(cx^6+bx^3+a)^2}\,dx$$

input
```
integrate((d*x)^m/(c*x^6+b*x^3+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x)^m/(c*x^6 + b*x^3 + a)^2, x)
```

3.251.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{(a+bx^3+cx^6)^2}\,dx = \int \frac{(d\,x)^m}{(c\,x^6+b\,x^3+a)^2}\,dx$$

input
```
int((d*x)^m/(a + b*x^3 + c*x^6)^2,x)
```

output
```
int((d*x)^m/(a + b*x^3 + c*x^6)^2, x)
```

3.252 $\int (dx)^m \left(a+bx^3+cx^6\right)^{3/2} dx$

3.252.1 Optimal result

Integrand size = 22, antiderivative size = 158

$$\int (dx)^m \left(a+bx^3+cx^6\right)^{3/2} dx=\frac{a(dx)^{1+m}\sqrt{a+bx^3+cx^6}\,\text{AppellF1}\left(\frac{1+m}{3},-\frac{3}{2},-\frac{3}{2},\frac{4+m}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{d(1+m)\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

output
```
a*(d*x)^(1+m)*AppellF1(1/3+1/3*m,-3/2,-3/2,4/3+1/3*m,-2*c*x^3/(b-(-4*a*c+b
^2)^(1/2)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))*(c*x^6+b*x^3+a)^(1/2)/d/(1+m)/
(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2))
)^(1/2)
```

3.252.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 357 vs. 2(158) = 316.

Time = 1.85 (sec) , antiderivative size = 357, normalized size of antiderivative = 2.26

$$\int (dx)^m \left(a+bx^3+cx^6\right)^{3/2} dx=\frac{x(dx)^m\sqrt{a+bx^3+cx^6}\Big(a(28+11m+m^2)\,\text{AppellF1}\left(\frac{1+m}{3},-\frac{1}{2},-\frac{1}{2},\frac{4+m}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2\cdots}}\right.}{}$$

input
```
Integrate[(d*x)^m*(a + b*x^3 + c*x^6)^(3/2),x]
```

output
```
(x*(d*x)^m*Sqrt[a + b*x^3 + c*x^6]*(a*(28 + 11*m + m^2)*AppellF1[(1 + m)/3
, -1/2, -1/2, (4 + m)/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b
 + Sqrt[b^2 - 4*a*c])] + (1 + m)*x^3*(b*(7 + m)*AppellF1[(4 + m)/3, -1/2,
-1/2, (7 + m)/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[
b^2 - 4*a*c])] + c*(4 + m)*x^3*AppellF1[(7 + m)/3, -1/2, -1/2, (10 + m)/3,
 (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]))
)/((1 + m)*(4 + m)*(7 + m)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqr
t[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*
a*c])])
```

3.252.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a+bx^3+cx^6\right)^{3/2} dx$$

$$\Big\downarrow \text{ 1721}$$

$$\frac{a\sqrt{a+bx^3+cx^6}\int (dx)^m \left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{3/2}\left(\frac{2cx^3}{b+\sqrt{b^2-4ac}}+1\right)^{3/2}dx}{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

$$\Big\downarrow \text{ 1012}$$

$$\frac{a(dx)^{m+1}\sqrt{a+bx^3+cx^6}\,\text{AppellF1}\left(\frac{m+1}{3},-\frac{3}{2},-\frac{3}{2},\frac{m+4}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[(d*x)^m*(a + b*x^3 + c*x^6)^(3/2),x]
```

output
```
(a*(d*x)^(1 + m)*Sqrt[a + b*x^3 + c*x^6]*AppellF1[(1 + m)/3, -3/2, -3/2, (
4 + m)/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4
*a*c])])/(d*(1 + m)*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (
2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])
```

3.252.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

3.252.4 Maple [F]

$$\int (dx)^m \left(c\,x^6+b\,x^3+a\right)^{\frac{3}{2}}dx$$

input
```
int((d*x)^m*(c*x^6+b*x^3+a)^(3/2),x)
```

output
```
int((d*x)^m*(c*x^6+b*x^3+a)^(3/2),x)
```

3.252.5 Fricas [F]

$$\int (dx)^m \left(a+bx^3+cx^6\right)^{3/2}\,dx = \int \left(cx^6+bx^3+a\right)^{\frac{3}{2}}(dx)^m\,dx$$

input
```
integrate((d*x)^m*(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
integral((c*x^6 + b*x^3 + a)^(3/2)*(d*x)^m, x)
```

3.252.6 Sympy [F]

$$\int (dx)^m \left(a+bx^3+cx^6\right)^{3/2} dx = \int (dx)^m \left(a+bx^3+cx^6\right)^{\frac{3}{2}} dx$$

input
```
integrate((d*x)**m*(c*x**6+b*x**3+a)**(3/2),x)
```

output
```
Integral((d*x)**m*(a + b*x**3 + c*x**6)**(3/2), x)
```

3.252.7 Maxima [F]

$$\int (dx)^m \left(a+bx^3+cx^6\right)^{3/2} dx = \int \left(cx^6+bx^3+a\right)^{\frac{3}{2}} (dx)^m dx$$

input
```
integrate((d*x)^m*(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)^(3/2)*(d*x)^m, x)
```

3.252.8 Giac [F]

$$\int (dx)^m \left(a+bx^3+cx^6\right)^{3/2} dx = \int \left(cx^6+bx^3+a\right)^{\frac{3}{2}} (dx)^m dx$$

input
```
integrate((d*x)^m*(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^(3/2)*(d*x)^m, x)
```

3.252.9 Mupad [F(-1)]

Timed out.

$$\int (dx)^m \left(a+bx^3+cx^6\right)^{3/2} \, dx = \int (d\,x)^m \left(c\,x^6+b\,x^3+a\right)^{3/2} dx$$

input
```
int((d*x)^m*(a + b*x^3 + c*x^6)^(3/2),x)
```

output
```
int((d*x)^m*(a + b*x^3 + c*x^6)^(3/2), x)
```

3.253 $\int (dx)^m \sqrt{a+bx^3+cx^6}\,dx$

3.253.1 Optimal result

Integrand size = 22, antiderivative size = 157

$$\int (dx)^m \sqrt{a+bx^3+cx^6}\,dx = \frac{(dx)^{1+m}\sqrt{a+bx^3+cx^6}\,\text{AppellF1}\left(\frac{1+m}{3},-\frac{1}{2},-\frac{1}{2},\frac{4+m}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{d(1+m)\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

output
```
(d*x)^(1+m)*AppellF1(1/3+1/3*m,-1/2,-1/2,4/3+1/3*m,-2*c*x^3/(b-(-4*a*c+b^2
)^(1/2)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))*(c*x^6+b*x^3+a)^(1/2)/d/(1+m)/(1
+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^
(1/2)
```

3.253.2 Mathematica [A] (verified)

Time = 1.54 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.15

$$\int (dx)^m \sqrt{a+bx^3+cx^6}\,dx = \frac{x(dx)^m\sqrt{a+bx^3+cx^6}\,\text{AppellF1}\left(\frac{1+m}{3},-\frac{1}{2},-\frac{1}{2},\frac{4+m}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)}{(1+m)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}}$$

input
```
Integrate[(d*x)^m*Sqrt[a + b*x^3 + c*x^6],x]
```

output
```
(x*(d*x)^m*Sqrt[a + b*x^3 + c*x^6]*AppellF1[(1 + m)/3, -1/2, -1/2, (4 + m)
/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])
])/((1 + m)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])
]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])
```

3.253.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \sqrt{a+bx^3+cx^6}\, dx$$

$$\Big\downarrow \text{ 1721}$$

$$\frac{\sqrt{a+bx^3+cx^6}\int (dx)^m\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{b+\sqrt{b^2-4ac}}+1}dx}{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

$$\Big\downarrow \text{ 1012}$$

$$\frac{(dx)^{m+1}\sqrt{a+bx^3+cx^6}\,\text{AppellF1}\left(\frac{m+1}{3},-\frac{1}{2},-\frac{1}{2},\frac{m+4}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[(d*x)^m*Sqrt[a + b*x^3 + c*x^6],x]
```

output
```
((d*x)^(1 + m)*Sqrt[a + b*x^3 + c*x^6]*AppellF1[(1 + m)/3, -1/2, -1/2, (4
+ m)/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a
*c])])/(d*(1 + m)*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*
c*x^3)/(b + Sqrt[b^2 - 4*a*c])])
```

3.253.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

3.253.4 Maple [F]

$$\int (dx)^m \sqrt{c\,x^6+b\,x^3+a}dx$$

input
```
int((d*x)^m*(c*x^6+b*x^3+a)^(1/2),x)
```

output
```
int((d*x)^m*(c*x^6+b*x^3+a)^(1/2),x)
```

3.253.5 Fricas [F]

$$\int (dx)^m\sqrt{a+bx^3+cx^6}\,dx = \int \sqrt{cx^6+bx^3+a}(dx)^m\,dx$$

input
```
integrate((d*x)^m*(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^6 + b*x^3 + a)*(d*x)^m, x)
```

3.253.6 Sympy [F]

$$\int (dx)^m\sqrt{a+bx^3+cx^6}\,dx=\int (dx)^m\sqrt{a+bx^3+cx^6}\,dx$$

input
```
integrate((d*x)**m*(c*x**6+b*x**3+a)**(1/2),x)
```

output
```
Integral((d*x)**m*sqrt(a + b*x**3 + c*x**6), x)
```

3.253.7 Maxima [F]

$$\int (dx)^m\sqrt{a+bx^3+cx^6}\,dx=\int \sqrt{cx^6+bx^3+a}(dx)^m\,dx$$

input
```
integrate((d*x)^m*(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^6 + b*x^3 + a)*(d*x)^m, x)
```

3.253.8 Giac [F]

$$\int (dx)^m\sqrt{a+bx^3+cx^6}\,dx=\int \sqrt{cx^6+bx^3+a}(dx)^m\,dx$$

input
```
integrate((d*x)^m*(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^6 + b*x^3 + a)*(d*x)^m, x)
```

3.253.9 Mupad [F(-1)]

Timed out.

$$\int (dx)^m \sqrt{a+bx^3+cx^6}\,dx=\int (d\,x)^m\,\sqrt{c\,x^6+b\,x^3+a}\,dx$$

input
```
int((d*x)^m*(a + b*x^3 + c*x^6)^(1/2),x)
```

output
```
int((d*x)^m*(a + b*x^3 + c*x^6)^(1/2), x)
```

3.254 $\int \frac{(dx)^m}{\sqrt{a+bx^3+cx^6}}\,dx$

3.254.1 Optimal result

Integrand size = 22, antiderivative size = 157

$$\int \frac{(dx)^m}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{(dx)^{1+m}\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{1+m}{3},\frac{1}{2},\frac{1}{2},\frac{4+m}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{d(1+m)\sqrt{a+bx^3+cx^6}}$$

output
```
(d*x)^(1+m)*AppellF1(1/3+1/3*m,1/2,1/2,4/3+1/3*m,-2*c*x^3/(b-(-4*a*c+b^2)^
(1/2)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))
^(1/2)*(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/d/(1+m)/(c*x^6+b*x^3+a)^(1
/2)
```

3.254.2 Mathematica [A] (verified)

Time = 1.74 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.15

$$\int \frac{(dx)^m}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{x(dx)^m\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{1+m}{3},\frac{1}{2},\frac{1}{2},\frac{4+m}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)}{(1+m)\sqrt{a+bx^3+cx^6}}$$

input
```
Integrate[(d*x)^m/Sqrt[a + b*x^3 + c*x^6],x]
```

output
```
(x*(d*x)^m*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]
*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[
(1 + m)/3, 1/2, 1/2, (4 + m)/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x
^3)/(-b + Sqrt[b^2 - 4*a*c])])/((1 + m)*Sqrt[a + b*x^3 + c*x^6])
```

3.254.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{\sqrt{a+bx^3+cx^6}}dx$$

$$\Big\downarrow \text{1721}$$

$$\frac{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\int \frac{(dx)^m}{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{b+\sqrt{b^2-4ac}}+1}}dx}{\sqrt{a+bx^3+cx^6}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{(dx)^{m+1}\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{m+1}{3},\frac{1}{2},\frac{1}{2},\frac{m+4}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{a+bx^3+cx^6}}$$

input
```
Int[(d*x)^m/Sqrt[a + b*x^3 + c*x^6],x]
```

output
```
((d*x)^(1 + m)*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x
^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/3, 1/2, 1/2, (4 + m)/3, (-2*
c*x^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(d*(1
 + m)*Sqrt[a + b*x^3 + c*x^6])
```

3.254.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

3.254.4 Maple [F]

$$\int \frac{(dx)^m}{\sqrt{c\,x^6+b\,x^3+a}}dx$$

input
```
int((d*x)^m/(c*x^6+b*x^3+a)^(1/2),x)
```

output
```
int((d*x)^m/(c*x^6+b*x^3+a)^(1/2),x)
```

3.254.5 Fricas [F]

$$\int \frac{(dx)^m}{\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{(dx)^m}{\sqrt{cx^6+bx^3+a}}\,dx$$

input
```
integrate((d*x)^m/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
integral((d*x)^m/sqrt(c*x^6 + b*x^3 + a), x)
```

3.254.6 Sympy [F]

$$\int \frac{(dx)^m}{\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{(dx)^m}{\sqrt{a+bx^3+cx^6}}\,dx$$

input
```
integrate((d*x)**m/(c*x**6+b*x**3+a)**(1/2),x)
```

output
```
Integral((d*x)**m/sqrt(a + b*x**3 + c*x**6), x)
```

3.254.7 Maxima [F]

$$\int \frac{(dx)^m}{\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{(dx)^m}{\sqrt{cx^6+bx^3+a}}\,dx$$

input
```
integrate((d*x)^m/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x)^m/sqrt(c*x^6 + b*x^3 + a), x)
```

3.254.8 Giac [F]

$$\int \frac{(dx)^m}{\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{(dx)^m}{\sqrt{cx^6+bx^3+a}}\,dx$$

input
```
integrate((d*x)^m/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x)^m/sqrt(c*x^6 + b*x^3 + a), x)
```

3.254.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{\sqrt{a+bx^3+cx^6}}\,dx=\int \frac{(d\,x)^m}{\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

input
```
int((d*x)^m/(a + b*x^3 + c*x^6)^(1/2),x)
```

output
```
int((d*x)^m/(a + b*x^3 + c*x^6)^(1/2), x)
```

3.255 $\int \frac{(dx)^m}{(a+bx^3+cx^6)^{3/2}}\,dx$

3.255.1 Optimal result

Integrand size = 22, antiderivative size = 160

$$\int \frac{(dx)^m}{(a+bx^3+cx^6)^{3/2}}\,dx=\frac{(dx)^{1+m}\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(\frac{1+m}{3},\frac{3}{2},\frac{3}{2},\frac{4+m}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\right.}{ad(1+m)\sqrt{a+bx^3+cx^6}}$$

output
```
(d*x)^(1+m)*AppellF1(1/3+1/3*m,3/2,3/2,4/3+1/3*m,-2*c*x^3/(b-(-4*a*c+b^2)^
(1/2)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))
^(1/2)*(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/d/(1+m)/(c*x^6+b*x^3+a)^
(1/2)
```

3.255.2 Mathematica [A] (verified)

Time = 6.93 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.38

$$\int \frac{(dx)^m}{(a+bx^3+cx^6)^{3/2}}\,dx=\frac{x(dx)^m\left(-b+\sqrt{b^2-4ac}-2cx^3\right)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}\right)^{3/2}\operatorname{AppellF1}}{\left(-b+\sqrt{b^2-4ac}\right)(1+m)\left(a+bx^3+cx^6\right.}$$

input
```
Integrate[(d*x)^m/(a + b*x^3 + c*x^6)^(3/2),x]
```

output
```
(x*(d*x)^m*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^3)*Sqrt[(b - Sqrt[b^2 - 4*a*c]
+ 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b
+ Sqrt[b^2 - 4*a*c]))^(3/2)*AppellF1[(1 + m)/3, 3/2, 3/2, (4 + m)/3, (-2*c
*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/((-b +
 Sqrt[b^2 - 4*a*c])*(1 + m)*(a + b*x^3 + c*x^6)^(3/2))
```

3.255.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{(a+bx^3+cx^6)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 1721}$$

$$\frac{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\int \frac{(dx)^m}{\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{3/2}\left(\frac{2cx^3}{b+\sqrt{b^2-4ac}}+1\right)^{3/2}}dx}{a\sqrt{a+bx^3+cx^6}}$$

$$\Big\downarrow \text{ 1012}$$

$$\frac{(dx)^{m+1}\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{m+1}{3},\frac{3}{2},\frac{3}{2},\frac{m+4}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{ad(m+1)\sqrt{a+bx^3+cx^6}}$$

input
```
Int[(d*x)^m/(a + b*x^3 + c*x^6)^(3/2),x]
```

output
```
((d*x)^(1 + m)*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x
^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/3, 3/2, 3/2, (4 + m)/3, (-2*
c*x^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(a*d*
(1 + m)*Sqrt[a + b*x^3 + c*x^6])
```

3.255.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

3.255.4 Maple [F]

$$\int \frac{(dx)^m}{(c\,x^6+b\,x^3+a)^{\frac{3}{2}}}dx$$

input
```
int((d*x)^m/(c*x^6+b*x^3+a)^(3/2),x)
```

output
```
int((d*x)^m/(c*x^6+b*x^3+a)^(3/2),x)
```

3.255.5 Fricas [F]

$$\int \frac{(dx)^m}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{(dx)^m}{(cx^6+bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)^m/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^6 + b*x^3 + a)*(d*x)^m/(c^2*x^12 + 2*b*c*x^9 + (b^2 + 2*
a*c)*x^6 + 2*a*b*x^3 + a^2), x)
```

3.255.6 Sympy [F]

$$\int \frac{(dx)^m}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{(dx)^m}{(a+bx^3+cx^6)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)**m/(c*x**6+b*x**3+a)**(3/2),x)
```

output
```
Integral((d*x)**m/(a + b*x**3 + c*x**6)**(3/2), x)
```

3.255.7 Maxima [F]

$$\int \frac{(dx)^m}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{(dx)^m}{(cx^6+bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)^m/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x)^m/(c*x^6 + b*x^3 + a)^(3/2), x)
```

3.255.8 Giac [F]

$$\int \frac{(dx)^m}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{(dx)^m}{(cx^6+bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)^m/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x)^m/(c*x^6 + b*x^3 + a)^(3/2), x)
```

3.255.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{(d\,x)^m}{(c\,x^6+b\,x^3+a)^{3/2}}\,dx$$

input
```
int((d*x)^m/(a + b*x^3 + c*x^6)^(3/2),x)
```

output
```
int((d*x)^m/(a + b*x^3 + c*x^6)^(3/2), x)
```

3.256 $\int (dx)^m (a+bx^3+cx^6)^p \, dx$

3.256.1 Optimal result

Integrand size = 20, antiderivative size = 155

$$\int (dx)^m (a+bx^3+cx^6)^p \, dx = \frac{(dx)^{1+m} \left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p} \left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p} (a+bx^3+cx^6)^p \text{AppellF1}\left(\frac{1+m}{3},-p,-p,\frac{4+m}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right.}{d(1+m)}$$

output
```
(d*x)^(1+m)*(c*x^6+b*x^3+a)^p*AppellF1(1/3+1/3*m,-p,-p,4/3+1/3*m,-2*c*x^3/
(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/d/(1+m)/((1+2*c*x^
3/(b-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^p)
```

3.256.2 Mathematica [A] (warning: unable to verify)

Time = 0.26 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.15

$$\int (dx)^m (a+bx^3+cx^6)^p \, dx = \frac{x(dx)^m \left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p} \left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p} (a+bx^3+cx^6)^p \text{AppellF1}\left(\frac{1+m}{3},-p,-p,\frac{4+m}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right.}{1+m}$$

input
```
Integrate[(d*x)^m*(a + b*x^3 + c*x^6)^p,x]
```

output
```
(x*(d*x)^m*(a + b*x^3 + c*x^6)^p*AppellF1[(1 + m)/3, -p, -p, (4 + m)/3, (-
2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/((1
 + m)*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*((b +
Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

3.256.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a+bx^3+cx^6\right)^p \, dx$$

$$\Big\downarrow \ 1721$$

$$\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p} \left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p} \left(a+bx^3+cx^6\right)^p \int (dx)^m \left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^p \left(\frac{2cx^3}{b+\sqrt{b^2-4}}\right.$$

$$\Big\downarrow \ 1012$$

$$\frac{(dx)^{m+1} \left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p} \left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p} \left(a+bx^3+cx^6\right)^p \text{AppellF1}\left(\frac{m+1}{3},-p,-p,\frac{m+4}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{}{b+}\right.}{d(m+1)}$$

input
```
Int[(d*x)^m*(a + b*x^3 + c*x^6)^p,x]
```

output
```
((d*x)^(1 + m)*(a + b*x^3 + c*x^6)^p*AppellF1[(1 + m)/3, -p, -p, (4 + m)/3
, (-2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])
/(d*(1 + m)*(1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b +
Sqrt[b^2 - 4*a*c]))^p)
```

3.256.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

3.256.4 Maple [F]

$$\int (dx)^m \left(c\,x^6+b\,x^3+a\right)^p dx$$

input
```
int((d*x)^m*(c*x^6+b*x^3+a)^p,x)
```

output
```
int((d*x)^m*(c*x^6+b*x^3+a)^p,x)
```

3.256.5 Fricas [F]

$$\int (dx)^m \left(a+bx^3+cx^6\right)^p \, dx = \int \left(cx^6+bx^3+a\right)^p (dx)^m \, dx$$

input
```
integrate((d*x)^m*(c*x^6+b*x^3+a)^p,x, algorithm="fricas")
```

output
```
integral((c*x^6 + b*x^3 + a)^p*(d*x)^m, x)
```

3.256.6 Sympy [F(-1)]

Timed out.

$$\int (dx)^m \left(a+bx^3+cx^6\right)^p \, dx = \text{Timed out}$$

input
```
integrate((d*x)**m*(c*x**6+b*x**3+a)**p,x)
```

output
```
Timed out
```

3.256.7 Maxima [F]

$$\int (dx)^m \left(a+bx^3+cx^6\right)^p \, dx = \int \left(cx^6+bx^3+a\right)^p (dx)^m \, dx$$

input
```
integrate((d*x)^m*(c*x^6+b*x^3+a)^p,x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p*(d*x)^m, x)
```

3.256.8 Giac [F]

$$\int (dx)^m \left(a+bx^3+cx^6\right)^p \, dx = \int \left(cx^6+bx^3+a\right)^p (dx)^m \, dx$$

input
```
integrate((d*x)^m*(c*x^6+b*x^3+a)^p,x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p*(d*x)^m, x)
```

3.256.9 Mupad [F(-1)]

Timed out.

$$\int (dx)^m \left(a+bx^3+cx^6\right)^p \, dx = \int (d\,x)^m \left(c\,x^6+b\,x^3+a\right)^p \, dx$$

input
```
int((d*x)^m*(a + b*x^3 + c*x^6)^p,x)
```

output
```
int((d*x)^m*(a + b*x^3 + c*x^6)^p, x)
```

3.257 $\int x^8(a+bx^3+cx^6)^p\,dx$

3.257.1 Optimal result

Integrand size = 18, antiderivative size = 224

$$\int x^8(a+bx^3+cx^6)^p\,dx = -\frac{b(2+p)\left(a+bx^3+cx^6\right)^{1+p}}{6c^2(1+p)(3+2p)}+\frac{x^3\left(a+bx^3+cx^6\right)^{1+p}}{3c(3+2p)} + \frac{2^p(2ac-b^2(2+p))\left(-\frac{b-\sqrt{b^2-4ac}+2cx^3}{\sqrt{b^2-4ac}}\right)^{-1-p}\left(a+bx^3+cx^6\right)^{1+p}\text{Hypergeometric2F1}\left(-p,1+p,2+p,\frac{b+\dots}{\dots}\right)}{3c^2\sqrt{b^2-4ac}(1+p)(3+2p)}$$

output
```
-1/6*b*(2+p)*(c*x^6+b*x^3+a)^(p+1)/c^2/(2*p^2+5*p+3)+1/3*x^3*(c*x^6+b*x^3+
a)^(p+1)/c/(3+2*p)+1/3*2^p*(2*a*c-b^2*(2+p))*(c*x^6+b*x^3+a)^(p+1)*hyperge
om([-p, p+1],[2+p],1/2*(b+2*c*x^3+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))*
((-b-2*c*x^3+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(-1-p)/c^2/(p+1)/(3+2
*p)/(-4*a*c+b^2)^(1/2)
```

3.257.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.33 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.72

$$\int x^8(a+bx^3+cx^6)^p\,dx = \frac{1}{9}x^9\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p\,\text{AppellF1}\left(3,-p,-p,4,-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)$$

input
```
Integrate[x^8*(a + b*x^3 + c*x^6)^p,x]
```

output
```
(x^9*(a + b*x^3 + c*x^6)^p*AppellF1[3, -p, -p, 4, (-2*c*x^3)/(b + Sqrt[b^2
 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(9*((b - Sqrt[b^2 - 4*a*c
] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)
/(b + Sqrt[b^2 - 4*a*c]))^p)
```

3.257.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1693, 1166, 25, 1160, 1096}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8 \left(a+bx^3+cx^6\right)^p \, dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int x^6 \left(cx^6+bx^3+a\right)^p dx^3$$

$$\Big\downarrow \text{1166}$$

$$\frac{1}{3}\left(\frac{\int -\left(\left(b(p+2)x^3+a\right)\left(cx^6+bx^3+a\right)^p\right)dx^3}{c(2p+3)}+\frac{x^3\left(a+bx^3+cx^6\right)^{p+1}}{c(2p+3)}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\left(\frac{x^3\left(a+bx^3+cx^6\right)^{p+1}}{c(2p+3)}-\frac{\int \left(b(p+2)x^3+a\right)\left(cx^6+bx^3+a\right)^p dx^3}{c(2p+3)}\right)$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{3}\left(\frac{x^3\left(a+bx^3+cx^6\right)^{p+1}}{c(2p+3)}-\frac{\frac{\left(2ac-b^2(p+2)\right)\int \left(cx^6+bx^3+a\right)^p dx^3}{2c}+\frac{b(p+2)\left(a+bx^3+cx^6\right)^{p+1}}{2c(p+1)}}{c(2p+3)}\right)$$

$$\Big\downarrow \text{1096}$$

$$\frac{1}{3}\left(\frac{x^3(a+bx^3+cx^6)^{p+1}}{c(2p+3)}-\frac{\frac{b(p+2)(a+bx^3+cx^6)^{p+1}}{2c(p+1)}-\frac{2^p(2ac-b^2(p+2))\left(-\frac{-\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}}\right)^{-p-1}(a+bx^3+cx^6)^{p+1}\text{Hypergeomet}}{c(p+1)\sqrt{b^2-4ac}}}{c(2p+3)}\right.$$

input
```
Int[x^8*(a + b*x^3 + c*x^6)^p,x]
```

output
```
((x^3*(a + b*x^3 + c*x^6)^(1 + p))/(c*(3 + 2*p)) - ((b*(2 + p)*(a + b*x^3
+ c*x^6)^(1 + p))/(2*c*(1 + p)) - (2^p*(2*a*c - b^2*(2 + p))*(-((b - Sqrt[
b^2 - 4*a*c] + 2*c*x^3)/Sqrt[b^2 - 4*a*c]))^(-1 - p)*(a + b*x^3 + c*x^6)^(
1 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, (b + Sqrt[b^2 - 4*a*c] + 2*c*x^
3)/(2*Sqrt[b^2 - 4*a*c])])/(c*Sqrt[b^2 - 4*a*c]*(1 + p)))/(c*(3 + 2*p)))/3
```

3.257.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1096
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] &&  !IntegerQ[4*p] &&  !IntegerQ[3*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.257.4 Maple [F]

$$\int x^{8} \left(c\,x^{6}+b\,x^{3}+a \right)^{p}d x$$

input
```
int(x^8*(c*x^6+b*x^3+a)^p,x)
```

output
```
int(x^8*(c*x^6+b*x^3+a)^p,x)
```

3.257.5 Fricas [F]

$$\int x^8 \left(a+bx^3+cx^6\right)^p \, dx = \int (cx^6+bx^3+a)^p x^8 \, dx$$

input
```
integrate(x^8*(c*x^6+b*x^3+a)^p,x, algorithm="fricas")
```

output
```
integral((c*x^6 + b*x^3 + a)^p*x^8, x)
```

3.257.6 Sympy [F(-1)]

Timed out.

$$\int x^8 \left(a+bx^3+cx^6\right)^p \, dx = \text{Timed out}$$

input
```
integrate(x**8*(c*x**6+b*x**3+a)**p,x)
```

output
```
Timed out
```

3.257.7 Maxima [F]

$$\int x^8(a+bx^3+cx^6)^p\,dx=\int (cx^6+bx^3+a)^p x^8\,dx$$

input
```
integrate(x^8*(c*x^6+b*x^3+a)^p,x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p*x^8, x)
```

3.257.8 Giac [F]

$$\int x^8(a+bx^3+cx^6)^p\,dx=\int (cx^6+bx^3+a)^p x^8\,dx$$

input
```
integrate(x^8*(c*x^6+b*x^3+a)^p,x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p*x^8, x)
```

3.257.9 Mupad [F(-1)]

Timed out.

$$\int x^8(a+bx^3+cx^6)^p\,dx=\int x^8\,(c\,x^6+b\,x^3+a)^p\,dx$$

input
```
int(x^8*(a + b*x^3 + c*x^6)^p,x)
```

output
```
int(x^8*(a + b*x^3 + c*x^6)^p, x)
```

3.258 $\int x^5(a+bx^3+cx^6)^p\,dx$

3.258.1 Optimal result

Integrand size = 18, antiderivative size = 161

$$\int x^5\left(a+bx^3+cx^6\right)^p\,dx=\frac{\left(a+bx^3+cx^6\right)^{1+p}}{6c(1+p)}+\frac{2^pb\left(-\frac{b-\sqrt{b^2-4ac}+2cx^3}{\sqrt{b^2-4ac}}\right)^{-1-p}\left(a+bx^3+cx^6\right)^{1+p}\text{Hypergeometric2F1}\left(-p,1+p,2+p,\frac{b+\sqrt{b^2-4ac}+2cx^3}{2\sqrt{b^2-4ac}}\right)}{3c\sqrt{b^2-4ac}(1+p)}$$

output
```
1/6*(c*x^6+b*x^3+a)^(p+1)/c/(p+1)+1/3*2^p*b*(c*x^6+b*x^3+a)^(p+1)*hypergeo
m([-p, p+1],[2+p],1/2*(b+2*c*x^3+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))*(
(-b-2*c*x^3+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(-1-p)/c/(p+1)/(-4*a*c
+b^2)^(1/2)
```

3.258.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.27 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.01

$$\int x^5\left(a+bx^3+cx^6\right)^p\,dx=\frac{1}{6}x^6\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p\text{AppellF1}\left(2,-p,-p,3,-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)$$

input
```
Integrate[x^5*(a + b*x^3 + c*x^6)^p,x]
```

output
```
(x^6*(a + b*x^3 + c*x^6)^p*AppellF1[2, -p, -p, 3, (-2*c*x^3)/(b + Sqrt[b^2
 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(6*((b - Sqrt[b^2 - 4*a*c
] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)
/(b + Sqrt[b^2 - 4*a*c]))^p)
```

3.258.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1693, 1160, 1096}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(a+bx^3+cx^6\right)^p \, dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int x^3 \left(cx^6+bx^3+a\right)^p dx^3$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{3}\left(\frac{\left(a+bx^3+cx^6\right)^{p+1}}{2c(p+1)}-\frac{b\int \left(cx^6+bx^3+a\right)^p dx^3}{2c}\right)$$

$$\Big\downarrow \text{1096}$$

$$\frac{1}{3}\left(\frac{b2^p\left(a+bx^3+cx^6\right)^{p+1}\left(-\frac{-\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}}\right)^{-p-1}\text{Hypergeometric2F1}\left(-p,p+1,p+2,\frac{2cx^3+b+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{c(p+1)\sqrt{b^2-4ac}}+\frac{(a$$

input
```
Int[x^5*(a + b*x^3 + c*x^6)^p,x]
```

output
```
((a + b*x^3 + c*x^6)^(1 + p)/(2*c*(1 + p)) + (2^p*b*(-((b - Sqrt[b^2 - 4*a
*c] + 2*c*x^3)/Sqrt[b^2 - 4*a*c]))^(-1 - p)*(a + b*x^3 + c*x^6)^(1 + p)*Hy
pergeometric2F1[-p, 1 + p, 2 + p, (b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(2*Sqr
t[b^2 - 4*a*c])])/(c*Sqrt[b^2 - 4*a*c]*(1 + p)))/3
```

3.258.3.1 Defintions of rubi rules used

rule 1096
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] &&  !IntegerQ[4*p] &&  !IntegerQ[3*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.258.4 Maple [F]

$$\int x^{5} \left(c\,x^{6}+b\,x^{3}+a\right)^{p}dx$$

input
```
int(x^5*(c*x^6+b*x^3+a)^p,x)
```

output
```
int(x^5*(c*x^6+b*x^3+a)^p,x)
```

3.258.5 Fricas [F]

$$\int x^5 \left(a+bx^3+cx^6\right)^p \, dx = \int (cx^6+bx^3+a)^p x^5 \, dx$$

input
```
integrate(x^5*(c*x^6+b*x^3+a)^p,x, algorithm="fricas")
```

output
```
integral((c*x^6 + b*x^3 + a)^p*x^5, x)
```

3.258.6 Sympy [F(-1)]

Timed out.

$$\int x^5 \left(a + bx^3 + cx^6\right)^p \, dx = \text{Timed out}$$

input
```
integrate(x**5*(c*x**6+b*x**3+a)**p,x)
```

output
```
Timed out
```

3.258.7 Maxima [F]

$$\int x^5 \left(a + bx^3 + cx^6\right)^p \, dx = \int \left(cx^6 + bx^3 + a\right)^p x^5 \, dx$$

input
```
integrate(x^5*(c*x^6+b*x^3+a)^p,x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p*x^5, x)
```

3.258.8 Giac [F]

$$\int x^5 \left(a + bx^3 + cx^6\right)^p \, dx = \int \left(cx^6 + bx^3 + a\right)^p x^5 \, dx$$

input
```
integrate(x^5*(c*x^6+b*x^3+a)^p,x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p*x^5, x)
```

3.258.9 Mupad [F(-1)]

Timed out.

$$\int x^5 (a + bx^3 + cx^6)^p \, dx = \int x^5 \left(c\,x^6 + b\,x^3 + a\right)^p \, dx$$

input
```
int(x^5*(a + b*x^3 + c*x^6)^p,x)
```

output
```
int(x^5*(a + b*x^3 + c*x^6)^p, x)
```

3.259 $\int x^2(a+bx^3+cx^6)^p\,dx$

3.259.1 Optimal result

Integrand size = 18, antiderivative size = 130

$$\int x^2(a+bx^3+cx^6)^p\,dx = -\frac{2^{1+p}\left(-\frac{b-\sqrt{b^2-4ac}+2cx^3}{\sqrt{b^2-4ac}}\right)^{-1-p}(a+bx^3+cx^6)^{1+p}\,\text{Hypergeometric2F1}\left(-p,1+p,2+p,\frac{b+\sqrt{b^2-4ac}+2cx^3}{2\sqrt{b^2-4ac}}\right)}{3\sqrt{b^2-4ac}(1+p)}$$

output
```
-1/3*2^(p+1)*(c*x^6+b*x^3+a)^(p+1)*hypergeom([-p, p+1],[2+p],1/2*(b+2*c*x^
3+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))*((-b-2*c*x^3+(-4*a*c+b^2)^(1/2))
/(-4*a*c+b^2)^(1/2))^(-1-p)/(p+1)/(-4*a*c+b^2)^(1/2)
```

3.259.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.06

$$\int x^2(a+bx^3+cx^6)^p\,dx = \frac{2^{-1+p}\left(b-\sqrt{b^2-4ac}+2cx^3\right)\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p\,\text{Hypergeometric2F1}\left(-p,1+p,2+p,\right.}{3c(1+p)}$$

input
```
Integrate[x^2*(a + b*x^3 + c*x^6)^p,x]
```

output
```
(2^(-1 + p)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)*(a + b*x^3 + c*x^6)^p*Hyperg
eometric2F1[-p, 1 + p, 2 + p, (-b + Sqrt[b^2 - 4*a*c] - 2*c*x^3)/(2*Sqrt[b
^2 - 4*a*c])])/(3*c*(1 + p)*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/Sqrt[b^2 -
4*a*c])^p)
```

3.259.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1690, 1096}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+bx^3+cx^6\right)^p \, dx$$

$$\Big\downarrow \text{1690}$$

$$\frac{1}{3}\int \left(cx^6+bx^3+a\right)^p dx^3$$

$$\Big\downarrow \text{1096}$$

$$-\frac{2^{p+1}\left(-\frac{-\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}}\right)^{-p-1}\left(a+bx^3+cx^6\right)^{p+1}\text{Hypergeometric2F1}\left(-p,p+1,p+2,\frac{2cx^3+b+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{3(p+1)\sqrt{b^2-4ac}}$$

input
```
Int[x^2*(a + b*x^3 + c*x^6)^p,x]
```

output
```
-1/3*(2^(1 + p)*(-((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/Sqrt[b^2 - 4*a*c]))^(
-1 - p)*(a + b*x^3 + c*x^6)^(1 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, (b
 + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(2*Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]
*(1 + p))
```

3.259.3.1 Defintions of rubi rules used

rule 1096
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] &&  !IntegerQ[4*p] &&  !IntegerQ[3*p]
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

3.259.4 Maple [F]

$$\int x^{2} \left(c\,x^{6}+b\,x^{3}+a\right)^{p}d x$$

input
```
int(x^2*(c*x^6+b*x^3+a)^p,x)
```

output
```
int(x^2*(c*x^6+b*x^3+a)^p,x)
```

3.259.5 Fricas [F]

$$\int x^2 \left(a+bx^3+cx^6\right)^p\,dx=\int \left(cx^6+bx^3+a\right)^p x^2\,dx$$

input
```
integrate(x^2*(c*x^6+b*x^3+a)^p,x, algorithm="fricas")
```

output
```
integral((c*x^6 + b*x^3 + a)^p*x^2, x)
```

3.259.6 Sympy [F(-1)]

Timed out.

$$\int x^2(a+bx^3+cx^6)^p\,dx = \text{Timed out}$$

input
```
integrate(x**2*(c*x**6+b*x**3+a)**p,x)
```

output
```
Timed out
```

3.259.7 Maxima [F]

$$\int x^2(a+bx^3+cx^6)^p\,dx = \int (cx^6+bx^3+a)^p x^2\,dx$$

input
```
integrate(x^2*(c*x^6+b*x^3+a)^p,x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p*x^2, x)
```

3.259.8 Giac [F]

$$\int x^2(a+bx^3+cx^6)^p\,dx = \int (cx^6+bx^3+a)^p x^2\,dx$$

input
```
integrate(x^2*(c*x^6+b*x^3+a)^p,x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p*x^2, x)
```

3.259.9 Mupad [F(-1)]

Timed out.

$$\int x^2 (a + bx^3 + cx^6)^p \, dx = \int x^2 \left(c\,x^6 + b\,x^3 + a\right)^p dx$$

input
```
int(x^2*(a + b*x^3 + c*x^6)^p,x)
```

output
```
int(x^2*(a + b*x^3 + c*x^6)^p, x)
```

3.260 $\int x^4(a+bx^3+cx^6)^p\,dx$

3.260.1 Optimal result

Integrand size = 18, antiderivative size = 138

$$\int x^4(a+bx^3+cx^6)^p\,dx=\frac{1}{5}x^5\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p\,\text{AppellF1}\left(\frac{5}{3},-p,-p,\frac{8}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)$$

output
```
1/5*x^5*(c*x^6+b*x^3+a)^p*AppellF1(5/3,-p,-p,8/3,-2*c*x^3/(b-(-4*a*c+b^2)^
(1/2)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/((1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2))
)^p)/((1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^p)
```

3.260.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.20

$$\int x^4(a+bx^3+cx^6)^p\,dx=\frac{1}{5}x^5\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p\,\text{AppellF1}\left(\frac{5}{3},-p,-p,\frac{8}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)$$

input
```
Integrate[x^4*(a + b*x^3 + c*x^6)^p,x]
```

output
```
(x^5*(a + b*x^3 + c*x^6)^p*AppellF1[5/3, -p, -p, 8/3, (-2*c*x^3)/(b + Sqrt
[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(5*((b - Sqrt[b^2 - 4
*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*
x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

3.260.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a+bx^3+cx^6\right)^p \, dx$$

$$\Big\downarrow \text{1721}$$

$$\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p} \left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p} \left(a+bx^3+cx^6\right)^p \int x^4 \left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^p \left(\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right.$$

$$\Big\downarrow \text{1012}$$

$$\frac{1}{5}x^5 \left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p} \left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p} \left(a+bx^3+cx^6\right)^p \text{AppellF1}\left(\frac{5}{3},-p,-p,\frac{8}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right.$$

input
```
Int[x^4*(a + b*x^3 + c*x^6)^p,x]
```

output
```
(x^5*(a + b*x^3 + c*x^6)^p*AppellF1[5/3, -p, -p, 8/3, (-2*c*x^3)/(b - Sqrt
[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(5*(1 + (2*c*x^3)/(b
- Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

3.260.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

3.260.4 Maple [F]

$$\int x^4 \left(c\,x^6+b\,x^3+a\right)^p dx$$

input
```
int(x^4*(c*x^6+b*x^3+a)^p,x)
```

output
```
int(x^4*(c*x^6+b*x^3+a)^p,x)
```

3.260.5 Fricas [F]

$$\int x^4 \left(a+bx^3+cx^6\right)^p \, dx = \int (cx^6+bx^3+a)^p x^4 \, dx$$

input
```
integrate(x^4*(c*x^6+b*x^3+a)^p,x, algorithm="fricas")
```

output
```
integral((c*x^6 + b*x^3 + a)^p*x^4, x)
```

3.260.6 Sympy [F(-1)]

Timed out.

$$\int x^4(a+bx^3+cx^6)^p\,dx = \text{Timed out}$$

input
```
integrate(x**4*(c*x**6+b*x**3+a)**p,x)
```

output
```
Timed out
```

3.260.7 Maxima [F]

$$\int x^4(a+bx^3+cx^6)^p\,dx = \int (cx^6+bx^3+a)^p x^4\,dx$$

input
```
integrate(x^4*(c*x^6+b*x^3+a)^p,x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p*x^4, x)
```

3.260.8 Giac [F]

$$\int x^4(a+bx^3+cx^6)^p\,dx = \int (cx^6+bx^3+a)^p x^4\,dx$$

input
```
integrate(x^4*(c*x^6+b*x^3+a)^p,x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p*x^4, x)
```

3.260.9 Mupad [F(-1)]

Timed out.

$$\int x^4 (a + bx^3 + cx^6)^p \, dx = \int x^4 \left(c\,x^6 + b\,x^3 + a\right)^p \, dx$$

input
```
int(x^4*(a + b*x^3 + c*x^6)^p,x)
```

output
```
int(x^4*(a + b*x^3 + c*x^6)^p, x)
```

3.261 $\int x^3(a+bx^3+cx^6)^p\,dx$

3.261.1 Optimal result

Integrand size = 18, antiderivative size = 138

$$\int x^3\left(a+bx^3+cx^6\right)^p\,dx=\frac{1}{4}x^4\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p\text{AppellF1}\left(\frac{4}{3},-p,-p,\frac{7}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)$$

output
```
1/4*x^4*(c*x^6+b*x^3+a)^p*AppellF1(4/3,-p,-p,7/3,-2*c*x^3/(b-(-4*a*c+b^2)^
(1/2)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/((1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2))
)^p)/((1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^p)
```

3.261.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.20

$$\int x^3\left(a+bx^3+cx^6\right)^p\,dx=\frac{1}{4}x^4\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p\text{AppellF1}\left(\frac{4}{3},-p,-p,\frac{7}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)$$

input
```
Integrate[x^3*(a + b*x^3 + c*x^6)^p,x]
```

output
```
(x^4*(a + b*x^3 + c*x^6)^p*AppellF1[4/3, -p, -p, 7/3, (-2*c*x^3)/(b + Sqrt
[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(4*((b - Sqrt[b^2 - 4
*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*
x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

3.261.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+bx^3+cx^6\right)^p \, dx$$

$$\Big\downarrow \text{1721}$$

$$\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p\int x^3\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^p\left(\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right.$$

$$\Big\downarrow \text{1012}$$

$$\frac{1}{4}x^4\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p \text{AppellF1}\left(\frac{4}{3},-p,-p,\frac{7}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right.$$

input
```
Int[x^3*(a + b*x^3 + c*x^6)^p,x]
```

output
```
(x^4*(a + b*x^3 + c*x^6)^p*AppellF1[4/3, -p, -p, 7/3, (-2*c*x^3)/(b - Sqrt
[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(4*(1 + (2*c*x^3)/(b
- Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

3.261.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

3.261.4 Maple [F]

$$\int x^{3} \left(c\,x^{6}+b\,x^{3}+a\right)^{p}d x$$

input
```
int(x^3*(c*x^6+b*x^3+a)^p,x)
```

output
```
int(x^3*(c*x^6+b*x^3+a)^p,x)
```

3.261.5 Fricas [F]

$$\int x^3 \left(a+bx^3+cx^6\right)^p \, dx = \int \left(cx^6+bx^3+a\right)^p x^3 \, dx$$

input
```
integrate(x^3*(c*x^6+b*x^3+a)^p,x, algorithm="fricas")
```

output
```
integral((c*x^6 + b*x^3 + a)^p*x^3, x)
```

3.261.6 Sympy [F(-1)]

Timed out.

$$\int x^3 (a + bx^3 + cx^6)^p \, dx = \text{Timed out}$$

input
```
integrate(x**3*(c*x**6+b*x**3+a)**p,x)
```

output
```
Timed out
```

3.261.7 Maxima [F]

$$\int x^3 (a + bx^3 + cx^6)^p \, dx = \int (cx^6 + bx^3 + a)^p x^3 \, dx$$

input
```
integrate(x^3*(c*x^6+b*x^3+a)^p,x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p*x^3, x)
```

3.261.8 Giac [F]

$$\int x^3 (a + bx^3 + cx^6)^p \, dx = \int (cx^6 + bx^3 + a)^p x^3 \, dx$$

input
```
integrate(x^3*(c*x^6+b*x^3+a)^p,x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p*x^3, x)
```

3.261.9 Mupad [F(-1)]

Timed out.

$$\int x^3(a+bx^3+cx^6)^p\,dx=\int x^3\,\left(c\,x^6+b\,x^3+a\right)^p\,dx$$

input
```
int(x^3*(a + b*x^3 + c*x^6)^p,x)
```

output
```
int(x^3*(a + b*x^3 + c*x^6)^p, x)
```

3.262 $\int x(a+bx^3+cx^6)^p\,dx$

3.262.1 Optimal result

Integrand size = 16, antiderivative size = 138

$$\int x(a+bx^3+cx^6)^p\,dx=\frac{1}{2}x^2\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p\operatorname{AppellF1}\left(\frac{2}{3},-p,-p,\frac{5}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)$$

output
```
1/2*x^2*(c*x^6+b*x^3+a)^p*AppellF1(2/3,-p,-p,5/3,-2*c*x^3/(b-(-4*a*c+b^2)^
(1/2)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/((1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2))
)^p)/((1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^p)
```

3.262.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.20

$$\int x(a+bx^3+cx^6)^p\,dx=\frac{1}{2}x^2\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p\operatorname{AppellF1}\left(\frac{2}{3},-p,-p,\frac{5}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)$$

input
```
Integrate[x*(a + b*x^3 + c*x^6)^p,x]
```

output
```
(x^2*(a + b*x^3 + c*x^6)^p*AppellF1[2/3, -p, -p, 5/3, (-2*c*x^3)/(b + Sqrt
[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(2*((b - Sqrt[b^2 - 4
*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*
x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

3.262.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+bx^3+cx^6\right)^p\,dx$$

$$\Big\downarrow \text{1721}$$

$$\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p\int x\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^p\left(\frac{2cx^3}{b+\sqrt{b^2-4ac}}+\right.$$

$$\Big\downarrow \text{1012}$$

$$\frac{1}{2}x^2\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p\text{AppellF1}\left(\frac{2}{3},-p,-p,\frac{5}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},\right.$$

input
```
Int[x*(a + b*x^3 + c*x^6)^p,x]
```

output
```
(x^2*(a + b*x^3 + c*x^6)^p*AppellF1[2/3, -p, -p, 5/3, (-2*c*x^3)/(b - Sqrt
[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(2*(1 + (2*c*x^3)/(b
- Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

3.262.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

3.262.4 Maple [F]

$$\int x(c\,x^{6}+b\,x^{3}+a)^{p}\,dx$$

input
```
int(x*(c*x^6+b*x^3+a)^p,x)
```

output
```
int(x*(c*x^6+b*x^3+a)^p,x)
```

3.262.5 Fricas [F]

$$\int x\left(a+bx^3+cx^6\right)^p\,dx=\int\left(cx^6+bx^3+a\right)^p x\,dx$$

input
```
integrate(x*(c*x^6+b*x^3+a)^p,x, algorithm="fricas")
```

output
```
integral((c*x^6 + b*x^3 + a)^p*x, x)
```

3.262.6 Sympy [F(-1)]

Timed out.

$$\int x(a+bx^3+cx^6)^p\,dx = \text{Timed out}$$

input
```
integrate(x*(c*x**6+b*x**3+a)**p,x)
```

output
```
Timed out
```

3.262.7 Maxima [F]

$$\int x(a+bx^3+cx^6)^p\,dx = \int (cx^6+bx^3+a)^p x\,dx$$

input
```
integrate(x*(c*x^6+b*x^3+a)^p,x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p*x, x)
```

3.262.8 Giac [F]

$$\int x(a+bx^3+cx^6)^p\,dx = \int (cx^6+bx^3+a)^p x\,dx$$

input
```
integrate(x*(c*x^6+b*x^3+a)^p,x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p*x, x)
```

3.262.9 Mupad [F(-1)]

Timed out.

$$\int x(a+bx^3+cx^6)^p \, dx = \int x \left(c\,x^6+b\,x^3+a\right)^p \,dx$$

input
```
int(x*(a + b*x^3 + c*x^6)^p,x)
```

output
```
int(x*(a + b*x^3 + c*x^6)^p, x)
```

3.263 $\int (a + bx^3 + cx^6)^p \, dx$

3.263.1 Optimal result

Integrand size = 14, antiderivative size = 133

$$\int (a + bx^3 + cx^6)^p \, dx = x\left(1 + \frac{2cx^3}{b - \sqrt{b^2 - 4ac}}\right)^{-p}\left(1 + \frac{2cx^3}{b + \sqrt{b^2 - 4ac}}\right)^{-p}(a + bx^3 + cx^6)^p \operatorname{AppellF1}\left(\frac{1}{3}, -p, -p, \frac{4}{3}, -\frac{2cx^3}{b - \sqrt{b^2 - 4ac}}, -\frac{2cx^3}{b + \sqrt{b^2 - 4ac}}\right)$$

output
```
x*(c*x^6+b*x^3+a)^p*AppellF1(1/3,-p,-p,4/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2))
,-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/((1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^p)/(
(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^p)
```

3.263.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.21

$$\int (a + bx^3 + cx^6)^p \, dx = x\left(\frac{b - \sqrt{b^2 - 4ac} + 2cx^3}{b - \sqrt{b^2 - 4ac}}\right)^{-p}\left(\frac{b + \sqrt{b^2 - 4ac} + 2cx^3}{b + \sqrt{b^2 - 4ac}}\right)^{-p}(a + bx^3 + cx^6)^p \operatorname{AppellF1}\left(\frac{1}{3}, -p, -p, \frac{4}{3}, -\frac{2cx^3}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^3}{-b + \sqrt{b^2 - 4ac}}\right)$$

input
```
Integrate[(a + b*x^3 + c*x^6)^p,x]
```

output
```
(x*(a + b*x^3 + c*x^6)^p*AppellF1[1/3, -p, -p, 4/3, (-2*c*x^3)/(b + Sqrt[b
^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(((b - Sqrt[b^2 - 4*a*c
] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)
/(b + Sqrt[b^2 - 4*a*c]))^p)
```

3.263.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1686, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3+cx^6\right)^p dx$$

$$\Big\downarrow \text{1686}$$

$$\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p\int\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^p\left(\frac{2cx^3}{b+\sqrt{b^2-4ac}}+\right.$$

$$\Big\downarrow \text{936}$$

$$x\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p \text{AppellF1}\left(\frac{1}{3},-p,-p,\frac{4}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\right.$$

input
```
Int[(a + b*x^3 + c*x^6)^p,x]
```

output
```
(x*(a + b*x^3 + c*x^6)^p*AppellF1[1/3, -p, -p, 4/3, (-2*c*x^3)/(b - Sqrt[b
^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/((1 + (2*c*x^3)/(b - Sq
rt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

3.263.3.1 Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1686
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^
IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*c*(x^n/(b + Rt[b^2
- 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4*a*c, 2])))^FracPar
t[p]))   Int[(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2*c*(x^n/(b - S
qrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n]
&& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

3.263.4 Maple [F]

$$\int (c\,x^{6}+b\,x^{3}+a)^{p}\,dx$$

input
```
int((c*x^6+b*x^3+a)^p,x)
```

output
```
int((c*x^6+b*x^3+a)^p,x)
```

3.263.5 Fricas [F]

$$\int (a+bx^3+cx^6)^p\,dx=\int (cx^6+bx^3+a)^p\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p,x, algorithm="fricas")
```

output
```
integral((c*x^6 + b*x^3 + a)^p, x)
```

3.263.6 Sympy [F(-1)]

Timed out.

$$\int (a+bx^3+cx^6)^p \, dx = \text{Timed out}$$

input
```
integrate((c*x**6+b*x**3+a)**p,x)
```

output
```
Timed out
```

3.263.7 Maxima [F]

$$\int (a+bx^3+cx^6)^p \, dx = \int (cx^6+bx^3+a)^p \, dx$$

input
```
integrate((c*x^6+b*x^3+a)^p,x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p, x)
```

3.263.8 Giac [F]

$$\int (a+bx^3+cx^6)^p \, dx = \int (cx^6+bx^3+a)^p \, dx$$

input
```
integrate((c*x^6+b*x^3+a)^p,x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p, x)
```

3.263.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^3+cx^6\right)^p\,dx=\int \left(c\,x^6+b\,x^3+a\right)^p\,dx$$

input
```
int((a + b*x^3 + c*x^6)^p,x)
```

output
```
int((a + b*x^3 + c*x^6)^p, x)
```

3.264 $\int \frac{(a+bx^3+cx^6)^p}{x}\,dx$

3.264.1 Optimal result

Integrand size = 18, antiderivative size = 157

$$\int \frac{(a+bx^3+cx^6)^p}{x}\,dx = \frac{2^{-1+2p}\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p\,\mathrm{AppellF1}\left(-2p,-p,-p,1-2p,-\frac{b-\sqrt{b^2-4ac}}{2cx^3}\right.}{3p}$$

output
```
1/3*2^(-1+2*p)*(c*x^6+b*x^3+a)^p*AppellF1(-2*p,-p,-p,1-2*p,1/2*(-b-(-4*a*c
+b^2)^(1/2))/c/x^3,1/2*(-b+(-4*a*c+b^2)^(1/2))/c/x^3)/p/(((b+2*c*x^3-(-4*a
*c+b^2)^(1/2))/c/x^3)^p)/(((b+2*c*x^3+(-4*a*c+b^2)^(1/2))/c/x^3)^p)
```

3.264.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3+cx^6)^p}{x}\,dx = \frac{2^{-1+2p}\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p\,\mathrm{AppellF1}\left(-2p,-p,-p,1-2p,-\frac{b+\sqrt{b^2-4ac}}{2cx^3}\right.}{3p}$$

input
```
Integrate[(a + b*x^3 + c*x^6)^p/x,x]
```

output
```
(2^(-1 + 2*p)*(a + b*x^3 + c*x^6)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, -1/2*(
b + Sqrt[b^2 - 4*a*c])/(c*x^3), (-b + Sqrt[b^2 - 4*a*c])/(2*c*x^3)])/(3*p*
((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(c*x^3))^p*((b + Sqrt[b^2 - 4*a*c] + 2*
c*x^3)/(c*x^3))^p)
```

3.264.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1693, 1178, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3+cx^6)^p}{x}\,dx$$

$$\Big\downarrow \ 1693$$

$$\frac{1}{3}\int \frac{(cx^6+bx^3+a)^p}{x^3}dx^3$$

$$\Big\downarrow \ 1178$$

$$-\frac{1}{3}4^p\left(\frac{1}{x^3}\right)^{2p}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p\int\left(\frac{b-\sqrt{b^2-4ac}}{2cx^3}+1\right.$$

$$\Big\downarrow \ 150$$

$$\frac{2^{2p-1}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p\,\text{AppellF1}\left(-2p,-p,-p,1-2p,-\frac{b-\sqrt{b^2-4ac}}{2cx^3},-\frac{b}{}\right.}{3p}$$

input
```
Int[(a + b*x^3 + c*x^6)^p/x,x]
```

output
```
(2^(-1 + 2*p)*(a + b*x^3 + c*x^6)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, -1/2*(
b - Sqrt[b^2 - 4*a*c])/(c*x^3), -1/2*(b + Sqrt[b^2 - 4*a*c])/(c*x^3)])/(3*
p*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(c*x^3))^p*((b + Sqrt[b^2 - 4*a*c] +
2*c*x^3)/(c*x^3))^p)
```

3.264.3.1 Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !In
tegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1178
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(-(1/(d + e*x))^(2*p))*((a +
b*x + c*x^2)^p/(e*(e*((b - q + 2*c*x)/(2*c*(d + e*x))))^p*(e*((b + q + 2*c*
x)/(2*c*(d + e*x))))^p))   Subst[Int[x^(-m - 2*(p + 1))*Simp[1 - (d - e*((b
 - q)/(2*c)))*x, x]^p*Simp[1 - (d - e*((b + q)/(2*c)))*x, x]^p, x], x, 1/(d
 + e*x)], x]] /; FreeQ[{a, b, c, d, e, p}, x] && ILtQ[m, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.264.4 Maple [F]

$$\int \frac{(c\,x^{6}+b\,x^{3}+a)^{p}}{x}dx$$

input
```
int((c*x^6+b*x^3+a)^p/x,x)
```

output
```
int((c*x^6+b*x^3+a)^p/x,x)
```

3.264.5 Fricas [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x}\,dx=\int \frac{(cx^6+bx^3+a)^p}{x}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x,x, algorithm="fricas")
```

output
```
integral((c*x^6 + b*x^3 + a)^p/x, x)
```

3.264.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^p}{x}\,dx = \text{Timed out}$$

input
```
integrate((c*x**6+b*x**3+a)**p/x,x)
```

output
```
Timed out
```

3.264.7 Maxima [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x}\,dx = \int \frac{(cx^6+bx^3+a)^p}{x}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x,x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p/x, x)
```

3.264.8 Giac [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x}\,dx = \int \frac{(cx^6+bx^3+a)^p}{x}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x,x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p/x, x)
```

3.264.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^p}{x}\,dx = \int \frac{(c\,x^6+b\,x^3+a)^p}{x}\,dx$$

input
```
int((a + b*x^3 + c*x^6)^p/x,x)
```

output
```
int((a + b*x^3 + c*x^6)^p/x, x)
```

3.265 $\int \frac{(a+bx^3+cx^6)^p}{x^2}\,dx$

3.265.1 Optimal result

Integrand size = 18, antiderivative size = 136

$$\int \frac{(a+bx^3+cx^6)^p}{x^2}\,dx = -\frac{\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p \operatorname{AppellF1}\left(-\frac{1}{3},-p,-p,\frac{2}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4}}\right.}{x}$$

output
```
-(c*x^6+b*x^3+a)^p*AppellF1(-1/3,-p,-p,2/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2))
,-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/x/((1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^p)
/((1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^p)
```

3.265.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.21

$$\int \frac{(a+bx^3+cx^6)^p}{x^2}\,dx = -\frac{\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p \operatorname{AppellF1}\left(-\frac{1}{3},-p,-p,\frac{2}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-}}\right.}{x}$$

input
```
Integrate[(a + b*x^3 + c*x^6)^p/x^2,x]
```

output
```
-(((a + b*x^3 + c*x^6)^p*AppellF1[-1/3, -p, -p, 2/3, (-2*c*x^3)/(b + Sqrt[
b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(x*((b - Sqrt[b^2 - 4*
a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x
^3)/(b + Sqrt[b^2 - 4*a*c]))^p))
```

3.265.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3+cx^6)^p}{x^2}\,dx$$

$$\Big\downarrow \text{ 1721}$$

$$\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^3+cx^6)^p\int \frac{\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^p\left(\frac{2cx^3}{b+\sqrt{b^2-4ac}}+1\right)^p}{x^2}dx$$

$$\Big\downarrow \text{ 1012}$$

$$-\frac{\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^3+cx^6)^p\operatorname{AppellF1}\left(-\frac{1}{3},-p,-p,\frac{2}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{x}$$

input
```
Int[(a + b*x^3 + c*x^6)^p/x^2,x]
```

output
```
-(((a + b*x^3 + c*x^6)^p*AppellF1[-1/3, -p, -p, 2/3, (-2*c*x^3)/(b - Sqrt[
b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(x*(1 + (2*c*x^3)/(b -
 Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p))
```

3.265.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

3.265.4 Maple [F]

$$\int \frac{(c\,x^{6}+b\,x^{3}+a)^{p}}{x^{2}}dx$$

input
```
int((c*x^6+b*x^3+a)^p/x^2,x)
```

output
```
int((c*x^6+b*x^3+a)^p/x^2,x)
```

3.265.5 Fricas [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x^2}\,dx=\int \frac{(cx^6+bx^3+a)^p}{x^2}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x^2,x, algorithm="fricas")
```

output
```
integral((c*x^6 + b*x^3 + a)^p/x^2, x)
```

3.265.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^p}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((c*x**6+b*x**3+a)**p/x**2,x)
```

output
```
Timed out
```

3.265.7 Maxima [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x^2}\,dx = \int \frac{(cx^6+bx^3+a)^p}{x^2}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x^2,x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p/x^2, x)
```

3.265.8 Giac [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x^2}\,dx = \int \frac{(cx^6+bx^3+a)^p}{x^2}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x^2,x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p/x^2, x)
```

3.265.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^p}{x^2}\,dx = \int \frac{(c\,x^6+b\,x^3+a)^p}{x^2}\,dx$$

input
```
int((a + b*x^3 + c*x^6)^p/x^2,x)
```

output
```
int((a + b*x^3 + c*x^6)^p/x^2, x)
```

3.266 $\int \frac{(a+bx^3+cx^6)^p}{x^3}\,dx$

3.266.1 Optimal result

Integrand size = 18, antiderivative size = 138

$$\int \frac{(a+bx^3+cx^6)^p}{x^3}\,dx = -\frac{\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p \operatorname{AppellF1}\left(-\frac{2}{3},-p,-p,\frac{1}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2x^2}$$

output
```
-1/2*(c*x^6+b*x^3+a)^p*AppellF1(-2/3,-p,-p,1/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1
/2)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/x^2/((1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2
)))^p)/((1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^p)
```

3.266.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.20

$$\int \frac{(a+bx^3+cx^6)^p}{x^3}\,dx = -\frac{\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p \operatorname{AppellF1}\left(-\frac{2}{3},-p,-p,\frac{1}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)}{2x^2}$$

input
```
Integrate[(a + b*x^3 + c*x^6)^p/x^3,x]
```

output
```
-1/2*((a + b*x^3 + c*x^6)^p*AppellF1[-2/3, -p, -p, 1/3, (-2*c*x^3)/(b + Sq
rt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(x^2*((b - Sqrt[b^2
 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] +
2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

3.266.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3+cx^6)^p}{x^3}dx$$

$$\Big\downarrow \text{1721}$$

$$\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^3+cx^6)^p\int \frac{\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^p\left(\frac{2cx^3}{b+\sqrt{b^2-4ac}}+1\right)^p}{x^3}dx$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^3+cx^6)^p\,\text{AppellF1}\left(-\frac{2}{3},-p,-p,\frac{1}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2x^2}$$

input
```
Int[(a + b*x^3 + c*x^6)^p/x^3,x]
```

output
```
-1/2*((a + b*x^3 + c*x^6)^p*AppellF1[-2/3, -p, -p, 1/3, (-2*c*x^3)/(b - Sq
rt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(x^2*(1 + (2*c*x^3)
/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

3.266.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

3.266.4 Maple [F]

$$\int \frac{(c\,x^6 + b\,x^3 + a)^p}{x^3}dx$$

input
```
int((c*x^6+b*x^3+a)^p/x^3,x)
```

output
```
int((c*x^6+b*x^3+a)^p/x^3,x)
```

3.266.5 Fricas [F]

$$\int \frac{(a + bx^3 + cx^6)^p}{x^3}\,dx = \int \frac{(cx^6 + bx^3 + a)^p}{x^3}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x^3,x, algorithm="fricas")
```

output
```
integral((c*x^6 + b*x^3 + a)^p/x^3, x)
```

3.266.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^p}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((c*x**6+b*x**3+a)**p/x**3,x)
```

output
```
Timed out
```

3.266.7 Maxima [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x^3}\,dx = \int \frac{(cx^6+bx^3+a)^p}{x^3}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x^3,x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p/x^3, x)
```

3.266.8 Giac [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x^3}\,dx = \int \frac{(cx^6+bx^3+a)^p}{x^3}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x^3,x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p/x^3, x)
```

3.266.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^p}{x^3}\,dx = \int \frac{(c\,x^6+b\,x^3+a)^p}{x^3}\,dx$$

input
```
int((a + b*x^3 + c*x^6)^p/x^3,x)
```

output
```
int((a + b*x^3 + c*x^6)^p/x^3, x)
```

3.267 $\int \frac{(a+bx^3+cx^6)^p}{x^4}\,dx$

3.267.1 Optimal result

Integrand size = 18, antiderivative size = 164

$$\int \frac{(a+bx^3+cx^6)^p}{x^4}\,dx = -\frac{4^p\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p\,\text{AppellF1}\left(1-2p,-p,-p,2(1-p),-\frac{b-\sqrt{b^2-}}{2cx^3}\right.}{3(1-2p)x^3}$$

output
```
-1/3*4^p*(c*x^6+b*x^3+a)^p*AppellF1(1-2*p,-p,-p,2-2*p,1/2*(-b-(-4*a*c+b^2)
^(1/2))/c/x^3,1/2*(-b+(-4*a*c+b^2)^(1/2))/c/x^3)/(1-2*p)/x^3/(((b+2*c*x^3-
(-4*a*c+b^2)^(1/2))/c/x^3)^p)/(((b+2*c*x^3+(-4*a*c+b^2)^(1/2))/c/x^3)^p)
```

3.267.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^3+cx^6)^p}{x^4}\,dx = \frac{4^p\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p\,\text{AppellF1}\left(1-2p,-p,-p,2-2p,-\frac{b+\sqrt{b^2-4ac}}{2cx^3},\right.}{3(-1+2p)x^3}$$

input
```
Integrate[(a + b*x^3 + c*x^6)^p/x^4,x]
```

output
```
(4^p*(a + b*x^3 + c*x^6)^p*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, -1/2*(b + Sq
rt[b^2 - 4*a*c])/(c*x^3), (-b + Sqrt[b^2 - 4*a*c])/(2*c*x^3)])/(3*(-1 + 2*
p)*x^3*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(c*x^3))^p*((b + Sqrt[b^2 - 4*a*
c] + 2*c*x^3)/(c*x^3))^p)
```

3.267.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1693, 1178, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3+cx^6)^p}{x^4} \, dx$$

$$\Big\downarrow \; 1693$$

$$\frac{1}{3}\int \frac{(cx^6+bx^3+a)^p}{x^6}dx^3$$

$$\Big\downarrow \; 1178$$

$$-\frac{1}{3}4^p\left(\frac{1}{x^3}\right)^{2p}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p\int\left(\frac{b-\sqrt{b^2-4ac}}{2cx^3}+$$

$$\Big\downarrow \; 150$$

$$-\frac{4^p\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p\operatorname{AppellF1}\left(1-2p,-p,-p,2-2p,-\frac{b-\sqrt{b^2-4ac}}{2cx^3},-\right.}{3(1-2p)x^3}$$

input
```
Int[(a + b*x^3 + c*x^6)^p/x^4,x]
```

output
```
-1/3*(4^p*(a + b*x^3 + c*x^6)^p*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, -1/2*(b
 - Sqrt[b^2 - 4*a*c])/(c*x^3), -1/2*(b + Sqrt[b^2 - 4*a*c])/(c*x^3)])/((1
- 2*p)*x^3*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(c*x^3))^p*((b + Sqrt[b^2 -
4*a*c] + 2*c*x^3)/(c*x^3))^p)
```

3.267.3.1 Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1178
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(-(1/(d + e*x))^(2*p))*((a +
b*x + c*x^2)^p/(e*(e*((b - q + 2*c*x)/(2*c*(d + e*x))))^p*(e*((b + q + 2*c*
x)/(2*c*(d + e*x))))^p))   Subst[Int[x^(-m - 2*(p + 1))*Simp[1 - (d - e*((b
 - q)/(2*c)))*x, x]^p*Simp[1 - (d - e*((b + q)/(2*c)))*x, x]^p, x], x, 1/(d
 + e*x)], x]] /; FreeQ[{a, b, c, d, e, p}, x] && ILtQ[m, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.267.4 Maple [F]

$$\int \frac{(c\,x^{6}+b\,x^{3}+a)^{p}}{x^{4}}dx$$

input
```
int((c*x^6+b*x^3+a)^p/x^4,x)
```

output
```
int((c*x^6+b*x^3+a)^p/x^4,x)
```

3.267.5 Fricas [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x^4}\,dx=\int \frac{(cx^6+bx^3+a)^p}{x^4}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x^4,x, algorithm="fricas")
```

output
```
integral((c*x^6 + b*x^3 + a)^p/x^4, x)
```

3.267.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^p}{x^4}\,dx = \text{Timed out}$$

input
```
integrate((c*x**6+b*x**3+a)**p/x**4,x)
```

output
```
Timed out
```

3.267.7 Maxima [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x^4}\,dx = \int \frac{(cx^6+bx^3+a)^p}{x^4}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x^4,x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p/x^4, x)
```

3.267.8 Giac [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x^4}\,dx = \int \frac{(cx^6+bx^3+a)^p}{x^4}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x^4,x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p/x^4, x)
```

3.267.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^p}{x^4}\,dx = \int \frac{(c\,x^6+b\,x^3+a)^p}{x^4}\,dx$$

input
```
int((a + b*x^3 + c*x^6)^p/x^4,x)
```

output
```
int((a + b*x^3 + c*x^6)^p/x^4, x)
```

3.268 $\int \frac{(a+bx^3+cx^6)^p}{x^5} \, dx$

3.268.1 Optimal result

Integrand size = 18, antiderivative size = 138

$$\int \frac{(a+bx^3+cx^6)^p}{x^5} \, dx = -\frac{\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p \operatorname{AppellF1}\left(-\frac{4}{3},-p,-p,-\frac{1}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4x^4}$$

output
```
-1/4*(c*x^6+b*x^3+a)^p*AppellF1(-4/3,-p,-p,-1/3,-2*c*x^3/(b-(-4*a*c+b^2)^(
1/2)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/x^4/((1+2*c*x^3/(b-(-4*a*c+b^2)^(1/
2)))^p)/((1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^p)
```

3.268.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.20

$$\int \frac{(a+bx^3+cx^6)^p}{x^5} \, dx = -\frac{\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p \operatorname{AppellF1}\left(-\frac{4}{3},-p,-p,-\frac{1}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)}{4x^4}$$

input
```
Integrate[(a + b*x^3 + c*x^6)^p/x^5,x]
```

output
```
-1/4*((a + b*x^3 + c*x^6)^p*AppellF1[-4/3, -p, -p, -1/3, (-2*c*x^3)/(b + S
qrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(x^4*((b - Sqrt[b^
2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] +
 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

3.268.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3+cx^6)^p}{x^5}\,dx$$

$$\Big\downarrow \text{1721}$$

$$\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^3+cx^6)^p\int \frac{\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^p\left(\frac{2cx^3}{b+\sqrt{b^2-4ac}}+1\right)^p}{x^5}dx$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^3+cx^6)^p\operatorname{AppellF1}\left(-\frac{4}{3},-p,-p,-\frac{1}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4x^4}$$

input
```
Int[(a + b*x^3 + c*x^6)^p/x^5,x]
```

output
```
-1/4*((a + b*x^3 + c*x^6)^p*AppellF1[-4/3, -p, -p, -1/3, (-2*c*x^3)/(b - S
qrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(x^4*(1 + (2*c*x^3
)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

3.268.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

3.268.4 Maple [F]

$$\int \frac{(c\,x^{6}+b\,x^{3}+a)^{p}}{x^{5}}dx$$

input
```
int((c*x^6+b*x^3+a)^p/x^5,x)
```

output
```
int((c*x^6+b*x^3+a)^p/x^5,x)
```

3.268.5 Fricas [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x^5}\,dx = \int \frac{(cx^6+bx^3+a)^p}{x^5}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x^5,x, algorithm="fricas")
```

output
```
integral((c*x^6 + b*x^3 + a)^p/x^5, x)
```

3.268.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^p}{x^5}\,dx = \text{Timed out}$$

input
```
integrate((c*x**6+b*x**3+a)**p/x**5,x)
```

output
```
Timed out
```

3.268.7 Maxima [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x^5}\,dx = \int \frac{(cx^6+bx^3+a)^p}{x^5}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x^5,x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p/x^5, x)
```

3.268.8 Giac [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x^5}\,dx = \int \frac{(cx^6+bx^3+a)^p}{x^5}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x^5,x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p/x^5, x)
```

3.268.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^p}{x^5}\,dx = \int \frac{(c\,x^6+b\,x^3+a)^p}{x^5}\,dx$$

input
```
int((a + b*x^3 + c*x^6)^p/x^5,x)
```

output
```
int((a + b*x^3 + c*x^6)^p/x^5, x)
```

3.269 $\int \frac{(a+bx^3+cx^6)^p}{x^6}\,dx$

3.269.1 Optimal result

Integrand size = 18, antiderivative size = 138

$$\int \frac{(a+bx^3+cx^6)^p}{x^6}\,dx = -\frac{\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p\,\text{AppellF1}\left(-\frac{5}{3},-p,-p,-\frac{2}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{5x^5}$$

output
```
-1/5*(c*x^6+b*x^3+a)^p*AppellF1(-5/3,-p,-p,-2/3,-2*c*x^3/(b-(-4*a*c+b^2)^(
1/2)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/x^5/((1+2*c*x^3/(b-(-4*a*c+b^2)^(1/
2)))^p)/((1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^p)
```

3.269.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.20

$$\int \frac{(a+bx^3+cx^6)^p}{x^6}\,dx = -\frac{\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p\,\text{AppellF1}\left(-\frac{5}{3},-p,-p,-\frac{2}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)}{5x^5}$$

input
```
Integrate[(a + b*x^3 + c*x^6)^p/x^6,x]
```

output
```
-1/5*((a + b*x^3 + c*x^6)^p*AppellF1[-5/3, -p, -p, -2/3, (-2*c*x^3)/(b + S
qrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(x^5*((b - Sqrt[b^
2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] +
 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

3.269.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3+cx^6)^p}{x^6}\,dx$$

$$\Big\downarrow \text{1721}$$

$$\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^3+cx^6)^p\int \frac{\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^p\left(\frac{2cx^3}{b+\sqrt{b^2-4ac}}+1\right)^p}{x^6}dx$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^3+cx^6)^p\,\text{AppellF1}\left(-\frac{5}{3},-p,-p,-\frac{2}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{5x^5}$$

input
```
Int[(a + b*x^3 + c*x^6)^p/x^6,x]
```

output
```
-1/5*((a + b*x^3 + c*x^6)^p*AppellF1[-5/3, -p, -p, -2/3, (-2*c*x^3)/(b - S
qrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(x^5*(1 + (2*c*x^3
)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

3.269.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

3.269.4 Maple [F]

$$\int \frac{(c\,x^6+b\,x^3+a)^p}{x^6}dx$$

input
```
int((c*x^6+b*x^3+a)^p/x^6,x)
```

output
```
int((c*x^6+b*x^3+a)^p/x^6,x)
```

3.269.5 Fricas [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x^6}\,dx=\int \frac{(cx^6+bx^3+a)^p}{x^6}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x^6,x, algorithm="fricas")
```

output
```
integral((c*x^6 + b*x^3 + a)^p/x^6, x)
```

3.269.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a + bx^3 + cx^6)^p}{x^6} \, dx = \text{Timed out}$$

input
```
integrate((c*x**6+b*x**3+a)**p/x**6,x)
```

output
```
Timed out
```

3.269.7 Maxima [F]

$$\int \frac{(a + bx^3 + cx^6)^p}{x^6} \, dx = \int \frac{(cx^6 + bx^3 + a)^p}{x^6} \, dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x^6,x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p/x^6, x)
```

3.269.8 Giac [F]

$$\int \frac{(a + bx^3 + cx^6)^p}{x^6} \, dx = \int \frac{(cx^6 + bx^3 + a)^p}{x^6} \, dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x^6,x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p/x^6, x)
```

3.269.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^p}{x^6}\,dx=\int \frac{(c\,x^6+b\,x^3+a)^p}{x^6}\,dx$$

input
```
int((a + b*x^3 + c*x^6)^p/x^6,x)
```

output
```
int((a + b*x^3 + c*x^6)^p/x^6, x)
```

3.270 $\int \frac{(a+bx^3+cx^6)^p}{x^7} dx$

3.270.1 Optimal result

Integrand size = 18, antiderivative size = 168

$$\int \frac{(a+bx^3+cx^6)^p}{x^7} dx = -\frac{2^{-1+2p}\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p \operatorname{AppellF1}\left(2(1-p),-p,-p,3-2p,-\frac{b-}{}\right.}{3(1-p)x^6}$$

output
```
-1/3*2^(-1+2*p)*(c*x^6+b*x^3+a)^p*AppellF1(2-2*p,-p,-p,3-2*p,1/2*(-b-(-4*a
*c+b^2)^(1/2))/c/x^3,1/2*(-b+(-4*a*c+b^2)^(1/2))/c/x^3)/(1-p)/x^6/(((b+2*c
*x^3-(-4*a*c+b^2)^(1/2))/c/x^3)^p)/(((b+2*c*x^3+(-4*a*c+b^2)^(1/2))/c/x^3)
^p)
```

3.270.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^3+cx^6)^p}{x^7} dx = \frac{2^{-1+2p}\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p \operatorname{AppellF1}\left(2-2p,-p,-p,3-2p,-\frac{b+\sqrt{b^2-}}{2cx^3}\right.}{3(-1+p)x^6}$$

input
```
Integrate[(a + b*x^3 + c*x^6)^p/x^7,x]
```

output
```
(2^(-1 + 2*p)*(a + b*x^3 + c*x^6)^p*AppellF1[2 - 2*p, -p, -p, 3 - 2*p, -1/
2*(b + Sqrt[b^2 - 4*a*c])/(c*x^3), (-b + Sqrt[b^2 - 4*a*c])/(2*c*x^3)])/(3
*(-1 + p)*x^6*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(c*x^3))^p*((b + Sqrt[b^2
 - 4*a*c] + 2*c*x^3)/(c*x^3))^p)
```

3.270.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1693, 1178, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3+cx^6)^p}{x^7}\,dx$$

$$\Big\downarrow \; 1693$$

$$\frac{1}{3}\int \frac{(cx^6+bx^3+a)^p}{x^9}dx^3$$

$$\Big\downarrow \; 1178$$

$$-\frac{1}{3}4^p\left(\frac{1}{x^3}\right)^{2p}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p\int\left(\frac{b-\sqrt{b^2-4ac}}{2cx^3}+\right.$$

$$\Big\downarrow \; 150$$

$$-\frac{2^{2p-1}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p\,\text{AppellF1}\left(2-2p,-p,-p,3-2p,-\frac{b-\sqrt{b^2-4ac}}{2cx^3}\right.}{3(1-p)x^6}$$

input
```
Int[(a + b*x^3 + c*x^6)^p/x^7,x]
```

output
```
-1/3*(2^(-1 + 2*p)*(a + b*x^3 + c*x^6)^p*AppellF1[2 - 2*p, -p, -p, 3 - 2*p
, -1/2*(b - Sqrt[b^2 - 4*a*c])/(c*x^3), -1/2*(b + Sqrt[b^2 - 4*a*c])/(c*x^
3)])/((1 - p)*x^6*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(c*x^3))^p*((b + Sqrt
[b^2 - 4*a*c] + 2*c*x^3)/(c*x^3))^p)
```

3.270.3.1 Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1178
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(-(1/(d + e*x))^(2*p))*((a +
b*x + c*x^2)^p/(e*(e*((b - q + 2*c*x)/(2*c*(d + e*x))))^p*(e*((b + q + 2*c*
x)/(2*c*(d + e*x))))^p))   Subst[Int[x^(-m - 2*(p + 1))*Simp[1 - (d - e*((b
 - q)/(2*c)))*x, x]^p*Simp[1 - (d - e*((b + q)/(2*c)))*x, x]^p, x], x, 1/(d
 + e*x)], x]] /; FreeQ[{a, b, c, d, e, p}, x] && ILtQ[m, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.270.4 Maple [F]

$$\int \frac{(c\,x^6+b\,x^3+a)^p}{x^7}dx$$

input
```
int((c*x^6+b*x^3+a)^p/x^7,x)
```

output
```
int((c*x^6+b*x^3+a)^p/x^7,x)
```

3.270.5 Fricas [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x^7}\,dx = \int \frac{(cx^6+bx^3+a)^p}{x^7}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x^7,x, algorithm="fricas")
```

output
```
integral((c*x^6 + b*x^3 + a)^p/x^7, x)
```

3.270.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^p}{x^7}\,dx = \text{Timed out}$$

input
```
integrate((c*x**6+b*x**3+a)**p/x**7,x)
```

output
```
Timed out
```

3.270.7 Maxima [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x^7}\,dx = \int \frac{(cx^6+bx^3+a)^p}{x^7}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x^7,x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p/x^7, x)
```

3.270.8 Giac [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x^7}\,dx = \int \frac{(cx^6+bx^3+a)^p}{x^7}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x^7,x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p/x^7, x)
```

3.270.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^p}{x^7}\,dx = \int \frac{(c\,x^6+b\,x^3+a)^p}{x^7}\,dx$$

input
```
int((a + b*x^3 + c*x^6)^p/x^7,x)
```

output
```
int((a + b*x^3 + c*x^6)^p/x^7, x)
```

3.271 $\int \frac{x^m}{1+2x^4+x^8}\,dx$

3.271.1 Optimal result

Integrand size = 16, antiderivative size = 32

$$\int \frac{x^m}{1+2x^4+x^8}\,dx=\frac{x^{1+m}\,\text{Hypergeometric2F1}\left(2,\frac{1+m}{4},\frac{5+m}{4},-x^4\right)}{1+m}$$

output
```
x^(1+m)*hypergeom([2, 1/4+1/4*m],[5/4+1/4*m],-x^4)/(1+m)
```

3.271.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{x^m}{1+2x^4+x^8}\,dx=\frac{x^{1+m}\,\text{Hypergeometric2F1}\left(2,\frac{1+m}{4},1+\frac{1+m}{4},-x^4\right)}{1+m}$$

input
```
Integrate[x^m/(1 + 2*x^4 + x^8),x]
```

output
```
(x^(1 + m)*Hypergeometric2F1[2, (1 + m)/4, 1 + (1 + m)/4, -x^4])/(1 + m)
```

3.271.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1380, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{x^8+2x^4+1}\,dx$$

$$\Big\downarrow \text{1380}$$

$$\int \frac{x^m}{(x^4+1)^2}dx$$

$$\Big\downarrow \text{888}$$

$$\frac{x^{m+1}\,\text{Hypergeometric2F1}\left(2,\frac{m+1}{4},\frac{m+5}{4},-x^4\right)}{m+1}$$

input
```
Int[x^m/(1 + 2*x^4 + x^8),x]
```

output
```
(x^(1 + m)*Hypergeometric2F1[2, (1 + m)/4, (5 + m)/4, -x^4])/(1 + m)
```

3.271.3.1 Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

3.271.4 Maple [F]

$$\int \frac{x^m}{x^8 + 2x^4 + 1} dx$$

input
```
int(x^m/(x^8+2*x^4+1),x)
```

output
```
int(x^m/(x^8+2*x^4+1),x)
```

3.271.5 Fricas [F]

$$\int \frac{x^m}{1 + 2x^4 + x^8} \, dx = \int \frac{x^m}{x^8 + 2\,x^4 + 1} \, dx$$

input
```
integrate(x^m/(x^8+2*x^4+1),x, algorithm="fricas")
```

output
```
integral(x^m/(x^8 + 2*x^4 + 1), x)
```

3.271.6 Sympy [F]

$$\int \frac{x^m}{1 + 2x^4 + x^8} \, dx = \int \frac{x^m}{(x^4 + 1)^2} \, dx$$

input
```
integrate(x**m/(x**8+2*x**4+1),x)
```

output
```
Integral(x**m/(x**4 + 1)**2, x)
```

3.271.7 Maxima [F]

$$\int \frac{x^m}{1+2x^4+x^8}\,dx = \int \frac{x^m}{x^8+2\,x^4+1}\,dx$$

input
```
integrate(x^m/(x^8+2*x^4+1),x, algorithm="maxima")
```

output
```
integrate(x^m/(x^8 + 2*x^4 + 1), x)
```

3.271.8 Giac [F]

$$\int \frac{x^m}{1+2x^4+x^8}\,dx = \int \frac{x^m}{x^8+2\,x^4+1}\,dx$$

input
```
integrate(x^m/(x^8+2*x^4+1),x, algorithm="giac")
```

output
```
integrate(x^m/(x^8 + 2*x^4 + 1), x)
```

3.271.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{1+2x^4+x^8}\,dx = \int \frac{x^m}{x^8+2\,x^4+1}\,dx$$

input
```
int(x^m/(2*x^4 + x^8 + 1),x)
```

output
```
int(x^m/(2*x^4 + x^8 + 1), x)
```

3.272 $\int \frac{x^9}{1+2x^4+x^8}\,dx$

3.272.1 Optimal result

Integrand size = 16, antiderivative size = 30

$$\int \frac{x^9}{1+2x^4+x^8}\,dx=\frac{3x^2}{4}-\frac{x^6}{4(1+x^4)}-\frac{3\arctan(x^2)}{4}$$

output
```
3/4*x^2-1/4*x^6/(x^4+1)-3/4*arctan(x^2)
```

3.272.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{x^9}{1+2x^4+x^8}\,dx=\frac{1}{4}\left(x^2\left(2+\frac{1}{1+x^4}\right)-3\arctan\left(x^2\right)\right)$$

input
```
Integrate[x^9/(1 + 2*x^4 + x^8),x]
```

output
```
(x^2*(2 + (1 + x^4)^(-1)) - 3*ArcTan[x^2])/4
```

3.272.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1380, 807, 252, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{x^8+2x^4+1} \, dx$$

$$\Big\downarrow \text{1380}$$

$$\int \frac{x^9}{\left(x^4+1\right)^2}dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \frac{x^8}{\left(x^4+1\right)^2}dx^2$$

$$\Big\downarrow \text{252}$$

$$\frac{1}{2}\left(\frac{3}{2}\int \frac{x^4}{x^4+1}dx^2-\frac{x^6}{2\left(x^4+1\right)}\right)$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{2}\left(\frac{3}{2}\left(x^2-\int \frac{1}{x^4+1}dx^2\right)-\frac{x^6}{2\left(x^4+1\right)}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\left(\frac{3}{2}\left(x^2-\arctan\left(x^2\right)\right)-\frac{x^6}{2\left(x^4+1\right)}\right)$$

input
```
Int[x^9/(1 + 2*x^4 + x^8),x]
```

output
```
(-1/2*x^6/(1 + x^4) + (3*(x^2 - ArcTan[x^2]))/2)/2
```

3.272.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

3.272.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{x^{2}}{2}+\frac{x^{2}}{4x^{4}+4}-\frac{3\arctan\left(x^{2}\right)}{4}$	25
risch	$\frac{x^{2}}{2}+\frac{x^{2}}{4x^{4}+4}-\frac{3\arctan\left(x^{2}\right)}{4}$	25
parallelrisch	$\frac{3i\ln\left(x^{2}-i\right)x^{4}-3i\ln\left(x^{2}+i\right)x^{4}+4x^{6}+3i\ln\left(x^{2}-i\right)-3i\ln\left(x^{2}+i\right)+6x^{2}}{8x^{4}+8}$	67

input
```
int(x^9/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2+1/4*x^2/(x^4+1)-3/4*arctan(x^2)
```

3.272.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int \frac{x^9}{1+2x^4+x^8}\,dx = \frac{2\,x^6+3\,x^2-3\,(x^4+1)\arctan{(x^2)}}{4\,(x^4+1)}$$

input
```
integrate(x^9/(x^8+2*x^4+1),x, algorithm="fricas")
```

output
```
1/4*(2*x^6 + 3*x^2 - 3*(x^4 + 1)*arctan(x^2))/(x^4 + 1)
```

3.272.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \frac{x^9}{1+2x^4+x^8}\,dx = \frac{x^2}{2}+\frac{x^2}{4x^4+4}-\frac{3\operatorname{atan}{(x^2)}}{4}$$

input
```
integrate(x**9/(x**8+2*x**4+1),x)
```

output
```
x**2/2 + x**2/(4*x**4 + 4) - 3*atan(x**2)/4
```

3.272.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{x^9}{1+2x^4+x^8}\,dx = \frac{1}{2}\,x^2+\frac{x^2}{4\,(x^4+1)}-\frac{3}{4}\,\arctan{(x^2)}$$

input
```
integrate(x^9/(x^8+2*x^4+1),x, algorithm="maxima")
```

output
```
1/2*x^2 + 1/4*x^2/(x^4 + 1) - 3/4*arctan(x^2)
```

3.272.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{x^9}{1+2x^4+x^8}\,dx = \frac{1}{2}\,x^2 + \frac{x^2}{4\,(x^4+1)} - \frac{3}{4}\,\arctan\left(x^2\right)$$

input
```
integrate(x^9/(x^8+2*x^4+1),x, algorithm="giac")
```

output
```
1/2*x^2 + 1/4*x^2/(x^4 + 1) - 3/4*arctan(x^2)
```

3.272.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

$$\int \frac{x^9}{1+2x^4+x^8}\,dx = \frac{x^2}{4\,\left(x^4+1\right)} - \frac{3\,\mathrm{atan}(x^2)}{4} + \frac{x^2}{2}$$

input
```
int(x^9/(2*x^4 + x^8 + 1),x)
```

output
```
x^2/(4*(x^4 + 1)) - (3*atan(x^2))/4 + x^2/2
```

3.273 $\int \frac{x^7}{1+2x^4+x^8} dx$

3.273.1 Optimal result

Integrand size = 16, antiderivative size = 22

$$\int \frac{x^7}{1+2x^4+x^8} dx = \frac{1}{4(1+x^4)} + \frac{1}{4}\log\left(1+x^4\right)$$

output
```
1/4/(x^4+1)+1/4*ln(x^4+1)
```

3.273.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{x^7}{1+2x^4+x^8} dx = \frac{1}{4}\left(\frac{1}{1+x^4} + \log\left(1+x^4\right)\right)$$

input
```
Integrate[x^7/(1 + 2*x^4 + x^8),x]
```

output
```
((1 + x^4)^(-1) + Log[1 + x^4])/4
```

3.273.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1380, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{x^8+2x^4+1}dx$$
$$\Big\downarrow \text{1380}$$
$$\int \frac{x^7}{(x^4+1)^2}dx$$
$$\Big\downarrow \text{798}$$
$$\frac{1}{4}\int \frac{x^4}{(x^4+1)^2}dx^4$$
$$\Big\downarrow \text{49}$$
$$\frac{1}{4}\int \left(\frac{1}{x^4+1}-\frac{1}{(x^4+1)^2}\right)dx^4$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{4}\left(\frac{1}{x^4+1}+\log\left(x^4+1\right)\right)$$

input
```
Int[x^7/(1 + 2*x^4 + x^8),x]
```

output
```
((1 + x^4)^(-1) + Log[1 + x^4])/4
```

3.273.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.273.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{1}{4x^4+4}+\frac{\ln(x^4+1)}{4}$	19
norman	$\frac{1}{4x^4+4}+\frac{\ln(x^4+1)}{4}$	19
risch	$\frac{1}{4x^4+4}+\frac{\ln(x^4+1)}{4}$	19
parallelrisch	$\frac{\ln(x^4+1)x^4+1+\ln(x^4+1)}{4x^4+4}$	28

input
```
int(x^7/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4/(x^4+1)+1/4*ln(x^4+1)
```

3.273.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{x^7}{1+2x^4+x^8}\,dx = \frac{(x^4+1)\log(x^4+1)+1}{4(x^4+1)}$$

input
```
integrate(x^7/(x^8+2*x^4+1),x, algorithm="fricas")
```

output
```
1/4*((x^4 + 1)*log(x^4 + 1) + 1)/(x^4 + 1)
```

3.273.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.68

$$\int \frac{x^7}{1+2x^4+x^8}\,dx = \frac{\log(x^4+1)}{4} + \frac{1}{4x^4+4}$$

input
```
integrate(x**7/(x**8+2*x**4+1),x)
```

output
```
log(x**4 + 1)/4 + 1/(4*x**4 + 4)
```

3.273.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{x^7}{1+2x^4+x^8}\,dx = \frac{1}{4(x^4+1)} + \frac{1}{4}\log(x^4+1)$$

input
```
integrate(x^7/(x^8+2*x^4+1),x, algorithm="maxima")
```

output
```
1/4/(x^4 + 1) + 1/4*log(x^4 + 1)
```

3.273.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{x^7}{1+2x^4+x^8}\,dx = \frac{1}{4(x^4+1)} + \frac{1}{4}\log(x^4+1)$$

input
```
integrate(x^7/(x^8+2*x^4+1),x, algorithm="giac")
```

output
```
1/4/(x^4 + 1) + 1/4*log(x^4 + 1)
```

3.273.9 Mupad [B] (verification not implemented)

Time = 8.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{x^7}{1+2x^4+x^8}\,dx = \frac{\ln(x^4+1)}{4} + \frac{1}{4\,(x^4+1)}$$

input
```
int(x^7/(2*x^4 + x^8 + 1),x)
```

output
```
log(x^4 + 1)/4 + 1/(4*(x^4 + 1))
```

3.274 $\int \frac{x^5}{1+2x^4+x^8}\,dx$

3.274.1 Optimal result

Integrand size = 16, antiderivative size = 23

$$\int \frac{x^5}{1+2x^4+x^8}\,dx=-\frac{x^2}{4\left(1+x^4\right)}+\frac{\arctan\left(x^2\right)}{4}$$

output
```
-1/4*x^2/(x^4+1)+1/4*arctan(x^2)
```

3.274.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{1+2x^4+x^8}\,dx=-\frac{x^2}{4\left(1+x^4\right)}+\frac{\arctan\left(x^2\right)}{4}$$

input
```
Integrate[x^5/(1 + 2*x^4 + x^8),x]
```

output
```
-1/4*x^2/(1 + x^4) + ArcTan[x^2]/4
```

3.274.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1380, 807, 252, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{x^8+2x^4+1}dx$$

$$\Big\downarrow \text{1380}$$

$$\int \frac{x^5}{(x^4+1)^2}dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \frac{x^4}{(x^4+1)^2}dx^2$$

$$\Big\downarrow \text{252}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^4+1}dx^2-\frac{x^2}{2(x^4+1)}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\left(\frac{\arctan(x^2)}{2}-\frac{x^2}{2(x^4+1)}\right)$$

input
```
Int[x^5/(1 + 2*x^4 + x^8),x]
```

output
```
(-1/2*x^2/(1 + x^4) + ArcTan[x^2]/2)/2
```

3.274.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

3.274.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result	size
default	$-\frac{x^2}{4(x^4+1)}+\frac{\arctan(x^2)}{4}$	20
risch	$-\frac{x^2}{4(x^4+1)}+\frac{\arctan(x^2)}{4}$	20
parallelrisch	$-\frac{i\ln(x^2-i)x^4-i\ln(x^2+i)x^4+i\ln(x^2-i)-i\ln(x^2+i)+2x^2}{8(x^4+1)}$	62

input
```
int(x^5/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/4*x^2/(x^4+1)+1/4*arctan(x^2)
```

3.274.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{x^5}{1+2x^4+x^8}\,dx = -\frac{x^2-(x^4+1)\arctan(x^2)}{4(x^4+1)}$$

input
```
integrate(x^5/(x^8+2*x^4+1),x, algorithm="fricas")
```

output
```
-1/4*(x^2 - (x^4 + 1)*arctan(x^2))/(x^4 + 1)
```

3.274.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int \frac{x^5}{1+2x^4+x^8}\,dx = -\frac{x^2}{4x^4+4}+\frac{\operatorname{atan}(x^2)}{4}$$

input
```
integrate(x**5/(x**8+2*x**4+1),x)
```

output
```
-x**2/(4*x**4 + 4) + atan(x**2)/4
```

3.274.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{x^5}{1+2x^4+x^8}\,dx = -\frac{x^2}{4(x^4+1)}+\frac{1}{4}\arctan(x^2)$$

input
```
integrate(x^5/(x^8+2*x^4+1),x, algorithm="maxima")
```

output
```
-1/4*x^2/(x^4 + 1) + 1/4*arctan(x^2)
```

3.274.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{x^5}{1+2x^4+x^8}\,dx = -\frac{x^2}{4\,(x^4+1)} + \frac{1}{4}\arctan\left(x^2\right)$$

input
```
integrate(x^5/(x^8+2*x^4+1),x, algorithm="giac")
```

output
```
-1/4*x^2/(x^4 + 1) + 1/4*arctan(x^2)
```

3.274.9 Mupad [B] (verification not implemented)

Time = 8.31 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x^5}{1+2x^4+x^8}\,dx = \frac{\operatorname{atan}(x^2)}{4} - \frac{x^2}{4\,(x^4+1)}$$

input
```
int(x^5/(2*x^4 + x^8 + 1),x)
```

output
```
atan(x^2)/4 - x^2/(4*(x^4 + 1))
```

3.275 $\int \frac{x^3}{1+2x^4+x^8}\,dx$

3.275.1 Optimal result

Integrand size = 16, antiderivative size = 11

$$\int \frac{x^3}{1+2x^4+x^8}\,dx = -\frac{1}{4(1+x^4)}$$

output
```
-1/4/(x^4+1)
```

3.275.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{1+2x^4+x^8}\,dx = -\frac{1}{4(1+x^4)}$$

input
```
Integrate[x^3/(1 + 2*x^4 + x^8),x]
```

output
```
-1/4*1/(1 + x^4)
```

3.275.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1380, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{x^8+2x^4+1}\,dx$$
$$\Big\downarrow \text{ 1380}$$
$$\int \frac{x^3}{\left(x^4+1\right)^2}dx$$
$$\Big\downarrow \text{ 793}$$
$$-\frac{1}{4\left(x^4+1\right)}$$

input
```
Int[x^3/(1 + 2*x^4 + x^8),x]
```

output
```
-1/4*1/(1 + x^4)
```

3.275.3.1 Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

3.275.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

method	result	size
gosper	$-\frac{1}{4(x^4+1)}$	10
default	$-\frac{1}{4(x^4+1)}$	10
norman	$-\frac{1}{4(x^4+1)}$	10
risch	$-\frac{1}{4(x^4+1)}$	10
parallelrisch	$-\frac{1}{4(x^4+1)}$	10

input
```
int(x^3/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/4/(x^4+1)
```

3.275.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{x^3}{1+2x^4+x^8}\,dx = -\frac{1}{4\,(x^4+1)}$$

input
```
integrate(x^3/(x^8+2*x^4+1),x, algorithm="fricas")
```

output
```
-1/4/(x^4 + 1)
```

3.275.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{x^3}{1+2x^4+x^8}\,dx = -\frac{1}{4x^4+4}$$

input
```
integrate(x**3/(x**8+2*x**4+1),x)
```

output
```
-1/(4*x**4 + 4)
```

3.275.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{x^3}{1+2x^4+x^8}\,dx = -\frac{1}{4\,(x^4+1)}$$

input
```
integrate(x^3/(x^8+2*x^4+1),x, algorithm="maxima")
```

output
```
-1/4/(x^4 + 1)
```

3.275.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{x^3}{1+2x^4+x^8}\,dx = -\frac{1}{4\,(x^4+1)}$$

input
```
integrate(x^3/(x^8+2*x^4+1),x, algorithm="giac")
```

output
```
-1/4/(x^4 + 1)
```

3.275.9 Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{1+2x^4+x^8}\,dx = -\frac{1}{4\,\left(x^4+1\right)}$$

input
```
int(x^3/(2*x^4 + x^8 + 1),x)
```

output
```
-1/(4*(x^4 + 1))
```

3.276 $\int \frac{x}{1+2x^4+x^8}\,dx$

3.276.1 Optimal result

Integrand size = 14, antiderivative size = 23

$$\int \frac{x}{1+2x^4+x^8}\,dx=\frac{x^2}{4\left(1+x^4\right)}+\frac{\arctan\left(x^2\right)}{4}$$

output
```
1/4*x^2/(x^4+1)+1/4*arctan(x^2)
```

3.276.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{x}{1+2x^4+x^8}\,dx=\frac{1}{4}\left(\frac{x^2}{1+x^4}+\arctan\left(x^2\right)\right)$$

input
```
Integrate[x/(1 + 2*x^4 + x^8),x]
```

output
```
(x^2/(1 + x^4) + ArcTan[x^2])/4
```

3.276.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1380, 807, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{x^8+2x^4+1}dx$$
$$\Big\downarrow \text{1380}$$
$$\int \frac{x}{(x^4+1)^2}dx$$
$$\Big\downarrow \text{807}$$
$$\frac{1}{2}\int \frac{1}{(x^4+1)^2}dx^2$$
$$\Big\downarrow \text{215}$$
$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^4+1}dx^2+\frac{x^2}{2(x^4+1)}\right)$$
$$\Big\downarrow \text{216}$$
$$\frac{1}{2}\left(\frac{\arctan(x^2)}{2}+\frac{x^2}{2(x^4+1)}\right)$$

input
```
Int[x/(1 + 2*x^4 + x^8),x]
```

output
```
(x^2/(2*(1 + x^4)) + ArcTan[x^2]/2)/2
```

3.276.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

3.276.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{x^2}{4x^4+4}+\frac{\arctan(x^2)}{4}$	20
risch	$\frac{x^2}{4x^4+4}+\frac{\arctan(x^2)}{4}$	20
parallelrisch	$-\frac{i\ln(x^2-i)x^4-i\ln(x^2+i)x^4+i\ln(x^2-i)-i\ln(x^2+i)-2x^2}{8(x^4+1)}$	62

input
```
int(x/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^2/(x^4+1)+1/4*arctan(x^2)
```

3.276.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x}{1+2x^4+x^8}\,dx=\frac{x^2+(x^4+1)\arctan(x^2)}{4(x^4+1)}$$

input
```
integrate(x/(x^8+2*x^4+1),x, algorithm="fricas")
```

output
```
1/4*(x^2 + (x^4 + 1)*arctan(x^2))/(x^4 + 1)
```

3.276.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int \frac{x}{1+2x^4+x^8}\,dx = \frac{x^2}{4x^4+4} + \frac{\operatorname{atan}(x^2)}{4}$$

input
```
integrate(x/(x**8+2*x**4+1),x)
```

output
```
x**2/(4*x**4 + 4) + atan(x**2)/4
```

3.276.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{x}{1+2x^4+x^8}\,dx = \frac{x^2}{4(x^4+1)} + \frac{1}{4}\arctan(x^2)$$

input
```
integrate(x/(x^8+2*x^4+1),x, algorithm="maxima")
```

output
```
1/4*x^2/(x^4 + 1) + 1/4*arctan(x^2)
```

3.276.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{x}{1+2x^4+x^8}\,dx = \frac{x^2}{4(x^4+1)} + \frac{1}{4}\arctan(x^2)$$

input
```
integrate(x/(x^8+2*x^4+1),x, algorithm="giac")
```

output
```
1/4*x^2/(x^4 + 1) + 1/4*arctan(x^2)
```

3.276.9 Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{x}{1+2x^4+x^8}\,dx = \frac{\operatorname{atan}(x^2)}{4} + \frac{x^2}{4\,(x^4+1)}$$

input
```
int(x/(2*x^4 + x^8 + 1),x)
```

output
```
atan(x^2)/4 + x^2/(4*(x^4 + 1))
```

3.277 $\int \frac{1}{x(1+2x^4+x^8)} dx$

3.277.1 Optimal result

Integrand size = 16, antiderivative size = 24

$$\int \frac{1}{x(1+2x^4+x^8)} dx = \frac{1}{4(1+x^4)} + \log(x) - \frac{1}{4}\log(1+x^4)$$

output
```
1/4/(x^4+1)+ln(x)-1/4*ln(x^4+1)
```

3.277.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(1+2x^4+x^8)} dx = \frac{1}{4(1+x^4)} + \log(x) - \frac{1}{4}\log(1+x^4)$$

input
```
Integrate[1/(x*(1 + 2*x^4 + x^8)),x]
```

output
```
1/(4*(1 + x^4)) + Log[x] - Log[1 + x^4]/4
```

3.277.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1380, 798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(x^8+2x^4+1)}dx$$
$$\Big\downarrow \text{1380}$$
$$\int \frac{1}{x(x^4+1)^2}dx$$
$$\Big\downarrow \text{798}$$
$$\frac{1}{4}\int \frac{1}{x^4(x^4+1)^2}dx^4$$
$$\Big\downarrow \text{54}$$
$$\frac{1}{4}\int \left(-\frac{1}{(x^4+1)^2}+\frac{1}{x^4}+\frac{1}{-x^4-1}\right)dx^4$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{4}\left(\frac{1}{x^4+1}+\log(x^4)-\log(x^4+1)\right)$$

input
```
Int[1/(x*(1 + 2*x^4 + x^8)),x]
```

output
```
((1 + x^4)^(-1) + Log[x^4] - Log[1 + x^4])/4
```

3.277.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.277.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{1}{4x^4+4}+\ln(x)-\frac{\ln(x^4+1)}{4}$	21
norman	$\frac{1}{4x^4+4}+\ln(x)-\frac{\ln(x^4+1)}{4}$	21
risch	$\frac{1}{4x^4+4}+\ln(x)-\frac{\ln(x^4+1)}{4}$	21
parallelrisch	$\frac{4\ln(x)x^4-\ln(x^4+1)x^4+1+4\ln(x)-\ln(x^4+1)}{4x^4+4}$	42

input
```
int(1/x/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4/(x^4+1)+ln(x)-1/4*ln(x^4+1)
```

3.277.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.33

$$\int \frac{1}{x(1+2x^4+x^8)}\,dx = -\frac{(x^4+1)\log(x^4+1)-4(x^4+1)\log(x)-1}{4(x^4+1)}$$

input
```
integrate(1/x/(x^8+2*x^4+1),x, algorithm="fricas")
```

output
```
-1/4*((x^4 + 1)*log(x^4 + 1) - 4*(x^4 + 1)*log(x) - 1)/(x^4 + 1)
```

3.277.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \frac{1}{x\left(1+2x^4+x^8\right)}\,dx = \log\left(x\right) - \frac{\log\left(x^4+1\right)}{4} + \frac{1}{4x^4+4}$$

input
```
integrate(1/x/(x**8+2*x**4+1),x)
```

output
```
log(x) - log(x**4 + 1)/4 + 1/(4*x**4 + 4)
```

3.277.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1+2x^4+x^8\right)}\,dx = \frac{1}{4\left(x^4+1\right)} - \frac{1}{4}\log\left(x^4+1\right) + \frac{1}{4}\log\left(x^4\right)$$

input
```
integrate(1/x/(x^8+2*x^4+1),x, algorithm="maxima")
```

output
```
1/4/(x^4 + 1) - 1/4*log(x^4 + 1) + 1/4*log(x^4)
```

3.277.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int \frac{1}{x\left(1+2x^4+x^8\right)}\,dx = \frac{x^4+2}{4\left(x^4+1\right)} - \frac{1}{4}\log\left(x^4+1\right) + \frac{1}{4}\log\left(x^4\right)$$

input
```
integrate(1/x/(x^8+2*x^4+1),x, algorithm="giac")
```

output
```
1/4*(x^4 + 2)/(x^4 + 1) - 1/4*log(x^4 + 1) + 1/4*log(x^4)
```

3.277.9 Mupad [B] (verification not implemented)

Time = 8.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{1}{x\left(1+2x^4+x^8\right)}\,dx = \ln\left(x\right) - \frac{\ln\left(x^4+1\right)}{4} + \frac{1}{4\left(x^4+1\right)}$$

input
```
int(1/(x*(2*x^4 + x^8 + 1)),x)
```

output
```
log(x) - log(x^4 + 1)/4 + 1/(4*(x^4 + 1))
```

3.278 $\int \frac{1}{x^3(1+2x^4+x^8)}\,dx$

3.278.1 Optimal result

Integrand size = 16, antiderivative size = 30

$$\int \frac{1}{x^3\left(1+2x^4+x^8\right)}\,dx=-\frac{3}{4x^2}+\frac{1}{4x^2\left(1+x^4\right)}-\frac{3\arctan\left(x^2\right)}{4}$$

output
```
-3/4/x^2+1/4/x^2/(x^4+1)-3/4*arctan(x^2)
```

3.278.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3\left(1+2x^4+x^8\right)}\,dx=-\frac{1}{2x^2}-\frac{x^2}{4\left(1+x^4\right)}+\frac{3}{4}\arctan\left(\frac{1}{x^2}\right)$$

input
```
Integrate[1/(x^3*(1 + 2*x^4 + x^8)),x]
```

output
```
-1/2*1/x^2 - x^2/(4*(1 + x^4)) + (3*ArcTan[x^(-2)])/4
```

3.278.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1380, 807, 253, 264, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (x^8+2x^4+1)} \, dx$$
$$\Big\downarrow \text{1380}$$
$$\int \frac{1}{x^3 (x^4+1)^2}dx$$
$$\Big\downarrow \text{807}$$
$$\frac{1}{2}\int \frac{1}{x^4 (x^4+1)^2}dx^2$$
$$\Big\downarrow \text{253}$$
$$\frac{1}{2}\left(\frac{3}{2}\int \frac{1}{x^4 (x^4+1)}dx^2+\frac{1}{2x^2 (x^4+1)}\right)$$
$$\Big\downarrow \text{264}$$
$$\frac{1}{2}\left(\frac{3}{2}\left(-\int \frac{1}{x^4+1}dx^2-\frac{1}{x^2}\right)+\frac{1}{2x^2 (x^4+1)}\right)$$
$$\Big\downarrow \text{216}$$
$$\frac{1}{2}\left(\frac{3}{2}\left(-\arctan \left(x^2\right)-\frac{1}{x^2}\right)+\frac{1}{2x^2 (x^4+1)}\right)$$

input
```
Int[1/(x^3*(1 + 2*x^4 + x^8)),x]
```

output
```
(1/(2*x^2*(1 + x^4)) + (3*(-x^(-2) - ArcTan[x^2]))/2)/2
```

3.278.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

3.278.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{1}{2x^{2}}-\frac{x^{2}}{4(x^{4}+1)}-\frac{3\arctan\left(x^{2}\right)}{4}$	25
risch	$\frac{-\frac{3x^{4}}{4}-\frac{1}{2}}{x^{2}(x^{4}+1)}-\frac{3\arctan\left(x^{2}\right)}{4}$	26
parallelrisch	$\frac{3i\ln\left(x^{2}-i\right)x^{6}-3i\ln\left(x^{2}+i\right)x^{6}-4+3i\ln\left(x^{2}-i\right)x^{2}-3i\ln\left(x^{2}+i\right)x^{2}-6x^{4}}{8x^{2}(x^{4}+1)}$	72

input
```
int(1/x^3/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2/x^2-1/4*x^2/(x^4+1)-3/4*arctan(x^2)
```

3.278.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^3 (1 + 2x^4 + x^8)} \, dx = -\frac{3 \, x^4 + 3 \, (x^6 + x^2) \arctan \left(x^2\right) + 2}{4 \, (x^6 + x^2)}$$

input
```
integrate(1/x^3/(x^8+2*x^4+1),x, algorithm="fricas")
```

output
```
-1/4*(3*x^4 + 3*(x^6 + x^2)*arctan(x^2) + 2)/(x^6 + x^2)
```

3.278.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^3 (1 + 2x^4 + x^8)} \, dx = \frac{-3x^4 - 2}{4x^6 + 4x^2} - \frac{3 \operatorname{atan} \left(x^2\right)}{4}$$

input
```
integrate(1/x**3/(x**8+2*x**4+1),x)
```

output
```
(-3*x**4 - 2)/(4*x**6 + 4*x**2) - 3*atan(x**2)/4
```

3.278.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^3 (1 + 2x^4 + x^8)} \, dx = -\frac{3 \, x^4 + 2}{4 \, (x^6 + x^2)} - \frac{3}{4} \, \arctan \left(x^2\right)$$

input
```
integrate(1/x^3/(x^8+2*x^4+1),x, algorithm="maxima")
```

output
```
-1/4*(3*x^4 + 2)/(x^6 + x^2) - 3/4*arctan(x^2)
```

3.278.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^3 (1 + 2x^4 + x^8)} \, dx = -\frac{3\,x^4 + 2}{4\,(x^6 + x^2)} - \frac{3}{4}\arctan\left(x^2\right)$$

input
```
integrate(1/x^3/(x^8+2*x^4+1),x, algorithm="giac")
```

output
```
-1/4*(3*x^4 + 2)/(x^6 + x^2) - 3/4*arctan(x^2)
```

3.278.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^3 (1 + 2x^4 + x^8)} \, dx = -\frac{3\,\mathrm{atan}(x^2)}{4} - \frac{\frac{3\,x^4}{4} + \frac{1}{2}}{x^6 + x^2}$$

input
```
int(1/(x^3*(2*x^4 + x^8 + 1)),x)
```

output
```
- (3*atan(x^2))/4 - ((3*x^4)/4 + 1/2)/(x^2 + x^6)
```

3.279 $\int \frac{1}{x^5(1+2x^4+x^8)}\,dx$

3.279.1 Optimal result

Integrand size = 16, antiderivative size = 33

$$\int \frac{1}{x^5\left(1+2x^4+x^8\right)}\,dx=-\frac{1}{4x^4}-\frac{1}{4\left(1+x^4\right)}-2\log(x)+\frac{1}{2}\log\left(1+x^4\right)$$

output
```
-1/4/x^4-1/4/(x^4+1)-2*ln(x)+1/2*ln(x^4+1)
```

3.279.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^5\left(1+2x^4+x^8\right)}\,dx=-\frac{1}{4x^4}-\frac{1}{4\left(1+x^4\right)}-2\log(x)+\frac{1}{2}\log\left(1+x^4\right)$$

input
```
Integrate[1/(x^5*(1 + 2*x^4 + x^8)),x]
```

output
```
-1/4*1/x^4 - 1/(4*(1 + x^4)) - 2*Log[x] + Log[1 + x^4]/2
```

3.279.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1380, 798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 (x^8+2x^4+1)} \, dx$$

$$\Big\downarrow \text{1380}$$

$$\int \frac{1}{x^5 (x^4+1)^2} dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4} \int \frac{1}{x^8 (x^4+1)^2} dx^4$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{4} \int \left(-\frac{2}{x^4}+\frac{1}{x^8}+\frac{2}{x^4+1}+\frac{1}{(x^4+1)^2}\right) dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(-\frac{1}{x^4+1}-\frac{1}{x^4}-2\log\left(x^4\right)+2\log\left(x^4+1\right)\right)$$

input
```
Int[1/(x^5*(1 + 2*x^4 + x^8)),x]
```

output
```
(-x^(-4) - (1 + x^4)^(-1) - 2*Log[x^4] + 2*Log[1 + x^4])/4
```

3.279.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.279.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

method	result	size
default	$-\frac{1}{4x^4}-\frac{1}{4(x^4+1)}-2\ln(x)+\frac{\ln(x^4+1)}{2}$	28
norman	$\frac{-\frac{1}{4}-\frac{x^4}{2}}{x^4(x^4+1)}-2\ln(x)+\frac{\ln(x^4+1)}{2}$	32
risch	$\frac{-\frac{1}{4}-\frac{x^4}{2}}{x^4(x^4+1)}-2\ln(x)+\frac{\ln(x^4+1)}{2}$	32
parallelrisch	$-\frac{8\ln(x)x^8-2\ln(x^4+1)x^8+1+8\ln(x)x^4-2\ln(x^4+1)x^4+2x^4}{4x^4(x^4+1)}$	56

input
```
int(1/x^5/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/4/x^4-1/4/(x^4+1)-2*ln(x)+1/2*ln(x^4+1)
```

3.279.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.33

$$\int \frac{1}{x^5(1+2x^4+x^8)}\,dx = -\frac{2\,x^4-2\,(x^8+x^4)\log(x^4+1)+8\,(x^8+x^4)\log(x)+1}{4\,(x^8+x^4)}$$

input
```
integrate(1/x^5/(x^8+2*x^4+1),x, algorithm="fricas")
```

output
```
-1/4*(2*x^4 - 2*(x^8 + x^4)*log(x^4 + 1) + 8*(x^8 + x^4)*log(x) + 1)/(x^8
+ x^4)
```

3.279.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^5 (1 + 2x^4 + x^8)} dx = \frac{-2x^4 - 1}{4x^8 + 4x^4} - 2\log(x) + \frac{\log(x^4 + 1)}{2}$$

input
```
integrate(1/x**5/(x**8+2*x**4+1),x)
```

output
```
(-2*x**4 - 1)/(4*x**8 + 4*x**4) - 2*log(x) + log(x**4 + 1)/2
```

3.279.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^5 (1 + 2x^4 + x^8)} dx = -\frac{2x^4 + 1}{4(x^8 + x^4)} + \frac{1}{2}\log(x^4 + 1) - \frac{1}{2}\log(x^4)$$

input
```
integrate(1/x^5/(x^8+2*x^4+1),x, algorithm="maxima")
```

output
```
-1/4*(2*x^4 + 1)/(x^8 + x^4) + 1/2*log(x^4 + 1) - 1/2*log(x^4)
```

3.279.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^5 (1 + 2x^4 + x^8)} dx = -\frac{2x^4 + 1}{4(x^8 + x^4)} + \frac{1}{2}\log(x^4 + 1) - \frac{1}{2}\log(x^4)$$

input
```
integrate(1/x^5/(x^8+2*x^4+1),x, algorithm="giac")
```

output
```
-1/4*(2*x^4 + 1)/(x^8 + x^4) + 1/2*log(x^4 + 1) - 1/2*log(x^4)
```

3.279.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^5\left(1+2x^4+x^8\right)}\,dx = \frac{\ln\left(x^4+1\right)}{2} - 2\,\ln\left(x\right) - \frac{\frac{x^4}{2}+\frac{1}{4}}{x^8+x^4}$$

input
```
int(1/(x^5*(2*x^4 + x^8 + 1)),x)
```

output
```
log(x^4 + 1)/2 - 2*log(x) - (x^4/2 + 1/4)/(x^4 + x^8)
```

3.280 $\int \frac{1}{x^7(1+2x^4+x^8)}\,dx$

3.280.1 Optimal result

Integrand size = 16, antiderivative size = 37

$$\int \frac{1}{x^7(1+2x^4+x^8)}\,dx = -\frac{5}{12x^6}+\frac{5}{4x^2}+\frac{1}{4x^6(1+x^4)}+\frac{5\arctan(x^2)}{4}$$

output
```
-5/12/x^6+5/4/x^2+1/4/x^6/(x^4+1)+5/4*arctan(x^2)
```

3.280.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^7(1+2x^4+x^8)}\,dx = -\frac{1}{6x^6}+\frac{1}{x^2}+\frac{x^2}{4(1+x^4)}-\frac{5}{4}\arctan\left(\frac{1}{x^2}\right)$$

input
```
Integrate[1/(x^7*(1 + 2*x^4 + x^8)),x]
```

output
```
-1/6*1/x^6 + x^(-2) + x^2/(4*(1 + x^4)) - (5*ArcTan[x^(-2)])/4
```

3.280.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1380, 807, 253, 264, 264, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7(x^8+2x^4+1)}dx$$
$$\Big\downarrow \text{1380}$$
$$\int \frac{1}{x^7(x^4+1)^2}dx$$
$$\Big\downarrow \text{807}$$
$$\frac{1}{2}\int \frac{1}{x^8(x^4+1)^2}dx^2$$
$$\Big\downarrow \text{253}$$
$$\frac{1}{2}\left(\frac{5}{2}\int \frac{1}{x^8(x^4+1)}dx^2+\frac{1}{2x^6(x^4+1)}\right)$$
$$\Big\downarrow \text{264}$$
$$\frac{1}{2}\left(\frac{5}{2}\left(-\int \frac{1}{x^4(x^4+1)}dx^2-\frac{1}{3x^6}\right)+\frac{1}{2x^6(x^4+1)}\right)$$
$$\Big\downarrow \text{264}$$
$$\frac{1}{2}\left(\frac{5}{2}\left(\int \frac{1}{x^4+1}dx^2-\frac{1}{3x^6}+\frac{1}{x^2}\right)+\frac{1}{2x^6(x^4+1)}\right)$$
$$\Big\downarrow \text{216}$$
$$\frac{1}{2}\left(\frac{5}{2}\left(\arctan\left(x^2\right)-\frac{1}{3x^6}+\frac{1}{x^2}\right)+\frac{1}{2x^6(x^4+1)}\right)$$

input
```
Int[1/(x^7*(1 + 2*x^4 + x^8)),x]
```

output
```
(1/(2*x^6*(1 + x^4)) + (5*(-1/3*1/x^6 + x^(-2) + ArcTan[x^2]))/2)/2
```

3.280.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

3.280.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.76

method	result	size
default	$-\frac{1}{6x^{6}}+\frac{1}{x^{2}}+\frac{x^{2}}{4x^{4}+4}+\frac{5\arctan\left(x^{2}\right)}{4}$	28
risch	$\frac{\frac{5}{4}x^{8}+\frac{5}{6}x^{4}-\frac{1}{6}}{x^{6}\left(x^{4}+1\right)}+\frac{5\arctan\left(x^{2}\right)}{4}$	31
parallelrisch	$-\frac{15i\ln\left(x^{2}-i\right)x^{10}-15i\ln\left(x^{2}+i\right)x^{10}+4+15i\ln\left(x^{2}-i\right)x^{6}-15i\ln\left(x^{2}+i\right)x^{6}-30x^{8}-20x^{4}}{24x^{6}\left(x^{4}+1\right)}$	77

input
```
int(1/x^7/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/6/x^6+1/x^2+1/4*x^2/(x^4+1)+5/4*arctan(x^2)
```

3.280.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^7 (1+2x^4+x^8)} dx = \frac{15 x^8 + 10 x^4 + 15 (x^{10}+x^6) \arctan(x^2) - 2}{12 (x^{10}+x^6)}$$

input
```
integrate(1/x^7/(x^8+2*x^4+1),x, algorithm="fricas")
```

output
```
1/12*(15*x^8 + 10*x^4 + 15*(x^10 + x^6)*arctan(x^2) - 2)/(x^10 + x^6)
```

3.280.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^7 (1+2x^4+x^8)} dx = \frac{5 \operatorname{atan}(x^2)}{4} + \frac{15x^8+10x^4-2}{12x^{10}+12x^6}$$

input
```
integrate(1/x**7/(x**8+2*x**4+1),x)
```

output
```
5*atan(x**2)/4 + (15*x**8 + 10*x**4 - 2)/(12*x**10 + 12*x**6)
```

3.280.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^7 (1+2x^4+x^8)} dx = \frac{15 x^8 + 10 x^4 - 2}{12 (x^{10}+x^6)} + \frac{5}{4} \arctan(x^2)$$

input
```
integrate(1/x^7/(x^8+2*x^4+1),x, algorithm="maxima")
```

output
```
1/12*(15*x^8 + 10*x^4 - 2)/(x^10 + x^6) + 5/4*arctan(x^2)
```

3.280.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^7 (1+2x^4+x^8)}\,dx = \frac{x^2}{4\,(x^4+1)} + \frac{6\,x^4-1}{6\,x^6} + \frac{5}{4}\arctan\left(x^2\right)$$

input
```
integrate(1/x^7/(x^8+2*x^4+1),x, algorithm="giac")
```

output
```
1/4*x^2/(x^4 + 1) + 1/6*(6*x^4 - 1)/x^6 + 5/4*arctan(x^2)
```

3.280.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^7 (1+2x^4+x^8)}\,dx = \frac{5\,\mathrm{atan}(x^2)}{4} + \frac{\frac{5\,x^8}{4}+\frac{5\,x^4}{6}-\frac{1}{6}}{x^6\,\left(x^4+1\right)}$$

input
```
int(1/(x^7*(2*x^4 + x^8 + 1)),x)
```

output
```
(5*atan(x^2))/4 + ((5*x^4)/6 + (5*x^8)/4 - 1/6)/(x^6*(x^4 + 1))
```

3.281 $\int \frac{x^8}{1+2x^4+x^8}\,dx$

3.281.1 Optimal result

Integrand size = 16, antiderivative size = 104

$$\int \frac{x^8}{1+2x^4+x^8}\,dx=\frac{5x}{4}-\frac{x^5}{4(1+x^4)}+\frac{5\arctan\left(1-\sqrt{2}x\right)}{8\sqrt{2}}-\frac{5\arctan\left(1+\sqrt{2}x\right)}{8\sqrt{2}}+\frac{5\log\left(1-\sqrt{2}x+x^2\right)}{16\sqrt{2}}-\frac{5\log\left(1+\sqrt{2}x+x^2\right)}{16\sqrt{2}}$$

output
```
5/4*x-1/4*x^5/(x^4+1)-5/16*arctan(-1+x*2^(1/2))*2^(1/2)-5/16*arctan(1+x*2^
(1/2))*2^(1/2)+5/32*ln(1+x^2-x*2^(1/2))*2^(1/2)-5/32*ln(1+x^2+x*2^(1/2))*2
^(1/2)
```

3.281.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90

$$\int \frac{x^8}{1+2x^4+x^8}\,dx=\frac{1}{32}\left(32x+\frac{8x}{1+x^4}+10\sqrt{2}\arctan\left(1-\sqrt{2}x\right)-10\sqrt{2}\arctan\left(1+\sqrt{2}x\right)+5\sqrt{2}\log\left(1-\sqrt{2}x+x^2\right)-5\sqrt{2}\log\left(1+\sqrt{2}x+x^2\right)\right)$$

input
```
Integrate[x^8/(1 + 2*x^4 + x^8),x]
```

output
```
(32*x + (8*x)/(1 + x^4) + 10*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] - 10*Sqrt[2]*Ar
cTan[1 + Sqrt[2]*x] + 5*Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] - 5*Sqrt[2]*Log[1
 + Sqrt[2]*x + x^2])/32
```

3.281.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.688$, Rules used = {1380, 817, 843, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{x^8+2x^4+1}\,dx$$

$$\Big\downarrow \text{1380}$$

$$\int \frac{x^8}{(x^4+1)^2}dx$$

$$\Big\downarrow \text{817}$$

$$\frac{5}{4}\int \frac{x^4}{x^4+1}dx-\frac{x^5}{4(x^4+1)}$$

$$\Big\downarrow \text{843}$$

$$\frac{5}{4}\left(x-\int \frac{1}{x^4+1}dx\right)-\frac{x^5}{4(x^4+1)}$$

$$\Big\downarrow \text{755}$$

$$\frac{5}{4}\left(-\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx-\frac{1}{2}\int \frac{x^2+1}{x^4+1}dx+x\right)-\frac{x^5}{4(x^4+1)}$$

$$\Big\downarrow \text{1476}$$

$$\frac{5}{4}\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{x^2-\sqrt{2}x+1}dx-\frac{1}{2}\int \frac{1}{x^2+\sqrt{2}x+1}dx\right)-\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx+x\right)-\frac{x^5}{4(x^4+1)}$$

$$\Big\downarrow \text{1082}$$

$$\frac{5}{4}\left(-\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx+\frac{1}{2}\left(\frac{\int \frac{1}{-\left(\sqrt{2}x+1\right)^2-1}d\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\left(1-\sqrt{2}x\right)^2-1}d\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)+x\right)-\frac{x^5}{4(x^4+1)}$$

$$\Big\downarrow \text{217}$$

$$\frac{5}{4}\left(-\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx+\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}\right)+x\right)-\frac{x^5}{4\left(x^4+1\right)}$$

$$\Big\downarrow \text{1479}$$

$$\frac{5}{4}\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}x+1\right)}{x^2+\sqrt{2}x+1}dx}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}\right)+x\right)-\frac{x^5}{4\left(x^4+1\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{5}{4}\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{2}x+1\right)}{x^2+\sqrt{2}x+1}dx}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}\right)+x\right)-\frac{x^5}{4\left(x^4+1\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{5}{4}\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}-\frac{1}{2}\int \frac{\sqrt{2}x+1}{x^2+\sqrt{2}x+1}dx\right)+\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}\right)+x\right)-\frac{x^5}{4\left(x^4+1\right)}$$

$$\Big\downarrow \text{1103}$$

$$\frac{5}{4}\left(\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(x^2-\sqrt{2}x+1\right)}{2\sqrt{2}}-\frac{\log\left(x^2+\sqrt{2}x+1\right)}{2\sqrt{2}}\right)+x\right)-\frac{x^5}{4\left(x^4+1\right)}$$

input
```
Int[x^8/(1 + 2*x^4 + x^8),x]
```

output
```
-1/4*x^5/(1 + x^4) + (5*(x + (ArcTan[1 - Sqrt[2]*x]/Sqrt[2] - ArcTan[1 + S
qrt[2]*x]/Sqrt[2])/2 + (Log[1 - Sqrt[2]*x + x^2]/(2*Sqrt[2]) - Log[1 + Sqr
t[2]*x + x^2]/(2*Sqrt[2]))/2))/4
```

3.281. $\int \frac{x^8}{1+2x^4+x^8}\,dx$

3.281.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.281. $\int \frac{x^8}{1+2x^4+x^8}\,dx$

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.281.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.33

method	result	size
risch	$x+\frac{x}{4x^4+4}-\frac{5\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+1\right)}\frac{\ln\left(x-\_R\right)}{\_R^3}\right)}{16}$	34
default	$x+\frac{x}{4x^4+4}-\frac{5\sqrt{2}\left(\ln\left(\frac{1+x^2+x\sqrt{2}}{1+x^2-x\sqrt{2}}\right)+2\arctan\left(x\sqrt{2}+1\right)+2\arctan\left(x\sqrt{2}-1\right)\right)}{32}$	64

input
```
int(x^8/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
x+1/4*x/(x^4+1)-5/16*sum(1/_R^3*ln(x-_R),_R=RootOf(_Z^4+1))
```

3.281.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.02

$$\int \frac{x^8}{1+2x^4+x^8}\,dx = \frac{32\,x^5 - 5\,\sqrt{2}((i+1)\,x^4+i+1)\log\left(2\,x+(i+1)\,\sqrt{2}\right) - 5\,\sqrt{2}(-(i-1)\,x^4-i+1)\log\left(2\,x-(i-1)\,\sqrt{2}\right.}{32}$$

input
```
integrate(x^8/(x^8+2*x^4+1),x, algorithm="fricas")
```

output
```
1/32*(32*x^5 - 5*sqrt(2)*((I + 1)*x^4 + I + 1)*log(2*x + (I + 1)*sqrt(2))
- 5*sqrt(2)*(-(I - 1)*x^4 - I + 1)*log(2*x - (I - 1)*sqrt(2)) - 5*sqrt(2)*
((I - 1)*x^4 + I - 1)*log(2*x + (I - 1)*sqrt(2)) - 5*sqrt(2)*(-(I + 1)*x^4
 - I - 1)*log(2*x - (I + 1)*sqrt(2)) + 40*x)/(x^4 + 1)
```

3.281.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.87

$$\int \frac{x^8}{1+2x^4+x^8}\,dx = x + \frac{x}{4x^4+4} + \frac{5\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{32} - \frac{5\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{32} - \frac{5\sqrt{2}\operatorname{atan}\left(\sqrt{2}x-1\right)}{16} - \frac{5\sqrt{2}\operatorname{atan}\left(\sqrt{2}x+1\right)}{16}$$

input
```
integrate(x**8/(x**8+2*x**4+1),x)
```

output
```
x + x/(4*x**4 + 4) + 5*sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 - 5*sqrt(2)*lo
g(x**2 + sqrt(2)*x + 1)/32 - 5*sqrt(2)*atan(sqrt(2)*x - 1)/16 - 5*sqrt(2)*
atan(sqrt(2)*x + 1)/16
```

3.281.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.80

$$\int \frac{x^8}{1+2x^4+x^8}\,dx = -\frac{5}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x+\sqrt{2}\right)\right) - \frac{5}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x-\sqrt{2}\right)\right) - \frac{5}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right) + \frac{5}{32}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right) + x + \frac{x}{4\,(x^4+1)}$$

input
```
integrate(x^8/(x^8+2*x^4+1),x, algorithm="maxima")
```

output
```
-5/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 5/16*sqrt(2)*arctan(1/
2*sqrt(2)*(2*x - sqrt(2))) - 5/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 5/32*
sqrt(2)*log(x^2 - sqrt(2)*x + 1) + x + 1/4*x/(x^4 + 1)
```

3.281.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.80

$$\int \frac{x^8}{1+2x^4+x^8}\,dx = -\frac{5}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x+\sqrt{2}\right)\right) - \frac{5}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x-\sqrt{2}\right)\right) - \frac{5}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right) + \frac{5}{32}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right) + x + \frac{x}{4\,(x^4+1)}$$

input
```
integrate(x^8/(x^8+2*x^4+1),x, algorithm="giac")
```

output
```
-5/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 5/16*sqrt(2)*arctan(1/
2*sqrt(2)*(2*x - sqrt(2))) - 5/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 5/32*
sqrt(2)*log(x^2 - sqrt(2)*x + 1) + x + 1/4*x/(x^4 + 1)
```

3.281.9 Mupad [B] (verification not implemented)

Time = 8.29 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.43

$$\int \frac{x^8}{1+2x^4+x^8}\,dx = x+\frac{x}{4\,(x^4+1)}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{5}{16}-\frac{5}{16}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{5}{16}+\frac{5}{16}\mathrm{i}\right)$$

input
```
int(x^8/(2*x^4 + x^8 + 1),x)
```

output
```
x - 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(5/16 + 5i/16) - 2^(1/2)*atan(2^(
1/2)*x*(1/2 + 1i/2))*(5/16 - 5i/16) + x/(4*(x^4 + 1))
```

3.282 $\int \frac{x^6}{1+2x^4+x^8}\,dx$

3.282.1 Optimal result

Integrand size = 16, antiderivative size = 99

$$\int \frac{x^6}{1+2x^4+x^8}\,dx = -\frac{x^3}{4(1+x^4)} - \frac{3\arctan\left(1-\sqrt{2}x\right)}{8\sqrt{2}} + \frac{3\arctan\left(1+\sqrt{2}x\right)}{8\sqrt{2}} + \frac{3\log\left(1-\sqrt{2}x+x^2\right)}{16\sqrt{2}} - \frac{3\log\left(1+\sqrt{2}x+x^2\right)}{16\sqrt{2}}$$

output
```
-1/4*x^3/(x^4+1)+3/16*arctan(-1+x*2^(1/2))*2^(1/2)+3/16*arctan(1+x*2^(1/2)
)*2^(1/2)+3/32*ln(1+x^2-x*2^(1/2))*2^(1/2)-3/32*ln(1+x^2+x*2^(1/2))*2^(1/2
)
```

3.282.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.94

$$\int \frac{x^6}{1+2x^4+x^8}\,dx = \frac{1}{32}\left(-\frac{8x^3}{1+x^4} - 6\sqrt{2}\arctan\left(1-\sqrt{2}x\right) + 6\sqrt{2}\arctan\left(1+\sqrt{2}x\right) + 3\sqrt{2}\log\left(1-\sqrt{2}x+x^2\right) - 3\sqrt{2}\log\left(1+\sqrt{2}x+x^2\right)\right)$$

input
```
Integrate[x^6/(1 + 2*x^4 + x^8),x]
```

output
```
((-8*x^3)/(1 + x^4) - 6*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] + 6*Sqrt[2]*ArcTan[1
 + Sqrt[2]*x] + 3*Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] - 3*Sqrt[2]*Log[1 + Sqr
t[2]*x + x^2])/32
```

3.282.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {1380, 817, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{x^8+2x^4+1}\,dx$$

$$\Big\downarrow \text{1380}$$

$$\int \frac{x^6}{\left(x^4+1\right)^2}dx$$

$$\Big\downarrow \text{817}$$

$$\frac{3}{4}\int \frac{x^2}{x^4+1}dx-\frac{x^3}{4\left(x^4+1\right)}$$

$$\Big\downarrow \text{826}$$

$$\frac{3}{4}\left(\frac{1}{2}\int \frac{x^2+1}{x^4+1}dx-\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx\right)-\frac{x^3}{4\left(x^4+1\right)}$$

$$\Big\downarrow \text{1476}$$

$$\frac{3}{4}\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2-\sqrt{2}x+1}dx+\frac{1}{2}\int \frac{1}{x^2+\sqrt{2}x+1}dx\right)-\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx\right)-\frac{x^3}{4\left(x^4+1\right)}$$

$$\Big\downarrow \text{1082}$$

$$\frac{3}{4}\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\left(1-\sqrt{2}x\right)^2-1}d\left(1-\sqrt{2}x\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\left(\sqrt{2}x+1\right)^2-1}d\left(\sqrt{2}x+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx\right)-\frac{x^3}{4\left(x^4+1\right)}$$

$$\Big\downarrow \text{217}$$

$$\frac{3}{4}\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx\right)-\frac{x^3}{4\left(x^4+1\right)}$$

$$\Big\downarrow \text{1479}$$

$$\frac{3}{4}\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}x+1\right)}{x^2+\sqrt{2}x+1}dx}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)\right)-\frac{x^3}{4\left(x^4+1\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{3}{4}\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{2}x+1\right)}{x^2+\sqrt{2}x+1}dx}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)\right)-\frac{x^3}{4\left(x^4+1\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{3}{4}\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}-\frac{1}{2}\int \frac{\sqrt{2}x+1}{x^2+\sqrt{2}x+1}dx\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)\right)-\frac{x^3}{4\left(x^4+1\right)}$$

$$\Big\downarrow \; 1103$$

$$\frac{3}{4}\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(x^2-\sqrt{2}x+1\right)}{2\sqrt{2}}-\frac{\log\left(x^2+\sqrt{2}x+1\right)}{2\sqrt{2}}\right)\right)-\frac{x^3}{4\left(x^4+1\right)}$$

input
```
Int[x^6/(1 + 2*x^4 + x^8),x]
```

output
```
-1/4*x^3/(1 + x^4) + (3*((-(ArcTan[1 - Sqrt[2]*x]/Sqrt[2]) + ArcTan[1 + Sq
rt[2]*x]/Sqrt[2])/2 + (Log[1 - Sqrt[2]*x + x^2]/(2*Sqrt[2]) - Log[1 + Sqrt
[2]*x + x^2]/(2*Sqrt[2]))/2))/4
```

3.282. $\int \frac{x^6}{1+2x^4+x^8}\,dx$

3.282.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

3.282. $\int \frac{x^6}{1+2x^4+x^8}\,dx$

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.282.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.35

method	result	size
risch	$-\frac{x^3}{4(x^4+1)}+\frac{3\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+1\right)}\frac{\ln\left(x-\_R\right)}{\_R}\right)}{16}$	35
default	$-\frac{x^3}{4(x^4+1)}+\frac{3\sqrt{2}\left(\ln\left(\frac{1+x^2-x\sqrt{2}}{1+x^2+x\sqrt{2}}\right)+2\arctan\left(x\sqrt{2}+1\right)+2\arctan\left(x\sqrt{2}-1\right)\right)}{32}$	65

input
```
int(x^6/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/4*x^3/(x^4+1)+3/16*sum(1/_R*ln(x-_R),_R=RootOf(_Z^4+1))
```

3.282.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.04

$$\int \frac{x^6}{1+2x^4+x^8}\,dx = -\frac{8\,x^3+3\,\sqrt{2}(-(i-1)\;x^4-i+1)\log\left(2\,x+(i+1)\;\sqrt{2}\right)+3\,\sqrt{2}((i+1)\;x^4+i+1)\log\left(2\,x-(i-1)\;\sqrt{\right.}}{32\,(x}$$

input
```
integrate(x^6/(x^8+2*x^4+1),x, algorithm="fricas")
```

output
```
-1/32*(8*x^3 + 3*sqrt(2)*(-(I - 1)*x^4 - I + 1)*log(2*x + (I + 1)*sqrt(2))
 + 3*sqrt(2)*((I + 1)*x^4 + I + 1)*log(2*x - (I - 1)*sqrt(2)) + 3*sqrt(2)*
(-(I + 1)*x^4 - I - 1)*log(2*x + (I - 1)*sqrt(2)) + 3*sqrt(2)*((I - 1)*x^4
 + I - 1)*log(2*x - (I + 1)*sqrt(2)))/(x^4 + 1)
```

3.282.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.91

$$\int \frac{x^6}{1+2x^4+x^8}\,dx = -\frac{x^3}{4x^4+4} + \frac{3\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{32} - \frac{3\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{32} + \frac{3\sqrt{2}\operatorname{atan}\left(\sqrt{2}x-1\right)}{16} + \frac{3\sqrt{2}\operatorname{atan}\left(\sqrt{2}x+1\right)}{16}$$

input
```
integrate(x**6/(x**8+2*x**4+1),x)
```

output
```
-x**3/(4*x**4 + 4) + 3*sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 - 3*sqrt(2)*lo
g(x**2 + sqrt(2)*x + 1)/32 + 3*sqrt(2)*atan(sqrt(2)*x - 1)/16 + 3*sqrt(2)*
atan(sqrt(2)*x + 1)/16
```

3.282.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.85

$$\int \frac{x^6}{1+2x^4+x^8}\,dx = -\frac{x^3}{4\,(x^4+1)} + \frac{3}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x+\sqrt{2}\right)\right) + \frac{3}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x-\sqrt{2}\right)\right) - \frac{3}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right) + \frac{3}{32}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)$$

input
```
integrate(x^6/(x^8+2*x^4+1),x, algorithm="maxima")
```

output
```
-1/4*x^3/(x^4 + 1) + 3/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 3/
16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 3/32*sqrt(2)*log(x^2 + sq
rt(2)*x + 1) + 3/32*sqrt(2)*log(x^2 - sqrt(2)*x + 1)
```

3.282.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.85

$$\int \frac{x^6}{1+2x^4+x^8}\,dx = -\frac{x^3}{4\,(x^4+1)} + \frac{3}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x+\sqrt{2}\right)\right) + \frac{3}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x-\sqrt{2}\right)\right) - \frac{3}{32}\,\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right) + \frac{3}{32}\,\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)$$

input
```
integrate(x^6/(x^8+2*x^4+1),x, algorithm="giac")
```

output
```
-1/4*x^3/(x^4 + 1) + 3/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 3/
16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 3/32*sqrt(2)*log(x^2 + sq
rt(2)*x + 1) + 3/32*sqrt(2)*log(x^2 - sqrt(2)*x + 1)
```

3.282.9 Mupad [B] (verification not implemented)

Time = 8.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.47

$$\int \frac{x^6}{1+2x^4+x^8}\,dx = -\frac{x^3}{4\,(x^4+1)} + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{3}{16}-\frac{3}{16}\mathrm{i}\right) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{3}{16}+\frac{3}{16}\mathrm{i}\right)$$

input
```
int(x^6/(2*x^4 + x^8 + 1),x)
```

output
```
2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(3/16 - 3i/16) + 2^(1/2)*atan(2^(1/2)
*x*(1/2 + 1i/2))*(3/16 + 3i/16) - x^3/(4*(x^4 + 1))
```

3.283 $\int \frac{x^4}{1+2x^4+x^8}\,dx$

3.283.1 Optimal result

Integrand size = 16, antiderivative size = 97

$$\int \frac{x^4}{1+2x^4+x^8}\,dx=-\frac{x}{4(1+x^4)}-\frac{\arctan\left(1-\sqrt{2}x\right)}{8\sqrt{2}}+\frac{\arctan\left(1+\sqrt{2}x\right)}{8\sqrt{2}}-\frac{\log\left(1-\sqrt{2}x+x^2\right)}{16\sqrt{2}}+\frac{\log\left(1+\sqrt{2}x+x^2\right)}{16\sqrt{2}}$$

output
```
-1/4*x/(x^4+1)+1/16*arctan(-1+x*2^(1/2))*2^(1/2)+1/16*arctan(1+x*2^(1/2))*
2^(1/2)-1/32*ln(1+x^2-x*2^(1/2))*2^(1/2)+1/32*ln(1+x^2+x*2^(1/2))*2^(1/2)
```

3.283.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.93

$$\int \frac{x^4}{1+2x^4+x^8}\,dx=\frac{1}{32}\left(-\frac{8x}{1+x^4}-2\sqrt{2}\arctan\left(1-\sqrt{2}x\right)+2\sqrt{2}\arctan\left(1+\sqrt{2}x\right)-\sqrt{2}\log\left(1-\sqrt{2}x+x^2\right)+\sqrt{2}\log\left(1+\sqrt{2}x+x^2\right)\right)$$

input
```
Integrate[x^4/(1 + 2*x^4 + x^8),x]
```

output
```
((-8*x)/(1 + x^4) - 2*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] + 2*Sqrt[2]*ArcTan[1 +
 Sqrt[2]*x] - Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] + Sqrt[2]*Log[1 + Sqrt[2]*x
 + x^2])/32
```

3.283.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {1380, 817, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{x^8+2x^4+1}\,dx$$

$$\Big\downarrow \text{1380}$$

$$\int \frac{x^4}{(x^4+1)^2}dx$$

$$\Big\downarrow \text{817}$$

$$\frac{1}{4}\int \frac{1}{x^4+1}dx-\frac{x}{4(x^4+1)}$$

$$\Big\downarrow \text{755}$$

$$\frac{1}{4}\left(\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx+\frac{1}{2}\int \frac{x^2+1}{x^4+1}dx\right)-\frac{x}{4(x^4+1)}$$

$$\Big\downarrow \text{1476}$$

$$\frac{1}{4}\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2-\sqrt{2}x+1}dx+\frac{1}{2}\int \frac{1}{x^2+\sqrt{2}x+1}dx\right)+\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx\right)-\frac{x}{4(x^4+1)}$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{4}\left(\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx+\frac{1}{2}\left(\frac{\int \frac{1}{-\left(1-\sqrt{2}x\right)^2-1}d(1-\sqrt{2}x)}{\sqrt{2}}-\frac{\int \frac{1}{-\left(\sqrt{2}x+1\right)^2-1}d(\sqrt{2}x+1)}{\sqrt{2}}\right)\right)-\frac{x}{4(x^4+1)}$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{4}\left(\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)\right)-\frac{x}{4(x^4+1)}$$

$$\Big\downarrow \text{1479}$$

$$\frac{1}{4}\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}x+1\right)}{x^2+\sqrt{2}x+1}dx}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)\right)-\frac{x}{4\left(x^4+1\right)}$$

$$\Big\downarrow \ 25$$

$$\frac{1}{4}\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}x+1\right)}{x^2+\sqrt{2}x+1}dx}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)\right)-\frac{x}{4\left(x^4+1\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{4}\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}x+1}{x^2+\sqrt{2}x+1}dx\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)\right)-\frac{x}{4\left(x^4+1\right)}$$

$$\Big\downarrow \ 1103$$

$$\frac{1}{4}\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(x^2+\sqrt{2}x+1\right)}{2\sqrt{2}}-\frac{\log\left(x^2-\sqrt{2}x+1\right)}{2\sqrt{2}}\right)\right)-\frac{x}{4\left(x^4+1\right)}$$

input
```
Int[x^4/(1 + 2*x^4 + x^8),x]
```

output
```
-1/4*x/(1 + x^4) + ((-(ArcTan[1 - Sqrt[2]*x]/Sqrt[2]) + ArcTan[1 + Sqrt[2]
*x]/Sqrt[2])/2 + (-1/2*Log[1 - Sqrt[2]*x + x^2]/Sqrt[2] + Log[1 + Sqrt[2]*
x + x^2]/(2*Sqrt[2]))/2)/4
```

3.283. $\int \frac{x^4}{1+2x^4+x^8}\, dx$

3.283.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

3.283. $\int \frac{x^4}{1+2x^4+x^8} \, dx$

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.283.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.34

method	result	size
risch	$-\frac{x}{4(x^4+1)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}(\_Z^4+1)}\frac{\ln(x-\_R)}{\_R^3}\right)}{16}$	33
default	$-\frac{x}{4(x^4+1)}+\frac{\sqrt{2}\left(\ln\left(\frac{1+x^2+x\sqrt{2}}{1+x^2-x\sqrt{2}}\right)+2\arctan\left(x\sqrt{2}+1\right)+2\arctan\left(x\sqrt{2}-1\right)\right)}{32}$	63

input
```
int(x^4/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/4*x/(x^4+1)+1/16*sum(1/_R^3*ln(x-_R),_R=RootOf(_Z^4+1))
```

3.283.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{1+2x^4+x^8}\,dx$$
$$=\frac{\sqrt{2}((i+1)\,x^4+i+1)\log\left(2\,x+(i+1)\,\sqrt{2}\right)+\sqrt{2}(-(i-1)\,x^4-i+1)\log\left(2\,x-(i-1)\,\sqrt{2}\right)+\sqrt{2}((i}{32\,(x^4+1)}$$

input
```
integrate(x^4/(x^8+2*x^4+1),x, algorithm="fricas")
```

output
```
1/32*(sqrt(2)*((I + 1)*x^4 + I + 1)*log(2*x + (I + 1)*sqrt(2)) + sqrt(2)*(
-(I - 1)*x^4 - I + 1)*log(2*x - (I - 1)*sqrt(2)) + sqrt(2)*((I - 1)*x^4 +
I - 1)*log(2*x + (I - 1)*sqrt(2)) + sqrt(2)*(-(I + 1)*x^4 - I - 1)*log(2*x
 - (I + 1)*sqrt(2)) - 8*x)/(x^4 + 1)
```

3.283.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.85

$$\int \frac{x^4}{1+2x^4+x^8}\,dx = -\frac{x}{4x^4+4} - \frac{\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{32} + \frac{\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{32} + \frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}x-1\right)}{16} + \frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}x+1\right)}{16}$$

input
```
integrate(x**4/(x**8+2*x**4+1),x)
```

output
```
-x/(4*x**4 + 4) - sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 + sqrt(2)*log(x**2
+ sqrt(2)*x + 1)/32 + sqrt(2)*atan(sqrt(2)*x - 1)/16 + sqrt(2)*atan(sqrt(2
)*x + 1)/16
```

3.283.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.85

$$\int \frac{x^4}{1+2x^4+x^8}\,dx = \frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right) + \frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right) + \frac{1}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right) - \frac{1}{32}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right) - \frac{x}{4\left(x^4+1\right)}$$

input
```
integrate(x^4/(x^8+2*x^4+1),x, algorithm="maxima")
```

output
```
1/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/16*sqrt(2)*arctan(1/2
*sqrt(2)*(2*x - sqrt(2))) + 1/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/32*s
qrt(2)*log(x^2 - sqrt(2)*x + 1) - 1/4*x/(x^4 + 1)
```

3.283.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.85

$$\int \frac{x^4}{1+2x^4+x^8}\,dx = \frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x+\sqrt{2}\right)\right)+\frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x-\sqrt{2}\right)\right) + \frac{1}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{1}{32}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)-\frac{x}{4\,(x^4+1)}$$

input
```
integrate(x^4/(x^8+2*x^4+1),x, algorithm="giac")
```

output
```
1/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/16*sqrt(2)*arctan(1/2
*sqrt(2)*(2*x - sqrt(2))) + 1/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/32*s
qrt(2)*log(x^2 - sqrt(2)*x + 1) - 1/4*x/(x^4 + 1)
```

3.283.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.46

$$\int \frac{x^4}{1+2x^4+x^8}\,dx = -\frac{x}{4\,(x^4+1)}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{16}+\frac{1}{16}\mathrm{i}\right) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{16}-\frac{1}{16}\mathrm{i}\right)$$

input
```
int(x^4/(2*x^4 + x^8 + 1),x)
```

output
```
2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(1/16 + 1i/16) + 2^(1/2)*atan(2^(1/2)
*x*(1/2 + 1i/2))*(1/16 - 1i/16) - x/(4*(x^4 + 1))
```

3.284 $\int \frac{x^2}{1+2x^4+x^8} dx$

3.284.1 Optimal result

Integrand size = 16, antiderivative size = 99

$$\int \frac{x^2}{1+2x^4+x^8} dx = \frac{x^3}{4(1+x^4)} - \frac{\arctan\left(1-\sqrt{2}x\right)}{8\sqrt{2}} + \frac{\arctan\left(1+\sqrt{2}x\right)}{8\sqrt{2}} + \frac{\log\left(1-\sqrt{2}x+x^2\right)}{16\sqrt{2}} - \frac{\log\left(1+\sqrt{2}x+x^2\right)}{16\sqrt{2}}$$

output
```
1/4*x^3/(x^4+1)+1/16*arctan(-1+x*2^(1/2))*2^(1/2)+1/16*arctan(1+x*2^(1/2))
*2^(1/2)+1/32*ln(1+x^2-x*2^(1/2))*2^(1/2)-1/32*ln(1+x^2+x*2^(1/2))*2^(1/2)
```

3.284.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.93

$$\int \frac{x^2}{1+2x^4+x^8} dx = \frac{1}{32}\left(\frac{8x^3}{1+x^4} - 2\sqrt{2}\arctan\left(1-\sqrt{2}x\right) + 2\sqrt{2}\arctan\left(1+\sqrt{2}x\right) + \sqrt{2}\log\left(1-\sqrt{2}x+x^2\right) - \sqrt{2}\log\left(1+\sqrt{2}x+x^2\right)\right)$$

input
```
Integrate[x^2/(1 + 2*x^4 + x^8),x]
```

output
```
((8*x^3)/(1 + x^4) - 2*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] + 2*Sqrt[2]*ArcTan[1
+ Sqrt[2]*x] + Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] - Sqrt[2]*Log[1 + Sqrt[2]*
x + x^2])/32
```

3.284.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {1380, 819, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{x^8+2x^4+1} \, dx$$

$$\Big\downarrow \text{1380}$$

$$\int \frac{x^2}{(x^4+1)^2}dx$$

$$\Big\downarrow \text{819}$$

$$\frac{1}{4}\int \frac{x^2}{x^4+1}dx+\frac{x^3}{4(x^4+1)}$$

$$\Big\downarrow \text{826}$$

$$\frac{1}{4}\left(\frac{1}{2}\int \frac{x^2+1}{x^4+1}dx-\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx\right)+\frac{x^3}{4(x^4+1)}$$

$$\Big\downarrow \text{1476}$$

$$\frac{1}{4}\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2-\sqrt{2}x+1}dx+\frac{1}{2}\int \frac{1}{x^2+\sqrt{2}x+1}dx\right)-\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx\right)+\frac{x^3}{4(x^4+1)}$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{4}\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\left(1-\sqrt{2}x\right)^2-1}d\left(1-\sqrt{2}x\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\left(\sqrt{2}x+1\right)^2-1}d\left(\sqrt{2}x+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx\right)+\frac{x^3}{4(x^4+1)}$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{4}\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx\right)+\frac{x^3}{4(x^4+1)}$$

$$\Big\downarrow \text{1479}$$

$$\frac{1}{4}\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}x+1\right)}{x^2+\sqrt{2}x+1}dx}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)\right)+\frac{x^3}{4\left(x^4+1\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{1}{4}\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{2}x+1\right)}{x^2+\sqrt{2}x+1}dx}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)\right)+\frac{x^3}{4\left(x^4+1\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{4}\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}-\frac{1}{2}\int \frac{\sqrt{2}x+1}{x^2+\sqrt{2}x+1}dx\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)\right)+\frac{x^3}{4\left(x^4+1\right)}$$

$$\Big\downarrow \; 1103$$

$$\frac{1}{4}\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(x^2-\sqrt{2}x+1\right)}{2\sqrt{2}}-\frac{\log\left(x^2+\sqrt{2}x+1\right)}{2\sqrt{2}}\right)\right)+\frac{x^3}{4\left(x^4+1\right)}$$

input
```
Int[x^2/(1 + 2*x^4 + x^8),x]
```

output
```
x^3/(4*(1 + x^4)) + ((-(ArcTan[1 - Sqrt[2]*x]/Sqrt[2]) + ArcTan[1 + Sqrt[2
]*x]/Sqrt[2])/2 + (Log[1 - Sqrt[2]*x + x^2]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*
x + x^2]/(2*Sqrt[2]))/2)/4
```

3.284. $\int \frac{x^2}{1+2x^4+x^8}\,dx$

3.284.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

3.284. $\int \frac{x^2}{1+2x^4+x^8} \, dx$

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.284.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.35

method	result	size
risch	$\frac{x^3}{4x^4+4}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^4+1\right)}\frac{\ln\left(x-\_R\right)}{\_R}\right)}{16}$	35
default	$\frac{x^3}{4x^4+4}+\frac{\sqrt{2}\left(\ln\left(\frac{1+x^2-x\sqrt{2}}{1+x^2+x\sqrt{2}}\right)+2\arctan\left(x\sqrt{2}+1\right)+2\arctan\left(x\sqrt{2}-1\right)\right)}{32}$	65

input
```
int(x^2/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^3/(x^4+1)+1/16*sum(1/_R*ln(x-_R),_R=RootOf(_Z^4+1))
```

3.284.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{1+2x^4+x^8}\,dx$$
$$=\frac{8\,x^3+\sqrt{2}((i-1)\;x^4+i-1)\log\left(2\,x+(i+1)\;\sqrt{2}\right)+\sqrt{2}(-(i+1)\;x^4-i-1)\log\left(2\,x-(i-1)\;\sqrt{2}\right)+}{32\,(x^4+1}$$

input
```
integrate(x^2/(x^8+2*x^4+1),x, algorithm="fricas")
```

output
```
1/32*(8*x^3 + sqrt(2)*((I - 1)*x^4 + I - 1)*log(2*x + (I + 1)*sqrt(2)) + s
qrt(2)*(-(I + 1)*x^4 - I - 1)*log(2*x - (I - 1)*sqrt(2)) + sqrt(2)*((I + 1
)*x^4 + I + 1)*log(2*x + (I - 1)*sqrt(2)) + sqrt(2)*(-(I - 1)*x^4 - I + 1)
*log(2*x - (I + 1)*sqrt(2)))/(x^4 + 1)
```

3.284.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.84

$$\int \frac{x^2}{1+2x^4+x^8}\,dx = \frac{x^3}{4x^4+4} + \frac{\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{32} - \frac{\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{32} + \frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x-1\right)}{16} + \frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x+1\right)}{16}$$

input
```
integrate(x**2/(x**8+2*x**4+1),x)
```

output
```
x**3/(4*x**4 + 4) + sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 - sqrt(2)*log(x**
2 + sqrt(2)*x + 1)/32 + sqrt(2)*atan(sqrt(2)*x - 1)/16 + sqrt(2)*atan(sqrt
(2)*x + 1)/16
```

3.284.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.85

$$\int \frac{x^2}{1+2x^4+x^8}\,dx = \frac{x^3}{4\,(x^4+1)} + \frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x+\sqrt{2}\right)\right) + \frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x-\sqrt{2}\right)\right) - \frac{1}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right) + \frac{1}{32}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)$$

input
```
integrate(x^2/(x^8+2*x^4+1),x, algorithm="maxima")
```

output
```
1/4*x^3/(x^4 + 1) + 1/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/1
6*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 1/32*sqrt(2)*log(x^2 + sqr
t(2)*x + 1) + 1/32*sqrt(2)*log(x^2 - sqrt(2)*x + 1)
```

3.284.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.85

$$\int \frac{x^2}{1+2x^4+x^8}\,dx = \frac{x^3}{4\,(x^4+1)} + \frac{1}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x+\sqrt{2}\right)\right) + \frac{1}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x-\sqrt{2}\right)\right) - \frac{1}{32}\,\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right) + \frac{1}{32}\,\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)$$

input
```
integrate(x^2/(x^8+2*x^4+1),x, algorithm="giac")
```

output
```
1/4*x^3/(x^4 + 1) + 1/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/1
6*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 1/32*sqrt(2)*log(x^2 + sqr
t(2)*x + 1) + 1/32*sqrt(2)*log(x^2 - sqrt(2)*x + 1)
```

3.284.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.46

$$\int \frac{x^2}{1+2x^4+x^8}\,dx = \frac{x^3}{4\,\left(x^4+1\right)} + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{1}{16}-\frac{1}{16}\mathrm{i}\right) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{1}{16}+\frac{1}{16}\mathrm{i}\right)$$

input
```
int(x^2/(2*x^4 + x^8 + 1),x)
```

output
```
2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(1/16 - 1i/16) + 2^(1/2)*atan(2^(1/2)
*x*(1/2 + 1i/2))*(1/16 + 1i/16) + x^3/(4*(x^4 + 1))
```

3.285 $\int \frac{1}{1+2x^4+x^8} dx$

3.285.1 Optimal result

Integrand size = 12, antiderivative size = 97

$$\int \frac{1}{1+2x^4+x^8} dx = \frac{x}{4(1+x^4)} - \frac{3\arctan\left(1-\sqrt{2}x\right)}{8\sqrt{2}} + \frac{3\arctan\left(1+\sqrt{2}x\right)}{8\sqrt{2}} - \frac{3\log\left(1-\sqrt{2}x+x^2\right)}{16\sqrt{2}} + \frac{3\log\left(1+\sqrt{2}x+x^2\right)}{16\sqrt{2}}$$

output
```
1/4*x/(x^4+1)+3/16*arctan(-1+x*2^(1/2))*2^(1/2)+3/16*arctan(1+x*2^(1/2))*2
^(1/2)-3/32*ln(1+x^2-x*2^(1/2))*2^(1/2)+3/32*ln(1+x^2+x*2^(1/2))*2^(1/2)
```

3.285.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.94

$$\int \frac{1}{1+2x^4+x^8} dx = \frac{1}{32}\left(\frac{8x}{1+x^4} - 6\sqrt{2}\arctan\left(1-\sqrt{2}x\right) + 6\sqrt{2}\arctan\left(1+\sqrt{2}x\right) - 3\sqrt{2}\log\left(1-\sqrt{2}x+x^2\right) + 3\sqrt{2}\log\left(1+\sqrt{2}x+x^2\right)\right)$$

input
```
Integrate[(1 + 2*x^4 + x^8)^(-1),x]
```

output
```
((8*x)/(1 + x^4) - 6*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] + 6*Sqrt[2]*ArcTan[1 +
Sqrt[2]*x] - 3*Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] + 3*Sqrt[2]*Log[1 + Sqrt[2
]*x + x^2])/32
```

3.285.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {1379, 749, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^8+2x^4+1}dx$$

$$\Big\downarrow \text{1379}$$

$$\int \frac{1}{(x^4+1)^2}dx$$

$$\Big\downarrow \text{749}$$

$$\frac{3}{4}\int \frac{1}{x^4+1}dx+\frac{x}{4(x^4+1)}$$

$$\Big\downarrow \text{755}$$

$$\frac{3}{4}\left(\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx+\frac{1}{2}\int \frac{x^2+1}{x^4+1}dx\right)+\frac{x}{4(x^4+1)}$$

$$\Big\downarrow \text{1476}$$

$$\frac{3}{4}\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2-\sqrt{2}x+1}dx+\frac{1}{2}\int \frac{1}{x^2+\sqrt{2}x+1}dx\right)+\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx\right)+\frac{x}{4(x^4+1)}$$

$$\Big\downarrow \text{1082}$$

$$\frac{3}{4}\left(\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx+\frac{1}{2}\left(\frac{\int \frac{1}{-\left(1-\sqrt{2}x\right)^2-1}d\left(1-\sqrt{2}x\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\left(\sqrt{2}x+1\right)^2-1}d\left(\sqrt{2}x+1\right)}{\sqrt{2}}\right)\right)+\frac{x}{4(x^4+1)}$$

$$\Big\downarrow \text{217}$$

$$\frac{3}{4}\left(\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)\right)+\frac{x}{4(x^4+1)}$$

$$\Big\downarrow \text{1479}$$

$$\frac{3}{4}\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}x+1\right)}{x^2+\sqrt{2}x+1}dx}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)\right)+\frac{x}{4\left(x^4+1\right)}$$

↓ 25

$$\frac{3}{4}\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}x+1\right)}{x^2+\sqrt{2}x+1}dx}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)\right)+\frac{x}{4\left(x^4+1\right)}$$

↓ 27

$$\frac{3}{4}\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}x+1}{x^2+\sqrt{2}x+1}dx\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)\right)+\frac{x}{4\left(x^4+1\right)}$$

↓ 1103

$$\frac{3}{4}\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(x^2+\sqrt{2}x+1\right)}{2\sqrt{2}}-\frac{\log\left(x^2-\sqrt{2}x+1\right)}{2\sqrt{2}}\right)\right)+\frac{x}{4\left(x^4+1\right)}$$

input
```
Int[(1 + 2*x^4 + x^8)^(-1),x]
```

output
```
x/(4*(1 + x^4)) + (3*((-(ArcTan[1 - Sqrt[2]*x]/Sqrt[2]) + ArcTan[1 + Sqrt[
2]*x]/Sqrt[2])/2 + (-1/2*Log[1 - Sqrt[2]*x + x^2]/Sqrt[2] + Log[1 + Sqrt[2
]*x + x^2]/(2*Sqrt[2]))/2))/4
```

3.285. $\int \frac{1}{1+2x^4+x^8}\,dx$

3.285.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1379
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/
c^p   Int[(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n
2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p] && NeQ[p, 1]
```

3.285. $\int \frac{1}{1+2x^4+x^8}\,dx$

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.285.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.34

method	result	size
risch	$\frac{x}{4x^4+4}+\frac{3\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+1\right)}\frac{\ln\left(x-\_R\right)}{\_R^3}\right)}{16}$	33
default	$\frac{x}{4x^4+4}+\frac{3\sqrt{2}\left(\ln\left(\frac{1+x^2+x\sqrt{2}}{1+x^2-x\sqrt{2}}\right)+2\arctan\left(x\sqrt{2}+1\right)+2\arctan\left(x\sqrt{2}-1\right)\right)}{32}$	63

input
```
int(1/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*x/(x^4+1)+3/16*sum(1/_R^3*ln(x-_R),_R=RootOf(_Z^4+1))
```

3.285.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.04

$$\int \frac{1}{1+2x^4+x^8}\,dx =$$

$$-\frac{3\sqrt{2}(-(i+1)\,x^4-i-1)\log\left(2\,x+(i+1)\,\sqrt{2}\right)+3\sqrt{2}((i-1)\,x^4+i-1)\log\left(2\,x-(i-1)\,\sqrt{2}\right)+3}{32\,(x}$$

input
```
integrate(1/(x^8+2*x^4+1),x, algorithm="fricas")
```

output
```
-1/32*(3*sqrt(2)*(-(I + 1)*x^4 - I - 1)*log(2*x + (I + 1)*sqrt(2)) + 3*sqr
t(2)*((I - 1)*x^4 + I - 1)*log(2*x - (I - 1)*sqrt(2)) + 3*sqrt(2)*(-(I - 1
)*x^4 - I + 1)*log(2*x + (I - 1)*sqrt(2)) + 3*sqrt(2)*((I + 1)*x^4 + I + 1
)*log(2*x - (I + 1)*sqrt(2)) - 8*x)/(x^4 + 1)
```

3.285.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.91

$$\int \frac{1}{1+2x^4+x^8}\,dx = \frac{x}{4x^4+4} - \frac{3\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{32} + \frac{3\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{32} + \frac{3\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x-1\right)}{16} + \frac{3\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x+1\right)}{16}$$

input
```
integrate(1/(x**8+2*x**4+1),x)
```

output
```
x/(4*x**4 + 4) - 3*sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 + 3*sqrt(2)*log(x*
*2 + sqrt(2)*x + 1)/32 + 3*sqrt(2)*atan(sqrt(2)*x - 1)/16 + 3*sqrt(2)*atan
(sqrt(2)*x + 1)/16
```

3.285.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.85

$$\int \frac{1}{1+2x^4+x^8}\,dx = \frac{3}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right) + \frac{3}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right) + \frac{3}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right) - \frac{3}{32}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right) + \frac{x}{4\left(x^4+1\right)}$$

input
```
integrate(1/(x^8+2*x^4+1),x, algorithm="maxima")
```

output
```
3/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 3/16*sqrt(2)*arctan(1/2
*sqrt(2)*(2*x - sqrt(2))) + 3/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 3/32*s
qrt(2)*log(x^2 - sqrt(2)*x + 1) + 1/4*x/(x^4 + 1)
```

3.285.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.85

$$\int \frac{1}{1+2x^4+x^8}\,dx = \frac{3}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x+\sqrt{2}\right)\right)+\frac{3}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x-\sqrt{2}\right)\right) + \frac{3}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{3}{32}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)+\frac{x}{4\,(x^4+1)}$$

input
```
integrate(1/(x^8+2*x^4+1),x, algorithm="giac")
```

output
```
3/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 3/16*sqrt(2)*arctan(1/2
*sqrt(2)*(2*x - sqrt(2))) + 3/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 3/32*s
qrt(2)*log(x^2 - sqrt(2)*x + 1) + 1/4*x/(x^4 + 1)
```

3.285.9 Mupad [B] (verification not implemented)

Time = 8.15 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.45

$$\int \frac{1}{1+2x^4+x^8}\,dx = \frac{x}{4\,(x^4+1)}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{3}{16}+\frac{3}{16}\mathrm{i}\right) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{3}{16}-\frac{3}{16}\mathrm{i}\right)$$

input
```
int(1/(2*x^4 + x^8 + 1),x)
```

output
```
2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(3/16 + 3i/16) + 2^(1/2)*atan(2^(1/2)
*x*(1/2 + 1i/2))*(3/16 - 3i/16) + x/(4*(x^4 + 1))
```

3.286 $\int \frac{1}{x^2(1+2x^4+x^8)}\,dx$

3.286.1 Optimal result

Integrand size = 16, antiderivative size = 106

$$\int \frac{1}{x^2(1+2x^4+x^8)}\,dx = -\frac{5}{4x}+\frac{1}{4x(1+x^4)}+\frac{5\arctan\left(1-\sqrt{2}x\right)}{8\sqrt{2}}-\frac{5\arctan\left(1+\sqrt{2}x\right)}{8\sqrt{2}}-\frac{5\log\left(1-\sqrt{2}x+x^2\right)}{16\sqrt{2}}+\frac{5\log\left(1+\sqrt{2}x+x^2\right)}{16\sqrt{2}}$$

output
```
-5/4/x+1/4/x/(x^4+1)-5/16*arctan(-1+x*2^(1/2))*2^(1/2)-5/16*arctan(1+x*2^(
1/2))*2^(1/2)-5/32*ln(1+x^2-x*2^(1/2))*2^(1/2)+5/32*ln(1+x^2+x*2^(1/2))*2^
(1/2)
```

3.286.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^2(1+2x^4+x^8)}\,dx = \frac{1}{32}\left(-\frac{32}{x}-\frac{8x^3}{1+x^4}+10\sqrt{2}\arctan\left(1-\sqrt{2}x\right)-10\sqrt{2}\arctan\left(1+\sqrt{2}x\right)-5\sqrt{2}\log\left(1-\sqrt{2}x+x^2\right)+5\sqrt{2}\log\left(1+\sqrt{2}x+x^2\right)\right)$$

input
```
Integrate[1/(x^2*(1 + 2*x^4 + x^8)),x]
```

output
```
(-32/x - (8*x^3)/(1 + x^4) + 10*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] - 10*Sqrt[2]
*ArcTan[1 + Sqrt[2]*x] - 5*Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] + 5*Sqrt[2]*Lo
g[1 + Sqrt[2]*x + x^2])/32
```

3.286.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.08, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.688, Rules used = {1380, 819, 847, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (x^8+2x^4+1)}dx$$

$$\Big\downarrow \text{1380}$$

$$\int \frac{1}{x^2 (x^4+1)^2}dx$$

$$\Big\downarrow \text{819}$$

$$\frac{5}{4}\int \frac{1}{x^2 (x^4+1)}dx+\frac{1}{4x (x^4+1)}$$

$$\Big\downarrow \text{847}$$

$$\frac{5}{4}\left(-\int \frac{x^2}{x^4+1}dx-\frac{1}{x}\right)+\frac{1}{4x (x^4+1)}$$

$$\Big\downarrow \text{826}$$

$$\frac{5}{4}\left(\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx-\frac{1}{2}\int \frac{x^2+1}{x^4+1}dx-\frac{1}{x}\right)+\frac{1}{4x (x^4+1)}$$

$$\Big\downarrow \text{1476}$$

$$\frac{5}{4}\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{x^2-\sqrt{2}x+1}dx-\frac{1}{2}\int \frac{1}{x^2+\sqrt{2}x+1}dx\right)+\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx-\frac{1}{x}\right)+\frac{1}{4x (x^4+1)}$$

$$\Big\downarrow \text{1082}$$

$$\frac{5}{4}\left(\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx+\frac{1}{2}\left(\frac{\int \frac{1}{-\left(\sqrt{2}x+1\right)^2-1}d\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\left(1-\sqrt{2}x\right)^2-1}d\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)-\frac{1}{x}\right)+$$
$$\frac{1}{4x (x^4+1)}$$

$$\Big\downarrow \text{217}$$

$$\frac{5}{4}\left(\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx+\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}\right)-\frac{1}{x}\right)+\frac{1}{4x\left(x^4+1\right)}$$

$$\Big\downarrow \text{1479}$$

$$\frac{5}{4}\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}x+1\right)}{x^2+\sqrt{2}x+1}dx}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}\right)-\frac{1}{x}\right)+\frac{1}{4x\left(x^4+1\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{5}{4}\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}x+1\right)}{x^2+\sqrt{2}x+1}dx}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}\right)-\frac{1}{x}\right)+\frac{1}{4x\left(x^4+1\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{5}{4}\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}x+1}{x^2+\sqrt{2}x+1}dx\right)+\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}\right)-\frac{1}{x}\right)+\frac{1}{4x\left(x^4+1\right)}$$

$$\Big\downarrow \text{1103}$$

$$\frac{5}{4}\left(\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(x^2+\sqrt{2}x+1\right)}{2\sqrt{2}}-\frac{\log\left(x^2-\sqrt{2}x+1\right)}{2\sqrt{2}}\right)-\frac{1}{x}\right)+\frac{1}{4x\left(x^4+1\right)}$$

input
```
Int[1/(x^2*(1 + 2*x^4 + x^8)),x]
```

output
```
1/(4*x*(1 + x^4)) + (5*(-x^(-1) + (ArcTan[1 - Sqrt[2]*x]/Sqrt[2] - ArcTan[
1 + Sqrt[2]*x]/Sqrt[2])/2 + (-1/2*Log[1 - Sqrt[2]*x + x^2]/Sqrt[2] + Log[1
 + Sqrt[2]*x + x^2]/(2*Sqrt[2]))/2))/4
```

3.286. $\int \frac{1}{x^2(1+2x^4+x^8)}\,dx$

3.286.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.286.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.39

method	result	size
risch	$\frac{-\frac{5x^4}{4}-1}{(x^4+1)x}+\frac{5\left(\sum_{\_R=\mathrm{RootOf}(\_Z^4+1)}\_R\ln\left(-\_R^3+x\right)\right)}{16}$	41
default	$-\frac{1}{x}-\frac{x^3}{4(x^4+1)}-\frac{5\sqrt{2}\left(\ln\left(\frac{1+x^2-x\sqrt{2}}{1+x^2+x\sqrt{2}}\right)+2\arctan\left(x\sqrt{2}+1\right)+2\arctan\left(x\sqrt{2}-1\right)\right)}{32}$	70

input
```
int(1/x^2/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
(-5/4*x^4-1)/(x^4+1)/x+5/16*sum(_R*ln(-_R^3+x),_R=RootOf(_Z^4+1))
```

3.286.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^2(1+2x^4+x^8)}\,dx =$$
$$-\frac{40\,x^4+5\,\sqrt{2}((i-1)\,x^5+(i-1)\,x)\log\left(2\,x+(i+1)\,\sqrt{2}\right)+5\,\sqrt{2}(-(i+1)\,x^5-(i+1)\,x)\log\left(2\,x-\right.}{}$$

input
```
integrate(1/x^2/(x^8+2*x^4+1),x, algorithm="fricas")
```

output
```
-1/32*(40*x^4 + 5*sqrt(2)*((I - 1)*x^5 + (I - 1)*x)*log(2*x + (I + 1)*sqrt
(2)) + 5*sqrt(2)*(-(I + 1)*x^5 - (I + 1)*x)*log(2*x - (I - 1)*sqrt(2)) + 5
*sqrt(2)*((I + 1)*x^5 + (I + 1)*x)*log(2*x + (I - 1)*sqrt(2)) + 5*sqrt(2)*
(-(I - 1)*x^5 - (I - 1)*x)*log(2*x - (I + 1)*sqrt(2)) + 32)/(x^5 + x)
```

3.286.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^2(1+2x^4+x^8)}\,dx = \frac{-5x^4-4}{4x^5+4x} - \frac{5\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{32} + \frac{5\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{32} - \frac{5\sqrt{2}\operatorname{atan}\left(\sqrt{2}x-1\right)}{16} - \frac{5\sqrt{2}\operatorname{atan}\left(\sqrt{2}x+1\right)}{16}$$

input
```
integrate(1/x**2/(x**8+2*x**4+1),x)
```

output
```
(-5*x**4 - 4)/(4*x**5 + 4*x) - 5*sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 + 5*
sqrt(2)*log(x**2 + sqrt(2)*x + 1)/32 - 5*sqrt(2)*atan(sqrt(2)*x - 1)/16 -
5*sqrt(2)*atan(sqrt(2)*x + 1)/16
```

3.286.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^2(1+2x^4+x^8)}\,dx = -\frac{5}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right) - \frac{5}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right) + \frac{5}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right) - \frac{5}{32}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right) - \frac{5x^4+4}{4(x^5+x)}$$

input
```
integrate(1/x^2/(x^8+2*x^4+1),x, algorithm="maxima")
```

output
```
-5/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 5/16*sqrt(2)*arctan(1/
2*sqrt(2)*(2*x - sqrt(2))) + 5/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 5/32*
sqrt(2)*log(x^2 - sqrt(2)*x + 1) - 1/4*(5*x^4 + 4)/(x^5 + x)
```

3.286.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^2(1+2x^4+x^8)}\,dx = -\frac{5}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right) - \frac{5}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right) + \frac{5}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right) - \frac{5}{32}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right) - \frac{5x^4+4}{4(x^5+x)}$$

input
```
integrate(1/x^2/(x^8+2*x^4+1),x, algorithm="giac")
```

output
```
-5/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 5/16*sqrt(2)*arctan(1/
2*sqrt(2)*(2*x - sqrt(2))) + 5/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 5/32*
sqrt(2)*log(x^2 - sqrt(2)*x + 1) - 1/4*(5*x^4 + 4)/(x^5 + x)
```

3.286.9 Mupad [B] (verification not implemented)

Time = 8.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.46

$$\int \frac{1}{x^2 (1 + 2x^4 + x^8)} \, dx = -\frac{\frac{5 x^4}{4} + 1}{x^5 + x} + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x \left(\frac{1}{2} - \frac{1}{2}\mathrm{i}\right)\right) \left(-\frac{5}{16} + \frac{5}{16}\mathrm{i}\right) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x \left(\frac{1}{2} + \frac{1}{2}\mathrm{i}\right)\right) \left(-\frac{5}{16} - \frac{5}{16}\mathrm{i}\right)$$

input
```
int(1/(x^2*(2*x^4 + x^8 + 1)),x)
```

output
```
- ((5*x^4)/4 + 1)/(x + x^5) - 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(5/16 -
 5i/16) - 2^(1/2)*atan(2^(1/2)*x*(1/2 + 1i/2))*(5/16 + 5i/16)
```

3.287 $\int \frac{1}{x^4(1+2x^4+x^8)}\,dx$

3.287.1 Optimal result

Integrand size = 16, antiderivative size = 106

$$\int \frac{1}{x^4(1+2x^4+x^8)}\,dx=-\frac{7}{12x^3}+\frac{1}{4x^3(1+x^4)}+\frac{7\arctan\left(1-\sqrt{2}x\right)}{8\sqrt{2}}-\frac{7\arctan\left(1+\sqrt{2}x\right)}{8\sqrt{2}}+\frac{7\log\left(1-\sqrt{2}x+x^2\right)}{16\sqrt{2}}-\frac{7\log\left(1+\sqrt{2}x+x^2\right)}{16\sqrt{2}}$$

output
```
-7/12/x^3+1/4/x^3/(x^4+1)-7/16*arctan(-1+x*2^(1/2))*2^(1/2)-7/16*arctan(1+
x*2^(1/2))*2^(1/2)+7/32*ln(1+x^2-x*2^(1/2))*2^(1/2)-7/32*ln(1+x^2+x*2^(1/2
))*2^(1/2)
```

3.287.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^4(1+2x^4+x^8)}\,dx=\frac{1}{96}\left(-\frac{32}{x^3}-\frac{24x}{1+x^4}+42\sqrt{2}\arctan\left(1-\sqrt{2}x\right)-42\sqrt{2}\arctan\left(1+\sqrt{2}x\right)+21\sqrt{2}\log\left(1-\sqrt{2}x+x^2\right)-21\sqrt{2}\log\left(1+\sqrt{2}x+x^2\right)\right)$$

input
```
Integrate[1/(x^4*(1 + 2*x^4 + x^8)),x]
```

output
```
(-32/x^3 - (24*x)/(1 + x^4) + 42*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] - 42*Sqrt[2
]*ArcTan[1 + Sqrt[2]*x] + 21*Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] - 21*Sqrt[2]
*Log[1 + Sqrt[2]*x + x^2])/96
```

3.287.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.09, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.688, Rules used = {1380, 819, 847, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\left(x^8+2x^4+1\right)}dx$$

$$\Big\downarrow \text{1380}$$

$$\int \frac{1}{x^4\left(x^4+1\right)^2}dx$$

$$\Big\downarrow \text{819}$$

$$\frac{7}{4}\int \frac{1}{x^4\left(x^4+1\right)}dx+\frac{1}{4x^3\left(x^4+1\right)}$$

$$\Big\downarrow \text{847}$$

$$\frac{7}{4}\left(-\int \frac{1}{x^4+1}dx-\frac{1}{3x^3}\right)+\frac{1}{4x^3\left(x^4+1\right)}$$

$$\Big\downarrow \text{755}$$

$$\frac{7}{4}\left(-\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx-\frac{1}{2}\int \frac{x^2+1}{x^4+1}dx-\frac{1}{3x^3}\right)+\frac{1}{4x^3\left(x^4+1\right)}$$

$$\Big\downarrow \text{1476}$$

$$\frac{7}{4}\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{x^2-\sqrt{2}x+1}dx-\frac{1}{2}\int \frac{1}{x^2+\sqrt{2}x+1}dx\right)-\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx-\frac{1}{3x^3}\right)+\frac{1}{4x^3\left(x^4+1\right)}$$

$$\Big\downarrow \text{1082}$$

$$\frac{7}{4}\left(-\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx+\frac{1}{2}\left(\frac{\int \frac{1}{-\left(\sqrt{2}x+1\right)^2-1}d\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\left(1-\sqrt{2}x\right)^2-1}d\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)-\frac{1}{3x^3}\right)+\frac{1}{4x^3\left(x^4+1\right)}$$

$$\Big\downarrow \ 217$$

$$\frac{7}{4}\left(-\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx+\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}\right)-\frac{1}{3x^3}\right)+\frac{1}{4x^3\left(x^4+1\right)}$$

$$\Big\downarrow \ 1479$$

$$\frac{7}{4}\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}x+1\right)}{x^2+\sqrt{2}x+1}dx}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}\right)-\frac{1}{3x^3}\right)+\frac{1}{4x^3\left(x^4+1\right)}$$

$$\Big\downarrow \ 25$$

$$\frac{7}{4}\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{2}x+1\right)}{x^2+\sqrt{2}x+1}dx}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}\right)-\frac{1}{3x^3}\right)+\frac{1}{4x^3\left(x^4+1\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{7}{4}\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}-\frac{1}{2}\int \frac{\sqrt{2}x+1}{x^2+\sqrt{2}x+1}dx\right)+\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}\right)-\frac{1}{3x^3}\right)+\frac{1}{4x^3\left(x^4+1\right)}$$

$$\Big\downarrow \ 1103$$

$$\frac{7}{4}\left(\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}\right)-\frac{1}{3x^3}+\frac{1}{2}\left(\frac{\log\left(x^2-\sqrt{2}x+1\right)}{2\sqrt{2}}-\frac{\log\left(x^2+\sqrt{2}x+1\right)}{2\sqrt{2}}\right)\right)+\frac{1}{4x^3\left(x^4+1\right)}$$

input
```
Int[1/(x^4*(1 + 2*x^4 + x^8)),x]
```

output
```
1/(4*x^3*(1 + x^4)) + (7*(-1/3*1/x^3 + (ArcTan[1 - Sqrt[2]*x]/Sqrt[2] - Ar
cTan[1 + Sqrt[2]*x]/Sqrt[2])/2 + (Log[1 - Sqrt[2]*x + x^2]/(2*Sqrt[2]) - L
og[1 + Sqrt[2]*x + x^2]/(2*Sqrt[2]))/2))/4
```

3.287. $\int \frac{1}{x^4(1+2x^4+x^8)}\,dx$

3.287.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.287.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.37

method	result	size
risch	$\frac{-\frac{7x^4}{12}-\frac{1}{3}}{x^3(x^4+1)}+\frac{7\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^4+1\right)}\_R\ln\left(x-\_R\right)\right)}{16}$	39
default	$-\frac{1}{3x^3}-\frac{x}{4(x^4+1)}-\frac{7\sqrt{2}\left(\ln\left(\frac{1+x^2+x\sqrt{2}}{1+x^2-x\sqrt{2}}\right)+2\arctan\left(x\sqrt{2}+1\right)+2\arctan\left(x\sqrt{2}-1\right)\right)}{32}$	68

input
```
int(1/x^4/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
(-7/12*x^4-1/3)/x^3/(x^4+1)+7/16*sum(_R*ln(x-_R),_R=RootOf(_Z^4+1))
```

3.287.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.15

$$\int \frac{1}{x^4 (1 + 2x^4 + x^8)} dx = -\frac{56 x^4 + 21 \sqrt{2}((i+1) x^7 + (i+1) x^3) \log (2x + (i+1) \sqrt{2}) + 21 \sqrt{2}(-(i-1) x^7 - (i-1) x^3) \log (2$$

input
```
integrate(1/x^4/(x^8+2*x^4+1),x, algorithm="fricas")
```

output
```
-1/96*(56*x^4 + 21*sqrt(2)*((I + 1)*x^7 + (I + 1)*x^3)*log(2*x + (I + 1)*s
qrt(2)) + 21*sqrt(2)*(-(I - 1)*x^7 - (I - 1)*x^3)*log(2*x - (I - 1)*sqrt(2
)) + 21*sqrt(2)*((I - 1)*x^7 + (I - 1)*x^3)*log(2*x + (I - 1)*sqrt(2)) + 2
1*sqrt(2)*(-(I + 1)*x^7 - (I + 1)*x^3)*log(2*x - (I + 1)*sqrt(2)) + 32)/(x
^7 + x^3)
```

3.287.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^4 (1 + 2x^4 + x^8)} dx = \frac{-7x^4 - 4}{12x^7 + 12x^3} + \frac{7\sqrt{2} \log (x^2 - \sqrt{2}x + 1)}{32} - \frac{7\sqrt{2} \log (x^2 + \sqrt{2}x + 1)}{32} - \frac{7\sqrt{2} \operatorname{atan} (\sqrt{2}x - 1)}{16} - \frac{7\sqrt{2} \operatorname{atan} (\sqrt{2}x + 1)}{16}$$

input
```
integrate(1/x**4/(x**8+2*x**4+1),x)
```

output
```
(-7*x**4 - 4)/(12*x**7 + 12*x**3) + 7*sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32
 - 7*sqrt(2)*log(x**2 + sqrt(2)*x + 1)/32 - 7*sqrt(2)*atan(sqrt(2)*x - 1)/
16 - 7*sqrt(2)*atan(sqrt(2)*x + 1)/16
```

3.287.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^4\left(1+2x^4+x^8\right)}\,dx = -\frac{7}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right) - \frac{7}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right) - \frac{7}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right) + \frac{7}{32}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right) - \frac{7x^4+4}{12\left(x^7+x^3\right)}$$

input
```
integrate(1/x^4/(x^8+2*x^4+1),x, algorithm="maxima")
```

output
```
-7/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 7/16*sqrt(2)*arctan(1/
2*sqrt(2)*(2*x - sqrt(2))) - 7/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 7/32*
sqrt(2)*log(x^2 - sqrt(2)*x + 1) - 1/12*(7*x^4 + 4)/(x^7 + x^3)
```

3.287.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^4\left(1+2x^4+x^8\right)}\,dx = -\frac{7}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right) - \frac{7}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right) - \frac{7}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right) + \frac{7}{32}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right) - \frac{x}{4\left(x^4+1\right)} - \frac{1}{3x^3}$$

input
```
integrate(1/x^4/(x^8+2*x^4+1),x, algorithm="giac")
```

output
```
-7/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 7/16*sqrt(2)*arctan(1/
2*sqrt(2)*(2*x - sqrt(2))) - 7/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 7/32*
sqrt(2)*log(x^2 - sqrt(2)*x + 1) - 1/4*x/(x^4 + 1) - 1/3/x^3
```

3.287.9 Mupad [B] (verification not implemented)

Time = 8.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.48

$$\int \frac{1}{x^4(1+2x^4+x^8)}\,dx = -\frac{\frac{7x^4}{12}+\frac{1}{3}}{x^7+x^3}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{7}{16}-\frac{7}{16}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{7}{16}+\frac{7}{16}\mathrm{i}\right)$$

input
```
int(1/(x^4*(2*x^4 + x^8 + 1)),x)
```

output
```
- 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(7/16 + 7i/16) - 2^(1/2)*atan(2^(1/
2)*x*(1/2 + 1i/2))*(7/16 - 7i/16) - ((7*x^4)/12 + 1/3)/(x^3 + x^7)
```

3.288 $\int \frac{1}{x^6(1+2x^4+x^8)} dx$

3.288.1 Optimal result

Integrand size = 16, antiderivative size = 113

$$\int \frac{1}{x^6(1+2x^4+x^8)} dx = -\frac{9}{20x^5}+\frac{9}{4x}+\frac{1}{4x^5(1+x^4)} - \frac{9\arctan\left(1-\sqrt{2}x\right)}{8\sqrt{2}}+\frac{9\arctan\left(1+\sqrt{2}x\right)}{8\sqrt{2}} + \frac{9\log\left(1-\sqrt{2}x+x^2\right)}{16\sqrt{2}}-\frac{9\log\left(1+\sqrt{2}x+x^2\right)}{16\sqrt{2}}$$

output
```
-9/20/x^5+9/4/x+1/4/x^5/(x^4+1)+9/16*arctan(-1+x*2^(1/2))*2^(1/2)+9/16*arc
tan(1+x*2^(1/2))*2^(1/2)+9/32*ln(1+x^2-x*2^(1/2))*2^(1/2)-9/32*ln(1+x^2+x*
2^(1/2))*2^(1/2)
```

3.288.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^6(1+2x^4+x^8)} dx = \frac{1}{160}\left(-\frac{32}{x^5}+\frac{320}{x}+\frac{40x^3}{1+x^4}-90\sqrt{2}\arctan\left(1-\sqrt{2}x\right) + 90\sqrt{2}\arctan\left(1+\sqrt{2}x\right)+45\sqrt{2}\log\left(1-\sqrt{2}x+x^2\right) - 45\sqrt{2}\log\left(1+\sqrt{2}x+x^2\right)\right)$$

input
```
Integrate[1/(x^6*(1 + 2*x^4 + x^8)),x]
```

output
```
(-32/x^5 + 320/x + (40*x^3)/(1 + x^4) - 90*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] +
 90*Sqrt[2]*ArcTan[1 + Sqrt[2]*x] + 45*Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] -
45*Sqrt[2]*Log[1 + Sqrt[2]*x + x^2])/160
```

3.288.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.05, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {1380, 819, 847, 847, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 (x^8+2x^4+1)} \, dx$$

$$\Big\downarrow \text{1380}$$

$$\int \frac{1}{x^6 (x^4+1)^2}dx$$

$$\Big\downarrow \text{819}$$

$$\frac{9}{4}\int \frac{1}{x^6 (x^4+1)}dx+\frac{1}{4x^5 (x^4+1)}$$

$$\Big\downarrow \text{847}$$

$$\frac{9}{4}\left(-\int \frac{1}{x^2 (x^4+1)}dx-\frac{1}{5x^5}\right)+\frac{1}{4x^5 (x^4+1)}$$

$$\Big\downarrow \text{847}$$

$$\frac{9}{4}\left(\int \frac{x^2}{x^4+1}dx-\frac{1}{5x^5}+\frac{1}{x}\right)+\frac{1}{4x^5 (x^4+1)}$$

$$\Big\downarrow \text{826}$$

$$\frac{9}{4}\left(-\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx+\frac{1}{2}\int \frac{x^2+1}{x^4+1}dx-\frac{1}{5x^5}+\frac{1}{x}\right)+\frac{1}{4x^5 (x^4+1)}$$

$$\Big\downarrow \text{1476}$$

$$\frac{9}{4}\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{x^2-\sqrt{2}x+1}dx+\frac{1}{2}\int\frac{1}{x^2+\sqrt{2}x+1}dx\right)-\frac{1}{2}\int\frac{1-x^2}{x^4+1}dx-\frac{1}{5x^5}+\frac{1}{x}\right)+\frac{1}{4x^5\left(x^4+1\right)}$$

$$\Big\downarrow \ 1082$$

$$\frac{9}{4}\left(-\frac{1}{2}\int\frac{1-x^2}{x^4+1}dx+\frac{1}{2}\left(\frac{\int\frac{1}{-\left(1-\sqrt{2}x\right)^2-1}d\left(1-\sqrt{2}x\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\left(\sqrt{2}x+1\right)^2-1}d\left(\sqrt{2}x+1\right)}{\sqrt{2}}\right)-\frac{1}{5x^5}+\frac{1}{x}\right)+\frac{1}{4x^5\left(x^4+1\right)}$$

$$\Big\downarrow \ 217$$

$$\frac{9}{4}\left(-\frac{1}{2}\int\frac{1-x^2}{x^4+1}dx+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)-\frac{1}{5x^5}+\frac{1}{x}\right)+\frac{1}{4x^5\left(x^4+1\right)}$$

$$\Big\downarrow \ 1479$$

$$\frac{9}{4}\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}+\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}x+1\right)}{x^2+\sqrt{2}x+1}dx}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)-\frac{1}{5x^5}+\frac{1}{x}\right)+\frac{1}{4x^5\left(x^4+1\right)}$$

$$\Big\downarrow \ 25$$

$$\frac{9}{4}\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}-\frac{\int\frac{\sqrt{2}\left(\sqrt{2}x+1\right)}{x^2+\sqrt{2}x+1}dx}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)-\frac{1}{5x^5}+\frac{1}{x}\right)+\frac{1}{4x^5\left(x^4+1\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{9}{4}\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}-\frac{1}{2}\int\frac{\sqrt{2}x+1}{x^2+\sqrt{2}x+1}dx\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)-\frac{1}{5x^5}+\frac{1}{x}\right)+\frac{1}{4x^5\left(x^4+1\right)}$$

$$\Big\downarrow \ 1103$$

3.288. $\int\frac{1}{x^6\left(1+2x^4+x^8\right)}\,dx$

$$\frac{9}{4}\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)-\frac{1}{5x^5}+\frac{1}{2}\left(\frac{\log\left(x^2-\sqrt{2}x+1\right)}{2\sqrt{2}}-\frac{\log\left(x^2+\sqrt{2}x+1\right)}{2\sqrt{2}}\right)+\frac{1}{x}\right)+\frac{1}{4x^5\left(x^4+1\right)}$$

input
```
Int[1/(x^6*(1 + 2*x^4 + x^8)),x]
```

output
```
1/(4*x^5*(1 + x^4)) + (9*(-1/5*1/x^5 + x^(-1) + (-(ArcTan[1 - Sqrt[2]*x]/S
qrt[2]) + ArcTan[1 + Sqrt[2]*x]/Sqrt[2])/2 + (Log[1 - Sqrt[2]*x + x^2]/(2*
Sqrt[2]) - Log[1 + Sqrt[2]*x + x^2]/(2*Sqrt[2]))/2))/4
```

3.288.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.288. $\int \frac{1}{x^6(1+2x^4+x^8)} \, dx$

3.288.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.39

method	result	size
risch	$\frac{\frac{9}{4}x^8+\frac{9}{5}x^4-\frac{1}{5}}{x^5(x^4+1)}+\frac{9\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+1\right)}\_R\ln\left(\_R^3+x\right)\right)}{16}$	44
default	$-\frac{1}{5x^5}+\frac{2}{x}+\frac{x^3}{4x^4+4}+\frac{9\sqrt{2}\left(\ln\left(\frac{1+x^2-x\sqrt{2}}{1+x^2+x\sqrt{2}}\right)+2\arctan\left(x\sqrt{2}+1\right)+2\arctan\left(x\sqrt{2}-1\right)\right)}{32}$	75

input
```
int(1/x^6/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
(9/4*x^8+9/5*x^4-1/5)/x^5/(x^4+1)+9/16*sum(_R*ln(_R^3+x),_R=RootOf(_Z^4+1)
)
```

3.288.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.12

$$\int \frac{1}{x^6(1+2x^4+x^8)}\,dx = \frac{360\,x^8+288\,x^4-45\,\sqrt{2}(-(i-1)\,x^9-(i-1)\,x^5)\log\left(2\,x+(i+1)\,\sqrt{2}\right)-45\,\sqrt{2}((i+1)\,x^9+(i+1)\,x^5}{}$$

input
```
integrate(1/x^6/(x^8+2*x^4+1),x, algorithm="fricas")
```

output
```
1/160*(360*x^8 + 288*x^4 - 45*sqrt(2)*(-(I - 1)*x^9 - (I - 1)*x^5)*log(2*x
 + (I + 1)*sqrt(2)) - 45*sqrt(2)*((I + 1)*x^9 + (I + 1)*x^5)*log(2*x - (I
- 1)*sqrt(2)) - 45*sqrt(2)*(-(I + 1)*x^9 - (I + 1)*x^5)*log(2*x + (I - 1)*
sqrt(2)) - 45*sqrt(2)*((I - 1)*x^9 + (I - 1)*x^5)*log(2*x - (I + 1)*sqrt(2
)) - 32)/(x^9 + x^5)
```

3.288.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^6(1+2x^4+x^8)}\,dx = \frac{9\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{32} - \frac{9\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{32} + \frac{9\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x-1\right)}{16} + \frac{9\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x+1\right)}{16} + \frac{45x^8+36x^4-4}{20x^9+20x^5}$$

input
```
integrate(1/x**6/(x**8+2*x**4+1),x)
```

output
```
9*sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 - 9*sqrt(2)*log(x**2 + sqrt(2)*x +
1)/32 + 9*sqrt(2)*atan(sqrt(2)*x - 1)/16 + 9*sqrt(2)*atan(sqrt(2)*x + 1)/1
6 + (45*x**8 + 36*x**4 - 4)/(20*x**9 + 20*x**5)
```

3.288.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^6(1+2x^4+x^8)}\,dx = \frac{9}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right) + \frac{9}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right) - \frac{9}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right) + \frac{9}{32}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right) + \frac{45x^8+36x^4-4}{20(x^9+x^5)}$$

input
```
integrate(1/x^6/(x^8+2*x^4+1),x, algorithm="maxima")
```

output
```
9/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 9/16*sqrt(2)*arctan(1/2
*sqrt(2)*(2*x - sqrt(2))) - 9/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 9/32*s
qrt(2)*log(x^2 - sqrt(2)*x + 1) + 1/20*(45*x^8 + 36*x^4 - 4)/(x^9 + x^5)
```

3.288.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^6 (1+2x^4+x^8)} \, dx = \frac{x^3}{4 (x^4+1)} + \frac{9}{16} \sqrt{2} \arctan \left(\frac{1}{2} \sqrt{2} \left(2x+\sqrt{2} \right) \right) + \frac{9}{16} \sqrt{2} \arctan \left(\frac{1}{2} \sqrt{2} \left(2x-\sqrt{2} \right) \right) - \frac{9}{32} \sqrt{2} \log \left(x^2+\sqrt{2}x+1 \right) + \frac{9}{32} \sqrt{2} \log \left(x^2-\sqrt{2}x+1 \right) + \frac{10x^4-1}{5x^5}$$

input
```
integrate(1/x^6/(x^8+2*x^4+1),x, algorithm="giac")
```

output
```
1/4*x^3/(x^4 + 1) + 9/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 9/1
6*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 9/32*sqrt(2)*log(x^2 + sqr
t(2)*x + 1) + 9/32*sqrt(2)*log(x^2 - sqrt(2)*x + 1) + 1/5*(10*x^4 - 1)/x^5
```

3.288.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.49

$$\int \frac{1}{x^6 (1+2x^4+x^8)} \, dx = \frac{\frac{9x^8}{4}+\frac{9x^4}{5}-\frac{1}{5}}{x^9+x^5} + \sqrt{2} \operatorname{atan}\left(\sqrt{2}\,x \left(\frac{1}{2}-\frac{1}{2}\mathrm{i} \right) \right) \left(\frac{9}{16}-\frac{9}{16}\mathrm{i} \right) + \sqrt{2} \operatorname{atan}\left(\sqrt{2}\,x \left(\frac{1}{2}+\frac{1}{2}\mathrm{i} \right) \right) \left(\frac{9}{16}+\frac{9}{16}\mathrm{i} \right)$$

input
```
int(1/(x^6*(2*x^4 + x^8 + 1)),x)
```

output
```
2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(9/16 - 9i/16) + 2^(1/2)*atan(2^(1/2)
*x*(1/2 + 1i/2))*(9/16 + 9i/16) + ((9*x^4)/5 + (9*x^8)/4 - 1/5)/(x^5 + x^9
)
```

3.289 $\int \frac{1}{x^8(1+2x^4+x^8)}\,dx$

3.289.1 Optimal result

Integrand size = 16, antiderivative size = 113

$$\int \frac{1}{x^8(1+2x^4+x^8)}\,dx = -\frac{11}{28x^7}+\frac{11}{12x^3}+\frac{1}{4x^7(1+x^4)} - \frac{11\arctan\left(1-\sqrt{2}x\right)}{8\sqrt{2}}+\frac{11\arctan\left(1+\sqrt{2}x\right)}{8\sqrt{2}} - \frac{11\log\left(1-\sqrt{2}x+x^2\right)}{16\sqrt{2}}+\frac{11\log\left(1+\sqrt{2}x+x^2\right)}{16\sqrt{2}}$$

output
```
-11/28/x^7+11/12/x^3+1/4/x^7/(x^4+1)+11/16*arctan(-1+x*2^(1/2))*2^(1/2)+11
/16*arctan(1+x*2^(1/2))*2^(1/2)-11/32*ln(1+x^2-x*2^(1/2))*2^(1/2)+11/32*ln
(1+x^2+x*2^(1/2))*2^(1/2)
```

3.289.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^8(1+2x^4+x^8)}\,dx = \frac{1}{672}\left(-\frac{96}{x^7}+\frac{448}{x^3}+\frac{168x}{1+x^4}-462\sqrt{2}\arctan\left(1-\sqrt{2}x\right) + 462\sqrt{2}\arctan\left(1+\sqrt{2}x\right)-231\sqrt{2}\log\left(1-\sqrt{2}x+x^2\right) + 231\sqrt{2}\log\left(1+\sqrt{2}x+x^2\right)\right)$$

input
```
Integrate[1/(x^8*(1 + 2*x^4 + x^8)),x]
```

output
```
(-96/x^7 + 448/x^3 + (168*x)/(1 + x^4) - 462*Sqrt[2]*ArcTan[1 - Sqrt[2]*x]
 + 462*Sqrt[2]*ArcTan[1 + Sqrt[2]*x] - 231*Sqrt[2]*Log[1 - Sqrt[2]*x + x^2
] + 231*Sqrt[2]*Log[1 + Sqrt[2]*x + x^2])/672
```

3.289.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.09, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {1380, 819, 847, 847, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^8 (x^8+2x^4+1)} dx$$

$$\Big\downarrow \text{1380}$$

$$\int \frac{1}{x^8 (x^4+1)^2} dx$$

$$\Big\downarrow \text{819}$$

$$\frac{11}{4} \int \frac{1}{x^8 (x^4+1)} dx + \frac{1}{4x^7 (x^4+1)}$$

$$\Big\downarrow \text{847}$$

$$\frac{11}{4} \left(-\int \frac{1}{x^4 (x^4+1)} dx - \frac{1}{7x^7}\right) + \frac{1}{4x^7 (x^4+1)}$$

$$\Big\downarrow \text{847}$$

$$\frac{11}{4} \left(\int \frac{1}{x^4+1} dx - \frac{1}{7x^7} + \frac{1}{3x^3}\right) + \frac{1}{4x^7 (x^4+1)}$$

$$\Big\downarrow \text{755}$$

$$\frac{11}{4} \left(\frac{1}{2} \int \frac{1-x^2}{x^4+1} dx + \frac{1}{2} \int \frac{x^2+1}{x^4+1} dx - \frac{1}{7x^7} + \frac{1}{3x^3}\right) + \frac{1}{4x^7 (x^4+1)}$$

$$\Big\downarrow \text{1476}$$

$$\frac{11}{4}\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{x^2-\sqrt{2}x+1}dx+\frac{1}{2}\int\frac{1}{x^2+\sqrt{2}x+1}dx\right)+\frac{1}{2}\int\frac{1-x^2}{x^4+1}dx-\frac{1}{7x^7}+\frac{1}{3x^3}\right)+\frac{1}{4x^7(x^4+1)}$$

$$\Big\downarrow \text{1082}$$

$$\frac{11}{4}\left(\frac{1}{2}\int\frac{1-x^2}{x^4+1}dx+\frac{1}{2}\left(\frac{\int\frac{1}{-\left(1-\sqrt{2}x\right)^2-1}d\left(1-\sqrt{2}x\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\left(\sqrt{2}x+1\right)^2-1}d\left(\sqrt{2}x+1\right)}{\sqrt{2}}\right)-\frac{1}{7x^7}+\frac{1}{3x^3}\right)+\frac{1}{4x^7(x^4+1)}$$

$$\Big\downarrow \text{217}$$

$$\frac{11}{4}\left(\frac{1}{2}\int\frac{1-x^2}{x^4+1}dx+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)-\frac{1}{7x^7}+\frac{1}{3x^3}\right)+\frac{1}{4x^7(x^4+1)}$$

$$\Big\downarrow \text{1479}$$

$$\frac{11}{4}\left(\frac{1}{2}\left(-\frac{\int-\frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}x+1\right)}{x^2+\sqrt{2}x+1}dx}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)-\frac{1}{7x^7}+\frac{1}{3x^3}\right)+\frac{1}{4x^7(x^4+1)}$$

$$\Big\downarrow \text{25}$$

$$\frac{11}{4}\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}x+1\right)}{x^2+\sqrt{2}x+1}dx}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)-\frac{1}{7x^7}+\frac{1}{3x^3}\right)+\frac{1}{4x^7(x^4+1)}$$

$$\Big\downarrow \text{27}$$

$$\frac{11}{4}\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}+\frac{1}{2}\int\frac{\sqrt{2}x+1}{x^2+\sqrt{2}x+1}dx\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)-\frac{1}{7x^7}+\frac{1}{3x^3}\right)+\frac{1}{4x^7(x^4+1)}$$

$$\Big\downarrow \text{1103}$$

3.289. $\int \frac{1}{x^8(1+2x^4+x^8)}\,dx$

$$\frac{11}{4}\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)-\frac{1}{7x^7}+\frac{1}{3x^3}+\frac{1}{2}\left(\frac{\log\left(x^2+\sqrt{2}x+1\right)}{2\sqrt{2}}-\frac{\log\left(x^2-\sqrt{2}x+1\right)}{2\sqrt{2}}\right)\right.$$
$$\frac{1}{4x^7\left(x^4+1\right)}$$

input
```
Int[1/(x^8*(1 + 2*x^4 + x^8)),x]
```

output
```
1/(4*x^7*(1 + x^4)) + (11*(-1/7*1/x^7 + 1/(3*x^3) + (-(ArcTan[1 - Sqrt[2]*
x]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*x]/Sqrt[2])/2 + (-1/2*Log[1 - Sqrt[2]*x +
 x^2]/Sqrt[2] + Log[1 + Sqrt[2]*x + x^2]/(2*Sqrt[2]))/2))/4
```

3.289.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.289. $\int \frac{1}{x^8(1+2x^4+x^8)}\,dx$

3.289.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.37

method	result	size
risch	$\frac{\frac{11}{12}x^8+\frac{11}{21}x^4-\frac{1}{7}}{x^7(x^4+1)}+\frac{11\left(\sum_{\_R=\mathrm{RootOf}(\_Z^4+1)}\_R\ln\left(x+\_R\right)\right)}{16}$	42
default	$-\frac{1}{7x^7}+\frac{2}{3x^3}+\frac{x}{4x^4+4}+\frac{11\sqrt{2}\left(\ln\left(\frac{1+x^2+x\sqrt{2}}{1+x^2-x\sqrt{2}}\right)+2\arctan\left(x\sqrt{2}+1\right)+2\arctan\left(x\sqrt{2}-1\right)\right)}{32}$	73

input
```
int(1/x^8/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
(11/12*x^8+11/21*x^4-1/7)/x^7/(x^4+1)+11/16*sum(_R*ln(x+_R),_R=RootOf(_Z^4
+1))
```

3.289.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.12

$$\int \frac{1}{x^8(1+2x^4+x^8)}\,dx = \frac{616\,x^8+352\,x^4-231\,\sqrt{2}(-(i+1)\,x^{11}-(i+1)\,x^7)\log\left(2\,x+(i+1)\,\sqrt{2}\right)-231\,\sqrt{2}((i-1)\,x^{11}+(i-1)$$

input
```
integrate(1/x^8/(x^8+2*x^4+1),x, algorithm="fricas")
```

output
```
1/672*(616*x^8 + 352*x^4 - 231*sqrt(2)*(-(I + 1)*x^11 - (I + 1)*x^7)*log(2
*x + (I + 1)*sqrt(2)) - 231*sqrt(2)*((I - 1)*x^11 + (I - 1)*x^7)*log(2*x -
 (I - 1)*sqrt(2)) - 231*sqrt(2)*(-(I - 1)*x^11 - (I - 1)*x^7)*log(2*x + (I
 - 1)*sqrt(2)) - 231*sqrt(2)*((I + 1)*x^11 + (I + 1)*x^7)*log(2*x - (I + 1
)*sqrt(2)) - 96)/(x^11 + x^7)
```

3.289.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^8(1+2x^4+x^8)}\,dx = -\frac{11\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{32} + \frac{11\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{32} + \frac{11\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x-1\right)}{16} + \frac{11\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x+1\right)}{16} + \frac{77x^8+44x^4-12}{84x^{11}+84x^7}$$

input
```
integrate(1/x**8/(x**8+2*x**4+1),x)
```

output
```
-11*sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 + 11*sqrt(2)*log(x**2 + sqrt(2)*x
 + 1)/32 + 11*sqrt(2)*atan(sqrt(2)*x - 1)/16 + 11*sqrt(2)*atan(sqrt(2)*x +
 1)/16 + (77*x**8 + 44*x**4 - 12)/(84*x**11 + 84*x**7)
```

3.289.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^8(1+2x^4+x^8)}\,dx = \frac{11}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right) + \frac{11}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right) + \frac{11}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right) - \frac{11}{32}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right) + \frac{77x^8+44x^4-12}{84(x^{11}+x^7)}$$

input
```
integrate(1/x^8/(x^8+2*x^4+1),x, algorithm="maxima")
```

output
```
11/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 11/16*sqrt(2)*arctan(1
/2*sqrt(2)*(2*x - sqrt(2))) + 11/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 11/
32*sqrt(2)*log(x^2 - sqrt(2)*x + 1) + 1/84*(77*x^8 + 44*x^4 - 12)/(x^11 +
x^7)
```

3.289.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^8 (1+2x^4+x^8)}\,dx = \frac{11}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right) + \frac{11}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right) + \frac{11}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right) - \frac{11}{32}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right) + \frac{x}{4(x^4+1)} + \frac{14x^4-3}{21x^7}$$

input
```
integrate(1/x^8/(x^8+2*x^4+1),x, algorithm="giac")
```

output
```
11/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 11/16*sqrt(2)*arctan(1
/2*sqrt(2)*(2*x - sqrt(2))) + 11/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 11/
32*sqrt(2)*log(x^2 - sqrt(2)*x + 1) + 1/4*x/(x^4 + 1) + 1/21*(14*x^4 - 3)/
x^7
```

3.289.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.49

$$\int \frac{1}{x^8 (1+2x^4+x^8)}\,dx = \frac{\frac{11x^8}{12}+\frac{11x^4}{21}-\frac{1}{7}}{x^{11}+x^7} + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{11}{16}+\frac{11}{16}\mathrm{i}\right) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{11}{16}-\frac{11}{16}\mathrm{i}\right)$$

input
```
int(1/(x^8*(2*x^4 + x^8 + 1)),x)
```

output
```
2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(11/16 + 11i/16) + 2^(1/2)*atan(2^(1/
2)*x*(1/2 + 1i/2))*(11/16 - 11i/16) + ((11*x^4)/21 + (11*x^8)/12 - 1/7)/(x
^7 + x^11)
```

3.290 $\int \frac{x^m}{1-2x^4+x^8}\,dx$

3.290.1 Optimal result

Integrand size = 16, antiderivative size = 30

$$\int \frac{x^m}{1-2x^4+x^8}\,dx = \frac{x^{1+m}\,\text{Hypergeometric2F1}\left(2,\frac{1+m}{4},\frac{5+m}{4},x^4\right)}{1+m}$$

output
```
x^(1+m)*hypergeom([2, 1/4+1/4*m],[5/4+1/4*m],x^4)/(1+m)
```

3.290.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{x^m}{1-2x^4+x^8}\,dx = \frac{x^{1+m}\,\text{Hypergeometric2F1}\left(2,\frac{1+m}{4},1+\frac{1+m}{4},x^4\right)}{1+m}$$

input
```
Integrate[x^m/(1 - 2*x^4 + x^8),x]
```

output
```
(x^(1 + m)*Hypergeometric2F1[2, (1 + m)/4, 1 + (1 + m)/4, x^4])/(1 + m)
```

3.290.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1380, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{x^8-2x^4+1}dx$$
$$\Big\downarrow \text{ 1380}$$
$$\int \frac{x^m}{(1-x^4)^2}dx$$
$$\Big\downarrow \text{ 888}$$
$$\frac{x^{m+1}\text{Hypergeometric2F1}\left(2,\frac{m+1}{4},\frac{m+5}{4},x^4\right)}{m+1}$$

input
```
Int[x^m/(1 - 2*x^4 + x^8),x]
```

output
```
(x^(1 + m)*Hypergeometric2F1[2, (1 + m)/4, (5 + m)/4, x^4])/(1 + m)
```

3.290.3.1 Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

3.290.4 Maple [F]

$$\int \frac{x^m}{x^8 - 2x^4 + 1} dx$$

input
```
int(x^m/(x^8-2*x^4+1),x)
```

output
```
int(x^m/(x^8-2*x^4+1),x)
```

3.290.5 Fricas [F]

$$\int \frac{x^m}{1 - 2x^4 + x^8} \, dx = \int \frac{x^m}{x^8 - 2\,x^4 + 1} \, dx$$

input
```
integrate(x^m/(x^8-2*x^4+1),x, algorithm="fricas")
```

output
```
integral(x^m/(x^8 - 2*x^4 + 1), x)
```

3.290.6 Sympy [F]

$$\int \frac{x^m}{1 - 2x^4 + x^8} \, dx = \int \frac{x^m}{(x - 1)^2 \, (x + 1)^2 \, (x^2 + 1)^2} \, dx$$

input
```
integrate(x**m/(x**8-2*x**4+1),x)
```

output
```
Integral(x**m/((x - 1)**2*(x + 1)**2*(x**2 + 1)**2), x)
```

3.290.7 Maxima [F]

$$\int \frac{x^m}{1-2x^4+x^8}\,dx = \int \frac{x^m}{x^8-2\,x^4+1}\,dx$$

input
```
integrate(x^m/(x^8-2*x^4+1),x, algorithm="maxima")
```

output
```
integrate(x^m/(x^8 - 2*x^4 + 1), x)
```

3.290.8 Giac [F]

$$\int \frac{x^m}{1-2x^4+x^8}\,dx = \int \frac{x^m}{x^8-2\,x^4+1}\,dx$$

input
```
integrate(x^m/(x^8-2*x^4+1),x, algorithm="giac")
```

output
```
integrate(x^m/(x^8 - 2*x^4 + 1), x)
```

3.290.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{1-2x^4+x^8}\,dx = \int \frac{x^m}{x^8-2\,x^4+1}\,dx$$

input
```
int(x^m/(x^8 - 2*x^4 + 1),x)
```

output
```
int(x^m/(x^8 - 2*x^4 + 1), x)
```

3.291 $\int \frac{x^9}{1-2x^4+x^8}\,dx$

3.291.1 Optimal result

Integrand size = 16, antiderivative size = 32

$$\int \frac{x^9}{1-2x^4+x^8}\,dx=\frac{3x^2}{4}+\frac{x^6}{4\,(1-x^4)}-\frac{3\text{arctanh}(x^2)}{4}$$

output
```
3/4*x^2+1/4*x^6/(-x^4+1)-3/4*arctanh(x^2)
```

3.291.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.22

$$\int \frac{x^9}{1-2x^4+x^8}\,dx=\frac{1}{8}\left(2x^2\left(2+\frac{1}{1-x^4}\right)+3\log\left(1-x^2\right)-3\log\left(1+x^2\right)\right)$$

input
```
Integrate[x^9/(1 - 2*x^4 + x^8),x]
```

output
```
(2*x^2*(2 + (1 - x^4)^(-1)) + 3*Log[1 - x^2] - 3*Log[1 + x^2])/8
```

3.291.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1380, 807, 252, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{x^8-2x^4+1}dx$$
$$\Big\downarrow \text{1380}$$
$$\int \frac{x^9}{(1-x^4)^2}dx$$
$$\Big\downarrow \text{807}$$
$$\frac{1}{2}\int \frac{x^8}{(1-x^4)^2}dx^2$$
$$\Big\downarrow \text{252}$$
$$\frac{1}{2}\left(\frac{x^6}{2(1-x^4)}-\frac{3}{2}\int \frac{x^4}{1-x^4}dx^2\right)$$
$$\Big\downarrow \text{262}$$
$$\frac{1}{2}\left(\frac{x^6}{2(1-x^4)}-\frac{3}{2}\left(\int \frac{1}{1-x^4}dx^2-x^2\right)\right)$$
$$\Big\downarrow \text{219}$$
$$\frac{1}{2}\left(\frac{x^6}{2(1-x^4)}-\frac{3}{2}\left(\text{arctanh}(x^2)-x^2\right)\right)$$

input
```
Int[x^9/(1 - 2*x^4 + x^8),x]
```

output
```
(x^6/(2*(1 - x^4)) - (3*(-x^2 + ArcTanh[x^2]))/2)/2
```

3.291.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

3.291.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.09

method	result	size
risch	$\frac{x^{2}}{2}-\frac{x^{2}}{4(x^{4}-1)}-\frac{3\ln(x^{2}+1)}{8}+\frac{3\ln(x^{2}-1)}{8}$	35
default	$\frac{x^{2}}{2}-\frac{1}{8(x^{2}+1)}-\frac{3\ln(x^{2}+1)}{8}-\frac{1}{8(x^{2}-1)}+\frac{3\ln(x^{2}-1)}{8}$	41
norman	$\frac{-\frac{3}{4}x^{2}+\frac{1}{2}x^{6}}{x^{4}-1}+\frac{3\ln(x-1)}{8}+\frac{3\ln(x+1)}{8}-\frac{3\ln(x^{2}+1)}{8}$	41
parallelrisch	$\frac{4x^{6}+3\ln(x-1)x^{4}+3\ln(x+1)x^{4}-3\ln(x^{2}+1)x^{4}-6x^{2}-3\ln(x-1)-3\ln(x+1)+3\ln(x^{2}+1)}{8x^{4}-8}$	70

input
```
int(x^9/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2-1/4*x^2/(x^4-1)-3/8*ln(x^2+1)+3/8*ln(x^2-1)
```

3.291.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.44

$$\int \frac{x^9}{1-2x^4+x^8}\,dx = \frac{4\,x^6 - 6\,x^2 - 3\,(x^4-1)\log\left(x^2+1\right) + 3\,(x^4-1)\log\left(x^2-1\right)}{8\,(x^4-1)}$$

input
```
integrate(x^9/(x^8-2*x^4+1),x, algorithm="fricas")
```

output
```
1/8*(4*x^6 - 6*x^2 - 3*(x^4 - 1)*log(x^2 + 1) + 3*(x^4 - 1)*log(x^2 - 1))/
(x^4 - 1)
```

3.291.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{x^9}{1-2x^4+x^8}\,dx = \frac{x^2}{2} - \frac{x^2}{4x^4-4} + \frac{3\log\left(x^2-1\right)}{8} - \frac{3\log\left(x^2+1\right)}{8}$$

input
```
integrate(x**9/(x**8-2*x**4+1),x)
```

output
```
x**2/2 - x**2/(4*x**4 - 4) + 3*log(x**2 - 1)/8 - 3*log(x**2 + 1)/8
```

3.291.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{x^9}{1-2x^4+x^8}\,dx = \frac{1}{2}\,x^2 - \frac{x^2}{4\,(x^4-1)} - \frac{3}{8}\,\log\left(x^2+1\right) + \frac{3}{8}\,\log\left(x^2-1\right)$$

input
```
integrate(x^9/(x^8-2*x^4+1),x, algorithm="maxima")
```

output
```
1/2*x^2 - 1/4*x^2/(x^4 - 1) - 3/8*log(x^2 + 1) + 3/8*log(x^2 - 1)
```

3.291.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.09

$$\int \frac{x^9}{1-2x^4+x^8}\,dx = \frac{1}{2}\,x^2 - \frac{x^2}{4\,(x^4-1)} - \frac{3}{8}\,\log\left(x^2+1\right) + \frac{3}{8}\,\log\left(\left|x^2-1\right|\right)$$

input
```
integrate(x^9/(x^8-2*x^4+1),x, algorithm="giac")
```

output
```
1/2*x^2 - 1/4*x^2/(x^4 - 1) - 3/8*log(x^2 + 1) + 3/8*log(abs(x^2 - 1))
```

3.291.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{x^9}{1-2x^4+x^8}\,dx = \frac{x^2}{2} - \frac{x^2}{4\,(x^4-1)} - \frac{3\,\mathrm{atanh}(x^2)}{4}$$

input
```
int(x^9/(x^8 - 2*x^4 + 1),x)
```

output
```
x^2/2 - x^2/(4*(x^4 - 1)) - (3*atanh(x^2))/4
```

3.292 $\int \frac{x^7}{1-2x^4+x^8} dx$

3.292.1 Optimal result

Integrand size = 16, antiderivative size = 26

$$\int \frac{x^7}{1-2x^4+x^8} dx = \frac{1}{4(1-x^4)} + \frac{1}{4}\log\left(1-x^4\right)$$

output
```
1/4/(-x^4+1)+1/4*ln(-x^4+1)
```

3.292.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^7}{1-2x^4+x^8} dx = -\frac{1}{4(-1+x^4)} + \frac{1}{4}\log\left(-1+x^4\right)$$

input
```
Integrate[x^7/(1 - 2*x^4 + x^8),x]
```

output
```
-1/4*1/(-1 + x^4) + Log[-1 + x^4]/4
```

3.292.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1380, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{x^8-2x^4+1}dx$$

$$\Big\downarrow \text{ 1380}$$

$$\int \frac{x^7}{(1-x^4)^2}dx$$

$$\Big\downarrow \text{ 798}$$

$$\frac{1}{4}\int \frac{x^4}{(1-x^4)^2}dx^4$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{4}\int \left(\frac{1}{x^4-1}+\frac{1}{(x^4-1)^2}\right)dx^4$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4}\left(\frac{1}{1-x^4}+\log\left(1-x^4\right)\right)$$

input
```
Int[x^7/(1 - 2*x^4 + x^8),x]
```

output
```
((1 - x^4)^(-1) + Log[1 - x^4])/4
```

3.292.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.292.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.73

method	result	size
default	$-\frac{1}{4(x^4-1)}+\frac{\ln(x^4-1)}{4}$	19
risch	$-\frac{1}{4(x^4-1)}+\frac{\ln(x^4-1)}{4}$	19
norman	$-\frac{1}{4(x^4-1)}+\frac{\ln(x-1)}{4}+\frac{\ln(x+1)}{4}+\frac{\ln(x^2+1)}{4}$	31
parallelrisch	$\frac{\ln(x-1)x^4+\ln(x+1)x^4+\ln(x^2+1)x^4-1-\ln(x-1)-\ln(x+1)-\ln(x^2+1)}{4x^4-4}$	58

input
```
int(x^7/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/4/(x^4-1)+1/4*ln(x^4-1)
```

3.292.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

$$\int \frac{x^7}{1-2x^4+x^8}\,dx = \frac{(x^4-1)\log(x^4-1)-1}{4(x^4-1)}$$

input
```
integrate(x^7/(x^8-2*x^4+1),x, algorithm="fricas")
```

output
```
1/4*((x^4 - 1)*log(x^4 - 1) - 1)/(x^4 - 1)
```

3.292.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.58

$$\int \frac{x^7}{1-2x^4+x^8}\,dx = \frac{\log\left(x^4-1\right)}{4} - \frac{1}{4x^4-4}$$

input
```
integrate(x**7/(x**8-2*x**4+1),x)
```

output
```
log(x**4 - 1)/4 - 1/(4*x**4 - 4)
```

3.292.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.69

$$\int \frac{x^7}{1-2x^4+x^8}\,dx = -\frac{1}{4\left(x^4-1\right)} + \frac{1}{4}\log\left(x^4-1\right)$$

input
```
integrate(x^7/(x^8-2*x^4+1),x, algorithm="maxima")
```

output
```
-1/4/(x^4 - 1) + 1/4*log(x^4 - 1)
```

3.292.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.73

$$\int \frac{x^7}{1-2x^4+x^8}\,dx = -\frac{1}{4\left(x^4-1\right)} + \frac{1}{4}\log\left(\left|x^4-1\right|\right)$$

input
```
integrate(x^7/(x^8-2*x^4+1),x, algorithm="giac")
```

output
```
-1/4/(x^4 - 1) + 1/4*log(abs(x^4 - 1))
```

3.292.9 Mupad [B] (verification not implemented)

Time = 8.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{x^7}{1-2x^4+x^8}\,dx = \frac{\ln(x^4-1)}{4} - \frac{1}{4(x^4-1)}$$

input
```
int(x^7/(x^8 - 2*x^4 + 1),x)
```

output
```
log(x^4 - 1)/4 - 1/(4*(x^4 - 1))
```

3.293 $\int \frac{x^5}{1-2x^4+x^8} \, dx$

3.293.1 Optimal result

Integrand size = 16, antiderivative size = 25

$$\int \frac{x^5}{1-2x^4+x^8} \, dx = \frac{x^2}{4(1-x^4)} - \frac{\text{arctanh}(x^2)}{4}$$

output
```
1/4*x^2/(-x^4+1)-1/4*arctanh(x^2)
```

3.293.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.32

$$\int \frac{x^5}{1-2x^4+x^8} \, dx = \frac{1}{8}\left(-\frac{2x^2}{-1+x^4} + \log(1-x^2) - \log(1+x^2)\right)$$

input
```
Integrate[x^5/(1 - 2*x^4 + x^8),x]
```

output
```
((-2*x^2)/(-1 + x^4) + Log[1 - x^2] - Log[1 + x^2])/8
```

3.293.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1380, 807, 252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{x^8-2x^4+1}dx$$
$$\Big\downarrow \text{1380}$$
$$\int \frac{x^5}{(1-x^4)^2}dx$$
$$\Big\downarrow \text{807}$$
$$\frac{1}{2}\int \frac{x^4}{(1-x^4)^2}dx^2$$
$$\Big\downarrow \text{252}$$
$$\frac{1}{2}\left(\frac{x^2}{2(1-x^4)}-\frac{1}{2}\int \frac{1}{1-x^4}dx^2\right)$$
$$\Big\downarrow \text{219}$$
$$\frac{1}{2}\left(\frac{x^2}{2(1-x^4)}-\frac{\text{arctanh}(x^2)}{2}\right)$$

input
```
Int[x^5/(1 - 2*x^4 + x^8),x]
```

output
```
(x^2/(2*(1 - x^4)) - ArcTanh[x^2]/2)/2
```

3.293.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

3.293.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.20

method	result	size
risch	$-\frac{x^{2}}{4(x^{4}-1)}+\frac{\ln(x^{2}-1)}{8}-\frac{\ln(x^{2}+1)}{8}$	30
norman	$-\frac{x^{2}}{4(x^{4}-1)}+\frac{\ln(x-1)}{8}+\frac{\ln(x+1)}{8}-\frac{\ln(x^{2}+1)}{8}$	34
default	$-\frac{1}{8(x^{2}+1)}-\frac{\ln(x^{2}+1)}{8}-\frac{1}{8(x^{2}-1)}+\frac{\ln(x^{2}-1)}{8}$	36
parallelrisch	$\frac{\ln(x-1)x^{4}+\ln(x+1)x^{4}-\ln(x^{2}+1)x^{4}-2x^{2}-\ln(x-1)-\ln(x+1)+\ln(x^{2}+1)}{8x^{4}-8}$	61

input
```
int(x^5/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/4*x^2/(x^4-1)+1/8*ln(x^2-1)-1/8*ln(x^2+1)
```

3.293.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. 2(19) = 38.

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.60

$$\int \frac{x^5}{1-2x^4+x^8}\,dx = -\frac{2\,x^2+(x^4-1)\log\left(x^2+1\right)-(x^4-1)\log\left(x^2-1\right)}{8\,(x^4-1)}$$

input
```
integrate(x^5/(x^8-2*x^4+1),x, algorithm="fricas")
```

output
```
-1/8*(2*x^2 + (x^4 - 1)*log(x^2 + 1) - (x^4 - 1)*log(x^2 - 1))/(x^4 - 1)
```

3.293.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{x^5}{1-2x^4+x^8}\,dx = -\frac{x^2}{4x^4-4}+\frac{\log\left(x^2-1\right)}{8}-\frac{\log\left(x^2+1\right)}{8}$$

input
```
integrate(x**5/(x**8-2*x**4+1),x)
```

output
```
-x**2/(4*x**4 - 4) + log(x**2 - 1)/8 - log(x**2 + 1)/8
```

3.293.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \frac{x^5}{1-2x^4+x^8}\,dx = -\frac{x^2}{4\,(x^4-1)}-\frac{1}{8}\,\log\left(x^2+1\right)+\frac{1}{8}\,\log\left(x^2-1\right)$$

input
```
integrate(x^5/(x^8-2*x^4+1),x, algorithm="maxima")
```

output
```
-1/4*x^2/(x^4 - 1) - 1/8*log(x^2 + 1) + 1/8*log(x^2 - 1)
```

3.293.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.20

$$\int \frac{x^5}{1-2x^4+x^8}\,dx = -\frac{x^2}{4\left(x^4-1\right)} - \frac{1}{8}\log\left(x^2+1\right) + \frac{1}{8}\log\left(\left|x^2-1\right|\right)$$

input
```
integrate(x^5/(x^8-2*x^4+1),x, algorithm="giac")
```

output
```
-1/4*x^2/(x^4 - 1) - 1/8*log(x^2 + 1) + 1/8*log(abs(x^2 - 1))
```

3.293.9 Mupad [B] (verification not implemented)

Time = 8.44 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{x^5}{1-2x^4+x^8}\,dx = -\frac{\operatorname{atanh}(x^2)}{4} - \frac{x^2}{4\left(x^4-1\right)}$$

input
```
int(x^5/(x^8 - 2*x^4 + 1),x)
```

output
```
- atanh(x^2)/4 - x^2/(4*(x^4 - 1))
```

3.294 $\int \frac{x^3}{1-2x^4+x^8} dx$

3.294.1 Optimal result

Integrand size = 16, antiderivative size = 13

$$\int \frac{x^3}{1-2x^4+x^8} \, dx = \frac{1}{4(1-x^4)}$$

output
```
1/4/(-x^4+1)
```

3.294.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{x^3}{1-2x^4+x^8} \, dx = -\frac{1}{4(-1+x^4)}$$

input
```
Integrate[x^3/(1 - 2*x^4 + x^8),x]
```

output
```
-1/4*1/(-1 + x^4)
```

3.294.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1380, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{x^8-2x^4+1} \, dx$$

$$\Big\downarrow \text{1380}$$

$$\int \frac{x^3}{(1-x^4)^2}dx$$

$$\Big\downarrow \text{793}$$

$$\frac{1}{4(1-x^4)}$$

input
```
Int[x^3/(1 - 2*x^4 + x^8),x]
```

output
```
1/(4*(1 - x^4))
```

3.294.3.1 Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

3.294.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
gosper	$-\frac{1}{4(x^4-1)}$	10
default	$-\frac{1}{4(x^4-1)}$	10
norman	$-\frac{1}{4(x^4-1)}$	10
risch	$-\frac{1}{4(x^4-1)}$	10
parallelrisch	$-\frac{1}{4(x^4-1)}$	10

input
```
int(x^3/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/4/(x^4-1)
```

3.294.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x^3}{1-2x^4+x^8}\,dx = -\frac{1}{4\left(x^4-1\right)}$$

input
```
integrate(x^3/(x^8-2*x^4+1),x, algorithm="fricas")
```

output
```
-1/4/(x^4 - 1)
```

3.294.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.62

$$\int \frac{x^3}{1-2x^4+x^8}\,dx = -\frac{1}{4x^4-4}$$

input
```
integrate(x**3/(x**8-2*x**4+1),x)
```

output
```
-1/(4*x**4 - 4)
```

3.294.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x^3}{1-2x^4+x^8}\,dx = -\frac{1}{4\,(x^4-1)}$$

input
```
integrate(x^3/(x^8-2*x^4+1),x, algorithm="maxima")
```

output
```
-1/4/(x^4 - 1)
```

3.294.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x^3}{1-2x^4+x^8}\,dx = -\frac{1}{4\,(x^4-1)}$$

input
```
integrate(x^3/(x^8-2*x^4+1),x, algorithm="giac")
```

output
```
-1/4/(x^4 - 1)
```

3.294.9 Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{x^3}{1-2x^4+x^8}\,dx = -\frac{1}{4\,(x^4-1)}$$

input
```
int(x^3/(x^8 - 2*x^4 + 1),x)
```

output
```
-1/(4*(x^4 - 1))
```

3.295 $\int \frac{x}{1-2x^4+x^8}\,dx$

3.295.1 Optimal result

Integrand size = 14, antiderivative size = 25

$$\int \frac{x}{1-2x^4+x^8}\,dx=\frac{x^2}{4(1-x^4)}+\frac{\operatorname{arctanh}(x^2)}{4}$$

output
```
1/4*x^2/(-x^4+1)+1/4*arctanh(x^2)
```

3.295.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.32

$$\int \frac{x}{1-2x^4+x^8}\,dx=\frac{1}{8}\left(-\frac{2x^2}{-1+x^4}-\log\left(1-x^2\right)+\log\left(1+x^2\right)\right)$$

input
```
Integrate[x/(1 - 2*x^4 + x^8),x]
```

output
```
((-2*x^2)/(-1 + x^4) - Log[1 - x^2] + Log[1 + x^2])/8
```

3.295.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1380, 807, 215, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{x^8-2x^4+1}dx$$
$$\Big\downarrow \text{1380}$$
$$\int \frac{x}{(1-x^4)^2}dx$$
$$\Big\downarrow \text{807}$$
$$\frac{1}{2}\int \frac{1}{(1-x^4)^2}dx^2$$
$$\Big\downarrow \text{215}$$
$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{1-x^4}dx^2+\frac{x^2}{2(1-x^4)}\right)$$
$$\Big\downarrow \text{219}$$
$$\frac{1}{2}\left(\frac{\operatorname{arctanh}(x^2)}{2}+\frac{x^2}{2(1-x^4)}\right)$$

input
```
Int[x/(1 - 2*x^4 + x^8),x]
```

output
```
(x^2/(2*(1 - x^4)) + ArcTanh[x^2]/2)/2
```

3.295.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

3.295.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.20

method	result	size
risch	$-\frac{x^{2}}{4\left(x^{4}-1\right)}-\frac{\ln\left(x^{2}-1\right)}{8}+\frac{\ln\left(x^{2}+1\right)}{8}$	30
norman	$-\frac{x^{2}}{4\left(x^{4}-1\right)}-\frac{\ln\left(x-1\right)}{8}-\frac{\ln\left(x+1\right)}{8}+\frac{\ln\left(x^{2}+1\right)}{8}$	34
default	$-\frac{1}{8\left(x^{2}+1\right)}+\frac{\ln\left(x^{2}+1\right)}{8}-\frac{1}{8\left(x^{2}-1\right)}-\frac{\ln\left(x^{2}-1\right)}{8}$	36
parallelrisch	$-\frac{\ln\left(x-1\right)x^{4}+\ln\left(x+1\right)x^{4}-\ln\left(x^{2}+1\right)x^{4}+2x^{2}-\ln\left(x-1\right)-\ln\left(x+1\right)+\ln\left(x^{2}+1\right)}{8\left(x^{4}-1\right)}$	61

input
```
int(x/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/4*x^2/(x^4-1)-1/8*ln(x^2-1)+1/8*ln(x^2+1)
```

3.295.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. 2(19) = 38.

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.60

$$\int \frac{x}{1-2x^4+x^8}\,dx = -\frac{2\,x^2-(x^4-1)\log(x^2+1)+(x^4-1)\log(x^2-1)}{8\,(x^4-1)}$$

input
```
integrate(x/(x^8-2*x^4+1),x, algorithm="fricas")
```

output
```
-1/8*(2*x^2 - (x^4 - 1)*log(x^2 + 1) + (x^4 - 1)*log(x^2 - 1))/(x^4 - 1)
```

3.295.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{x}{1-2x^4+x^8}\,dx = -\frac{x^2}{4x^4-4}-\frac{\log(x^2-1)}{8}+\frac{\log(x^2+1)}{8}$$

input
```
integrate(x/(x**8-2*x**4+1),x)
```

output
```
-x**2/(4*x**4 - 4) - log(x**2 - 1)/8 + log(x**2 + 1)/8
```

3.295.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \frac{x}{1-2x^4+x^8}\,dx = -\frac{x^2}{4\,(x^4-1)}+\frac{1}{8}\log\left(x^2+1\right)-\frac{1}{8}\log\left(x^2-1\right)$$

input
```
integrate(x/(x^8-2*x^4+1),x, algorithm="maxima")
```

output
```
-1/4*x^2/(x^4 - 1) + 1/8*log(x^2 + 1) - 1/8*log(x^2 - 1)
```

3.295.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.20

$$\int \frac{x}{1-2x^4+x^8}\,dx = -\frac{x^2}{4(x^4-1)} + \frac{1}{8}\log\left(x^2+1\right) - \frac{1}{8}\log\left(\left|x^2-1\right|\right)$$

input
```
integrate(x/(x^8-2*x^4+1),x, algorithm="giac")
```

output
```
-1/4*x^2/(x^4 - 1) + 1/8*log(x^2 + 1) - 1/8*log(abs(x^2 - 1))
```

3.295.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{x}{1-2x^4+x^8}\,dx = \frac{\operatorname{atanh}(x^2)}{4} - \frac{x^2}{4\,(x^4-1)}$$

input
```
int(x/(x^8 - 2*x^4 + 1),x)
```

output
```
atanh(x^2)/4 - x^2/(4*(x^4 - 1))
```

3.296 $\int \frac{1}{x(1-2x^4+x^8)}\,dx$

3.296.1 Optimal result

Integrand size = 16, antiderivative size = 28

$$\int \frac{1}{x(1-2x^4+x^8)}\,dx = \frac{1}{4(1-x^4)} + \log(x) - \frac{1}{4}\log\left(1-x^4\right)$$

output
```
1/4/(-x^4+1)+ln(x)-1/4*ln(-x^4+1)
```

3.296.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{x(1-2x^4+x^8)}\,dx = -\frac{1}{4(-1+x^4)} + \log(x) - \frac{1}{4}\log\left(1-x^4\right)$$

input
```
Integrate[1/(x*(1 - 2*x^4 + x^8)),x]
```

output
```
-1/4*1/(-1 + x^4) + Log[x] - Log[1 - x^4]/4
```

3.296.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1380, 798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(x^8-2x^4+1)}dx$$

$$\Big\downarrow \text{1380}$$

$$\int \frac{1}{x(1-x^4)^2}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{1}{x^4(1-x^4)^2}dx^4$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{4}\int \left(\frac{1}{(x^4-1)^2}+\frac{1}{x^4}+\frac{1}{1-x^4}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\frac{1}{1-x^4}+\log\left(x^4\right)-\log\left(1-x^4\right)\right)$$

input
```
Int[1/(x*(1 - 2*x^4 + x^8)),x]
```

output
```
((1 - x^4)^(-1) + Log[x^4] - Log[1 - x^4])/4
```

3.296.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.296.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

method	result	size
risch	$-\frac{1}{4(x^4-1)}+\ln(x)-\frac{\ln(x^4-1)}{4}$	21
norman	$-\frac{1}{4(x^4-1)}-\frac{\ln(x-1)}{4}-\frac{\ln(x+1)}{4}-\frac{\ln(x^2+1)}{4}+\ln(x)$	33
default	$\ln(x)+\frac{1}{16x+16}-\frac{\ln(x+1)}{4}-\frac{\ln(x^2+1)}{4}+\frac{1}{8x^2+8}-\frac{1}{16(x-1)}-\frac{\ln(x-1)}{4}$	47
parallelrisch	$\frac{4\ln(x)x^4-\ln(x-1)x^4-\ln(x+1)x^4-\ln(x^2+1)x^4-1-4\ln(x)+\ln(x-1)+\ln(x+1)+\ln(x^2+1)}{4x^4-4}$	66

input
```
int(1/x/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/4/(x^4-1)+ln(x)-1/4*ln(x^4-1)
```

3.296.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.14

$$\int \frac{1}{x(1-2x^4+x^8)}\,dx = -\frac{(x^4-1)\log(x^4-1)-4(x^4-1)\log(x)+1}{4(x^4-1)}$$

input
```
integrate(1/x/(x^8-2*x^4+1),x, algorithm="fricas")
```

output
```
-1/4*((x^4 - 1)*log(x^4 - 1) - 4*(x^4 - 1)*log(x) + 1)/(x^4 - 1)
```

3.296.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.68

$$\int \frac{1}{x\left(1-2x^4+x^8\right)}\,dx=\log\left(x\right)-\frac{\log\left(x^4-1\right)}{4}-\frac{1}{4x^4-4}$$

input
```
integrate(1/x/(x**8-2*x**4+1),x)
```

output
```
log(x) - log(x**4 - 1)/4 - 1/(4*x**4 - 4)
```

3.296.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{1}{x\left(1-2x^4+x^8\right)}\,dx=-\frac{1}{4\left(x^4-1\right)}-\frac{1}{4}\log\left(x^4-1\right)+\frac{1}{4}\log\left(x^4\right)$$

input
```
integrate(1/x/(x^8-2*x^4+1),x, algorithm="maxima")
```

output
```
-1/4/(x^4 - 1) - 1/4*log(x^4 - 1) + 1/4*log(x^4)
```

3.296.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{x\left(1-2x^4+x^8\right)}\,dx=\frac{x^4-2}{4\left(x^4-1\right)}+\frac{1}{4}\log\left(x^4\right)-\frac{1}{4}\log\left(\left|x^4-1\right|\right)$$

input
```
integrate(1/x/(x^8-2*x^4+1),x, algorithm="giac")
```

output
```
1/4*(x^4 - 2)/(x^4 - 1) + 1/4*log(x^4) - 1/4*log(abs(x^4 - 1))
```

3.296.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \frac{1}{x(1-2x^4+x^8)}\,dx = \ln(x) - \frac{\ln(x^4-1)}{4} - \frac{1}{4(x^4-1)}$$

input
```
int(1/(x*(x^8 - 2*x^4 + 1)),x)
```

output
```
log(x) - log(x^4 - 1)/4 - 1/(4*(x^4 - 1))
```

3.297 $\int \frac{1}{x^3(1-2x^4+x^8)}\,dx$

3.297.1 Optimal result

Integrand size = 16, antiderivative size = 32

$$\int \frac{1}{x^3(1-2x^4+x^8)}\,dx = -\frac{3}{4x^2}+\frac{1}{4x^2(1-x^4)}+\frac{3\text{arctanh}(x^2)}{4}$$

output
```
-3/4/x^2+1/4/x^2/(-x^4+1)+3/4*arctanh(x^2)
```

3.297.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.28

$$\int \frac{1}{x^3(1-2x^4+x^8)}\,dx = \frac{1}{8}\left(\frac{4-6x^4}{x^2(-1+x^4)}-3\log\left(1-x^2\right)+3\log\left(1+x^2\right)\right)$$

input
```
Integrate[1/(x^3*(1 - 2*x^4 + x^8)),x]
```

output
```
((4 - 6*x^4)/(x^2*(-1 + x^4)) - 3*Log[1 - x^2] + 3*Log[1 + x^2])/8
```

3.297.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1380, 807, 253, 264, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (x^8-2x^4+1)} \, dx$$

$$\Big\downarrow \text{1380}$$

$$\int \frac{1}{x^3 (1-x^4)^2}dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \frac{1}{x^4 (1-x^4)^2}dx^2$$

$$\Big\downarrow \text{253}$$

$$\frac{1}{2}\left(\frac{3}{2}\int \frac{1}{x^4 (1-x^4)}dx^2+\frac{1}{2x^2 (1-x^4)}\right)$$

$$\Big\downarrow \text{264}$$

$$\frac{1}{2}\left(\frac{3}{2}\left(\int \frac{1}{1-x^4}dx^2-\frac{1}{x^2}\right)+\frac{1}{2x^2 (1-x^4)}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{3}{2}\left(\text{arctanh}(x^2)-\frac{1}{x^2}\right)+\frac{1}{2x^2 (1-x^4)}\right)$$

input
```
Int[1/(x^3*(1 - 2*x^4 + x^8)),x]
```

output
```
(1/(2*x^2*(1 - x^4)) + (3*(-x^(-2) + ArcTanh[x^2]))/2)/2
```

3.297.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

3.297.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12

method	result	size
risch	$\frac{\frac{1}{2}-\frac{3x^4}{4}}{x^2(x^4-1)}-\frac{3\ln(x^2-1)}{8}+\frac{3\ln(x^2+1)}{8}$	36
norman	$\frac{\frac{1}{2}-\frac{3x^4}{4}}{x^2(x^4-1)}-\frac{3\ln(x-1)}{8}-\frac{3\ln(x+1)}{8}+\frac{3\ln(x^2+1)}{8}$	40
default	$-\frac{1}{2x^2}+\frac{1}{16x+16}-\frac{3\ln(x+1)}{8}+\frac{3\ln(x^2+1)}{8}-\frac{1}{8(x^2+1)}-\frac{1}{16(x-1)}-\frac{3\ln(x-1)}{8}$	50
parallelrisch	$-\frac{3\ln(x-1)x^6+3\ln(x+1)x^6-3\ln(x^2+1)x^6-4+6x^4-3\ln(x-1)x^2-3\ln(x+1)x^2+3\ln(x^2+1)x^2}{8x^2(x^4-1)}$	78

input
```
int(1/x^3/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
(1/2-3/4*x^4)/x^2/(x^4-1)-3/8*ln(x^2-1)+3/8*ln(x^2+1)
```

3.297.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(24) = 48.

Time = 0.23 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.69

$$\int \frac{1}{x^3\left(1-2x^4+x^8\right)}\,dx = -\frac{6\,x^4-3\left(x^6-x^2\right)\log\left(x^2+1\right)+3\left(x^6-x^2\right)\log\left(x^2-1\right)-4}{8\left(x^6-x^2\right)}$$

input
```
integrate(1/x^3/(x^8-2*x^4+1),x, algorithm="fricas")
```

output
```
-1/8*(6*x^4 - 3*(x^6 - x^2)*log(x^2 + 1) + 3*(x^6 - x^2)*log(x^2 - 1) - 4)
/(x^6 - x^2)
```

3.297.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12

$$\int \frac{1}{x^3\left(1-2x^4+x^8\right)}\,dx = \frac{2-3x^4}{4x^6-4x^2}-\frac{3\log\left(x^2-1\right)}{8}+\frac{3\log\left(x^2+1\right)}{8}$$

input
```
integrate(1/x**3/(x**8-2*x**4+1),x)
```

output
```
(2 - 3*x**4)/(4*x**6 - 4*x**2) - 3*log(x**2 - 1)/8 + 3*log(x**2 + 1)/8
```

3.297.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.16

$$\int \frac{1}{x^3(1-2x^4+x^8)}\,dx = -\frac{3\,x^4-2}{4\,(x^6-x^2)} + \frac{3}{8}\,\log\left(x^2+1\right) - \frac{3}{8}\,\log\left(x^2-1\right)$$

input
```
integrate(1/x^3/(x^8-2*x^4+1),x, algorithm="maxima")
```

output
```
-1/4*(3*x^4 - 2)/(x^6 - x^2) + 3/8*log(x^2 + 1) - 3/8*log(x^2 - 1)
```

3.297.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.19

$$\int \frac{1}{x^3(1-2x^4+x^8)}\,dx = -\frac{3\,x^4-2}{4\,(x^6-x^2)} + \frac{3}{8}\,\log\left(x^2+1\right) - \frac{3}{8}\,\log\left(\left|x^2-1\right|\right)$$

input
```
integrate(1/x^3/(x^8-2*x^4+1),x, algorithm="giac")
```

output
```
-1/4*(3*x^4 - 2)/(x^6 - x^2) + 3/8*log(x^2 + 1) - 3/8*log(abs(x^2 - 1))
```

3.297.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^3(1-2x^4+x^8)}\,dx = \frac{3\,\mathrm{atanh}(x^2)}{4} + \frac{\frac{3\,x^4}{4}-\frac{1}{2}}{x^2-x^6}$$

input
```
int(1/(x^3*(x^8 - 2*x^4 + 1)),x)
```

output
```
(3*atanh(x^2))/4 + ((3*x^4)/4 - 1/2)/(x^2 - x^6)
```

3.298 $\int \frac{1}{x^5(1-2x^4+x^8)}\,dx$

3.298.1 Optimal result

Integrand size = 16, antiderivative size = 37

$$\int \frac{1}{x^5\left(1-2x^4+x^8\right)}\,dx=-\frac{1}{4x^4}+\frac{1}{4\left(1-x^4\right)}+2\log(x)-\frac{1}{2}\log\left(1-x^4\right)$$

output
```
-1/4/x^4+1/4/(-x^4+1)+2*ln(x)-1/2*ln(-x^4+1)
```

3.298.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^5\left(1-2x^4+x^8\right)}\,dx=-\frac{1}{4x^4}-\frac{1}{4\left(-1+x^4\right)}+2\log(x)-\frac{1}{2}\log\left(1-x^4\right)$$

input
```
Integrate[1/(x^5*(1 - 2*x^4 + x^8)),x]
```

output
```
-1/4*1/x^4 - 1/(4*(-1 + x^4)) + 2*Log[x] - Log[1 - x^4]/2
```

3.298.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1380, 798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 (x^8-2x^4+1)}dx$$

$$\Big\downarrow \ 1380$$

$$\int \frac{1}{x^5 (1-x^4)^2}dx$$

$$\Big\downarrow \ 798$$

$$\frac{1}{4}\int \frac{1}{x^8 (1-x^4)^2}dx^4$$

$$\Big\downarrow \ 54$$

$$\frac{1}{4}\int \left(\frac{2}{x^4}+\frac{1}{x^8}-\frac{2}{x^4-1}+\frac{1}{(x^4-1)^2}\right)dx^4$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{4}\left(\frac{1}{1-x^4}-\frac{1}{x^4}+2\log\left(x^4\right)-2\log\left(1-x^4\right)\right)$$

input
```
Int[1/(x^5*(1 - 2*x^4 + x^8)),x]
```

output
```
(-x^(-4) + (1 - x^4)^(-1) + 2*Log[x^4] - 2*Log[1 - x^4])/4
```

3.298.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.298.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

method	result	size
risch	$\frac{\frac{1}{4}-\frac{x^4}{2}}{x^4(x^4-1)}+2\ln(x)-\frac{\ln(x^4-1)}{2}$	32
norman	$\frac{\frac{1}{4}-\frac{x^4}{2}}{x^4(x^4-1)}+2\ln(x)-\frac{\ln(x-1)}{2}-\frac{\ln(x+1)}{2}-\frac{\ln(x^2+1)}{2}$	44
default	$-\frac{1}{4x^4}+2\ln(x)+\frac{1}{16x+16}-\frac{\ln(x+1)}{2}-\frac{\ln(x^2+1)}{2}+\frac{1}{8x^2+8}-\frac{1}{16(x-1)}-\frac{\ln(x-1)}{2}$	54
parallelrisch	$\frac{8\ln(x)x^8-2\ln(x-1)x^8-2\ln(x+1)x^8-2\ln(x^2+1)x^8+1-8\ln(x)x^4+2\ln(x-1)x^4+2\ln(x+1)x^4+2\ln(x^2+1)x^4-2x^4}{4x^4(x^4-1)}$	92

input
```
int(1/x^5/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
(1/4-1/2*x^4)/x^4/(x^4-1)+2*ln(x)-1/2*ln(x^4-1)
```

3.298.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.35

$$\int \frac{1}{x^5(1-2x^4+x^8)}\,dx = -\frac{2x^4+2(x^8-x^4)\log(x^4-1)-8(x^8-x^4)\log(x)-1}{4(x^8-x^4)}$$

input
```
integrate(1/x^5/(x^8-2*x^4+1),x, algorithm="fricas")
```

output
```
-1/4*(2*x^4 + 2*(x^8 - x^4)*log(x^4 - 1) - 8*(x^8 - x^4)*log(x) - 1)/(x^8
- x^4)
```

3.298.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^5 (1-2x^4+x^8)}\,dx = \frac{1-2x^4}{4x^8-4x^4} + 2\log(x) - \frac{\log(x^4-1)}{2}$$

input
```
integrate(1/x**5/(x**8-2*x**4+1),x)
```

output
```
(1 - 2*x**4)/(4*x**8 - 4*x**4) + 2*log(x) - log(x**4 - 1)/2
```

3.298.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^5 (1-2x^4+x^8)}\,dx = -\frac{2\,x^4-1}{4\,(x^8-x^4)} - \frac{1}{2}\,\log\left(x^4-1\right) + \frac{1}{2}\,\log\left(x^4\right)$$

input
```
integrate(1/x^5/(x^8-2*x^4+1),x, algorithm="maxima")
```

output
```
-1/4*(2*x^4 - 1)/(x^8 - x^4) - 1/2*log(x^4 - 1) + 1/2*log(x^4)
```

3.298.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^5 (1-2x^4+x^8)}\,dx = -\frac{2\,x^4-1}{4\,(x^8-x^4)} + \frac{1}{2}\,\log\left(x^4\right) - \frac{1}{2}\,\log\left(\left|x^4-1\right|\right)$$

input
```
integrate(1/x^5/(x^8-2*x^4+1),x, algorithm="giac")
```

output
```
-1/4*(2*x^4 - 1)/(x^8 - x^4) + 1/2*log(x^4) - 1/2*log(abs(x^4 - 1))
```

3.298.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^5\left(1-2x^4+x^8\right)}\,dx = 2\,\ln\left(x\right)-\frac{\ln\left(x^4-1\right)}{2}+\frac{\frac{x^4}{2}-\frac{1}{4}}{x^4-x^8}$$

input
```
int(1/(x^5*(x^8 - 2*x^4 + 1)),x)
```

output
```
2*log(x) - log(x^4 - 1)/2 + (x^4/2 - 1/4)/(x^4 - x^8)
```

3.299 $\int \frac{1}{x^7(1-2x^4+x^8)}\,dx$

3.299.1 Optimal result

Integrand size = 16, antiderivative size = 39

$$\int \frac{1}{x^7(1-2x^4+x^8)}\,dx = -\frac{5}{12x^6}-\frac{5}{4x^2}+\frac{1}{4x^6(1-x^4)}+\frac{5\mathrm{arctanh}(x^2)}{4}$$

output
```
-5/12/x^6-5/4/x^2+1/4/x^6/(-x^4+1)+5/4*arctanh(x^2)
```

3.299.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.26

$$\int \frac{1}{x^7(1-2x^4+x^8)}\,dx = -\frac{1}{6x^6}-\frac{1}{x^2}-\frac{x^2}{4(-1+x^4)}-\frac{5}{8}\log\left(1-x^2\right)+\frac{5}{8}\log\left(1+x^2\right)$$

input
```
Integrate[1/(x^7*(1 - 2*x^4 + x^8)),x]
```

output
```
-1/6*1/x^6 - x^(-2) - x^2/(4*(-1 + x^4)) - (5*Log[1 - x^2])/8 + (5*Log[1 +
 x^2])/8
```

3.299.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1380, 807, 253, 264, 264, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7 (x^8-2x^4+1)}dx$$
$$\Big\downarrow \text{1380}$$
$$\int \frac{1}{x^7 (1-x^4)^2}dx$$
$$\Big\downarrow \text{807}$$
$$\frac{1}{2}\int \frac{1}{x^8 (1-x^4)^2}dx^2$$
$$\Big\downarrow \text{253}$$
$$\frac{1}{2}\left(\frac{5}{2}\int \frac{1}{x^8 (1-x^4)}dx^2+\frac{1}{2x^6 (1-x^4)}\right)$$
$$\Big\downarrow \text{264}$$
$$\frac{1}{2}\left(\frac{5}{2}\left(\int \frac{1}{x^4 (1-x^4)}dx^2-\frac{1}{3x^6}\right)+\frac{1}{2x^6 (1-x^4)}\right)$$
$$\Big\downarrow \text{264}$$
$$\frac{1}{2}\left(\frac{5}{2}\left(\int \frac{1}{1-x^4}dx^2-\frac{1}{3x^6}-\frac{1}{x^2}\right)+\frac{1}{2x^6 (1-x^4)}\right)$$
$$\Big\downarrow \text{219}$$
$$\frac{1}{2}\left(\frac{5}{2}\left(\text{arctanh}(x^2)-\frac{1}{3x^6}-\frac{1}{x^2}\right)+\frac{1}{2x^6 (1-x^4)}\right)$$

input
```
Int[1/(x^7*(1 - 2*x^4 + x^8)),x]
```

output
```
(1/(2*x^6*(1 - x^4)) + (5*(-1/3*1/x^6 - x^(-2) + ArcTanh[x^2]))/2)/2
```

3.299.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

3.299.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

method	result	size
risch	$\frac{\frac{1}{6}+\frac{5}{6}x^4-\frac{5}{4}x^8}{x^6(x^4-1)}+\frac{5\ln(x^2+1)}{8}-\frac{5\ln(x^2-1)}{8}$	41
norman	$\frac{\frac{1}{6}+\frac{5}{6}x^4-\frac{5}{4}x^8}{x^6(x^4-1)}-\frac{5\ln(x-1)}{8}-\frac{5\ln(x+1)}{8}+\frac{5\ln(x^2+1)}{8}$	45
default	$-\frac{1}{6x^6}-\frac{1}{x^2}+\frac{1}{16x+16}-\frac{5\ln(x+1)}{8}+\frac{5\ln(x^2+1)}{8}-\frac{1}{8(x^2+1)}-\frac{1}{16(x-1)}-\frac{5\ln(x-1)}{8}$	55
parallelrisch	$-\frac{15\ln(x-1)x^{10}+15\ln(x+1)x^{10}-15\ln(x^2+1)x^{10}-4+30x^8-15\ln(x-1)x^6-15\ln(x+1)x^6+15\ln(x^2+1)x^6-20x^4}{24x^6(x^4-1)}$	83

input
```
int(1/x^7/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
(1/6+5/6*x^4-5/4*x^8)/x^6/(x^4-1)+5/8*ln(x^2+1)-5/8*ln(x^2-1)
```

3.299.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(29) = 58.

Time = 0.23 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.51

$$\int \frac{1}{x^7 (1 - 2x^4 + x^8)} \, dx = -\frac{30\, x^8 - 20\, x^4 - 15\, (x^{10} - x^6) \log (x^2 + 1) + 15\, (x^{10} - x^6) \log (x^2 - 1) - 4}{24\, (x^{10} - x^6)}$$

input
```
integrate(1/x^7/(x^8-2*x^4+1),x, algorithm="fricas")
```

output
```
-1/24*(30*x^8 - 20*x^4 - 15*(x^10 - x^6)*log(x^2 + 1) + 15*(x^10 - x^6)*lo
g(x^2 - 1) - 4)/(x^10 - x^6)
```

3.299.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^7 (1 - 2x^4 + x^8)} \, dx = -\frac{5 \log (x^2 - 1)}{8} + \frac{5 \log (x^2 + 1)}{8} + \frac{-15x^8 + 10x^4 + 2}{12x^{10} - 12x^6}$$

input
```
integrate(1/x**7/(x**8-2*x**4+1),x)
```

output
```
-5*log(x**2 - 1)/8 + 5*log(x**2 + 1)/8 + (-15*x**8 + 10*x**4 + 2)/(12*x**1
0 - 12*x**6)
```

3.299.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^7 (1-2x^4+x^8)}\,dx = -\frac{15\,x^8-10\,x^4-2}{12\,(x^{10}-x^6)} + \frac{5}{8}\,\log\left(x^2+1\right) - \frac{5}{8}\,\log\left(x^2-1\right)$$

input
```
integrate(1/x^7/(x^8-2*x^4+1),x, algorithm="maxima")
```

output
```
-1/12*(15*x^8 - 10*x^4 - 2)/(x^10 - x^6) + 5/8*log(x^2 + 1) - 5/8*log(x^2
- 1)
```

3.299.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^7 (1-2x^4+x^8)}\,dx = -\frac{x^2}{4\,(x^4-1)} - \frac{6\,x^4+1}{6\,x^6} + \frac{5}{8}\,\log\left(x^2+1\right) - \frac{5}{8}\,\log\left(\left|x^2-1\right|\right)$$

input
```
integrate(1/x^7/(x^8-2*x^4+1),x, algorithm="giac")
```

output
```
-1/4*x^2/(x^4 - 1) - 1/6*(6*x^4 + 1)/x^6 + 5/8*log(x^2 + 1) - 5/8*log(abs(
x^2 - 1))
```

3.299.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^7 (1-2x^4+x^8)}\,dx = \frac{5\,\mathrm{atanh}(x^2)}{4} - \frac{-\frac{5\,x^8}{4}+\frac{5\,x^4}{6}+\frac{1}{6}}{x^6-x^{10}}$$

input
```
int(1/(x^7*(x^8 - 2*x^4 + 1)),x)
```

output
```
(5*atanh(x^2))/4 - ((5*x^4)/6 - (5*x^8)/4 + 1/6)/(x^6 - x^10)
```

3.300 $\int \frac{x^8}{1-2x^4+x^8}\,dx$

3.300.1 Optimal result

Integrand size = 16, antiderivative size = 34

$$\int \frac{x^8}{1-2x^4+x^8}\,dx=\frac{5x}{4}+\frac{x^5}{4\left(1-x^4\right)}-\frac{5\arctan(x)}{8}-\frac{5\text{arctanh}(x)}{8}$$

output
```
5/4*x+1/4*x^5/(-x^4+1)-5/8*arctan(x)-5/8*arctanh(x)
```

3.300.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12

$$\int \frac{x^8}{1-2x^4+x^8}\,dx=x-\frac{x}{4\left(-1+x^4\right)}-\frac{5\arctan(x)}{8}+\frac{5}{16}\log(1-x)-\frac{5}{16}\log(1+x)$$

input
```
Integrate[x^8/(1 - 2*x^4 + x^8),x]
```

output
```
x - x/(4*(-1 + x^4)) - (5*ArcTan[x])/8 + (5*Log[1 - x])/16 - (5*Log[1 + x]
)/16
```

3.300.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1380, 817, 843, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{x^8-2x^4+1}dx$$
$$\Big\downarrow \text{ 1380}$$
$$\int \frac{x^8}{(1-x^4)^2}dx$$
$$\Big\downarrow \text{ 817}$$
$$\frac{x^5}{4(1-x^4)}-\frac{5}{4}\int \frac{x^4}{1-x^4}dx$$
$$\Big\downarrow \text{ 843}$$
$$\frac{x^5}{4(1-x^4)}-\frac{5}{4}\left(\int \frac{1}{1-x^4}dx-x\right)$$
$$\Big\downarrow \text{ 756}$$
$$\frac{x^5}{4(1-x^4)}-\frac{5}{4}\left(\frac{1}{2}\int \frac{1}{1-x^2}dx+\frac{1}{2}\int \frac{1}{x^2+1}dx-x\right)$$
$$\Big\downarrow \text{ 216}$$
$$\frac{x^5}{4(1-x^4)}-\frac{5}{4}\left(\frac{1}{2}\int \frac{1}{1-x^2}dx+\frac{\arctan(x)}{2}-x\right)$$
$$\Big\downarrow \text{ 219}$$
$$\frac{x^5}{4(1-x^4)}-\frac{5}{4}\left(\frac{\arctan(x)}{2}+\frac{\text{arctanh}(x)}{2}-x\right)$$

input
```
Int[x^8/(1 - 2*x^4 + x^8),x]
```

output
```
x^5/(4*(1 - x^4)) - (5*(-x + ArcTan[x]/2 + ArcTanh[x]/2))/4
```

3.300.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

3.300.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

method	result	size
risch	$x-\frac{x}{4(x^4-1)}+\frac{5\ln(x-1)}{16}-\frac{5\arctan(x)}{8}-\frac{5\ln(x+1)}{16}$	29
default	$x-\frac{1}{16(x+1)}-\frac{5\ln(x+1)}{16}+\frac{x}{8x^2+8}-\frac{5\arctan(x)}{8}-\frac{1}{16(x-1)}+\frac{5\ln(x-1)}{16}$	43
parallelrisch	$\frac{5i\ln(x-i)x^4-5i\ln(x+i)x^4+5\ln(x-1)x^4-5\ln(x+1)x^4+16x^5-5i\ln(x-i)+5i\ln(x+i)-5\ln(x-1)+5\ln(x+1)-20x}{16x^4-16}$	87

input
```
int(x^8/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
x-1/4*x/(x^4-1)+5/16*ln(x-1)-5/8*arctan(x)-5/16*ln(x+1)
```

3.300.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(24) = 48.

Time = 0.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.44

$$\int \frac{x^8}{1-2x^4+x^8}\,dx = \frac{16\,x^5-10\,(x^4-1)\arctan(x)-5\,(x^4-1)\log(x+1)+5\,(x^4-1)\log(x-1)-20\,x}{16\,(x^4-1)}$$

input
```
integrate(x^8/(x^8-2*x^4+1),x, algorithm="fricas")
```

output
```
1/16*(16*x^5 - 10*(x^4 - 1)*arctan(x) - 5*(x^4 - 1)*log(x + 1) + 5*(x^4 -
1)*log(x - 1) - 20*x)/(x^4 - 1)
```

3.300.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{x^8}{1-2x^4+x^8}\,dx = x-\frac{x}{4x^4-4}+\frac{5\log(x-1)}{16}-\frac{5\log(x+1)}{16}-\frac{5\,\mathrm{atan}(x)}{8}$$

input
```
integrate(x**8/(x**8-2*x**4+1),x)
```

output
```
x - x/(4*x**4 - 4) + 5*log(x - 1)/16 - 5*log(x + 1)/16 - 5*atan(x)/8
```

3.300.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int \frac{x^8}{1-2x^4+x^8}\,dx = x - \frac{x}{4\,(x^4-1)} - \frac{5}{8}\arctan(x) - \frac{5}{16}\log(x+1) + \frac{5}{16}\log(x-1)$$

input
```
integrate(x^8/(x^8-2*x^4+1),x, algorithm="maxima")
```

output
```
x - 1/4*x/(x^4 - 1) - 5/8*arctan(x) - 5/16*log(x + 1) + 5/16*log(x - 1)
```

3.300.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{x^8}{1-2x^4+x^8}\,dx = x - \frac{x}{4\,(x^4-1)} - \frac{5}{8}\arctan(x) - \frac{5}{16}\log(|x+1|) + \frac{5}{16}\log(|x-1|)$$

input
```
integrate(x^8/(x^8-2*x^4+1),x, algorithm="giac")
```

output
```
x - 1/4*x/(x^4 - 1) - 5/8*arctan(x) - 5/16*log(abs(x + 1)) + 5/16*log(abs(
x - 1))
```

3.300.9 Mupad [B] (verification not implemented)

Time = 8.37 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{x^8}{1-2x^4+x^8}\,dx = x - \frac{5\,\mathrm{atan}(x)}{8} - \frac{x}{4\,(x^4-1)} + \frac{\mathrm{atan}(x\,1\mathrm{i})\,5\mathrm{i}}{8}$$

input
```
int(x^8/(x^8 - 2*x^4 + 1),x)
```

output
```
x + (atan(x*1i)*5i)/8 - (5*atan(x))/8 - x/(4*(x^4 - 1))
```

3.301 $\int \frac{x^6}{1-2x^4+x^8} dx$

3.301.1 Optimal result

Integrand size = 16, antiderivative size = 29

$$\int \frac{x^6}{1-2x^4+x^8} \, dx = \frac{x^3}{4(1-x^4)} + \frac{3\arctan(x)}{8} - \frac{3\text{arctanh}(x)}{8}$$

output
```
1/4*x^3/(-x^4+1)+3/8*arctan(x)-3/8*arctanh(x)
```

3.301.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.21

$$\int \frac{x^6}{1-2x^4+x^8} \, dx = \frac{1}{16}\left(-\frac{4x^3}{-1+x^4} + 6\arctan(x) + 3\log(1-x) - 3\log(1+x)\right)$$

input
```
Integrate[x^6/(1 - 2*x^4 + x^8),x]
```

output
```
((-4*x^3)/(-1 + x^4) + 6*ArcTan[x] + 3*Log[1 - x] - 3*Log[1 + x])/16
```

3.301.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.17, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1380, 817, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{x^8-2x^4+1}dx$$
$$\Big\downarrow \text{1380}$$
$$\int \frac{x^6}{(1-x^4)^2}dx$$
$$\Big\downarrow \text{817}$$
$$\frac{x^3}{4(1-x^4)}-\frac{3}{4}\int \frac{x^2}{1-x^4}dx$$
$$\Big\downarrow \text{827}$$
$$\frac{x^3}{4(1-x^4)}-\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{1-x^2}dx-\frac{1}{2}\int \frac{1}{x^2+1}dx\right)$$
$$\Big\downarrow \text{216}$$
$$\frac{x^3}{4(1-x^4)}-\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{1-x^2}dx-\frac{\arctan(x)}{2}\right)$$
$$\Big\downarrow \text{219}$$
$$\frac{x^3}{4(1-x^4)}-\frac{3}{4}\left(\frac{\text{arctanh}(x)}{2}-\frac{\arctan(x)}{2}\right)$$

input
```
Int[x^6/(1 - 2*x^4 + x^8),x]
```

output
```
x^3/(4*(1 - x^4)) - (3*(-1/2*ArcTan[x] + ArcTanh[x]/2))/4
```

3.301.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

3.301.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

method	result	size
risch	$-\frac{x^{3}}{4(x^{4}-1)}+\frac{3\arctan(x)}{8}+\frac{3\ln(x-1)}{16}-\frac{3\ln(x+1)}{16}$	30
default	$-\frac{1}{16(x+1)}-\frac{3\ln(x+1)}{16}-\frac{x}{8(x^{2}+1)}+\frac{3\arctan(x)}{8}-\frac{1}{16(x-1)}+\frac{3\ln(x-1)}{16}$	42
parallelrisch	$\frac{-3i\ln(x-i)x^{4}+3i\ln(x+i)x^{4}+3\ln(x-1)x^{4}-3\ln(x+1)x^{4}-4x^{3}+3i\ln(x-i)-3i\ln(x+i)-3\ln(x-1)+3\ln(x+1)}{16x^{4}-16}$	84

input
```
int(x^6/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/4*x^3/(x^4-1)+3/8*arctan(x)+3/16*ln(x-1)-3/16*ln(x+1)
```

3.301.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(21) = 42.

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.59

$$\int \frac{x^6}{1-2x^4+x^8}\,dx = -\frac{4\,x^3-6\,(x^4-1)\arctan(x)+3\,(x^4-1)\log(x+1)-3\,(x^4-1)\log(x-1)}{16\,(x^4-1)}$$

input
```
integrate(x^6/(x^8-2*x^4+1),x, algorithm="fricas")
```

output
```
-1/16*(4*x^3 - 6*(x^4 - 1)*arctan(x) + 3*(x^4 - 1)*log(x + 1) - 3*(x^4 - 1
)*log(x - 1))/(x^4 - 1)
```

3.301.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.10

$$\int \frac{x^6}{1-2x^4+x^8}\,dx = -\frac{x^3}{4x^4-4}+\frac{3\log(x-1)}{16}-\frac{3\log(x+1)}{16}+\frac{3\,\mathrm{atan}(x)}{8}$$

input
```
integrate(x**6/(x**8-2*x**4+1),x)
```

output
```
-x**3/(4*x**4 - 4) + 3*log(x - 1)/16 - 3*log(x + 1)/16 + 3*atan(x)/8
```

3.301.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{x^6}{1-2x^4+x^8}\,dx = -\frac{x^3}{4\,(x^4-1)}+\frac{3}{8}\arctan(x)-\frac{3}{16}\log(x+1)+\frac{3}{16}\log(x-1)$$

input
```
integrate(x^6/(x^8-2*x^4+1),x, algorithm="maxima")
```

output
```
-1/4*x^3/(x^4 - 1) + 3/8*arctan(x) - 3/16*log(x + 1) + 3/16*log(x - 1)
```

3.301.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{x^6}{1-2x^4+x^8}\,dx = -\frac{x^3}{4\,(x^4-1)} + \frac{3}{8}\,\arctan\left(x\right) - \frac{3}{16}\,\log\left(\left|x+1\right|\right) + \frac{3}{16}\,\log\left(\left|x-1\right|\right)$$

input
```
integrate(x^6/(x^8-2*x^4+1),x, algorithm="giac")
```

output
```
-1/4*x^3/(x^4 - 1) + 3/8*arctan(x) - 3/16*log(abs(x + 1)) + 3/16*log(abs(x
 - 1))
```

3.301.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{x^6}{1-2x^4+x^8}\,dx = \frac{3\,\mathrm{atan}(x)}{8} - \frac{3\,\mathrm{atanh}(x)}{8} - \frac{x^3}{4\,\left(x^4-1\right)}$$

input
```
int(x^6/(x^8 - 2*x^4 + 1),x)
```

output
```
(3*atan(x))/8 - (3*atanh(x))/8 - x^3/(4*(x^4 - 1))
```

3.302 $\int \frac{x^4}{1-2x^4+x^8}\,dx$

3.302.1 Optimal result

Integrand size = 16, antiderivative size = 27

$$\int \frac{x^4}{1-2x^4+x^8}\,dx=\frac{x}{4(1-x^4)}-\frac{\arctan(x)}{8}-\frac{\operatorname{arctanh}(x)}{8}$$

output
```
1/4*x/(-x^4+1)-1/8*arctan(x)-1/8*arctanh(x)
```

3.302.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int \frac{x^4}{1-2x^4+x^8}\,dx=\frac{1}{16}\left(-\frac{4x}{-1+x^4}-2\arctan(x)+\log(1-x)-\log(1+x)\right)$$

input
```
Integrate[x^4/(1 - 2*x^4 + x^8),x]
```

output
```
((-4*x)/(-1 + x^4) - 2*ArcTan[x] + Log[1 - x] - Log[1 + x])/16
```

3.302.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1380, 817, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{x^8-2x^4+1}dx$$
$$\Big\downarrow \text{1380}$$
$$\int \frac{x^4}{(1-x^4)^2}dx$$
$$\Big\downarrow \text{817}$$
$$\frac{x}{4(1-x^4)}-\frac{1}{4}\int \frac{1}{1-x^4}dx$$
$$\Big\downarrow \text{756}$$
$$\frac{1}{4}\left(-\frac{1}{2}\int \frac{1}{1-x^2}dx-\frac{1}{2}\int \frac{1}{x^2+1}dx\right)+\frac{x}{4(1-x^4)}$$
$$\Big\downarrow \text{216}$$
$$\frac{1}{4}\left(-\frac{1}{2}\int \frac{1}{1-x^2}dx-\frac{\arctan(x)}{2}\right)+\frac{x}{4(1-x^4)}$$
$$\Big\downarrow \text{219}$$
$$\frac{1}{4}\left(-\frac{\arctan(x)}{2}-\frac{\text{arctanh}(x)}{2}\right)+\frac{x}{4(1-x^4)}$$

input
```
Int[x^4/(1 - 2*x^4 + x^8),x]
```

output
```
x/(4*(1 - x^4)) + (-1/2*ArcTan[x] - ArcTanh[x]/2)/4
```

3.302.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

3.302.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

method	result	size
risch	$-\frac{x}{4(x^4-1)}-\frac{\arctan(x)}{8}-\frac{\ln(x+1)}{16}+\frac{\ln(x-1)}{16}$	28
default	$-\frac{1}{16(x+1)}-\frac{\ln(x+1)}{16}+\frac{x}{8x^2+8}-\frac{\arctan(x)}{8}-\frac{1}{16(x-1)}+\frac{\ln(x-1)}{16}$	42
parallelrisch	$-\frac{i\ln(x+i)x^4-i\ln(x-i)x^4-\ln(x-1)x^4+\ln(x+1)x^4-i\ln(x+i)+i\ln(x-i)+\ln(x-1)-\ln(x+1)+4x}{16(x^4-1)}$	79

input
```
int(x^4/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/4*x/(x^4-1)-1/8*arctan(x)-1/16*ln(x+1)+1/16*ln(x-1)
```

3.302.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(19) = 38$.

Time = 0.24 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.59

$$\int \frac{x^4}{1-2x^4+x^8}\,dx = -\frac{2\,(x^4-1)\arctan(x)+(x^4-1)\log(x+1)-(x^4-1)\log(x-1)+4\,x}{16\,(x^4-1)}$$

input
```
integrate(x^4/(x^8-2*x^4+1),x, algorithm="fricas")
```

output
```
-1/16*(2*(x^4 - 1)*arctan(x) + (x^4 - 1)*log(x + 1) - (x^4 - 1)*log(x - 1)
 + 4*x)/(x^4 - 1)
```

3.302.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{x^4}{1-2x^4+x^8}\,dx = -\frac{x}{4x^4-4}+\frac{\log(x-1)}{16}-\frac{\log(x+1)}{16}-\frac{\operatorname{atan}(x)}{8}$$

input
```
integrate(x**4/(x**8-2*x**4+1),x)
```

output
```
-x/(4*x**4 - 4) + log(x - 1)/16 - log(x + 1)/16 - atan(x)/8
```

3.302.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{1-2x^4+x^8}\,dx = -\frac{x}{4\,(x^4-1)}-\frac{1}{8}\arctan(x)-\frac{1}{16}\log(x+1)+\frac{1}{16}\log(x-1)$$

input
```
integrate(x^4/(x^8-2*x^4+1),x, algorithm="maxima")
```

output
```
-1/4*x/(x^4 - 1) - 1/8*arctan(x) - 1/16*log(x + 1) + 1/16*log(x - 1)
```

3.302.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^4}{1-2x^4+x^8}\,dx = -\frac{x}{4\left(x^4-1\right)} - \frac{1}{8}\arctan\left(x\right) - \frac{1}{16}\log\left(\left|x+1\right|\right) + \frac{1}{16}\log\left(\left|x-1\right|\right)$$

input
```
integrate(x^4/(x^8-2*x^4+1),x, algorithm="giac")
```

output
```
-1/4*x/(x^4 - 1) - 1/8*arctan(x) - 1/16*log(abs(x + 1)) + 1/16*log(abs(x -
 1))
```

3.302.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{x^4}{1-2x^4+x^8}\,dx = -\frac{\operatorname{atan}(x)}{8} - \frac{\operatorname{atanh}(x)}{8} - \frac{x}{4\left(x^4-1\right)}$$

input
```
int(x^4/(x^8 - 2*x^4 + 1),x)
```

output
```
- atan(x)/8 - atanh(x)/8 - x/(4*(x^4 - 1))
```

3.303 $\int \frac{x^2}{1-2x^4+x^8} dx$

3.303.1 Optimal result

Integrand size = 16, antiderivative size = 29

$$\int \frac{x^2}{1-2x^4+x^8} dx = \frac{x^3}{4(1-x^4)} - \frac{\arctan(x)}{8} + \frac{\operatorname{arctanh}(x)}{8}$$

output
```
1/4*x^3/(-x^4+1)-1/8*arctan(x)+1/8*arctanh(x)
```

3.303.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.14

$$\int \frac{x^2}{1-2x^4+x^8} dx = \frac{1}{16}\left(-\frac{4x^3}{-1+x^4} - 2\arctan(x) - \log(1-x) + \log(1+x)\right)$$

input
```
Integrate[x^2/(1 - 2*x^4 + x^8),x]
```

output
```
((-4*x^3)/(-1 + x^4) - 2*ArcTan[x] - Log[1 - x] + Log[1 + x])/16
```

3.303.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.17, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1380, 819, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{x^8-2x^4+1}dx$$

$$\Big\downarrow \text{1380}$$

$$\int \frac{x^2}{(1-x^4)^2}dx$$

$$\Big\downarrow \text{819}$$

$$\frac{1}{4}\int \frac{x^2}{1-x^4}dx+\frac{x^3}{4(1-x^4)}$$

$$\Big\downarrow \text{827}$$

$$\frac{1}{4}\left(\frac{1}{2}\int \frac{1}{1-x^2}dx-\frac{1}{2}\int \frac{1}{x^2+1}dx\right)+\frac{x^3}{4(1-x^4)}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{4}\left(\frac{1}{2}\int \frac{1}{1-x^2}dx-\frac{\arctan(x)}{2}\right)+\frac{x^3}{4(1-x^4)}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{4}\left(\frac{\text{arctanh}(x)}{2}-\frac{\arctan(x)}{2}\right)+\frac{x^3}{4(1-x^4)}$$

input
```
Int[x^2/(1 - 2*x^4 + x^8),x]
```

output
```
x^3/(4*(1 - x^4)) + (-1/2*ArcTan[x] + ArcTanh[x]/2)/4
```

3.303.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

3.303.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

method	result	size
risch	$-\frac{x^3}{4(x^4-1)}-\frac{\arctan(x)}{8}-\frac{\ln(x-1)}{16}+\frac{\ln(x+1)}{16}$	30
default	$-\frac{1}{16(x+1)}+\frac{\ln(x+1)}{16}-\frac{x}{8(x^2+1)}-\frac{\arctan(x)}{8}-\frac{1}{16(x-1)}-\frac{\ln(x-1)}{16}$	42
parallelrisch	$-\frac{-i\ln(x-i)x^4+i\ln(x+i)x^4+\ln(x-1)x^4-\ln(x+1)x^4+4x^3+i\ln(x-i)-i\ln(x+i)-\ln(x-1)+\ln(x+1)}{16(x^4-1)}$	81

input
```
int(x^2/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/4*x^3/(x^4-1)-1/8*arctan(x)-1/16*ln(x-1)+1/16*ln(x+1)
```

3.303.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. 2(21) = 42.

Time = 0.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.55

$$\int \frac{x^2}{1-2x^4+x^8}\,dx = -\frac{4\,x^3+2\,(x^4-1)\arctan(x)-(x^4-1)\log(x+1)+(x^4-1)\log(x-1)}{16\,(x^4-1)}$$

input
```
integrate(x^2/(x^8-2*x^4+1),x, algorithm="fricas")
```

output
```
-1/16*(4*x^3 + 2*(x^4 - 1)*arctan(x) - (x^4 - 1)*log(x + 1) + (x^4 - 1)*lo
g(x - 1))/(x^4 - 1)
```

3.303.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{x^2}{1-2x^4+x^8}\,dx = -\frac{x^3}{4x^4-4} - \frac{\log(x-1)}{16} + \frac{\log(x+1)}{16} - \frac{\operatorname{atan}(x)}{8}$$

input
```
integrate(x**2/(x**8-2*x**4+1),x)
```

output
```
-x**3/(4*x**4 - 4) - log(x - 1)/16 + log(x + 1)/16 - atan(x)/8
```

3.303.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{1-2x^4+x^8}\,dx = -\frac{x^3}{4\,(x^4-1)} - \frac{1}{8}\arctan(x) + \frac{1}{16}\log(x+1) - \frac{1}{16}\log(x-1)$$

input
```
integrate(x^2/(x^8-2*x^4+1),x, algorithm="maxima")
```

output
```
-1/4*x^3/(x^4 - 1) - 1/8*arctan(x) + 1/16*log(x + 1) - 1/16*log(x - 1)
```

3.303.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{x^2}{1-2x^4+x^8}\,dx = -\frac{x^3}{4\left(x^4-1\right)} - \frac{1}{8}\arctan\left(x\right) + \frac{1}{16}\log\left(\left|x+1\right|\right) - \frac{1}{16}\log\left(\left|x-1\right|\right)$$

input
```
integrate(x^2/(x^8-2*x^4+1),x, algorithm="giac")
```

output
```
-1/4*x^3/(x^4 - 1) - 1/8*arctan(x) + 1/16*log(abs(x + 1)) - 1/16*log(abs(x
 - 1))
```

3.303.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{x^2}{1-2x^4+x^8}\,dx = \frac{\operatorname{atanh}(x)}{8} - \frac{\operatorname{atan}(x)}{8} - \frac{x^3}{4\left(x^4-1\right)}$$

input
```
int(x^2/(x^8 - 2*x^4 + 1),x)
```

output
```
atanh(x)/8 - atan(x)/8 - x^3/(4*(x^4 - 1))
```

3.304 $\int \frac{1}{1-2x^4+x^8}\,dx$

3.304.1 Optimal result

Integrand size = 12, antiderivative size = 27

$$\int \frac{1}{1-2x^4+x^8}\,dx = \frac{x}{4(1-x^4)} + \frac{3\arctan(x)}{8} + \frac{3\text{arctanh}(x)}{8}$$

output
```
1/4*x/(-x^4+1)+3/8*arctan(x)+3/8*arctanh(x)
```

3.304.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.22

$$\int \frac{1}{1-2x^4+x^8}\,dx = \frac{1}{16}\left(-\frac{4x}{-1+x^4} + 6\arctan(x) - 3\log(1-x) + 3\log(1+x)\right)$$

input
```
Integrate[(1 - 2*x^4 + x^8)^(-1),x]
```

output
```
((-4*x)/(-1 + x^4) + 6*ArcTan[x] - 3*Log[1 - x] + 3*Log[1 + x])/16
```

3.304.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {1379, 749, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^8-2x^4+1}dx$$

$$\Big\downarrow \text{1379}$$

$$\int \frac{1}{(x^4-1)^2}dx$$

$$\Big\downarrow \text{749}$$

$$\frac{x}{4(1-x^4)}-\frac{3}{4}\int \frac{1}{x^4-1}dx$$

$$\Big\downarrow \text{756}$$

$$\frac{x}{4(1-x^4)}-\frac{3}{4}\left(-\frac{1}{2}\int \frac{1}{1-x^2}dx-\frac{1}{2}\int \frac{1}{x^2+1}dx\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{x}{4(1-x^4)}-\frac{3}{4}\left(-\frac{1}{2}\int \frac{1}{1-x^2}dx-\frac{\arctan(x)}{2}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{x}{4(1-x^4)}-\frac{3}{4}\left(-\frac{\arctan(x)}{2}-\frac{\text{arctanh}(x)}{2}\right)$$

input
```
Int[(1 - 2*x^4 + x^8)^(-1),x]
```

output
```
x/(4*(1 - x^4)) - (3*(-1/2*ArcTan[x] - ArcTanh[x]/2))/4
```

3.304.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 1379
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/
c^p   Int[(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n
2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p] && NeQ[p, 1]
```

3.304.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

method	result	size
risch	$-\frac{x}{4(x^4-1)}+\frac{3\ln(x+1)}{16}-\frac{3\ln(x-1)}{16}+\frac{3\arctan(x)}{8}$	28
default	$-\frac{1}{16(x+1)}+\frac{3\ln(x+1)}{16}+\frac{x}{8x^2+8}+\frac{3\arctan(x)}{8}-\frac{1}{16(x-1)}-\frac{3\ln(x-1)}{16}$	42
parallelrisch	$-\frac{3i\ln(x-i)x^4-3i\ln(x+i)x^4+3\ln(x-1)x^4-3\ln(x+1)x^4-3i\ln(x-i)+3i\ln(x+i)-3\ln(x-1)+3\ln(x+1)+4x}{16(x^4-1)}$	82

input
```
int(1/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/4*x/(x^4-1)+3/16*ln(x+1)-3/16*ln(x-1)+3/8*arctan(x)
```

3.304.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. 2(19) = 38.

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.63

$$\int \frac{1}{1-2x^4+x^8}\,dx = \frac{6\left(x^4-1\right)\arctan\left(x\right)+3\left(x^4-1\right)\log\left(x+1\right)-3\left(x^4-1\right)\log\left(x-1\right)-4\,x}{16\left(x^4-1\right)}$$

input
```
integrate(1/(x^8-2*x^4+1),x, algorithm="fricas")
```

output
```
1/16*(6*(x^4 - 1)*arctan(x) + 3*(x^4 - 1)*log(x + 1) - 3*(x^4 - 1)*log(x -
 1) - 4*x)/(x^4 - 1)
```

3.304.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int \frac{1}{1-2x^4+x^8}\,dx = -\frac{x}{4x^4-4}-\frac{3\log\left(x-1\right)}{16}+\frac{3\log\left(x+1\right)}{16}+\frac{3\,\mathrm{atan}\left(x\right)}{8}$$

input
```
integrate(1/(x**8-2*x**4+1),x)
```

output
```
-x/(4*x**4 - 4) - 3*log(x - 1)/16 + 3*log(x + 1)/16 + 3*atan(x)/8
```

3.304.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{1-2x^4+x^8}\,dx = -\frac{x}{4\left(x^4-1\right)}+\frac{3}{8}\arctan\left(x\right)+\frac{3}{16}\log\left(x+1\right)-\frac{3}{16}\log\left(x-1\right)$$

input
```
integrate(1/(x^8-2*x^4+1),x, algorithm="maxima")
```

output
```
-1/4*x/(x^4 - 1) + 3/8*arctan(x) + 3/16*log(x + 1) - 3/16*log(x - 1)
```

3.304.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{1}{1-2x^4+x^8}\,dx = -\frac{x}{4\,(x^4-1)} + \frac{3}{8}\,\arctan\left(x\right) + \frac{3}{16}\,\log\left(|x+1|\right) - \frac{3}{16}\,\log\left(|x-1|\right)$$

input
```
integrate(1/(x^8-2*x^4+1),x, algorithm="giac")
```

output
```
-1/4*x/(x^4 - 1) + 3/8*arctan(x) + 3/16*log(abs(x + 1)) - 3/16*log(abs(x -
 1))
```

3.304.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{1}{1-2x^4+x^8}\,dx = \frac{3\,\mathrm{atan}(x)}{8} + \frac{3\,\mathrm{atanh}(x)}{8} - \frac{x}{4\,(x^4-1)}$$

input
```
int(1/(x^8 - 2*x^4 + 1),x)
```

output
```
(3*atan(x))/8 + (3*atanh(x))/8 - x/(4*(x^4 - 1))
```

3.305 $\int \frac{1}{x^2(1-2x^4+x^8)}\,dx$

3.305.1 Optimal result

Integrand size = 16, antiderivative size = 36

$$\int \frac{1}{x^2(1-2x^4+x^8)}\,dx = -\frac{5}{4x}+\frac{1}{4x(1-x^4)}-\frac{5\arctan(x)}{8}+\frac{5\text{arctanh}(x)}{8}$$

output
```
-5/4/x+1/4/x/(-x^4+1)-5/8*arctan(x)+5/8*arctanh(x)
```

3.305.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2(1-2x^4+x^8)}\,dx = \frac{1}{16}\left(-\frac{16}{x}-\frac{4x^3}{-1+x^4}-10\arctan(x)-5\log(1-x)+5\log(1+x)\right)$$

input
```
Integrate[1/(x^2*(1 - 2*x^4 + x^8)),x]
```

output
```
(-16/x - (4*x^3)/(-1 + x^4) - 10*ArcTan[x] - 5*Log[1 - x] + 5*Log[1 + x])/
16
```

3.305.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1380, 819, 847, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (x^8-2x^4+1)} \, dx$$

$$\Big\downarrow \text{1380}$$

$$\int \frac{1}{x^2 (1-x^4)^2}dx$$

$$\Big\downarrow \text{819}$$

$$\frac{5}{4} \int \frac{1}{x^2 (1-x^4)}dx+\frac{1}{4x (1-x^4)}$$

$$\Big\downarrow \text{847}$$

$$\frac{5}{4} \left(\int \frac{x^2}{1-x^4}dx-\frac{1}{x}\right)+\frac{1}{4x (1-x^4)}$$

$$\Big\downarrow \text{827}$$

$$\frac{5}{4} \left(\frac{1}{2} \int \frac{1}{1-x^2}dx-\frac{1}{2} \int \frac{1}{x^2+1}dx-\frac{1}{x}\right)+\frac{1}{4x (1-x^4)}$$

$$\Big\downarrow \text{216}$$

$$\frac{5}{4} \left(\frac{1}{2} \int \frac{1}{1-x^2}dx-\frac{\arctan(x)}{2}-\frac{1}{x}\right)+\frac{1}{4x (1-x^4)}$$

$$\Big\downarrow \text{219}$$

$$\frac{5}{4} \left(-\frac{\arctan(x)}{2}+\frac{\text{arctanh}(x)}{2}-\frac{1}{x}\right)+\frac{1}{4x (1-x^4)}$$

input
```
Int[1/(x^2*(1 - 2*x^4 + x^8)),x]
```

output
```
1/(4*x*(1 - x^4)) + (5*(-x^(-1) - ArcTan[x]/2 + ArcTanh[x]/2))/4
```

3.305.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

3.305.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

method	result
risch	$\frac{-\frac{5x^4}{4}+1}{(x^4-1)x}+\frac{5\ln(x+1)}{16}-\frac{5\ln(x-1)}{16}-\frac{5\arctan(x)}{8}$
default	$-\frac{1}{x}-\frac{1}{16(x+1)}+\frac{5\ln(x+1)}{16}-\frac{x}{8(x^2+1)}-\frac{5\arctan(x)}{8}-\frac{1}{16(x-1)}-\frac{5\ln(x-1)}{16}$
parallelrisch	$-\frac{-5i\ln(x-i)x^5+5i\ln(x+i)x^5+5\ln(x-1)x^5-5\ln(x+1)x^5-16+20x^4+5i\ln(x-i)x-5i\ln(x+i)x-5\ln(x-1)x+5\ln(x+1)x}{16x(x^4-1)}$

input
```
int(1/x^2/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
(-5/4*x^4+1)/(x^4-1)/x+5/16*ln(x+1)-5/16*ln(x-1)-5/8*arctan(x)
```

3.305.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. 2(26) = 52.

Time = 0.24 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.53

$$\int \frac{1}{x^2(1-2x^4+x^8)}\,dx = -\frac{20\,x^4+10\,(x^5-x)\arctan(x)-5\,(x^5-x)\log(x+1)+5\,(x^5-x)\log(x-1)-16}{16\,(x^5-x)}$$

input
```
integrate(1/x^2/(x^8-2*x^4+1),x, algorithm="fricas")
```

output
```
-1/16*(20*x^4 + 10*(x^5 - x)*arctan(x) - 5*(x^5 - x)*log(x + 1) + 5*(x^5 -
 x)*log(x - 1) - 16)/(x^5 - x)
```

3.305.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^2(1-2x^4+x^8)}\,dx = \frac{4-5x^4}{4x^5-4x} - \frac{5\log(x-1)}{16} + \frac{5\log(x+1)}{16} - \frac{5\operatorname{atan}(x)}{8}$$

input
```
integrate(1/x**2/(x**8-2*x**4+1),x)
```

output
```
(4 - 5*x**4)/(4*x**5 - 4*x) - 5*log(x - 1)/16 + 5*log(x + 1)/16 - 5*atan(x
)/8
```

3.305.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^2(1-2x^4+x^8)}\,dx = -\frac{5\,x^4-4}{4\,(x^5-x)} - \frac{5}{8}\arctan(x) + \frac{5}{16}\log(x+1) - \frac{5}{16}\log(x-1)$$

input
```
integrate(1/x^2/(x^8-2*x^4+1),x, algorithm="maxima")
```

output
```
-1/4*(5*x^4 - 4)/(x^5 - x) - 5/8*arctan(x) + 5/16*log(x + 1) - 5/16*log(x
- 1)
```

3.305.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^2(1-2x^4+x^8)}\,dx = -\frac{5\,x^4-4}{4\,(x^5-x)} - \frac{5}{8}\arctan(x) + \frac{5}{16}\log(|x+1|) - \frac{5}{16}\log(|x-1|)$$

input
```
integrate(1/x^2/(x^8-2*x^4+1),x, algorithm="giac")
```

output
```
-1/4*(5*x^4 - 4)/(x^5 - x) - 5/8*arctan(x) + 5/16*log(abs(x + 1)) - 5/16*l
og(abs(x - 1))
```

3.305.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.72

$$\int \frac{1}{x^2 (1 - 2x^4 + x^8)} \, dx = \frac{5 \operatorname{atanh}(x)}{8} - \frac{5 \operatorname{atan}(x)}{8} + \frac{\frac{5 x^4}{4} - 1}{x - x^5}$$

input
```
int(1/(x^2*(x^8 - 2*x^4 + 1)),x)
```

output
```
(5*atanh(x))/8 - (5*atan(x))/8 + ((5*x^4)/4 - 1)/(x - x^5)
```

3.306 $\int \frac{1}{x^4(1-2x^4+x^8)}\,dx$

3.306.1 Optimal result

Integrand size = 16, antiderivative size = 36

$$\int \frac{1}{x^4(1-2x^4+x^8)}\,dx = -\frac{7}{12x^3}+\frac{1}{4x^3(1-x^4)}+\frac{7\arctan(x)}{8}+\frac{7\text{arctanh}(x)}{8}$$

output
```
-7/12/x^3+1/4/x^3/(-x^4+1)+7/8*arctan(x)+7/8*arctanh(x)
```

3.306.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^4(1-2x^4+x^8)}\,dx = \frac{1}{48}\left(-\frac{16}{x^3}-\frac{12x}{-1+x^4}+42\arctan(x)-21\log(1-x)+21\log(1+x)\right)$$

input
```
Integrate[1/(x^4*(1 - 2*x^4 + x^8)),x]
```

output
```
(-16/x^3 - (12*x)/(-1 + x^4) + 42*ArcTan[x] - 21*Log[1 - x] + 21*Log[1 + x
])/48
```

3.306.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1380, 819, 847, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (x^8 - 2x^4 + 1)} \, dx$$

$$\Big\downarrow \text{1380}$$

$$\int \frac{1}{x^4 (1 - x^4)^2} dx$$

$$\Big\downarrow \text{819}$$

$$\frac{7}{4} \int \frac{1}{x^4 (1 - x^4)} dx + \frac{1}{4x^3 (1 - x^4)}$$

$$\Big\downarrow \text{847}$$

$$\frac{7}{4} \left(\int \frac{1}{1 - x^4} dx - \frac{1}{3x^3} \right) + \frac{1}{4x^3 (1 - x^4)}$$

$$\Big\downarrow \text{756}$$

$$\frac{7}{4} \left(\frac{1}{2} \int \frac{1}{1 - x^2} dx + \frac{1}{2} \int \frac{1}{x^2 + 1} dx - \frac{1}{3x^3} \right) + \frac{1}{4x^3 (1 - x^4)}$$

$$\Big\downarrow \text{216}$$

$$\frac{7}{4} \left(\frac{1}{2} \int \frac{1}{1 - x^2} dx + \frac{\arctan(x)}{2} - \frac{1}{3x^3} \right) + \frac{1}{4x^3 (1 - x^4)}$$

$$\Big\downarrow \text{219}$$

$$\frac{7}{4} \left(\frac{\arctan(x)}{2} + \frac{\operatorname{arctanh}(x)}{2} - \frac{1}{3x^3} \right) + \frac{1}{4x^3 (1 - x^4)}$$

input
```
Int[1/(x^4*(1 - 2*x^4 + x^8)),x]
```

output
```
1/(4*x^3*(1 - x^4)) + (7*(-1/3*1/x^3 + ArcTan[x]/2 + ArcTanh[x]/2))/4
```

3.306.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

3.306.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

method	result
risch	$\frac{-\frac{7x^4}{12}+\frac{1}{3}}{x^3(x^4-1)}-\frac{7\ln(x-1)}{16}+\frac{7\ln(x+1)}{16}+\frac{7\arctan(x)}{8}$
default	$-\frac{1}{3x^3}-\frac{1}{16(x+1)}+\frac{7\ln(x+1)}{16}+\frac{x}{8x^2+8}+\frac{7\arctan(x)}{8}-\frac{1}{16(x-1)}-\frac{7\ln(x-1)}{16}$
parallelrisch	$-\frac{21i\ln(x-i)x^7-21i\ln(x+i)x^7+21\ln(x-1)x^7-21\ln(x+1)x^7-16-21i\ln(x-i)x^3+21i\ln(x+i)x^3-21\ln(x-1)x^3+21\ln(x+1)}{48x^3(x^4-1)}$

input
```
int(1/x^4/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
(-7/12*x^4+1/3)/x^3/(x^4-1)-7/16*ln(x-1)+7/16*ln(x+1)+7/8*arctan(x)
```

3.306.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(26) = 52.

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.75

$$\int \frac{1}{x^4(1-2x^4+x^8)}\,dx = -\frac{28\,x^4-42\,(x^7-x^3)\arctan(x)-21\,(x^7-x^3)\log(x+1)+21\,(x^7-x^3)\log(x-1)-16}{48\,(x^7-x^3)}$$

input
```
integrate(1/x^4/(x^8-2*x^4+1),x, algorithm="fricas")
```

output
```
-1/48*(28*x^4 - 42*(x^7 - x^3)*arctan(x) - 21*(x^7 - x^3)*log(x + 1) + 21*
(x^7 - x^3)*log(x - 1) - 16)/(x^7 - x^3)
```

3.306.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^4 (1-2x^4+x^8)}\,dx = \frac{4-7x^4}{12x^7-12x^3} - \frac{7\log(x-1)}{16} + \frac{7\log(x+1)}{16} + \frac{7\,\mathrm{atan}(x)}{8}$$

input
```
integrate(1/x**4/(x**8-2*x**4+1),x)
```

output
```
(4 - 7*x**4)/(12*x**7 - 12*x**3) - 7*log(x - 1)/16 + 7*log(x + 1)/16 + 7*a
tan(x)/8
```

3.306.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^4 (1-2x^4+x^8)}\,dx = -\frac{7\,x^4-4}{12\,(x^7-x^3)} + \frac{7}{8}\,\arctan(x) + \frac{7}{16}\,\log(x+1) - \frac{7}{16}\,\log(x-1)$$

input
```
integrate(1/x^4/(x^8-2*x^4+1),x, algorithm="maxima")
```

output
```
-1/12*(7*x^4 - 4)/(x^7 - x^3) + 7/8*arctan(x) + 7/16*log(x + 1) - 7/16*log
(x - 1)
```

3.306.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^4 (1-2x^4+x^8)}\,dx = -\frac{x}{4\,(x^4-1)} - \frac{1}{3\,x^3} + \frac{7}{8}\,\arctan(x) + \frac{7}{16}\,\log(|x+1|) - \frac{7}{16}\,\log(|x-1|)$$

input
```
integrate(1/x^4/(x^8-2*x^4+1),x, algorithm="giac")
```

output
```
-1/4*x/(x^4 - 1) - 1/3/x^3 + 7/8*arctan(x) + 7/16*log(abs(x + 1)) - 7/16*l
og(abs(x - 1))
```

3.306.9 Mupad [B] (verification not implemented)

Time = 8.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^4 (1-2x^4+x^8)} \, dx = \frac{7 \operatorname{atan}(x)}{8} + \frac{7 \operatorname{atanh}(x)}{8} + \frac{\frac{7 x^4}{12} - \frac{1}{3}}{x^3 - x^7}$$

input
```
int(1/(x^4*(x^8 - 2*x^4 + 1)),x)
```

output
```
(7*atan(x))/8 + (7*atanh(x))/8 + ((7*x^4)/12 - 1/3)/(x^3 - x^7)
```

3.307 $\int \frac{1}{x^6(1-2x^4+x^8)}\,dx$

3.307.1 Optimal result

Integrand size = 16, antiderivative size = 43

$$\int \frac{1}{x^6\left(1-2x^4+x^8\right)}\,dx=-\frac{9}{20x^5}-\frac{9}{4x}+\frac{1}{4x^5\left(1-x^4\right)}-\frac{9\arctan(x)}{8}+\frac{9\text{arctanh}(x)}{8}$$

output
```
-9/20/x^5-9/4/x+1/4/x^5/(-x^4+1)-9/8*arctan(x)+9/8*arctanh(x)
```

3.307.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.19

$$\int \frac{1}{x^6\left(1-2x^4+x^8\right)}\,dx=-\frac{1}{5x^5}-\frac{2}{x}-\frac{x^3}{4\left(-1+x^4\right)}-\frac{9\arctan(x)}{8}-\frac{9}{16}\log(1-x)+\frac{9}{16}\log(1+x)$$

input
```
Integrate[1/(x^6*(1 - 2*x^4 + x^8)),x]
```

output
```
-1/5*1/x^5 - 2/x - x^3/(4*(-1 + x^4)) - (9*ArcTan[x])/8 - (9*Log[1 - x])/1
6 + (9*Log[1 + x])/16
```

3.307.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1380, 819, 847, 847, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 (x^8-2 x^4+1)} \, dx$$

$$\Big\downarrow \text{1380}$$

$$\int \frac{1}{x^6 (1-x^4)^2}dx$$

$$\Big\downarrow \text{819}$$

$$\frac{9}{4} \int \frac{1}{x^6 (1-x^4)}dx+\frac{1}{4 x^5 (1-x^4)}$$

$$\Big\downarrow \text{847}$$

$$\frac{9}{4} \left(\int \frac{1}{x^2 (1-x^4)}dx-\frac{1}{5 x^5}\right)+\frac{1}{4 x^5 (1-x^4)}$$

$$\Big\downarrow \text{847}$$

$$\frac{9}{4} \left(\int \frac{x^2}{1-x^4}dx-\frac{1}{5 x^5}-\frac{1}{x}\right)+\frac{1}{4 x^5 (1-x^4)}$$

$$\Big\downarrow \text{827}$$

$$\frac{9}{4} \left(\frac{1}{2} \int \frac{1}{1-x^2}dx-\frac{1}{2} \int \frac{1}{x^2+1}dx-\frac{1}{5 x^5}-\frac{1}{x}\right)+\frac{1}{4 x^5 (1-x^4)}$$

$$\Big\downarrow \text{216}$$

$$\frac{9}{4} \left(\frac{1}{2} \int \frac{1}{1-x^2}dx-\frac{\arctan (x)}{2}-\frac{1}{5 x^5}-\frac{1}{x}\right)+\frac{1}{4 x^5 (1-x^4)}$$

$$\Big\downarrow \text{219}$$

$$\frac{9}{4} \left(-\frac{\arctan (x)}{2}+\frac{\text{arctanh}(x)}{2}-\frac{1}{5 x^5}-\frac{1}{x}\right)+\frac{1}{4 x^5 (1-x^4)}$$

input
```
Int[1/(x^6*(1 - 2*x^4 + x^8)),x]
```

output
```
1/(4*x^5*(1 - x^4)) + (9*(-1/5*1/x^5 - x^(-1) - ArcTan[x]/2 + ArcTanh[x]/2
))/4
```

3.307.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

3.307. $\int \frac{1}{x^6(1-2x^4+x^8)} \, dx$

3.307.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

method	result
risch	$\frac{-\frac{9}{4}x^8+\frac{9}{5}x^4+\frac{1}{5}}{x^5(x^4-1)}-\frac{9\ln(x-1)}{16}+\frac{9\ln(x+1)}{16}-\frac{9\arctan(x)}{8}$
default	$-\frac{1}{5x^5}-\frac{2}{x}-\frac{1}{16(x+1)}+\frac{9\ln(x+1)}{16}-\frac{x}{8(x^2+1)}-\frac{9\arctan(x)}{8}-\frac{1}{16(x-1)}-\frac{9\ln(x-1)}{16}$
parallelrisch	$-\frac{-45i\ln(x-i)x^9+45i\ln(x+i)x^9+45\ln(x-1)x^9-45\ln(x+1)x^9-16+180x^8+45i\ln(x-i)x^5-45i\ln(x+i)x^5-45\ln(x-1)x^5+4}{80x^5(x^4-1)}$

input
```
int(1/x^6/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
(-9/4*x^8+9/5*x^4+1/5)/x^5/(x^4-1)-9/16*ln(x-1)+9/16*ln(x+1)-9/8*arctan(x)
```

3.307.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(31) = 62.

Time = 0.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.58

$$\int \frac{1}{x^6(1-2x^4+x^8)}\,dx = -\frac{180\,x^8-144\,x^4+90\,(x^9-x^5)\arctan(x)-45\,(x^9-x^5)\log(x+1)+45\,(x^9-x^5)\log(x-1)-16}{80\,(x^9-x^5)}$$

input
```
integrate(1/x^6/(x^8-2*x^4+1),x, algorithm="fricas")
```

output
```
-1/80*(180*x^8 - 144*x^4 + 90*(x^9 - x^5)*arctan(x) - 45*(x^9 - x^5)*log(x
 + 1) + 45*(x^9 - x^5)*log(x - 1) - 16)/(x^9 - x^5)
```

3.307.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.02

$$\int \frac{1}{x^6(1-2x^4+x^8)}\,dx = -\frac{9\log(x-1)}{16} + \frac{9\log(x+1)}{16} - \frac{9\,\mathrm{atan}(x)}{8} + \frac{-45x^8+36x^4+4}{20x^9-20x^5}$$

input
```
integrate(1/x**6/(x**8-2*x**4+1),x)
```

output
```
-9*log(x - 1)/16 + 9*log(x + 1)/16 - 9*atan(x)/8 + (-45*x**8 + 36*x**4 + 4
)/(20*x**9 - 20*x**5)
```

3.307.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int \frac{1}{x^6(1-2x^4+x^8)}\,dx = -\frac{45\,x^8-36\,x^4-4}{20\,(x^9-x^5)} - \frac{9}{8}\arctan(x) + \frac{9}{16}\log(x+1) - \frac{9}{16}\log(x-1)$$

input
```
integrate(1/x^6/(x^8-2*x^4+1),x, algorithm="maxima")
```

output
```
-1/20*(45*x^8 - 36*x^4 - 4)/(x^9 - x^5) - 9/8*arctan(x) + 9/16*log(x + 1)
- 9/16*log(x - 1)
```

3.307.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^6(1-2x^4+x^8)}\,dx = -\frac{x^3}{4\,(x^4-1)} - \frac{10\,x^4+1}{5\,x^5} - \frac{9}{8}\arctan(x) + \frac{9}{16}\log(|x+1|) - \frac{9}{16}\log(|x-1|)$$

input
```
integrate(1/x^6/(x^8-2*x^4+1),x, algorithm="giac")
```

output
```
-1/4*x^3/(x^4 - 1) - 1/5*(10*x^4 + 1)/x^5 - 9/8*arctan(x) + 9/16*log(abs(x
 + 1)) - 9/16*log(abs(x - 1))
```

3.307.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^6 (1 - 2x^4 + x^8)} \, dx = \frac{9 \operatorname{atanh}(x)}{8} - \frac{9 \operatorname{atan}(x)}{8} - \frac{-\frac{9 x^8}{4} + \frac{9 x^4}{5} + \frac{1}{5}}{x^5 - x^9}$$

input
```
int(1/(x^6*(x^8 - 2*x^4 + 1)),x)
```

output
```
(9*atanh(x))/8 - (9*atan(x))/8 - ((9*x^4)/5 - (9*x^8)/4 + 1/5)/(x^5 - x^9)
```

3.308 $\int \frac{1}{x^8(1-2x^4+x^8)}\,dx$

3.308.1 Optimal result

Integrand size = 16, antiderivative size = 43

$$\int \frac{1}{x^8\left(1-2x^4+x^8\right)}\,dx=-\frac{11}{28x^7}-\frac{11}{12x^3}+\frac{1}{4x^7\left(1-x^4\right)}+\frac{11\arctan(x)}{8}+\frac{11\text{arctanh}(x)}{8}$$

output
```
-11/28/x^7-11/12/x^3+1/4/x^7/(-x^4+1)+11/8*arctan(x)+11/8*arctanh(x)
```

3.308.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^8\left(1-2x^4+x^8\right)}\,dx=\frac{1}{336}\left(-\frac{48}{x^7}-\frac{224}{x^3}-\frac{84x}{-1+x^4}+462\arctan(x)-231\log(1-x)+231\log(1+x)\right)$$

input
```
Integrate[1/(x^8*(1 - 2*x^4 + x^8)),x]
```

output
```
(-48/x^7 - 224/x^3 - (84*x)/(-1 + x^4) + 462*ArcTan[x] - 231*Log[1 - x] +
231*Log[1 + x])/336
```

3.308.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1380, 819, 847, 847, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^8 (x^8-2x^4+1)}dx$$
$$\Big\downarrow \text{1380}$$
$$\int \frac{1}{x^8 (1-x^4)^2}dx$$
$$\Big\downarrow \text{819}$$
$$\frac{11}{4}\int \frac{1}{x^8 (1-x^4)}dx+\frac{1}{4x^7 (1-x^4)}$$
$$\Big\downarrow \text{847}$$
$$\frac{11}{4}\left(\int \frac{1}{x^4 (1-x^4)}dx-\frac{1}{7x^7}\right)+\frac{1}{4x^7 (1-x^4)}$$
$$\Big\downarrow \text{847}$$
$$\frac{11}{4}\left(\int \frac{1}{1-x^4}dx-\frac{1}{7x^7}-\frac{1}{3x^3}\right)+\frac{1}{4x^7 (1-x^4)}$$
$$\Big\downarrow \text{756}$$
$$\frac{11}{4}\left(\frac{1}{2}\int \frac{1}{1-x^2}dx+\frac{1}{2}\int \frac{1}{x^2+1}dx-\frac{1}{7x^7}-\frac{1}{3x^3}\right)+\frac{1}{4x^7 (1-x^4)}$$
$$\Big\downarrow \text{216}$$
$$\frac{11}{4}\left(\frac{1}{2}\int \frac{1}{1-x^2}dx+\frac{\arctan(x)}{2}-\frac{1}{7x^7}-\frac{1}{3x^3}\right)+\frac{1}{4x^7 (1-x^4)}$$
$$\Big\downarrow \text{219}$$
$$\frac{11}{4}\left(\frac{\arctan(x)}{2}+\frac{\text{arctanh}(x)}{2}-\frac{1}{7x^7}-\frac{1}{3x^3}\right)+\frac{1}{4x^7 (1-x^4)}$$

input
```
Int[1/(x^8*(1 - 2*x^4 + x^8)),x]
```

output
```
1/(4*x^7*(1 - x^4)) + (11*(-1/7*1/x^7 - 1/(3*x^3) + ArcTan[x]/2 + ArcTanh[
x]/2))/4
```

3.308.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

3.308.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

method	result
risch	$\frac{-\frac{11}{12}x^8+\frac{11}{21}x^4+\frac{1}{7}}{x^7(x^4-1)}+\frac{11\arctan(x)}{8}-\frac{11\ln(x-1)}{16}+\frac{11\ln(x+1)}{16}$
default	$-\frac{1}{7x^7}-\frac{2}{3x^3}-\frac{1}{16(x+1)}+\frac{11\ln(x+1)}{16}+\frac{x}{8x^2+8}+\frac{11\arctan(x)}{8}-\frac{1}{16(x-1)}-\frac{11\ln(x-1)}{16}$
parallelrisch	$-\frac{231i\ln(x-i)x^{11}-231i\ln(x+i)x^{11}+231\ln(x-1)x^{11}-231\ln(x+1)x^{11}-48-231i\ln(x-i)x^7+231i\ln(x+i)x^7-231\ln(x-1)x^7}{336x^7(x^4-1)}$

input
```
int(1/x^8/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
(-11/12*x^8+11/21*x^4+1/7)/x^7/(x^4-1)+11/8*arctan(x)-11/16*ln(x-1)+11/16*
ln(x+1)
```

3.308.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(31) = 62.

Time = 0.26 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.58

$$\int \frac{1}{x^8(1-2x^4+x^8)}\,dx = -\frac{308\,x^8-176\,x^4-462\,(x^{11}-x^7)\arctan(x)-231\,(x^{11}-x^7)\log(x+1)+231\,(x^{11}-x^7)\log(x-1)-}{336\,(x^{11}-x^7)}$$

input
```
integrate(1/x^8/(x^8-2*x^4+1),x, algorithm="fricas")
```

output
```
-1/336*(308*x^8 - 176*x^4 - 462*(x^11 - x^7)*arctan(x) - 231*(x^11 - x^7)*
log(x + 1) + 231*(x^11 - x^7)*log(x - 1) - 48)/(x^11 - x^7)
```

3.308.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.02

$$\int \frac{1}{x^8 (1-2x^4+x^8)}\,dx = -\frac{11\log{(x-1)}}{16} + \frac{11\log{(x+1)}}{16} + \frac{11\,\mathrm{atan}\,(x)}{8} + \frac{-77x^8+44x^4+12}{84x^{11}-84x^7}$$

input
```
integrate(1/x**8/(x**8-2*x**4+1),x)
```

output
```
-11*log(x - 1)/16 + 11*log(x + 1)/16 + 11*atan(x)/8 + (-77*x**8 + 44*x**4
+ 12)/(84*x**11 - 84*x**7)
```

3.308.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int \frac{1}{x^8 (1-2x^4+x^8)}\,dx = -\frac{77\,x^8-44\,x^4-12}{84\,(x^{11}-x^7)} + \frac{11}{8}\arctan{(x)} + \frac{11}{16}\log{(x+1)} - \frac{11}{16}\log{(x-1)}$$

input
```
integrate(1/x^8/(x^8-2*x^4+1),x, algorithm="maxima")
```

output
```
-1/84*(77*x^8 - 44*x^4 - 12)/(x^11 - x^7) + 11/8*arctan(x) + 11/16*log(x +
 1) - 11/16*log(x - 1)
```

3.308.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^8 (1-2x^4+x^8)}\,dx = -\frac{x}{4\,(x^4-1)} - \frac{14\,x^4+3}{21\,x^7} + \frac{11}{8}\arctan{(x)} + \frac{11}{16}\log{(|x+1|)} - \frac{11}{16}\log{(|x-1|)}$$

input
```
integrate(1/x^8/(x^8-2*x^4+1),x, algorithm="giac")
```

output
```
-1/4*x/(x^4 - 1) - 1/21*(14*x^4 + 3)/x^7 + 11/8*arctan(x) + 11/16*log(abs(
x + 1)) - 11/16*log(abs(x - 1))
```

3.308.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^8 (1-2x^4+x^8)}\,dx = \frac{11\,\mathrm{atan}(x)}{8} + \frac{11\,\mathrm{atanh}(x)}{8} - \frac{-\frac{11\,x^8}{12} + \frac{11\,x^4}{21} + \frac{1}{7}}{x^7 - x^{11}}$$

input
```
int(1/(x^8*(x^8 - 2*x^4 + 1)),x)
```

output
```
(11*atan(x))/8 + (11*atanh(x))/8 - ((11*x^4)/21 - (11*x^8)/12 + 1/7)/(x^7
- x^11)
```

3.309 $\int \frac{x^m}{a+bx^4+cx^8}\,dx$

3.309.1 Optimal result

Integrand size = 18, antiderivative size = 163

$$\int \frac{x^m}{a+bx^4+cx^8}\,dx = \frac{2cx^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{4},\frac{5+m}{4},-\frac{2cx^4}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)(1+m)} - \frac{2cx^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{4},\frac{5+m}{4},-\frac{2cx^4}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)(1+m)}$$

output
```
2*c*x^(1+m)*hypergeom([1, 1/4+1/4*m],[5/4+1/4*m],-2*c*x^4/(b-(-4*a*c+b^2)^
(1/2)))/(1+m)/(b-(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)-2*c*x^(1+m)*hyperg
eom([1, 1/4+1/4*m],[5/4+1/4*m],-2*c*x^4/(b+(-4*a*c+b^2)^(1/2)))/(1+m)/(-4*
a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))
```

3.309.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.50

$$\int \frac{x^m}{a+bx^4+cx^8}\,dx = \frac{x^m\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{\text{Hypergeometric2F1}\left(-m,-m,1-m,-\frac{\#1}{x-\#1}\right)\left(\frac{x}{x-\#1}\right)^{-m}}{b\#1^3+2c\#1^7}\&\right]}{4m}$$

input
```
Integrate[x^m/(a + b*x^4 + c*x^8),x]
```

output
```
(x^m*RootSum[a + b*#1^4 + c*#1^8 & , Hypergeometric2F1[-m, -m, 1 - m, -(#1
/(x - #1))]/((x/(x - #1))^m*(b*#1^3 + 2*c*#1^7)) & ])/(4*m)
```

3.309.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1711, 27, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{a+bx^4+cx^8}\,dx$$

$$\Big\downarrow \text{1711}$$

$$\frac{c\int \frac{2x^m}{2cx^4+b-\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}-\frac{c\int \frac{2x^m}{2cx^4+b+\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2c\int \frac{x^m}{2cx^4+b-\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}-\frac{2c\int \frac{x^m}{2cx^4+b+\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{888}$$

$$\frac{2cx^{m+1}\text{Hypergeometric2F1}\left(1,\frac{m+1}{4},\frac{m+5}{4},-\frac{2cx^4}{b-\sqrt{b^2-4ac}}\right)}{(m+1)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2cx^{m+1}\text{Hypergeometric2F1}\left(1,\frac{m+1}{4},\frac{m+5}{4},-\frac{2cx^4}{b+\sqrt{b^2-4ac}}\right)}{(m+1)\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}$$

input
```
Int[x^m/(a + b*x^4 + c*x^8),x]
```

output
```
(2*c*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, (-2*c*x^4)/(b -
Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])*(1 + m)) -
 (2*c*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, (-2*c*x^4)/(b +
 Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*(1 + m))
```

3.309.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1711
```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[c/q   Int[(d*x)^m/(b/2 - q/2 + c
*x^n), x], x] - Simp[c/q   Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; Free
Q[{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

3.309.4 Maple [F]

$$\int \frac{x^m}{c\,x^8+b\,x^4+a}dx$$

input
```
int(x^m/(c*x^8+b*x^4+a),x)
```

output
```
int(x^m/(c*x^8+b*x^4+a),x)
```

3.309.5 Fricas [F]

$$\int \frac{x^m}{a+bx^4+cx^8}\,dx=\int \frac{x^m}{cx^8+bx^4+a}\,dx$$

input
```
integrate(x^m/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
integral(x^m/(c*x^8 + b*x^4 + a), x)
```

3.309.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^m}{a+bx^4+cx^8}\,dx = \text{Timed out}$$

input
```
integrate(x**m/(c*x**8+b*x**4+a),x)
```

output
```
Timed out
```

3.309.7 Maxima [F]

$$\int \frac{x^m}{a+bx^4+cx^8}\,dx = \int \frac{x^m}{cx^8+bx^4+a}\,dx$$

input
```
integrate(x^m/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
integrate(x^m/(c*x^8 + b*x^4 + a), x)
```

3.309.8 Giac [F]

$$\int \frac{x^m}{a+bx^4+cx^8}\,dx = \int \frac{x^m}{cx^8+bx^4+a}\,dx$$

input
```
integrate(x^m/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
integrate(x^m/(c*x^8 + b*x^4 + a), x)
```

3.309.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{a+bx^4+cx^8}\,dx=\int \frac{x^m}{c\,x^8+b\,x^4+a}\,dx$$

input
```
int(x^m/(a + b*x^4 + c*x^8),x)
```

output
```
int(x^m/(a + b*x^4 + c*x^8), x)
```

3.310 $\int \frac{x^{11}}{a+bx^4+cx^8}\,dx$

3.310.1 Optimal result

Integrand size = 18, antiderivative size = 81

$$\int \frac{x^{11}}{a+bx^4+cx^8}\,dx=\frac{x^4}{4c}-\frac{(b^2-2ac)\,\text{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx^4+cx^8\right)}{8c^2}$$

output
```
1/4*x^4/c-1/8*b*ln(c*x^8+b*x^4+a)/c^2-1/4*(-2*a*c+b^2)*arctanh((2*c*x^4+b)
/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(1/2)
```

3.310.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.96

$$\int \frac{x^{11}}{a+bx^4+cx^8}\,dx=\frac{2cx^4+\frac{2(b^2-2ac)\arctan\left(\frac{b+2cx^4}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}-b\log\left(a+bx^4+cx^8\right)}{8c^2}$$

input
```
Integrate[x^11/(a + b*x^4 + c*x^8),x]
```

output
```
(2*c*x^4 + (2*(b^2 - 2*a*c)*ArcTan[(b + 2*c*x^4)/Sqrt[-b^2 + 4*a*c]])/Sqrt
[-b^2 + 4*a*c] - b*Log[a + b*x^4 + c*x^8])/(8*c^2)
```

3.310.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1693, 1143, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{a+bx^4+cx^8} \, dx$$

$$\Big\downarrow \text{ 1693}$$

$$\frac{1}{4} \int \frac{x^8}{cx^8+bx^4+a}dx^4$$

$$\Big\downarrow \text{ 1143}$$

$$\frac{1}{4} \int \left(\frac{1}{c}-\frac{bx^4+a}{c\,(cx^8+bx^4+a)}\right)dx^4$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4}\left(-\frac{(b^2-2ac)\,\text{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx^4+cx^8\right)}{2c^2}+\frac{x^4}{c}\right)$$

input
```
Int[x^11/(a + b*x^4 + c*x^8),x]
```

output
```
(x^4/c - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]])/(c^2*Sqr
t[b^2 - 4*a*c]) - (b*Log[a + b*x^4 + c*x^8])/(2*c^2))/4
```

3.310.3.1 Defintions of rubi rules used

rule 1143
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x)^m/(a + b*x + c*x^2), x], x] /; FreeQ[{a,
b, c, d, e}, x] && IGtQ[m, 1]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.310.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.02

method	result
default	$\frac{x^4}{4c}+\frac{-\frac{b\ln\left(c\,x^8+b\,x^4+a\right)}{2c}+\frac{2\left(-a+\frac{b^2}{2c}\right)\arctan\left(\frac{2c\,x^4+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{4c}$
risch	$\frac{x^4}{4c}-\frac{\ln\left(\left(-8a^2c^2+6a\,b^2c-b^4+\sqrt{-(4ac-b^2)(2ac-b^2)^2}\,b\right)x^4+2\sqrt{-(4ac-b^2)(2ac-b^2)^2}\,a\right)ab}{2c(4ac-b^2)}+\frac{\ln\left(\left(-8a^2c^2+6a\,b^2c-b^4+\sqrt{-(\right.\right.}{}$

input
```
int(x^11/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4/c+1/4/c*(-1/2*b/c*ln(c*x^8+b*x^4+a)+2*(-a+1/2/c*b^2)/(4*a*c-b^2)^(
1/2)*arctan((2*c*x^4+b)/(4*a*c-b^2)^(1/2)))
```

3.310.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 254, normalized size of antiderivative = 3.14

$$\int \frac{x^{11}}{a+bx^4+cx^8}\,dx = \left[\frac{2\,(b^2c-4\,ac^2)x^4-(b^2-2\,ac)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^8+2\,bcx^4+b^2-2\,ac+(2\,cx^4+b)\sqrt{b^2-4\,ac}}{cx^8+bx^4+a}\right)-(b^3-4\,abc)\log\left(cx^8\right.}{8\,(b^2c^2-4\,ac^3)}\right.$$

input
```
integrate(x^11/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
[1/8*(2*(b^2*c - 4*a*c^2)*x^4 - (b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2
*x^8 + 2*b*c*x^4 + b^2 - 2*a*c + (2*c*x^4 + b)*sqrt(b^2 - 4*a*c))/(c*x^8 +
 b*x^4 + a)) - (b^3 - 4*a*b*c)*log(c*x^8 + b*x^4 + a))/(b^2*c^2 - 4*a*c^3)
, 1/8*(2*(b^2*c - 4*a*c^2)*x^4 - 2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*arctan
(-(2*c*x^4 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^3 - 4*a*b*c)*log(c*
x^8 + b*x^4 + a))/(b^2*c^2 - 4*a*c^3)]
```

3.310.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. 2(73) = 146.

Time = 2.70 (sec) , antiderivative size = 316, normalized size of antiderivative = 3.90

$$\int \frac{x^{11}}{a+bx^4+cx^8}\,dx = \left(-\frac{b}{8c^2} - \frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{8c^2\cdot(4ac-b^2)}\right)\log\left(x^4 + \frac{-ab-16ac^2\left(-\frac{b}{8c^2}-\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{8c^2\cdot(4ac-b^2)}\right)+4b^2c\left(-\frac{b}{8c^2}-\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{8c^2\cdot(4ac-b^2)}\right)}{2ac-b^2}\right) + \left(-\frac{b}{8c^2} + \frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{8c^2\cdot(4ac-b^2)}\right)\log\left(x^4 + \frac{-ab-16ac^2\left(-\frac{b}{8c^2}+\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{8c^2\cdot(4ac-b^2)}\right)+4b^2c\left(-\frac{b}{8c^2}+\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{8c^2\cdot(4ac-b^2)}\right)}{2ac-b^2}\right) + \frac{x^4}{4c}$$

input
```
integrate(x**11/(c*x**8+b*x**4+a),x)
```

output
```
(-b/(8*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(8*c**2*(4*a*c - b**2)))
*log(x**4 + (-a*b - 16*a*c**2*(-b/(8*c**2) - sqrt(-4*a*c + b**2)*(2*a*c -
b**2)/(8*c**2*(4*a*c - b**2))) + 4*b**2*c*(-b/(8*c**2) - sqrt(-4*a*c + b**
2)*(2*a*c - b**2)/(8*c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + (-b/(8*c**2)
 + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(8*c**2*(4*a*c - b**2)))*log(x**4 +
(-a*b - 16*a*c**2*(-b/(8*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(8*c**
2*(4*a*c - b**2))) + 4*b**2*c*(-b/(8*c**2) + sqrt(-4*a*c + b**2)*(2*a*c -
b**2)/(8*c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + x**4/(4*c)
```

3.310.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{11}}{a+bx^4+cx^8}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^11/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.310.8 Giac [A] (verification not implemented)

Time = 1.56 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.93

$$\int \frac{x^{11}}{a+bx^4+cx^8}\,dx = \frac{x^4}{4\,c} - \frac{b\log\left(cx^8+bx^4+a\right)}{8\,c^2} + \frac{\left(b^2-2\,ac\right)\arctan\left(\frac{2\,cx^4+b}{\sqrt{-b^2+4\,ac}}\right)}{4\,\sqrt{-b^2+4\,ac}c^2}$$

input
```
integrate(x^11/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
1/4*x^4/c - 1/8*b*log(c*x^8 + b*x^4 + a)/c^2 + 1/4*(b^2 - 2*a*c)*arctan((2
*c*x^4 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

3.310.9 Mupad [B] (verification not implemented)

Time = 8.92 (sec) , antiderivative size = 3916, normalized size of antiderivative = 48.35

$$\int \frac{x^{11}}{a+bx^4+cx^8}\,dx = \text{Too large to display}$$

input
```
int(x^11/(a + b*x^4 + c*x^8),x)
```

output
```
x^4/(4*c) + (log(a + b*x^4 + c*x^8)*(4*b^3 - 16*a*b*c))/(2*(64*a*c^3 - 16*
b^2*c^2)) - (atan((8*c^4*x^4*(((a*c - b^2)*(((((2*a*c - b^2)*((((448*b^4*c
^6 - 384*a*b^2*c^7)/c^4 + (256*b^3*c^4*(4*b^3 - 16*a*b*c))/(64*a*c^3 - 16*
b^2*c^2))*(2*a*c - b^2))/(8*c^2*(4*a*c - b^2)^(1/2)) + (32*b^3*c^2*(4*b^3
- 16*a*b*c)*(2*a*c - b^2))/((4*a*c - b^2)^(1/2)*(64*a*c^3 - 16*b^2*c^2))))
/(8*c^2*(4*a*c - b^2)^(1/2)) + (4*b^3*(4*b^3 - 16*a*b*c)*(2*a*c - b^2)^2)/
((4*a*c - b^2)*(64*a*c^3 - 16*b^2*c^2)))*(2*a*c - b^2))/(8*c^2*(4*a*c - b^
2)^(1/2)) - ((4*b^3 - 16*a*b*c)*(((4*b^3 - 16*a*b*c)*((((448*b^4*c^6 - 384
*a*b^2*c^7)/c^4 + (256*b^3*c^4*(4*b^3 - 16*a*b*c))/(64*a*c^3 - 16*b^2*c^2)
)*(2*a*c - b^2))/(8*c^2*(4*a*c - b^2)^(1/2)) + (32*b^3*c^2*(4*b^3 - 16*a*b
*c)*(2*a*c - b^2))/((4*a*c - b^2)^(1/2)*(64*a*c^3 - 16*b^2*c^2))))/(2*(64*
a*c^3 - 16*b^2*c^2)) + ((2*a*c - b^2)*((144*b^5*c^4 - 240*a*b^3*c^5 + 96*a
^2*b*c^6)/c^4 + ((4*b^3 - 16*a*b*c)*((448*b^4*c^6 - 384*a*b^2*c^7)/c^4 + (
256*b^3*c^4*(4*b^3 - 16*a*b*c))/(64*a*c^3 - 16*b^2*c^2)))/(2*(64*a*c^3 - 1
6*b^2*c^2))))/(8*c^2*(4*a*c - b^2)^(1/2))))/(2*(64*a*c^3 - 16*b^2*c^2)) +
(((8*a^3*c^5 - 20*b^6*c^2 + 48*a*b^4*c^3 - 36*a^2*b^2*c^4)/c^4 - ((4*b^3 -
 16*a*b*c)*((144*b^5*c^4 - 240*a*b^3*c^5 + 96*a^2*b*c^6)/c^4 + ((4*b^3 - 1
6*a*b*c)*((448*b^4*c^6 - 384*a*b^2*c^7)/c^4 + (256*b^3*c^4*(4*b^3 - 16*a*b
*c))/(64*a*c^3 - 16*b^2*c^2)))/(2*(64*a*c^3 - 16*b^2*c^2))))/(2*(64*a*c^3
- 16*b^2*c^2)))*(2*a*c - b^2))/(8*c^2*(4*a*c - b^2)^(1/2)) + (b^3*(4*b^...
```

3.310. $\int \frac{x^{11}}{a+bx^4+cx^8}\,dx$

3.311 $\int \frac{x^9}{a+bx^4+cx^8} dx$

3.311.1 Optimal result

Integrand size = 18, antiderivative size = 192

$$\int \frac{x^9}{a+bx^4+cx^8} dx = \frac{x^2}{2c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
1/2*x^2/c-1/4*arctan(x^2*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b+
(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))/c^(3/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/
2)-1/4*arctan(x^2*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-2*a*c
+b^2)/(-4*a*c+b^2)^(1/2))/c^(3/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.311.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.09

$$\int \frac{x^9}{a+bx^4+cx^8} dx = \frac{2\sqrt{c}x^2 - \frac{\sqrt{2}\left(-b^2+2ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}\left(b^2-2ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}}{4c^{3/2}}$$

input
```
Integrate[x^9/(a + b*x^4 + c*x^8),x]
```

output
```
(2*Sqrt[c]*x^2 - (Sqrt[2]*(-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqr
t[2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b
- Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*(b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTa
n[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*S
qrt[b + Sqrt[b^2 - 4*a*c]]))/(4*c^(3/2))
```

3.311.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1695, 1442, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{a+bx^4+cx^8} \, dx$$

$$\Big\downarrow \text{1695}$$

$$\frac{1}{2}\int \frac{x^8}{cx^8+bx^4+a}dx^2$$

$$\Big\downarrow \text{1442}$$

$$\frac{1}{2}\left(\frac{x^2}{c}-\frac{\int \frac{bx^4+a}{cx^8+bx^4+a}dx^2}{c}\right)$$

$$\Big\downarrow \text{1480}$$

$$\frac{1}{2}\left(\frac{x^2}{c}-\frac{\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^4+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx^2+\frac{1}{2}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\int \frac{1}{cx^4+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx^2}{c}\right)$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{2}\left(\frac{x^2}{c}-\frac{\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{c}\right)$$

input
```
Int[x^9/(a + b*x^4 + c*x^8),x]
```

output
```
(x^2/c - (((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x
^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a
*c]]) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^2
)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c
]]))/c)/2
```

3.311.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1442
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
x] - Simp[d^4/(c*(m + 4*p + 1))   Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1695
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b
*x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c,
p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.311.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.72

method	result
risch	$\frac{x^2}{2c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\left(16a^2c^3-8b^2ac^2+b^4c\right)\_Z^4+\left(12a^2bc^2-7ab^3c+b^5\right)\_Z^2+a^3c^2\right)}\_R\ln\left(\left(-a^2c^2+ab^2c\right)x^2+\left(-4abc^2+b^3c\right)\_R^3+(}{4c}$
default	$\frac{x^2}{2c}+\frac{\left(-b\sqrt{-4ac+b^2}+2ac-b^2\right)\sqrt{2}\arctan\left(\frac{cx^2\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\sqrt{-4ac+b^2}\,c\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\left(b^2-2ac-b\sqrt{-4ac+b^2}\right)\sqrt{2}\,\mathrm{arctanh}\left(\frac{cx^2\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\sqrt{-4ac+b^2}\,c\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}$

input
```
int(x^9/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2/c+1/4/c*sum(_R*ln((-a^2*c^2+a*b^2*c)*x^2+(-4*a*b*c^2+b^3*c)*_R^3+(
2*a^2*c^2-4*a*b^2*c+b^4)*_R),_R=RootOf((16*a^2*c^3-8*a*b^2*c^2+b^4*c)*_Z^4
+(12*a^2*b*c^2-7*a*b^3*c+b^5)*_Z^2+a^3*c^2))
```

3.311.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1071 vs. 2(150) = 300.

Time = 0.26 (sec) , antiderivative size = 1071, normalized size of antiderivative = 5.58

$$\int \frac{x^9}{a+bx^4+cx^8}\,dx = -\frac{\sqrt{\frac{1}{2}}c\sqrt{-\frac{b^3-3\,abc+(b^2c^3-4\,ac^4)\sqrt{\frac{b^4-2\,ab^2c+a^2c^2}{b^2c^6-4\,ac^7}}}{b^2c^3-4\,ac^4}}\log\left(-(ab^2-a^2c)x^2+\frac{1}{2}\sqrt{\frac{1}{2}}\left(b^4-5\,ab^2c+4\,a^2c^2-(b^3c^3-4\right.\right.}{}$$

input
```
integrate(x^9/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
-1/4*(sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2
*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-(a*b^2
 - a^2*c)*x^2 + 1/2*sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 - (b^3*c^3 - 4*
a*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3
 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6
 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) - sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c + (
b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/
(b^2*c^3 - 4*a*c^4))*log(-(a*b^2 - a^2*c)*x^2 - 1/2*sqrt(1/2)*(b^4 - 5*a*b
^2*c + 4*a^2*c^2 - (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/
(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4
 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) + sqrt
(1/2)*c*sqrt(-(b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c +
 a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-(a*b^2 - a^2*c)*
x^2 + 1/2*sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 + (b^3*c^3 - 4*a*b*c^4)*s
qrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3*a*b*c
 - (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7
)))/(b^2*c^3 - 4*a*c^4))) - sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c - (b^2*c^3 -
4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 -
 4*a*c^4))*log(-(a*b^2 - a^2*c)*x^2 - 1/2*sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a
^2*c^2 + (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^...
```

3.311.6 Sympy [A] (verification not implemented)

Time = 2.81 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.70

$$\int \frac{x^9}{a+bx^4+cx^8}\,dx$$
$$= \operatorname{RootSum}\left(t^4\cdot\left(4096a^2c^5-2048ab^2c^4+256b^4c^3\right)+t^2\cdot\left(192a^2bc^2-112ab^3c+16b^5\right)+a^3,\left(t\mapsto t\log\left(x\right.\right.\right.$$
$$+\frac{x^2}{2c}$$

input
```
integrate(x**9/(c*x**8+b*x**4+a),x)
```

output
```
RootSum(_t**4*(4096*a**2*c**5 - 2048*a*b**2*c**4 + 256*b**4*c**3) + _t**2*
(192*a**2*b*c**2 - 112*a*b**3*c + 16*b**5) + a**3, Lambda(_t, _t*log(x**2
+ (256*_t**3*a*b*c**4 - 64*_t**3*b**3*c**3 - 8*_t*a**2*c**2 + 16*_t*a*b**2
*c - 4*_t*b**4)/(a**2*c - a*b**2)))) + x**2/(2*c)
```

3.311.7 Maxima [F]

$$\int \frac{x^9}{a+bx^4+cx^8}\,dx = \int \frac{x^9}{cx^8+bx^4+a}\,dx$$

input
```
integrate(x^9/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
1/2*x^2/c - integrate((b*x^4 + a)*x/(c*x^8 + b*x^4 + a), x)/c
```

3.311.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2043 vs. $2(150) = 300$.

Time = 1.92 (sec) , antiderivative size = 2043, normalized size of antiderivative = 10.64

$$\int \frac{x^9}{a+bx^4+cx^8}\,dx = \text{Too large to display}$$

input
```
integrate(x^9/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
1/2*x^2/c + 1/8*(2*a*b^3*c^3 - 8*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*a*b*c^3 - (sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*b^5 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^
3*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 2*b^5*c + 16*sqrt(
2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a*b^2*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2
 + 16*a*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 32*a
^2*b*c^3 + 2*(b^2 - 4*a*c)*b^3*c - 8*(b^2 - 4*a*c)*a*b*c^2)*x^4*abs(c) + (
2*b^4*c^3 - 8*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*b^4*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*
c^3 - 2*(b^2 - 4*a*c)*b^2*c^3)*x^4 - (sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a*b^4 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c - 2*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 2*a*b^4*c + 16*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^3*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^2*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 16*a...
```

3.311.9 Mupad [B] (verification not implemented)

Time = 9.23 (sec) , antiderivative size = 5659, normalized size of antiderivative = 29.47

$$\int \frac{x^9}{a+bx^4+cx^8}\,dx = \text{Too large to display}$$

input
```
int(x^9/(a + b*x^4 + c*x^8),x)
```

output
```
atan(((((16*(a*b^8 + 4*a^5*c^4 - 8*a^2*b^6*c + 20*a^3*b^4*c^2 - 16*a^4*b^2
*c^3))/c^2 + (((16*(32*a*b^7*c^3 - 256*a^4*b*c^6 - 256*a^2*b^5*c^4 + 576*a
^3*b^3*c^5))/c^2 - ((256*a*b^6*c^6 - 2048*a^2*b^4*c^7 + 4096*a^3*b^2*c^8)*
(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*
a*c - b^2)^3)^(1/2)))/(2*c^2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(b^5
 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c
- b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (4*x^2
*(64*a^4*b*c^5 + 16*a^2*b^5*c^3 - 80*a^3*b^3*c^4))/c^2)*(-(b^5 + b^2*(-(4*
a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1
/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))*(-(b^5 + b^2*(-(4*a
*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/
2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (4*x^2*(a^2*b^6 - 2
*a^5*c^3 - 5*a^3*b^4*c + 6*a^4*b^2*c^2))/c^2)*(-(b^5 + b^2*(-(4*a*c - b^2)
^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(
16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i - (((16*(a*b^8 + 4*a^5*c^4
- 8*a^2*b^6*c + 20*a^3*b^4*c^2 - 16*a^4*b^2*c^3))/c^2 + (((16*(32*a*b^7*c^
3 - 256*a^4*b*c^6 - 256*a^2*b^5*c^4 + 576*a^3*b^3*c^5))/c^2 - ((256*a*b^6*
c^6 - 2048*a^2*b^4*c^7 + 4096*a^3*b^2*c^8)*(b^5 + b^2*(-(4*a*c - b^2)^3)^(
1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2)))/(2*c^2*(1
6*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1...
```

3.312 $\int \frac{x^7}{a+bx^4+cx^8}\,dx$

3.312.1 Optimal result

Integrand size = 18, antiderivative size = 63

$$\int \frac{x^7}{a+bx^4+cx^8}\,dx=\frac{b\text{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^4+cx^8\right)}{8c}$$

output
```
1/8*ln(c*x^8+b*x^4+a)/c+1/4*b*arctanh((2*c*x^4+b)/(-4*a*c+b^2)^(1/2))/c/(-
4*a*c+b^2)^(1/2)
```

3.312.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.98

$$\int \frac{x^7}{a+bx^4+cx^8}\,dx=\frac{-\frac{2b\arctan\left(\frac{b+2cx^4}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+\log\left(a+bx^4+cx^8\right)}{8c}$$

input
```
Integrate[x^7/(a + b*x^4 + c*x^8),x]
```

output
```
((-2*b*ArcTan[(b + 2*c*x^4)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + Log[
a + b*x^4 + c*x^8])/(8*c)
```

3.312.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1693, 1142, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{a+bx^4+cx^8}dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{4}\int \frac{x^4}{cx^8+bx^4+a}dx^4$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{4}\left(\frac{\int \frac{2cx^4+b}{cx^8+bx^4+a}dx^4}{2c}-\frac{b\int \frac{1}{cx^8+bx^4+a}dx^4}{2c}\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{4}\left(\frac{b\int \frac{1}{-x^8+b^2-4ac}d(2cx^4+b)}{c}+\frac{\int \frac{2cx^4+b}{cx^8+bx^4+a}dx^4}{2c}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{4}\left(\frac{\int \frac{2cx^4+b}{cx^8+bx^4+a}dx^4}{2c}+\frac{b\,\text{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{4}\left(\frac{b\,\text{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}+\frac{\log(a+bx^4+cx^8)}{2c}\right)$$

input
```
Int[x^7/(a + b*x^4 + c*x^8),x]
```

output
```
((b*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 - 4*a*c]) + Log[
a + b*x^4 + c*x^8]/(2*c))/4
```

3.312.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.312.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

method	result
default	$\frac{\ln\left(c\,x^{8}+b\,x^{4}+a\right)}{8c}-\frac{b\arctan\left(\frac{2c\,x^{4}+b}{\sqrt{4ac-b^{2}}}\right)}{4c\sqrt{4ac-b^{2}}}$
risch	$\frac{\ln\left(\left(-4abc+b^{3}+\sqrt{-b^{2}(4ac-b^{2})}\,b\right)x^{4}+2\sqrt{-b^{2}(4ac-b^{2})}\,a\right)a}{8ac-2b^{2}}-\frac{\ln\left(\left(-4abc+b^{3}+\sqrt{-b^{2}(4ac-b^{2})}\,b\right)x^{4}+2\sqrt{-b^{2}(4ac-b^{2})}\,a\right)b^{2}}{8c(4ac-b^{2})}+$

input
```
int(x^7/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/8*ln(c*x^8+b*x^4+a)/c-1/4*b/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x^4+b)/(4*a*
c-b^2)^(1/2))
```

3.312.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 197, normalized size of antiderivative = 3.13

$$\int \frac{x^7}{a+bx^4+cx^8}\,dx = \left[\frac{\sqrt{b^2-4\,ac}b\log\left(\frac{2\,c^2x^8+2\,bcx^4+b^2-2\,ac+(2\,cx^4+b)\sqrt{b^2-4\,ac}}{cx^8+bx^4+a}\right)+(b^2-4\,ac)\log\left(cx^8+bx^4+a\right)}{8\,(b^2c-4\,ac^2)}, \frac{2\sqrt{-b^2+4\,ac}b\,\mathrm{ar}}{}\right.$$

input
```
integrate(x^7/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
[1/8*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^8 + 2*b*c*x^4 + b^2 - 2*a*c + (2*c*
x^4 + b)*sqrt(b^2 - 4*a*c))/(c*x^8 + b*x^4 + a)) + (b^2 - 4*a*c)*log(c*x^8
 + b*x^4 + a))/(b^2*c - 4*a*c^2), 1/8*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-(2*c
*x^4 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^2 - 4*a*c)*log(c*x^8 + b*
x^4 + a))/(b^2*c - 4*a*c^2)]
```

3.312.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. 2(54) = 108.

Time = 1.50 (sec) , antiderivative size = 223, normalized size of antiderivative = 3.54

$$\int \frac{x^7}{a+bx^4+cx^8}\,dx = \left(-\frac{b\sqrt{-4ac+b^2}}{8c\,(4ac-b^2)}+\frac{1}{8c}\right)\log\left(x^4+\frac{-16ac\left(-\frac{b\sqrt{-4ac+b^2}}{8c(4ac-b^2)}+\frac{1}{8c}\right)+2a+4b^2\left(-\frac{b\sqrt{-4ac+b^2}}{8c(4ac-b^2)}+\frac{1}{8c}\right)}{b}\right) + \left(\frac{b\sqrt{-4ac+b^2}}{8c\,(4ac-b^2)}+\frac{1}{8c}\right)\log\left(x^4+\frac{-16ac\left(\frac{b\sqrt{-4ac+b^2}}{8c(4ac-b^2)}+\frac{1}{8c}\right)+2a+4b^2\left(\frac{b\sqrt{-4ac+b^2}}{8c(4ac-b^2)}+\frac{1}{8c}\right)}{b}\right)$$

input
```
integrate(x**7/(c*x**8+b*x**4+a),x)
```

output
```
(-b*sqrt(-4*a*c + b**2)/(8*c*(4*a*c - b**2)) + 1/(8*c))*log(x**4 + (-16*a*
c*(-b*sqrt(-4*a*c + b**2)/(8*c*(4*a*c - b**2)) + 1/(8*c)) + 2*a + 4*b**2*(
-b*sqrt(-4*a*c + b**2)/(8*c*(4*a*c - b**2)) + 1/(8*c)))/b) + (b*sqrt(-4*a*
c + b**2)/(8*c*(4*a*c - b**2)) + 1/(8*c))*log(x**4 + (-16*a*c*(b*sqrt(-4*a
*c + b**2)/(8*c*(4*a*c - b**2)) + 1/(8*c)) + 2*a + 4*b**2*(b*sqrt(-4*a*c +
 b**2)/(8*c*(4*a*c - b**2)) + 1/(8*c)))/b)
```

3.312.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^7}{a+bx^4+cx^8}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^7/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.312.8 Giac [A] (verification not implemented)

Time = 1.52 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.94

$$\int \frac{x^7}{a+bx^4+cx^8}\,dx = -\frac{b\arctan\left(\frac{2\,cx^4+b}{\sqrt{-b^2+4\,ac}}\right)}{4\,\sqrt{-b^2+4\,ac}\,c} + \frac{\log\left(cx^8+bx^4+a\right)}{8\,c}$$

input
```
integrate(x^7/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
-1/4*b*arctan((2*c*x^4 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c) + 1
/8*log(c*x^8 + b*x^4 + a)/c
```

3.312.9 Mupad [B] (verification not implemented)

Time = 9.03 (sec) , antiderivative size = 2654, normalized size of antiderivative = 42.13

$$\int \frac{x^7}{a + bx^4 + cx^8} \, dx = \text{Too large to display}$$

input
```
int(x^7/(a + b*x^4 + c*x^8),x)
```

output
```
(log(a + b*x^4 + c*x^8)*(16*a*c - 4*b^2))/(2*(64*a*c^2 - 16*b^2*c)) - (b*a
tan((8*x^4*(((a*c - b^2)*(((((16*a*c - 4*b^2)*((b*(448*b^3*c^3 - (256*b^3*
c^4*(16*a*c - 4*b^2))/(64*a*c^2 - 16*b^2*c)))/(8*c*(4*a*c - b^2)^(1/2)) -
(32*b^4*c^3*(16*a*c - 4*b^2))/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2)^(1/2)))
)/(2*(64*a*c^2 - 16*b^2*c)) - (b*(144*b^3*c^2 - ((448*b^3*c^3 - (256*b^3*c
^4*(16*a*c - 4*b^2))/(64*a*c^2 - 16*b^2*c))*(16*a*c - 4*b^2))/(2*(64*a*c^2
 - 16*b^2*c))))/(8*c*(4*a*c - b^2)^(1/2)))*(16*a*c - 4*b^2))/(2*(64*a*c^2
- 16*b^2*c)) - (b*((b*((b*(448*b^3*c^3 - (256*b^3*c^4*(16*a*c - 4*b^2))/(6
4*a*c^2 - 16*b^2*c)))/(8*c*(4*a*c - b^2)^(1/2)) - (32*b^4*c^3*(16*a*c - 4*
b^2))/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2)^(1/2))))/(8*c*(4*a*c - b^2)^(1/
2)) - (4*b^5*c^2*(16*a*c - 4*b^2))/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2))))
/(8*c*(4*a*c - b^2)^(1/2)) + (b*(20*b^3*c - ((144*b^3*c^2 - ((448*b^3*c^3
- (256*b^3*c^4*(16*a*c - 4*b^2))/(64*a*c^2 - 16*b^2*c))*(16*a*c - 4*b^2))/
(2*(64*a*c^2 - 16*b^2*c)))*(16*a*c - 4*b^2))/(2*(64*a*c^2 - 16*b^2*c))))/(
8*c*(4*a*c - b^2)^(1/2)) + (b^6*c*(16*a*c - 4*b^2))/(2*(64*a*c^2 - 16*b^2*
c)*(4*a*c - b^2)^(3/2))))/(8*a^3*c^2) + ((b^3 - 3*a*b*c)*(b^7/(8*(4*a*c -
b^2)^2) + b^3 - ((20*b^3*c - ((144*b^3*c^2 - ((448*b^3*c^3 - (256*b^3*c^4*
(16*a*c - 4*b^2))/(64*a*c^2 - 16*b^2*c))*(16*a*c - 4*b^2))/(2*(64*a*c^2 -
16*b^2*c)))*(16*a*c - 4*b^2))/(2*(64*a*c^2 - 16*b^2*c)))*(16*a*c - 4*b^2))
/(2*(64*a*c^2 - 16*b^2*c)) + ((16*a*c - 4*b^2)*((b*((b*(448*b^3*c^3 - (...
```

3.313 $\int \frac{x^5}{a+bx^4+cx^8} dx$

3.313.1 Optimal result

Integrand size = 18, antiderivative size = 159

$$\int \frac{x^5}{a+bx^4+cx^8}\,dx = -\frac{\sqrt{b-\sqrt{b^2-4ac}}\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}} + \frac{\sqrt{b+\sqrt{b^2-4ac}}\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

output
```
-1/4*arctan(x^2*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b-(-4*a*c+b
^2)^(1/2))^(1/2)*2^(1/2)/c^(1/2)/(-4*a*c+b^2)^(1/2)+1/4*arctan(x^2*2^(1/2)
*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2
)/c^(1/2)/(-4*a*c+b^2)^(1/2)
```

3.313.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.08

$$\int \frac{x^5}{a+bx^4+cx^8}\,dx = \frac{\left(-b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)+\sqrt{b-\sqrt{b^2-4ac}}\sqrt{b+\sqrt{b^2-4ac}}\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}$$

input
```
Integrate[x^5/(a + b*x^4 + c*x^8),x]
```

output
```
((-b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 -
 4*a*c]]] + Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*ArcTan
[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*Sq
rt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]])
```

3.313.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1695, 1450, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{a+bx^4+cx^8} \, dx$$

$$\Big\downarrow \text{1695}$$

$$\frac{1}{2}\int \frac{x^4}{cx^8+bx^4+a}dx^2$$

$$\Big\downarrow \text{1450}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^4+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx^2+\frac{1}{2}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\int \frac{1}{cx^4+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx^2\right)$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{2}\left(\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}\right)$$

input
```
Int[x^5/(a + b*x^4 + c*x^8),x]
```

output
```
(((1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2
 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((1 + b/Sqrt[
b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(
Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/2
```

3.313.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1450
```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^2/2)*(b/q + 1)   Int[(d*x)^(m - 2)/(b/
2 + q/2 + c*x^2), x], x] - Simp[(d^2/2)*(b/q - 1)   Int[(d*x)^(m - 2)/(b/2
- q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] &&
 GeQ[m, 2]
```

rule 1695
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b
*x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c,
p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.313.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.66

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(16a^2c^3-8b^2ac^2+b^4c\right)\_Z^4+\left(-4abc+b^3\right)\_Z^2+a\right)}\_R\ln\left(\left(\left(-4ac^2+b^2c\right)\_R^2+b\right)x^2+\left(4abc^2-b^3c\right)\_R^3+\left(2ac-b^2\right)\_ \cdots\right.\right.}{4}$
default	$2c\left(\frac{\left(b+\sqrt{-4ac+b^2}\right)\sqrt{2}\,\arctan\left(\frac{c\,x^2\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{8c\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\left(-b+\sqrt{-4ac+b^2}\right)\sqrt{2}\,\mathrm{arctanh}\left(\frac{c\,x^2\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{8c\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)$

input
```
int(x^5/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(_R*ln(((-4*a*c^2+b^2*c)*_R^2+b)*x^2+(4*a*b*c^2-b^3*c)*_R^3+(2*a*c-
b^2)*_R),_R=RootOf((16*a^2*c^3-8*a*b^2*c^2+b^4*c)*_Z^4+(-4*a*b*c+b^3)*_Z^2
+a))
```

3.313.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 567 vs. $2(119) = 238$.

Time = 0.26 (sec) , antiderivative size = 567, normalized size of antiderivative = 3.57

$$\int \frac{x^5}{a+bx^4+cx^8}\,dx = \frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(x^2+\frac{\sqrt{\frac{1}{2}}(b^2c-4\,ac^2)\sqrt{-\frac{b+\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}}{\sqrt{b^2c^2-4\,ac^3}}\right) - \frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(x^2-\frac{\sqrt{\frac{1}{2}}(b^2c-4\,ac^2)\sqrt{-\frac{b+\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}}{\sqrt{b^2c^2-4\,ac^3}}\right) - \frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{-\frac{b-\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(x^2+\frac{\sqrt{\frac{1}{2}}(b^2c-4\,ac^2)\sqrt{-\frac{b-\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}}{\sqrt{b^2c^2-4\,ac^3}}\right) + \frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{-\frac{b-\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(x^2-\frac{\sqrt{\frac{1}{2}}(b^2c-4\,ac^2)\sqrt{-\frac{b-\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}}{\sqrt{b^2c^2-4\,ac^3}}\right)$$

input
```
integrate(x^5/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
1/4*sqrt(1/2)*sqrt(-(b + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c
 - 4*a*c^2))*log(x^2 + sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b + (b^2*c - 4*a
*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3))
 - 1/4*sqrt(1/2)*sqrt(-(b + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^
2*c - 4*a*c^2))*log(x^2 - sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b + (b^2*c -
4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^
3)) - 1/4*sqrt(1/2)*sqrt(-(b - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/
(b^2*c - 4*a*c^2))*log(x^2 + sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b - (b^2*c
 - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a
*c^3)) + 1/4*sqrt(1/2)*sqrt(-(b - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3
))/(b^2*c - 4*a*c^2))*log(x^2 - sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b - (b^
2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 -
4*a*c^3))
```

3.313.6 Sympy [A] (verification not implemented)

Time = 1.43 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.48

$$\int \frac{x^5}{a+bx^4+cx^8}\,dx$$
$$= \mathrm{RootSum}\left(t^4\cdot\left(4096a^2c^3-2048ab^2c^2+256b^4c\right)+t^2\left(-64abc+16b^3\right)+a,\left(t\mapsto t\log\left(512t^3ac^2-128t^3b^2\right.\right.\right.$$

input
```
integrate(x**5/(c*x**8+b*x**4+a),x)
```

output
```
RootSum(_t**4*(4096*a**2*c**3 - 2048*a*b**2*c**2 + 256*b**4*c) + _t**2*(-6
4*a*b*c + 16*b**3) + a, Lambda(_t, _t*log(512*_t**3*a*c**2 - 128*_t**3*b**
2*c - 4*_t*b + x**2)))
```

3.313.7 Maxima [F]

$$\int \frac{x^5}{a+bx^4+cx^8}\,dx = \int \frac{x^5}{cx^8+bx^4+a}\,dx$$

input
```
integrate(x^5/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
integrate(x^5/(c*x^8 + b*x^4 + a), x)
```

3.313.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1034 vs. 2(119) = 238.

Time = 2.04 (sec) , antiderivative size = 1034, normalized size of antiderivative = 6.50

$$\int \frac{x^5}{a+bx^4+cx^8}\,dx$$

$$= \frac{\left(\sqrt{2}\sqrt{bc+\sqrt{b^2-4\,ac}c}b^4 - 8\sqrt{2}\sqrt{bc+\sqrt{b^2-4\,ac}c}ab^2c - 2\sqrt{2}\sqrt{bc+\sqrt{b^2-4\,ac}c}b^3c - 2\,b^4c + 16\sqrt{2}\sqrt{}\right.}{}$$

$$+ \frac{\left(\sqrt{2}\sqrt{bc-\sqrt{b^2-4\,ac}c}b^4 - 8\sqrt{2}\sqrt{bc-\sqrt{b^2-4\,ac}c}ab^2c - 2\sqrt{2}\sqrt{bc-\sqrt{b^2-4\,ac}c}b^3c + 2\,b^4c + 16\sqrt{2}\right.}{}$$

input
```
integrate(x^5/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
1/8*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3
*c - 2*b^4*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*b^2*c^2 + 16*a*b^2*c^2 - 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a*c^3 - 32*a^2*c^3 + 8*a*b*c^3 + sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*b^2*c + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*b*c^2 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2 + 2*(b^2
 - 4*a*c)*b*c^2)*x^4*arctan(2*sqrt(1/2)*x^2/sqrt((b + sqrt(b^2 - 4*a*c))/c
))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^
2 - 4*a^2*c^3)*abs(c)) + 1/8*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4
- 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*b^3*c + 2*b^4*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^2*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqr
t(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - 16*a*b^2*c^2 - 2*b^3*c^2 -
4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 + 32*a^2*c^3 + 8*a*b*c^3 +
 sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 - 4*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sq...
```

3.313.9 Mupad [B] (verification not implemented)

Time = 9.18 (sec) , antiderivative size = 1220, normalized size of antiderivative = 7.67

$$\int \frac{x^5}{a+bx^4+cx^8}\,dx$$
$$=\operatorname{atan}\left(\frac{}{8\,b^4\,\sqrt{\frac{\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-b^3+4\,a\,b\,c}{512\,a^2\,c^3-256\,a\,b^2\,c^2+32\,b^4\,c}}+128\,b^5\,c\left(\frac{\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-b^3+4\,a\,b\,c}{512\,a^2\,c^3-256\,a\,b^2\,c^2+32\,b^4\,c}\right)^{3/2}+6\right.$$

input
```
int(x^5/(a + b*x^4 + c*x^8),x)
```

output
```
atan((x^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)*1i + b^3*
x^2*1i - a*b*c*x^2*4i)/(8*b^4*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*
b^4*c)^(1/2) - b^3 + 4*a*b*c)/(32*b^4*c + 512*a^2*c^3 - 256*a*b^2*c^2))^(1
/2) + 128*b^5*c*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) -
 b^3 + 4*a*b*c)/(32*b^4*c + 512*a^2*c^3 - 256*a*b^2*c^2))^(3/2) + 64*a^2*c
^2*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*
c)/(32*b^4*c + 512*a^2*c^3 - 256*a*b^2*c^2))^(1/2) - 1024*a*b^3*c^2*(((b^6
 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(32*b^
4*c + 512*a^2*c^3 - 256*a*b^2*c^2))^(3/2) + 2048*a^2*b*c^3*(((b^6 - 64*a^3
*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(32*b^4*c + 512
*a^2*c^3 - 256*a*b^2*c^2))^(3/2) - 48*a*b^2*c*(((b^6 - 64*a^3*c^3 + 48*a^2
*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(32*b^4*c + 512*a^2*c^3 - 25
6*a*b^2*c^2))^(1/2)))*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(
1/2) - b^3 + 4*a*b*c)/(32*b^4*c + 512*a^2*c^3 - 256*a*b^2*c^2))^(1/2)*2i -
 atan((x^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)*1i - b^3
*x^2*1i + a*b*c*x^2*4i)/(8*b^4*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2
 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(32*b^4*c + 512*a^2*c^3 - 256*a*b^2*c^2))^
(1/2) + 128*b^5*c*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c
)^(1/2) - 4*a*b*c)/(32*b^4*c + 512*a^2*c^3 - 256*a*b^2*c^2))^(3/2) + 64*a^
2*c^2*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) -...
```

3.314 $\int \frac{x^3}{a+bx^4+cx^8}\,dx$

3.314.1 Optimal result

Integrand size = 18, antiderivative size = 38

$$\int \frac{x^3}{a+bx^4+cx^8}\,dx=-\frac{\operatorname{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{2\sqrt{b^2-4ac}}$$

output
```
-1/2*arctanh((2*c*x^4+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)
```

3.314.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11

$$\int \frac{x^3}{a+bx^4+cx^8}\,dx=\frac{\arctan\left(\frac{b+2cx^4}{\sqrt{-b^2+4ac}}\right)}{2\sqrt{-b^2+4ac}}$$

input
```
Integrate[x^3/(a + b*x^4 + c*x^8),x]
```

output
```
ArcTan[(b + 2*c*x^4)/Sqrt[-b^2 + 4*a*c]]/(2*Sqrt[-b^2 + 4*a*c])
```

3.314.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1690, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{a+bx^4+cx^8}\,dx$$
$$\Big\downarrow \text{1690}$$
$$\frac{1}{4}\int \frac{1}{cx^8+bx^4+a}dx^4$$
$$\Big\downarrow \text{1083}$$
$$-\frac{1}{2}\int \frac{1}{-x^8+b^2-4ac}d(2cx^4+b)$$
$$\Big\downarrow \text{219}$$
$$-\frac{\operatorname{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{2\sqrt{b^2-4ac}}$$

input
```
Int[x^3/(a + b*x^4 + c*x^8),x]
```

output
```
-1/2*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]]/Sqrt[b^2 - 4*a*c]
```

3.314.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

3.314.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{\arctan\left(\frac{2c\,x^{4}+b}{\sqrt{4ac-b^{2}}}\right)}{2\sqrt{4ac-b^{2}}}$	37
risch	$-\frac{\ln\left(\left(-b+\sqrt{-4ac+b^{2}}\right)x^{4}-2a\right)}{4\sqrt{-4ac+b^{2}}}+\frac{\ln\left(\left(b+\sqrt{-4ac+b^{2}}\right)x^{4}+2a\right)}{4\sqrt{-4ac+b^{2}}}$	70

input
```
int(x^3/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^4+b)/(4*a*c-b^2)^(1/2))
```

3.314.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 129, normalized size of antiderivative = 3.39

$$\int \frac{x^3}{a+bx^4+cx^8}\,dx = \left[\frac{\log\left(\frac{2\,c^2x^8+2\,bcx^4+b^2-2\,ac-(2\,cx^4+b)\sqrt{b^2-4\,ac}}{cx^8+bx^4+a}\right)}{4\,\sqrt{b^2-4\,ac}},\ -\frac{\sqrt{-b^2+4\,ac}\arctan\left(-\frac{(2\,cx^4+b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)}{2\,(b^2-4\,ac)}\right]$$

input
```
integrate(x^3/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
[1/4*log((2*c^2*x^8 + 2*b*c*x^4 + b^2 - 2*a*c - (2*c*x^4 + b)*sqrt(b^2 - 4
*a*c))/(c*x^8 + b*x^4 + a))/sqrt(b^2 - 4*a*c), -1/2*sqrt(-b^2 + 4*a*c)*arc
tan(-(2*c*x^4 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c))/(b^2 - 4*a*c)]
```

3.314.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(36) = 72$.

Time = 0.44 (sec) , antiderivative size = 131, normalized size of antiderivative = 3.45

$$\int \frac{x^3}{a+bx^4+cx^8}\,dx = -\frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(x^4+\frac{-4ac\sqrt{-\frac{1}{4ac-b^2}}+b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)}{4} + \frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(x^4+\frac{4ac\sqrt{-\frac{1}{4ac-b^2}}-b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)}{4}$$

input
```
integrate(x**3/(c*x**8+b*x**4+a),x)
```

output
```
-sqrt(-1/(4*a*c - b**2))*log(x**4 + (-4*a*c*sqrt(-1/(4*a*c - b**2)) + b**2
*sqrt(-1/(4*a*c - b**2)) + b)/(2*c))/4 + sqrt(-1/(4*a*c - b**2))*log(x**4
+ (4*a*c*sqrt(-1/(4*a*c - b**2)) - b**2*sqrt(-1/(4*a*c - b**2)) + b)/(2*c)
)/4
```

3.314.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{a+bx^4+cx^8}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.314.8 Giac [A] (verification not implemented)

Time = 1.66 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

$$\int \frac{x^3}{a+bx^4+cx^8}\,dx=\frac{\arctan\left(\frac{2\,cx^4+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}}$$

input
```
integrate(x^3/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
1/2*arctan((2*c*x^4 + b)/sqrt(-b^2 + 4*a*c))/sqrt(-b^2 + 4*a*c)
```

3.314.9 Mupad [B] (verification not implemented)

Time = 8.21 (sec) , antiderivative size = 260, normalized size of antiderivative = 6.84

$$\int \frac{x^3}{a+bx^4+cx^8}\,dx=$$

$$-\frac{\mathrm{atan}\left(\frac{(4\,a\,c-b^2)^2\left(\frac{\left(\frac{4\,a\,c^4}{4\,a\,c-b^2}-\frac{4\,a\,b^2\,c^4}{(4\,a\,c-b^2)^2}\right)(b^3-3\,a\,b\,c)}{8\,a^3\,c^2\,\sqrt{4\,a\,c-b^2}}-x^4\left(\frac{\left(\frac{2\,c^4}{\sqrt{4\,a\,c-b^2}}-\frac{6\,b^2\,c^4}{(4\,a\,c-b^2)^{3/2}}\right)(a\,c-b^2)}{8\,a^3\,c^2}-\frac{(b^3-3\,a\,b\,c)\left(\frac{6\,b\,c^4}{4\,a\,c-b^2}-\frac{2\,b^3\,c^4}{(4\,a\,c-b^2)^2}\right.}{8\,a^3\,c^2\,\sqrt{4\,a\,c-b^2}}\right.\right.}{2\,c^4}\right)}{2\,\sqrt{4\,a\,c-b^2}}$$

input
```
int(x^3/(a + b*x^4 + c*x^8),x)
```

output
```
-atan(((4*a*c - b^2)^2*((((4*a*c^4)/(4*a*c - b^2) - (4*a*b^2*c^4)/(4*a*c -
 b^2)^2)*(b^3 - 3*a*b*c))/(8*a^3*c^2*(4*a*c - b^2)^(1/2)) - x^4*((((2*c^4)
/(4*a*c - b^2)^(1/2) - (6*b^2*c^4)/(4*a*c - b^2)^(3/2))*(a*c - b^2))/(8*a^
3*c^2) - ((b^3 - 3*a*b*c)*((6*b*c^4)/(4*a*c - b^2) - (2*b^3*c^4)/(4*a*c -
b^2)^2))/(8*a^3*c^2*(4*a*c - b^2)^(1/2))) + (b*c^2*(a*c - b^2))/(a^2*(4*a*
c - b^2)^(3/2))))/(2*c^4))/(2*(4*a*c - b^2)^(1/2))
```

3.315 $\int \frac{x}{a+bx^4+cx^8}\,dx$

3.315.1 Optimal result

Integrand size = 16, antiderivative size = 154

$$\int \frac{x}{a+bx^4+cx^8}\,dx=\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
1/2*arctan(x^2*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*2^(1/
2)/(-4*a*c+b^2)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*arctan(x^2*2^(1/2)*
c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*2^(1/2)/(-4*a*c+b^2)^(1/2)/(
b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.315.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.86

$$\int \frac{x}{a+bx^4+cx^8}\,dx=\frac{\sqrt{c}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}}$$

input
```
Integrate[x/(a + b*x^4 + c*x^8),x]
```

output
```
(Sqrt[c]*(ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/Sqrt[b
 - Sqrt[b^2 - 4*a*c]] - ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4
*a*c]]]/Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(Sqrt[2]*Sqrt[b^2 - 4*a*c])
```

3.315.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1695, 1406, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{a+bx^4+cx^8}\,dx$$

$$\Big\downarrow \text{1695}$$

$$\frac{1}{2}\int \frac{1}{cx^8+bx^4+a}dx^2$$

$$\Big\downarrow \text{1406}$$

$$\frac{1}{2}\left(\frac{c\int \frac{1}{cx^4+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx^2}{\sqrt{b^2-4ac}}-\frac{c\int \frac{1}{cx^4+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx^2}{\sqrt{b^2-4ac}}\right)$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{2}\left(\frac{\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}\right)$$

input
```
Int[x/(a + b*x^4 + c*x^8),x]
```

output
```
((Sqrt[2]*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]
])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*Sqrt[c]*ArcT
an[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*
Sqrt[b + Sqrt[b^2 - 4*a*c]]))/2
```

3.315.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

rule 1695
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b
*x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c,
p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.315.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.64

method	result
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\left(16a^3c^2-8a^2b^2c+b^4a\right)\_Z^4+\left(-4abc+b^3\right)\_Z^2+c\right)}\_R\ln\left(\left(\left(4abc-b^3\right)\_R^2-c\right)x^2+\left(4cb\,a^2-a\,b^3\right)\_R^3-2ac\_R\right)\right)}{4}$
default	$2c\left(-\frac{\sqrt{2}\,\arctan\left(\frac{c\,x^2\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{c\,x^2\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)$

input
```
int(x/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(_R*ln(((4*a*b*c-b^3)*_R^2-c)*x^2+(4*a^2*b*c-a*b^3)*_R^3-2*a*c*_R),
_R=RootOf((16*a^3*c^2-8*a^2*b^2*c+a*b^4)*_Z^4+(-4*a*b*c+b^3)*_Z^2+c))
```

3.315.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 619 vs. $2(119) = 238$.

Time = 0.28 (sec) , antiderivative size = 619, normalized size of antiderivative = 4.02

$$\int \frac{x}{a+bx^4+cx^8}\,dx = -\frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\log\left(cx^2 + \frac{1}{2}\sqrt{\frac{1}{2}}\left(b^2-4\,ac-\frac{ab^3-4\,a^2bc}{\sqrt{a^2b^2-4\,a^3c}}\right)\sqrt{-\frac{b+\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\right) + \frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\log\left(cx^2 - \frac{1}{2}\sqrt{\frac{1}{2}}\left(b^2-4\,ac-\frac{ab^3-4\,a^2bc}{\sqrt{a^2b^2-4\,a^3c}}\right)\sqrt{-\frac{b+\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\right) - \frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{-\frac{b-\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\log\left(cx^2 + \frac{1}{2}\sqrt{\frac{1}{2}}\left(b^2-4\,ac+\frac{ab^3-4\,a^2bc}{\sqrt{a^2b^2-4\,a^3c}}\right)\sqrt{-\frac{b-\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\right) + \frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{-\frac{b-\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\log\left(cx^2 - \frac{1}{2}\sqrt{\frac{1}{2}}\left(b^2-4\,ac+\frac{ab^3-4\,a^2bc}{\sqrt{a^2b^2-4\,a^3c}}\right)\sqrt{-\frac{b-\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\right)$$

input
```
integrate(x/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
-1/4*sqrt(1/2)*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^
2 - 4*a^2*c))*log(c*x^2 + 1/2*sqrt(1/2)*(b^2 - 4*a*c - (a*b^3 - 4*a^2*b*c)
/sqrt(a^2*b^2 - 4*a^3*c))*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^
3*c))/(a*b^2 - 4*a^2*c))) + 1/4*sqrt(1/2)*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqr
t(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(c*x^2 - 1/2*sqrt(1/2)*(b^2 -
4*a*c - (a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt(-(b + (a*b^2 - 4
*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))) - 1/4*sqrt(1/2)*sqrt(
-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(c*
x^2 + 1/2*sqrt(1/2)*(b^2 - 4*a*c + (a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^
3*c))*sqrt(-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2
*c))) + 1/4*sqrt(1/2)*sqrt(-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c)
)/(a*b^2 - 4*a^2*c))*log(c*x^2 - 1/2*sqrt(1/2)*(b^2 - 4*a*c + (a*b^3 - 4*a
^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt(-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2
 - 4*a^3*c))/(a*b^2 - 4*a^2*c)))
```

3.315.6 Sympy [A] (verification not implemented)

Time = 2.17 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.57

$$\int \frac{x}{a+bx^4+cx^8}\,dx$$
$$= \text{RootSum}\left(t^4\cdot\left(4096a^3c^2-2048a^2b^2c+256ab^4\right)+t^2\left(-64abc+16b^3\right)+c,\left(t\mapsto t\log\left(x^2+\frac{256t^3a^2bc-$$

input
```
integrate(x/(c*x**8+b*x**4+a),x)
```

output
```
RootSum(_t**4*(4096*a**3*c**2 - 2048*a**2*b**2*c + 256*a*b**4) + _t**2*(-6
4*a*b*c + 16*b**3) + c, Lambda(_t, _t*log(x**2 + (256*_t**3*a**2*b*c - 64*
_t**3*a*b**3 + 8*_t*a*c - 4*_t*b**2)/c)))
```

3.315.7 Maxima [F]

$$\int \frac{x}{a+bx^4+cx^8}\,dx=\int \frac{x}{cx^8+bx^4+a}\,dx$$

input
```
integrate(x/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
integrate(x/(c*x^8 + b*x^4 + a), x)
```

3.315.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1028 vs. 2(119) = 238.

Time = 1.89 (sec) , antiderivative size = 1028, normalized size of antiderivative = 6.68

$$\int \frac{x}{a+bx^4+cx^8}\,dx$$
$$= \frac{\left(\sqrt{2}\sqrt{bc+\sqrt{b^2-4\,ac}c}b^4-8\,\sqrt{2}\sqrt{bc+\sqrt{b^2-4\,ac}c}ab^2c-2\,\sqrt{2}\sqrt{bc+\sqrt{b^2-4\,ac}c}b^3c-2\,b^4c+16\,\sqrt{2}\sqrt{\cdots}\right.}{\cdots}$$
$$+\frac{\left(\sqrt{2}\sqrt{bc-\sqrt{b^2-4\,ac}c}b^4-8\,\sqrt{2}\sqrt{bc-\sqrt{b^2-4\,ac}c}ab^2c-2\,\sqrt{2}\sqrt{bc-\sqrt{b^2-4\,ac}c}b^3c+2\,b^4c+16\,\sqrt{2}\cdots\right.}{\cdots}$$

input
```
integrate(x/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
1/8*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3
*c - 2*b^4*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*b^2*c^2 + 16*a*b^2*c^2 - 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a*c^3 - 32*a^2*c^3 + 8*a*b*c^3 + sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*b^2*c + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*b*c^2 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2 + 2*(b^2
 - 4*a*c)*b*c^2)*arctan(2*sqrt(1/2)*x^2/sqrt((b + sqrt(b^2 - 4*a*c))/c))/(
(a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 -
4*a^2*c^3)*abs(c)) + 1/8*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 - 8*
sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*b^3*c + 2*b^4*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^2*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - 16*a*b^2*c^2 - 2*b^3*c^2 - 4*sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 + 32*a^2*c^3 + 8*a*b*c^3 + sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 - 4*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sqrt(b...
```

3.315.9 Mupad [B] (verification not implemented)

Time = 8.84 (sec) , antiderivative size = 1105, normalized size of antiderivative = 7.18

$$\int \frac{x}{a+bx^4+cx^8}\,dx$$

$$= \operatorname{atan}\left(\frac{b^4\,x}{128\,a^2\,b^5\left(-\frac{b^3+\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-4\,a\,b\,c}{512\,a^3\,c^2-256\,a^2\,b^2\,c+32\,a\,b^4}\right)^{3/2}-64\,a^3\,c^2\sqrt{-\frac{b^3+\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-}{512\,a^3\,c^2-256\,a^2\,b^2\,c+32\,a\,b^4}}}\right.$$

input

```
int(x/(a + b*x^4 + c*x^8),x)
```

output

```
atan((b^4*x^2*1i + b*x^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^
(1/2)*1i + a^2*c^2*x^2*8i - a*b^2*c*x^2*6i)/(128*a^2*b^5*(-(b^3 + (b^6 - 6
4*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(32*a*b^4 + 512*
a^3*c^2 - 256*a^2*b^2*c))^(3/2) - 64*a^3*c^2*(-(b^3 + (b^6 - 64*a^3*c^3 +
48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(32*a*b^4 + 512*a^3*c^2 - 25
6*a^2*b^2*c))^(1/2) + 16*a^2*b^2*c*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2
*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(32*a*b^4 + 512*a^3*c^2 - 256*a^2*b^2*
c))^(1/2) - 1024*a^3*b^3*c*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 1
2*a*b^4*c)^(1/2) - 4*a*b*c)/(32*a*b^4 + 512*a^3*c^2 - 256*a^2*b^2*c))^(3/2
) + 2048*a^4*b*c^2*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*
c)^(1/2) - 4*a*b*c)/(32*a*b^4 + 512*a^3*c^2 - 256*a^2*b^2*c))^(3/2)))*(-(b
^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(32
*a*b^4 + 512*a^3*c^2 - 256*a^2*b^2*c))^(1/2)*2i + atan((b^4*x^2*1i - b*x^2
*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)*1i + a^2*c^2*x^2*8
i - a*b^2*c*x^2*6i)/(128*a^2*b^5*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12
*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(32*a*b^4 + 512*a^3*c^2 - 256*a^2*b^2*c))
^(3/2) - 64*a^3*c^2*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/
2) - b^3 + 4*a*b*c)/(32*a*b^4 + 512*a^3*c^2 - 256*a^2*b^2*c))^(1/2) + 16*a
^2*b^2*c*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 +
4*a*b*c)/(32*a*b^4 + 512*a^3*c^2 - 256*a^2*b^2*c))^(1/2) - 1024*a^3*b^3...
```

3.316 $\int \frac{1}{x(a+bx^4+cx^8)} dx$

3.316.1 Optimal result

Integrand size = 18, antiderivative size = 69

$$\int \frac{1}{x(a+bx^4+cx^8)} dx = \frac{b\operatorname{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4a\sqrt{b^2-4ac}} + \frac{\log(x)}{a} - \frac{\log(a+bx^4+cx^8)}{8a}$$

output
```
ln(x)/a-1/8*ln(c*x^8+b*x^4+a)/a+1/4*b*arctanh((2*c*x^4+b)/(-4*a*c+b^2)^(1/
2))/a/(-4*a*c+b^2)^(1/2)
```

3.316.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int \frac{1}{x(a+bx^4+cx^8)} dx = \frac{\log(x)}{a} - \frac{\operatorname{RootSum}\left[a+b\#1^4+c\#1^8\&, \frac{b\log(x-\#1)+c\log(x-\#1)\#1^4}{b+2c\#1^4}\&\right]}{4a}$$

input
```
Integrate[1/(x*(a + b*x^4 + c*x^8)),x]
```

output
```
Log[x]/a - RootSum[a + b*#1^4 + c*#1^8 & , (b*Log[x - #1] + c*Log[x - #1]*
#1^4)/(b + 2*c*#1^4) & ]/(4*a)
```

3.316.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1693, 1144, 25, 1142, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+bx^4+cx^8\right)}dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{4}\int \frac{1}{x^4\left(cx^8+bx^4+a\right)}dx^4$$

$$\Big\downarrow \text{1144}$$

$$\frac{1}{4}\left(\frac{\int -\frac{cx^4+b}{cx^8+bx^4+a}dx^4}{a}+\frac{\log\left(x^4\right)}{a}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4}\left(\frac{\log\left(x^4\right)}{a}-\frac{\int \frac{cx^4+b}{cx^8+bx^4+a}dx^4}{a}\right)$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{4}\left(\frac{\log\left(x^4\right)}{a}-\frac{\frac{1}{2}b\int \frac{1}{cx^8+bx^4+a}dx^4+\frac{1}{2}\int \frac{2cx^4+b}{cx^8+bx^4+a}dx^4}{a}\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{4}\left(\frac{\log\left(x^4\right)}{a}-\frac{\frac{1}{2}\int \frac{2cx^4+b}{cx^8+bx^4+a}dx^4-b\int \frac{1}{-x^8+b^2-4ac}d\left(2cx^4+b\right)}{a}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{4}\left(\frac{\log\left(x^4\right)}{a}-\frac{\frac{1}{2}\int \frac{2cx^4+b}{cx^8+bx^4+a}dx^4-\frac{b\,\text{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}}{a}\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{4}\left(\frac{\log(x^4)}{a}-\frac{\frac{1}{2}\log(a+bx^4+cx^8)-\frac{b\operatorname{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}}{a}\right)$$

input
```
Int[1/(x*(a + b*x^4 + c*x^8)),x]
```

output
```
(Log[x^4]/a - (-((b*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4
*a*c]) + Log[a + b*x^4 + c*x^8]/2)/a)/4
```

3.316.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1144
```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Simp[e*(Log[RemoveContent[d + e*x, x]]/(c*d^2 - b*d*e + a*e^2)), x] + S
imp[1/(c*d^2 - b*d*e + a*e^2)   Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2),
x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.316.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

method	result	size
default	$\frac{\ln(x)}{a}-\frac{\frac{\ln\left(c\,x^{8}+b\,x^{4}+a\right)}{4}+\frac{b\arctan\left(\frac{2c\,x^{4}+b}{\sqrt{4ac-b^{2}}}\right)}{2\sqrt{4ac-b^{2}}}}{2a}$	66
risch	$\frac{\ln(x)}{a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4c\,a^{2}-b^{2}a\right)\_Z^{2}+\left(4ac-b^{2}\right)\_Z+c\right)}\_R\ln\left(\left(\left(18ac-5b^{2}\right)\_R+9c\right)x^{4}-ab\_R+4b\right)\right)}{4}$	77

input
```
int(1/x/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
ln(x)/a-1/2/a*(1/4*ln(c*x^8+b*x^4+a)+1/2*b/(4*a*c-b^2)^(1/2)*arctan((2*c*x
^4+b)/(4*a*c-b^2)^(1/2)))
```

3.316.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 223, normalized size of antiderivative = 3.23

$$\int \frac{1}{x\left(a+bx^4+cx^8\right)}\,dx$$
$$=\left[\frac{\sqrt{b^2-4\,ac}b\log\left(\frac{2\,c^2x^8+2\,bcx^4+b^2-2\,ac+\left(2\,cx^4+b\right)\sqrt{b^2-4\,ac}}{cx^8+bx^4+a}\right)-\left(b^2-4\,ac\right)\log\left(cx^8+bx^4+a\right)+8\left(b^2-4\,ac\right)\log}{8\left(ab^2-4\,a^2c\right)}\right.$$

input
```
integrate(1/x/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
[1/8*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^8 + 2*b*c*x^4 + b^2 - 2*a*c + (2*c*
x^4 + b)*sqrt(b^2 - 4*a*c))/(c*x^8 + b*x^4 + a)) - (b^2 - 4*a*c)*log(c*x^8
 + b*x^4 + a) + 8*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c), 1/8*(2*sqrt(-b^
2 + 4*a*c)*b*arctan(-(2*c*x^4 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^
2 - 4*a*c)*log(c*x^8 + b*x^4 + a) + 8*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2
*c)]
```

3.316.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(60) = 120$.

Time = 99.71 (sec) , antiderivative size = 253, normalized size of antiderivative = 3.67

$$\int \frac{1}{x(a+bx^4+cx^8)}\,dx = \left(-\frac{b\sqrt{-4ac+b^2}}{8a(4ac-b^2)} - \frac{1}{8a}\right)\log\left(x^4 + \frac{-16a^2c\left(-\frac{b\sqrt{-4ac+b^2}}{8a(4ac-b^2)} - \frac{1}{8a}\right) + 4ab^2\left(-\frac{b\sqrt{-4ac+b^2}}{8a(4ac-b^2)} - \frac{1}{8a}\right) - 2ac + b^2}{bc}\right) + \left(\frac{b\sqrt{-4ac+b^2}}{8a(4ac-b^2)} - \frac{1}{8a}\right)\log\left(x^4 + \frac{-16a^2c\left(\frac{b\sqrt{-4ac+b^2}}{8a(4ac-b^2)} - \frac{1}{8a}\right) + 4ab^2\left(\frac{b\sqrt{-4ac+b^2}}{8a(4ac-b^2)} - \frac{1}{8a}\right) - 2ac + b^2}{bc}\right) + \frac{\log(x)}{a}$$

input
```
integrate(1/x/(c*x**8+b*x**4+a),x)
```

output
```
(-b*sqrt(-4*a*c + b**2)/(8*a*(4*a*c - b**2)) - 1/(8*a))*log(x**4 + (-16*a*
*2*c*(-b*sqrt(-4*a*c + b**2)/(8*a*(4*a*c - b**2)) - 1/(8*a)) + 4*a*b**2*(-
b*sqrt(-4*a*c + b**2)/(8*a*(4*a*c - b**2)) - 1/(8*a)) - 2*a*c + b**2)/(b*c
)) + (b*sqrt(-4*a*c + b**2)/(8*a*(4*a*c - b**2)) - 1/(8*a))*log(x**4 + (-1
6*a**2*c*(b*sqrt(-4*a*c + b**2)/(8*a*(4*a*c - b**2)) - 1/(8*a)) + 4*a*b**2
*(b*sqrt(-4*a*c + b**2)/(8*a*(4*a*c - b**2)) - 1/(8*a)) - 2*a*c + b**2)/(b
*c)) + log(x)/a
```

3.316.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\,(a + bx^4 + cx^8)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.316.8 Giac [A] (verification not implemented)

Time = 1.54 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99

$$\int \frac{1}{x\,(a + bx^4 + cx^8)}\,dx = -\frac{b \arctan\left(\frac{2\,cx^4+b}{\sqrt{-b^2+4\,ac}}\right)}{4\,\sqrt{-b^2+4\,aca}} - \frac{\log\left(cx^8 + bx^4 + a\right)}{8\,a} + \frac{\log\left(x^4\right)}{4\,a}$$

input
```
integrate(1/x/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
-1/4*b*arctan((2*c*x^4 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a) - 1
/8*log(c*x^8 + b*x^4 + a)/a + 1/4*log(x^4)/a
```

3.316.9 Mupad [B] (verification not implemented)

Time = 8.98 (sec) , antiderivative size = 1690, normalized size of antiderivative = 24.49

$$\int \frac{1}{x\,(a + bx^4 + cx^8)}\,dx = \text{Too large to display}$$

input
```
int(1/(x*(a + b*x^4 + c*x^8)),x)
```

output
```
log(x)/a + (log(a + b*x^4 + c*x^8)*(16*a*c - 4*b^2))/(2*(16*a*b^2 - 64*a^2
*c)) - (b*atan((4*(4*a*c - b^2)^2*(5*b^6 - 18*a^3*c^3 + 61*a^2*b^2*c^2 - 3
4*a*b^4*c)*((b^9*c^4)/(128*a^4*(4*a*c - b^2)^(5/2)) + (2*b^5*c^4*(16*a*c -
 4*b^2)^4)/((16*a*b^2 - 64*a^2*c)^4*(4*a*c - b^2)^(1/2)) - (b*(16*a*c - 4*
b^2)^3*(256*b^4*c^4 - (128*a*b^4*c^4*(16*a*c - 4*b^2))/(16*a*b^2 - 64*a^2*
c)))/(16*a*(16*a*b^2 - 64*a^2*c)^3*(4*a*c - b^2)^(1/2)) + (b^3*(16*a*c - 4
*b^2)*(256*b^4*c^4 - (128*a*b^4*c^4*(16*a*c - 4*b^2))/(16*a*b^2 - 64*a^2*c
)))/(256*a^3*(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)^(3/2)) - (3*b^7*c^4*(16*a
*c - 4*b^2)^2)/(4*a^2*(16*a*b^2 - 64*a^2*c)^2*(4*a*c - b^2)^(3/2))))/(b^4*
c^8*(81*a*c - 20*b^2)) + (128*a^5*x^4*(((5*b^5 + 23*a^2*b*c^2 - 24*a*b^3*c
)*(((576*b^3*c^5 - ((1280*b^5*c^4 - 4608*a*b^3*c^5)*(16*a*c - 4*b^2))/(2*(
16*a*b^2 - 64*a^2*c)))*(16*a*c - 4*b^2)^4)/(16*(16*a*b^2 - 64*a^2*c)^4) +
(b^4*(576*b^3*c^5 - ((1280*b^5*c^4 - 4608*a*b^3*c^5)*(16*a*c - 4*b^2))/(2*
(16*a*b^2 - 64*a^2*c))))/(4096*a^4*(4*a*c - b^2)^2) + (b^2*(1280*b^5*c^4 -
 4608*a*b^3*c^5)*(16*a*c - 4*b^2)^3)/(128*a^2*(16*a*b^2 - 64*a^2*c)^3*(4*a
*c - b^2)) - (3*b^2*(576*b^3*c^5 - ((1280*b^5*c^4 - 4608*a*b^3*c^5)*(16*a*
c - 4*b^2))/(2*(16*a*b^2 - 64*a^2*c)))*(16*a*c - 4*b^2)^2)/(128*a^2*(16*a*
b^2 - 64*a^2*c)^2*(4*a*c - b^2)) - (b^4*(1280*b^5*c^4 - 4608*a*b^3*c^5)*(1
6*a*c - 4*b^2))/(2048*a^4*(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)^2)))/(32*a^5
*c^4*(81*a*c - 20*b^2)) + ((5*b^6 - 18*a^3*c^3 + 61*a^2*b^2*c^2 - 34*a*...
```

3.316. $\int \frac{1}{x(a+bx^4+cx^8)}\,dx$

3.317 $\int \frac{1}{x^3(a+bx^4+cx^8)}\,dx$

3.317.1 Optimal result

Integrand size = 18, antiderivative size = 184

$$\int \frac{1}{x^3\left(a+bx^4+cx^8\right)}\,dx=-\frac{1}{2ax^2}-\frac{\sqrt{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
-1/2/a/x^2-1/4*arctan(x^2*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^
(1/2)*(1+b/(-4*a*c+b^2)^(1/2))/a*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*
arctan(x^2*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(1-b/(-4*
a*c+b^2)^(1/2))/a*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.317.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.41

$$\int \frac{1}{x^3\left(a+bx^4+cx^8\right)}\,dx=-\frac{1}{2ax^2}-\frac{\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{b\log(x-\#1)+c\log(x-\#1)\#1^4}{b\#1^2+2c\#1^6}\&\right]}{4a}$$

input
```
Integrate[1/(x^3*(a + b*x^4 + c*x^8)),x]
```

output
```
-1/2*1/(a*x^2) - RootSum[a + b*#1^4 + c*#1^8 & , (b*Log[x - #1] + c*Log[x
- #1]*#1^4)/(b*#1^2 + 2*c*#1^6) & ]/(4*a)
```

3.317.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1695, 1443, 25, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a+bx^4+cx^8)} \, dx$$

$$\Big\downarrow \text{1695}$$

$$\frac{1}{2}\int \frac{1}{x^4 (cx^8+bx^4+a)}dx^2$$

$$\Big\downarrow \text{1443}$$

$$\frac{1}{2}\left(\frac{\int -\frac{cx^4+b}{cx^8+bx^4+a}dx^2}{a}-\frac{1}{ax^2}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(-\frac{\int \frac{cx^4+b}{cx^8+bx^4+a}dx^2}{a}-\frac{1}{ax^2}\right)$$

$$\Big\downarrow \text{1480}$$

$$\frac{1}{2}\left(-\frac{\frac{1}{2}c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\int \frac{1}{cx^4+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx^2+\frac{1}{2}c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^4+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx^2}{a}-\frac{1}{ax^2}\right)$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{2}\left(-\frac{\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}}{a}-\frac{1}{ax^2}\right)$$

input
```
Int[1/(x^3*(a + b*x^4 + c*x^8)),x]
```

output
```
(-(1/(a*x^2)) - ((Sqrt[c]*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c
]*x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]])
 + (Sqrt[c]*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b
+ Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/a)/2
```

3.317.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1443
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Sim
p[1/(a*d^2*(m + 1))   Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*
x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1695
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b
*x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c,
p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.317.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.86

method	result
default	$-\frac{2c\left(\frac{\left(-b+\sqrt{-4ac+b^2}\right)\sqrt{2}\,\arctan\left(\frac{c\,x^2\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{8\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\left(b+\sqrt{-4ac+b^2}\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{c\,x^2\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{8\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{a}-\frac{1}{2a\,x^2}$
risch	$-\frac{1}{2a\,x^2}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\left(16a^5c^2-8a^4b^2c+b^4a^3\right)\_Z^4+\left(12a^2b\,c^2-7a\,b^3c+b^5\right)\_Z^2+c^3\right)}\_R\ln\left(\left(\left(-72a^5c^2+38a^4b^2c-5b^4a^3\right)\_R^4+\right.\right.\right.}{4}$

input
```
int(1/x^3/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-2/a*c*(1/8*(-b+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*arctan(c*x^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2
))-1/8*(b+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*arctanh(c*x^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))
-1/2/a/x^2
```

3.317.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1134 vs. 2(141) = 282.

Time = 0.28 (sec) , antiderivative size = 1134, normalized size of antiderivative = 6.16

$$\int \frac{1}{x^3\left(a+bx^4+cx^8\right)}\,dx = -\frac{\sqrt{\frac{1}{2}}ax^2\sqrt{-\frac{b^3-3\,abc+(a^3b^2-4\,a^4c)\sqrt{\frac{b^4-2\,ab^2c+a^2c^2}{a^6b^2-4\,a^7c}}}{a^3b^2-4\,a^4c}}\log\left(-(b^2c^2-ac^3)x^2+\frac{1}{2}\sqrt{\frac{1}{2}}\left(b^5-5\,ab^3c+4\,a^2bc^2-(a^3b^4\right.\right.}{}$$

input
```
integrate(1/x^3/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
-1/4*(sqrt(1/2)*a*x^2*sqrt(-(b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4
 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-(b
^2*c^2 - a*c^3)*x^2 + 1/2*sqrt(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 - (a^3*
b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 -
 4*a^7*c)))*sqrt(-(b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2
*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) - sqrt(1/2)*a*x^
2*sqrt(-(b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c
^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-(b^2*c^2 - a*c^3)*x^2
- 1/2*sqrt(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 - (a^3*b^4 - 6*a^4*b^2*c +
8*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(b
^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b
^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) + sqrt(1/2)*a*x^2*sqrt(-(b^3 - 3*a*b
*c - (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7
*c)))/(a^3*b^2 - 4*a^4*c))*log(-(b^2*c^2 - a*c^3)*x^2 + 1/2*sqrt(1/2)*(b^5
 - 5*a*b^3*c + 4*a^2*b*c^2 + (a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*sqrt((b^4
 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(b^3 - 3*a*b*c - (a^3*
b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3
*b^2 - 4*a^4*c))) - sqrt(1/2)*a*x^2*sqrt(-(b^3 - 3*a*b*c - (a^3*b^2 - 4*a^
4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a
^4*c))*log(-(b^2*c^2 - a*c^3)*x^2 - 1/2*sqrt(1/2)*(b^5 - 5*a*b^3*c + 4*...
```

3.317.6 Sympy [A] (verification not implemented)

Time = 149.53 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^3\left(a+bx^4+cx^8\right)}\,dx$$
$$= \operatorname{RootSum}\left(t^4\cdot\left(4096a^5c^2-2048a^4b^2c+256a^3b^4\right)+t^2\cdot\left(192a^2bc^2-112ab^3c+16b^5\right)+c^3,\left(t\mapsto t\log\left(x\right.\right.\right.$$
$$-\frac{1}{2ax^2}$$

input
```
integrate(1/x**3/(c*x**8+b*x**4+a),x)
```

output
```
RootSum(_t**4*(4096*a**5*c**2 - 2048*a**4*b**2*c + 256*a**3*b**4) + _t**2*
(192*a**2*b*c**2 - 112*a*b**3*c + 16*b**5) + c**3, Lambda(_t, _t*log(x**2
+ (-512*_t**3*a**5*c**2 + 384*_t**3*a**4*b**2*c - 64*_t**3*a**3*b**4 - 20*
_t*a**2*b*c**2 + 20*_t*a*b**3*c - 4*_t*b**5)/(a*c**3 - b**2*c**2)))) - 1/(
2*a*x**2)
```

3.317.7 Maxima [F]

$$\int \frac{1}{x^3(a+bx^4+cx^8)}\,dx = \int \frac{1}{(cx^8+bx^4+a)x^3}\,dx$$

input
```
integrate(1/x^3/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
-integrate((c*x^4 + b)*x/(c*x^8 + b*x^4 + a), x)/a - 1/2/(a*x^2)
```

3.317.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2055 vs. 2(141) = 282.

Time = 1.87 (sec) , antiderivative size = 2055, normalized size of antiderivative = 11.17

$$\int \frac{1}{x^3(a+bx^4+cx^8)}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^3/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
-1/8*(2*a*b^4*c^2 - 8*a^2*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^2*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b^3*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a*b^2*c^2 - 2*(b^2 - 4*a*c)*a*b^2*c^2 + (sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b^4*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 -
 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 2*b^4*c^2 + 16*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^3 + 16
*a*b^2*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^4 - 32*a^2*c^4
+ 2*(b^2 - 4*a*c)*b^2*c^2 - 8*(b^2 - 4*a*c)*a*c^3)*x^4*abs(a) + (2*a*b^3*c
^3 - 8*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a*b^3*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b
^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3
 - 2*(b^2 - 4*a*c)*a*b*c^3)*x^4 + (sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*b^5 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 2*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 2*b^5*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^2*b*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*
c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 16*a*b^3*c^2 - ...
```

3.317.9 Mupad [B] (verification not implemented)

Time = 8.97 (sec) , antiderivative size = 5451, normalized size of antiderivative = 29.62

$$\int \frac{1}{x^3 (a + bx^4 + cx^8)} \, dx = \text{Too large to display}$$

input
```
int(1/(x^3*(a + b*x^4 + c*x^8)),x)
```

output
```
- atan((((64*a^10*c^8 + ((-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*
c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2
- 8*a^4*b^2*c)))^(1/2)*(((-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*
c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2
- 8*a^4*b^2*c)))^(1/2)*(4096*a^12*b^6*c^4 - 32768*a^13*b^4*c^5 + 65536*a^1
4*b^2*c^6) + x^2*(16384*a^13*b*c^7 - 1024*a^10*b^7*c^4 + 9216*a^11*b^5*c^5
 - 24576*a^12*b^3*c^6))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c
^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 -
 8*a^4*b^2*c)))^(1/2) + 4096*a^12*b*c^7 + 512*a^10*b^5*c^5 - 3072*a^11*b^3
*c^6) + x^2*(512*a^11*c^8 - 64*a^8*b^6*c^5 + 448*a^9*b^4*c^6 - 896*a^10*b^
2*c^7))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c -
 a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^
(1/2) + 16*a^8*b^4*c^6 - 64*a^9*b^2*c^7)*(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/
2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))*1i)/(32*(a^3
*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)) - ((64*a^10*c^8 + ((-(b^5 + b^2*(-(4*a*c
 - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2)
)/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(((-(b^5 + b^2*(-(4*a*c
 - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2)
)/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(4096*a^12*b^6*c^4 - 32
768*a^13*b^4*c^5 + 65536*a^14*b^2*c^6) - x^2*(16384*a^13*b*c^7 - 1024*a...
```

3.318 $\int \frac{1}{x^5(a+bx^4+cx^8)}\,dx$

3.318.1 Optimal result

Integrand size = 18, antiderivative size = 89

$$\int \frac{1}{x^5(a+bx^4+cx^8)}\,dx = -\frac{1}{4ax^4}-\frac{(b^2-2ac)\,\mathrm{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4a^2\sqrt{b^2-4ac}} - \frac{b\log(x)}{a^2}+\frac{b\log(a+bx^4+cx^8)}{8a^2}$$

output
```
-1/4/a/x^4-b*ln(x)/a^2+1/8*b*ln(c*x^8+b*x^4+a)/a^2-1/4*(-2*a*c+b^2)*arctan
h((2*c*x^4+b)/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(1/2)
```

3.318.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.03

$$\begin{aligned}\int \frac{1}{x^5(a+bx^4+cx^8)}\,dx &= -\frac{1}{4ax^4}-\frac{b\log(x)}{a^2} \\ &+ \frac{\mathrm{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{b^2\log(x-\#1)-ac\log(x-\#1)+bc\log(x-\#1)\#1^4}{b+2c\#1^4}\&\right]}{4a^2}\end{aligned}$$

input
```
Integrate[1/(x^5*(a + b*x^4 + c*x^8)),x]
```

output
```
-1/4*1/(a*x^4) - (b*Log[x])/a^2 + RootSum[a + b*#1^4 + c*#1^8 & , (b^2*Log
[x - #1] - a*c*Log[x - #1] + b*c*Log[x - #1]*#1^4)/(b + 2*c*#1^4) & ]/(4*a
^2)
```

3.318.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1693, 1145, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 \left(a+bx^4+cx^8\right)} \, dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{4} \int \frac{1}{x^8 \left(cx^8+bx^4+a\right)}dx^4$$

$$\Big\downarrow \text{1145}$$

$$\frac{1}{4} \left(\frac{\int -\frac{cx^4+b}{x^4(cx^8+bx^4+a)}dx^4}{a}-\frac{1}{ax^4}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4} \left(-\frac{\int \frac{cx^4+b}{x^4(cx^8+bx^4+a)}dx^4}{a}-\frac{1}{ax^4}\right)$$

$$\Big\downarrow \text{1200}$$

$$\frac{1}{4} \left(-\frac{\int \left(\frac{b}{ax^4}+\frac{-bcx^4-b^2+ac}{a(cx^8+bx^4+a)}\right)dx^4}{a}-\frac{1}{ax^4}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4} \left(-\frac{\frac{(b^2-2ac)\text{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}-\frac{b\log(a+bx^4+cx^8)}{2a}+\frac{b\log(x^4)}{a}}{a}-\frac{1}{ax^4}\right)$$

input
```
Int[1/(x^5*(a + b*x^4 + c*x^8)),x]
```

output
```
(-(1/(a*x^4)) - (((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]])/
(a*Sqrt[b^2 - 4*a*c]) + (b*Log[x^4])/a - (b*Log[a + b*x^4 + c*x^8])/(2*a))
/a)/4
```

3.318.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1145
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp
[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[m, -1]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.318.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.94

method	result
default	$-\frac{1}{4a\,x^{4}}-\frac{b\ln(x)}{a^{2}}-\frac{-\frac{b\ln\left(c\,x^{8}+b\,x^{4}+a\right)}{4}+\frac{\left(ac-\frac{b^{2}}{2}\right)\arctan\left(\frac{2c\,x^{4}+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{2a^{2}}$
risch	$-\frac{1}{4a\,x^{4}}-\frac{b\ln(x)}{a^{2}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4a^{3}c-a^{2}b^{2}\right)\_Z^{2}+\left(-4abc+b^{3}\right)\_Z+c^{2}\right)}\_R\ln\left(\left(\left(18a^{3}c-5a^{2}b^{2}\right)\_R^{2}-8\_Rabc+4c^{2}\right)x^{4}-a\right.\right.}{4}$

input
```
int(1/x^5/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4/a/x^4-b*ln(x)/a^2-1/2/a^2*(-1/4*b*ln(c*x^8+b*x^4+a)+(a*c-1/2*b^2)/(4*
a*c-b^2)^(1/2)*arctan((2*c*x^4+b)/(4*a*c-b^2)^(1/2)))
```

3.318.5 Fricas [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 293, normalized size of antiderivative = 3.29

$$\int \frac{1}{x^5(a+bx^4+cx^8)}\,dx$$
$$= \left[-\frac{(b^2-2\,ac)\sqrt{b^2-4\,ac}x^4\log\left(\frac{2\,c^2x^8+2\,bcx^4+b^2-2\,ac+(2\,cx^4+b)\sqrt{b^2-4\,ac}}{cx^8+bx^4+a}\right)-(b^3-4\,abc)x^4\log\left(cx^8+bx^4+a\right)+}{8\,(a^2b^2-4\,a^3c)x^4}\right.$$
$$-\frac{2\,(b^2-2\,ac)\sqrt{-b^2+4\,ac}x^4\arctan\left(-\frac{(2\,cx^4+b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)-(b^3-4\,abc)x^4\log\left(cx^8+bx^4+a\right)+8\,(b^3-}{8\,(a^2b^2-4\,a^3c)x^4}$$

input
```
integrate(1/x^5/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
[-1/8*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*x^4*log((2*c^2*x^8 + 2*b*c*x^4 + b^
2 - 2*a*c + (2*c*x^4 + b)*sqrt(b^2 - 4*a*c))/(c*x^8 + b*x^4 + a)) - (b^3 -
 4*a*b*c)*x^4*log(c*x^8 + b*x^4 + a) + 8*(b^3 - 4*a*b*c)*x^4*log(x) + 2*a*
b^2 - 8*a^2*c)/((a^2*b^2 - 4*a^3*c)*x^4), -1/8*(2*(b^2 - 2*a*c)*sqrt(-b^2
+ 4*a*c)*x^4*arctan(-(2*c*x^4 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^
3 - 4*a*b*c)*x^4*log(c*x^8 + b*x^4 + a) + 8*(b^3 - 4*a*b*c)*x^4*log(x) + 2
*a*b^2 - 8*a^2*c)/((a^2*b^2 - 4*a^3*c)*x^4)]
```

3.318.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^5 (a + bx^4 + cx^8)} dx = \text{Timed out}$$

input
```
integrate(1/x**5/(c*x**8+b*x**4+a),x)
```

output
```
Timed out
```

3.318.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^5 (a + bx^4 + cx^8)} dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x^5/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.318.8 Giac [A] (verification not implemented)

Time = 1.50 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^5 (a + bx^4 + cx^8)} dx = \frac{b \log (cx^8 + bx^4 + a)}{8 a^2} - \frac{b \log (x^4)}{4 a^2} + \frac{(b^2 - 2 ac) \arctan \left(\frac{2 cx^4 + b}{\sqrt{-b^2 + 4 ac}}\right)}{4 \sqrt{-b^2 + 4 ac} a^2} + \frac{bx^4 - a}{4 a^2 x^4}$$

input
```
integrate(1/x^5/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
1/8*b*log(c*x^8 + b*x^4 + a)/a^2 - 1/4*b*log(x^4)/a^2 + 1/4*(b^2 - 2*a*c)*
arctan((2*c*x^4 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^2) + 1/4*(b
*x^4 - a)/(a^2*x^4)
```

3.318.9 Mupad [B] (verification not implemented)

Time = 9.22 (sec) , antiderivative size = 8817, normalized size of antiderivative = 99.07

$$\int \frac{1}{x^5 (a + bx^4 + cx^8)} dx = \text{Too large to display}$$

input
```
int(1/(x^5*(a + b*x^4 + c*x^8)),x)
```

output
```
(atan((4*a^5*(4*a*c - b^2)^2*(5*b^7 - 23*a^3*b*c^3 + 66*a^2*b^3*c^2 - 35*a
*b^5*c)*(((4*b^3 - 16*a*b*c)*((((((((256*a^4*b^5*c^4 - 256*a^5*b^3*c^5)/a^
5 - (128*a*b^4*c^4*(4*b^3 - 16*a*b*c))/(64*a^3*c - 16*a^2*b^2))*(2*a*c - b
^2))/(8*a^2*(4*a*c - b^2)^(1/2)) - (16*b^4*c^4*(4*b^3 - 16*a*b*c)*(2*a*c -
 b^2))/(a*(4*a*c - b^2)^(1/2)*(64*a^3*c - 16*a^2*b^2)))*(2*a*c - b^2))/(8*
a^2*(4*a*c - b^2)^(1/2)) - (2*b^4*c^4*(4*b^3 - 16*a*b*c)*(2*a*c - b^2)^2)/
(a^3*(4*a*c - b^2)*(64*a^3*c - 16*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c
- b^2)^(1/2)) - (b^4*c^4*(4*b^3 - 16*a*b*c)*(2*a*c - b^2)^3)/(4*a^5*(4*a*c
 - b^2)^(3/2)*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2)) - ((4
*b^3 - 16*a*b*c)*(((4*b^3 - 16*a*b*c)*(((4*b^3 - 16*a*b*c)*((((256*a^4*b^5
*c^4 - 256*a^5*b^3*c^5)/a^5 - (128*a*b^4*c^4*(4*b^3 - 16*a*b*c))/(64*a^3*c
 - 16*a^2*b^2))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2)) - (16*b^4*c^4*(
4*b^3 - 16*a*b*c)*(2*a*c - b^2))/(a*(4*a*c - b^2)^(1/2)*(64*a^3*c - 16*a^2
*b^2))))/(2*(64*a^3*c - 16*a^2*b^2)) + (((256*a^3*b^4*c^5 - 96*a^4*b^2*c^6
)/a^5 + ((4*b^3 - 16*a*b*c)*((256*a^4*b^5*c^4 - 256*a^5*b^3*c^5)/a^5 - (12
8*a*b^4*c^4*(4*b^3 - 16*a*b*c))/(64*a^3*c - 16*a^2*b^2)))/(2*(64*a^3*c - 1
6*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2))))/(2*(64*a^3*c - 1
6*a^2*b^2)) - (((16*a^3*b*c^7 - 96*a^2*b^3*c^6)/a^5 - ((4*b^3 - 16*a*b*c)*
((256*a^3*b^4*c^5 - 96*a^4*b^2*c^6)/a^5 + ((4*b^3 - 16*a*b*c)*((256*a^4*b^
5*c^4 - 256*a^5*b^3*c^5)/a^5 - (128*a*b^4*c^4*(4*b^3 - 16*a*b*c))/(64*a...
```

3.319 $\int \frac{x^{10}}{a+bx^4+cx^8} \, dx$

3.319.1 Optimal result

Integrand size = 18, antiderivative size = 381

$$\begin{aligned}\int \frac{x^{10}}{a+bx^4+cx^8} \, dx = {} & \frac{x^3}{3c} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\ 2^{3/4}c^{7/4}\sqrt[4]{-b-\sqrt{b^2-4ac}}} \\ & - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\ 2^{3/4}c^{7/4}\sqrt[4]{-b+\sqrt{b^2-4ac}}} \\ & + \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\ 2^{3/4}c^{7/4}\sqrt[4]{-b-\sqrt{b^2-4ac}}} \\ & + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\ 2^{3/4}c^{7/4}\sqrt[4]{-b+\sqrt{b^2-4ac}}}\end{aligned}$$

output
```
1/3*x^3/c-1/4*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(b+(
-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*2^(1/4)/c^(7/4)/(-b-(-4*a*c+b^2)^(1/2))^(1
/4)+1/4*arctanh(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(b+(-2*a*
c+b^2)/(-4*a*c+b^2)^(1/2))*2^(1/4)/c^(7/4)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)-1
/4*arctan(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(b+(2*a*c-b^2)/
(-4*a*c+b^2)^(1/2))*2^(1/4)/c^(7/4)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)+1/4*arct
anh(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(b+(2*a*c-b^2)/(-4*a*
c+b^2)^(1/2))*2^(1/4)/c^(7/4)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)
```

3.319.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.18

$$\int \frac{x^{10}}{a+bx^4+cx^8}\,dx = \frac{4x^3-3\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{a\log(x-\#1)+b\log(x-\#1)\#1^4}{b\#1+2c\#1^5}\&\right]}{12c}$$

input
```
Integrate[x^10/(a + b*x^4 + c*x^8),x]
```

output
```
(4*x^3 - 3*RootSum[a + b*#1^4 + c*#1^8 & , (a*Log[x - #1] + b*Log[x - #1]*
#1^4)/(b*#1 + 2*c*#1^5) & ])/(12*c)
```

3.319.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 344, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {1703, 27, 1834, 27, 827, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{10}}{a+bx^4+cx^8}\,dx$$

$$\Big\downarrow \text{ 1703}$$

$$\frac{x^3}{3c}-\frac{\int \frac{3x^2(bx^4+a)}{cx^8+bx^4+a}dx}{3c}$$

$$\Big\downarrow \ 27$$

$$\frac{x^3}{3c}-\frac{\int \frac{x^2(bx^4+a)}{cx^8+bx^4+a}dx}{c}$$

$$\Big\downarrow \ 1834$$

$$\frac{x^3}{3c}-\frac{\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{2x^2}{2cx^4+b-\sqrt{b^2-4ac}}dx+\frac{1}{2}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\int \frac{2x^2}{2cx^4+b+\sqrt{b^2-4ac}}dx}{c}$$

$$\Big\downarrow \ 27$$

$$\frac{x^3}{3c}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{x^2}{2cx^4+b-\sqrt{b^2-4ac}}dx+\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\int \frac{x^2}{2cx^4+b+\sqrt{b^2-4ac}}dx}{c}$$

$$\Big\downarrow \ 827$$

$$\frac{x^3}{3c}-$$

$$\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\left(\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x^2+\sqrt{-b-\sqrt{b^2-4ac}}}dx}{2\sqrt{2}\sqrt{c}}-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}dx}{2\sqrt{2}\sqrt{c}}\right)+\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x^2+\sqrt{\sqrt{b^2-4ac}-b}}dx}{2\sqrt{2}\sqrt{c}}-\frac{\int}{}\right.}{c}$$

$$\Big\downarrow \ 218$$

$$\frac{x^3}{3c}-$$

$$\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\ 2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}dx}{2\sqrt{2}\sqrt{c}}\right)+\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}}}\right.}{2\ 2^{3/4}c^{3/4}\sqrt[4]{\sqrt{b^2-4}}}\right.}{c}$$

$$\Big\downarrow \ 221$$

$$\frac{x^3}{3c}-$$

$$\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\ 2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\ 2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)+\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{\arctan\left(\frac{}{\sqrt[4]{\sqrt{}}}\right.}{2\ 2^{3/4}c^{3/4}\sqrt[4]{}}\right.}{c}$$

input
```
Int[x^10/(a + b*x^4 + c*x^8),x]
```

3.319. $\int \frac{x^{10}}{a+bx^4+cx^8}\,dx$

output
```
x^3/(3*c) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*(ArcTan[(2^(1/4)*c^(1/4
)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b - Sqrt[b^2 - 4
*a*c])^(1/4)) - ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)
]/(2*2^(3/4)*c^(3/4)*(-b - Sqrt[b^2 - 4*a*c])^(1/4))) + (b - (b^2 - 2*a*c)
/Sqrt[b^2 - 4*a*c])*(ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(
1/4)]/(2*2^(3/4)*c^(3/4)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) - ArcTanh[(2^(1/4
)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b + Sqrt
[b^2 - 4*a*c])^(1/4))))/c
```

3.319.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 1703
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(2*n - 1)*(d*x)^(m - 2*n + 1)*((a + b*x^n + c*x^(2*n))^(
p + 1)/(c*(m + 2*n*p + 1))), x] - Simp[d^(2*n)/(c*(m + 2*n*p + 1))   Int[(d
*x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x
^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && N
eQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0
] && IntegerQ[p]
```

rule 1834
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 +
 (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2
 - (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ
[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n
, 0]
```

3.319.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.17

method	result	size
default	$\frac{x^3}{3c}-\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+\_Z^4b+a\right)}\frac{\left(\_R^6b+\_R^2a\right)\ln\left(x-\_R\right)}{2\_R^7c+\_R^3b}}{4c}$	63
risch	$\frac{x^3}{3c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+\_Z^4b+a\right)}\frac{\left(-\_R^6b-\_R^2a\right)\ln\left(x-\_R\right)}{2\_R^7c+\_R^3b}}{4c}$	65

input
```
int(x^10/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3/c-1/4/c*sum((_R^6*b+_R^2*a)/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_
Z^8*c+_Z^4*b+a))
```

3.319.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7003 vs. 2(299) = 598.

Time = 1.02 (sec) , antiderivative size = 7003, normalized size of antiderivative = 18.38

$$\int \frac{x^{10}}{a+bx^4+cx^8}\,dx=\text{Too large to display}$$

input
```
integrate(x^10/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.319.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{10}}{a+bx^4+cx^8}\,dx = \text{Timed out}$$

input
```
integrate(x**10/(c*x**8+b*x**4+a),x)
```

output
```
Timed out
```

3.319.7 Maxima [F]

$$\int \frac{x^{10}}{a+bx^4+cx^8}\,dx = \int \frac{x^{10}}{cx^8+bx^4+a}\,dx$$

input
```
integrate(x^10/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
1/3*x^3/c - integrate((b*x^6 + a*x^2)/(c*x^8 + b*x^4 + a), x)/c
```

3.319.8 Giac [F]

$$\int \frac{x^{10}}{a+bx^4+cx^8}\,dx = \int \frac{x^{10}}{cx^8+bx^4+a}\,dx$$

input
```
integrate(x^10/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
integrate(x^10/(c*x^8 + b*x^4 + a), x)
```

3.319.9 Mupad [B] (verification not implemented)

Time = 9.51 (sec) , antiderivative size = 12709, normalized size of antiderivative = 33.36

$$\int \frac{x^{10}}{a+bx^4+cx^8}\,dx = \text{Too large to display}$$

input
```
int(x^10/(a + b*x^4 + c*x^8),x)
```

output
```
atan(((((8192*a^6*b*c^6 - 256*a^3*b^7*c^3 + 2560*a^4*b^5*c^4 - 8192*a^5*b^
3*c^5)/c^3 - (4*x*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 +
 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b
^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b
^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8
 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*(8192*a^6*c^8 - 256*a^3*b^6*
c^5 + 2560*a^4*b^4*c^6 - 8192*a^5*b^2*c^7))/c^3)*(-(b^11 + b^6*(-(4*a*c -
b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4
*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(
-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^
4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/
4) + (4*x*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 + b^6*(-(4*a
*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 28
0*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*
c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(2
56*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10))
)^(1/4)*1i - (((8192*a^6*b*c^6 - 256*a^3*b^7*c^3 + 2560*a^4*b^5*c^4 - 8192
*a^5*b^3*c^5)/c^3 + (4*x*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*
b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*
a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/...
```

3.320 $\int \frac{x^8}{a+bx^4+cx^8}\,dx$

3.320.1 Optimal result

Integrand size = 18, antiderivative size = 376

$$\begin{aligned}\int \frac{x^8}{a+bx^4+cx^8}\,dx &= \frac{x}{c}+\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} \\ &+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}} \\ &+\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} \\ &+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}\end{aligned}$$

output
```
x/c+1/4*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(b+(-2*a*c
+b^2)/(-4*a*c+b^2)^(1/2))*2^(3/4)/c^(5/4)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)+1/
4*arctanh(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(b+(-2*a*c+b^2)
/(-4*a*c+b^2)^(1/2))*2^(3/4)/c^(5/4)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)+1/4*arc
tan(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(b+(2*a*c-b^2)/(-4*a*
c+b^2)^(1/2))*2^(3/4)/c^(5/4)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)+1/4*arctanh(2^
(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(b+(2*a*c-b^2)/(-4*a*c+b^2)
^(1/2))*2^(3/4)/c^(5/4)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)
```

3.320.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.19

$$\int \frac{x^8}{a+bx^4+cx^8}\,dx=\frac{x}{c}-\frac{\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{a\log(x-\#1)+b\log(x-\#1)\#1^4}{b\#1^3+2c\#1^7}\&\right]}{4c}$$

input
```
Integrate[x^8/(a + b*x^4 + c*x^8),x]
```

output
```
x/c - RootSum[a + b*#1^4 + c*#1^8 & , (a*Log[x - #1] + b*Log[x - #1]*#1^4)
/(b*#1^3 + 2*c*#1^7) & ]/(4*c)
```

3.320.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 337, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1703, 1752, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{a+bx^4+cx^8}\,dx$$

$$\Big\downarrow \text{1703}$$

$$\frac{x}{c}-\frac{\int \frac{bx^4+a}{cx^8+bx^4+a}dx}{c}$$

$$\Big\downarrow \; 1752$$

$$\frac{x}{c}-\frac{\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^4+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\int \frac{1}{cx^4+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{c}$$

$$\Big\downarrow \; 756$$

$$\frac{x}{c}-\frac{\frac{1}{2}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\left(-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}dx}{\sqrt{-\sqrt{b^2-4ac}-b}}-\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x^2+\sqrt{-b-\sqrt{b^2-4ac}}}dx}{\sqrt{-\sqrt{b^2-4ac}-b}}\right)+\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(-\frac{\int \frac{1}{\sqrt{\sqrt{b^2-4ac}-b}-\sqrt{2}\sqrt{c}x^2}\cdots}{\sqrt{\sqrt{b^2-4ac}-b}}\cdots\right.}{c}$$

$$\Big\downarrow \; 218$$

$$\frac{x}{c}-\frac{\frac{1}{2}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\left(-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}dx}{\sqrt{-\sqrt{b^2-4ac}-b}}-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}\right)+\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(-\frac{\int \frac{1}{\sqrt{\sqrt{b^2-4ac}-b-\cdots}}\cdots}{\sqrt{\sqrt{b^2-4ac}\cdots}}\cdots\right.}{c}$$

$$\Big\downarrow \; 221$$

$$\frac{x}{c}-\frac{\frac{1}{2}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\left(-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}\right)+\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(-\frac{\arctan\cdots}{\sqrt[4]{2}\cdots}\cdots\right.}{c}$$

input
```
Int[x^8/(a + b*x^4 + c*x^8),x]
```

output
```
x/c - (((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*(-(ArcTan[(2^(1/4)*c^(1/4)*x
)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b - Sqrt[b^2 - 4*a*c]
)^(3/4))) - ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2
^(1/4)*c^(1/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4))))/2 + ((b - (b^2 - 2*a*c)/S
qrt[b^2 - 4*a*c])*(-(ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(
1/4)]/(2^(1/4)*c^(1/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))) - ArcTanh[(2^(1/4)
*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b + Sqrt[b^
2 - 4*a*c])^(3/4))))/2)/c
```

3.320. $\int \frac{x^8}{a+bx^4+cx^8}\,dx$

3.320.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 1703
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(2*n - 1)*(d*x)^(m - 2*n + 1)*((a + b*x^n + c*x^(2*n))^(
p + 1)/(c*(m + 2*n*p + 1))), x] - Simp[d^(2*n)/(c*(m + 2*n*p + 1))   Int[(d
*x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x
^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && N
eQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0
] && IntegerQ[p]
```

rule 1752
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

3.320.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.16

method	result	size
default	$\frac{x}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{8}+\_Z^{4}b+a\right)}\frac{\left(-\_R^{4}b-a\right)\ln\left(x-\_R\right)}{2\_R^{7}c+\_R^{3}b}}{4c}$	59
risch	$\frac{x}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{8}+\_Z^{4}b+a\right)}\frac{\left(-\_R^{4}b-a\right)\ln\left(x-\_R\right)}{2\_R^{7}c+\_R^{3}b}}{4c}$	59

input
```
int(x^8/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
x/c+1/4/c*sum((-_R^4*b-a)/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*c+_Z^4
*b+a))
```

3.320.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4001 vs. $2(296) = 592$.

Time = 0.41 (sec) , antiderivative size = 4001, normalized size of antiderivative = 10.64

$$\int \frac{x^8}{a+bx^4+cx^8}\,dx = \text{Too large to display}$$

input
```
integrate(x^8/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
1/4*(c*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*
a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2
*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)
))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*log((a*b^4 - 3*a^2*b^2*c + a^3*c
^2)*x + 1/2*(b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3 - (b^5*c^5 - 8*a
*b^3*c^6 + 16*a^2*b*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^
2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13
)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b
^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^
3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/
(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))) - c*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*
a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6
*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*
c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^
7)))*log((a*b^4 - 3*a^2*b^2*c + a^3*c^2)*x - 1/2*(b^6 - 7*a*b^4*c + 13*a^2
*b^2*c^2 - 4*a^3*c^3 - (b^5*c^5 - 8*a*b^3*c^6 + 16*a^2*b*c^7)*sqrt((b^8 -
6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4
*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b
^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*
b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*...
```

3.320.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^8}{a + bx^4 + cx^8} \, dx = \text{Timed out}$$

input
```
integrate(x**8/(c*x**8+b*x**4+a),x)
```

output
```
Timed out
```

3.320.7 Maxima [F]

$$\int \frac{x^8}{a+bx^4+cx^8}\,dx = \int \frac{x^8}{cx^8+bx^4+a}\,dx$$

input
```
integrate(x^8/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
x/c - integrate((b*x^4 + a)/(c*x^8 + b*x^4 + a), x)/c
```

3.320.8 Giac [F]

$$\int \frac{x^8}{a+bx^4+cx^8}\,dx = \int \frac{x^8}{cx^8+bx^4+a}\,dx$$

input
```
integrate(x^8/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
integrate(x^8/(c*x^8 + b*x^4 + a), x)
```

3.320.9 Mupad [B] (verification not implemented)

Time = 9.70 (sec) , antiderivative size = 10382, normalized size of antiderivative = 27.61

$$\int \frac{x^8}{a+bx^4+cx^8}\,dx = \text{Too large to display}$$

input
```
int(x^8/(a + b*x^4 + c*x^8),x)
```

output
```
atan(((((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c - (4*x
*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 1
20*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c
*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96
*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4
- 2048*a^4*b^3*c^5))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c
^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) -
 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*
c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) - (4*x*(a^4
*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2)
+ 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2
)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^
4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)
*1i - (((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c + (4*x
*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 1
20*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c
*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96
*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4
- 2048*a^4*b^3*c^5))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c
^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2...
```

3.320. $\int \frac{x^8}{a+bx^4+cx^8}\,dx$

3.321 $\int \frac{x^6}{a+bx^4+cx^8}\,dx$

3.321.1 Optimal result

Integrand size = 18, antiderivative size = 325

$$\begin{aligned}\int \frac{x^6}{a+bx^4+cx^8}\,dx = &-\frac{\left(-b-\sqrt{b^2-4ac}\right)^{3/4}\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}c^{3/4}\sqrt{b^2-4ac}}\\ &+\frac{\left(-b+\sqrt{b^2-4ac}\right)^{3/4}\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}c^{3/4}\sqrt{b^2-4ac}}\\ &+\frac{\left(-b-\sqrt{b^2-4ac}\right)^{3/4}\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}c^{3/4}\sqrt{b^2-4ac}}\\ &-\frac{\left(-b+\sqrt{b^2-4ac}\right)^{3/4}\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}c^{3/4}\sqrt{b^2-4ac}}\end{aligned}$$

output
```
-1/4*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(-b-(-4*a*c+b
^2)^(1/2))^(3/4)*2^(1/4)/c^(3/4)/(-4*a*c+b^2)^(1/2)+1/4*arctanh(2^(1/4)*c^
(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(-b-(-4*a*c+b^2)^(1/2))^(3/4)*2^(1/
4)/c^(3/4)/(-4*a*c+b^2)^(1/2)+1/4*arctan(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2
)^(1/2))^(1/4))*(-b+(-4*a*c+b^2)^(1/2))^(3/4)*2^(1/4)/c^(3/4)/(-4*a*c+b^2)
^(1/2)-1/4*arctanh(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(-b+(-
4*a*c+b^2)^(1/2))^(3/4)*2^(1/4)/c^(3/4)/(-4*a*c+b^2)^(1/2)
```

3.321.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.14

$$\int \frac{x^6}{a+bx^4+cx^8}\,dx=\frac{1}{4}\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{\log(x-\#1)\#1^3}{b+2c\#1^4}\&\right]$$

input
```
Integrate[x^6/(a + b*x^4 + c*x^8),x]
```

output
```
RootSum[a + b*#1^4 + c*#1^8 & , (Log[x - #1]*#1^3)/(b + 2*c*#1^4) & ]/4
```

3.321.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 314, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1710, 27, 827, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{a+bx^4+cx^8}\,dx$$
$$\Big\downarrow \text{1710}$$
$$\frac{1}{2}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{2x^2}{2cx^4+b-\sqrt{b^2-4ac}}dx+\frac{1}{2}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\int \frac{2x^2}{2cx^4+b+\sqrt{b^2-4ac}}dx$$
$$\Big\downarrow \text{27}$$
$$\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{x^2}{2cx^4+b-\sqrt{b^2-4ac}}dx+\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\int \frac{x^2}{2cx^4+b+\sqrt{b^2-4ac}}dx$$

$$\Big\downarrow \; 827$$

$$\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x^2+\sqrt{-b-\sqrt{b^2-4ac}}}dx}{2\sqrt{2}\sqrt{c}}-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}dx}{2\sqrt{2}\sqrt{c}}\right)+\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x^2+\sqrt{\sqrt{b^2-4ac}-b}}dx}{2\sqrt{2}\sqrt{c}}-\frac{\int \frac{1}{\sqrt{\sqrt{b^2-4ac}-b}-\sqrt{2}\sqrt{c}x^2}dx}{2\sqrt{2}\sqrt{c}}\right)$$

$$\Big\downarrow \; 218$$

$$\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}dx}{2\sqrt{2}\sqrt{c}}\right)+\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{\int \frac{1}{\sqrt{\sqrt{b^2-4ac}-b}-\sqrt{2}\sqrt{c}x^2}dx}{2\sqrt{2}\sqrt{c}}\right)$$

$$\Big\downarrow \; 221$$

$$\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)+\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)$$

input
```
Int[x^6/(a + b*x^4 + c*x^8),x]
```

3.321. $\int \frac{x^6}{a+bx^4+cx^8}\,dx$

output
```
(1 + b/Sqrt[b^2 - 4*a*c])*(ArcTan[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a
*c])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) - ArcTanh[(
2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b
- Sqrt[b^2 - 4*a*c])^(1/4))) + (1 - b/Sqrt[b^2 - 4*a*c])*(ArcTan[(2^(1/4)*
c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b + Sqrt[b
^2 - 4*a*c])^(1/4)) - ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])
^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)))
```

3.321.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 1710
```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^n/2)*(b/q + 1)   Int[(d*x)^(m
- n)/(b/2 + q/2 + c*x^n), x], x] - Simp[(d^n/2)*(b/q - 1)   Int[(d*x)^(m -
n)/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

3.321. $\int \frac{x^6}{a+bx^4+cx^8}\,dx$

3.321.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.13

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{8}+\_Z^{4}b+a\right)}\frac{\_R^{6}\ln\left(x-\_R\right)}{2\_R^{7}c+\_R^{3}b}\right)}{4}$	43
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{8}+\_Z^{4}b+a\right)}\frac{\_R^{6}\ln\left(x-\_R\right)}{2\_R^{7}c+\_R^{3}b}\right)}{4}$	43

input
```
int(x^6/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(_R^6/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

3.321.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4433 vs. 2(245) = 490.

Time = 0.33 (sec) , antiderivative size = 4433, normalized size of antiderivative = 13.64

$$\int \frac{x^6}{a+bx^4+cx^8}\,dx = \text{Too large to display}$$

input
```
integrate(x^6/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
1/4*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*
c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2
*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*log(1/2*sqrt(1
/2)*(b^7 - 9*a*b^5*c + 24*a^2*b^3*c^2 - 16*a^3*b*c^3 - (b^8*c^3 - 14*a*b^6
*c^4 + 72*a^2*b^4*c^5 - 160*a^3*b^2*c^6 + 128*a^4*c^7)*sqrt((b^4 - 2*a*b^2
*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqr
t(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sq
rt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 -
64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*sqrt(-(b^3 - 3*a*b*c
+ (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b
^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*
c^4 + 16*a^2*c^5)) - (a^2*b^2 - a^3*c)*x) - 1/4*sqrt(sqrt(1/2)*sqrt(-(b^3
- 3*a*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a
^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3
- 8*a*b^2*c^4 + 16*a^2*c^5)))*log(-1/2*sqrt(1/2)*(b^7 - 9*a*b^5*c + 24*a^2
*b^3*c^2 - 16*a^3*b*c^3 - (b^8*c^3 - 14*a*b^6*c^4 + 72*a^2*b^4*c^5 - 160*a
^3*b^2*c^6 + 128*a^4*c^7)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a
*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*
b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2
)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8...
```

3.321.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^6}{a + bx^4 + cx^8} \, dx = \text{Timed out}$$

input
```
integrate(x**6/(c*x**8+b*x**4+a),x)
```

output
```
Timed out
```

3.321.7 Maxima [F]

$$\int \frac{x^6}{a + bx^4 + cx^8} \, dx = \int \frac{x^6}{cx^8 + bx^4 + a} \, dx$$

input
```
integrate(x^6/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
integrate(x^6/(c*x^8 + b*x^4 + a), x)
```

3.321.8 Giac [F]

$$\int \frac{x^6}{a + bx^4 + cx^8} \, dx = \int \frac{x^6}{cx^8 + bx^4 + a} \, dx$$

input
```
integrate(x^6/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
integrate(x^6/(c*x^8 + b*x^4 + a), x)
```

3.321.9 Mupad [B] (verification not implemented)

Time = 9.34 (sec) , antiderivative size = 8033, normalized size of antiderivative = 24.72

$$\int \frac{x^6}{a + bx^4 + cx^8} \, dx = \text{Too large to display}$$

input
```
int(x^6/(a + b*x^4 + c*x^8),x)
```

output
```
atan((((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c
^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^
3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(3/4)*(4096*a^5*c^5
 + 256*a^3*b^4*c^3 - 2048*a^4*b^2*c^4 + x*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^
(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5
)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256
*a^3*b^2*c^6)))^(1/4)*(32768*a^5*c^6 + 2048*a^3*b^4*c^4 - 16384*a^4*b^2*c^
5)) + x*(4*a^3*b^3*c - 12*a^4*b*c^2))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2
) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1
/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3
*b^2*c^6)))^(1/4)*1i - ((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c
^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256
*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(3
/4)*(4096*a^5*c^5 + 256*a^3*b^4*c^3 - 2048*a^4*b^2*c^4 - x*(-(b^7 + b^2*(-
(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*
(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*
a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*(32768*a^5*c^6 + 2048*a^3*b^4*c^4 -
 16384*a^4*b^2*c^5)) - x*(4*a^3*b^3*c - 12*a^4*b*c^2))*(-(b^7 + b^2*(-(4*a
*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4
*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a...
```

3.321. $\int \frac{x^6}{a+bx^4+cx^8}\,dx$

3.322 $\int \frac{x^4}{a+bx^4+cx^8} \, dx$

3.322.1 Optimal result

Integrand size = 18, antiderivative size = 325

$$\int \frac{x^4}{a+bx^4+cx^8} \, dx = \frac{\sqrt[4]{-b-\sqrt{b^2-4ac}}\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}} - \frac{\sqrt[4]{-b+\sqrt{b^2-4ac}}\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}} + \frac{\sqrt[4]{-b-\sqrt{b^2-4ac}}\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}} - \frac{\sqrt[4]{-b+\sqrt{b^2-4ac}}\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}}$$

output
```
1/4*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(-b-(-4*a*c+b^
2)^(1/2))^(1/4)*2^(3/4)/c^(1/4)/(-4*a*c+b^2)^(1/2)+1/4*arctanh(2^(1/4)*c^(
1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(-b-(-4*a*c+b^2)^(1/2))^(1/4)*2^(3/4
)/c^(1/4)/(-4*a*c+b^2)^(1/2)-1/4*arctan(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)
^(1/2))^(1/4))*(-b+(-4*a*c+b^2)^(1/2))^(1/4)*2^(3/4)/c^(1/4)/(-4*a*c+b^2)^
(1/2)-1/4*arctanh(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(-b+(-4
*a*c+b^2)^(1/2))^(1/4)*2^(3/4)/c^(1/4)/(-4*a*c+b^2)^(1/2)
```

3.322.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.13

$$\int \frac{x^4}{a+bx^4+cx^8}\,dx = \frac{1}{4}\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{\log(x-\#1)\#1}{b+2c\#1^4}\&\right]$$

input
```
Integrate[x^4/(a + b*x^4 + c*x^8),x]
```

output
```
RootSum[a + b*#1^4 + c*#1^8 & , (Log[x - #1]*#1)/(b + 2*c*#1^4) & ]/4
```

3.322.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 312, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1710, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{a+bx^4+cx^8}\,dx$$

$$\Big\downarrow \text{1710}$$

$$\frac{1}{2}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^4+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\int \frac{1}{cx^4+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx$$

$$\Big\downarrow \ 756$$

$$\frac{1}{2}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}dx}{\sqrt{-\sqrt{b^2-4ac}-b}}-\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x^2+\sqrt{-b-\sqrt{b^2-4ac}}}dx}{\sqrt{-\sqrt{b^2-4ac}-b}}\right)+\frac{1}{2}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(-\frac{\int \frac{1}{\sqrt{\sqrt{b^2-4ac}-b}-\sqrt{2}\sqrt{c}x^2}dx}{\sqrt{\sqrt{b^2-4ac}-b}}-\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x^2+\sqrt{\sqrt{b^2-4ac}-b}}dx}{\sqrt{\sqrt{b^2-4ac}-b}}\right)$$

$$\Big\downarrow \ 218$$

$$\frac{1}{2}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}dx}{\sqrt{-\sqrt{b^2-4ac}-b}}-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}\right)+\frac{1}{2}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(-\frac{\int \frac{1}{\sqrt{\sqrt{b^2-4ac}-b}-\sqrt{2}\sqrt{c}x^2}dx}{\sqrt{\sqrt{b^2-4ac}-b}}-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}\right)$$

$$\Big\downarrow \ 221$$

$$\frac{1}{2}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}\right)+\frac{1}{2}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}\right)$$

input
```
Int[x^4/(a + b*x^4 + c*x^8),x]
```

3.322. $\int \frac{x^4}{a+bx^4+cx^8}\,dx$

output
```
((1 + b/Sqrt[b^2 - 4*a*c])*(-(ArcTan[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 -
4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4))) - ArcTanh
[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b
- Sqrt[b^2 - 4*a*c])^(3/4))))/2 + ((1 - b/Sqrt[b^2 - 4*a*c])*(-(ArcTan[(2^
(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b + Sq
rt[b^2 - 4*a*c])^(3/4))) - ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*
a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))))/2
```

3.322.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 1710
```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^n/2)*(b/q + 1)   Int[(d*x)^(m
- n)/(b/2 + q/2 + c*x^n), x], x] - Simp[(d^n/2)*(b/q - 1)   Int[(d*x)^(m -
n)/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

3.322.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.13

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+\_Z^4 b+a\right)}\frac{\_R^4 \ln\left(x-\_R\right)}{2\_R^7 c+\_R^3 b}\right)}{4}$	43
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+\_Z^4 b+a\right)}\frac{\_R^4 \ln\left(x-\_R\right)}{2\_R^7 c+\_R^3 b}\right)}{4}$	43

input
```
int(x^4/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(_R^4/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

3.322.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2141 vs. 2(245) = 490.

Time = 0.30 (sec) , antiderivative size = 2141, normalized size of antiderivative = 6.59

$$\int \frac{x^4}{a+bx^4+cx^8}\,dx = \text{Too large to display}$$

input
```
integrate(x^4/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
1/4*sqrt(sqrt(1/2)*sqrt(-(b + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*
c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 +
16*a^2*c^3)))*log(x + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*sqrt(sqrt(1/2)*sq
rt(-(b + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 +
48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/sqrt(b^
6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) - 1/4*sqrt(sqrt(1/2)*
sqrt(-(b + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3
+ 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))*log(x
 - (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*sqrt(sqrt(1/2)*sqrt(-(b + (b^4*c - 8
*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64
*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/sqrt(b^6*c^2 - 12*a*b^4*c^
3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) + 1/4*sqrt(-sqrt(1/2)*sqrt(-(b + (b^4*c
- 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 -
 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))*log(x + (b^4*c - 8*a*b^
2*c^2 + 16*a^2*c^3)*sqrt(-sqrt(1/2)*sqrt(-(b + (b^4*c - 8*a*b^2*c^2 + 16*a
^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c
 - 8*a*b^2*c^2 + 16*a^2*c^3)))/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^
4 - 64*a^3*c^5)) - 1/4*sqrt(-sqrt(1/2)*sqrt(-(b + (b^4*c - 8*a*b^2*c^2 + 1
6*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^
4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))*log(x - (b^4*c - 8*a*b^2*c^2 + 16*a^2...
```

3.322.6 Sympy [A] (verification not implemented)

Time = 3.23 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.39

$$\int \frac{x^4}{a+bx^4+cx^8}\,dx$$
$$= \text{RootSum}\left(t^8\cdot\left(16777216a^4c^5-16777216a^3b^2c^4+6291456a^2b^4c^3-1048576ab^6c^2+65536b^8c\right)+t^4\cdot\left(409\right.\right.$$

input
```
integrate(x**4/(c*x**8+b*x**4+a),x)
```

output
```
RootSum(_t**8*(16777216*a**4*c**5 - 16777216*a**3*b**2*c**4 + 6291456*a**2
*b**4*c**3 - 1048576*a*b**6*c**2 + 65536*b**8*c) + _t**4*(4096*a**2*b*c**2
 - 2048*a*b**3*c + 256*b**5) + a, Lambda(_t, _t*log(-32768*_t**5*a**2*c**3
 + 16384*_t**5*a*b**2*c**2 - 2048*_t**5*b**4*c - 4*_t*b + x)))
```

3.322.7 Maxima [F]

$$\int \frac{x^4}{a+bx^4+cx^8}\,dx=\int \frac{x^4}{cx^8+bx^4+a}\,dx$$

input
```
integrate(x^4/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
integrate(x^4/(c*x^8 + b*x^4 + a), x)
```

3.322.8 Giac [F]

$$\int \frac{x^4}{a+bx^4+cx^8}\,dx=\int \frac{x^4}{cx^8+bx^4+a}\,dx$$

input
```
integrate(x^4/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
integrate(x^4/(c*x^8 + b*x^4 + a), x)
```

3.322.9 Mupad [B] (verification not implemented)

Time = 9.49 (sec) , antiderivative size = 8169, normalized size of antiderivative = 25.14

$$\int \frac{x^4}{a+bx^4+cx^8}\,dx=\text{Too large to display}$$

input
```
int(x^4/(a + b*x^4 + c*x^8),x)
```

output
```
- atan((((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(51
2*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4))
)^(1/4)*(((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(5
12*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)
))^(1/4)*(262144*a^5*c^7 - 4096*a^2*b^6*c^4 + 49152*a^3*b^4*c^5 - 196608*a
^4*b^2*c^6) + x*(16384*a^4*b*c^6 + 1024*a^2*b^5*c^4 - 8192*a^3*b^3*c^5))*(
-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c +
 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3/4) +
64*a^3*b*c^4 - 16*a^2*b^3*c^3) - x*(8*a^3*c^4 - 4*a^2*b^2*c^3))*(-(b^5 + (
-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*
c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*1i - ((-(b^
5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256
*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(((-(b
^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 25
6*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(2621
44*a^5*c^7 - 4096*a^2*b^6*c^4 + 49152*a^3*b^4*c^5 - 196608*a^4*b^2*c^6) -
x*(16384*a^4*b*c^6 + 1024*a^2*b^5*c^4 - 8192*a^3*b^3*c^5))*(-(b^5 + (-(4*a
*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 -
 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3/4) + 64*a^3*b*c^4 -
 16*a^2*b^3*c^3) + x*(8*a^3*c^4 - 4*a^2*b^2*c^3))*(-(b^5 + (-(4*a*c - b...
```

3.322. $\int \frac{x^4}{a+bx^4+cx^8}\,dx$

3.323 $\int \frac{x^2}{a+bx^4+cx^8}\,dx$

3.323.1 Optimal result

Integrand size = 18, antiderivative size = 315

$$\begin{aligned}\int \frac{x^2}{a+bx^4+cx^8}\,dx = &-\frac{\sqrt[4]{c}\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2^{3/4}\sqrt{b^2-4ac}\sqrt[4]{-b-\sqrt{b^2-4ac}}}\\
&+\frac{\sqrt[4]{c}\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2^{3/4}\sqrt{b^2-4ac}\sqrt[4]{-b+\sqrt{b^2-4ac}}}\\
&+\frac{\sqrt[4]{c}\,\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2^{3/4}\sqrt{b^2-4ac}\sqrt[4]{-b-\sqrt{b^2-4ac}}}\\
&-\frac{\sqrt[4]{c}\,\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2^{3/4}\sqrt{b^2-4ac}\sqrt[4]{-b+\sqrt{b^2-4ac}}}\end{aligned}$$

output
```
-1/2*c^(1/4)*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(1/
4)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)/(-4*a*c+b^2)^(1/2)+1/2*c^(1/4)*arctanh(2^
(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(1/4)/(-b-(-4*a*c+b^2)^(1
/2))^(1/4)/(-4*a*c+b^2)^(1/2)+1/2*c^(1/4)*arctan(2^(1/4)*c^(1/4)*x/(-b+(-4
*a*c+b^2)^(1/2))^(1/4))*2^(1/4)/(-4*a*c+b^2)^(1/2)/(-b+(-4*a*c+b^2)^(1/2))
^(1/4)-1/2*c^(1/4)*arctanh(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4)
)*2^(1/4)/(-4*a*c+b^2)^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)
```

3.323.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.14

$$\int \frac{x^2}{a+bx^4+cx^8}\,dx = \frac{1}{4}\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{\log(x-\#1)}{b\#1+2c\#1^5}\&\right]$$

input
```
Integrate[x^2/(a + b*x^4 + c*x^8),x]
```

output
```
RootSum[a + b*#1^4 + c*#1^8 & , Log[x - #1]/(b*#1 + 2*c*#1^5) & ]/4
```

3.323.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 309, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1711, 27, 827, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{a+bx^4+cx^8}\,dx$$

$$\Big\downarrow \text{1711}$$

$$\frac{c\int \frac{2x^2}{2cx^4+b-\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}-\frac{c\int \frac{2x^2}{2cx^4+b+\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2c\int \frac{x^2}{2cx^4+b-\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}-\frac{2c\int \frac{x^2}{2cx^4+b+\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \; 827$$

$$\frac{2c\left(\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x^2+\sqrt{\sqrt{b^2-4ac}-b}}dx}{2\sqrt{2}\sqrt{c}}-\frac{\int \frac{1}{\sqrt{\sqrt{b^2-4ac}-b}-\sqrt{2}\sqrt{c}x^2}dx}{2\sqrt{2}\sqrt{c}}\right)}{\sqrt{b^2-4ac}}-\frac{2c\left(\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x^2+\sqrt{-b-\sqrt{b^2-4ac}}}dx}{2\sqrt{2}\sqrt{c}}-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}dx}{2\sqrt{2}\sqrt{c}}\right)}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \; 218$$

$$\frac{2c\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{\int \frac{1}{\sqrt{\sqrt{b^2-4ac}-b}-\sqrt{2}\sqrt{c}x^2}dx}{2\sqrt{2}\sqrt{c}}\right)}{\sqrt{b^2-4ac}}-\frac{2c\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}dx}{2\sqrt{2}\sqrt{c}}\right)}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \; 221$$

$$\frac{2c\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}}-\frac{2c\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}}$$

3.323. $\int \frac{x^2}{a+bx^4+cx^8}\,dx$

input
```
Int[x^2/(a + b*x^4 + c*x^8),x]
```

output
```
(-2*c*(ArcTan[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/
4)*c^(3/4)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) - ArcTanh[(2^(1/4)*c^(1/4)*x)/(
-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b - Sqrt[b^2 - 4*a*c])
^(1/4))))/Sqrt[b^2 - 4*a*c] + (2*c*(ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[
b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) -
ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(
3/4)*(-b + Sqrt[b^2 - 4*a*c])^(1/4))))/Sqrt[b^2 - 4*a*c]
```

3.323.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 1711
```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[c/q   Int[(d*x)^m/(b/2 - q/2 + c
*x^n), x], x] - Simp[c/q   Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; Free
Q[{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

3.323.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.14

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+\_Z^4b+a\right)}\frac{\_R^2\ln\left(x-\_R\right)}{2\_R^7c+\_R^3b}\right)}{4}$	43
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+\_Z^4b+a\right)}\frac{\_R^2\ln\left(x-\_R\right)}{2\_R^7c+\_R^3b}\right)}{4}$	43

input
```
int(x^2/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(_R^2/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

3.323.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3193 vs. 2(245) = 490.

Time = 0.29 (sec) , antiderivative size = 3193, normalized size of antiderivative = 10.14

$$\int \frac{x^2}{a+bx^4+cx^8}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
-1/4*sqrt(sqrt(1/2)*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2
*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c +
 16*a^3*c^2)))*log(1/2*sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a^2*c^2 - (a*b^7 -
12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c
+ 48*a^4*b^2*c^2 - 64*a^5*c^3))*sqrt(sqrt(1/2)*sqrt(-(b + (a*b^4 - 8*a^2*b
^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c
^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c
+ 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/
(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)) + c*x) + 1/4*sqrt(sqrt(1/2)*sqrt(-(b +
 (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b
^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))*log(-1/2*sqrt(1
/2)*(b^4 - 8*a*b^2*c + 16*a^2*c^2 - (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2
 - 64*a^4*b*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3
))*sqrt(sqrt(1/2)*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b
^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 1
6*a^3*c^2)))*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 -
12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3
*c^2)) + c*x) + 1/4*sqrt(-sqrt(1/2)*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c + 16*a
^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4
 - 8*a^2*b^2*c + 16*a^3*c^2)))*log(1/2*sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*...
```

3.323.6 Sympy [A] (verification not implemented)

Time = 3.03 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.55

$$\int \frac{x^2}{a + bx^4 + cx^8} \, dx$$
$$= \text{RootSum}\left(t^8 \cdot \left(16777216a^5c^4 - 16777216a^4b^2c^3 + 6291456a^3b^4c^2 - 1048576a^2b^6c + 65536ab^8\right) + t^4 \cdot \left(40\right.\right.$$

input
```
integrate(x**2/(c*x**8+b*x**4+a),x)
```

output
```
RootSum(_t**8*(16777216*a**5*c**4 - 16777216*a**4*b**2*c**3 + 6291456*a**3
*b**4*c**2 - 1048576*a**2*b**6*c + 65536*a*b**8) + _t**4*(4096*a**2*b*c**2
 - 2048*a*b**3*c + 256*b**5) + c, Lambda(_t, _t*log(x + (1048576*_t**7*a**
4*b*c**3 - 786432*_t**7*a**3*b**3*c**2 + 196608*_t**7*a**2*b**5*c - 16384*
_t**7*a*b**7 - 512*_t**3*a**2*c**2 + 384*_t**3*a*b**2*c - 64*_t**3*b**4)/c
)))
```

3.323.7 Maxima [F]

$$\int \frac{x^2}{a + bx^4 + cx^8} \, dx = \int \frac{x^2}{cx^8 + bx^4 + a} \, dx$$

input
```
integrate(x^2/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
integrate(x^2/(c*x^8 + b*x^4 + a), x)
```

3.323.8 Giac [F]

$$\int \frac{x^2}{a + bx^4 + cx^8} \, dx = \int \frac{x^2}{cx^8 + bx^4 + a} \, dx$$

input
```
integrate(x^2/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
integrate(x^2/(c*x^8 + b*x^4 + a), x)
```

3.323.9 Mupad [B] (verification not implemented)

Time = 8.96 (sec) , antiderivative size = 6067, normalized size of antiderivative = 19.26

$$\int \frac{x^2}{a + bx^4 + cx^8} \, dx = \text{Too large to display}$$

input
```
int(x^2/(a + b*x^4 + c*x^8),x)
```

output
```
2*atan((((-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(51
2*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3))
)^(3/4)*(256*a*b^5*c^4 + 4096*a^3*b*c^6 - x*(-(b^5 - (-(4*a*c - b^2)^5)^(1
/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c +
 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*(32768*a^4*c^7 - 1024*a*b^6*c^4
 + 10240*a^2*b^4*c^5 - 32768*a^3*b^2*c^6)*1i - 2048*a^2*b^3*c^5)*1i - 4*a*
b*c^5*x)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(51
2*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3))
)^(1/4) - ((-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(
512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3
)))^(3/4)*(256*a*b^5*c^4 + 4096*a^3*b*c^6 + x*(-(b^5 - (-(4*a*c - b^2)^5)^
(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c
 + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*(32768*a^4*c^7 - 1024*a*b^6*c
^4 + 10240*a^2*b^4*c^5 - 32768*a^3*b^2*c^6)*1i - 2048*a^2*b^3*c^5)*1i + 4*
a*b*c^5*x)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(
512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3
)))^(1/4))/(((-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)
/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c
^3)))^(3/4)*(256*a*b^5*c^4 + 4096*a^3*b*c^6 - x*(-(b^5 - (-(4*a*c - b^2)^5
)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*...
```

3.323. $\int \frac{x^2}{a+bx^4+cx^8}\,dx$

3.324 $\int \frac{1}{a+bx^4+cx^8}\,dx$

3.324.1 Optimal result

Integrand size = 14, antiderivative size = 315

$$\begin{aligned}\int \frac{1}{a+bx^4+cx^8}\,dx &= \frac{c^{3/4}\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} \\ &\quad - \frac{c^{3/4}\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}} \\ &\quad + \frac{c^{3/4}\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} \\ &\quad - \frac{c^{3/4}\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}\end{aligned}$$

output
```
1/2*c^(3/4)*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(3/4
)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)/(-4*a*c+b^2)^(1/2)+1/2*c^(3/4)*arctanh(2^(
1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(3/4)/(-b-(-4*a*c+b^2)^(1/
2))^(3/4)/(-4*a*c+b^2)^(1/2)-1/2*c^(3/4)*arctan(2^(1/4)*c^(1/4)*x/(-b+(-4*
a*c+b^2)^(1/2))^(1/4))*2^(3/4)/(-4*a*c+b^2)^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^
(3/4)-1/2*c^(3/4)*arctanh(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))
*2^(3/4)/(-4*a*c+b^2)^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)
```

3.324.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.14

$$\int \frac{1}{a+bx^4+cx^8}\,dx=\frac{1}{4}\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{\log(x-\#1)}{b\#1^3+2c\#1^7}\&\right]$$

input
```
Integrate[(a + b*x^4 + c*x^8)^(-1),x]
```

output
```
RootSum[a + b*#1^4 + c*#1^8 & , Log[x - #1]/(b*#1^3 + 2*c*#1^7) & ]/4
```

3.324.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1685, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+bx^4+cx^8}\,dx$$

$$\Big\downarrow \text{1685}$$

$$\frac{c\int \frac{1}{cx^4+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx}{\sqrt{b^2-4ac}}-\frac{c\int \frac{1}{cx^4+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{756}$$

$$\frac{c\left(-\frac{\int \frac{1}{\sqrt{\sqrt{b^2-4ac}-b-\sqrt{2}\sqrt{c}x^2}}dx}{\sqrt{\sqrt{b^2-4ac}-b}}-\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x^2+\sqrt{\sqrt{b^2-4ac}-b}}dx}{\sqrt{\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}}-\frac{c\left(-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}-\sqrt{2}\sqrt{c}x^2}}dx}{\sqrt{-\sqrt{b^2-4ac}-b}}-\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x^2+\sqrt{-b-\sqrt{b^2-4ac}}}dx}{\sqrt{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}}$$

$\downarrow$ 218

$$\frac{c\left(-\frac{\int \frac{1}{\sqrt{\sqrt{b^2-4ac}-b-\sqrt{2}\sqrt{c}x^2}}dx}{\sqrt{\sqrt{b^2-4ac}-b}}-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}\right)}{\sqrt{b^2-4ac}}-\frac{c\left(-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}-\sqrt{2}\sqrt{c}x^2}}dx}{\sqrt{-\sqrt{b^2-4ac}-b}}-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}\right)}{\sqrt{b^2-4ac}}$$

$\downarrow$ 221

$$\frac{c\left(-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}\right)}{\sqrt{b^2-4ac}}-\frac{c\left(-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}\right)}{\sqrt{b^2-4ac}}$$

input
```
Int[(a + b*x^4 + c*x^8)^(-1),x]
```

3.324. $\int \frac{1}{a+bx^4+cx^8}\,dx$

output
```
-((c*(-(ArcTan[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4
)*c^(1/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4))) - ArcTanh[(2^(1/4)*c^(1/4)*x)/(
-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b - Sqrt[b^2 - 4*a*c])^(
3/4))))/Sqrt[b^2 - 4*a*c]) + (c*(-(ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b
^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))) - Ar
cTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)
*(-b + Sqrt[b^2 - 4*a*c])^(3/4))))/Sqrt[b^2 - 4*a*c]
```

3.324.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 1685
```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^n), x], x] - Simp[
c/q   Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2,
2*n] && NeQ[b^2 - 4*a*c, 0]
```

3.324.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.13

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{8}+\_Z^{4}b+a\right)}\frac{\ln\left(x-\_R\right)}{2\_R^{7}c+\_R^{3}b}\right)}{4}$	40
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{8}+\_Z^{4}b+a\right)}\frac{\ln\left(x-\_R\right)}{2\_R^{7}c+\_R^{3}b}\right)}{4}$	40

input
```
int(1/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(1/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

3.324.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3125 vs. 2(245) = 490.

Time = 0.33 (sec) , antiderivative size = 3125, normalized size of antiderivative = 9.92

$$\int \frac{1}{a+bx^4+cx^8}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
1/4*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*
c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2
*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))*log(-(b^2*c -
a*c^2)*x + 1/2*(b^4 - 5*a*b^2*c + 4*a^2*c^2 - (a^3*b^5 - 8*a^4*b^3*c + 16*
a^5*b*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a
^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (a^3*b^4
- 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12
*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^
5*c^2)))) - 1/4*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (a^3*b^4 - 8*a^4*b^2
*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c
+ 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))*lo
g(-(b^2*c - a*c^2)*x - 1/2*(b^4 - 5*a*b^2*c + 4*a^2*c^2 - (a^3*b^5 - 8*a^4
*b^3*c + 16*a^5*b*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*
b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c
 + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(
a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b
^2*c + 16*a^5*c^2)))) + 1/4*sqrt(-sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (a^3*b^
4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 -
12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*
a^5*c^2)))*log(-(b^2*c - a*c^2)*x + 1/2*(b^4 - 5*a*b^2*c + 4*a^2*c^2 - ...
```

3.324.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{a+bx^4+cx^8}\,dx = \text{Timed out}$$

input
```
integrate(1/(c*x**8+b*x**4+a),x)
```

output
```
Timed out
```

3.324.7 Maxima [F]

$$\int \frac{1}{a+bx^4+cx^8}\,dx=\int \frac{1}{cx^8+bx^4+a}\,dx$$

input
```
integrate(1/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
integrate(1/(c*x^8 + b*x^4 + a), x)
```

3.324.8 Giac [F]

$$\int \frac{1}{a+bx^4+cx^8}\,dx=\int \frac{1}{cx^8+bx^4+a}\,dx$$

input
```
integrate(1/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
integrate(1/(c*x^8 + b*x^4 + a), x)
```

3.324.9 Mupad [B] (verification not implemented)

Time = 9.42 (sec) , antiderivative size = 10337, normalized size of antiderivative = 32.82

$$\int \frac{1}{a+bx^4+cx^8}\,dx=\text{Too large to display}$$

input
```
int(1/(a + b*x^4 + c*x^8),x)
```

output
```
- atan((((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3
*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*
c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(64*a*c^7 +
 ((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 -
11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 1
6*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(4096*a*b^7*c^4 -
262144*a^4*b*c^7 - 49152*a^2*b^5*c^5 + 196608*a^3*b^3*c^6) + x*(1024*b^7*c
^4 - 11264*a*b^5*c^5 - 49152*a^3*b*c^7 + 40960*a^2*b^3*c^6))*(-(b^7 + b^2*
(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*
c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 9
6*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(3/4) - 16*b^2*c^6) + 8*c^7*x)*(-(b^7 +
 b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c
 - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*
c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*1i - ((-(b^7 + b^2*(-(4*a*c
- b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*
c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4
*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(64*a*c^7 + ((-(b^7 + b^2*(-(4*a*c - b^2)^
5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2
)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 -
256*a^6*b^2*c^3)))^(1/4)*(4096*a*b^7*c^4 - 262144*a^4*b*c^7 - 49152*a^2...
```

3.324. $\int \frac{1}{a+bx^4+cx^8}\,dx$

3.325 $\int \frac{1}{x^2(a+bx^4+cx^8)}\,dx$

3.325.1 Optimal result

Integrand size = 18, antiderivative size = 363

$$\begin{aligned}\int \frac{1}{x^2(a+bx^4+cx^8)}\,dx = &-\frac{1}{ax}-\frac{\sqrt[4]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\ 2^{3/4}a\sqrt[4]{-b-\sqrt{b^2-4ac}}}\\ &-\frac{\sqrt[4]{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\ 2^{3/4}a\sqrt[4]{-b+\sqrt{b^2-4ac}}}\\ &+\frac{\sqrt[4]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\ 2^{3/4}a\sqrt[4]{-b-\sqrt{b^2-4ac}}}\\ &+\frac{\sqrt[4]{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\ 2^{3/4}a\sqrt[4]{-b+\sqrt{b^2-4ac}}}\end{aligned}$$

output
```
-1/a/x-1/4*c^(1/4)*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))
*(1-b/(-4*a*c+b^2)^(1/2))*2^(1/4)/a/(-b-(-4*a*c+b^2)^(1/2))^(1/4)+1/4*c^(1
/4)*arctanh(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(1-b/(-4*a*c+
b^2)^(1/2))*2^(1/4)/a/(-b-(-4*a*c+b^2)^(1/2))^(1/4)-1/4*c^(1/4)*arctan(2^(
1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(1+b/(-4*a*c+b^2)^(1/2))*2^(
1/4)/a/(-b+(-4*a*c+b^2)^(1/2))^(1/4)+1/4*c^(1/4)*arctanh(2^(1/4)*c^(1/4)*x
/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(1+b/(-4*a*c+b^2)^(1/2))*2^(1/4)/a/(-b+(-4
*a*c+b^2)^(1/2))^(1/4)
```

3.325.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.20

$$\int \frac{1}{x^2 (a+bx^4+cx^8)} \, dx = -\frac{1}{ax} - \frac{\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{b\log(x-\#1)+c\log(x-\#1)\#1^4}{b\#1+2c\#1^5}\&\right]}{4a}$$

input
```
Integrate[1/(x^2*(a + b*x^4 + c*x^8)),x]
```

output
```
-(1/(a*x)) - RootSum[a + b*#1^4 + c*#1^8 & , (b*Log[x - #1] + c*Log[x - #1
]*#1^4)/(b*#1 + 2*c*#1^5) & ]/(4*a)
```

3.325.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 330, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {1704, 25, 1834, 27, 827, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^4+cx^8)} \, dx$$

$$\Big\downarrow \text{ 1704}$$

$$\frac{\int -\frac{x^2(cx^4+b)}{cx^8+bx^4+a}dx}{a}-\frac{1}{ax}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int \frac{x^2(cx^4+b)}{cx^8+bx^4+a}dx}{a}-\frac{1}{ax}$$

$$\Big\downarrow \text{ 1834}$$

$$-\frac{\frac{1}{2}c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\int \frac{2x^2}{2cx^4+b-\sqrt{b^2-4ac}}dx+\frac{1}{2}c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{2x^2}{2cx^4+b+\sqrt{b^2-4ac}}dx}{a}-\frac{1}{ax}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\int \frac{x^2}{2cx^4+b-\sqrt{b^2-4ac}}dx+c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{x^2}{2cx^4+b+\sqrt{b^2-4ac}}dx}{a}-\frac{1}{ax}$$

$$\Big\downarrow \text{ 827}$$

$$-\frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x^2+\sqrt{-b-\sqrt{b^2-4ac}}}dx}{2\sqrt{2}\sqrt{c}}-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}dx}{2\sqrt{2}\sqrt{c}}\right)+c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x^2+\sqrt{\sqrt{b^2-4ac}-b}}dx}{2\sqrt{2}\sqrt{c}} \cdots\right.}{a} \cdots$$

$$\frac{1}{ax}$$

$$\Big\downarrow \text{ 218}$$

$$-\frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}dx}{2\sqrt{2}\sqrt{c}}\right)+c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\cdots}{\sqrt[4]{\sqrt{b^2-\cdots}}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{\sqrt{b^2\cdots}}}\cdots\right.}{a} \cdots$$

$$\frac{1}{ax}$$

$$\Big\downarrow \text{ 221}$$

3.325. $\int \frac{1}{x^2(a+bx^4+cx^8)}\,dx$

$$-\frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)+c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{\arctan\left(\frac{\cdots}{\cdots}\right)}{2\,2^{3/4}c^{3/\cdots}}\cdots\right.}{a}$$

$$\frac{1}{ax}$$

input
```
Int[1/(x^2*(a + b*x^4 + c*x^8)),x]
```

output
```
-(1/(a*x)) - (c*(1 - b/Sqrt[b^2 - 4*a*c])*(ArcTan[(2^(1/4)*c^(1/4)*x)/(-b
- Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b - Sqrt[b^2 - 4*a*c])^(1
/4)) - ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3
/4)*c^(3/4)*(-b - Sqrt[b^2 - 4*a*c])^(1/4))) + c*(1 + b/Sqrt[b^2 - 4*a*c])
*(ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^
(3/4)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) - ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b +
Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b + Sqrt[b^2 - 4*a*c])^(1/4
))))/a
```

3.325.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 1704
```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Simp[1/(a*d^n*(m + 1))   Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1)
+ c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{
a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] &&
LtQ[m, -1] && IntegerQ[p]
```

rule 1834
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 +
 (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2
 - (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ
[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n
, 0]
```

3.325.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.17

method	result
default	$-\frac{\sum_{\_R=\operatorname{RootOf}\left(c\_Z^{8}+\_Z^{4}b+a\right)}\frac{\left(\_R^{6}c+b\_R^{2}\right)\ln\left(x-\_R\right)}{2\_R^{7}c+\_R^{3}b}}{4a}-\frac{1}{ax}$
risch	$-\frac{1}{ax}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\left(256a^{9}c^{4}-256b^{2}c^{3}a^{8}+96b^{4}c^{2}a^{7}-16b^{6}c\,a^{6}+b^{8}a^{5}\right)\_Z^{8}+\left(80a^{4}b\,c^{4}-120a^{3}b^{3}c^{3}+61a^{2}b^{5}c^{2}-13c\,b^{7}a+b^{9}\right)\_Z^{4}+c^{5}\right)}\right.}{}$

input
```
int(1/x^2/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4/a*sum((_R^6*c+_R^2*b)/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*c+_Z^
4*b+a))-1/a/x
```

3.325. $\int \frac{1}{x^2(a+bx^4+cx^8)}\,dx$

3.325.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5758 vs. 2(281) = 562.

Time = 0.53 (sec) , antiderivative size = 5758, normalized size of antiderivative = 15.86

$$\int \frac{1}{x^2 (a + bx^4 + cx^8)} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.325.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a + bx^4 + cx^8)} \, dx = \text{Timed out}$$

input
```
integrate(1/x**2/(c*x**8+b*x**4+a),x)
```

output
```
Timed out
```

3.325.7 Maxima [F]

$$\int \frac{1}{x^2 (a + bx^4 + cx^8)} \, dx = \int \frac{1}{(cx^8 + bx^4 + a)x^2} \, dx$$

input
```
integrate(1/x^2/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
-integrate((c*x^6 + b*x^2)/(c*x^8 + b*x^4 + a), x)/a - 1/(a*x)
```

3.325.8 Giac [F]

$$\int \frac{1}{x^2\left(a+bx^4+cx^8\right)}\,dx=\int \frac{1}{(cx^8+bx^4+a)x^2}\,dx$$

input
```
integrate(1/x^2/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
integrate(1/((c*x^8 + b*x^4 + a)*x^2), x)
```

3.325.9 Mupad [B] (verification not implemented)

Time = 9.01 (sec) , antiderivative size = 10509, normalized size of antiderivative = 28.95

$$\int \frac{1}{x^2\left(a+bx^4+cx^8\right)}\,dx=\text{Too large to display}$$

input
```
int(1/(x^2*(a + b*x^4 + c*x^8)),x)
```

output
```
2*atan((((-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5
*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3
*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^
6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(4096*a^15*c^8 - x*(-(b^9
+ b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b
^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*
c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4
*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^8 + 1024*a^12*b^8*c^4 - 1228
8*a^13*b^6*c^5 + 51200*a^14*b^4*c^6 - 81920*a^15*b^2*c^7)*1i + 256*a^11*b^
8*c^4 - 2816*a^12*b^6*c^5 + 10496*a^13*b^4*c^6 - 14336*a^14*b^2*c^7)*1i +
4*a^11*b*c^8*x)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*
a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^
7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16
*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4) - ((-(b^9 + b^4*(-(
4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 +
a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^
5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 25
6*a^8*b^2*c^3)))^(3/4)*(4096*a^15*c^8 + x*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^
(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c
 - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(51...
```

3.326 $\int \frac{1}{x^4(a+bx^4+cx^8)}\,dx$

3.326.1 Optimal result

Integrand size = 18, antiderivative size = 365

$$\begin{aligned}\int \frac{1}{x^4(a+bx^4+cx^8)}\,dx = &-\frac{1}{3ax^3}+\frac{c^{3/4}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}a\left(-b-\sqrt{b^2-4ac}\right)^{3/4}}\\
&+\frac{c^{3/4}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}a\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}\\
&+\frac{c^{3/4}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}a\left(-b-\sqrt{b^2-4ac}\right)^{3/4}}\\
&+\frac{c^{3/4}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}a\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}\end{aligned}$$

output
```
-1/3/a/x^3+1/4*c^(3/4)*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1
/4))*(1-b/(-4*a*c+b^2)^(1/2))*2^(3/4)/a/(-b-(-4*a*c+b^2)^(1/2))^(3/4)+1/4*
c^(3/4)*arctanh(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(1-b/(-4*
a*c+b^2)^(1/2))*2^(3/4)/a/(-b-(-4*a*c+b^2)^(1/2))^(3/4)+1/4*c^(3/4)*arctan
(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(1+b/(-4*a*c+b^2)^(1/2))
*2^(3/4)/a/(-b+(-4*a*c+b^2)^(1/2))^(3/4)+1/4*c^(3/4)*arctanh(2^(1/4)*c^(1/
4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(1+b/(-4*a*c+b^2)^(1/2))*2^(3/4)/a/(-b
+(-4*a*c+b^2)^(1/2))^(3/4)
```

3.326.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.21

$$\int \frac{1}{x^4 (a+bx^4+cx^8)} dx = -\frac{1}{3ax^3} - \frac{\text{RootSum}\left[a+b\#1^4+c\#1^8\&, \frac{b\log(x-\#1)+c\log(x-\#1)\#1^4}{b\#1^3+2c\#1^7}\&\right]}{4a}$$

input
```
Integrate[1/(x^4*(a + b*x^4 + c*x^8)),x]
```

output
```
-1/3*1/(a*x^3) - RootSum[a + b*#1^4 + c*#1^8 & , (b*Log[x - #1] + c*Log[x
- #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ]/(4*a)
```

3.326.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 330, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1704, 27, 1752, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a+bx^4+cx^8)} dx$$

$$\Big\downarrow \text{1704}$$

$$\frac{\int -\frac{3(cx^4+b)}{cx^8+bx^4+a}dx}{3a}-\frac{1}{3ax^3}$$

$\Big\downarrow$ 27

$$-\frac{\int \frac{cx^4+b}{cx^8+bx^4+a}dx}{a}-\frac{1}{3ax^3}$$

$\Big\downarrow$ 1752

$$-\frac{\frac{1}{2}c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\int \frac{1}{cx^4+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^4+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{a}-\frac{1}{3ax^3}$$

$\Big\downarrow$ 756

$$-\frac{\frac{1}{2}c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}dx}{\sqrt{-\sqrt{b^2-4ac}-b}}-\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x^2+\sqrt{-b-\sqrt{b^2-4ac}}}dx}{\sqrt{-\sqrt{b^2-4ac}-b}}\right)+\frac{1}{2}c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(-\frac{\int \frac{1}{\sqrt{\sqrt{b^2-4ac}-b-\sqrt{2}\ldots}}}{\sqrt{\sqrt{b^2-4ac}-\ldots}}\right.}{a}$$

$$\frac{1}{3ax^3}$$

$\Big\downarrow$ 218

$$-\frac{\frac{1}{2}c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}dx}{\sqrt{-\sqrt{b^2-4ac}-b}}-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}\right)+\frac{1}{2}c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(-\frac{\int \frac{1}{\sqrt{\sqrt{b^2-4a\ldots}}}}{\sqrt{\sqrt{b^2\ldots}}}\right.}{a}$$

$$\frac{1}{3ax^3}$$

$\Big\downarrow$ 221

$$-\frac{\frac{1}{2}c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}\right)+\frac{1}{2}c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(-\ldots\right.}{a}$$

$$\frac{1}{3ax^3}$$

input
```
Int[1/(x^4*(a + b*x^4 + c*x^8)),x]
```

3.326. $\int \frac{1}{x^4(a+bx^4+cx^8)}\,dx$

output
```
-1/3*1/(a*x^3) - ((c*(1 - b/Sqrt[b^2 - 4*a*c])*(-(ArcTan[(2^(1/4)*c^(1/4)*
x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b - Sqrt[b^2 - 4*a*c
])^(3/4))) - ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(
2^(1/4)*c^(1/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4))))/2 + (c*(1 + b/Sqrt[b^2 -
 4*a*c])*(-(ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2^
(1/4)*c^(1/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))) - ArcTanh[(2^(1/4)*c^(1/4)*
x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b + Sqrt[b^2 - 4*a*c
])^(3/4))))/2)/a
```

3.326.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 1704
```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Simp[1/(a*d^n*(m + 1))   Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1)
+ c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{
a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] &&
LtQ[m, -1] && IntegerQ[p]
```

rule 1752
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

3.326.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.17

method	result
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{8}+\_Z^{4}b+a\right)}\frac{\left(-\_R^{4}c-b\right)\ln\left(x-\_R\right)}{2\_R^{7}c+\_R^{3}b}}{4a}-\frac{1}{3a\,x^{3}}$
risch	$-\frac{1}{3a\,x^{3}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(256c^{4}a^{11}-256a^{10}b^{2}c^{3}+96a^{9}b^{4}c^{2}-16a^{8}b^{6}c+a^{7}b^{8}\right)\_Z^{8}+\left(-112b\,c^{5}a^{5}+280b^{3}c^{4}a^{4}-231c^{3}b^{5}a^{3}+86b^{7}c^{2}a^{2}-15b^{9}c\right.\right.}\right.}{}$

input
```
int(1/x^4/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4/a*sum((-_R^4*c-b)/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*c+_Z^4*b+a
))-1/3/a/x^3
```

3.326.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5030 vs. 2(281) = 562.

Time = 0.62 (sec) , antiderivative size = 5030, normalized size of antiderivative = 13.78

$$\int \frac{1}{x^4\left(a+bx^4+cx^8\right)}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^4/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.326.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a + bx^4 + cx^8)} dx = \text{Timed out}$$

input
```
integrate(1/x**4/(c*x**8+b*x**4+a),x)
```

output
```
Timed out
```

3.326.7 Maxima [F]

$$\int \frac{1}{x^4 (a + bx^4 + cx^8)} dx = \int \frac{1}{(cx^8 + bx^4 + a)x^4} dx$$

input
```
integrate(1/x^4/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
-integrate((c*x^4 + b)/(c*x^8 + b*x^4 + a), x)/a - 1/3/(a*x^3)
```

3.326.8 Giac [F]

$$\int \frac{1}{x^4 (a + bx^4 + cx^8)} dx = \int \frac{1}{(cx^8 + bx^4 + a)x^4} dx$$

input
```
integrate(1/x^4/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
integrate(1/((c*x^8 + b*x^4 + a)*x^4), x)
```

3.326.9 Mupad [B] (verification not implemented)

Time = 10.98 (sec) , antiderivative size = 16497, normalized size of antiderivative = 45.20

$$\int \frac{1}{x^4\left(a+bx^4+cx^8\right)}\,dx = \text{Too large to display}$$

input
```
int(1/(x^4*(a + b*x^4 + c*x^8)),x)
```

output
```
2*atan(-(((-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*
b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(
1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(
4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^
9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x*(81920*a^15*b*c^8 + 1024*a^11*b^
9*c^4 - 13312*a^12*b^7*c^5 + 62464*a^13*b^5*c^6 - 122880*a^14*b^3*c^7) - (
-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 2
31*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a
*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2
)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 -
 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8 + 4096*a^13*b^8*c^4 - 53248*a^
14*b^6*c^5 + 245760*a^15*b^4*c^6 - 458752*a^16*b^2*c^7)*1i)*(-(b^11 + b^6*
(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^
3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^
2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(
512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2
*c^3)))^(3/4)*1i - 128*a^11*b*c^9 - 16*a^9*b^5*c^7 + 96*a^10*b^3*c^8)*1i +
 x*(8*a^10*c^10 - 4*a^9*b^2*c^9))*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) -
 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*
c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^...
```

3.327 $\int \frac{x^m}{1+x^4+x^8} \, dx$

3.327.1 Optimal result

Integrand size = 14, antiderivative size = 127

$$\int \frac{x^m}{1+x^4+x^8} \, dx = \frac{2x^{1+m} \operatorname{Hypergeometric2F1}\left(1, \frac{1+m}{4}, \frac{5+m}{4}, -\frac{2x^4}{1-i\sqrt{3}}\right)}{\sqrt{3}\left(i+\sqrt{3}\right)(1+m)} - \frac{2x^{1+m} \operatorname{Hypergeometric2F1}\left(1, \frac{1+m}{4}, \frac{5+m}{4}, -\frac{2x^4}{1+i\sqrt{3}}\right)}{\sqrt{3}\left(i-\sqrt{3}\right)(1+m)}$$

output
```
-2/3*x^(1+m)*hypergeom([1, 1/4+1/4*m],[5/4+1/4*m],-2*x^4/(1+I*3^(1/2)))/(1
+m)/(I-3^(1/2))*3^(1/2)+2/3*x^(1+m)*hypergeom([1, 1/4+1/4*m],[5/4+1/4*m],-
2*x^4/(1-I*3^(1/2)))/(1+m)*3^(1/2)/(3^(1/2)+I)
```

3.327.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 1.06 (sec) , antiderivative size = 488, normalized size of antiderivative = 3.84

$$\int \frac{x^m}{1+x^4+x^8} \, dx = \frac{x^m \left(-\frac{i\left(\left(\frac{x}{-\sqrt[3]{-1}+x}\right)^{-m} \operatorname{Hypergeometric2F1}\left(-m,-m,1-m,\frac{\sqrt[3]{-1}}{\sqrt[3]{-1}-x}\right)+\left(\frac{x}{-(-1)^{2/3}+x}\right)^{-m} \operatorname{Hypergeometric2F1}\left(-m,-m,1-m,\frac{(-}{(-1}\right.\right.}{} \right.}{}$$

input
```
Integrate[x^m/(1 + x^4 + x^8),x]
```

output
```
(x^m*(((-I)*(Hypergeometric2F1[-m, -m, 1 - m, (-1)^(1/3)/((-1)^(1/3) - x)]
/(x/(-(-1)^(1/3) + x))^m + Hypergeometric2F1[-m, -m, 1 - m, (-1)^(2/3)/((-
1)^(2/3) - x)]/(x/(-(-1)^(2/3) + x))^m - Hypergeometric2F1[-m, -m, 1 - m,
(-1)^(1/3)/((-1)^(1/3) + x)]/(x/((-1)^(1/3) + x))^m - Hypergeometric2F1[-m
, -m, 1 - m, (-1)^(2/3)/((-1)^(2/3) + x)]/(x/((-1)^(2/3) + x))^m))/Sqrt[3]
 + RootSum[1 - #1^2 + #1^4 & , Hypergeometric2F1[-m, -m, 1 - m, -(#1/(x -
#1))]/((x/(x - #1))^m*(-#1 + 2*#1^3)) & ] - RootSum[1 - #1^2 + #1^4 & , (m
*x^2 + m^2*x^2 + 2*m*x*#1 + m^2*x*#1 + (2*Hypergeometric2F1[-m, -m, 1 - m,
 -(#1/(x - #1))]*#1^2)/(x/(x - #1))^m + (3*m*Hypergeometric2F1[-m, -m, 1 -
 m, -(#1/(x - #1))]*#1^2)/(x/(x - #1))^m + (m^2*Hypergeometric2F1[-m, -m,
1 - m, -(#1/(x - #1))]*#1^2)/(x/(x - #1))^m + (m*#1^2)/(x/#1)^m)/(-#1 + 2*
#1^3) & ]/(2 + 3*m + m^2)))/(4*m)
```

3.327.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1711, 27, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{x^8+x^4+1} \, dx$$
$$\Big\downarrow \ 1711$$
$$\frac{i \int \frac{2x^m}{2x^4+i\sqrt{3}+1}dx}{\sqrt{3}}-\frac{i \int \frac{2x^m}{2x^4-i\sqrt{3}+1}dx}{\sqrt{3}}$$
$$\Big\downarrow \ 27$$
$$\frac{2i \int \frac{x^m}{2x^4+i\sqrt{3}+1}dx}{\sqrt{3}}-\frac{2i \int \frac{x^m}{2x^4-i\sqrt{3}+1}dx}{\sqrt{3}}$$
$$\Big\downarrow \ 888$$
$$\frac{2ix^{m+1} \text{Hypergeometric2F1}\left(1,\frac{m+1}{4},\frac{m+5}{4},-\frac{2x^4}{1+i\sqrt{3}}\right)}{\sqrt{3}\left(1+i\sqrt{3}\right)(m+1)}-\frac{2ix^{m+1} \text{Hypergeometric2F1}\left(1,\frac{m+1}{4},\frac{m+5}{4},-\frac{2x^4}{1-i\sqrt{3}}\right)}{\sqrt{3}\left(1-i\sqrt{3}\right)(m+1)}$$

input
```
Int[x^m/(1 + x^4 + x^8),x]
```

output
```
((-2*I)*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, (-2*x^4)/(1 -
 I*Sqrt[3])])/(Sqrt[3]*(1 - I*Sqrt[3])*(1 + m)) + ((2*I)*x^(1 + m)*Hyperge
ometric2F1[1, (1 + m)/4, (5 + m)/4, (-2*x^4)/(1 + I*Sqrt[3])])/(Sqrt[3]*(1
 + I*Sqrt[3])*(1 + m))
```

3.327.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1711
```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[c/q   Int[(d*x)^m/(b/2 - q/2 + c
*x^n), x], x] - Simp[c/q   Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; Free
Q[{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

3.327.4 Maple [F]

$$\int \frac{x^m}{x^8 + x^4 + 1} dx$$

input
```
int(x^m/(x^8+x^4+1),x)
```

output
```
int(x^m/(x^8+x^4+1),x)
```

3.327.5 Fricas [F]

$$\int \frac{x^m}{1+x^4+x^8}\,dx = \int \frac{x^m}{x^8+x^4+1}\,dx$$

input
```
integrate(x^m/(x^8+x^4+1),x, algorithm="fricas")
```

output
```
integral(x^m/(x^8 + x^4 + 1), x)
```

3.327.6 Sympy [F]

$$\int \frac{x^m}{1+x^4+x^8}\,dx = \int \frac{x^m}{(x^2-x+1)\,(x^2+x+1)\,(x^4-x^2+1)}\,dx$$

input
```
integrate(x**m/(x**8+x**4+1),x)
```

output
```
Integral(x**m/((x**2 - x + 1)*(x**2 + x + 1)*(x**4 - x**2 + 1)), x)
```

3.327.7 Maxima [F]

$$\int \frac{x^m}{1+x^4+x^8}\,dx = \int \frac{x^m}{x^8+x^4+1}\,dx$$

input
```
integrate(x^m/(x^8+x^4+1),x, algorithm="maxima")
```

output
```
integrate(x^m/(x^8 + x^4 + 1), x)
```

3.327.8 Giac [F]

$$\int \frac{x^m}{1+x^4+x^8}\,dx=\int \frac{x^m}{x^8+x^4+1}\,dx$$

input
```
integrate(x^m/(x^8+x^4+1),x, algorithm="giac")
```

output
```
integrate(x^m/(x^8 + x^4 + 1), x)
```

3.327.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{1+x^4+x^8}\,dx=\int \frac{x^m}{x^8+x^4+1}\,dx$$

input
```
int(x^m/(x^4 + x^8 + 1),x)
```

output
```
int(x^m/(x^4 + x^8 + 1), x)
```

3.328 $\int \frac{x^{11}}{1+x^4+x^8} \, dx$

3.328.1 Optimal result

Integrand size = 14, antiderivative size = 44

$$\int \frac{x^{11}}{1+x^4+x^8} \, dx = \frac{x^4}{4} - \frac{\arctan\left(\frac{1+2x^4}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{8}\log\left(1+x^4+x^8\right)$$

output
```
1/4*x^4-1/8*ln(x^8+x^4+1)-1/12*arctan(1/3*(2*x^4+1)*3^(1/2))*3^(1/2)
```

3.328.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{x^{11}}{1+x^4+x^8} \, dx = \frac{x^4}{4} - \frac{\arctan\left(\frac{1+2x^4}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{8}\log\left(1+x^4+x^8\right)$$

input
```
Integrate[x^11/(1 + x^4 + x^8),x]
```

output
```
x^4/4 - ArcTan[(1 + 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) - Log[1 + x^4 + x^8]/8
```

3.328.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1693, 1143, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{x^8+x^4+1} \, dx$$
$$\Big\downarrow \ 1693$$
$$\frac{1}{4}\int \frac{x^8}{x^8+x^4+1}dx^4$$
$$\Big\downarrow \ 1143$$
$$\frac{1}{4}\int \left(1-\frac{x^4+1}{x^8+x^4+1}\right)dx^4$$
$$\Big\downarrow \ 2009$$
$$\frac{1}{4}\left(-\frac{\arctan\left(\frac{2x^4+1}{\sqrt{3}}\right)}{\sqrt{3}}+x^4-\frac{1}{2}\log\left(x^8+x^4+1\right)\right)$$

input
```
Int[x^11/(1 + x^4 + x^8),x]
```

output
```
(x^4 - ArcTan[(1 + 2*x^4)/Sqrt[3]]/Sqrt[3] - Log[1 + x^4 + x^8]/2)/4
```

3.328.3.1 Defintions of rubi rules used

rule 1143
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x)^m/(a + b*x + c*x^2), x], x] /; FreeQ[{a,
b, c, d, e}, x] && IGtQ[m, 1]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.328.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{x^{4}}{4}-\frac{\ln\left(x^{8}+x^{4}+1\right)}{8}-\frac{\arctan\left(\frac{\left(2x^{4}+1\right)\sqrt{3}}{3}\right)\sqrt{3}}{12}$	36
risch	$\frac{x^{4}}{4}-\frac{\ln\left(4x^{8}+4x^{4}+4\right)}{8}-\frac{\arctan\left(\frac{\left(2x^{4}+1\right)\sqrt{3}}{3}\right)\sqrt{3}}{12}$	40

input
```
int(x^11/(x^8+x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4-1/8*ln(x^8+x^4+1)-1/12*arctan(1/3*(2*x^4+1)*3^(1/2))*3^(1/2)
```

3.328.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.80

$$\int \frac{x^{11}}{1+x^4+x^8}\,dx=\frac{1}{4}\,x^{4}-\frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^{4}+1)\right)-\frac{1}{8}\,\log\left(x^{8}+x^{4}+1\right)$$

input
```
integrate(x^11/(x^8+x^4+1),x, algorithm="fricas")
```

output
```
1/4*x^4 - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) - 1/8*log(x^8 + x^4
 + 1)
```

3.328.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int \frac{x^{11}}{1+x^4+x^8}\,dx = \frac{x^4}{4} - \frac{\log\left(x^8+x^4+1\right)}{8} - \frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^4}{3}+\frac{\sqrt{3}}{3}\right)}{12}$$

input
```
integrate(x**11/(x**8+x**4+1),x)
```

output
```
x**4/4 - log(x**8 + x**4 + 1)/8 - sqrt(3)*atan(2*sqrt(3)*x**4/3 + sqrt(3)/
3)/12
```

3.328.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.80

$$\int \frac{x^{11}}{1+x^4+x^8}\,dx = \frac{1}{4}x^4 - \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^4+1)\right) - \frac{1}{8}\log\left(x^8+x^4+1\right)$$

input
```
integrate(x^11/(x^8+x^4+1),x, algorithm="maxima")
```

output
```
1/4*x^4 - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) - 1/8*log(x^8 + x^4
 + 1)
```

3.328.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.80

$$\int \frac{x^{11}}{1+x^4+x^8}\,dx = \frac{1}{4}x^4 - \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^4+1)\right) - \frac{1}{8}\log\left(x^8+x^4+1\right)$$

input
```
integrate(x^11/(x^8+x^4+1),x, algorithm="giac")
```

output
```
1/4*x^4 - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) - 1/8*log(x^8 + x^4
 + 1)
```

3.328.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

$$\int \frac{x^{11}}{1+x^4+x^8}\,dx = \frac{x^4}{4} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x^4}{3} + \frac{\sqrt{3}}{3}\right)}{12} - \frac{\ln\left(x^8+x^4+1\right)}{8}$$

input

```
int(x^11/(x^4 + x^8 + 1),x)
```

output

```
x^4/4 - (3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^4)/3))/12 - log(x^4 + x^8 +
 1)/8
```

3.329 $\int \frac{x^9}{1+x^4+x^8} \, dx$

3.329.1 Optimal result

Integrand size = 14, antiderivative size = 54

$$\int \frac{x^9}{1+x^4+x^8} \, dx = \frac{x^2}{2} + \frac{\arctan\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{\arctan\left(\frac{1+2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}$$

output
```
1/2*x^2+1/6*arctan(1/3*(-2*x^2+1)*3^(1/2))*3^(1/2)-1/6*arctan(1/3*(2*x^2+1
)*3^(1/2))*3^(1/2)
```

3.329.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.81

$$\int \frac{x^9}{1+x^4+x^8} \, dx = \frac{x^2}{2} - \frac{(i+\sqrt{3}) \arctan\left(\frac{1}{2}(-i+\sqrt{3})\, x^2\right)}{2\sqrt{6+6i\sqrt{3}}} - \frac{(-i+\sqrt{3}) \arctan\left(\frac{1}{2}(i+\sqrt{3})\, x^2\right)}{2\sqrt{6-6i\sqrt{3}}}$$

input
```
Integrate[x^9/(1 + x^4 + x^8),x]
```

output
```
x^2/2 - ((I + Sqrt[3])*ArcTan[((-I + Sqrt[3])*x^2)/2])/(2*Sqrt[6 + (6*I)*S
qrt[3]]) - ((-I + Sqrt[3])*ArcTan[((I + Sqrt[3])*x^2)/2])/(2*Sqrt[6 - (6*I
)*Sqrt[3]])
```

3.329.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {1695, 1442, 1475, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{x^8+x^4+1} \, dx$$

$$\Big\downarrow \text{1695}$$

$$\frac{1}{2} \int \frac{x^8}{x^8+x^4+1}dx^2$$

$$\Big\downarrow \text{1442}$$

$$\frac{1}{2} \left(x^2-\int \frac{x^4+1}{x^8+x^4+1}dx^2\right)$$

$$\Big\downarrow \text{1475}$$

$$\frac{1}{2} \left(-\frac{1}{2} \int \frac{1}{x^4-x^2+1}dx^2-\frac{1}{2} \int \frac{1}{x^4+x^2+1}dx^2+x^2\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2} \left(\int \frac{1}{-x^4-3}d\left(2 x^2-1\right)+\int \frac{1}{-x^4-3}d\left(2 x^2+1\right)+x^2\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2} \left(-\frac{\arctan \left(\frac{2 x^2-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\arctan \left(\frac{2 x^2+1}{\sqrt{3}}\right)}{\sqrt{3}}+x^2\right)$$

input
```
Int[x^9/(1 + x^4 + x^8),x]
```

output
```
(x^2 - ArcTan[(-1 + 2*x^2)/Sqrt[3]]/Sqrt[3] - ArcTan[(1 + 2*x^2)/Sqrt[3]]/
Sqrt[3])/2
```

3.329.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1442
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
x] - Simp[d^4/(c*(m + 4*p + 1))   Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 1695
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b
*x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c,
p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.329. $\int \frac{x^9}{1+x^4+x^8} \, dx$

3.329.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{x^2}{2}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x^2-1)\sqrt{3}}{3}\right)}{6}-\frac{\arctan\left(\frac{(2x^2+1)\sqrt{3}}{3}\right)\sqrt{3}}{6}$	43
risch	$\frac{x^2}{2}-\frac{\sqrt{3}\,\arctan\left(\frac{x^2\sqrt{3}}{3}\right)}{6}-\frac{\sqrt{3}\,\arctan\left(\frac{x^6\sqrt{3}}{3}+\frac{2x^2\sqrt{3}}{3}\right)}{6}$	44

input
```
int(x^9/(x^8+x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2-1/6*3^(1/2)*arctan(1/3*(2*x^2-1)*3^(1/2))-1/6*arctan(1/3*(2*x^2+1)
*3^(1/2))*3^(1/2)
```

3.329.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.74

$$\int \frac{x^9}{1+x^4+x^8}\,dx=\frac{1}{2}\,x^2-\frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}x^2\right)-\frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(x^6+2\,x^2)\right)$$

input
```
integrate(x^9/(x^8+x^4+1),x, algorithm="fricas")
```

output
```
1/2*x^2 - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*x^2) - 1/6*sqrt(3)*arctan(1/3*sqr
t(3)*(x^6 + 2*x^2))
```

3.329.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

$$\int \frac{x^9}{1+x^4+x^8}\,dx=\frac{x^2}{2}+\frac{\sqrt{3}\left(-2\,\mathrm{atan}\left(\frac{\sqrt{3}x^2}{3}\right)-2\,\mathrm{atan}\left(\frac{\sqrt{3}x^6}{3}+\frac{2\sqrt{3}x^2}{3}\right)\right)}{12}$$

input
```
integrate(x**9/(x**8+x**4+1),x)
```

output
```
x**2/2 + sqrt(3)*(-2*atan(sqrt(3)*x**2/3) - 2*atan(sqrt(3)*x**6/3 + 2*sqrt
(3)*x**2/3))/12
```

3.329.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int \frac{x^9}{1+x^4+x^8}\,dx = \frac{1}{2}\,x^2 - \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^2+1)\right) - \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^2-1)\right)$$

input
```
integrate(x^9/(x^8+x^4+1),x, algorithm="maxima")
```

output
```
1/2*x^2 - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) - 1/6*sqrt(3)*arctan
(1/3*sqrt(3)*(2*x^2 - 1))
```

3.329.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int \frac{x^9}{1+x^4+x^8}\,dx = \frac{1}{2}\,x^2 - \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^2+1)\right) - \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^2-1)\right)$$

input
```
integrate(x^9/(x^8+x^4+1),x, algorithm="giac")
```

output
```
1/2*x^2 - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) - 1/6*sqrt(3)*arctan
(1/3*sqrt(3)*(2*x^2 - 1))
```

3.329.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80

$$\int \frac{x^9}{1+x^4+x^8}\,dx = \frac{x^2}{2} - \frac{\sqrt{3}\left(2\,\mathrm{atan}\left(\frac{\sqrt{3}\,x^6}{3}+\frac{2\,\sqrt{3}\,x^2}{3}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{3}\,x^2}{3}\right)\right)}{12}$$

input
```
int(x^9/(x^4 + x^8 + 1),x)
```

output
```
x^2/2 - (3^(1/2)*(2*atan((2*3^(1/2)*x^2)/3 + (3^(1/2)*x^6)/3) + 2*atan((3^
(1/2)*x^2)/3)))/12
```

3.330 $\int \frac{x^7}{1+x^4+x^8} dx$

3.330.1 Optimal result

Integrand size = 14, antiderivative size = 37

$$\int \frac{x^7}{1+x^4+x^8} dx = -\frac{\arctan\left(\frac{1+2x^4}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{1}{8}\log\left(1+x^4+x^8\right)$$

output
```
1/8*ln(x^8+x^4+1)-1/12*arctan(1/3*(2*x^4+1)*3^(1/2))*3^(1/2)
```

3.330.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{x^7}{1+x^4+x^8} dx = -\frac{\arctan\left(\frac{1+2x^4}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{1}{8}\log\left(1+x^4+x^8\right)$$

input
```
Integrate[x^7/(1 + x^4 + x^8),x]
```

output
```
-1/4*ArcTan[(1 + 2*x^4)/Sqrt[3]]/Sqrt[3] + Log[1 + x^4 + x^8]/8
```

3.330.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {1693, 1142, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{x^8+x^4+1}dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{4}\int \frac{x^4}{x^8+x^4+1}dx^4$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{4}\left(\frac{1}{2}\int \frac{2x^4+1}{x^8+x^4+1}dx^4-\frac{1}{2}\int \frac{1}{x^8+x^4+1}dx^4\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{4}\left(\int \frac{1}{-x^8-3}d\left(2x^4+1\right)+\frac{1}{2}\int \frac{2x^4+1}{x^8+x^4+1}dx^4\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{4}\left(\frac{1}{2}\int \frac{2x^4+1}{x^8+x^4+1}dx^4-\frac{\arctan\left(\frac{2x^4+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{4}\left(\frac{1}{2}\log\left(x^8+x^4+1\right)-\frac{\arctan\left(\frac{2x^4+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)$$

input
```
Int[x^7/(1 + x^4 + x^8),x]
```

output
```
(-(ArcTan[(1 + 2*x^4)/Sqrt[3]]/Sqrt[3]) + Log[1 + x^4 + x^8]/2)/4
```

3.330.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.330.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{\ln\left(x^{8}+x^{4}+1\right)}{8}-\frac{\arctan\left(\frac{\left(2x^{4}+1\right)\sqrt{3}}{3}\right)\sqrt{3}}{12}$	31
risch	$\frac{\ln\left(4x^{8}+4x^{4}+4\right)}{8}-\frac{\arctan\left(\frac{\left(2x^{4}+1\right)\sqrt{3}}{3}\right)\sqrt{3}}{12}$	35

input
```
int(x^7/(x^8+x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/8*ln(x^8+x^4+1)-1/12*arctan(1/3*(2*x^4+1)*3^(1/2))*3^(1/2)
```

3.330.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{x^7}{1+x^4+x^8}\,dx = -\frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^4+1)\right) + \frac{1}{8}\,\log\left(x^8+x^4+1\right)$$

input
```
integrate(x^7/(x^8+x^4+1),x, algorithm="fricas")
```

output
```
-1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) + 1/8*log(x^8 + x^4 + 1)
```

3.330.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{x^7}{1+x^4+x^8}\,dx = \frac{\log\left(x^8+x^4+1\right)}{8} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^4}{3}+\frac{\sqrt{3}}{3}\right)}{12}$$

input
```
integrate(x**7/(x**8+x**4+1),x)
```

output
```
log(x**8 + x**4 + 1)/8 - sqrt(3)*atan(2*sqrt(3)*x**4/3 + sqrt(3)/3)/12
```

3.330.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{x^7}{1+x^4+x^8}\,dx = -\frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^4+1)\right) + \frac{1}{8}\,\log\left(x^8+x^4+1\right)$$

input
```
integrate(x^7/(x^8+x^4+1),x, algorithm="maxima")
```

output
```
-1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) + 1/8*log(x^8 + x^4 + 1)
```

3.330.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{x^7}{1+x^4+x^8}\,dx = -\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x^4+1)\right) + \frac{1}{8}\log\left(x^8+x^4+1\right)$$

input
```
integrate(x^7/(x^8+x^4+1),x, algorithm="giac")
```

output
```
-1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) + 1/8*log(x^8 + x^4 + 1)
```

3.330.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

$$\int \frac{x^7}{1+x^4+x^8}\,dx = \frac{\ln\left(x^8+x^4+1\right)}{8} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x^4}{3}+\frac{\sqrt{3}}{3}\right)}{12}$$

input
```
int(x^7/(x^4 + x^8 + 1),x)
```

output
```
log(x^4 + x^8 + 1)/8 - (3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^4)/3))/12
```

3.331 $\int \frac{x^5}{1+x^4+x^8}\,dx$

3.331.1 Optimal result

Integrand size = 14, antiderivative size = 75

$$\int \frac{x^5}{1+x^4+x^8}\,dx=-\frac{\arctan\left(\frac{1-2x^2}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{\arctan\left(\frac{1+2x^2}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{8}\log\left(1-x^2+x^4\right)-\frac{1}{8}\log\left(1+x^2+x^4\right)$$

output
```
1/8*ln(x^4-x^2+1)-1/8*ln(x^4+x^2+1)-1/12*arctan(1/3*(-2*x^2+1)*3^(1/2))*3^
(1/2)+1/12*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)
```

3.331.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.25

$$\int \frac{x^5}{1+x^4+x^8}\,dx=\frac{\sqrt{1-i\sqrt{3}}\left(-i+\sqrt{3}\right)\arctan\left(\frac{1}{2}\left(-i+\sqrt{3}\right)x^2\right)+\sqrt{1+i\sqrt{3}}\left(i+\sqrt{3}\right)\arctan\left(\frac{1}{2}\left(i+\sqrt{3}\right)x^2\right)}{4\sqrt{6}}$$

input
```
Integrate[x^5/(1 + x^4 + x^8),x]
```

output
```
(Sqrt[1 - I*Sqrt[3]]*(-I + Sqrt[3])*ArcTan[((-I + Sqrt[3])*x^2)/2] + Sqrt[
1 + I*Sqrt[3]]*(I + Sqrt[3])*ArcTan[((I + Sqrt[3])*x^2)/2])/(4*Sqrt[6])
```

3.331.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {1695, 1447, 1475, 1083, 217, 1478, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{x^8+x^4+1}dx$$

$$\Big\downarrow \text{1695}$$

$$\frac{1}{2}\int \frac{x^4}{x^8+x^4+1}dx^2$$

$$\Big\downarrow \text{1447}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{x^4+1}{x^8+x^4+1}dx^2-\frac{1}{2}\int \frac{1-x^4}{x^8+x^4+1}dx^2\right)$$

$$\Big\downarrow \text{1475}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^4-x^2+1}dx^2+\frac{1}{2}\int \frac{1}{x^4+x^2+1}dx^2\right)-\frac{1}{2}\int \frac{1-x^4}{x^8+x^4+1}dx^2\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-\int \frac{1}{-x^4-3}d(2x^2-1)-\int \frac{1}{-x^4-3}d(2x^2+1)\right)-\frac{1}{2}\int \frac{1-x^4}{x^8+x^4+1}dx^2\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{2x^2-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)-\frac{1}{2}\int \frac{1-x^4}{x^8+x^4+1}dx^2\right)$$

$$\Big\downarrow \text{1478}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\int -\frac{1-2x^2}{x^4-x^2+1}dx^2+\frac{1}{2}\int -\frac{2x^2+1}{x^4+x^2+1}dx^2\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x^2-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1-2x^2}{x^4-x^2+1}dx^2-\frac{1}{2}\int \frac{2x^2+1}{x^4+x^2+1}dx^2\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x^2-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{2x^2-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)+\frac{1}{2}\left(\frac{1}{2}\log\left(x^4-x^2+1\right)-\frac{1}{2}\log\left(x^4+x^2+1\right)\right)\right)$$

input
```
Int[x^5/(1 + x^4 + x^8),x]
```

output
```
((ArcTan[(-1 + 2*x^2)/Sqrt[3]]/Sqrt[3] + ArcTan[(1 + 2*x^2)/Sqrt[3]]/Sqrt[
3])/2 + (Log[1 - x^2 + x^4]/2 - Log[1 + x^2 + x^4]/2)/2)/2
```

3.331.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1447
```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Simp[1/2   Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Simp[1/2
 Int[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b
^2 - 4*a*c, 0] && PosQ[a*c]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

rule 1695
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b
*x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c,
p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.331.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{\ln(x^4-x^2+1)}{8}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x^2-1)\sqrt{3}}{3}\right)}{12}-\frac{\ln(x^4+x^2+1)}{8}+\frac{\arctan\left(\frac{(2x^2+1)\sqrt{3}}{3}\right)\sqrt{3}}{12}$	62
risch	$-\frac{\ln(4x^4+4x^2+4)}{8}+\frac{\arctan\left(\frac{(2x^2+1)\sqrt{3}}{3}\right)\sqrt{3}}{12}+\frac{\ln(4x^4-4x^2+4)}{8}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x^2-1)\sqrt{3}}{3}\right)}{12}$	68

input
```
int(x^5/(x^8+x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/8*ln(x^4-x^2+1)+1/12*3^(1/2)*arctan(1/3*(2*x^2-1)*3^(1/2))-1/8*ln(x^4+x^
2+1)+1/12*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)
```

3.331.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.81

$$\int \frac{x^5}{1+x^4+x^8}\,dx = \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2+1)\right) + \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2-1)\right) - \frac{1}{8}\log\left(x^4+x^2+1\right) + \frac{1}{8}\log\left(x^4-x^2+1\right)$$

input
```
integrate(x^5/(x^8+x^4+1),x, algorithm="fricas")
```

output
```
1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*sqrt(3)*arctan(1/3*sqr
t(3)*(2*x^2 - 1)) - 1/8*log(x^4 + x^2 + 1) + 1/8*log(x^4 - x^2 + 1)
```

3.331.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.01

$$\int \frac{x^5}{1+x^4+x^8}\,dx = \frac{\log(x^4-x^2+1)}{8} - \frac{\log(x^4+x^2+1)}{8} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^2}{3} - \frac{\sqrt{3}}{3}\right)}{12} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^2}{3} + \frac{\sqrt{3}}{3}\right)}{12}$$

input
```
integrate(x**5/(x**8+x**4+1),x)
```

output
```
log(x**4 - x**2 + 1)/8 - log(x**4 + x**2 + 1)/8 + sqrt(3)*atan(2*sqrt(3)*x
**2/3 - sqrt(3)/3)/12 + sqrt(3)*atan(2*sqrt(3)*x**2/3 + sqrt(3)/3)/12
```

3.331.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.81

$$\int \frac{x^5}{1+x^4+x^8}\,dx = \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2+1)\right) + \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2-1)\right) - \frac{1}{8}\log\left(x^4+x^2+1\right) + \frac{1}{8}\log\left(x^4-x^2+1\right)$$

input
```
integrate(x^5/(x^8+x^4+1),x, algorithm="maxima")
```

output
```
1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*sqrt(3)*arctan(1/3*sqr
t(3)*(2*x^2 - 1)) - 1/8*log(x^4 + x^2 + 1) + 1/8*log(x^4 - x^2 + 1)
```

3.331.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.81

$$\int \frac{x^5}{1+x^4+x^8}\,dx = \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x^2+1)\right) + \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x^2-1)\right) - \frac{1}{8}\log\left(x^4+x^2+1\right) + \frac{1}{8}\log\left(x^4-x^2+1\right)$$

input
```
integrate(x^5/(x^8+x^4+1),x, algorithm="giac")
```

output
```
1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*sqrt(3)*arctan(1/3*sqr
t(3)*(2*x^2 - 1)) - 1/8*log(x^4 + x^2 + 1) + 1/8*log(x^4 - x^2 + 1)
```

3.331.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.68

$$\int \frac{x^5}{1+x^4+x^8}\,dx = \operatorname{atanh}\left(\frac{2\,x^2}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) + \operatorname{atanh}\left(\frac{2\,x^2}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)$$

input
```
int(x^5/(x^4 + x^8 + 1),x)
```

output
```
atanh((2*x^2)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/12 + 1/4) + atanh((2*x^2)/(3
^(1/2)*1i + 1))*((3^(1/2)*1i)/12 - 1/4)
```

3.332 $\int \frac{x^3}{1+x^4+x^8} dx$

3.332.1 Optimal result

Integrand size = 14, antiderivative size = 23

$$\int \frac{x^3}{1+x^4+x^8} dx = \frac{\arctan\left(\frac{1+2x^4}{\sqrt{3}}\right)}{2\sqrt{3}}$$

output
```
1/6*arctan(1/3*(2*x^4+1)*3^(1/2))*3^(1/2)
```

3.332.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{1+x^4+x^8} dx = \frac{\arctan\left(\frac{1+2x^4}{\sqrt{3}}\right)}{2\sqrt{3}}$$

input
```
Integrate[x^3/(1 + x^4 + x^8),x]
```

output
```
ArcTan[(1 + 2*x^4)/Sqrt[3]]/(2*Sqrt[3])
```

3.332.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1690, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{x^8+x^4+1}dx$$

$$\Big\downarrow \text{1690}$$

$$\frac{1}{4}\int \frac{1}{x^8+x^4+1}dx^4$$

$$\Big\downarrow \text{1083}$$

$$-\frac{1}{2}\int \frac{1}{-x^8-3}d(2x^4+1)$$

$$\Big\downarrow \text{217}$$

$$\frac{\arctan\left(\frac{2x^4+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

input
```
Int[x^3/(1 + x^4 + x^8),x]
```

output
```
ArcTan[(1 + 2*x^4)/Sqrt[3]]/(2*Sqrt[3])
```

3.332.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

3.332.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{\arctan\left(\frac{(2x^4+1)\sqrt{3}}{3}\right)\sqrt{3}}{6}$	19
risch	$\frac{\arctan\left(\frac{(2x^4+1)\sqrt{3}}{3}\right)\sqrt{3}}{6}$	19

input
```
int(x^3/(x^8+x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/6*arctan(1/3*(2*x^4+1)*3^(1/2))*3^(1/2)
```

3.332.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{1+x^4+x^8}\,dx = \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^4+1)\right)$$

input
```
integrate(x^3/(x^8+x^4+1),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1))
```

3.332.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \frac{x^3}{1+x^4+x^8}\,dx = \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^4}{3}+\frac{\sqrt{3}}{3}\right)}{6}$$

input
```
integrate(x**3/(x**8+x**4+1),x)
```

output
```
sqrt(3)*atan(2*sqrt(3)*x**4/3 + sqrt(3)/3)/6
```

3.332.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{1+x^4+x^8}\,dx = \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x^4+1)\right)$$

input
```
integrate(x^3/(x^8+x^4+1),x, algorithm="maxima")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1))
```

3.332.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{1+x^4+x^8}\,dx = \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x^4+1)\right)$$

input
```
integrate(x^3/(x^8+x^4+1),x, algorithm="giac")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1))
```

3.332.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{x^3}{1+x^4+x^8}\,dx = \frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\,\left(\frac{2\,x^4}{3}+\frac{1}{3}\right)\right)}{6}$$

input
```
int(x^3/(x^4 + x^8 + 1),x)
```

output
```
(3^(1/2)*atan(3^(1/2)*((2*x^4)/3 + 1/3)))/6
```

3.333 $\int \frac{x}{1+x^4+x^8} \, dx$

3.333.1 Optimal result

Integrand size = 12, antiderivative size = 75

$$\int \frac{x}{1+x^4+x^8} \, dx = -\frac{\arctan\left(\frac{1-2x^2}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{\arctan\left(\frac{1+2x^2}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{8}\log\left(1-x^2+x^4\right) + \frac{1}{8}\log\left(1+x^2+x^4\right)$$

output
```
-1/8*ln(x^4-x^2+1)+1/8*ln(x^4+x^2+1)-1/12*arctan(1/3*(-2*x^2+1)*3^(1/2))*3
^(1/2)+1/12*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)
```

3.333.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.05

$$\int \frac{x}{1+x^4+x^8} \, dx = \frac{i\left(\sqrt{1-i\sqrt{3}}\arctan\left(\frac{1}{2}\left(-i+\sqrt{3}\right)x^2\right) - \sqrt{1+i\sqrt{3}}\arctan\left(\frac{1}{2}\left(i+\sqrt{3}\right)x^2\right)\right)}{2\sqrt{6}}$$

input
```
Integrate[x/(1 + x^4 + x^8),x]
```

output
```
((I/2)*(Sqrt[1 - I*Sqrt[3]]*ArcTan[((-I + Sqrt[3])*x^2)/2] - Sqrt[1 + I*Sq
rt[3]]*ArcTan[((I + Sqrt[3])*x^2)/2]))/Sqrt[6]
```

3.333.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {1695, 1407, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{x^8+x^4+1} \, dx$$

$$\Big\downarrow \ 1695$$

$$\frac{1}{2}\int \frac{1}{x^8+x^4+1}dx^2$$

$$\Big\downarrow \ 1407$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1-x^2}{x^4-x^2+1}dx^2+\frac{1}{2}\int \frac{x^2+1}{x^4+x^2+1}dx^2\right)$$

$$\Big\downarrow \ 1142$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^4-x^2+1}dx^2-\frac{1}{2}\int -\frac{1-2x^2}{x^4-x^2+1}dx^2\right)+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^4+x^2+1}dx^2+\frac{1}{2}\int \frac{2x^2+1}{x^4+x^2+1}dx^2\right)\right)$$

$$\Big\downarrow \ 25$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^4-x^2+1}dx^2+\frac{1}{2}\int \frac{1-2x^2}{x^4-x^2+1}dx^2\right)+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^4+x^2+1}dx^2+\frac{1}{2}\int \frac{2x^2+1}{x^4+x^2+1}dx^2\right)\right)$$

$$\Big\downarrow \ 1083$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1-2x^2}{x^4-x^2+1}dx^2-\int \frac{1}{-x^4-3}d(2x^2-1)\right)+\frac{1}{2}\left(\frac{1}{2}\int \frac{2x^2+1}{x^4+x^2+1}dx^2-\int \frac{1}{-x^4-3}d(2x^2+1)\right)\right)$$

$$\Big\downarrow \ 217$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1-2x^2}{x^4-x^2+1}dx^2+\frac{\arctan\left(\frac{2x^2-1}{\sqrt{3}}\right)}{\sqrt{3}}\right)+\frac{1}{2}\left(\frac{1}{2}\int \frac{2x^2+1}{x^4+x^2+1}dx^2+\frac{\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)\right)$$

$$\Big\downarrow \ 1103$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{2x^2-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}\log\left(x^4-x^2+1\right)\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(x^4+x^2+1\right)\right)\right)$$

input
```
Int[x/(1 + x^4 + x^8),x]
```

output
```
((ArcTan[(-1 + 2*x^2)/Sqrt[3]]/Sqrt[3] - Log[1 - x^2 + x^4]/2)/2 + (ArcTan
[(1 + 2*x^2)/Sqrt[3]]/Sqrt[3] + Log[1 + x^2 + x^4]/2)/2)/2
```

3.333.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1407
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   Int[(r - x)/(q - r*
x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(r + x)/(q + r*x + x^2), x], x]]]
 /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

rule 1695
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b
*x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c,
p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.333.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{\ln(x^4-x^2+1)}{8}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x^2-1)\sqrt{3}}{3}\right)}{12}+\frac{\ln(x^4+x^2+1)}{8}+\frac{\arctan\left(\frac{(2x^2+1)\sqrt{3}}{3}\right)\sqrt{3}}{12}$	62
risch	$-\frac{\ln(4x^4-4x^2+4)}{8}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x^2-1)\sqrt{3}}{3}\right)}{12}+\frac{\ln(4x^4+4x^2+4)}{8}+\frac{\arctan\left(\frac{(2x^2+1)\sqrt{3}}{3}\right)\sqrt{3}}{12}$	68

input
```
int(x/(x^8+x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/8*ln(x^4-x^2+1)+1/12*3^(1/2)*arctan(1/3*(2*x^2-1)*3^(1/2))+1/8*ln(x^4+x
^2+1)+1/12*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)
```

3.333.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.81

$$\int \frac{x}{1+x^4+x^8}\,dx = \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2+1)\right)+\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2-1)\right) + \frac{1}{8}\log\left(x^4+x^2+1\right)-\frac{1}{8}\log\left(x^4-x^2+1\right)$$

input
```
integrate(x/(x^8+x^4+1),x, algorithm="fricas")
```

output
```
1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*sqrt(3)*arctan(1/3*sqr
t(3)*(2*x^2 - 1)) + 1/8*log(x^4 + x^2 + 1) - 1/8*log(x^4 - x^2 + 1)
```

3.333.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.01

$$\int \frac{x}{1+x^4+x^8}\,dx = -\frac{\log\left(x^4-x^2+1\right)}{8}+\frac{\log\left(x^4+x^2+1\right)}{8} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^2}{3}-\frac{\sqrt{3}}{3}\right)}{12}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^2}{3}+\frac{\sqrt{3}}{3}\right)}{12}$$

input
```
integrate(x/(x**8+x**4+1),x)
```

output
```
-log(x**4 - x**2 + 1)/8 + log(x**4 + x**2 + 1)/8 + sqrt(3)*atan(2*sqrt(3)*
x**2/3 - sqrt(3)/3)/12 + sqrt(3)*atan(2*sqrt(3)*x**2/3 + sqrt(3)/3)/12
```

3.333.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.81

$$\int \frac{x}{1+x^4+x^8}\,dx = \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2+1)\right)+\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2-1)\right) + \frac{1}{8}\log\left(x^4+x^2+1\right)-\frac{1}{8}\log\left(x^4-x^2+1\right)$$

input
```
integrate(x/(x^8+x^4+1),x, algorithm="maxima")
```

output
```
1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*sqrt(3)*arctan(1/3*sqr
t(3)*(2*x^2 - 1)) + 1/8*log(x^4 + x^2 + 1) - 1/8*log(x^4 - x^2 + 1)
```

3.333.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.81

$$\int \frac{x}{1+x^4+x^8}\,dx = \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2+1)\right)+\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2-1)\right) + \frac{1}{8}\log\left(x^4+x^2+1\right)-\frac{1}{8}\log\left(x^4-x^2+1\right)$$

input
```
integrate(x/(x^8+x^4+1),x, algorithm="giac")
```

output
```
1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*sqrt(3)*arctan(1/3*sqr
t(3)*(2*x^2 - 1)) + 1/8*log(x^4 + x^2 + 1) - 1/8*log(x^4 - x^2 + 1)
```

3.333.9 Mupad [B] (verification not implemented)

Time = 8.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.68

$$\int \frac{x}{1+x^4+x^8}\,dx = \operatorname{atan}\left(\frac{\sqrt{3}\,x^2}{2}-\frac{x^2\,1\mathrm{i}}{2}\right)\,\left(\frac{\sqrt{3}}{12}+\frac{1}{4}\mathrm{i}\right)+\operatorname{atan}\left(\frac{\sqrt{3}\,x^2}{2}+\frac{x^2\,1\mathrm{i}}{2}\right)\,\left(\frac{\sqrt{3}}{12}-\frac{1}{4}\mathrm{i}\right)$$

input
```
int(x/(x^4 + x^8 + 1),x)
```

output
```
atan((3^(1/2)*x^2)/2 - (x^2*1i)/2)*(3^(1/2)/12 + 1i/4) + atan((3^(1/2)*x^2
)/2 + (x^2*1i)/2)*(3^(1/2)/12 - 1i/4)
```

3.334 $\int \frac{1}{x(1+x^4+x^8)}\,dx$

3.334.1 Optimal result

Integrand size = 14, antiderivative size = 39

$$\int \frac{1}{x(1+x^4+x^8)}\,dx=-\frac{\arctan\left(\frac{1+2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\log(x)-\frac{1}{8}\log\left(1+x^4+x^8\right)$$

output
```
ln(x)-1/8*ln(x^8+x^4+1)-1/12*arctan(1/3*(2*x^4+1)*3^(1/2))*3^(1/2)
```

3.334.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 133, normalized size of antiderivative = 3.41

$$\int \frac{1}{x(1+x^4+x^8)}\,dx=\frac{1}{24}\left(2\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)-2\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right)+24\log(x)-\sqrt{3}\left(i+\sqrt{3}\right)\log\left(i+\sqrt{3}-2ix^2\right)-\sqrt{3}\left(-i+\sqrt{3}\right)\log\left(-i+\sqrt{3}+2ix^2\right)-3\log\left(1-x+x^2\right)-3\log\left(1+x+x^2\right)\right)$$

input
```
Integrate[1/(x*(1 + x^4 + x^8)),x]
```

output
```
(2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]
] + 24*Log[x] - Sqrt[3]*(I + Sqrt[3])*Log[I + Sqrt[3] - (2*I)*x^2] - Sqrt[
3]*(-I + Sqrt[3])*Log[-I + Sqrt[3] + (2*I)*x^2] - 3*Log[1 - x + x^2] - 3*L
og[1 + x + x^2])/24
```

3.334.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1693, 1144, 25, 1142, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(x^8+x^4+1\right)}dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{4}\int \frac{1}{x^4\left(x^8+x^4+1\right)}dx^4$$

$$\Big\downarrow \text{1144}$$

$$\frac{1}{4}\left(\int -\frac{x^4+1}{x^8+x^4+1}dx^4+\log\left(x^4\right)\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4}\left(\log\left(x^4\right)-\int \frac{x^4+1}{x^8+x^4+1}dx^4\right)$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{4}\left(-\frac{1}{2}\int \frac{1}{x^8+x^4+1}dx^4-\frac{1}{2}\int \frac{2x^4+1}{x^8+x^4+1}dx^4+\log\left(x^4\right)\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{4}\left(\int \frac{1}{-x^8-3}d\left(2x^4+1\right)-\frac{1}{2}\int \frac{2x^4+1}{x^8+x^4+1}dx^4+\log\left(x^4\right)\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{4}\left(-\frac{1}{2}\int \frac{2x^4+1}{x^8+x^4+1}dx^4-\frac{\arctan\left(\frac{2x^4+1}{\sqrt{3}}\right)}{\sqrt{3}}+\log\left(x^4\right)\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{4}\left(-\frac{\arctan\left(\frac{2x^4+1}{\sqrt{3}}\right)}{\sqrt{3}}+\log\left(x^4\right)-\frac{1}{2}\log\left(x^8+x^4+1\right)\right)$$

input
```
Int[1/(x*(1 + x^4 + x^8)),x]
```

output
```
(-(ArcTan[(1 + 2*x^4)/Sqrt[3]]/Sqrt[3]) + Log[x^4] - Log[1 + x^4 + x^8]/2)
/4
```

3.334.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1144
```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Simp[e*(Log[RemoveContent[d + e*x, x]]/(c*d^2 - b*d*e + a*e^2)), x] + S
imp[1/(c*d^2 - b*d*e + a*e^2)   Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2),
x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.334.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

method	result
risch	$\ln(x)-\frac{\ln(x^8+x^4+1)}{8}-\frac{\sqrt{3}\arctan\left(\frac{2\left(x^4+\frac{1}{2}\right)\sqrt{3}}{3}\right)}{12}$
default	$\ln(x)-\frac{\ln(x^2-x+1)}{8}+\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12}-\frac{\ln(x^2+x+1)}{8}-\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{12}-\frac{\ln(x^4-x^2+1)}{8}-$ $\sqrt{3}$ arctan(

input
```
int(1/x/(x^8+x^4+1),x,method=_RETURNVERBOSE)
```

output
```
ln(x)-1/8*ln(x^8+x^4+1)-1/12*3^(1/2)*arctan(2/3*(x^4+1/2)*3^(1/2))
```

3.334.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{1}{x(1+x^4+x^8)}\,dx = -\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^4+1)\right)-\frac{1}{8}\log\left(x^8+x^4+1\right)+\log(x)$$

input
```
integrate(1/x/(x^8+x^4+1),x, algorithm="fricas")
```

output
```
-1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) - 1/8*log(x^8 + x^4 + 1) + l
og(x)
```

3.334.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{1}{x(1+x^4+x^8)}\,dx = \log(x) - \frac{\log(x^8+x^4+1)}{8} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^4}{3}+\frac{\sqrt{3}}{3}\right)}{12}$$

input
```
integrate(1/x/(x**8+x**4+1),x)
```

output
```
log(x) - log(x**8 + x**4 + 1)/8 - sqrt(3)*atan(2*sqrt(3)*x**4/3 + sqrt(3)/
3)/12
```

3.334.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \frac{1}{x(1+x^4+x^8)}\,dx = -\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^4+1)\right) - \frac{1}{8}\log(x^8+x^4+1) + \frac{1}{4}\log(x^4)$$

input
```
integrate(1/x/(x^8+x^4+1),x, algorithm="maxima")
```

output
```
-1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) - 1/8*log(x^8 + x^4 + 1) + 1
/4*log(x^4)
```

3.334.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \frac{1}{x(1+x^4+x^8)}\,dx = -\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^4+1)\right) - \frac{1}{8}\log(x^8+x^4+1) + \frac{1}{4}\log(x^4)$$

input
```
integrate(1/x/(x^8+x^4+1),x, algorithm="giac")
```

output
```
-1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) - 1/8*log(x^8 + x^4 + 1) + 1
/4*log(x^4)
```

3.334.9 Mupad [B] (verification not implemented)

Time = 8.20 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

$$\int \frac{1}{x\left(1+x^4+x^8\right)}\,dx = \ln(x) - \frac{\ln\left(x^8+x^4+1\right)}{8} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x^4}{3}+\frac{\sqrt{3}}{3}\right)}{12}$$

input
```
int(1/(x*(x^4 + x^8 + 1)),x)
```

output
```
log(x) - log(x^4 + x^8 + 1)/8 - (3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^4)/
3))/12
```

3.335 $\int \frac{1}{x^3(1+x^4+x^8)}\,dx$

3.335.1 Optimal result

Integrand size = 14, antiderivative size = 54

$$\int \frac{1}{x^3(1+x^4+x^8)}\,dx=-\frac{1}{2x^2}+\frac{\arctan\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\arctan\left(\frac{1+2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}$$

output
```
-1/2/x^2+1/6*arctan(1/3*(-2*x^2+1)*3^(1/2))*3^(1/2)-1/6*arctan(1/3*(2*x^2+
1)*3^(1/2))*3^(1/2)
```

3.335.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.03 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.85

$$\int \frac{1}{x^3(1+x^4+x^8)}\,dx=\frac{1}{12}\left(-\frac{6}{x^2}-2\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)+2\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right)\right.$$
$$\left.-i\sqrt{3}\log\left(i+\sqrt{3}-2ix^2\right)+i\sqrt{3}\log\left(-i+\sqrt{3}+2ix^2\right)\right)$$

input
```
Integrate[1/(x^3*(1 + x^4 + x^8)),x]
```

output
```
(-6/x^2 - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 2*Sqrt[3]*ArcTan[(1 + 2*x
)/Sqrt[3]] - I*Sqrt[3]*Log[I + Sqrt[3] - (2*I)*x^2] + I*Sqrt[3]*Log[-I + S
qrt[3] + (2*I)*x^2])/12
```

3.335.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1695, 1443, 25, 1475, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (x^8+x^4+1)} \, dx$$
$$\Big\downarrow \text{1695}$$
$$\frac{1}{2} \int \frac{1}{x^4 (x^8+x^4+1)}dx^2$$
$$\Big\downarrow \text{1443}$$
$$\frac{1}{2} \left(\int -\frac{x^4+1}{x^8+x^4+1}dx^2-\frac{1}{x^2}\right)$$
$$\Big\downarrow \text{25}$$
$$\frac{1}{2} \left(-\int \frac{x^4+1}{x^8+x^4+1}dx^2-\frac{1}{x^2}\right)$$
$$\Big\downarrow \text{1475}$$
$$\frac{1}{2} \left(-\frac{1}{2} \int \frac{1}{x^4-x^2+1}dx^2-\frac{1}{2} \int \frac{1}{x^4+x^2+1}dx^2-\frac{1}{x^2}\right)$$
$$\Big\downarrow \text{1083}$$
$$\frac{1}{2} \left(\int \frac{1}{-x^4-3}d\left(2 x^2-1\right)+\int \frac{1}{-x^4-3}d\left(2 x^2+1\right)-\frac{1}{x^2}\right)$$
$$\Big\downarrow \text{217}$$
$$\frac{1}{2} \left(-\frac{\arctan \left(\frac{2 x^2-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\arctan \left(\frac{2 x^2+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{x^2}\right)$$

input
```
Int[1/(x^3*(1 + x^4 + x^8)),x]
```

output
```
(-x^(-2) - ArcTan[(-1 + 2*x^2)/Sqrt[3]]/Sqrt[3] - ArcTan[(1 + 2*x^2)/Sqrt[
3]]/Sqrt[3])/2
```

3.335.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1443
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Sim
p[1/(a*d^2*(m + 1))   Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*
x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 1695
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b
*x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c,
p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.335. $\int \frac{1}{x^3(1+x^4+x^8)}\,dx$

3.335.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

method	result	size
risch	$-\frac{1}{2x^2}-\frac{\sqrt{3}\,\arctan\left(\frac{x^6\sqrt{3}}{3}+\frac{2x^2\sqrt{3}}{3}\right)}{6}-\frac{\sqrt{3}\,\arctan\left(\frac{x^2\sqrt{3}}{3}\right)}{6}$	44
default	$-\frac{1}{2x^2}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}+\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{6}-\frac{\sqrt{3}\,\arctan\left(\frac{\left(2x^2-1\right)\sqrt{3}}{3}\right)}{6}$	57

input
```
int(1/x^3/(x^8+x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2/x^2-1/6*3^(1/2)*arctan(1/3*x^6*3^(1/2)+2/3*x^2*3^(1/2))-1/6*3^(1/2)*a
rctan(1/3*x^2*3^(1/2))
```

3.335.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^3\left(1+x^4+x^8\right)}\,dx=-\frac{\sqrt{3}x^2\arctan\left(\frac{1}{3}\sqrt{3}x^2\right)+\sqrt{3}x^2\arctan\left(\frac{1}{3}\sqrt{3}(x^6+2\,x^2)\right)+3}{6\,x^2}$$

input
```
integrate(1/x^3/(x^8+x^4+1),x, algorithm="fricas")
```

output
```
-1/6*(sqrt(3)*x^2*arctan(1/3*sqrt(3)*x^2) + sqrt(3)*x^2*arctan(1/3*sqrt(3)
*(x^6 + 2*x^2)) + 3)/x^2
```

3.335.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

$$\int \frac{1}{x^3\left(1+x^4+x^8\right)}\,dx=\frac{\sqrt{3}\left(-2\,\mathrm{atan}\left(\frac{\sqrt{3}x^2}{3}\right)-2\,\mathrm{atan}\left(\frac{\sqrt{3}x^6}{3}+\frac{2\sqrt{3}x^2}{3}\right)\right)}{12}-\frac{1}{2x^2}$$

input
```
integrate(1/x**3/(x**8+x**4+1),x)
```

output
```
sqrt(3)*(-2*atan(sqrt(3)*x**2/3) - 2*atan(sqrt(3)*x**6/3 + 2*sqrt(3)*x**2/
3))/12 - 1/(2*x**2)
```

3.335.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^3(1+x^4+x^8)}\,dx = -\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2+1)\right) - \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2-1)\right) - \frac{1}{2x^2}$$

input
```
integrate(1/x^3/(x^8+x^4+1),x, algorithm="maxima")
```

output
```
-1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) - 1/6*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x^2 - 1)) - 1/2/x^2
```

3.335.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^3(1+x^4+x^8)}\,dx = -\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2+1)\right) - \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2-1)\right) - \frac{1}{2x^2}$$

input
```
integrate(1/x^3/(x^8+x^4+1),x, algorithm="giac")
```

output
```
-1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) - 1/6*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x^2 - 1)) - 1/2/x^2
```

3.335.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80

$$\int \frac{1}{x^3(1+x^4+x^8)}\,dx = -\frac{\sqrt{3}\left(2\,\mathrm{atan}\left(\frac{\sqrt{3}\,x^6}{3}+\frac{2\sqrt{3}\,x^2}{3}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{3}\,x^2}{3}\right)\right)}{12} - \frac{1}{2x^2}$$

input
```
int(1/(x^3*(x^4 + x^8 + 1)),x)
```

output
```
- (3^(1/2)*(2*atan((2*3^(1/2)*x^2)/3 + (3^(1/2)*x^6)/3) + 2*atan((3^(1/2)*
x^2)/3)))/12 - 1/(2*x^2)
```

3.336 $\int \frac{1}{x^5(1+x^4+x^8)}\,dx$

3.336.1 Optimal result

Integrand size = 14, antiderivative size = 48

$$\int \frac{1}{x^5(1+x^4+x^8)}\,dx = -\frac{1}{4x^4}-\frac{\arctan\left(\frac{1+2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}-\log(x)+\frac{1}{8}\log\left(1+x^4+x^8\right)$$

output
```
-1/4/x^4-ln(x)+1/8*ln(x^8+x^4+1)-1/12*arctan(1/3*(2*x^4+1)*3^(1/2))*3^(1/2
)
```

3.336.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.83

$$\begin{aligned}\int \frac{1}{x^5(1+x^4+x^8)}\,dx &= \frac{1}{24}\left(-\frac{6}{x^4}+2\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)-2\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right)\right.\\ &\quad -24\log(x)+\sqrt{3}\left(-i+\sqrt{3}\right)\log\left(i+\sqrt{3}-2ix^2\right)\\ &\quad +\sqrt{3}\left(i+\sqrt{3}\right)\log\left(-i+\sqrt{3}+2ix^2\right)+3\log\left(1-x+x^2\right)\\ &\quad \left.+3\log\left(1+x+x^2\right)\right)\end{aligned}$$

input
```
Integrate[1/(x^5*(1 + x^4 + x^8)),x]
```

output
```
(-6/x^4 + 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x
)/Sqrt[3]] - 24*Log[x] + Sqrt[3]*(-I + Sqrt[3])*Log[I + Sqrt[3] - (2*I)*x^
2] + Sqrt[3]*(I + Sqrt[3])*Log[-I + Sqrt[3] + (2*I)*x^2] + 3*Log[1 - x + x
^2] + 3*Log[1 + x + x^2])/24
```

3.336.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {1693, 1145, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 (x^8+x^4+1)} \, dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{4} \int \frac{1}{x^8 (x^8+x^4+1)} dx^4$$

$$\Big\downarrow \text{1145}$$

$$\frac{1}{4} \left(\int -\frac{x^4+1}{x^4 (x^8+x^4+1)} dx^4 - \frac{1}{x^4}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4} \left(-\int \frac{x^4+1}{x^4 (x^8+x^4+1)} dx^4 - \frac{1}{x^4}\right)$$

$$\Big\downarrow \text{1200}$$

$$\frac{1}{4} \left(-\int \left(\frac{1}{x^4} - \frac{x^4}{x^8+x^4+1}\right) dx^4 - \frac{1}{x^4}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4} \left(-\frac{\arctan\left(\frac{2x^4+1}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{x^4} - \log\left(x^4\right) + \frac{1}{2}\log\left(x^8+x^4+1\right)\right)$$

input
```
Int[1/(x^5*(1 + x^4 + x^8)),x]
```

output
```
(-x^(-4) - ArcTan[(1 + 2*x^4)/Sqrt[3]]/Sqrt[3] - Log[x^4] + Log[1 + x^4 +
x^8]/2)/4
```

3.336.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1145
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp
[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[m, -1]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.336.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.79

method	result
risch	$-\frac{1}{4x^4}-\ln\left(x\right)+\frac{\ln\left(x^8+x^4+1\right)}{8}-\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x^4+\frac{1}{2}\right)\sqrt{3}}{3}\right)}{12}$
default	$-\frac{1}{4x^4}-\ln\left(x\right)+\frac{\ln\left(x^2-x+1\right)}{8}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12}+\frac{\ln\left(x^2+x+1\right)}{8}-\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{12}+\frac{\ln\left(x^4-x^2+1\right)}{8}-$

input
```
int(1/x^5/(x^8+x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/4/x^4-ln(x)+1/8*ln(x^8+x^4+1)-1/12*3^(1/2)*arctan(2/3*(x^4+1/2)*3^(1/2)
)
```

3.336.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.02

$$\int \frac{1}{x^5 (1 + x^4 + x^8)} \, dx = -\frac{2\sqrt{3}x^4 \arctan\left(\frac{1}{3}\sqrt{3}(2x^4 + 1)\right) - 3x^4 \log\left(x^8 + x^4 + 1\right) + 24x^4 \log(x) + 6}{24x^4}$$

input
```
integrate(1/x^5/(x^8+x^4+1),x, algorithm="fricas")
```

output
```
-1/24*(2*sqrt(3)*x^4*arctan(1/3*sqrt(3)*(2*x^4 + 1)) - 3*x^4*log(x^8 + x^4
 + 1) + 24*x^4*log(x) + 6)/x^4
```

3.336.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^5 (1 + x^4 + x^8)} \, dx = -\log(x) + \frac{\log(x^8 + x^4 + 1)}{8} - \frac{\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}x^4}{3} + \frac{\sqrt{3}}{3}\right)}{12} - \frac{1}{4x^4}$$

input
```
integrate(1/x**5/(x**8+x**4+1),x)
```

output
```
-log(x) + log(x**8 + x**4 + 1)/8 - sqrt(3)*atan(2*sqrt(3)*x**4/3 + sqrt(3)
/3)/12 - 1/(4*x**4)
```

3.336.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^5 (1+x^4+x^8)} dx = -\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^4+1)\right) - \frac{1}{4x^4} + \frac{1}{8}\log\left(x^8+x^4+1\right) - \frac{1}{4}\log\left(x^4\right)$$

input
```
integrate(1/x^5/(x^8+x^4+1),x, algorithm="maxima")
```

output
```
-1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) - 1/4/x^4 + 1/8*log(x^8 + x^
4 + 1) - 1/4*log(x^4)
```

3.336.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^5 (1+x^4+x^8)} dx = -\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^4+1)\right) + \frac{x^4-1}{4x^4} + \frac{1}{8}\log\left(x^8+x^4+1\right) - \frac{1}{4}\log\left(x^4\right)$$

input
```
integrate(1/x^5/(x^8+x^4+1),x, algorithm="giac")
```

output
```
-1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) + 1/4*(x^4 - 1)/x^4 + 1/8*lo
g(x^8 + x^4 + 1) - 1/4*log(x^4)
```

3.336.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^5 (1+x^4+x^8)} dx = \frac{\ln\left(x^8+x^4+1\right)}{8} - \ln(x) - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^4}{3}+\frac{\sqrt{3}}{3}\right)}{12} - \frac{1}{4x^4}$$

input
```
int(1/(x^5*(x^4 + x^8 + 1)),x)
```

output
```
log(x^4 + x^8 + 1)/8 - log(x) - (3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^4)/
3))/12 - 1/(4*x^4)
```

3.337 $\int \frac{1}{x^7(1+x^4+x^8)}\,dx$

3.337.1 Optimal result

Integrand size = 14, antiderivative size = 89

$$\int \frac{1}{x^7(1+x^4+x^8)}\,dx = -\frac{1}{6x^6}+\frac{1}{2x^2}-\frac{\arctan\left(\frac{1-2x^2}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{\arctan\left(\frac{1+2x^2}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{8}\log\left(1-x^2+x^4\right)-\frac{1}{8}\log\left(1+x^2+x^4\right)$$

output
```
-1/6/x^6+1/2/x^2+1/8*ln(x^4-x^2+1)-1/8*ln(x^4+x^2+1)-1/12*arctan(1/3*(-2*x
^2+1)*3^(1/2))*3^(1/2)+1/12*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)
```

3.337.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^7(1+x^4+x^8)}\,dx = \frac{1}{24}\left(-\frac{4}{x^6}+\frac{12}{x^2}-2\sqrt{3}\arctan\left(\frac{1-2x}{\sqrt{3}}\right)-2\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right)-2\sqrt{3}\arctan\left(\frac{1-2x^2}{\sqrt{3}}\right)-3\log\left(1-x+x^2\right)-3\log\left(1+x+x^2\right)+3\log\left(1-x^2+x^4\right)\right)$$

input
```
Integrate[1/(x^7*(1 + x^4 + x^8)),x]
```

output
```
(-4/x^6 + 12/x^2 - 2*Sqrt[3]*ArcTan[(1 - 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[
(1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 - 2*x^2)/Sqrt[3]] - 3*Log[1 - x +
 x^2] - 3*Log[1 + x + x^2] + 3*Log[1 - x^2 + x^4])/24
```

3.337.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {1695, 1443, 27, 1604, 1447, 1475, 1083, 217, 1478, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7 \left(x^8+x^4+1\right)} \, dx$$

$$\Big\downarrow \text{1695}$$

$$\frac{1}{2} \int \frac{1}{x^8 \left(x^8+x^4+1\right)}dx^2$$

$$\Big\downarrow \text{1443}$$

$$\frac{1}{2} \left(\frac{1}{3} \int -\frac{3 \left(x^4+1\right)}{x^4 \left(x^8+x^4+1\right)}dx^2-\frac{1}{3 x^6}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2} \left(-\int \frac{x^4+1}{x^4 \left(x^8+x^4+1\right)}dx^2-\frac{1}{3 x^6}\right)$$

$$\Big\downarrow \text{1604}$$

$$\frac{1}{2} \left(\int \frac{x^4}{x^8+x^4+1}dx^2-\frac{1}{3 x^6}+\frac{1}{x^2}\right)$$

$$\Big\downarrow \text{1447}$$

$$\frac{1}{2} \left(-\frac{1}{2} \int \frac{1-x^4}{x^8+x^4+1}dx^2+\frac{1}{2} \int \frac{x^4+1}{x^8+x^4+1}dx^2-\frac{1}{3 x^6}+\frac{1}{x^2}\right)$$

$$\Big\downarrow \text{1475}$$

$$\frac{1}{2} \left(\frac{1}{2} \left(\frac{1}{2} \int \frac{1}{x^4-x^2+1}dx^2+\frac{1}{2} \int \frac{1}{x^4+x^2+1}dx^2\right)-\frac{1}{2} \int \frac{1-x^4}{x^8+x^4+1}dx^2-\frac{1}{3 x^6}+\frac{1}{x^2}\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-\int \frac{1}{-x^4-3}d(2x^2-1)-\int \frac{1}{-x^4-3}d(2x^2+1)\right)-\frac{1}{2}\int \frac{1-x^4}{x^8+x^4+1}dx^2-\frac{1}{3x^6}+\frac{1}{x^2}\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(-\frac{1}{2}\int \frac{1-x^4}{x^8+x^4+1}dx^2+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x^2-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)-\frac{1}{3x^6}+\frac{1}{x^2}\right)$$

$$\Big\downarrow \text{1478}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\int -\frac{1-2x^2}{x^4-x^2+1}dx^2+\frac{1}{2}\int -\frac{2x^2+1}{x^4+x^2+1}dx^2\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x^2-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)-\frac{1}{3x^6}+\frac{1}{x^2}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1-2x^2}{x^4-x^2+1}dx^2-\frac{1}{2}\int \frac{2x^2+1}{x^4+x^2+1}dx^2\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x^2-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)-\frac{1}{3x^6}+\frac{1}{x^2}\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{2x^2-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)-\frac{1}{3x^6}+\frac{1}{x^2}+\frac{1}{2}\left(\frac{1}{2}\log\left(x^4-x^2+1\right)-\frac{1}{2}\log\left(x^4+x^2+1\right)\right)\right)$$

input
```
Int[1/(x^7*(1 + x^4 + x^8)),x]
```

output
```
(-1/3*1/x^6 + x^(-2) + (ArcTan[(-1 + 2*x^2)/Sqrt[3]]/Sqrt[3] + ArcTan[(1 +
 2*x^2)/Sqrt[3]]/Sqrt[3])/2 + (Log[1 - x^2 + x^4]/2 - Log[1 + x^2 + x^4]/2
)/2)/2
```

3.337.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1443
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Sim
p[1/(a*d^2*(m + 1))   Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*
x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1447
```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Simp[1/2   Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Simp[1/2
 Int[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b
^2 - 4*a*c, 0] && PosQ[a*c]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

3.337. $\int \frac{1}{x^7(1+x^4+x^8)}\,dx$

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1695
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b
*x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c,
p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.337.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.78

method	result
risch	$\frac{\frac{x^4}{2}-\frac{1}{6}}{x^6}-\frac{\ln(x^4+x^2+1)}{8}+\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x^2+\frac{1}{2}\right)\sqrt{3}}{3}\right)}{12}+\frac{\ln(x^4-x^2+1)}{8}+\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x^2-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{12}$
default	$-\frac{1}{6x^6}+\frac{1}{2x^2}-\frac{\ln(x^2-x+1)}{8}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12}-\frac{\ln(x^2+x+1)}{8}-\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{12}+\frac{\ln(x^4-x^2+1)}{8}+\frac{\sqrt{3}\,\mathrm{a}}{}$

input
```
int(1/x^7/(x^8+x^4+1),x,method=_RETURNVERBOSE)
```

output
```
(1/2*x^4-1/6)/x^6-1/8*ln(x^4+x^2+1)+1/12*3^(1/2)*arctan(2/3*(x^2+1/2)*3^(1
/2))+1/8*ln(x^4-x^2+1)+1/12*3^(1/2)*arctan(2/3*(x^2-1/2)*3^(1/2))
```

3.337.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^7\left(1+x^4+x^8\right)}\,dx = \frac{2\sqrt{3}x^6\arctan\left(\frac{1}{3}\sqrt{3}(2\,x^2+1)\right)+2\sqrt{3}x^6\arctan\left(\frac{1}{3}\sqrt{3}(2\,x^2-1)\right)-3\,x^6\log\left(x^4+x^2+1\right)+3\,x^6\log\left(x^4-\right.}{24\,x^6}$$

input
```
integrate(1/x^7/(x^8+x^4+1),x, algorithm="fricas")
```

output
```
1/24*(2*sqrt(3)*x^6*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 2*sqrt(3)*x^6*arctan
(1/3*sqrt(3)*(2*x^2 - 1)) - 3*x^6*log(x^4 + x^2 + 1) + 3*x^6*log(x^4 - x^2
 + 1) + 12*x^4 - 4)/x^6
```

3.337.6 Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.99

$$\int \frac{1}{x^7 (1+x^4+x^8)} dx = \frac{\log (x^4 - x^2 + 1)}{8} - \frac{\log (x^4 + x^2 + 1)}{8} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^2}{3} - \frac{\sqrt{3}}{3}\right)}{12} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^2}{3} + \frac{\sqrt{3}}{3}\right)}{12} + \frac{3x^4 - 1}{6x^6}$$

input
```
integrate(1/x**7/(x**8+x**4+1),x)
```

output
```
log(x**4 - x**2 + 1)/8 - log(x**4 + x**2 + 1)/8 + sqrt(3)*atan(2*sqrt(3)*x
**2/3 - sqrt(3)/3)/12 + sqrt(3)*atan(2*sqrt(3)*x**2/3 + sqrt(3)/3)/12 + (3
*x**4 - 1)/(6*x**6)
```

3.337.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^7 (1+x^4+x^8)} dx = \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2+1)\right) + \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2-1)\right) + \frac{3x^4-1}{6x^6} - \frac{1}{8}\log\left(x^4+x^2+1\right) + \frac{1}{8}\log\left(x^4-x^2+1\right)$$

input
```
integrate(1/x^7/(x^8+x^4+1),x, algorithm="maxima")
```

output
```
1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*sqrt(3)*arctan(1/3*sqr
t(3)*(2*x^2 - 1)) + 1/6*(3*x^4 - 1)/x^6 - 1/8*log(x^4 + x^2 + 1) + 1/8*log
(x^4 - x^2 + 1)
```

3.337.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^7\left(1+x^4+x^8\right)}\,dx=\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2+1)\right)+\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2-1)\right)+\frac{3x^4-1}{6x^6}-\frac{1}{8}\log\left(x^4+x^2+1\right)+\frac{1}{8}\log\left(x^4-x^2+1\right)$$

input
```
integrate(1/x^7/(x^8+x^4+1),x, algorithm="giac")
```

output
```
1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*sqrt(3)*arctan(1/3*sqr
t(3)*(2*x^2 - 1)) + 1/6*(3*x^4 - 1)/x^6 - 1/8*log(x^4 + x^2 + 1) + 1/8*log
(x^4 - x^2 + 1)
```

3.337.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.70

$$\int \frac{1}{x^7\left(1+x^4+x^8\right)}\,dx=\operatorname{atanh}\left(\frac{2x^2}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)+\operatorname{atanh}\left(\frac{2x^2}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)+\frac{\frac{x^4}{2}-\frac{1}{6}}{x^6}$$

input
```
int(1/(x^7*(x^4 + x^8 + 1)),x)
```

output
```
atanh((2*x^2)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/12 + 1/4) + atanh((2*x^2)/(3
^(1/2)*1i + 1))*((3^(1/2)*1i)/12 - 1/4) + (x^4/2 - 1/6)/x^6
```

3.338 $\int \frac{x^8}{1+x^4+x^8}\,dx$

3.338.1 Optimal result

Integrand size = 14, antiderivative size = 141

$$\int \frac{x^8}{1+x^4+x^8}\,dx = x+\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\arctan\left(\sqrt{3}-2x\right)-\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}} -\frac{1}{4}\arctan\left(\sqrt{3}+2x\right)+\frac{1}{8}\log\left(1-x+x^2\right)-\frac{1}{8}\log\left(1+x+x^2\right) +\frac{\log\left(1-\sqrt{3}x+x^2\right)}{8\sqrt{3}}-\frac{\log\left(1+\sqrt{3}x+x^2\right)}{8\sqrt{3}}$$

output
```
x-1/4*arctan(2*x-3^(1/2))-1/4*arctan(2*x+3^(1/2))+1/8*ln(x^2-x+1)-1/8*ln(x
^2+x+1)+1/12*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)-1/12*arctan(1/3*(1+2*x)*3
^(1/2))*3^(1/2)+1/24*ln(1+x^2-x*3^(1/2))*3^(1/2)-1/24*ln(1+x^2+x*3^(1/2))*
3^(1/2)
```

3.338.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.99

$$\int \frac{x^8}{1+x^4+x^8}\,dx = -\frac{i\arctan\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x\right)}{\sqrt{-6+6i\sqrt{3}}}+\frac{i\arctan\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x\right)}{\sqrt{-6-6i\sqrt{3}}} +\frac{1}{24}\left(24x-2\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)-2\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right) +3\log\left(1-x+x^2\right)-3\log\left(1+x+x^2\right)\right)$$

input
```
Integrate[x^8/(1 + x^4 + x^8),x]
```

output
```
((-I)*ArcTan[((1 - I*Sqrt[3])*x)/2])/Sqrt[-6 + (6*I)*Sqrt[3]] + (I*ArcTan[
((1 + I*Sqrt[3])*x)/2])/Sqrt[-6 - (6*I)*Sqrt[3]] + (24*x - 2*Sqrt[3]*ArcTa
n[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + 3*Log[1 - x
+ x^2] - 3*Log[1 + x + x^2])/24
```

3.338.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.22, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {1703, 1749, 1407, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{x^8+x^4+1}dx$$

$$\Big\downarrow \text{1703}$$

$$x-\int \frac{x^4+1}{x^8+x^4+1}dx$$

$$\Big\downarrow \text{1749}$$

$$-\frac{1}{2}\int \frac{1}{x^4-x^2+1}dx-\frac{1}{2}\int \frac{1}{x^4+x^2+1}dx+x$$

$$\Big\downarrow \text{1407}$$

$$\frac{1}{2}\left(-\frac{1}{2}\int \frac{1-x}{x^2-x+1}dx-\frac{1}{2}\int \frac{x+1}{x^2+x+1}dx\right)+\frac{1}{2}\left(-\frac{\int \frac{\sqrt{3}-x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}-\frac{\int \frac{x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}\right)+x$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\int -\frac{1-2x}{x^2-x+1}dx-\frac{1}{2}\int \frac{1}{x^2-x+1}dx\right)+\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{x^2+x+1}dx-\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx\right)\right)+\frac{1}{2}\left(-\frac{\frac{1}{2}\sqrt{3}\int \frac{1}{x^2-\sqrt{3}x+1}dx-\frac{1}{2}\int -\frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}-\frac{\frac{1}{2}\sqrt{3}\int \frac{1}{x^2+\sqrt{3}x+1}dx+\frac{1}{2}\int \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}\right)+x$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-\frac{1}{2}\int\frac{1}{x^2-x+1}dx-\frac{1}{2}\int\frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{2}\left(-\frac{1}{2}\int\frac{1}{x^2+x+1}dx-\frac{1}{2}\int\frac{2x+1}{x^2+x+1}dx\right)\right)+\frac{1}{2}\left(-\frac{\frac{1}{2}\sqrt{3}\int\frac{1}{x^2-\sqrt{3}x+1}dx+\frac{1}{2}\int\frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}-\frac{\frac{1}{2}\sqrt{3}\int\frac{1}{x^2+\sqrt{3}x+1}dx+\frac{1}{2}\int\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}\right)+x$$

$\Big\downarrow$ 1083

$$\frac{1}{2}\left(\frac{1}{2}\left(\int\frac{1}{-(2x-1)^2-3}d(2x-1)-\frac{1}{2}\int\frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{2}\left(\int\frac{1}{-(2x+1)^2-3}d(2x+1)-\frac{1}{2}\int\frac{2x+1}{x^2+x+1}dx\right)\right)+\frac{1}{2}\left(-\frac{\frac{1}{2}\int\frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx-\sqrt{3}\int\frac{1}{-\left(2x-\sqrt{3}\right)^2-1}d\left(2x-\sqrt{3}\right)}{2\sqrt{3}}-\frac{\frac{1}{2}\int\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx-\sqrt{3}\int\frac{1}{-\left(2x+\sqrt{3}\right)^2-1}d\left(2x+\sqrt{3}\right)}{2\sqrt{3}}\right)+x$$

$\Big\downarrow$ 217

$$\frac{1}{2}\left(\frac{1}{2}\left(-\frac{1}{2}\int\frac{1-2x}{x^2-x+1}dx-\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}\right)+\frac{1}{2}\left(-\frac{1}{2}\int\frac{2x+1}{x^2+x+1}dx-\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)\right)+\frac{1}{2}\left(-\frac{\frac{1}{2}\int\frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx-\sqrt{3}\arctan\left(\sqrt{3}-2x\right)}{2\sqrt{3}}-\frac{\frac{1}{2}\int\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx+\sqrt{3}\arctan\left(2x+\sqrt{3}\right)}{2\sqrt{3}}\right)+x$$

$\Big\downarrow$ 1103

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\log\left(x^2-x+1\right)-\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}\right)+\frac{1}{2}\left(-\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}\log\left(x^2+x+1\right)\right)\right)+\frac{1}{2}\left(-\frac{-\sqrt{3}\arctan\left(\sqrt{3}-2x\right)-\frac{1}{2}\log\left(x^2-\sqrt{3}x+1\right)}{2\sqrt{3}}-\frac{\sqrt{3}\arctan\left(2x+\sqrt{3}\right)+\frac{1}{2}\log\left(x^2+\sqrt{3}x+1\right)}{2\sqrt{3}}\right)+x$$

input
```
Int[x^8/(1 + x^4 + x^8),x]
```

output
```
x + ((-(ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3]) + Log[1 - x + x^2]/2)/2 + (-(A
rcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3]) - Log[1 + x + x^2]/2)/2)/2 + (-1/2*(-(Sq
rt[3]*ArcTan[Sqrt[3] - 2*x]) - Log[1 - Sqrt[3]*x + x^2]/2)/Sqrt[3] - (Sqrt
[3]*ArcTan[Sqrt[3] + 2*x] + Log[1 + Sqrt[3]*x + x^2]/2)/(2*Sqrt[3]))/2
```

3.338. $\int \frac{x^8}{1+x^4+x^8}\,dx$

3.338.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1407
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   Int[(r - x)/(q - r*
x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(r + x)/(q + r*x + x^2), x], x]]]
 /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

rule 1703
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(2*n - 1)*(d*x)^(m - 2*n + 1)*((a + b*x^n + c*x^(2*n))^(
p + 1)/(c*(m + 2*n*p + 1))), x] - Simp[d^(2*n)/(c*(m + 2*n*p + 1))   Int[(d
*x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x
^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && N
eQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0
] && IntegerQ[p]
```

3.338. $\int \frac{x^8}{1+x^4+x^8} \, dx$

rule 1749
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e
+ q*x^(n/2) + x^n, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x^(n/2) +
 x^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[2*(d/e) - b/c,
0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

3.338.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.65

method	result
risch	$x - \frac{\ln(4x^2+4x+4)}{8} - \frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{12} + \frac{\left(\sum_{\_R=\operatorname{RootOf}\left(9\_Z^4+3\_Z^2+1\right)} \_R\ln\left(3\_R^3-\_R+x\right)\right)}{4} + \frac{\ln(4x^2-4x+4}{8}$
default	$x + \frac{\ln(x^2-x+1)}{8} - \frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12} - \frac{\ln(x^2+x+1)}{8} - \frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{12} + \frac{\ln\left(1+x^2-x\sqrt{3}\right)\sqrt{3}}{24} - \frac{\arctan\left(2x-\right.}{4}$

input
```
int(x^8/(x^8+x^4+1),x,method=_RETURNVERBOSE)
```

output
```
x-1/8*ln(4*x^2+4*x+4)-1/12*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/4*sum(_R*
ln(3*_R^3-_R+x),_R=RootOf(9*_Z^4+3*_Z^2+1))+1/8*ln(4*x^2-4*x+4)-1/12*3^(1/
2)*arctan(1/3*(2*x-1)*3^(1/2))
```

3.338.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.49

$$\int \frac{x^8}{1+x^4+x^8}\,dx = \frac{1}{24}\sqrt{6}\sqrt{i\sqrt{3}-1}\log\left(\sqrt{6}\sqrt{i\sqrt{3}-1}\left(i\sqrt{3}-3\right)+12\,x\right) - \frac{1}{24}\sqrt{6}\sqrt{i\sqrt{3}-1}\log\left(\sqrt{6}\sqrt{i\sqrt{3}-1}\left(-i\sqrt{3}+3\right)+12\,x\right) - \frac{1}{24}\sqrt{6}\sqrt{-i\sqrt{3}-1}\log\left(\sqrt{6}\left(i\sqrt{3}+3\right)\sqrt{-i\sqrt{3}-1}+12\,x\right) + \frac{1}{24}\sqrt{6}\sqrt{-i\sqrt{3}-1}\log\left(\sqrt{6}\sqrt{-i\sqrt{3}-1}\left(-i\sqrt{3}-3\right)+12\,x\right) - \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x+1)\right) - \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right) + x - \frac{1}{8}\log\left(x^2+x+1\right) + \frac{1}{8}\log\left(x^2-x+1\right)$$

input
```
integrate(x^8/(x^8+x^4+1),x, algorithm="fricas")
```

output
```
1/24*sqrt(6)*sqrt(I*sqrt(3) - 1)*log(sqrt(6)*sqrt(I*sqrt(3) - 1)*(I*sqrt(3
) - 3) + 12*x) - 1/24*sqrt(6)*sqrt(I*sqrt(3) - 1)*log(sqrt(6)*sqrt(I*sqrt(
3) - 1)*(-I*sqrt(3) + 3) + 12*x) - 1/24*sqrt(6)*sqrt(-I*sqrt(3) - 1)*log(s
qrt(6)*(I*sqrt(3) + 3)*sqrt(-I*sqrt(3) - 1) + 12*x) + 1/24*sqrt(6)*sqrt(-I
*sqrt(3) - 1)*log(sqrt(6)*sqrt(-I*sqrt(3) - 1)*(-I*sqrt(3) - 3) + 12*x) -
1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x - 1)) + x - 1/8*log(x^2 + x + 1) + 1/8*log(x^2 - x + 1)
```

3.338. $\int \frac{x^8}{1+x^4+x^8}\,dx$

3.338.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.36

$$\begin{aligned}\int \frac{x^8}{1+x^4+x^8}\,dx = x &+ \left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x-1-\frac{\sqrt{3}i}{3}-9216\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^5\right)\\ &+ \left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x-1-9216\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^5+\frac{\sqrt{3}i}{3}\right)\\ &+ \left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x+1-\frac{\sqrt{3}i}{3}-9216\left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^5\right)\\ &+ \left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x+1-9216\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^5+\frac{\sqrt{3}i}{3}\right)\\ &+ \operatorname{RootSum}\left(2304t^4+48t^2+1,\left(t\mapsto t\log\left(-9216t^5-8t+x\right)\right)\right)\end{aligned}$$

input
```
integrate(x**8/(x**8+x**4+1),x)
```

output
```
x + (1/8 + sqrt(3)*I/24)*log(x - 1 - sqrt(3)*I/3 - 9216*(1/8 + sqrt(3)*I/2
4)**5) + (1/8 - sqrt(3)*I/24)*log(x - 1 - 9216*(1/8 - sqrt(3)*I/24)**5 + s
qrt(3)*I/3) + (-1/8 + sqrt(3)*I/24)*log(x + 1 - sqrt(3)*I/3 - 9216*(-1/8 +
 sqrt(3)*I/24)**5) + (-1/8 - sqrt(3)*I/24)*log(x + 1 - 9216*(-1/8 - sqrt(3
)*I/24)**5 + sqrt(3)*I/3) + RootSum(2304*_t**4 + 48*_t**2 + 1, Lambda(_t,
_t*log(-9216*_t**5 - 8*_t + x)))
```

3.338.7 Maxima [F]

$$\int \frac{x^8}{1+x^4+x^8}\,dx = \int \frac{x^8}{x^8+x^4+1}\,dx$$

input
```
integrate(x^8/(x^8+x^4+1),x, algorithm="maxima")
```

output
```
-1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x - 1)) + x - 1/2*integrate(1/(x^4 - x^2 + 1), x) - 1/8*log(x^2 + x
 + 1) + 1/8*log(x^2 - x + 1)
```

3.338.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.77

$$\int \frac{x^8}{1+x^4+x^8}\,dx = -\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) -\frac{1}{24}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)+\frac{1}{24}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right) +x-\frac{1}{4}\arctan\left(2x+\sqrt{3}\right)-\frac{1}{4}\arctan\left(2x-\sqrt{3}\right) -\frac{1}{8}\log\left(x^2+x+1\right)+\frac{1}{8}\log\left(x^2-x+1\right)$$

input
```
integrate(x^8/(x^8+x^4+1),x, algorithm="giac")
```

output
```
-1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x - 1)) - 1/24*sqrt(3)*log(x^2 + sqrt(3)*x + 1) + 1/24*sqrt(3)*log(
x^2 - sqrt(3)*x + 1) + x - 1/4*arctan(2*x + sqrt(3)) - 1/4*arctan(2*x - sq
rt(3)) - 1/8*log(x^2 + x + 1) + 1/8*log(x^2 - x + 1)
```

3.338.9 Mupad [B] (verification not implemented)

Time = 8.36 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.71

$$\int \frac{x^8}{1+x^4+x^8}\,dx = x-\operatorname{atan}\left(\frac{2x}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) -\operatorname{atan}\left(\frac{2x}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) -\operatorname{atan}\left(\frac{x\,2\mathrm{i}}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{12}+\frac{1}{4}\mathrm{i}\right)-\operatorname{atan}\left(\frac{x\,2\mathrm{i}}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{12}-\frac{1}{4}\mathrm{i}\right)$$

input
```
int(x^8/(x^4 + x^8 + 1),x)
```

output
```
x - atan((2*x)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/12 - 1/4) - atan((2*x)/(3^(
1/2)*1i + 1))*((3^(1/2)*1i)/12 + 1/4) - atan((x*2i)/(3^(1/2)*1i - 1))*(3^(
1/2)/12 + 1i/4) - atan((x*2i)/(3^(1/2)*1i + 1))*(3^(1/2)/12 - 1i/4)
```

3.339 $\int \frac{x^6}{1+x^4+x^8} \, dx$

3.339.1 Optimal result

Integrand size = 14, antiderivative size = 88

$$\int \frac{x^6}{1+x^4+x^8} \, dx = -\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\log\left(1-\sqrt{3}x+x^2\right)}{4\sqrt{3}} - \frac{\log\left(1+\sqrt{3}x+x^2\right)}{4\sqrt{3}}$$

output
```
-1/6*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/6*arctan(1/3*(1+2*x)*3^(1/2))*3
^(1/2)+1/12*ln(1+x^2-x*3^(1/2))*3^(1/2)-1/12*ln(1+x^2+x*3^(1/2))*3^(1/2)
```

3.339.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.77

$$\int \frac{x^6}{1+x^4+x^8} \, dx = \frac{2\arctan\left(\frac{-1+2x}{\sqrt{3}}\right) + 2\arctan\left(\frac{1+2x}{\sqrt{3}}\right) + \log\left(-1+\sqrt{3}x-x^2\right) - \log\left(1+\sqrt{3}x+x^2\right)}{4\sqrt{3}}$$

input
```
Integrate[x^6/(1 + x^4 + x^8),x]
```

output
```
(2*ArcTan[(-1 + 2*x)/Sqrt[3]] + 2*ArcTan[(1 + 2*x)/Sqrt[3]] + Log[-1 + Sqr
t[3]*x - x^2] - Log[1 + Sqrt[3]*x + x^2])/(4*Sqrt[3])
```

3.339.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1708, 1475, 1083, 217, 1478, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{x^8+x^4+1} \, dx$$

$$\Big\downarrow \ 1708$$

$$\frac{1}{2}\int \frac{x^2+1}{x^4+x^2+1}dx-\frac{1}{2}\int \frac{1-x^2}{x^4-x^2+1}dx$$

$$\Big\downarrow \ 1475$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2-x+1}dx+\frac{1}{2}\int \frac{1}{x^2+x+1}dx\right)-\frac{1}{2}\int \frac{1-x^2}{x^4-x^2+1}dx$$

$$\Big\downarrow \ 1083$$

$$\frac{1}{2}\left(-\int \frac{1}{-(2x-1)^2-3}d(2x-1)-\int \frac{1}{-(2x+1)^2-3}d(2x+1)\right)-\frac{1}{2}\int \frac{1-x^2}{x^4-x^2+1}dx$$

$$\Big\downarrow \ 217$$

$$\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)-\frac{1}{2}\int \frac{1-x^2}{x^4-x^2+1}dx$$

$$\Big\downarrow \ 1478$$

$$\frac{1}{2}\left(\frac{\int -\frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}+\frac{\int -\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)$$

$$\Big\downarrow \ 25$$

$$\frac{1}{2}\left(-\frac{\int \frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}-\frac{\int \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)$$

$$\Big\downarrow \ 1103$$

$$\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)+\frac{1}{2}\left(\frac{\log\left(x^2-\sqrt{3}x+1\right)}{2\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}x+1\right)}{2\sqrt{3}}\right)$$

input
```
Int[x^6/(1 + x^4 + x^8),x]
```

output
```
(ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3] + ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3])/2
 + (Log[1 - Sqrt[3]*x + x^2]/(2*Sqrt[3]) - Log[1 + Sqrt[3]*x + x^2]/(2*Sqr
t[3]))/2
```

3.339.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

rule 1708
```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, -Simp[1/(2*c*r)   Int[x^
(m - 3*(n/2))*((q - r*x^(n/2))/(q - r*x^(n/2) + x^n)), x], x] + Simp[1/(2*c
*r)   Int[x^(m - 3*(n/2))*((q + r*x^(n/2))/(q + r*x^(n/2) + x^n)), x], x]]]
 /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n/2,
 0] && IGtQ[m, 0] && GeQ[m, 3*(n/2)] && LtQ[m, 2*n] && NegQ[b^2 - 4*a*c]
```

3.339.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.76

method	result	size
default	$\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}+\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{6}+\frac{\ln\left(1+x^{2}-x\sqrt{3}\right)\sqrt{3}}{12}-\frac{\ln\left(1+x^{2}+x\sqrt{3}\right)\sqrt{3}}{12}$	67
risch	$\frac{\ln\left(1+x^{2}-x\sqrt{3}\right)\sqrt{3}}{12}-\frac{\ln\left(1+x^{2}+x\sqrt{3}\right)\sqrt{3}}{12}+\frac{\sqrt{3}\,\arctan\left(\frac{x\sqrt{3}}{3}\right)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{x^{3}\sqrt{3}}{3}+\frac{2x\sqrt{3}}{3}\right)}{6}$	68

input
```
int(x^6/(x^8+x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))+1/6*arctan(1/3*(1+2*x)*3^(1/2))*3^
(1/2)+1/12*ln(1+x^2-x*3^(1/2))*3^(1/2)-1/12*ln(1+x^2+x*3^(1/2))*3^(1/2)
```

3.339.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.80

$$\int \frac{x^6}{1+x^4+x^8}\,dx = \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(x^3+2x)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x\right) + \frac{1}{12}\sqrt{3}\log\left(\frac{x^4+5x^2-2\sqrt{3}(x^3+x)+1}{x^4-x^2+1}\right)$$

input
```
integrate(x^6/(x^8+x^4+1),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*(x^3 + 2*x)) + 1/6*sqrt(3)*arctan(1/3*sqrt(
3)*x) + 1/12*sqrt(3)*log((x^4 + 5*x^2 - 2*sqrt(3)*(x^3 + x) + 1)/(x^4 - x^
2 + 1))
```

3.339.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.93

$$\int \frac{x^6}{1+x^4+x^8}\,dx = \frac{\sqrt{3}\cdot\left(2\,\mathrm{atan}\left(\frac{\sqrt{3}x}{3}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{3}x^3}{3}+\frac{2\sqrt{3}x}{3}\right)\right)}{12} + \frac{\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)}{12} - \frac{\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)}{12}$$

input
```
integrate(x**6/(x**8+x**4+1),x)
```

output
```
sqrt(3)*(2*atan(sqrt(3)*x/3) + 2*atan(sqrt(3)*x**3/3 + 2*sqrt(3)*x/3))/12
+ sqrt(3)*log(x**2 - sqrt(3)*x + 1)/12 - sqrt(3)*log(x**2 + sqrt(3)*x + 1)
/12
```

3.339.7 Maxima [F]

$$\int \frac{x^6}{1+x^4+x^8}\,dx = \int \frac{x^6}{x^8+x^4+1}\,dx$$

input
```
integrate(x^6/(x^8+x^4+1),x, algorithm="maxima")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) + 1/2*integrate((x^2 - 1)/(x^4 - x^2 + 1), x)
```

3.339.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.75

$$\int \frac{x^6}{1+x^4+x^8}\,dx = \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x+1)\right) + \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right) - \frac{1}{12}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right) + \frac{1}{12}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)$$

input
```
integrate(x^6/(x^8+x^4+1),x, algorithm="giac")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) - 1/12*sqrt(3)*log(x^2 + sqrt(3)*x + 1) + 1/12*sqrt(3)*log(x^2
 - sqrt(3)*x + 1)
```

3.339.9 Mupad [B] (verification not implemented)

Time = 8.42 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.43

$$\int \frac{x^6}{1+x^4+x^8}\,dx = -\frac{\sqrt{3}\left(\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3\left(\frac{2x^2}{3}-\frac{2}{3}\right)}\right)+\operatorname{atanh}\left(\frac{2\sqrt{3}\,x}{3\left(\frac{2x^2}{3}+\frac{2}{3}\right)}\right)\right)}{6}$$

input
```
int(x^6/(x^4 + x^8 + 1),x)
```

output
```
-(3^(1/2)*(atan((2*3^(1/2)*x)/(3*((2*x^2)/3 - 2/3))) + atanh((2*3^(1/2)*x)
/(3*((2*x^2)/3 + 2/3)))))/6
```

3.340 $\int \frac{x^4}{1+x^4+x^8}\,dx$

3.340.1 Optimal result

Integrand size = 14, antiderivative size = 140

$$\int \frac{x^4}{1+x^4+x^8}\,dx = \frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{4}\arctan\left(\sqrt{3}-2x\right) - \frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{1}{4}\arctan\left(\sqrt{3}+2x\right) - \frac{1}{8}\log\left(1-x+x^2\right) + \frac{1}{8}\log\left(1+x+x^2\right) + \frac{\log\left(1-\sqrt{3}x+x^2\right)}{8\sqrt{3}} - \frac{\log\left(1+\sqrt{3}x+x^2\right)}{8\sqrt{3}}$$

output
```
1/4*arctan(2*x-3^(1/2))+1/4*arctan(2*x+3^(1/2))-1/8*ln(x^2-x+1)+1/8*ln(x^2
+x+1)+1/12*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)-1/12*arctan(1/3*(1+2*x)*3^(
1/2))*3^(1/2)+1/24*ln(1+x^2-x*3^(1/2))*3^(1/2)-1/24*ln(1+x^2+x*3^(1/2))*3^
(1/2)
```

3.340.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.96

$$\int \frac{x^4}{1+x^4+x^8}\,dx = \frac{1}{24}\left(-2i\sqrt{-6+6i\sqrt{3}}\arctan\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x\right) + 2i\sqrt{-6-6i\sqrt{3}}\arctan\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x\right) - 2\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right) - 2\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right) - 3\log\left(1-x+x^2\right) + 3\log\left(1+x+x^2\right)\right)$$

input
```
Integrate[x^4/(1 + x^4 + x^8),x]
```

output
```
((-2*I)*Sqrt[-6 + (6*I)*Sqrt[3]]*ArcTan[((1 - I*Sqrt[3])*x)/2] + (2*I)*Sqr
t[-6 - (6*I)*Sqrt[3]]*ArcTan[((1 + I*Sqrt[3])*x)/2] - 2*Sqrt[3]*ArcTan[(-1
 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 3*Log[1 - x + x^2
] + 3*Log[1 + x + x^2])/24
```

3.340.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {1709, 1447, 1475, 1083, 217, 1478, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{x^8+x^4+1}\,dx$$

$$\Big\downarrow \text{1709}$$

$$\frac{1}{2}\int \frac{x^2}{x^4-x^2+1}dx-\frac{1}{2}\int \frac{x^2}{x^4+x^2+1}dx$$

$$\Big\downarrow \text{1447}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{x^2+1}{x^4-x^2+1}dx-\frac{1}{2}\int \frac{1-x^2}{x^4-x^2+1}dx\right)+\frac{1}{2}\left(\frac{1}{2}\int \frac{1-x^2}{x^4+x^2+1}dx-\frac{1}{2}\int \frac{x^2+1}{x^4+x^2+1}dx\right)$$

$$\Big\downarrow \text{1475}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2-\sqrt{3}x+1}dx+\frac{1}{2}\int \frac{1}{x^2+\sqrt{3}x+1}dx\right)-\frac{1}{2}\int \frac{1-x^2}{x^4-x^2+1}dx\right)+\frac{1}{2}\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{x^2-x+1}dx-\frac{1}{2}\int \frac{1}{x^2+x+1}dx\right)+\frac{1}{2}\int \frac{1-x^2}{x^4+x^2+1}dx\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1-x^2}{x^4+x^2+1}dx+\frac{1}{2}\left(\int \frac{1}{-(2x-1)^2-3}d(2x-1)+\int \frac{1}{-(2x+1)^2-3}d(2x+1)\right)\right)+\frac{1}{2}\left(\frac{1}{2}\left(-\int \frac{1}{-\left(2x-\sqrt{3}\right)^2-1}d\left(2x-\sqrt{3}\right)-\int \frac{1}{-\left(2x+\sqrt{3}\right)^2-1}d\left(2x+\sqrt{3}\right)\right)-\frac{1}{2}\int \frac{1-x^2}{x^4-x^2+1}dx\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\arctan\left(2x+\sqrt{3}\right)-\arctan\left(\sqrt{3}-2x\right)\right)-\frac{1}{2}\int\frac{1-x^2}{x^4-x^2+1}dx\right)+\frac{1}{2}\left(\frac{1}{2}\int\frac{1-x^2}{x^4+x^2+1}dx+\frac{1}{2}\left(-\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)\right)$$

$$\Big\downarrow \text{1478}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-\frac{1}{2}\int-\frac{1-2x}{x^2-x+1}dx-\frac{1}{2}\int-\frac{2x+1}{x^2+x+1}dx\right)+\frac{1}{2}\left(-\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)\right)+\frac{1}{2}\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}+\frac{\int-\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}\right)+\frac{1}{2}\left(\arctan\left(2x+\sqrt{3}\right)-\arctan\left(\sqrt{3}-2x\right)\right)\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1-2x}{x^2-x+1}dx+\frac{1}{2}\int\frac{2x+1}{x^2+x+1}dx\right)+\frac{1}{2}\left(-\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)\right)+\frac{1}{2}\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}-\frac{\int\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}\right)+\frac{1}{2}\left(\arctan\left(2x+\sqrt{3}\right)-\arctan\left(\sqrt{3}-2x\right)\right)\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)+\frac{1}{2}\left(\frac{1}{2}\log\left(x^2+x+1\right)-\frac{1}{2}\log\left(x^2-x+1\right)\right)\right)+\frac{1}{2}\left(\frac{1}{2}\left(\arctan\left(2x+\sqrt{3}\right)-\arctan\left(\sqrt{3}-2x\right)\right)+\frac{1}{2}\left(\frac{\log\left(x^2-\sqrt{3}x+1\right)}{2\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}x+1\right)}{2\sqrt{3}}\right)\right)$$

input
```
Int[x^4/(1 + x^4 + x^8),x]
```

output
```
((-(ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3]) - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3
])/2 + (-1/2*Log[1 - x + x^2] + Log[1 + x + x^2]/2)/2)/2 + ((-ArcTan[Sqrt[
3] - 2*x] + ArcTan[Sqrt[3] + 2*x])/2 + (Log[1 - Sqrt[3]*x + x^2]/(2*Sqrt[3
]) - Log[1 + Sqrt[3]*x + x^2]/(2*Sqrt[3]))/2)/2
```

3.340. $\int \frac{x^4}{1+x^4+x^8}\,dx$

3.340.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1447
```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Simp[1/2   Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Simp[1/2
 Int[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b
^2 - 4*a*c, 0] && PosQ[a*c]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

3.340. $\int \frac{x^4}{1+x^4+x^8} \, dx$

rule 1709
```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*r)   Int[x^(
m - n/2)/(q - r*x^(n/2) + x^n), x], x] - Simp[1/(2*c*r)   Int[x^(m - n/2)/(
q + r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && IGtQ[m, 0] && GeQ[m, n/2] && LtQ[m, 3*
(n/2)] && NegQ[b^2 - 4*a*c]
```

3.340.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.64

method	result
risch	$-\frac{\ln(4x^2-4x+4)}{8}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(9\_Z^4+3\_Z^2+1\right)}\_R\ln\left(6\_R^3+\_R+x\right)\right)}{4}+\frac{\ln(4x^2+4x+4)}{8}$
default	$-\frac{\ln(x^2-x+1)}{8}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12}+\frac{\ln(x^2+x+1)}{8}-\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{12}-\frac{\sqrt{3}\left(-\frac{\ln\left(1+x^2-x\sqrt{3}\right)}{2}-\sqrt{3}\,\arctan\left(2x-\right.\right.}{12}$

input
```
int(x^4/(x^8+x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/8*ln(4*x^2-4*x+4)-1/12*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))+1/4*sum(_R*l
n(6*_R^3+_R+x),_R=RootOf(9*_Z^4+3*_Z^2+1))+1/8*ln(4*x^2+4*x+4)-1/12*arctan
(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

3.340.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.41

$$\int \frac{x^4}{1+x^4+x^8}\,dx = \frac{1}{24}\sqrt{6}\sqrt{i\sqrt{3}-1}\log\left(i\sqrt{6}\sqrt{3}\sqrt{i\sqrt{3}-1}+6x\right) - \frac{1}{24}\sqrt{6}\sqrt{i\sqrt{3}-1}\log\left(-i\sqrt{6}\sqrt{3}\sqrt{i\sqrt{3}-1}+6x\right) - \frac{1}{24}\sqrt{6}\sqrt{-i\sqrt{3}-1}\log\left(i\sqrt{6}\sqrt{3}\sqrt{-i\sqrt{3}-1}+6x\right) + \frac{1}{24}\sqrt{6}\sqrt{-i\sqrt{3}-1}\log\left(-i\sqrt{6}\sqrt{3}\sqrt{-i\sqrt{3}-1}+6x\right) - \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{8}\log\left(x^2+x+1\right) - \frac{1}{8}\log\left(x^2-x+1\right)$$

input
```
integrate(x^4/(x^8+x^4+1),x, algorithm="fricas")
```

output
```
1/24*sqrt(6)*sqrt(I*sqrt(3) - 1)*log(I*sqrt(6)*sqrt(3)*sqrt(I*sqrt(3) - 1)
 + 6*x) - 1/24*sqrt(6)*sqrt(I*sqrt(3) - 1)*log(-I*sqrt(6)*sqrt(3)*sqrt(I*s
qrt(3) - 1) + 6*x) - 1/24*sqrt(6)*sqrt(-I*sqrt(3) - 1)*log(I*sqrt(6)*sqrt(
3)*sqrt(-I*sqrt(3) - 1) + 6*x) + 1/24*sqrt(6)*sqrt(-I*sqrt(3) - 1)*log(-I*
sqrt(6)*sqrt(3)*sqrt(-I*sqrt(3) - 1) + 6*x) - 1/12*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/8*log(x^2
+ x + 1) - 1/8*log(x^2 - x + 1)
```

3.340.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.41

$$\int \frac{x^4}{1+x^4+x^8}\,dx = \left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x-\frac{1}{2}+\frac{\sqrt{3}i}{6}-18432\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^5\right) + \left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x-\frac{1}{2}-18432\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^5-\frac{\sqrt{3}i}{6}\right) + \left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x+\frac{1}{2}+\frac{\sqrt{3}i}{6}-18432\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^5\right) + \left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x+\frac{1}{2}-18432\left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^5-\frac{\sqrt{3}i}{6}\right) + \mathrm{RootSum}\left(2304t^4+48t^2+1,\left(t\mapsto t\log\left(-18432t^5-4t+x\right)\right)\right)$$

input
```
integrate(x**4/(x**8+x**4+1),x)
```

output
```
(1/8 - sqrt(3)*I/24)*log(x - 1/2 + sqrt(3)*I/6 - 18432*(1/8 - sqrt(3)*I/24
)**5) + (1/8 + sqrt(3)*I/24)*log(x - 1/2 - 18432*(1/8 + sqrt(3)*I/24)**5 -
 sqrt(3)*I/6) + (-1/8 - sqrt(3)*I/24)*log(x + 1/2 + sqrt(3)*I/6 - 18432*(-
1/8 - sqrt(3)*I/24)**5) + (-1/8 + sqrt(3)*I/24)*log(x + 1/2 - 18432*(-1/8
+ sqrt(3)*I/24)**5 - sqrt(3)*I/6) + RootSum(2304*_t**4 + 48*_t**2 + 1, Lam
bda(_t, _t*log(-18432*_t**5 - 4*_t + x)))
```

3.340.7 Maxima [F]

$$\int \frac{x^4}{1+x^4+x^8}\,dx = \int \frac{x^4}{x^8+x^4+1}\,dx$$

input
```
integrate(x^4/(x^8+x^4+1),x, algorithm="maxima")
```

output
```
-1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x - 1)) + 1/2*integrate(x^2/(x^4 - x^2 + 1), x) + 1/8*log(x^2 + x +
 1) - 1/8*log(x^2 - x + 1)
```

3.340.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.77

$$\int \frac{x^4}{1+x^4+x^8}\,dx = -\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{1}{24}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right) + \frac{1}{24}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right) + \frac{1}{4}\arctan\left(2x+\sqrt{3}\right) + \frac{1}{4}\arctan\left(2x-\sqrt{3}\right) + \frac{1}{8}\log\left(x^2+x+1\right) - \frac{1}{8}\log\left(x^2-x+1\right)$$

input
```
integrate(x^4/(x^8+x^4+1),x, algorithm="giac")
```

output
```
-1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x - 1)) - 1/24*sqrt(3)*log(x^2 + sqrt(3)*x + 1) + 1/24*sqrt(3)*log(
x^2 - sqrt(3)*x + 1) + 1/4*arctan(2*x + sqrt(3)) + 1/4*arctan(2*x - sqrt(3
)) + 1/8*log(x^2 + x + 1) - 1/8*log(x^2 - x + 1)
```

3.340.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.71

$$\int \frac{x^4}{1+x^4+x^8}\,dx = -\mathrm{atan}\left(\frac{2x}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) - \mathrm{atan}\left(\frac{2x}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) - \mathrm{atan}\left(\frac{x\,2\mathrm{i}}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{12}-\frac{1}{4}\mathrm{i}\right) - \mathrm{atan}\left(\frac{x\,2\mathrm{i}}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{12}+\frac{1}{4}\mathrm{i}\right)$$

input
```
int(x^4/(x^4 + x^8 + 1),x)
```

output
```
- atan((2*x)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/12 + 1/4) - atan((2*x)/(3^(1/
2)*1i + 1))*((3^(1/2)*1i)/12 - 1/4) - atan((x*2i)/(3^(1/2)*1i - 1))*(3^(1/
2)/12 - 1i/4) - atan((x*2i)/(3^(1/2)*1i + 1))*(3^(1/2)/12 + 1i/4)
```

3.341 $\int \frac{x^2}{1+x^4+x^8} dx$

3.341.1 Optimal result

Integrand size = 14, antiderivative size = 140

$$\int \frac{x^2}{1+x^4+x^8} dx = \frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{4}\arctan\left(\sqrt{3}-2x\right) - \frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{1}{4}\arctan\left(\sqrt{3}+2x\right) + \frac{1}{8}\log\left(1-x+x^2\right) - \frac{1}{8}\log\left(1+x+x^2\right) - \frac{\log\left(1-\sqrt{3}x+x^2\right)}{8\sqrt{3}} + \frac{\log\left(1+\sqrt{3}x+x^2\right)}{8\sqrt{3}}$$

output
```
1/4*arctan(2*x-3^(1/2))+1/4*arctan(2*x+3^(1/2))+1/8*ln(x^2-x+1)-1/8*ln(x^2
+x+1)+1/12*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)-1/12*arctan(1/3*(1+2*x)*3^(
1/2))*3^(1/2)-1/24*ln(1+x^2-x*3^(1/2))*3^(1/2)+1/24*ln(1+x^2+x*3^(1/2))*3^
(1/2)
```

3.341.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.96

$$\int \frac{x^2}{1+x^4+x^8} dx = \frac{1}{48}\left(4i\sqrt{-6-6i\sqrt{3}}\arctan\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x\right) - 4i\sqrt{-6+6i\sqrt{3}}\arctan\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x\right) - 4\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right) - 4\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right) + 6\log\left(1-x+x^2\right) - 6\log\left(1+x+x^2\right)\right)$$

input
```
Integrate[x^2/(1 + x^4 + x^8),x]
```

output
```
((4*I)*Sqrt[-6 - (6*I)*Sqrt[3]]*ArcTan[((1 - I*Sqrt[3])*x)/2] - (4*I)*Sqrt
[-6 + (6*I)*Sqrt[3]]*ArcTan[((1 + I*Sqrt[3])*x)/2] - 4*Sqrt[3]*ArcTan[(-1
+ 2*x)/Sqrt[3]] - 4*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + 6*Log[1 - x + x^2]
 - 6*Log[1 + x + x^2])/48
```

3.341.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.22, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1709, 1407, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{x^8+x^4+1}dx$$

$$\Big\downarrow \text{1709}$$

$$\frac{1}{2}\int \frac{1}{x^4-x^2+1}dx-\frac{1}{2}\int \frac{1}{x^4+x^2+1}dx$$

$$\Big\downarrow \text{1407}$$

$$\frac{1}{2}\left(-\frac{1}{2}\int \frac{1-x}{x^2-x+1}dx-\frac{1}{2}\int \frac{x+1}{x^2+x+1}dx\right)+\frac{1}{2}\left(\frac{\int \frac{\sqrt{3}-x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}+\frac{\int \frac{x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}\right)$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\int -\frac{1-2x}{x^2-x+1}dx-\frac{1}{2}\int \frac{1}{x^2-x+1}dx\right)+\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{x^2+x+1}dx-\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx\right)\right)+\frac{1}{2}\left(\frac{\frac{1}{2}\sqrt{3}\int \frac{1}{x^2-\sqrt{3}x+1}dx-\frac{1}{2}\int -\frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}+\frac{\frac{1}{2}\sqrt{3}\int \frac{1}{x^2+\sqrt{3}x+1}dx+\frac{1}{2}\int \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{x^2-x+1}dx-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{x^2+x+1}dx-\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx\right)\right)+\frac{1}{2}\left(\frac{\frac{1}{2}\sqrt{3}\int \frac{1}{x^2-\sqrt{3}x+1}dx+\frac{1}{2}\int \frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}+\frac{\frac{1}{2}\sqrt{3}\int \frac{1}{x^2+\sqrt{3}x+1}dx+\frac{1}{2}\int \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}\right)$$

$$\downarrow 1083$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\int \frac{1}{-(2x-1)^2-3}d(2x-1)-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{2}\left(\int \frac{1}{-(2x+1)^2-3}d(2x+1)-\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx\right.\right.$$
$$\frac{1}{2}\left(\frac{\frac{1}{2}\int \frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx-\sqrt{3}\int \frac{1}{-\left(2x-\sqrt{3}\right)^2-1}d\left(2x-\sqrt{3}\right)}{2\sqrt{3}}+\frac{\frac{1}{2}\int \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx-\sqrt{3}\int \frac{1}{-\left(2x+\sqrt{3}\right)^2-1}d\left(2x+\sqrt{3}\right)}{2\sqrt{3}}\right)$$

$$\downarrow 217$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx-\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}\right)+\frac{1}{2}\left(-\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx-\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)\right)+$$
$$\frac{1}{2}\left(\frac{\frac{1}{2}\int \frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx-\sqrt{3}\arctan\left(\sqrt{3}-2x\right)}{2\sqrt{3}}+\frac{\frac{1}{2}\int \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx+\sqrt{3}\arctan\left(2x+\sqrt{3}\right)}{2\sqrt{3}}\right)$$

$$\downarrow 1103$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\log\left(x^2-x+1\right)-\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}\right)+\frac{1}{2}\left(-\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}\log\left(x^2+x+1\right)\right)\right)+$$
$$\frac{1}{2}\left(\frac{-\sqrt{3}\arctan\left(\sqrt{3}-2x\right)-\frac{1}{2}\log\left(x^2-\sqrt{3}x+1\right)}{2\sqrt{3}}+\frac{\sqrt{3}\arctan\left(2x+\sqrt{3}\right)+\frac{1}{2}\log\left(x^2+\sqrt{3}x+1\right)}{2\sqrt{3}}\right)$$

input
```
Int[x^2/(1 + x^4 + x^8),x]
```

output
```
((-(ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3]) + Log[1 - x + x^2]/2)/2 + (-(ArcTa
n[(1 + 2*x)/Sqrt[3]]/Sqrt[3]) - Log[1 + x + x^2]/2)/2)/2 + ((-(Sqrt[3]*Arc
Tan[Sqrt[3] - 2*x]) - Log[1 - Sqrt[3]*x + x^2]/2)/(2*Sqrt[3]) + (Sqrt[3]*A
rcTan[Sqrt[3] + 2*x] + Log[1 + Sqrt[3]*x + x^2]/2)/(2*Sqrt[3]))/2
```

3.341.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

3.341. $\int \frac{x^2}{1+x^4+x^8}\,dx$

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1407
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   Int[(r - x)/(q - r*
x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(r + x)/(q + r*x + x^2), x], x]]]
 /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

rule 1709
```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*r)   Int[x^(
m - n/2)/(q - r*x^(n/2) + x^n), x], x] - Simp[1/(2*c*r)   Int[x^(m - n/2)/(
q + r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && IGtQ[m, 0] && GeQ[m, n/2] && LtQ[m, 3*
(n/2)] && NegQ[b^2 - 4*a*c]
```

3.341.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.64

method	result
risch	$\frac{\ln\left(4x^2-4x+4\right)}{8}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12}-\frac{\ln\left(4x^2+4x+4\right)}{8}-\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{12}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(9\_Z^4+3\_Z^2+1\right)}\_R\ln\right.}{4}$
default	$\frac{\ln\left(x^2-x+1\right)}{8}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12}-\frac{\ln\left(x^2+x+1\right)}{8}-\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{12}-\frac{\ln\left(1+x^2-x\sqrt{3}\right)\sqrt{3}}{24}+\frac{\arctan\left(2x-\sqrt{3}\right)}{4}$

3.341. $\int \frac{x^2}{1+x^4+x^8}\,dx$

input
```
int(x^2/(x^8+x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/8*ln(4*x^2-4*x+4)-1/12*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))-1/8*ln(4*x^2+
4*x+4)-1/12*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/4*sum(_R*ln(-3*_R^3+_R+x
),_R=RootOf(9*_Z^4+3*_Z^2+1))
```

3.341.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.49

$$\begin{aligned}\int \frac{x^2}{1+x^4+x^8}\,dx =& -\frac{1}{24}\sqrt{6}\sqrt{i\sqrt{3}-1}\log\left(\sqrt{6}\sqrt{i\sqrt{3}-1}\left(i\sqrt{3}-3\right)+12\,x\right)\\ &+\frac{1}{24}\sqrt{6}\sqrt{i\sqrt{3}-1}\log\left(\sqrt{6}\sqrt{i\sqrt{3}-1}\left(-i\sqrt{3}+3\right)+12\,x\right)\\ &+\frac{1}{24}\sqrt{6}\sqrt{-i\sqrt{3}-1}\log\left(\sqrt{6}\left(i\sqrt{3}+3\right)\sqrt{-i\sqrt{3}-1}+12\,x\right)\\ &-\frac{1}{24}\sqrt{6}\sqrt{-i\sqrt{3}-1}\log\left(\sqrt{6}\sqrt{-i\sqrt{3}-1}\left(-i\sqrt{3}-3\right)+12\,x\right)\\ &-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x+1)\right)-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right)\\ &-\frac{1}{8}\log\left(x^2+x+1\right)+\frac{1}{8}\log\left(x^2-x+1\right)\end{aligned}$$

input
```
integrate(x^2/(x^8+x^4+1),x, algorithm="fricas")
```

output
```
-1/24*sqrt(6)*sqrt(I*sqrt(3) - 1)*log(sqrt(6)*sqrt(I*sqrt(3) - 1)*(I*sqrt(
3) - 3) + 12*x) + 1/24*sqrt(6)*sqrt(I*sqrt(3) - 1)*log(sqrt(6)*sqrt(I*sqrt
(3) - 1)*(-I*sqrt(3) + 3) + 12*x) + 1/24*sqrt(6)*sqrt(-I*sqrt(3) - 1)*log(
sqrt(6)*(I*sqrt(3) + 3)*sqrt(-I*sqrt(3) - 1) + 12*x) - 1/24*sqrt(6)*sqrt(-
I*sqrt(3) - 1)*log(sqrt(6)*sqrt(-I*sqrt(3) - 1)*(-I*sqrt(3) - 3) + 12*x) -
 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x - 1)) - 1/8*log(x^2 + x + 1) + 1/8*log(x^2 - x + 1)
```

3.341.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.53

$$\int \frac{x^2}{1+x^4+x^8}\,dx = \left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x+442368\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^7-192\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^3\right) + \left(-\frac{1}{8} + \frac{\sqrt{3}i}{24}\right)\log\left(x-192\left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^3+442368\left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^7\right) + \left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x+442368\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^7-192\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^3\right) + \left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x-192\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^3+442368\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^7\right) + \operatorname{RootSum}\left(2304t^4+48t^2+1,\left(t\mapsto t\log\left(442368t^7-192t^3+x\right)\right)\right)$$

input
```
integrate(x**2/(x**8+x**4+1),x)
```

output
```
(-1/8 - sqrt(3)*I/24)*log(x + 442368*(-1/8 - sqrt(3)*I/24)**7 - 192*(-1/8
- sqrt(3)*I/24)**3) + (-1/8 + sqrt(3)*I/24)*log(x - 192*(-1/8 + sqrt(3)*I/
24)**3 + 442368*(-1/8 + sqrt(3)*I/24)**7) + (1/8 - sqrt(3)*I/24)*log(x + 4
42368*(1/8 - sqrt(3)*I/24)**7 - 192*(1/8 - sqrt(3)*I/24)**3) + (1/8 + sqrt
(3)*I/24)*log(x - 192*(1/8 + sqrt(3)*I/24)**3 + 442368*(1/8 + sqrt(3)*I/24
)**7) + RootSum(2304*_t**4 + 48*_t**2 + 1, Lambda(_t, _t*log(442368*_t**7
- 192*_t**3 + x)))
```

3.341.7 Maxima [F]

$$\int \frac{x^2}{1+x^4+x^8}\,dx = \int \frac{x^2}{x^8+x^4+1}\,dx$$

input
```
integrate(x^2/(x^8+x^4+1),x, algorithm="maxima")
```

output
```
-1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x - 1)) + 1/2*integrate(1/(x^4 - x^2 + 1), x) - 1/8*log(x^2 + x + 1
) + 1/8*log(x^2 - x + 1)
```

3.341.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.77

$$\begin{aligned}\int \frac{x^2}{1+x^4+x^8}\,dx = & -\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) \\ & + \frac{1}{24}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right) - \frac{1}{24}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right) \\ & + \frac{1}{4}\arctan\left(2x+\sqrt{3}\right) + \frac{1}{4}\arctan\left(2x-\sqrt{3}\right) \\ & - \frac{1}{8}\log\left(x^2+x+1\right) + \frac{1}{8}\log\left(x^2-x+1\right)\end{aligned}$$

input
```
integrate(x^2/(x^8+x^4+1),x, algorithm="giac")
```

output
```
-1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x - 1)) + 1/24*sqrt(3)*log(x^2 + sqrt(3)*x + 1) - 1/24*sqrt(3)*log(
x^2 - sqrt(3)*x + 1) + 1/4*arctan(2*x + sqrt(3)) + 1/4*arctan(2*x - sqrt(3
)) - 1/8*log(x^2 + x + 1) + 1/8*log(x^2 - x + 1)
```

3.341.9 Mupad [B] (verification not implemented)

Time = 8.31 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.69

$$\int \frac{x^2}{1+x^4+x^8}\,dx = \operatorname{atan}\left(\frac{2\,x}{-1+\sqrt{3}\,1\mathrm{i}}\right)\,\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)+\operatorname{atan}\left(\frac{2\,x}{1+\sqrt{3}\,1\mathrm{i}}\right)\,\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) - \operatorname{atan}\left(\frac{x\,2\mathrm{i}}{-1+\sqrt{3}\,1\mathrm{i}}\right)\,\left(\frac{\sqrt{3}}{12}+\frac{1}{4}\mathrm{i}\right)-\operatorname{atan}\left(\frac{x\,2\mathrm{i}}{1+\sqrt{3}\,1\mathrm{i}}\right)\,\left(\frac{\sqrt{3}}{12}-\frac{1}{4}\mathrm{i}\right)$$

input
```
int(x^2/(x^4 + x^8 + 1),x)
```

output
```
atan((2*x)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/12 - 1/4) + atan((2*x)/(3^(1/2)
*1i + 1))*((3^(1/2)*1i)/12 + 1/4) - atan((x*2i)/(3^(1/2)*1i - 1))*(3^(1/2)
/12 + 1i/4) - atan((x*2i)/(3^(1/2)*1i + 1))*(3^(1/2)/12 - 1i/4)
```

3.342 $\int \frac{1}{1+x^4+x^8}\,dx$

3.342.1 Optimal result

Integrand size = 10, antiderivative size = 88

$$\int \frac{1}{1+x^4+x^8}\,dx = -\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{\log\left(1-\sqrt{3}x+x^2\right)}{4\sqrt{3}}+\frac{\log\left(1+\sqrt{3}x+x^2\right)}{4\sqrt{3}}$$

output
```
-1/6*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/6*arctan(1/3*(1+2*x)*3^(1/2))*3
^(1/2)-1/12*ln(1+x^2-x*3^(1/2))*3^(1/2)+1/12*ln(1+x^2+x*3^(1/2))*3^(1/2)
```

3.342.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.77

$$\int \frac{1}{1+x^4+x^8}\,dx = \frac{2\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)+2\arctan\left(\frac{1+2x}{\sqrt{3}}\right)-\log\left(-1+\sqrt{3}x-x^2\right)+\log\left(1+\sqrt{3}x+x^2\right)}{4\sqrt{3}}$$

input
```
Integrate[(1 + x^4 + x^8)^(-1),x]
```

output
```
(2*ArcTan[(-1 + 2*x)/Sqrt[3]] + 2*ArcTan[(1 + 2*x)/Sqrt[3]] - Log[-1 + Sqr
t[3]*x - x^2] + Log[1 + Sqrt[3]*x + x^2])/(4*Sqrt[3])
```

3.342.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {1684, 1475, 1083, 217, 1478, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^8+x^4+1} \, dx$$

$$\Big\downarrow \text{1684}$$

$$\frac{1}{2} \int \frac{1-x^2}{x^4-x^2+1}dx+\frac{1}{2} \int \frac{x^2+1}{x^4+x^2+1}dx$$

$$\Big\downarrow \text{1475}$$

$$\frac{1}{2} \left(\frac{1}{2} \int \frac{1}{x^2-x+1}dx+\frac{1}{2} \int \frac{1}{x^2+x+1}dx\right)+\frac{1}{2} \int \frac{1-x^2}{x^4-x^2+1}dx$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2} \int \frac{1-x^2}{x^4-x^2+1}dx+\frac{1}{2} \left(-\int \frac{1}{-(2x-1)^2-3}d(2x-1)-\int \frac{1}{-(2x+1)^2-3}d(2x+1)\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2} \int \frac{1-x^2}{x^4-x^2+1}dx+\frac{1}{2} \left(\frac{\arctan \left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan \left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)$$

$$\Big\downarrow \text{1478}$$

$$\frac{1}{2} \left(-\frac{\int -\frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}-\frac{\int -\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}\right)+\frac{1}{2} \left(\frac{\arctan \left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan \left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2} \left(\frac{\int \frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}+\frac{\int \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}\right)+\frac{1}{2} \left(\frac{\arctan \left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan \left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2} \left(\frac{\arctan \left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan \left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)+\frac{1}{2} \left(\frac{\log \left(x^2+\sqrt{3}x+1\right)}{2\sqrt{3}}-\frac{\log \left(x^2-\sqrt{3}x+1\right)}{2\sqrt{3}}\right)$$

input
```
Int[(1 + x^4 + x^8)^(-1),x]
```

output
```
(ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3] + ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3])/2
 + (-1/2*Log[1 - Sqrt[3]*x + x^2]/Sqrt[3] + Log[1 + Sqrt[3]*x + x^2]/(2*Sq
rt[3]))/2
```

3.342.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

rule 1684
```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   Int[(r - x^(
n/2))/(q - r*x^(n/2) + x^n), x], x] + Simp[1/(2*c*q*r)   Int[(r + x^(n/2))/
(q + r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && N
eQ[b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && NegQ[b^2 - 4*a*c]
```

3.342.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.76

method	result	size
default	$\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}+\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{6}-\frac{\ln\left(1+x^2-x\sqrt{3}\right)\sqrt{3}}{12}+\frac{\ln\left(1+x^2+x\sqrt{3}\right)\sqrt{3}}{12}$	67
risch	$-\frac{\ln\left(1+x^2-x\sqrt{3}\right)\sqrt{3}}{12}+\frac{\ln\left(1+x^2+x\sqrt{3}\right)\sqrt{3}}{12}+\frac{\sqrt{3}\,\arctan\left(\frac{x\sqrt{3}}{3}\right)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{x^3\sqrt{3}}{3}+\frac{2x\sqrt{3}}{3}\right)}{6}$	68

input
```
int(1/(x^8+x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))+1/6*arctan(1/3*(1+2*x)*3^(1/2))*3^
(1/2)-1/12*ln(1+x^2-x*3^(1/2))*3^(1/2)+1/12*ln(1+x^2+x*3^(1/2))*3^(1/2)
```

3.342.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.80

$$\int \frac{1}{1+x^4+x^8}\,dx = \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(x^3+2\,x)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x\right) + \frac{1}{12}\sqrt{3}\log\left(\frac{x^4+5\,x^2+2\,\sqrt{3}(x^3+x)+1}{x^4-x^2+1}\right)$$

input
```
integrate(1/(x^8+x^4+1),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*(x^3 + 2*x)) + 1/6*sqrt(3)*arctan(1/3*sqrt(
3)*x) + 1/12*sqrt(3)*log((x^4 + 5*x^2 + 2*sqrt(3)*(x^3 + x) + 1)/(x^4 - x^
2 + 1))
```

3.342.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.93

$$\int \frac{1}{1+x^4+x^8}\,dx = \frac{\sqrt{3}\cdot\left(2\,\mathrm{atan}\left(\frac{\sqrt{3}x}{3}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{3}x^3}{3}+\frac{2\sqrt{3}x}{3}\right)\right)}{12} - \frac{\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)}{12} + \frac{\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)}{12}$$

input
```
integrate(1/(x**8+x**4+1),x)
```

output
```
sqrt(3)*(2*atan(sqrt(3)*x/3) + 2*atan(sqrt(3)*x**3/3 + 2*sqrt(3)*x/3))/12
- sqrt(3)*log(x**2 - sqrt(3)*x + 1)/12 + sqrt(3)*log(x**2 + sqrt(3)*x + 1)
/12
```

3.342.7 Maxima [F]

$$\int \frac{1}{1+x^4+x^8}\,dx = \int \frac{1}{x^8+x^4+1}\,dx$$

input
```
integrate(1/(x^8+x^4+1),x, algorithm="maxima")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) - 1/2*integrate((x^2 - 1)/(x^4 - x^2 + 1), x)
```

3.342.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.75

$$\int \frac{1}{1+x^4+x^8}\,dx = \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x+1)\right) + \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right) + \frac{1}{12}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right) - \frac{1}{12}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)$$

input
```
integrate(1/(x^8+x^4+1),x, algorithm="giac")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) + 1/12*sqrt(3)*log(x^2 + sqrt(3)*x + 1) - 1/12*sqrt(3)*log(x^2
 - sqrt(3)*x + 1)
```

3.342.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.45

$$\int \frac{1}{1+x^4+x^8}\,dx = -\frac{\sqrt{3}\left(\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3\left(\frac{2\,x^2}{3}-\frac{2}{3}\right)}\right)-\operatorname{atanh}\left(\frac{2\sqrt{3}\,x}{3\left(\frac{2\,x^2}{3}+\frac{2}{3}\right)}\right)\right)}{6}$$

input
```
int(1/(x^4 + x^8 + 1),x)
```

output
```
-(3^(1/2)*(atan((2*3^(1/2)*x)/(3*((2*x^2)/3 - 2/3))) - atanh((2*3^(1/2)*x)
/(3*((2*x^2)/3 + 2/3)))))/6
```

3.343 $\int \frac{1}{x^2(1+x^4+x^8)} dx$

3.343.1 Optimal result

Integrand size = 14, antiderivative size = 145

$$\int \frac{1}{x^2(1+x^4+x^8)} dx = -\frac{1}{x}+\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\arctan\left(\sqrt{3}-2x\right)-\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}} -\frac{1}{4}\arctan\left(\sqrt{3}+2x\right)-\frac{1}{8}\log\left(1-x+x^2\right)+\frac{1}{8}\log\left(1+x+x^2\right) -\frac{\log\left(1-\sqrt{3}x+x^2\right)}{8\sqrt{3}}+\frac{\log\left(1+\sqrt{3}x+x^2\right)}{8\sqrt{3}}$$

output
```
-1/x-1/4*arctan(2*x-3^(1/2))-1/4*arctan(2*x+3^(1/2))-1/8*ln(x^2-x+1)+1/8*l
n(x^2+x+1)+1/12*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)-1/12*arctan(1/3*(1+2*x
)*3^(1/2))*3^(1/2)-1/24*ln(1+x^2-x*3^(1/2))*3^(1/2)+1/24*ln(1+x^2+x*3^(1/2
))*3^(1/2)
```

3.343.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^2(1+x^4+x^8)}\,dx = \frac{1}{24}\left(-\frac{24}{x}+2i\sqrt{-6+6i\sqrt{3}}\arctan\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x\right)\right.$$
$$-2i\sqrt{-6-6i\sqrt{3}}\arctan\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x\right)$$
$$-2\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)-2\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right)$$
$$\left.-3\log\left(1-x+x^2\right)+3\log\left(1+x+x^2\right)\right)$$

input
```
Integrate[1/(x^2*(1 + x^4 + x^8)),x]
```

output
```
(-24/x + (2*I)*Sqrt[-6 + (6*I)*Sqrt[3]]*ArcTan[((1 - I*Sqrt[3])*x)/2] - (2
*I)*Sqrt[-6 - (6*I)*Sqrt[3]]*ArcTan[((1 + I*Sqrt[3])*x)/2] - 2*Sqrt[3]*Arc
Tan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 3*Log[1 -
x + x^2] + 3*Log[1 + x + x^2])/24
```

3.343.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.14, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {1704, 25, 1830, 1447, 1475, 1083, 217, 1478, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(x^8+x^4+1)}\,dx$$
$$\Big\downarrow \text{1704}$$
$$\int -\frac{x^2(x^4+1)}{x^8+x^4+1}dx-\frac{1}{x}$$
$$\Big\downarrow \text{25}$$
$$-\int \frac{x^2(x^4+1)}{x^8+x^4+1}dx-\frac{1}{x}$$
$$\Big\downarrow \text{1830}$$
$$-\frac{1}{2}\int \frac{x^2}{x^4-x^2+1}dx-\frac{1}{2}\int \frac{x^2}{x^4+x^2+1}dx-\frac{1}{x}$$

$$\Big\downarrow \text{1447}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1-x^2}{x^4-x^2+1}dx-\frac{1}{2}\int \frac{x^2+1}{x^4-x^2+1}dx\right)+\frac{1}{2}\left(\frac{1}{2}\int \frac{1-x^2}{x^4+x^2+1}dx-\frac{1}{2}\int \frac{x^2+1}{x^4+x^2+1}dx\right)-\frac{1}{x}$$

$$\Big\downarrow \text{1475}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{x^2-\sqrt{3}x+1}dx-\frac{1}{2}\int \frac{1}{x^2+\sqrt{3}x+1}dx\right)+\frac{1}{2}\int \frac{1-x^2}{x^4-x^2+1}dx\right)+\frac{1}{2}\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{x^2-x+1}dx-\frac{1}{2}\int \frac{1}{x^2+x+1}dx\right)+\frac{1}{2}\int \frac{1-x^2}{x^4+x^2+1}dx\right)-\frac{1}{x}$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1-x^2}{x^4+x^2+1}dx+\frac{1}{2}\left(\int \frac{1}{-(2x-1)^2-3}d(2x-1)+\int \frac{1}{-(2x+1)^2-3}d(2x+1)\right)\right)+\frac{1}{2}\left(\frac{1}{2}\int \frac{1-x^2}{x^4-x^2+1}dx+\frac{1}{2}\left(\int \frac{1}{-\left(2x-\sqrt{3}\right)^2-1}d\left(2x-\sqrt{3}\right)+\int \frac{1}{-\left(2x+\sqrt{3}\right)^2-1}d\left(2x+\sqrt{3}\right)\right)\right)-\frac{1}{x}$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1-x^2}{x^4-x^2+1}dx+\frac{1}{2}\left(\arctan\left(\sqrt{3}-2x\right)-\arctan\left(2x+\sqrt{3}\right)\right)\right)+\frac{1}{2}\left(\frac{1}{2}\int \frac{1-x^2}{x^4+x^2+1}dx+\frac{1}{2}\left(-\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)\right)-\frac{1}{x}$$

$$\Big\downarrow \text{1478}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-\frac{1}{2}\int -\frac{1-2x}{x^2-x+1}dx-\frac{1}{2}\int -\frac{2x+1}{x^2+x+1}dx\right)+\frac{1}{2}\left(-\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)\right)+\frac{1}{2}\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}-\frac{\int -\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}\right)+\frac{1}{2}\left(\arctan\left(\sqrt{3}-2x\right)-\arctan\left(2x+\sqrt{3}\right)\right)\right)-\frac{1}{x}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx+\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx\right)+\frac{1}{2}\left(-\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)\right)+\frac{1}{2}\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}+\frac{\int \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}\right)+\frac{1}{2}\left(\arctan\left(\sqrt{3}-2x\right)-\arctan\left(2x+\sqrt{3}\right)\right)\right)-\frac{1}{x}$$

3.343. $\int \frac{1}{x^2(1+x^4+x^8)}\,dx$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)+\frac{1}{2}\left(\frac{1}{2}\log\left(x^2+x+1\right)-\frac{1}{2}\log\left(x^2-x+1\right)\right)\right)+\frac{1}{2}\left(\frac{1}{2}\left(\arctan\left(\sqrt{3}-2x\right)-\arctan\left(2x+\sqrt{3}\right)\right)+\frac{1}{2}\left(\frac{\log\left(x^2+\sqrt{3}x+1\right)}{2\sqrt{3}}-\frac{\log\left(x^2-\sqrt{3}x+1\right)}{2\sqrt{3}}\right)\right)-\frac{1}{x}$$

input
```
Int[1/(x^2*(1 + x^4 + x^8)),x]
```

output
```
-x^(-1) + ((-(ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3]) - ArcTan[(1 + 2*x)/Sqrt[
3]]/Sqrt[3])/2 + (-1/2*Log[1 - x + x^2] + Log[1 + x + x^2]/2)/2)/2 + ((Arc
Tan[Sqrt[3] - 2*x] - ArcTan[Sqrt[3] + 2*x])/2 + (-1/2*Log[1 - Sqrt[3]*x +
x^2]/Sqrt[3] + Log[1 + Sqrt[3]*x + x^2]/(2*Sqrt[3]))/2)/2
```

3.343.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1447
```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Simp[1/2   Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Simp[1/2
 Int[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b
^2 - 4*a*c, 0] && PosQ[a*c]
```

rule 1475

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 1478

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

rule 1704

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Simp[1/(a*d^n*(m + 1))   Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1)
+ c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{
a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] &&
LtQ[m, -1] && IntegerQ[p]
```

rule 1830

```
Int[(((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
(c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[a*c, 2]}, With[{r = Rt[2*c*q -
 b*c, 2]}, Simp[c/(2*q*r)   Int[(f*x)^m*(Simp[d*r - (c*d - e*q)*x^(n/2), x]
/(q - r*x^(n/2) + c*x^n)), x], x] + Simp[c/(2*q*r)   Int[(f*x)^m*(Simp[d*r
+ (c*d - e*q)*x^(n/2), x]/(q + r*x^(n/2) + c*x^n)), x], x]] /;  !LtQ[2*c*q
- b*c, 0]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[n2, 2*n] && LtQ[b^2 - 4*a
*c, 0] && IntegersQ[m, n/2] && LtQ[0, m, n] && PosQ[a*c]
```

3.343. $\int \frac{1}{x^2(1+x^4+x^8)} \, dx$

3.343.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.13 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.66

method	result
risch	$-\frac{1}{x}+\frac{\ln(4x^2+4x+4)}{8}-\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{12}-\frac{\ln(4x^2-4x+4)}{8}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12}+\frac{\left(\sum_{\_R=\mathrm{RootOf}(9\_Z^4+3\_Z^2+\cdots}\right.}{}$
default	$-\frac{\ln(x^2-x+1)}{8}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12}+\frac{\ln(x^2+x+1)}{8}-\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{12}-\frac{1}{x}+\frac{\sqrt{3}\left(-\frac{\ln\left(1+x^2-x\sqrt{3}\right)}{2}-\sqrt{3}\,\mathrm{arctan}\cdots\right.}{12}$

input
```
int(1/x^2/(x^8+x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/x+1/8*ln(4*x^2+4*x+4)-1/12*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-1/8*ln(4
*x^2-4*x+4)-1/12*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))+1/4*sum(_R*ln(-6*_R^3
-_R+x),_R=RootOf(9*_Z^4+3*_Z^2+1))
```

3.343.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.44

$$\int \frac{1}{x^2(1+x^4+x^8)}\,dx = -\frac{\sqrt{6}x\sqrt{i\sqrt{3}-1}\log\left(i\sqrt{6}\sqrt{3}\sqrt{i\sqrt{3}-1}+6x\right)-\sqrt{6}x\sqrt{i\sqrt{3}-1}\log\left(-i\sqrt{6}\sqrt{3}\sqrt{i\sqrt{3}-1}+6x\right)-\sqrt{\cdots}}{\cdots}$$

input
```
integrate(1/x^2/(x^8+x^4+1),x, algorithm="fricas")
```

output
```
-1/24*(sqrt(6)*x*sqrt(I*sqrt(3) - 1)*log(I*sqrt(6)*sqrt(3)*sqrt(I*sqrt(3)
- 1) + 6*x) - sqrt(6)*x*sqrt(I*sqrt(3) - 1)*log(-I*sqrt(6)*sqrt(3)*sqrt(I*
sqrt(3) - 1) + 6*x) - sqrt(6)*x*sqrt(-I*sqrt(3) - 1)*log(I*sqrt(6)*sqrt(3)
*sqrt(-I*sqrt(3) - 1) + 6*x) + sqrt(6)*x*sqrt(-I*sqrt(3) - 1)*log(-I*sqrt(
6)*sqrt(3)*sqrt(-I*sqrt(3) - 1) + 6*x) + 2*sqrt(3)*x*arctan(1/3*sqrt(3)*(2
*x + 1)) + 2*sqrt(3)*x*arctan(1/3*sqrt(3)*(2*x - 1)) - 3*x*log(x^2 + x + 1
) + 3*x*log(x^2 - x + 1) + 24)/x
```

3.343.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.50

$$\int \frac{1}{x^2\left(1+x^4+x^8\right)}\,dx$$
$$= \left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x-442368\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^7-384\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^3\right)$$
$$+\left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x-384\left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^3-442368\left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^7\right)$$
$$+\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x-442368\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^7-384\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^3\right)$$
$$+\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x-384\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^3-442368\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^7\right)$$
$$+\operatorname{RootSum}\left(2304t^4+48t^2+1,\left(t\mapsto t\log\left(-442368t^7-384t^3+x\right)\right)\right)-\frac{1}{x}$$

input
```
integrate(1/x**2/(x**8+x**4+1),x)
```

output
```
(-1/8 - sqrt(3)*I/24)*log(x - 442368*(-1/8 - sqrt(3)*I/24)**7 - 384*(-1/8
- sqrt(3)*I/24)**3) + (-1/8 + sqrt(3)*I/24)*log(x - 384*(-1/8 + sqrt(3)*I/
24)**3 - 442368*(-1/8 + sqrt(3)*I/24)**7) + (1/8 - sqrt(3)*I/24)*log(x - 4
42368*(1/8 - sqrt(3)*I/24)**7 - 384*(1/8 - sqrt(3)*I/24)**3) + (1/8 + sqrt
(3)*I/24)*log(x - 384*(1/8 + sqrt(3)*I/24)**3 - 442368*(1/8 + sqrt(3)*I/24
)**7) + RootSum(2304*_t**4 + 48*_t**2 + 1, Lambda(_t, _t*log(-442368*_t**7
 - 384*_t**3 + x))) - 1/x
```

3.343.7 Maxima [F]

$$\int \frac{1}{x^2(1+x^4+x^8)}\,dx = \int \frac{1}{(x^8+x^4+1)x^2}\,dx$$

input
```
integrate(1/x^2/(x^8+x^4+1),x, algorithm="maxima")
```

output
```
-1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x - 1)) - 1/x - 1/2*integrate(x^2/(x^4 - x^2 + 1), x) + 1/8*log(x^2
 + x + 1) - 1/8*log(x^2 - x + 1)
```

3.343.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.78

$$\begin{aligned}\int \frac{1}{x^2(1+x^4+x^8)}\,dx =& -\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) \\ &+ \frac{1}{24}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right) - \frac{1}{24}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right) \\ &- \frac{1}{x} - \frac{1}{4}\arctan\left(2x+\sqrt{3}\right) - \frac{1}{4}\arctan\left(2x-\sqrt{3}\right) \\ &+ \frac{1}{8}\log\left(x^2+x+1\right) - \frac{1}{8}\log\left(x^2-x+1\right)\end{aligned}$$

input
```
integrate(1/x^2/(x^8+x^4+1),x, algorithm="giac")
```

output
```
-1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x - 1)) + 1/24*sqrt(3)*log(x^2 + sqrt(3)*x + 1) - 1/24*sqrt(3)*log(
x^2 - sqrt(3)*x + 1) - 1/x - 1/4*arctan(2*x + sqrt(3)) - 1/4*arctan(2*x -
sqrt(3)) + 1/8*log(x^2 + x + 1) - 1/8*log(x^2 - x + 1)
```

3.343.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.70

$$\int \frac{1}{x^2 (1+x^4+x^8)} \, dx = \operatorname{atan}\left(\frac{2x}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) + \operatorname{atan}\left(\frac{2x}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) - \operatorname{atan}\left(\frac{x\,2\mathrm{i}}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{12}-\frac{1}{4}\mathrm{i}\right) - \operatorname{atan}\left(\frac{x\,2\mathrm{i}}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{12}+\frac{1}{4}\mathrm{i}\right) - \frac{1}{x}$$

input
```
int(1/(x^2*(x^4 + x^8 + 1)),x)
```

output
```
atan((2*x)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/12 + 1/4) + atan((2*x)/(3^(1/2)
*1i + 1))*((3^(1/2)*1i)/12 - 1/4) - atan((x*2i)/(3^(1/2)*1i - 1))*(3^(1/2)
/12 - 1i/4) - atan((x*2i)/(3^(1/2)*1i + 1))*(3^(1/2)/12 + 1i/4) - 1/x
```

3.344 $\int \frac{1}{x^4(1+x^4+x^8)} dx$

3.344.1 Optimal result

Integrand size = 14, antiderivative size = 147

$$\int \frac{1}{x^4(1+x^4+x^8)} dx = -\frac{1}{3x^3} + \frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{1}{4}\arctan\left(\sqrt{3}-2x\right) - \frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{4}\arctan\left(\sqrt{3}+2x\right) + \frac{1}{8}\log\left(1-x+x^2\right) - \frac{1}{8}\log\left(1+x+x^2\right) + \frac{\log\left(1-\sqrt{3}x+x^2\right)}{8\sqrt{3}} - \frac{\log\left(1+\sqrt{3}x+x^2\right)}{8\sqrt{3}}$$

output
```
-1/3/x^3-1/4*arctan(2*x-3^(1/2))-1/4*arctan(2*x+3^(1/2))+1/8*ln(x^2-x+1)-1
/8*ln(x^2+x+1)+1/12*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)-1/12*arctan(1/3*(1
+2*x)*3^(1/2))*3^(1/2)+1/24*ln(1+x^2-x*3^(1/2))*3^(1/2)-1/24*ln(1+x^2+x*3^
(1/2))*3^(1/2)
```

3.344.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.01

$$\int \frac{1}{x^4(1+x^4+x^8)}\,dx = \frac{1}{24}\left(-\frac{8}{x^3}-\frac{4i\arctan\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x\right)}{\sqrt{\frac{1}{6}i\left(i+\sqrt{3}\right)}}+\frac{4i\arctan\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x\right)}{\sqrt{-\frac{1}{6}i\left(-i+\sqrt{3}\right)}} - 2\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)-2\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right) + 3\log\left(1-x+x^2\right)-3\log\left(1+x+x^2\right)\right)$$

input
```
Integrate[1/(x^4*(1 + x^4 + x^8)),x]
```

output
```
(-8/x^3 - ((4*I)*ArcTan[((1 - I*Sqrt[3])*x)/2])/Sqrt[(I/6)*(I + Sqrt[3])]
+ ((4*I)*ArcTan[((1 + I*Sqrt[3])*x)/2])/Sqrt[(-1/6*I)*(-I + Sqrt[3])] - 2*
Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] +
 3*Log[1 - x + x^2] - 3*Log[1 + x + x^2])/24
```

3.344.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.21, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {1704, 27, 1749, 1407, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4(x^8+x^4+1)}dx$$

$$\Big\downarrow \text{1704}$$

$$\frac{1}{3}\int -\frac{3(x^4+1)}{x^8+x^4+1}dx-\frac{1}{3x^3}$$

$$\Big\downarrow \text{27}$$

$$-\int \frac{x^4+1}{x^8+x^4+1}dx-\frac{1}{3x^3}$$

$$\Big\downarrow \text{1749}$$

$$-\frac{1}{2}\int \frac{1}{x^4-x^2+1}dx-\frac{1}{2}\int \frac{1}{x^4+x^2+1}dx-\frac{1}{3x^3}$$

$$\Big\downarrow \text{1407}$$

$$\frac{1}{2}\left(-\frac{1}{2}\int \frac{1-x}{x^2-x+1}dx-\frac{1}{2}\int \frac{x+1}{x^2+x+1}dx\right)+\frac{1}{2}\left(-\frac{\int \frac{\sqrt{3}-x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}-\frac{\int \frac{x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}\right)-\frac{1}{3x^3}$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\int -\frac{1-2x}{x^2-x+1}dx-\frac{1}{2}\int \frac{1}{x^2-x+1}dx\right)+\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{x^2+x+1}dx-\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx\right)\right)+ \frac{1}{2}\left(-\frac{\frac{1}{2}\sqrt{3}\int \frac{1}{x^2-\sqrt{3}x+1}dx-\frac{1}{2}\int -\frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}-\frac{\frac{1}{2}\sqrt{3}\int \frac{1}{x^2+\sqrt{3}x+1}dx+\frac{1}{2}\int \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}\right)-\frac{1}{3x^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{x^2-x+1}dx-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{x^2+x+1}dx-\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx\right)\right)+ \frac{1}{2}\left(-\frac{\frac{1}{2}\sqrt{3}\int \frac{1}{x^2-\sqrt{3}x+1}dx+\frac{1}{2}\int \frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}-\frac{\frac{1}{2}\sqrt{3}\int \frac{1}{x^2+\sqrt{3}x+1}dx+\frac{1}{2}\int \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}\right)-\frac{1}{3x^3}$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\int \frac{1}{-(2x-1)^2-3}d(2x-1)-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{2}\left(\int \frac{1}{-(2x+1)^2-3}d(2x+1)-\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx\right)\right)+ \frac{1}{2}\left(-\frac{\frac{1}{2}\int \frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx-\sqrt{3}\int \frac{1}{-\left(2x-\sqrt{3}\right)^2-1}d\left(2x-\sqrt{3}\right)}{2\sqrt{3}}-\frac{\frac{1}{2}\int \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx-\sqrt{3}\int \frac{1}{-\left(2x+\sqrt{3}\right)^2-1}d\left(2x+\sqrt{3}\right)}{2\sqrt{3}}\right)- \frac{1}{3x^3}$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx-\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}\right)+\frac{1}{2}\left(-\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx-\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)\right)+ \frac{1}{2}\left(-\frac{\frac{1}{2}\int \frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx-\sqrt{3}\arctan\left(\sqrt{3}-2x\right)}{2\sqrt{3}}-\frac{\frac{1}{2}\int \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx+\sqrt{3}\arctan\left(2x+\sqrt{3}\right)}{2\sqrt{3}}\right)-\frac{1}{3x^3}$$

$$\Big\downarrow \text{1103}$$

3.344. $\int \frac{1}{x^4(1+x^4+x^8)}\,dx$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\log\left(x^2-x+1\right)-\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}\right)+\frac{1}{2}\left(-\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}\log\left(x^2+x+1\right)\right)\right)+\frac{1}{2}\left(-\frac{-\sqrt{3}\arctan\left(\sqrt{3}-2x\right)-\frac{1}{2}\log\left(x^2-\sqrt{3}x+1\right)}{2\sqrt{3}}-\frac{\sqrt{3}\arctan\left(2x+\sqrt{3}\right)+\frac{1}{2}\log\left(x^2+\sqrt{3}x+1\right)}{2\sqrt{3}}\right)-\frac{1}{3x^3}$$

input
```
Int[1/(x^4*(1 + x^4 + x^8)),x]
```

output
```
-1/3*1/x^3 + ((-(ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3]) + Log[1 - x + x^2]/2)
/2 + (-(ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3]) - Log[1 + x + x^2]/2)/2)/2 + (-
1/2*(-(Sqrt[3]*ArcTan[Sqrt[3] - 2*x]) - Log[1 - Sqrt[3]*x + x^2]/2)/Sqrt[3
] - (Sqrt[3]*ArcTan[Sqrt[3] + 2*x] + Log[1 + Sqrt[3]*x + x^2]/2)/(2*Sqrt[3
]))/2
```

3.344.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1407
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   Int[(r - x)/(q - r*
x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(r + x)/(q + r*x + x^2), x], x]]]
 /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

rule 1704
```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Simp[1/(a*d^n*(m + 1))   Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1)
+ c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{
a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] &&
LtQ[m, -1] && IntegerQ[p]
```

rule 1749
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e
+ q*x^(n/2) + x^n, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x^(n/2) +
 x^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[2*(d/e) - b/c,
0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

3.344.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.63

method	result
risch	$-\frac{1}{3x^3}-\frac{\ln(4x^2+4x+4)}{8}-\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{12}+\frac{\left(\sum_{\_R=\mathrm{RootOf}(9\_Z^4+3\_Z^2+1)}\_R\ln\left(3\_R^3-\_R+x\right)\right)}{4}+\frac{\ln(x^2-x}{8}$
default	$\frac{\ln(x^2-x+1)}{8}-\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12}-\frac{\ln(x^2+x+1)}{8}-\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{12}-\frac{1}{3x^3}+\frac{\ln\left(1+x^2-x\sqrt{3}\right)\sqrt{3}}{24}-\frac{\arctan(2}{4}$

input
```
int(1/x^4/(x^8+x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/3/x^3-1/8*ln(4*x^2+4*x+4)-1/12*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/4*
sum(_R*ln(3*_R^3-_R+x),_R=RootOf(9*_Z^4+3*_Z^2+1))+1/8*ln(x^2-x+1)-1/12*3^
(1/2)*arctan(2/3*(x-1/2)*3^(1/2))
```

3.344.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.61

$$\int \frac{1}{x^4 (1 + x^4 + x^8)} \, dx$$
$$= \frac{\sqrt{6}x^3\sqrt{i\sqrt{3}-1}\log\left(\sqrt{6}\sqrt{i\sqrt{3}-1}(i\sqrt{3}-3)+12\,x\right)-\sqrt{6}x^3\sqrt{i\sqrt{3}-1}\log\left(\sqrt{6}\sqrt{i\sqrt{3}-1}(-i\sqrt{3}+3\right.}{}$$

input
```
integrate(1/x^4/(x^8+x^4+1),x, algorithm="fricas")
```

output
```
1/24*(sqrt(6)*x^3*sqrt(I*sqrt(3) - 1)*log(sqrt(6)*sqrt(I*sqrt(3) - 1)*(I*s
qrt(3) - 3) + 12*x) - sqrt(6)*x^3*sqrt(I*sqrt(3) - 1)*log(sqrt(6)*sqrt(I*s
qrt(3) - 1)*(-I*sqrt(3) + 3) + 12*x) - sqrt(6)*x^3*sqrt(-I*sqrt(3) - 1)*lo
g(sqrt(6)*(I*sqrt(3) + 3)*sqrt(-I*sqrt(3) - 1) + 12*x) + sqrt(6)*x^3*sqrt(
-I*sqrt(3) - 1)*log(sqrt(6)*sqrt(-I*sqrt(3) - 1)*(-I*sqrt(3) - 3) + 12*x)
- 2*sqrt(3)*x^3*arctan(1/3*sqrt(3)*(2*x + 1)) - 2*sqrt(3)*x^3*arctan(1/3*s
qrt(3)*(2*x - 1)) - 3*x^3*log(x^2 + x + 1) + 3*x^3*log(x^2 - x + 1) - 8)/x
^3
```

3.344.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.34

$$\int \frac{1}{x^4(1+x^4+x^8)}\,dx = \left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x-1-\frac{\sqrt{3}i}{3}-9216\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^5\right) + \left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x-1-9216\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^5+\frac{\sqrt{3}i}{3}\right) + \left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x+1-\frac{\sqrt{3}i}{3}-9216\left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^5\right) + \left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x+1-9216\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^5+\frac{\sqrt{3}i}{3}\right) + \operatorname{RootSum}\left(2304t^4+48t^2+1,\left(t\mapsto t\log\left(-9216t^5-8t+x\right)\right)\right) - \frac{1}{3x^3}$$

input
```
integrate(1/x**4/(x**8+x**4+1),x)
```

output
```
(1/8 + sqrt(3)*I/24)*log(x - 1 - sqrt(3)*I/3 - 9216*(1/8 + sqrt(3)*I/24)**
5) + (1/8 - sqrt(3)*I/24)*log(x - 1 - 9216*(1/8 - sqrt(3)*I/24)**5 + sqrt(
3)*I/3) + (-1/8 + sqrt(3)*I/24)*log(x + 1 - sqrt(3)*I/3 - 9216*(-1/8 + sqr
t(3)*I/24)**5) + (-1/8 - sqrt(3)*I/24)*log(x + 1 - 9216*(-1/8 - sqrt(3)*I/
24)**5 + sqrt(3)*I/3) + RootSum(2304*_t**4 + 48*_t**2 + 1, Lambda(_t, _t*l
og(-9216*_t**5 - 8*_t + x))) - 1/(3*x**3)
```

3.344.7 Maxima [F]

$$\int \frac{1}{x^4(1+x^4+x^8)}\,dx = \int \frac{1}{(x^8+x^4+1)x^4}\,dx$$

input
```
integrate(1/x^4/(x^8+x^4+1),x, algorithm="maxima")
```

output
```
-1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x - 1)) - 1/3/x^3 - 1/2*integrate(1/(x^4 - x^2 + 1), x) - 1/8*log(x
^2 + x + 1) + 1/8*log(x^2 - x + 1)
```

3.344.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.77

$$
\begin{aligned}
\int \frac{1}{x^4\left(1+x^4+x^8\right)}\,dx = &-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x+1)\right)-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right)\\
&-\frac{1}{24}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)+\frac{1}{24}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)\\
&-\frac{1}{3\,x^3}-\frac{1}{4}\arctan\left(2\,x+\sqrt{3}\right)-\frac{1}{4}\arctan\left(2\,x-\sqrt{3}\right)\\
&-\frac{1}{8}\log\left(x^2+x+1\right)+\frac{1}{8}\log\left(x^2-x+1\right)
\end{aligned}
$$

input
```
integrate(1/x^4/(x^8+x^4+1),x, algorithm="giac")
```

output
```
-1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x - 1)) - 1/24*sqrt(3)*log(x^2 + sqrt(3)*x + 1) + 1/24*sqrt(3)*log(
x^2 - sqrt(3)*x + 1) - 1/3/x^3 - 1/4*arctan(2*x + sqrt(3)) - 1/4*arctan(2*
x - sqrt(3)) - 1/8*log(x^2 + x + 1) + 1/8*log(x^2 - x + 1)
```

3.344.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.71

$$
\begin{aligned}
\int \frac{1}{x^4\left(1+x^4+x^8\right)}\,dx = &-\mathrm{atan}\left(\frac{2\,x}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)\\
&-\mathrm{atan}\left(\frac{2\,x}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)\\
&-\mathrm{atan}\left(\frac{x\,2\mathrm{i}}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{12}+\frac{1}{4}\mathrm{i}\right)\\
&-\mathrm{atan}\left(\frac{x\,2\mathrm{i}}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{12}-\frac{1}{4}\mathrm{i}\right)-\frac{1}{3\,x^3}
\end{aligned}
$$

input
```
int(1/(x^4*(x^4 + x^8 + 1)),x)
```

output
```
- atan((2*x)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/12 - 1/4) - atan((2*x)/(3^(1/
2)*1i + 1))*((3^(1/2)*1i)/12 + 1/4) - atan((x*2i)/(3^(1/2)*1i - 1))*(3^(1/
2)/12 + 1i/4) - atan((x*2i)/(3^(1/2)*1i + 1))*(3^(1/2)/12 - 1i/4) - 1/(3*x
^3)
```

3.344. $\int \frac{1}{x^4(1+x^4+x^8)}\,dx$

3.345 $\int \frac{1}{x^6(1+x^4+x^8)} dx$

3.345.1 Optimal result

Integrand size = 14, antiderivative size = 98

$$\int \frac{1}{x^6(1+x^4+x^8)} dx = -\frac{1}{5x^5}+\frac{1}{x}-\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\log\left(1-\sqrt{3}x+x^2\right)}{4\sqrt{3}}-\frac{\log\left(1+\sqrt{3}x+x^2\right)}{4\sqrt{3}}$$

output
```
-1/5/x^5+1/x-1/6*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/6*arctan(1/3*(1+2*x
)*3^(1/2))*3^(1/2)+1/12*ln(1+x^2-x*3^(1/2))*3^(1/2)-1/12*ln(1+x^2+x*3^(1/2
))*3^(1/2)
```

3.345.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^6(1+x^4+x^8)} dx = \frac{1}{60}\left(-\frac{12}{x^5}+\frac{60}{x}+10\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)+10\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right)+5\sqrt{3}\log\left(-1+\sqrt{3}x-x^2\right)-5\sqrt{3}\log\left(1+\sqrt{3}x+x^2\right)\right)$$

input
```
Integrate[1/(x^6*(1 + x^4 + x^8)),x]
```

output
```
(-12/x^5 + 60/x + 10*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 10*Sqrt[3]*ArcTa
n[(1 + 2*x)/Sqrt[3]] + 5*Sqrt[3]*Log[-1 + Sqrt[3]*x - x^2] - 5*Sqrt[3]*Log
[1 + Sqrt[3]*x + x^2])/60
```

3.345.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {1704, 27, 1828, 1708, 1475, 1083, 217, 1478, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 (x^8+x^4+1)} \, dx$$

$$\Big\downarrow \text{1704}$$

$$\frac{1}{5} \int -\frac{5(x^4+1)}{x^2 (x^8+x^4+1)} dx - \frac{1}{5x^5}$$

$$\Big\downarrow \text{27}$$

$$-\int \frac{x^4+1}{x^2 (x^8+x^4+1)} dx - \frac{1}{5x^5}$$

$$\Big\downarrow \text{1828}$$

$$\int \frac{x^6}{x^8+x^4+1} dx - \frac{1}{5x^5} + \frac{1}{x}$$

$$\Big\downarrow \text{1708}$$

$$-\frac{1}{2} \int \frac{1-x^2}{x^4-x^2+1} dx + \frac{1}{2} \int \frac{x^2+1}{x^4+x^2+1} dx - \frac{1}{5x^5} + \frac{1}{x}$$

$$\Big\downarrow \text{1475}$$

$$\frac{1}{2} \left(\frac{1}{2} \int \frac{1}{x^2-x+1} dx + \frac{1}{2} \int \frac{1}{x^2+x+1} dx \right) - \frac{1}{2} \int \frac{1-x^2}{x^4-x^2+1} dx - \frac{1}{5x^5} + \frac{1}{x}$$

$$\Big\downarrow \text{1083}$$

$$-\frac{1}{2} \int \frac{1-x^2}{x^4-x^2+1} dx + \frac{1}{2} \left(-\int \frac{1}{-(2x-1)^2-3} d(2x-1) - \int \frac{1}{-(2x+1)^2-3} d(2x+1) \right) - \frac{1}{5x^5} + \frac{1}{x}$$

$$\Big\downarrow \text{217}$$

$$-\frac{1}{2}\int \frac{1-x^2}{x^4-x^2+1}dx+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)-\frac{1}{5x^5}+\frac{1}{x}$$

$\big\downarrow$ 1478

$$\frac{1}{2}\left(\frac{\int -\frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}+\frac{\int -\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)-\frac{1}{5x^5}+\frac{1}{x}$$

$\big\downarrow$ 25

$$\frac{1}{2}\left(-\frac{\int \frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}-\frac{\int \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)-\frac{1}{5x^5}+\frac{1}{x}$$

$\big\downarrow$ 1103

$$\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)-\frac{1}{5x^5}+\frac{1}{2}\left(\frac{\log\left(x^2-\sqrt{3}x+1\right)}{2\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}x+1\right)}{2\sqrt{3}}\right)+\frac{1}{x}$$

input
```
Int[1/(x^6*(1 + x^4 + x^8)),x]
```

output
```
-1/5*1/x^5 + x^(-1) + (ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3] + ArcTan[(1 + 2*
x)/Sqrt[3]]/Sqrt[3])/2 + (Log[1 - Sqrt[3]*x + x^2]/(2*Sqrt[3]) - Log[1 + S
qrt[3]*x + x^2]/(2*Sqrt[3]))/2
```

3.345.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

3.345. $\int \frac{1}{x^6(1+x^4+x^8)}\,dx$

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

rule 1704
```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Simp[1/(a*d^n*(m + 1))   Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1)
+ c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{
a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] &&
LtQ[m, -1] && IntegerQ[p]
```

rule 1708
```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, -Simp[1/(2*c*r)   Int[x^
(m - 3*(n/2))*((q - r*x^(n/2))/(q - r*x^(n/2) + x^n)), x], x] + Simp[1/(2*c
*r)   Int[x^(m - 3*(n/2))*((q + r*x^(n/2))/(q + r*x^(n/2) + x^n)), x], x]]]
 /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n/2,
 0] && IGtQ[m, 0] && GeQ[m, 3*(n/2)] && LtQ[m, 2*n] && NegQ[b^2 - 4*a*c]
```

3.345. $\int \frac{1}{x^6(1+x^4+x^8)}\,dx$

rule 1828
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Simp[1/(a*f^n*(m + 1))   Int[(f*x)^(m +
 n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) -
c*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x
] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Int
egerQ[p]
```

3.345.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.77

method	result	size
default	$-\frac{1}{5x^5}+\frac{1}{x}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}+\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{6}+\frac{\ln\left(1+x^2-x\sqrt{3}\right)\sqrt{3}}{12}-\frac{\ln\left(1+x^2+x\sqrt{3}\right)\sqrt{3}}{12}$	75
risch	$\frac{x^4-\frac{1}{5}}{x^5}+\frac{\ln\left(1+x^2-x\sqrt{3}\right)\sqrt{3}}{12}-\frac{\ln\left(1+x^2+x\sqrt{3}\right)\sqrt{3}}{12}+\frac{\sqrt{3}\,\arctan\left(\frac{x\sqrt{3}}{3}\right)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{x^3\sqrt{3}}{3}+\frac{2x\sqrt{3}}{3}\right)}{6}$	77

input
```
int(1/x^6/(x^8+x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/5/x^5+1/x+1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))+1/6*arctan(1/3*(1+2*x
)*3^(1/2))*3^(1/2)+1/12*ln(1+x^2-x*3^(1/2))*3^(1/2)-1/12*ln(1+x^2+x*3^(1/2
))*3^(1/2)
```

3.345.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^6\left(1+x^4+x^8\right)}\,dx$$
$$=\frac{10\,\sqrt{3}x^5\arctan\left(\frac{1}{3}\,\sqrt{3}(x^3+2\,x)\right)+10\,\sqrt{3}x^5\arctan\left(\frac{1}{3}\,\sqrt{3}x\right)+5\,\sqrt{3}x^5\log\left(\frac{x^4+5\,x^2-2\,\sqrt{3}(x^3+x)+1}{x^4-x^2+1}\right)+60\,x^4-}{60\,x^5}$$

input
```
integrate(1/x^6/(x^8+x^4+1),x, algorithm="fricas")
```

output
```
1/60*(10*sqrt(3)*x^5*arctan(1/3*sqrt(3)*(x^3 + 2*x)) + 10*sqrt(3)*x^5*arct
an(1/3*sqrt(3)*x) + 5*sqrt(3)*x^5*log((x^4 + 5*x^2 - 2*sqrt(3)*(x^3 + x) +
 1)/(x^4 - x^2 + 1)) + 60*x^4 - 12)/x^5
```

3.345.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^6\left(1+x^4+x^8\right)}\,dx=\frac{\sqrt{3}\cdot\left(2\operatorname{atan}\left(\frac{\sqrt{3}x}{3}\right)+2\operatorname{atan}\left(\frac{\sqrt{3}x^3}{3}+\frac{2\sqrt{3}x}{3}\right)\right)}{12}+\frac{\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)}{12}-\frac{\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)}{12}+\frac{5x^4-1}{5x^5}$$

input
```
integrate(1/x**6/(x**8+x**4+1),x)
```

output
```
sqrt(3)*(2*atan(sqrt(3)*x/3) + 2*atan(sqrt(3)*x**3/3 + 2*sqrt(3)*x/3))/12
+ sqrt(3)*log(x**2 - sqrt(3)*x + 1)/12 - sqrt(3)*log(x**2 + sqrt(3)*x + 1)
/12 + (5*x**4 - 1)/(5*x**5)
```

3.345.7 Maxima [F]

$$\int \frac{1}{x^6\left(1+x^4+x^8\right)}\,dx=\int \frac{1}{(x^8+x^4+1)x^6}\,dx$$

input
```
integrate(1/x^6/(x^8+x^4+1),x, algorithm="maxima")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) + 1/5*(5*x^4 - 1)/x^5 + 1/2*integrate((x^2 - 1)/(x^4 - x^2 + 1
), x)
```

3.345.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.02

$$\int \frac{1}{x^6\left(1+x^4+x^8\right)}\,dx=\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{24}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)+\frac{1}{24}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)+\frac{5x^4-1}{5x^5}+\frac{1}{4}\arctan\left(2x+\sqrt{3}\right)+\frac{1}{4}\arctan\left(2x-\sqrt{3}\right)$$

input
```
integrate(1/x^6/(x^8+x^4+1),x, algorithm="giac")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) - 1/24*sqrt(3)*log(x^2 + sqrt(3)*x + 1) + 1/24*sqrt(3)*log(x^2
 - sqrt(3)*x + 1) + 1/5*(5*x^4 - 1)/x^5 + 1/4*arctan(2*x + sqrt(3)) + 1/4*
arctan(2*x - sqrt(3))
```

3.345.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.53

$$\int \frac{1}{x^6\,(1+x^4+x^8)}\,dx=\frac{x^4-\frac{1}{5}}{x^5}-\frac{\sqrt{3}\,\mathrm{atanh}\left(\frac{2\,\sqrt{3}\,x}{3\,\left(\frac{2\,x^2}{3}+\frac{2}{3}\right)}\right)}{6}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\,\sqrt{3}\,x}{3\,\left(\frac{2\,x^2}{3}-\frac{2}{3}\right)}\right)}{6}$$

input
```
int(1/(x^6*(x^4 + x^8 + 1)),x)
```

output
```
(x^4 - 1/5)/x^5 - (3^(1/2)*atanh((2*3^(1/2)*x)/(3*((2*x^2)/3 + 2/3))))/6 -
 (3^(1/2)*atan((2*3^(1/2)*x)/(3*((2*x^2)/3 - 2/3))))/6
```

3.346 $\int \frac{1}{x^8(1+x^4+x^8)}\,dx$

3.346.1 Optimal result

Integrand size = 14, antiderivative size = 154

$$\int \frac{1}{x^8(1+x^4+x^8)}\,dx = -\frac{1}{7x^7}+\frac{1}{3x^3}+\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\arctan\left(\sqrt{3}-2x\right)-\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\arctan\left(\sqrt{3}+2x\right)-\frac{1}{8}\log\left(1-x+x^2\right)+\frac{1}{8}\log\left(1+x+x^2\right)+\frac{\log\left(1-\sqrt{3}x+x^2\right)}{8\sqrt{3}}-\frac{\log\left(1+\sqrt{3}x+x^2\right)}{8\sqrt{3}}$$

output
```
-1/7/x^7+1/3/x^3+1/4*arctan(2*x-3^(1/2))+1/4*arctan(2*x+3^(1/2))-1/8*ln(x^
2-x+1)+1/8*ln(x^2+x+1)+1/12*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)-1/12*arcta
n(1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/24*ln(1+x^2-x*3^(1/2))*3^(1/2)-1/24*ln(1+
x^2+x*3^(1/2))*3^(1/2)
```

3.346.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^8(1+x^4+x^8)}\,dx = -\frac{1}{7x^7}+\frac{1}{3x^3}+\frac{(i+\sqrt{3})\arctan\left(\frac{1}{2}(1-i\sqrt{3})\,x\right)}{2\sqrt{-6+6i\sqrt{3}}} + \frac{(-i+\sqrt{3})\arctan\left(\frac{1}{2}(1+i\sqrt{3})\,x\right)}{2\sqrt{-6-6i\sqrt{3}}} - \frac{\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{8}\log\left(1-x+x^2\right)+\frac{1}{8}\log\left(1+x+x^2\right)$$

input
```
Integrate[1/(x^8*(1 + x^4 + x^8)),x]
```

output
```
-1/7*1/x^7 + 1/(3*x^3) + ((I + Sqrt[3])*ArcTan[((1 - I*Sqrt[3])*x)/2])/(2*
Sqrt[-6 + (6*I)*Sqrt[3]]) + ((-I + Sqrt[3])*ArcTan[((1 + I*Sqrt[3])*x)/2])
/(2*Sqrt[-6 - (6*I)*Sqrt[3]]) - ArcTan[(-1 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) - A
rcTan[(1 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) - Log[1 - x + x^2]/8 + Log[1 + x + x^
2]/8
```

3.346.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.13, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {1704, 27, 1828, 27, 1709, 1447, 1475, 1083, 217, 1478, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^8(x^8+x^4+1)}dx$$

$$\Big\downarrow \text{1704}$$

$$\frac{1}{7}\int -\frac{7(x^4+1)}{x^4(x^8+x^4+1)}dx-\frac{1}{7x^7}$$

$$\Big\downarrow \text{27}$$

$$-\int \frac{x^4+1}{x^4(x^8+x^4+1)}dx-\frac{1}{7x^7}$$

$$\Big\downarrow \text{1828}$$

$$\frac{1}{3}\int \frac{3x^4}{x^8+x^4+1}dx-\frac{1}{7x^7}+\frac{1}{3x^3}$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{x^4}{x^8+x^4+1}dx-\frac{1}{7x^7}+\frac{1}{3x^3}$$

$$\Big\downarrow \text{ 1709}$$

$$\frac{1}{2}\int \frac{x^2}{x^4-x^2+1}dx-\frac{1}{2}\int \frac{x^2}{x^4+x^2+1}dx-\frac{1}{7x^7}+\frac{1}{3x^3}$$

$$\Big\downarrow \text{ 1447}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{x^2+1}{x^4-x^2+1}dx-\frac{1}{2}\int \frac{1-x^2}{x^4-x^2+1}dx\right)+\frac{1}{2}\left(\frac{1}{2}\int \frac{1-x^2}{x^4+x^2+1}dx-\frac{1}{2}\int \frac{x^2+1}{x^4+x^2+1}dx\right)-\frac{1}{7x^7}+\frac{1}{3x^3}$$

$$\Big\downarrow \text{ 1475}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2-\sqrt{3}x+1}dx+\frac{1}{2}\int \frac{1}{x^2+\sqrt{3}x+1}dx\right)-\frac{1}{2}\int \frac{1-x^2}{x^4-x^2+1}dx\right)+\frac{1}{2}\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{x^2-x+1}dx-\frac{1}{2}\int \frac{1}{x^2+x+1}dx\right)+\frac{1}{2}\int \frac{1-x^2}{x^4+x^2+1}dx\right)-\frac{1}{7x^7}+\frac{1}{3x^3}$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1-x^2}{x^4+x^2+1}dx+\frac{1}{2}\left(\int \frac{1}{-(2x-1)^2-3}d(2x-1)+\int \frac{1}{-(2x+1)^2-3}d(2x+1)\right)\right)+\frac{1}{2}\left(\frac{1}{2}\left(-\int \frac{1}{-\left(2x-\sqrt{3}\right)^2-1}d\left(2x-\sqrt{3}\right)-\int \frac{1}{-\left(2x+\sqrt{3}\right)^2-1}d\left(2x+\sqrt{3}\right)\right)-\frac{1}{2}\int \frac{1-x^2}{x^4-x^2+1}dx\right)-\frac{1}{7x^7}+\frac{1}{3x^3}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\arctan\left(2x+\sqrt{3}\right)-\arctan\left(\sqrt{3}-2x\right)\right)-\frac{1}{2}\int \frac{1-x^2}{x^4-x^2+1}dx\right)+\frac{1}{2}\left(\frac{1}{2}\int \frac{1-x^2}{x^4+x^2+1}dx+\frac{1}{2}\left(-\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)\right)-\frac{1}{7x^7}+\frac{1}{3x^3}$$

$$\Big\downarrow \text{ 1478}$$

3.346. $\int \frac{1}{x^8(1+x^4+x^8)}\,dx$

$$\frac{1}{2}\left(\frac{1}{2}\left(-\frac{1}{2}\int -\frac{1-2x}{x^2-x+1}dx-\frac{1}{2}\int -\frac{2x+1}{x^2+x+1}dx\right)+\frac{1}{2}\left(-\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)\right)+\frac{1}{2}\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}+\frac{\int -\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}\right)+\frac{1}{2}\left(\arctan\left(2x+\sqrt{3}\right)-\arctan\left(\sqrt{3}-2x\right)\right)\right)-\frac{1}{7x^7}+\frac{1}{3x^3}$$

$$\Big\downarrow \ 25$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx+\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx\right)+\frac{1}{2}\left(-\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)\right)+\frac{1}{2}\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}-\frac{\int \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}\right)+\frac{1}{2}\left(\arctan\left(2x+\sqrt{3}\right)-\arctan\left(\sqrt{3}-2x\right)\right)\right)-\frac{1}{7x^7}+\frac{1}{3x^3}$$

$$\Big\downarrow \ 1103$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)+\frac{1}{2}\left(\frac{1}{2}\log\left(x^2+x+1\right)-\frac{1}{2}\log\left(x^2-x+1\right)\right)\right)+\frac{1}{2}\left(\frac{1}{2}\left(\arctan\left(2x+\sqrt{3}\right)-\arctan\left(\sqrt{3}-2x\right)\right)+\frac{1}{2}\left(\frac{\log\left(x^2-\sqrt{3}x+1\right)}{2\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}x+1\right)}{2\sqrt{3}}\right)\right)-\frac{1}{7x^7}+\frac{1}{3x^3}$$

input
```
Int[1/(x^8*(1 + x^4 + x^8)),x]
```

output
```
-1/7*1/x^7 + 1/(3*x^3) + ((-(ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3]) - ArcTan[
(1 + 2*x)/Sqrt[3]]/Sqrt[3])/2 + (-1/2*Log[1 - x + x^2] + Log[1 + x + x^2]/
2)/2)/2 + ((-ArcTan[Sqrt[3] - 2*x] + ArcTan[Sqrt[3] + 2*x])/2 + (Log[1 - S
qrt[3]*x + x^2]/(2*Sqrt[3]) - Log[1 + Sqrt[3]*x + x^2]/(2*Sqrt[3]))/2)/2
```

3.346. $\int \frac{1}{x^8(1+x^4+x^8)}\,dx$

3.346.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1447
```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Simp[1/2   Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Simp[1/2
 Int[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b
^2 - 4*a*c, 0] && PosQ[a*c]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

3.346. $\int \frac{1}{x^8(1+x^4+x^8)}\,dx$

rule 1704
```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Simp[1/(a*d^n*(m + 1))   Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1)
+ c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{
a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] &&
LtQ[m, -1] && IntegerQ[p]
```

rule 1709
```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*r)   Int[x^(
m - n/2)/(q - r*x^(n/2) + x^n), x], x] - Simp[1/(2*c*r)   Int[x^(m - n/2)/(
q + r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && IGtQ[m, 0] && GeQ[m, n/2] && LtQ[m, 3*
(n/2)] && NegQ[b^2 - 4*a*c]
```

rule 1828
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Simp[1/(a*f^n*(m + 1))   Int[(f*x)^(m +
 n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) -
c*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x
] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Int
egerQ[p]
```

3.346.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.14 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.62

method	result
risch	$\frac{\frac{x^4}{3}-\frac{1}{7}}{x^7}+\frac{\ln\left(4x^2+4x+4\right)}{8}-\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{12}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(9\_Z^4+3\_Z^2+1\right)}\_R\ln\left(6\_R^3+\_R+x\right)\right)}{4}-\frac{\ln\left(x^2-x\right.}{8}$
default	$-\frac{1}{7x^7}+\frac{1}{3x^3}-\frac{\ln\left(x^2-x+1\right)}{8}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12}+\frac{\ln\left(x^2+x+1\right)}{8}-\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{12}-\frac{\sqrt{3}\left(-\frac{\ln\left(1+x^2-x\sqrt{3}\right)}{2}\right.}{1}$

input
```
int(1/x^8/(x^8+x^4+1),x,method=_RETURNVERBOSE)
```

output
```
(1/3*x^4-1/7)/x^7+1/8*ln(4*x^2+4*x+4)-1/12*arctan(1/3*(1+2*x)*3^(1/2))*3^(
1/2)+1/4*sum(_R*ln(6*_R^3+_R+x),_R=RootOf(9*_Z^4+3*_Z^2+1))-1/8*ln(x^2-x+1
)-1/12*3^(1/2)*arctan(2/3*(x-1/2)*3^(1/2))
```

3.346.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.51

$$\int \frac{1}{x^8 (1 + x^4 + x^8)} dx$$
$$= \frac{7\sqrt{6}x^7\sqrt{i\sqrt{3}-1}\log\left(i\sqrt{6}\sqrt{3}\sqrt{i\sqrt{3}-1}+6x\right) - 7\sqrt{6}x^7\sqrt{i\sqrt{3}-1}\log\left(-i\sqrt{6}\sqrt{3}\sqrt{i\sqrt{3}-1}+6x\right) -}{}$$

input
```
integrate(1/x^8/(x^8+x^4+1),x, algorithm="fricas")
```

output
```
1/168*(7*sqrt(6)*x^7*sqrt(I*sqrt(3) - 1)*log(I*sqrt(6)*sqrt(3)*sqrt(I*sqrt
(3) - 1) + 6*x) - 7*sqrt(6)*x^7*sqrt(I*sqrt(3) - 1)*log(-I*sqrt(6)*sqrt(3)
*sqrt(I*sqrt(3) - 1) + 6*x) - 7*sqrt(6)*x^7*sqrt(-I*sqrt(3) - 1)*log(I*sqr
t(6)*sqrt(3)*sqrt(-I*sqrt(3) - 1) + 6*x) + 7*sqrt(6)*x^7*sqrt(-I*sqrt(3) -
 1)*log(-I*sqrt(6)*sqrt(3)*sqrt(-I*sqrt(3) - 1) + 6*x) - 14*sqrt(3)*x^7*ar
ctan(1/3*sqrt(3)*(2*x + 1)) - 14*sqrt(3)*x^7*arctan(1/3*sqrt(3)*(2*x - 1))
 + 21*x^7*log(x^2 + x + 1) - 21*x^7*log(x^2 - x + 1) + 56*x^4 - 24)/x^7
```

3.346.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.36

$$\int \frac{1}{x^8(1+x^4+x^8)}\,dx = \left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x-\frac{1}{2}+\frac{\sqrt{3}i}{6}-18432\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^5\right) + \left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x-\frac{1}{2}-18432\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^5-\frac{\sqrt{3}i}{6}\right) + \left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x+\frac{1}{2}+\frac{\sqrt{3}i}{6}-18432\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^5\right) + \left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x+\frac{1}{2}-18432\left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^5-\frac{\sqrt{3}i}{6}\right) + \operatorname{RootSum}\left(2304t^4+48t^2+1,\left(t\mapsto t\log\left(-18432t^5-4t+x\right)\right)\right) + \frac{7x^4-3}{21x^7}$$

input
```
integrate(1/x**8/(x**8+x**4+1),x)
```

output
```
(1/8 - sqrt(3)*I/24)*log(x - 1/2 + sqrt(3)*I/6 - 18432*(1/8 - sqrt(3)*I/24
)**5) + (1/8 + sqrt(3)*I/24)*log(x - 1/2 - 18432*(1/8 + sqrt(3)*I/24)**5 -
 sqrt(3)*I/6) + (-1/8 - sqrt(3)*I/24)*log(x + 1/2 + sqrt(3)*I/6 - 18432*(-
1/8 - sqrt(3)*I/24)**5) + (-1/8 + sqrt(3)*I/24)*log(x + 1/2 - 18432*(-1/8
+ sqrt(3)*I/24)**5 - sqrt(3)*I/6) + RootSum(2304*_t**4 + 48*_t**2 + 1, Lam
bda(_t, _t*log(-18432*_t**5 - 4*_t + x))) + (7*x**4 - 3)/(21*x**7)
```

3.346.7 Maxima [F]

$$\int \frac{1}{x^8(1+x^4+x^8)}\,dx = \int \frac{1}{(x^8+x^4+1)x^8}\,dx$$

input
```
integrate(1/x^8/(x^8+x^4+1),x, algorithm="maxima")
```

output
```
-1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x - 1)) + 1/21*(7*x^4 - 3)/x^7 + 1/2*integrate(x^2/(x^4 - x^2 + 1),
 x) + 1/8*log(x^2 + x + 1) - 1/8*log(x^2 - x + 1)
```

3.346.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^8(1+x^4+x^8)}\,dx = -\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{1}{24}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right) + \frac{1}{24}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right) + \frac{7x^4-3}{21x^7} + \frac{1}{4}\arctan\left(2x+\sqrt{3}\right) + \frac{1}{4}\arctan\left(2x-\sqrt{3}\right) + \frac{1}{8}\log\left(x^2+x+1\right) - \frac{1}{8}\log\left(x^2-x+1\right)$$

input
```
integrate(1/x^8/(x^8+x^4+1),x, algorithm="giac")
```

output
```
-1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x - 1)) - 1/24*sqrt(3)*log(x^2 + sqrt(3)*x + 1) + 1/24*sqrt(3)*log(
x^2 - sqrt(3)*x + 1) + 1/21*(7*x^4 - 3)/x^7 + 1/4*arctan(2*x + sqrt(3)) +
1/4*arctan(2*x - sqrt(3)) + 1/8*log(x^2 + x + 1) - 1/8*log(x^2 - x + 1)
```

3.346.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.71

$$\int \frac{1}{x^8(1+x^4+x^8)}\,dx = \frac{\frac{x^4}{3}-\frac{1}{7}}{x^7} - \mathrm{atan}\left(\frac{2x}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) - \mathrm{atan}\left(\frac{x\,2\mathrm{i}}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{12}-\frac{1}{4}\mathrm{i}\right) - \mathrm{atan}\left(\frac{x\,2\mathrm{i}}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{12}+\frac{1}{4}\mathrm{i}\right) - \mathrm{atan}\left(\frac{2x}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)$$

input
```
int(1/(x^8*(x^4 + x^8 + 1)),x)
```

output
```
(x^4/3 - 1/7)/x^7 - atan((2*x)/(3^(1/2)*1i + 1))*((3^(1/2)*1i)/12 - 1/4) -
 atan((x*2i)/(3^(1/2)*1i - 1))*(3^(1/2)/12 - 1i/4) - atan((x*2i)/(3^(1/2)*
1i + 1))*(3^(1/2)/12 + 1i/4) - atan((2*x)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/
12 + 1/4)
```

3.346. $\int \frac{1}{x^8(1+x^4+x^8)}\,dx$

3.347 $\int \frac{x^m}{1-x^4+x^8} dx$

3.347.1 Optimal result

Integrand size = 16, antiderivative size = 127

$$\int \frac{x^m}{1-x^4+x^8} dx = \frac{2x^{1+m}\text{Hypergeometric2F1}\left(1,\frac{1+m}{4},\frac{5+m}{4},\frac{2x^4}{1-i\sqrt{3}}\right)}{\sqrt{3}\left(i+\sqrt{3}\right)(1+m)} - \frac{2x^{1+m}\text{Hypergeometric2F1}\left(1,\frac{1+m}{4},\frac{5+m}{4},\frac{2x^4}{1+i\sqrt{3}}\right)}{\sqrt{3}\left(i-\sqrt{3}\right)(1+m)}$$

output
```
-2/3*x^(1+m)*hypergeom([1, 1/4+1/4*m],[5/4+1/4*m],2*x^4/(1+I*3^(1/2)))/(1+
m)/(I-3^(1/2))*3^(1/2)+2/3*x^(1+m)*hypergeom([1, 1/4+1/4*m],[5/4+1/4*m],2*
x^4/(1-I*3^(1/2)))/(1+m)*3^(1/2)/(3^(1/2)+I)
```

3.347.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.62

$$\int \frac{x^m}{1-x^4+x^8} dx = \frac{x^m\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{\text{Hypergeometric2F1}\left(-m,-m,1-m,-\frac{\#1}{x-\#1}\right)\left(\frac{x}{x-\#1}\right)^{-m}}{-\#1^3+2\#1^7}\&\right]}{4m}$$

input
```
Integrate[x^m/(1 - x^4 + x^8),x]
```

output
```
(x^m*RootSum[1 - #1^4 + #1^8 & , Hypergeometric2F1[-m, -m, 1 - m, -(#1/(x
- #1))]/((x/(x - #1))^m*(-#1^3 + 2*#1^7)) & ])/(4*m)
```

3.347.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1711, 27, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{x^8-x^4+1}dx$$

$$\Big\downarrow \text{1711}$$

$$\frac{i\int -\frac{2x^m}{-2x^4-i\sqrt{3}+1}dx}{\sqrt{3}}-\frac{i\int -\frac{2x^m}{-2x^4+i\sqrt{3}+1}dx}{\sqrt{3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2i\int \frac{x^m}{-2x^4+i\sqrt{3}+1}dx}{\sqrt{3}}-\frac{2i\int \frac{x^m}{-2x^4-i\sqrt{3}+1}dx}{\sqrt{3}}$$

$$\Big\downarrow \text{888}$$

$$\frac{2ix^{m+1}\text{Hypergeometric2F1}\left(1,\frac{m+1}{4},\frac{m+5}{4},\frac{2x^4}{1+i\sqrt{3}}\right)}{\sqrt{3}\left(1+i\sqrt{3}\right)(m+1)}-\frac{2ix^{m+1}\text{Hypergeometric2F1}\left(1,\frac{m+1}{4},\frac{m+5}{4},\frac{2x^4}{1-i\sqrt{3}}\right)}{\sqrt{3}\left(1-i\sqrt{3}\right)(m+1)}$$

input
```
Int[x^m/(1 - x^4 + x^8),x]
```

output
```
((-2*I)*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, (2*x^4)/(1 -
I*Sqrt[3])])/(Sqrt[3]*(1 - I*Sqrt[3])*(1 + m)) + ((2*I)*x^(1 + m)*Hypergeo
metric2F1[1, (1 + m)/4, (5 + m)/4, (2*x^4)/(1 + I*Sqrt[3])])/(Sqrt[3]*(1 +
 I*Sqrt[3])*(1 + m))
```

3.347.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1711
```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[c/q   Int[(d*x)^m/(b/2 - q/2 + c
*x^n), x], x] - Simp[c/q   Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; Free
Q[{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

3.347.4 Maple [F]

$$\int \frac{x^m}{x^8 - x^4 + 1} dx$$

input
```
int(x^m/(x^8-x^4+1),x)
```

output
```
int(x^m/(x^8-x^4+1),x)
```

3.347.5 Fricas [F]

$$\int \frac{x^m}{1 - x^4 + x^8} dx = \int \frac{x^m}{x^8 - x^4 + 1} dx$$

input
```
integrate(x^m/(x^8-x^4+1),x, algorithm="fricas")
```

output
```
integral(x^m/(x^8 - x^4 + 1), x)
```

3.347.6 Sympy [F]

$$\int \frac{x^m}{1-x^4+x^8}\,dx=\int \frac{x^m}{x^8-x^4+1}\,dx$$

input
```
integrate(x**m/(x**8-x**4+1),x)
```

output
```
Integral(x**m/(x**8 - x**4 + 1), x)
```

3.347.7 Maxima [F]

$$\int \frac{x^m}{1-x^4+x^8}\,dx=\int \frac{x^m}{x^8-x^4+1}\,dx$$

input
```
integrate(x^m/(x^8-x^4+1),x, algorithm="maxima")
```

output
```
integrate(x^m/(x^8 - x^4 + 1), x)
```

3.347.8 Giac [F]

$$\int \frac{x^m}{1-x^4+x^8}\,dx=\int \frac{x^m}{x^8-x^4+1}\,dx$$

input
```
integrate(x^m/(x^8-x^4+1),x, algorithm="giac")
```

output
```
integrate(x^m/(x^8 - x^4 + 1), x)
```

3.347.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{1-x^4+x^8}\,dx = \int \frac{x^m}{x^8-x^4+1}\,dx$$

input
```
int(x^m/(x^8 - x^4 + 1),x)
```

output
```
int(x^m/(x^8 - x^4 + 1), x)
```

3.348 $\int \frac{x^{11}}{1-x^4+x^8} \, dx$

3.348.1 Optimal result

Integrand size = 16, antiderivative size = 46

$$\int \frac{x^{11}}{1-x^4+x^8} \, dx = \frac{x^4}{4}+\frac{\arctan\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{8}\log\left(1-x^4+x^8\right)$$

output
```
1/4*x^4+1/8*ln(x^8-x^4+1)+1/12*arctan(1/3*(-2*x^4+1)*3^(1/2))*3^(1/2)
```

3.348.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{x^{11}}{1-x^4+x^8} \, dx = \frac{x^4}{4}-\frac{\arctan\left(\frac{-1+2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{8}\log\left(1-x^4+x^8\right)$$

input
```
Integrate[x^11/(1 - x^4 + x^8),x]
```

output
```
x^4/4 - ArcTan[(-1 + 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[1 - x^4 + x^8]/8
```

3.348.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1693, 1143, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{x^8-x^4+1}dx$$

$$\Big\downarrow \text{ 1693}$$

$$\frac{1}{4}\int \frac{x^8}{x^8-x^4+1}dx^4$$

$$\Big\downarrow \text{ 1143}$$

$$\frac{1}{4}\int \left(1-\frac{1-x^4}{x^8-x^4+1}\right)dx^4$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4}\left(\frac{\arctan\left(\frac{1-2x^4}{\sqrt{3}}\right)}{\sqrt{3}}+x^4+\frac{1}{2}\log\left(x^8-x^4+1\right)\right)$$

input
```
Int[x^11/(1 - x^4 + x^8),x]
```

output
```
(x^4 + ArcTan[(1 - 2*x^4)/Sqrt[3]]/Sqrt[3] + Log[1 - x^4 + x^8]/2)/4
```

3.348.3.1 Defintions of rubi rules used

rule 1143
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x)^m/(a + b*x + c*x^2), x], x] /; FreeQ[{a,
b, c, d, e}, x] && IGtQ[m, 1]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.348.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{x^4}{4}+\frac{\ln(x^8-x^4+1)}{8}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x^4-1)\sqrt{3}}{3}\right)}{12}$	38
risch	$\frac{x^4}{4}+\frac{\ln(4x^8-4x^4+4)}{8}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x^4-1)\sqrt{3}}{3}\right)}{12}$	40

input
```
int(x^11/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4+1/8*ln(x^8-x^4+1)-1/12*3^(1/2)*arctan(1/3*(2*x^4-1)*3^(1/2))
```

3.348.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \frac{x^{11}}{1-x^4+x^8}\,dx = \frac{1}{4}\,x^4 - \frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^4-1)\right) + \frac{1}{8}\,\log\left(x^8-x^4+1\right)$$

input
```
integrate(x^11/(x^8-x^4+1),x, algorithm="fricas")
```

output
```
1/4*x^4 - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/8*log(x^8 - x^4
 + 1)
```

3.348.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int \frac{x^{11}}{1-x^4+x^8}\,dx = \frac{x^4}{4} + \frac{\log\left(x^8-x^4+1\right)}{8} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^4}{3}-\frac{\sqrt{3}}{3}\right)}{12}$$

input
```
integrate(x**11/(x**8-x**4+1),x)
```

output
```
x**4/4 + log(x**8 - x**4 + 1)/8 - sqrt(3)*atan(2*sqrt(3)*x**4/3 - sqrt(3)/
3)/12
```

3.348.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \frac{x^{11}}{1-x^4+x^8}\,dx = \frac{1}{4}\,x^4 - \frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^4-1)\right) + \frac{1}{8}\,\log\left(x^8-x^4+1\right)$$

input
```
integrate(x^11/(x^8-x^4+1),x, algorithm="maxima")
```

output
```
1/4*x^4 - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/8*log(x^8 - x^4
 + 1)
```

3.348.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \frac{x^{11}}{1-x^4+x^8}\,dx = \frac{1}{4}\,x^4 - \frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^4-1)\right) + \frac{1}{8}\,\log\left(x^8-x^4+1\right)$$

input
```
integrate(x^11/(x^8-x^4+1),x, algorithm="giac")
```

output
```
1/4*x^4 - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/8*log(x^8 - x^4
 + 1)
```

3.348.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{x^{11}}{1-x^4+x^8}\,dx = \frac{\ln\left(x^8-x^4+1\right)}{8} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}\,x^4}{3}\right)}{12} + \frac{x^4}{4}$$

input
```
int(x^11/(x^8 - x^4 + 1),x)
```

output
```
log(x^8 - x^4 + 1)/8 + (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^4)/3))/12 +
x^4/4
```

3.349 $\int \frac{x^9}{1-x^4+x^8}\,dx$

3.349.1 Optimal result

Integrand size = 16, antiderivative size = 57

$$\int \frac{x^9}{1-x^4+x^8}\,dx=\frac{x^2}{2}+\frac{\log\left(1-\sqrt{3}x^2+x^4\right)}{4\sqrt{3}}-\frac{\log\left(1+\sqrt{3}x^2+x^4\right)}{4\sqrt{3}}$$

output
```
1/2*x^2+1/12*ln(1+x^4-x^2*3^(1/2))*3^(1/2)-1/12*ln(1+x^4+x^2*3^(1/2))*3^(1
/2)
```

3.349.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96

$$\int \frac{x^9}{1-x^4+x^8}\,dx=\frac{1}{12}\left(6x^2+\sqrt{3}\log\left(-1+\sqrt{3}x^2-x^4\right)-\sqrt{3}\log\left(1+\sqrt{3}x^2+x^4\right)\right)$$

input
```
Integrate[x^9/(1 - x^4 + x^8),x]
```

output
```
(6*x^2 + Sqrt[3]*Log[-1 + Sqrt[3]*x^2 - x^4] - Sqrt[3]*Log[1 + Sqrt[3]*x^2
 + x^4])/12
```

3.349.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1695, 1442, 1478, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{x^8-x^4+1}dx$$
$$\Big\downarrow \text{1695}$$
$$\frac{1}{2}\int \frac{x^8}{x^8-x^4+1}dx^2$$
$$\Big\downarrow \text{1442}$$
$$\frac{1}{2}\left(x^2-\int \frac{1-x^4}{x^8-x^4+1}dx^2\right)$$
$$\Big\downarrow \text{1478}$$
$$\frac{1}{2}\left(\frac{\int -\frac{\sqrt{3}-2x^2}{x^4-\sqrt{3}x^2+1}dx^2}{2\sqrt{3}}+\frac{\int -\frac{2x^2+\sqrt{3}}{x^4+\sqrt{3}x^2+1}dx^2}{2\sqrt{3}}+x^2\right)$$
$$\Big\downarrow \text{25}$$
$$\frac{1}{2}\left(-\frac{\int \frac{\sqrt{3}-2x^2}{x^4-\sqrt{3}x^2+1}dx^2}{2\sqrt{3}}-\frac{\int \frac{2x^2+\sqrt{3}}{x^4+\sqrt{3}x^2+1}dx^2}{2\sqrt{3}}+x^2\right)$$
$$\Big\downarrow \text{1103}$$
$$\frac{1}{2}\left(x^2+\frac{\log\left(x^4-\sqrt{3}x^2+1\right)}{2\sqrt{3}}-\frac{\log\left(x^4+\sqrt{3}x^2+1\right)}{2\sqrt{3}}\right)$$

input
```
Int[x^9/(1 - x^4 + x^8),x]
```

output
```
(x^2 + Log[1 - Sqrt[3]*x^2 + x^4]/(2*Sqrt[3]) - Log[1 + Sqrt[3]*x^2 + x^4]
/(2*Sqrt[3]))/2
```

3.349.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1442
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
x] - Simp[d^4/(c*(m + 4*p + 1))   Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

rule 1695
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b
*x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c,
p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.349.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.77

method	result	size
default	$\frac{x^2}{2}+\frac{\ln\left(1+x^4-x^2\sqrt{3}\right)\sqrt{3}}{12}-\frac{\ln\left(1+x^4+x^2\sqrt{3}\right)\sqrt{3}}{12}$	44
risch	$\frac{x^2}{2}+\frac{\ln\left(1+x^4-x^2\sqrt{3}\right)\sqrt{3}}{12}-\frac{\ln\left(1+x^4+x^2\sqrt{3}\right)\sqrt{3}}{12}$	44

input
```
int(x^9/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2+1/12*ln(1+x^4-x^2*3^(1/2))*3^(1/2)-1/12*ln(1+x^4+x^2*3^(1/2))*3^(1
/2)
```

3.349.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82

$$\int \frac{x^9}{1-x^4+x^8}\,dx = \frac{1}{2}\,x^2 + \frac{1}{12}\,\sqrt{3}\log\left(\frac{x^8+5\,x^4-2\,\sqrt{3}(x^6+x^2)+1}{x^8-x^4+1}\right)$$

input
```
integrate(x^9/(x^8-x^4+1),x, algorithm="fricas")
```

output
```
1/2*x^2 + 1/12*sqrt(3)*log((x^8 + 5*x^4 - 2*sqrt(3)*(x^6 + x^2) + 1)/(x^8
- x^4 + 1))
```

3.349.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.84

$$\int \frac{x^9}{1-x^4+x^8}\,dx = \frac{x^2}{2} + \frac{\sqrt{3}\log\left(x^4-\sqrt{3}x^2+1\right)}{12} - \frac{\sqrt{3}\log\left(x^4+\sqrt{3}x^2+1\right)}{12}$$

input
```
integrate(x**9/(x**8-x**4+1),x)
```

output
```
x**2/2 + sqrt(3)*log(x**4 - sqrt(3)*x**2 + 1)/12 - sqrt(3)*log(x**4 + sqrt
(3)*x**2 + 1)/12
```

3.349.7 Maxima [F]

$$\int \frac{x^9}{1-x^4+x^8}\,dx = \int \frac{x^9}{x^8-x^4+1}\,dx$$

input
```
integrate(x^9/(x^8-x^4+1),x, algorithm="maxima")
```

output
```
1/2*x^2 + integrate((x^4 - 1)*x/(x^8 - x^4 + 1), x)
```

3.349.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. 2(43) = 86.

Time = 0.34 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.74

$$\int \frac{x^9}{1-x^4+x^8}\,dx = \frac{1}{2}\,x^2 + \frac{1}{4}\left(x^4-1\right)\arctan\left(2\,x^2+\sqrt{3}\right) + \frac{1}{4}\left(x^4-1\right)\arctan\left(2\,x^2-\sqrt{3}\right) + \frac{1}{24}\left(\sqrt{3}x^4-\sqrt{3}\right)\log\left(x^4+\sqrt{3}x^2+1\right) - \frac{1}{24}\left(\sqrt{3}x^4-\sqrt{3}\right)\log\left(x^4-\sqrt{3}x^2+1\right)$$

input
```
integrate(x^9/(x^8-x^4+1),x, algorithm="giac")
```

output
```
1/2*x^2 + 1/4*(x^4 - 1)*arctan(2*x^2 + sqrt(3)) + 1/4*(x^4 - 1)*arctan(2*x
^2 - sqrt(3)) + 1/24*(sqrt(3)*x^4 - sqrt(3))*log(x^4 + sqrt(3)*x^2 + 1) -
1/24*(sqrt(3)*x^4 - sqrt(3))*log(x^4 - sqrt(3)*x^2 + 1)
```

3.349.9 Mupad [B] (verification not implemented)

Time = 8.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.51

$$\int \frac{x^9}{1-x^4+x^8}\,dx = \frac{x^2}{2} - \frac{\sqrt{3}\,\mathrm{atanh}\left(\frac{2\sqrt{3}\,x^2}{9\left(\frac{2\,x^4}{9}+\frac{2}{9}\right)}\right)}{6}$$

input
```
int(x^9/(x^8 - x^4 + 1),x)
```

output
```
x^2/2 - (3^(1/2)*atanh((2*3^(1/2)*x^2)/(9*((2*x^4)/9 + 2/9))))/6
```

3.350 $\int \frac{x^7}{1-x^4+x^8}\,dx$

3.350.1 Optimal result

Integrand size = 16, antiderivative size = 39

$$\int \frac{x^7}{1-x^4+x^8}\,dx = -\frac{\arctan\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{1}{8}\log\left(1-x^4+x^8\right)$$

output
```
1/8*ln(x^8-x^4+1)-1/12*arctan(1/3*(-2*x^4+1)*3^(1/2))*3^(1/2)
```

3.350.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{x^7}{1-x^4+x^8}\,dx = \frac{\arctan\left(\frac{-1+2x^4}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{1}{8}\log\left(1-x^4+x^8\right)$$

input
```
Integrate[x^7/(1 - x^4 + x^8),x]
```

output
```
ArcTan[(-1 + 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[1 - x^4 + x^8]/8
```

3.350.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1693, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{x^8-x^4+1} \, dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{4}\int \frac{x^4}{x^8-x^4+1}dx^4$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{4}\left(\frac{1}{2}\int \frac{1}{x^8-x^4+1}dx^4+\frac{1}{2}\int -\frac{1-2x^4}{x^8-x^4+1}dx^4\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4}\left(\frac{1}{2}\int \frac{1}{x^8-x^4+1}dx^4-\frac{1}{2}\int \frac{1-2x^4}{x^8-x^4+1}dx^4\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{4}\left(-\int \frac{1}{-x^8-3}d(2x^4-1)-\frac{1}{2}\int \frac{1-2x^4}{x^8-x^4+1}dx^4\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{4}\left(\frac{\arctan\left(\frac{2x^4-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}\int \frac{1-2x^4}{x^8-x^4+1}dx^4\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{4}\left(\frac{\arctan\left(\frac{2x^4-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(x^8-x^4+1\right)\right)$$

input
```
Int[x^7/(1 - x^4 + x^8),x]
```

output
```
(ArcTan[(-1 + 2*x^4)/Sqrt[3]]/Sqrt[3] + Log[1 - x^4 + x^8]/2)/4
```

3.350.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.350.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{\ln(x^8-x^4+1)}{8}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x^4-1)\sqrt{3}}{3}\right)}{12}$	33
risch	$\frac{\ln(4x^8-4x^4+4)}{8}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x^4-1)\sqrt{3}}{3}\right)}{12}$	35

input
```
int(x^7/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/8*ln(x^8-x^4+1)+1/12*3^(1/2)*arctan(1/3*(2*x^4-1)*3^(1/2))
```

3.350.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{x^7}{1-x^4+x^8}\,dx = \frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^4-1)\right) + \frac{1}{8}\,\log\left(x^8-x^4+1\right)$$

input
```
integrate(x^7/(x^8-x^4+1),x, algorithm="fricas")
```

output
```
1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/8*log(x^8 - x^4 + 1)
```

3.350.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{x^7}{1-x^4+x^8}\,dx = \frac{\log\left(x^8-x^4+1\right)}{8} + \frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^4}{3}-\frac{\sqrt{3}}{3}\right)}{12}$$

input
```
integrate(x**7/(x**8-x**4+1),x)
```

output
```
log(x**8 - x**4 + 1)/8 + sqrt(3)*atan(2*sqrt(3)*x**4/3 - sqrt(3)/3)/12
```

3.350.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{x^7}{1-x^4+x^8}\,dx = \frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^4-1)\right) + \frac{1}{8}\,\log\left(x^8-x^4+1\right)$$

input
```
integrate(x^7/(x^8-x^4+1),x, algorithm="maxima")
```

output
```
1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/8*log(x^8 - x^4 + 1)
```

3.350.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{x^7}{1-x^4+x^8}\,dx = \frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^4-1)\right)+\frac{1}{8}\,\log\left(x^8-x^4+1\right)$$

input
```
integrate(x^7/(x^8-x^4+1),x, algorithm="giac")
```

output
```
1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/8*log(x^8 - x^4 + 1)
```

3.350.9 Mupad [B] (verification not implemented)

Time = 8.23 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

$$\int \frac{x^7}{1-x^4+x^8}\,dx = \frac{\ln\left(x^8-x^4+1\right)}{8}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\,\sqrt{3}\,x^4}{3}\right)}{12}$$

input
```
int(x^7/(x^8 - x^4 + 1),x)
```

output
```
log(x^8 - x^4 + 1)/8 - (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^4)/3))/12
```

3.351 $\int \frac{x^5}{1-x^4+x^8}\,dx$

3.351.1 Optimal result

Integrand size = 16, antiderivative size = 82

$$\int \frac{x^5}{1-x^4+x^8}\,dx = -\frac{1}{4}\arctan\left(\sqrt{3}-2x^2\right)+\frac{1}{4}\arctan\left(\sqrt{3}+2x^2\right)+\frac{\log\left(1-\sqrt{3}x^2+x^4\right)}{8\sqrt{3}}-\frac{\log\left(1+\sqrt{3}x^2+x^4\right)}{8\sqrt{3}}$$

output
```
1/4*arctan(2*x^2-3^(1/2))+1/4*arctan(2*x^2+3^(1/2))+1/24*ln(1+x^4-x^2*3^(1
/2))*3^(1/2)-1/24*ln(1+x^4+x^2*3^(1/2))*3^(1/2)
```

3.351.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.20

$$\int \frac{x^5}{1-x^4+x^8}\,dx = \frac{\sqrt{-1-i\sqrt{3}}\left(i+\sqrt{3}\right)\arctan\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x^2\right)+\sqrt{-1+i\sqrt{3}}\left(-i+\sqrt{3}\right)\arctan\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x^2\right)}{4\sqrt{6}}$$

input
```
Integrate[x^5/(1 - x^4 + x^8),x]
```

output
```
(Sqrt[-1 - I*Sqrt[3]]*(I + Sqrt[3])*ArcTan[((1 - I*Sqrt[3])*x^2)/2] + Sqrt
[-1 + I*Sqrt[3]]*(-I + Sqrt[3])*ArcTan[((1 + I*Sqrt[3])*x^2)/2])/(4*Sqrt[6
])
```

3.351.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.10, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1695, 1447, 1475, 1083, 217, 1478, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{x^8-x^4+1}\,dx$$

$$\Big\downarrow \text{1695}$$

$$\frac{1}{2}\int \frac{x^4}{x^8-x^4+1}dx^2$$

$$\Big\downarrow \text{1447}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{x^4+1}{x^8-x^4+1}dx^2-\frac{1}{2}\int \frac{1-x^4}{x^8-x^4+1}dx^2\right)$$

$$\Big\downarrow \text{1475}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^4-\sqrt{3}x^2+1}dx^2+\frac{1}{2}\int \frac{1}{x^4+\sqrt{3}x^2+1}dx^2\right)-\frac{1}{2}\int \frac{1-x^4}{x^8-x^4+1}dx^2\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-\int \frac{1}{-x^4-1}d\left(2x^2-\sqrt{3}\right)-\int \frac{1}{-x^4-1}d\left(2x^2+\sqrt{3}\right)\right)-\frac{1}{2}\int \frac{1-x^4}{x^8-x^4+1}dx^2\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\arctan\left(2x^2+\sqrt{3}\right)-\arctan\left(\sqrt{3}-2x^2\right)\right)-\frac{1}{2}\int \frac{1-x^4}{x^8-x^4+1}dx^2\right)$$

$$\Big\downarrow \text{1478}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{3}-2x^2}{x^4-\sqrt{3}x^2+1}dx^2}{2\sqrt{3}}+\frac{\int -\frac{2x^2+\sqrt{3}}{x^4+\sqrt{3}x^2+1}dx^2}{2\sqrt{3}}\right)+\frac{1}{2}\left(\arctan\left(2x^2+\sqrt{3}\right)-\arctan\left(\sqrt{3}-2x^2\right)\right)\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{3}-2x^2}{x^4-\sqrt{3}x^2+1}dx^2}{2\sqrt{3}}-\frac{\int \frac{2x^2+\sqrt{3}}{x^4+\sqrt{3}x^2+1}dx^2}{2\sqrt{3}}\right)+\frac{1}{2}\left(\arctan\left(2x^2+\sqrt{3}\right)-\arctan\left(\sqrt{3}-2x^2\right)\right)\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\arctan\left(2x^2+\sqrt{3}\right)-\arctan\left(\sqrt{3}-2x^2\right)\right)+\frac{1}{2}\left(\frac{\log\left(x^4-\sqrt{3}x^2+1\right)}{2\sqrt{3}}-\frac{\log\left(x^4+\sqrt{3}x^2+1\right)}{2\sqrt{3}}\right)\right)$$

input
```
Int[x^5/(1 - x^4 + x^8),x]
```

output
```
((-ArcTan[Sqrt[3] - 2*x^2] + ArcTan[Sqrt[3] + 2*x^2])/2 + (Log[1 - Sqrt[3]
*x^2 + x^4]/(2*Sqrt[3]) - Log[1 + Sqrt[3]*x^2 + x^4]/(2*Sqrt[3]))/2)/2
```

3.351.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1447
```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Simp[1/2   Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Simp[1/2
 Int[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b
^2 - 4*a*c, 0] && PosQ[a*c]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

rule 1695
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b
*x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c,
p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.351.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.39

method	result	size
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(9\_Z^4+3\_Z^2+1\right)}\_R\ln\left(6\_R^3+x^2+\_R\right)\right)}{4}$	32
default	$-\frac{\sqrt{3}\left(-\frac{\ln\left(1+x^4-x^2\sqrt{3}\right)}{2}-\sqrt{3}\arctan\left(2x^2-\sqrt{3}\right)\right)}{12}-\frac{\sqrt{3}\left(\frac{\ln\left(1+x^4+x^2\sqrt{3}\right)}{2}-\sqrt{3}\arctan\left(2x^2+\sqrt{3}\right)\right)}{12}$	77

input
```
int(x^5/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(_R*ln(6*_R^3+x^2+_R),_R=RootOf(9*_Z^4+3*_Z^2+1))
```

3.351.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.87

$$\int \frac{x^5}{1-x^4+x^8}\,dx = \frac{1}{24}\sqrt{6}\sqrt{i\sqrt{3}-1}\log\left(6x^2+i\sqrt{6}\sqrt{3}\sqrt{i\sqrt{3}-1}\right) - \frac{1}{24}\sqrt{6}\sqrt{i\sqrt{3}-1}\log\left(6x^2-i\sqrt{6}\sqrt{3}\sqrt{i\sqrt{3}-1}\right) - \frac{1}{24}\sqrt{6}\sqrt{-i\sqrt{3}-1}\log\left(6x^2+i\sqrt{6}\sqrt{3}\sqrt{-i\sqrt{3}-1}\right) + \frac{1}{24}\sqrt{6}\sqrt{-i\sqrt{3}-1}\log\left(6x^2-i\sqrt{6}\sqrt{3}\sqrt{-i\sqrt{3}-1}\right)$$

input
```
integrate(x^5/(x^8-x^4+1),x, algorithm="fricas")
```

output
```
1/24*sqrt(6)*sqrt(I*sqrt(3) - 1)*log(6*x^2 + I*sqrt(6)*sqrt(3)*sqrt(I*sqrt
(3) - 1)) - 1/24*sqrt(6)*sqrt(I*sqrt(3) - 1)*log(6*x^2 - I*sqrt(6)*sqrt(3)
*sqrt(I*sqrt(3) - 1)) - 1/24*sqrt(6)*sqrt(-I*sqrt(3) - 1)*log(6*x^2 + I*sq
rt(6)*sqrt(3)*sqrt(-I*sqrt(3) - 1)) + 1/24*sqrt(6)*sqrt(-I*sqrt(3) - 1)*lo
g(6*x^2 - I*sqrt(6)*sqrt(3)*sqrt(-I*sqrt(3) - 1))
```

3.351.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.85

$$\int \frac{x^5}{1-x^4+x^8}\,dx = \frac{\sqrt{3}\log\left(x^4-\sqrt{3}x^2+1\right)}{24} - \frac{\sqrt{3}\log\left(x^4+\sqrt{3}x^2+1\right)}{24} + \frac{\operatorname{atan}\left(2x^2-\sqrt{3}\right)}{4} + \frac{\operatorname{atan}\left(2x^2+\sqrt{3}\right)}{4}$$

input
```
integrate(x**5/(x**8-x**4+1),x)
```

output
```
sqrt(3)*log(x**4 - sqrt(3)*x**2 + 1)/24 - sqrt(3)*log(x**4 + sqrt(3)*x**2
+ 1)/24 + atan(2*x**2 - sqrt(3))/4 + atan(2*x**2 + sqrt(3))/4
```

3.351.7 Maxima [F]

$$\int \frac{x^5}{1-x^4+x^8}\,dx = \int \frac{x^5}{x^8-x^4+1}\,dx$$

input
```
integrate(x^5/(x^8-x^4+1),x, algorithm="maxima")
```

output
```
integrate(x^5/(x^8 - x^4 + 1), x)
```

3.351.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.93

$$\int \frac{x^5}{1-x^4+x^8}\,dx = \frac{1}{24}\sqrt{3}x^4\log\left(x^4+\sqrt{3}x^2+1\right) - \frac{1}{24}\sqrt{3}x^4\log\left(x^4-\sqrt{3}x^2+1\right) + \frac{1}{4}\,x^4\arctan\left(2\,x^2+\sqrt{3}\right) + \frac{1}{4}\,x^4\arctan\left(2\,x^2-\sqrt{3}\right)$$

input
```
integrate(x^5/(x^8-x^4+1),x, algorithm="giac")
```

output
```
1/24*sqrt(3)*x^4*log(x^4 + sqrt(3)*x^2 + 1) - 1/24*sqrt(3)*x^4*log(x^4 - s
qrt(3)*x^2 + 1) + 1/4*x^4*arctan(2*x^2 + sqrt(3)) + 1/4*x^4*arctan(2*x^2 -
 sqrt(3))
```

3.351.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.65

$$\int \frac{x^5}{1-x^4+x^8}\,dx = -\mathrm{atan}\left(\frac{2\,x^2}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) - \mathrm{atan}\left(\frac{2\,x^2}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)$$

input
```
int(x^5/(x^8 - x^4 + 1),x)
```

output
```
- atan((2*x^2)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/12 + 1/4) - atan((2*x^2)/(3
^(1/2)*1i + 1))*((3^(1/2)*1i)/12 - 1/4)
```

3.352 $\int \frac{x^3}{1-x^4+x^8} dx$

3.352.1 Optimal result

Integrand size = 16, antiderivative size = 23

$$\int \frac{x^3}{1-x^4+x^8} \, dx = -\frac{\arctan\left(\frac{1-2x^4}{\sqrt{3}}\right)}{2\sqrt{3}}$$

output
```
-1/6*arctan(1/3*(-2*x^4+1)*3^(1/2))*3^(1/2)
```

3.352.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{1-x^4+x^8} \, dx = \frac{\arctan\left(\frac{-1+2x^4}{\sqrt{3}}\right)}{2\sqrt{3}}$$

input
```
Integrate[x^3/(1 - x^4 + x^8),x]
```

output
```
ArcTan[(-1 + 2*x^4)/Sqrt[3]]/(2*Sqrt[3])
```

3.352.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1690, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{x^8-x^4+1}dx$$
$$\Big\downarrow \text{1690}$$
$$\frac{1}{4}\int \frac{1}{x^8-x^4+1}dx^4$$
$$\Big\downarrow \text{1083}$$
$$-\frac{1}{2}\int \frac{1}{-x^8-3}d(2x^4-1)$$
$$\Big\downarrow \text{217}$$
$$\frac{\arctan\left(\frac{2x^4-1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

input
```
Int[x^3/(1 - x^4 + x^8),x]
```

output
```
ArcTan[(-1 + 2*x^4)/Sqrt[3]]/(2*Sqrt[3])
```

3.352.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

3.352.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{\sqrt{3}\,\arctan\left(\frac{\left(2x^4-1\right)\sqrt{3}}{3}\right)}{6}$	19
risch	$\frac{\sqrt{3}\,\arctan\left(\frac{\left(2x^4-1\right)\sqrt{3}}{3}\right)}{6}$	19

input
```
int(x^3/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/6*3^(1/2)*arctan(1/3*(2*x^4-1)*3^(1/2))
```

3.352.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{1-x^4+x^8}\,dx = \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^4-1)\right)$$

input
```
integrate(x^3/(x^8-x^4+1),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1))
```

3.352.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \frac{x^3}{1-x^4+x^8}\,dx = \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^4}{3}-\frac{\sqrt{3}}{3}\right)}{6}$$

input
```
integrate(x**3/(x**8-x**4+1),x)
```

output
```
sqrt(3)*atan(2*sqrt(3)*x**4/3 - sqrt(3)/3)/6
```

3.352.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{1-x^4+x^8}\,dx = \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^4-1)\right)$$

input
```
integrate(x^3/(x^8-x^4+1),x, algorithm="maxima")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1))
```

3.352.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{1-x^4+x^8}\,dx = \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^4-1)\right)$$

input
```
integrate(x^3/(x^8-x^4+1),x, algorithm="giac")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1))
```

3.352.9 Mupad [B] (verification not implemented)

Time = 8.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{x^3}{1-x^4+x^8}\,dx = \frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x^4}{3}-\frac{1}{3}\right)\right)}{6}$$

input
```
int(x^3/(x^8 - x^4 + 1),x)
```

output
```
(3^(1/2)*atan(3^(1/2)*((2*x^4)/3 - 1/3)))/6
```

3.353 $\int \frac{x}{1-x^4+x^8} dx$

3.353.1 Optimal result

Integrand size = 14, antiderivative size = 82

$$\int \frac{x}{1-x^4+x^8} dx = -\frac{1}{4}\arctan\left(\sqrt{3}-2x^2\right)+\frac{1}{4}\arctan\left(\sqrt{3}+2x^2\right) - \frac{\log\left(1-\sqrt{3}x^2+x^4\right)}{8\sqrt{3}}+\frac{\log\left(1+\sqrt{3}x^2+x^4\right)}{8\sqrt{3}}$$

output
```
1/4*arctan(2*x^2-3^(1/2))+1/4*arctan(2*x^2+3^(1/2))-1/24*ln(1+x^4-x^2*3^(1
/2))*3^(1/2)+1/24*ln(1+x^4+x^2*3^(1/2))*3^(1/2)
```

3.353.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.01

$$\int \frac{x}{1-x^4+x^8} dx = \frac{i\left(\sqrt{-1-i\sqrt{3}}\arctan\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x^2\right)-\sqrt{-1+i\sqrt{3}}\arctan\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x^2\right)\right)}{2\sqrt{6}}$$

input
```
Integrate[x/(1 - x^4 + x^8),x]
```

output
```
((I/2)*(Sqrt[-1 - I*Sqrt[3]]*ArcTan[((1 - I*Sqrt[3])*x^2)/2] - Sqrt[-1 + I
*Sqrt[3]]*ArcTan[((1 + I*Sqrt[3])*x^2)/2]))/Sqrt[6]
```

3.353.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.23, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1695, 1407, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{x^8-x^4+1}\,dx$$

$$\Big\downarrow \text{1695}$$

$$\frac{1}{2}\int \frac{1}{x^8-x^4+1}dx^2$$

$$\Big\downarrow \text{1407}$$

$$\frac{1}{2}\left(\frac{\int \frac{\sqrt{3}-x^2}{x^4-\sqrt{3}x^2+1}dx^2}{2\sqrt{3}}+\frac{\int \frac{x^2+\sqrt{3}}{x^4+\sqrt{3}x^2+1}dx^2}{2\sqrt{3}}\right)$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{2}\left(\frac{\frac{1}{2}\sqrt{3}\int \frac{1}{x^4-\sqrt{3}x^2+1}dx^2-\frac{1}{2}\int -\frac{\sqrt{3}-2x^2}{x^4-\sqrt{3}x^2+1}dx^2}{2\sqrt{3}}+\frac{\frac{1}{2}\sqrt{3}\int \frac{1}{x^4+\sqrt{3}x^2+1}dx^2+\frac{1}{2}\int \frac{2x^2+\sqrt{3}}{x^4+\sqrt{3}x^2+1}dx^2}{2\sqrt{3}}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{\frac{1}{2}\sqrt{3}\int \frac{1}{x^4-\sqrt{3}x^2+1}dx^2+\frac{1}{2}\int \frac{\sqrt{3}-2x^2}{x^4-\sqrt{3}x^2+1}dx^2}{2\sqrt{3}}+\frac{\frac{1}{2}\sqrt{3}\int \frac{1}{x^4+\sqrt{3}x^2+1}dx^2+\frac{1}{2}\int \frac{2x^2+\sqrt{3}}{x^4+\sqrt{3}x^2+1}dx^2}{2\sqrt{3}}\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(\frac{\frac{1}{2}\int \frac{\sqrt{3}-2x^2}{x^4-\sqrt{3}x^2+1}dx^2-\sqrt{3}\int \frac{1}{-x^4-1}d\left(2x^2-\sqrt{3}\right)}{2\sqrt{3}}+\frac{\frac{1}{2}\int \frac{2x^2+\sqrt{3}}{x^4+\sqrt{3}x^2+1}dx^2-\sqrt{3}\int \frac{1}{-x^4-1}d\left(2x^2+\sqrt{3}\right)}{2\sqrt{3}}\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(\frac{\frac{1}{2}\int \frac{\sqrt{3}-2x^2}{x^4-\sqrt{3}x^2+1}dx^2-\sqrt{3}\arctan\left(\sqrt{3}-2x^2\right)}{2\sqrt{3}}+\frac{\frac{1}{2}\int \frac{2x^2+\sqrt{3}}{x^4+\sqrt{3}x^2+1}dx^2+\sqrt{3}\arctan\left(2x^2+\sqrt{3}\right)}{2\sqrt{3}}\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\left(\frac{-\sqrt{3}\arctan\left(\sqrt{3}-2x^2\right)-\frac{1}{2}\log\left(x^4-\sqrt{3}x^2+1\right)}{2\sqrt{3}}+\frac{\sqrt{3}\arctan\left(2x^2+\sqrt{3}\right)+\frac{1}{2}\log\left(x^4+\sqrt{3}x^2+1\right)}{2\sqrt{3}}\right)$$

input
```
Int[x/(1 - x^4 + x^8),x]
```

output
```
((-(Sqrt[3]*ArcTan[Sqrt[3] - 2*x^2]) - Log[1 - Sqrt[3]*x^2 + x^4]/2)/(2*Sq
rt[3]) + (Sqrt[3]*ArcTan[Sqrt[3] + 2*x^2] + Log[1 + Sqrt[3]*x^2 + x^4]/2)/
(2*Sqrt[3]))/2
```

3.353.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1407
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   Int[(r - x)/(q - r*
x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(r + x)/(q + r*x + x^2), x], x]]
 /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

rule 1695
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b
*x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c,
p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.353.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.39

method	result	size
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(9\_Z^4+3\_Z^2+1\right)}\_R\ln\left(-3\_R^3+x^2+\_R\right)\right)}{4}$	32
default	$\frac{\arctan\left(2x^2-\sqrt{3}\right)}{4}+\frac{\arctan\left(2x^2+\sqrt{3}\right)}{4}-\frac{\ln\left(1+x^4-x^2\sqrt{3}\right)\sqrt{3}}{24}+\frac{\ln\left(1+x^4+x^2\sqrt{3}\right)\sqrt{3}}{24}$	65

input
```
int(x/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(_R*ln(-3*_R^3+x^2+_R),_R=RootOf(9*_Z^4+3*_Z^2+1))
```

3.353.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.01

$$\begin{aligned}\int \frac{x}{1-x^4+x^8}\,dx =& -\frac{1}{24}\sqrt{6}\sqrt{i\sqrt{3}-1}\log\left(12x^2+\sqrt{6}\sqrt{i\sqrt{3}-1}\left(i\sqrt{3}-3\right)\right)\\&+\frac{1}{24}\sqrt{6}\sqrt{i\sqrt{3}-1}\log\left(12x^2+\sqrt{6}\sqrt{i\sqrt{3}-1}\left(-i\sqrt{3}+3\right)\right)\\&+\frac{1}{24}\sqrt{6}\sqrt{-i\sqrt{3}-1}\log\left(12x^2+\sqrt{6}\left(i\sqrt{3}+3\right)\sqrt{-i\sqrt{3}-1}\right)\\&-\frac{1}{24}\sqrt{6}\sqrt{-i\sqrt{3}-1}\log\left(12x^2+\sqrt{6}\sqrt{-i\sqrt{3}-1}\left(-i\sqrt{3}-3\right)\right)\end{aligned}$$

input
```
integrate(x/(x^8-x^4+1),x, algorithm="fricas")
```

output
```
-1/24*sqrt(6)*sqrt(I*sqrt(3) - 1)*log(12*x^2 + sqrt(6)*sqrt(I*sqrt(3) - 1)
*(I*sqrt(3) - 3)) + 1/24*sqrt(6)*sqrt(I*sqrt(3) - 1)*log(12*x^2 + sqrt(6)*
sqrt(I*sqrt(3) - 1)*(-I*sqrt(3) + 3)) + 1/24*sqrt(6)*sqrt(-I*sqrt(3) - 1)*
log(12*x^2 + sqrt(6)*(I*sqrt(3) + 3)*sqrt(-I*sqrt(3) - 1)) - 1/24*sqrt(6)*
sqrt(-I*sqrt(3) - 1)*log(12*x^2 + sqrt(6)*sqrt(-I*sqrt(3) - 1)*(-I*sqrt(3)
 - 3))
```

3.353.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.85

$$\int \frac{x}{1-x^4+x^8}\,dx = -\frac{\sqrt{3}\log\left(x^4-\sqrt{3}x^2+1\right)}{24}+\frac{\sqrt{3}\log\left(x^4+\sqrt{3}x^2+1\right)}{24}+\frac{\operatorname{atan}\left(2x^2-\sqrt{3}\right)}{4}+\frac{\operatorname{atan}\left(2x^2+\sqrt{3}\right)}{4}$$

input
```
integrate(x/(x**8-x**4+1),x)
```

output
```
-sqrt(3)*log(x**4 - sqrt(3)*x**2 + 1)/24 + sqrt(3)*log(x**4 + sqrt(3)*x**2
 + 1)/24 + atan(2*x**2 - sqrt(3))/4 + atan(2*x**2 + sqrt(3))/4
```

3.353.7 Maxima [F]

$$\int \frac{x}{1-x^4+x^8}\,dx = \int \frac{x}{x^8-x^4+1}\,dx$$

input
```
integrate(x/(x^8-x^4+1),x, algorithm="maxima")
```

output
```
integrate(x/(x^8 - x^4 + 1), x)
```

3.353.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.78

$$\int \frac{x}{1-x^4+x^8}\,dx = \frac{1}{24}\sqrt{3}\log\left(x^4+\sqrt{3}x^2+1\right) - \frac{1}{24}\sqrt{3}\log\left(x^4-\sqrt{3}x^2+1\right) + \frac{1}{4}\arctan\left(2\,x^2+\sqrt{3}\right) + \frac{1}{4}\arctan\left(2\,x^2-\sqrt{3}\right)$$

input
```
integrate(x/(x^8-x^4+1),x, algorithm="giac")
```

output
```
1/24*sqrt(3)*log(x^4 + sqrt(3)*x^2 + 1) - 1/24*sqrt(3)*log(x^4 - sqrt(3)*x
^2 + 1) + 1/4*arctan(2*x^2 + sqrt(3)) + 1/4*arctan(2*x^2 - sqrt(3))
```

3.353.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.65

$$\int \frac{x}{1-x^4+x^8}\,dx = -\mathrm{atan}\left(-\frac{x^2}{2}+\frac{\sqrt{3}\,x^2\,1\mathrm{i}}{2}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) - \mathrm{atan}\left(\frac{x^2}{2}+\frac{\sqrt{3}\,x^2\,1\mathrm{i}}{2}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)$$

input
```
int(x/(x^8 - x^4 + 1),x)
```

output
```
- atan((3^(1/2)*x^2*1i)/2 - x^2/2)*((3^(1/2)*1i)/12 + 1/4) - atan((3^(1/2)
*x^2*1i)/2 + x^2/2)*((3^(1/2)*1i)/12 - 1/4)
```

3.354 $\int \frac{1}{x(1-x^4+x^8)}\,dx$

3.354.1 Optimal result

Integrand size = 16, antiderivative size = 41

$$\int \frac{1}{x(1-x^4+x^8)}\,dx=-\frac{\arctan\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\log(x)-\frac{1}{8}\log\left(1-x^4+x^8\right)$$

output
```
ln(x)-1/8*ln(x^8-x^4+1)-1/12*arctan(1/3*(-2*x^4+1)*3^(1/2))*3^(1/2)
```

3.354.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.34

$$\int \frac{1}{x(1-x^4+x^8)}\,dx=\log(x)-\frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{-\log(x-\#1)+\log(x-\#1)\#1^4}{-1+2\#1^4}\&\right]$$

input
```
Integrate[1/(x*(1 - x^4 + x^8)),x]
```

output
```
Log[x] - RootSum[1 - #1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)/(-1
 + 2*#1^4) & ]/4
```

3.354.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1693, 1144, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(x^8-x^4+1\right)}dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{4}\int \frac{1}{x^4\left(x^8-x^4+1\right)}dx^4$$

$$\Big\downarrow \text{1144}$$

$$\frac{1}{4}\left(\int \frac{1-x^4}{x^8-x^4+1}dx^4+\log\left(x^4\right)\right)$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{4}\left(\frac{1}{2}\int \frac{1}{x^8-x^4+1}dx^4-\frac{1}{2}\int -\frac{1-2x^4}{x^8-x^4+1}dx^4+\log\left(x^4\right)\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4}\left(\frac{1}{2}\int \frac{1}{x^8-x^4+1}dx^4+\frac{1}{2}\int \frac{1-2x^4}{x^8-x^4+1}dx^4+\log\left(x^4\right)\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{4}\left(-\int \frac{1}{-x^8-3}d(2x^4-1)+\frac{1}{2}\int \frac{1-2x^4}{x^8-x^4+1}dx^4+\log\left(x^4\right)\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{4}\left(\frac{1}{2}\int \frac{1-2x^4}{x^8-x^4+1}dx^4+\frac{\arctan\left(\frac{2x^4-1}{\sqrt{3}}\right)}{\sqrt{3}}+\log\left(x^4\right)\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{4}\left(\frac{\arctan\left(\frac{2x^4-1}{\sqrt{3}}\right)}{\sqrt{3}}+\log\left(x^4\right)-\frac{1}{2}\log\left(x^8-x^4+1\right)\right)$$

input
```
Int[1/(x*(1 - x^4 + x^8)),x]
```

output
```
(ArcTan[(-1 + 2*x^4)/Sqrt[3]]/Sqrt[3] + Log[x^4] - Log[1 - x^4 + x^8]/2)/4
```

3.354.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1144
```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Simp[e*(Log[RemoveContent[d + e*x, x]]/(c*d^2 - b*d*e + a*e^2)), x] + S
imp[1/(c*d^2 - b*d*e + a*e^2)   Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2),
x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.354. $\int \frac{1}{x(1-x^4+x^8)}\,dx$

3.354.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

method	result	size
risch	$\ln(x)-\frac{\ln(x^8-x^4+1)}{8}+\frac{\sqrt{3}\arctan\left(\frac{2\left(x^4-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{12}$	33
default	$\ln(x)-\frac{\ln(x^8-x^4+1)}{8}+\frac{\sqrt{3}\arctan\left(\frac{(2x^4-1)\sqrt{3}}{3}\right)}{12}$	35

input
```
int(1/x/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
ln(x)-1/8*ln(x^8-x^4+1)+1/12*3^(1/2)*arctan(2/3*(x^4-1/2)*3^(1/2))
```

3.354.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{1}{x(1-x^4+x^8)}\,dx = \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^4-1)\right)-\frac{1}{8}\log\left(x^8-x^4+1\right)+\log(x)$$

input
```
integrate(1/x/(x^8-x^4+1),x, algorithm="fricas")
```

output
```
1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*log(x^8 - x^4 + 1) + lo
g(x)
```

3.354.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(1-x^4+x^8)}\,dx = \log(x)-\frac{\log(x^8-x^4+1)}{8}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^4}{3}-\frac{\sqrt{3}}{3}\right)}{12}$$

input
```
integrate(1/x/(x**8-x**4+1),x)
```

output
```
log(x) - log(x**8 - x**4 + 1)/8 + sqrt(3)*atan(2*sqrt(3)*x**4/3 - sqrt(3)/
3)/12
```

3.354.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

$$\int \frac{1}{x\left(1-x^4+x^8\right)}\,dx = \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4-1\right)\right)-\frac{1}{8}\log\left(x^8-x^4+1\right)+\frac{1}{4}\log\left(x^4\right)$$

input
```
integrate(1/x/(x^8-x^4+1),x, algorithm="maxima")
```

output
```
1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*log(x^8 - x^4 + 1) + 1/
4*log(x^4)
```

3.354.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

$$\int \frac{1}{x\left(1-x^4+x^8\right)}\,dx = \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4-1\right)\right)-\frac{1}{8}\log\left(x^8-x^4+1\right)+\frac{1}{4}\log\left(x^4\right)$$

input
```
integrate(1/x/(x^8-x^4+1),x, algorithm="giac")
```

output
```
1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*log(x^8 - x^4 + 1) + 1/
4*log(x^4)
```

3.354.9 Mupad [B] (verification not implemented)

Time = 8.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int \frac{1}{x\left(1-x^4+x^8\right)}\,dx = \ln(x)-\frac{\ln\left(x^8-x^4+1\right)}{8}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}\,x^4}{3}\right)}{12}$$

input
```
int(1/(x*(x^8 - x^4 + 1)),x)
```

output
```
log(x) - log(x^8 - x^4 + 1)/8 - (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^4)/
3))/12
```

3.355 $\int \frac{1}{x^3(1-x^4+x^8)}\,dx$

3.355.1 Optimal result

Integrand size = 16, antiderivative size = 57

$$\int \frac{1}{x^3(1-x^4+x^8)}\,dx = -\frac{1}{2x^2} - \frac{\log\left(1-\sqrt{3}x^2+x^4\right)}{4\sqrt{3}} + \frac{\log\left(1+\sqrt{3}x^2+x^4\right)}{4\sqrt{3}}$$

output
```
-1/2/x^2-1/12*ln(1+x^4-x^2*3^(1/2))*3^(1/2)+1/12*ln(1+x^4+x^2*3^(1/2))*3^(
1/2)
```

3.355.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^3(1-x^4+x^8)}\,dx = \frac{1}{12}\left(-\frac{6}{x^2} - \sqrt{3}\log\left(-1+\sqrt{3}x^2-x^4\right) + \sqrt{3}\log\left(1+\sqrt{3}x^2+x^4\right)\right)$$

input
```
Integrate[1/(x^3*(1 - x^4 + x^8)),x]
```

output
```
(-6/x^2 - Sqrt[3]*Log[-1 + Sqrt[3]*x^2 - x^4] + Sqrt[3]*Log[1 + Sqrt[3]*x^
2 + x^4])/12
```

3.355.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1695, 1443, 1478, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (x^8 - x^4 + 1)} \, dx$$

$$\Big\downarrow \text{1695}$$

$$\frac{1}{2} \int \frac{1}{x^4 (x^8 - x^4 + 1)} dx^2$$

$$\Big\downarrow \text{1443}$$

$$\frac{1}{2} \left(\int \frac{1 - x^4}{x^8 - x^4 + 1} dx^2 - \frac{1}{x^2} \right)$$

$$\Big\downarrow \text{1478}$$

$$\frac{1}{2} \left(-\frac{\int -\frac{\sqrt{3} - 2x^2}{x^4 - \sqrt{3}x^2 + 1} dx^2}{2\sqrt{3}} - \frac{\int -\frac{2x^2 + \sqrt{3}}{x^4 + \sqrt{3}x^2 + 1} dx^2}{2\sqrt{3}} - \frac{1}{x^2} \right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2} \left(\frac{\int \frac{\sqrt{3} - 2x^2}{x^4 - \sqrt{3}x^2 + 1} dx^2}{2\sqrt{3}} + \frac{\int \frac{2x^2 + \sqrt{3}}{x^4 + \sqrt{3}x^2 + 1} dx^2}{2\sqrt{3}} - \frac{1}{x^2} \right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2} \left(-\frac{1}{x^2} - \frac{\log \left(x^4 - \sqrt{3}x^2 + 1 \right)}{2\sqrt{3}} + \frac{\log \left(x^4 + \sqrt{3}x^2 + 1 \right)}{2\sqrt{3}} \right)$$

input
```
Int[1/(x^3*(1 - x^4 + x^8)),x]
```

output
```
(-x^(-2) - Log[1 - Sqrt[3]*x^2 + x^4]/(2*Sqrt[3]) + Log[1 + Sqrt[3]*x^2 +
x^4]/(2*Sqrt[3]))/2
```

3.355.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1443
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Sim
p[1/(a*d^2*(m + 1))   Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*
x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

rule 1695
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b
*x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c,
p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.355.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.77

method	result	size
default	$-\frac{1}{2x^2}-\frac{\ln\left(1+x^4-x^2\sqrt{3}\right)\sqrt{3}}{12}+\frac{\ln\left(1+x^4+x^2\sqrt{3}\right)\sqrt{3}}{12}$	44
risch	$-\frac{1}{2x^2}-\frac{\ln\left(1+x^4-x^2\sqrt{3}\right)\sqrt{3}}{12}+\frac{\ln\left(1+x^4+x^2\sqrt{3}\right)\sqrt{3}}{12}$	44

input
```
int(1/x^3/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2/x^2-1/12*ln(1+x^4-x^2*3^(1/2))*3^(1/2)+1/12*ln(1+x^4+x^2*3^(1/2))*3^(
1/2)
```

3.355.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^3 (1 - x^4 + x^8)} dx = \frac{\sqrt{3}x^2 \log \left(\frac{x^8+5x^4+2\sqrt{3}(x^6+x^2)+1}{x^8-x^4+1}\right) - 6}{12 x^2}$$

input
```
integrate(1/x^3/(x^8-x^4+1),x, algorithm="fricas")
```

output
```
1/12*(sqrt(3)*x^2*log((x^8 + 5*x^4 + 2*sqrt(3)*(x^6 + x^2) + 1)/(x^8 - x^4
 + 1)) - 6)/x^2
```

3.355.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^3 (1 - x^4 + x^8)} dx = -\frac{\sqrt{3} \log (x^4 - \sqrt{3}x^2 + 1)}{12} + \frac{\sqrt{3} \log (x^4 + \sqrt{3}x^2 + 1)}{12} - \frac{1}{2x^2}$$

input
```
integrate(1/x**3/(x**8-x**4+1),x)
```

output
```
-sqrt(3)*log(x**4 - sqrt(3)*x**2 + 1)/12 + sqrt(3)*log(x**4 + sqrt(3)*x**2
 + 1)/12 - 1/(2*x**2)
```

3.355.7 Maxima [F]

$$\int \frac{1}{x^3 (1 - x^4 + x^8)} dx = \int \frac{1}{(x^8 - x^4 + 1)x^3} dx$$

input
```
integrate(1/x^3/(x^8-x^4+1),x, algorithm="maxima")
```

output
```
-1/2/x^2 - integrate((x^4 - 1)*x/(x^8 - x^4 + 1), x)
```

3.355.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. 2(43) = 86.

Time = 0.33 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.74

$$\int \frac{1}{x^3(1-x^4+x^8)}\,dx = -\frac{1}{4}\left(x^4-1\right)\arctan\left(2\,x^2+\sqrt{3}\right)-\frac{1}{4}\left(x^4-1\right)\arctan\left(2\,x^2-\sqrt{3}\right) - \frac{1}{24}\left(\sqrt{3}x^4-\sqrt{3}\right)\log\left(x^4+\sqrt{3}x^2+1\right) + \frac{1}{24}\left(\sqrt{3}x^4-\sqrt{3}\right)\log\left(x^4-\sqrt{3}x^2+1\right)-\frac{1}{2\,x^2}$$

input
```
integrate(1/x^3/(x^8-x^4+1),x, algorithm="giac")
```

output
```
-1/4*(x^4 - 1)*arctan(2*x^2 + sqrt(3)) - 1/4*(x^4 - 1)*arctan(2*x^2 - sqrt
(3)) - 1/24*(sqrt(3)*x^4 - sqrt(3))*log(x^4 + sqrt(3)*x^2 + 1) + 1/24*(sqr
t(3)*x^4 - sqrt(3))*log(x^4 - sqrt(3)*x^2 + 1) - 1/2/x^2
```

3.355.9 Mupad [B] (verification not implemented)

Time = 8.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.51

$$\int \frac{1}{x^3(1-x^4+x^8)}\,dx = \frac{\sqrt{3}\,\mathrm{atanh}\left(\frac{2\sqrt{3}\,x^2}{9\left(\frac{2\,x^4}{9}+\frac{2}{9}\right)}\right)}{6}-\frac{1}{2\,x^2}$$

input
```
int(1/(x^3*(x^8 - x^4 + 1)),x)
```

output
```
(3^(1/2)*atanh((2*3^(1/2)*x^2)/(9*((2*x^4)/9 + 2/9))))/6 - 1/(2*x^2)
```

3.356 $\int \frac{1}{x^5(1-x^4+x^8)}\,dx$

3.356.1 Optimal result

Integrand size = 16, antiderivative size = 48

$$\int \frac{1}{x^5(1-x^4+x^8)}\,dx=-\frac{1}{4x^4}+\frac{\arctan\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\log(x)-\frac{1}{8}\log\left(1-x^4+x^8\right)$$

output
```
-1/4/x^4+ln(x)-1/8*ln(x^8-x^4+1)+1/12*arctan(1/3*(-2*x^4+1)*3^(1/2))*3^(1/
2)
```

3.356.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^5(1-x^4+x^8)}\,dx=-\frac{1}{4x^4}+\log(x)-\frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{\log(x-\#1)\#1^4}{-1+2\#1^4}\&\right]$$

input
```
Integrate[1/(x^5*(1 - x^4 + x^8)),x]
```

output
```
-1/4*1/x^4 + Log[x] - RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1^4)/(-1 +
 2*#1^4) & ]/4
```

3.356.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1693, 1145, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 (x^8-x^4+1)} \, dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{4} \int \frac{1}{x^8 (x^8-x^4+1)}dx^4$$

$$\Big\downarrow \text{1145}$$

$$\frac{1}{4} \left(\int \frac{1-x^4}{x^4 (x^8-x^4+1)}dx^4-\frac{1}{x^4}\right)$$

$$\Big\downarrow \text{1200}$$

$$\frac{1}{4} \left(\int \left(\frac{1}{x^4}-\frac{x^4}{x^8-x^4+1}\right)dx^4-\frac{1}{x^4}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4} \left(\frac{\arctan \left(\frac{1-2 x^4}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{x^4}+\log \left(x^4\right)-\frac{1}{2} \log \left(x^8-x^4+1\right)\right)$$

input
```
Int[1/(x^5*(1 - x^4 + x^8)),x]
```

output
```
(-x^(-4) + ArcTan[(1 - 2*x^4)/Sqrt[3]]/Sqrt[3] + Log[x^4] - Log[1 - x^4 + 
x^8]/2)/4
```

3.356.3.1 Defintions of rubi rules used

rule 1145
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp
[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[m, -1]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.356.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.79

method	result	size
risch	$-\frac{1}{4x^4}+\ln(x)-\frac{\ln(x^8-x^4+1)}{8}-\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x^4-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{12}$	38
default	$-\frac{1}{4x^4}+\ln(x)-\frac{\ln(x^8-x^4+1)}{8}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x^4-1)\sqrt{3}}{3}\right)}{12}$	40

input
```
int(1/x^5/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/4/x^4+ln(x)-1/8*ln(x^8-x^4+1)-1/12*3^(1/2)*arctan(2/3*(x^4-1/2)*3^(1/2)
)
```

3.356.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^5 (1-x^4+x^8)} \, dx = -\frac{2 \sqrt{3} x^4 \arctan \left(\frac{1}{3} \sqrt{3}(2 x^4-1)\right)+3 x^4 \log \left(x^8-x^4+1\right)-24 x^4 \log (x)+6}{24 x^4}$$

input
```
integrate(1/x^5/(x^8-x^4+1),x, algorithm="fricas")
```

output
```
-1/24*(2*sqrt(3)*x^4*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 3*x^4*log(x^8 - x^4
 + 1) - 24*x^4*log(x) + 6)/x^4
```

3.356.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^5 (1-x^4+x^8)} \, dx = \log (x) - \frac{\log (x^8-x^4+1)}{8} - \frac{\sqrt{3} \operatorname{atan} \left(\frac{2\sqrt{3}x^4}{3} - \frac{\sqrt{3}}{3}\right)}{12} - \frac{1}{4x^4}$$

input
```
integrate(1/x**5/(x**8-x**4+1),x)
```

output
```
log(x) - log(x**8 - x**4 + 1)/8 - sqrt(3)*atan(2*sqrt(3)*x**4/3 - sqrt(3)/
3)/12 - 1/(4*x**4)
```

3.356.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^5 (1-x^4+x^8)} \, dx = -\frac{1}{12} \sqrt{3} \arctan \left(\frac{1}{3} \sqrt{3}(2 x^4-1)\right) - \frac{1}{4 x^4} - \frac{1}{8} \log \left(x^8-x^4+1\right) + \frac{1}{4} \log \left(x^4\right)$$

input
```
integrate(1/x^5/(x^8-x^4+1),x, algorithm="maxima")
```

output
```
-1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/4/x^4 - 1/8*log(x^8 - x^
4 + 1) + 1/4*log(x^4)
```

3.356.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^5(1-x^4+x^8)}\,dx = -\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^4-1)\right) - \frac{x^4+1}{4x^4} - \frac{1}{8}\log\left(x^8-x^4+1\right) + \frac{1}{4}\log\left(x^4\right)$$

input
```
integrate(1/x^5/(x^8-x^4+1),x, algorithm="giac")
```

output
```
-1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/4*(x^4 + 1)/x^4 - 1/8*lo
g(x^8 - x^4 + 1) + 1/4*log(x^4)
```

3.356.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^5(1-x^4+x^8)}\,dx = \ln(x) - \frac{\ln(x^8-x^4+1)}{8} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}\,x^4}{3}\right)}{12} - \frac{1}{4x^4}$$

input
```
int(1/(x^5*(x^8 - x^4 + 1)),x)
```

output
```
log(x) - log(x^8 - x^4 + 1)/8 + (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^4)/
3))/12 - 1/(4*x^4)
```

3.357 $\int \frac{1}{x^7(1-x^4+x^8)}\,dx$

3.357.1 Optimal result

Integrand size = 16, antiderivative size = 96

$$\int \frac{1}{x^7\left(1-x^4+x^8\right)}\,dx = -\frac{1}{6x^6}-\frac{1}{2x^2}+\frac{1}{4}\arctan\left(\sqrt{3}-2x^2\right)-\frac{1}{4}\arctan\left(\sqrt{3}+2x^2\right) -\frac{\log\left(1-\sqrt{3}x^2+x^4\right)}{8\sqrt{3}}+\frac{\log\left(1+\sqrt{3}x^2+x^4\right)}{8\sqrt{3}}$$

output
```
-1/6/x^6-1/2/x^2-1/4*arctan(2*x^2-3^(1/2))-1/4*arctan(2*x^2+3^(1/2))-1/24*
ln(1+x^4-x^2*3^(1/2))*3^(1/2)+1/24*ln(1+x^4+x^2*3^(1/2))*3^(1/2)
```

3.357.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.58

$$\int \frac{1}{x^7\left(1-x^4+x^8\right)}\,dx = -\frac{1}{6x^6}-\frac{1}{2x^2}-\frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{\log(x-\#1)\#1^2}{-1+2\#1^4}\&\right]$$

input
```
Integrate[1/(x^7*(1 - x^4 + x^8)),x]
```

output
```
-1/6*1/x^6 - 1/(2*x^2) - RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1^2)/(-
1 + 2*#1^4) & ]/4
```

3.357.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.688, Rules used = {1695, 1443, 27, 1604, 1447, 1475, 1083, 217, 1478, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7 (x^8-x^4+1)}dx$$

$$\Big\downarrow \text{1695}$$

$$\frac{1}{2}\int \frac{1}{x^8 (x^8-x^4+1)}dx^2$$

$$\Big\downarrow \text{1443}$$

$$\frac{1}{2}\left(\frac{1}{3}\int \frac{3(1-x^4)}{x^4 (x^8-x^4+1)}dx^2-\frac{1}{3x^6}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\int \frac{1-x^4}{x^4 (x^8-x^4+1)}dx^2-\frac{1}{3x^6}\right)$$

$$\Big\downarrow \text{1604}$$

$$\frac{1}{2}\left(-\int \frac{x^4}{x^8-x^4+1}dx^2-\frac{1}{3x^6}-\frac{1}{x^2}\right)$$

$$\Big\downarrow \text{1447}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1-x^4}{x^8-x^4+1}dx^2-\frac{1}{2}\int \frac{x^4+1}{x^8-x^4+1}dx^2-\frac{1}{3x^6}-\frac{1}{x^2}\right)$$

$$\Big\downarrow \text{1475}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{x^4-\sqrt{3}x^2+1}dx^2-\frac{1}{2}\int \frac{1}{x^4+\sqrt{3}x^2+1}dx^2\right)+\frac{1}{2}\int \frac{1-x^4}{x^8-x^4+1}dx^2-\frac{1}{3x^6}-\frac{1}{x^2}\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\int \frac{1}{-x^4-1}d\left(2x^2-\sqrt{3}\right)+\int \frac{1}{-x^4-1}d\left(2x^2+\sqrt{3}\right)\right)+\frac{1}{2}\int \frac{1-x^4}{x^8-x^4+1}dx^2-\frac{1}{3x^6}-\frac{1}{x^2}\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1-x^4}{x^8-x^4+1}dx^2+\frac{1}{2}\left(\arctan\left(\sqrt{3}-2x^2\right)-\arctan\left(2x^2+\sqrt{3}\right)\right)-\frac{1}{3x^6}-\frac{1}{x^2}\right)$$

$$\Big\downarrow \text{1478}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{3}-2x^2}{x^4-\sqrt{3}x^2+1}dx^2}{2\sqrt{3}}-\frac{\int -\frac{2x^2+\sqrt{3}}{x^4+\sqrt{3}x^2+1}dx^2}{2\sqrt{3}}\right)+\frac{1}{2}\left(\arctan\left(\sqrt{3}-2x^2\right)-\arctan\left(2x^2+\sqrt{3}\right)\right)-\frac{1}{3x^6}-\frac{1}{x^2}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{3}-2x^2}{x^4-\sqrt{3}x^2+1}dx^2}{2\sqrt{3}}+\frac{\int \frac{2x^2+\sqrt{3}}{x^4+\sqrt{3}x^2+1}dx^2}{2\sqrt{3}}\right)+\frac{1}{2}\left(\arctan\left(\sqrt{3}-2x^2\right)-\arctan\left(2x^2+\sqrt{3}\right)\right)-\frac{1}{3x^6}-\frac{1}{x^2}\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\arctan\left(\sqrt{3}-2x^2\right)-\arctan\left(2x^2+\sqrt{3}\right)\right)-\frac{1}{3x^6}-\frac{1}{x^2}+\frac{1}{2}\left(\frac{\log\left(x^4+\sqrt{3}x^2+1\right)}{2\sqrt{3}}-\frac{\log\left(x^4-\sqrt{3}x^2+1\right)}{2\sqrt{3}}\right)\right)$$

input
```
Int[1/(x^7*(1 - x^4 + x^8)),x]
```

output
```
(-1/3*1/x^6 - x^(-2) + (ArcTan[Sqrt[3] - 2*x^2] - ArcTan[Sqrt[3] + 2*x^2])
/2 + (-1/2*Log[1 - Sqrt[3]*x^2 + x^4]/Sqrt[3] + Log[1 + Sqrt[3]*x^2 + x^4]
/(2*Sqrt[3]))/2)/2
```

3.357.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1443
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Sim
p[1/(a*d^2*(m + 1))   Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*
x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1447
```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Simp[1/2   Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Simp[1/2
 Int[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b
^2 - 4*a*c, 0] && PosQ[a*c]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

3.357. $\int \frac{1}{x^7(1-x^4+x^8)}\,dx$

rule 1695
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b
*x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c,
p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.357.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.48

method	result	size
risch	$\frac{-\frac{x^4}{2}-\frac{1}{6}}{x^6}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(9\_Z^4+3\_Z^2+1\right)}\_R\ln\left(-6\_R^3+x^2-\_R\right)\right)}{4}$	46
default	$-\frac{1}{6x^6}-\frac{1}{2x^2}+\frac{\sqrt{3}\left(-\frac{\ln\left(1+x^4-x^2\sqrt{3}\right)}{2}-\sqrt{3}\arctan\left(2x^2-\sqrt{3}\right)\right)}{12}+\frac{\sqrt{3}\left(\frac{\ln\left(1+x^4+x^2\sqrt{3}\right)}{2}-\sqrt{3}\arctan\left(2x^2+\sqrt{3}\right)\right)}{12}$	87

input
```
int(1/x^7/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
(-1/2*x^4-1/6)/x^6+1/4*sum(_R*ln(-6*_R^3+x^2-_R),_R=RootOf(9*_Z^4+3*_Z^2+1
))
```

3.357.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.81

$$\int \frac{1}{x^7(1-x^4+x^8)}\,dx = -\frac{\sqrt{6}x^6\sqrt{i\sqrt{3}-1}\log\left(6x^2+i\sqrt{6}\sqrt{3}\sqrt{i\sqrt{3}-1}\right)-\sqrt{6}x^6\sqrt{i\sqrt{3}-1}\log\left(6x^2-i\sqrt{6}\sqrt{3}\sqrt{i\sqrt{3}-1}\right)-$$

input
```
integrate(1/x^7/(x^8-x^4+1),x, algorithm="fricas")
```

output
```
-1/24*(sqrt(6)*x^6*sqrt(I*sqrt(3) - 1)*log(6*x^2 + I*sqrt(6)*sqrt(3)*sqrt(
I*sqrt(3) - 1)) - sqrt(6)*x^6*sqrt(I*sqrt(3) - 1)*log(6*x^2 - I*sqrt(6)*sq
rt(3)*sqrt(I*sqrt(3) - 1)) - sqrt(6)*x^6*sqrt(-I*sqrt(3) - 1)*log(6*x^2 +
I*sqrt(6)*sqrt(3)*sqrt(-I*sqrt(3) - 1)) + sqrt(6)*x^6*sqrt(-I*sqrt(3) - 1)
*log(6*x^2 - I*sqrt(6)*sqrt(3)*sqrt(-I*sqrt(3) - 1)) + 12*x^4 + 4)/x^6
```

3.357.6 Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^7(1-x^4+x^8)}\,dx = -\frac{\sqrt{3}\log\left(x^4-\sqrt{3}x^2+1\right)}{24} + \frac{\sqrt{3}\log\left(x^4+\sqrt{3}x^2+1\right)}{24} - \frac{\operatorname{atan}\left(2x^2-\sqrt{3}\right)}{4} - \frac{\operatorname{atan}\left(2x^2+\sqrt{3}\right)}{4} + \frac{-3x^4-1}{6x^6}$$

input
```
integrate(1/x**7/(x**8-x**4+1),x)
```

output
```
-sqrt(3)*log(x**4 - sqrt(3)*x**2 + 1)/24 + sqrt(3)*log(x**4 + sqrt(3)*x**2
 + 1)/24 - atan(2*x**2 - sqrt(3))/4 - atan(2*x**2 + sqrt(3))/4 + (-3*x**4
- 1)/(6*x**6)
```

3.357.7 Maxima [F]

$$\int \frac{1}{x^7(1-x^4+x^8)}\,dx = \int \frac{1}{(x^8-x^4+1)x^7}\,dx$$

input
```
integrate(1/x^7/(x^8-x^4+1),x, algorithm="maxima")
```

output
```
-1/6*(3*x^4 + 1)/x^6 - integrate(x^5/(x^8 - x^4 + 1), x)
```

3.357.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.58

$$\int \frac{1}{x^7(1-x^4+x^8)}\,dx = -\frac{1}{12}\sqrt{3}x^4\log\left(x^4+\sqrt{3}x^2+1\right) + \frac{1}{12}\sqrt{3}x^4\log\left(x^4-\sqrt{3}x^2+1\right) - \frac{3x^4+1}{6x^6}$$

input
```
integrate(1/x^7/(x^8-x^4+1),x, algorithm="giac")
```

output
```
-1/12*sqrt(3)*x^4*log(x^4 + sqrt(3)*x^2 + 1) + 1/12*sqrt(3)*x^4*log(x^4 -
sqrt(3)*x^2 + 1) - 1/6*(3*x^4 + 1)/x^6
```

3.357.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.66

$$\int \frac{1}{x^7(1-x^4+x^8)}\,dx = \operatorname{atan}\left(\frac{2x^2}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) + \operatorname{atan}\left(\frac{2x^2}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) - \frac{\frac{x^4}{2}+\frac{1}{6}}{x^6}$$

input
```
int(1/(x^7*(x^8 - x^4 + 1)),x)
```

output
```
atan((2*x^2)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/12 + 1/4) + atan((2*x^2)/(3^(
1/2)*1i + 1))*((3^(1/2)*1i)/12 - 1/4) - (x^4/2 + 1/6)/x^6
```

3.358 $\int \frac{x^8}{1-x^4+x^8}\,dx$

3.358.1 Optimal result

Integrand size = 16, antiderivative size = 356

$$\begin{aligned}\int \frac{x^8}{1-x^4+x^8}\,dx &= x+\frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}-\frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}\\
&\quad+\frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)\\
&\quad+\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)\\
&\quad+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)\\
&\quad-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(1+\sqrt{2+\sqrt{3}}x+x^2\right)\end{aligned}$$

output
```
x-1/8*ln(1+x^2-x*(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))+1/8*
ln(1+x^2+x*(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))+1/4*arctan
((-2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))/(3/2*2^(1/2)-1/
2*6^(1/2))-1/4*arctan((2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/
2)))/(3/2*2^(1/2)-1/2*6^(1/2))+1/8*ln(1+x^2-x*(1/2*6^(1/2)+1/2*2^(1/2)))*(
1/2*2^(1/2)+1/6*6^(1/2))-1/8*ln(1+x^2+x*(1/2*6^(1/2)+1/2*2^(1/2)))*(1/2*2^
(1/2)+1/6*6^(1/2))-1/4*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-
1/2*2^(1/2)))/(3/2*2^(1/2)+1/2*6^(1/2))+1/4*arctan((2*x+1/2*6^(1/2)+1/2*2^
(1/2))/(1/2*6^(1/2)-1/2*2^(1/2)))/(3/2*2^(1/2)+1/2*6^(1/2))
```

3.358.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.17

$$\int \frac{x^8}{1-x^4+x^8} \, dx = x+\frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{-\log(x-\#1)+\log(x-\#1)\#1^4}{-\#1^3+2\#1^7}\&\right]$$

input
```
Integrate[x^8/(1 - x^4 + x^8),x]
```

output
```
x + RootSum[1 - #1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)/(-#1^3 +
 2*#1^7) & ]/4
```

3.358.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 348, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {1703, 1751, 25, 1483, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{x^8-x^4+1} \, dx$$

$$\Big\downarrow \text{1703}$$

$$x-\int \frac{1-x^4}{x^8-x^4+1}dx$$

$$\Big\downarrow \text{1751}$$

$$\frac{\int -\frac{\sqrt{3}-2x^2}{x^4-\sqrt{3}x^2+1}dx}{2\sqrt{3}}+\frac{\int -\frac{2x^2+\sqrt{3}}{x^4+\sqrt{3}x^2+1}dx}{2\sqrt{3}}+x$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\sqrt{3}-2x^2}{x^4-\sqrt{3}x^2+1}dx}{2\sqrt{3}}-\frac{\int \frac{2x^2+\sqrt{3}}{x^4+\sqrt{3}x^2+1}dx}{2\sqrt{3}}+x$$

$$\Big\downarrow \text{1483}$$

$$-\frac{\frac{\int \frac{\left(2-\sqrt{3}\right)x+\sqrt{3\left(2-\sqrt{3}\right)}}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\int \frac{\sqrt{3\left(2-\sqrt{3}\right)}-\left(2-\sqrt{3}\right)x}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}-\frac{\frac{\int \frac{\sqrt{3\left(2+\sqrt{3}\right)}-\left(2+\sqrt{3}\right)x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{\int \frac{\left(2+\sqrt{3}\right)x+\sqrt{3\left(2+\sqrt{3}\right)}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}+x$$

$$\Big\downarrow \; 1142$$

$$-\frac{\frac{\frac{1}{2}\sqrt{2+\sqrt{3}}\int \frac{1}{x^2-\sqrt{2-\sqrt{3}}x+1}dx+\frac{1}{2}\left(2-\sqrt{3}\right)\int -\frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\frac{1}{2}\sqrt{2+\sqrt{3}}\int \frac{1}{x^2+\sqrt{2-\sqrt{3}}x+1}dx-\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}-\frac{\frac{-\frac{1}{2}\sqrt{2-\sqrt{3}}\int \frac{1}{x^2-\sqrt{2+\sqrt{3}}x+1}dx-\frac{1}{2}\left(2+\sqrt{3}\right)\int -\frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(2+\sqrt{3}\right)\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx-\frac{1}{2}\sqrt{2-\sqrt{3}}\int \frac{1}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}+x$$

$$\Big\downarrow \; 25$$

$$-\frac{\frac{\frac{1}{2}\sqrt{2+\sqrt{3}}\int \frac{1}{x^2-\sqrt{2-\sqrt{3}}x+1}dx-\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\frac{1}{2}\sqrt{2+\sqrt{3}}\int \frac{1}{x^2+\sqrt{2-\sqrt{3}}x+1}dx-\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}-\frac{\frac{\frac{1}{2}\left(2+\sqrt{3}\right)\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx-\frac{1}{2}\sqrt{2-\sqrt{3}}\int \frac{1}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(2+\sqrt{3}\right)\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx-\frac{1}{2}\sqrt{2-\sqrt{3}}\int \frac{1}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}+x$$

$$\Big\downarrow \; 1083$$

$$-\frac{\frac{-\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx-\sqrt{2+\sqrt{3}}\int \frac{1}{-\left(2x-\sqrt{2-\sqrt{3}}\right)^2-\sqrt{3}-2}d\left(2x-\sqrt{2-\sqrt{3}}\right)}{2\sqrt{2-\sqrt{3}}}+\frac{-\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx-\sqrt{2+\sqrt{3}}\int \frac{\cdots}{-\left(2x+\sqrt{\cdots}\right.}\cdots}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}\cdots \frac{\frac{\frac{1}{2}\left(2+\sqrt{3}\right)\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx+\sqrt{2-\sqrt{3}}\int \frac{1}{-\left(2x-\sqrt{2+\sqrt{3}}\right)^2+\sqrt{3}-2}d\left(2x-\sqrt{2+\sqrt{3}}\right)}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(2+\sqrt{3}\right)\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx+\sqrt{2-\sqrt{3}}\int \frac{1}{-\left(2x+\sqrt{2+\sqrt{3}}\cdots\right.}\cdots}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}\cdots x$$

$$\Big\downarrow \; 217$$

$$-\frac{\frac{\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}-\frac{\frac{\frac{1}{2}\left(2+\sqrt{3}\right)\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx-\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(2+\sqrt{3}\right)\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx-\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}+x$$

$$\Big\downarrow \; 1103$$

3.358. $\int \frac{x^8}{1-x^4+x^8}\,dx$

$$-\frac{\frac{\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)+\frac{1}{2}\left(2-\sqrt{3}\right)\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{2\sqrt{2-\sqrt{3}}}+\frac{\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{2}\left(2-\sqrt{3}\right)\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}-\frac{\frac{-\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)-\frac{1}{2}\left(2+\sqrt{3}\right)\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(2+\sqrt{3}\right)\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)-\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}+x$$

input
```
Int[x^8/(1 - x^4 + x^8),x]
```

output
```
x - ((ArcTan[(-Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]] + ((2 - Sqrt[3]
)*Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2])/2)/(2*Sqrt[2 - Sqrt[3]]) + (ArcTan[(
Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]] - ((2 - Sqrt[3])*Log[1 + Sqrt[
2 - Sqrt[3]]*x + x^2])/2)/(2*Sqrt[2 - Sqrt[3]]))/(2*Sqrt[3]) - ((-ArcTan[(
-Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]] - ((2 + Sqrt[3])*Log[1 - Sqrt
[2 + Sqrt[3]]*x + x^2])/2)/(2*Sqrt[2 + Sqrt[3]]) + (-ArcTan[(Sqrt[2 + Sqrt
[3]] + 2*x)/Sqrt[2 - Sqrt[3]]] + ((2 + Sqrt[3])*Log[1 + Sqrt[2 + Sqrt[3]]*
x + x^2])/2)/(2*Sqrt[2 + Sqrt[3]]))/(2*Sqrt[3])
```

3.358.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

3.358. $\int \frac{x^8}{1-x^4+x^8}\,dx$

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 1703
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(2*n - 1)*(d*x)^(m - 2*n + 1)*((a + b*x^n + c*x^(2*n))^(
p + 1)/(c*(m + 2*n*p + 1))), x] - Simp[d^(2*n)/(c*(m + 2*n*p + 1))   Int[(d
*x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x
^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && N
eQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0
] && IntegerQ[p]
```

rule 1751
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x
^(n/2))/Simp[d/e + q*x^(n/2) - x^n, x], x], x] + Simp[e/(2*c*q)   Int[(q +
2*x^(n/2))/Simp[d/e - q*x^(n/2) - x^n, x], x], x]] /; FreeQ[{a, b, c, d, e}
, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGt
Q[n/2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

3.358.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.12

method	result	size
default	$x+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{8}-\_Z^{4}+1\right)}\frac{\left(\_R^{4}-1\right)\ln\left(x-\_R\right)}{2\_R^{7}-\_R^{3}}\right)}{4}$	44
risch	$x+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{8}-\_Z^{4}+1\right)}\frac{\left(\_R^{4}-1\right)\ln\left(x-\_R\right)}{2\_R^{7}-\_R^{3}}\right)}{4}$	44

input
```
int(x^8/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
x+1/4*sum((_R^4-1)/(2*_R^7-_R^3)*ln(x-_R),_R=RootOf(_Z^8-_Z^4+1))
```

3.358.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.17

$$\int \frac{x^8}{1-x^4+x^8}\,dx = -\frac{1}{24}\sqrt{6}\sqrt{\sqrt{2}\sqrt{-i\sqrt{3}+1}}\log\left(\sqrt{6}\sqrt{\sqrt{2}\sqrt{-i\sqrt{3}+1}}\left(i\sqrt{3}+3\right)+12x\right) - \frac{1}{24}\sqrt{6}\sqrt{-\sqrt{2}\sqrt{-i\sqrt{3}+1}}\log\left(\sqrt{6}\sqrt{-\sqrt{2}\sqrt{-i\sqrt{3}+1}}\left(i\sqrt{3}+3\right)+12x\right) + \frac{1}{24}\sqrt{6}\sqrt{\sqrt{2}\sqrt{i\sqrt{3}+1}}\log\left(\sqrt{6}\sqrt{\sqrt{2}\sqrt{i\sqrt{3}+1}}\left(i\sqrt{3}-3\right)+12x\right) + \frac{1}{24}\sqrt{6}\sqrt{-\sqrt{2}\sqrt{i\sqrt{3}+1}}\log\left(\sqrt{6}\sqrt{-\sqrt{2}\sqrt{i\sqrt{3}+1}}\left(i\sqrt{3}-3\right)+12x\right) - \frac{1}{24}\sqrt{6}\sqrt{\sqrt{2}\sqrt{i\sqrt{3}+1}}\log\left(\sqrt{6}\sqrt{\sqrt{2}\sqrt{i\sqrt{3}+1}}\left(-i\sqrt{3}+3\right)+12x\right) - \frac{1}{24}\sqrt{6}\sqrt{-\sqrt{2}\sqrt{i\sqrt{3}+1}}\log\left(\sqrt{6}\sqrt{-\sqrt{2}\sqrt{i\sqrt{3}+1}}\left(-i\sqrt{3}+3\right)+12x\right) + \frac{1}{24}\sqrt{6}\sqrt{\sqrt{2}\sqrt{-i\sqrt{3}+1}}\log\left(\sqrt{6}\sqrt{\sqrt{2}\sqrt{-i\sqrt{3}+1}}\left(-i\sqrt{3}-3\right)+12x\right) + \frac{1}{24}\sqrt{6}\sqrt{-\sqrt{2}\sqrt{-i\sqrt{3}+1}}\log\left(\sqrt{6}\sqrt{-\sqrt{2}\sqrt{-i\sqrt{3}+1}}\left(-i\sqrt{3}-3\right)+12x\right) + x$$

input
```
integrate(x^8/(x^8-x^4+1),x, algorithm="fricas")
```

3.358. $\int \frac{x^8}{1-x^4+x^8}\,dx$

output
```
-1/24*sqrt(6)*sqrt(sqrt(2)*sqrt(-I*sqrt(3) + 1))*log(sqrt(6)*sqrt(sqrt(2)*
sqrt(-I*sqrt(3) + 1))*(I*sqrt(3) + 3) + 12*x) - 1/24*sqrt(6)*sqrt(-sqrt(2)
*sqrt(-I*sqrt(3) + 1))*log(sqrt(6)*sqrt(-sqrt(2)*sqrt(-I*sqrt(3) + 1))*(I*
sqrt(3) + 3) + 12*x) + 1/24*sqrt(6)*sqrt(sqrt(2)*sqrt(I*sqrt(3) + 1))*log(
sqrt(6)*sqrt(sqrt(2)*sqrt(I*sqrt(3) + 1))*(I*sqrt(3) - 3) + 12*x) + 1/24*s
qrt(6)*sqrt(-sqrt(2)*sqrt(I*sqrt(3) + 1))*log(sqrt(6)*sqrt(-sqrt(2)*sqrt(I
*sqrt(3) + 1))*(I*sqrt(3) - 3) + 12*x) - 1/24*sqrt(6)*sqrt(sqrt(2)*sqrt(I*
sqrt(3) + 1))*log(sqrt(6)*sqrt(sqrt(2)*sqrt(I*sqrt(3) + 1))*(-I*sqrt(3) +
3) + 12*x) - 1/24*sqrt(6)*sqrt(-sqrt(2)*sqrt(I*sqrt(3) + 1))*log(sqrt(6)*s
qrt(-sqrt(2)*sqrt(I*sqrt(3) + 1))*(-I*sqrt(3) + 3) + 12*x) + 1/24*sqrt(6)*
sqrt(sqrt(2)*sqrt(-I*sqrt(3) + 1))*log(sqrt(6)*sqrt(sqrt(2)*sqrt(-I*sqrt(3
) + 1))*(-I*sqrt(3) - 3) + 12*x) + 1/24*sqrt(6)*sqrt(-sqrt(2)*sqrt(-I*sqrt
(3) + 1))*log(sqrt(6)*sqrt(-sqrt(2)*sqrt(-I*sqrt(3) + 1))*(-I*sqrt(3) - 3)
 + 12*x) + x
```

3.358.6 Sympy [A] (verification not implemented)

Time = 1.49 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.07

$$\int \frac{x^8}{1-x^4+x^8}\,dx = x + \operatorname{RootSum}\left(5308416t^8 - 2304t^4 + 1, \left(t \mapsto t\log\left(9216t^5 - 8t + x\right)\right)\right)$$

input
```
integrate(x**8/(x**8-x**4+1),x)
```

output
```
x + RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(9216*_t**5 -
 8*_t + x)))
```

3.358.7 Maxima [F]

$$\int \frac{x^8}{1-x^4+x^8}\,dx = \int \frac{x^8}{x^8-x^4+1}\,dx$$

input
```
integrate(x^8/(x^8-x^4+1),x, algorithm="maxima")
```

output
```
x + integrate((x^4 - 1)/(x^8 - x^4 + 1), x)
```

3.358.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.71

$$\begin{aligned}\int \frac{x^8}{1-x^4+x^8}\,dx = &-\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)\\ &-\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)\\ &-\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)\\ &-\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)\\ &-\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)\\ &+\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)\\ &-\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)\\ &+\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)+x\end{aligned}$$

input
```
integrate(x^8/(x^8-x^4+1),x, algorithm="giac")
```

output
```
-1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sq
rt(2))) - 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqr
t(6) + sqrt(2))) - 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt
(2))/(sqrt(6) - sqrt(2))) - 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(
6) - sqrt(2))/(sqrt(6) - sqrt(2))) - 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 +
1/2*x*(sqrt(6) + sqrt(2)) + 1) + 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*
x*(sqrt(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x*(s
qrt(6) - sqrt(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(
6) - sqrt(2)) + 1) + x
```

3.358.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.59

$$\int \frac{x^8}{1-x^4+x^8}\,dx = x+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x}{\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}+\frac{\sqrt{3}\,x\,1\mathrm{i}}{\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}\right)\,\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}\,1\mathrm{i}}{12} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\,1\mathrm{i}}{\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}-\frac{\sqrt{3}\,x}{\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}\right)\,\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}{12} - \frac{2^{3/4}\,\sqrt{3}\,\mathrm{atan}\left(\frac{2^{1/4}\,x}{2\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}-\frac{2^{1/4}\,\sqrt{3}\,x\,1\mathrm{i}}{2\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}\,1\mathrm{i}}{12} - \frac{2^{3/4}\,\sqrt{3}\,\mathrm{atan}\left(\frac{2^{1/4}\,x\,1\mathrm{i}}{2\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}+\frac{2^{1/4}\,\sqrt{3}\,x}{2\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}{12}$$

input
```
int(x^8/(x^8 - x^4 + 1),x)
```

output
```
x + (3^(1/2)*atan(x/(8 - 3^(1/2)*8i)^(1/4) + (3^(1/2)*x*1i)/(8 - 3^(1/2)*8
i)^(1/4))*(8 - 3^(1/2)*8i)^(1/4)*1i)/12 + (3^(1/2)*atan((x*1i)/(8 - 3^(1/2
)*8i)^(1/4) - (3^(1/2)*x)/(8 - 3^(1/2)*8i)^(1/4))*(8 - 3^(1/2)*8i)^(1/4))/
12 - (2^(3/4)*3^(1/2)*atan((2^(1/4)*x)/(2*(3^(1/2)*1i + 1)^(1/4)) - (2^(1/
4)*3^(1/2)*x*1i)/(2*(3^(1/2)*1i + 1)^(1/4)))*(3^(1/2)*1i + 1)^(1/4)*1i)/12
 - (2^(3/4)*3^(1/2)*atan((2^(1/4)*x*1i)/(2*(3^(1/2)*1i + 1)^(1/4)) + (2^(1
/4)*3^(1/2)*x)/(2*(3^(1/2)*1i + 1)^(1/4)))*(3^(1/2)*1i + 1)^(1/4))/12
```

3.359 $\int \frac{x^6}{1-x^4+x^8} dx$

3.359.1 Optimal result

Integrand size = 16, antiderivative size = 275

$$\int \frac{x^6}{1-x^4+x^8} dx = -\frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}} - \frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}} + \frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}} + \frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}} + \frac{\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)}{4\sqrt{6}} - \frac{\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)}{4\sqrt{6}} + \frac{\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)}{4\sqrt{6}} - \frac{\log\left(1+\sqrt{2+\sqrt{3}}x+x^2\right)}{4\sqrt{6}}$$

output
```
-1/12*arctan((-2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))*6^(
1/2)+1/12*arctan((2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))*
6^(1/2)-1/12*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/2
)))*6^(1/2)+1/12*arctan((2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(
1/2)))*6^(1/2)+1/24*ln(1+x^2-x*(1/2*6^(1/2)-1/2*2^(1/2)))*6^(1/2)-1/24*ln(
1+x^2+x*(1/2*6^(1/2)-1/2*2^(1/2)))*6^(1/2)+1/24*ln(1+x^2-x*(1/2*6^(1/2)+1/
2*2^(1/2)))*6^(1/2)-1/24*ln(1+x^2+x*(1/2*6^(1/2)+1/2*2^(1/2)))*6^(1/2)
```

3.359.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.15

$$\int \frac{x^6}{1-x^4+x^8}\,dx = \frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{\log(x-\#1)\#1^3}{-1+2\#1^4}\&\right]$$

input
```
Integrate[x^6/(1 - x^4 + x^8),x]
```

output
```
RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1^3)/(-1 + 2*#1^4) & ]/4
```

3.359.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.50, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1708, 1483, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{x^8-x^4+1}\,dx$$

$$\Big\downarrow \text{1708}$$

$$\frac{\int \frac{\sqrt{3}x^2+1}{x^4+\sqrt{3}x^2+1}dx}{2\sqrt{3}}-\frac{\int \frac{1-\sqrt{3}x^2}{x^4-\sqrt{3}x^2+1}dx}{2\sqrt{3}}$$

$$\Big\downarrow \text{1483}$$

$$\frac{\frac{\int \frac{\sqrt{2-\sqrt{3}}-\left(1-\sqrt{3}\right)x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\int \frac{\left(1-\sqrt{3}\right)x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}-\frac{\frac{\int \frac{\sqrt{2+\sqrt{3}}-\left(1+\sqrt{3}\right)x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{\int \frac{\left(1+\sqrt{3}\right)x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}$$

$$\Big\downarrow \text{1142}$$

$$\frac{\frac{\frac{\int \frac{1}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{\sqrt{2}}-\frac{1}{2}\left(1-\sqrt{3}\right)\int -\frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\frac{\int \frac{1}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{\sqrt{2}}+\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}-\frac{\frac{-\frac{\int \frac{1}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{\sqrt{2}}-\frac{1}{2}\left(1+\sqrt{3}\right)\int -\frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx-\frac{\int \frac{1}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{\sqrt{2}}}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}$$

$\downarrow$ 25

$$\frac{\frac{\frac{\int \frac{1}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{\sqrt{2}}+\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\frac{\int \frac{1}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{\sqrt{2}}+\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}-\frac{\frac{\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx-\frac{\int \frac{1}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{\sqrt{2}}}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx-\frac{\int \frac{1}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{\sqrt{2}}}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}$$

$\downarrow$ 1083

$$\frac{\frac{\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx-\sqrt{2}\int \frac{1}{-\left(2x-\sqrt{2-\sqrt{3}}\right)^2-\sqrt{3}-2}d\left(2x-\sqrt{2-\sqrt{3}}\right)}{2\sqrt{2-\sqrt{3}}}+\frac{\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx-\sqrt{2}\int \frac{1}{-\left(2x+\sqrt{2-\sqrt{3}}\right)^2-\sqrt{3}-2}d}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}$$

$$\frac{\frac{\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx+\sqrt{2}\int \frac{1}{-\left(2x-\sqrt{2+\sqrt{3}}\right)^2+\sqrt{3}-2}d\left(2x-\sqrt{2+\sqrt{3}}\right)}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx+\sqrt{2}\int \frac{1}{-\left(2x+\sqrt{2+\sqrt{3}}\right)^2+\sqrt{3}-2}d}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}$$

$\downarrow$ 217

$$\frac{\frac{\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx+\sqrt{\frac{2}{2+\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{2-\sqrt{3}}}+\frac{\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx+\sqrt{\frac{2}{2+\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}-\frac{\frac{\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx-\sqrt{\frac{2}{2-\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx-\sqrt{\frac{2}{2-\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}$$

$\downarrow$ 1103

$$\frac{\frac{\sqrt{\frac{2}{2+\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{2}\left(1-\sqrt{3}\right)\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{2\sqrt{2-\sqrt{3}}}+\frac{\sqrt{\frac{2}{2+\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)+\frac{1}{2}\left(1-\sqrt{3}\right)\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}-\frac{\frac{-\sqrt{\frac{2}{2-\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)-\frac{1}{2}\left(1+\sqrt{3}\right)\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(1+\sqrt{3}\right)\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)-\sqrt{\frac{2}{2-\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}$$

input
```
Int[x^6/(1 - x^4 + x^8),x]
```

3.359. $\int \frac{x^6}{1-x^4+x^8}\,dx$

output
```
((Sqrt[2/(2 + Sqrt[3])]*ArcTan[(-Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]
]] - ((1 - Sqrt[3])*Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2])/2)/(2*Sqrt[2 - Sqr
t[3]]) + (Sqrt[2/(2 + Sqrt[3])]*ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 +
Sqrt[3]]] + ((1 - Sqrt[3])*Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2])/2)/(2*Sqrt[
2 - Sqrt[3]]))/(2*Sqrt[3]) - ((-(Sqrt[2/(2 - Sqrt[3])]*ArcTan[(-Sqrt[2 + S
qrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]) - ((1 + Sqrt[3])*Log[1 - Sqrt[2 + Sqrt[
3]]*x + x^2])/2)/(2*Sqrt[2 + Sqrt[3]]) + (-(Sqrt[2/(2 - Sqrt[3])]*ArcTan[(
Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]) + ((1 + Sqrt[3])*Log[1 + Sqrt
[2 + Sqrt[3]]*x + x^2])/2)/(2*Sqrt[2 + Sqrt[3]]))/(2*Sqrt[3])
```

3.359.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

3.359. $\int \frac{x^6}{1-x^4+x^8} \, dx$

rule 1708
```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, -Simp[1/(2*c*r)   Int[x^
(m - 3*(n/2))*((q - r*x^(n/2))/(q - r*x^(n/2) + x^n)), x], x] + Simp[1/(2*c
*r)   Int[x^(m - 3*(n/2))*((q + r*x^(n/2))/(q + r*x^(n/2) + x^n)), x], x]]]
 /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n/2,
 0] && IGtQ[m, 0] && GeQ[m, 3*(n/2)] && LtQ[m, 2*n] && NegQ[b^2 - 4*a*c]
```

3.359.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.12

method	result	size
default	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(9\_Z^4+1\right)}\_R\ln\left(9x\_R^3-3\_R^2+x^2\right)\right)}{4}$	32
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(9\_Z^4+1\right)}\_R\ln\left(9x\_R^3-3\_R^2+x^2\right)\right)}{4}$	32

input
```
int(x^6/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(_R*ln(9*_R^3*x-3*_R^2+x^2),_R=RootOf(9*_Z^4+1))
```

3.359.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.37

$$\begin{aligned}\int \frac{x^6}{1-x^4+x^8}\,dx =& \left(\frac{1}{24}i-\frac{1}{24}\right)\sqrt{3}\sqrt{2}\log\left((3i+3)\sqrt{3}\sqrt{2}x+6x^2+6i\right)\\ &-\left(\frac{1}{24}i+\frac{1}{24}\right)\sqrt{3}\sqrt{2}\log\left(-(3i-3)\sqrt{3}\sqrt{2}x+6x^2-6i\right)\\ &+\left(\frac{1}{24}i+\frac{1}{24}\right)\sqrt{3}\sqrt{2}\log\left((3i-3)\sqrt{3}\sqrt{2}x+6x^2-6i\right)\\ &-\left(\frac{1}{24}i-\frac{1}{24}\right)\sqrt{3}\sqrt{2}\log\left(-(3i+3)\sqrt{3}\sqrt{2}x+6x^2+6i\right)\end{aligned}$$

input
```
integrate(x^6/(x^8-x^4+1),x, algorithm="fricas")
```

output
```
(1/24*I - 1/24)*sqrt(3)*sqrt(2)*log((3*I + 3)*sqrt(3)*sqrt(2)*x + 6*x^2 +
6*I) - (1/24*I + 1/24)*sqrt(3)*sqrt(2)*log(-(3*I - 3)*sqrt(3)*sqrt(2)*x +
6*x^2 - 6*I) + (1/24*I + 1/24)*sqrt(3)*sqrt(2)*log((3*I - 3)*sqrt(3)*sqrt(
2)*x + 6*x^2 - 6*I) - (1/24*I - 1/24)*sqrt(3)*sqrt(2)*log(-(3*I + 3)*sqrt(
3)*sqrt(2)*x + 6*x^2 + 6*I)
```

3.359.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.60

$$\int \frac{x^6}{1-x^4+x^8}\,dx = \frac{\sqrt{6}\cdot\left(2\,\mathrm{atan}\left(\frac{\sqrt{6}x}{3}-\frac{1}{3}\right)+2\,\mathrm{atan}\left(\sqrt{6}x^3-4x^2+2\sqrt{6}x-3\right)\right)}{24} + \frac{\sqrt{6}\cdot\left(2\,\mathrm{atan}\left(\frac{\sqrt{6}x}{3}+\frac{1}{3}\right)+2\,\mathrm{atan}\left(\sqrt{6}x^3+4x^2+2\sqrt{6}x+3\right)\right)}{24} + \frac{\sqrt{6}\log\left(x^4-\sqrt{6}x^3+3x^2-\sqrt{6}x+1\right)}{24} - \frac{\sqrt{6}\log\left(x^4+\sqrt{6}x^3+3x^2+\sqrt{6}x+1\right)}{24}$$

input
```
integrate(x**6/(x**8-x**4+1),x)
```

output
```
sqrt(6)*(2*atan(sqrt(6)*x/3 - 1/3) + 2*atan(sqrt(6)*x**3 - 4*x**2 + 2*sqrt
(6)*x - 3))/24 + sqrt(6)*(2*atan(sqrt(6)*x/3 + 1/3) + 2*atan(sqrt(6)*x**3
+ 4*x**2 + 2*sqrt(6)*x + 3))/24 + sqrt(6)*log(x**4 - sqrt(6)*x**3 + 3*x**2
 - sqrt(6)*x + 1)/24 - sqrt(6)*log(x**4 + sqrt(6)*x**3 + 3*x**2 + sqrt(6)*
x + 1)/24
```

3.359.7 Maxima [F]

$$\int \frac{x^6}{1-x^4+x^8}\,dx = \int \frac{x^6}{x^8-x^4+1}\,dx$$

input
```
integrate(x^6/(x^8-x^4+1),x, algorithm="maxima")
```

output
```
integrate(x^6/(x^8 - x^4 + 1), x)
```

3.359.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.75

$$\begin{aligned}\int \frac{x^6}{1-x^4+x^8}\,dx =& \frac{1}{12}\sqrt{6}\arctan\left(\frac{4\,x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4\,x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)\\ &+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4\,x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4\,x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)\\ &-\frac{1}{24}\sqrt{6}\log\left(x^2+\frac{1}{2}\,x\left(\sqrt{6}+\sqrt{2}\right)+1\right)\\ &+\frac{1}{24}\sqrt{6}\log\left(x^2-\frac{1}{2}\,x\left(\sqrt{6}+\sqrt{2}\right)+1\right)\\ &-\frac{1}{24}\sqrt{6}\log\left(x^2+\frac{1}{2}\,x\left(\sqrt{6}-\sqrt{2}\right)+1\right)\\ &+\frac{1}{24}\sqrt{6}\log\left(x^2-\frac{1}{2}\,x\left(\sqrt{6}-\sqrt{2}\right)+1\right)\end{aligned}$$

input
```
integrate(x^6/(x^8-x^4+1),x, algorithm="giac")
```

output
```
1/12*sqrt(6)*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt(2))) + 1/12*
sqrt(6)*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6) + sqrt(2))) + 1/12*sqrt(
6)*arctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2))) + 1/12*sqrt(6)*ar
ctan((4*x - sqrt(6) - sqrt(2))/(sqrt(6) - sqrt(2))) - 1/24*sqrt(6)*log(x^2
 + 1/2*x*(sqrt(6) + sqrt(2)) + 1) + 1/24*sqrt(6)*log(x^2 - 1/2*x*(sqrt(6)
+ sqrt(2)) + 1) - 1/24*sqrt(6)*log(x^2 + 1/2*x*(sqrt(6) - sqrt(2)) + 1) +
1/24*sqrt(6)*log(x^2 - 1/2*x*(sqrt(6) - sqrt(2)) + 1)
```

3.359.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.19

$$\int \frac{x^6}{1-x^4+x^8}\,dx = \sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x\,\left(\frac{1}{3}+\frac{1}{3}\mathrm{i}\right)}{\frac{2\,x^2}{3}-\frac{2}{3}\mathrm{i}}\right)\,\left(-\frac{1}{12}+\frac{1}{12}\mathrm{i}\right) + \sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x\,\left(\frac{1}{3}-\frac{1}{3}\mathrm{i}\right)}{\frac{2\,x^2}{3}+\frac{2}{3}\mathrm{i}}\right)\,\left(-\frac{1}{12}-\frac{1}{12}\mathrm{i}\right)$$

input
```
int(x^6/(x^8 - x^4 + 1),x)
```

output
```
- 6^(1/2)*atan((6^(1/2)*x*(1/3 + 1i/3))/((2*x^2)/3 - 2i/3))*(1/12 - 1i/12)
 - 6^(1/2)*atan((6^(1/2)*x*(1/3 - 1i/3))/((2*x^2)/3 + 2i/3))*(1/12 + 1i/12
)
```

3.360 $\int \frac{x^4}{1-x^4+x^8} dx$

3.360.1 Optimal result

Integrand size = 16, antiderivative size = 347

$$\int \frac{x^4}{1-x^4+x^8} dx = \frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}} - \frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}} + \frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}} + \frac{\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}} + \frac{\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)}{8\sqrt{3\left(2+\sqrt{3}\right)}} - \frac{\log\left(1+\sqrt{2+\sqrt{3}}x+x^2\right)}{8\sqrt{3\left(2+\sqrt{3}\right)}}$$

output
```
-1/4*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/2)))/(3/2
*2^(1/2)-1/2*6^(1/2))+1/4*arctan((2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2
)-1/2*2^(1/2)))/(3/2*2^(1/2)-1/2*6^(1/2))-1/8*ln(1+x^2-x*(1/2*6^(1/2)-1/2*
2^(1/2)))/(3/2*2^(1/2)-1/2*6^(1/2))+1/8*ln(1+x^2+x*(1/2*6^(1/2)-1/2*2^(1/2
)))/(3/2*2^(1/2)-1/2*6^(1/2))+1/4*arctan((-2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1
/2*6^(1/2)+1/2*2^(1/2)))/(3/2*2^(1/2)+1/2*6^(1/2))-1/4*arctan((2*x+1/2*6^(
1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))/(3/2*2^(1/2)+1/2*6^(1/2))+1/8
*ln(1+x^2-x*(1/2*6^(1/2)+1/2*2^(1/2)))/(3/2*2^(1/2)+1/2*6^(1/2))-1/8*ln(1+
x^2+x*(1/2*6^(1/2)+1/2*2^(1/2)))/(3/2*2^(1/2)+1/2*6^(1/2))
```

3.360.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.11

$$\int \frac{x^4}{1-x^4+x^8}\,dx = \frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{\log(x-\#1)\#1}{-1+2\#1^4}\&\right]$$

input
```
Integrate[x^4/(1 - x^4 + x^8),x]
```

output
```
RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1)/(-1 + 2*#1^4) & ]/4
```

3.360.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1709, 1447, 1475, 1083, 217, 1478, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{x^8-x^4+1}dx$$

$$\Big\downarrow \text{1709}$$

$$\frac{\int \frac{x^2}{x^4-\sqrt{3}x^2+1}dx}{2\sqrt{3}}-\frac{\int \frac{x^2}{x^4+\sqrt{3}x^2+1}dx}{2\sqrt{3}}$$

$$\Big\downarrow \text{1447}$$

$$\frac{\frac{1}{2}\int \frac{x^2+1}{x^4-\sqrt{3}x^2+1}dx-\frac{1}{2}\int \frac{1-x^2}{x^4-\sqrt{3}x^2+1}dx}{2\sqrt{3}}-\frac{\frac{1}{2}\int \frac{x^2+1}{x^4+\sqrt{3}x^2+1}dx-\frac{1}{2}\int \frac{1-x^2}{x^4+\sqrt{3}x^2+1}dx}{2\sqrt{3}}$$

$$\Big\downarrow \text{1475}$$

$$\frac{\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2-\sqrt{2+\sqrt{3}}x+1}dx+\frac{1}{2}\int \frac{1}{x^2+\sqrt{2+\sqrt{3}}x+1}dx\right)-\frac{1}{2}\int \frac{1-x^2}{x^4-\sqrt{3}x^2+1}dx}{2\sqrt{3}}-\frac{\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2-\sqrt{2-\sqrt{3}}x+1}dx+\frac{1}{2}\int \frac{1}{x^2+\sqrt{2-\sqrt{3}}x+1}dx\right)-\frac{1}{2}\int \frac{1-x^2}{x^4+\sqrt{3}x^2+1}dx}{2\sqrt{3}}$$

$$\Big\downarrow \text{1083}$$

$$\frac{\frac{1}{2}\left(-\int \frac{1}{-\left(2x-\sqrt{2+\sqrt{3}}\right)^2+\sqrt{3}-2}d\left(2x-\sqrt{2+\sqrt{3}}\right)-\int \frac{1}{-\left(2x+\sqrt{2+\sqrt{3}}\right)^2+\sqrt{3}-2}d\left(2x+\sqrt{2+\sqrt{3}}\right)\right)-\frac{1}{2}\int \frac{1-x^2}{x^4-\sqrt{3}x^2+1}dx}{2\sqrt{3}}$$

$$\frac{\frac{1}{2}\left(-\int \frac{1}{-\left(2x-\sqrt{2-\sqrt{3}}\right)^2-\sqrt{3}-2}d\left(2x-\sqrt{2-\sqrt{3}}\right)-\int \frac{1}{-\left(2x+\sqrt{2-\sqrt{3}}\right)^2-\sqrt{3}-2}d\left(2x+\sqrt{2-\sqrt{3}}\right)\right)-\frac{1}{2}\int \frac{1-x^2}{x^4+\sqrt{3}x^2+1}dx}{2\sqrt{3}}$$

$\downarrow$ 217

$$\frac{\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2-\sqrt{3}}}+\frac{\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2-\sqrt{3}}}\right)-\frac{1}{2}\int \frac{1-x^2}{x^4-\sqrt{3}x^2+1}dx}{2\sqrt{3}}-\frac{\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{2+\sqrt{3}}}+\frac{\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{2}\int \frac{1-x^2}{x^4+\sqrt{3}x^2+1}dx}{2\sqrt{3}}$$

$\downarrow$ 1478

$$\frac{\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{\int -\frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2-\sqrt{3}}}+\frac{\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3}}-\frac{\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\int -\frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{2+\sqrt{3}}}+\frac{\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3}}$$

$\downarrow$ 25

$$\frac{\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}-\frac{\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2-\sqrt{3}}}+\frac{\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3}}-\frac{\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}-\frac{\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{2+\sqrt{3}}}+\frac{\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3}}$$

$\downarrow$ 1103

3.360. $\int \frac{x^4}{1-x^4+x^8}\,dx$

$$\frac{\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2-\sqrt{3}}}+\frac{\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2-\sqrt{3}}}\right)+\frac{1}{2}\left(\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{2\sqrt{2+\sqrt{3}}}-\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)}{2\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3}}-\frac{\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{2+\sqrt{3}}}+\frac{\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{2+\sqrt{3}}}\right)+\frac{1}{2}\left(\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{2\sqrt{2-\sqrt{3}}}-\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{2\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3}}$$

input
```
Int[x^4/(1 - x^4 + x^8),x]
```

output
```
-1/2*((ArcTan[(-Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/Sqrt[2 + Sqrt[
3]] + ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/Sqrt[2 + Sqrt[3]
])/2 + (Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2]/(2*Sqrt[2 - Sqrt[3]]) - Log[1 +
 Sqrt[2 - Sqrt[3]]*x + x^2]/(2*Sqrt[2 - Sqrt[3]]))/2)/Sqrt[3] + ((ArcTan[(
-Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/Sqrt[2 - Sqrt[3]] + ArcTan[(S
qrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/Sqrt[2 - Sqrt[3]])/2 + (Log[1 -
 Sqrt[2 + Sqrt[3]]*x + x^2]/(2*Sqrt[2 + Sqrt[3]]) - Log[1 + Sqrt[2 + Sqrt[
3]]*x + x^2]/(2*Sqrt[2 + Sqrt[3]]))/2)/(2*Sqrt[3])
```

3.360.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1447
```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Simp[1/2   Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Simp[1/2
 Int[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b
^2 - 4*a*c, 0] && PosQ[a*c]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

rule 1709
```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*r)   Int[x^(
m - n/2)/(q - r*x^(n/2) + x^n), x], x] - Simp[1/(2*c*r)   Int[x^(m - n/2)/(
q + r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && IGtQ[m, 0] && GeQ[m, n/2] && LtQ[m, 3*
(n/2)] && NegQ[b^2 - 4*a*c]
```

3.360.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.12

method	result	size
default	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^8-\_Z^4+1\right)}\frac{\_R^4\ln\left(x-\_R\right)}{2\_R^7-\_R^3}\right)}{4}$	40
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^8-\_Z^4+1\right)}\frac{\_R^4\ln\left(x-\_R\right)}{2\_R^7-\_R^3}\right)}{4}$	40

3.360. $\int \frac{x^4}{1-x^4+x^8}\,dx$

input
```
int(x^4/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(_R^4/(2*_R^7-_R^3)*ln(x-_R),_R=RootOf(_Z^8-_Z^4+1))
```

3.360.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.13

$$\begin{aligned}\int \frac{x^4}{1-x^4+x^8}\,dx = & -\frac{1}{24}\sqrt{6}\sqrt{\sqrt{2}\sqrt{i\sqrt{3}+1}}\log\left(i\sqrt{6}\sqrt{3}\sqrt{\sqrt{2}\sqrt{i\sqrt{3}+1}}+6x\right)\\
& +\frac{1}{24}\sqrt{6}\sqrt{\sqrt{2}\sqrt{i\sqrt{3}+1}}\log\left(-i\sqrt{6}\sqrt{3}\sqrt{\sqrt{2}\sqrt{i\sqrt{3}+1}}+6x\right)\\
& -\frac{1}{24}\sqrt{6}\sqrt{-\sqrt{2}\sqrt{i\sqrt{3}+1}}\log\left(i\sqrt{6}\sqrt{3}\sqrt{-\sqrt{2}\sqrt{i\sqrt{3}+1}}+6x\right)\\
& +\frac{1}{24}\sqrt{6}\sqrt{-\sqrt{2}\sqrt{i\sqrt{3}+1}}\log\left(-i\sqrt{6}\sqrt{3}\sqrt{-\sqrt{2}\sqrt{i\sqrt{3}+1}}+6x\right)\\
& +\frac{1}{24}\sqrt{6}\sqrt{\sqrt{2}\sqrt{-i\sqrt{3}+1}}\log\left(i\sqrt{6}\sqrt{3}\sqrt{\sqrt{2}\sqrt{-i\sqrt{3}+1}}+6x\right)\\
& -\frac{1}{24}\sqrt{6}\sqrt{\sqrt{2}\sqrt{-i\sqrt{3}+1}}\log\left(-i\sqrt{6}\sqrt{3}\sqrt{\sqrt{2}\sqrt{-i\sqrt{3}+1}}+6x\right)\\
& +\frac{1}{24}\sqrt{6}\sqrt{-\sqrt{2}\sqrt{-i\sqrt{3}+1}}\log\left(i\sqrt{6}\sqrt{3}\sqrt{-\sqrt{2}\sqrt{-i\sqrt{3}+1}}+6x\right)\\
& -\frac{1}{24}\sqrt{6}\sqrt{-\sqrt{2}\sqrt{-i\sqrt{3}+1}}\log\left(-i\sqrt{6}\sqrt{3}\sqrt{-\sqrt{2}\sqrt{-i\sqrt{3}+1}}+6x\right)\end{aligned}$$

input
```
integrate(x^4/(x^8-x^4+1),x, algorithm="fricas")
```

output
```
-1/24*sqrt(6)*sqrt(sqrt(2)*sqrt(I*sqrt(3) + 1))*log(I*sqrt(6)*sqrt(3)*sqrt
(sqrt(2)*sqrt(I*sqrt(3) + 1)) + 6*x) + 1/24*sqrt(6)*sqrt(sqrt(2)*sqrt(I*sq
rt(3) + 1))*log(-I*sqrt(6)*sqrt(3)*sqrt(sqrt(2)*sqrt(I*sqrt(3) + 1)) + 6*x
) - 1/24*sqrt(6)*sqrt(-sqrt(2)*sqrt(I*sqrt(3) + 1))*log(I*sqrt(6)*sqrt(3)*
sqrt(-sqrt(2)*sqrt(I*sqrt(3) + 1)) + 6*x) + 1/24*sqrt(6)*sqrt(-sqrt(2)*sqr
t(I*sqrt(3) + 1))*log(-I*sqrt(6)*sqrt(3)*sqrt(-sqrt(2)*sqrt(I*sqrt(3) + 1)
) + 6*x) + 1/24*sqrt(6)*sqrt(sqrt(2)*sqrt(-I*sqrt(3) + 1))*log(I*sqrt(6)*s
qrt(3)*sqrt(sqrt(2)*sqrt(-I*sqrt(3) + 1)) + 6*x) - 1/24*sqrt(6)*sqrt(sqrt(
2)*sqrt(-I*sqrt(3) + 1))*log(-I*sqrt(6)*sqrt(3)*sqrt(sqrt(2)*sqrt(-I*sqrt(
3) + 1)) + 6*x) + 1/24*sqrt(6)*sqrt(-sqrt(2)*sqrt(-I*sqrt(3) + 1))*log(I*s
qrt(6)*sqrt(3)*sqrt(-sqrt(2)*sqrt(-I*sqrt(3) + 1)) + 6*x) - 1/24*sqrt(6)*s
qrt(-sqrt(2)*sqrt(-I*sqrt(3) + 1))*log(-I*sqrt(6)*sqrt(3)*sqrt(-sqrt(2)*sq
rt(-I*sqrt(3) + 1)) + 6*x)
```

3.360.6 Sympy [A] (verification not implemented)

Time = 1.48 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.07

$$\int \frac{x^4}{1-x^4+x^8}\,dx = \text{RootSum}\left(5308416t^8-2304t^4+1,\left(t\mapsto t\log\left(-18432t^5+4t+x\right)\right)\right)$$

input
```
integrate(x**4/(x**8-x**4+1),x)
```

output
```
RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(-18432*_t**5 + 4
*_t + x)))
```

3.360.7 Maxima [F]

$$\int \frac{x^4}{1-x^4+x^8}\,dx = \int \frac{x^4}{x^8-x^4+1}\,dx$$

input
```
integrate(x^4/(x^8-x^4+1),x, algorithm="maxima")
```

output
```
integrate(x^4/(x^8 - x^4 + 1), x)
```

3.360.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.73

$$
\begin{aligned}
\int \frac{x^4}{1-x^4+x^8}\,dx = {} & \frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right) \\
& + \frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right) \\
& + \frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) \\
& + \frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) \\
& + \frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right) \\
& - \frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right) \\
& + \frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right) \\
& - \frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)
\end{aligned}
$$

input
```
integrate(x^4/(x^8-x^4+1),x, algorithm="giac")
```

output
```
1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqr
t(2))) + 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt
(6) + sqrt(2))) + 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt(
2))/(sqrt(6) - sqrt(2))) + 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6
) - sqrt(2))/(sqrt(6) - sqrt(2))) + 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1
/2*x*(sqrt(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x
*(sqrt(6) + sqrt(2)) + 1) + 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 + 1/2*x*(sq
rt(6) - sqrt(2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6
) - sqrt(2)) + 1)
```

3.360.9 Mupad [B] (verification not implemented)

Time = 8.25 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.37

$$\int \frac{x^4}{1-x^4+x^8}\,dx$$
$$= -\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\,(8-\sqrt{3}\,8\mathrm{i})^{1/4}}{2\left(\frac{\sqrt{3}\,\sqrt{8-\sqrt{3}\,8\mathrm{i}}\,1\mathrm{i}}{4}+\frac{\sqrt{8-\sqrt{3}\,8\mathrm{i}}}{4}\right)}+\frac{\sqrt{3}\,x\,(8-\sqrt{3}\,8\mathrm{i})^{1/4}\,1\mathrm{i}}{2\left(\frac{\sqrt{3}\,\sqrt{8-\sqrt{3}\,8\mathrm{i}}\,1\mathrm{i}}{4}+\frac{\sqrt{8-\sqrt{3}\,8\mathrm{i}}}{4}\right)}\right)(8-\sqrt{3}\,8\mathrm{i})^{1/4}\,1\mathrm{i}}{12}$$
$$-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\,(8-\sqrt{3}\,8\mathrm{i})^{1/4}\,1\mathrm{i}}{2\left(\frac{\sqrt{3}\,\sqrt{8-\sqrt{3}\,8\mathrm{i}}\,1\mathrm{i}}{4}+\frac{\sqrt{8-\sqrt{3}\,8\mathrm{i}}}{4}\right)}-\frac{\sqrt{3}\,x\,(8-\sqrt{3}\,8\mathrm{i})^{1/4}}{2\left(\frac{\sqrt{3}\,\sqrt{8-\sqrt{3}\,8\mathrm{i}}\,1\mathrm{i}}{4}+\frac{\sqrt{8-\sqrt{3}\,8\mathrm{i}}}{4}\right)}\right)(8-\sqrt{3}\,8\mathrm{i})^{1/4}}{12}$$
$$+\frac{2^{3/4}\sqrt{3}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\,(1+\sqrt{3}\,1\mathrm{i})^{1/4}}{2\left(\frac{\sqrt{2}\,\sqrt{1+\sqrt{3}\,1\mathrm{i}}}{2}-\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{1+\sqrt{3}\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}-\frac{2^{3/4}\sqrt{3}\,x\,(1+\sqrt{3}\,1\mathrm{i})^{1/4}\,1\mathrm{i}}{2\left(\frac{\sqrt{2}\,\sqrt{1+\sqrt{3}\,1\mathrm{i}}}{2}-\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{1+\sqrt{3}\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}\right)(1+\sqrt{3}\,1\mathrm{i})^{1/4}\,1\mathrm{i}}{12}$$
$$+\frac{2^{3/4}\sqrt{3}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\,(1+\sqrt{3}\,1\mathrm{i})^{1/4}\,1\mathrm{i}}{2\left(\frac{\sqrt{2}\,\sqrt{1+\sqrt{3}\,1\mathrm{i}}}{2}-\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{1+\sqrt{3}\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}+\frac{2^{3/4}\sqrt{3}\,x\,(1+\sqrt{3}\,1\mathrm{i})^{1/4}}{2\left(\frac{\sqrt{2}\,\sqrt{1+\sqrt{3}\,1\mathrm{i}}}{2}-\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{1+\sqrt{3}\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}\right)(1+\sqrt{3}\,1\mathrm{i})^{1/4}}{12}$$

input
```
int(x^4/(x^8 - x^4 + 1),x)
```

output
```
(2^(3/4)*3^(1/2)*atan((2^(3/4)*x*(3^(1/2)*1i + 1)^(1/4))/(2*((2^(1/2)*(3^(
1/2)*1i + 1)^(1/2))/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i + 1)^(1/2)*1i)/2)) -
(2^(3/4)*3^(1/2)*x*(3^(1/2)*1i + 1)^(1/4)*1i)/(2*((2^(1/2)*(3^(1/2)*1i + 1
)^(1/2))/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i + 1)^(1/2)*1i)/2)))*(3^(1/2)*1i
+ 1)^(1/4)*1i)/12 - (3^(1/2)*atan((x*(8 - 3^(1/2)*8i)^(1/4)*1i)/(2*((3^(1/
2)*(8 - 3^(1/2)*8i)^(1/2)*1i)/4 + (8 - 3^(1/2)*8i)^(1/2)/4)) - (3^(1/2)*x*
(8 - 3^(1/2)*8i)^(1/4))/(2*((3^(1/2)*(8 - 3^(1/2)*8i)^(1/2)*1i)/4 + (8 - 3
^(1/2)*8i)^(1/2)/4)))*(8 - 3^(1/2)*8i)^(1/4))/12 - (3^(1/2)*atan((x*(8 - 3
^(1/2)*8i)^(1/4))/(2*((3^(1/2)*(8 - 3^(1/2)*8i)^(1/2)*1i)/4 + (8 - 3^(1/2)
*8i)^(1/2)/4)) + (3^(1/2)*x*(8 - 3^(1/2)*8i)^(1/4)*1i)/(2*((3^(1/2)*(8 - 3
^(1/2)*8i)^(1/2)*1i)/4 + (8 - 3^(1/2)*8i)^(1/2)/4)))*(8 - 3^(1/2)*8i)^(1/4
)*1i)/12 + (2^(3/4)*3^(1/2)*atan((2^(3/4)*x*(3^(1/2)*1i + 1)^(1/4)*1i)/(2*
((2^(1/2)*(3^(1/2)*1i + 1)^(1/2))/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i + 1)^(1
/2)*1i)/2)) + (2^(3/4)*3^(1/2)*x*(3^(1/2)*1i + 1)^(1/4))/(2*((2^(1/2)*(3^(
1/2)*1i + 1)^(1/2))/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i + 1)^(1/2)*1i)/2)))*(
3^(1/2)*1i + 1)^(1/4))/12
```

3.361 $\int \frac{x^2}{1-x^4+x^8} dx$

3.361.1 Optimal result

Integrand size = 16, antiderivative size = 355

$$\begin{aligned}\int \frac{x^2}{1-x^4+x^8} dx &= \frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right) \\ &\quad - \frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right) \\ &\quad - \frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right) \\ &\quad + \frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right) \\ &\quad + \frac{\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}} \\ &\quad - \frac{\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)}{8\sqrt{3\left(2+\sqrt{3}\right)}} + \frac{\log\left(1+\sqrt{2+\sqrt{3}}x+x^2\right)}{8\sqrt{3\left(2+\sqrt{3}\right)}}\end{aligned}$$

output
```
1/4*arctan((-2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))*(1/2*
2^(1/2)-1/6*6^(1/2))-1/4*arctan((2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)
+1/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))+1/8*ln(1+x^2-x*(1/2*6^(1/2)-1/2*2
^(1/2)))/(3/2*2^(1/2)-1/2*6^(1/2))-1/8*ln(1+x^2+x*(1/2*6^(1/2)-1/2*2^(1/2)
))/(3/2*2^(1/2)-1/2*6^(1/2))-1/4*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/
2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)+1/6*6^(1/2))+1/4*arctan((2*x+1/2*6^(1
/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)+1/6*6^(1/2))-1/8*
ln(1+x^2-x*(1/2*6^(1/2)+1/2*2^(1/2)))/(3/2*2^(1/2)+1/2*6^(1/2))+1/8*ln(1+x
^2+x*(1/2*6^(1/2)+1/2*2^(1/2)))/(3/2*2^(1/2)+1/2*6^(1/2))
```

3.361.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.11

$$\int \frac{x^2}{1-x^4+x^8}\,dx = \frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{\log(x-\#1)}{-\#1+2\#1^5}\&\right]$$

input
```
Integrate[x^2/(1 - x^4 + x^8),x]
```

output
```
RootSum[1 - #1^4 + #1^8 & , Log[x - #1]/(-#1 + 2*#1^5) & ]/4
```

3.361.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1709, 1407, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{x^8-x^4+1}\,dx$$

$$\Big\downarrow \ 1709$$

$$\frac{\int \frac{1}{x^4-\sqrt{3}x^2+1}dx}{2\sqrt{3}}-\frac{\int \frac{1}{x^4+\sqrt{3}x^2+1}dx}{2\sqrt{3}}$$

$$\Big\downarrow \ 1407$$

$$\frac{\frac{\int \frac{\sqrt{2+\sqrt{3}}-x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{\int \frac{x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}-\frac{\frac{\int \frac{\sqrt{2-\sqrt{3}}-x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\int \frac{x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}$$

$\Big\downarrow$ 1142

$$\frac{\frac{\frac{1}{2}\sqrt{2+\sqrt{3}}\int \frac{1}{x^2-\sqrt{2+\sqrt{3}}x+1}dx-\frac{1}{2}\int -\frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\sqrt{2+\sqrt{3}}\int \frac{1}{x^2+\sqrt{2+\sqrt{3}}x+1}dx+\frac{1}{2}\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}-\frac{\frac{\frac{1}{2}\sqrt{2-\sqrt{3}}\int \frac{1}{x^2-\sqrt{2-\sqrt{3}}x+1}dx-\frac{1}{2}\int -\frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\frac{1}{2}\sqrt{2-\sqrt{3}}\int \frac{1}{x^2+\sqrt{2-\sqrt{3}}x+1}dx+\frac{1}{2}\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}$$

$\Big\downarrow$ 25

$$\frac{\frac{\frac{1}{2}\sqrt{2+\sqrt{3}}\int \frac{1}{x^2-\sqrt{2+\sqrt{3}}x+1}dx+\frac{1}{2}\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\sqrt{2+\sqrt{3}}\int \frac{1}{x^2+\sqrt{2+\sqrt{3}}x+1}dx+\frac{1}{2}\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}-\frac{\frac{\frac{1}{2}\sqrt{2-\sqrt{3}}\int \frac{1}{x^2-\sqrt{2-\sqrt{3}}x+1}dx+\frac{1}{2}\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\frac{1}{2}\sqrt{2-\sqrt{3}}\int \frac{1}{x^2+\sqrt{2-\sqrt{3}}x+1}dx+\frac{1}{2}\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}$$

$\Big\downarrow$ 1083

$$\frac{\frac{\frac{1}{2}\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx-\sqrt{2+\sqrt{3}}\int \frac{1}{-\left(2x-\sqrt{2+\sqrt{3}}\right)^2+\sqrt{3}-2}d\left(2x-\sqrt{2+\sqrt{3}}\right)}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx-\sqrt{2+\sqrt{3}}\int \frac{1}{-\left(2x+\sqrt{2+\sqrt{3}}\right)^2+\sqrt{3}-2}d\Big(2x+\cdots}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}$$

$$\frac{\frac{\frac{1}{2}\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx-\sqrt{2-\sqrt{3}}\int \frac{1}{-\left(2x-\sqrt{2-\sqrt{3}}\right)^2-\sqrt{3}-2}d\left(2x-\sqrt{2-\sqrt{3}}\right)}{2\sqrt{2-\sqrt{3}}}+\frac{\frac{1}{2}\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx-\sqrt{2-\sqrt{3}}\int \frac{1}{-\left(2x+\sqrt{2-\sqrt{3}}\right)^2-\sqrt{3}-2}d\Big(2x+\cdots}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}$$

$\Big\downarrow$ 217

$$\frac{\frac{\frac{1}{2}\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx+\sqrt{\frac{2+\sqrt{3}}{2-\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx+\sqrt{\frac{2+\sqrt{3}}{2-\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}-\frac{\frac{\frac{1}{2}\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx+\sqrt{\frac{2-\sqrt{3}}{2+\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{2-\sqrt{3}}}+\frac{\frac{1}{2}\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx+\sqrt{\frac{2-\sqrt{3}}{2+\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}$$

$\Big\downarrow$ 1103

3.361. $\int \frac{x^2}{1-x^4+x^8}\,dx$

$$\frac{\frac{\sqrt{\frac{2+\sqrt{3}}{2-\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)-\frac{1}{2}\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{2\sqrt{2+\sqrt{3}}}+\frac{\sqrt{\frac{2+\sqrt{3}}{2-\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)+\frac{1}{2}\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}-\frac{\frac{\sqrt{\frac{2-\sqrt{3}}{2+\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{2}\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{2\sqrt{2-\sqrt{3}}}+\frac{\sqrt{\frac{2-\sqrt{3}}{2+\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)+\frac{1}{2}\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}$$

input
```
Int[x^2/(1 - x^4 + x^8),x]
```

output
```
-1/2*((Sqrt[(2 - Sqrt[3])/(2 + Sqrt[3])]*ArcTan[(-Sqrt[2 - Sqrt[3]] + 2*x)
/Sqrt[2 + Sqrt[3]]] - Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2]/2)/(2*Sqrt[2 - Sq
rt[3]]) + (Sqrt[(2 - Sqrt[3])/(2 + Sqrt[3])]*ArcTan[(Sqrt[2 - Sqrt[3]] + 2
*x)/Sqrt[2 + Sqrt[3]]] + Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2]/2)/(2*Sqrt[2 -
 Sqrt[3]]))/Sqrt[3] + ((Sqrt[(2 + Sqrt[3])/(2 - Sqrt[3])]*ArcTan[(-Sqrt[2
+ Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]] - Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2]/
2)/(2*Sqrt[2 + Sqrt[3]]) + (Sqrt[(2 + Sqrt[3])/(2 - Sqrt[3])]*ArcTan[(Sqrt
[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]] + Log[1 + Sqrt[2 + Sqrt[3]]*x + x^
2]/2)/(2*Sqrt[2 + Sqrt[3]]))/(2*Sqrt[3])
```

3.361.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1407
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   Int[(r - x)/(q - r*
x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(r + x)/(q + r*x + x^2), x], x]]]
 /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

rule 1709
```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*r)   Int[x^(
m - n/2)/(q - r*x^(n/2) + x^n), x], x] - Simp[1/(2*c*r)   Int[x^(m - n/2)/(
q + r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && IGtQ[m, 0] && GeQ[m, n/2] && LtQ[m, 3*
(n/2)] && NegQ[b^2 - 4*a*c]
```

3.361.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.11

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}-\_Z^{4}+1\right)}\frac{\_R^{2}\ln\left(x-\_R\right)}{2\_R^{7}-\_R^{3}}\right)}{4}$	40
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}-\_Z^{4}+1\right)}\frac{\_R^{2}\ln\left(x-\_R\right)}{2\_R^{7}-\_R^{3}}\right)}{4}$	40

input
```
int(x^2/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(_R^2/(2*_R^7-_R^3)*ln(x-_R),_R=RootOf(_Z^8-_Z^4+1))
```

3.361.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 545, normalized size of antiderivative = 1.54

$$\int \frac{x^2}{1-x^4+x^8}\,dx$$
$$= -\frac{1}{24}\sqrt{6}\sqrt{\sqrt{2}\sqrt{i\sqrt{3}+1}}\log\left(\sqrt{6}\left(i\sqrt{3}\sqrt{2}+3\sqrt{2}\right)\sqrt{\sqrt{2}\sqrt{i\sqrt{3}+1}}\sqrt{i\sqrt{3}+1}+24x\right)$$
$$+\frac{1}{24}\sqrt{6}\sqrt{\sqrt{2}\sqrt{i\sqrt{3}+1}}\log\left(\sqrt{6}\left(-i\sqrt{3}\sqrt{2}-3\sqrt{2}\right)\sqrt{\sqrt{2}\sqrt{i\sqrt{3}+1}}\sqrt{i\sqrt{3}+1}+24x\right)$$
$$+\frac{1}{24}\sqrt{6}\sqrt{-\sqrt{2}\sqrt{i\sqrt{3}+1}}\log\left(\sqrt{6}\left(i\sqrt{3}\sqrt{2}+3\sqrt{2}\right)\sqrt{-\sqrt{2}\sqrt{i\sqrt{3}+1}}\sqrt{i\sqrt{3}+1}+24x\right)$$
$$-\frac{1}{24}\sqrt{6}\sqrt{-\sqrt{2}\sqrt{i\sqrt{3}+1}}\log\left(\sqrt{6}\left(-i\sqrt{3}\sqrt{2}-3\sqrt{2}\right)\sqrt{-\sqrt{2}\sqrt{i\sqrt{3}+1}}\sqrt{i\sqrt{3}+1}+24x\right)$$
$$+\frac{1}{24}\sqrt{6}\sqrt{\sqrt{2}\sqrt{-i\sqrt{3}+1}}\log\left(\sqrt{6}\left(i\sqrt{3}\sqrt{2}-3\sqrt{2}\right)\sqrt{\sqrt{2}\sqrt{-i\sqrt{3}+1}}\sqrt{-i\sqrt{3}+1}+24x\right)$$
$$-\frac{1}{24}\sqrt{6}\sqrt{\sqrt{2}\sqrt{-i\sqrt{3}+1}}\log\left(\sqrt{6}\left(-i\sqrt{3}\sqrt{2}+3\sqrt{2}\right)\sqrt{\sqrt{2}\sqrt{-i\sqrt{3}+1}}\sqrt{-i\sqrt{3}+1}+24x\right)$$
$$-\frac{1}{24}\sqrt{6}\sqrt{-\sqrt{2}\sqrt{-i\sqrt{3}+1}}\log\left(\sqrt{6}\left(i\sqrt{3}\sqrt{2}-3\sqrt{2}\right)\sqrt{-\sqrt{2}\sqrt{-i\sqrt{3}+1}}\sqrt{-i\sqrt{3}+1}+24x\right)$$
$$+\frac{1}{24}\sqrt{6}\sqrt{-\sqrt{2}\sqrt{-i\sqrt{3}+1}}\log\left(\sqrt{6}\left(-i\sqrt{3}\sqrt{2}+3\sqrt{2}\right)\sqrt{-\sqrt{2}\sqrt{-i\sqrt{3}+1}}\sqrt{-i\sqrt{3}+1}+24x\right)$$

3.361. $\int \frac{x^2}{1-x^4+x^8}\,dx$

input
```
integrate(x^2/(x^8-x^4+1),x, algorithm="fricas")
```

output
```
-1/24*sqrt(6)*sqrt(sqrt(2)*sqrt(I*sqrt(3) + 1))*log(sqrt(6)*(I*sqrt(3)*sqr
t(2) + 3*sqrt(2))*sqrt(sqrt(2)*sqrt(I*sqrt(3) + 1))*sqrt(I*sqrt(3) + 1) +
24*x) + 1/24*sqrt(6)*sqrt(sqrt(2)*sqrt(I*sqrt(3) + 1))*log(sqrt(6)*(-I*sqr
t(3)*sqrt(2) - 3*sqrt(2))*sqrt(sqrt(2)*sqrt(I*sqrt(3) + 1))*sqrt(I*sqrt(3)
 + 1) + 24*x) + 1/24*sqrt(6)*sqrt(-sqrt(2)*sqrt(I*sqrt(3) + 1))*log(sqrt(6
)*(I*sqrt(3)*sqrt(2) + 3*sqrt(2))*sqrt(-sqrt(2)*sqrt(I*sqrt(3) + 1))*sqrt(
I*sqrt(3) + 1) + 24*x) - 1/24*sqrt(6)*sqrt(-sqrt(2)*sqrt(I*sqrt(3) + 1))*l
og(sqrt(6)*(-I*sqrt(3)*sqrt(2) - 3*sqrt(2))*sqrt(-sqrt(2)*sqrt(I*sqrt(3) +
 1))*sqrt(I*sqrt(3) + 1) + 24*x) + 1/24*sqrt(6)*sqrt(sqrt(2)*sqrt(-I*sqrt(
3) + 1))*log(sqrt(6)*(I*sqrt(3)*sqrt(2) - 3*sqrt(2))*sqrt(sqrt(2)*sqrt(-I*
sqrt(3) + 1))*sqrt(-I*sqrt(3) + 1) + 24*x) - 1/24*sqrt(6)*sqrt(sqrt(2)*sqr
t(-I*sqrt(3) + 1))*log(sqrt(6)*(-I*sqrt(3)*sqrt(2) + 3*sqrt(2))*sqrt(sqrt(
2)*sqrt(-I*sqrt(3) + 1))*sqrt(-I*sqrt(3) + 1) + 24*x) - 1/24*sqrt(6)*sqrt(
-sqrt(2)*sqrt(-I*sqrt(3) + 1))*log(sqrt(6)*(I*sqrt(3)*sqrt(2) - 3*sqrt(2))
*sqrt(-sqrt(2)*sqrt(-I*sqrt(3) + 1))*sqrt(-I*sqrt(3) + 1) + 24*x) + 1/24*s
qrt(6)*sqrt(-sqrt(2)*sqrt(-I*sqrt(3) + 1))*log(sqrt(6)*(-I*sqrt(3)*sqrt(2)
 + 3*sqrt(2))*sqrt(-sqrt(2)*sqrt(-I*sqrt(3) + 1))*sqrt(-I*sqrt(3) + 1) + 2
4*x)
```

3.361.6 Sympy [A] (verification not implemented)

Time = 1.44 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.07

$$\int \frac{x^2}{1-x^4+x^8}\,dx = \text{RootSum}\left(5308416t^8-2304t^4+1,\left(t\mapsto t\log\left(-442368t^7-192t^3+x\right)\right)\right)$$

input
```
integrate(x**2/(x**8-x**4+1),x)
```

output
```
RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(-442368*_t**7 -
192*_t**3 + x)))
```

3.361.7 Maxima [F]

$$\int \frac{x^2}{1-x^4+x^8}\,dx = \int \frac{x^2}{x^8-x^4+1}\,dx$$

input
```
integrate(x^2/(x^8-x^4+1),x, algorithm="maxima")
```

output
```
integrate(x^2/(x^8 - x^4 + 1), x)
```

3.361.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.71

$$\begin{aligned}\int \frac{x^2}{1-x^4+x^8}\,dx = {} & \frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4\,x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right) \\ & + \frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4\,x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right) \\ & + \frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4\,x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) \\ & + \frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4\,x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) \\ & - \frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}\,x\left(\sqrt{6}+\sqrt{2}\right)+1\right) \\ & + \frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}\,x\left(\sqrt{6}+\sqrt{2}\right)+1\right) \\ & - \frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}\,x\left(\sqrt{6}-\sqrt{2}\right)+1\right) \\ & + \frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}\,x\left(\sqrt{6}-\sqrt{2}\right)+1\right)\end{aligned}$$

input
```
integrate(x^2/(x^8-x^4+1),x, algorithm="giac")
```

output
```
1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqr
t(2))) + 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt
(6) + sqrt(2))) + 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt(
2))/(sqrt(6) - sqrt(2))) + 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6
) - sqrt(2))/(sqrt(6) - sqrt(2))) - 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1
/2*x*(sqrt(6) + sqrt(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x
*(sqrt(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 + 1/2*x*(sq
rt(6) - sqrt(2)) + 1) + 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6
) - sqrt(2)) + 1)
```

3.361.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.81

$$\int \frac{x^2}{1-x^4+x^8}\,dx = -\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}{2\left(1+\sqrt{3}\,1\mathrm{i}\right)}+\frac{\sqrt{3}\,x\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}\,1\mathrm{i}}{2\left(1+\sqrt{3}\,1\mathrm{i}\right)}\right)\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}\,1\mathrm{i}}{12} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}\,1\mathrm{i}}{2\left(1+\sqrt{3}\,1\mathrm{i}\right)}-\frac{\sqrt{3}\,x\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}{2\left(1+\sqrt{3}\,1\mathrm{i}\right)}\right)\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}{12} - \frac{2^{3/4}\sqrt{3}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}{2\left(-1+\sqrt{3}\,1\mathrm{i}\right)}-\frac{2^{3/4}\sqrt{3}\,x\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}\,1\mathrm{i}}{2\left(-1+\sqrt{3}\,1\mathrm{i}\right)}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}\,1\mathrm{i}}{12} + \frac{2^{3/4}\sqrt{3}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}\,1\mathrm{i}}{2\left(-1+\sqrt{3}\,1\mathrm{i}\right)}+\frac{2^{3/4}\sqrt{3}\,x\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}{2\left(-1+\sqrt{3}\,1\mathrm{i}\right)}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}{12}$$

input
```
int(x^2/(x^8 - x^4 + 1),x)
```

output
```
(3^(1/2)*atan((x*(8 - 3^(1/2)*8i)^(1/4)*1i)/(2*(3^(1/2)*1i + 1)) - (3^(1/2
)*x*(8 - 3^(1/2)*8i)^(1/4))/(2*(3^(1/2)*1i + 1)))*(8 - 3^(1/2)*8i)^(1/4))/
12 - (3^(1/2)*atan((x*(8 - 3^(1/2)*8i)^(1/4))/(2*(3^(1/2)*1i + 1)) + (3^(1
/2)*x*(8 - 3^(1/2)*8i)^(1/4)*1i)/(2*(3^(1/2)*1i + 1)))*(8 - 3^(1/2)*8i)^(1
/4)*1i)/12 - (2^(3/4)*3^(1/2)*atan((2^(3/4)*x*(3^(1/2)*1i + 1)^(1/4))/(2*(
3^(1/2)*1i - 1)) - (2^(3/4)*3^(1/2)*x*(3^(1/2)*1i + 1)^(1/4)*1i)/(2*(3^(1/
2)*1i - 1)))*(3^(1/2)*1i + 1)^(1/4)*1i)/12 + (2^(3/4)*3^(1/2)*atan((2^(3/4
)*x*(3^(1/2)*1i + 1)^(1/4)*1i)/(2*(3^(1/2)*1i - 1)) + (2^(3/4)*3^(1/2)*x*(
3^(1/2)*1i + 1)^(1/4))/(2*(3^(1/2)*1i - 1)))*(3^(1/2)*1i + 1)^(1/4))/12
```

3.361. $\int \frac{x^2}{1-x^4+x^8}\,dx$

3.362 $\int \frac{1}{1-x^4+x^8} dx$

3.362.1 Optimal result

Integrand size = 12, antiderivative size = 275

$$\int \frac{1}{1-x^4+x^8} dx = -\frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}} - \frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}} + \frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}} + \frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}} - \frac{\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)}{4\sqrt{6}} + \frac{\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)}{4\sqrt{6}} - \frac{\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)}{4\sqrt{6}} + \frac{\log\left(1+\sqrt{2+\sqrt{3}}x+x^2\right)}{4\sqrt{6}}$$

output
```
-1/12*arctan((-2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))*6^(
1/2)+1/12*arctan((2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))*
6^(1/2)-1/12*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/2
)))*6^(1/2)+1/12*arctan((2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(
1/2)))*6^(1/2)-1/24*ln(1+x^2-x*(1/2*6^(1/2)-1/2*2^(1/2)))*6^(1/2)+1/24*ln(
1+x^2+x*(1/2*6^(1/2)-1/2*2^(1/2)))*6^(1/2)-1/24*ln(1+x^2-x*(1/2*6^(1/2)+1/
2*2^(1/2)))*6^(1/2)+1/24*ln(1+x^2+x*(1/2*6^(1/2)+1/2*2^(1/2)))*6^(1/2)
```

3.362.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.15

$$\int \frac{1}{1-x^4+x^8}\,dx = \frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{\log(x-\#1)}{-\#1^3+2\#1^7}\&\right]$$

input
```
Integrate[(1 - x^4 + x^8)^(-1),x]
```

output
```
RootSum[1 - #1^4 + #1^8 & , Log[x - #1]/(-#1^3 + 2*#1^7) & ]/4
```

3.362.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.49, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {1684, 1483, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^8-x^4+1}dx$$

$$\Big\downarrow \text{1684}$$

$$\frac{\int \frac{\sqrt{3}-x^2}{x^4-\sqrt{3}x^2+1}dx}{2\sqrt{3}}+\frac{\int \frac{x^2+\sqrt{3}}{x^4+\sqrt{3}x^2+1}dx}{2\sqrt{3}}$$

$$\Big\downarrow \text{1483}$$

$$\frac{\frac{\int \frac{\left(1-\sqrt{3}\right)x+\sqrt{3\left(2-\sqrt{3}\right)}}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\int \frac{\sqrt{3\left(2-\sqrt{3}\right)}-\left(1-\sqrt{3}\right)x}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+\frac{\frac{\int \frac{\sqrt{3\left(2+\sqrt{3}\right)}-\left(1+\sqrt{3}\right)x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{\int \frac{\left(1+\sqrt{3}\right)x+\sqrt{3\left(2+\sqrt{3}\right)}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}$$

$$\Big\downarrow \text{1142}$$

$$\frac{\frac{\frac{\int \frac{1}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{\sqrt{2}}+\frac{1}{2}\left(1-\sqrt{3}\right)\int -\frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\frac{\int \frac{1}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{\sqrt{2}}-\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+\frac{\frac{\frac{\int \frac{1}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{\sqrt{2}}-\frac{1}{2}\left(1+\sqrt{3}\right)\int -\frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{\int \frac{1}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{\sqrt{2}}+\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}$$

$\downarrow$ 25

$$\frac{\frac{\frac{\int \frac{1}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{\sqrt{2}}-\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\frac{\int \frac{1}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{\sqrt{2}}-\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+\frac{\frac{\frac{\int \frac{1}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{\sqrt{2}}+\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{\int \frac{1}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{\sqrt{2}}+\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}$$

$\downarrow$ 1083

$$\frac{\frac{-\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx-\sqrt{2}\int \frac{1}{-\left(2x-\sqrt{2-\sqrt{3}}\right)^2-\sqrt{3}-2}d\left(2x-\sqrt{2-\sqrt{3}}\right)}{2\sqrt{2-\sqrt{3}}}+\frac{-\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx-\sqrt{2}\int \frac{1}{-\left(2x+\sqrt{2-\sqrt{3}}\right)^2-\sqrt{3}\cdots}}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}$$

$$\frac{\frac{\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx-\sqrt{2}\int \frac{1}{-\left(2x-\sqrt{2+\sqrt{3}}\right)^2+\sqrt{3}-2}d\left(2x-\sqrt{2+\sqrt{3}}\right)}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx-\sqrt{2}\int \frac{1}{-\left(2x+\sqrt{2+\sqrt{3}}\right)^2+\sqrt{3}-2}d\cdots}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}$$

$\downarrow$ 217

$$\frac{\frac{\sqrt{\frac{2}{2+\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\sqrt{\frac{2}{2+\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+\frac{\frac{\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx+\sqrt{\frac{2}{2-\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx+\sqrt{\frac{2}{2-\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}$$

$\downarrow$ 1103

$$\frac{\frac{\sqrt{\frac{2}{2+\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)+\frac{1}{2}\left(1-\sqrt{3}\right)\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{2\sqrt{2-\sqrt{3}}}+\frac{\sqrt{\frac{2}{2+\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{2}\left(1-\sqrt{3}\right)\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+\frac{\frac{\sqrt{\frac{2}{2-\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)-\frac{1}{2}\left(1+\sqrt{3}\right)\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{2\sqrt{2+\sqrt{3}}}+\frac{\sqrt{\frac{2}{2-\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)+\frac{1}{2}\left(1+\sqrt{3}\right)\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}$$

3.362. $\int \frac{1}{1-x^4+x^8}\,dx$

input
```
Int[(1 - x^4 + x^8)^(-1),x]
```

output
```
((Sqrt[2/(2 + Sqrt[3])]*ArcTan[(-Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]
]] + ((1 - Sqrt[3])*Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2])/2)/(2*Sqrt[2 - Sqr
t[3]]) + (Sqrt[2/(2 + Sqrt[3])]*ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 +
Sqrt[3]]] - ((1 - Sqrt[3])*Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2])/2)/(2*Sqrt[
2 - Sqrt[3]]))/(2*Sqrt[3]) + ((Sqrt[2/(2 - Sqrt[3])]*ArcTan[(-Sqrt[2 + Sqr
t[3]] + 2*x)/Sqrt[2 - Sqrt[3]]] - ((1 + Sqrt[3])*Log[1 - Sqrt[2 + Sqrt[3]]
*x + x^2])/2)/(2*Sqrt[2 + Sqrt[3]]) + (Sqrt[2/(2 - Sqrt[3])]*ArcTan[(Sqrt[
2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]] + ((1 + Sqrt[3])*Log[1 + Sqrt[2 + S
qrt[3]]*x + x^2])/2)/(2*Sqrt[2 + Sqrt[3]]))/(2*Sqrt[3])
```

3.362.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 1684
```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   Int[(r - x^(
n/2))/(q - r*x^(n/2) + x^n), x], x] + Simp[1/(2*c*q*r)   Int[(r + x^(n/2))/
(q + r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && N
eQ[b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && NegQ[b^2 - 4*a*c]
```

3.362.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.11

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(9\_Z^4+1\right)}\_R\ln\left(3\_R^2+3\_Rx+x^2\right)\right)}{4}$	30
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(9\_Z^4+1\right)}\_R\ln\left(3\_R^2+3\_Rx+x^2\right)\right)}{4}$	30

input
```
int(1/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(_R*ln(3*_R^2+3*_R*x+x^2),_R=RootOf(9*_Z^4+1))
```

3.362.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.37

$$\int \frac{1}{1-x^4+x^8}\,dx = \left(\frac{1}{24}i+\frac{1}{24}\right)\sqrt{3}\sqrt{2}\log\left((3i+3)\sqrt{3}\sqrt{2}x+6x^2+6i\right) - \left(\frac{1}{24}i-\frac{1}{24}\right)\sqrt{3}\sqrt{2}\log\left(-(3i-3)\sqrt{3}\sqrt{2}x+6x^2-6i\right) + \left(\frac{1}{24}i-\frac{1}{24}\right)\sqrt{3}\sqrt{2}\log\left((3i-3)\sqrt{3}\sqrt{2}x+6x^2-6i\right) - \left(\frac{1}{24}i+\frac{1}{24}\right)\sqrt{3}\sqrt{2}\log\left(-(3i+3)\sqrt{3}\sqrt{2}x+6x^2+6i\right)$$

input
```
integrate(1/(x^8-x^4+1),x, algorithm="fricas")
```

output
```
(1/24*I + 1/24)*sqrt(3)*sqrt(2)*log((3*I + 3)*sqrt(3)*sqrt(2)*x + 6*x^2 +
6*I) - (1/24*I - 1/24)*sqrt(3)*sqrt(2)*log(-(3*I - 3)*sqrt(3)*sqrt(2)*x +
6*x^2 - 6*I) + (1/24*I - 1/24)*sqrt(3)*sqrt(2)*log((3*I - 3)*sqrt(3)*sqrt(
2)*x + 6*x^2 - 6*I) - (1/24*I + 1/24)*sqrt(3)*sqrt(2)*log(-(3*I + 3)*sqrt(
3)*sqrt(2)*x + 6*x^2 + 6*I)
```

3.362.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.60

$$\int \frac{1}{1-x^4+x^8}\,dx = \frac{\sqrt{6}\cdot\left(2\operatorname{atan}\left(\frac{\sqrt{6}x}{3}-\frac{1}{3}\right)+2\operatorname{atan}\left(\sqrt{6}x^3-4x^2+2\sqrt{6}x-3\right)\right)}{24} + \frac{\sqrt{6}\cdot\left(2\operatorname{atan}\left(\frac{\sqrt{6}x}{3}+\frac{1}{3}\right)+2\operatorname{atan}\left(\sqrt{6}x^3+4x^2+2\sqrt{6}x+3\right)\right)}{24} - \frac{\sqrt{6}\log\left(x^4-\sqrt{6}x^3+3x^2-\sqrt{6}x+1\right)}{24} + \frac{\sqrt{6}\log\left(x^4+\sqrt{6}x^3+3x^2+\sqrt{6}x+1\right)}{24}$$

input
```
integrate(1/(x**8-x**4+1),x)
```

output
```
sqrt(6)*(2*atan(sqrt(6)*x/3 - 1/3) + 2*atan(sqrt(6)*x**3 - 4*x**2 + 2*sqrt
(6)*x - 3))/24 + sqrt(6)*(2*atan(sqrt(6)*x/3 + 1/3) + 2*atan(sqrt(6)*x**3
+ 4*x**2 + 2*sqrt(6)*x + 3))/24 - sqrt(6)*log(x**4 - sqrt(6)*x**3 + 3*x**2
 - sqrt(6)*x + 1)/24 + sqrt(6)*log(x**4 + sqrt(6)*x**3 + 3*x**2 + sqrt(6)*
x + 1)/24
```

3.362.7 Maxima [F]

$$\int \frac{1}{1-x^4+x^8}\,dx = \int \frac{1}{x^8-x^4+1}\,dx$$

input
```
integrate(1/(x^8-x^4+1),x, algorithm="maxima")
```

output
```
integrate(1/(x^8 - x^4 + 1), x)
```

3.362.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.75

$$\begin{aligned}\int \frac{1}{1-x^4+x^8}\,dx &= \frac{1}{12}\sqrt{6}\arctan\left(\frac{4\,x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4\,x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)\\
&\quad+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4\,x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4\,x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)\\
&\quad+\frac{1}{24}\sqrt{6}\log\left(x^2+\frac{1}{2}\,x\left(\sqrt{6}+\sqrt{2}\right)+1\right)\\
&\quad-\frac{1}{24}\sqrt{6}\log\left(x^2-\frac{1}{2}\,x\left(\sqrt{6}+\sqrt{2}\right)+1\right)\\
&\quad+\frac{1}{24}\sqrt{6}\log\left(x^2+\frac{1}{2}\,x\left(\sqrt{6}-\sqrt{2}\right)+1\right)\\
&\quad-\frac{1}{24}\sqrt{6}\log\left(x^2-\frac{1}{2}\,x\left(\sqrt{6}-\sqrt{2}\right)+1\right)\end{aligned}$$

input
```
integrate(1/(x^8-x^4+1),x, algorithm="giac")
```

output
```
1/12*sqrt(6)*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt(2))) + 1/12*
sqrt(6)*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6) + sqrt(2))) + 1/12*sqrt(
6)*arctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2))) + 1/12*sqrt(6)*ar
ctan((4*x - sqrt(6) - sqrt(2))/(sqrt(6) - sqrt(2))) + 1/24*sqrt(6)*log(x^2
 + 1/2*x*(sqrt(6) + sqrt(2)) + 1) - 1/24*sqrt(6)*log(x^2 - 1/2*x*(sqrt(6)
+ sqrt(2)) + 1) + 1/24*sqrt(6)*log(x^2 + 1/2*x*(sqrt(6) - sqrt(2)) + 1) -
1/24*sqrt(6)*log(x^2 - 1/2*x*(sqrt(6) - sqrt(2)) + 1)
```

3.362.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.19

$$\int \frac{1}{1-x^4+x^8}\,dx = \sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x\,\left(\frac{1}{3}+\frac{1}{3}\mathrm{i}\right)}{\frac{2\,x^2}{3}-\frac{2}{3}\mathrm{i}}\right)\,\left(-\frac{1}{12}-\frac{1}{12}\mathrm{i}\right) + \sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x\,\left(\frac{1}{3}-\frac{1}{3}\mathrm{i}\right)}{\frac{2\,x^2}{3}+\frac{2}{3}\mathrm{i}}\right)\,\left(-\frac{1}{12}+\frac{1}{12}\mathrm{i}\right)$$

input
```
int(1/(x^8 - x^4 + 1),x)
```

output
```
- 6^(1/2)*atan((6^(1/2)*x*(1/3 + 1i/3))/((2*x^2)/3 - 2i/3))*(1/12 + 1i/12)
 - 6^(1/2)*atan((6^(1/2)*x*(1/3 - 1i/3))/((2*x^2)/3 + 2i/3))*(1/12 - 1i/12
)
```

3.363 $\int \frac{1}{x^2(1-x^4+x^8)}\,dx$

3.363.1 Optimal result

Integrand size = 16, antiderivative size = 360

$$\begin{aligned}\int \frac{1}{x^2(1-x^4+x^8)}\,dx = &-\frac{1}{x}+\frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}\\
&-\frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}\\
&+\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)\\
&-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)\\
&-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)\\
&+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(1+\sqrt{2+\sqrt{3}}x+x^2\right)\end{aligned}$$

output
```
-1/x+1/8*ln(1+x^2-x*(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))-1
/8*ln(1+x^2+x*(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))+1/4*arc
tan((-2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))/(3/2*2^(1/2)
-1/2*6^(1/2))-1/4*arctan((2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^
(1/2)))/(3/2*2^(1/2)-1/2*6^(1/2))-1/8*ln(1+x^2-x*(1/2*6^(1/2)+1/2*2^(1/2))
)*(1/2*2^(1/2)+1/6*6^(1/2))+1/8*ln(1+x^2+x*(1/2*6^(1/2)+1/2*2^(1/2)))*(1/2
*2^(1/2)+1/6*6^(1/2))-1/4*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/
2)-1/2*2^(1/2)))/(3/2*2^(1/2)+1/2*6^(1/2))+1/4*arctan((2*x+1/2*6^(1/2)+1/2
*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/2)))/(3/2*2^(1/2)+1/2*6^(1/2))
```

3.363.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.17

$$\int \frac{1}{x^2(1-x^4+x^8)}\,dx = -\frac{1}{x}-\frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{-\log(x-\#1)+\log(x-\#1)\#1^4}{-\#1+2\#1^5}\&\right]$$

input
```
Integrate[1/(x^2*(1 - x^4 + x^8)),x]
```

output
```
-x^(-1) - RootSum[1 - #1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)/(-
#1 + 2*#1^5) & ]/4
```

3.363.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.01, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.688, Rules used = {1704, 1830, 1602, 25, 1483, 27, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(x^8-x^4+1)}dx$$

$$\Big\downarrow \text{1704}$$

$$\int \frac{x^2\left(1-x^4\right)}{x^8-x^4+1}dx-\frac{1}{x}$$

$$\Big\downarrow \text{ 1830}$$

$$\frac{\int \frac{x^2\left(\sqrt{3}-2x^2\right)}{x^4-\sqrt{3}x^2+1}dx}{2\sqrt{3}}+\frac{\int \frac{x^2\left(2x^2+\sqrt{3}\right)}{x^4+\sqrt{3}x^2+1}dx}{2\sqrt{3}}-\frac{1}{x}$$

$$\Big\downarrow \text{ 1602}$$

$$\frac{-\int -\frac{2-\sqrt{3}x^2}{x^4-\sqrt{3}x^2+1}dx-2x}{2\sqrt{3}}+\frac{2x-\int \frac{\sqrt{3}x^2+2}{x^4+\sqrt{3}x^2+1}dx}{2\sqrt{3}}-\frac{1}{x}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{2-\sqrt{3}x^2}{x^4-\sqrt{3}x^2+1}dx-2x}{2\sqrt{3}}+\frac{2x-\int \frac{\sqrt{3}x^2+2}{x^4+\sqrt{3}x^2+1}dx}{2\sqrt{3}}-\frac{1}{x}$$

$$\Big\downarrow \text{ 1483}$$

$$\frac{-\frac{\int \frac{2\sqrt{2-\sqrt{3}}-\left(2-\sqrt{3}\right)x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}-\frac{\int \frac{\sqrt{2-\sqrt{3}}\left(\sqrt{2-\sqrt{3}}x+2\right)}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+2x}{2\sqrt{3}}+\frac{\frac{\int \frac{2\sqrt{2+\sqrt{3}}-\left(2+\sqrt{3}\right)x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{\int \frac{\sqrt{2+\sqrt{3}}\left(\sqrt{2+\sqrt{3}}x+2\right)}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}-2x}{2\sqrt{3}}-\frac{1}{x}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{-\frac{\int \frac{2\sqrt{2-\sqrt{3}}-\left(2-\sqrt{3}\right)x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}-\frac{1}{2}\int \frac{\sqrt{2-\sqrt{3}}x+2}{x^2+\sqrt{2-\sqrt{3}}x+1}dx+2x}{2\sqrt{3}}+\frac{\frac{\int \frac{2\sqrt{2+\sqrt{3}}-\left(2+\sqrt{3}\right)x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{1}{2}\int \frac{\sqrt{2+\sqrt{3}}x+2}{x^2+\sqrt{2+\sqrt{3}}x+1}dx-2x}{2\sqrt{3}}-\frac{1}{x}$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{-\frac{\frac{1}{2}\sqrt{2+\sqrt{3}}\int \frac{1}{x^2-\sqrt{2-\sqrt{3}}x+1}dx-\frac{1}{2}\left(2-\sqrt{3}\right)\int -\frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{1}{2}\left(-\frac{1}{2}\left(2+\sqrt{3}\right)\int \frac{1}{x^2+\sqrt{2-\sqrt{3}}x+1}dx-\frac{1}{2}\sqrt{2-\sqrt{3}}\int \frac{2x+\sqrt{\cdots}}{x^2+\sqrt{2-\cdots}}\cdots\right.}{2\sqrt{3}}$$

$$\frac{\frac{\frac{1}{2}\sqrt{2-\sqrt{3}}\int \frac{1}{x^2-\sqrt{2+\sqrt{3}}x+1}dx-\frac{1}{2}\left(2+\sqrt{3}\right)\int -\frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{1}{2}\left(\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{1}{x^2+\sqrt{2+\sqrt{3}}x+1}dx+\frac{1}{2}\sqrt{2+\sqrt{3}}\int \frac{2x+\sqrt{2+\sqrt{\cdots}}}{x^2+\sqrt{2+\sqrt{3}}x\cdots}\cdots\right.}{2\sqrt{3}}$$

$$\frac{1}{x}$$

3.363. $\int \frac{1}{x^2\left(1-x^4+x^8\right)}\,dx$

$\Big\downarrow$ 25

$$\frac{-\frac{\frac{1}{2}\sqrt{2+\sqrt{3}}\int \frac{1}{x^2-\sqrt{2-\sqrt{3}}x+1}dx+\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{1}{2}\left(-\frac{1}{2}\left(2+\sqrt{3}\right)\int \frac{1}{x^2+\sqrt{2-\sqrt{3}}x+1}dx-\frac{1}{2}\sqrt{2-\sqrt{3}}\int \frac{2x+\sqrt{2-}}{x^2+\sqrt{2-\surd}}\right.}{2\sqrt{3}}$$

$$\frac{\frac{\frac{1}{2}\sqrt{2-\sqrt{3}}\int \frac{1}{x^2-\sqrt{2+\sqrt{3}}x+1}dx+\frac{1}{2}\left(2+\sqrt{3}\right)\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{1}{2}\left(\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{1}{x^2+\sqrt{2+\sqrt{3}}x+1}dx+\frac{1}{2}\sqrt{2+\sqrt{3}}\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+}\right.}{2\sqrt{3}}$$

$$\frac{1}{x}$$

$\Big\downarrow$ 1083

$$\frac{-\frac{\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx-\sqrt{2+\sqrt{3}}\int \frac{1}{-\left(2x-\sqrt{2-\sqrt{3}}\right)^2-\sqrt{3}-2}d\left(2x-\sqrt{2-\sqrt{3}}\right)}{2\sqrt{2-\sqrt{3}}}+\frac{1}{2}\left(\left(2+\sqrt{3}\right)\int \frac{1}{-\left(2x+\sqrt{2-\sqrt{3}}\right)^2-\sqrt{3}-2}d\left(2x+\right.\right.}{2\sqrt{3}}$$

$$\frac{\frac{\frac{1}{2}\left(2+\sqrt{3}\right)\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx-\sqrt{2-\sqrt{3}}\int \frac{1}{-\left(2x-\sqrt{2+\sqrt{3}}\right)^2+\sqrt{3}-2}d\left(2x-\sqrt{2+\sqrt{3}}\right)}{2\sqrt{2+\sqrt{3}}}+\frac{1}{2}\left(\frac{1}{2}\sqrt{2+\sqrt{3}}\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx-\left(2-\sqrt{3}\right)\right.}{2\sqrt{3}}$$

$$\frac{1}{x}$$

$\Big\downarrow$ 217

$$\frac{-\frac{\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx+\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{2-\sqrt{3}}}+\frac{1}{2}\left(-\frac{1}{2}\sqrt{2-\sqrt{3}}\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx-\sqrt{2+\sqrt{3}}\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right.\right.}{2\sqrt{3}}$$

$$\frac{\frac{\frac{1}{2}\left(2+\sqrt{3}\right)\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx+\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{2+\sqrt{3}}}+\frac{1}{2}\left(\frac{1}{2}\sqrt{2+\sqrt{3}}\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx+\sqrt{2-\sqrt{3}}\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)\right)}{2\sqrt{3}}$$

$$\frac{1}{x}$$

$\Big\downarrow$ 1103

$$\frac{-\frac{\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{2}\left(2-\sqrt{3}\right)\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{2\sqrt{2-\sqrt{3}}}+\frac{1}{2}\left(-\sqrt{2+\sqrt{3}}\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{2}\sqrt{2-\sqrt{3}}\log\left(x^2+\sqrt{2}\right.\right.}{2\sqrt{3}}$$

$$\frac{\frac{\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)-\frac{1}{2}\left(2+\sqrt{3}\right)\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{2\sqrt{2+\sqrt{3}}}+\frac{1}{2}\left(\sqrt{2-\sqrt{3}}\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)+\frac{1}{2}\sqrt{2+\sqrt{3}}\log\left(x^2+\sqrt{2+\surd}\right.\right.}{2\sqrt{3}}$$

$$\frac{1}{x}$$

3.363. $\int \frac{1}{x^2(1-x^4+x^8)}\,dx$

input
```
Int[1/(x^2*(1 - x^4 + x^8)),x]
```

output
```
-x^(-1) + (2*x - (ArcTan[(-Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]] - (
(2 - Sqrt[3])*Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2])/2)/(2*Sqrt[2 - Sqrt[3]])
 + (-(Sqrt[2 + Sqrt[3]]*ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]
]) - (Sqrt[2 - Sqrt[3]]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2])/2)/2)/(2*Sqrt[
3]) + (-2*x + (ArcTan[(-Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]] - ((2
+ Sqrt[3])*Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2])/2)/(2*Sqrt[2 + Sqrt[3]]) +
(Sqrt[2 - Sqrt[3]]*ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]] + (
Sqrt[2 + Sqrt[3]]*Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/2)/2)/(2*Sqrt[3])
```

3.363.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

rule 1704
```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Simp[1/(a*d^n*(m + 1))   Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1)
+ c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{
a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] &&
LtQ[m, -1] && IntegerQ[p]
```

rule 1830
```
Int[(((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
(c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[a*c, 2]}, With[{r = Rt[2*c*q -
 b*c, 2]}, Simp[c/(2*q*r)   Int[(f*x)^m*(Simp[d*r - (c*d - e*q)*x^(n/2), x]
/(q - r*x^(n/2) + c*x^n)), x], x] + Simp[c/(2*q*r)   Int[(f*x)^m*(Simp[d*r
+ (c*d - e*q)*x^(n/2), x]/(q + r*x^(n/2) + c*x^n)), x], x]] /;  !LtQ[2*c*q
- b*c, 0]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[n2, 2*n] && LtQ[b^2 - 4*a
*c, 0] && IntegersQ[m, n/2] && LtQ[0, m, n] && PosQ[a*c]
```

3.363. $\int \frac{1}{x^2(1-x^4+x^8)} \, dx$

3.363.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.11

method	result	size
risch	$-\frac{1}{x}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(81\_Z^{8}-9\_Z^{4}+1\right)}\_R\ln\left(-27\_R^{7}+6\_R^{3}+x\right)\right)}{4}$	40
default	$-\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}-\_Z^{4}+1\right)}\frac{\left(\_R^{6}-\_R^{2}\right)\ln\left(x-\_R\right)}{2\_R^{7}-\_R^{3}}\right)}{4}-\frac{1}{x}$	52

input
```
int(1/x^2/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/x+1/4*sum(_R*ln(-27*_R^7+6*_R^3+x),_R=RootOf(81*_Z^8-9*_Z^4+1))
```

3.363.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 555, normalized size of antiderivative = 1.54

$$\int \frac{1}{x^2(1-x^4+x^8)}\,dx = -\frac{\sqrt{6}x\sqrt{\sqrt{2}\sqrt{i\sqrt{3}+1}}\log\left(\sqrt{6}(i\sqrt{3}\sqrt{2}-3\sqrt{2})\sqrt{\sqrt{2}\sqrt{i\sqrt{3}+1}}\sqrt{i\sqrt{3}+1}+24x\right)-\sqrt{6}x\sqrt{\sqrt{2}\sqrt{i\sqrt{3}}}}{}$$

input
```
integrate(1/x^2/(x^8-x^4+1),x, algorithm="fricas")
```

output
```
-1/24*(sqrt(6)*x*sqrt(sqrt(2)*sqrt(I*sqrt(3) + 1))*log(sqrt(6)*(I*sqrt(3)*
sqrt(2) - 3*sqrt(2))*sqrt(sqrt(2)*sqrt(I*sqrt(3) + 1))*sqrt(I*sqrt(3) + 1)
 + 24*x) - sqrt(6)*x*sqrt(sqrt(2)*sqrt(I*sqrt(3) + 1))*log(sqrt(6)*(-I*sqr
t(3)*sqrt(2) + 3*sqrt(2))*sqrt(sqrt(2)*sqrt(I*sqrt(3) + 1))*sqrt(I*sqrt(3)
 + 1) + 24*x) - sqrt(6)*x*sqrt(-sqrt(2)*sqrt(I*sqrt(3) + 1))*log(sqrt(6)*(
I*sqrt(3)*sqrt(2) - 3*sqrt(2))*sqrt(-sqrt(2)*sqrt(I*sqrt(3) + 1))*sqrt(I*s
qrt(3) + 1) + 24*x) + sqrt(6)*x*sqrt(-sqrt(2)*sqrt(I*sqrt(3) + 1))*log(sqr
t(6)*(-I*sqrt(3)*sqrt(2) + 3*sqrt(2))*sqrt(-sqrt(2)*sqrt(I*sqrt(3) + 1))*s
qrt(I*sqrt(3) + 1) + 24*x) - sqrt(6)*x*sqrt(sqrt(2)*sqrt(-I*sqrt(3) + 1))*
log(sqrt(6)*(I*sqrt(3)*sqrt(2) + 3*sqrt(2))*sqrt(sqrt(2)*sqrt(-I*sqrt(3) +
 1))*sqrt(-I*sqrt(3) + 1) + 24*x) + sqrt(6)*x*sqrt(sqrt(2)*sqrt(-I*sqrt(3)
 + 1))*log(sqrt(6)*(-I*sqrt(3)*sqrt(2) - 3*sqrt(2))*sqrt(sqrt(2)*sqrt(-I*s
qrt(3) + 1))*sqrt(-I*sqrt(3) + 1) + 24*x) + sqrt(6)*x*sqrt(-sqrt(2)*sqrt(-
I*sqrt(3) + 1))*log(sqrt(6)*(I*sqrt(3)*sqrt(2) + 3*sqrt(2))*sqrt(-sqrt(2)*
sqrt(-I*sqrt(3) + 1))*sqrt(-I*sqrt(3) + 1) + 24*x) - sqrt(6)*x*sqrt(-sqrt(
2)*sqrt(-I*sqrt(3) + 1))*log(sqrt(6)*(-I*sqrt(3)*sqrt(2) - 3*sqrt(2))*sqrt
(-sqrt(2)*sqrt(-I*sqrt(3) + 1))*sqrt(-I*sqrt(3) + 1) + 24*x) + 24)/x
```

3.363.6 Sympy [A] (verification not implemented)

Time = 1.47 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.08

$$\int \frac{1}{x^2 (1 - x^4 + x^8)} \, dx$$
$$= \operatorname{RootSum}\left(5308416t^8 - 2304t^4 + 1, \left(t \mapsto t \log\left(-442368t^7 + 384t^3 + x\right)\right)\right) - \frac{1}{x}$$

input
```
integrate(1/x**2/(x**8-x**4+1),x)
```

output
```
RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(-442368*_t**7 +
384*_t**3 + x))) - 1/x
```

3.363.7 Maxima [F]

$$\int \frac{1}{x^2 (1 - x^4 + x^8)} \, dx = \int \frac{1}{(x^8 - x^4 + 1)x^2} \, dx$$

input
```
integrate(1/x^2/(x^8-x^4+1),x, algorithm="maxima")
```

output
```
-1/x - integrate((x^6 - x^2)/(x^8 - x^4 + 1), x)
```

3.363.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.72

$$\begin{aligned}\int \frac{1}{x^2 (1 - x^4 + x^8)} \, dx = & -\frac{1}{24} \left(\sqrt{6} + 3\sqrt{2}\right) \arctan\left(\frac{4x + \sqrt{6} - \sqrt{2}}{\sqrt{6} + \sqrt{2}}\right) \\ & - \frac{1}{24} \left(\sqrt{6} + 3\sqrt{2}\right) \arctan\left(\frac{4x - \sqrt{6} + \sqrt{2}}{\sqrt{6} + \sqrt{2}}\right) \\ & - \frac{1}{24} \left(\sqrt{6} - 3\sqrt{2}\right) \arctan\left(\frac{4x + \sqrt{6} + \sqrt{2}}{\sqrt{6} - \sqrt{2}}\right) \\ & - \frac{1}{24} \left(\sqrt{6} - 3\sqrt{2}\right) \arctan\left(\frac{4x - \sqrt{6} - \sqrt{2}}{\sqrt{6} - \sqrt{2}}\right) \\ & + \frac{1}{48} \left(\sqrt{6} + 3\sqrt{2}\right) \log\left(x^2 + \frac{1}{2} x\left(\sqrt{6} + \sqrt{2}\right) + 1\right) \\ & - \frac{1}{48} \left(\sqrt{6} + 3\sqrt{2}\right) \log\left(x^2 - \frac{1}{2} x\left(\sqrt{6} + \sqrt{2}\right) + 1\right) \\ & + \frac{1}{48} \left(\sqrt{6} - 3\sqrt{2}\right) \log\left(x^2 + \frac{1}{2} x\left(\sqrt{6} - \sqrt{2}\right) + 1\right) \\ & - \frac{1}{48} \left(\sqrt{6} - 3\sqrt{2}\right) \log\left(x^2 - \frac{1}{2} x\left(\sqrt{6} - \sqrt{2}\right) + 1\right) - \frac{1}{x}\end{aligned}$$

input
```
integrate(1/x^2/(x^8-x^4+1),x, algorithm="giac")
```

output
```
-1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sq
rt(2))) - 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqr
t(6) + sqrt(2))) - 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt
(2))/(sqrt(6) - sqrt(2))) - 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(
6) - sqrt(2))/(sqrt(6) - sqrt(2))) + 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 +
1/2*x*(sqrt(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*
x*(sqrt(6) + sqrt(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x*(s
qrt(6) - sqrt(2)) + 1) - 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(
6) - sqrt(2)) + 1) - 1/x
```

3.363.9 Mupad [B] (verification not implemented)

Time = 8.31 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.70

$$\int \frac{1}{x^2(1-x^4+x^8)}\,dx = -\frac{1}{x} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}{2\left(-1+\sqrt{3}\,1\mathrm{i}\right)}+\frac{\sqrt{3}\,x\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}1\mathrm{i}}{2\left(-1+\sqrt{3}\,1\mathrm{i}\right)}\right)\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}1\mathrm{i}}{12} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}1\mathrm{i}}{2\left(-1+\sqrt{3}\,1\mathrm{i}\right)}-\frac{\sqrt{3}\,x\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}{2\left(-1+\sqrt{3}\,1\mathrm{i}\right)}\right)\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}{12} + \frac{2^{3/4}\sqrt{3}\,\mathrm{atan}\left(\frac{2^{3/4}x}{2\left(1+\sqrt{3}\,1\mathrm{i}\right)^{3/4}}-\frac{2^{3/4}\sqrt{3}\,x\,1\mathrm{i}}{2\left(1+\sqrt{3}\,1\mathrm{i}\right)^{3/4}}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}1\mathrm{i}}{12} - \frac{2^{3/4}\sqrt{3}\,\mathrm{atan}\left(\frac{2^{3/4}x\,1\mathrm{i}}{2\left(1+\sqrt{3}\,1\mathrm{i}\right)^{3/4}}+\frac{2^{3/4}\sqrt{3}\,x}{2\left(1+\sqrt{3}\,1\mathrm{i}\right)^{3/4}}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}{12}$$

input
```
int(1/(x^2*(x^8 - x^4 + 1)),x)
```

output
```
(3^(1/2)*atan((x*(8 - 3^(1/2)*8i)^(1/4))/(2*(3^(1/2)*1i - 1)) + (3^(1/2)*x
*(8 - 3^(1/2)*8i)^(1/4)*1i)/(2*(3^(1/2)*1i - 1)))*(8 - 3^(1/2)*8i)^(1/4)*1
i)/12 - 1/x - (3^(1/2)*atan((x*(8 - 3^(1/2)*8i)^(1/4)*1i)/(2*(3^(1/2)*1i -
 1)) - (3^(1/2)*x*(8 - 3^(1/2)*8i)^(1/4))/(2*(3^(1/2)*1i - 1)))*(8 - 3^(1/
2)*8i)^(1/4))/12 + (2^(3/4)*3^(1/2)*atan((2^(3/4)*x)/(2*(3^(1/2)*1i + 1)^(
3/4)) - (2^(3/4)*3^(1/2)*x*1i)/(2*(3^(1/2)*1i + 1)^(3/4)))*(3^(1/2)*1i + 1
)^(1/4)*1i)/12 - (2^(3/4)*3^(1/2)*atan((2^(3/4)*x*1i)/(2*(3^(1/2)*1i + 1)^
(3/4)) + (2^(3/4)*3^(1/2)*x)/(2*(3^(1/2)*1i + 1)^(3/4)))*(3^(1/2)*1i + 1)^
(1/4))/12
```

3.364 $\int \frac{1}{x^4(1-x^4+x^8)} dx$

3.364.1 Optimal result

Integrand size = 16, antiderivative size = 370

$$\begin{aligned}\int \frac{1}{x^4(1-x^4+x^8)} dx = &-\frac{1}{3x^3} - \frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right) \\ &+ \frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right) \\ &+ \frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right) \\ &- \frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right) \\ &+ \frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right) \\ &- \frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right) \\ &- \frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right) \\ &+ \frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(1+\sqrt{2+\sqrt{3}}x+x^2\right)\end{aligned}$$

output
```
-1/3/x^3+1/4*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/2
)))*(1/2*2^(1/2)-1/6*6^(1/2))-1/4*arctan((2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/
2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))+1/8*ln(1+x^2-x*(1/2*6^(1
/2)-1/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))-1/8*ln(1+x^2+x*(1/2*6^(1/2)-1/
2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))-1/4*arctan((-2*x+1/2*6^(1/2)-1/2*2^(
1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))*(1/2*2^(1/2)+1/6*6^(1/2))+1/4*arctan((2*x
+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))*(1/2*2^(1/2)+1/6*6^(1
/2))-1/8*ln(1+x^2-x*(1/2*6^(1/2)+1/2*2^(1/2)))*(1/2*2^(1/2)+1/6*6^(1/2))+1
/8*ln(1+x^2+x*(1/2*6^(1/2)+1/2*2^(1/2)))*(1/2*2^(1/2)+1/6*6^(1/2))
```

3.364.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.18

$$\int \frac{1}{x^4 (1-x^4+x^8)} \, dx = -\frac{1}{3x^3} - \frac{1}{4}\text{RootSum}\left[1-\#1^4 + \#1^8\&, \frac{-\log(x-\#1)+\log(x-\#1)\#1^4}{-\#1^3+2\#1^7}\&\right]$$

input
```
Integrate[1/(x^4*(1 - x^4 + x^8)),x]
```

output
```
-1/3*1/x^3 - RootSum[1 - #1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)
/(-#1^3 + 2*#1^7) & ]/4
```

3.364.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 354, normalized size of antiderivative = 0.96, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {1704, 27, 1751, 25, 1483, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (x^8-x^4+1)} \, dx$$

$$\Big\downarrow \text{1704}$$

$$\frac{1}{3}\int \frac{3\left(1-x^4\right)}{x^8-x^4+1}dx-\frac{1}{3x^3}$$

$$\Big\downarrow \; 27$$

$$\int \frac{1-x^4}{x^8-x^4+1}dx-\frac{1}{3x^3}$$

$$\Big\downarrow \; 1751$$

$$-\frac{\int -\frac{\sqrt{3}-2x^2}{x^4-\sqrt{3}x^2+1}dx}{2\sqrt{3}}-\frac{\int -\frac{2x^2+\sqrt{3}}{x^4+\sqrt{3}x^2+1}dx}{2\sqrt{3}}-\frac{1}{3x^3}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{\sqrt{3}-2x^2}{x^4-\sqrt{3}x^2+1}dx}{2\sqrt{3}}+\frac{\int \frac{2x^2+\sqrt{3}}{x^4+\sqrt{3}x^2+1}dx}{2\sqrt{3}}-\frac{1}{3x^3}$$

$$\Big\downarrow \; 1483$$

$$\frac{\frac{\int \frac{\left(2-\sqrt{3}\right)x+\sqrt{3\left(2-\sqrt{3}\right)}}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\int \frac{\sqrt{3\left(2-\sqrt{3}\right)}-\left(2-\sqrt{3}\right)x}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+\frac{\frac{\int \frac{\sqrt{3\left(2+\sqrt{3}\right)}-\left(2+\sqrt{3}\right)x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{\int \frac{\left(2+\sqrt{3}\right)x+\sqrt{3\left(2+\sqrt{3}\right)}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}-\frac{1}{3x^3}$$

$$\Big\downarrow \; 1142$$

$$\frac{\frac{\frac{1}{2}\sqrt{2+\sqrt{3}}\int \frac{1}{x^2-\sqrt{2-\sqrt{3}}x+1}dx+\frac{1}{2}\left(2-\sqrt{3}\right)\int -\frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\frac{1}{2}\sqrt{2+\sqrt{3}}\int \frac{1}{x^2+\sqrt{2-\sqrt{3}}x+1}dx-\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+\frac{\frac{-\frac{1}{2}\sqrt{2-\sqrt{3}}\int \frac{1}{x^2-\sqrt{2+\sqrt{3}}x+1}dx-\frac{1}{2}\left(2+\sqrt{3}\right)\int -\frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(2+\sqrt{3}\right)\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx-\frac{1}{2}\sqrt{2-\sqrt{3}}\int \frac{1}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}-\frac{1}{3x^3}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\frac{1}{2}\sqrt{2+\sqrt{3}}\int \frac{1}{x^2-\sqrt{2-\sqrt{3}}x+1}dx-\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\frac{1}{2}\sqrt{2+\sqrt{3}}\int \frac{1}{x^2+\sqrt{2-\sqrt{3}}x+1}dx-\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+\frac{\frac{\frac{1}{2}\left(2+\sqrt{3}\right)\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx-\frac{1}{2}\sqrt{2-\sqrt{3}}\int \frac{1}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(2+\sqrt{3}\right)\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx-\frac{1}{2}\sqrt{2-\sqrt{3}}\int \frac{1}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}-\frac{1}{3x^3}$$

$$\Big\downarrow \; 1083$$

3.364. $\int \frac{1}{x^4\left(1-x^4+x^8\right)}\,dx$

$$\frac{\frac{-\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx-\sqrt{2+\sqrt{3}}\int \frac{1}{-\left(2x-\sqrt{2-\sqrt{3}}\right)^2-\sqrt{3}-2}d\left(2x-\sqrt{2-\sqrt{3}}\right)}{2\sqrt{2-\sqrt{3}}}+\frac{-\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx-\sqrt{2+\sqrt{3}}\int \frac{}{-\left(2x+\sqrt{2-\right.}}}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}$$

$$\frac{\frac{\frac{1}{2}\left(2+\sqrt{3}\right)\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx+\sqrt{2-\sqrt{3}}\int \frac{1}{-\left(2x-\sqrt{2+\sqrt{3}}\right)^2+\sqrt{3}-2}d\left(2x-\sqrt{2+\sqrt{3}}\right)}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(2+\sqrt{3}\right)\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx+\sqrt{2-\sqrt{3}}\int \frac{1}{-\left(2x+\sqrt{2+\sqrt{3}\right.}}}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}$$

$$\frac{1}{3x^3}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+\frac{\frac{\frac{1}{2}\left(2+\sqrt{3}\right)\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx-\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(2+\sqrt{3}\right)\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx-\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}-\frac{1}{3x^3}$$

$$\Big\downarrow \text{1103}$$

$$\frac{\frac{\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)+\frac{1}{2}\left(2-\sqrt{3}\right)\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{2\sqrt{2-\sqrt{3}}}+\frac{\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{2}\left(2-\sqrt{3}\right)\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+\frac{\frac{-\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)-\frac{1}{2}\left(2+\sqrt{3}\right)\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(2+\sqrt{3}\right)\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)-\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}-\frac{1}{3x^3}$$

input
```
Int[1/(x^4*(1 - x^4 + x^8)),x]
```

output
```
-1/3*1/x^3 + ((ArcTan[(-Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]] + ((2
- Sqrt[3])*Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2])/2)/(2*Sqrt[2 - Sqrt[3]]) +
(ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]] - ((2 - Sqrt[3])*Log[
1 + Sqrt[2 - Sqrt[3]]*x + x^2])/2)/(2*Sqrt[2 - Sqrt[3]]))/(2*Sqrt[3]) + ((
-ArcTan[(-Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]] - ((2 + Sqrt[3])*Log
[1 - Sqrt[2 + Sqrt[3]]*x + x^2])/2)/(2*Sqrt[2 + Sqrt[3]]) + (-ArcTan[(Sqrt
[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]] + ((2 + Sqrt[3])*Log[1 + Sqrt[2 +
Sqrt[3]]*x + x^2])/2)/(2*Sqrt[2 + Sqrt[3]]))/(2*Sqrt[3])
```

3.364. $\int \frac{1}{x^4(1-x^4+x^8)}\,dx$

3.364.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 1704
```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Simp[1/(a*d^n*(m + 1))   Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1)
+ c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{
a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] &&
LtQ[m, -1] && IntegerQ[p]
```

rule 1751
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x
^(n/2))/Simp[d/e + q*x^(n/2) - x^n, x], x], x] + Simp[e/(2*c*q)   Int[(q +
2*x^(n/2))/Simp[d/e - q*x^(n/2) - x^n, x], x], x]] /; FreeQ[{a, b, c, d, e}
, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGt
Q[n/2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

3.364.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.10

method	result	size
risch	$-\frac{1}{3x^3}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(81\_Z^8-9\_Z^4+1\right)}\_R\ln\left(-9\_R^5+2\_R+x\right)\right)}{4}$	38
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)}\frac{\left(-\_R^4+1\right)\ln\left(x-\_R\right)}{2\_R^7-\_R^3}\right)}{4}-\frac{1}{3x^3}$	50

input
```
int(1/x^4/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/3/x^3+1/4*sum(_R*ln(-9*_R^5+2*_R+x),_R=RootOf(81*_Z^8-9*_Z^4+1))
```

3.364.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^4(1-x^4+x^8)}\,dx$$
$$= \frac{\sqrt{6}x^3\sqrt{\sqrt{2}\sqrt{-i\sqrt{3}+1}}\log\left(\sqrt{6}\sqrt{\sqrt{2}\sqrt{-i\sqrt{3}+1}}(i\sqrt{3}+3)+12x\right)+\sqrt{6}x^3\sqrt{-\sqrt{2}\sqrt{-i\sqrt{3}+1}}\log\left(\right.}{}$$

input
```
integrate(1/x^4/(x^8-x^4+1),x, algorithm="fricas")
```

output
```
1/24*(sqrt(6)*x^3*sqrt(sqrt(2)*sqrt(-I*sqrt(3) + 1))*log(sqrt(6)*sqrt(sqrt
(2)*sqrt(-I*sqrt(3) + 1))*(I*sqrt(3) + 3) + 12*x) + sqrt(6)*x^3*sqrt(-sqrt
(2)*sqrt(-I*sqrt(3) + 1))*log(sqrt(6)*sqrt(-sqrt(2)*sqrt(-I*sqrt(3) + 1))*
(I*sqrt(3) + 3) + 12*x) - sqrt(6)*x^3*sqrt(sqrt(2)*sqrt(I*sqrt(3) + 1))*lo
g(sqrt(6)*sqrt(sqrt(2)*sqrt(I*sqrt(3) + 1))*(I*sqrt(3) - 3) + 12*x) - sqrt
(6)*x^3*sqrt(-sqrt(2)*sqrt(I*sqrt(3) + 1))*log(sqrt(6)*sqrt(-sqrt(2)*sqrt(
I*sqrt(3) + 1))*(I*sqrt(3) - 3) + 12*x) + sqrt(6)*x^3*sqrt(sqrt(2)*sqrt(I*
sqrt(3) + 1))*log(sqrt(6)*sqrt(sqrt(2)*sqrt(I*sqrt(3) + 1))*(-I*sqrt(3) +
3) + 12*x) + sqrt(6)*x^3*sqrt(-sqrt(2)*sqrt(I*sqrt(3) + 1))*log(sqrt(6)*sq
rt(-sqrt(2)*sqrt(I*sqrt(3) + 1))*(-I*sqrt(3) + 3) + 12*x) - sqrt(6)*x^3*sq
rt(sqrt(2)*sqrt(-I*sqrt(3) + 1))*log(sqrt(6)*sqrt(sqrt(2)*sqrt(-I*sqrt(3)
+ 1))*(-I*sqrt(3) - 3) + 12*x) - sqrt(6)*x^3*sqrt(-sqrt(2)*sqrt(-I*sqrt(3)
 + 1))*log(sqrt(6)*sqrt(-sqrt(2)*sqrt(-I*sqrt(3) + 1))*(-I*sqrt(3) - 3) +
12*x) - 8)/x^3
```

3.364.6 Sympy [A] (verification not implemented)

Time = 1.49 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.08

$$\int \frac{1}{x^4(1-x^4+x^8)}\,dx = \operatorname{RootSum}\left(5308416t^8-2304t^4+1,\left(t\mapsto t\log\left(-9216t^5+8t+x\right)\right)\right) - \frac{1}{3x^3}$$

input
```
integrate(1/x**4/(x**8-x**4+1),x)
```

output
```
RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(-9216*_t**5 + 8*
_t + x))) - 1/(3*x**3)
```

3.364.7 Maxima [F]

$$\int \frac{1}{x^4(1-x^4+x^8)}\,dx = \int \frac{1}{(x^8-x^4+1)x^4}\,dx$$

input
```
integrate(1/x^4/(x^8-x^4+1),x, algorithm="maxima")
```

output
```
-1/3/x^3 - integrate((x^4 - 1)/(x^8 - x^4 + 1), x)
```

3.364.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.70

$$\int \frac{1}{x^4(1-x^4+x^8)}\,dx = \frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right) + \frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right) + \frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) + \frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) + \frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right) - \frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right) + \frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right) - \frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right) - \frac{1}{3x^3}$$

input
```
integrate(1/x^4/(x^8-x^4+1),x, algorithm="giac")
```

output
```
1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqr
t(2))) + 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt
(6) + sqrt(2))) + 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt(
2))/(sqrt(6) - sqrt(2))) + 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6
) - sqrt(2))/(sqrt(6) - sqrt(2))) + 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 + 1
/2*x*(sqrt(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*x
*(sqrt(6) + sqrt(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x*(sq
rt(6) - sqrt(2)) + 1) - 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6
) - sqrt(2)) + 1) - 1/3/x^3
```

3.364.9 Mupad [B] (verification not implemented)

Time = 8.35 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.58

$$\int \frac{1}{x^4\,(1-x^4+x^8)}\,dx = -\frac{1}{3\,x^3} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x}{\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}+\frac{\sqrt{3}\,x\,1\mathrm{i}}{\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}\right)\,\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}\,1\mathrm{i}}{12} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\,1\mathrm{i}}{\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}-\frac{\sqrt{3}\,x}{\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}\right)\,\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}{12} + \frac{2^{3/4}\,\sqrt{3}\,\mathrm{atan}\left(\frac{2^{1/4}\,x}{2\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}-\frac{2^{1/4}\,\sqrt{3}\,x\,1\mathrm{i}}{2\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}\,1\mathrm{i}}{12} + \frac{2^{3/4}\,\sqrt{3}\,\mathrm{atan}\left(\frac{2^{1/4}\,x\,1\mathrm{i}}{2\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}+\frac{2^{1/4}\,\sqrt{3}\,x}{2\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}{12}$$

input
```
int(1/(x^4*(x^8 - x^4 + 1)),x)
```

output
```
(2^(3/4)*3^(1/2)*atan((2^(1/4)*x)/(2*(3^(1/2)*1i + 1)^(1/4)) - (2^(1/4)*3^
(1/2)*x*1i)/(2*(3^(1/2)*1i + 1)^(1/4)))*(3^(1/2)*1i + 1)^(1/4)*1i)/12 - (3
^(1/2)*atan(x/(8 - 3^(1/2)*8i)^(1/4) + (3^(1/2)*x*1i)/(8 - 3^(1/2)*8i)^(1/
4))*(8 - 3^(1/2)*8i)^(1/4)*1i)/12 - (3^(1/2)*atan((x*1i)/(8 - 3^(1/2)*8i)^
(1/4) - (3^(1/2)*x)/(8 - 3^(1/2)*8i)^(1/4))*(8 - 3^(1/2)*8i)^(1/4))/12 - 1
/(3*x^3) + (2^(3/4)*3^(1/2)*atan((2^(1/4)*x*1i)/(2*(3^(1/2)*1i + 1)^(1/4))
 + (2^(1/4)*3^(1/2)*x)/(2*(3^(1/2)*1i + 1)^(1/4)))*(3^(1/2)*1i + 1)^(1/4))
/12
```

3.365 $\int \frac{1}{x^6(1-x^4+x^8)}\,dx$

3.365.1 Optimal result

Integrand size = 16, antiderivative size = 287

$$\begin{aligned}\int \frac{1}{x^6(1-x^4+x^8)}\,dx = & -\frac{1}{5x^5}-\frac{1}{x}+\frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}} \\ & -\frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}} \\ & -\frac{\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)}{4\sqrt{6}}+\frac{\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)}{4\sqrt{6}} \\ & -\frac{\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)}{4\sqrt{6}}+\frac{\log\left(1+\sqrt{2+\sqrt{3}}x+x^2\right)}{4\sqrt{6}}\end{aligned}$$

output
```
-1/5/x^5-1/x+1/12*arctan((-2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2
^(1/2)))*6^(1/2)-1/12*arctan((2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/
2*2^(1/2)))*6^(1/2)+1/12*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2
)-1/2*2^(1/2)))*6^(1/2)-1/12*arctan((2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(
1/2)-1/2*2^(1/2)))*6^(1/2)-1/24*ln(1+x^2-x*(1/2*6^(1/2)-1/2*2^(1/2)))*6^(1
/2)+1/24*ln(1+x^2+x*(1/2*6^(1/2)-1/2*2^(1/2)))*6^(1/2)-1/24*ln(1+x^2-x*(1/
2*6^(1/2)+1/2*2^(1/2)))*6^(1/2)+1/24*ln(1+x^2+x*(1/2*6^(1/2)+1/2*2^(1/2)))
*6^(1/2)
```

3.365.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.19

$$\int \frac{1}{x^6 (1-x^4+x^8)} \, dx = -\frac{1}{5x^5} - \frac{1}{x} - \frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&, \frac{\log(x-\#1)\#1^3}{-1+2\#1^4}\&\right]$$

input
```
Integrate[1/(x^6*(1 - x^4 + x^8)),x]
```

output
```
-1/5*1/x^5 - x^(-1) - RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1^3)/(-1 +
 2*#1^4) & ]/4
```

3.365.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.48, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {1704, 27, 1828, 1708, 1483, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 (x^8-x^4+1)} \, dx$$

$$\Big\downarrow \text{1704}$$

$$\frac{1}{5}\int \frac{5(1-x^4)}{x^2 (x^8-x^4+1)}dx - \frac{1}{5x^5}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{1-x^4}{x^2 (x^8-x^4+1)}dx - \frac{1}{5x^5}$$

$$\Big\downarrow \text{1828}$$

$$-\int \frac{x^6}{x^8-x^4+1}dx - \frac{1}{5x^5} - \frac{1}{x}$$

$$\Big\downarrow \text{1708}$$

$$\frac{\int \frac{1-\sqrt{3}x^2}{x^4-\sqrt{3}x^2+1}dx}{2\sqrt{3}} - \frac{\int \frac{\sqrt{3}x^2+1}{x^4+\sqrt{3}x^2+1}dx}{2\sqrt{3}} - \frac{1}{5x^5} - \frac{1}{x}$$

$$\downarrow \textcolor{blue}{1483}$$

$$-\frac{\frac{\int \frac{\sqrt{2-\sqrt{3}}-\left(1-\sqrt{3}\right)x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\int \frac{\left(1-\sqrt{3}\right)x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+\frac{\frac{\int \frac{\sqrt{2+\sqrt{3}}-\left(1+\sqrt{3}\right)x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{\int \frac{\left(1+\sqrt{3}\right)x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}-\frac{1}{5x^5}-\frac{1}{x}$$

$$\downarrow \textcolor{blue}{1142}$$

$$-\frac{\frac{\frac{\int \frac{1}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{\sqrt{2}}-\frac{1}{2}\left(1-\sqrt{3}\right)\int -\frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\frac{\int \frac{1}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{\sqrt{2}}+\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+\frac{\frac{-\frac{\int \frac{1}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{\sqrt{2}}-\frac{1}{2}\left(1+\sqrt{3}\right)\int -\frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx-\frac{\int \frac{1}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{\sqrt{2}}}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}-\frac{1}{5x^5}-\frac{1}{x}$$

$$\downarrow \textcolor{blue}{25}$$

$$-\frac{\frac{\frac{\int \frac{1}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{\sqrt{2}}+\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\frac{\int \frac{1}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{\sqrt{2}}+\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+\frac{\frac{\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx-\frac{\int \frac{1}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{\sqrt{2}}}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx-\frac{\int \frac{1}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{\sqrt{2}}}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}-\frac{1}{5x^5}-\frac{1}{x}$$

$$\downarrow \textcolor{blue}{1083}$$

$$-\frac{\frac{\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx-\sqrt{2}\int \frac{1}{-\left(2x-\sqrt{2-\sqrt{3}}\right)^2-\sqrt{3}-2}d\left(2x-\sqrt{2-\sqrt{3}}\right)}{2\sqrt{2-\sqrt{3}}}+\frac{\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx-\sqrt{2}\int \frac{1}{-\left(2x+\sqrt{2-\sqrt{3}}\right)^2-\sqrt{3}-\cdots}}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}$$

$$\frac{\frac{\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx+\sqrt{2}\int \frac{1}{-\left(2x-\sqrt{2+\sqrt{3}}\right)^2+\sqrt{3}-2}d\left(2x-\sqrt{2+\sqrt{3}}\right)}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx+\sqrt{2}\int \frac{1}{-\left(2x+\sqrt{2+\sqrt{3}}\right)^2+\sqrt{3}-2}d\cdots}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}$$

$$\frac{1}{5x^5}-\frac{1}{x}$$

$$\downarrow \textcolor{blue}{217}$$

$$-\frac{\frac{\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx+\sqrt{\frac{2}{2+\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{2-\sqrt{3}}}+\frac{\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx+\sqrt{\frac{2}{2+\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+\frac{\frac{\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx-\sqrt{\frac{2}{2-\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx-\sqrt{\frac{2}{2-\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}-\frac{1}{5x^5}-\frac{1}{x}$$

3.365. $\int \frac{1}{x^6(1-x^4+x^8)}\,dx$

$$\Big\downarrow \; 1103$$

$$-\frac{\frac{\sqrt{\frac{2}{2+\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{2}\left(1-\sqrt{3}\right)\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{2\sqrt{2-\sqrt{3}}}+\frac{\sqrt{\frac{2}{2+\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)+\frac{1}{2}\left(1-\sqrt{3}\right)\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+\frac{\frac{-\sqrt{\frac{2}{2-\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)-\frac{1}{2}\left(1+\sqrt{3}\right)\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(1+\sqrt{3}\right)\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)-\sqrt{\frac{2}{2-\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}-\frac{1}{5x^5}-\frac{1}{x}$$

input
```
Int[1/(x^6*(1 - x^4 + x^8)),x]
```

output
```
-1/5*1/x^5 - x^(-1) - ((Sqrt[2/(2 + Sqrt[3])]*ArcTan[(-Sqrt[2 - Sqrt[3]] +
 2*x)/Sqrt[2 + Sqrt[3]]] - ((1 - Sqrt[3])*Log[1 - Sqrt[2 - Sqrt[3]]*x + x^
2])/2)/(2*Sqrt[2 - Sqrt[3]]) + (Sqrt[2/(2 + Sqrt[3])]*ArcTan[(Sqrt[2 - Sqr
t[3]] + 2*x)/Sqrt[2 + Sqrt[3]]] + ((1 - Sqrt[3])*Log[1 + Sqrt[2 - Sqrt[3]]
*x + x^2])/2)/(2*Sqrt[2 - Sqrt[3]]))/(2*Sqrt[3]) + ((-(Sqrt[2/(2 - Sqrt[3]
)]*ArcTan[(-Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]) - ((1 + Sqrt[3])*
Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2])/2)/(2*Sqrt[2 + Sqrt[3]]) + (-(Sqrt[2/(
2 - Sqrt[3])]*ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]) + ((1 +
 Sqrt[3])*Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/2)/(2*Sqrt[2 + Sqrt[3]]))/(2
*Sqrt[3])
```

3.365.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 1704
```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Simp[1/(a*d^n*(m + 1))   Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1)
+ c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{
a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] &&
LtQ[m, -1] && IntegerQ[p]
```

rule 1708
```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, -Simp[1/(2*c*r)   Int[x^
(m - 3*(n/2))*((q - r*x^(n/2))/(q - r*x^(n/2) + x^n)), x], x] + Simp[1/(2*c
*r)   Int[x^(m - 3*(n/2))*((q + r*x^(n/2))/(q + r*x^(n/2) + x^n)), x], x]]]
 /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n/2,
 0] && IGtQ[m, 0] && GeQ[m, 3*(n/2)] && LtQ[m, 2*n] && NegQ[b^2 - 4*a*c]
```

3.365. $\int \frac{1}{x^6(1-x^4+x^8)}\,dx$

rule 1828
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Simp[1/(a*f^n*(m + 1))   Int[(f*x)^(m +
 n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) -
c*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x
] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Int
egerQ[p]
```

3.365.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.15

method	result	size
default	$-\frac{1}{5x^5}-\frac{1}{x}-\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(9\_Z^4+1\right)}\_R\ln\left(9x\_R^3-3\_R^2+x^2\right)\right)}{4}$	43
risch	$\frac{-x^4-\frac{1}{5}}{x^5}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(9\_Z^4+1\right)}\_R\ln\left(-9x\_R^3-3\_R^2+x^2\right)\right)}{4}$	44

input
```
int(1/x^6/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/5/x^5-1/x-1/4*sum(_R*ln(9*_R^3*x-3*_R^2+x^2),_R=RootOf(9*_Z^4+1))
```

3.365.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.43

$$\int \frac{1}{x^6(1-x^4+x^8)}\,dx$$
$$= \frac{-(5i-5)\sqrt{3}\sqrt{2}x^5\log\left((3i+3)\sqrt{3}\sqrt{2}x+6x^2+6i\right)+(5i+5)\sqrt{3}\sqrt{2}x^5\log\left(-(3i-3)\sqrt{3}\sqrt{2}x+6x^2\right.}{}$$

input
```
integrate(1/x^6/(x^8-x^4+1),x, algorithm="fricas")
```

output
```
1/120*(-(5*I - 5)*sqrt(3)*sqrt(2)*x^5*log((3*I + 3)*sqrt(3)*sqrt(2)*x + 6*
x^2 + 6*I) + (5*I + 5)*sqrt(3)*sqrt(2)*x^5*log(-(3*I - 3)*sqrt(3)*sqrt(2)*
x + 6*x^2 - 6*I) - (5*I + 5)*sqrt(3)*sqrt(2)*x^5*log((3*I - 3)*sqrt(3)*sqr
t(2)*x + 6*x^2 - 6*I) + (5*I - 5)*sqrt(3)*sqrt(2)*x^5*log(-(3*I + 3)*sqrt(
3)*sqrt(2)*x + 6*x^2 + 6*I) - 120*x^4 - 24)/x^5
```

3.365.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.63

$$\int \frac{1}{x^6(1-x^4+x^8)}\,dx = \frac{\sqrt{6}\left(-2\,\mathrm{atan}\left(\frac{\sqrt{6}x}{3}-\frac{1}{3}\right)-2\,\mathrm{atan}\left(\sqrt{6}x^3-4x^2+2\sqrt{6}x-3\right)\right)}{24} + \frac{\sqrt{6}\left(-2\,\mathrm{atan}\left(\frac{\sqrt{6}x}{3}+\frac{1}{3}\right)-2\,\mathrm{atan}\left(\sqrt{6}x^3+4x^2+2\sqrt{6}x+3\right)\right)}{24} - \frac{\sqrt{6}\log\left(x^4-\sqrt{6}x^3+3x^2-\sqrt{6}x+1\right)}{24} + \frac{\sqrt{6}\log\left(x^4+\sqrt{6}x^3+3x^2+\sqrt{6}x+1\right)}{24} + \frac{-5x^4-1}{5x^5}$$

input
```
integrate(1/x**6/(x**8-x**4+1),x)
```

output
```
sqrt(6)*(-2*atan(sqrt(6)*x/3 - 1/3) - 2*atan(sqrt(6)*x**3 - 4*x**2 + 2*sqr
t(6)*x - 3))/24 + sqrt(6)*(-2*atan(sqrt(6)*x/3 + 1/3) - 2*atan(sqrt(6)*x**
3 + 4*x**2 + 2*sqrt(6)*x + 3))/24 - sqrt(6)*log(x**4 - sqrt(6)*x**3 + 3*x*
*2 - sqrt(6)*x + 1)/24 + sqrt(6)*log(x**4 + sqrt(6)*x**3 + 3*x**2 + sqrt(6
)*x + 1)/24 + (-5*x**4 - 1)/(5*x**5)
```

3.365.7 Maxima [F]

$$\int \frac{1}{x^6(1-x^4+x^8)}\,dx = \int \frac{1}{(x^8-x^4+1)x^6}\,dx$$

input
```
integrate(1/x^6/(x^8-x^4+1),x, algorithm="maxima")
```

output
```
-1/5*(5*x^4 + 1)/x^5 - integrate(x^6/(x^8 - x^4 + 1), x)
```

3.365.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^6(1-x^4+x^8)}\,dx = -\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right) - \frac{1}{12}\sqrt{6}\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right) - \frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) - \frac{1}{12}\sqrt{6}\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) + \frac{1}{24}\sqrt{6}\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right) - \frac{1}{24}\sqrt{6}\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right) + \frac{1}{24}\sqrt{6}\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right) - \frac{1}{24}\sqrt{6}\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right) - \frac{5x^4+1}{5x^5}$$

input
```
integrate(1/x^6/(x^8-x^4+1),x, algorithm="giac")
```

output
```
-1/12*sqrt(6)*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt(2))) - 1/12
*sqrt(6)*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6) + sqrt(2))) - 1/12*sqrt
(6)*arctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2))) - 1/12*sqrt(6)*a
rctan((4*x - sqrt(6) - sqrt(2))/(sqrt(6) - sqrt(2))) + 1/24*sqrt(6)*log(x^
2 + 1/2*x*(sqrt(6) + sqrt(2)) + 1) - 1/24*sqrt(6)*log(x^2 - 1/2*x*(sqrt(6)
 + sqrt(2)) + 1) + 1/24*sqrt(6)*log(x^2 + 1/2*x*(sqrt(6) - sqrt(2)) + 1) -
 1/24*sqrt(6)*log(x^2 - 1/2*x*(sqrt(6) - sqrt(2)) + 1) - 1/5*(5*x^4 + 1)/x
^5
```

3.365.9 Mupad [B] (verification not implemented)

Time = 8.34 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.22

$$\int \frac{1}{x^6\left(1-x^4+x^8\right)}\,dx = -\frac{x^4+\frac{1}{5}}{x^5}+\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x\,\left(\frac{1}{3}+\frac{1}{3}\mathrm{i}\right)}{\frac{2\,x^2}{3}-\frac{2}{3}\mathrm{i}}\right)\,\left(\frac{1}{12}-\frac{1}{12}\mathrm{i}\right) + \sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x\,\left(\frac{1}{3}-\frac{1}{3}\mathrm{i}\right)}{\frac{2\,x^2}{3}+\frac{2}{3}\mathrm{i}}\right)\,\left(\frac{1}{12}+\frac{1}{12}\mathrm{i}\right)$$

input
```
int(1/(x^6*(x^8 - x^4 + 1)),x)
```

output
```
6^(1/2)*atan((6^(1/2)*x*(1/3 + 1i/3))/((2*x^2)/3 - 2i/3))*(1/12 - 1i/12) +
 6^(1/2)*atan((6^(1/2)*x*(1/3 - 1i/3))/((2*x^2)/3 + 2i/3))*(1/12 + 1i/12)
- (x^4 + 1/5)/x^5
```

3.366 $\int \frac{1}{x^8(1-x^4+x^8)}\,dx$

3.366.1 Optimal result

Integrand size = 16, antiderivative size = 377

$$\begin{aligned}\int \frac{1}{x^8(1-x^4+x^8)}\,dx &= -\frac{1}{7x^7}-\frac{1}{3x^3}-\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)\\
&+\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)\\
&+\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)\\
&-\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)\\
&+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)\\
&-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)\\
&-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)\\
&+\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(1+\sqrt{2+\sqrt{3}}x+x^2\right)\end{aligned}$$

output
```
-1/7/x^7-1/3/x^3-1/4*arctan((-2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/
2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))+1/4*arctan((2*x+1/2*6^(1/2)-1/2*2^(1
/2))/(1/2*6^(1/2)+1/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))-1/8*ln(1+x^2-x*(
1/2*6^(1/2)+1/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))+1/8*ln(1+x^2+x*(1/2*6^
(1/2)+1/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))+1/4*arctan((-2*x+1/2*6^(1/2)
+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)+1/6*6^(1/2))-1/4*arc
tan((2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)+
1/6*6^(1/2))+1/8*ln(1+x^2-x*(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)+1/6*6^
(1/2))-1/8*ln(1+x^2+x*(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)+1/6*6^(1/2))
```

3.366.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.14

$$\int \frac{1}{x^8(1-x^4+x^8)}\,dx = -\frac{1}{7x^7}-\frac{1}{3x^3}-\frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{\log(x-\#1)\#1}{-1+2\#1^4}\&\right]$$

input
```
Integrate[1/(x^8*(1 - x^4 + x^8)),x]
```

output
```
-1/7*1/x^7 - 1/(3*x^3) - RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1)/(-1
+ 2*#1^4) & ]/4
```

3.366.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {1704, 27, 1828, 27, 1709, 1447, 1475, 1083, 217, 1478, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^8(x^8-x^4+1)}\,dx$$

$$\Big\downarrow \text{1704}$$

$$\frac{1}{7}\int \frac{7(1-x^4)}{x^4(x^8-x^4+1)}dx-\frac{1}{7x^7}$$

$$\Big\downarrow \; 27$$

$$\int \frac{1-x^4}{x^4\left(x^8-x^4+1\right)}dx-\frac{1}{7x^7}$$

$$\Big\downarrow \; 1828$$

$$-\frac{1}{3}\int \frac{3x^4}{x^8-x^4+1}dx-\frac{1}{7x^7}-\frac{1}{3x^3}$$

$$\Big\downarrow \; 27$$

$$-\int \frac{x^4}{x^8-x^4+1}dx-\frac{1}{7x^7}-\frac{1}{3x^3}$$

$$\Big\downarrow \; 1709$$

$$-\frac{\int \frac{x^2}{x^4-\sqrt{3}x^2+1}dx}{2\sqrt{3}}+\frac{\int \frac{x^2}{x^4+\sqrt{3}x^2+1}dx}{2\sqrt{3}}-\frac{1}{7x^7}-\frac{1}{3x^3}$$

$$\Big\downarrow \; 1447$$

$$-\frac{\frac{1}{2}\int \frac{x^2+1}{x^4-\sqrt{3}x^2+1}dx-\frac{1}{2}\int \frac{1-x^2}{x^4-\sqrt{3}x^2+1}dx}{2\sqrt{3}}+\frac{\frac{1}{2}\int \frac{x^2+1}{x^4+\sqrt{3}x^2+1}dx-\frac{1}{2}\int \frac{1-x^2}{x^4+\sqrt{3}x^2+1}dx}{2\sqrt{3}}-\frac{1}{7x^7}-\frac{1}{3x^3}$$

$$\Big\downarrow \; 1475$$

$$-\frac{\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2-\sqrt{2+\sqrt{3}}x+1}dx+\frac{1}{2}\int \frac{1}{x^2+\sqrt{2+\sqrt{3}}x+1}dx\right)-\frac{1}{2}\int \frac{1-x^2}{x^4-\sqrt{3}x^2+1}dx}{2\sqrt{3}}+$$
$$\frac{\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2-\sqrt{2-\sqrt{3}}x+1}dx+\frac{1}{2}\int \frac{1}{x^2+\sqrt{2-\sqrt{3}}x+1}dx\right)-\frac{1}{2}\int \frac{1-x^2}{x^4+\sqrt{3}x^2+1}dx}{2\sqrt{3}}-\frac{1}{7x^7}-\frac{1}{3x^3}$$

$$\Big\downarrow \; 1083$$

$$\frac{\frac{1}{2}\left(-\int \frac{1}{-\left(2x-\sqrt{2-\sqrt{3}}\right)^2-\sqrt{3}-2}d\left(2x-\sqrt{2-\sqrt{3}}\right)-\int \frac{1}{-\left(2x+\sqrt{2-\sqrt{3}}\right)^2-\sqrt{3}-2}d\left(2x+\sqrt{2-\sqrt{3}}\right)\right)-\frac{1}{2}\int \frac{1-x^2}{x^4+\sqrt{3}x^2+1}dx}{2\sqrt{3}}$$
$$\frac{\frac{1}{2}\left(-\int \frac{1}{-\left(2x-\sqrt{2+\sqrt{3}}\right)^2+\sqrt{3}-2}d\left(2x-\sqrt{2+\sqrt{3}}\right)-\int \frac{1}{-\left(2x+\sqrt{2+\sqrt{3}}\right)^2+\sqrt{3}-2}d\left(2x+\sqrt{2+\sqrt{3}}\right)\right)-\frac{1}{2}\int \frac{1-x^2}{x^4-\sqrt{3}x^2+1}dx}{2\sqrt{3}}$$
$$\frac{1}{7x^7}-\frac{1}{3x^3}$$

$$\Big\downarrow \; 217$$

3.366. $\int \frac{1}{x^8\left(1-x^4+x^8\right)}\,dx$

$$-\frac{\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2-\sqrt{3}}}+\frac{\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2-\sqrt{3}}}\right)-\frac{1}{2}\int\frac{1-x^2}{x^4-\sqrt{3}x^2+1}dx}{2\sqrt{3}}+\frac{\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{2+\sqrt{3}}}+\frac{\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{2}\int\frac{1-x^2}{x^4+\sqrt{3}x^2+1}dx}{2\sqrt{3}}-\frac{1}{7x^7}-\frac{1}{3x^3}$$

$$\Big\downarrow \ 1478$$

$$\frac{\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\int-\frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{2+\sqrt{3}}}+\frac{\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3}}-\frac{\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{\int-\frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2-\sqrt{3}}}+\frac{\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3}}-\frac{1}{7x^7}-\frac{1}{3x^3}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}-\frac{\int\frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{2+\sqrt{3}}}+\frac{\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3}}-\frac{\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}-\frac{\int\frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2-\sqrt{3}}}+\frac{\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3}}-\frac{1}{7x^7}-\frac{1}{3x^3}$$

$$\Big\downarrow \ 1103$$

$$\frac{\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{2+\sqrt{3}}}+\frac{\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{2+\sqrt{3}}}\right)+\frac{1}{2}\left(\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{2\sqrt{2-\sqrt{3}}}-\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{2\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3}}-\frac{\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2-\sqrt{3}}}+\frac{\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2-\sqrt{3}}}\right)+\frac{1}{2}\left(\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{2\sqrt{2+\sqrt{3}}}-\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)}{2\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3}}-\frac{1}{7x^7}-\frac{1}{3x^3}$$

3.366. $\int \frac{1}{x^8(1-x^4+x^8)}\,dx$

input
```
Int[1/(x^8*(1 - x^4 + x^8)),x]
```

output
```
-1/7*1/x^7 - 1/(3*x^3) + ((ArcTan[(-Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt
[3]]]/Sqrt[2 + Sqrt[3]] + ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3
]]]/Sqrt[2 + Sqrt[3]])/2 + (Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2]/(2*Sqrt[2 -
 Sqrt[3]]) - Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2]/(2*Sqrt[2 - Sqrt[3]]))/2)/
(2*Sqrt[3]) - ((ArcTan[(-Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/Sqrt[
2 - Sqrt[3]] + ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/Sqrt[2
- Sqrt[3]])/2 + (Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2]/(2*Sqrt[2 + Sqrt[3]])
- Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2]/(2*Sqrt[2 + Sqrt[3]]))/2)/(2*Sqrt[3])
```

3.366.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1447
```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Simp[1/2   Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Simp[1/2
 Int[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b
^2 - 4*a*c, 0] && PosQ[a*c]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

rule 1704
```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Simp[1/(a*d^n*(m + 1))   Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1)
+ c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{
a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] &&
LtQ[m, -1] && IntegerQ[p]
```

rule 1709
```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*r)   Int[x^(
m - n/2)/(q - r*x^(n/2) + x^n), x], x] - Simp[1/(2*c*r)   Int[x^(m - n/2)/(
q + r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && IGtQ[m, 0] && GeQ[m, n/2] && LtQ[m, 3*
(n/2)] && NegQ[b^2 - 4*a*c]
```

rule 1828
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Simp[1/(a*f^n*(m + 1))   Int[(f*x)^(m +
 n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) -
c*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x
] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Int
egerQ[p]
```

3.366. $\int \frac{1}{x^8(1-x^4+x^8)}\,dx$

3.366.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.12

method	result	size
risch	$\frac{-\frac{x^4}{3}-\frac{1}{7}}{x^7}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(81\_Z^8-9\_Z^4+1\right)}\_R\ln\left(18\_R^5-\_R+x\right)\right)}{4}$	44
default	$-\frac{1}{7x^7}-\frac{1}{3x^3}-\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)}\frac{\_R^4\ln\left(x-\_R\right)}{2\_R^7-\_R^3}\right)}{4}$	51

input
```
int(1/x^8/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
(-1/3*x^4-1/7)/x^7+1/4*sum(_R*ln(18*_R^5-_R+x),_R=RootOf(81*_Z^8-9*_Z^4+1)
)
```

3.366.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 428, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^8\left(1-x^4+x^8\right)}\,dx$$
$$= \frac{7\sqrt{6}x^7\sqrt{\sqrt{2}\sqrt{i\sqrt{3}+1}}\log\left(i\sqrt{6}\sqrt{3}\sqrt{\sqrt{2}\sqrt{i\sqrt{3}+1}}+6x\right)-7\sqrt{6}x^7\sqrt{\sqrt{2}\sqrt{i\sqrt{3}+1}}\log\left(-i\sqrt{6}\sqrt{3}\sqrt{\right.}$$

input
```
integrate(1/x^8/(x^8-x^4+1),x, algorithm="fricas")
```

output
```
1/168*(7*sqrt(6)*x^7*sqrt(sqrt(2)*sqrt(I*sqrt(3) + 1))*log(I*sqrt(6)*sqrt(
3)*sqrt(sqrt(2)*sqrt(I*sqrt(3) + 1)) + 6*x) - 7*sqrt(6)*x^7*sqrt(sqrt(2)*s
qrt(I*sqrt(3) + 1))*log(-I*sqrt(6)*sqrt(3)*sqrt(sqrt(2)*sqrt(I*sqrt(3) + 1
)) + 6*x) + 7*sqrt(6)*x^7*sqrt(-sqrt(2)*sqrt(I*sqrt(3) + 1))*log(I*sqrt(6)
*sqrt(3)*sqrt(-sqrt(2)*sqrt(I*sqrt(3) + 1)) + 6*x) - 7*sqrt(6)*x^7*sqrt(-s
qrt(2)*sqrt(I*sqrt(3) + 1))*log(-I*sqrt(6)*sqrt(3)*sqrt(-sqrt(2)*sqrt(I*sq
rt(3) + 1)) + 6*x) - 7*sqrt(6)*x^7*sqrt(sqrt(2)*sqrt(-I*sqrt(3) + 1))*log(
I*sqrt(6)*sqrt(3)*sqrt(sqrt(2)*sqrt(-I*sqrt(3) + 1)) + 6*x) + 7*sqrt(6)*x^
7*sqrt(sqrt(2)*sqrt(-I*sqrt(3) + 1))*log(-I*sqrt(6)*sqrt(3)*sqrt(sqrt(2)*s
qrt(-I*sqrt(3) + 1)) + 6*x) - 7*sqrt(6)*x^7*sqrt(-sqrt(2)*sqrt(-I*sqrt(3)
+ 1))*log(I*sqrt(6)*sqrt(3)*sqrt(-sqrt(2)*sqrt(-I*sqrt(3) + 1)) + 6*x) + 7
*sqrt(6)*x^7*sqrt(-sqrt(2)*sqrt(-I*sqrt(3) + 1))*log(-I*sqrt(6)*sqrt(3)*sq
rt(-sqrt(2)*sqrt(-I*sqrt(3) + 1)) + 6*x) - 56*x^4 - 24)/x^7
```

3.366.6 Sympy [A] (verification not implemented)

Time = 1.46 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.10

$$\int \frac{1}{x^8 (1 - x^4 + x^8)} \, dx = \text{RootSum}\left(5308416t^8 - 2304t^4 + 1, \left(t \mapsto t \log\left(18432t^5 - 4t + x\right)\right)\right) + \frac{-7x^4 - 3}{21x^7}$$

input
```
integrate(1/x**8/(x**8-x**4+1),x)
```

output
```
RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(18432*_t**5 - 4*
_t + x))) + (-7*x**4 - 3)/(21*x**7)
```

3.366.7 Maxima [F]

$$\int \frac{1}{x^8 (1 - x^4 + x^8)} \, dx = \int \frac{1}{(x^8 - x^4 + 1)x^8} \, dx$$

input
```
integrate(1/x^8/(x^8-x^4+1),x, algorithm="maxima")
```

output
```
-1/21*(7*x^4 + 3)/x^7 - integrate(x^4/(x^8 - x^4 + 1), x)
```

3.366.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.70

$$\int \frac{1}{x^8(1-x^4+x^8)}\,dx = -\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right) - \frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right) - \frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) - \frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) - \frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right) + \frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right) - \frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right) + \frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right) - \frac{7x^4+3}{21x^7}$$

input
```
integrate(1/x^8/(x^8-x^4+1),x, algorithm="giac")
```

output
```
-1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sq
rt(2))) - 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqr
t(6) + sqrt(2))) - 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt
(2))/(sqrt(6) - sqrt(2))) - 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(
6) - sqrt(2))/(sqrt(6) - sqrt(2))) - 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 +
1/2*x*(sqrt(6) + sqrt(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*
x*(sqrt(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 + 1/2*x*(s
qrt(6) - sqrt(2)) + 1) + 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(
6) - sqrt(2)) + 1) - 1/21*(7*x^4 + 3)/x^7
```

3.366.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 486, normalized size of antiderivative = 1.29

$$\int \frac{1}{x^8(1-x^4+x^8)}\,dx$$

$$= -\frac{\frac{x^4}{3}+\frac{1}{7}}{x^7} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}{2\left(\frac{\sqrt{3}\,\sqrt{8-\sqrt{3}\,8\mathrm{i}}\,1\mathrm{i}}{4}+\frac{\sqrt{8-\sqrt{3}\,8\mathrm{i}}}{4}\right)}+\frac{\sqrt{3}\,x\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}\,1\mathrm{i}}{2\left(\frac{\sqrt{3}\,\sqrt{8-\sqrt{3}\,8\mathrm{i}}\,1\mathrm{i}}{4}+\frac{\sqrt{8-\sqrt{3}\,8\mathrm{i}}}{4}\right)}\right)\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}\,1\mathrm{i}}{12}$$

$$+ \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}\,1\mathrm{i}}{2\left(\frac{\sqrt{3}\,\sqrt{8-\sqrt{3}\,8\mathrm{i}}\,1\mathrm{i}}{4}+\frac{\sqrt{8-\sqrt{3}\,8\mathrm{i}}}{4}\right)}-\frac{\sqrt{3}\,x\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}{2\left(\frac{\sqrt{3}\,\sqrt{8-\sqrt{3}\,8\mathrm{i}}\,1\mathrm{i}}{4}+\frac{\sqrt{8-\sqrt{3}\,8\mathrm{i}}}{4}\right)}\right)\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}{12}$$

$$- \frac{2^{3/4}\,\sqrt{3}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}{2\left(\frac{\sqrt{2}\,\sqrt{1+\sqrt{3}\,1\mathrm{i}}}{2}-\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{1+\sqrt{3}\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}-\frac{2^{3/4}\,\sqrt{3}\,x\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}\,1\mathrm{i}}{2\left(\frac{\sqrt{2}\,\sqrt{1+\sqrt{3}\,1\mathrm{i}}}{2}-\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{1+\sqrt{3}\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}\,1\mathrm{i}}{12}$$

$$- \frac{2^{3/4}\,\sqrt{3}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}\,1\mathrm{i}}{2\left(\frac{\sqrt{2}\,\sqrt{1+\sqrt{3}\,1\mathrm{i}}}{2}-\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{1+\sqrt{3}\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}+\frac{2^{3/4}\,\sqrt{3}\,x\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}{2\left(\frac{\sqrt{2}\,\sqrt{1+\sqrt{3}\,1\mathrm{i}}}{2}-\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{1+\sqrt{3}\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}{12}$$

input
```
int(1/(x^8*(x^8 - x^4 + 1)),x)
```

output
```
(3^(1/2)*atan((x*(8 - 3^(1/2)*8i)^(1/4))/(2*((3^(1/2)*(8 - 3^(1/2)*8i)^(1/
2)*1i)/4 + (8 - 3^(1/2)*8i)^(1/2)/4)) + (3^(1/2)*x*(8 - 3^(1/2)*8i)^(1/4)*
1i)/(2*((3^(1/2)*(8 - 3^(1/2)*8i)^(1/2)*1i)/4 + (8 - 3^(1/2)*8i)^(1/2)/4))
)*(8 - 3^(1/2)*8i)^(1/4)*1i)/12 - (x^4/3 + 1/7)/x^7 + (3^(1/2)*atan((x*(8
- 3^(1/2)*8i)^(1/4)*1i)/(2*((3^(1/2)*(8 - 3^(1/2)*8i)^(1/2)*1i)/4 + (8 - 3
^(1/2)*8i)^(1/2)/4)) - (3^(1/2)*x*(8 - 3^(1/2)*8i)^(1/4))/(2*((3^(1/2)*(8
- 3^(1/2)*8i)^(1/2)*1i)/4 + (8 - 3^(1/2)*8i)^(1/2)/4)))*(8 - 3^(1/2)*8i)^(
1/4))/12 - (2^(3/4)*3^(1/2)*atan((2^(3/4)*x*(3^(1/2)*1i + 1)^(1/4))/(2*((2
^(1/2)*(3^(1/2)*1i + 1)^(1/2))/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i + 1)^(1/2)
*1i)/2)) - (2^(3/4)*3^(1/2)*x*(3^(1/2)*1i + 1)^(1/4)*1i)/(2*((2^(1/2)*(3^(
1/2)*1i + 1)^(1/2))/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i + 1)^(1/2)*1i)/2)))*(
3^(1/2)*1i + 1)^(1/4)*1i)/12 - (2^(3/4)*3^(1/2)*atan((2^(3/4)*x*(3^(1/2)*1
i + 1)^(1/4)*1i)/(2*((2^(1/2)*(3^(1/2)*1i + 1)^(1/2))/2 - (2^(1/2)*3^(1/2)
*(3^(1/2)*1i + 1)^(1/2)*1i)/2)) + (2^(3/4)*3^(1/2)*x*(3^(1/2)*1i + 1)^(1/4
))/(2*((2^(1/2)*(3^(1/2)*1i + 1)^(1/2))/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i +
 1)^(1/2)*1i)/2)))*(3^(1/2)*1i + 1)^(1/4))/12
```

3.367 $\int \frac{x^m}{1+3x^4+x^8} dx$

3.367.1 Optimal result

Integrand size = 16, antiderivative size = 117

$$\int \frac{x^m}{1+3x^4+x^8} dx = \frac{2x^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{4},\frac{5+m}{4},-\frac{2x^4}{3-\sqrt{5}}\right)}{\sqrt{5}\left(3-\sqrt{5}\right)(1+m)} - \frac{2x^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{4},\frac{5+m}{4},-\frac{2x^4}{3+\sqrt{5}}\right)}{\sqrt{5}\left(3+\sqrt{5}\right)(1+m)}$$

output
```
2/5*x^(1+m)*hypergeom([1, 1/4+1/4*m],[5/4+1/4*m],-2*x^4/(3-5^(1/2)))/(1+m)
/(3-5^(1/2))*5^(1/2)-2/5*x^(1+m)*hypergeom([1, 1/4+1/4*m],[5/4+1/4*m],-2*x
^4/(3+5^(1/2)))/(1+m)*5^(1/2)/(3+5^(1/2))
```

3.367.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.68

$$\int \frac{x^m}{1+3x^4+x^8} dx = \frac{x^m \text{RootSum}\left[1+3\#1^4+\#1^8\&,\frac{\text{Hypergeometric2F1}\left(-m,-m,1-m,-\frac{\#1}{x-\#1}\right)\left(\frac{x}{x-\#1}\right)^{-m}}{3\#1^3+2\#1^7}\&\right]}{4m}$$

input
```
Integrate[x^m/(1 + 3*x^4 + x^8),x]
```

output
```
(x^m*RootSum[1 + 3*#1^4 + #1^8 & , Hypergeometric2F1[-m, -m, 1 - m, -(#1/(
x - #1))]/((x/(x - #1))^m*(3*#1^3 + 2*#1^7)) & ])/(4*m)
```

3.367.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1711, 27, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{x^8+3x^4+1}\,dx$$

$$\Big\downarrow \text{1711}$$

$$\frac{\int \frac{2x^m}{2x^4-\sqrt{5}+3}dx}{\sqrt{5}}-\frac{\int \frac{2x^m}{2x^4+\sqrt{5}+3}dx}{\sqrt{5}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\int \frac{x^m}{2x^4-\sqrt{5}+3}dx}{\sqrt{5}}-\frac{2\int \frac{x^m}{2x^4+\sqrt{5}+3}dx}{\sqrt{5}}$$

$$\Big\downarrow \text{888}$$

$$\frac{2x^{m+1}\text{Hypergeometric2F1}\left(1,\frac{m+1}{4},\frac{m+5}{4},-\frac{2x^4}{3-\sqrt{5}}\right)}{\sqrt{5}\left(3-\sqrt{5}\right)(m+1)}-\frac{2x^{m+1}\text{Hypergeometric2F1}\left(1,\frac{m+1}{4},\frac{m+5}{4},-\frac{2x^4}{3+\sqrt{5}}\right)}{\sqrt{5}\left(3+\sqrt{5}\right)(m+1)}$$

input
```
Int[x^m/(1 + 3*x^4 + x^8),x]
```

output
```
(2*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, (-2*x^4)/(3 - Sqrt
[5])])/(Sqrt[5]*(3 - Sqrt[5])*(1 + m)) - (2*x^(1 + m)*Hypergeometric2F1[1,
 (1 + m)/4, (5 + m)/4, (-2*x^4)/(3 + Sqrt[5])])/(Sqrt[5]*(3 + Sqrt[5])*(1
+ m))
```

3.367.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1711
```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[c/q   Int[(d*x)^m/(b/2 - q/2 + c
*x^n), x], x] - Simp[c/q   Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; Free
Q[{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

3.367.4 Maple [F]

$$\int \frac{x^m}{x^8+3x^4+1}dx$$

input
```
int(x^m/(x^8+3*x^4+1),x)
```

output
```
int(x^m/(x^8+3*x^4+1),x)
```

3.367.5 Fricas [F]

$$\int \frac{x^m}{1+3x^4+x^8}\,dx=\int \frac{x^m}{x^8+3\,x^4+1}\,dx$$

input
```
integrate(x^m/(x^8+3*x^4+1),x, algorithm="fricas")
```

output
```
integral(x^m/(x^8 + 3*x^4 + 1), x)
```

3.367.6 Sympy [F]

$$\int \frac{x^m}{1+3x^4+x^8}\,dx = \int \frac{x^m}{x^8+3x^4+1}\,dx$$

input
```
integrate(x**m/(x**8+3*x**4+1),x)
```

output
```
Integral(x**m/(x**8 + 3*x**4 + 1), x)
```

3.367.7 Maxima [F]

$$\int \frac{x^m}{1+3x^4+x^8}\,dx = \int \frac{x^m}{x^8+3\,x^4+1}\,dx$$

input
```
integrate(x^m/(x^8+3*x^4+1),x, algorithm="maxima")
```

output
```
integrate(x^m/(x^8 + 3*x^4 + 1), x)
```

3.367.8 Giac [F]

$$\int \frac{x^m}{1+3x^4+x^8}\,dx = \int \frac{x^m}{x^8+3\,x^4+1}\,dx$$

input
```
integrate(x^m/(x^8+3*x^4+1),x, algorithm="giac")
```

output
```
integrate(x^m/(x^8 + 3*x^4 + 1), x)
```

3.367.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{1+3x^4+x^8}\,dx = \int \frac{x^m}{x^8+3\,x^4+1}\,dx$$

input
```
int(x^m/(3*x^4 + x^8 + 1),x)
```

output
```
int(x^m/(3*x^4 + x^8 + 1), x)
```

3.368 $\int \frac{x^{11}}{1+3x^4+x^8} dx$

3.368.1 Optimal result

Integrand size = 16, antiderivative size = 62

$$\int \frac{x^{11}}{1+3x^4+x^8} dx = \frac{x^4}{4} - \frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(3-\sqrt{5}+2x^4\right) - \frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(3+\sqrt{5}+2x^4\right)$$

output
```
1/4*x^4-1/40*ln(2*x^4-5^(1/2)+3)*(15-7*5^(1/2))-1/40*ln(2*x^4+5^(1/2)+3)*(
15+7*5^(1/2))
```

3.368.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.92

$$\int \frac{x^{11}}{1+3x^4+x^8} dx = \frac{1}{40}\left(10x^4+\left(-15+7\sqrt{5}\right)\log\left(-3+\sqrt{5}-2x^4\right) - \left(15+7\sqrt{5}\right)\log\left(3+\sqrt{5}+2x^4\right)\right)$$

input
```
Integrate[x^11/(1 + 3*x^4 + x^8),x]
```

output
```
(10*x^4 + (-15 + 7*Sqrt[5])*Log[-3 + Sqrt[5] - 2*x^4] - (15 + 7*Sqrt[5])*L
og[3 + Sqrt[5] + 2*x^4])/40
```

3.368.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1693, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{x^8+3x^4+1}\,dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{4}\int \frac{x^8}{x^8+3x^4+1}dx^4$$

$$\Big\downarrow \text{1141}$$

$$\frac{1}{4}\int \left(-\frac{15+7\sqrt{5}}{5\left(2x^4+\sqrt{5}+3\right)}+1-\frac{15-7\sqrt{5}}{5\left(2x^4-\sqrt{5}+3\right)}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(x^4-\frac{1}{10}\left(15-7\sqrt{5}\right)\log\left(2x^4-\sqrt{5}+3\right)-\frac{1}{10}\left(15+7\sqrt{5}\right)\log\left(2x^4+\sqrt{5}+3\right)\right)$$

input
```
Int[x^11/(1 + 3*x^4 + x^8),x]
```

output
```
(x^4 - ((15 - 7*Sqrt[5])*Log[3 - Sqrt[5] + 2*x^4])/10 - ((15 + 7*Sqrt[5])*
Log[3 + Sqrt[5] + 2*x^4])/10)/4
```

3.368.3.1 Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.368.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.61

method	result	size
default	$\frac{x^4}{4}-\frac{3\ln\left(x^8+3x^4+1\right)}{8}-\frac{7\,\text{arctanh}\left(\frac{\left(2x^4+3\right)\sqrt{5}}{5}\right)\sqrt{5}}{20}$	38
risch	$\frac{x^4}{4}-\frac{3\ln\left(2x^4-\sqrt{5}+3\right)}{8}+\frac{7\ln\left(2x^4-\sqrt{5}+3\right)\sqrt{5}}{40}-\frac{3\ln\left(2x^4+\sqrt{5}+3\right)}{8}-\frac{7\ln\left(2x^4+\sqrt{5}+3\right)\sqrt{5}}{40}$	69

input
```
int(x^11/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4-3/8*ln(x^8+3*x^4+1)-7/20*arctanh(1/5*(2*x^4+3)*5^(1/2))*5^(1/2)
```

3.368.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00

$$\int \frac{x^{11}}{1+3x^4+x^8}\,dx = \frac{1}{4}\,x^4+\frac{7}{40}\,\sqrt{5}\log\left(\frac{2\,x^8+6\,x^4-\sqrt{5}(2\,x^4+3)+7}{x^8+3\,x^4+1}\right) - \frac{3}{8}\,\log\left(x^8+3\,x^4+1\right)$$

input
```
integrate(x^11/(x^8+3*x^4+1),x, algorithm="fricas")
```

output
```
1/4*x^4 + 7/40*sqrt(5)*log((2*x^8 + 6*x^4 - sqrt(5)*(2*x^4 + 3) + 7)/(x^8
+ 3*x^4 + 1)) - 3/8*log(x^8 + 3*x^4 + 1)
```

3.368.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.97

$$\int \frac{x^{11}}{1+3x^4+x^8}\,dx = \frac{x^4}{4} + \left(-\frac{3}{8} + \frac{7\sqrt{5}}{40}\right)\log\left(x^4 - \frac{\sqrt{5}}{2} + \frac{3}{2}\right) + \left(-\frac{7\sqrt{5}}{40} - \frac{3}{8}\right)\log\left(x^4 + \frac{\sqrt{5}}{2} + \frac{3}{2}\right)$$

input
```
integrate(x**11/(x**8+3*x**4+1),x)
```

output
```
x**4/4 + (-3/8 + 7*sqrt(5)/40)*log(x**4 - sqrt(5)/2 + 3/2) + (-7*sqrt(5)/4
0 - 3/8)*log(x**4 + sqrt(5)/2 + 3/2)
```

3.368.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.81

$$\int \frac{x^{11}}{1+3x^4+x^8}\,dx = \frac{1}{4}\,x^4 + \frac{7}{40}\,\sqrt{5}\log\left(\frac{2\,x^4 - \sqrt{5} + 3}{2\,x^4 + \sqrt{5} + 3}\right) - \frac{3}{8}\,\log\left(x^8 + 3\,x^4 + 1\right)$$

input
```
integrate(x^11/(x^8+3*x^4+1),x, algorithm="maxima")
```

output
```
1/4*x^4 + 7/40*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3)) -
3/8*log(x^8 + 3*x^4 + 1)
```

3.368.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.81

$$\int \frac{x^{11}}{1+3x^4+x^8}\,dx = \frac{1}{4}\,x^4 + \frac{7}{40}\,\sqrt{5}\log\left(\frac{2\,x^4 - \sqrt{5} + 3}{2\,x^4 + \sqrt{5} + 3}\right) - \frac{3}{8}\,\log\left(x^8 + 3\,x^4 + 1\right)$$

input
```
integrate(x^11/(x^8+3*x^4+1),x, algorithm="giac")
```

output
```
1/4*x^4 + 7/40*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3)) -
3/8*log(x^8 + 3*x^4 + 1)
```

3.368.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.03

$$\int \frac{x^{11}}{1+3x^4+x^8}\,dx = \frac{7\sqrt{5}\,\ln\left(x^4-\frac{\sqrt{5}}{2}+\frac{3}{2}\right)}{40}-\frac{3\,\ln\left(x^4+\frac{\sqrt{5}}{2}+\frac{3}{2}\right)}{8}-\frac{3\,\ln\left(x^4-\frac{\sqrt{5}}{2}+\frac{3}{2}\right)}{8}-\frac{7\sqrt{5}\,\ln\left(x^4+\frac{\sqrt{5}}{2}+\frac{3}{2}\right)}{40}+\frac{x^4}{4}$$

input

```
int(x^11/(3*x^4 + x^8 + 1),x)
```

output

```
(7*5^(1/2)*log(x^4 - 5^(1/2)/2 + 3/2))/40 - (3*log(5^(1/2)/2 + x^4 + 3/2))
/8 - (3*log(x^4 - 5^(1/2)/2 + 3/2))/8 - (7*5^(1/2)*log(5^(1/2)/2 + x^4 + 3
/2))/40 + x^4/4
```

3.369 $\int \frac{x^9}{1+3x^4+x^8} dx$

3.369.1 Optimal result

Integrand size = 16, antiderivative size = 90

$$\int \frac{x^9}{1+3x^4+x^8} dx = \frac{x^2}{2} - \frac{1}{2}\sqrt{\frac{1}{5}\left(9+4\sqrt{5}\right)}\arctan\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right) + \frac{1}{2}\sqrt{\frac{1}{5}\left(9-4\sqrt{5}\right)}\arctan\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)$$

output
```
1/2*x^2+1/2*arctan(x^2*(1/2+1/2*5^(1/2)))*(1-2/5*5^(1/2))-1/2*arctan(x^2*2
^(1/2)/(3+5^(1/2))^(1/2))*(1+2/5*5^(1/2))
```

3.369.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.08

$$\int \frac{x^9}{1+3x^4+x^8} dx = \frac{1}{40}\left(20x^2 - \sqrt{6-2\sqrt{5}}\left(15+7\sqrt{5}\right)\arctan\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right) + \sqrt{2\left(3+\sqrt{5}\right)}\left(-15+7\sqrt{5}\right)\arctan\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)\right)$$

input
```
Integrate[x^9/(1 + 3*x^4 + x^8),x]
```

output
```
(20*x^2 - Sqrt[6 - 2*Sqrt[5]]*(15 + 7*Sqrt[5])*ArcTan[Sqrt[2/(3 + Sqrt[5])
]*x^2] + Sqrt[2*(3 + Sqrt[5])]*(-15 + 7*Sqrt[5])*ArcTan[Sqrt[(3 + Sqrt[5])
/2]*x^2])/40
```

3.369.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1695, 1442, 1480, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{x^8+3x^4+1}dx$$

$$\Big\downarrow \text{1695}$$

$$\frac{1}{2}\int \frac{x^8}{x^8+3x^4+1}dx^2$$

$$\Big\downarrow \text{1442}$$

$$\frac{1}{2}\left(x^2-\int \frac{3x^4+1}{x^8+3x^4+1}dx^2\right)$$

$$\Big\downarrow \text{1480}$$

$$\frac{1}{2}\left(-\frac{1}{10}\left(15-7\sqrt{5}\right)\int \frac{1}{x^4+\frac{1}{2}\left(3-\sqrt{5}\right)}dx^2-\frac{1}{10}\left(15+7\sqrt{5}\right)\int \frac{1}{x^4+\frac{1}{2}\left(3+\sqrt{5}\right)}dx^2+x^2\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\left(-\frac{\left(15+7\sqrt{5}\right)\arctan\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)}{5\sqrt{2\left(3+\sqrt{5}\right)}}-\frac{1}{10}\left(15-7\sqrt{5}\right)\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\arctan\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)+x^2\right)$$

input
```
Int[x^9/(1 + 3*x^4 + x^8),x]
```

output
```
(x^2 - ((15 + 7*Sqrt[5])*ArcTan[Sqrt[2/(3 + Sqrt[5])]*x^2])/(5*Sqrt[2*(3 +
 Sqrt[5])]) - ((15 - 7*Sqrt[5])*Sqrt[(3 + Sqrt[5])/2]*ArcTan[Sqrt[(3 + Sqr
t[5])/2]*x^2])/10)/2
```

3.369.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1442
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
x] - Simp[d^4/(c*(m + 4*p + 1))   Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1695
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b
*x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c,
p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.369.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.47

method	result	size
risch	$\frac{x^2}{2}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(25\_Z^4+90\_Z^2+1\right)}\_R\ln\left(15\_R^3+8x^2+47\_R\right)\right)}{4}$	42
default	$\frac{x^2}{2}-\frac{\left(7+3\sqrt{5}\right)\sqrt{5}\,\arctan\left(\frac{4x^2}{2\sqrt{5}+2}\right)}{5\left(2\sqrt{5}+2\right)}-\frac{\left(-7+3\sqrt{5}\right)\sqrt{5}\,\arctan\left(\frac{4x^2}{2\sqrt{5}-2}\right)}{5\left(2\sqrt{5}-2\right)}$	79

3.369. $\int \frac{x^9}{1+3x^4+x^8}\,dx$

input
```
int(x^9/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2+1/4*sum(_R*ln(15*_R^3+8*x^2+47*_R),_R=RootOf(25*_Z^4+90*_Z^2+1))
```

3.369.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 152 vs. 2(50) = 100.

Time = 0.27 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.69

$$\begin{aligned}\int \frac{x^9}{1+3x^4+x^8}\,dx &= \frac{1}{2}\,x^2+\frac{1}{20}\,\sqrt{5}\sqrt{4\,\sqrt{5}-9}\log\left(2\,x^2+\sqrt{4\,\sqrt{5}-9}\left(\sqrt{5}+3\right)\right)\\&\quad-\frac{1}{20}\,\sqrt{5}\sqrt{4\,\sqrt{5}-9}\log\left(2\,x^2-\sqrt{4\,\sqrt{5}-9}\left(\sqrt{5}+3\right)\right)\\&\quad+\frac{1}{20}\,\sqrt{5}\sqrt{-4\,\sqrt{5}-9}\log\left(2\,x^2+\left(\sqrt{5}-3\right)\sqrt{-4\,\sqrt{5}-9}\right)\\&\quad-\frac{1}{20}\,\sqrt{5}\sqrt{-4\,\sqrt{5}-9}\log\left(2\,x^2-\left(\sqrt{5}-3\right)\sqrt{-4\,\sqrt{5}-9}\right)\end{aligned}$$

input
```
integrate(x^9/(x^8+3*x^4+1),x, algorithm="fricas")
```

output
```
1/2*x^2 + 1/20*sqrt(5)*sqrt(4*sqrt(5) - 9)*log(2*x^2 + sqrt(4*sqrt(5) - 9)
*(sqrt(5) + 3)) - 1/20*sqrt(5)*sqrt(4*sqrt(5) - 9)*log(2*x^2 - sqrt(4*sqrt
(5) - 9)*(sqrt(5) + 3)) + 1/20*sqrt(5)*sqrt(-4*sqrt(5) - 9)*log(2*x^2 + (s
qrt(5) - 3)*sqrt(-4*sqrt(5) - 9)) - 1/20*sqrt(5)*sqrt(-4*sqrt(5) - 9)*log(
2*x^2 - (sqrt(5) - 3)*sqrt(-4*sqrt(5) - 9))
```

3.369.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.60

$$\int \frac{x^9}{1+3x^4+x^8}\,dx = \frac{x^2}{2}+2\cdot\left(\frac{1}{4}-\frac{\sqrt{5}}{10}\right)\operatorname{atan}\left(\frac{2x^2}{-1+\sqrt{5}}\right)-2\left(\frac{\sqrt{5}}{10}+\frac{1}{4}\right)\operatorname{atan}\left(\frac{2x^2}{1+\sqrt{5}}\right)$$

input
```
integrate(x**9/(x**8+3*x**4+1),x)
```

output
```
x**2/2 + 2*(1/4 - sqrt(5)/10)*atan(2*x**2/(-1 + sqrt(5))) - 2*(sqrt(5)/10
+ 1/4)*atan(2*x**2/(1 + sqrt(5)))
```

3.369.7 Maxima [F]

$$\int \frac{x^9}{1+3x^4+x^8}\,dx = \int \frac{x^9}{x^8+3\,x^4+1}\,dx$$

input
```
integrate(x^9/(x^8+3*x^4+1),x, algorithm="maxima")
```

output
```
1/2*x^2 - integrate((3*x^4 + 1)*x/(x^8 + 3*x^4 + 1), x)
```

3.369.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.73

$$\int \frac{x^9}{1+3x^4+x^8}\,dx = \frac{1}{2}\,x^2 - \frac{1}{20}\left(3\,x^4\left(\sqrt{5}-5\right)+\sqrt{5}-5\right)\arctan\left(\frac{2\,x^2}{\sqrt{5}+1}\right) - \frac{1}{20}\left(3\,x^4\left(\sqrt{5}+5\right)+\sqrt{5}+5\right)\arctan\left(\frac{2\,x^2}{\sqrt{5}-1}\right)$$

input
```
integrate(x^9/(x^8+3*x^4+1),x, algorithm="giac")
```

output
```
1/2*x^2 - 1/20*(3*x^4*(sqrt(5) - 5) + sqrt(5) - 5)*arctan(2*x^2/(sqrt(5) +
 1)) - 1/20*(3*x^4*(sqrt(5) + 5) + sqrt(5) + 5)*arctan(2*x^2/(sqrt(5) - 1)
)
```

3.369.9 Mupad [B] (verification not implemented)

Time = 8.39 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.44

$$\int \frac{x^9}{1+3x^4+x^8}\,dx = 2\,\mathrm{atanh}\left(\frac{1280\,x^2\,\sqrt{\frac{\sqrt{5}}{20}-\frac{9}{80}}}{64\,\sqrt{5}-192}+\frac{768\,\sqrt{5}\,x^2\,\sqrt{\frac{\sqrt{5}}{20}-\frac{9}{80}}}{64\,\sqrt{5}-192}\right)\sqrt{\frac{\sqrt{5}}{20}-\frac{9}{80}} - 2\,\mathrm{atanh}\left(\frac{1280\,x^2\,\sqrt{-\frac{\sqrt{5}}{20}-\frac{9}{80}}}{64\,\sqrt{5}+192} - \frac{768\,\sqrt{5}\,x^2\,\sqrt{-\frac{\sqrt{5}}{20}-\frac{9}{80}}}{64\,\sqrt{5}+192}\right)\sqrt{-\frac{\sqrt{5}}{20}-\frac{9}{80}}+\frac{x^2}{2}$$

input
```
int(x^9/(3*x^4 + x^8 + 1),x)
```

output
```
2*atanh((1280*x^2*(5^(1/2)/20 - 9/80)^(1/2))/(64*5^(1/2) - 192) + (768*5^(
1/2)*x^2*(5^(1/2)/20 - 9/80)^(1/2))/(64*5^(1/2) - 192))*(5^(1/2)/20 - 9/80
)^(1/2) - 2*atanh((1280*x^2*(- 5^(1/2)/20 - 9/80)^(1/2))/(64*5^(1/2) + 192
) - (768*5^(1/2)*x^2*(- 5^(1/2)/20 - 9/80)^(1/2))/(64*5^(1/2) + 192))*(- 5
^(1/2)/20 - 9/80)^(1/2) + x^2/2
```

3.369. $\int \frac{x^9}{1+3x^4+x^8}\,dx$

3.370 $\int \frac{x^7}{1+3x^4+x^8} dx$

3.370.1 Optimal result

Integrand size = 16, antiderivative size = 55

$$\int \frac{x^7}{1+3x^4+x^8} dx = \frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(3-\sqrt{5}+2x^4\right)+\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(3+\sqrt{5}+2x^4\right)$$

output
```
1/40*ln(2*x^4-5^(1/2)+3)*(5-3*5^(1/2))+1/40*ln(2*x^4+5^(1/2)+3)*(5+3*5^(1/
2))
```

3.370.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int \frac{x^7}{1+3x^4+x^8} dx = \frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(-3+\sqrt{5}-2x^4\right)+\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(3+\sqrt{5}+2x^4\right)$$

input
```
Integrate[x^7/(1 + 3*x^4 + x^8),x]
```

output
```
((5 - 3*Sqrt[5])*Log[-3 + Sqrt[5] - 2*x^4])/40 + ((5 + 3*Sqrt[5])*Log[3 +
Sqrt[5] + 2*x^4])/40
```

3.370.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1693, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{x^8+3x^4+1}dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{4}\int \frac{x^4}{x^8+3x^4+1}dx^4$$

$$\Big\downarrow \text{1141}$$

$$\frac{1}{4}\int \left(\frac{5+3\sqrt{5}}{5\left(2x^4+\sqrt{5}+3\right)}+\frac{5-3\sqrt{5}}{5\left(2x^4-\sqrt{5}+3\right)}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\frac{1}{10}\left(5-3\sqrt{5}\right)\log\left(2x^4-\sqrt{5}+3\right)+\frac{1}{10}\left(5+3\sqrt{5}\right)\log\left(2x^4+\sqrt{5}+3\right)\right)$$

input
```
Int[x^7/(1 + 3*x^4 + x^8),x]
```

output
```
(((5 - 3*Sqrt[5])*Log[3 - Sqrt[5] + 2*x^4])/10 + ((5 + 3*Sqrt[5])*Log[3 +
Sqrt[5] + 2*x^4])/10)/4
```

3.370.3.1 Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.370.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.60

method	result	size
default	$\frac{\ln\left(x^{8}+3x^{4}+1\right)}{8}+\frac{3\,\operatorname{arctanh}\left(\frac{\left(2x^{4}+3\right)\sqrt{5}}{5}\right)\sqrt{5}}{20}$	33
risch	$\frac{\ln\left(2x^{4}+\sqrt{5}+3\right)}{8}+\frac{3\ln\left(2x^{4}+\sqrt{5}+3\right)\sqrt{5}}{40}+\frac{\ln\left(2x^{4}-\sqrt{5}+3\right)}{8}-\frac{3\ln\left(2x^{4}-\sqrt{5}+3\right)\sqrt{5}}{40}$	64

input
```
int(x^7/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/8*ln(x^8+3*x^4+1)+3/20*arctanh(1/5*(2*x^4+3)*5^(1/2))*5^(1/2)
```

3.370.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int \frac{x^7}{1+3x^4+x^8}\,dx = \frac{3}{40}\sqrt{5}\log\left(\frac{2\,x^{8}+6\,x^{4}+\sqrt{5}(2\,x^{4}+3)+7}{x^{8}+3\,x^{4}+1}\right)+\frac{1}{8}\log\left(x^{8}+3\,x^{4}+1\right)$$

input
```
integrate(x^7/(x^8+3*x^4+1),x, algorithm="fricas")
```

output
```
3/40*sqrt(5)*log((2*x^8 + 6*x^4 + sqrt(5)*(2*x^4 + 3) + 7)/(x^8 + 3*x^4 +
1)) + 1/8*log(x^8 + 3*x^4 + 1)
```

3.370.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int \frac{x^7}{1+3x^4+x^8}\,dx = \left(\frac{1}{8} - \frac{3\sqrt{5}}{40}\right)\log\left(x^4 - \frac{\sqrt{5}}{2} + \frac{3}{2}\right) + \left(\frac{1}{8} + \frac{3\sqrt{5}}{40}\right)\log\left(x^4 + \frac{\sqrt{5}}{2} + \frac{3}{2}\right)$$

input
```
integrate(x**7/(x**8+3*x**4+1),x)
```

output
```
(1/8 - 3*sqrt(5)/40)*log(x**4 - sqrt(5)/2 + 3/2) + (1/8 + 3*sqrt(5)/40)*lo
g(x**4 + sqrt(5)/2 + 3/2)
```

3.370.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.82

$$\int \frac{x^7}{1+3x^4+x^8}\,dx = -\frac{3}{40}\sqrt{5}\log\left(\frac{2\,x^4 - \sqrt{5} + 3}{2\,x^4 + \sqrt{5} + 3}\right) + \frac{1}{8}\log\left(x^8 + 3\,x^4 + 1\right)$$

input
```
integrate(x^7/(x^8+3*x^4+1),x, algorithm="maxima")
```

output
```
-3/40*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3)) + 1/8*log(x
^8 + 3*x^4 + 1)
```

3.370.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.82

$$\int \frac{x^7}{1+3x^4+x^8}\,dx = -\frac{3}{40}\sqrt{5}\log\left(\frac{2\,x^4 - \sqrt{5} + 3}{2\,x^4 + \sqrt{5} + 3}\right) + \frac{1}{8}\log\left(x^8 + 3\,x^4 + 1\right)$$

input
```
integrate(x^7/(x^8+3*x^4+1),x, algorithm="giac")
```

output
```
-3/40*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3)) + 1/8*log(x
^8 + 3*x^4 + 1)
```

3.370.9 Mupad [B] (verification not implemented)

Time = 8.42 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.07

$$\int \frac{x^7}{1+3x^4+x^8}\,dx = \frac{\ln\left(x^4-\frac{\sqrt{5}}{2}+\frac{3}{2}\right)}{8}+\frac{\ln\left(x^4+\frac{\sqrt{5}}{2}+\frac{3}{2}\right)}{8} - \frac{3\sqrt{5}\,\ln\left(x^4-\frac{\sqrt{5}}{2}+\frac{3}{2}\right)}{40}+\frac{3\sqrt{5}\,\ln\left(x^4+\frac{\sqrt{5}}{2}+\frac{3}{2}\right)}{40}$$

input
```
int(x^7/(3*x^4 + x^8 + 1),x)
```

output
```
log(x^4 - 5^(1/2)/2 + 3/2)/8 + log(5^(1/2)/2 + x^4 + 3/2)/8 - (3*5^(1/2)*l
og(x^4 - 5^(1/2)/2 + 3/2))/40 + (3*5^(1/2)*log(5^(1/2)/2 + x^4 + 3/2))/40
```

3.371 $\int \frac{x^5}{1+3x^4+x^8}\,dx$

3.371.1 Optimal result

Integrand size = 16, antiderivative size = 81

$$\int \frac{x^5}{1+3x^4+x^8}\,dx = \frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\arctan\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right) - \frac{1}{2}\sqrt{\frac{1}{10}\left(3-\sqrt{5}\right)}\arctan\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)$$

output
```
-1/2*arctan(x^2*(1/2+1/2*5^(1/2)))*(1/2-1/10*5^(1/2))+1/2*arctan(x^2*2^(1/
2)/(3+5^(1/2))^(1/2))*(1/2+1/10*5^(1/2))
```

3.371.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.93

$$\int \frac{x^5}{1+3x^4+x^8}\,dx = \frac{2\sqrt{5}\arctan\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right) + (5-3\sqrt{5})\arctan\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)}{10\sqrt{6-2\sqrt{5}}}$$

input
```
Integrate[x^5/(1 + 3*x^4 + x^8),x]
```

output
```
(2*Sqrt[5]*ArcTan[Sqrt[2/(3 + Sqrt[5])]*x^2] + (5 - 3*Sqrt[5])*ArcTan[Sqrt
[(3 + Sqrt[5])/2]*x^2])/(10*Sqrt[6 - 2*Sqrt[5]])
```

3.371.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.22, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1695, 1450, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{x^8+3x^4+1}dx$$

$$\Big\downarrow \text{1695}$$

$$\frac{1}{2}\int \frac{x^4}{x^8+3x^4+1}dx^2$$

$$\Big\downarrow \text{1450}$$

$$\frac{1}{2}\left(\frac{1}{10}\left(5-3\sqrt{5}\right)\int \frac{1}{x^4+\frac{1}{2}\left(3-\sqrt{5}\right)}dx^2+\frac{1}{10}\left(5+3\sqrt{5}\right)\int \frac{1}{x^4+\frac{1}{2}\left(3+\sqrt{5}\right)}dx^2\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\left(\frac{\left(5+3\sqrt{5}\right)\arctan\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)}{5\sqrt{2\left(3+\sqrt{5}\right)}}+\frac{1}{10}\left(5-3\sqrt{5}\right)\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\arctan\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)\right)$$

input
```
Int[x^5/(1 + 3*x^4 + x^8),x]
```

output
```
(((5 + 3*Sqrt[5])*ArcTan[Sqrt[2/(3 + Sqrt[5])]*x^2])/(5*Sqrt[2*(3 + Sqrt[5
])]) + ((5 - 3*Sqrt[5])*Sqrt[(3 + Sqrt[5])/2]*ArcTan[Sqrt[(3 + Sqrt[5])/2]
*x^2])/10)/2
```

3.371.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1450
```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^2/2)*(b/q + 1)   Int[(d*x)^(m - 2)/(b/
2 + q/2 + c*x^2), x], x] - Simp[(d^2/2)*(b/q - 1)   Int[(d*x)^(m - 2)/(b/2
- q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] &&
 GeQ[m, 2]
```

rule 1695
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b
*x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c,
p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.371.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.42

method	result	size
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(25\_Z^4+15\_Z^2+1\right)}\_R\ln\left(-10\_R^3+x^2-3\_R\right)\right)}{4}$	34
default	$\frac{\left(3+\sqrt{5}\right)\sqrt{5}\,\arctan\left(\frac{4x^2}{2\sqrt{5}+2}\right)}{10+10\sqrt{5}}+\frac{\left(\sqrt{5}-3\right)\sqrt{5}\,\arctan\left(\frac{4x^2}{2\sqrt{5}-2}\right)}{-10+10\sqrt{5}}$	70

input
```
int(x^5/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(_R*ln(-10*_R^3+x^2-3*_R),_R=RootOf(25*_Z^4+15*_Z^2+1))
```

3.371.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(41) = 82.

Time = 0.26 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.77

$$\int \frac{x^5}{1+3x^4+x^8}\,dx = -\frac{1}{40}\sqrt{10}\sqrt{\sqrt{5}-3}\log\left(10\,x^2+\sqrt{10}\sqrt{5}\sqrt{\sqrt{5}-3}\right) + \frac{1}{40}\sqrt{10}\sqrt{\sqrt{5}-3}\log\left(10\,x^2-\sqrt{10}\sqrt{5}\sqrt{\sqrt{5}-3}\right) + \frac{1}{40}\sqrt{10}\sqrt{-\sqrt{5}-3}\log\left(10\,x^2+\sqrt{10}\sqrt{5}\sqrt{-\sqrt{5}-3}\right) - \frac{1}{40}\sqrt{10}\sqrt{-\sqrt{5}-3}\log\left(10\,x^2-\sqrt{10}\sqrt{5}\sqrt{-\sqrt{5}-3}\right)$$

input
```
integrate(x^5/(x^8+3*x^4+1),x, algorithm="fricas")
```

output
```
-1/40*sqrt(10)*sqrt(sqrt(5) - 3)*log(10*x^2 + sqrt(10)*sqrt(5)*sqrt(sqrt(5
) - 3)) + 1/40*sqrt(10)*sqrt(sqrt(5) - 3)*log(10*x^2 - sqrt(10)*sqrt(5)*sq
rt(sqrt(5) - 3)) + 1/40*sqrt(10)*sqrt(-sqrt(5) - 3)*log(10*x^2 + sqrt(10)*
sqrt(5)*sqrt(-sqrt(5) - 3)) - 1/40*sqrt(10)*sqrt(-sqrt(5) - 3)*log(10*x^2
- sqrt(10)*sqrt(5)*sqrt(-sqrt(5) - 3))
```

3.371.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.60

$$\int \frac{x^5}{1+3x^4+x^8}\,dx = -2\cdot\left(\frac{1}{8}-\frac{\sqrt{5}}{40}\right)\operatorname{atan}\left(\frac{2x^2}{-1+\sqrt{5}}\right)+2\left(\frac{\sqrt{5}}{40}+\frac{1}{8}\right)\operatorname{atan}\left(\frac{2x^2}{1+\sqrt{5}}\right)$$

input
```
integrate(x**5/(x**8+3*x**4+1),x)
```

output
```
-2*(1/8 - sqrt(5)/40)*atan(2*x**2/(-1 + sqrt(5))) + 2*(sqrt(5)/40 + 1/8)*a
tan(2*x**2/(1 + sqrt(5)))
```

3.371.7 Maxima [F]

$$\int \frac{x^5}{1+3x^4+x^8}\,dx = \int \frac{x^5}{x^8+3\,x^4+1}\,dx$$

input
```
integrate(x^5/(x^8+3*x^4+1),x, algorithm="maxima")
```

output
```
integrate(x^5/(x^8 + 3*x^4 + 1), x)
```

3.371.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.58

$$\int \frac{x^5}{1+3x^4+x^8}\,dx = \frac{1}{20}\,x^4\left(\sqrt{5}-5\right)\arctan\left(\frac{2\,x^2}{\sqrt{5}+1}\right)+\frac{1}{20}\,x^4\left(\sqrt{5}+5\right)\arctan\left(\frac{2\,x^2}{\sqrt{5}-1}\right)$$

input
```
integrate(x^5/(x^8+3*x^4+1),x, algorithm="giac")
```

output
```
1/20*x^4*(sqrt(5) - 5)*arctan(2*x^2/(sqrt(5) + 1)) + 1/20*x^4*(sqrt(5) + 5
)*arctan(2*x^2/(sqrt(5) - 1))
```

3.371.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.44

$$\int \frac{x^5}{1+3x^4+x^8}\,dx = 2\,\mathrm{atanh}\left(\frac{60\,x^2\sqrt{\frac{\sqrt{5}}{160}-\frac{3}{160}}}{\sqrt{5}+3}+\frac{28\,\sqrt{5}\,x^2\sqrt{\frac{\sqrt{5}}{160}-\frac{3}{160}}}{\sqrt{5}+3}\right)\sqrt{\frac{\sqrt{5}}{160}-\frac{3}{160}} - 2\,\mathrm{atanh}\left(\frac{60\,x^2\sqrt{-\frac{\sqrt{5}}{160}-\frac{3}{160}}}{\sqrt{5}-3} - \frac{28\,\sqrt{5}\,x^2\sqrt{-\frac{\sqrt{5}}{160}-\frac{3}{160}}}{\sqrt{5}-3}\right)\sqrt{-\frac{\sqrt{5}}{160}-\frac{3}{160}}$$

input
```
int(x^5/(3*x^4 + x^8 + 1),x)
```

output
```
2*atanh((60*x^2*(5^(1/2)/160 - 3/160)^(1/2))/(5^(1/2) + 3) + (28*5^(1/2)*x
^2*(5^(1/2)/160 - 3/160)^(1/2))/(5^(1/2) + 3))*(5^(1/2)/160 - 3/160)^(1/2)
 - 2*atanh((60*x^2*(- 5^(1/2)/160 - 3/160)^(1/2))/(5^(1/2) - 3) - (28*5^(1
/2)*x^2*(- 5^(1/2)/160 - 3/160)^(1/2))/(5^(1/2) - 3))*(- 5^(1/2)/160 - 3/1
60)^(1/2)
```

3.371. $\int \frac{x^5}{1+3x^4+x^8}\,dx$

3.372 $\int \frac{x^3}{1+3x^4+x^8} dx$

3.372.1 Optimal result

Integrand size = 16, antiderivative size = 23

$$\int \frac{x^3}{1+3x^4+x^8} dx = -\frac{\operatorname{arctanh}\left(\frac{3+2x^4}{\sqrt{5}}\right)}{2\sqrt{5}}$$

output
```
-1/10*arctanh(1/5*(2*x^4+3)*5^(1/2))*5^(1/2)
```

3.372.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.65

$$\int \frac{x^3}{1+3x^4+x^8} dx = \frac{\log\left(-3+\sqrt{5}-2x^4\right)-\log\left(3+\sqrt{5}+2x^4\right)}{4\sqrt{5}}$$

input
```
Integrate[x^3/(1 + 3*x^4 + x^8),x]
```

output
```
(Log[-3 + Sqrt[5] - 2*x^4] - Log[3 + Sqrt[5] + 2*x^4])/(4*Sqrt[5])
```

3.372.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1690, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{x^8+3x^4+1}dx$$

$$\Big\downarrow \text{1690}$$

$$\frac{1}{4}\int \frac{1}{x^8+3x^4+1}dx^4$$

$$\Big\downarrow \text{1081}$$

$$\frac{1}{4}\int \left(\frac{2}{\sqrt{5}\left(2x^4-\sqrt{5}+3\right)}-\frac{2}{\sqrt{5}\left(2x^4+\sqrt{5}+3\right)}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\frac{\log\left(2x^4-\sqrt{5}+3\right)}{\sqrt{5}}-\frac{\log\left(2x^4+\sqrt{5}+3\right)}{\sqrt{5}}\right)$$

input
```
Int[x^3/(1 + 3*x^4 + x^8),x]
```

output
```
(Log[3 - Sqrt[5] + 2*x^4]/Sqrt[5] - Log[3 + Sqrt[5] + 2*x^4]/Sqrt[5])/4
```

3.372.3.1 Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.372.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{\operatorname{arctanh}\left(\frac{(2x^4+3)\sqrt{5}}{5}\right)\sqrt{5}}{10}$	19
risch	$\frac{\ln\left(2x^4-\sqrt{5}+3\right)\sqrt{5}}{20}-\frac{\ln\left(2x^4+\sqrt{5}+3\right)\sqrt{5}}{20}$	36

input
```
int(x^3/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/10*arctanh(1/5*(2*x^4+3)*5^(1/2))*5^(1/2)
```

3.372.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. 2(18) = 36.

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.87

$$\int \frac{x^3}{1+3x^4+x^8}\,dx = \frac{1}{20}\sqrt{5}\log\left(\frac{2x^8+6x^4-\sqrt{5}(2x^4+3)+7}{x^8+3x^4+1}\right)$$

input
```
integrate(x^3/(x^8+3*x^4+1),x, algorithm="fricas")
```

output
```
1/20*sqrt(5)*log((2*x^8 + 6*x^4 - sqrt(5)*(2*x^4 + 3) + 7)/(x^8 + 3*x^4 +
1))
```

3.372.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.83

$$\int \frac{x^3}{1+3x^4+x^8}\,dx = \frac{\sqrt{5}\log\left(x^4-\frac{\sqrt{5}}{2}+\frac{3}{2}\right)}{20} - \frac{\sqrt{5}\log\left(x^4+\frac{\sqrt{5}}{2}+\frac{3}{2}\right)}{20}$$

input
```
integrate(x**3/(x**8+3*x**4+1),x)
```

output
```
sqrt(5)*log(x**4 - sqrt(5)/2 + 3/2)/20 - sqrt(5)*log(x**4 + sqrt(5)/2 + 3/
2)/20
```

3.372.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.35

$$\int \frac{x^3}{1+3x^4+x^8}\,dx = \frac{1}{20}\sqrt{5}\log\left(\frac{2x^4-\sqrt{5}+3}{2x^4+\sqrt{5}+3}\right)$$

input
```
integrate(x^3/(x^8+3*x^4+1),x, algorithm="maxima")
```

output
```
1/20*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3))
```

3.372.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.35

$$\int \frac{x^3}{1+3x^4+x^8}\,dx = \frac{1}{20}\sqrt{5}\log\left(\frac{2x^4-\sqrt{5}+3}{2x^4+\sqrt{5}+3}\right)$$

input
```
integrate(x^3/(x^8+3*x^4+1),x, algorithm="giac")
```

output
```
1/20*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3))
```

3.372.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.30

$$\int \frac{x^3}{1+3x^4+x^8}\,dx = \frac{\sqrt{5}\,\mathrm{atanh}\left(\frac{8\sqrt{5}\,x^4+3\sqrt{5}}{18\,x^4+7}\right)}{10}$$

input
```
int(x^3/(3*x^4 + x^8 + 1),x)
```

output
```
(5^(1/2)*atanh((3*5^(1/2) + 8*5^(1/2)*x^4)/(18*x^4 + 7)))/10
```

3.373 $\int \frac{x}{1+3x^4+x^8} dx$

3.373.1 Optimal result

Integrand size = 14, antiderivative size = 75

$$\int \frac{x}{1+3x^4+x^8} \, dx = -\frac{\arctan\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)}{\sqrt{10\left(3+\sqrt{5}\right)}}+\frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\arctan\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)$$

output
```
1/2*arctan(x^2*(1/2+1/2*5^(1/2)))*(1/2+1/10*5^(1/2))-arctan(x^2*2^(1/2)/(3
+5^(1/2))^(1/2))/(5+5^(1/2))
```

3.373.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.99

$$\int \frac{x}{1+3x^4+x^8} \, dx = \frac{\arctan\left(\sqrt{\frac{2}{3-\sqrt{5}}}x^2\right)}{\sqrt{10\left(3-\sqrt{5}\right)}}-\frac{\arctan\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)}{\sqrt{10\left(3+\sqrt{5}\right)}}$$

input
```
Integrate[x/(1 + 3*x^4 + x^8),x]
```

output
```
ArcTan[Sqrt[2/(3 - Sqrt[5])]*x^2]/Sqrt[10*(3 - Sqrt[5])] - ArcTan[Sqrt[2/(
3 + Sqrt[5])]*x^2]/Sqrt[10*(3 + Sqrt[5])]
```

3.373.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1695, 1406, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{x^8+3x^4+1}\,dx$$

$$\Big\downarrow \text{1695}$$

$$\frac{1}{2}\int \frac{1}{x^8+3x^4+1}dx^2$$

$$\Big\downarrow \text{1406}$$

$$\frac{1}{2}\left(\frac{\int \frac{1}{x^4+\frac{1}{2}\left(3-\sqrt{5}\right)}dx^2}{\sqrt{5}}-\frac{\int \frac{1}{x^4+\frac{1}{2}\left(3+\sqrt{5}\right)}dx^2}{\sqrt{5}}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\left(\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\arctan\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)-\sqrt{\frac{2}{5\left(3+\sqrt{5}\right)}}\arctan\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)\right)$$

input
```
Int[x/(1 + 3*x^4 + x^8),x]
```

output
```
(-(Sqrt[2/(5*(3 + Sqrt[5]))]*ArcTan[Sqrt[2/(3 + Sqrt[5])]*x^2]) + Sqrt[(3
+ Sqrt[5])/10]*ArcTan[Sqrt[(3 + Sqrt[5])/2]*x^2])/2
```

3.373.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

rule 1695
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b
*x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c,
p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.373.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.45

method	result	size
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(25\_Z^4+15\_Z^2+1\right)}\_R\ln\left(-15\_R^3+x^2-7\_R\right)\right)}{4}$	34
default	$-\frac{2\sqrt{5}\,\arctan\left(\frac{4x^2}{2\sqrt{5}+2}\right)}{5\left(2\sqrt{5}+2\right)}+\frac{2\sqrt{5}\,\arctan\left(\frac{4x^2}{2\sqrt{5}-2}\right)}{5\left(2\sqrt{5}-2\right)}$	60

input
```
int(x/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(_R*ln(-15*_R^3+x^2-7*_R),_R=RootOf(25*_Z^4+15*_Z^2+1))
```

3.373.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(43) = 86$.

Time = 0.25 (sec) , antiderivative size = 159, normalized size of antiderivative = 2.12

$$\begin{aligned}\int \frac{x}{1+3x^4+x^8}\,dx = &-\frac{1}{40}\sqrt{10}\sqrt{\sqrt{5}-3}\log\left(20\,x^2+\sqrt{10}\left(3\sqrt{5}+5\right)\sqrt{\sqrt{5}-3}\right)\\ &+\frac{1}{40}\sqrt{10}\sqrt{\sqrt{5}-3}\log\left(20\,x^2-\sqrt{10}\left(3\sqrt{5}+5\right)\sqrt{\sqrt{5}-3}\right)\\ &+\frac{1}{40}\sqrt{10}\sqrt{-\sqrt{5}-3}\log\left(20\,x^2+\sqrt{10}\left(3\sqrt{5}-5\right)\sqrt{-\sqrt{5}-3}\right)\\ &-\frac{1}{40}\sqrt{10}\sqrt{-\sqrt{5}-3}\log\left(20\,x^2-\sqrt{10}\left(3\sqrt{5}-5\right)\sqrt{-\sqrt{5}-3}\right)\end{aligned}$$

input
```
integrate(x/(x^8+3*x^4+1),x, algorithm="fricas")
```

output
```
-1/40*sqrt(10)*sqrt(sqrt(5) - 3)*log(20*x^2 + sqrt(10)*(3*sqrt(5) + 5)*sqr
t(sqrt(5) - 3)) + 1/40*sqrt(10)*sqrt(sqrt(5) - 3)*log(20*x^2 - sqrt(10)*(3
*sqrt(5) + 5)*sqrt(sqrt(5) - 3)) + 1/40*sqrt(10)*sqrt(-sqrt(5) - 3)*log(20
*x^2 + sqrt(10)*(3*sqrt(5) - 5)*sqrt(-sqrt(5) - 3)) - 1/40*sqrt(10)*sqrt(-
sqrt(5) - 3)*log(20*x^2 - sqrt(10)*(3*sqrt(5) - 5)*sqrt(-sqrt(5) - 3))
```

3.373.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.65

$$\int \frac{x}{1+3x^4+x^8}\,dx = 2\left(\frac{\sqrt{5}}{40}+\frac{1}{8}\right)\operatorname{atan}\left(\frac{2x^2}{-1+\sqrt{5}}\right)-2\cdot\left(\frac{1}{8}-\frac{\sqrt{5}}{40}\right)\operatorname{atan}\left(\frac{2x^2}{1+\sqrt{5}}\right)$$

input
```
integrate(x/(x**8+3*x**4+1),x)
```

output
```
2*(sqrt(5)/40 + 1/8)*atan(2*x**2/(-1 + sqrt(5))) - 2*(1/8 - sqrt(5)/40)*at
an(2*x**2/(1 + sqrt(5)))
```

3.373.7 Maxima [F]

$$\int \frac{x}{1+3x^4+x^8}\,dx = \int \frac{x}{x^8+3\,x^4+1}\,dx$$

input
```
integrate(x/(x^8+3*x^4+1),x, algorithm="maxima")
```

output
```
integrate(x/(x^8 + 3*x^4 + 1), x)
```

3.373.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.55

$$\int \frac{x}{1+3x^4+x^8}\,dx = \frac{1}{20}\left(\sqrt{5}-5\right)\arctan\left(\frac{2\,x^2}{\sqrt{5}+1}\right) + \frac{1}{20}\left(\sqrt{5}+5\right)\arctan\left(\frac{2\,x^2}{\sqrt{5}-1}\right)$$

input
```
integrate(x/(x^8+3*x^4+1),x, algorithm="giac")
```

output
```
1/20*(sqrt(5) - 5)*arctan(2*x^2/(sqrt(5) + 1)) + 1/20*(sqrt(5) + 5)*arctan
(2*x^2/(sqrt(5) - 1))
```

3.373.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.67

$$\int \frac{x}{1+3x^4+x^8}\,dx = 2\,\mathrm{atanh}\left(\frac{160\,x^2\sqrt{\frac{\sqrt{5}}{160}-\frac{3}{160}}}{8\,\sqrt{5}-18} - \frac{72\,\sqrt{5}\,x^2\sqrt{\frac{\sqrt{5}}{160}-\frac{3}{160}}}{8\,\sqrt{5}-18}\right)\sqrt{\frac{\sqrt{5}}{160}-\frac{3}{160}} - 2\,\mathrm{atanh}\left(\frac{160\,x^2\sqrt{-\frac{\sqrt{5}}{160}-\frac{3}{160}}}{8\,\sqrt{5}+18} + \frac{72\,\sqrt{5}\,x^2\sqrt{-\frac{\sqrt{5}}{160}-\frac{3}{160}}}{8\,\sqrt{5}+18}\right)\sqrt{-\frac{\sqrt{5}}{160}-\frac{3}{160}}$$

input
```
int(x/(3*x^4 + x^8 + 1),x)
```

output
```
2*atanh((160*x^2*(5^(1/2)/160 - 3/160)^(1/2))/(8*5^(1/2) - 18) - (72*5^(1/
2)*x^2*(5^(1/2)/160 - 3/160)^(1/2))/(8*5^(1/2) - 18))*(5^(1/2)/160 - 3/160
)^(1/2) - 2*atanh((160*x^2*(- 5^(1/2)/160 - 3/160)^(1/2))/(8*5^(1/2) + 18)
 + (72*5^(1/2)*x^2*(- 5^(1/2)/160 - 3/160)^(1/2))/(8*5^(1/2) + 18))*(- 5^(
1/2)/160 - 3/160)^(1/2)
```

3.373. $\int \frac{x}{1+3x^4+x^8} \, dx$

3.374 $\int \frac{1}{x(1+3x^4+x^8)}\,dx$

3.374.1 Optimal result

Integrand size = 16, antiderivative size = 57

$$\int \frac{1}{x\left(1+3x^4+x^8\right)}\,dx = \log(x)-\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(3-\sqrt{5}+2x^4\right)-\frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(3+\sqrt{5}+2x^4\right)$$

output
```
ln(x)-1/40*ln(2*x^4+5^(1/2)+3)*(5-3*5^(1/2))-1/40*ln(2*x^4-5^(1/2)+3)*(5+3
*5^(1/2))
```

3.374.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96

$$\int \frac{1}{x\left(1+3x^4+x^8\right)}\,dx = \log(x)+\frac{1}{40}\left(-5-3\sqrt{5}\right)\log\left(-3+\sqrt{5}-2x^4\right)+\frac{1}{40}\left(-5+3\sqrt{5}\right)\log\left(3+\sqrt{5}+2x^4\right)$$

input
```
Integrate[1/(x*(1 + 3*x^4 + x^8)),x]
```

output
```
Log[x] + ((-5 - 3*Sqrt[5])*Log[-3 + Sqrt[5] - 2*x^4])/40 + ((-5 + 3*Sqrt[5
])*Log[3 + Sqrt[5] + 2*x^4])/40
```

3.374.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.30, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1693, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(x^8+3x^4+1\right)}dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{4}\int \frac{1}{x^4\left(x^8+3x^4+1\right)}dx^4$$

$$\Big\downarrow \text{1141}$$

$$\frac{1}{4}\int \left(\frac{2}{\left(5-3\sqrt{5}\right)x^4-7\sqrt{5}+15}+\frac{1}{x^4}+\frac{4}{\sqrt{5}\left(3+\sqrt{5}\right)\left(2x^4+\sqrt{5}+3\right)}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\log\left(x^4\right)+\frac{2\log\left(2x^4+\sqrt{5}+3\right)}{\sqrt{5}\left(3+\sqrt{5}\right)}+\frac{2\log\left(\left(5-3\sqrt{5}\right)x^4-7\sqrt{5}+15\right)}{5-3\sqrt{5}}\right)$$

input
```
Int[1/(x*(1 + 3*x^4 + x^8)),x]
```

output
```
(Log[x^4] + (2*Log[3 + Sqrt[5] + 2*x^4])/(Sqrt[5]*(3 + Sqrt[5])) + (2*Log[
15 - 7*Sqrt[5] + (5 - 3*Sqrt[5])*x^4])/(5 - 3*Sqrt[5]))/4
```

3.374.3.1 Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.374.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.61

method	result	size
default	$\ln(x)-\frac{\ln(x^8+3x^4+1)}{8}+\frac{3\,\text{arctanh}\left(\frac{(2x^4+3)\sqrt{5}}{5}\right)\sqrt{5}}{20}$	35
risch	$\ln(x)-\frac{\ln\left(3x^4+\frac{9}{2}+\frac{3\sqrt{5}}{2}\right)}{8}+\frac{3\ln\left(3x^4+\frac{9}{2}+\frac{3\sqrt{5}}{2}\right)\sqrt{5}}{40}-\frac{3\ln\left(3x^4-\frac{3\sqrt{5}}{2}+\frac{9}{2}\right)\sqrt{5}}{40}-\frac{\ln\left(3x^4-\frac{3\sqrt{5}}{2}+\frac{9}{2}\right)}{8}$	70

input
```
int(1/x/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
ln(x)-1/8*ln(x^8+3*x^4+1)+3/20*arctanh(1/5*(2*x^4+3)*5^(1/2))*5^(1/2)
```

3.374.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.02

$$\int \frac{1}{x(1+3x^4+x^8)}\,dx = \frac{3}{40}\sqrt{5}\log\left(\frac{2\,x^8+6\,x^4+\sqrt{5}(2\,x^4+3)+7}{x^8+3\,x^4+1}\right) - \frac{1}{8}\log\left(x^8+3\,x^4+1\right)+\log\left(x\right)$$

input
```
integrate(1/x/(x^8+3*x^4+1),x, algorithm="fricas")
```

output
```
3/40*sqrt(5)*log((2*x^8 + 6*x^4 + sqrt(5)*(2*x^4 + 3) + 7)/(x^8 + 3*x^4 +
1)) - 1/8*log(x^8 + 3*x^4 + 1) + log(x)
```

3.374.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.02

$$\int \frac{1}{x(1+3x^4+x^8)}\,dx = \log(x) + \left(-\frac{3\sqrt{5}}{40} - \frac{1}{8}\right)\log\left(x^4 - \frac{\sqrt{5}}{2} + \frac{3}{2}\right) + \left(-\frac{1}{8} + \frac{3\sqrt{5}}{40}\right)\log\left(x^4 + \frac{\sqrt{5}}{2} + \frac{3}{2}\right)$$

input
```
integrate(1/x/(x**8+3*x**4+1),x)
```

output
```
log(x) + (-3*sqrt(5)/40 - 1/8)*log(x**4 - sqrt(5)/2 + 3/2) + (-1/8 + 3*sqr
t(5)/40)*log(x**4 + sqrt(5)/2 + 3/2)
```

3.374.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int \frac{1}{x(1+3x^4+x^8)}\,dx = -\frac{3}{40}\sqrt{5}\log\left(\frac{2x^4-\sqrt{5}+3}{2x^4+\sqrt{5}+3}\right) - \frac{1}{8}\log\left(x^8+3x^4+1\right) + \frac{1}{4}\log\left(x^4\right)$$

input
```
integrate(1/x/(x^8+3*x^4+1),x, algorithm="maxima")
```

output
```
-3/40*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3)) - 1/8*log(x
^8 + 3*x^4 + 1) + 1/4*log(x^4)
```

3.374.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int \frac{1}{x(1+3x^4+x^8)}\,dx = -\frac{3}{40}\sqrt{5}\log\left(\frac{2x^4-\sqrt{5}+3}{2x^4+\sqrt{5}+3}\right) - \frac{1}{8}\log\left(x^8+3x^4+1\right) + \frac{1}{4}\log\left(x^4\right)$$

input
```
integrate(1/x/(x^8+3*x^4+1),x, algorithm="giac")
```

output
```
-3/40*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3)) - 1/8*log(x
^8 + 3*x^4 + 1) + 1/4*log(x^4)
```

3.374.9 Mupad [B] (verification not implemented)

Time = 8.37 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.74

$$\int \frac{1}{x(1+3x^4+x^8)}\,dx = \ln(x) - \ln\left(x^4 - \frac{\sqrt{5}}{2} + \frac{3}{2}\right)\left(\frac{3\sqrt{5}}{40} + \frac{1}{8}\right) + \ln\left(x^4 + \frac{\sqrt{5}}{2} + \frac{3}{2}\right)\left(\frac{3\sqrt{5}}{40} - \frac{1}{8}\right)$$

input
```
int(1/(x*(3*x^4 + x^8 + 1)),x)
```

output
```
log(x) - log(x^4 - 5^(1/2)/2 + 3/2)*((3*5^(1/2))/40 + 1/8) + log(5^(1/2)/2
 + x^4 + 3/2)*((3*5^(1/2))/40 - 1/8)
```

3.375 $\int \frac{1}{x^3(1+3x^4+x^8)}\,dx$

3.375.1 Optimal result

Integrand size = 16, antiderivative size = 89

$$\int \frac{1}{x^3(1+3x^4+x^8)}\,dx=-\frac{1}{2x^2}+\frac{1}{2}\sqrt{\frac{1}{5}\left(9-4\sqrt{5}\right)}\arctan\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)-\frac{\left(3+\sqrt{5}\right)^{3/2}\arctan\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)}{4\sqrt{10}}$$

output
```
-1/2/x^2-1/40*arctan(x^2*(1/2+1/2*5^(1/2)))*(3+5^(1/2))^(3/2)*10^(1/2)+1/2
*arctan(x^2*2^(1/2)/(3+5^(1/2))^(1/2))*(1-2/5*5^(1/2))
```

3.375.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.73

$$\int \frac{1}{x^3(1+3x^4+x^8)}\,dx=-\frac{1}{2x^2}-\frac{1}{4}\text{RootSum}\left[1+3\#1^4+\#1^8\&,\frac{3\log(x-\#1)+\log(x-\#1)\#1^4}{3\#1^2+2\#1^6}\&\right]$$

input
```
Integrate[1/(x^3*(1 + 3*x^4 + x^8)),x]
```

output
```
-1/2*1/x^2 - RootSum[1 + 3*#1^4 + #1^8 & , (3*Log[x - #1] + Log[x - #1]*#1
^4)/(3*#1^2 + 2*#1^6) & ]/4
```

3.375.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1695, 1443, 25, 1480, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \left(x^8+3 x^4+1\right)} \, dx$$

$$\Big\downarrow \text{ 1695}$$

$$\frac{1}{2} \int \frac{1}{x^4 \left(x^8+3 x^4+1\right)}dx^2$$

$$\Big\downarrow \text{ 1443}$$

$$\frac{1}{2} \left(\int -\frac{x^4+3}{x^8+3 x^4+1}dx^2-\frac{1}{x^2}\right)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{2} \left(-\int \frac{x^4+3}{x^8+3 x^4+1}dx^2-\frac{1}{x^2}\right)$$

$$\Big\downarrow \text{ 1480}$$

$$\frac{1}{2} \left(-\frac{1}{10} \left(5+3 \sqrt{5}\right) \int \frac{1}{x^4+\frac{1}{2} \left(3-\sqrt{5}\right)}dx^2-\frac{1}{10} \left(5-3 \sqrt{5}\right) \int \frac{1}{x^4+\frac{1}{2} \left(3+\sqrt{5}\right)}dx^2-\frac{1}{x^2}\right)$$

$$\Big\downarrow \text{ 216}$$

$$\frac{1}{2} \left(-\frac{\left(5-3 \sqrt{5}\right) \arctan \left(\sqrt{\frac{2}{3+\sqrt{5}}} x^2\right)}{5 \sqrt{2 \left(3+\sqrt{5}\right)}}-\frac{1}{10} \sqrt{\frac{1}{2} \left(3+\sqrt{5}\right)} \left(5+3 \sqrt{5}\right) \arctan \left(\sqrt{\frac{1}{2} \left(3+\sqrt{5}\right)} x^2\right)-\frac{1}{x^2}\right)$$

input
```
Int[1/(x^3*(1 + 3*x^4 + x^8)),x]
```

output
```
(-x^(-2) - ((5 - 3*Sqrt[5])*ArcTan[Sqrt[2/(3 + Sqrt[5])]*x^2])/(5*Sqrt[2*(
3 + Sqrt[5])]) - (Sqrt[(3 + Sqrt[5])/2]*(5 + 3*Sqrt[5])*ArcTan[Sqrt[(3 + S
qrt[5])/2]*x^2])/10)/2
```

3.375.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1443
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Sim
p[1/(a*d^2*(m + 1))   Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*
x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1695
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b
*x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c,
p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.375. $\int \frac{1}{x^3(1+3x^4+x^8)}\,dx$

3.375.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.47

method	result	size
risch	$-\frac{1}{2x^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(25\_Z^4+90\_Z^2+1\right)}\_R\ln\left(35\_R^3+8x^2+123\_R\right)\right)}{4}$	42
default	$-\frac{1}{2x^2}-\frac{\left(\sqrt{5}-3\right)\sqrt{5}\arctan\left(\frac{4x^2}{2\sqrt{5}+2}\right)}{5\left(2\sqrt{5}+2\right)}-\frac{\left(3+\sqrt{5}\right)\sqrt{5}\arctan\left(\frac{4x^2}{2\sqrt{5}-2}\right)}{5\left(2\sqrt{5}-2\right)}$	75

input
```
int(1/x^3/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2/x^2+1/4*sum(_R*ln(35*_R^3+8*x^2+123*_R),_R=RootOf(25*_Z^4+90*_Z^2+1))
```

3.375.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(53) = 106.

Time = 0.25 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.92

$$\int \frac{1}{x^3\left(1+3x^4+x^8\right)}\,dx$$
$$= \frac{\sqrt{5}x^2\sqrt{4\sqrt{5}-9}\log\left(2x^2+\sqrt{4\sqrt{5}-9}(3\sqrt{5}+7)\right)-\sqrt{5}x^2\sqrt{4\sqrt{5}-9}\log\left(2x^2-\sqrt{4\sqrt{5}-9}(3\sqrt{5}+\right.}{}$$

input
```
integrate(1/x^3/(x^8+3*x^4+1),x, algorithm="fricas")
```

output
```
1/20*(sqrt(5)*x^2*sqrt(4*sqrt(5) - 9)*log(2*x^2 + sqrt(4*sqrt(5) - 9)*(3*s
qrt(5) + 7)) - sqrt(5)*x^2*sqrt(4*sqrt(5) - 9)*log(2*x^2 - sqrt(4*sqrt(5)
- 9)*(3*sqrt(5) + 7)) + sqrt(5)*x^2*sqrt(-4*sqrt(5) - 9)*log(2*x^2 + (3*sq
rt(5) - 7)*sqrt(-4*sqrt(5) - 9)) - sqrt(5)*x^2*sqrt(-4*sqrt(5) - 9)*log(2*
x^2 - (3*sqrt(5) - 7)*sqrt(-4*sqrt(5) - 9)) - 10)/x^2
```

3.375.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.63

$$\int \frac{1}{x^3(1+3x^4+x^8)}\,dx = -2\left(\frac{\sqrt{5}}{10}+\frac{1}{4}\right)\operatorname{atan}\left(\frac{2x^2}{-1+\sqrt{5}}\right)+2\cdot\left(\frac{1}{4}-\frac{\sqrt{5}}{10}\right)\operatorname{atan}\left(\frac{2x^2}{1+\sqrt{5}}\right)-\frac{1}{2x^2}$$

input
```
integrate(1/x**3/(x**8+3*x**4+1),x)
```

output
```
-2*(sqrt(5)/10 + 1/4)*atan(2*x**2/(-1 + sqrt(5))) + 2*(1/4 - sqrt(5)/10)*a
tan(2*x**2/(1 + sqrt(5))) - 1/(2*x**2)
```

3.375.7 Maxima [F]

$$\int \frac{1}{x^3(1+3x^4+x^8)}\,dx = \int \frac{1}{(x^8+3\,x^4+1)x^3}\,dx$$

input
```
integrate(1/x^3/(x^8+3*x^4+1),x, algorithm="maxima")
```

output
```
-1/2/x^2 - integrate((x^4 + 3)*x/(x^8 + 3*x^4 + 1), x)
```

3.375.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^3(1+3x^4+x^8)}\,dx = -\frac{1}{20}\left(x^4\left(\sqrt{5}-5\right)+3\sqrt{5}-15\right)\arctan\left(\frac{2\,x^2}{\sqrt{5}+1}\right)-\frac{1}{20}\left(x^4\left(\sqrt{5}+5\right)+3\sqrt{5}+15\right)\arctan\left(\frac{2\,x^2}{\sqrt{5}-1}\right)-\frac{1}{2\,x^2}$$

input
```
integrate(1/x^3/(x^8+3*x^4+1),x, algorithm="giac")
```

output
```
-1/20*(x^4*(sqrt(5) - 5) + 3*sqrt(5) - 15)*arctan(2*x^2/(sqrt(5) + 1)) - 1
/20*(x^4*(sqrt(5) + 5) + 3*sqrt(5) + 15)*arctan(2*x^2/(sqrt(5) - 1)) - 1/2
/x^2
```

3.375.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.46

$$\int \frac{1}{x^3(1+3x^4+x^8)}\,dx = 2\,\mathrm{atanh}\left(\frac{26880\,x^2\,\sqrt{-\frac{\sqrt{5}}{20}-\frac{9}{80}}}{3520\,\sqrt{5}+7872} + \frac{12032\,\sqrt{5}\,x^2\,\sqrt{-\frac{\sqrt{5}}{20}-\frac{9}{80}}}{3520\,\sqrt{5}+7872}\right)\sqrt{-\frac{\sqrt{5}}{20}-\frac{9}{80}} - 2\,\mathrm{atanh}\left(\frac{26880\,x^2\,\sqrt{\frac{\sqrt{5}}{20}-\frac{9}{80}}}{3520\,\sqrt{5}-7872} - \frac{12032\,\sqrt{5}\,x^2\,\sqrt{\frac{\sqrt{5}}{20}-\frac{9}{80}}}{3520\,\sqrt{5}-7872}\right)\sqrt{\frac{\sqrt{5}}{20}-\frac{9}{80}} - \frac{1}{2\,x^2}$$

input
```
int(1/(x^3*(3*x^4 + x^8 + 1)),x)
```

output
```
2*atanh((26880*x^2*(- 5^(1/2)/20 - 9/80)^(1/2))/(3520*5^(1/2) + 7872) + (1
2032*5^(1/2)*x^2*(- 5^(1/2)/20 - 9/80)^(1/2))/(3520*5^(1/2) + 7872))*(- 5^
(1/2)/20 - 9/80)^(1/2) - 2*atanh((26880*x^2*(5^(1/2)/20 - 9/80)^(1/2))/(35
20*5^(1/2) - 7872) - (12032*5^(1/2)*x^2*(5^(1/2)/20 - 9/80)^(1/2))/(3520*5
^(1/2) - 7872))*(5^(1/2)/20 - 9/80)^(1/2) - 1/(2*x^2)
```

3.376 $\int \frac{1}{x^5(1+3x^4+x^8)}\,dx$

3.376.1 Optimal result

Integrand size = 16, antiderivative size = 66

$$\int \frac{1}{x^5\left(1+3x^4+x^8\right)}\,dx=-\frac{1}{4x^4}-3\log(x)+\frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(3-\sqrt{5}+2x^4\right)+\frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(3+\sqrt{5}+2x^4\right)$$

output
```
-1/4/x^4-3*ln(x)+1/40*ln(2*x^4+5^(1/2)+3)*(15-7*5^(1/2))+1/40*ln(2*x^4-5^(
1/2)+3)*(15+7*5^(1/2))
```

3.376.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^5\left(1+3x^4+x^8\right)}\,dx=\frac{1}{40}\left(-\frac{10}{x^4}-120\log(x)+\left(15+7\sqrt{5}\right)\log\left(-3+\sqrt{5}-2x^4\right)+\left(15-7\sqrt{5}\right)\log\left(3+\sqrt{5}+2x^4\right)\right)$$

input
```
Integrate[1/(x^5*(1 + 3*x^4 + x^8)),x]
```

output
```
(-10/x^4 - 120*Log[x] + (15 + 7*Sqrt[5])*Log[-3 + Sqrt[5] - 2*x^4] + (15 -
 7*Sqrt[5])*Log[3 + Sqrt[5] + 2*x^4])/40
```

3.376.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.29, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1693, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 (x^8+3 x^4+1)} \, dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{4} \int \frac{1}{x^8 (x^8+3 x^4+1)}dx^4$$

$$\Big\downarrow \text{1141}$$

$$\frac{1}{4} \int \left(-\frac{2}{\left(15-7 \sqrt{5}\right) x^4+2 \left(20-9 \sqrt{5}\right)}-\frac{3}{x^4}+\frac{1}{x^8}-\frac{8}{\sqrt{5} \left(3+\sqrt{5}\right)^2 \left(2 x^4+\sqrt{5}+3\right)}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4} \left(-\frac{1}{x^4}-3 \log \left(x^4\right)-\frac{4 \log \left(2 x^4+\sqrt{5}+3\right)}{\sqrt{5} \left(3+\sqrt{5}\right)^2}-\frac{2 \log \left(-\left(\left(15-7 \sqrt{5}\right) x^4\right)-2 \left(20-9 \sqrt{5}\right)\right)}{15-7 \sqrt{5}}\right)$$

input
```
Int[1/(x^5*(1 + 3*x^4 + x^8)),x]
```

output
```
(-x^(-4) - 3*Log[x^4] - (4*Log[3 + Sqrt[5] + 2*x^4])/(Sqrt[5]*(3 + Sqrt[5]
)^2) - (2*Log[-2*(20 - 9*Sqrt[5]) - (15 - 7*Sqrt[5])*x^4])/(15 - 7*Sqrt[5]
))/4
```

3.376.3.1 Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.376.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.64

method	result
default	$-\frac{1}{4x^4}-3\ln\left(x\right)+\frac{3\ln\left(x^8+3x^4+1\right)}{8}-\frac{7\,\mathrm{arctanh}\left(\frac{\left(2x^4+3\right)\sqrt{5}}{5}\right)\sqrt{5}}{20}$
risch	$-\frac{1}{4x^4}-3\ln\left(x\right)+\frac{3\ln\left(7x^4+\frac{21}{2}-\frac{7\sqrt{5}}{2}\right)}{8}+\frac{7\ln\left(7x^4+\frac{21}{2}-\frac{7\sqrt{5}}{2}\right)\sqrt{5}}{40}+\frac{3\ln\left(7x^4+\frac{21}{2}+\frac{7\sqrt{5}}{2}\right)}{8}-\frac{7\ln\left(7x^4+\frac{21}{2}+\frac{7\sqrt{5}}{2}\right)\sqrt{5}}{40}$

input
```
int(1/x^5/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/4/x^4-3*ln(x)+3/8*ln(x^8+3*x^4+1)-7/20*arctanh(1/5*(2*x^4+3)*5^(1/2))*5
^(1/2)
```

3.376.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.15

$$\int \frac{1}{x^5\left(1+3x^4+x^8\right)}\,dx = \frac{7\sqrt{5}x^4\log\left(\frac{2\,x^8+6\,x^4-\sqrt{5}(2\,x^4+3)+7}{x^8+3\,x^4+1}\right)+15\,x^4\log\left(x^8+3\,x^4+1\right)-120\,x^4\log\left(x\right)-10}{40\,x^4}$$

input
```
integrate(1/x^5/(x^8+3*x^4+1),x, algorithm="fricas")
```

output
```
1/40*(7*sqrt(5)*x^4*log((2*x^8 + 6*x^4 - sqrt(5)*(2*x^4 + 3) + 7)/(x^8 + 3
*x^4 + 1)) + 15*x^4*log(x^8 + 3*x^4 + 1) - 120*x^4*log(x) - 10)/x^4
```

3.376.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.98

$$\int \frac{1}{x^5 (1+3x^4+x^8)} \, dx = -3\log(x) + \left(\frac{3}{8} + \frac{7\sqrt{5}}{40}\right)\log\left(x^4 - \frac{\sqrt{5}}{2} + \frac{3}{2}\right) + \left(\frac{3}{8} - \frac{7\sqrt{5}}{40}\right)\log\left(x^4 + \frac{\sqrt{5}}{2} + \frac{3}{2}\right) - \frac{1}{4x^4}$$

input
```
integrate(1/x**5/(x**8+3*x**4+1),x)
```

output
```
-3*log(x) + (3/8 + 7*sqrt(5)/40)*log(x**4 - sqrt(5)/2 + 3/2) + (3/8 - 7*sq
rt(5)/40)*log(x**4 + sqrt(5)/2 + 3/2) - 1/(4*x**4)
```

3.376.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^5 (1+3x^4+x^8)} \, dx = \frac{7}{40}\sqrt{5}\log\left(\frac{2x^4 - \sqrt{5} + 3}{2x^4 + \sqrt{5} + 3}\right) - \frac{1}{4x^4} + \frac{3}{8}\log\left(x^8 + 3x^4 + 1\right) - \frac{3}{4}\log\left(x^4\right)$$

input
```
integrate(1/x^5/(x^8+3*x^4+1),x, algorithm="maxima")
```

output
```
7/40*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3)) - 1/4/x^4 +
3/8*log(x^8 + 3*x^4 + 1) - 3/4*log(x^4)
```

3.376.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^5 (1+3x^4+x^8)} \, dx = \frac{7}{40}\sqrt{5}\log\left(\frac{2x^4 - \sqrt{5} + 3}{2x^4 + \sqrt{5} + 3}\right) + \frac{3x^4 - 1}{4x^4} + \frac{3}{8}\log\left(x^8 + 3x^4 + 1\right) - \frac{3}{4}\log\left(x^4\right)$$

input
```
integrate(1/x^5/(x^8+3*x^4+1),x, algorithm="giac")
```

output
```
7/40*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3)) + 1/4*(3*x^4
 - 1)/x^4 + 3/8*log(x^8 + 3*x^4 + 1) - 3/4*log(x^4)
```

3.376.9 Mupad [B] (verification not implemented)

Time = 8.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^5 (1+3x^4+x^8)} \, dx = \ln \left(x^4-\frac{\sqrt{5}}{2}+\frac{3}{2}\right) \left(\frac{7\sqrt{5}}{40}+\frac{3}{8}\right)-\frac{1}{4\,x^4} - 3\ln(x)-\ln\left(x^4+\frac{\sqrt{5}}{2}+\frac{3}{2}\right)\left(\frac{7\sqrt{5}}{40}-\frac{3}{8}\right)$$

input
```
int(1/(x^5*(3*x^4 + x^8 + 1)),x)
```

output
```
log(x^4 - 5^(1/2)/2 + 3/2)*((7*5^(1/2))/40 + 3/8) - 1/(4*x^4) - 3*log(x) -
 log(5^(1/2)/2 + x^4 + 3/2)*((7*5^(1/2))/40 - 3/8)
```

3.377 $\int \frac{1}{x^7(1+3x^4+x^8)}\,dx$

3.377.1 Optimal result

Integrand size = 16, antiderivative size = 97

$$\int \frac{1}{x^7(1+3x^4+x^8)}\,dx = -\frac{1}{6x^6}+\frac{3}{2x^2}-\frac{1}{2}\sqrt{\frac{1}{10}\left(123-55\sqrt{5}\right)}\arctan\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)+\frac{1}{2}\sqrt{\frac{1}{10}\left(123+55\sqrt{5}\right)}\arctan\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)$$

output
```
-1/6/x^6+3/2/x^2-1/2*arctan(x^2*2^(1/2)/(3+5^(1/2))^(1/2))*(5/2-11/10*5^(1
/2))+1/2*arctan(x^2*(1/2+1/2*5^(1/2)))*(5/2+11/10*5^(1/2))
```

3.377.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.75

$$\int \frac{1}{x^7(1+3x^4+x^8)}\,dx = -\frac{1}{6x^6}+\frac{3}{2x^2}+\frac{1}{4}\text{RootSum}\left[1+3\#1^4+\#1^8\&,\frac{8\log(x-\#1)+3\log(x-\#1)\#1^4}{3\#1^2+2\#1^6}\&\right]$$

input
```
Integrate[1/(x^7*(1 + 3*x^4 + x^8)),x]
```

output
```
-1/6*1/x^6 + 3/(2*x^2) + RootSum[1 + 3*#1^4 + #1^8 & , (8*Log[x - #1] + 3*
Log[x - #1]*#1^4)/(3*#1^2 + 2*#1^6) & ]/4
```

3.377.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1695, 1443, 27, 1604, 1480, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7\left(x^8+3x^4+1\right)}dx$$

$$\Big\downarrow \text{1695}$$

$$\frac{1}{2}\int \frac{1}{x^8\left(x^8+3x^4+1\right)}dx^2$$

$$\Big\downarrow \text{1443}$$

$$\frac{1}{2}\left(\frac{1}{3}\int -\frac{3\left(x^4+3\right)}{x^4\left(x^8+3x^4+1\right)}dx^2-\frac{1}{3x^6}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(-\int \frac{x^4+3}{x^4\left(x^8+3x^4+1\right)}dx^2-\frac{1}{3x^6}\right)$$

$$\Big\downarrow \text{1604}$$

$$\frac{1}{2}\left(\int \frac{3x^4+8}{x^8+3x^4+1}dx^2-\frac{1}{3x^6}+\frac{3}{x^2}\right)$$

$$\Big\downarrow \text{1480}$$

$$\frac{1}{2}\left(\frac{1}{10}\left(15+7\sqrt{5}\right)\int \frac{1}{x^4+\frac{1}{2}\left(3-\sqrt{5}\right)}dx^2+\frac{1}{10}\left(15-7\sqrt{5}\right)\int \frac{1}{x^4+\frac{1}{2}\left(3+\sqrt{5}\right)}dx^2-\frac{1}{3x^6}+\frac{3}{x^2}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\left(\frac{\left(15-7\sqrt{5}\right)\arctan\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)}{5\sqrt{2\left(3+\sqrt{5}\right)}}+\frac{1}{10}\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\left(15+7\sqrt{5}\right)\arctan\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)-\frac{1}{3x^6}+\frac{3}{x^2}\right)$$

input
```
Int[1/(x^7*(1 + 3*x^4 + x^8)),x]
```

output
```
(-1/3*1/x^6 + 3/x^2 + ((15 - 7*Sqrt[5])*ArcTan[Sqrt[2/(3 + Sqrt[5])]*x^2])
/(5*Sqrt[2*(3 + Sqrt[5])]) + (Sqrt[(3 + Sqrt[5])/2]*(15 + 7*Sqrt[5])*ArcTa
n[Sqrt[(3 + Sqrt[5])/2]*x^2])/10)/2
```

3.377.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1443
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Sim
p[1/(a*d^2*(m + 1))   Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*
x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1695
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b
*x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c,
p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.377.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.49

method	result	size
risch	$\frac{\frac{3x^4}{2}-\frac{1}{6}}{x^6}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(25\_Z^4+615\_Z^2+1\right)}\_R\ln\left(-90\_R^3+55x^2-2207\_R\right)\right)}{4}$	48
default	$-\frac{1}{6x^6}+\frac{3}{2x^2}+\frac{\left(-7+3\sqrt{5}\right)\sqrt{5}\,\arctan\left(\frac{4x^2}{2\sqrt{5}+2}\right)}{10+10\sqrt{5}}+\frac{\left(7+3\sqrt{5}\right)\sqrt{5}\,\arctan\left(\frac{4x^2}{2\sqrt{5}-2}\right)}{-10+10\sqrt{5}}$	84

input
```
int(1/x^7/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
(3/2*x^4-1/6)/x^6+1/4*sum(_R*ln(-90*_R^3+55*x^2-2207*_R),_R=RootOf(25*_Z^4
+615*_Z^2+1))
```

3.377.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. 2(55) = 110.

Time = 0.25 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.96

$$\int \frac{1}{x^7\left(1+3x^4+x^8\right)}\,dx = -\frac{3\sqrt{10}x^6\sqrt{55\sqrt{5}-123}\log\left(10\,x^2+\sqrt{10}\sqrt{55\sqrt{5}-123}\left(9\sqrt{5}+20\right)\right)-3\sqrt{10}x^6\sqrt{55\sqrt{5}-123}\log\left(10\right.}{}$$

input
```
integrate(1/x^7/(x^8+3*x^4+1),x, algorithm="fricas")
```

output
```
-1/120*(3*sqrt(10)*x^6*sqrt(55*sqrt(5) - 123)*log(10*x^2 + sqrt(10)*sqrt(5
5*sqrt(5) - 123)*(9*sqrt(5) + 20)) - 3*sqrt(10)*x^6*sqrt(55*sqrt(5) - 123)
*log(10*x^2 - sqrt(10)*sqrt(55*sqrt(5) - 123)*(9*sqrt(5) + 20)) - 3*sqrt(1
0)*x^6*sqrt(-55*sqrt(5) - 123)*log(10*x^2 + sqrt(10)*(9*sqrt(5) - 20)*sqrt
(-55*sqrt(5) - 123)) + 3*sqrt(10)*x^6*sqrt(-55*sqrt(5) - 123)*log(10*x^2 -
 sqrt(10)*(9*sqrt(5) - 20)*sqrt(-55*sqrt(5) - 123)) - 180*x^4 + 20)/x^6
```

3.377.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.67

$$\int \frac{1}{x^7 (1+3x^4+x^8)} \, dx = 2 \cdot \left(\frac{11\sqrt{5}}{40} + \frac{5}{8}\right) \operatorname{atan}\left(\frac{2x^2}{-1+\sqrt{5}}\right) - 2 \cdot \left(\frac{5}{8} - \frac{11\sqrt{5}}{40}\right) \operatorname{atan}\left(\frac{2x^2}{1+\sqrt{5}}\right) + \frac{9x^4-1}{6x^6}$$

input
```
integrate(1/x**7/(x**8+3*x**4+1),x)
```

output
```
2*(11*sqrt(5)/40 + 5/8)*atan(2*x**2/(-1 + sqrt(5))) - 2*(5/8 - 11*sqrt(5)/
40)*atan(2*x**2/(1 + sqrt(5))) + (9*x**4 - 1)/(6*x**6)
```

3.377.7 Maxima [F]

$$\int \frac{1}{x^7 (1+3x^4+x^8)} \, dx = \int \frac{1}{(x^8+3\,x^4+1)x^7} \, dx$$

input
```
integrate(1/x^7/(x^8+3*x^4+1),x, algorithm="maxima")
```

output
```
1/6*(9*x^4 - 1)/x^6 + integrate((3*x^4 + 8)*x/(x^8 + 3*x^4 + 1), x)
```

3.377.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^7(1+3x^4+x^8)}\,dx = \frac{1}{20}\left(3\,x^4\left(\sqrt{5}-5\right)+8\,\sqrt{5}-40\right)\arctan\left(\frac{2\,x^2}{\sqrt{5}+1}\right) + \frac{1}{20}\left(3\,x^4\left(\sqrt{5}+5\right)+8\,\sqrt{5}+40\right)\arctan\left(\frac{2\,x^2}{\sqrt{5}-1}\right) + \frac{9\,x^4-1}{6\,x^6}$$

input
```
integrate(1/x^7/(x^8+3*x^4+1),x, algorithm="giac")
```

output
```
1/20*(3*x^4*(sqrt(5) - 5) + 8*sqrt(5) - 40)*arctan(2*x^2/(sqrt(5) + 1)) +
1/20*(3*x^4*(sqrt(5) + 5) + 8*sqrt(5) + 40)*arctan(2*x^2/(sqrt(5) - 1)) +
1/6*(9*x^4 - 1)/x^6
```

3.377.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.40

$$\int \frac{1}{x^7(1+3x^4+x^8)}\,dx = 2\,\mathrm{atanh}\left(\frac{3327500\,x^2\,\sqrt{\frac{11\,\sqrt{5}}{32}-\frac{123}{160}}}{1140425\,\sqrt{5}-2550075} - \frac{1488300\,\sqrt{5}\,x^2\,\sqrt{\frac{11\,\sqrt{5}}{32}-\frac{123}{160}}}{1140425\,\sqrt{5}-2550075}\right)\sqrt{\frac{11\,\sqrt{5}}{32}-\frac{123}{160}} - 2\,\mathrm{atanh}\left(\frac{3327500\,x^2\,\sqrt{-\frac{11\,\sqrt{5}}{32}-\frac{123}{160}}}{1140425\,\sqrt{5}+2550075} + \frac{1488300\,\sqrt{5}\,x^2\,\sqrt{-\frac{11\,\sqrt{5}}{32}-\frac{123}{160}}}{1140425\,\sqrt{5}+2550075}\right)\sqrt{-\frac{11\,\sqrt{5}}{32}-\frac{123}{160}} + \frac{\frac{3\,x^4}{2}-\frac{1}{6}}{x^6}$$

input
```
int(1/(x^7*(3*x^4 + x^8 + 1)),x)
```

output
```
2*atanh((3327500*x^2*((11*5^(1/2))/32 - 123/160)^(1/2))/(1140425*5^(1/2) -
 2550075) - (1488300*5^(1/2)*x^2*((11*5^(1/2))/32 - 123/160)^(1/2))/(11404
25*5^(1/2) - 2550075))*((11*5^(1/2))/32 - 123/160)^(1/2) - 2*atanh((332750
0*x^2*(- (11*5^(1/2))/32 - 123/160)^(1/2))/(1140425*5^(1/2) + 2550075) + (
1488300*5^(1/2)*x^2*(- (11*5^(1/2))/32 - 123/160)^(1/2))/(1140425*5^(1/2)
+ 2550075))*(- (11*5^(1/2))/32 - 123/160)^(1/2) + ((3*x^4)/2 - 1/6)/x^6
```

3.378 $\int \frac{x^8}{1+3x^4+x^8} \, dx$

3.378.1 Optimal result

Integrand size = 16, antiderivative size = 460

$$\int \frac{x^8}{1+3x^4+x^8}\,dx = x-\frac{\sqrt[4]{123-55\sqrt{5}}\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}} + \frac{\sqrt[4]{123-55\sqrt{5}}\arctan\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}} + \frac{\sqrt[4]{123+55\sqrt{5}}\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}} - \frac{\sqrt[4]{123+55\sqrt{5}}\arctan\left(1+\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}} - \frac{\sqrt[4]{123-55\sqrt{5}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{4\,2^{3/4}\sqrt{5}} + \frac{\sqrt[4]{123-55\sqrt{5}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{4\,2^{3/4}\sqrt{5}} + \frac{\sqrt[4]{123+55\sqrt{5}}\log\left(\sqrt{2\left(3+\sqrt{5}\right)}-2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+2x^2\right)}{4\,2^{3/4}\sqrt{5}} - \frac{\sqrt[4]{123+55\sqrt{5}}\log\left(\sqrt{2\left(3+\sqrt{5}\right)}+2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+2x^2\right)}{4\,2^{3/4}\sqrt{5}}$$

output
```
x+1/20*arctan(-1+2^(3/4)*x/(3-5^(1/2))^(1/4))*(123-55*5^(1/2))^(1/4)*2^(1/
4)*5^(1/2)+1/20*arctan(1+2^(3/4)*x/(3-5^(1/2))^(1/4))*(123-55*5^(1/2))^(1/
4)*2^(1/4)*5^(1/2)-1/40*ln(2*x^2-2*2^(1/4)*x*(3-5^(1/2))^(1/4)+5^(1/2)-1)*
(123-55*5^(1/2))^(1/4)*2^(1/4)*5^(1/2)+1/40*ln(2*x^2+2*2^(1/4)*x*(3-5^(1/2
))^(1/4)+5^(1/2)-1)*(123-55*5^(1/2))^(1/4)*2^(1/4)*5^(1/2)-1/20*arctan(-1+
2^(3/4)*x/(3+5^(1/2))^(1/4))*(123+55*5^(1/2))^(1/4)*2^(1/4)*5^(1/2)-1/20*a
rctan(1+2^(3/4)*x/(3+5^(1/2))^(1/4))*(123+55*5^(1/2))^(1/4)*2^(1/4)*5^(1/2
)+1/40*ln(2*x^2-2*2^(1/4)*x*(3+5^(1/2))^(1/4)+5^(1/2)+1)*(123+55*5^(1/2))^
(1/4)*2^(1/4)*5^(1/2)-1/40*ln(2*x^2+2*2^(1/4)*x*(3+5^(1/2))^(1/4)+5^(1/2)+
1)*(123+55*5^(1/2))^(1/4)*2^(1/4)*5^(1/2)
```

3.378.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.13

$$\int \frac{x^8}{1+3x^4+x^8}\,dx = x-\frac{1}{4}\text{RootSum}\left[1+3\#1^4+\#1^8\&,\frac{\log(x-\#1)+3\log(x-\#1)\#1^4}{3\#1^3+2\#1^7}\&\right]$$

input
```
Integrate[x^8/(1 + 3*x^4 + x^8),x]
```

output
```
x - RootSum[1 + 3*#1^4 + #1^8 & , (Log[x - #1] + 3*Log[x - #1]*#1^4)/(3*#1
^3 + 2*#1^7) & ]/4
```

3.378.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 480, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {1703, 1752, 755, 27, 1476, 1082, 217, 1479, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{x^8+3x^4+1}dx$$

$$\Big\downarrow \text{1703}$$

$$x-\int \frac{3x^4+1}{x^8+3x^4+1}dx$$

$$\Big\downarrow \text{1752}$$

$$-\frac{1}{10}\left(15-7\sqrt{5}\right)\int \frac{1}{x^4+\frac{1}{2}\left(3-\sqrt{5}\right)}dx-\frac{1}{10}\left(15+7\sqrt{5}\right)\int \frac{1}{x^4+\frac{1}{2}\left(3+\sqrt{5}\right)}dx+x$$

$$\Big\downarrow \text{755}$$

$$-\frac{1}{10}\left(15-7\sqrt{5}\right)\left(\frac{\int \frac{2\left(\sqrt{3-\sqrt{5}}-\sqrt{2}x^2\right)}{2x^4-\sqrt{5}+3}dx}{2\sqrt{3-\sqrt{5}}}+\frac{\int \frac{2\left(\sqrt{2}x^2+\sqrt{3-\sqrt{5}}\right)}{2x^4-\sqrt{5}+3}dx}{2\sqrt{3-\sqrt{5}}}\right)-\frac{1}{10}\left(15+7\sqrt{5}\right)\left(\frac{\int \frac{2\left(\sqrt{3+\sqrt{5}}-\sqrt{2}x^2\right)}{2x^4+\sqrt{5}+3}dx}{2\sqrt{3+\sqrt{5}}}+\frac{\int \frac{2\left(\sqrt{2}x^2+\sqrt{3+\sqrt{5}}\right)}{2x^4+\sqrt{5}+3}dx}{2\sqrt{3+\sqrt{5}}}\right)+x$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{10}\left(15-7\sqrt{5}\right)\left(\frac{\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{2x^4-\sqrt{5}+3}dx}{\sqrt{3-\sqrt{5}}}+\frac{\int \frac{\sqrt{2}x^2+\sqrt{3-\sqrt{5}}}{2x^4-\sqrt{5}+3}dx}{\sqrt{3-\sqrt{5}}}\right)-\frac{1}{10}\left(15+7\sqrt{5}\right)\left(\frac{\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{2x^4+\sqrt{5}+3}dx}{\sqrt{3+\sqrt{5}}}+\frac{\int \frac{\sqrt{2}x^2+\sqrt{3+\sqrt{5}}}{2x^4+\sqrt{5}+3}dx}{\sqrt{3+\sqrt{5}}}\right)+x$$

$$\Big\downarrow \text{1476}$$

$$-\frac{1}{10}\left(15-7\sqrt{5}\right)\left(\frac{\frac{\int \frac{1}{x^2-\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}}dx}{2\sqrt{2}}+\frac{\int \frac{1}{x^2+\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}}dx}{2\sqrt{2}}}{\sqrt{3-\sqrt{5}}}+\frac{\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{2x^4-\sqrt{5}+3}dx}{\sqrt{3-\sqrt{5}}}\right)-\frac{1}{10}\left(15+7\sqrt{5}\right)\left(\frac{\frac{\int \frac{1}{x^2-\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\sqrt{2}}+\frac{\int \frac{1}{x^2+\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\sqrt{2}}}{\sqrt{3+\sqrt{5}}}+\frac{\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{2x^4+\sqrt{5}+3}dx}{\sqrt{3+\sqrt{5}}}\right)+x$$

$$\Big\downarrow \text{1082}$$

3.378. $\int \frac{x^8}{1+3x^4+x^8}\,dx$

$$-\frac{1}{10}\left(15-7\sqrt{5}\right)\left(\frac{\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{2x^4-\sqrt{5}+3}dx}{\sqrt{3-\sqrt{5}}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)^2-1}d\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}-\frac{\int \frac{1}{-\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)^2-1}d\left(\frac{2^{3/4}x}{\sqrt[4]{3-\cdots}}\right.}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}\cdots}{\sqrt{3-\sqrt{5}}}\right.\cdots$$

$$\frac{1}{10}\left(15+7\sqrt{5}\right)\left(\frac{\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{2x^4+\sqrt{5}+3}dx}{\sqrt{3+\sqrt{5}}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)^2-1}d\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\int \frac{1}{-\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)^2-1}d\left(\frac{2^{3/4}x}{\sqrt[4]{3+\cdots}}\right.}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}\cdots}{\sqrt{3+\sqrt{5}}}\right.\cdots$$

$$x$$

$$\downarrow 217$$

$$-\frac{1}{10}\left(15-7\sqrt{5}\right)\left(\frac{\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{2x^4-\sqrt{5}+3}dx}{\sqrt{3-\sqrt{5}}}+\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}}{\sqrt{3-\sqrt{5}}}\right)-$$

$$\frac{1}{10}\left(15+7\sqrt{5}\right)\left(\frac{\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{2x^4+\sqrt{5}+3}dx}{\sqrt{3+\sqrt{5}}}+\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{\sqrt{3+\sqrt{5}}}\right)+x$$

3.378. $\int \frac{x^8}{1+3x^4+x^8}\,dx$

$\downarrow$ 1479

$$-\frac{1}{10}\left(15-7\sqrt{5}\right)\left(\frac{-\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\int -\frac{\sqrt[4]{2\left(3-\sqrt{5}\right)}-2x}{x^2-\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}}dx-\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\int -\frac{2x+\sqrt[4]{2\left(\cdots\right.}}{x^2+\sqrt[4]{2\left(3-\sqrt{5}\right.}\cdots}}{\sqrt{3-\sqrt{5}}}\right.$$

$$\frac{1}{10}\left(15+7\sqrt{5}\right)\left(\frac{-\frac{\int -\frac{\sqrt[4]{2\left(3+\sqrt{5}\right)}-2x}{x^2-\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\int -\frac{2x+\sqrt[4]{2\left(3+\sqrt{5}\right)}}{x^2+\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{\sqrt{3+\sqrt{5}}}+\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\cdots\right.}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{\sqrt{\cdots}}\right.$$

x

$\downarrow$ 25

3.378. $\int \frac{x^8}{1+3x^4+x^8}\,dx$

$$-\frac{1}{10}\left(15-7\sqrt{5}\right)\left(\frac{\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\int\frac{\sqrt[4]{2\left(3-\sqrt{5}\right)}-2x}{x^2-\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}}dx+\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\int\frac{2x+\sqrt[4]{2\left(3-\sqrt{\right.}}}{x^2+\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\right.}}}{\sqrt{3-\sqrt{5}}}\right.$$

$$\frac{1}{10}\left(15+7\sqrt{5}\right)\left(\frac{\frac{\int\frac{\sqrt[4]{2\left(3+\sqrt{5}\right)}-2x}{x^2-\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}+\frac{\int\frac{2x+\sqrt[4]{2\left(3+\sqrt{5}\right)}}{x^2+\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{\sqrt{3+\sqrt{5}}}+\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-}{\sqrt{3+}}\right.$$

$$x$$

$$\Big\downarrow \text{1103}$$

$$-\frac{1}{10}\left(15-7\sqrt{5}\right)\left(\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}}{\sqrt{3-\sqrt{5}}}+\frac{\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x\right.}{}\right.$$

$$\frac{1}{10}\left(15+7\sqrt{5}\right)\left(\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{\sqrt{3+\sqrt{5}}}+\frac{\frac{\log\left(2x^2+2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\log\left(2x\right.}{}}{\sqrt{3+\sqrt{5}}}\right.$$

$$x$$

3.378. $\int \frac{x^8}{1+3x^4+x^8}\,dx$

input
```
Int[x^8/(1 + 3*x^4 + x^8),x]
```

output
```
x - ((15 - 7*Sqrt[5])*((-(ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)]/(2^(
3/4)*(3 - Sqrt[5])^(1/4))) + ArcTan[1 + (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)]/(
2^(3/4)*(3 - Sqrt[5])^(1/4)))/Sqrt[3 - Sqrt[5]] + (-1/4*(((3 + Sqrt[5])/2)
^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])] - 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2]) +
 (((3 + Sqrt[5])/2)^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])] + 2*(2*(3 - Sqrt[5]))^
(1/4)*x + 2*x^2])/4)/Sqrt[3 - Sqrt[5]]))/10 - ((15 + 7*Sqrt[5])*((-(ArcTan
[1 - (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)]/(2^(3/4)*(3 + Sqrt[5])^(1/4))) + Arc
Tan[1 + (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)]/(2^(3/4)*(3 + Sqrt[5])^(1/4)))/Sq
rt[3 + Sqrt[5]] + (-1/2*Log[Sqrt[2*(3 + Sqrt[5])] - 2*(2*(3 + Sqrt[5]))^(1
/4)*x + 2*x^2]/(2^(3/4)*(3 + Sqrt[5])^(1/4)) + Log[Sqrt[2*(3 + Sqrt[5])] +
 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2]/(2*2^(3/4)*(3 + Sqrt[5])^(1/4)))/Sqr
t[3 + Sqrt[5]]))/10
```

3.378.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1703
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(2*n - 1)*(d*x)^(m - 2*n + 1)*((a + b*x^n + c*x^(2*n))^(
p + 1)/(c*(m + 2*n*p + 1))), x] - Simp[d^(2*n)/(c*(m + 2*n*p + 1))   Int[(d
*x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x
^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && N
eQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0
] && IntegerQ[p]
```

rule 1752
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

3.378.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.10

method	result	size
default	$x+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}+3\_Z^{4}+1\right)}\frac{\left(-3\_R^{4}-1\right)\ln\left(x-\_R\right)}{2\_R^{7}+3\_R^{3}}\right)}{4}$	46
risch	$x+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}+3\_Z^{4}+1\right)}\frac{\left(-3\_R^{4}-1\right)\ln\left(x-\_R\right)}{2\_R^{7}+3\_R^{3}}\right)}{4}$	46

input
```
int(x^8/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
x+1/4*sum((-3*_R^4-1)/(2*_R^7+3*_R^3)*ln(x-_R),_R=RootOf(_Z^8+3*_Z^4+1))
```

3.378. $\int \frac{x^8}{1+3x^4+x^8}\,dx$

3.378.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 422, normalized size of antiderivative = 0.92

$$
\begin{aligned}
\int \frac{x^8}{1+3x^4+x^8}\,dx
&= \frac{1}{40}\sqrt{10}\sqrt{\sqrt{2}\sqrt{55\sqrt{5}-123}}\log\left(\sqrt{10}\sqrt{\sqrt{2}\sqrt{55\sqrt{5}-123}}\left(3\sqrt{5}+5\right)+20\,x\right)\\
&\quad-\frac{1}{40}\sqrt{10}\sqrt{\sqrt{2}\sqrt{55\sqrt{5}-123}}\log\left(-\sqrt{10}\sqrt{\sqrt{2}\sqrt{55\sqrt{5}-123}}\left(3\sqrt{5}+5\right)+20\,x\right)\\
&\quad+\frac{1}{40}\sqrt{10}\sqrt{-\sqrt{2}\sqrt{55\sqrt{5}-123}}\log\left(\sqrt{10}\sqrt{-\sqrt{2}\sqrt{55\sqrt{5}-123}}\left(3\sqrt{5}+5\right)+20\,x\right)\\
&\quad-\frac{1}{40}\sqrt{10}\sqrt{-\sqrt{2}\sqrt{55\sqrt{5}-123}}\log\left(-\sqrt{10}\sqrt{-\sqrt{2}\sqrt{55\sqrt{5}-123}}\left(3\sqrt{5}+5\right)+20\,x\right)\\
&\quad-\frac{1}{40}\sqrt{10}\sqrt{\sqrt{2}\sqrt{-55\sqrt{5}-123}}\log\left(\sqrt{10}\sqrt{\sqrt{2}\sqrt{-55\sqrt{5}-123}}\left(3\sqrt{5}-5\right)+20\,x\right)\\
&\quad+\frac{1}{40}\sqrt{10}\sqrt{\sqrt{2}\sqrt{-55\sqrt{5}-123}}\log\left(-\sqrt{10}\sqrt{\sqrt{2}\sqrt{-55\sqrt{5}-123}}\left(3\sqrt{5}-5\right)+20\,x\right)\\
&\quad-\frac{1}{40}\sqrt{10}\sqrt{-\sqrt{2}\sqrt{-55\sqrt{5}-123}}\log\left(\sqrt{10}\sqrt{-\sqrt{2}\sqrt{-55\sqrt{5}-123}}\left(3\sqrt{5}-5\right)+20\,x\right)\\
&\quad+\frac{1}{40}\sqrt{10}\sqrt{-\sqrt{2}\sqrt{-55\sqrt{5}-123}}\log\left(-\sqrt{10}\sqrt{-\sqrt{2}\sqrt{-55\sqrt{5}-123}}\left(3\sqrt{5}-5\right)+20\,x\right)+x
\end{aligned}
$$

input
```
integrate(x^8/(x^8+3*x^4+1),x, algorithm="fricas")
```

output
```
1/40*sqrt(10)*sqrt(sqrt(2)*sqrt(55*sqrt(5) - 123))*log(sqrt(10)*sqrt(sqrt(
2)*sqrt(55*sqrt(5) - 123))*(3*sqrt(5) + 5) + 20*x) - 1/40*sqrt(10)*sqrt(sq
rt(2)*sqrt(55*sqrt(5) - 123))*log(-sqrt(10)*sqrt(sqrt(2)*sqrt(55*sqrt(5) -
 123))*(3*sqrt(5) + 5) + 20*x) + 1/40*sqrt(10)*sqrt(-sqrt(2)*sqrt(55*sqrt(
5) - 123))*log(sqrt(10)*sqrt(-sqrt(2)*sqrt(55*sqrt(5) - 123))*(3*sqrt(5) +
 5) + 20*x) - 1/40*sqrt(10)*sqrt(-sqrt(2)*sqrt(55*sqrt(5) - 123))*log(-sqr
t(10)*sqrt(-sqrt(2)*sqrt(55*sqrt(5) - 123))*(3*sqrt(5) + 5) + 20*x) - 1/40
*sqrt(10)*sqrt(sqrt(2)*sqrt(-55*sqrt(5) - 123))*log(sqrt(10)*sqrt(sqrt(2)*
sqrt(-55*sqrt(5) - 123))*(3*sqrt(5) - 5) + 20*x) + 1/40*sqrt(10)*sqrt(sqrt
(2)*sqrt(-55*sqrt(5) - 123))*log(-sqrt(10)*sqrt(sqrt(2)*sqrt(-55*sqrt(5) -
 123))*(3*sqrt(5) - 5) + 20*x) - 1/40*sqrt(10)*sqrt(-sqrt(2)*sqrt(-55*sqrt
(5) - 123))*log(sqrt(10)*sqrt(-sqrt(2)*sqrt(-55*sqrt(5) - 123))*(3*sqrt(5)
 - 5) + 20*x) + 1/40*sqrt(10)*sqrt(-sqrt(2)*sqrt(-55*sqrt(5) - 123))*log(-
sqrt(10)*sqrt(-sqrt(2)*sqrt(-55*sqrt(5) - 123))*(3*sqrt(5) - 5) + 20*x) +
x
```

3.378.6 Sympy [A] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.06

$$\int \frac{x^8}{1+3x^4+x^8}\,dx$$
$$= x + \mathrm{RootSum}\left(40960000t^8 + 787200t^4 + 1, \left(t \mapsto t\log\left(\frac{15360t^5}{11} + \frac{1288t}{55} + x\right)\right)\right)$$

input
```
integrate(x**8/(x**8+3*x**4+1),x)
```

output
```
x + RootSum(40960000*_t**8 + 787200*_t**4 + 1, Lambda(_t, _t*log(15360*_t*
*5/11 + 1288*_t/55 + x)))
```

3.378.7 Maxima [F]

$$\int \frac{x^8}{1+3x^4+x^8}\,dx = \int \frac{x^8}{x^8+3\,x^4+1}\,dx$$

input
```
integrate(x^8/(x^8+3*x^4+1),x, algorithm="maxima")
```

output
```
x - integrate((3*x^4 + 1)/(x^8 + 3*x^4 + 1), x)
```

3.378.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.52

$$\int \frac{x^8}{1+3x^4+x^8}\,dx = -\frac{1}{80}\left(\pi+4\arctan\left(x\sqrt{\sqrt{5}-1}+1\right)\right)\sqrt{25\sqrt{5}+55} + \frac{1}{80}\left(\pi+4\arctan\left(-x\sqrt{\sqrt{5}-1}+1\right)\right)\sqrt{25\sqrt{5}+55} + \frac{1}{80}\left(\pi+4\arctan\left(x\sqrt{\sqrt{5}+1}-1\right)\right)\sqrt{25\sqrt{5}-55} - \frac{1}{80}\left(\pi+4\arctan\left(-x\sqrt{\sqrt{5}+1}-1\right)\right)\sqrt{25\sqrt{5}-55} - \frac{1}{40}\sqrt{25\sqrt{5}+55}\log\left(722500\left(x+\sqrt{\sqrt{5}+1}\right)^2+722500\,x^2\right) + \frac{1}{40}\sqrt{25\sqrt{5}+55}\log\left(722500\left(x-\sqrt{\sqrt{5}+1}\right)^2+722500\,x^2\right) + \frac{1}{40}\sqrt{25\sqrt{5}-55}\log\left(2992900\left(x+\sqrt{\sqrt{5}-1}\right)^2+2992900\,x^2\right) - \frac{1}{40}\sqrt{25\sqrt{5}-55}\log\left(2992900\left(x-\sqrt{\sqrt{5}-1}\right)^2+2992900\,x^2\right)+x$$

input
```
integrate(x^8/(x^8+3*x^4+1),x, algorithm="giac")
```

output
```
-1/80*(pi + 4*arctan(x*sqrt(sqrt(5) - 1) + 1))*sqrt(25*sqrt(5) + 55) + 1/8
0*(pi + 4*arctan(-x*sqrt(sqrt(5) - 1) + 1))*sqrt(25*sqrt(5) + 55) + 1/80*(
pi + 4*arctan(x*sqrt(sqrt(5) + 1) - 1))*sqrt(25*sqrt(5) - 55) - 1/80*(pi +
 4*arctan(-x*sqrt(sqrt(5) + 1) - 1))*sqrt(25*sqrt(5) - 55) - 1/40*sqrt(25*
sqrt(5) + 55)*log(722500*(x + sqrt(sqrt(5) + 1))^2 + 722500*x^2) + 1/40*sq
rt(25*sqrt(5) + 55)*log(722500*(x - sqrt(sqrt(5) + 1))^2 + 722500*x^2) + 1
/40*sqrt(25*sqrt(5) - 55)*log(2992900*(x + sqrt(sqrt(5) - 1))^2 + 2992900*
x^2) - 1/40*sqrt(25*sqrt(5) - 55)*log(2992900*(x - sqrt(sqrt(5) - 1))^2 +
2992900*x^2) + x
```

3.378.9 Mupad [B] (verification not implemented)

Time = 8.31 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.47

$$\int \frac{x^8}{1+3x^4+x^8}\,dx$$
$$= x - \frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{3\,2^{1/4}\,x}{2\left(-55\sqrt{5}-123\right)^{1/4}}+\frac{2^{1/4}\sqrt{5}\,x}{2\left(-55\sqrt{5}-123\right)^{1/4}}\right)\left(-55\sqrt{5}-123\right)^{1/4}}{20}$$
$$+ \frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{3\,2^{1/4}\,x}{2\left(55\sqrt{5}-123\right)^{1/4}}-\frac{2^{1/4}\sqrt{5}\,x}{2\left(55\sqrt{5}-123\right)^{1/4}}\right)\left(55\sqrt{5}-123\right)^{1/4}}{20}$$
$$+ \frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{2^{1/4}\,x\,3\mathrm{i}}{2\left(-55\sqrt{5}-123\right)^{1/4}}+\frac{2^{1/4}\sqrt{5}\,x\,1\mathrm{i}}{2\left(-55\sqrt{5}-123\right)^{1/4}}\right)\left(-55\sqrt{5}-123\right)^{1/4}1\mathrm{i}}{20}$$
$$- \frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{2^{1/4}\,x\,3\mathrm{i}}{2\left(55\sqrt{5}-123\right)^{1/4}}-\frac{2^{1/4}\sqrt{5}\,x\,1\mathrm{i}}{2\left(55\sqrt{5}-123\right)^{1/4}}\right)\left(55\sqrt{5}-123\right)^{1/4}1\mathrm{i}}{20}$$

input
```
int(x^8/(3*x^4 + x^8 + 1),x)
```

output
```
x - (2^(3/4)*5^(1/2)*atan((3*2^(1/4)*x)/(2*(- 55*5^(1/2) - 123)^(1/4)) + (
2^(1/4)*5^(1/2)*x)/(2*(- 55*5^(1/2) - 123)^(1/4)))*(- 55*5^(1/2) - 123)^(1
/4))/20 + (2^(3/4)*5^(1/2)*atan((3*2^(1/4)*x)/(2*(55*5^(1/2) - 123)^(1/4))
 - (2^(1/4)*5^(1/2)*x)/(2*(55*5^(1/2) - 123)^(1/4)))*(55*5^(1/2) - 123)^(1
/4))/20 + (2^(3/4)*5^(1/2)*atan((2^(1/4)*x*3i)/(2*(- 55*5^(1/2) - 123)^(1/
4)) + (2^(1/4)*5^(1/2)*x*1i)/(2*(- 55*5^(1/2) - 123)^(1/4)))*(- 55*5^(1/2)
 - 123)^(1/4)*1i)/20 - (2^(3/4)*5^(1/2)*atan((2^(1/4)*x*3i)/(2*(55*5^(1/2)
 - 123)^(1/4)) - (2^(1/4)*5^(1/2)*x*1i)/(2*(55*5^(1/2) - 123)^(1/4)))*(55*
5^(1/2) - 123)^(1/4)*1i)/20
```

3.379 $\int \frac{x^6}{1+3x^4+x^8} \, dx$

3.379.1 Optimal result

Integrand size = 16, antiderivative size = 431

$$\int \frac{x^6}{1+3x^4+x^8}\,dx = \frac{\sqrt[4]{9-4\sqrt{5}}\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\sqrt{10}} - \frac{\sqrt[4]{9-4\sqrt{5}}\arctan\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\sqrt{10}} - \frac{\left(3+\sqrt{5}\right)^{3/4}\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{4\sqrt[4]{2}\sqrt{5}} + \frac{\left(3+\sqrt{5}\right)^{3/4}\arctan\left(1+\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{4\sqrt[4]{2}\sqrt{5}} - \frac{\sqrt[4]{9-4\sqrt{5}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{4\sqrt{10}} + \frac{\sqrt[4]{9-4\sqrt{5}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{4\sqrt{10}} + \frac{\left(3+\sqrt{5}\right)^{3/4}\log\left(\sqrt{2\left(3+\sqrt{5}\right)}-2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+2x^2\right)}{8\sqrt[4]{2}\sqrt{5}} - \frac{\left(3+\sqrt{5}\right)^{3/4}\log\left(\sqrt{2\left(3+\sqrt{5}\right)}+2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+2x^2\right)}{8\sqrt[4]{2}\sqrt{5}}$$

output
```
-1/40*arctan(-1+2^(3/4)*x/(3-5^(1/2))^(1/4))*(3-5^(1/2))^(3/4)*2^(3/4)*5^(
1/2)-1/40*arctan(1+2^(3/4)*x/(3-5^(1/2))^(1/4))*(3-5^(1/2))^(3/4)*2^(3/4)*
5^(1/2)-1/80*ln(2*x^2-2*2^(1/4)*x*(3-5^(1/2))^(1/4)+5^(1/2)-1)*(3-5^(1/2))
^(3/4)*2^(3/4)*5^(1/2)+1/80*ln(2*x^2+2*2^(1/4)*x*(3-5^(1/2))^(1/4)+5^(1/2)
-1)*(3-5^(1/2))^(3/4)*2^(3/4)*5^(1/2)+1/40*arctan(-1+2^(3/4)*x/(3+5^(1/2))
^(1/4))*(3+5^(1/2))^(3/4)*2^(3/4)*5^(1/2)+1/40*arctan(1+2^(3/4)*x/(3+5^(1/
2))^(1/4))*(3+5^(1/2))^(3/4)*2^(3/4)*5^(1/2)+1/80*ln(2*x^2-2*2^(1/4)*x*(3+
5^(1/2))^(1/4)+5^(1/2)+1)*(3+5^(1/2))^(3/4)*2^(3/4)*5^(1/2)-1/80*ln(2*x^2+
2*2^(1/4)*x*(3+5^(1/2))^(1/4)+5^(1/2)+1)*(3+5^(1/2))^(3/4)*2^(3/4)*5^(1/2)
```

3.379.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.10

$$\int \frac{x^6}{1+3x^4+x^8}\,dx = \frac{1}{4}\text{RootSum}\left[1+3\#1^4+\#1^8\&,\frac{\log(x-\#1)\#1^3}{3+2\#1^4}\&\right]$$

input
```
Integrate[x^6/(1 + 3*x^4 + x^8),x]
```

output
```
RootSum[1 + 3*#1^4 + #1^8 & , (Log[x - #1]*#1^3)/(3 + 2*#1^4) & ]/4
```

3.379.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {1710, 27, 826, 1476, 1082, 217, 1479, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{x^8+3x^4+1}\,dx$$

$$\Big\downarrow \text{1710}$$

$$\frac{1}{10}\left(5-3\sqrt{5}\right)\int \frac{2x^2}{2x^4-\sqrt{5}+3}dx+\frac{1}{10}\left(5+3\sqrt{5}\right)\int \frac{2x^2}{2x^4+\sqrt{5}+3}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\left(5-3\sqrt{5}\right)\int\frac{x^2}{2x^4-\sqrt{5}+3}dx+\frac{1}{5}\left(5+3\sqrt{5}\right)\int\frac{x^2}{2x^4+\sqrt{5}+3}dx$$

$$\downarrow 826$$

$$\frac{1}{5}\left(5-3\sqrt{5}\right)\left(\frac{\int\frac{\sqrt{2}x^2+\sqrt{3-\sqrt{5}}}{2x^4-\sqrt{5}+3}dx}{2\sqrt{2}}-\frac{\int\frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{2x^4-\sqrt{5}+3}dx}{2\sqrt{2}}\right)+\frac{1}{5}\left(5+3\sqrt{5}\right)\left(\frac{\int\frac{\sqrt{2}x^2+\sqrt{3+\sqrt{5}}}{2x^4+\sqrt{5}+3}dx}{2\sqrt{2}}-\frac{\int\frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{2x^4+\sqrt{5}+3}dx}{2\sqrt{2}}\right)$$

$$\downarrow 1476$$

$$\frac{1}{5}\left(5-3\sqrt{5}\right)\left(\frac{\frac{\int\frac{1}{x^2-\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}}dx}{2\sqrt{2}}+\frac{\int\frac{1}{x^2+\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}}dx}{2\sqrt{2}}}{2\sqrt{2}}-\frac{\int\frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{2x^4-\sqrt{5}+3}dx}{2\sqrt{2}}\right)+\frac{1}{5}\left(5+3\sqrt{5}\right)\left(\frac{\frac{\int\frac{1}{x^2-\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\sqrt{2}}+\frac{\int\frac{1}{x^2+\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\sqrt{2}}}{2\sqrt{2}}-\frac{\int\frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{2x^4+\sqrt{5}+3}dx}{2\sqrt{2}}\right)$$

$$\downarrow 1082$$

3.379. $\int \frac{x^6}{1+3x^4+x^8}\,dx$

$$\frac{1}{5}\left(5-3\sqrt{5}\right)\left(\frac{\frac{\int \frac{1}{-\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)^2-1}d\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}-\frac{\int \frac{1}{-\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)^2-1}d\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{2x^4-\sqrt{5}+3}}{2\sqrt{2}}\right.$$

$$\frac{1}{5}\left(5+3\sqrt{5}\right)\left(\frac{\frac{\int \frac{1}{-\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)^2-1}d\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\int \frac{1}{-\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)^2-1}d\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{2x^4+\sqrt{5}+3}}{2\sqrt{2}}\right.$$

$$\Big\downarrow \; 217$$

$$\frac{1}{5}\left(5-3\sqrt{5}\right)\left(\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{2x^4-\sqrt{5}+3}dx}{2\sqrt{2}}\right)+$$

$$\frac{1}{5}\left(5+3\sqrt{5}\right)\left(\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{2x^4+\sqrt{5}+3}dx}{2\sqrt{2}}\right)$$

3.379. $\int \frac{x^6}{1+3x^4+x^8}\,dx$

$$\Big\downarrow \; 1479$$

$$\frac{1}{5}\left(5-3\sqrt{5}\right)\left(\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}}{2\sqrt{2}}-\frac{-\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\int -\frac{\sqrt[4]{2\left(3-\sqrt{5}\right)}-2x}{x^2-\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3-\cdots\right.}}\cdots}{\cdots}\right.$$

$$\frac{1}{5}\left(5+3\sqrt{5}\right)\left(\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{2\sqrt{2}}-\frac{-\frac{\int -\frac{\sqrt[4]{2\left(3+\sqrt{5}\right)}-2x}{x^2-\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\;2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\int -\frac{2x\cdots}{x^2+\sqrt[4]{2\cdots}}\cdots}{2\cdots}}{2\sqrt{2}}\right.$$

$$\Big\downarrow \; 25$$

$$\frac{1}{5}\left(5-3\sqrt{5}\right)\left(\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}}{2\sqrt{2}}-\frac{\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\int \frac{\sqrt[4]{2\left(3-\sqrt{5}\right)}-2x}{x^2-\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}}\cdots}{\cdots}\right.$$

$$\frac{1}{5}\left(5+3\sqrt{5}\right)\left(\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{2\sqrt{2}}-\frac{\frac{\int \frac{\sqrt[4]{2\left(3+\sqrt{5}\right)}-2x}{x^2-\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\;2^{3/4}\sqrt[4]{3+\sqrt{5}}}+\frac{\int \frac{2x+\sqrt[4]{2\cdots}}{x^2+\sqrt[4]{2\left(3+\cdots\right.}}\cdots}{2\;2^{3/4}\cdots}}{2\sqrt{2}}\right.$$

3.379. $\int \frac{x^6}{1+3x^4+x^8}\,dx$

$\Big\downarrow$ 1103

$$\frac{1}{5}\left(5-3\sqrt{5}\right)\left(\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}}{2\sqrt{2}}-\frac{\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\cdots\right.}{\cdots}\right.$$

$$\frac{1}{5}\left(5+3\sqrt{5}\right)\left(\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{2\sqrt{2}}-\frac{\frac{\log\left(2x^2+2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{2\,2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\log\left(2x^2-\cdots\right.}{\cdots}}{2\sqrt{2}}\right.$$

input
```
Int[x^6/(1 + 3*x^4 + x^8),x]
```

output
```
((5 - 3*Sqrt[5])*((-(ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)]/(2^(3/4)*
(3 - Sqrt[5])^(1/4))) + ArcTan[1 + (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)]/(2^(3/
4)*(3 - Sqrt[5])^(1/4)))/(2*Sqrt[2]) - (-1/4*(((3 + Sqrt[5])/2)^(1/4)*Log[
Sqrt[2*(3 - Sqrt[5])] - 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2]) + (((3 + Sqr
t[5])/2)^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])] + 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2
*x^2])/4)/(2*Sqrt[2])))/5 + ((5 + 3*Sqrt[5])*((-(ArcTan[1 - (2^(3/4)*x)/(3
 + Sqrt[5])^(1/4)]/(2^(3/4)*(3 + Sqrt[5])^(1/4))) + ArcTan[1 + (2^(3/4)*x)
/(3 + Sqrt[5])^(1/4)]/(2^(3/4)*(3 + Sqrt[5])^(1/4)))/(2*Sqrt[2]) - (-1/2*L
og[Sqrt[2*(3 + Sqrt[5])] - 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2]/(2^(3/4)*(
3 + Sqrt[5])^(1/4)) + Log[Sqrt[2*(3 + Sqrt[5])] + 2*(2*(3 + Sqrt[5]))^(1/4
)*x + 2*x^2]/(2*2^(3/4)*(3 + Sqrt[5])^(1/4)))/(2*Sqrt[2])))/5
```

3.379. $\int \frac{x^6}{1+3x^4+x^8}\,dx$

3.379.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.379. $\int \frac{x^6}{1+3x^4+x^8}\,dx$

rule 1710
```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^n/2)*(b/q + 1)   Int[(d*x)^(m
- n)/(b/2 + q/2 + c*x^n), x], x] - Simp[(d^n/2)*(b/q - 1)   Int[(d*x)^(m -
n)/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

3.379.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.09

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}+3\_Z^{4}+1\right)}\frac{\_R^{6}\ln\left(x-\_R\right)}{2\_R^{7}+3\_R^{3}}\right)}{4}$	40
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}+3\_Z^{4}+1\right)}\frac{\_R^{6}\ln\left(x-\_R\right)}{2\_R^{7}+3\_R^{3}}\right)}{4}$	40

input
```
int(x^6/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(_R^6/(2*_R^7+3*_R^3)*ln(x-_R),_R=RootOf(_Z^8+3*_Z^4+1))
```

3.379.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 361, normalized size of antiderivative = 0.84

$$\int \frac{x^6}{1+3x^4+x^8}\,dx$$
$$= \frac{1}{20}\sqrt{5}\sqrt{-\sqrt{4\sqrt{5}-9}}\log\left(\sqrt{4\sqrt{5}-9}\left(3\sqrt{5}+7\right)\sqrt{-\sqrt{4\sqrt{5}-9}}+2x\right)$$
$$-\frac{1}{20}\sqrt{5}\sqrt{-\sqrt{4\sqrt{5}-9}}\log\left(-\sqrt{4\sqrt{5}-9}\left(3\sqrt{5}+7\right)\sqrt{-\sqrt{4\sqrt{5}-9}}+2x\right)$$
$$+\frac{1}{20}\sqrt{5}\sqrt{-\sqrt{-4\sqrt{5}-9}}\log\left(\left(3\sqrt{5}-7\right)\sqrt{-4\sqrt{5}-9}\sqrt{-\sqrt{-4\sqrt{5}-9}}+2x\right)$$
$$-\frac{1}{20}\sqrt{5}\sqrt{-\sqrt{-4\sqrt{5}-9}}\log\left(-\left(3\sqrt{5}-7\right)\sqrt{-4\sqrt{5}-9}\sqrt{-\sqrt{-4\sqrt{5}-9}}+2x\right)$$
$$-\frac{1}{20}\sqrt{5}\left(4\sqrt{5}-9\right)^{\frac{1}{4}}\log\left(\left(4\sqrt{5}-9\right)^{\frac{3}{4}}\left(3\sqrt{5}+7\right)+2x\right)$$
$$+\frac{1}{20}\sqrt{5}\left(4\sqrt{5}-9\right)^{\frac{1}{4}}\log\left(-\left(4\sqrt{5}-9\right)^{\frac{3}{4}}\left(3\sqrt{5}+7\right)+2x\right)$$
$$-\frac{1}{20}\sqrt{5}\left(-4\sqrt{5}-9\right)^{\frac{1}{4}}\log\left(\left(3\sqrt{5}-7\right)\left(-4\sqrt{5}-9\right)^{\frac{3}{4}}+2x\right)$$
$$+\frac{1}{20}\sqrt{5}\left(-4\sqrt{5}-9\right)^{\frac{1}{4}}\log\left(-\left(3\sqrt{5}-7\right)\left(-4\sqrt{5}-9\right)^{\frac{3}{4}}+2x\right)$$

input
```
integrate(x^6/(x^8+3*x^4+1),x, algorithm="fricas")
```

output
```
1/20*sqrt(5)*sqrt(-sqrt(4*sqrt(5) - 9))*log(sqrt(4*sqrt(5) - 9)*(3*sqrt(5)
 + 7)*sqrt(-sqrt(4*sqrt(5) - 9)) + 2*x) - 1/20*sqrt(5)*sqrt(-sqrt(4*sqrt(5
) - 9))*log(-sqrt(4*sqrt(5) - 9)*(3*sqrt(5) + 7)*sqrt(-sqrt(4*sqrt(5) - 9)
) + 2*x) + 1/20*sqrt(5)*sqrt(-sqrt(-4*sqrt(5) - 9))*log((3*sqrt(5) - 7)*sq
rt(-4*sqrt(5) - 9)*sqrt(-sqrt(-4*sqrt(5) - 9)) + 2*x) - 1/20*sqrt(5)*sqrt(
-sqrt(-4*sqrt(5) - 9))*log(-(3*sqrt(5) - 7)*sqrt(-4*sqrt(5) - 9)*sqrt(-sqr
t(-4*sqrt(5) - 9)) + 2*x) - 1/20*sqrt(5)*(4*sqrt(5) - 9)^(1/4)*log((4*sqrt
(5) - 9)^(3/4)*(3*sqrt(5) + 7) + 2*x) + 1/20*sqrt(5)*(4*sqrt(5) - 9)^(1/4)
*log(-(4*sqrt(5) - 9)^(3/4)*(3*sqrt(5) + 7) + 2*x) - 1/20*sqrt(5)*(-4*sqrt
(5) - 9)^(1/4)*log((3*sqrt(5) - 7)*(-4*sqrt(5) - 9)^(3/4) + 2*x) + 1/20*sq
rt(5)*(-4*sqrt(5) - 9)^(1/4)*log(-(3*sqrt(5) - 7)*(-4*sqrt(5) - 9)^(3/4) +
 2*x)
```

3.379.6 Sympy [A] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.06

$$\int \frac{x^6}{1+3x^4+x^8}\,dx$$
$$= \text{RootSum}\left(40960000t^8+115200t^4+1,\left(t\mapsto t\log\left(-1792000t^7-4920t^3+x\right)\right)\right)$$

input
```
integrate(x**6/(x**8+3*x**4+1),x)
```

output
```
RootSum(40960000*_t**8 + 115200*_t**4 + 1, Lambda(_t, _t*log(-1792000*_t**
7 - 4920*_t**3 + x)))
```

3.379.7 Maxima [F]

$$\int \frac{x^6}{1+3x^4+x^8}\,dx=\int \frac{x^6}{x^8+3\,x^4+1}\,dx$$

input
```
integrate(x^6/(x^8+3*x^4+1),x, algorithm="maxima")
```

output
```
integrate(x^6/(x^8 + 3*x^4 + 1), x)
```

3.379.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.55

$$\begin{aligned}\int \frac{x^6}{1+3x^4+x^8}\,dx =& \frac{1}{80}\left(\pi+4\arctan\left(x\sqrt{\sqrt{5}-1}-1\right)\right)\sqrt{10\sqrt{5}+20}\\ &-\frac{1}{80}\left(\pi+4\arctan\left(-x\sqrt{\sqrt{5}-1}-1\right)\right)\sqrt{10\sqrt{5}+20}\\ &-\frac{1}{80}\left(\pi+4\arctan\left(x\sqrt{\sqrt{5}+1}+1\right)\right)\sqrt{10\sqrt{5}-20}\\ &+\frac{1}{80}\left(\pi+4\arctan\left(-x\sqrt{\sqrt{5}+1}+1\right)\right)\sqrt{10\sqrt{5}-20}\\ &-\frac{1}{40}\sqrt{10\sqrt{5}+20}\log\left(400\left(x+\sqrt{\sqrt{5}+1}\right)^2+400\,x^2\right)\\ &+\frac{1}{40}\sqrt{10\sqrt{5}+20}\log\left(400\left(x-\sqrt{\sqrt{5}+1}\right)^2+400\,x^2\right)\\ &+\frac{1}{40}\sqrt{10\sqrt{5}-20}\log\left(10000\left(x+\sqrt{\sqrt{5}-1}\right)^2+10000\,x^2\right)\\ &-\frac{1}{40}\sqrt{10\sqrt{5}-20}\log\left(10000\left(x-\sqrt{\sqrt{5}-1}\right)^2+10000\,x^2\right)\end{aligned}$$

input
```
integrate(x^6/(x^8+3*x^4+1),x, algorithm="giac")
```

output
```
1/80*(pi + 4*arctan(x*sqrt(sqrt(5) - 1) - 1))*sqrt(10*sqrt(5) + 20) - 1/80
*(pi + 4*arctan(-x*sqrt(sqrt(5) - 1) - 1))*sqrt(10*sqrt(5) + 20) - 1/80*(p
i + 4*arctan(x*sqrt(sqrt(5) + 1) + 1))*sqrt(10*sqrt(5) - 20) + 1/80*(pi +
4*arctan(-x*sqrt(sqrt(5) + 1) + 1))*sqrt(10*sqrt(5) - 20) - 1/40*sqrt(10*s
qrt(5) + 20)*log(400*(x + sqrt(sqrt(5) + 1))^2 + 400*x^2) + 1/40*sqrt(10*s
qrt(5) + 20)*log(400*(x - sqrt(sqrt(5) + 1))^2 + 400*x^2) + 1/40*sqrt(10*s
qrt(5) - 20)*log(10000*(x + sqrt(sqrt(5) - 1))^2 + 10000*x^2) - 1/40*sqrt(
10*sqrt(5) - 20)*log(10000*(x - sqrt(sqrt(5) - 1))^2 + 10000*x^2)
```

3.379.9 Mupad [B] (verification not implemented)

Time = 8.33 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.35

$$\int \frac{x^6}{1+3x^4+x^8}\,dx = \frac{\sqrt{5}\,\mathrm{atan}\left(\frac{16\,x\,\left(-4\,\sqrt{5}-9\right)^{1/4}}{8\,\sqrt{5}+24}\right)\,\left(-4\,\sqrt{5}-9\right)^{1/4}}{10} + \frac{\sqrt{5}\,\mathrm{atan}\left(\frac{16\,x\,\left(4\,\sqrt{5}-9\right)^{1/4}}{8\,\sqrt{5}-24}\right)\,\left(4\,\sqrt{5}-9\right)^{1/4}}{10} + \frac{\sqrt{5}\,\mathrm{atan}\left(\frac{x\,\left(-4\,\sqrt{5}-9\right)^{1/4}\,16\mathrm{i}}{8\,\sqrt{5}+24}\right)\,\left(-4\,\sqrt{5}-9\right)^{1/4}\,1\mathrm{i}}{10} + \frac{\sqrt{5}\,\mathrm{atan}\left(\frac{x\,\left(4\,\sqrt{5}-9\right)^{1/4}\,16\mathrm{i}}{8\,\sqrt{5}-24}\right)\,\left(4\,\sqrt{5}-9\right)^{1/4}\,1\mathrm{i}}{10}$$

input
```
int(x^6/(3*x^4 + x^8 + 1),x)
```

output
```
(5^(1/2)*atan((16*x*(- 4*5^(1/2) - 9)^(1/4))/(8*5^(1/2) + 24))*(- 4*5^(1/2
) - 9)^(1/4))/10 + (5^(1/2)*atan((16*x*(4*5^(1/2) - 9)^(1/4))/(8*5^(1/2) -
 24))*(4*5^(1/2) - 9)^(1/4))/10 + (5^(1/2)*atan((x*(- 4*5^(1/2) - 9)^(1/4)
*16i)/(8*5^(1/2) + 24))*(- 4*5^(1/2) - 9)^(1/4)*1i)/10 + (5^(1/2)*atan((x*
(4*5^(1/2) - 9)^(1/4)*16i)/(8*5^(1/2) - 24))*(4*5^(1/2) - 9)^(1/4)*1i)/10
```

3.380 $\int \frac{x^4}{1+3x^4+x^8}\,dx$

3.380.1 Optimal result

Integrand size = 16, antiderivative size = 451

$$\int \frac{x^4}{1+3x^4+x^8}\,dx = \frac{\sqrt[4]{3-\sqrt{5}}\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}} - \frac{\sqrt[4]{3-\sqrt{5}}\arctan\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}} - \frac{\sqrt[4]{3+\sqrt{5}}\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}} + \frac{\sqrt[4]{3+\sqrt{5}}\arctan\left(1+\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}} + \frac{\sqrt[4]{3-\sqrt{5}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{4\,2^{3/4}\sqrt{5}} - \frac{\sqrt[4]{3-\sqrt{5}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{4\,2^{3/4}\sqrt{5}} - \frac{\sqrt[4]{3+\sqrt{5}}\log\left(\sqrt{2\left(3+\sqrt{5}\right)}-2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+2x^2\right)}{4\,2^{3/4}\sqrt{5}} + \frac{\sqrt[4]{3+\sqrt{5}}\log\left(\sqrt{2\left(3+\sqrt{5}\right)}+2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+2x^2\right)}{4\,2^{3/4}\sqrt{5}}$$

output
```
-1/20*arctan(-1+2^(3/4)*x/(3-5^(1/2))^(1/4))*(3-5^(1/2))^(1/4)*2^(1/4)*5^(
1/2)-1/20*arctan(1+2^(3/4)*x/(3-5^(1/2))^(1/4))*(3-5^(1/2))^(1/4)*2^(1/4)*
5^(1/2)+1/40*ln(2*x^2-2*2^(1/4)*x*(3-5^(1/2))^(1/4)+5^(1/2)-1)*(3-5^(1/2))
^(1/4)*2^(1/4)*5^(1/2)-1/40*ln(2*x^2+2*2^(1/4)*x*(3-5^(1/2))^(1/4)+5^(1/2)
-1)*(3-5^(1/2))^(1/4)*2^(1/4)*5^(1/2)+1/20*arctan(-1+2^(3/4)*x/(3+5^(1/2))
^(1/4))*(3+5^(1/2))^(1/4)*2^(1/4)*5^(1/2)+1/20*arctan(1+2^(3/4)*x/(3+5^(1/
2))^(1/4))*(3+5^(1/2))^(1/4)*2^(1/4)*5^(1/2)-1/40*ln(2*x^2-2*2^(1/4)*x*(3+
5^(1/2))^(1/4)+5^(1/2)+1)*(3+5^(1/2))^(1/4)*2^(1/4)*5^(1/2)+1/40*ln(2*x^2+
2*2^(1/4)*x*(3+5^(1/2))^(1/4)+5^(1/2)+1)*(3+5^(1/2))^(1/4)*2^(1/4)*5^(1/2)
```

3.380.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.09

$$\int \frac{x^4}{1+3x^4+x^8}\,dx=\frac{1}{4}\text{RootSum}\left[1+3\#1^4+\#1^8\&,\frac{\log(x-\#1)\#1}{3+2\#1^4}\&\right]$$

input
```
Integrate[x^4/(1 + 3*x^4 + x^8),x]
```

output
```
RootSum[1 + 3*#1^4 + #1^8 & , (Log[x - #1]*#1)/(3 + 2*#1^4) & ]/4
```

3.380.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {1710, 755, 27, 1476, 1082, 217, 1479, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{x^8+3x^4+1}\,dx$$

$$\Big\downarrow \text{1710}$$

$$\frac{1}{10}\left(5-3\sqrt{5}\right)\int \frac{1}{x^4+\frac{1}{2}\left(3-\sqrt{5}\right)}dx+\frac{1}{10}\left(5+3\sqrt{5}\right)\int \frac{1}{x^4+\frac{1}{2}\left(3+\sqrt{5}\right)}dx$$

$$\Big\downarrow \text{755}$$

$$\frac{1}{10}\left(5-3\sqrt{5}\right)\left(\frac{\int\frac{2\left(\sqrt{3-\sqrt{5}}-\sqrt{2}x^2\right)}{2x^4-\sqrt{5}+3}dx}{2\sqrt{3-\sqrt{5}}}+\frac{\int\frac{2\left(\sqrt{2}x^2+\sqrt{3-\sqrt{5}}\right)}{2x^4-\sqrt{5}+3}dx}{2\sqrt{3-\sqrt{5}}}\right)+\frac{1}{10}\left(5+3\sqrt{5}\right)\left(\frac{\int\frac{2\left(\sqrt{3+\sqrt{5}}-\sqrt{2}x^2\right)}{2x^4+\sqrt{5}+3}dx}{2\sqrt{3+\sqrt{5}}}+\frac{\int\frac{2\left(\sqrt{2}x^2+\sqrt{3+\sqrt{5}}\right)}{2x^4+\sqrt{5}+3}dx}{2\sqrt{3+\sqrt{5}}}\right)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{10}\left(5-3\sqrt{5}\right)\left(\frac{\int\frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{2x^4-\sqrt{5}+3}dx}{\sqrt{3-\sqrt{5}}}+\frac{\int\frac{\sqrt{2}x^2+\sqrt{3-\sqrt{5}}}{2x^4-\sqrt{5}+3}dx}{\sqrt{3-\sqrt{5}}}\right)+\frac{1}{10}\left(5+3\sqrt{5}\right)\left(\frac{\int\frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{2x^4+\sqrt{5}+3}dx}{\sqrt{3+\sqrt{5}}}+\frac{\int\frac{\sqrt{2}x^2+\sqrt{3+\sqrt{5}}}{2x^4+\sqrt{5}+3}dx}{\sqrt{3+\sqrt{5}}}\right)$$

$$\Big\downarrow \; 1476$$

$$\frac{1}{10}\left(5-3\sqrt{5}\right)\left(\frac{\frac{\int\frac{1}{x^2-\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}}dx}{2\sqrt{2}}+\frac{\int\frac{1}{x^2+\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}}dx}{2\sqrt{2}}}{\sqrt{3-\sqrt{5}}}+\frac{\int\frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{2x^4-\sqrt{5}+3}dx}{\sqrt{3-\sqrt{5}}}\right)+\frac{1}{10}\left(5+3\sqrt{5}\right)\left(\frac{\frac{\int\frac{1}{x^2-\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\sqrt{2}}+\frac{\int\frac{1}{x^2+\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\sqrt{2}}}{\sqrt{3+\sqrt{5}}}+\frac{\int\frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{2x^4+\sqrt{5}+3}dx}{\sqrt{3+\sqrt{5}}}\right)$$

$$\Big\downarrow \; 1082$$

3.380. $\int \frac{x^4}{1+3x^4+x^8}\,dx$

$$\frac{1}{10}\left(5-3\sqrt{5}\right)\left(\frac{\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{2x^4-\sqrt{5}+3}dx}{\sqrt{3-\sqrt{5}}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)^2-1}d\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}-\frac{\int \frac{1}{-\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)^2-1}d\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\cdots\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}\cdots}{\sqrt{3-\sqrt{5}}}\right.$$

$$\frac{1}{10}\left(5+3\sqrt{5}\right)\left(\frac{\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{2x^4+\sqrt{5}+3}dx}{\sqrt{3+\sqrt{5}}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)^2-1}d\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\int \frac{1}{-\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)^2-1}d\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\cdots\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}\cdots}{\sqrt{3+\sqrt{5}}}\right.$$

$$\Big\downarrow \; 217$$

$$\frac{1}{10}\left(5-3\sqrt{5}\right)\left(\frac{\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{2x^4-\sqrt{5}+3}dx}{\sqrt{3-\sqrt{5}}}+\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}}{\sqrt{3-\sqrt{5}}}\right)+$$

$$\frac{1}{10}\left(5+3\sqrt{5}\right)\left(\frac{\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{2x^4+\sqrt{5}+3}dx}{\sqrt{3+\sqrt{5}}}+\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{\sqrt{3+\sqrt{5}}}\right)$$

3.380. $\int \frac{x^4}{1+3x^4+x^8}\,dx$

$$\downarrow 1479$$

$$\frac{1}{10}\left(5-3\sqrt{5}\right)\left(\frac{-\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\int -\frac{\sqrt[4]{2\left(3-\sqrt{5}\right)}-2x}{x^2-\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}}dx-\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\int -\frac{2x+\sqrt[4]{2\left(3-\right.}}{x^2+\sqrt[4]{2\left(3-\sqrt{5}\right)}x}}{\sqrt{3-\sqrt{5}}}\right.$$

$$\frac{1}{10}\left(5+3\sqrt{5}\right)\left(\frac{-\frac{\int -\frac{\sqrt[4]{2\left(3+\sqrt{5}\right)}-2x}{x^2-\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\int -\frac{2x+\sqrt[4]{2\left(3+\sqrt{5}\right)}}{x^2+\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{\sqrt{3+\sqrt{5}}}+\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{\sqrt{3}}\right.$$

$$\downarrow 25$$

$$\frac{1}{10}\left(5-3\sqrt{5}\right)\left(\frac{\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\int \frac{\sqrt[4]{2\left(3-\sqrt{5}\right)}-2x}{x^2-\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}}dx+\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\int \frac{2x+\sqrt[4]{2\left(3-\sqrt{5}\right)}}{x^2+\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(\right.}}}{\sqrt{3-\sqrt{5}}}\right.$$

$$\frac{1}{10}\left(5+3\sqrt{5}\right)\left(\frac{\frac{\int \frac{\sqrt[4]{2\left(3+\sqrt{5}\right)}-2x}{x^2-\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}+\frac{\int \frac{2x+\sqrt[4]{2\left(3+\sqrt{5}\right)}}{x^2+\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{\sqrt{3+\sqrt{5}}}+\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\ldots}{\sqrt{3+\sqrt{}}}\right.$$

3.380. $\int \frac{x^4}{1+3x^4+x^8}\,dx$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{10}\left(5-3\sqrt{5}\right)\left(\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}}{\sqrt{3-\sqrt{5}}}+\frac{\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\right.}{}\right.$$

$$\frac{1}{10}\left(5+3\sqrt{5}\right)\left(\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{\sqrt{3+\sqrt{5}}}+\frac{\frac{\log\left(2x^2+2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\log\left(2x^2\right.}{}}{\sqrt{3+\sqrt{5}}}\right.$$

input
```
Int[x^4/(1 + 3*x^4 + x^8),x]
```

output
```
((5 - 3*Sqrt[5])*((-(ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)]/(2^(3/4)*
(3 - Sqrt[5])^(1/4))) + ArcTan[1 + (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)]/(2^(3/
4)*(3 - Sqrt[5])^(1/4)))/Sqrt[3 - Sqrt[5]] + (-1/4*(((3 + Sqrt[5])/2)^(1/4
)*Log[Sqrt[2*(3 - Sqrt[5])] - 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2]) + (((3
 + Sqrt[5])/2)^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])] + 2*(2*(3 - Sqrt[5]))^(1/4)
*x + 2*x^2])/4)/Sqrt[3 - Sqrt[5]]))/10 + ((5 + 3*Sqrt[5])*((-(ArcTan[1 - (
2^(3/4)*x)/(3 + Sqrt[5])^(1/4)]/(2^(3/4)*(3 + Sqrt[5])^(1/4))) + ArcTan[1
+ (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)]/(2^(3/4)*(3 + Sqrt[5])^(1/4)))/Sqrt[3 +
 Sqrt[5]] + (-1/2*Log[Sqrt[2*(3 + Sqrt[5])] - 2*(2*(3 + Sqrt[5]))^(1/4)*x
+ 2*x^2]/(2^(3/4)*(3 + Sqrt[5])^(1/4)) + Log[Sqrt[2*(3 + Sqrt[5])] + 2*(2*
(3 + Sqrt[5]))^(1/4)*x + 2*x^2]/(2*2^(3/4)*(3 + Sqrt[5])^(1/4)))/Sqrt[3 +
Sqrt[5]]))/10
```

3.380. $\int \frac{x^4}{1+3x^4+x^8}\,dx$

3.380.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.380. $\int \frac{x^4}{1+3x^4+x^8}\,dx$

rule 1710
```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^n/2)*(b/q + 1)   Int[(d*x)^(m
- n)/(b/2 + q/2 + c*x^n), x], x] - Simp[(d^n/2)*(b/q - 1)   Int[(d*x)^(m -
n)/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

3.380.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.09

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^8+3\_Z^4+1\right)}\frac{\_R^4\ln\left(x-\_R\right)}{2\_R^7+3\_R^3}\right)}{4}$	40
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^8+3\_Z^4+1\right)}\frac{\_R^4\ln\left(x-\_R\right)}{2\_R^7+3\_R^3}\right)}{4}$	40

input
```
int(x^4/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(_R^4/(2*_R^7+3*_R^3)*ln(x-_R),_R=RootOf(_Z^8+3*_Z^4+1))
```

3.380.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 373, normalized size of antiderivative = 0.83

$$\int \frac{x^4}{1+3x^4+x^8}\,dx = -\frac{1}{40}\sqrt{10}\sqrt{\sqrt{2}\sqrt{\sqrt{5}-3}}\log\left(\sqrt{10}\sqrt{5}\sqrt{\sqrt{2}\sqrt{\sqrt{5}-3}}+10\,x\right)$$
$$+\frac{1}{40}\sqrt{10}\sqrt{\sqrt{2}\sqrt{\sqrt{5}-3}}\log\left(-\sqrt{10}\sqrt{5}\sqrt{\sqrt{2}\sqrt{\sqrt{5}-3}}+10\,x\right)$$
$$-\frac{1}{40}\sqrt{10}\sqrt{-\sqrt{2}\sqrt{\sqrt{5}-3}}\log\left(\sqrt{10}\sqrt{5}\sqrt{-\sqrt{2}\sqrt{\sqrt{5}-3}}+10\,x\right)$$
$$+\frac{1}{40}\sqrt{10}\sqrt{-\sqrt{2}\sqrt{\sqrt{5}-3}}\log\left(-\sqrt{10}\sqrt{5}\sqrt{-\sqrt{2}\sqrt{\sqrt{5}-3}}+10\,x\right)$$
$$+\frac{1}{40}\sqrt{10}\sqrt{\sqrt{2}\sqrt{-\sqrt{5}-3}}\log\left(\sqrt{10}\sqrt{5}\sqrt{\sqrt{2}\sqrt{-\sqrt{5}-3}}+10\,x\right)$$
$$-\frac{1}{40}\sqrt{10}\sqrt{\sqrt{2}\sqrt{-\sqrt{5}-3}}\log\left(-\sqrt{10}\sqrt{5}\sqrt{\sqrt{2}\sqrt{-\sqrt{5}-3}}+10\,x\right)$$
$$+\frac{1}{40}\sqrt{10}\sqrt{-\sqrt{2}\sqrt{-\sqrt{5}-3}}\log\left(\sqrt{10}\sqrt{5}\sqrt{-\sqrt{2}\sqrt{-\sqrt{5}-3}}+10\,x\right)$$
$$-\frac{1}{40}\sqrt{10}\sqrt{-\sqrt{2}\sqrt{-\sqrt{5}-3}}\log\left(-\sqrt{10}\sqrt{5}\sqrt{-\sqrt{2}\sqrt{-\sqrt{5}-3}}+10\,x\right)$$

input
```
integrate(x^4/(x^8+3*x^4+1),x, algorithm="fricas")
```

output
```
-1/40*sqrt(10)*sqrt(sqrt(2)*sqrt(sqrt(5) - 3))*log(sqrt(10)*sqrt(5)*sqrt(s
qrt(2)*sqrt(sqrt(5) - 3)) + 10*x) + 1/40*sqrt(10)*sqrt(sqrt(2)*sqrt(sqrt(5
) - 3))*log(-sqrt(10)*sqrt(5)*sqrt(sqrt(2)*sqrt(sqrt(5) - 3)) + 10*x) - 1/
40*sqrt(10)*sqrt(-sqrt(2)*sqrt(sqrt(5) - 3))*log(sqrt(10)*sqrt(5)*sqrt(-sq
rt(2)*sqrt(sqrt(5) - 3)) + 10*x) + 1/40*sqrt(10)*sqrt(-sqrt(2)*sqrt(sqrt(5
) - 3))*log(-sqrt(10)*sqrt(5)*sqrt(-sqrt(2)*sqrt(sqrt(5) - 3)) + 10*x) + 1
/40*sqrt(10)*sqrt(sqrt(2)*sqrt(-sqrt(5) - 3))*log(sqrt(10)*sqrt(5)*sqrt(sq
rt(2)*sqrt(-sqrt(5) - 3)) + 10*x) - 1/40*sqrt(10)*sqrt(sqrt(2)*sqrt(-sqrt(
5) - 3))*log(-sqrt(10)*sqrt(5)*sqrt(sqrt(2)*sqrt(-sqrt(5) - 3)) + 10*x) +
1/40*sqrt(10)*sqrt(-sqrt(2)*sqrt(-sqrt(5) - 3))*log(sqrt(10)*sqrt(5)*sqrt(
-sqrt(2)*sqrt(-sqrt(5) - 3)) + 10*x) - 1/40*sqrt(10)*sqrt(-sqrt(2)*sqrt(-s
qrt(5) - 3))*log(-sqrt(10)*sqrt(5)*sqrt(-sqrt(2)*sqrt(-sqrt(5) - 3)) + 10*
x)
```

3.380.6 Sympy [A] (verification not implemented)

Time = 0.86 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.05

$$\int \frac{x^4}{1+3x^4+x^8}\,dx = \text{RootSum}\left(40960000t^8+19200t^4+1,\left(t\mapsto t\log\left(-51200t^5-12t+x\right)\right)\right)$$

input
```
integrate(x**4/(x**8+3*x**4+1),x)
```

output
```
RootSum(40960000*_t**8 + 19200*_t**4 + 1, Lambda(_t, _t*log(-51200*_t**5 -
 12*_t + x)))
```

3.380.7 Maxima [F]

$$\int \frac{x^4}{1+3x^4+x^8}\,dx = \int \frac{x^4}{x^8+3\,x^4+1}\,dx$$

input
```
integrate(x^4/(x^8+3*x^4+1),x, algorithm="maxima")
```

output
```
integrate(x^4/(x^8 + 3*x^4 + 1), x)
```

3.380.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.53

$$\begin{aligned}\int \frac{x^4}{1+3x^4+x^8}\,dx = {} & \frac{1}{80}\left(\pi+4\arctan\left(x\sqrt{\sqrt{5}-1}+1\right)\right)\sqrt{5\sqrt{5}+5} \\ & -\frac{1}{80}\left(\pi+4\arctan\left(-x\sqrt{\sqrt{5}-1}+1\right)\right)\sqrt{5\sqrt{5}+5} \\ & -\frac{1}{80}\left(\pi+4\arctan\left(x\sqrt{\sqrt{5}+1}-1\right)\right)\sqrt{5\sqrt{5}-5} \\ & +\frac{1}{80}\left(\pi+4\arctan\left(-x\sqrt{\sqrt{5}+1}-1\right)\right)\sqrt{5\sqrt{5}-5} \\ & +\frac{1}{40}\sqrt{5\sqrt{5}+5}\log\left(625\left(x+\sqrt{\sqrt{5}+1}\right)^2+625\,x^2\right) \\ & -\frac{1}{40}\sqrt{5\sqrt{5}+5}\log\left(625\left(x-\sqrt{\sqrt{5}+1}\right)^2+625\,x^2\right) \\ & -\frac{1}{40}\sqrt{5\sqrt{5}-5}\log\left(4225\left(x+\sqrt{\sqrt{5}-1}\right)^2+4225\,x^2\right) \\ & +\frac{1}{40}\sqrt{5\sqrt{5}-5}\log\left(4225\left(x-\sqrt{\sqrt{5}-1}\right)^2+4225\,x^2\right)\end{aligned}$$

input
```
integrate(x^4/(x^8+3*x^4+1),x, algorithm="giac")
```

output
```
1/80*(pi + 4*arctan(x*sqrt(sqrt(5) - 1) + 1))*sqrt(5*sqrt(5) + 5) - 1/80*(
pi + 4*arctan(-x*sqrt(sqrt(5) - 1) + 1))*sqrt(5*sqrt(5) + 5) - 1/80*(pi +
4*arctan(x*sqrt(sqrt(5) + 1) - 1))*sqrt(5*sqrt(5) - 5) + 1/80*(pi + 4*arct
an(-x*sqrt(sqrt(5) + 1) - 1))*sqrt(5*sqrt(5) - 5) + 1/40*sqrt(5*sqrt(5) +
5)*log(625*(x + sqrt(sqrt(5) + 1))^2 + 625*x^2) - 1/40*sqrt(5*sqrt(5) + 5)
*log(625*(x - sqrt(sqrt(5) + 1))^2 + 625*x^2) - 1/40*sqrt(5*sqrt(5) - 5)*l
og(4225*(x + sqrt(sqrt(5) - 1))^2 + 4225*x^2) + 1/40*sqrt(5*sqrt(5) - 5)*l
og(4225*(x - sqrt(sqrt(5) - 1))^2 + 4225*x^2)
```

3.380.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 454, normalized size of antiderivative = 1.01

$$\int \frac{x^4}{1+3x^4+x^8}\,dx$$
$$= \frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{3\,2^{3/4}\,x\left(-\sqrt{5}-3\right)^{1/4}}{2\left(\frac{3\sqrt{2}\sqrt{-\sqrt{5}-3}}{2}-\frac{\sqrt{2}\sqrt{5}\sqrt{-\sqrt{5}-3}}{2}\right)}-\frac{2^{3/4}\sqrt{5}\,x\left(-\sqrt{5}-3\right)^{1/4}}{2\left(\frac{3\sqrt{2}\sqrt{-\sqrt{5}-3}}{2}-\frac{\sqrt{2}\sqrt{5}\sqrt{-\sqrt{5}-3}}{2}\right)}\right)\left(-\sqrt{5}-3\right)^{1/4}}{20}$$
$$-\frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\left(-\sqrt{5}-3\right)^{1/4}3\mathrm{i}}{2\left(\frac{3\sqrt{2}\sqrt{-\sqrt{5}-3}}{2}-\frac{\sqrt{2}\sqrt{5}\sqrt{-\sqrt{5}-3}}{2}\right)}-\frac{2^{3/4}\sqrt{5}\,x\left(-\sqrt{5}-3\right)^{1/4}1\mathrm{i}}{2\left(\frac{3\sqrt{2}\sqrt{-\sqrt{5}-3}}{2}-\frac{\sqrt{2}\sqrt{5}\sqrt{-\sqrt{5}-3}}{2}\right)}\right)\left(-\sqrt{5}-3\right)^{1/4}1\mathrm{i}}{20}$$
$$-\frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{3\,2^{3/4}\,x\left(\sqrt{5}-3\right)^{1/4}}{2\left(\frac{3\sqrt{2}\sqrt{\sqrt{5}-3}}{2}+\frac{\sqrt{2}\sqrt{5}\sqrt{\sqrt{5}-3}}{2}\right)}+\frac{2^{3/4}\sqrt{5}\,x\left(\sqrt{5}-3\right)^{1/4}}{2\left(\frac{3\sqrt{2}\sqrt{\sqrt{5}-3}}{2}+\frac{\sqrt{2}\sqrt{5}\sqrt{\sqrt{5}-3}}{2}\right)}\right)\left(\sqrt{5}-3\right)^{1/4}}{20}$$
$$+\frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\left(\sqrt{5}-3\right)^{1/4}3\mathrm{i}}{2\left(\frac{3\sqrt{2}\sqrt{\sqrt{5}-3}}{2}+\frac{\sqrt{2}\sqrt{5}\sqrt{\sqrt{5}-3}}{2}\right)}+\frac{2^{3/4}\sqrt{5}\,x\left(\sqrt{5}-3\right)^{1/4}1\mathrm{i}}{2\left(\frac{3\sqrt{2}\sqrt{\sqrt{5}-3}}{2}+\frac{\sqrt{2}\sqrt{5}\sqrt{\sqrt{5}-3}}{2}\right)}\right)\left(\sqrt{5}-3\right)^{1/4}1\mathrm{i}}{20}$$

input
```
int(x^4/(3*x^4 + x^8 + 1),x)
```

output
```
(2^(3/4)*5^(1/2)*atan((3*2^(3/4)*x*(- 5^(1/2) - 3)^(1/4))/(2*((3*2^(1/2)*(
- 5^(1/2) - 3)^(1/2))/2 - (2^(1/2)*5^(1/2)*(- 5^(1/2) - 3)^(1/2))/2)) - (2
^(3/4)*5^(1/2)*x*(- 5^(1/2) - 3)^(1/4))/(2*((3*2^(1/2)*(- 5^(1/2) - 3)^(1/
2))/2 - (2^(1/2)*5^(1/2)*(- 5^(1/2) - 3)^(1/2))/2)))*(- 5^(1/2) - 3)^(1/4)
)/20 - (2^(3/4)*5^(1/2)*atan((2^(3/4)*x*(- 5^(1/2) - 3)^(1/4)*3i)/(2*((3*2
^(1/2)*(- 5^(1/2) - 3)^(1/2))/2 - (2^(1/2)*5^(1/2)*(- 5^(1/2) - 3)^(1/2))/
2)) - (2^(3/4)*5^(1/2)*x*(- 5^(1/2) - 3)^(1/4)*1i)/(2*((3*2^(1/2)*(- 5^(1/
2) - 3)^(1/2))/2 - (2^(1/2)*5^(1/2)*(- 5^(1/2) - 3)^(1/2))/2)))*(- 5^(1/2)
 - 3)^(1/4)*1i)/20 - (2^(3/4)*5^(1/2)*atan((3*2^(3/4)*x*(5^(1/2) - 3)^(1/4
))/(2*((3*2^(1/2)*(5^(1/2) - 3)^(1/2))/2 + (2^(1/2)*5^(1/2)*(5^(1/2) - 3)^
(1/2))/2)) + (2^(3/4)*5^(1/2)*x*(5^(1/2) - 3)^(1/4))/(2*((3*2^(1/2)*(5^(1/
2) - 3)^(1/2))/2 + (2^(1/2)*5^(1/2)*(5^(1/2) - 3)^(1/2))/2)))*(5^(1/2) - 3
)^(1/4))/20 + (2^(3/4)*5^(1/2)*atan((2^(3/4)*x*(5^(1/2) - 3)^(1/4)*3i)/(2*
((3*2^(1/2)*(5^(1/2) - 3)^(1/2))/2 + (2^(1/2)*5^(1/2)*(5^(1/2) - 3)^(1/2))
/2)) + (2^(3/4)*5^(1/2)*x*(5^(1/2) - 3)^(1/4)*1i)/(2*((3*2^(1/2)*(5^(1/2)
- 3)^(1/2))/2 + (2^(1/2)*5^(1/2)*(5^(1/2) - 3)^(1/2))/2)))*(5^(1/2) - 3)^(
1/4)*1i)/20
```

3.381 $\int \frac{x^2}{1+3x^4+x^8} \, dx$

3.381.1 Optimal result

Integrand size = 16, antiderivative size = 427

$$\int \frac{x^2}{1+3x^4+x^8}\,dx = -\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\sqrt{5}\sqrt[4]{2\left(3-\sqrt{5}\right)}}+\frac{\arctan\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\sqrt{5}\sqrt[4]{2\left(3-\sqrt{5}\right)}}+\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\sqrt{5}\sqrt[4]{2\left(3+\sqrt{5}\right)}}-\frac{\arctan\left(1+\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\sqrt{5}\sqrt[4]{2\left(3+\sqrt{5}\right)}}+\frac{\log\left(\sqrt{2\left(3-\sqrt{5}\right)}-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{4\sqrt{5}\sqrt[4]{2\left(3-\sqrt{5}\right)}}-\frac{\log\left(\sqrt{2\left(3-\sqrt{5}\right)}+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{4\sqrt{5}\sqrt[4]{2\left(3-\sqrt{5}\right)}}-\frac{\log\left(\sqrt{2\left(3+\sqrt{5}\right)}-2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+2x^2\right)}{4\sqrt{5}\sqrt[4]{2\left(3+\sqrt{5}\right)}}+\frac{\log\left(\sqrt{2\left(3+\sqrt{5}\right)}+2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+2x^2\right)}{4\sqrt{5}\sqrt[4]{2\left(3+\sqrt{5}\right)}}$$

output
```
1/20*arctan(-1+2^(3/4)*x/(3-5^(1/2))^(1/4))*2^(3/4)/(3-5^(1/2))^(1/4)*5^(1
/2)+1/20*arctan(1+2^(3/4)*x/(3-5^(1/2))^(1/4))*2^(3/4)/(3-5^(1/2))^(1/4)*5
^(1/2)+1/40*ln(2*x^2-2*2^(1/4)*x*(3-5^(1/2))^(1/4)+5^(1/2)-1)*2^(3/4)/(3-5
^(1/2))^(1/4)*5^(1/2)-1/40*ln(2*x^2+2*2^(1/4)*x*(3-5^(1/2))^(1/4)+5^(1/2)-
1)*2^(3/4)/(3-5^(1/2))^(1/4)*5^(1/2)-1/20*arctan(-1+2^(3/4)*x/(3+5^(1/2))^
(1/4))*2^(3/4)*5^(1/2)/(3+5^(1/2))^(1/4)-1/20*arctan(1+2^(3/4)*x/(3+5^(1/2
))^(1/4))*2^(3/4)*5^(1/2)/(3+5^(1/2))^(1/4)-1/40*ln(2*x^2-2*2^(1/4)*x*(3+5
^(1/2))^(1/4)+5^(1/2)+1)*2^(3/4)*5^(1/2)/(3+5^(1/2))^(1/4)+1/40*ln(2*x^2+2
*2^(1/4)*x*(3+5^(1/2))^(1/4)+5^(1/2)+1)*2^(3/4)*5^(1/2)/(3+5^(1/2))^(1/4)
```

3.381.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.09

$$\int \frac{x^2}{1+3x^4+x^8}\,dx=\frac{1}{4}\text{RootSum}\left[1+3\#1^4+\#1^8\&,\frac{\log(x-\#1)}{3\#1+2\#1^5}\&\right]$$

input
```
Integrate[x^2/(1 + 3*x^4 + x^8),x]
```

output
```
RootSum[1 + 3*#1^4 + #1^8 & , Log[x - #1]/(3*#1 + 2*#1^5) & ]/4
```

3.381.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 451, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {1711, 27, 826, 1476, 1082, 217, 1479, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{x^8+3x^4+1}\,dx$$

$$\Big\downarrow \text{ 1711}$$

$$\frac{\int \frac{2x^2}{2x^4-\sqrt{5}+3}dx}{\sqrt{5}}-\frac{\int \frac{2x^2}{2x^4+\sqrt{5}+3}dx}{\sqrt{5}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2\int \frac{x^2}{2x^4-\sqrt{5}+3}dx}{\sqrt{5}}-\frac{2\int \frac{x^2}{2x^4+\sqrt{5}+3}dx}{\sqrt{5}}$$

$$\Big\downarrow \text{826}$$

$$\frac{2\left(\frac{\int \frac{\sqrt{2}x^2+\sqrt{3-\sqrt{5}}}{2x^4-\sqrt{5}+3}dx}{2\sqrt{2}}-\frac{\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{2x^4-\sqrt{5}+3}dx}{2\sqrt{2}}\right)}{\sqrt{5}}-\frac{2\left(\frac{\int \frac{\sqrt{2}x^2+\sqrt{3+\sqrt{5}}}{2x^4+\sqrt{5}+3}dx}{2\sqrt{2}}-\frac{\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{2x^4+\sqrt{5}+3}dx}{2\sqrt{2}}\right)}{\sqrt{5}}$$

$$\Big\downarrow \text{1476}$$

$$\frac{2\left(\frac{\frac{\int \frac{1}{x^2-\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}}dx}{2\sqrt{2}}+\frac{\int \frac{1}{x^2+\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}}dx}{2\sqrt{2}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{2x^4-\sqrt{5}+3}dx}{2\sqrt{2}}\right)}{\sqrt{5}}-$$

$$\frac{2\left(\frac{\frac{\int \frac{1}{x^2-\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\sqrt{2}}+\frac{\int \frac{1}{x^2+\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\sqrt{2}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{2x^4+\sqrt{5}+3}dx}{2\sqrt{2}}\right)}{\sqrt{5}}$$

$$\Big\downarrow \text{1082}$$

3.381. $\int \frac{x^2}{1+3x^4+x^8}\, dx$

$$\frac{2\left(\frac{\frac{\int \frac{1}{-\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)^2-1}d\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}-\frac{\int \frac{1}{-\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)^2-1}d\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{2x^4-\sqrt{5}+3}dx}{2\sqrt{2}}\right)}{\sqrt{5}}-$$

$$\frac{2\left(\frac{\frac{\int \frac{1}{-\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)^2-1}d\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\int \frac{1}{-\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)^2-1}d\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{2x^4+\sqrt{5}+3}dx}{2\sqrt{2}}\right)}{\sqrt{5}}$$

$$\downarrow 217$$

3.381. $\int \frac{x^2}{1+3x^4+x^8}\,dx$

$$\frac{2\left(\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}}{2\sqrt{2}}-\frac{\int\frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{2x^4-\sqrt{5}+3}dx}{2\sqrt{2}}\right)}{\sqrt{5}}-$$

$$\frac{2\left(\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{2\sqrt{2}}-\frac{\int\frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{2x^4+\sqrt{5}+3}dx}{2\sqrt{2}}\right)}{\sqrt{5}}$$

$\downarrow$ 1479

$$\frac{2\left(\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}}{2\sqrt{2}}-\frac{-\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\int-\frac{\sqrt[4]{2\left(3-\sqrt{5}\right)}-2x}{x^2-\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}}dx-\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\cdots\right.}}{2\sqrt{2}}\right.}{\sqrt{5}}$$

$$\frac{2\left(\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{2\sqrt{2}}-\frac{-\frac{\int-\frac{\sqrt[4]{2\left(3+\sqrt{5}\right)}-2x}{x^2-\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\,2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\int-\frac{2x+\sqrt[4]{2\left(3+\sqrt{5}\right)}}{x^2+\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\cdots\right.}}}{2\,2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{2\sqrt{2}}\right.}{\sqrt{5}}$$

3.381. $\int\frac{x^2}{1+3x^4+x^8}\,dx$

$\downarrow$ 25

$$\frac{2\left(\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}}{2\sqrt{2}}-\frac{\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\int\frac{\sqrt[4]{2\left(3-\sqrt{5}\right)}-2x}{x^2-\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}}dx+\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}}{2\sqrt{2}}\right.}{\sqrt{5}}$$

$$\frac{2\left(\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{2\sqrt{2}}-\frac{\frac{\int\frac{\sqrt[4]{2\left(3+\sqrt{5}\right)}-2x}{x^2-\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}+\frac{\int\frac{2x+\sqrt[4]{2\left(3+\sqrt{5}\right)}}{x^2+\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{2\sqrt{2}}\right.}{\sqrt{5}}$$

$\downarrow$ 1103

3.381. $\int \frac{x^2}{1+3x^4+x^8}\,dx$

$$\frac{2\left(\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}}{2\sqrt{2}}-\frac{\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)-\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\right.}}{2\sqrt{2}}\right.}{\sqrt{5}}$$

$$\frac{2\left(\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{2\sqrt{2}}-\frac{\frac{\log\left(2x^2+2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{2\,2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\log\left(2x^2-2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{2\left(3+\right.}\right.}{2\,2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{2\sqrt{2}}\right.}{\sqrt{5}}$$

input

```
Int[x^2/(1 + 3*x^4 + x^8),x]
```

output

```
(2*((-(ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)]/(2^(3/4)*(3 - Sqrt[5])^
(1/4))) + ArcTan[1 + (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)]/(2^(3/4)*(3 - Sqrt[5
])^(1/4)))/(2*Sqrt[2]) - (-1/4*(((3 + Sqrt[5])/2)^(1/4)*Log[Sqrt[2*(3 - Sq
rt[5])] - 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2]) + (((3 + Sqrt[5])/2)^(1/4)
*Log[Sqrt[2*(3 - Sqrt[5])] + 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2])/4)/(2*S
qrt[2])))/Sqrt[5] - (2*((-(ArcTan[1 - (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)]/(2^
(3/4)*(3 + Sqrt[5])^(1/4))) + ArcTan[1 + (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)]/
(2^(3/4)*(3 + Sqrt[5])^(1/4)))/(2*Sqrt[2]) - (-1/2*Log[Sqrt[2*(3 + Sqrt[5]
)] - 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2]/(2^(3/4)*(3 + Sqrt[5])^(1/4)) +
Log[Sqrt[2*(3 + Sqrt[5])] + 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2]/(2*2^(3/4
)*(3 + Sqrt[5])^(1/4)))/(2*Sqrt[2])))/Sqrt[5]
```

3.381. $\int \frac{x^2}{1+3x^4+x^8}\,dx$

3.381.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.381. $\int \frac{x^2}{1+3x^4+x^8} \, dx$

rule 1711
```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[c/q   Int[(d*x)^m/(b/2 - q/2 + c
*x^n), x], x] - Simp[c/q   Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; Free
Q[{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

3.381.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.09

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}+3\_Z^{4}+1\right)}\frac{\_R^{2}\ln\left(x-\_R\right)}{2\_R^{7}+3\_R^{3}}\right)}{4}$	40
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}+3\_Z^{4}+1\right)}\frac{\_R^{2}\ln\left(x-\_R\right)}{2\_R^{7}+3\_R^{3}}\right)}{4}$	40

input
```
int(x^2/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(_R^2/(2*_R^7+3*_R^3)*ln(x-_R),_R=RootOf(_Z^8+3*_Z^4+1))
```

3.381.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 525, normalized size of antiderivative = 1.23

$$\int \frac{x^2}{1+3x^4+x^8}\,dx$$
$$= -\frac{1}{40}\sqrt{10}\sqrt{\sqrt{2}\sqrt{\sqrt{5}-3}}\log\left(\sqrt{10}\left(3\sqrt{5}\sqrt{2}+5\sqrt{2}\right)\sqrt{\sqrt{2}\sqrt{\sqrt{5}-3}}\sqrt{\sqrt{5}-3}+40\,x\right)$$
$$+\frac{1}{40}\sqrt{10}\sqrt{\sqrt{2}\sqrt{\sqrt{5}-3}}\log\left(-\sqrt{10}\left(3\sqrt{5}\sqrt{2}+5\sqrt{2}\right)\sqrt{\sqrt{2}\sqrt{\sqrt{5}-3}}\sqrt{\sqrt{5}-3}+40\,x\right)$$
$$+\frac{1}{40}\sqrt{10}\sqrt{-\sqrt{2}\sqrt{\sqrt{5}-3}}\log\left(\sqrt{10}\left(3\sqrt{5}\sqrt{2}+5\sqrt{2}\right)\sqrt{-\sqrt{2}\sqrt{\sqrt{5}-3}}\sqrt{\sqrt{5}-3}+40\,x\right)$$
$$-\frac{1}{40}\sqrt{10}\sqrt{-\sqrt{2}\sqrt{\sqrt{5}-3}}\log\left(-\sqrt{10}\left(3\sqrt{5}\sqrt{2}+5\sqrt{2}\right)\sqrt{-\sqrt{2}\sqrt{\sqrt{5}-3}}\sqrt{\sqrt{5}-3}+40\,x\right)$$
$$+\frac{1}{40}\sqrt{10}\sqrt{\sqrt{2}\sqrt{-\sqrt{5}-3}}\log\left(\sqrt{10}\left(3\sqrt{5}\sqrt{2}-5\sqrt{2}\right)\sqrt{\sqrt{2}\sqrt{-\sqrt{5}-3}}\sqrt{-\sqrt{5}-3}+40\,x\right)$$
$$-\frac{1}{40}\sqrt{10}\sqrt{\sqrt{2}\sqrt{-\sqrt{5}-3}}\log\left(-\sqrt{10}\left(3\sqrt{5}\sqrt{2}-5\sqrt{2}\right)\sqrt{\sqrt{2}\sqrt{-\sqrt{5}-3}}\sqrt{-\sqrt{5}-3}+40\,x\right)$$
$$-\frac{1}{40}\sqrt{10}\sqrt{-\sqrt{2}\sqrt{-\sqrt{5}-3}}\log\left(\sqrt{10}\left(3\sqrt{5}\sqrt{2}-5\sqrt{2}\right)\sqrt{-\sqrt{2}\sqrt{-\sqrt{5}-3}}\sqrt{-\sqrt{5}-3}+40\,x\right)$$
$$+\frac{1}{40}\sqrt{10}\sqrt{-\sqrt{2}\sqrt{-\sqrt{5}-3}}\log\left(-\sqrt{10}\left(3\sqrt{5}\sqrt{2}-5\sqrt{2}\right)\sqrt{-\sqrt{2}\sqrt{-\sqrt{5}-3}}\sqrt{-\sqrt{5}-3}+40\,x\right)$$

input
```
integrate(x^2/(x^8+3*x^4+1),x, algorithm="fricas")
```

output
```
-1/40*sqrt(10)*sqrt(sqrt(2)*sqrt(sqrt(5) - 3))*log(sqrt(10)*(3*sqrt(5)*sqr
t(2) + 5*sqrt(2))*sqrt(sqrt(2)*sqrt(sqrt(5) - 3))*sqrt(sqrt(5) - 3) + 40*x
) + 1/40*sqrt(10)*sqrt(sqrt(2)*sqrt(sqrt(5) - 3))*log(-sqrt(10)*(3*sqrt(5)
*sqrt(2) + 5*sqrt(2))*sqrt(sqrt(2)*sqrt(sqrt(5) - 3))*sqrt(sqrt(5) - 3) +
40*x) + 1/40*sqrt(10)*sqrt(-sqrt(2)*sqrt(sqrt(5) - 3))*log(sqrt(10)*(3*sqr
t(5)*sqrt(2) + 5*sqrt(2))*sqrt(-sqrt(2)*sqrt(sqrt(5) - 3))*sqrt(sqrt(5) -
3) + 40*x) - 1/40*sqrt(10)*sqrt(-sqrt(2)*sqrt(sqrt(5) - 3))*log(-sqrt(10)*
(3*sqrt(5)*sqrt(2) + 5*sqrt(2))*sqrt(-sqrt(2)*sqrt(sqrt(5) - 3))*sqrt(sqrt
(5) - 3) + 40*x) + 1/40*sqrt(10)*sqrt(sqrt(2)*sqrt(-sqrt(5) - 3))*log(sqrt
(10)*(3*sqrt(5)*sqrt(2) - 5*sqrt(2))*sqrt(sqrt(2)*sqrt(-sqrt(5) - 3))*sqrt
(-sqrt(5) - 3) + 40*x) - 1/40*sqrt(10)*sqrt(sqrt(2)*sqrt(-sqrt(5) - 3))*lo
g(-sqrt(10)*(3*sqrt(5)*sqrt(2) - 5*sqrt(2))*sqrt(sqrt(2)*sqrt(-sqrt(5) - 3
))*sqrt(-sqrt(5) - 3) + 40*x) - 1/40*sqrt(10)*sqrt(-sqrt(2)*sqrt(-sqrt(5)
- 3))*log(sqrt(10)*(3*sqrt(5)*sqrt(2) - 5*sqrt(2))*sqrt(-sqrt(2)*sqrt(-sqr
t(5) - 3))*sqrt(-sqrt(5) - 3) + 40*x) + 1/40*sqrt(10)*sqrt(-sqrt(2)*sqrt(-
sqrt(5) - 3))*log(-sqrt(10)*(3*sqrt(5)*sqrt(2) - 5*sqrt(2))*sqrt(-sqrt(2)*
sqrt(-sqrt(5) - 3))*sqrt(-sqrt(5) - 3) + 40*x)
```

3.381.6 Sympy [A] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.06

$$\int \frac{x^2}{1+3x^4+x^8}\,dx$$
$$= \text{RootSum}\left(40960000t^8+19200t^4+1,\left(t\mapsto t\log\left(-6144000t^7-2240t^3+x\right)\right)\right)$$

input
```
integrate(x**2/(x**8+3*x**4+1),x)
```

output
```
RootSum(40960000*_t**8 + 19200*_t**4 + 1, Lambda(_t, _t*log(-6144000*_t**7
 - 2240*_t**3 + x)))
```

3.381.7 Maxima [F]

$$\int \frac{x^2}{1+3x^4+x^8}\,dx = \int \frac{x^2}{x^8+3\,x^4+1}\,dx$$

input
```
integrate(x^2/(x^8+3*x^4+1),x, algorithm="maxima")
```

output
```
integrate(x^2/(x^8 + 3*x^4 + 1), x)
```

3.381.8 Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.56

$$\begin{aligned}\int \frac{x^2}{1+3x^4+x^8}\,dx = {} & \frac{1}{80}\left(\pi+4\,\arctan\left(x\sqrt{\sqrt{5}+1}-1\right)\right)\sqrt{5\,\sqrt{5}+5} \\ & -\frac{1}{80}\left(\pi+4\,\arctan\left(-x\sqrt{\sqrt{5}+1}-1\right)\right)\sqrt{5\,\sqrt{5}+5} \\ & -\frac{1}{80}\left(\pi+4\,\arctan\left(x\sqrt{\sqrt{5}-1}+1\right)\right)\sqrt{5\,\sqrt{5}-5} \\ & +\frac{1}{80}\left(\pi+4\,\arctan\left(-x\sqrt{\sqrt{5}-1}+1\right)\right)\sqrt{5\,\sqrt{5}-5} \\ & +\frac{1}{40}\sqrt{5\,\sqrt{5}-5}\log\left(16900\left(x+\sqrt{\sqrt{5}+1}\right)^2+16900\,x^2\right) \\ & -\frac{1}{40}\sqrt{5\,\sqrt{5}-5}\log\left(16900\left(x-\sqrt{\sqrt{5}+1}\right)^2+16900\,x^2\right) \\ & -\frac{1}{40}\sqrt{5\,\sqrt{5}+5}\log\left(2500\left(x+\sqrt{\sqrt{5}-1}\right)^2+2500\,x^2\right) \\ & +\frac{1}{40}\sqrt{5\,\sqrt{5}+5}\log\left(2500\left(x-\sqrt{\sqrt{5}-1}\right)^2+2500\,x^2\right)\end{aligned}$$

input
```
integrate(x^2/(x^8+3*x^4+1),x, algorithm="giac")
```

output
```
1/80*(pi + 4*arctan(x*sqrt(sqrt(5) + 1) - 1))*sqrt(5*sqrt(5) + 5) - 1/80*(
pi + 4*arctan(-x*sqrt(sqrt(5) + 1) - 1))*sqrt(5*sqrt(5) + 5) - 1/80*(pi +
4*arctan(x*sqrt(sqrt(5) - 1) + 1))*sqrt(5*sqrt(5) - 5) + 1/80*(pi + 4*arct
an(-x*sqrt(sqrt(5) - 1) + 1))*sqrt(5*sqrt(5) - 5) + 1/40*sqrt(5*sqrt(5) -
5)*log(16900*(x + sqrt(sqrt(5) + 1))^2 + 16900*x^2) - 1/40*sqrt(5*sqrt(5)
- 5)*log(16900*(x - sqrt(sqrt(5) + 1))^2 + 16900*x^2) - 1/40*sqrt(5*sqrt(5
) + 5)*log(2500*(x + sqrt(sqrt(5) - 1))^2 + 2500*x^2) + 1/40*sqrt(5*sqrt(5
) + 5)*log(2500*(x - sqrt(sqrt(5) - 1))^2 + 2500*x^2)
```

3.381.9 Mupad [B] (verification not implemented)

Time = 8.29 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.64

$$\int \frac{x^2}{1+3x^4+x^8}\,dx = \frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{7\,2^{3/4}\,x\,\left(\sqrt{5}-3\right)^{1/4}}{2\,\left(3\,\sqrt{5}-7\right)}-\frac{3\,2^{3/4}\,\sqrt{5}\,x\,\left(\sqrt{5}-3\right)^{1/4}}{2\,\left(3\,\sqrt{5}-7\right)}\right)\,\left(\sqrt{5}-3\right)^{1/4}}{20} + \frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\,\left(\sqrt{5}-3\right)^{1/4}\,7\mathrm{i}}{2\,\left(3\,\sqrt{5}-7\right)}-\frac{2^{3/4}\,\sqrt{5}\,x\,\left(\sqrt{5}-3\right)^{1/4}\,3\mathrm{i}}{2\,\left(3\,\sqrt{5}-7\right)}\right)\,\left(\sqrt{5}-3\right)^{1/4}\,1\mathrm{i}}{20} + \frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{7\,2^{3/4}\,x\,\left(-\sqrt{5}-3\right)^{1/4}}{2\,\left(3\,\sqrt{5}+7\right)}+\frac{3\,2^{3/4}\,\sqrt{5}\,x\,\left(-\sqrt{5}-3\right)^{1/4}}{2\,\left(3\,\sqrt{5}+7\right)}\right)\,\left(-\sqrt{5}-3\right)^{1/4}}{20} + \frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\,\left(-\sqrt{5}-3\right)^{1/4}\,7\mathrm{i}}{2\,\left(3\,\sqrt{5}+7\right)}+\frac{2^{3/4}\,\sqrt{5}\,x\,\left(-\sqrt{5}-3\right)^{1/4}\,3\mathrm{i}}{2\,\left(3\,\sqrt{5}+7\right)}\right)\,\left(-\sqrt{5}-3\right)^{1/4}\,1\mathrm{i}}{20}$$

input
```
int(x^2/(3*x^4 + x^8 + 1),x)
```

output
```
(2^(3/4)*5^(1/2)*atan((7*2^(3/4)*x*(5^(1/2) - 3)^(1/4))/(2*(3*5^(1/2) - 7)
) - (3*2^(3/4)*5^(1/2)*x*(5^(1/2) - 3)^(1/4))/(2*(3*5^(1/2) - 7)))*(5^(1/2
) - 3)^(1/4))/20 + (2^(3/4)*5^(1/2)*atan((2^(3/4)*x*(5^(1/2) - 3)^(1/4)*7i
)/(2*(3*5^(1/2) - 7)) - (2^(3/4)*5^(1/2)*x*(5^(1/2) - 3)^(1/4)*3i)/(2*(3*5
^(1/2) - 7)))*(5^(1/2) - 3)^(1/4)*1i)/20 + (2^(3/4)*5^(1/2)*atan((7*2^(3/4
)*x*(- 5^(1/2) - 3)^(1/4))/(2*(3*5^(1/2) + 7)) + (3*2^(3/4)*5^(1/2)*x*(- 5
^(1/2) - 3)^(1/4))/(2*(3*5^(1/2) + 7)))*(- 5^(1/2) - 3)^(1/4))/20 + (2^(3/
4)*5^(1/2)*atan((2^(3/4)*x*(- 5^(1/2) - 3)^(1/4)*7i)/(2*(3*5^(1/2) + 7)) +
 (2^(3/4)*5^(1/2)*x*(- 5^(1/2) - 3)^(1/4)*3i)/(2*(3*5^(1/2) + 7)))*(- 5^(1
/2) - 3)^(1/4)*1i)/20
```

3.381. $\int \frac{x^2}{1+3x^4+x^8} \, dx$

3.382 $\int \frac{1}{1+3x^4+x^8} \, dx$

3.382.1 Optimal result

Integrand size = 12, antiderivative size = 414

$$\int \frac{1}{1+3x^4+x^8}\,dx = -\frac{\sqrt[4]{9+4\sqrt{5}}\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\sqrt{10}} + \frac{\sqrt[4]{9+4\sqrt{5}}\arctan\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\sqrt{10}} + \frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{\sqrt{5}\left(2\left(3+\sqrt{5}\right)\right)^{3/4}} - \frac{\arctan\left(1+\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{\sqrt{5}\left(2\left(3+\sqrt{5}\right)\right)^{3/4}} - \frac{\sqrt[4]{9+4\sqrt{5}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{4\sqrt{10}} + \frac{\sqrt[4]{9+4\sqrt{5}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{4\sqrt{10}} + \frac{\log\left(\sqrt{2\left(3+\sqrt{5}\right)}-2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+2x^2\right)}{2\sqrt{5}\left(2\left(3+\sqrt{5}\right)\right)^{3/4}} - \frac{\log\left(\sqrt{2\left(3+\sqrt{5}\right)}+2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+2x^2\right)}{2\sqrt{5}\left(2\left(3+\sqrt{5}\right)\right)^{3/4}}$$

output
```
-1/20*arctan(-1+x*(5^(1/2)-1)^(1/2))*(-20+10*5^(1/2))^(1/2)-1/20*arctan(1+
x*(5^(1/2)-1)^(1/2))*(-20+10*5^(1/2))^(1/2)+1/40*ln(1+2*x^2+5^(1/2)-2*x*(5
^(1/2)+1)^(1/2))*(-20+10*5^(1/2))^(1/2)-1/40*ln(1+2*x^2+5^(1/2)+2*x*(5^(1/
2)+1)^(1/2))*(-20+10*5^(1/2))^(1/2)+1/20*arctan(-1+x*(5^(1/2)+1)^(1/2))*(2
0+10*5^(1/2))^(1/2)+1/20*arctan(1+x*(5^(1/2)+1)^(1/2))*(20+10*5^(1/2))^(1/
2)-1/40*ln(-1+2*x^2+5^(1/2)-2*x*(5^(1/2)-1)^(1/2))*(20+10*5^(1/2))^(1/2)+1
/40*ln(-1+2*x^2+5^(1/2)+2*x*(5^(1/2)-1)^(1/2))*(20+10*5^(1/2))^(1/2)
```

3.382.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.10

$$\int \frac{1}{1+3x^4+x^8}\,dx = \frac{1}{4}\text{RootSum}\left[1+3\#1^4+\#1^8\&,\frac{\log(x-\#1)}{3\#1^3+2\#1^7}\&\right]$$

input
```
Integrate[(1 + 3*x^4 + x^8)^(-1),x]
```

output
```
RootSum[1 + 3*#1^4 + #1^8 & , Log[x - #1]/(3*#1^3 + 2*#1^7) & ]/4
```

3.382.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.13, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {1685, 755, 27, 1476, 1082, 217, 1479, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^8+3x^4+1}dx$$

$$\Big\downarrow \text{1685}$$

$$\frac{\int \frac{1}{x^4+\frac{1}{2}\left(3-\sqrt{5}\right)}dx}{\sqrt{5}}-\frac{\int \frac{1}{x^4+\frac{1}{2}\left(3+\sqrt{5}\right)}dx}{\sqrt{5}}$$

$$\Big\downarrow \text{755}$$

$$\frac{\frac{\int \frac{2\left(\sqrt{3-\sqrt{5}}-\sqrt{2}x^2\right)}{2x^4-\sqrt{5}+3}dx}{2\sqrt{3-\sqrt{5}}}+\frac{\int \frac{2\left(\sqrt{2}x^2+\sqrt{3-\sqrt{5}}\right)}{2x^4-\sqrt{5}+3}dx}{2\sqrt{3-\sqrt{5}}}}{\sqrt{5}}-\frac{\frac{\int \frac{2\left(\sqrt{3+\sqrt{5}}-\sqrt{2}x^2\right)}{2x^4+\sqrt{5}+3}dx}{2\sqrt{3+\sqrt{5}}}+\frac{\int \frac{2\left(\sqrt{2}x^2+\sqrt{3+\sqrt{5}}\right)}{2x^4+\sqrt{5}+3}dx}{2\sqrt{3+\sqrt{5}}}}{\sqrt{5}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{2x^4-\sqrt{5}+3}dx}{\sqrt{3-\sqrt{5}}}+\frac{\int \frac{\sqrt{2}x^2+\sqrt{3-\sqrt{5}}}{2x^4-\sqrt{5}+3}dx}{\sqrt{3-\sqrt{5}}}}{\sqrt{5}}-\frac{\frac{\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{2x^4+\sqrt{5}+3}dx}{\sqrt{3+\sqrt{5}}}+\frac{\int \frac{\sqrt{2}x^2+\sqrt{3+\sqrt{5}}}{2x^4+\sqrt{5}+3}dx}{\sqrt{3+\sqrt{5}}}}{\sqrt{5}}$$

$$\Big\downarrow \text{1476}$$

$$\frac{\frac{\frac{\int \frac{1}{x^2-\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}}dx}{2\sqrt{2}}+\frac{\int \frac{1}{x^2+\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}}dx}{2\sqrt{2}}}{\sqrt{3-\sqrt{5}}}+\frac{\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{2x^4-\sqrt{5}+3}dx}{\sqrt{3-\sqrt{5}}}}{\sqrt{5}}-\frac{\frac{\frac{\int \frac{1}{x^2-\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\sqrt{2}}+\frac{\int \frac{1}{x^2+\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\sqrt{2}}}{\sqrt{3+\sqrt{5}}}+\frac{\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{2x^4+\sqrt{5}+3}dx}{\sqrt{3+\sqrt{5}}}}{\sqrt{5}}$$

$$\Big\downarrow \text{1082}$$

$$\frac{\frac{\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{2x^4-\sqrt{5}+3}dx}{\sqrt{3-\sqrt{5}}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)^2-1}d\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}-\frac{\int \frac{1}{-\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)^2-1}d\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}}{\sqrt{3-\sqrt{5}}}}{\sqrt{5}}-\frac{\frac{\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{2x^4+\sqrt{5}+3}dx}{\sqrt{3+\sqrt{5}}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)^2-1}d\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\int \frac{1}{-\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)^2-1}d\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{\sqrt{3+\sqrt{5}}}}{\sqrt{5}}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{2x^4-\sqrt{5}+3}dx}{\sqrt{3-\sqrt{5}}}+\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}}{\sqrt{3-\sqrt{5}}}}{\sqrt{5}}-\frac{\frac{\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{2x^4+\sqrt{5}+3}dx}{\sqrt{3+\sqrt{5}}}+\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{\sqrt{3+\sqrt{5}}}}{\sqrt{5}}$$

$$\Big\downarrow \text{1479}$$

3.382. $\int \frac{1}{1+3x^4+x^8}\,dx$

$$\frac{\frac{-\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\int -\frac{\sqrt[4]{2\left(3-\sqrt{5}\right)}-2x}{x^2-\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}}dx-\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\int -\frac{2x+\sqrt[4]{2\left(3-\sqrt{5}\right)}}{x^2+\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}}dx}{\sqrt{3-\sqrt{5}}}+\frac{\frac{\arctan\left(\cdots\right)}{2^{3/4}\cdots}\cdots}{\cdots}}{\sqrt{5}}$$

$$\frac{\frac{-\frac{\int -\frac{\sqrt[4]{2\left(3+\sqrt{5}\right)}-2x}{x^2-\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\int -\frac{2x+\sqrt[4]{2\left(3+\sqrt{5}\right)}}{x^2+\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{\sqrt{3+\sqrt{5}}}+\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}\cdots}{\sqrt{3+\sqrt{5}}}}{\sqrt{5}}$$

$\downarrow$ 25

$$\frac{\frac{\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\int \frac{\sqrt[4]{2\left(3-\sqrt{5}\right)}-2x}{x^2-\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}}dx+\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\int \frac{2x+\sqrt[4]{2\left(3-\sqrt{5}\right)}}{x^2+\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}}dx}{\sqrt{3-\sqrt{5}}}+\frac{\frac{\arctan\left(\frac{2^{3/4}\cdots}{\sqrt[4]{3-\cdots}}\cdots\right)}{2^{3/4}\sqrt[4]{3-\cdots}}\cdots}{\cdots}}{\sqrt{5}}$$

$$\frac{\frac{\frac{\int \frac{\sqrt[4]{2\left(3+\sqrt{5}\right)}-2x}{x^2-\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}+\frac{\int \frac{2x+\sqrt[4]{2\left(3+\sqrt{5}\right)}}{x^2+\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{\sqrt{3+\sqrt{5}}}+\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{\sqrt{3+\sqrt{5}}}}{\sqrt{5}}$$

$\downarrow$ 1103

3.382. $\int \frac{1}{1+3x^4+x^8}\,dx$

$$\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}}{\sqrt{3-\sqrt{5}}}+\frac{\frac{\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)-\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{\cdots}\right.}}{\sqrt{3-\sqrt{5}}}}{\sqrt{5}}$$

$$\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{\sqrt{3+\sqrt{5}}}+\frac{\frac{\log\left(2x^2+2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\log\left(2x^2-2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{2\left(3+\sqrt{5}\right)}\cdots\right.}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{\sqrt{3+\sqrt{5}}}}{\sqrt{5}}$$

input
```
Int[(1 + 3*x^4 + x^8)^(-1),x]
```

output
```
((-(ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)]/(2^(3/4)*(3 - Sqrt[5])^(1/
4))) + ArcTan[1 + (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)]/(2^(3/4)*(3 - Sqrt[5])^
(1/4)))/Sqrt[3 - Sqrt[5]] + (-1/4*(((3 + Sqrt[5])/2)^(1/4)*Log[Sqrt[2*(3 -
 Sqrt[5])] - 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2]) + (((3 + Sqrt[5])/2)^(1
/4)*Log[Sqrt[2*(3 - Sqrt[5])] + 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2])/4)/S
qrt[3 - Sqrt[5]])/Sqrt[5] - ((-(ArcTan[1 - (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)
]/(2^(3/4)*(3 + Sqrt[5])^(1/4))) + ArcTan[1 + (2^(3/4)*x)/(3 + Sqrt[5])^(1
/4)]/(2^(3/4)*(3 + Sqrt[5])^(1/4)))/Sqrt[3 + Sqrt[5]] + (-1/2*Log[Sqrt[2*(
3 + Sqrt[5])] - 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2]/(2^(3/4)*(3 + Sqrt[5]
)^(1/4)) + Log[Sqrt[2*(3 + Sqrt[5])] + 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2
]/(2*2^(3/4)*(3 + Sqrt[5])^(1/4)))/Sqrt[3 + Sqrt[5]])/Sqrt[5]
```

3.382.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1685
```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^n), x], x] - Simp[
c/q   Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2,
2*n] && NeQ[b^2 - 4*a*c, 0]
```

3.382. $\int \frac{1}{1+3x^4+x^8}\,dx$

3.382.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.09

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}+3\_Z^{4}+1\right)}\frac{\ln\left(x-\_R\right)}{2\_R^{7}+3\_R^{3}}\right)}{4}$	37
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}+3\_Z^{4}+1\right)}\frac{\ln\left(x-\_R\right)}{2\_R^{7}+3\_R^{3}}\right)}{4}$	37

input
```
int(1/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(1/(2*_R^7+3*_R^3)*ln(x-_R),_R=RootOf(_Z^8+3*_Z^4+1))
```

3.382.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 309, normalized size of antiderivative = 0.75

$$\begin{aligned}\int \frac{1}{1+3x^4+x^8}\,dx = &-\frac{1}{20}\sqrt{5}\sqrt{-\sqrt{4\sqrt{5}-9}}\log\left(\left(\sqrt{5}+3\right)\sqrt{-\sqrt{4\sqrt{5}-9}}+2x\right)\\ &+\frac{1}{20}\sqrt{5}\sqrt{-\sqrt{4\sqrt{5}-9}}\log\left(-\left(\sqrt{5}+3\right)\sqrt{-\sqrt{4\sqrt{5}-9}}+2x\right)\\ &-\frac{1}{20}\sqrt{5}\sqrt{-\sqrt{-4\sqrt{5}-9}}\log\left(\left(\sqrt{5}-3\right)\sqrt{-\sqrt{-4\sqrt{5}-9}}+2x\right)\\ &+\frac{1}{20}\sqrt{5}\sqrt{-\sqrt{-4\sqrt{5}-9}}\log\left(-\left(\sqrt{5}-3\right)\sqrt{-\sqrt{-4\sqrt{5}-9}}+2x\right)\\ &-\frac{1}{20}\sqrt{5}\left(4\sqrt{5}-9\right)^{\frac{1}{4}}\log\left(\left(4\sqrt{5}-9\right)^{\frac{1}{4}}\left(\sqrt{5}+3\right)+2x\right)\\ &+\frac{1}{20}\sqrt{5}\left(4\sqrt{5}-9\right)^{\frac{1}{4}}\log\left(-\left(4\sqrt{5}-9\right)^{\frac{1}{4}}\left(\sqrt{5}+3\right)+2x\right)\\ &-\frac{1}{20}\sqrt{5}\left(-4\sqrt{5}-9\right)^{\frac{1}{4}}\log\left(\left(\sqrt{5}-3\right)\left(-4\sqrt{5}-9\right)^{\frac{1}{4}}+2x\right)\\ &+\frac{1}{20}\sqrt{5}\left(-4\sqrt{5}-9\right)^{\frac{1}{4}}\log\left(-\left(\sqrt{5}-3\right)\left(-4\sqrt{5}-9\right)^{\frac{1}{4}}+2x\right)\end{aligned}$$

input
```
integrate(1/(x^8+3*x^4+1),x, algorithm="fricas")
```

output
```
-1/20*sqrt(5)*sqrt(-sqrt(4*sqrt(5) - 9))*log((sqrt(5) + 3)*sqrt(-sqrt(4*sq
rt(5) - 9)) + 2*x) + 1/20*sqrt(5)*sqrt(-sqrt(4*sqrt(5) - 9))*log(-(sqrt(5)
 + 3)*sqrt(-sqrt(4*sqrt(5) - 9)) + 2*x) - 1/20*sqrt(5)*sqrt(-sqrt(-4*sqrt(
5) - 9))*log((sqrt(5) - 3)*sqrt(-sqrt(-4*sqrt(5) - 9)) + 2*x) + 1/20*sqrt(
5)*sqrt(-sqrt(-4*sqrt(5) - 9))*log(-(sqrt(5) - 3)*sqrt(-sqrt(-4*sqrt(5) -
9)) + 2*x) - 1/20*sqrt(5)*(4*sqrt(5) - 9)^(1/4)*log((4*sqrt(5) - 9)^(1/4)*
(sqrt(5) + 3) + 2*x) + 1/20*sqrt(5)*(4*sqrt(5) - 9)^(1/4)*log(-(4*sqrt(5)
- 9)^(1/4)*(sqrt(5) + 3) + 2*x) - 1/20*sqrt(5)*(-4*sqrt(5) - 9)^(1/4)*log(
(sqrt(5) - 3)*(-4*sqrt(5) - 9)^(1/4) + 2*x) + 1/20*sqrt(5)*(-4*sqrt(5) - 9
)^(1/4)*log(-(sqrt(5) - 3)*(-4*sqrt(5) - 9)^(1/4) + 2*x)
```

3.382.6 Sympy [A] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.06

$$\int \frac{1}{1+3x^4+x^8}\,dx$$
$$= \text{RootSum}\left(40960000t^8+115200t^4+1,\left(t\mapsto t\log\left(-9600t^5-\frac{47t}{2}+x\right)\right)\right)$$

input
```
integrate(1/(x**8+3*x**4+1),x)
```

output
```
RootSum(40960000*_t**8 + 115200*_t**4 + 1, Lambda(_t, _t*log(-9600*_t**5 -
 47*_t/2 + x)))
```

3.382.7 Maxima [F]

$$\int \frac{1}{1+3x^4+x^8}\,dx = \int \frac{1}{x^8+3\,x^4+1}\,dx$$

input
```
integrate(1/(x^8+3*x^4+1),x, algorithm="maxima")
```

output
```
integrate(1/(x^8 + 3*x^4 + 1), x)
```

3.382.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.58

$$\int \frac{1}{1+3x^4+x^8}\,dx = \frac{1}{80}\left(\pi+4\arctan\left(x\sqrt{\sqrt{5}+1}+1\right)\right)\sqrt{10\sqrt{5}+20} - \frac{1}{80}\left(\pi+4\arctan\left(-x\sqrt{\sqrt{5}+1}+1\right)\right)\sqrt{10\sqrt{5}+20} - \frac{1}{80}\left(\pi+4\arctan\left(x\sqrt{\sqrt{5}-1}-1\right)\right)\sqrt{10\sqrt{5}-20} + \frac{1}{80}\left(\pi+4\arctan\left(-x\sqrt{\sqrt{5}-1}-1\right)\right)\sqrt{10\sqrt{5}-20} - \frac{1}{40}\sqrt{10\sqrt{5}-20}\log\left(10000\left(x+\sqrt{\sqrt{5}+1}\right)^2+10000\,x^2\right) + \frac{1}{40}\sqrt{10\sqrt{5}-20}\log\left(10000\left(x-\sqrt{\sqrt{5}+1}\right)^2+10000\,x^2\right) + \frac{1}{40}\sqrt{10\sqrt{5}+20}\log\left(400\left(x+\sqrt{\sqrt{5}-1}\right)^2+400\,x^2\right) - \frac{1}{40}\sqrt{10\sqrt{5}+20}\log\left(400\left(x-\sqrt{\sqrt{5}-1}\right)^2+400\,x^2\right)$$

input
```
integrate(1/(x^8+3*x^4+1),x, algorithm="giac")
```

output
```
1/80*(pi + 4*arctan(x*sqrt(sqrt(5) + 1) + 1))*sqrt(10*sqrt(5) + 20) - 1/80
*(pi + 4*arctan(-x*sqrt(sqrt(5) + 1) + 1))*sqrt(10*sqrt(5) + 20) - 1/80*(p
i + 4*arctan(x*sqrt(sqrt(5) - 1) - 1))*sqrt(10*sqrt(5) - 20) + 1/80*(pi +
4*arctan(-x*sqrt(sqrt(5) - 1) - 1))*sqrt(10*sqrt(5) - 20) - 1/40*sqrt(10*s
qrt(5) - 20)*log(10000*(x + sqrt(sqrt(5) + 1))^2 + 10000*x^2) + 1/40*sqrt(
10*sqrt(5) - 20)*log(10000*(x - sqrt(sqrt(5) + 1))^2 + 10000*x^2) + 1/40*s
qrt(10*sqrt(5) + 20)*log(400*(x + sqrt(sqrt(5) - 1))^2 + 400*x^2) - 1/40*s
qrt(10*sqrt(5) + 20)*log(400*(x - sqrt(sqrt(5) - 1))^2 + 400*x^2)
```

3.382.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 403, normalized size of antiderivative = 0.97

$$\int \frac{1}{1+3x^4+x^8}\,dx$$
$$= \frac{\sqrt{5}\,\mathrm{atan}\left(\frac{144\,x\,\left(-4\,\sqrt{5}-9\right)^{1/4}}{24\,\sqrt{5}\,\sqrt{-4\,\sqrt{5}-9}+56\,\sqrt{-4\,\sqrt{5}-9}}+\frac{64\,\sqrt{5}\,x\,\left(-4\,\sqrt{5}-9\right)^{1/4}}{24\,\sqrt{5}\,\sqrt{-4\,\sqrt{5}-9}+56\,\sqrt{-4\,\sqrt{5}-9}}\right)\,\left(-4\,\sqrt{5}-9\right)^{1/4}}{10}$$
$$+\frac{\sqrt{5}\,\mathrm{atan}\left(\frac{144\,x\,\left(4\,\sqrt{5}-9\right)^{1/4}}{24\,\sqrt{5}\,\sqrt{4\,\sqrt{5}-9}-56\,\sqrt{4\,\sqrt{5}-9}}-\frac{64\,\sqrt{5}\,x\,\left(4\,\sqrt{5}-9\right)^{1/4}}{24\,\sqrt{5}\,\sqrt{4\,\sqrt{5}-9}-56\,\sqrt{4\,\sqrt{5}-9}}\right)\,\left(4\,\sqrt{5}-9\right)^{1/4}}{10}$$
$$-\frac{\sqrt{5}\,\mathrm{atan}\left(\frac{x\,\left(-4\,\sqrt{5}-9\right)^{1/4}\,144\mathrm{i}}{24\,\sqrt{5}\,\sqrt{-4\,\sqrt{5}-9}+56\,\sqrt{-4\,\sqrt{5}-9}}+\frac{\sqrt{5}\,x\,\left(-4\,\sqrt{5}-9\right)^{1/4}\,64\mathrm{i}}{24\,\sqrt{5}\,\sqrt{-4\,\sqrt{5}-9}+56\,\sqrt{-4\,\sqrt{5}-9}}\right)\,\left(-4\,\sqrt{5}-9\right)^{1/4}\,1\mathrm{i}}{10}$$
$$-\frac{\sqrt{5}\,\mathrm{atan}\left(\frac{x\,\left(4\,\sqrt{5}-9\right)^{1/4}\,144\mathrm{i}}{24\,\sqrt{5}\,\sqrt{4\,\sqrt{5}-9}-56\,\sqrt{4\,\sqrt{5}-9}}-\frac{\sqrt{5}\,x\,\left(4\,\sqrt{5}-9\right)^{1/4}\,64\mathrm{i}}{24\,\sqrt{5}\,\sqrt{4\,\sqrt{5}-9}-56\,\sqrt{4\,\sqrt{5}-9}}\right)\,\left(4\,\sqrt{5}-9\right)^{1/4}\,1\mathrm{i}}{10}$$

input
```
int(1/(3*x^4 + x^8 + 1),x)
```

output
```
(5^(1/2)*atan((144*x*(- 4*5^(1/2) - 9)^(1/4))/(24*5^(1/2)*(- 4*5^(1/2) - 9
)^(1/2) + 56*(- 4*5^(1/2) - 9)^(1/2)) + (64*5^(1/2)*x*(- 4*5^(1/2) - 9)^(1
/4))/(24*5^(1/2)*(- 4*5^(1/2) - 9)^(1/2) + 56*(- 4*5^(1/2) - 9)^(1/2)))*(-
 4*5^(1/2) - 9)^(1/4))/10 + (5^(1/2)*atan((144*x*(4*5^(1/2) - 9)^(1/4))/(2
4*5^(1/2)*(4*5^(1/2) - 9)^(1/2) - 56*(4*5^(1/2) - 9)^(1/2)) - (64*5^(1/2)*
x*(4*5^(1/2) - 9)^(1/4))/(24*5^(1/2)*(4*5^(1/2) - 9)^(1/2) - 56*(4*5^(1/2)
 - 9)^(1/2)))*(4*5^(1/2) - 9)^(1/4))/10 - (5^(1/2)*atan((x*(- 4*5^(1/2) -
9)^(1/4)*144i)/(24*5^(1/2)*(- 4*5^(1/2) - 9)^(1/2) + 56*(- 4*5^(1/2) - 9)^
(1/2)) + (5^(1/2)*x*(- 4*5^(1/2) - 9)^(1/4)*64i)/(24*5^(1/2)*(- 4*5^(1/2)
- 9)^(1/2) + 56*(- 4*5^(1/2) - 9)^(1/2)))*(- 4*5^(1/2) - 9)^(1/4)*1i)/10 -
 (5^(1/2)*atan((x*(4*5^(1/2) - 9)^(1/4)*144i)/(24*5^(1/2)*(4*5^(1/2) - 9)^
(1/2) - 56*(4*5^(1/2) - 9)^(1/2)) - (5^(1/2)*x*(4*5^(1/2) - 9)^(1/4)*64i)/
(24*5^(1/2)*(4*5^(1/2) - 9)^(1/2) - 56*(4*5^(1/2) - 9)^(1/2)))*(4*5^(1/2)
- 9)^(1/4)*1i)/10
```

3.383 $\int \frac{1}{x^2(1+3x^4+x^8)}\,dx$

3.383.1 Optimal result

Integrand size = 16, antiderivative size = 416

$$\int \frac{1}{x^2(1+3x^4+x^8)}\,dx = -\frac{1}{x}+\frac{\left(3+\sqrt{5}\right)^{5/4}\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{4\ 2^{3/4}\sqrt{5}}-\frac{\left(3+\sqrt{5}\right)^{5/4}\arctan\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{4\ 2^{3/4}\sqrt{5}}$$

$$-\frac{1}{20}\sqrt[4]{6150-2750\sqrt{5}}\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)+\frac{1}{20}\sqrt[4]{6150-2750\sqrt{5}}\arctan\left(1+\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)-\frac{(3+\sqrt{}}{}$$

output
```
-1/x+1/20*arctan(-1+2^(3/4)*x/(3+5^(1/2))^(1/4))*(6150-2750*5^(1/2))^(1/4)
+1/20*arctan(1+2^(3/4)*x/(3+5^(1/2))^(1/4))*(6150-2750*5^(1/2))^(1/4)+1/40
*ln(2*x^2-2*2^(1/4)*x*(3+5^(1/2))^(1/4)+5^(1/2)+1)*(6150-2750*5^(1/2))^(1/
4)-1/40*ln(2*x^2+2*2^(1/4)*x*(3+5^(1/2))^(1/4)+5^(1/2)+1)*(6150-2750*5^(1/
2))^(1/4)-1/20*arctan(-1+2^(3/4)*x/(3-5^(1/2))^(1/4))*(246+110*5^(1/2))^(1
/4)*5^(1/2)-1/20*arctan(1+2^(3/4)*x/(3-5^(1/2))^(1/4))*(246+110*5^(1/2))^(
1/4)*5^(1/2)-1/40*ln(2*x^2-2*2^(1/4)*x*(3-5^(1/2))^(1/4)+5^(1/2)-1)*(246+1
10*5^(1/2))^(1/4)*5^(1/2)+1/40*ln(2*x^2+2*2^(1/4)*x*(3-5^(1/2))^(1/4)+5^(1
/2)-1)*(246+110*5^(1/2))^(1/4)*5^(1/2)
```

3.383.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.15

$$\int \frac{1}{x^2 (1+3x^4+x^8)} \, dx = -\frac{1}{x} - \frac{1}{4}\text{RootSum}\left[1+3\#1^4 + \#1^8\&, \frac{3\log(x-\#1)+\log(x-\#1)\#1^4}{3\#1+2\#1^5}\&\right]$$

input
```
Integrate[1/(x^2*(1 + 3*x^4 + x^8)),x]
```

output
```
-x^(-1) - RootSum[1 + 3*#1^4 + #1^8 & , (3*Log[x - #1] + Log[x - #1]*#1^4)
/(3*#1 + 2*#1^5) & ]/4
```

3.383.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 468, normalized size of antiderivative = 1.12, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.688, Rules used = {1704, 25, 1834, 27, 826, 1476, 1082, 217, 1479, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (x^8+3x^4+1)} \, dx$$

$$\Big\downarrow \; 1704$$

$$\int -\frac{x^2 (x^4+3)}{x^8+3x^4+1} dx - \frac{1}{x}$$

$$\Big\downarrow \; 25$$

$$-\int \frac{x^2 (x^4+3)}{x^8+3x^4+1} dx - \frac{1}{x}$$

$$\Big\downarrow \; 1834$$

$$-\frac{1}{10}\left(5+3\sqrt{5}\right)\int \frac{2x^2}{2x^4-\sqrt{5}+3} dx - \frac{1}{10}\left(5-3\sqrt{5}\right)\int \frac{2x^2}{2x^4+\sqrt{5}+3} dx - \frac{1}{x}$$

$$\Big\downarrow \; 27$$

$$-\frac{1}{5}\left(5+3\sqrt{5}\right)\int \frac{x^2}{2x^4-\sqrt{5}+3}dx-\frac{1}{5}\left(5-3\sqrt{5}\right)\int \frac{x^2}{2x^4+\sqrt{5}+3}dx-\frac{1}{x}$$

$$\Big\downarrow \text{ 826}$$

$$-\frac{1}{5}\left(5+3\sqrt{5}\right)\left(\frac{\int \frac{\sqrt{2}x^2+\sqrt{3-\sqrt{5}}}{2x^4-\sqrt{5}+3}dx}{2\sqrt{2}}-\frac{\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{2x^4-\sqrt{5}+3}dx}{2\sqrt{2}}\right)-\frac{1}{5}\left(5-3\sqrt{5}\right)\left(\frac{\int \frac{\sqrt{2}x^2+\sqrt{3+\sqrt{5}}}{2x^4+\sqrt{5}+3}dx}{2\sqrt{2}}-\frac{\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{2x^4+\sqrt{5}+3}dx}{2\sqrt{2}}\right)-\frac{1}{x}$$

$$\Big\downarrow \text{ 1476}$$

$$-\frac{1}{5}\left(5+3\sqrt{5}\right)\left(\frac{\frac{\int \frac{1}{x^2-\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}}dx}{2\sqrt{2}}+\frac{\int \frac{1}{x^2+\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}}dx}{2\sqrt{2}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{2x^4-\sqrt{5}+3}dx}{2\sqrt{2}}\right)-\frac{1}{5}\left(5-3\sqrt{5}\right)\left(\frac{\frac{\int \frac{1}{x^2-\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\sqrt{2}}+\frac{\int \frac{1}{x^2+\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\sqrt{2}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{2x^4+\sqrt{5}+3}dx}{2\sqrt{2}}\right)-\frac{1}{x}$$

$$\Big\downarrow \text{ 1082}$$

3.383. $\int \frac{1}{x^2(1+3x^4+x^8)}\,dx$

$$-\frac{1}{5}\left(5+3\sqrt{5}\right)\left(\frac{\frac{\int \frac{1}{-\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)^2-1}d\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}-\frac{\int \frac{1}{-\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)^2-1}d\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}}{2x^4-\sqrt{5}+3}}{2\sqrt{2}}\right.$$

$$\frac{1}{5}\left(5-3\sqrt{5}\right)\left(\frac{\frac{\int \frac{1}{-\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)^2-1}d\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\int \frac{1}{-\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)^2-1}d\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{2x^4+\sqrt{5}+3}}{2\sqrt{2}}\right.$$

$$\frac{1}{x}$$

$$\Big\downarrow \; 217$$

3.383. $\int \frac{1}{x^2(1+3x^4+x^8)}\,dx$

$$-\frac{1}{5}\left(5+3\sqrt{5}\right)\left(\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}}{2\sqrt{2}}-\frac{\int\frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{2x^4-\sqrt{5}+3}dx}{2\sqrt{2}}\right)-$$

$$\frac{1}{5}\left(5-3\sqrt{5}\right)\left(\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{2\sqrt{2}}-\frac{\int\frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{2x^4+\sqrt{5}+3}dx}{2\sqrt{2}}\right)-\frac{1}{x}$$

$\downarrow$ 1479

$$-\frac{1}{5}\left(5+3\sqrt{5}\right)\left(\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}}{2\sqrt{2}}-\frac{-\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\int-\frac{\sqrt[4]{2\left(3-\sqrt{5}\right)}-2x}{x^2-\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3 \ldots\right.}}}{\ldots}\right.$$

$$\frac{1}{5}\left(5-3\sqrt{5}\right)\left(\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{2\sqrt{2}}-\frac{-\frac{\int-\frac{\sqrt[4]{2\left(3+\sqrt{5}\right)}-2x}{x^2-\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\,2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\int-\frac{2x \ldots}{x^2+\sqrt[4]{2 \ldots}}}{2 \ldots}}{2\sqrt{2}}\right.$$

$$\frac{1}{x}$$

$\downarrow$ 25

3.383. $\int\frac{1}{x^2\left(1+3x^4+x^8\right)}\,dx$

$$-\frac{1}{5}\left(5+3\sqrt{5}\right)\left(\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}}{2\sqrt{2}}-\frac{\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\int\frac{\sqrt[4]{2\left(3-\sqrt{5}\right)}-2x}{x^2-\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}}\right.$$

$$\frac{1}{5}\left(5-3\sqrt{5}\right)\left(\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{2\sqrt{2}}-\frac{\frac{\int\frac{\sqrt[4]{2\left(3+\sqrt{5}\right)}-2x}{x^2-\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\,2^{3/4}\sqrt[4]{3+\sqrt{5}}}+\frac{\int\frac{2x+\sqrt[4]{2}}{x^2+\sqrt[4]{2\left(3+\right.}}}{2\,2^{3/4}}}{2\sqrt{2}}\right.$$

$$\frac{1}{x}$$

$$\Big\downarrow \text{1103}$$

$$-\frac{1}{5}\left(5+3\sqrt{5}\right)\left(\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}}{2\sqrt{2}}-\frac{\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\right.}{}\right.$$

$$\frac{1}{5}\left(5-3\sqrt{5}\right)\left(\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{2\sqrt{2}}-\frac{\frac{\log\left(2x^2+2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{2\,2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\log\left(2x^2-\right.}{}}{2\sqrt{2}}\right.$$

$$\frac{1}{x}$$

3.383. $\int \frac{1}{x^2(1+3x^4+x^8)}\,dx$

input
```
Int[1/(x^2*(1 + 3*x^4 + x^8)),x]
```

output
```
-x^(-1) - ((5 + 3*Sqrt[5])*((-(ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)]
/(2^(3/4)*(3 - Sqrt[5])^(1/4))) + ArcTan[1 + (2^(3/4)*x)/(3 - Sqrt[5])^(1/
4)]/(2^(3/4)*(3 - Sqrt[5])^(1/4)))/(2*Sqrt[2]) - (-1/4*(((3 + Sqrt[5])/2)^
(1/4)*Log[Sqrt[2*(3 - Sqrt[5])] - 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2]) +
(((3 + Sqrt[5])/2)^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])] + 2*(2*(3 - Sqrt[5]))^(
1/4)*x + 2*x^2])/4)/(2*Sqrt[2])))/5 - ((5 - 3*Sqrt[5])*((-(ArcTan[1 - (2^(
3/4)*x)/(3 + Sqrt[5])^(1/4)]/(2^(3/4)*(3 + Sqrt[5])^(1/4))) + ArcTan[1 + (
2^(3/4)*x)/(3 + Sqrt[5])^(1/4)]/(2^(3/4)*(3 + Sqrt[5])^(1/4)))/(2*Sqrt[2])
 - (-1/2*Log[Sqrt[2*(3 + Sqrt[5])] - 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2]/
(2^(3/4)*(3 + Sqrt[5])^(1/4)) + Log[Sqrt[2*(3 + Sqrt[5])] + 2*(2*(3 + Sqrt
[5]))^(1/4)*x + 2*x^2]/(2*2^(3/4)*(3 + Sqrt[5])^(1/4)))/(2*Sqrt[2])))/5
```

3.383.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1704
```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Simp[1/(a*d^n*(m + 1))   Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1)
+ c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{
a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] &&
LtQ[m, -1] && IntegerQ[p]
```

rule 1834
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 +
 (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2
 - (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ
[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n
, 0]
```

3.383. $\int \frac{1}{x^2(1+3x^4+x^8)}\,dx$

3.383.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.10

method	result	size
risch	$-\frac{1}{x}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(625\_Z^{8}+3075\_Z^{4}+1\right)}\_R\ln\left(1175\_R^{7}+5778\_R^{3}+11x\right)\right)}{4}$	42
default	$-\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}+3\_Z^{4}+1\right)}\frac{\left(\_R^{6}+3\_R^{2}\right)\ln\left(x-\_R\right)}{2\_R^{7}+3\_R^{3}}\right)}{4}-\frac{1}{x}$	52

input
```
int(1/x^2/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/x+1/4*sum(_R*ln(1175*_R^7+5778*_R^3+11*x),_R=RootOf(625*_Z^8+3075*_Z^4+
1))
```

3.383.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 559 vs. 2(270) = 540.

Time = 0.26 (sec) , antiderivative size = 559, normalized size of antiderivative = 1.34

$$\int \frac{1}{x^2(1+3x^4+x^8)}\,dx = \frac{\sqrt{10}x\sqrt{\sqrt{2}\sqrt{55\sqrt{5}-123}}\log\left(\sqrt{10}(47\sqrt{5}\sqrt{2}+105\sqrt{2})\sqrt{\sqrt{2}\sqrt{55\sqrt{5}-123}}\sqrt{55\sqrt{5}-123}+40\,x\right)-}{}$$

input
```
integrate(1/x^2/(x^8+3*x^4+1),x, algorithm="fricas")
```

output
```
1/40*(sqrt(10)*x*sqrt(sqrt(2)*sqrt(55*sqrt(5) - 123))*log(sqrt(10)*(47*sqr
t(5)*sqrt(2) + 105*sqrt(2))*sqrt(sqrt(2)*sqrt(55*sqrt(5) - 123))*sqrt(55*s
qrt(5) - 123) + 40*x) - sqrt(10)*x*sqrt(sqrt(2)*sqrt(55*sqrt(5) - 123))*lo
g(-sqrt(10)*(47*sqrt(5)*sqrt(2) + 105*sqrt(2))*sqrt(sqrt(2)*sqrt(55*sqrt(5
) - 123))*sqrt(55*sqrt(5) - 123) + 40*x) - sqrt(10)*x*sqrt(-sqrt(2)*sqrt(5
5*sqrt(5) - 123))*log(sqrt(10)*(47*sqrt(5)*sqrt(2) + 105*sqrt(2))*sqrt(-sq
rt(2)*sqrt(55*sqrt(5) - 123))*sqrt(55*sqrt(5) - 123) + 40*x) + sqrt(10)*x*
sqrt(-sqrt(2)*sqrt(55*sqrt(5) - 123))*log(-sqrt(10)*(47*sqrt(5)*sqrt(2) +
105*sqrt(2))*sqrt(-sqrt(2)*sqrt(55*sqrt(5) - 123))*sqrt(55*sqrt(5) - 123)
+ 40*x) - sqrt(10)*x*sqrt(sqrt(2)*sqrt(-55*sqrt(5) - 123))*log(sqrt(10)*(4
7*sqrt(5)*sqrt(2) - 105*sqrt(2))*sqrt(sqrt(2)*sqrt(-55*sqrt(5) - 123))*sqr
t(-55*sqrt(5) - 123) + 40*x) + sqrt(10)*x*sqrt(sqrt(2)*sqrt(-55*sqrt(5) -
123))*log(-sqrt(10)*(47*sqrt(5)*sqrt(2) - 105*sqrt(2))*sqrt(sqrt(2)*sqrt(-
55*sqrt(5) - 123))*sqrt(-55*sqrt(5) - 123) + 40*x) + sqrt(10)*x*sqrt(-sqrt
(2)*sqrt(-55*sqrt(5) - 123))*log(sqrt(10)*(47*sqrt(5)*sqrt(2) - 105*sqrt(2
))*sqrt(-sqrt(2)*sqrt(-55*sqrt(5) - 123))*sqrt(-55*sqrt(5) - 123) + 40*x)
- sqrt(10)*x*sqrt(-sqrt(2)*sqrt(-55*sqrt(5) - 123))*log(-sqrt(10)*(47*sqrt
(5)*sqrt(2) - 105*sqrt(2))*sqrt(-sqrt(2)*sqrt(-55*sqrt(5) - 123))*sqrt(-55
*sqrt(5) - 123) + 40*x) - 40)/x
```

3.383.6 Sympy [A] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.08

$$\int \frac{1}{x^2(1+3x^4+x^8)}\,dx = \text{RootSum}\left(40960000t^8+787200t^4+1,\left(t\mapsto t\log\left(\frac{19251200t^7}{11}+\frac{369792t^3}{11}+x\right)\right)\right) - \frac{1}{x}$$

input
```
integrate(1/x**2/(x**8+3*x**4+1),x)
```

output
```
RootSum(40960000*_t**8 + 787200*_t**4 + 1, Lambda(_t, _t*log(19251200*_t**
7/11 + 369792*_t**3/11 + x))) - 1/x
```

3.383.7 Maxima [F]

$$\int \frac{1}{x^2 (1 + 3x^4 + x^8)} \, dx = \int \frac{1}{(x^8 + 3\,x^4 + 1)x^2} \, dx$$

input
```
integrate(1/x^2/(x^8+3*x^4+1),x, algorithm="maxima")
```

output
```
-1/x - integrate((x^6 + 3*x^2)/(x^8 + 3*x^4 + 1), x)
```

3.383.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.59

$$\begin{aligned}\int \frac{1}{x^2 (1 + 3x^4 + x^8)} \, dx = & -\frac{1}{80} \left(\pi + 4 \arctan\left(x\sqrt{\sqrt{5}+1} - 1\right)\right)\sqrt{25\,\sqrt{5}+55} \\ & + \frac{1}{80} \left(\pi + 4 \arctan\left(-x\sqrt{\sqrt{5}+1} - 1\right)\right)\sqrt{25\,\sqrt{5}+55} \\ & + \frac{1}{80} \left(\pi + 4 \arctan\left(x\sqrt{\sqrt{5}-1} + 1\right)\right)\sqrt{25\,\sqrt{5}-55} \\ & - \frac{1}{80} \left(\pi + 4 \arctan\left(-x\sqrt{\sqrt{5}-1} + 1\right)\right)\sqrt{25\,\sqrt{5}-55} \\ & - \frac{1}{40}\sqrt{25\,\sqrt{5}-55} \log\left(748225\left(x + \sqrt{\sqrt{5}+1}\right)^2 + 748225\,x^2\right) \\ & + \frac{1}{40}\sqrt{25\,\sqrt{5}-55} \log\left(748225\left(x - \sqrt{\sqrt{5}+1}\right)^2 + 748225\,x^2\right) \\ & + \frac{1}{40}\sqrt{25\,\sqrt{5}+55} \log\left(180625\left(x + \sqrt{\sqrt{5}-1}\right)^2 + 180625\,x^2\right) \\ & - \frac{1}{40}\sqrt{25\,\sqrt{5}+55} \log\left(180625\left(x - \sqrt{\sqrt{5}-1}\right)^2 + 180625\,x^2\right) \\ & - \frac{1}{x}\end{aligned}$$

input
```
integrate(1/x^2/(x^8+3*x^4+1),x, algorithm="giac")
```

output
```
-1/80*(pi + 4*arctan(x*sqrt(sqrt(5) + 1) - 1))*sqrt(25*sqrt(5) + 55) + 1/8
0*(pi + 4*arctan(-x*sqrt(sqrt(5) + 1) - 1))*sqrt(25*sqrt(5) + 55) + 1/80*(
pi + 4*arctan(x*sqrt(sqrt(5) - 1) + 1))*sqrt(25*sqrt(5) - 55) - 1/80*(pi +
 4*arctan(-x*sqrt(sqrt(5) - 1) + 1))*sqrt(25*sqrt(5) - 55) - 1/40*sqrt(25*
sqrt(5) - 55)*log(748225*(x + sqrt(sqrt(5) + 1))^2 + 748225*x^2) + 1/40*sq
rt(25*sqrt(5) - 55)*log(748225*(x - sqrt(sqrt(5) + 1))^2 + 748225*x^2) + 1
/40*sqrt(25*sqrt(5) + 55)*log(180625*(x + sqrt(sqrt(5) - 1))^2 + 180625*x^
2) - 1/40*sqrt(25*sqrt(5) + 55)*log(180625*(x - sqrt(sqrt(5) - 1))^2 + 180
625*x^2) - 1/x
```

3.383.9 Mupad [B] (verification not implemented)

Time = 8.25 (sec) , antiderivative size = 292, normalized size of antiderivative = 0.70

$$\int \frac{1}{x^2 (1+3x^4+x^8)}\,dx = -\frac{1}{x} - \frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{2585\,2^{3/4}\,x\left(-55\sqrt{5}-123\right)^{1/4}}{2\left(3025\sqrt{5}+6765\right)}+\frac{1155\,2^{3/4}\sqrt{5}\,x\left(-55\sqrt{5}-123\right)^{1/4}}{2\left(3025\sqrt{5}+6765\right)}\right)\left(-55\sqrt{5}-123\right)^{1/4}}{20} - \frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{2585\,2^{3/4}\,x\left(55\sqrt{5}-123\right)^{1/4}}{2\left(3025\sqrt{5}-6765\right)}-\frac{1155\,2^{3/4}\sqrt{5}\,x\left(55\sqrt{5}-123\right)^{1/4}}{2\left(3025\sqrt{5}-6765\right)}\right)\left(55\sqrt{5}-123\right)^{1/4}}{20} - \frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\left(-55\sqrt{5}-123\right)^{1/4}\,2585\mathrm{i}}{2\left(3025\sqrt{5}+6765\right)}+\frac{2^{3/4}\sqrt{5}\,x\left(-55\sqrt{5}-123\right)^{1/4}\,1155\mathrm{i}}{2\left(3025\sqrt{5}+6765\right)}\right)\left(-55\sqrt{5}-123\right)^{1/4}\,1\mathrm{i}}{20} - \frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\left(55\sqrt{5}-123\right)^{1/4}\,2585\mathrm{i}}{2\left(3025\sqrt{5}-6765\right)}-\frac{2^{3/4}\sqrt{5}\,x\left(55\sqrt{5}-123\right)^{1/4}\,1155\mathrm{i}}{2\left(3025\sqrt{5}-6765\right)}\right)\left(55\sqrt{5}-123\right)^{1/4}\,1\mathrm{i}}{20}$$

input
```
int(1/(x^2*(3*x^4 + x^8 + 1)),x)
```

output
```
- 1/x - (2^(3/4)*5^(1/2)*atan((2585*2^(3/4)*x*(- 55*5^(1/2) - 123)^(1/4))/
(2*(3025*5^(1/2) + 6765)) + (1155*2^(3/4)*5^(1/2)*x*(- 55*5^(1/2) - 123)^(
1/4))/(2*(3025*5^(1/2) + 6765)))*(- 55*5^(1/2) - 123)^(1/4))/20 - (2^(3/4)
*5^(1/2)*atan((2585*2^(3/4)*x*(55*5^(1/2) - 123)^(1/4))/(2*(3025*5^(1/2) -
 6765)) - (1155*2^(3/4)*5^(1/2)*x*(55*5^(1/2) - 123)^(1/4))/(2*(3025*5^(1/
2) - 6765)))*(55*5^(1/2) - 123)^(1/4))/20 - (2^(3/4)*5^(1/2)*atan((2^(3/4)
*x*(- 55*5^(1/2) - 123)^(1/4)*2585i)/(2*(3025*5^(1/2) + 6765)) + (2^(3/4)*
5^(1/2)*x*(- 55*5^(1/2) - 123)^(1/4)*1155i)/(2*(3025*5^(1/2) + 6765)))*(-
55*5^(1/2) - 123)^(1/4)*1i)/20 - (2^(3/4)*5^(1/2)*atan((2^(3/4)*x*(55*5^(1
/2) - 123)^(1/4)*2585i)/(2*(3025*5^(1/2) - 6765)) - (2^(3/4)*5^(1/2)*x*(55
*5^(1/2) - 123)^(1/4)*1155i)/(2*(3025*5^(1/2) - 6765)))*(55*5^(1/2) - 123)
^(1/4)*1i)/20
```

3.383. $\int \frac{1}{x^2(1+3x^4+x^8)}\,dx$

3.384 $\int \frac{1}{x^4(1+3x^4+x^8)} \, dx$

3.384.1 Optimal result

Integrand size = 16, antiderivative size = 466

$$\int \frac{1}{x^4\left(1+3x^4+x^8\right)}\,dx = -\frac{1}{3x^3}+\frac{\sqrt[4]{843+377\sqrt{5}}\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}}$$
$$-\frac{\sqrt[4]{843+377\sqrt{5}}\arctan\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}}$$
$$-\frac{\sqrt[4]{843-377\sqrt{5}}\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}}$$
$$+\frac{\sqrt[4]{843-377\sqrt{5}}\arctan\left(1+\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}}$$
$$+\frac{\sqrt[4]{843+377\sqrt{5}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{4\,2^{3/4}\sqrt{5}}$$
$$-\frac{\sqrt[4]{843+377\sqrt{5}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{4\,2^{3/4}\sqrt{5}}$$
$$-\frac{\sqrt[4]{843-377\sqrt{5}}\log\left(\sqrt{2\left(3+\sqrt{5}\right)}-2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+2x^2\right)}{4\,2^{3/4}\sqrt{5}}$$
$$+\frac{\sqrt[4]{843-377\sqrt{5}}\log\left(\sqrt{2\left(3+\sqrt{5}\right)}+2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+2x^2\right)}{4\,2^{3/4}\sqrt{5}}$$

output
```
-1/3/x^3+1/20*arctan(-1+2^(3/4)*x/(3+5^(1/2))^(1/4))*(843-377*5^(1/2))^(1/
4)*2^(1/4)*5^(1/2)+1/20*arctan(1+2^(3/4)*x/(3+5^(1/2))^(1/4))*(843-377*5^(
1/2))^(1/4)*2^(1/4)*5^(1/2)-1/40*ln(2*x^2-2*2^(1/4)*x*(3+5^(1/2))^(1/4)+5^
(1/2)+1)*(843-377*5^(1/2))^(1/4)*2^(1/4)*5^(1/2)+1/40*ln(2*x^2+2*2^(1/4)*x
*(3+5^(1/2))^(1/4)+5^(1/2)+1)*(843-377*5^(1/2))^(1/4)*2^(1/4)*5^(1/2)-1/20
*arctan(-1+2^(3/4)*x/(3-5^(1/2))^(1/4))*(843+377*5^(1/2))^(1/4)*2^(1/4)*5^
(1/2)-1/20*arctan(1+2^(3/4)*x/(3-5^(1/2))^(1/4))*(843+377*5^(1/2))^(1/4)*2
^(1/4)*5^(1/2)+1/40*ln(2*x^2-2*2^(1/4)*x*(3-5^(1/2))^(1/4)+5^(1/2)-1)*(843
+377*5^(1/2))^(1/4)*2^(1/4)*5^(1/2)-1/40*ln(2*x^2+2*2^(1/4)*x*(3-5^(1/2))^
(1/4)+5^(1/2)-1)*(843+377*5^(1/2))^(1/4)*2^(1/4)*5^(1/2)
```

3.384.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.14

$$\int \frac{1}{x^4(1+3x^4+x^8)}\,dx = -\frac{1}{3x^3} - \frac{1}{4}\text{RootSum}\Big[1+3\#1^4 + \#1^8\&, \frac{3\log(x-\#1)+\log(x-\#1)\#1^4}{3\#1^3+2\#1^7}\&\Big]$$

input
```
Integrate[1/(x^4*(1 + 3*x^4 + x^8)),x]
```

output
```
-1/3*1/x^3 - RootSum[1 + 3*#1^4 + #1^8 & , (3*Log[x - #1] + Log[x - #1]*#1
^4)/(3*#1^3 + 2*#1^7) & ]/4
```

3.384.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 486, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.688$, Rules used = {1704, 27, 1752, 755, 27, 1476, 1082, 217, 1479, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4(x^8+3x^4+1)}\,dx$$

$$\Big\downarrow \text{1704}$$

$$\frac{1}{3}\int -\frac{3\left(x^4+3\right)}{x^8+3x^4+1}dx-\frac{1}{3x^3}$$

$$\Big\downarrow \; 27$$

$$-\int \frac{x^4+3}{x^8+3x^4+1}dx-\frac{1}{3x^3}$$

$$\Big\downarrow \; 1752$$

$$-\frac{1}{10}\left(5+3\sqrt{5}\right)\int \frac{1}{x^4+\frac{1}{2}\left(3-\sqrt{5}\right)}dx-\frac{1}{10}\left(5-3\sqrt{5}\right)\int \frac{1}{x^4+\frac{1}{2}\left(3+\sqrt{5}\right)}dx-\frac{1}{3x^3}$$

$$\Big\downarrow \; 755$$

$$-\frac{1}{10}\left(5+3\sqrt{5}\right)\left(\frac{\int \frac{2\left(\sqrt{3-\sqrt{5}}-\sqrt{2}x^2\right)}{2x^4-\sqrt{5}+3}dx}{2\sqrt{3-\sqrt{5}}}+\frac{\int \frac{2\left(\sqrt{2}x^2+\sqrt{3-\sqrt{5}}\right)}{2x^4-\sqrt{5}+3}dx}{2\sqrt{3-\sqrt{5}}}\right)-\frac{1}{10}\left(5-3\sqrt{5}\right)\left(\frac{\int \frac{2\left(\sqrt{3+\sqrt{5}}-\sqrt{2}x^2\right)}{2x^4+\sqrt{5}+3}dx}{2\sqrt{3+\sqrt{5}}}+\frac{\int \frac{2\left(\sqrt{2}x^2+\sqrt{3+\sqrt{5}}\right)}{2x^4+\sqrt{5}+3}dx}{2\sqrt{3+\sqrt{5}}}\right)-\frac{1}{3x^3}$$

$$\Big\downarrow \; 27$$

$$-\frac{1}{10}\left(5+3\sqrt{5}\right)\left(\frac{\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{2x^4-\sqrt{5}+3}dx}{\sqrt{3-\sqrt{5}}}+\frac{\int \frac{\sqrt{2}x^2+\sqrt{3-\sqrt{5}}}{2x^4-\sqrt{5}+3}dx}{\sqrt{3-\sqrt{5}}}\right)-\frac{1}{10}\left(5-3\sqrt{5}\right)\left(\frac{\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{2x^4+\sqrt{5}+3}dx}{\sqrt{3+\sqrt{5}}}+\frac{\int \frac{\sqrt{2}x^2+\sqrt{3+\sqrt{5}}}{2x^4+\sqrt{5}+3}dx}{\sqrt{3+\sqrt{5}}}\right)-\frac{1}{3x^3}$$

$$\Big\downarrow \; 1476$$

$$-\frac{1}{10}\left(5+3\sqrt{5}\right)\left(\frac{\frac{\int \frac{1}{x^2-\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}}dx}{2\sqrt{2}}+\frac{\int \frac{1}{x^2+\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}}dx}{2\sqrt{2}}}{\sqrt{3-\sqrt{5}}}+\frac{\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{2x^4-\sqrt{5}+3}dx}{\sqrt{3-\sqrt{5}}}\right)-\frac{1}{10}\left(5-3\sqrt{5}\right)\left(\frac{\frac{\int \frac{1}{x^2-\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\sqrt{2}}+\frac{\int \frac{1}{x^2+\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\sqrt{2}}}{\sqrt{3+\sqrt{5}}}+\frac{\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{2x^4+\sqrt{5}+3}dx}{\sqrt{3+\sqrt{5}}}\right)-\frac{1}{3x^3}$$

3.384. $\int \frac{1}{x^4\left(1+3x^4+x^8\right)}\,dx$

$\downarrow$ 1082

$$-\frac{1}{10}\left(5+3\sqrt{5}\right)\left(\frac{\int\frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{2x^4-\sqrt{5}+3}dx}{\sqrt{3-\sqrt{5}}}+\frac{\frac{\int\frac{1}{-\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)^2-1}d\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}-\frac{\int\frac{1}{-\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)^2-1}d\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{}}}\right.}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}}{\sqrt{3-\sqrt{5}}}\right.$$

$$\frac{1}{10}\left(5-3\sqrt{5}\right)\left(\frac{\int\frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{2x^4+\sqrt{5}+3}dx}{\sqrt{3+\sqrt{5}}}+\frac{\frac{\int\frac{1}{-\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)^2-1}d\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\int\frac{1}{-\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)^2-1}d\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right.}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{\sqrt{3+\sqrt{5}}}\right.$$

$$\frac{1}{3x^3}$$

$\downarrow$ 217

3.384. $\int\frac{1}{x^4(1+3x^4+x^8)}\,dx$

$$-\frac{1}{10}\left(5+3\sqrt{5}\right)\left(\frac{\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{2x^4-\sqrt{5}+3}dx}{\sqrt{3-\sqrt{5}}}+\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}}{\sqrt{3-\sqrt{5}}}\right)-$$

$$\frac{1}{10}\left(5-3\sqrt{5}\right)\left(\frac{\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{2x^4+\sqrt{5}+3}dx}{\sqrt{3+\sqrt{5}}}+\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{\sqrt{3+\sqrt{5}}}\right)-\frac{1}{3x^3}$$

$\downarrow$ 1479

$$-\frac{1}{10}\left(5+3\sqrt{5}\right)\left(\frac{-\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\int -\frac{\sqrt[4]{2\left(3-\sqrt{5}\right)}-2x}{x^2-\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}}dx-\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\int -\frac{2x+\sqrt[4]{2\left(3\right.}}{x^2+\sqrt[4]{2\left(3-\sqrt{5}\right.}}}{\sqrt{3-\sqrt{5}}}\right.$$

$$\frac{1}{10}\left(5-3\sqrt{5}\right)\left(\frac{-\frac{\int -\frac{\sqrt[4]{2\left(3+\sqrt{5}\right)}-2x}{x^2-\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\int -\frac{2x+\sqrt[4]{2\left(3+\sqrt{5}\right)}}{x^2+\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{\sqrt{3+\sqrt{5}}}+\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{\sqrt{3}}\right.$$

$$\frac{1}{3x^3}$$

$\downarrow$ 25

3.384. $\int \frac{1}{x^4(1+3x^4+x^8)}\,dx$

$$-\frac{1}{10}\left(5+3\sqrt{5}\right)\left(\frac{\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\int\frac{\sqrt[4]{2\left(3-\sqrt{5}\right)}-2x}{x^2-\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}}dx+\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\int\frac{2x+\sqrt[4]{2\left(3-\sqrt{5}\right.}}{x^2+\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{}}}{\sqrt{3-\sqrt{5}}}\right.$$

$$\frac{1}{10}\left(5-3\sqrt{5}\right)\left(\frac{\frac{\int\frac{\sqrt[4]{2\left(3+\sqrt{5}\right)}-2x}{x^2-\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}+\frac{\int\frac{2x+\sqrt[4]{2\left(3+\sqrt{5}\right)}}{x^2+\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{\sqrt{3+\sqrt{5}}}+\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\ldots}{\sqrt{3+\ldots}}\right.$$

$$\frac{1}{3x^3}$$

$$\Big\downarrow \text{1103}$$

$$-\frac{1}{10}\left(5+3\sqrt{5}\right)\left(\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}}{\sqrt{3-\sqrt{5}}}+\frac{\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x\ldots\right.}{}\right.$$

$$\frac{1}{10}\left(5-3\sqrt{5}\right)\left(\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{\sqrt{3+\sqrt{5}}}+\frac{\frac{\log\left(2x^2+2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\log\left(2x^2\ldots\right.}{}}{\sqrt{3+\sqrt{5}}}\right.$$

$$\frac{1}{3x^3}$$

3.384. $\int \frac{1}{x^4(1+3x^4+x^8)}\,dx$

input
```
Int[1/(x^4*(1 + 3*x^4 + x^8)),x]
```

output
```
-1/3*1/x^3 - ((5 + 3*Sqrt[5])*((-(ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/
4)]/(2^(3/4)*(3 - Sqrt[5])^(1/4))) + ArcTan[1 + (2^(3/4)*x)/(3 - Sqrt[5])^
(1/4)]/(2^(3/4)*(3 - Sqrt[5])^(1/4)))/Sqrt[3 - Sqrt[5]] + (-1/4*(((3 + Sqr
t[5])/2)^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])] - 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2
*x^2]) + (((3 + Sqrt[5])/2)^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])] + 2*(2*(3 - Sq
rt[5]))^(1/4)*x + 2*x^2])/4)/Sqrt[3 - Sqrt[5]]))/10 - ((5 - 3*Sqrt[5])*((-
(ArcTan[1 - (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)]/(2^(3/4)*(3 + Sqrt[5])^(1/4))
) + ArcTan[1 + (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)]/(2^(3/4)*(3 + Sqrt[5])^(1/
4)))/Sqrt[3 + Sqrt[5]] + (-1/2*Log[Sqrt[2*(3 + Sqrt[5])] - 2*(2*(3 + Sqrt[
5]))^(1/4)*x + 2*x^2]/(2^(3/4)*(3 + Sqrt[5])^(1/4)) + Log[Sqrt[2*(3 + Sqrt
[5])] + 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2]/(2*2^(3/4)*(3 + Sqrt[5])^(1/4
)))/Sqrt[3 + Sqrt[5]]))/10
```

3.384.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1704
```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Simp[1/(a*d^n*(m + 1))   Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1)
+ c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{
a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] &&
LtQ[m, -1] && IntegerQ[p]
```

rule 1752
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

3.384. $\int \frac{1}{x^4(1+3x^4+x^8)}\,dx$

3.384.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.09

method	result	size
risch	$-\frac{1}{3x^3}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(625\_Z^8+21075\_Z^4+1\right)}\_R\ln\left(175\_R^5+5778\_R+377x\right)\right)}{4}$	40
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^8+3\_Z^4+1\right)}\frac{\left(-\_R^4-3\right)\ln\left(x-\_R\right)}{2\_R^7+3\_R^3}\right)}{4}-\frac{1}{3x^3}$	50

input
```
int(1/x^4/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/3/x^3+1/4*sum(_R*ln(175*_R^5+5778*_R+377*x),_R=RootOf(625*_Z^8+21075*_Z
^4+1))
```

3.384.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 451, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^4\left(1+3x^4+x^8\right)}\,dx$$
$$= \frac{3\sqrt{10}x^3\sqrt{\sqrt{2}\sqrt{377\sqrt{5}-843}}\log\left(\sqrt{10}\sqrt{\sqrt{2}\sqrt{377\sqrt{5}-843}}(7\sqrt{5}+15)+20\,x\right)-3\sqrt{10}x^3\sqrt{\sqrt{2}\sqrt{377}}}{}$$

input
```
integrate(1/x^4/(x^8+3*x^4+1),x, algorithm="fricas")
```

output
```
1/120*(3*sqrt(10)*x^3*sqrt(sqrt(2)*sqrt(377*sqrt(5) - 843))*log(sqrt(10)*s
qrt(sqrt(2)*sqrt(377*sqrt(5) - 843))*(7*sqrt(5) + 15) + 20*x) - 3*sqrt(10)
*x^3*sqrt(sqrt(2)*sqrt(377*sqrt(5) - 843))*log(-sqrt(10)*sqrt(sqrt(2)*sqrt
(377*sqrt(5) - 843))*(7*sqrt(5) + 15) + 20*x) + 3*sqrt(10)*x^3*sqrt(-sqrt(
2)*sqrt(377*sqrt(5) - 843))*log(sqrt(10)*sqrt(-sqrt(2)*sqrt(377*sqrt(5) -
843))*(7*sqrt(5) + 15) + 20*x) - 3*sqrt(10)*x^3*sqrt(-sqrt(2)*sqrt(377*sqr
t(5) - 843))*log(-sqrt(10)*sqrt(-sqrt(2)*sqrt(377*sqrt(5) - 843))*(7*sqrt(
5) + 15) + 20*x) - 3*sqrt(10)*x^3*sqrt(sqrt(2)*sqrt(-377*sqrt(5) - 843))*l
og(sqrt(10)*sqrt(sqrt(2)*sqrt(-377*sqrt(5) - 843))*(7*sqrt(5) - 15) + 20*x
) + 3*sqrt(10)*x^3*sqrt(sqrt(2)*sqrt(-377*sqrt(5) - 843))*log(-sqrt(10)*sq
rt(sqrt(2)*sqrt(-377*sqrt(5) - 843))*(7*sqrt(5) - 15) + 20*x) - 3*sqrt(10)
*x^3*sqrt(-sqrt(2)*sqrt(-377*sqrt(5) - 843))*log(sqrt(10)*sqrt(-sqrt(2)*sq
rt(-377*sqrt(5) - 843))*(7*sqrt(5) - 15) + 20*x) + 3*sqrt(10)*x^3*sqrt(-sq
rt(2)*sqrt(-377*sqrt(5) - 843))*log(-sqrt(10)*sqrt(-sqrt(2)*sqrt(-377*sqrt
(5) - 843))*(7*sqrt(5) - 15) + 20*x) - 40)/x^3
```

3.384.6 Sympy [A] (verification not implemented)

Time = 0.96 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.07

$$\int \frac{1}{x^4(1+3x^4+x^8)}\,dx = \operatorname{RootSum}\left(40960000t^8+5395200t^4+1,\left(t\mapsto t\log\left(\frac{179200t^5}{377}+\frac{23112t}{377}+x\right)\right)\right) - \frac{1}{3x^3}$$

input
```
integrate(1/x**4/(x**8+3*x**4+1),x)
```

output
```
RootSum(40960000*_t**8 + 5395200*_t**4 + 1, Lambda(_t, _t*log(179200*_t**5
/377 + 23112*_t/377 + x))) - 1/(3*x**3)
```

3.384.7 Maxima [F]

$$\int \frac{1}{x^4 (1 + 3x^4 + x^8)} \, dx = \int \frac{1}{(x^8 + 3 \, x^4 + 1)x^4} \, dx$$

input
```
integrate(1/x^4/(x^8+3*x^4+1),x, algorithm="maxima")
```

output
```
-1/3/x^3 - integrate((x^4 + 3)/(x^8 + 3*x^4 + 1), x)
```

3.384.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.52

$$\int \frac{1}{x^4\left(1+3x^4+x^8\right)}\,dx = -\frac{1}{80}\left(\pi+4\arctan\left(x\sqrt{\sqrt{5}+1}+1\right)\right)\sqrt{65\sqrt{5}+145} + \frac{1}{80}\left(\pi+4\arctan\left(-x\sqrt{\sqrt{5}+1}+1\right)\right)\sqrt{65\sqrt{5}+145} + \frac{1}{80}\left(\pi+4\arctan\left(x\sqrt{\sqrt{5}-1}-1\right)\right)\sqrt{65\sqrt{5}-145} - \frac{1}{80}\left(\pi+4\arctan\left(-x\sqrt{\sqrt{5}-1}-1\right)\right)\sqrt{65\sqrt{5}-145} + \frac{1}{40}\sqrt{65\sqrt{5}-145}\log\left(93122500\left(x+\sqrt{\sqrt{5}+1}\right)^2 + 93122500\,x^2\right) - \frac{1}{40}\sqrt{65\sqrt{5}-145}\log\left(93122500\left(x-\sqrt{\sqrt{5}+1}\right)^2 + 93122500\,x^2\right) - \frac{1}{40}\sqrt{65\sqrt{5}+145}\log\left(53728900\left(x+\sqrt{\sqrt{5}-1}\right)^2 + 53728900\,x^2\right) + \frac{1}{40}\sqrt{65\sqrt{5}+145}\log\left(53728900\left(x-\sqrt{\sqrt{5}-1}\right)^2 + 53728900\,x^2\right) - \frac{1}{3\,x^3}$$

input
```
integrate(1/x^4/(x^8+3*x^4+1),x, algorithm="giac")
```

output
```
-1/80*(pi + 4*arctan(x*sqrt(sqrt(5) + 1) + 1))*sqrt(65*sqrt(5) + 145) + 1/
80*(pi + 4*arctan(-x*sqrt(sqrt(5) + 1) + 1))*sqrt(65*sqrt(5) + 145) + 1/80
*(pi + 4*arctan(x*sqrt(sqrt(5) - 1) - 1))*sqrt(65*sqrt(5) - 145) - 1/80*(p
i + 4*arctan(-x*sqrt(sqrt(5) - 1) - 1))*sqrt(65*sqrt(5) - 145) + 1/40*sqrt
(65*sqrt(5) - 145)*log(93122500*(x + sqrt(sqrt(5) + 1))^2 + 93122500*x^2)
- 1/40*sqrt(65*sqrt(5) - 145)*log(93122500*(x - sqrt(sqrt(5) + 1))^2 + 931
22500*x^2) - 1/40*sqrt(65*sqrt(5) + 145)*log(53728900*(x + sqrt(sqrt(5) -
1))^2 + 53728900*x^2) + 1/40*sqrt(65*sqrt(5) + 145)*log(53728900*(x - sqrt
(sqrt(5) - 1))^2 + 53728900*x^2) - 1/3/x^3
```

3.384.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 492, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^4(1+3x^4+x^8)}\,dx$$
$$= \frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{46371\,2^{3/4}\,x\,\left(377\sqrt{5}-843\right)^{1/4}}{2\left(3393\sqrt{2}\sqrt{377\sqrt{5}-843}-1508\sqrt{2}\sqrt{5}\sqrt{377\sqrt{5}-843}\right)}-\frac{20735\,2^{3/4}\sqrt{5}\,x\,\left(377\sqrt{5}-843\right)^{1/4}}{2\left(3393\sqrt{2}\sqrt{377\sqrt{5}-843}-1508\sqrt{2}\sqrt{5}\sqrt{377\sqrt{5}-843}\right)}\right)\,(3}{20}$$
$$-\frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{46371\,2^{3/4}\,x\,\left(-377\sqrt{5}-843\right)^{1/4}}{2\left(3393\sqrt{2}\sqrt{-377\sqrt{5}-843}+1508\sqrt{2}\sqrt{5}\sqrt{-377\sqrt{5}-843}\right)}+\frac{20735\,2^{3/4}\sqrt{5}\,x\,\left(-377\sqrt{5}-843\right)^{1/4}}{2\left(3393\sqrt{2}\sqrt{-377\sqrt{5}-843}+1508\sqrt{2}\sqrt{5}\sqrt{-377\sqrt{5}-8}\right.}}{20}$$
$$-\frac{1}{3\,x^3}$$
$$+\frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\,\left(-377\sqrt{5}-843\right)^{1/4}\,46371\mathrm{i}}{2\left(3393\sqrt{2}\sqrt{-377\sqrt{5}-843}+1508\sqrt{2}\sqrt{5}\sqrt{-377\sqrt{5}-843}\right)}+\frac{2^{3/4}\sqrt{5}\,x\,\left(-377\sqrt{5}-843\right)^{1/4}\,20735\mathrm{i}}{2\left(3393\sqrt{2}\sqrt{-377\sqrt{5}-843}+1508\sqrt{2}\sqrt{5}\sqrt{-377\sqrt{5}-8}\right.}}{20}$$
$$-\frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\,\left(377\sqrt{5}-843\right)^{1/4}\,46371\mathrm{i}}{2\left(3393\sqrt{2}\sqrt{377\sqrt{5}-843}-1508\sqrt{2}\sqrt{5}\sqrt{377\sqrt{5}-843}\right)}-\frac{2^{3/4}\sqrt{5}\,x\,\left(377\sqrt{5}-843\right)^{1/4}\,20735\mathrm{i}}{2\left(3393\sqrt{2}\sqrt{377\sqrt{5}-843}-1508\sqrt{2}\sqrt{5}\sqrt{377\sqrt{5}-843}\right)}\right)}{20}$$

input
```
int(1/(x^4*(3*x^4 + x^8 + 1)),x)
```

output
```
(2^(3/4)*5^(1/2)*atan((46371*2^(3/4)*x*(377*5^(1/2) - 843)^(1/4))/(2*(3393
*2^(1/2)*(377*5^(1/2) - 843)^(1/2) - 1508*2^(1/2)*5^(1/2)*(377*5^(1/2) - 8
43)^(1/2))) - (20735*2^(3/4)*5^(1/2)*x*(377*5^(1/2) - 843)^(1/4))/(2*(3393
*2^(1/2)*(377*5^(1/2) - 843)^(1/2) - 1508*2^(1/2)*5^(1/2)*(377*5^(1/2) - 8
43)^(1/2))))*(377*5^(1/2) - 843)^(1/4))/20 - (2^(3/4)*5^(1/2)*atan((46371*
2^(3/4)*x*(- 377*5^(1/2) - 843)^(1/4))/(2*(3393*2^(1/2)*(- 377*5^(1/2) - 8
43)^(1/2) + 1508*2^(1/2)*5^(1/2)*(- 377*5^(1/2) - 843)^(1/2))) + (20735*2^
(3/4)*5^(1/2)*x*(- 377*5^(1/2) - 843)^(1/4))/(2*(3393*2^(1/2)*(- 377*5^(1/
2) - 843)^(1/2) + 1508*2^(1/2)*5^(1/2)*(- 377*5^(1/2) - 843)^(1/2))))*(- 3
77*5^(1/2) - 843)^(1/4))/20 - 1/(3*x^3) + (2^(3/4)*5^(1/2)*atan((2^(3/4)*x
*(- 377*5^(1/2) - 843)^(1/4)*46371i)/(2*(3393*2^(1/2)*(- 377*5^(1/2) - 843
)^(1/2) + 1508*2^(1/2)*5^(1/2)*(- 377*5^(1/2) - 843)^(1/2))) + (2^(3/4)*5^
(1/2)*x*(- 377*5^(1/2) - 843)^(1/4)*20735i)/(2*(3393*2^(1/2)*(- 377*5^(1/2
) - 843)^(1/2) + 1508*2^(1/2)*5^(1/2)*(- 377*5^(1/2) - 843)^(1/2))))*(- 37
7*5^(1/2) - 843)^(1/4)*1i)/20 - (2^(3/4)*5^(1/2)*atan((2^(3/4)*x*(377*5^(1
/2) - 843)^(1/4)*46371i)/(2*(3393*2^(1/2)*(377*5^(1/2) - 843)^(1/2) - 1508
*2^(1/2)*5^(1/2)*(377*5^(1/2) - 843)^(1/2))) - (2^(3/4)*5^(1/2)*x*(377*5^(
1/2) - 843)^(1/4)*20735i)/(2*(3393*2^(1/2)*(377*5^(1/2) - 843)^(1/2) - 150
8*2^(1/2)*5^(1/2)*(377*5^(1/2) - 843)^(1/2))))*(377*5^(1/2) - 843)^(1/4)*1
i)/20
```

3.384. $\int \frac{1}{x^4(1+3x^4+x^8)} \, dx$

3.385 $\int \frac{x^m}{1-3x^4+x^8}\,dx$

3.385.1 Optimal result

Integrand size = 16, antiderivative size = 117

$$\int \frac{x^m}{1-3x^4+x^8}\,dx = \frac{2x^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{4},\frac{5+m}{4},\frac{2x^4}{3-\sqrt{5}}\right)}{\sqrt{5}\left(3-\sqrt{5}\right)(1+m)} - \frac{2x^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{4},\frac{5+m}{4},\frac{2x^4}{3+\sqrt{5}}\right)}{\sqrt{5}\left(3+\sqrt{5}\right)(1+m)}$$

output
```
2/5*x^(1+m)*hypergeom([1, 1/4+1/4*m],[5/4+1/4*m],2*x^4/(3-5^(1/2)))/(1+m)/
(3-5^(1/2))*5^(1/2)-2/5*x^(1+m)*hypergeom([1, 1/4+1/4*m],[5/4+1/4*m],2*x^4
/(3+5^(1/2)))/(1+m)*5^(1/2)/(3+5^(1/2))
```

3.385.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.60 (sec) , antiderivative size = 575, normalized size of antiderivative = 4.91

$$\int \frac{x^m}{1-3x^4+x^8}\,dx = \frac{x^m\left(-\text{RootSum}\left[-1-\#1^2+\#1^4\&,\frac{\text{Hypergeometric2F1}\left(-m,-m,1-m,-\frac{\#1}{x-\#1}\right)\left(\frac{x}{x-\#1}\right)^{-m}}{-\#1+2\#1^3}\&\right]+\frac{\text{RootSum}\left[-1-\right.}{}\right.}{}$$

input
```
Integrate[x^m/(1 - 3*x^4 + x^8),x]
```

output
```
(x^m*(-RootSum[-1 - #1^2 + #1^4 & , Hypergeometric2F1[-m, -m, 1 - m, -(#1/
(x - #1))]/((x/(x - #1))^m*(-#1 + 2*#1^3)) & ] + (RootSum[-1 - #1^2 + #1^4
 & , (m*x^2 + m^2*x^2 + 2*m*x*#1 + m^2*x*#1 + (2*Hypergeometric2F1[-m, -m,
 1 - m, -(#1/(x - #1))]*#1^2)/(x/(x - #1))^m + (3*m*Hypergeometric2F1[-m,
-m, 1 - m, -(#1/(x - #1))]*#1^2)/(x/(x - #1))^m + (m^2*Hypergeometric2F1[-
m, -m, 1 - m, -(#1/(x - #1))]*#1^2)/(x/(x - #1))^m + (m*#1^2)/(x/#1)^m)/(-
#1 + 2*#1^3) & ] - (2 + 3*m + m^2)*RootSum[-1 + #1^2 + #1^4 & , Hypergeome
tric2F1[-m, -m, 1 - m, -(#1/(x - #1))]/((x/(x - #1))^m*(#1 + 2*#1^3)) & ]
- RootSum[-1 + #1^2 + #1^4 & , (m*x^2 + m^2*x^2 + 2*m*x*#1 + m^2*x*#1 + (2
*Hypergeometric2F1[-m, -m, 1 - m, -(#1/(x - #1))]*#1^2)/(x/(x - #1))^m + (
3*m*Hypergeometric2F1[-m, -m, 1 - m, -(#1/(x - #1))]*#1^2)/(x/(x - #1))^m
+ (m^2*Hypergeometric2F1[-m, -m, 1 - m, -(#1/(x - #1))]*#1^2)/(x/(x - #1))
^m + (m*#1^2)/(x/#1)^m)/(#1 + 2*#1^3) & ])/(2 + 3*m + m^2)))/(4*m)
```

3.385.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1711, 27, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{x^8-3x^4+1}\,dx$$

$$\Big\downarrow \text{1711}$$

$$\frac{\int -\frac{2x^m}{-2x^4+\sqrt{5}+3}dx}{\sqrt{5}}-\frac{\int -\frac{2x^m}{-2x^4-\sqrt{5}+3}dx}{\sqrt{5}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\int \frac{x^m}{-2x^4-\sqrt{5}+3}dx}{\sqrt{5}}-\frac{2\int \frac{x^m}{-2x^4+\sqrt{5}+3}dx}{\sqrt{5}}$$

$$\Big\downarrow \text{888}$$

$$\frac{2x^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{4},\frac{m+5}{4},\frac{2x^4}{3-\sqrt{5}}\right)}{\sqrt{5}\,(3-\sqrt{5})\,(m+1)} - \frac{2x^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{4},\frac{m+5}{4},\frac{2x^4}{3+\sqrt{5}}\right)}{\sqrt{5}\,(3+\sqrt{5})\,(m+1)}$$

input
```
Int[x^m/(1 - 3*x^4 + x^8),x]
```

output
```
(2*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, (2*x^4)/(3 - Sqrt[
5])])/(Sqrt[5]*(3 - Sqrt[5])*(1 + m)) - (2*x^(1 + m)*Hypergeometric2F1[1,
(1 + m)/4, (5 + m)/4, (2*x^4)/(3 + Sqrt[5])])/(Sqrt[5]*(3 + Sqrt[5])*(1 +
m))
```

3.385.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1711
```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[c/q   Int[(d*x)^m/(b/2 - q/2 + c
*x^n), x], x] - Simp[c/q   Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; Free
Q[{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

3.385.4 Maple [F]

$$\int \frac{x^m}{x^8-3x^4+1}dx$$

input
```
int(x^m/(x^8-3*x^4+1),x)
```

output
```
int(x^m/(x^8-3*x^4+1),x)
```

3.385.5 Fricas [F]

$$\int \frac{x^m}{1-3x^4+x^8}\,dx = \int \frac{x^m}{x^8-3\,x^4+1}\,dx$$

input
```
integrate(x^m/(x^8-3*x^4+1),x, algorithm="fricas")
```

output
```
integral(x^m/(x^8 - 3*x^4 + 1), x)
```

3.385.6 Sympy [F]

$$\int \frac{x^m}{1-3x^4+x^8}\,dx = \int \frac{x^m}{(x^4-x^2-1)\,(x^4+x^2-1)}\,dx$$

input
```
integrate(x**m/(x**8-3*x**4+1),x)
```

output
```
Integral(x**m/((x**4 - x**2 - 1)*(x**4 + x**2 - 1)), x)
```

3.385.7 Maxima [F]

$$\int \frac{x^m}{1-3x^4+x^8}\,dx = \int \frac{x^m}{x^8-3\,x^4+1}\,dx$$

input
```
integrate(x^m/(x^8-3*x^4+1),x, algorithm="maxima")
```

output
```
integrate(x^m/(x^8 - 3*x^4 + 1), x)
```

3.385.8 Giac [F]

$$\int \frac{x^m}{1-3x^4+x^8}\,dx=\int \frac{x^m}{x^8-3\,x^4+1}\,dx$$

input
```
integrate(x^m/(x^8-3*x^4+1),x, algorithm="giac")
```

output
```
integrate(x^m/(x^8 - 3*x^4 + 1), x)
```

3.385.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{1-3x^4+x^8}\,dx=\int \frac{x^m}{x^8-3\,x^4+1}\,dx$$

input
```
int(x^m/(x^8 - 3*x^4 + 1),x)
```

output
```
int(x^m/(x^8 - 3*x^4 + 1), x)
```

3.386 $\int \frac{x^{11}}{1-3x^4+x^8}\,dx$

3.386.1 Optimal result

Integrand size = 16, antiderivative size = 62

$$\int \frac{x^{11}}{1-3x^4+x^8}\,dx = \frac{x^4}{4}+\frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(3-\sqrt{5}-2x^4\right)+\frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(3+\sqrt{5}-2x^4\right)$$

output
```
1/4*x^4+1/40*ln(-2*x^4-5^(1/2)+3)*(15-7*5^(1/2))+1/40*ln(-2*x^4+5^(1/2)+3)
*(15+7*5^(1/2))
```

3.386.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.90

$$\int \frac{x^{11}}{1-3x^4+x^8}\,dx = \frac{1}{40}\left(10x^4+\left(15+7\sqrt{5}\right)\log\left(3+\sqrt{5}-2x^4\right)+\left(15-7\sqrt{5}\right)\log\left(-3+\sqrt{5}+2x^4\right)\right)$$

input
```
Integrate[x^11/(1 - 3*x^4 + x^8),x]
```

output
```
(10*x^4 + (15 + 7*Sqrt[5])*Log[3 + Sqrt[5] - 2*x^4] + (15 - 7*Sqrt[5])*Log
[-3 + Sqrt[5] + 2*x^4])/40
```

3.386.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1693, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{x^8-3x^4+1}dx$$
$$\Big\downarrow \text{1693}$$
$$\frac{1}{4}\int \frac{x^8}{x^8-3x^4+1}dx^4$$
$$\Big\downarrow \text{1141}$$
$$\frac{1}{4}\int \left(-\frac{15+7\sqrt{5}}{5\left(-2x^4+\sqrt{5}+3\right)}+1-\frac{15-7\sqrt{5}}{5\left(-2x^4-\sqrt{5}+3\right)}\right)dx^4$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{4}\left(x^4+\frac{1}{10}\left(15-7\sqrt{5}\right)\log\left(-2x^4-\sqrt{5}+3\right)+\frac{1}{10}\left(15+7\sqrt{5}\right)\log\left(-2x^4+\sqrt{5}+3\right)\right)$$

input
```
Int[x^11/(1 - 3*x^4 + x^8),x]
```

output
```
(x^4 + ((15 - 7*Sqrt[5])*Log[3 - Sqrt[5] - 2*x^4])/10 + ((15 + 7*Sqrt[5])*
Log[3 + Sqrt[5] - 2*x^4])/10)/4
```

3.386.3.1 Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.386.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.61

method	result	size
default	$\frac{x^4}{4}+\frac{3\ln\left(x^8-3x^4+1\right)}{8}-\frac{7\sqrt{5}\,\operatorname{arctanh}\left(\frac{\left(2x^4-3\right)\sqrt{5}}{5}\right)}{20}$	38
risch	$\frac{x^4}{4}+\frac{3\ln\left(2x^4-\sqrt{5}-3\right)}{8}+\frac{7\ln\left(2x^4-\sqrt{5}-3\right)\sqrt{5}}{40}+\frac{3\ln\left(2x^4+\sqrt{5}-3\right)}{8}-\frac{7\ln\left(2x^4+\sqrt{5}-3\right)\sqrt{5}}{40}$	69

input
```
int(x^11/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4+3/8*ln(x^8-3*x^4+1)-7/20*5^(1/2)*arctanh(1/5*(2*x^4-3)*5^(1/2))
```

3.386.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00

$$\int \frac{x^{11}}{1-3x^4+x^8}\,dx = \frac{1}{4}\,x^4+\frac{7}{40}\,\sqrt{5}\log\left(\frac{2\,x^8-6\,x^4-\sqrt{5}(2\,x^4-3)+7}{x^8-3\,x^4+1}\right)+\frac{3}{8}\,\log\left(x^8-3\,x^4+1\right)$$

input
```
integrate(x^11/(x^8-3*x^4+1),x, algorithm="fricas")
```

output
```
1/4*x^4 + 7/40*sqrt(5)*log((2*x^8 - 6*x^4 - sqrt(5)*(2*x^4 - 3) + 7)/(x^8
- 3*x^4 + 1)) + 3/8*log(x^8 - 3*x^4 + 1)
```

3.386.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.94

$$\int \frac{x^{11}}{1-3x^4+x^8}\,dx = \frac{x^4}{4} + \left(\frac{3}{8} + \frac{7\sqrt{5}}{40}\right)\log\left(x^4 - \frac{3}{2} - \frac{\sqrt{5}}{2}\right) + \left(\frac{3}{8} - \frac{7\sqrt{5}}{40}\right)\log\left(x^4 - \frac{3}{2} + \frac{\sqrt{5}}{2}\right)$$

input
```
integrate(x**11/(x**8-3*x**4+1),x)
```

output
```
x**4/4 + (3/8 + 7*sqrt(5)/40)*log(x**4 - 3/2 - sqrt(5)/2) + (3/8 - 7*sqrt(
5)/40)*log(x**4 - 3/2 + sqrt(5)/2)
```

3.386.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.81

$$\int \frac{x^{11}}{1-3x^4+x^8}\,dx = \frac{1}{4}\,x^4 + \frac{7}{40}\,\sqrt{5}\log\left(\frac{2\,x^4 - \sqrt{5} - 3}{2\,x^4 + \sqrt{5} - 3}\right) + \frac{3}{8}\,\log\left(x^8 - 3\,x^4 + 1\right)$$

input
```
integrate(x^11/(x^8-3*x^4+1),x, algorithm="maxima")
```

output
```
1/4*x^4 + 7/40*sqrt(5)*log((2*x^4 - sqrt(5) - 3)/(2*x^4 + sqrt(5) - 3)) +
3/8*log(x^8 - 3*x^4 + 1)
```

3.386.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.85

$$\int \frac{x^{11}}{1-3x^4+x^8}\,dx = \frac{1}{4}\,x^4 + \frac{7}{40}\,\sqrt{5}\log\left(\frac{|2\,x^4 - \sqrt{5} - 3|}{|2\,x^4 + \sqrt{5} - 3|}\right) + \frac{3}{8}\,\log\left(|x^8 - 3\,x^4 + 1|\right)$$

input
```
integrate(x^11/(x^8-3*x^4+1),x, algorithm="giac")
```

output
```
1/4*x^4 + 7/40*sqrt(5)*log(abs(2*x^4 - sqrt(5) - 3)/abs(2*x^4 + sqrt(5) -
3)) + 3/8*log(abs(x^8 - 3*x^4 + 1))
```

3.386.9 Mupad [B] (verification not implemented)

Time = 8.45 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.03

$$\int \frac{x^{11}}{1-3x^4+x^8}\,dx = \frac{3\,\ln\left(x^4-\frac{\sqrt{5}}{2}-\frac{3}{2}\right)}{8}+\frac{3\,\ln\left(x^4+\frac{\sqrt{5}}{2}-\frac{3}{2}\right)}{8} + \frac{7\,\sqrt{5}\,\ln\left(x^4-\frac{\sqrt{5}}{2}-\frac{3}{2}\right)}{40}-\frac{7\,\sqrt{5}\,\ln\left(x^4+\frac{\sqrt{5}}{2}-\frac{3}{2}\right)}{40}+\frac{x^4}{4}$$

input
```
int(x^11/(x^8 - 3*x^4 + 1),x)
```

output
```
(3*log(x^4 - 5^(1/2)/2 - 3/2))/8 + (3*log(5^(1/2)/2 + x^4 - 3/2))/8 + (7*5
^(1/2)*log(x^4 - 5^(1/2)/2 - 3/2))/40 - (7*5^(1/2)*log(5^(1/2)/2 + x^4 - 3
/2))/40 + x^4/4
```

3.387 $\int \frac{x^9}{1-3x^4+x^8}\,dx$

3.387.1 Optimal result

Integrand size = 16, antiderivative size = 90

$$\int \frac{x^9}{1-3x^4+x^8}\,dx=\frac{x^2}{2}-\frac{1}{2}\sqrt{\frac{1}{5}\left(9+4\sqrt{5}\right)}\text{arctanh}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)+\frac{1}{2}\sqrt{\frac{1}{5}\left(9-4\sqrt{5}\right)}\text{arctanh}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)$$

output
```
1/2*x^2+1/2*arctanh(x^2*(1/2+1/2*5^(1/2)))*(1-2/5*5^(1/2))-1/2*arctanh(x^2
*2^(1/2)/(3+5^(1/2))^(1/2))*(1+2/5*5^(1/2))
```

3.387.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.14

$$\int \frac{x^9}{1-3x^4+x^8}\,dx=\frac{1}{20}\left(10x^2+\left(-5+2\sqrt{5}\right)\log\left(-1+\sqrt{5}-2x^2\right)+\left(5+2\sqrt{5}\right)\log\left(1+\sqrt{5}-2x^2\right)+\left(5-2\sqrt{5}\right)\log\left(-1+\sqrt{5}+2x^2\right)-\left(5+2\sqrt{5}\right)\log\left(1+\sqrt{5}+2x^2\right)\right)$$

input
```
Integrate[x^9/(1 - 3*x^4 + x^8),x]
```

output
```
(10*x^2 + (-5 + 2*Sqrt[5])*Log[-1 + Sqrt[5] - 2*x^2] + (5 + 2*Sqrt[5])*Log
[1 + Sqrt[5] - 2*x^2] + (5 - 2*Sqrt[5])*Log[-1 + Sqrt[5] + 2*x^2] - (5 + 2
*Sqrt[5])*Log[1 + Sqrt[5] + 2*x^2])/20
```

3.387.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1695, 1442, 1480, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{x^8-3x^4+1} \, dx$$

$$\Big\downarrow \text{1695}$$

$$\frac{1}{2}\int \frac{x^8}{x^8-3x^4+1}dx^2$$

$$\Big\downarrow \text{1442}$$

$$\frac{1}{2}\left(x^2-\int \frac{1-3x^4}{x^8-3x^4+1}dx^2\right)$$

$$\Big\downarrow \text{1480}$$

$$\frac{1}{2}\left(\frac{1}{10}\left(15+7\sqrt{5}\right)\int \frac{1}{x^4+\frac{1}{2}\left(-3-\sqrt{5}\right)}dx^2+\frac{1}{10}\left(15-7\sqrt{5}\right)\int \frac{1}{x^4+\frac{1}{2}\left(-3+\sqrt{5}\right)}dx^2+x^2\right)$$

$$\Big\downarrow \text{220}$$

$$\frac{1}{2}\left(-\frac{\left(15+7\sqrt{5}\right)\text{arctanh}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)}{5\sqrt{2\left(3+\sqrt{5}\right)}}-\frac{1}{10}\left(15-7\sqrt{5}\right)\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\text{arctanh}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)+x^2\right)$$

input
```
Int[x^9/(1 - 3*x^4 + x^8),x]
```

output
```
(x^2 - ((15 + 7*Sqrt[5])*ArcTanh[Sqrt[2/(3 + Sqrt[5])]*x^2])/(5*Sqrt[2*(3
+ Sqrt[5])]) - ((15 - 7*Sqrt[5])*Sqrt[(3 + Sqrt[5])/2]*ArcTanh[Sqrt[(3 + S
qrt[5])/2]*x^2])/10)/2
```

3.387.3.1 Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1442
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
x] - Simp[d^4/(c*(m + 4*p + 1))   Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1695
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b
*x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c,
p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.387.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.74

method	result
default	$\frac{x^{2}}{2}-\frac{\ln\left(x^{4}+x^{2}-1\right)}{4}-\frac{\operatorname{arctanh}\left(\frac{\left(2x^{2}+1\right)\sqrt{5}}{5}\right)\sqrt{5}}{5}+\frac{\ln\left(x^{4}-x^{2}-1\right)}{4}-\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{\left(2x^{2}-1\right)\sqrt{5}}{5}\right)}{5}$
risch	$\frac{x^{2}}{2}-\frac{\ln\left(2x^{2}-\sqrt{5}+1\right)}{4}+\frac{\ln\left(2x^{2}-\sqrt{5}+1\right)\sqrt{5}}{10}-\frac{\ln\left(2x^{2}+\sqrt{5}+1\right)}{4}-\frac{\ln\left(2x^{2}+\sqrt{5}+1\right)\sqrt{5}}{10}+\frac{\ln\left(2x^{2}-\sqrt{5}-1\right)}{4}+\frac{\ln\left(2x^{2}-\sqrt{5}-\right.}{10}$

input
```
int(x^9/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2-1/4*ln(x^4+x^2-1)-1/5*arctanh(1/5*(2*x^2+1)*5^(1/2))*5^(1/2)+1/4*l
n(x^4-x^2-1)-1/5*5^(1/2)*arctanh(1/5*(2*x^2-1)*5^(1/2))
```

3.387.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(50) = 100.

Time = 0.25 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.27

$$\int \frac{x^9}{1-3x^4+x^8}\,dx = \frac{1}{2}\,x^2 + \frac{1}{10}\,\sqrt{5}\log\left(\frac{2\,x^4+2\,x^2-\sqrt{5}(2\,x^2+1)+3}{x^4+x^2-1}\right) + \frac{1}{10}\,\sqrt{5}\log\left(\frac{2\,x^4-2\,x^2-\sqrt{5}(2\,x^2-1)+3}{x^4-x^2-1}\right) - \frac{1}{4}\,\log\left(x^4+x^2-1\right) + \frac{1}{4}\,\log\left(x^4-x^2-1\right)$$

input
```
integrate(x^9/(x^8-3*x^4+1),x, algorithm="fricas")
```

output
```
1/2*x^2 + 1/10*sqrt(5)*log((2*x^4 + 2*x^2 - sqrt(5)*(2*x^2 + 1) + 3)/(x^4
+ x^2 - 1)) + 1/10*sqrt(5)*log((2*x^4 - 2*x^2 - sqrt(5)*(2*x^2 - 1) + 3)/(
x^4 - x^2 - 1)) - 1/4*log(x^4 + x^2 - 1) + 1/4*log(x^4 - x^2 - 1)
```

3.387.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. 2(63) = 126.

Time = 0.20 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.89

$$\int \frac{x^9}{1-3x^4+x^8}\,dx = \frac{x^2}{2} + \left(-\frac{1}{4}-\frac{\sqrt{5}}{10}\right)\log\left(x^2-\frac{47}{8}-\frac{47\sqrt{5}}{20}-120\left(-\frac{1}{4}-\frac{\sqrt{5}}{10}\right)^3\right) + \left(-\frac{1}{4}+\frac{\sqrt{5}}{10}\right)\log\left(x^2-\frac{47}{8}-120\left(-\frac{1}{4}+\frac{\sqrt{5}}{10}\right)^3+\frac{47\sqrt{5}}{20}\right) + \left(\frac{1}{4}-\frac{\sqrt{5}}{10}\right)\log\left(x^2-\frac{47\sqrt{5}}{20}-120\left(\frac{1}{4}-\frac{\sqrt{5}}{10}\right)^3+\frac{47}{8}\right) + \left(\frac{\sqrt{5}}{10}+\frac{1}{4}\right)\log\left(x^2-120\left(\frac{\sqrt{5}}{10}+\frac{1}{4}\right)^3+\frac{47\sqrt{5}}{20}+\frac{47}{8}\right)$$

input
```
integrate(x**9/(x**8-3*x**4+1),x)
```

output
```
x**2/2 + (-1/4 - sqrt(5)/10)*log(x**2 - 47/8 - 47*sqrt(5)/20 - 120*(-1/4 -
 sqrt(5)/10)**3) + (-1/4 + sqrt(5)/10)*log(x**2 - 47/8 - 120*(-1/4 + sqrt(
5)/10)**3 + 47*sqrt(5)/20) + (1/4 - sqrt(5)/10)*log(x**2 - 47*sqrt(5)/20 -
 120*(1/4 - sqrt(5)/10)**3 + 47/8) + (sqrt(5)/10 + 1/4)*log(x**2 - 120*(sq
rt(5)/10 + 1/4)**3 + 47*sqrt(5)/20 + 47/8)
```

3.387.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.02

$$\int \frac{x^9}{1-3x^4+x^8}\,dx = \frac{1}{2}\,x^2 + \frac{1}{10}\,\sqrt{5}\log\left(\frac{2\,x^2-\sqrt{5}+1}{2\,x^2+\sqrt{5}+1}\right) + \frac{1}{10}\,\sqrt{5}\log\left(\frac{2\,x^2-\sqrt{5}-1}{2\,x^2+\sqrt{5}-1}\right) - \frac{1}{4}\,\log\left(x^4+x^2-1\right) + \frac{1}{4}\,\log\left(x^4-x^2-1\right)$$

input
```
integrate(x^9/(x^8-3*x^4+1),x, algorithm="maxima")
```

output
```
1/2*x^2 + 1/10*sqrt(5)*log((2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1)) +
1/10*sqrt(5)*log((2*x^2 - sqrt(5) - 1)/(2*x^2 + sqrt(5) - 1)) - 1/4*log(x^
4 + x^2 - 1) + 1/4*log(x^4 - x^2 - 1)
```

3.387.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.08

$$\int \frac{x^9}{1-3x^4+x^8}\,dx = \frac{1}{2}\,x^2 + \frac{1}{10}\,\sqrt{5}\log\left(\frac{\left|2\,x^2-\sqrt{5}+1\right|}{2\,x^2+\sqrt{5}+1}\right) + \frac{1}{10}\,\sqrt{5}\log\left(\frac{\left|2\,x^2-\sqrt{5}-1\right|}{\left|2\,x^2+\sqrt{5}-1\right|}\right) - \frac{1}{4}\,\log\left(\left|x^4+x^2-1\right|\right) + \frac{1}{4}\,\log\left(\left|x^4-x^2-1\right|\right)$$

input
```
integrate(x^9/(x^8-3*x^4+1),x, algorithm="giac")
```

output
```
1/2*x^2 + 1/10*sqrt(5)*log(abs(2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1))
 + 1/10*sqrt(5)*log(abs(2*x^2 - sqrt(5) - 1)/abs(2*x^2 + sqrt(5) - 1)) - 1
/4*log(abs(x^4 + x^2 - 1)) + 1/4*log(abs(x^4 - x^2 - 1))
```

3.387.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00

$$\int \frac{x^9}{1-3x^4+x^8}\,dx = \frac{x^2}{2} - \operatorname{atanh}\left(\frac{64\,x^2}{64\,\sqrt{5}+192} + \frac{64\,\sqrt{5}\,x^2}{64\,\sqrt{5}+192}\right)\left(\frac{\sqrt{5}}{5}+\frac{1}{2}\right) - \operatorname{atanh}\left(\frac{64\,x^2}{64\,\sqrt{5}-192} - \frac{64\,\sqrt{5}\,x^2}{64\,\sqrt{5}-192}\right)\left(\frac{\sqrt{5}}{5}-\frac{1}{2}\right)$$

input
```
int(x^9/(x^8 - 3*x^4 + 1),x)
```

output
```
x^2/2 - atanh((64*x^2)/(64*5^(1/2) + 192) + (64*5^(1/2)*x^2)/(64*5^(1/2) +
 192))*(5^(1/2)/5 + 1/2) - atanh((64*x^2)/(64*5^(1/2) - 192) - (64*5^(1/2)
*x^2)/(64*5^(1/2) - 192))*(5^(1/2)/5 - 1/2)
```

3.388 $\int \frac{x^7}{1-3x^4+x^8}\,dx$

3.388.1 Optimal result

Integrand size = 16, antiderivative size = 55

$$\int \frac{x^7}{1-3x^4+x^8}\,dx=\frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(3-\sqrt{5}-2x^4\right)+\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(3+\sqrt{5}-2x^4\right)$$

output
```
1/40*ln(-2*x^4-5^(1/2)+3)*(5-3*5^(1/2))+1/40*ln(-2*x^4+5^(1/2)+3)*(5+3*5^(
1/2))
```

3.388.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int \frac{x^7}{1-3x^4+x^8}\,dx=\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(3+\sqrt{5}-2x^4\right)+\frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(-3+\sqrt{5}+2x^4\right)$$

input
```
Integrate[x^7/(1 - 3*x^4 + x^8),x]
```

output
```
((5 + 3*Sqrt[5])*Log[3 + Sqrt[5] - 2*x^4])/40 + ((5 - 3*Sqrt[5])*Log[-3 +
Sqrt[5] + 2*x^4])/40
```

3.388.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1693, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{x^8-3x^4+1}dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{4}\int \frac{x^4}{x^8-3x^4+1}dx^4$$

$$\Big\downarrow \text{1141}$$

$$\frac{1}{4}\int \left(-\frac{5+3\sqrt{5}}{5\left(-2x^4+\sqrt{5}+3\right)}-\frac{5-3\sqrt{5}}{5\left(-2x^4-\sqrt{5}+3\right)}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\frac{1}{10}\left(5-3\sqrt{5}\right)\log\left(-2x^4-\sqrt{5}+3\right)+\frac{1}{10}\left(5+3\sqrt{5}\right)\log\left(-2x^4+\sqrt{5}+3\right)\right)$$

input
```
Int[x^7/(1 - 3*x^4 + x^8),x]
```

output
```
(((5 - 3*Sqrt[5])*Log[3 - Sqrt[5] - 2*x^4])/10 + ((5 + 3*Sqrt[5])*Log[3 +
Sqrt[5] - 2*x^4])/10)/4
```

3.388.3.1 Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.388.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.60

method	result	size
default	$\frac{\ln(x^8-3x^4+1)}{8}-\frac{3\sqrt{5}\,\operatorname{arctanh}\left(\frac{(2x^4-3)\sqrt{5}}{5}\right)}{20}$	33
risch	$\frac{\ln\left(2x^4-\sqrt{5}-3\right)}{8}+\frac{3\ln\left(2x^4-\sqrt{5}-3\right)\sqrt{5}}{40}+\frac{\ln\left(2x^4+\sqrt{5}-3\right)}{8}-\frac{3\ln\left(2x^4+\sqrt{5}-3\right)\sqrt{5}}{40}$	64

input
```
int(x^7/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/8*ln(x^8-3*x^4+1)-3/20*5^(1/2)*arctanh(1/5*(2*x^4-3)*5^(1/2))
```

3.388.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.04

$$\int \frac{x^7}{1-3x^4+x^8}\,dx = \frac{3}{40}\sqrt{5}\log\left(\frac{2\,x^8-6\,x^4-\sqrt{5}(2\,x^4-3)+7}{x^8-3\,x^4+1}\right)+\frac{1}{8}\log\left(x^8-3\,x^4+1\right)$$

input
```
integrate(x^7/(x^8-3*x^4+1),x, algorithm="fricas")
```

output
```
3/40*sqrt(5)*log((2*x^8 - 6*x^4 - sqrt(5)*(2*x^4 - 3) + 7)/(x^8 - 3*x^4 +
1)) + 1/8*log(x^8 - 3*x^4 + 1)
```

3.388.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int \frac{x^7}{1-3x^4+x^8}\,dx = \left(\frac{1}{8}+\frac{3\sqrt{5}}{40}\right)\log\left(x^4-\frac{3}{2}-\frac{\sqrt{5}}{2}\right)+\left(\frac{1}{8}-\frac{3\sqrt{5}}{40}\right)\log\left(x^4-\frac{3}{2}+\frac{\sqrt{5}}{2}\right)$$

input
```
integrate(x**7/(x**8-3*x**4+1),x)
```

output
```
(1/8 + 3*sqrt(5)/40)*log(x**4 - 3/2 - sqrt(5)/2) + (1/8 - 3*sqrt(5)/40)*lo
g(x**4 - 3/2 + sqrt(5)/2)
```

3.388.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.82

$$\int \frac{x^7}{1-3x^4+x^8}\,dx = \frac{3}{40}\,\sqrt{5}\log\left(\frac{2\,x^4-\sqrt{5}-3}{2\,x^4+\sqrt{5}-3}\right)+\frac{1}{8}\,\log\left(x^8-3\,x^4+1\right)$$

input
```
integrate(x^7/(x^8-3*x^4+1),x, algorithm="maxima")
```

output
```
3/40*sqrt(5)*log((2*x^4 - sqrt(5) - 3)/(2*x^4 + sqrt(5) - 3)) + 1/8*log(x^
8 - 3*x^4 + 1)
```

3.388.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.87

$$\int \frac{x^7}{1-3x^4+x^8}\,dx = \frac{3}{40}\,\sqrt{5}\log\left(\frac{|2\,x^4-\sqrt{5}-3|}{|2\,x^4+\sqrt{5}-3|}\right)+\frac{1}{8}\,\log\left(|x^8-3\,x^4+1|\right)$$

input
```
integrate(x^7/(x^8-3*x^4+1),x, algorithm="giac")
```

output
```
3/40*sqrt(5)*log(abs(2*x^4 - sqrt(5) - 3)/abs(2*x^4 + sqrt(5) - 3)) + 1/8*
log(abs(x^8 - 3*x^4 + 1))
```

3.388.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.07

$$\int \frac{x^7}{1-3x^4+x^8}\,dx = \frac{\ln\left(x^4-\frac{\sqrt{5}}{2}-\frac{3}{2}\right)}{8}+\frac{\ln\left(x^4+\frac{\sqrt{5}}{2}-\frac{3}{2}\right)}{8}+\frac{3\sqrt{5}\,\ln\left(x^4-\frac{\sqrt{5}}{2}-\frac{3}{2}\right)}{40}-\frac{3\sqrt{5}\,\ln\left(x^4+\frac{\sqrt{5}}{2}-\frac{3}{2}\right)}{40}$$

input
```
int(x^7/(x^8 - 3*x^4 + 1),x)
```

output
```
log(x^4 - 5^(1/2)/2 - 3/2)/8 + log(5^(1/2)/2 + x^4 - 3/2)/8 + (3*5^(1/2)*l
og(x^4 - 5^(1/2)/2 - 3/2))/40 - (3*5^(1/2)*log(5^(1/2)/2 + x^4 - 3/2))/40
```

3.389 $\int \frac{x^5}{1-3x^4+x^8}\,dx$

3.389.1 Optimal result

Integrand size = 16, antiderivative size = 81

$$\int \frac{x^5}{1-3x^4+x^8}\,dx = -\frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\operatorname{arctanh}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right) + \frac{1}{2}\sqrt{\frac{1}{10}\left(3-\sqrt{5}\right)}\operatorname{arctanh}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)$$

output
```
1/2*arctanh(x^2*(1/2+1/2*5^(1/2)))*(1/2-1/10*5^(1/2))-1/2*arctanh(x^2*2^(1
/2)/(3+5^(1/2))^(1/2))*(1/2+1/10*5^(1/2))
```

3.389.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.12

$$\int \frac{x^5}{1-3x^4+x^8}\,dx = \frac{1}{40}\left(\left(-5+\sqrt{5}\right)\log\left(-1+\sqrt{5}-2x^2\right)+\left(5+\sqrt{5}\right)\log\left(1+\sqrt{5}-2x^2\right) - \left(-5+\sqrt{5}\right)\log\left(-1+\sqrt{5}+2x^2\right)-\left(5+\sqrt{5}\right)\log\left(1+\sqrt{5}+2x^2\right)\right)$$

input
```
Integrate[x^5/(1 - 3*x^4 + x^8),x]
```

output
```
((-5 + Sqrt[5])*Log[-1 + Sqrt[5] - 2*x^2] + (5 + Sqrt[5])*Log[1 + Sqrt[5]
- 2*x^2] - (-5 + Sqrt[5])*Log[-1 + Sqrt[5] + 2*x^2] - (5 + Sqrt[5])*Log[1
+ Sqrt[5] + 2*x^2])/40
```

3.389.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.22, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1695, 1450, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{x^8-3x^4+1} \, dx$$

$$\Big\downarrow \; 1695$$

$$\frac{1}{2}\int \frac{x^4}{x^8-3x^4+1}dx^2$$

$$\Big\downarrow \; 1450$$

$$\frac{1}{2}\left(\frac{1}{10}\left(5+3\sqrt{5}\right)\int \frac{1}{x^4+\frac{1}{2}\left(-3-\sqrt{5}\right)}dx^2+\frac{1}{10}\left(5-3\sqrt{5}\right)\int \frac{1}{x^4+\frac{1}{2}\left(-3+\sqrt{5}\right)}dx^2\right)$$

$$\Big\downarrow \; 220$$

$$\frac{1}{2}\left(-\frac{\left(5+3\sqrt{5}\right)\operatorname{arctanh}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)}{5\sqrt{2\left(3+\sqrt{5}\right)}}-\frac{1}{10}\left(5-3\sqrt{5}\right)\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\operatorname{arctanh}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)\right)$$

input
```
Int[x^5/(1 - 3*x^4 + x^8),x]
```

output
```
(-1/5*((5 + 3*Sqrt[5])*ArcTanh[Sqrt[2/(3 + Sqrt[5])]*x^2])/Sqrt[2*(3 + Sqr
t[5])] - ((5 - 3*Sqrt[5])*Sqrt[(3 + Sqrt[5])/2]*ArcTanh[Sqrt[(3 + Sqrt[5])
/2]*x^2])/10)/2
```

3.389.3.1 Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1450
```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^2/2)*(b/q + 1)   Int[(d*x)^(m - 2)/(b/
2 + q/2 + c*x^2), x], x] - Simp[(d^2/2)*(b/q - 1)   Int[(d*x)^(m - 2)/(b/2
- q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] &&
 GeQ[m, 2]
```

rule 1695
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b
*x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c,
p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.389.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.77

method	result
default	$-\frac{\ln(x^4+x^2-1)}{8}-\frac{\operatorname{arctanh}\left(\frac{(2x^2+1)\sqrt{5}}{5}\right)\sqrt{5}}{20}+\frac{\ln(x^4-x^2-1)}{8}-\frac{\sqrt{5}\operatorname{arctanh}\left(\frac{(2x^2-1)\sqrt{5}}{5}\right)}{20}$
risch	$\frac{\ln\left(2x^2-\sqrt{5}-1\right)}{8}+\frac{\ln\left(2x^2-\sqrt{5}-1\right)\sqrt{5}}{40}+\frac{\ln\left(2x^2+\sqrt{5}-1\right)}{8}-\frac{\ln\left(2x^2+\sqrt{5}-1\right)\sqrt{5}}{40}-\frac{\ln\left(2x^2-\sqrt{5}+1\right)}{8}+\frac{\ln\left(2x^2-\sqrt{5}+1\right)\sqrt{5}}{40}$

input
```
int(x^5/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/8*ln(x^4+x^2-1)-1/20*arctanh(1/5*(2*x^2+1)*5^(1/2))*5^(1/2)+1/8*ln(x^4-
x^2-1)-1/20*5^(1/2)*arctanh(1/5*(2*x^2-1)*5^(1/2))
```

3.389.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. 2(41) = 82.

Time = 0.26 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.35

$$\int \frac{x^5}{1-3x^4+x^8}\,dx = \frac{1}{40}\sqrt{5}\log\left(\frac{2\,x^4+2\,x^2-\sqrt{5}(2\,x^2+1)+3}{x^4+x^2-1}\right) + \frac{1}{40}\sqrt{5}\log\left(\frac{2\,x^4-2\,x^2-\sqrt{5}(2\,x^2-1)+3}{x^4-x^2-1}\right) - \frac{1}{8}\log\left(x^4+x^2-1\right)+\frac{1}{8}\log\left(x^4-x^2-1\right)$$

input
```
integrate(x^5/(x^8-3*x^4+1),x, algorithm="fricas")
```

output
```
1/40*sqrt(5)*log((2*x^4 + 2*x^2 - sqrt(5)*(2*x^2 + 1) + 3)/(x^4 + x^2 - 1)
) + 1/40*sqrt(5)*log((2*x^4 - 2*x^2 - sqrt(5)*(2*x^2 - 1) + 3)/(x^4 - x^2
- 1)) - 1/8*log(x^4 + x^2 - 1) + 1/8*log(x^4 - x^2 - 1)
```

3.389.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(58) = 116$.

Time = 0.20 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.04

$$\begin{aligned}\int \frac{x^5}{1-3x^4+x^8}\,dx = &\left(-\frac{1}{8}-\frac{\sqrt{5}}{40}\right)\log\left(x^2-\frac{3}{2}-\frac{3\sqrt{5}}{10}-640\left(-\frac{1}{8}-\frac{\sqrt{5}}{40}\right)^3\right)\\&+\left(-\frac{1}{8}+\frac{\sqrt{5}}{40}\right)\log\left(x^2-\frac{3}{2}-640\left(-\frac{1}{8}+\frac{\sqrt{5}}{40}\right)^3+\frac{3\sqrt{5}}{10}\right)\\&+\left(\frac{1}{8}-\frac{\sqrt{5}}{40}\right)\log\left(x^2-\frac{3\sqrt{5}}{10}-640\left(\frac{1}{8}-\frac{\sqrt{5}}{40}\right)^3+\frac{3}{2}\right)\\&+\left(\frac{\sqrt{5}}{40}+\frac{1}{8}\right)\log\left(x^2-640\left(\frac{\sqrt{5}}{40}+\frac{1}{8}\right)^3+\frac{3\sqrt{5}}{10}+\frac{3}{2}\right)\end{aligned}$$

input
```
integrate(x**5/(x**8-3*x**4+1),x)
```

output
```
(-1/8 - sqrt(5)/40)*log(x**2 - 3/2 - 3*sqrt(5)/10 - 640*(-1/8 - sqrt(5)/40
)**3) + (-1/8 + sqrt(5)/40)*log(x**2 - 3/2 - 640*(-1/8 + sqrt(5)/40)**3 +
3*sqrt(5)/10) + (1/8 - sqrt(5)/40)*log(x**2 - 3*sqrt(5)/10 - 640*(1/8 - sq
rt(5)/40)**3 + 3/2) + (sqrt(5)/40 + 1/8)*log(x**2 - 640*(sqrt(5)/40 + 1/8)
**3 + 3*sqrt(5)/10 + 3/2)
```

3.389.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. 2(41) = 82.

Time = 0.27 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.07

$$\int \frac{x^5}{1-3x^4+x^8}\,dx = \frac{1}{40}\,\sqrt{5}\log\left(\frac{2\,x^2-\sqrt{5}+1}{2\,x^2+\sqrt{5}+1}\right)+\frac{1}{40}\,\sqrt{5}\log\left(\frac{2\,x^2-\sqrt{5}-1}{2\,x^2+\sqrt{5}-1}\right) - \frac{1}{8}\,\log\left(x^4+x^2-1\right)+\frac{1}{8}\,\log\left(x^4-x^2-1\right)$$

input
```
integrate(x^5/(x^8-3*x^4+1),x, algorithm="maxima")
```

output
```
1/40*sqrt(5)*log((2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1)) + 1/40*sqrt(
5)*log((2*x^2 - sqrt(5) - 1)/(2*x^2 + sqrt(5) - 1)) - 1/8*log(x^4 + x^2 -
1) + 1/8*log(x^4 - x^2 - 1)
```

3.389.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(41) = 82.

Time = 0.32 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.14

$$\int \frac{x^5}{1-3x^4+x^8}\,dx = \frac{1}{40}\,\sqrt{5}\log\left(\frac{\left|2\,x^2-\sqrt{5}+1\right|}{2\,x^2+\sqrt{5}+1}\right)+\frac{1}{40}\,\sqrt{5}\log\left(\frac{\left|2\,x^2-\sqrt{5}-1\right|}{\left|2\,x^2+\sqrt{5}-1\right|}\right) - \frac{1}{8}\,\log\left(\left|x^4+x^2-1\right|\right)+\frac{1}{8}\,\log\left(\left|x^4-x^2-1\right|\right)$$

input
```
integrate(x^5/(x^8-3*x^4+1),x, algorithm="giac")
```

output
```
1/40*sqrt(5)*log(abs(2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1)) + 1/40*sq
rt(5)*log(abs(2*x^2 - sqrt(5) - 1)/abs(2*x^2 + sqrt(5) - 1)) - 1/8*log(abs
(x^4 + x^2 - 1)) + 1/8*log(abs(x^4 - x^2 - 1))
```

3.389.9 Mupad [B] (verification not implemented)

Time = 8.37 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.95

$$\int \frac{x^5}{1-3x^4+x^8}\,dx = -\mathrm{atanh}\left(\frac{4\,x^2}{\sqrt{5}-3}-\frac{2\,\sqrt{5}\,x^2}{\sqrt{5}-3}\right)\,\left(\frac{\sqrt{5}}{20}+\frac{1}{4}\right) - \mathrm{atanh}\left(\frac{4\,x^2}{\sqrt{5}+3}+\frac{2\,\sqrt{5}\,x^2}{\sqrt{5}+3}\right)\,\left(\frac{\sqrt{5}}{20}-\frac{1}{4}\right)$$

input
```
int(x^5/(x^8 - 3*x^4 + 1),x)
```

output
```
- atanh((4*x^2)/(5^(1/2) - 3) - (2*5^(1/2)*x^2)/(5^(1/2) - 3))*(5^(1/2)/20
 + 1/4) - atanh((4*x^2)/(5^(1/2) + 3) + (2*5^(1/2)*x^2)/(5^(1/2) + 3))*(5^
(1/2)/20 - 1/4)
```

3.390 $\int \frac{x^3}{1-3x^4+x^8} dx$

3.390.1 Optimal result

Integrand size = 16, antiderivative size = 23

$$\int \frac{x^3}{1-3x^4+x^8} dx = \frac{\operatorname{arctanh}\left(\frac{3-2x^4}{\sqrt{5}}\right)}{2\sqrt{5}}$$

output
```
1/10*arctanh(1/5*(-2*x^4+3)*5^(1/2))*5^(1/2)
```

3.390.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.65

$$\int \frac{x^3}{1-3x^4+x^8} dx = \frac{\log\left(3+\sqrt{5}-2x^4\right)-\log\left(-3+\sqrt{5}+2x^4\right)}{4\sqrt{5}}$$

input
```
Integrate[x^3/(1 - 3*x^4 + x^8),x]
```

output
```
(Log[3 + Sqrt[5] - 2*x^4] - Log[-3 + Sqrt[5] + 2*x^4])/(4*Sqrt[5])
```

3.390.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1690, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{x^8-3x^4+1}dx$$

$$\Big\downarrow \text{ 1690}$$

$$\frac{1}{4}\int \frac{1}{x^8-3x^4+1}dx^4$$

$$\Big\downarrow \text{ 1081}$$

$$\frac{1}{4}\int \left(\frac{2}{\sqrt{5}\left(-2x^4-\sqrt{5}+3\right)}-\frac{2}{\sqrt{5}\left(-2x^4+\sqrt{5}+3\right)}\right)dx^4$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4}\left(\frac{\log\left(-2x^4+\sqrt{5}+3\right)}{\sqrt{5}}-\frac{\log\left(-2x^4-\sqrt{5}+3\right)}{\sqrt{5}}\right)$$

input
```
Int[x^3/(1 - 3*x^4 + x^8),x]
```

output
```
(-(Log[3 - Sqrt[5] - 2*x^4]/Sqrt[5]) + Log[3 + Sqrt[5] - 2*x^4]/Sqrt[5])/4
```

3.390.3.1 Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.390.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{(2x^4-3)\sqrt{5}}{5}\right)}{10}$	19
risch	$\frac{\ln\left(2x^4-\sqrt{5}-3\right)\sqrt{5}}{20}-\frac{\ln\left(2x^4+\sqrt{5}-3\right)\sqrt{5}}{20}$	36

input
```
int(x^3/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/10*5^(1/2)*arctanh(1/5*(2*x^4-3)*5^(1/2))
```

3.390.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. 2(18) = 36.

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.87

$$\int \frac{x^3}{1-3x^4+x^8}\,dx = \frac{1}{20}\sqrt{5}\log\left(\frac{2\,x^8-6\,x^4-\sqrt{5}(2\,x^4-3)+7}{x^8-3\,x^4+1}\right)$$

input
```
integrate(x^3/(x^8-3*x^4+1),x, algorithm="fricas")
```

output
```
1/20*sqrt(5)*log((2*x^8 - 6*x^4 - sqrt(5)*(2*x^4 - 3) + 7)/(x^8 - 3*x^4 +
1))
```

3.390.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.83

$$\int \frac{x^3}{1-3x^4+x^8}\,dx=\frac{\sqrt{5}\log\left(x^4-\frac{3}{2}-\frac{\sqrt{5}}{2}\right)}{20}-\frac{\sqrt{5}\log\left(x^4-\frac{3}{2}+\frac{\sqrt{5}}{2}\right)}{20}$$

input
```
integrate(x**3/(x**8-3*x**4+1),x)
```

output
```
sqrt(5)*log(x**4 - 3/2 - sqrt(5)/2)/20 - sqrt(5)*log(x**4 - 3/2 + sqrt(5)/
2)/20
```

3.390.7 Maxima [F(-1)]

Timed out.

$$\int \frac{x^3}{1-3x^4+x^8}\,dx=\text{Timed out}$$

input
```
integrate(x^3/(x^8-3*x^4+1),x, algorithm="maxima")
```

output
```
Timed out
```

3.390.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.43

$$\int \frac{x^3}{1-3x^4+x^8}\,dx=\frac{1}{20}\sqrt{5}\log\left(\frac{|2x^4-\sqrt{5}-3|}{|2x^4+\sqrt{5}-3|}\right)$$

input
```
integrate(x^3/(x^8-3*x^4+1),x, algorithm="giac")
```

output
```
1/20*sqrt(5)*log(abs(2*x^4 - sqrt(5) - 3)/abs(2*x^4 + sqrt(5) - 3))
```

3.390.9 Mupad [B] (verification not implemented)

Time = 8.64 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.30

$$\int \frac{x^3}{1-3x^4+x^8}\,dx = \frac{\sqrt{5}\,\mathrm{atanh}\left(\frac{3\,\sqrt{5}-8\,\sqrt{5}\,x^4}{18\,x^4-7}\right)}{10}$$

input
```
int(x^3/(x^8 - 3*x^4 + 1),x)
```

output
```
(5^(1/2)*atanh((3*5^(1/2) - 8*5^(1/2)*x^4)/(18*x^4 - 7)))/10
```

3.391 $\int \frac{x}{1-3x^4+x^8} \, dx$

3.391.1 Optimal result

Integrand size = 14, antiderivative size = 75

$$\int \frac{x}{1-3x^4+x^8} \, dx = -\frac{\operatorname{arctanh}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)}{\sqrt{10\left(3+\sqrt{5}\right)}}+\frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\operatorname{arctanh}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)$$

output
```
1/2*arctanh(x^2*(1/2+1/2*5^(1/2)))*(1/2+1/10*5^(1/2))-arctanh(x^2*2^(1/2)/
(3+5^(1/2))^(1/2))/(5+5^(1/2))
```

3.391.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.21

$$\int \frac{x}{1-3x^4+x^8} \, dx = \frac{1}{40}\left(-\left(\left(5+\sqrt{5}\right)\log\left(-1+\sqrt{5}-2x^2\right)\right)-\left(-5+\sqrt{5}\right)\log\left(1+\sqrt{5}-2x^2\right)+\left(5+\sqrt{5}\right)\log\left(-1+\sqrt{5}+2x^2\right)+\left(-5+\sqrt{5}\right)\log\left(1+\sqrt{5}+2x^2\right)\right)$$

input
```
Integrate[x/(1 - 3*x^4 + x^8),x]
```

output
```
(-((5 + Sqrt[5])*Log[-1 + Sqrt[5] - 2*x^2]) - (-5 + Sqrt[5])*Log[1 + Sqrt[
5] - 2*x^2] + (5 + Sqrt[5])*Log[-1 + Sqrt[5] + 2*x^2] + (-5 + Sqrt[5])*Log
[1 + Sqrt[5] + 2*x^2])/40
```

3.391.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1695, 1406, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{x^8-3x^4+1}dx$$

$$\Big\downarrow \text{1695}$$

$$\frac{1}{2}\int \frac{1}{x^8-3x^4+1}dx^2$$

$$\Big\downarrow \text{1406}$$

$$\frac{1}{2}\left(\frac{\int \frac{1}{x^4+\frac{1}{2}\left(-3-\sqrt{5}\right)}dx^2}{\sqrt{5}}-\frac{\int \frac{1}{x^4+\frac{1}{2}\left(-3+\sqrt{5}\right)}dx^2}{\sqrt{5}}\right)$$

$$\Big\downarrow \text{220}$$

$$\frac{1}{2}\left(\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\text{arctanh}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)-\sqrt{\frac{2}{5\left(3+\sqrt{5}\right)}}\text{arctanh}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)\right)$$

input
```
Int[x/(1 - 3*x^4 + x^8),x]
```

output
```
(-(Sqrt[2/(5*(3 + Sqrt[5]))]*ArcTanh[Sqrt[2/(3 + Sqrt[5])]*x^2]) + Sqrt[(3
 + Sqrt[5])/10]*ArcTanh[Sqrt[(3 + Sqrt[5])/2]*x^2])/2
```

3.391.3.1 Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

rule 1695
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b
*x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c,
p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.391.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.83

method	result
default	$-\frac{\ln(x^4+x^2-1)}{8}+\frac{\operatorname{arctanh}\left(\frac{(2x^2+1)\sqrt{5}}{5}\right)\sqrt{5}}{20}+\frac{\ln(x^4-x^2-1)}{8}+\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{(2x^2-1)\sqrt{5}}{5}\right)}{20}$
risch	$\frac{\ln\left(2x^2+\sqrt{5}-1\right)}{8}+\frac{\ln\left(2x^2+\sqrt{5}-1\right)\sqrt{5}}{40}+\frac{\ln\left(2x^2-\sqrt{5}-1\right)}{8}-\frac{\ln\left(2x^2-\sqrt{5}-1\right)\sqrt{5}}{40}-\frac{\ln\left(2x^2+\sqrt{5}+1\right)}{8}+\frac{\ln\left(2x^2+\sqrt{5}+1\right)\sqrt{5}}{40}$

input
```
int(x/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/8*ln(x^4+x^2-1)+1/20*arctanh(1/5*(2*x^2+1)*5^(1/2))*5^(1/2)+1/8*ln(x^4-
x^2-1)+1/20*5^(1/2)*arctanh(1/5*(2*x^2-1)*5^(1/2))
```

3.391.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(43) = 86.

Time = 0.25 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.43

$$\int \frac{x}{1-3x^4+x^8}\,dx = \frac{1}{40}\sqrt{5}\log\left(\frac{2\,x^4+2\,x^2+\sqrt{5}(2\,x^2+1)+3}{x^4+x^2-1}\right) + \frac{1}{40}\sqrt{5}\log\left(\frac{2\,x^4-2\,x^2+\sqrt{5}(2\,x^2-1)+3}{x^4-x^2-1}\right) - \frac{1}{8}\log\left(x^4+x^2-1\right)+\frac{1}{8}\log\left(x^4-x^2-1\right)$$

input
```
integrate(x/(x^8-3*x^4+1),x, algorithm="fricas")
```

output
```
1/40*sqrt(5)*log((2*x^4 + 2*x^2 + sqrt(5)*(2*x^2 + 1) + 3)/(x^4 + x^2 - 1)
) + 1/40*sqrt(5)*log((2*x^4 - 2*x^2 + sqrt(5)*(2*x^2 - 1) + 3)/(x^4 - x^2
- 1)) - 1/8*log(x^4 + x^2 - 1) + 1/8*log(x^4 - x^2 - 1)
```

3.391.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(53) = 106$.

Time = 0.19 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.20

$$\int \frac{x}{1-3x^4+x^8}\,dx = \left(\frac{\sqrt{5}}{40}+\frac{1}{8}\right)\log\left(x^2-\frac{7}{2}-\frac{7\sqrt{5}}{10}+960\left(\frac{\sqrt{5}}{40}+\frac{1}{8}\right)^3\right) + \left(\frac{1}{8}-\frac{\sqrt{5}}{40}\right)\log\left(x^2-\frac{7}{2}+960\left(\frac{1}{8}-\frac{\sqrt{5}}{40}\right)^3+\frac{7\sqrt{5}}{10}\right) + \left(-\frac{1}{8}+\frac{\sqrt{5}}{40}\right)\log\left(x^2-\frac{7\sqrt{5}}{10}+960\left(-\frac{1}{8}+\frac{\sqrt{5}}{40}\right)^3+\frac{7}{2}\right) + \left(-\frac{1}{8}-\frac{\sqrt{5}}{40}\right)\log\left(x^2+960\left(-\frac{1}{8}-\frac{\sqrt{5}}{40}\right)^3+\frac{7\sqrt{5}}{10}+\frac{7}{2}\right)$$

input
```
integrate(x/(x**8-3*x**4+1),x)
```

output
```
(sqrt(5)/40 + 1/8)*log(x**2 - 7/2 - 7*sqrt(5)/10 + 960*(sqrt(5)/40 + 1/8)*
*3) + (1/8 - sqrt(5)/40)*log(x**2 - 7/2 + 960*(1/8 - sqrt(5)/40)**3 + 7*sq
rt(5)/10) + (-1/8 + sqrt(5)/40)*log(x**2 - 7*sqrt(5)/10 + 960*(-1/8 + sqrt
(5)/40)**3 + 7/2) + (-1/8 - sqrt(5)/40)*log(x**2 + 960*(-1/8 - sqrt(5)/40)
**3 + 7*sqrt(5)/10 + 7/2)
```

3.391.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(43) = 86$.

Time = 0.27 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.16

$$\int \frac{x}{1-3x^4+x^8}\,dx = -\frac{1}{40}\sqrt{5}\log\left(\frac{2\,x^2-\sqrt{5}+1}{2\,x^2+\sqrt{5}+1}\right) - \frac{1}{40}\sqrt{5}\log\left(\frac{2\,x^2-\sqrt{5}-1}{2\,x^2+\sqrt{5}-1}\right) - \frac{1}{8}\log\left(x^4+x^2-1\right) + \frac{1}{8}\log\left(x^4-x^2-1\right)$$

input
```
integrate(x/(x^8-3*x^4+1),x, algorithm="maxima")
```

output
```
-1/40*sqrt(5)*log((2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1)) - 1/40*sqrt
(5)*log((2*x^2 - sqrt(5) - 1)/(2*x^2 + sqrt(5) - 1)) - 1/8*log(x^4 + x^2 -
 1) + 1/8*log(x^4 - x^2 - 1)
```

3.391.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(43) = 86$.

Time = 0.32 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.23

$$\int \frac{x}{1-3x^4+x^8}\,dx = -\frac{1}{40}\sqrt{5}\log\left(\frac{|2\,x^2-\sqrt{5}+1|}{2\,x^2+\sqrt{5}+1}\right) - \frac{1}{40}\sqrt{5}\log\left(\frac{|2\,x^2-\sqrt{5}-1|}{|2\,x^2+\sqrt{5}-1|}\right) - \frac{1}{8}\log\left(|x^4+x^2-1|\right) + \frac{1}{8}\log\left(|x^4-x^2-1|\right)$$

input
```
integrate(x/(x^8-3*x^4+1),x, algorithm="giac")
```

output
```
-1/40*sqrt(5)*log(abs(2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1)) - 1/40*s
qrt(5)*log(abs(2*x^2 - sqrt(5) - 1)/abs(2*x^2 + sqrt(5) - 1)) - 1/8*log(ab
s(x^4 + x^2 - 1)) + 1/8*log(abs(x^4 - x^2 - 1))
```

3.391.9 Mupad [B] (verification not implemented)

Time = 8.56 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.11

$$\int \frac{x}{1-3x^4+x^8}\,dx = \operatorname{atanh}\left(\frac{29\,x^2}{8\,\sqrt{5}-18}-\frac{13\,\sqrt{5}\,x^2}{8\,\sqrt{5}-18}\right)\,\left(\frac{\sqrt{5}}{20}-\frac{1}{4}\right) + \operatorname{atanh}\left(\frac{29\,x^2}{8\,\sqrt{5}+18}+\frac{13\,\sqrt{5}\,x^2}{8\,\sqrt{5}+18}\right)\,\left(\frac{\sqrt{5}}{20}+\frac{1}{4}\right)$$

input
```
int(x/(x^8 - 3*x^4 + 1),x)
```

output
```
atanh((29*x^2)/(8*5^(1/2) - 18) - (13*5^(1/2)*x^2)/(8*5^(1/2) - 18))*(5^(1
/2)/20 - 1/4) + atanh((29*x^2)/(8*5^(1/2) + 18) + (13*5^(1/2)*x^2)/(8*5^(1
/2) + 18))*(5^(1/2)/20 + 1/4)
```

3.392 $\int \frac{1}{x(1-3x^4+x^8)}\,dx$

3.392.1 Optimal result

Integrand size = 16, antiderivative size = 57

$$\int \frac{1}{x(1-3x^4+x^8)}\,dx = \log(x) - \frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(3-\sqrt{5}-2x^4\right) - \frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(3+\sqrt{5}-2x^4\right)$$

output
```
ln(x)-1/40*ln(-2*x^4+5^(1/2)+3)*(5-3*5^(1/2))-1/40*ln(-2*x^4-5^(1/2)+3)*(5
+3*5^(1/2))
```

3.392.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96

$$\int \frac{1}{x(1-3x^4+x^8)}\,dx = \log(x) + \frac{1}{40}\left(-5+3\sqrt{5}\right)\log\left(3+\sqrt{5}-2x^4\right) + \frac{1}{40}\left(-5-3\sqrt{5}\right)\log\left(-3+\sqrt{5}+2x^4\right)$$

input
```
Integrate[1/(x*(1 - 3*x^4 + x^8)),x]
```

output
```
Log[x] + ((-5 + 3*Sqrt[5])*Log[3 + Sqrt[5] - 2*x^4])/40 + ((-5 - 3*Sqrt[5]
)*Log[-3 + Sqrt[5] + 2*x^4])/40
```

3.392.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.25, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1693, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(x^8-3x^4+1)}dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{4}\int \frac{1}{x^4(x^8-3x^4+1)}dx^4$$

$$\Big\downarrow \text{1141}$$

$$\frac{1}{4}\int \left(-\frac{4}{\sqrt{5}\left(3+\sqrt{5}\right)\left(-2x^4+\sqrt{5}+3\right)}+\frac{1}{x^4}+\frac{4}{\sqrt{5}\left(3-\sqrt{5}\right)\left(-2x^4-\sqrt{5}+3\right)}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\log\left(x^4\right)-\frac{2\log\left(-2x^4-\sqrt{5}+3\right)}{\sqrt{5}\left(3-\sqrt{5}\right)}+\frac{2\log\left(-2x^4+\sqrt{5}+3\right)}{\sqrt{5}\left(3+\sqrt{5}\right)}\right)$$

input
```
Int[1/(x*(1 - 3*x^4 + x^8)),x]
```

output
```
(Log[x^4] - (2*Log[3 - Sqrt[5] - 2*x^4])/(Sqrt[5]*(3 - Sqrt[5])) + (2*Log[
3 + Sqrt[5] - 2*x^4])/(Sqrt[5]*(3 + Sqrt[5])))/4
```

3.392.3.1 Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.392.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.12

method	result	size
default	$\ln(x)-\frac{\ln(x^4-x^2-1)}{8}-\frac{3\sqrt{5}\,\operatorname{arctanh}\left(\frac{(2x^2-1)\sqrt{5}}{5}\right)}{20}-\frac{\ln(x^4+x^2-1)}{8}+\frac{3\,\operatorname{arctanh}\left(\frac{(2x^2+1)\sqrt{5}}{5}\right)\sqrt{5}}{20}$	64
risch	$\ln(x)-\frac{\ln\left(3x^4-\frac{9}{2}-\frac{3\sqrt{5}}{2}\right)}{8}+\frac{3\ln\left(3x^4-\frac{9}{2}-\frac{3\sqrt{5}}{2}\right)\sqrt{5}}{40}-\frac{3\ln\left(3x^4+\frac{3\sqrt{5}}{2}-\frac{9}{2}\right)\sqrt{5}}{40}-\frac{\ln\left(3x^4+\frac{3\sqrt{5}}{2}-\frac{9}{2}\right)}{8}$	70

input
```
int(1/x/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
ln(x)-1/8*ln(x^4-x^2-1)-3/20*5^(1/2)*arctanh(1/5*(2*x^2-1)*5^(1/2))-1/8*ln
(x^4+x^2-1)+3/20*arctanh(1/5*(2*x^2+1)*5^(1/2))*5^(1/2)
```

3.392.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.04

$$\int \frac{1}{x(1-3x^4+x^8)}\,dx=\frac{3}{40}\sqrt{5}\log\left(\frac{2x^8-6x^4-\sqrt{5}(2x^4-3)+7}{x^8-3x^4+1}\right)-\frac{1}{8}\log\left(x^8-3x^4+1\right)+\log(x)$$

input
```
integrate(1/x/(x^8-3*x^4+1),x, algorithm="fricas")
```

output
```
3/40*sqrt(5)*log((2*x^8 - 6*x^4 - sqrt(5)*(2*x^4 - 3) + 7)/(x^8 - 3*x^4 +
1)) - 1/8*log(x^8 - 3*x^4 + 1) + log(x)
```

3.392.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.02

$$\int \frac{1}{x(1-3x^4+x^8)}\,dx = \log(x) + \left(-\frac{1}{8}+\frac{3\sqrt{5}}{40}\right)\log\left(x^4-\frac{3}{2}-\frac{\sqrt{5}}{2}\right) + \left(-\frac{3\sqrt{5}}{40}-\frac{1}{8}\right)\log\left(x^4-\frac{3}{2}+\frac{\sqrt{5}}{2}\right)$$

input
```
integrate(1/x/(x**8-3*x**4+1),x)
```

output
```
log(x) + (-1/8 + 3*sqrt(5)/40)*log(x**4 - 3/2 - sqrt(5)/2) + (-3*sqrt(5)/4
0 - 1/8)*log(x**4 - 3/2 + sqrt(5)/2)
```

3.392.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int \frac{1}{x(1-3x^4+x^8)}\,dx = \frac{3}{40}\sqrt{5}\log\left(\frac{2x^4-\sqrt{5}-3}{2x^4+\sqrt{5}-3}\right) - \frac{1}{8}\log\left(x^8-3x^4+1\right) + \frac{1}{4}\log\left(x^4\right)$$

input
```
integrate(1/x/(x^8-3*x^4+1),x, algorithm="maxima")
```

output
```
3/40*sqrt(5)*log((2*x^4 - sqrt(5) - 3)/(2*x^4 + sqrt(5) - 3)) - 1/8*log(x^
8 - 3*x^4 + 1) + 1/4*log(x^4)
```

3.392.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.95

$$\int \frac{1}{x(1-3x^4+x^8)}\,dx = \frac{3}{40}\sqrt{5}\log\left(\frac{|2x^4-\sqrt{5}-3|}{|2x^4+\sqrt{5}-3|}\right) + \frac{1}{4}\log\left(x^4\right) - \frac{1}{8}\log\left(|x^8-3x^4+1|\right)$$

input
```
integrate(1/x/(x^8-3*x^4+1),x, algorithm="giac")
```

output
```
3/40*sqrt(5)*log(abs(2*x^4 - sqrt(5) - 3)/abs(2*x^4 + sqrt(5) - 3)) + 1/4*
log(x^4) - 1/8*log(abs(x^8 - 3*x^4 + 1))
```

3.392.9 Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.74

$$\int \frac{1}{x(1-3x^4+x^8)}\,dx = \ln(x) + \ln\left(x^4-\frac{\sqrt{5}}{2}-\frac{3}{2}\right)\left(\frac{3\sqrt{5}}{40}-\frac{1}{8}\right) - \ln\left(x^4+\frac{\sqrt{5}}{2}-\frac{3}{2}\right)\left(\frac{3\sqrt{5}}{40}+\frac{1}{8}\right)$$

input
```
int(1/(x*(x^8 - 3*x^4 + 1)),x)
```

output
```
log(x) + log(x^4 - 5^(1/2)/2 - 3/2)*((3*5^(1/2))/40 - 1/8) - log(5^(1/2)/2
 + x^4 - 3/2)*((3*5^(1/2))/40 + 1/8)
```

3.393 $\int \frac{1}{x^3(1-3x^4+x^8)}\,dx$

3.393.1 Optimal result

Integrand size = 16, antiderivative size = 89

$$\int \frac{1}{x^3(1-3x^4+x^8)}\,dx=-\frac{1}{2x^2}-\frac{1}{2}\sqrt{\frac{1}{5}\left(9-4\sqrt{5}\right)}\text{arctanh}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)+\frac{\left(3+\sqrt{5}\right)^{3/2}\text{arctanh}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)}{4\sqrt{10}}$$

output
```
-1/2/x^2+1/40*arctanh(x^2*(1/2+1/2*5^(1/2)))*(3+5^(1/2))^(3/2)*10^(1/2)-1/
2*arctanh(x^2*2^(1/2)/(3+5^(1/2))^(1/2))*(1-2/5*5^(1/2))
```

3.393.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.16

$$\int \frac{1}{x^3(1-3x^4+x^8)}\,dx=\frac{1}{20}\left(-\frac{10}{x^2}-\left(5+2\sqrt{5}\right)\log\left(-1+\sqrt{5}-2x^2\right)+\left(5-2\sqrt{5}\right)\log\left(1+\sqrt{5}-2x^2\right)+\left(5+2\sqrt{5}\right)\log\left(-1+\sqrt{5}+2x^2\right)+\left(-5+2\sqrt{5}\right)\log\left(1+\sqrt{5}+2x^2\right)\right)$$

input
```
Integrate[1/(x^3*(1 - 3*x^4 + x^8)),x]
```

output
```
(-10/x^2 - (5 + 2*Sqrt[5])*Log[-1 + Sqrt[5] - 2*x^2] + (5 - 2*Sqrt[5])*Log
[1 + Sqrt[5] - 2*x^2] + (5 + 2*Sqrt[5])*Log[-1 + Sqrt[5] + 2*x^2] + (-5 +
2*Sqrt[5])*Log[1 + Sqrt[5] + 2*x^2])/20
```

3.393.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.17, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1695, 1443, 1480, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (x^8-3x^4+1)} dx$$

$$\Big\downarrow \text{1695}$$

$$\frac{1}{2}\int \frac{1}{x^4 (x^8-3x^4+1)}dx^2$$

$$\Big\downarrow \text{1443}$$

$$\frac{1}{2}\left(\int \frac{3-x^4}{x^8-3x^4+1}dx^2-\frac{1}{x^2}\right)$$

$$\Big\downarrow \text{1480}$$

$$\frac{1}{2}\left(-\frac{1}{10}\left(5-3\sqrt{5}\right)\int \frac{1}{x^4+\frac{1}{2}\left(-3-\sqrt{5}\right)}dx^2-\frac{1}{10}\left(5+3\sqrt{5}\right)\int \frac{1}{x^4+\frac{1}{2}\left(-3+\sqrt{5}\right)}dx^2-\frac{1}{x^2}\right)$$

$$\Big\downarrow \text{220}$$

$$\frac{1}{2}\left(\frac{\left(5-3\sqrt{5}\right)\text{arctanh}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)}{5\sqrt{2\left(3+\sqrt{5}\right)}}+\frac{1}{10}\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\left(5+3\sqrt{5}\right)\text{arctanh}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)-\frac{1}{x^2}\right)$$

input
```
Int[1/(x^3*(1 - 3*x^4 + x^8)),x]
```

output
```
(-x^(-2) + ((5 - 3*Sqrt[5])*ArcTanh[Sqrt[2/(3 + Sqrt[5])]*x^2])/(5*Sqrt[2*
(3 + Sqrt[5])]) + (Sqrt[(3 + Sqrt[5])/2]*(5 + 3*Sqrt[5])*ArcTanh[Sqrt[(3 +
 Sqrt[5])/2]*x^2])/10)/2
```

3.393.3.1 Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1443
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Sim
p[1/(a*d^2*(m + 1))   Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*
x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1695
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b
*x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c,
p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.393.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.75

method	result
default	$-\frac{1}{2x^2}+\frac{\ln(x^4-x^2-1)}{4}+\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{(2x^2-1)\sqrt{5}}{5}\right)}{5}-\frac{\ln(x^4+x^2-1)}{4}+\frac{\operatorname{arctanh}\left(\frac{(2x^2+1)\sqrt{5}}{5}\right)\sqrt{5}}{5}$
risch	$-\frac{1}{2x^2}+\frac{\ln\left(4x^2-2+2\sqrt{5}\right)}{4}+\frac{\ln\left(4x^2-2+2\sqrt{5}\right)\sqrt{5}}{10}+\frac{\ln\left(4x^2-2-2\sqrt{5}\right)}{4}-\frac{\ln\left(4x^2-2-2\sqrt{5}\right)\sqrt{5}}{10}-\frac{\ln\left(4x^2+2+2\sqrt{5}\right)}{4}+\ln($

input
```
int(1/x^3/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2/x^2+1/4*ln(x^4-x^2-1)+1/5*5^(1/2)*arctanh(1/5*(2*x^2-1)*5^(1/2))-1/4*
ln(x^4+x^2-1)+1/5*arctanh(1/5*(2*x^2+1)*5^(1/2))*5^(1/2)
```

3.393.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. 2(53) = 106.

Time = 0.24 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.40

$$\int \frac{1}{x^3 (1-3x^4+x^8)}\,dx = \frac{2\sqrt{5}x^2\log\left(\frac{2x^4+2x^2+\sqrt{5}(2x^2+1)+3}{x^4+x^2-1}\right)+2\sqrt{5}x^2\log\left(\frac{2x^4-2x^2+\sqrt{5}(2x^2-1)+3}{x^4-x^2-1}\right)-5x^2\log\left(x^4+x^2-1\right)+5x^2\log}{20x^2}$$

input
```
integrate(1/x^3/(x^8-3*x^4+1),x, algorithm="fricas")
```

output
```
1/20*(2*sqrt(5)*x^2*log((2*x^4 + 2*x^2 + sqrt(5)*(2*x^2 + 1) + 3)/(x^4 + x
^2 - 1)) + 2*sqrt(5)*x^2*log((2*x^4 - 2*x^2 + sqrt(5)*(2*x^2 - 1) + 3)/(x^
4 - x^2 - 1)) - 5*x^2*log(x^4 + x^2 - 1) + 5*x^2*log(x^4 - x^2 - 1) - 10)/
x^2
```

3.393.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. 2(70) = 140.

Time = 0.22 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.93

$$\begin{aligned}\int \frac{1}{x^3 (1-3x^4+x^8)}\,dx = {} & \left(\frac{\sqrt{5}}{10}+\frac{1}{4}\right)\log\left(x^2-\frac{123}{8}-\frac{123\sqrt{5}}{20}+280\left(\frac{\sqrt{5}}{10}+\frac{1}{4}\right)^3\right) \\ & +\left(\frac{1}{4}-\frac{\sqrt{5}}{10}\right)\log\left(x^2-\frac{123}{8}+280\left(\frac{1}{4}-\frac{\sqrt{5}}{10}\right)^3+\frac{123\sqrt{5}}{20}\right) \\ & +\left(-\frac{1}{4}+\frac{\sqrt{5}}{10}\right)\log\left(x^2-\frac{123\sqrt{5}}{20}+280\left(-\frac{1}{4}+\frac{\sqrt{5}}{10}\right)^3+\frac{123}{8}\right) \\ & +\left(-\frac{1}{4}-\frac{\sqrt{5}}{10}\right)\log\left(x^2+280\left(-\frac{1}{4}-\frac{\sqrt{5}}{10}\right)^3+\frac{123\sqrt{5}}{20}+\frac{123}{8}\right) \\ & -\frac{1}{2x^2}\end{aligned}$$

input
```
integrate(1/x**3/(x**8-3*x**4+1),x)
```

output
```
(sqrt(5)/10 + 1/4)*log(x**2 - 123/8 - 123*sqrt(5)/20 + 280*(sqrt(5)/10 + 1
/4)**3) + (1/4 - sqrt(5)/10)*log(x**2 - 123/8 + 280*(1/4 - sqrt(5)/10)**3
+ 123*sqrt(5)/20) + (-1/4 + sqrt(5)/10)*log(x**2 - 123*sqrt(5)/20 + 280*(-
1/4 + sqrt(5)/10)**3 + 123/8) + (-1/4 - sqrt(5)/10)*log(x**2 + 280*(-1/4 -
 sqrt(5)/10)**3 + 123*sqrt(5)/20 + 123/8) - 1/(2*x**2)
```

3.393.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^3(1-3x^4+x^8)}\,dx = -\frac{1}{10}\sqrt{5}\log\left(\frac{2\,x^2-\sqrt{5}+1}{2\,x^2+\sqrt{5}+1}\right)-\frac{1}{10}\sqrt{5}\log\left(\frac{2\,x^2-\sqrt{5}-1}{2\,x^2+\sqrt{5}-1}\right) -\frac{1}{2\,x^2}-\frac{1}{4}\log\left(x^4+x^2-1\right)+\frac{1}{4}\log\left(x^4-x^2-1\right)$$

input
```
integrate(1/x^3/(x^8-3*x^4+1),x, algorithm="maxima")
```

output
```
-1/10*sqrt(5)*log((2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1)) - 1/10*sqrt
(5)*log((2*x^2 - sqrt(5) - 1)/(2*x^2 + sqrt(5) - 1)) - 1/2/x^2 - 1/4*log(x
^4 + x^2 - 1) + 1/4*log(x^4 - x^2 - 1)
```

3.393.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^3(1-3x^4+x^8)}\,dx = -\frac{1}{10}\sqrt{5}\log\left(\frac{|2\,x^2-\sqrt{5}+1|}{2\,x^2+\sqrt{5}+1}\right)-\frac{1}{10}\sqrt{5}\log\left(\frac{|2\,x^2-\sqrt{5}-1|}{|2\,x^2+\sqrt{5}-1|}\right) -\frac{1}{2\,x^2}-\frac{1}{4}\log\left(|x^4+x^2-1|\right)+\frac{1}{4}\log\left(|x^4-x^2-1|\right)$$

input
```
integrate(1/x^3/(x^8-3*x^4+1),x, algorithm="giac")
```

output
```
-1/10*sqrt(5)*log(abs(2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1)) - 1/10*s
qrt(5)*log(abs(2*x^2 - sqrt(5) - 1)/abs(2*x^2 + sqrt(5) - 1)) - 1/2/x^2 -
1/4*log(abs(x^4 + x^2 - 1)) + 1/4*log(abs(x^4 - x^2 - 1))
```

3.393.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.99

$$\int \frac{1}{x^3 (1 - 3x^4 + x^8)} dx = \operatorname{atanh}\left(\frac{12736 x^2}{3520 \sqrt{5} - 7872} - \frac{5696 \sqrt{5} x^2}{3520 \sqrt{5} - 7872}\right) \left(\frac{\sqrt{5}}{5} - \frac{1}{2}\right) + \operatorname{atanh}\left(\frac{12736 x^2}{3520 \sqrt{5} + 7872} + \frac{5696 \sqrt{5} x^2}{3520 \sqrt{5} + 7872}\right) \left(\frac{\sqrt{5}}{5} + \frac{1}{2}\right) - \frac{1}{2 x^2}$$

input
```
int(1/(x^3*(x^8 - 3*x^4 + 1)),x)
```

output
```
atanh((12736*x^2)/(3520*5^(1/2) - 7872) - (5696*5^(1/2)*x^2)/(3520*5^(1/2)
 - 7872))*(5^(1/2)/5 - 1/2) + atanh((12736*x^2)/(3520*5^(1/2) + 7872) + (5
696*5^(1/2)*x^2)/(3520*5^(1/2) + 7872))*(5^(1/2)/5 + 1/2) - 1/(2*x^2)
```

3.394 $\int \frac{1}{x^5(1-3x^4+x^8)}\,dx$

3.394.1 Optimal result

Integrand size = 16, antiderivative size = 66

$$\int \frac{1}{x^5\left(1-3x^4+x^8\right)}\,dx=-\frac{1}{4x^4}+3\log(x)-\frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(3-\sqrt{5}-2x^4\right)-\frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(3+\sqrt{5}-2x^4\right)$$

output
```
-1/4/x^4+3*ln(x)-1/40*ln(-2*x^4+5^(1/2)+3)*(15-7*5^(1/2))-1/40*ln(-2*x^4-5
^(1/2)+3)*(15+7*5^(1/2))
```

3.394.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^5\left(1-3x^4+x^8\right)}\,dx=\frac{1}{40}\left(-\frac{10}{x^4}+120\log(x)+\left(-15+7\sqrt{5}\right)\log\left(3+\sqrt{5}-2x^4\right)-\left(15+7\sqrt{5}\right)\log\left(-3+\sqrt{5}+2x^4\right)\right)$$

input
```
Integrate[1/(x^5*(1 - 3*x^4 + x^8)),x]
```

output
```
(-10/x^4 + 120*Log[x] + (-15 + 7*Sqrt[5])*Log[3 + Sqrt[5] - 2*x^4] - (15 +
 7*Sqrt[5])*Log[-3 + Sqrt[5] + 2*x^4])/40
```

3.394.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.18, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1693, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 (x^8-3x^4+1)} \, dx$$

$$\Big\downarrow \; 1693$$

$$\frac{1}{4}\int \frac{1}{x^8 (x^8-3x^4+1)}dx^4$$

$$\Big\downarrow \; 1141$$

$$\frac{1}{4}\int \left(-\frac{8}{\sqrt{5} \left(3+\sqrt{5}\right)^2 \left(-2x^4+\sqrt{5}+3\right)}+\frac{3}{x^4}+\frac{1}{x^8}+\frac{8}{\sqrt{5} \left(3-\sqrt{5}\right)^2 \left(-2x^4-\sqrt{5}+3\right)}\right)dx^4$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{4} \left(-\frac{1}{x^4}+3 \log \left(x^4\right)-\frac{4 \log \left(-2x^4-\sqrt{5}+3\right)}{\sqrt{5} \left(3-\sqrt{5}\right)^2}+\frac{4 \log \left(-2x^4+\sqrt{5}+3\right)}{\sqrt{5} \left(3+\sqrt{5}\right)^2}\right)$$

input
```
Int[1/(x^5*(1 - 3*x^4 + x^8)),x]
```

output
```
(-x^(-4) + 3*Log[x^4] - (4*Log[3 - Sqrt[5] - 2*x^4])/(Sqrt[5]*(3 - Sqrt[5]
)^2) + (4*Log[3 + Sqrt[5] - 2*x^4])/(Sqrt[5]*(3 + Sqrt[5])^2))/4
```

3.394.3.1 Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.394.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.08

method	result
default	$-\frac{1}{4x^4}+3\ln(x)-\frac{3\ln(x^4-x^2-1)}{8}-\frac{7\sqrt{5}\,\operatorname{arctanh}\left(\frac{(2x^2-1)\sqrt{5}}{5}\right)}{20}-\frac{3\ln(x^4+x^2-1)}{8}+\frac{7\,\operatorname{arctanh}\left(\frac{(2x^2+1)\sqrt{5}}{5}\right)\sqrt{5}}{20}$
risch	$-\frac{1}{4x^4}+3\ln(x)-\frac{3\ln\left(7x^4-\frac{21}{2}-\frac{7\sqrt{5}}{2}\right)}{8}+\frac{7\ln\left(7x^4-\frac{21}{2}-\frac{7\sqrt{5}}{2}\right)\sqrt{5}}{40}-\frac{3\ln\left(7x^4-\frac{21}{2}+\frac{7\sqrt{5}}{2}\right)}{8}-\frac{7\ln\left(7x^4-\frac{21}{2}+\frac{7\sqrt{5}}{2}\right)\sqrt{5}}{40}$

input
```
int(1/x^5/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/4/x^4+3*ln(x)-3/8*ln(x^4-x^2-1)-7/20*5^(1/2)*arctanh(1/5*(2*x^2-1)*5^(1
/2))-3/8*ln(x^4+x^2-1)+7/20*arctanh(1/5*(2*x^2+1)*5^(1/2))*5^(1/2)
```

3.394.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.15

$$\int \frac{1}{x^5\left(1-3x^4+x^8\right)}\,dx = \frac{7\,\sqrt{5}x^4\log\left(\frac{2\,x^8-6\,x^4-\sqrt{5}(2\,x^4-3)+7}{x^8-3\,x^4+1}\right)-15\,x^4\log\left(x^8-3\,x^4+1\right)+120\,x^4\log\left(x\right)-10}{40\,x^4}$$

input
```
integrate(1/x^5/(x^8-3*x^4+1),x, algorithm="fricas")
```

output
```
1/40*(7*sqrt(5)*x^4*log((2*x^8 - 6*x^4 - sqrt(5)*(2*x^4 - 3) + 7)/(x^8 - 3
*x^4 + 1)) - 15*x^4*log(x^8 - 3*x^4 + 1) + 120*x^4*log(x) - 10)/x^4
```

3.394.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^5(1-3x^4+x^8)}\,dx = 3\log(x) + \left(-\frac{3}{8}+\frac{7\sqrt{5}}{40}\right)\log\left(x^4-\frac{3}{2}-\frac{\sqrt{5}}{2}\right) + \left(-\frac{7\sqrt{5}}{40}-\frac{3}{8}\right)\log\left(x^4-\frac{3}{2}+\frac{\sqrt{5}}{2}\right) - \frac{1}{4x^4}$$

input
```
integrate(1/x**5/(x**8-3*x**4+1),x)
```

output
```
3*log(x) + (-3/8 + 7*sqrt(5)/40)*log(x**4 - 3/2 - sqrt(5)/2) + (-7*sqrt(5)
/40 - 3/8)*log(x**4 - 3/2 + sqrt(5)/2) - 1/(4*x**4)
```

3.394.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^5(1-3x^4+x^8)}\,dx = \frac{7}{40}\sqrt{5}\log\left(\frac{2\,x^4-\sqrt{5}-3}{2\,x^4+\sqrt{5}-3}\right) - \frac{1}{4\,x^4} - \frac{3}{8}\log\left(x^8-3\,x^4+1\right) + \frac{3}{4}\log\left(x^4\right)$$

input
```
integrate(1/x^5/(x^8-3*x^4+1),x, algorithm="maxima")
```

output
```
7/40*sqrt(5)*log((2*x^4 - sqrt(5) - 3)/(2*x^4 + sqrt(5) - 3)) - 1/4/x^4 -
3/8*log(x^8 - 3*x^4 + 1) + 3/4*log(x^4)
```

3.394.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^5(1-3x^4+x^8)}\,dx = \frac{7}{40}\sqrt{5}\log\left(\frac{|2\,x^4-\sqrt{5}-3|}{|2\,x^4+\sqrt{5}-3|}\right) - \frac{3\,x^4+1}{4\,x^4} + \frac{3}{4}\log\left(x^4\right) - \frac{3}{8}\log\left(|x^8-3\,x^4+1|\right)$$

input
```
integrate(1/x^5/(x^8-3*x^4+1),x, algorithm="giac")
```

output
```
7/40*sqrt(5)*log(abs(2*x^4 - sqrt(5) - 3)/abs(2*x^4 + sqrt(5) - 3)) - 1/4*
(3*x^4 + 1)/x^4 + 3/4*log(x^4) - 3/8*log(abs(x^8 - 3*x^4 + 1))
```

3.394.9 Mupad [B] (verification not implemented)

Time = 8.59 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^5 (1-3x^4+x^8)}\,dx = 3\,\ln(x) - \frac{1}{4\,x^4} + \ln\left(x^4 - \frac{\sqrt{5}}{2} - \frac{3}{2}\right)\left(\frac{7\,\sqrt{5}}{40} - \frac{3}{8}\right) - \ln\left(x^4 + \frac{\sqrt{5}}{2} - \frac{3}{2}\right)\left(\frac{7\,\sqrt{5}}{40} + \frac{3}{8}\right)$$

input
```
int(1/(x^5*(x^8 - 3*x^4 + 1)),x)
```

output
```
3*log(x) - 1/(4*x^4) + log(x^4 - 5^(1/2)/2 - 3/2)*((7*5^(1/2))/40 - 3/8) -
 log(5^(1/2)/2 + x^4 - 3/2)*((7*5^(1/2))/40 + 3/8)
```

3.395 $\int \frac{1}{x^7(1-3x^4+x^8)}\,dx$

3.395.1 Optimal result

Integrand size = 16, antiderivative size = 97

$$\int \frac{1}{x^7\left(1-3x^4+x^8\right)}\,dx = -\frac{1}{6x^6}-\frac{3}{2x^2}-\frac{1}{2}\sqrt{\frac{1}{10}\left(123-55\sqrt{5}\right)}\operatorname{arctanh}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)+\frac{1}{2}\sqrt{\frac{1}{10}\left(123+55\sqrt{5}\right)}\operatorname{arctanh}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)$$

output
```
-1/6/x^6-3/2/x^2-1/2*arctanh(x^2*2^(1/2)/(3+5^(1/2))^(1/2))*(5/2-11/10*5^(
1/2))+1/2*arctanh(x^2*(1/2+1/2*5^(1/2)))*(5/2+11/10*5^(1/2))
```

3.395.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^7\left(1-3x^4+x^8\right)}\,dx = \frac{1}{120}\left(-\frac{20}{x^6}-\frac{180}{x^2}-3\left(25+11\sqrt{5}\right)\log\left(-1+\sqrt{5}-2x^2\right)+3\left(25-11\sqrt{5}\right)\log\left(1+\sqrt{5}-2x^2\right)+3\left(25+11\sqrt{5}\right)\log\left(-1+\sqrt{5}+2x^2\right)+3\left(-25+11\sqrt{5}\right)\log\left(1+\sqrt{5}+2x^2\right)\right)$$

input
```
Integrate[1/(x^7*(1 - 3*x^4 + x^8)),x]
```

output
```
(-20/x^6 - 180/x^2 - 3*(25 + 11*Sqrt[5])*Log[-1 + Sqrt[5] - 2*x^2] + 3*(25
 - 11*Sqrt[5])*Log[1 + Sqrt[5] - 2*x^2] + 3*(25 + 11*Sqrt[5])*Log[-1 + Sqr
t[5] + 2*x^2] + 3*(-25 + 11*Sqrt[5])*Log[1 + Sqrt[5] + 2*x^2])/120
```

3.395.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1695, 1443, 27, 1604, 25, 1480, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7 \left(x^8-3 x^4+1\right)} \, dx$$

$$\Big\downarrow \text{1695}$$

$$\frac{1}{2} \int \frac{1}{x^8 \left(x^8-3 x^4+1\right)}dx^2$$

$$\Big\downarrow \text{1443}$$

$$\frac{1}{2} \left(\frac{1}{3} \int \frac{3 \left(3-x^4\right)}{x^4 \left(x^8-3 x^4+1\right)}dx^2-\frac{1}{3 x^6}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2} \left(\int \frac{3-x^4}{x^4 \left(x^8-3 x^4+1\right)}dx^2-\frac{1}{3 x^6}\right)$$

$$\Big\downarrow \text{1604}$$

$$\frac{1}{2} \left(-\int -\frac{8-3 x^4}{x^8-3 x^4+1}dx^2-\frac{1}{3 x^6}-\frac{3}{x^2}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2} \left(\int \frac{8-3 x^4}{x^8-3 x^4+1}dx^2-\frac{1}{3 x^6}-\frac{3}{x^2}\right)$$

$$\Big\downarrow \text{1480}$$

$$\frac{1}{2}\left(-\frac{1}{10}\left(15-7\sqrt{5}\right)\int \frac{1}{x^4+\frac{1}{2}\left(-3-\sqrt{5}\right)}dx^2-\frac{1}{10}\left(15+7\sqrt{5}\right)\int \frac{1}{x^4+\frac{1}{2}\left(-3+\sqrt{5}\right)}dx^2-\frac{1}{3x^6}-\frac{3}{x^2}\right)$$

$$\Big\downarrow \text{220}$$

$$\frac{1}{2}\left(\frac{\left(15-7\sqrt{5}\right)\text{arctanh}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)}{5\sqrt{2\left(3+\sqrt{5}\right)}}+\frac{1}{10}\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\left(15+7\sqrt{5}\right)\text{arctanh}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)-\frac{1}{3x^6}-\frac{3}{x^2}\right)$$

input
```
Int[1/(x^7*(1 - 3*x^4 + x^8)),x]
```

output
```
(-1/3*1/x^6 - 3/x^2 + ((15 - 7*Sqrt[5])*ArcTanh[Sqrt[2/(3 + Sqrt[5])]*x^2]
)/(5*Sqrt[2*(3 + Sqrt[5])]) + (Sqrt[(3 + Sqrt[5])/2]*(15 + 7*Sqrt[5])*ArcT
anh[Sqrt[(3 + Sqrt[5])/2]*x^2])/10)/2
```

3.395.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1443
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Sim
p[1/(a*d^2*(m + 1))   Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*
x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

3.395. $\int \frac{1}{x^7(1-3x^4+x^8)}\,dx$

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1695
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b
*x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c,
p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.395.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.74

method	result
default	$-\frac{1}{6x^{6}}-\frac{3}{2x^{2}}+\frac{5\ln\left(x^{4}-x^{2}-1\right)}{8}+\frac{11\sqrt{5}\,\operatorname{arctanh}\left(\frac{\left(2x^{2}-1\right)\sqrt{5}}{5}\right)}{20}-\frac{5\ln\left(x^{4}+x^{2}-1\right)}{8}+\frac{11\,\operatorname{arctanh}\left(\frac{\left(2x^{2}+1\right)\sqrt{5}}{5}\right)\sqrt{5}}{20}$
risch	$\frac{-\frac{3x^{4}}{2}-\frac{1}{6}}{x^{6}}+\frac{5\ln\left(11x^{2}-\frac{11}{2}+\frac{11\sqrt{5}}{2}\right)}{8}+\frac{11\ln\left(11x^{2}-\frac{11}{2}+\frac{11\sqrt{5}}{2}\right)\sqrt{5}}{40}+\frac{5\ln\left(11x^{2}-\frac{11}{2}-\frac{11\sqrt{5}}{2}\right)}{8}-\frac{11\ln\left(11x^{2}-\frac{11}{2}-\frac{11\sqrt{5}}{2}\right)\sqrt{5}}{40}-$

input
```
int(1/x^7/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/6/x^6-3/2/x^2+5/8*ln(x^4-x^2-1)+11/20*5^(1/2)*arctanh(1/5*(2*x^2-1)*5^(
1/2))-5/8*ln(x^4+x^2-1)+11/20*arctanh(1/5*(2*x^2+1)*5^(1/2))*5^(1/2)
```

3.395.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. $2(55) = 110$.

Time = 0.25 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.34

$$\int \frac{1}{x^7 (1 - 3x^4 + x^8)}\, dx$$
$$= \frac{33\sqrt{5}x^6 \log\left(\frac{2\,x^4+2\,x^2+\sqrt{5}(2\,x^2+1)+3}{x^4+x^2-1}\right) + 33\sqrt{5}x^6 \log\left(\frac{2\,x^4-2\,x^2+\sqrt{5}(2\,x^2-1)+3}{x^4-x^2-1}\right) - 75\,x^6 \log\left(x^4+x^2-1\right) + 75\,x}{120\,x^6}$$

input
```
integrate(1/x^7/(x^8-3*x^4+1),x, algorithm="fricas")
```

output
```
1/120*(33*sqrt(5)*x^6*log((2*x^4 + 2*x^2 + sqrt(5)*(2*x^2 + 1) + 3)/(x^4 +
 x^2 - 1)) + 33*sqrt(5)*x^6*log((2*x^4 - 2*x^2 + sqrt(5)*(2*x^2 - 1) + 3)/
(x^4 - x^2 - 1)) - 75*x^6*log(x^4 + x^2 - 1) + 75*x^6*log(x^4 - x^2 - 1) -
 180*x^4 - 20)/x^6
```

3.395.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(75) = 150$.

Time = 0.24 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.05

$$\int \frac{1}{x^7(1-3x^4+x^8)}\,dx = \left(\frac{11\sqrt{5}}{40}+\frac{5}{8}\right)\log\left(x^2-\frac{2207}{22}-\frac{2207\sqrt{5}}{50}+\frac{1152\left(\frac{11\sqrt{5}}{40}+\frac{5}{8}\right)^3}{11}\right) + \left(\frac{5}{8} - \frac{11\sqrt{5}}{40}\right)\log\left(x^2-\frac{2207}{22}+\frac{1152\left(\frac{5}{8}-\frac{11\sqrt{5}}{40}\right)^3}{11}+\frac{2207\sqrt{5}}{50}\right) + \left(-\frac{5}{8} + \frac{11\sqrt{5}}{40}\right)\log\left(x^2-\frac{2207\sqrt{5}}{50}+\frac{1152\left(-\frac{5}{8}+\frac{11\sqrt{5}}{40}\right)^3}{11}+\frac{2207}{22}\right) + \left(-\frac{5}{8} - \frac{11\sqrt{5}}{40}\right)\log\left(x^2+\frac{1152\left(-\frac{5}{8}-\frac{11\sqrt{5}}{40}\right)^3}{11}+\frac{2207\sqrt{5}}{50}+\frac{2207}{22}\right) + \frac{-9x^4-1}{6x^6}$$

input
```
integrate(1/x**7/(x**8-3*x**4+1),x)
```

output
```
(11*sqrt(5)/40 + 5/8)*log(x**2 - 2207/22 - 2207*sqrt(5)/50 + 1152*(11*sqrt
(5)/40 + 5/8)**3/11) + (5/8 - 11*sqrt(5)/40)*log(x**2 - 2207/22 + 1152*(5/
8 - 11*sqrt(5)/40)**3/11 + 2207*sqrt(5)/50) + (-5/8 + 11*sqrt(5)/40)*log(x
**2 - 2207*sqrt(5)/50 + 1152*(-5/8 + 11*sqrt(5)/40)**3/11 + 2207/22) + (-5
/8 - 11*sqrt(5)/40)*log(x**2 + 1152*(-5/8 - 11*sqrt(5)/40)**3/11 + 2207*sq
rt(5)/50 + 2207/22) + (-9*x**4 - 1)/(6*x**6)
```

3.395. $\int \frac{1}{x^7(1-3x^4+x^8)}\,dx$

3.395.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.02

$$\int \frac{1}{x^7(1-3x^4+x^8)}\,dx = -\frac{11}{40}\sqrt{5}\log\left(\frac{2\,x^2-\sqrt{5}+1}{2\,x^2+\sqrt{5}+1}\right)-\frac{11}{40}\sqrt{5}\log\left(\frac{2\,x^2-\sqrt{5}-1}{2\,x^2+\sqrt{5}-1}\right) - \frac{9\,x^4+1}{6\,x^6}-\frac{5}{8}\log\left(x^4+x^2-1\right)+\frac{5}{8}\log\left(x^4-x^2-1\right)$$

input
```
integrate(1/x^7/(x^8-3*x^4+1),x, algorithm="maxima")
```

output
```
-11/40*sqrt(5)*log((2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1)) - 11/40*sq
rt(5)*log((2*x^2 - sqrt(5) - 1)/(2*x^2 + sqrt(5) - 1)) - 1/6*(9*x^4 + 1)/x
^6 - 5/8*log(x^4 + x^2 - 1) + 5/8*log(x^4 - x^2 - 1)
```

3.395.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^7(1-3x^4+x^8)}\,dx = -\frac{11}{40}\sqrt{5}\log\left(\frac{\left|2\,x^2-\sqrt{5}+1\right|}{2\,x^2+\sqrt{5}+1}\right)-\frac{11}{40}\sqrt{5}\log\left(\frac{\left|2\,x^2-\sqrt{5}-1\right|}{\left|2\,x^2+\sqrt{5}-1\right|}\right) - \frac{9\,x^4+1}{6\,x^6}-\frac{5}{8}\log\left(\left|x^4+x^2-1\right|\right)+\frac{5}{8}\log\left(\left|x^4-x^2-1\right|\right)$$

input
```
integrate(1/x^7/(x^8-3*x^4+1),x, algorithm="giac")
```

output
```
-11/40*sqrt(5)*log(abs(2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1)) - 11/40
*sqrt(5)*log(abs(2*x^2 - sqrt(5) - 1)/abs(2*x^2 + sqrt(5) - 1)) - 1/6*(9*x
^4 + 1)/x^6 - 5/8*log(abs(x^4 + x^2 - 1)) + 5/8*log(abs(x^4 - x^2 - 1))
```

3.395.9 Mupad [B] (verification not implemented)

Time = 8.59 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.98

$$\int \frac{1}{x^7 (1-3x^4+x^8)} \, dx = \operatorname{atanh}\left(\frac{4126100\, x^2}{1140425\sqrt{5}-2550075} - \frac{1845250\sqrt{5}\, x^2}{1140425\sqrt{5}-2550075}\right)\left(\frac{11\sqrt{5}}{20}-\frac{5}{4}\right) + \operatorname{atanh}\left(\frac{4126100\, x^2}{1140425\sqrt{5}+2550075} + \frac{1845250\sqrt{5}\, x^2}{1140425\sqrt{5}+2550075}\right)\left(\frac{11\sqrt{5}}{20}+\frac{5}{4}\right) - \frac{\frac{3x^4}{2}+\frac{1}{6}}{x^6}$$

input
```
int(1/(x^7*(x^8 - 3*x^4 + 1)),x)
```

output
```
atanh((4126100*x^2)/(1140425*5^(1/2) - 2550075) - (1845250*5^(1/2)*x^2)/(1
140425*5^(1/2) - 2550075))*((11*5^(1/2))/20 - 5/4) + atanh((4126100*x^2)/(
1140425*5^(1/2) + 2550075) + (1845250*5^(1/2)*x^2)/(1140425*5^(1/2) + 2550
075))*((11*5^(1/2))/20 + 5/4) - ((3*x^4)/2 + 1/6)/x^6
```

3.396 $\int \frac{x^8}{1-3x^4+x^8} dx$

3.396.1 Optimal result

Integrand size = 16, antiderivative size = 170

$$\int \frac{x^8}{1-3x^4+x^8} dx = x - \frac{\sqrt[4]{\frac{1}{2}\left(123+55\sqrt{5}\right)}\arctan\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}} + \frac{\sqrt[4]{984-440\sqrt{5}}\arctan\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{4\sqrt{5}} - \frac{\sqrt[4]{\frac{1}{2}\left(123+55\sqrt{5}\right)}\text{arctanh}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}} + \frac{\sqrt[4]{984-440\sqrt{5}}\text{arctanh}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{4\sqrt{5}}$$

output
```
x+1/20*arctan(1/2*x*(3+5^(1/2))^(1/4)*2^(3/4))*(984-440*5^(1/2))^(1/4)*5^(
1/2)+1/20*arctanh(1/2*x*(3+5^(1/2))^(1/4)*2^(3/4))*(984-440*5^(1/2))^(1/4)
*5^(1/2)-1/20*arctan(2^(1/4)*x*(1/(3+5^(1/2)))^(1/4))*(123+55*5^(1/2))^(1/
4)*2^(3/4)*5^(1/2)-1/20*arctanh(2^(1/4)*x*(1/(3+5^(1/2)))^(1/4))*(123+55*5
^(1/2))^(1/4)*2^(3/4)*5^(1/2)
```

3.396.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.94

$$\int \frac{x^8}{1-3x^4+x^8}\,dx = x+\frac{\left(-2+\sqrt{5}\right)\arctan\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)}{\sqrt{10\left(-1+\sqrt{5}\right)}}-\frac{\left(2+\sqrt{5}\right)\arctan\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}}+\frac{\left(-2+\sqrt{5}\right)\text{arctanh}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)}{\sqrt{10\left(-1+\sqrt{5}\right)}}-\frac{\left(2+\sqrt{5}\right)\text{arctanh}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}}$$

input
```
Integrate[x^8/(1 - 3*x^4 + x^8),x]
```

output
```
x + ((-2 + Sqrt[5])*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x])/Sqrt[10*(-1 + Sqrt[5
])] - ((2 + Sqrt[5])*ArcTan[Sqrt[2/(1 + Sqrt[5])]*x])/Sqrt[10*(1 + Sqrt[5]
)] + ((-2 + Sqrt[5])*ArcTanh[Sqrt[2/(-1 + Sqrt[5])]*x])/Sqrt[10*(-1 + Sqrt
[5])] - ((2 + Sqrt[5])*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*x])/Sqrt[10*(1 + Sqrt
[5])]
```

3.396.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1703, 1752, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{x^8-3x^4+1}dx$$

$$\Big\downarrow \text{1703}$$

$$x-\int \frac{1-3x^4}{x^8-3x^4+1}dx$$

$$\Big\downarrow \text{1752}$$

$$\frac{1}{10}\left(15+7\sqrt{5}\right)\int \frac{1}{x^4+\frac{1}{2}\left(-3-\sqrt{5}\right)}dx+\frac{1}{10}\left(15-7\sqrt{5}\right)\int \frac{1}{x^4+\frac{1}{2}\left(-3+\sqrt{5}\right)}dx+x$$

$$\Big\downarrow \text{756}$$

$$\frac{1}{10}\left(15-7\sqrt{5}\right)\left(-\frac{\int \frac{1}{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}dx}{\sqrt{3-\sqrt{5}}}-\frac{\int \frac{1}{\sqrt{2}x^2+\sqrt{3-\sqrt{5}}}dx}{\sqrt{3-\sqrt{5}}}\right)+\frac{1}{10}\left(15+7\sqrt{5}\right)\left(-\frac{\int \frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}dx}{\sqrt{3+\sqrt{5}}}-\frac{\int \frac{1}{\sqrt{2}x^2+\sqrt{3+\sqrt{5}}}dx}{\sqrt{3+\sqrt{5}}}\right)+x$$

$\downarrow$ 216

$$\frac{1}{10}\left(15-7\sqrt{5}\right)\left(-\frac{\int \frac{1}{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}dx}{\sqrt{3-\sqrt{5}}}-\frac{\arctan\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{\sqrt[4]{2}\left(3-\sqrt{5}\right)^{3/4}}\right)+\frac{1}{10}\left(15+7\sqrt{5}\right)\left(-\frac{\int \frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}dx}{\sqrt{3+\sqrt{5}}}-\frac{\arctan\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt[4]{2}\left(3+\sqrt{5}\right)^{3/4}}\right)+x$$

$\downarrow$ 219

$$\frac{1}{10}\left(15+7\sqrt{5}\right)\left(-\frac{\arctan\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt[4]{2}\left(3+\sqrt{5}\right)^{3/4}}-\frac{\text{arctanh}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt[4]{2}\left(3+\sqrt{5}\right)^{3/4}}\right)+\frac{1}{10}\left(15-7\sqrt{5}\right)\left(-\frac{\arctan\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{\sqrt[4]{2}\left(3-\sqrt{5}\right)^{3/4}}-\frac{\text{arctanh}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{\sqrt[4]{2}\left(3-\sqrt{5}\right)^{3/4}}\right)+x$$

input
```
Int[x^8/(1 - 3*x^4 + x^8),x]
```

output
```
x + ((15 + 7*Sqrt[5])*(-(ArcTan[(2/(3 + Sqrt[5]))^(1/4)*x]/(2^(1/4)*(3 + S
qrt[5])^(3/4))) - ArcTanh[(2/(3 + Sqrt[5]))^(1/4)*x]/(2^(1/4)*(3 + Sqrt[5]
)^(3/4))))/10 + ((15 - 7*Sqrt[5])*(-(ArcTan[((3 + Sqrt[5])/2)^(1/4)*x]/(2^
(1/4)*(3 - Sqrt[5])^(3/4))) - ArcTanh[((3 + Sqrt[5])/2)^(1/4)*x]/(2^(1/4)*
(3 - Sqrt[5])^(3/4))))/10
```

3.396. $\int \frac{x^8}{1-3x^4+x^8}\,dx$

3.396.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 1703
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(2*n - 1)*(d*x)^(m - 2*n + 1)*((a + b*x^n + c*x^(2*n))^(
p + 1)/(c*(m + 2*n*p + 1))), x] - Simp[d^(2*n)/(c*(m + 2*n*p + 1))   Int[(d
*x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x
^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && N
eQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0
] && IntegerQ[p]
```

rule 1752
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

3.396. $\int \frac{x^8}{1-3x^4+x^8} \, dx$

3.396.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.41

method	result
risch	$x+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(25\_Z^{4}+55\_Z^{2}-1\right)}\_R\ln\left(15\_R^{3}+29\_R+5x\right)\right)}{4}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(25\_Z^{4}-55\_Z^{2}-1\right)}\_R\ln\left(-15\_R^{3}+\right.\right.}{4}$
default	$x-\frac{\left(2+\sqrt{5}\right)\sqrt{5}\,\mathrm{arctanh}\left(\frac{2x}{\sqrt{2\sqrt{5}+2}}\right)}{5\sqrt{2\sqrt{5}+2}}+\frac{\left(\sqrt{5}-2\right)\sqrt{5}\,\arctan\left(\frac{2x}{\sqrt{2\sqrt{5}-2}}\right)}{5\sqrt{2\sqrt{5}-2}}-\frac{\left(2+\sqrt{5}\right)\sqrt{5}\,\arctan\left(\frac{2x}{\sqrt{2\sqrt{5}+2}}\right)}{5\sqrt{2\sqrt{5}+2}}+\frac{\left(\sqrt{5}-2\right)\sqrt{5}\,\mathrm{arc}}{5\sqrt{2}}$

input
```
int(x^8/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
x+1/4*sum(_R*ln(15*_R^3+29*_R+5*x),_R=RootOf(25*_Z^4+55*_Z^2-1))+1/4*sum(_
R*ln(-15*_R^3+29*_R+5*x),_R=RootOf(25*_Z^4-55*_Z^2-1))
```

3.396.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 318 vs. 2(118) = 236.

Time = 0.25 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.87

$$\begin{aligned}\int \frac{x^8}{1-3x^4+x^8}\,dx &= \frac{1}{40}\sqrt{10}\sqrt{5\sqrt{5}-11}\log\left(\sqrt{10}\sqrt{5\sqrt{5}-11}\left(3\sqrt{5}+5\right)+20\,x\right)\\ &\quad-\frac{1}{40}\sqrt{10}\sqrt{5\sqrt{5}-11}\log\left(-\sqrt{10}\sqrt{5\sqrt{5}-11}\left(3\sqrt{5}+5\right)+20\,x\right)\\ &\quad-\frac{1}{40}\sqrt{10}\sqrt{5\sqrt{5}+11}\log\left(\sqrt{10}\sqrt{5\sqrt{5}+11}\left(3\sqrt{5}-5\right)+20\,x\right)\\ &\quad+\frac{1}{40}\sqrt{10}\sqrt{5\sqrt{5}+11}\log\left(-\sqrt{10}\sqrt{5\sqrt{5}+11}\left(3\sqrt{5}-5\right)+20\,x\right)\\ &\quad+\frac{1}{40}\sqrt{10}\sqrt{-5\sqrt{5}+11}\log\left(\sqrt{10}\left(3\sqrt{5}+5\right)\sqrt{-5\sqrt{5}+11}+20\,x\right)\\ &\quad-\frac{1}{40}\sqrt{10}\sqrt{-5\sqrt{5}+11}\log\left(-\sqrt{10}\left(3\sqrt{5}+5\right)\sqrt{-5\sqrt{5}+11}+20\,x\right)\\ &\quad-\frac{1}{40}\sqrt{10}\sqrt{-5\sqrt{5}-11}\log\left(\sqrt{10}\left(3\sqrt{5}-5\right)\sqrt{-5\sqrt{5}-11}+20\,x\right)\\ &\quad+\frac{1}{40}\sqrt{10}\sqrt{-5\sqrt{5}-11}\log\left(-\sqrt{10}\left(3\sqrt{5}-5\right)\sqrt{-5\sqrt{5}-11}+20\,x\right)\\ &\quad+x\end{aligned}$$

input
```
integrate(x^8/(x^8-3*x^4+1),x, algorithm="fricas")
```

output
```
1/40*sqrt(10)*sqrt(5*sqrt(5) - 11)*log(sqrt(10)*sqrt(5*sqrt(5) - 11)*(3*sq
rt(5) + 5) + 20*x) - 1/40*sqrt(10)*sqrt(5*sqrt(5) - 11)*log(-sqrt(10)*sqrt
(5*sqrt(5) - 11)*(3*sqrt(5) + 5) + 20*x) - 1/40*sqrt(10)*sqrt(5*sqrt(5) +
11)*log(sqrt(10)*sqrt(5*sqrt(5) + 11)*(3*sqrt(5) - 5) + 20*x) + 1/40*sqrt(
10)*sqrt(5*sqrt(5) + 11)*log(-sqrt(10)*sqrt(5*sqrt(5) + 11)*(3*sqrt(5) - 5
) + 20*x) + 1/40*sqrt(10)*sqrt(-5*sqrt(5) + 11)*log(sqrt(10)*(3*sqrt(5) +
5)*sqrt(-5*sqrt(5) + 11) + 20*x) - 1/40*sqrt(10)*sqrt(-5*sqrt(5) + 11)*log
(-sqrt(10)*(3*sqrt(5) + 5)*sqrt(-5*sqrt(5) + 11) + 20*x) - 1/40*sqrt(10)*s
qrt(-5*sqrt(5) - 11)*log(sqrt(10)*(3*sqrt(5) - 5)*sqrt(-5*sqrt(5) - 11) +
20*x) + 1/40*sqrt(10)*sqrt(-5*sqrt(5) - 11)*log(-sqrt(10)*(3*sqrt(5) - 5)*
sqrt(-5*sqrt(5) - 11) + 20*x) + x
```

3.396.6 Sympy [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.34

$$\int \frac{x^8}{1-3x^4+x^8}\,dx = x + \operatorname{RootSum}\left(6400t^4 - 880t^2 - 1, \left(t \mapsto t\log\left(-\frac{15360t^5}{11} + \frac{1288t}{55} + x\right)\right)\right) + \operatorname{RootSum}\left(6400t^4 + 880t^2 - 1, \left(t \mapsto t\log\left(-\frac{15360t^5}{11} + \frac{1288t}{55} + x\right)\right)\right)$$

input
```
integrate(x**8/(x**8-3*x**4+1),x)
```

output
```
x + RootSum(6400*_t**4 - 880*_t**2 - 1, Lambda(_t, _t*log(-15360*_t**5/11
+ 1288*_t/55 + x))) + RootSum(6400*_t**4 + 880*_t**2 - 1, Lambda(_t, _t*lo
g(-15360*_t**5/11 + 1288*_t/55 + x)))
```

3.396.7 Maxima [F]

$$\int \frac{x^8}{1-3x^4+x^8}\,dx = \int \frac{x^8}{x^8-3\,x^4+1}\,dx$$

input
```
integrate(x^8/(x^8-3*x^4+1),x, algorithm="maxima")
```

output
```
x + 1/2*integrate((2*x^2 + 1)/(x^4 - x^2 - 1), x) - 1/2*integrate((2*x^2 -
 1)/(x^4 + x^2 - 1), x)
```

3.396.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.87

$$\begin{aligned}\int \frac{x^8}{1-3x^4+x^8}\,dx = &-\frac{1}{20}\sqrt{50\sqrt{5}+110}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right)\\ &+\frac{1}{20}\sqrt{50\sqrt{5}-110}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right)\\ &-\frac{1}{40}\sqrt{50\sqrt{5}+110}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)\\ &+\frac{1}{40}\sqrt{50\sqrt{5}+110}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)\\ &+\frac{1}{40}\sqrt{50\sqrt{5}-110}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)\\ &-\frac{1}{40}\sqrt{50\sqrt{5}-110}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)+x\end{aligned}$$

input
```
integrate(x^8/(x^8-3*x^4+1),x, algorithm="giac")
```

output
```
-1/20*sqrt(50*sqrt(5) + 110)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) + 1/20*sqrt
(50*sqrt(5) - 110)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/40*sqrt(50*sqrt(5
) + 110)*log(abs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(50*sqrt(5) + 11
0)*log(abs(x - sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(50*sqrt(5) - 110)*log
(abs(x + sqrt(1/2*sqrt(5) - 1/2))) - 1/40*sqrt(50*sqrt(5) - 110)*log(abs(x
 - sqrt(1/2*sqrt(5) - 1/2))) + x
```

3.396.9 Mupad [B] (verification not implemented)

Time = 8.57 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.45

$$\int \frac{x^8}{1-3x^4+x^8}\,dx = x - \frac{\operatorname{atan}\left(\frac{x\sqrt{-50\sqrt{5}-110}\,55\mathrm{i}}{2\left(275\sqrt{5}+605\right)}+\frac{\sqrt{5}\,x\sqrt{-50\sqrt{5}-110}\,33\mathrm{i}}{2\left(275\sqrt{5}+605\right)}\right)\sqrt{-50\sqrt{5}-110}\,1\mathrm{i}}{20} - \frac{\operatorname{atan}\left(\frac{x\sqrt{110-50\sqrt{5}}\,55\mathrm{i}}{2\left(275\sqrt{5}-605\right)}-\frac{\sqrt{5}\,x\sqrt{110-50\sqrt{5}}\,33\mathrm{i}}{2\left(275\sqrt{5}-605\right)}\right)\sqrt{110-50\sqrt{5}}\,1\mathrm{i}}{20} + \frac{\operatorname{atan}\left(\frac{x\sqrt{50\sqrt{5}-110}\,55\mathrm{i}}{2\left(275\sqrt{5}-605\right)}-\frac{\sqrt{5}\,x\sqrt{50\sqrt{5}-110}\,33\mathrm{i}}{2\left(275\sqrt{5}-605\right)}\right)\sqrt{50\sqrt{5}-110}\,1\mathrm{i}}{20} + \frac{\operatorname{atan}\left(\frac{x\sqrt{50\sqrt{5}+110}\,55\mathrm{i}}{2\left(275\sqrt{5}+605\right)}+\frac{\sqrt{5}\,x\sqrt{50\sqrt{5}+110}\,33\mathrm{i}}{2\left(275\sqrt{5}+605\right)}\right)\sqrt{50\sqrt{5}+110}\,1\mathrm{i}}{20}$$

input
```
int(x^8/(x^8 - 3*x^4 + 1),x)
```

output
```
x - (atan((x*(- 50*5^(1/2) - 110)^(1/2)*55i)/(2*(275*5^(1/2) + 605)) + (5^
(1/2)*x*(- 50*5^(1/2) - 110)^(1/2)*33i)/(2*(275*5^(1/2) + 605)))*(- 50*5^(
1/2) - 110)^(1/2)*1i)/20 - (atan((x*(110 - 50*5^(1/2))^(1/2)*55i)/(2*(275*
5^(1/2) - 605)) - (5^(1/2)*x*(110 - 50*5^(1/2))^(1/2)*33i)/(2*(275*5^(1/2)
 - 605)))*(110 - 50*5^(1/2))^(1/2)*1i)/20 + (atan((x*(50*5^(1/2) - 110)^(1
/2)*55i)/(2*(275*5^(1/2) - 605)) - (5^(1/2)*x*(50*5^(1/2) - 110)^(1/2)*33i
)/(2*(275*5^(1/2) - 605)))*(50*5^(1/2) - 110)^(1/2)*1i)/20 + (atan((x*(50*
5^(1/2) + 110)^(1/2)*55i)/(2*(275*5^(1/2) + 605)) + (5^(1/2)*x*(50*5^(1/2)
 + 110)^(1/2)*33i)/(2*(275*5^(1/2) + 605)))*(50*5^(1/2) + 110)^(1/2)*1i)/2
0
```

3.397 $\int \frac{x^6}{1-3x^4+x^8} dx$

3.397.1 Optimal result

Integrand size = 16, antiderivative size = 167

$$\int \frac{x^6}{1-3x^4+x^8} dx = \frac{\left(3+\sqrt{5}\right)^{3/4} \arctan\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\ 2^{3/4}\sqrt{5}} - \frac{\sqrt[4]{144-64\sqrt{5}} \arctan\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{4\sqrt{5}} - \frac{\left(3+\sqrt{5}\right)^{3/4} \operatorname{arctanh}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\ 2^{3/4}\sqrt{5}} + \frac{\sqrt[4]{144-64\sqrt{5}} \operatorname{arctanh}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{4\sqrt{5}}$$

output
```
-1/20*arctan(1/2*x*(3+5^(1/2))^(1/4)*2^(3/4))*(144-64*5^(1/2))^(1/4)*5^(1/
2)+1/20*arctanh(1/2*x*(3+5^(1/2))^(1/4)*2^(3/4))*(144-64*5^(1/2))^(1/4)*5^
(1/2)+1/20*arctan(2^(1/4)*x*(1/(3+5^(1/2)))^(1/4))*(3+5^(1/2))^(3/4)*2^(1/
4)*5^(1/2)-1/20*arctanh(2^(1/4)*x*(1/(3+5^(1/2)))^(1/4))*(3+5^(1/2))^(3/4)
*2^(1/4)*5^(1/2)
```

3.397.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.96

$$\int \frac{x^6}{1-3x^4+x^8}\,dx = \frac{\frac{\left(-3+\sqrt{5}\right)\arctan\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)}{\sqrt{-1+\sqrt{5}}}+\frac{\left(3+\sqrt{5}\right)\arctan\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{1+\sqrt{5}}}-\frac{\left(-3+\sqrt{5}\right)\text{arctanh}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)}{\sqrt{-1+\sqrt{5}}}-\frac{\left(3+\sqrt{5}\right)\text{arctanh}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{1+\sqrt{5}}}}{2\sqrt{10}}$$

input
```
Integrate[x^6/(1 - 3*x^4 + x^8),x]
```

output
```
(((-3 + Sqrt[5])*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x])/Sqrt[-1 + Sqrt[5]] + ((
3 + Sqrt[5])*ArcTan[Sqrt[2/(1 + Sqrt[5])]*x])/Sqrt[1 + Sqrt[5]] - ((-3 + S
qrt[5])*ArcTanh[Sqrt[2/(-1 + Sqrt[5])]*x])/Sqrt[-1 + Sqrt[5]] - ((3 + Sqrt
[5])*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*x])/Sqrt[1 + Sqrt[5]])/(2*Sqrt[10])
```

3.397.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1710, 27, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{x^8-3x^4+1}\,dx$$

$$\Big\downarrow \text{1710}$$

$$\frac{1}{10}\left(5-3\sqrt{5}\right)\int -\frac{2x^2}{-2x^4-\sqrt{5}+3}dx+\frac{1}{10}\left(5+3\sqrt{5}\right)\int -\frac{2x^2}{-2x^4+\sqrt{5}+3}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{5}\left(5-3\sqrt{5}\right)\int \frac{x^2}{-2x^4-\sqrt{5}+3}dx-\frac{1}{5}\left(5+3\sqrt{5}\right)\int \frac{x^2}{-2x^4+\sqrt{5}+3}dx$$

$$\Big\downarrow \text{827}$$

$$-\frac{1}{5}\left(5-3\sqrt{5}\right)\left(\frac{\int \frac{1}{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}dx}{2\sqrt{2}}-\frac{\int \frac{1}{\sqrt{2}x^2+\sqrt{3-\sqrt{5}}}dx}{2\sqrt{2}}\right)-\frac{1}{5}\left(5+3\sqrt{5}\right)\left(\frac{\int \frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}dx}{2\sqrt{2}}-\frac{\int \frac{1}{\sqrt{2}x^2+\sqrt{3+\sqrt{5}}}dx}{2\sqrt{2}}\right)$$

$$\Big\downarrow \text{216}$$

$$-\frac{1}{5}\left(5-3\sqrt{5}\right)\left(\frac{\int \frac{1}{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}dx}{2\sqrt{2}}-\frac{\arctan\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\ 2^{3/4}\sqrt[4]{3-\sqrt{5}}}\right)-\frac{1}{5}\left(5+3\sqrt{5}\right)\left(\frac{\int \frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}dx}{2\sqrt{2}}-\frac{\arctan\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}\right)$$

$$\Big\downarrow \text{219}$$

$$-\frac{1}{5}\left(5+3\sqrt{5}\right)\left(\frac{\text{arctanh}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\arctan\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}\right)-\frac{1}{5}\left(5-3\sqrt{5}\right)\left(\frac{\text{arctanh}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\ 2^{3/4}\sqrt[4]{3-\sqrt{5}}}-\frac{\arctan\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\ 2^{3/4}\sqrt[4]{3-\sqrt{5}}}\right)$$

input
```
Int[x^6/(1 - 3*x^4 + x^8),x]
```

output
```
-1/5*((5 + 3*Sqrt[5])*(-1/2*ArcTan[(2/(3 + Sqrt[5]))^(1/4)*x]/(2^(3/4)*(3
+ Sqrt[5])^(1/4)) + ArcTanh[(2/(3 + Sqrt[5]))^(1/4)*x]/(2*2^(3/4)*(3 + Sqr
t[5])^(1/4)))) - ((5 - 3*Sqrt[5])*(-1/2*ArcTan[((3 + Sqrt[5])/2)^(1/4)*x]/
(2^(3/4)*(3 - Sqrt[5])^(1/4)) + ArcTanh[((3 + Sqrt[5])/2)^(1/4)*x]/(2*2^(3
/4)*(3 - Sqrt[5])^(1/4))))/5
```

3.397. $\int \frac{x^6}{1-3x^4+x^8}\,dx$

3.397.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 1710
```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^n/2)*(b/q + 1)   Int[(d*x)^(m
- n)/(b/2 + q/2 + c*x^n), x], x] - Simp[(d^n/2)*(b/q - 1)   Int[(d*x)^(m -
n)/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

3.397.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.41

method	result
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(25\_Z^4-20\_Z^2-1\right)} \_R\ln\left(5\_R^3-7\_R+2x\right)\right)}{4}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(25\_Z^4+20\_Z^2-1\right)} \_R\ln\left(5\_R^3+7\_R+2x\right)\right.}{4}$
default	$-\frac{\left(3+\sqrt{5}\right)\sqrt{5}\,\operatorname{arctanh}\left(\frac{2x}{\sqrt{2\sqrt{5}+2}}\right)}{10\sqrt{2\sqrt{5}+2}}+\frac{\left(\sqrt{5}-3\right)\sqrt{5}\,\arctan\left(\frac{2x}{\sqrt{2\sqrt{5}-2}}\right)}{10\sqrt{2\sqrt{5}-2}}+\frac{\left(3+\sqrt{5}\right)\sqrt{5}\,\arctan\left(\frac{2x}{\sqrt{2\sqrt{5}+2}}\right)}{10\sqrt{2\sqrt{5}+2}}-\frac{\left(\sqrt{5}-3\right)\sqrt{5}\,\text{arctan}}{10\sqrt{2\sqrt{}}}$

input
```
int(x^6/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(_R*ln(5*_R^3-7*_R+2*x),_R=RootOf(25*_Z^4-20*_Z^2-1))+1/4*sum(_R*ln
(5*_R^3+7*_R+2*x),_R=RootOf(25*_Z^4+20*_Z^2-1))
```

3.397.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 261 vs. $2(113) = 226$.

Time = 0.26 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.56

$$\begin{aligned}\int \frac{x^6}{1-3x^4+x^8}\,dx = &-\frac{1}{20}\sqrt{5}\sqrt{\sqrt{5}+2}\log\left(\sqrt{\sqrt{5}+2}\left(\sqrt{5}-1\right)+2x\right)\\&+\frac{1}{20}\sqrt{5}\sqrt{\sqrt{5}+2}\log\left(-\sqrt{\sqrt{5}+2}\left(\sqrt{5}-1\right)+2x\right)\\&+\frac{1}{20}\sqrt{5}\sqrt{\sqrt{5}-2}\log\left(\left(\sqrt{5}+1\right)\sqrt{\sqrt{5}-2}+2x\right)\\&-\frac{1}{20}\sqrt{5}\sqrt{\sqrt{5}-2}\log\left(-\left(\sqrt{5}+1\right)\sqrt{\sqrt{5}-2}+2x\right)\\&-\frac{1}{20}\sqrt{5}\sqrt{-\sqrt{5}+2}\log\left(\left(\sqrt{5}+1\right)\sqrt{-\sqrt{5}+2}+2x\right)\\&+\frac{1}{20}\sqrt{5}\sqrt{-\sqrt{5}+2}\log\left(-\left(\sqrt{5}+1\right)\sqrt{-\sqrt{5}+2}+2x\right)\\&+\frac{1}{20}\sqrt{5}\sqrt{-\sqrt{5}-2}\log\left(\left(\sqrt{5}-1\right)\sqrt{-\sqrt{5}-2}+2x\right)\\&-\frac{1}{20}\sqrt{5}\sqrt{-\sqrt{5}-2}\log\left(-\left(\sqrt{5}-1\right)\sqrt{-\sqrt{5}-2}+2x\right)\end{aligned}$$

input
```
integrate(x^6/(x^8-3*x^4+1),x, algorithm="fricas")
```

output
```
-1/20*sqrt(5)*sqrt(sqrt(5) + 2)*log(sqrt(sqrt(5) + 2)*(sqrt(5) - 1) + 2*x)
 + 1/20*sqrt(5)*sqrt(sqrt(5) + 2)*log(-sqrt(sqrt(5) + 2)*(sqrt(5) - 1) + 2
*x) + 1/20*sqrt(5)*sqrt(sqrt(5) - 2)*log((sqrt(5) + 1)*sqrt(sqrt(5) - 2) +
 2*x) - 1/20*sqrt(5)*sqrt(sqrt(5) - 2)*log(-(sqrt(5) + 1)*sqrt(sqrt(5) - 2
) + 2*x) - 1/20*sqrt(5)*sqrt(-sqrt(5) + 2)*log((sqrt(5) + 1)*sqrt(-sqrt(5)
 + 2) + 2*x) + 1/20*sqrt(5)*sqrt(-sqrt(5) + 2)*log(-(sqrt(5) + 1)*sqrt(-sq
rt(5) + 2) + 2*x) + 1/20*sqrt(5)*sqrt(-sqrt(5) - 2)*log((sqrt(5) - 1)*sqrt
(-sqrt(5) - 2) + 2*x) - 1/20*sqrt(5)*sqrt(-sqrt(5) - 2)*log(-(sqrt(5) - 1)
*sqrt(-sqrt(5) - 2) + 2*x)
```

3.397.6 Sympy [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.32

$$\int \frac{x^6}{1-3x^4+x^8}\,dx = \text{RootSum}\left(6400t^4-320t^2-1,\left(t\mapsto t\log\left(-1792000t^7+4920t^3+x\right)\right)\right) + \text{RootSum}\left(6400t^4+320t^2-1,\left(t\mapsto t\log\left(-1792000t^7+4920t^3+x\right)\right)\right)$$

input
```
integrate(x**6/(x**8-3*x**4+1),x)
```

output
```
RootSum(6400*_t**4 - 320*_t**2 - 1, Lambda(_t, _t*log(-1792000*_t**7 + 492
0*_t**3 + x))) + RootSum(6400*_t**4 + 320*_t**2 - 1, Lambda(_t, _t*log(-17
92000*_t**7 + 4920*_t**3 + x)))
```

3.397.7 Maxima [F]

$$\int \frac{x^6}{1-3x^4+x^8}\,dx = \int \frac{x^6}{x^8-3\,x^4+1}\,dx$$

input
```
integrate(x^6/(x^8-3*x^4+1),x, algorithm="maxima")
```

output
```
integrate(x^6/(x^8 - 3*x^4 + 1), x)
```

3.397.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.88

$$\int \frac{x^6}{1-3x^4+x^8}\,dx = \frac{1}{10}\sqrt{5\sqrt{5}+10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right) - \frac{1}{10}\sqrt{5\sqrt{5}-10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right) - \frac{1}{20}\sqrt{5\sqrt{5}+10}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right) + \frac{1}{20}\sqrt{5\sqrt{5}+10}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right) + \frac{1}{20}\sqrt{5\sqrt{5}-10}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right) - \frac{1}{20}\sqrt{5\sqrt{5}-10}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)$$

input
```
integrate(x^6/(x^8-3*x^4+1),x, algorithm="giac")
```

output
```
1/10*sqrt(5*sqrt(5) + 10)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) - 1/10*sqrt(5*
sqrt(5) - 10)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/20*sqrt(5*sqrt(5) + 10
)*log(abs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/20*sqrt(5*sqrt(5) + 10)*log(ab
s(x - sqrt(1/2*sqrt(5) + 1/2))) + 1/20*sqrt(5*sqrt(5) - 10)*log(abs(x + sq
rt(1/2*sqrt(5) - 1/2))) - 1/20*sqrt(5*sqrt(5) - 10)*log(abs(x - sqrt(1/2*s
qrt(5) - 1/2)))
```

3.397.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.88

$$\int \frac{x^6}{1-3x^4+x^8}\,dx = \frac{\sqrt{5}\,\mathrm{atan}\left(\frac{16\,x\,\sqrt{2-\sqrt{5}}}{8\,\sqrt{5}-24}\right)\,\sqrt{\sqrt{5}-2}\,1\mathrm{i}}{10} + \frac{\sqrt{5}\,\mathrm{atan}\left(\frac{16\,x\,\sqrt{-\sqrt{5}-2}}{8\,\sqrt{5}+24}\right)\,\sqrt{\sqrt{5}+2}\,1\mathrm{i}}{10} + \frac{\sqrt{5}\,\mathrm{atan}\left(\frac{x\,\sqrt{2-\sqrt{5}}\,16\mathrm{i}}{8\,\sqrt{5}-24}\right)\,\sqrt{2-\sqrt{5}}\,1\mathrm{i}}{10} + \frac{\sqrt{5}\,\mathrm{atan}\left(\frac{x\,\sqrt{-\sqrt{5}-2}\,16\mathrm{i}}{8\,\sqrt{5}+24}\right)\,\sqrt{-\sqrt{5}-2}\,1\mathrm{i}}{10}$$

input
```
int(x^6/(x^8 - 3*x^4 + 1),x)
```

output
```
(5^(1/2)*atan((16*x*(2 - 5^(1/2))^(1/2))/(8*5^(1/2) - 24))*(5^(1/2) - 2)^(
1/2)*1i)/10 + (5^(1/2)*atan((16*x*(- 5^(1/2) - 2)^(1/2))/(8*5^(1/2) + 24))
*(5^(1/2) + 2)^(1/2)*1i)/10 + (5^(1/2)*atan((x*(2 - 5^(1/2))^(1/2)*16i)/(8
*5^(1/2) - 24))*(2 - 5^(1/2))^(1/2)*1i)/10 + (5^(1/2)*atan((x*(- 5^(1/2) -
 2)^(1/2)*16i)/(8*5^(1/2) + 24))*(- 5^(1/2) - 2)^(1/2)*1i)/10
```

3.398 $\int \frac{x^4}{1-3x^4+x^8}\,dx$

3.398.1 Optimal result

Integrand size = 16, antiderivative size = 173

$$\int \frac{x^4}{1-3x^4+x^8}\,dx = -\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\arctan\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}} + \frac{\sqrt[4]{\frac{1}{2}\left(3-\sqrt{5}\right)}\arctan\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{5}} - \frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\operatorname{arctanh}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}} + \frac{\sqrt[4]{\frac{1}{2}\left(3-\sqrt{5}\right)}\operatorname{arctanh}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{5}}$$

output
```
1/20*arctan(1/2*x*(3+5^(1/2))^(1/4)*2^(3/4))*(3-5^(1/2))^(1/4)*2^(3/4)*5^(
1/2)+1/20*arctanh(1/2*x*(3+5^(1/2))^(1/4)*2^(3/4))*(3-5^(1/2))^(1/4)*2^(3/
4)*5^(1/2)-1/20*arctan(2^(1/4)*x*(1/(3+5^(1/2)))^(1/4))*(3+5^(1/2))^(1/4)*
2^(3/4)*5^(1/2)-1/20*arctanh(2^(1/4)*x*(1/(3+5^(1/2)))^(1/4))*(3+5^(1/2))^
(1/4)*2^(3/4)*5^(1/2)
```

3.398.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.76

$$\int \frac{x^4}{1-3x^4+x^8}\,dx = \frac{\sqrt{-1+\sqrt{5}}\arctan\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)-\sqrt{1+\sqrt{5}}\arctan\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)+\sqrt{-1+\sqrt{5}}\text{arctanh}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)-\sqrt{1}}{2\sqrt{10}}$$

input
```
Integrate[x^4/(1 - 3*x^4 + x^8),x]
```

output
```
(Sqrt[-1 + Sqrt[5]]*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x] - Sqrt[1 + Sqrt[5]]*A
rcTan[Sqrt[2/(1 + Sqrt[5])]*x] + Sqrt[-1 + Sqrt[5]]*ArcTanh[Sqrt[2/(-1 + S
qrt[5])]*x] - Sqrt[1 + Sqrt[5]]*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*x])/(2*Sqrt[
10])
```

3.398.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1710, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{x^8-3x^4+1}\,dx$$

$$\Big\downarrow \; 1710$$

$$\frac{1}{10}\left(5+3\sqrt{5}\right)\int \frac{1}{x^4+\frac{1}{2}\left(-3-\sqrt{5}\right)}dx+\frac{1}{10}\left(5-3\sqrt{5}\right)\int \frac{1}{x^4+\frac{1}{2}\left(-3+\sqrt{5}\right)}dx$$

$$\Big\downarrow \; 756$$

$$\frac{1}{10}\left(5-3\sqrt{5}\right)\left(-\frac{\int \frac{1}{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}dx}{\sqrt{3-\sqrt{5}}}-\frac{\int \frac{1}{\sqrt{2}x^2+\sqrt{3-\sqrt{5}}}dx}{\sqrt{3-\sqrt{5}}}\right)+\frac{1}{10}\left(5+3\sqrt{5}\right)\left(-\frac{\int \frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}dx}{\sqrt{3+\sqrt{5}}}-\frac{\int \frac{1}{\sqrt{2}x^2+\sqrt{3+\sqrt{5}}}dx}{\sqrt{3+\sqrt{5}}}\right)$$

$$\Big\downarrow \; 216$$

$$\frac{1}{10}\left(5-3\sqrt{5}\right)\left(-\frac{\int \frac{1}{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}dx}{\sqrt{3-\sqrt{5}}}-\frac{\arctan\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{\sqrt[4]{2}\left(3-\sqrt{5}\right)^{3/4}}\right)+\frac{1}{10}\left(5+3\sqrt{5}\right)\left(-\frac{\int \frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}dx}{\sqrt{3+\sqrt{5}}}-\frac{\arctan\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt[4]{2}\left(3+\sqrt{5}\right)^{3/4}}\right)$$

$$\Big\downarrow \; 219$$

$$\frac{1}{10}\left(5+3\sqrt{5}\right)\left(-\frac{\arctan\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt[4]{2}\left(3+\sqrt{5}\right)^{3/4}}-\frac{\operatorname{arctanh}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt[4]{2}\left(3+\sqrt{5}\right)^{3/4}}\right)+\frac{1}{10}\left(5-3\sqrt{5}\right)\left(-\frac{\arctan\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{\sqrt[4]{2}\left(3-\sqrt{5}\right)^{3/4}}-\frac{\operatorname{arctanh}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{\sqrt[4]{2}\left(3-\sqrt{5}\right)^{3/4}}\right)$$

input
```
Int[x^4/(1 - 3*x^4 + x^8),x]
```

output
```
((5 + 3*Sqrt[5])*(-(ArcTan[(2/(3 + Sqrt[5]))^(1/4)*x]/(2^(1/4)*(3 + Sqrt[5
])^(3/4))) - ArcTanh[(2/(3 + Sqrt[5]))^(1/4)*x]/(2^(1/4)*(3 + Sqrt[5])^(3/
4))))/10 + ((5 - 3*Sqrt[5])*(-(ArcTan[((3 + Sqrt[5])/2)^(1/4)*x]/(2^(1/4)*
(3 - Sqrt[5])^(3/4))) - ArcTanh[((3 + Sqrt[5])/2)^(1/4)*x]/(2^(1/4)*(3 - S
qrt[5])^(3/4))))/10
```

3.398.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

3.398. $\int \frac{x^4}{1-3x^4+x^8}\,dx$

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 1710
```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^n/2)*(b/q + 1)   Int[(d*x)^(m
- n)/(b/2 + q/2 + c*x^n), x], x] - Simp[(d^n/2)*(b/q - 1)   Int[(d*x)^(m -
n)/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

3.398.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.35

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(25\_Z^4+5\_Z^2-1\right)}\_R\ln\left(10\_R^3+\_R+x\right)\right)}{4}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(25\_Z^4-5\_Z^2-1\right)}\_R\ln\left(-10\_R^3+\_R+x\right)\right)}{4}$
default	$-\frac{\left(\sqrt{5}+1\right)\sqrt{5}\,\operatorname{arctanh}\left(\frac{2x}{\sqrt{2\sqrt{5}+2}}\right)}{10\sqrt{2\sqrt{5}+2}}+\frac{\sqrt{5}\left(\sqrt{5}-1\right)\arctan\left(\frac{2x}{\sqrt{2\sqrt{5}-2}}\right)}{10\sqrt{2\sqrt{5}-2}}-\frac{\left(\sqrt{5}+1\right)\sqrt{5}\,\arctan\left(\frac{2x}{\sqrt{2\sqrt{5}+2}}\right)}{10\sqrt{2\sqrt{5}+2}}+\frac{\sqrt{5}\left(\sqrt{5}-1\right)\operatorname{arctan}}{10\sqrt{2\sqrt{5}}}$

input
```
int(x^4/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(_R*ln(10*_R^3+_R+x),_R=RootOf(25*_Z^4+5*_Z^2-1))+1/4*sum(_R*ln(-10
*_R^3+_R+x),_R=RootOf(25*_Z^4-5*_Z^2-1))
```

3.398.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(119) = 238$.

Time = 0.28 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.55

$$\begin{aligned}\int \frac{x^4}{1-3x^4+x^8}\,dx = & -\frac{1}{40}\sqrt{10}\sqrt{\sqrt{5}+1}\log\left(\sqrt{10}\sqrt{5}\sqrt{\sqrt{5}+1}+10\,x\right) \\ & +\frac{1}{40}\sqrt{10}\sqrt{\sqrt{5}+1}\log\left(-\sqrt{10}\sqrt{5}\sqrt{\sqrt{5}+1}+10\,x\right) \\ & +\frac{1}{40}\sqrt{10}\sqrt{\sqrt{5}-1}\log\left(\sqrt{10}\sqrt{5}\sqrt{\sqrt{5}-1}+10\,x\right) \\ & -\frac{1}{40}\sqrt{10}\sqrt{\sqrt{5}-1}\log\left(-\sqrt{10}\sqrt{5}\sqrt{\sqrt{5}-1}+10\,x\right) \\ & +\frac{1}{40}\sqrt{10}\sqrt{-\sqrt{5}+1}\log\left(\sqrt{10}\sqrt{5}\sqrt{-\sqrt{5}+1}+10\,x\right) \\ & -\frac{1}{40}\sqrt{10}\sqrt{-\sqrt{5}+1}\log\left(-\sqrt{10}\sqrt{5}\sqrt{-\sqrt{5}+1}+10\,x\right) \\ & -\frac{1}{40}\sqrt{10}\sqrt{-\sqrt{5}-1}\log\left(\sqrt{10}\sqrt{5}\sqrt{-\sqrt{5}-1}+10\,x\right) \\ & +\frac{1}{40}\sqrt{10}\sqrt{-\sqrt{5}-1}\log\left(-\sqrt{10}\sqrt{5}\sqrt{-\sqrt{5}-1}+10\,x\right)\end{aligned}$$

input
```
integrate(x^4/(x^8-3*x^4+1),x, algorithm="fricas")
```

output
```
-1/40*sqrt(10)*sqrt(sqrt(5) + 1)*log(sqrt(10)*sqrt(5)*sqrt(sqrt(5) + 1) +
10*x) + 1/40*sqrt(10)*sqrt(sqrt(5) + 1)*log(-sqrt(10)*sqrt(5)*sqrt(sqrt(5)
 + 1) + 10*x) + 1/40*sqrt(10)*sqrt(sqrt(5) - 1)*log(sqrt(10)*sqrt(5)*sqrt(
sqrt(5) - 1) + 10*x) - 1/40*sqrt(10)*sqrt(sqrt(5) - 1)*log(-sqrt(10)*sqrt(
5)*sqrt(sqrt(5) - 1) + 10*x) + 1/40*sqrt(10)*sqrt(-sqrt(5) + 1)*log(sqrt(1
0)*sqrt(5)*sqrt(-sqrt(5) + 1) + 10*x) - 1/40*sqrt(10)*sqrt(-sqrt(5) + 1)*l
og(-sqrt(10)*sqrt(5)*sqrt(-sqrt(5) + 1) + 10*x) - 1/40*sqrt(10)*sqrt(-sqrt
(5) - 1)*log(sqrt(10)*sqrt(5)*sqrt(-sqrt(5) - 1) + 10*x) + 1/40*sqrt(10)*s
qrt(-sqrt(5) - 1)*log(-sqrt(10)*sqrt(5)*sqrt(-sqrt(5) - 1) + 10*x)
```

3.398.6 Sympy [A] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.28

$$\int \frac{x^4}{1-3x^4+x^8}\,dx = \operatorname{RootSum}\left(6400t^4-80t^2-1,\left(t\mapsto t\log\left(-51200t^5+12t+x\right)\right)\right) + \operatorname{RootSum}\left(6400t^4+80t^2-1,\left(t\mapsto t\log\left(-51200t^5+12t+x\right)\right)\right)$$

input
```
integrate(x**4/(x**8-3*x**4+1),x)
```

output
```
RootSum(6400*_t**4 - 80*_t**2 - 1, Lambda(_t, _t*log(-51200*_t**5 + 12*_t
+ x))) + RootSum(6400*_t**4 + 80*_t**2 - 1, Lambda(_t, _t*log(-51200*_t**5
 + 12*_t + x)))
```

3.398.7 Maxima [F]

$$\int \frac{x^4}{1-3x^4+x^8}\,dx = \int \frac{x^4}{x^8-3\,x^4+1}\,dx$$

input
```
integrate(x^4/(x^8-3*x^4+1),x, algorithm="maxima")
```

output
```
integrate(x^4/(x^8 - 3*x^4 + 1), x)
```

3.398.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.85

$$\int \frac{x^4}{1-3x^4+x^8}\,dx = -\frac{1}{20}\sqrt{10\sqrt{5}+10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right) + \frac{1}{20}\sqrt{10\sqrt{5}-10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right) - \frac{1}{40}\sqrt{10\sqrt{5}+10}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right) + \frac{1}{40}\sqrt{10\sqrt{5}+10}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right) + \frac{1}{40}\sqrt{10\sqrt{5}-10}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right) - \frac{1}{40}\sqrt{10\sqrt{5}-10}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)$$

input
```
integrate(x^4/(x^8-3*x^4+1),x, algorithm="giac")
```

output
```
-1/20*sqrt(10*sqrt(5) + 10)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) + 1/20*sqrt(
10*sqrt(5) - 10)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/40*sqrt(10*sqrt(5)
+ 10)*log(abs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(10*sqrt(5) + 10)*l
og(abs(x - sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(10*sqrt(5) - 10)*log(abs(
x + sqrt(1/2*sqrt(5) - 1/2))) - 1/40*sqrt(10*sqrt(5) - 10)*log(abs(x - sqr
t(1/2*sqrt(5) - 1/2)))
```

3.398.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.55

$$\int \frac{x^4}{1-3x^4+x^8}\,dx = \frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{-\sqrt{5}-1}\,1\mathrm{i}}{2\left(\sqrt{5}-1\right)}-\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{-\sqrt{5}-1}\,3\mathrm{i}}{10\left(\sqrt{5}-1\right)}\right)\sqrt{-\sqrt{5}-1}\,1\mathrm{i}}{20} + \frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{1-\sqrt{5}}\,1\mathrm{i}}{2\left(\sqrt{5}+1\right)}+\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{1-\sqrt{5}}\,3\mathrm{i}}{10\left(\sqrt{5}+1\right)}\right)\sqrt{1-\sqrt{5}}\,1\mathrm{i}}{20} - \frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{\sqrt{5}+1}\,1\mathrm{i}}{2\left(\sqrt{5}-1\right)}-\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{\sqrt{5}+1}\,3\mathrm{i}}{10\left(\sqrt{5}-1\right)}\right)\sqrt{\sqrt{5}+1}\,1\mathrm{i}}{20} - \frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{\sqrt{5}-1}\,1\mathrm{i}}{2\left(\sqrt{5}+1\right)}+\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{\sqrt{5}-1}\,3\mathrm{i}}{10\left(\sqrt{5}+1\right)}\right)\sqrt{\sqrt{5}-1}\,1\mathrm{i}}{20}$$

input
```
int(x^4/(x^8 - 3*x^4 + 1),x)
```

output
```
(10^(1/2)*atan((10^(1/2)*x*(- 5^(1/2) - 1)^(1/2)*1i)/(2*(5^(1/2) - 1)) - (
5^(1/2)*10^(1/2)*x*(- 5^(1/2) - 1)^(1/2)*3i)/(10*(5^(1/2) - 1)))*(- 5^(1/2
) - 1)^(1/2)*1i)/20 + (10^(1/2)*atan((10^(1/2)*x*(1 - 5^(1/2))^(1/2)*1i)/(
2*(5^(1/2) + 1)) + (5^(1/2)*10^(1/2)*x*(1 - 5^(1/2))^(1/2)*3i)/(10*(5^(1/2
) + 1)))*(1 - 5^(1/2))^(1/2)*1i)/20 - (10^(1/2)*atan((10^(1/2)*x*(5^(1/2)
+ 1)^(1/2)*1i)/(2*(5^(1/2) - 1)) - (5^(1/2)*10^(1/2)*x*(5^(1/2) + 1)^(1/2)
*3i)/(10*(5^(1/2) - 1)))*(5^(1/2) + 1)^(1/2)*1i)/20 - (10^(1/2)*atan((10^(
1/2)*x*(5^(1/2) - 1)^(1/2)*1i)/(2*(5^(1/2) + 1)) + (5^(1/2)*10^(1/2)*x*(5^
(1/2) - 1)^(1/2)*3i)/(10*(5^(1/2) + 1)))*(5^(1/2) - 1)^(1/2)*1i)/20
```

3.399 $\int \frac{x^2}{1-3x^4+x^8}\,dx$

3.399.1 Optimal result

Integrand size = 16, antiderivative size = 145

$$\begin{aligned}\int \frac{x^2}{1-3x^4+x^8}\,dx = {} & \frac{1}{20}\sqrt{-10+10\sqrt{5}}\arctan\left(\frac{1}{2}\sqrt{-2+2\sqrt{5}}x\right) \\ & -\frac{1}{20}\sqrt{10+10\sqrt{5}}\arctan\left(\frac{1}{2}\sqrt{2+2\sqrt{5}}x\right) \\ & -\frac{1}{20}\sqrt{-10+10\sqrt{5}}\,\text{arctanh}\left(\frac{1}{2}\sqrt{-2+2\sqrt{5}}x\right) \\ & +\frac{1}{20}\sqrt{10+10\sqrt{5}}\,\text{arctanh}\left(\frac{1}{2}\sqrt{2+2\sqrt{5}}x\right)\end{aligned}$$

output
```
1/20*arctan(1/2*x*(-2+2*5^(1/2))^(1/2))*(-10+10*5^(1/2))^(1/2)-1/20*arctan
h(1/2*x*(-2+2*5^(1/2))^(1/2))*(-10+10*5^(1/2))^(1/2)-1/20*arctan(1/2*x*(2+
2*5^(1/2))^(1/2))*(10+10*5^(1/2))^(1/2)+1/20*arctanh(1/2*x*(2+2*5^(1/2))^(
1/2))*(10+10*5^(1/2))^(1/2)
```

3.399.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.90

$$\int \frac{x^2}{1-3x^4+x^8}\,dx = -\frac{\arctan\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)}{\sqrt{10\left(-1+\sqrt{5}\right)}} + \frac{\arctan\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}} + \frac{\operatorname{arctanh}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)}{\sqrt{10\left(-1+\sqrt{5}\right)}} - \frac{\operatorname{arctanh}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}}$$

input
```
Integrate[x^2/(1 - 3*x^4 + x^8),x]
```

output
```
-(ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x]/Sqrt[10*(-1 + Sqrt[5])]) + ArcTan[Sqrt[
2/(1 + Sqrt[5])]*x]/Sqrt[10*(1 + Sqrt[5])] + ArcTanh[Sqrt[2/(-1 + Sqrt[5])
]*x]/Sqrt[10*(-1 + Sqrt[5])] - ArcTanh[Sqrt[2/(1 + Sqrt[5])]*x]/Sqrt[10*(1
 + Sqrt[5])]
```

3.399.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.19, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1711, 27, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{x^8-3x^4+1}dx$$

$$\Big\downarrow \; 1711$$

$$\frac{\int -\frac{2x^2}{-2x^4+\sqrt{5}+3}dx}{\sqrt{5}} - \frac{\int -\frac{2x^2}{-2x^4-\sqrt{5}+3}dx}{\sqrt{5}}$$

$$\Big\downarrow \; 27$$

$$\frac{2\int \frac{x^2}{-2x^4-\sqrt{5}+3}dx}{\sqrt{5}} - \frac{2\int \frac{x^2}{-2x^4+\sqrt{5}+3}dx}{\sqrt{5}}$$

$$\Big\downarrow \; 827$$

$$\frac{2\left(\frac{\int \frac{1}{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}dx}{2\sqrt{2}}-\frac{\int \frac{1}{\sqrt{2}x^2+\sqrt{3-\sqrt{5}}}dx}{2\sqrt{2}}\right)}{\sqrt{5}}-\frac{2\left(\frac{\int \frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}dx}{2\sqrt{2}}-\frac{\int \frac{1}{\sqrt{2}x^2+\sqrt{3+\sqrt{5}}}dx}{2\sqrt{2}}\right)}{\sqrt{5}}$$

$$\Big\downarrow \text{216}$$

$$\frac{2\left(\frac{\int \frac{1}{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}dx}{2\sqrt{2}}-\frac{\arctan\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\ 2^{3/4}\sqrt[4]{3-\sqrt{5}}}\right)}{\sqrt{5}}-\frac{2\left(\frac{\int \frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}dx}{2\sqrt{2}}-\frac{\arctan\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}\right)}{\sqrt{5}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\left(\frac{\text{arctanh}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\ 2^{3/4}\sqrt[4]{3-\sqrt{5}}}-\frac{\arctan\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\ 2^{3/4}\sqrt[4]{3-\sqrt{5}}}\right)}{\sqrt{5}}-\frac{2\left(\frac{\text{arctanh}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\arctan\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}\right)}{\sqrt{5}}$$

input
```
Int[x^2/(1 - 3*x^4 + x^8),x]
```

output
```
(-2*(-1/2*ArcTan[(2/(3 + Sqrt[5]))^(1/4)*x]/(2^(3/4)*(3 + Sqrt[5])^(1/4))
+ ArcTanh[(2/(3 + Sqrt[5]))^(1/4)*x]/(2*2^(3/4)*(3 + Sqrt[5])^(1/4))))/Sqr
t[5] + (2*(-1/2*ArcTan[((3 + Sqrt[5])/2)^(1/4)*x]/(2^(3/4)*(3 - Sqrt[5])^(
1/4)) + ArcTanh[((3 + Sqrt[5])/2)^(1/4)*x]/(2*2^(3/4)*(3 - Sqrt[5])^(1/4))
))/Sqrt[5]
```

3.399. $\int \frac{x^2}{1-3x^4+x^8}\,dx$

3.399.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 1711
```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[c/q   Int[(d*x)^m/(b/2 - q/2 + c
*x^n), x], x] - Simp[c/q   Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; Free
Q[{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

3.399.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.44

method	result
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(25\_Z^4-5\_Z^2-1\right)}\_R\ln\left(-5\_R^3+3\_R+x\right)\right)}{4}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(25\_Z^4+5\_Z^2-1\right)}\_R\ln\left(-5\_R^3-3\_R+x\right)\right.}{4}$
default	$-\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{2x}{\sqrt{2\sqrt{5}+2}}\right)}{5\sqrt{2\sqrt{5}+2}}-\frac{\sqrt{5}\,\arctan\left(\frac{2x}{\sqrt{2\sqrt{5}-2}}\right)}{5\sqrt{2\sqrt{5}-2}}+\frac{\sqrt{5}\,\arctan\left(\frac{2x}{\sqrt{2\sqrt{5}+2}}\right)}{5\sqrt{2\sqrt{5}+2}}+\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{2x}{\sqrt{2\sqrt{5}-2}}\right)}{5\sqrt{2\sqrt{5}-2}}$

3.399. $\int \frac{x^2}{1-3x^4+x^8}\,dx$

input
```
int(x^2/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(_R*ln(-5*_R^3+3*_R+x),_R=RootOf(25*_Z^4-5*_Z^2-1))+1/4*sum(_R*ln(-
5*_R^3-3*_R+x),_R=RootOf(25*_Z^4+5*_Z^2-1))
```

3.399.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 285 vs. $2(97) = 194$.

Time = 0.28 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.97

$$\begin{aligned}\int \frac{x^2}{1-3x^4+x^8}\,dx = &-\frac{1}{40}\sqrt{10}\sqrt{\sqrt{5}-1}\log\left(\sqrt{10}\left(\sqrt{5}+5\right)\sqrt{\sqrt{5}-1}+20\,x\right)\\&+\frac{1}{40}\sqrt{10}\sqrt{\sqrt{5}-1}\log\left(-\sqrt{10}\left(\sqrt{5}+5\right)\sqrt{\sqrt{5}-1}+20\,x\right)\\&-\frac{1}{40}\sqrt{10}\sqrt{\sqrt{5}+1}\log\left(\sqrt{10}\sqrt{\sqrt{5}+1}\left(\sqrt{5}-5\right)+20\,x\right)\\&+\frac{1}{40}\sqrt{10}\sqrt{\sqrt{5}+1}\log\left(-\sqrt{10}\sqrt{\sqrt{5}+1}\left(\sqrt{5}-5\right)+20\,x\right)\\&+\frac{1}{40}\sqrt{10}\sqrt{-\sqrt{5}+1}\log\left(\sqrt{10}\left(\sqrt{5}+5\right)\sqrt{-\sqrt{5}+1}+20\,x\right)\\&-\frac{1}{40}\sqrt{10}\sqrt{-\sqrt{5}+1}\log\left(-\sqrt{10}\left(\sqrt{5}+5\right)\sqrt{-\sqrt{5}+1}+20\,x\right)\\&+\frac{1}{40}\sqrt{10}\sqrt{-\sqrt{5}-1}\log\left(\sqrt{10}\left(\sqrt{5}-5\right)\sqrt{-\sqrt{5}-1}+20\,x\right)\\&-\frac{1}{40}\sqrt{10}\sqrt{-\sqrt{5}-1}\log\left(-\sqrt{10}\left(\sqrt{5}-5\right)\sqrt{-\sqrt{5}-1}+20\,x\right)\end{aligned}$$

input
```
integrate(x^2/(x^8-3*x^4+1),x, algorithm="fricas")
```

output
```
-1/40*sqrt(10)*sqrt(sqrt(5) - 1)*log(sqrt(10)*(sqrt(5) + 5)*sqrt(sqrt(5) -
 1) + 20*x) + 1/40*sqrt(10)*sqrt(sqrt(5) - 1)*log(-sqrt(10)*(sqrt(5) + 5)*
sqrt(sqrt(5) - 1) + 20*x) - 1/40*sqrt(10)*sqrt(sqrt(5) + 1)*log(sqrt(10)*s
qrt(sqrt(5) + 1)*(sqrt(5) - 5) + 20*x) + 1/40*sqrt(10)*sqrt(sqrt(5) + 1)*l
og(-sqrt(10)*sqrt(sqrt(5) + 1)*(sqrt(5) - 5) + 20*x) + 1/40*sqrt(10)*sqrt(
-sqrt(5) + 1)*log(sqrt(10)*(sqrt(5) + 5)*sqrt(-sqrt(5) + 1) + 20*x) - 1/40
*sqrt(10)*sqrt(-sqrt(5) + 1)*log(-sqrt(10)*(sqrt(5) + 5)*sqrt(-sqrt(5) + 1
) + 20*x) + 1/40*sqrt(10)*sqrt(-sqrt(5) - 1)*log(sqrt(10)*(sqrt(5) - 5)*sq
rt(-sqrt(5) - 1) + 20*x) - 1/40*sqrt(10)*sqrt(-sqrt(5) - 1)*log(-sqrt(10)*
(sqrt(5) - 5)*sqrt(-sqrt(5) - 1) + 20*x)
```

3.399.6 Sympy [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.37

$$\int \frac{x^2}{1-3x^4+x^8}\,dx = \operatorname{RootSum}\left(6400t^4-80t^2-1,\left(t\mapsto t\log\left(6144000t^7-2240t^3+x\right)\right)\right) + \operatorname{RootSum}\left(6400t^4+80t^2-1,\left(t\mapsto t\log\left(6144000t^7-2240t^3+x\right)\right)\right)$$

input
```
integrate(x**2/(x**8-3*x**4+1),x)
```

output
```
RootSum(6400*_t**4 - 80*_t**2 - 1, Lambda(_t, _t*log(6144000*_t**7 - 2240*
_t**3 + x))) + RootSum(6400*_t**4 + 80*_t**2 - 1, Lambda(_t, _t*log(614400
0*_t**7 - 2240*_t**3 + x)))
```

3.399.7 Maxima [F]

$$\int \frac{x^2}{1-3x^4+x^8}\,dx = \int \frac{x^2}{x^8-3\,x^4+1}\,dx$$

input
```
integrate(x^2/(x^8-3*x^4+1),x, algorithm="maxima")
```

output
```
integrate(x^2/(x^8 - 3*x^4 + 1), x)
```

3.399.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.01

$$\int \frac{x^2}{1-3x^4+x^8}\,dx = \frac{1}{20}\sqrt{10\sqrt{5}-10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right) - \frac{1}{20}\sqrt{10\sqrt{5}+10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right) - \frac{1}{40}\sqrt{10\sqrt{5}-10}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right) + \frac{1}{40}\sqrt{10\sqrt{5}-10}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right) + \frac{1}{40}\sqrt{10\sqrt{5}+10}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right) - \frac{1}{40}\sqrt{10\sqrt{5}+10}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)$$

input
```
integrate(x^2/(x^8-3*x^4+1),x, algorithm="giac")
```

output
```
1/20*sqrt(10*sqrt(5) - 10)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) - 1/20*sqrt(1
0*sqrt(5) + 10)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/40*sqrt(10*sqrt(5) -
 10)*log(abs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(10*sqrt(5) - 10)*lo
g(abs(x - sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(10*sqrt(5) + 10)*log(abs(x
 + sqrt(1/2*sqrt(5) - 1/2))) - 1/40*sqrt(10*sqrt(5) + 10)*log(abs(x - sqrt
(1/2*sqrt(5) - 1/2)))
```

3.399.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.86

$$\int \frac{x^2}{1-3x^4+x^8}\,dx = \frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{\sqrt{5}-1}\,3\mathrm{i}}{2\left(3\sqrt{5}-7\right)}-\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{\sqrt{5}-1}\,7\mathrm{i}}{10\left(3\sqrt{5}-7\right)}\right)\sqrt{\sqrt{5}-1}\,1\mathrm{i}}{20} - \frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{\sqrt{5}+1}\,3\mathrm{i}}{2\left(3\sqrt{5}+7\right)}+\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{\sqrt{5}+1}\,7\mathrm{i}}{10\left(3\sqrt{5}+7\right)}\right)\sqrt{\sqrt{5}+1}\,1\mathrm{i}}{20} + \frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{1-\sqrt{5}}\,3\mathrm{i}}{2\left(3\sqrt{5}-7\right)}-\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{1-\sqrt{5}}\,7\mathrm{i}}{10\left(3\sqrt{5}-7\right)}\right)\sqrt{1-\sqrt{5}}\,1\mathrm{i}}{20} - \frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{-\sqrt{5}-1}\,3\mathrm{i}}{2\left(3\sqrt{5}+7\right)}+\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{-\sqrt{5}-1}\,7\mathrm{i}}{10\left(3\sqrt{5}+7\right)}\right)\sqrt{-\sqrt{5}-1}\,1\mathrm{i}}{20}$$

input
```
int(x^2/(x^8 - 3*x^4 + 1),x)
```

output
```
(10^(1/2)*atan((10^(1/2)*x*(5^(1/2) - 1)^(1/2)*3i)/(2*(3*5^(1/2) - 7)) - (
5^(1/2)*10^(1/2)*x*(5^(1/2) - 1)^(1/2)*7i)/(10*(3*5^(1/2) - 7)))*(5^(1/2)
- 1)^(1/2)*1i)/20 - (10^(1/2)*atan((10^(1/2)*x*(5^(1/2) + 1)^(1/2)*3i)/(2*
(3*5^(1/2) + 7)) + (5^(1/2)*10^(1/2)*x*(5^(1/2) + 1)^(1/2)*7i)/(10*(3*5^(1
/2) + 7)))*(5^(1/2) + 1)^(1/2)*1i)/20 + (10^(1/2)*atan((10^(1/2)*x*(1 - 5^
(1/2))^(1/2)*3i)/(2*(3*5^(1/2) - 7)) - (5^(1/2)*10^(1/2)*x*(1 - 5^(1/2))^(
1/2)*7i)/(10*(3*5^(1/2) - 7)))*(1 - 5^(1/2))^(1/2)*1i)/20 - (10^(1/2)*atan
((10^(1/2)*x*(- 5^(1/2) - 1)^(1/2)*3i)/(2*(3*5^(1/2) + 7)) + (5^(1/2)*10^(
1/2)*x*(- 5^(1/2) - 1)^(1/2)*7i)/(10*(3*5^(1/2) + 7)))*(- 5^(1/2) - 1)^(1/
2)*1i)/20
```

3.400 $\int \frac{1}{1-3x^4+x^8}\,dx$

3.400.1 Optimal result

Integrand size = 12, antiderivative size = 169

$$\int \frac{1}{1-3x^4+x^8}\,dx = -\frac{\arctan\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt[4]{2}\sqrt{5}\left(3+\sqrt{5}\right)^{3/4}}+\frac{\left(3+\sqrt{5}\right)^{3/4}\arctan\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\ 2^{3/4}\sqrt{5}} - \frac{\operatorname{arctanh}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt[4]{2}\sqrt{5}\left(3+\sqrt{5}\right)^{3/4}}+\frac{\left(3+\sqrt{5}\right)^{3/4}\operatorname{arctanh}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\ 2^{3/4}\sqrt{5}}$$

output
```
-1/10*arctan(2^(1/4)*x*(1/(3+5^(1/2)))^(1/4))*2^(3/4)*5^(1/2)/(3+5^(1/2))^
(3/4)-1/10*arctanh(2^(1/4)*x*(1/(3+5^(1/2)))^(1/4))*2^(3/4)*5^(1/2)/(3+5^(
1/2))^(3/4)+1/10*arctan(1/2*x*(3+5^(1/2))^(1/4)*2^(3/4))*(9+4*5^(1/2))^(1/
4)*5^(1/2)+1/10*arctanh(1/2*x*(3+5^(1/2))^(1/4)*2^(3/4))*(9+4*5^(1/2))^(1/
4)*5^(1/2)
```

3.400.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.95

$$\int \frac{1}{1-3x^4+x^8}\,dx = \frac{\frac{\left(1+\sqrt{5}\right)\arctan\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)}{\sqrt{-1+\sqrt{5}}} - \frac{\left(-1+\sqrt{5}\right)\arctan\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{1+\sqrt{5}}} + \frac{\left(1+\sqrt{5}\right)\text{arctanh}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)}{\sqrt{-1+\sqrt{5}}} - \frac{\left(-1+\sqrt{5}\right)\text{arctanh}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{1+\sqrt{5}}}}{2\sqrt{10}}$$

input
```
Integrate[(1 - 3*x^4 + x^8)^(-1),x]
```

output
```
(((1 + Sqrt[5])*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x])/Sqrt[-1 + Sqrt[5]] - ((-
1 + Sqrt[5])*ArcTan[Sqrt[2/(1 + Sqrt[5])]*x])/Sqrt[1 + Sqrt[5]] + ((1 + Sq
rt[5])*ArcTanh[Sqrt[2/(-1 + Sqrt[5])]*x])/Sqrt[-1 + Sqrt[5]] - ((-1 + Sqrt
[5])*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*x])/Sqrt[1 + Sqrt[5]])/(2*Sqrt[10])
```

3.400.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1685, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^8-3x^4+1}\,dx$$

$$\Big\downarrow \text{ 1685}$$

$$\frac{\int \frac{1}{x^4+\frac{1}{2}\left(-3-\sqrt{5}\right)}dx}{\sqrt{5}} - \frac{\int \frac{1}{x^4+\frac{1}{2}\left(-3+\sqrt{5}\right)}dx}{\sqrt{5}}$$

$$\Big\downarrow \text{ 756}$$

$$\frac{-\frac{\int \frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}dx}{\sqrt{3+\sqrt{5}}} - \frac{\int \frac{1}{\sqrt{2}x^2+\sqrt{3+\sqrt{5}}}dx}{\sqrt{3+\sqrt{5}}}}{\sqrt{5}} - \frac{-\frac{\int \frac{1}{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}dx}{\sqrt{3-\sqrt{5}}} - \frac{\int \frac{1}{\sqrt{2}x^2+\sqrt{3-\sqrt{5}}}dx}{\sqrt{3-\sqrt{5}}}}{\sqrt{5}}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{-\frac{\int \frac{1}{\sqrt{3+\sqrt{5}-\sqrt{2}x^2}}dx}{\sqrt{3+\sqrt{5}}}-\frac{\arctan\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt[4]{2}\left(3+\sqrt{5}\right)^{3/4}}}{\sqrt{5}}-\frac{-\frac{\int \frac{1}{\sqrt{3-\sqrt{5}-\sqrt{2}x^2}}dx}{\sqrt{3-\sqrt{5}}}-\frac{\arctan\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{\sqrt[4]{2}\left(3-\sqrt{5}\right)^{3/4}}}{\sqrt{5}}$$

$$\Big\downarrow \text{219}$$

$$\frac{-\frac{\arctan\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt[4]{2}\left(3+\sqrt{5}\right)^{3/4}}-\frac{\text{arctanh}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt[4]{2}\left(3+\sqrt{5}\right)^{3/4}}}{\sqrt{5}}-\frac{-\frac{\arctan\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{\sqrt[4]{2}\left(3-\sqrt{5}\right)^{3/4}}-\frac{\text{arctanh}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{\sqrt[4]{2}\left(3-\sqrt{5}\right)^{3/4}}}{\sqrt{5}}$$

input
```
Int[(1 - 3*x^4 + x^8)^(-1),x]
```

output
```
(-(ArcTan[(2/(3 + Sqrt[5]))^(1/4)*x]/(2^(1/4)*(3 + Sqrt[5])^(3/4))) - ArcT
anh[(2/(3 + Sqrt[5]))^(1/4)*x]/(2^(1/4)*(3 + Sqrt[5])^(3/4)))/Sqrt[5] - (-
(ArcTan[((3 + Sqrt[5])/2)^(1/4)*x]/(2^(1/4)*(3 - Sqrt[5])^(3/4))) - ArcTan
h[((3 + Sqrt[5])/2)^(1/4)*x]/(2^(1/4)*(3 - Sqrt[5])^(3/4)))/Sqrt[5]
```

3.400.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 1685
```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^n), x], x] - Simp[
c/q   Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2,
2*n] && NeQ[b^2 - 4*a*c, 0]
```

3.400.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.40

method	result
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(25\_Z^4+20\_Z^2-1\right)}\_R\ln\left(-15\_R^3-11\_R+2x\right)\right)}{4}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(25\_Z^4-20\_Z^2-1\right)}\_R\ln\left(15\_R^3-11\_R\right.\right.}{4}$
default	$-\frac{\sqrt{5}\left(\sqrt{5}-1\right)\operatorname{arctanh}\left(\frac{2x}{\sqrt{2\sqrt{5}+2}}\right)}{10\sqrt{2\sqrt{5}+2}}+\frac{\left(\sqrt{5}+1\right)\sqrt{5}\arctan\left(\frac{2x}{\sqrt{2\sqrt{5}-2}}\right)}{10\sqrt{2\sqrt{5}-2}}-\frac{\sqrt{5}\left(\sqrt{5}-1\right)\arctan\left(\frac{2x}{\sqrt{2\sqrt{5}+2}}\right)}{10\sqrt{2\sqrt{5}+2}}+\frac{\left(\sqrt{5}+1\right)\sqrt{5}\operatorname{arctan}}{10\sqrt{2\sqrt{5}}}$

input
```
int(1/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(_R*ln(-15*_R^3-11*_R+2*x),_R=RootOf(25*_Z^4+20*_Z^2-1))+1/4*sum(_R
*ln(15*_R^3-11*_R+2*x),_R=RootOf(25*_Z^4-20*_Z^2-1))
```

3.400.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 261 vs. 2(113) = 226.

Time = 0.27 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.54

$$\int \frac{1}{1-3x^4+x^8}\,dx = -\frac{1}{20}\sqrt{5}\sqrt{\sqrt{5}-2}\log\left(\left(\sqrt{5}+3\right)\sqrt{\sqrt{5}-2}+2\,x\right) + \frac{1}{20}\sqrt{5}\sqrt{\sqrt{5}-2}\log\left(-\left(\sqrt{5}+3\right)\sqrt{\sqrt{5}-2}+2\,x\right) - \frac{1}{20}\sqrt{5}\sqrt{\sqrt{5}+2}\log\left(\sqrt{\sqrt{5}+2}\left(\sqrt{5}-3\right)+2\,x\right) + \frac{1}{20}\sqrt{5}\sqrt{\sqrt{5}+2}\log\left(-\sqrt{\sqrt{5}+2}\left(\sqrt{5}-3\right)+2\,x\right) - \frac{1}{20}\sqrt{5}\sqrt{-\sqrt{5}+2}\log\left(\left(\sqrt{5}+3\right)\sqrt{-\sqrt{5}+2}+2\,x\right) + \frac{1}{20}\sqrt{5}\sqrt{-\sqrt{5}+2}\log\left(-\left(\sqrt{5}+3\right)\sqrt{-\sqrt{5}+2}+2\,x\right) - \frac{1}{20}\sqrt{5}\sqrt{-\sqrt{5}-2}\log\left(\left(\sqrt{5}-3\right)\sqrt{-\sqrt{5}-2}+2\,x\right) + \frac{1}{20}\sqrt{5}\sqrt{-\sqrt{5}-2}\log\left(-\left(\sqrt{5}-3\right)\sqrt{-\sqrt{5}-2}+2\,x\right)$$

input
```
integrate(1/(x^8-3*x^4+1),x, algorithm="fricas")
```

output
```
-1/20*sqrt(5)*sqrt(sqrt(5) - 2)*log((sqrt(5) + 3)*sqrt(sqrt(5) - 2) + 2*x)
 + 1/20*sqrt(5)*sqrt(sqrt(5) - 2)*log(-(sqrt(5) + 3)*sqrt(sqrt(5) - 2) + 2
*x) - 1/20*sqrt(5)*sqrt(sqrt(5) + 2)*log(sqrt(sqrt(5) + 2)*(sqrt(5) - 3) +
 2*x) + 1/20*sqrt(5)*sqrt(sqrt(5) + 2)*log(-sqrt(sqrt(5) + 2)*(sqrt(5) - 3
) + 2*x) - 1/20*sqrt(5)*sqrt(-sqrt(5) + 2)*log((sqrt(5) + 3)*sqrt(-sqrt(5)
 + 2) + 2*x) + 1/20*sqrt(5)*sqrt(-sqrt(5) + 2)*log(-(sqrt(5) + 3)*sqrt(-sq
rt(5) + 2) + 2*x) - 1/20*sqrt(5)*sqrt(-sqrt(5) - 2)*log((sqrt(5) - 3)*sqrt
(-sqrt(5) - 2) + 2*x) + 1/20*sqrt(5)*sqrt(-sqrt(5) - 2)*log(-(sqrt(5) - 3)
*sqrt(-sqrt(5) - 2) + 2*x)
```

3.400. $\int \frac{1}{1-3x^4+x^8}\,dx$

3.400.6 Sympy [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.31

$$\int \frac{1}{1-3x^4+x^8}\,dx = \text{RootSum}\left(6400t^4-320t^2-1,\left(t\mapsto t\log\left(9600t^5-\frac{47t}{2}+x\right)\right)\right) + \text{RootSum}\left(6400t^4+320t^2-1,\left(t\mapsto t\log\left(9600t^5-\frac{47t}{2}+x\right)\right)\right)$$

input
```
integrate(1/(x**8-3*x**4+1),x)
```

output
```
RootSum(6400*_t**4 - 320*_t**2 - 1, Lambda(_t, _t*log(9600*_t**5 - 47*_t/2
 + x))) + RootSum(6400*_t**4 + 320*_t**2 - 1, Lambda(_t, _t*log(9600*_t**5
 - 47*_t/2 + x)))
```

3.400.7 Maxima [F]

$$\int \frac{1}{1-3x^4+x^8}\,dx = \int \frac{1}{x^8-3\,x^4+1}\,dx$$

input
```
integrate(1/(x^8-3*x^4+1),x, algorithm="maxima")
```

output
```
integrate(1/(x^8 - 3*x^4 + 1), x)
```

3.400.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.87

$$\int \frac{1}{1-3x^4+x^8}\,dx = -\frac{1}{10}\sqrt{5\sqrt{5}-10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right) + \frac{1}{10}\sqrt{5\sqrt{5}+10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right) - \frac{1}{20}\sqrt{5\sqrt{5}-10}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right) + \frac{1}{20}\sqrt{5\sqrt{5}-10}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right) + \frac{1}{20}\sqrt{5\sqrt{5}+10}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right) - \frac{1}{20}\sqrt{5\sqrt{5}+10}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)$$

input
```
integrate(1/(x^8-3*x^4+1),x, algorithm="giac")
```

output
```
-1/10*sqrt(5*sqrt(5) - 10)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) + 1/10*sqrt(5
*sqrt(5) + 10)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/20*sqrt(5*sqrt(5) - 1
0)*log(abs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/20*sqrt(5*sqrt(5) - 10)*log(a
bs(x - sqrt(1/2*sqrt(5) + 1/2))) + 1/20*sqrt(5*sqrt(5) + 10)*log(abs(x + s
qrt(1/2*sqrt(5) - 1/2))) - 1/20*sqrt(5*sqrt(5) + 10)*log(abs(x - sqrt(1/2*
sqrt(5) - 1/2)))
```

3.400.9 Mupad [B] (verification not implemented)

Time = 8.38 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.45

$$\int \frac{1}{1-3x^4+x^8}\,dx = -\frac{\sqrt{5}\,\mathrm{atan}\left(\frac{x\sqrt{2-\sqrt{5}}\,144\mathrm{i}}{104\sqrt{5}-232}-\frac{\sqrt{5}\,x\sqrt{2-\sqrt{5}}\,64\mathrm{i}}{104\sqrt{5}-232}\right)\sqrt{2-\sqrt{5}}\,1\mathrm{i}}{10} + \frac{\sqrt{5}\,\mathrm{atan}\left(\frac{x\sqrt{-\sqrt{5}-2}\,144\mathrm{i}}{104\sqrt{5}+232}+\frac{\sqrt{5}\,x\sqrt{-\sqrt{5}-2}\,64\mathrm{i}}{104\sqrt{5}+232}\right)\sqrt{-\sqrt{5}-2}\,1\mathrm{i}}{10} + \frac{\sqrt{5}\,\mathrm{atan}\left(\frac{x\sqrt{\sqrt{5}-2}\,144\mathrm{i}}{104\sqrt{5}-232}-\frac{\sqrt{5}\,x\sqrt{\sqrt{5}-2}\,64\mathrm{i}}{104\sqrt{5}-232}\right)\sqrt{\sqrt{5}-2}\,1\mathrm{i}}{10} - \frac{\sqrt{5}\,\mathrm{atan}\left(\frac{x\sqrt{\sqrt{5}+2}\,144\mathrm{i}}{104\sqrt{5}+232}+\frac{\sqrt{5}\,x\sqrt{\sqrt{5}+2}\,64\mathrm{i}}{104\sqrt{5}+232}\right)\sqrt{\sqrt{5}+2}\,1\mathrm{i}}{10}$$

input
```
int(1/(x^8 - 3*x^4 + 1),x)
```

output
```
(5^(1/2)*atan((x*(- 5^(1/2) - 2)^(1/2)*144i)/(104*5^(1/2) + 232) + (5^(1/2
)*x*(- 5^(1/2) - 2)^(1/2)*64i)/(104*5^(1/2) + 232))*(- 5^(1/2) - 2)^(1/2)*
1i)/10 - (5^(1/2)*atan((x*(2 - 5^(1/2))^(1/2)*144i)/(104*5^(1/2) - 232) -
(5^(1/2)*x*(2 - 5^(1/2))^(1/2)*64i)/(104*5^(1/2) - 232))*(2 - 5^(1/2))^(1/
2)*1i)/10 + (5^(1/2)*atan((x*(5^(1/2) - 2)^(1/2)*144i)/(104*5^(1/2) - 232)
 - (5^(1/2)*x*(5^(1/2) - 2)^(1/2)*64i)/(104*5^(1/2) - 232))*(5^(1/2) - 2)^
(1/2)*1i)/10 - (5^(1/2)*atan((x*(5^(1/2) + 2)^(1/2)*144i)/(104*5^(1/2) + 2
32) + (5^(1/2)*x*(5^(1/2) + 2)^(1/2)*64i)/(104*5^(1/2) + 232))*(5^(1/2) +
2)^(1/2)*1i)/10
```

3.401 $\int \frac{1}{x^2(1-3x^4+x^8)}\,dx$

3.401.1 Optimal result

Integrand size = 16, antiderivative size = 172

$$\int \frac{1}{x^2(1-3x^4+x^8)}\,dx=-\frac{1}{x}+\frac{\sqrt[4]{984-440\sqrt{5}}\arctan\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{4\sqrt{5}}-\frac{\left(3+\sqrt{5}\right)^{5/4}\arctan\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{4\sqrt[4]{2}\sqrt{5}}-\frac{\sqrt[4]{984-440\sqrt{5}}\,\text{arctanh}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{4\sqrt{5}}+\frac{\left(3+\sqrt{5}\right)^{5/4}\text{arctanh}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{4\sqrt[4]{2}\sqrt{5}}$$

output
```
-1/x+1/20*arctan(2^(1/4)*x*(1/(3+5^(1/2)))^(1/4))*(984-440*5^(1/2))^(1/4)*
5^(1/2)-1/20*arctanh(2^(1/4)*x*(1/(3+5^(1/2)))^(1/4))*(984-440*5^(1/2))^(1
/4)*5^(1/2)-1/40*arctan(1/2*x*(3+5^(1/2))^(1/4)*2^(3/4))*(3+5^(1/2))^(5/4)
*2^(3/4)*5^(1/2)+1/40*arctanh(1/2*x*(3+5^(1/2))^(1/4)*2^(3/4))*(3+5^(1/2))
^(5/4)*2^(3/4)*5^(1/2)
```

3.401.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.01

$$\int \frac{1}{x^2(1-3x^4+x^8)}\,dx = -\frac{1}{x} - \frac{(3+\sqrt{5})\arctan\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)}{2\sqrt{10(-1+\sqrt{5})}} - \frac{(-3+\sqrt{5})\arctan\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{2\sqrt{10(1+\sqrt{5})}} + \frac{(3+\sqrt{5})\operatorname{arctanh}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)}{2\sqrt{10(-1+\sqrt{5})}} + \frac{(-3+\sqrt{5})\operatorname{arctanh}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{2\sqrt{10(1+\sqrt{5})}}$$

input
```
Integrate[1/(x^2*(1 - 3*x^4 + x^8)),x]
```

output
```
-x^(-1) - ((3 + Sqrt[5])*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x])/(2*Sqrt[10*(-1
+ Sqrt[5])]) - ((-3 + Sqrt[5])*ArcTan[Sqrt[2/(1 + Sqrt[5])]*x])/(2*Sqrt[10
*(1 + Sqrt[5])]) + ((3 + Sqrt[5])*ArcTanh[Sqrt[2/(-1 + Sqrt[5])]*x])/(2*Sq
rt[10*(-1 + Sqrt[5])]) + ((-3 + Sqrt[5])*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*x])
/(2*Sqrt[10*(1 + Sqrt[5])])
```

3.401.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1704, 1834, 27, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(x^8-3x^4+1)}\,dx$$

$$\Big\downarrow \; 1704$$

$$\int \frac{x^2(3-x^4)}{x^8-3x^4+1}dx - \frac{1}{x}$$

$$\Big\downarrow \; 1834$$

$$-\frac{1}{10}\left(5+3\sqrt{5}\right)\int -\frac{2x^2}{-2x^4-\sqrt{5}+3}dx-\frac{1}{10}\left(5-3\sqrt{5}\right)\int -\frac{2x^2}{-2x^4+\sqrt{5}+3}dx-\frac{1}{x}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{5}\left(5+3\sqrt{5}\right)\int \frac{x^2}{-2x^4-\sqrt{5}+3}dx+\frac{1}{5}\left(5-3\sqrt{5}\right)\int \frac{x^2}{-2x^4+\sqrt{5}+3}dx-\frac{1}{x}$$

$$\Big\downarrow \; 827$$

$$\frac{1}{5}\left(5+3\sqrt{5}\right)\left(\frac{\int \frac{1}{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}dx}{2\sqrt{2}}-\frac{\int \frac{1}{\sqrt{2}x^2+\sqrt{3-\sqrt{5}}}dx}{2\sqrt{2}}\right)+\frac{1}{5}\left(5-3\sqrt{5}\right)\left(\frac{\int \frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}dx}{2\sqrt{2}}-\frac{\int \frac{1}{\sqrt{2}x^2+\sqrt{3+\sqrt{5}}}dx}{2\sqrt{2}}\right)-\frac{1}{x}$$

$$\Big\downarrow \; 216$$

$$\frac{1}{5}\left(5+3\sqrt{5}\right)\left(\frac{\int \frac{1}{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}dx}{2\sqrt{2}}-\frac{\arctan\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\ 2^{3/4}\sqrt[4]{3-\sqrt{5}}}\right)+\frac{1}{5}\left(5-3\sqrt{5}\right)\left(\frac{\int \frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}dx}{2\sqrt{2}}-\frac{\arctan\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}\right)-\frac{1}{x}$$

$$\Big\downarrow \; 219$$

$$\frac{1}{5}\left(5-3\sqrt{5}\right)\left(\frac{\text{arctanh}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\arctan\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}\right)+\frac{1}{5}\left(5+3\sqrt{5}\right)\left(\frac{\text{arctanh}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\ 2^{3/4}\sqrt[4]{3-\sqrt{5}}}-\frac{\arctan\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\ 2^{3/4}\sqrt[4]{3-\sqrt{5}}}\right)-\frac{1}{x}$$

input
```
Int[1/(x^2*(1 - 3*x^4 + x^8)),x]
```

3.401. $\int \frac{1}{x^2(1-3x^4+x^8)}\,dx$

output
```
-x^(-1) + ((5 - 3*Sqrt[5])*(-1/2*ArcTan[(2/(3 + Sqrt[5]))^(1/4)*x]/(2^(3/4
)*(3 + Sqrt[5])^(1/4)) + ArcTanh[(2/(3 + Sqrt[5]))^(1/4)*x]/(2*2^(3/4)*(3
+ Sqrt[5])^(1/4))))/5 + ((5 + 3*Sqrt[5])*(-1/2*ArcTan[((3 + Sqrt[5])/2)^(1
/4)*x]/(2^(3/4)*(3 - Sqrt[5])^(1/4)) + ArcTanh[((3 + Sqrt[5])/2)^(1/4)*x]/
(2*2^(3/4)*(3 - Sqrt[5])^(1/4))))/5
```

3.401.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 1704
```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Simp[1/(a*d^n*(m + 1))   Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1)
+ c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{
a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] &&
LtQ[m, -1] && IntegerQ[p]
```

rule 1834
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 +
 (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2
 - (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ
[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n
, 0]
```

3.401.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.42

method	result
risch	$-\frac{1}{x}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(25\_Z^4-55\_Z^2-1\right)}\_R\ln\left(-20\_R^3+47\_R+5x\right)\right)}{4}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(25\_Z^4+55\_Z^2-1\right)}\_R\ln\left(-20\_\right.\right.}{4}$
default	$\frac{\left(\sqrt{5}-3\right)\sqrt{5}\,\operatorname{arctanh}\left(\frac{2x}{\sqrt{2\sqrt{5}+2}}\right)}{10\sqrt{2\sqrt{5}+2}}-\frac{\left(3+\sqrt{5}\right)\sqrt{5}\,\arctan\left(\frac{2x}{\sqrt{2\sqrt{5}-2}}\right)}{10\sqrt{2\sqrt{5}-2}}-\frac{1}{x}-\frac{\left(\sqrt{5}-3\right)\sqrt{5}\,\arctan\left(\frac{2x}{\sqrt{2\sqrt{5}+2}}\right)}{10\sqrt{2\sqrt{5}+2}}+\frac{\left(3+\sqrt{5}\right)\sqrt{5}\,\operatorname{ar}}{10\sqrt{}}$

input
```
int(1/x^2/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/x+1/4*sum(_R*ln(-20*_R^3+47*_R+5*x),_R=RootOf(25*_Z^4-55*_Z^2-1))+1/4*s
um(_R*ln(-20*_R^3-47*_R+5*x),_R=RootOf(25*_Z^4+55*_Z^2-1))
```

3.401.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 327 vs. 2(118) = 236.

Time = 0.28 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.90

$$\int \frac{1}{x^2\left(1-3x^4+x^8\right)}\,dx =$$
$$-\frac{\sqrt{10}x\sqrt{5\sqrt{5}-11}\log\left(\sqrt{10}\sqrt{5\sqrt{5}-11}\left(2\sqrt{5}+5\right)+10\,x\right)-\sqrt{10}x\sqrt{5\sqrt{5}-11}\log\left(-\sqrt{10}\sqrt{5\sqrt{5}-\right.}{}$$

input
```
integrate(1/x^2/(x^8-3*x^4+1),x, algorithm="fricas")
```

output
```
-1/40*(sqrt(10)*x*sqrt(5*sqrt(5) - 11)*log(sqrt(10)*sqrt(5*sqrt(5) - 11)*(
2*sqrt(5) + 5) + 10*x) - sqrt(10)*x*sqrt(5*sqrt(5) - 11)*log(-sqrt(10)*sqr
t(5*sqrt(5) - 11)*(2*sqrt(5) + 5) + 10*x) + sqrt(10)*x*sqrt(5*sqrt(5) + 11
)*log(sqrt(10)*sqrt(5*sqrt(5) + 11)*(2*sqrt(5) - 5) + 10*x) - sqrt(10)*x*s
qrt(5*sqrt(5) + 11)*log(-sqrt(10)*sqrt(5*sqrt(5) + 11)*(2*sqrt(5) - 5) + 1
0*x) - sqrt(10)*x*sqrt(-5*sqrt(5) + 11)*log(sqrt(10)*(2*sqrt(5) + 5)*sqrt(
-5*sqrt(5) + 11) + 10*x) + sqrt(10)*x*sqrt(-5*sqrt(5) + 11)*log(-sqrt(10)*
(2*sqrt(5) + 5)*sqrt(-5*sqrt(5) + 11) + 10*x) - sqrt(10)*x*sqrt(-5*sqrt(5)
 - 11)*log(sqrt(10)*(2*sqrt(5) - 5)*sqrt(-5*sqrt(5) - 11) + 10*x) + sqrt(1
0)*x*sqrt(-5*sqrt(5) - 11)*log(-sqrt(10)*(2*sqrt(5) - 5)*sqrt(-5*sqrt(5) -
 11) + 10*x) + 40)/x
```

3.401.6 Sympy [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.37

$$\begin{aligned}\int \frac{1}{x^2(1-3x^4+x^8)}\,dx &= \operatorname{RootSum}\left(6400t^4-880t^2-1,\left(t\mapsto t\log\left(\frac{19251200t^7}{11}-\frac{369792t^3}{11}+x\right)\right)\right)\\ &\quad+\operatorname{RootSum}\left(6400t^4+880t^2-1,\left(t\mapsto t\log\left(\frac{19251200t^7}{11}-\frac{369792t^3}{11}+x\right)\right)\right)-\frac{1}{x}\end{aligned}$$

input
```
integrate(1/x**2/(x**8-3*x**4+1),x)
```

output
```
RootSum(6400*_t**4 - 880*_t**2 - 1, Lambda(_t, _t*log(19251200*_t**7/11 -
369792*_t**3/11 + x))) + RootSum(6400*_t**4 + 880*_t**2 - 1, Lambda(_t, _t
*log(19251200*_t**7/11 - 369792*_t**3/11 + x))) - 1/x
```

3.401.7 Maxima [F]

$$\int \frac{1}{x^2(1-3x^4+x^8)}\,dx = \int \frac{1}{(x^8-3\,x^4+1)x^2}\,dx$$

input
```
integrate(1/x^2/(x^8-3*x^4+1),x, algorithm="maxima")
```

output
```
-1/x - 1/2*integrate((x^2 + 2)/(x^4 + x^2 - 1), x) - 1/2*integrate((x^2 -
2)/(x^4 - x^2 - 1), x)
```

3.401.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^2(1-3x^4+x^8)}\,dx = \frac{1}{20}\sqrt{50\sqrt{5}-110}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right) - \frac{1}{20}\sqrt{50\sqrt{5}+110}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right) - \frac{1}{40}\sqrt{50\sqrt{5}-110}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right) + \frac{1}{40}\sqrt{50\sqrt{5}-110}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right) + \frac{1}{40}\sqrt{50\sqrt{5}+110}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right) - \frac{1}{40}\sqrt{50\sqrt{5}+110}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right) - \frac{1}{x}$$

input
```
integrate(1/x^2/(x^8-3*x^4+1),x, algorithm="giac")
```

output
```
1/20*sqrt(50*sqrt(5) - 110)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) - 1/20*sqrt(
50*sqrt(5) + 110)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/40*sqrt(50*sqrt(5)
 - 110)*log(abs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(50*sqrt(5) - 110
)*log(abs(x - sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(50*sqrt(5) + 110)*log(
abs(x + sqrt(1/2*sqrt(5) - 1/2))) - 1/40*sqrt(50*sqrt(5) + 110)*log(abs(x
- sqrt(1/2*sqrt(5) - 1/2))) - 1/x
```

3.401.9 Mupad [B] (verification not implemented)

Time = 8.31 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.45

$$\int \frac{1}{x^2(1-3x^4+x^8)}\,dx$$
$$= -\frac{1}{x} - \frac{\operatorname{atan}\left(\frac{x\sqrt{-50\sqrt{5}-110}\,1155\mathrm{i}}{2\left(3025\sqrt{5}+6765\right)} + \frac{\sqrt{5}\,x\sqrt{-50\sqrt{5}-110}\,517\mathrm{i}}{2\left(3025\sqrt{5}+6765\right)}\right)\sqrt{-50\sqrt{5}-110}\,1\mathrm{i}}{20}$$
$$+ \frac{\operatorname{atan}\left(\frac{x\sqrt{110-50\sqrt{5}}\,1155\mathrm{i}}{2\left(3025\sqrt{5}-6765\right)} - \frac{\sqrt{5}\,x\sqrt{110-50\sqrt{5}}\,517\mathrm{i}}{2\left(3025\sqrt{5}-6765\right)}\right)\sqrt{110-50\sqrt{5}}\,1\mathrm{i}}{20}$$
$$+ \frac{\operatorname{atan}\left(\frac{x\sqrt{50\sqrt{5}-110}\,1155\mathrm{i}}{2\left(3025\sqrt{5}-6765\right)} - \frac{\sqrt{5}\,x\sqrt{50\sqrt{5}-110}\,517\mathrm{i}}{2\left(3025\sqrt{5}-6765\right)}\right)\sqrt{50\sqrt{5}-110}\,1\mathrm{i}}{20}$$
$$- \frac{\operatorname{atan}\left(\frac{x\sqrt{50\sqrt{5}+110}\,1155\mathrm{i}}{2\left(3025\sqrt{5}+6765\right)} + \frac{\sqrt{5}\,x\sqrt{50\sqrt{5}+110}\,517\mathrm{i}}{2\left(3025\sqrt{5}+6765\right)}\right)\sqrt{50\sqrt{5}+110}\,1\mathrm{i}}{20}$$

input
```
int(1/(x^2*(x^8 - 3*x^4 + 1)),x)
```

output
```
(atan((x*(110 - 50*5^(1/2))^(1/2)*1155i)/(2*(3025*5^(1/2) - 6765)) - (5^(1
/2)*x*(110 - 50*5^(1/2))^(1/2)*517i)/(2*(3025*5^(1/2) - 6765)))*(110 - 50*
5^(1/2))^(1/2)*1i)/20 - (atan((x*(- 50*5^(1/2) - 110)^(1/2)*1155i)/(2*(302
5*5^(1/2) + 6765)) + (5^(1/2)*x*(- 50*5^(1/2) - 110)^(1/2)*517i)/(2*(3025*
5^(1/2) + 6765)))*(- 50*5^(1/2) - 110)^(1/2)*1i)/20 + (atan((x*(50*5^(1/2)
 - 110)^(1/2)*1155i)/(2*(3025*5^(1/2) - 6765)) - (5^(1/2)*x*(50*5^(1/2) -
110)^(1/2)*517i)/(2*(3025*5^(1/2) - 6765)))*(50*5^(1/2) - 110)^(1/2)*1i)/2
0 - (atan((x*(50*5^(1/2) + 110)^(1/2)*1155i)/(2*(3025*5^(1/2) + 6765)) + (
5^(1/2)*x*(50*5^(1/2) + 110)^(1/2)*517i)/(2*(3025*5^(1/2) + 6765)))*(50*5^
(1/2) + 110)^(1/2)*1i)/20 - 1/x
```

3.402 $\int \frac{1}{x^4(1-3x^4+x^8)}\,dx$

3.402.1 Optimal result

Integrand size = 16, antiderivative size = 182

$$\begin{aligned}\int \frac{1}{x^4\left(1-3x^4+x^8\right)}\,dx = &-\frac{1}{3x^3}-\frac{\sqrt[4]{\frac{1}{2}\left(843-377\sqrt{5}\right)}\arctan\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}}\\&+\frac{\left(3+\sqrt{5}\right)^{7/4}\arctan\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{4\,2^{3/4}\sqrt{5}}\\&-\frac{\sqrt[4]{\frac{1}{2}\left(843-377\sqrt{5}\right)}\operatorname{arctanh}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}}\\&+\frac{\left(3+\sqrt{5}\right)^{7/4}\operatorname{arctanh}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{4\,2^{3/4}\sqrt{5}}\end{aligned}$$

output
```
-1/3/x^3-1/20*arctan(2^(1/4)*x*(1/(3+5^(1/2)))^(1/4))*(843-377*5^(1/2))^(1
/4)*2^(3/4)*5^(1/2)-1/20*arctanh(2^(1/4)*x*(1/(3+5^(1/2)))^(1/4))*(843-377
*5^(1/2))^(1/4)*2^(3/4)*5^(1/2)+1/20*arctan(1/2*x*(3+5^(1/2))^(1/4)*2^(3/4
))*(843+377*5^(1/2))^(1/4)*2^(3/4)*5^(1/2)+1/20*arctanh(1/2*x*(3+5^(1/2))^
(1/4)*2^(3/4))*(843+377*5^(1/2))^(1/4)*2^(3/4)*5^(1/2)
```

3.402.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^4 (1-3x^4+x^8)}\,dx = -\frac{1}{3x^3}+\frac{\left(2+\sqrt{5}\right)\arctan\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)}{\sqrt{10\left(-1+\sqrt{5}\right)}} - \frac{\left(-2+\sqrt{5}\right)\arctan\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}} + \frac{\left(2+\sqrt{5}\right)\operatorname{arctanh}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)}{\sqrt{10\left(-1+\sqrt{5}\right)}} - \frac{\left(-2+\sqrt{5}\right)\operatorname{arctanh}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}}$$

input
```
Integrate[1/(x^4*(1 - 3*x^4 + x^8)),x]
```

output
```
-1/3*1/x^3 + ((2 + Sqrt[5])*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x])/Sqrt[10*(-1
+ Sqrt[5])] - ((-2 + Sqrt[5])*ArcTan[Sqrt[2/(1 + Sqrt[5])]*x])/Sqrt[10*(1
+ Sqrt[5])] + ((2 + Sqrt[5])*ArcTanh[Sqrt[2/(-1 + Sqrt[5])]*x])/Sqrt[10*(-
1 + Sqrt[5])] - ((-2 + Sqrt[5])*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*x])/Sqrt[10*
(1 + Sqrt[5])]
```

3.402.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1704, 27, 1752, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (x^8-3x^4+1)}\,dx$$

$$\Big\downarrow \; 1704$$

$$\frac{1}{3}\int \frac{3(3-x^4)}{x^8-3x^4+1}dx-\frac{1}{3x^3}$$

$$\Big\downarrow \; 27$$

$$\int \frac{3-x^4}{x^8-3x^4+1}dx-\frac{1}{3x^3}$$

$$\Big\downarrow \text{1752}$$

$$-\frac{1}{10}\left(5-3\sqrt{5}\right)\int \frac{1}{x^4+\frac{1}{2}\left(-3-\sqrt{5}\right)}dx-\frac{1}{10}\left(5+3\sqrt{5}\right)\int \frac{1}{x^4+\frac{1}{2}\left(-3+\sqrt{5}\right)}dx-\frac{1}{3x^3}$$

$$\Big\downarrow \text{756}$$

$$-\frac{1}{10}\left(5+3\sqrt{5}\right)\left(-\frac{\int \frac{1}{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}dx}{\sqrt{3-\sqrt{5}}}-\frac{\int \frac{1}{\sqrt{2}x^2+\sqrt{3-\sqrt{5}}}dx}{\sqrt{3-\sqrt{5}}}\right)-\frac{1}{10}\left(5-3\sqrt{5}\right)\left(-\frac{\int \frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}dx}{\sqrt{3+\sqrt{5}}}-\frac{\int \frac{1}{\sqrt{2}x^2+\sqrt{3+\sqrt{5}}}dx}{\sqrt{3+\sqrt{5}}}\right)-\frac{1}{3x^3}$$

$$\Big\downarrow \text{216}$$

$$-\frac{1}{10}\left(5+3\sqrt{5}\right)\left(-\frac{\int \frac{1}{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}dx}{\sqrt{3-\sqrt{5}}}-\frac{\arctan\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{\sqrt[4]{2}\left(3-\sqrt{5}\right)^{3/4}}\right)-\frac{1}{10}\left(5-3\sqrt{5}\right)\left(-\frac{\int \frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}dx}{\sqrt{3+\sqrt{5}}}-\frac{\arctan\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt[4]{2}\left(3+\sqrt{5}\right)^{3/4}}\right)-\frac{1}{3x^3}$$

$$\Big\downarrow \text{219}$$

$$-\frac{1}{10}\left(5-3\sqrt{5}\right)\left(-\frac{\arctan\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt[4]{2}\left(3+\sqrt{5}\right)^{3/4}}-\frac{\text{arctanh}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt[4]{2}\left(3+\sqrt{5}\right)^{3/4}}\right)-\frac{1}{10}\left(5+3\sqrt{5}\right)\left(-\frac{\arctan\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{\sqrt[4]{2}\left(3-\sqrt{5}\right)^{3/4}}-\frac{\text{arctanh}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{\sqrt[4]{2}\left(3-\sqrt{5}\right)^{3/4}}\right)-\frac{1}{3x^3}$$

input
```
Int[1/(x^4*(1 - 3*x^4 + x^8)),x]
```

3.402. $\int \frac{1}{x^4(1-3x^4+x^8)}\,dx$

output
```
-1/3*1/x^3 - ((5 - 3*Sqrt[5])*(-(ArcTan[(2/(3 + Sqrt[5]))^(1/4)*x]/(2^(1/4
)*(3 + Sqrt[5])^(3/4))) - ArcTanh[(2/(3 + Sqrt[5]))^(1/4)*x]/(2^(1/4)*(3 +
 Sqrt[5])^(3/4))))/10 - ((5 + 3*Sqrt[5])*(-(ArcTan[((3 + Sqrt[5])/2)^(1/4)
*x]/(2^(1/4)*(3 - Sqrt[5])^(3/4))) - ArcTanh[((3 + Sqrt[5])/2)^(1/4)*x]/(2
^(1/4)*(3 - Sqrt[5])^(3/4))))/10
```

3.402.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 1704
```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Simp[1/(a*d^n*(m + 1))   Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1)
+ c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{
a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] &&
LtQ[m, -1] && IntegerQ[p]
```

rule 1752
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

3.402.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.40

method	result
risch	$-\frac{1}{3x^3}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(25\_Z^4+145\_Z^2-1\right)}\_R\ln\left(-35\_R^3-199\_R+13x\right)\right)}{4}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(25\_Z^4-145\_Z^2-1\right)}\_R\ln\left(3\right.\right.}{4}$
default	$-\frac{\left(\sqrt{5}-2\right)\sqrt{5}\,\operatorname{arctanh}\left(\frac{2x}{\sqrt{2\sqrt{5}+2}}\right)}{5\sqrt{2\sqrt{5}+2}}+\frac{\left(2+\sqrt{5}\right)\sqrt{5}\,\arctan\left(\frac{2x}{\sqrt{2\sqrt{5}-2}}\right)}{5\sqrt{2\sqrt{5}-2}}-\frac{\left(\sqrt{5}-2\right)\sqrt{5}\,\arctan\left(\frac{2x}{\sqrt{2\sqrt{5}+2}}\right)}{5\sqrt{2\sqrt{5}+2}}+\frac{\left(2+\sqrt{5}\right)\sqrt{5}\,\operatorname{arctar}}{5\sqrt{2\sqrt{5}}}$

input
```
int(1/x^4/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/3/x^3+1/4*sum(_R*ln(-35*_R^3-199*_R+13*x),_R=RootOf(25*_Z^4+145*_Z^2-1)
)+1/4*sum(_R*ln(35*_R^3-199*_R+13*x),_R=RootOf(25*_Z^4-145*_Z^2-1))
```

3.402.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 347 vs. 2(128) = 256.

Time = 0.28 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.91

$$\int \frac{1}{x^4\left(1-3x^4+x^8\right)}\,dx = -\frac{3\sqrt{10}x^3\sqrt{13\sqrt{5}-29}\log\left(\sqrt{10}\sqrt{13\sqrt{5}-29}\left(7\sqrt{5}+15\right)+20\,x\right)-3\sqrt{10}x^3\sqrt{13\sqrt{5}-29}\log\left(-\sqrt{10}\right.}{}$$

input
```
integrate(1/x^4/(x^8-3*x^4+1),x, algorithm="fricas")
```

output
```
-1/120*(3*sqrt(10)*x^3*sqrt(13*sqrt(5) - 29)*log(sqrt(10)*sqrt(13*sqrt(5)
- 29)*(7*sqrt(5) + 15) + 20*x) - 3*sqrt(10)*x^3*sqrt(13*sqrt(5) - 29)*log(
-sqrt(10)*sqrt(13*sqrt(5) - 29)*(7*sqrt(5) + 15) + 20*x) - 3*sqrt(10)*x^3*
sqrt(13*sqrt(5) + 29)*log(sqrt(10)*sqrt(13*sqrt(5) + 29)*(7*sqrt(5) - 15)
+ 20*x) + 3*sqrt(10)*x^3*sqrt(13*sqrt(5) + 29)*log(-sqrt(10)*sqrt(13*sqrt(
5) + 29)*(7*sqrt(5) - 15) + 20*x) + 3*sqrt(10)*x^3*sqrt(-13*sqrt(5) + 29)*
log(sqrt(10)*(7*sqrt(5) + 15)*sqrt(-13*sqrt(5) + 29) + 20*x) - 3*sqrt(10)*
x^3*sqrt(-13*sqrt(5) + 29)*log(-sqrt(10)*(7*sqrt(5) + 15)*sqrt(-13*sqrt(5)
 + 29) + 20*x) - 3*sqrt(10)*x^3*sqrt(-13*sqrt(5) - 29)*log(sqrt(10)*(7*sqr
t(5) - 15)*sqrt(-13*sqrt(5) - 29) + 20*x) + 3*sqrt(10)*x^3*sqrt(-13*sqrt(5
) - 29)*log(-sqrt(10)*(7*sqrt(5) - 15)*sqrt(-13*sqrt(5) - 29) + 20*x) + 40
)/x^3
```

3.402.6 Sympy [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.35

$$\int \frac{1}{x^4(1-3x^4+x^8)}\,dx = \operatorname{RootSum}\left(6400t^4-2320t^2-1,\left(t\mapsto t\log\left(\frac{179200t^5}{377}-\frac{23112t}{377}+x\right)\right)\right) + \operatorname{RootSum}\left(6400t^4+2320t^2-1,\left(t\mapsto t\log\left(\frac{179200t^5}{377}-\frac{23112t}{377}+x\right)\right)\right) - \frac{1}{3x^3}$$

input
```
integrate(1/x**4/(x**8-3*x**4+1),x)
```

output
```
RootSum(6400*_t**4 - 2320*_t**2 - 1, Lambda(_t, _t*log(179200*_t**5/377 -
23112*_t/377 + x))) + RootSum(6400*_t**4 + 2320*_t**2 - 1, Lambda(_t, _t*l
og(179200*_t**5/377 - 23112*_t/377 + x))) - 1/(3*x**3)
```

3.402.7 Maxima [F]

$$\int \frac{1}{x^4(1-3x^4+x^8)}\,dx = \int \frac{1}{(x^8-3\,x^4+1)x^4}\,dx$$

input
```
integrate(1/x^4/(x^8-3*x^4+1),x, algorithm="maxima")
```

output
```
-1/3/x^3 - 1/2*integrate((2*x^2 + 3)/(x^4 + x^2 - 1), x) + 1/2*integrate((
2*x^2 - 3)/(x^4 - x^2 - 1), x)
```

3.402.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^4(1-3x^4+x^8)}\,dx = -\frac{1}{20}\sqrt{130\sqrt{5}-290}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right) + \frac{1}{20}\sqrt{130\sqrt{5}+290}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right) - \frac{1}{40}\sqrt{130\sqrt{5}-290}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right) + \frac{1}{40}\sqrt{130\sqrt{5}-290}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right) + \frac{1}{40}\sqrt{130\sqrt{5}+290}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right) - \frac{1}{40}\sqrt{130\sqrt{5}+290}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right) - \frac{1}{3\,x^3}$$

input
```
integrate(1/x^4/(x^8-3*x^4+1),x, algorithm="giac")
```

output
```
-1/20*sqrt(130*sqrt(5) - 290)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) + 1/20*sqr
t(130*sqrt(5) + 290)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/40*sqrt(130*sqr
t(5) - 290)*log(abs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(130*sqrt(5)
- 290)*log(abs(x - sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(130*sqrt(5) + 290
)*log(abs(x + sqrt(1/2*sqrt(5) - 1/2))) - 1/40*sqrt(130*sqrt(5) + 290)*log
(abs(x - sqrt(1/2*sqrt(5) - 1/2))) - 1/3/x^3
```

3.402.9 Mupad [B] (verification not implemented)

Time = 8.53 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.47

$$\int \frac{1}{x^4(1-3x^4+x^8)}\,dx$$
$$= \frac{\operatorname{atan}\left(\frac{x\sqrt{-130\sqrt{5}-290}\,20735\mathrm{i}}{2\left(87841\sqrt{5}+196417\right)}+\frac{\sqrt{5}\,x\sqrt{-130\sqrt{5}-290}\,46371\mathrm{i}}{10\left(87841\sqrt{5}+196417\right)}\right)\sqrt{-130\sqrt{5}-290}\,1\mathrm{i}}{20}$$
$$+\frac{\operatorname{atan}\left(\frac{x\sqrt{290-130\sqrt{5}}\,20735\mathrm{i}}{2\left(87841\sqrt{5}-196417\right)}-\frac{\sqrt{5}\,x\sqrt{290-130\sqrt{5}}\,46371\mathrm{i}}{10\left(87841\sqrt{5}-196417\right)}\right)\sqrt{290-130\sqrt{5}}\,1\mathrm{i}}{20}-\frac{1}{3x^3}$$
$$-\frac{\sqrt{10}\operatorname{atan}\left(\frac{\sqrt{10}\,x\sqrt{13\sqrt{5}-29}\,20735\mathrm{i}}{2\left(87841\sqrt{5}-196417\right)}-\frac{\sqrt{5}\sqrt{10}\,x\sqrt{13\sqrt{5}-29}\,46371\mathrm{i}}{10\left(87841\sqrt{5}-196417\right)}\right)\sqrt{13\sqrt{5}-29}\,1\mathrm{i}}{20}$$
$$-\frac{\sqrt{10}\operatorname{atan}\left(\frac{\sqrt{10}\,x\sqrt{13\sqrt{5}+29}\,20735\mathrm{i}}{2\left(87841\sqrt{5}+196417\right)}+\frac{\sqrt{5}\sqrt{10}\,x\sqrt{13\sqrt{5}+29}\,46371\mathrm{i}}{10\left(87841\sqrt{5}+196417\right)}\right)\sqrt{13\sqrt{5}+29}\,1\mathrm{i}}{20}$$

input
```
int(1/(x^4*(x^8 - 3*x^4 + 1)),x)
```

output
```
(atan((x*(- 130*5^(1/2) - 290)^(1/2)*20735i)/(2*(87841*5^(1/2) + 196417))
+ (5^(1/2)*x*(- 130*5^(1/2) - 290)^(1/2)*46371i)/(10*(87841*5^(1/2) + 1964
17)))*(- 130*5^(1/2) - 290)^(1/2)*1i)/20 + (atan((x*(290 - 130*5^(1/2))^(1
/2)*20735i)/(2*(87841*5^(1/2) - 196417)) - (5^(1/2)*x*(290 - 130*5^(1/2))^
(1/2)*46371i)/(10*(87841*5^(1/2) - 196417)))*(290 - 130*5^(1/2))^(1/2)*1i)
/20 - 1/(3*x^3) - (10^(1/2)*atan((10^(1/2)*x*(13*5^(1/2) - 29)^(1/2)*20735
i)/(2*(87841*5^(1/2) - 196417)) - (5^(1/2)*10^(1/2)*x*(13*5^(1/2) - 29)^(1
/2)*46371i)/(10*(87841*5^(1/2) - 196417)))*(13*5^(1/2) - 29)^(1/2)*1i)/20
- (10^(1/2)*atan((10^(1/2)*x*(13*5^(1/2) + 29)^(1/2)*20735i)/(2*(87841*5^(
1/2) + 196417)) + (5^(1/2)*10^(1/2)*x*(13*5^(1/2) + 29)^(1/2)*46371i)/(10*
(87841*5^(1/2) + 196417)))*(13*5^(1/2) + 29)^(1/2)*1i)/20
```

3.403 $\int \frac{1}{x^6(1-3x^4+x^8)} dx$

3.403.1 Optimal result

Integrand size = 16, antiderivative size = 173

$$\int \frac{1}{x^6(1-3x^4+x^8)} dx = -\frac{1}{5x^5} - \frac{3}{x} + \frac{\sqrt[4]{2889-1292\sqrt{5}}\arctan\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}} - \frac{\sqrt[4]{2889+1292\sqrt{5}}\arctan\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{5}} - \frac{\sqrt[4]{2889-1292\sqrt{5}}\text{arctanh}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}} + \frac{\sqrt[4]{2889+1292\sqrt{5}}\text{arctanh}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{5}}$$

output
```
-1/5/x^5-3/x+1/10*arctan(2^(1/4)*x*(1/(3+5^(1/2)))^(1/4))*(2889-1292*5^(1/
2))^(1/4)*5^(1/2)-1/10*arctanh(2^(1/4)*x*(1/(3+5^(1/2)))^(1/4))*(2889-1292
*5^(1/2))^(1/4)*5^(1/2)-1/10*arctan(1/2*x*(3+5^(1/2))^(1/4)*2^(3/4))*(2889
+1292*5^(1/2))^(1/4)*5^(1/2)+1/10*arctanh(1/2*x*(3+5^(1/2))^(1/4)*2^(3/4))
*(2889+1292*5^(1/2))^(1/4)*5^(1/2)
```

3.403.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^6(1-3x^4+x^8)}\,dx = -\frac{1}{5x^5}-\frac{3}{x}+\frac{\left(-7-3\sqrt{5}\right)\arctan\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)}{2\sqrt{10\left(-1+\sqrt{5}\right)}} + \frac{\left(7-3\sqrt{5}\right)\arctan\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{2\sqrt{10\left(1+\sqrt{5}\right)}} - \frac{\left(-7-3\sqrt{5}\right)\text{arctanh}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)}{2\sqrt{10\left(-1+\sqrt{5}\right)}} - \frac{\left(7-3\sqrt{5}\right)\text{arctanh}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{2\sqrt{10\left(1+\sqrt{5}\right)}}$$

input
```
Integrate[1/(x^6*(1 - 3*x^4 + x^8)),x]
```

output
```
-1/5*1/x^5 - 3/x + ((-7 - 3*Sqrt[5])*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x])/(2*
Sqrt[10*(-1 + Sqrt[5])]) + ((7 - 3*Sqrt[5])*ArcTan[Sqrt[2/(1 + Sqrt[5])]*x
])/(2*Sqrt[10*(1 + Sqrt[5])]) - ((-7 - 3*Sqrt[5])*ArcTanh[Sqrt[2/(-1 + Sqr
t[5])]*x])/(2*Sqrt[10*(-1 + Sqrt[5])]) - ((7 - 3*Sqrt[5])*ArcTanh[Sqrt[2/(
1 + Sqrt[5])]*x])/(2*Sqrt[10*(1 + Sqrt[5])])
```

3.403.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {1704, 27, 1828, 25, 1834, 27, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6(x^8-3x^4+1)}dx$$

$$\Big\downarrow \text{1704}$$

$$\frac{1}{5}\int \frac{5(3-x^4)}{x^2(x^8-3x^4+1)}dx-\frac{1}{5x^5}$$

$$\Big\downarrow \ 27$$

$$\int \frac{3-x^4}{x^2\left(x^8-3x^4+1\right)}dx-\frac{1}{5x^5}$$

$$\Big\downarrow \ 1828$$

$$-\int -\frac{x^2\left(8-3x^4\right)}{x^8-3x^4+1}dx-\frac{1}{5x^5}-\frac{3}{x}$$

$$\Big\downarrow \ 25$$

$$\int \frac{x^2\left(8-3x^4\right)}{x^8-3x^4+1}dx-\frac{1}{5x^5}-\frac{3}{x}$$

$$\Big\downarrow \ 1834$$

$$-\frac{1}{10}\left(15+7\sqrt{5}\right)\int -\frac{2x^2}{-2x^4-\sqrt{5}+3}dx-\frac{1}{10}\left(15-7\sqrt{5}\right)\int -\frac{2x^2}{-2x^4+\sqrt{5}+3}dx-\frac{1}{5x^5}-\frac{3}{x}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{5}\left(15+7\sqrt{5}\right)\int \frac{x^2}{-2x^4-\sqrt{5}+3}dx+\frac{1}{5}\left(15-7\sqrt{5}\right)\int \frac{x^2}{-2x^4+\sqrt{5}+3}dx-\frac{1}{5x^5}-\frac{3}{x}$$

$$\Big\downarrow \ 827$$

$$\frac{1}{5}\left(15+7\sqrt{5}\right)\left(\frac{\int \frac{1}{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}dx}{2\sqrt{2}}-\frac{\int \frac{1}{\sqrt{2}x^2+\sqrt{3-\sqrt{5}}}dx}{2\sqrt{2}}\right)+\frac{1}{5}\left(15-7\sqrt{5}\right)\left(\frac{\int \frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}dx}{2\sqrt{2}}-\frac{\int \frac{1}{\sqrt{2}x^2+\sqrt{3+\sqrt{5}}}dx}{2\sqrt{2}}\right)-\frac{1}{5x^5}-\frac{3}{x}$$

$$\Big\downarrow \ 216$$

$$\frac{1}{5}\left(15+7\sqrt{5}\right)\left(\frac{\int \frac{1}{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}dx}{2\sqrt{2}}-\frac{\arctan\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\ 2^{3/4}\sqrt[4]{3-\sqrt{5}}}\right)+\frac{1}{5}\left(15-7\sqrt{5}\right)\left(\frac{\int \frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}dx}{2\sqrt{2}}-\frac{\arctan\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}\right)-\frac{1}{5x^5}-\frac{3}{x}$$

$$\Big\downarrow \ 219$$

3.403. $\int \frac{1}{x^6(1-3x^4+x^8)}\,dx$

$$\frac{1}{5}\left(15-7\sqrt{5}\right)\left(\frac{\operatorname{arctanh}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\,2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\arctan\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\,2^{3/4}\sqrt[4]{3+\sqrt{5}}}\right)+\frac{1}{5}\left(15+7\sqrt{5}\right)\left(\frac{\operatorname{arctanh}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\,2^{3/4}\sqrt[4]{3-\sqrt{5}}}-\frac{\arctan\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\,2^{3/4}\sqrt[4]{3-\sqrt{5}}}\right)-\frac{1}{5x^5}-\frac{3}{x}$$

input
```
Int[1/(x^6*(1 - 3*x^4 + x^8)),x]
```

output
```
-1/5*1/x^5 - 3/x + ((15 - 7*Sqrt[5])*(-1/2*ArcTan[(2/(3 + Sqrt[5]))^(1/4)*
x]/(2^(3/4)*(3 + Sqrt[5])^(1/4)) + ArcTanh[(2/(3 + Sqrt[5]))^(1/4)*x]/(2*2
^(3/4)*(3 + Sqrt[5])^(1/4))))/5 + ((15 + 7*Sqrt[5])*(-1/2*ArcTan[((3 + Sqr
t[5])/2)^(1/4)*x]/(2^(3/4)*(3 - Sqrt[5])^(1/4)) + ArcTanh[((3 + Sqrt[5])/2
)^(1/4)*x]/(2*2^(3/4)*(3 - Sqrt[5])^(1/4))))/5
```

3.403.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 1704
```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Simp[1/(a*d^n*(m + 1))   Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1)
+ c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{
a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] &&
LtQ[m, -1] && IntegerQ[p]
```

rule 1828
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Simp[1/(a*f^n*(m + 1))   Int[(f*x)^(m +
 n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) -
c*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x
] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Int
egerQ[p]
```

rule 1834
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 +
 (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2
 - (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ
[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n
, 0]
```

3.403.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.46

method	result
risch	$\frac{-3x^4-\frac{1}{5}}{x^5}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(25\_Z^4+380\_Z^2-1\right)}\_R\ln\left(-55\_R^3-843\_R+34x\right)\right)}{4}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(25\_Z^4-380\_Z^2-1\right)}\_R\ln\right.}{4}$
default	$-\frac{1}{5x^5}-\frac{3}{x}+\frac{\left(-7+3\sqrt{5}\right)\sqrt{5}\,\mathrm{arctanh}\left(\frac{2x}{\sqrt{2\sqrt{5}+2}}\right)}{10\sqrt{2\sqrt{5}+2}}-\frac{\left(7+3\sqrt{5}\right)\sqrt{5}\,\arctan\left(\frac{2x}{\sqrt{2\sqrt{5}-2}}\right)}{10\sqrt{2\sqrt{5}-2}}-\frac{\left(-7+3\sqrt{5}\right)\sqrt{5}\,\arctan\left(\frac{2x}{\sqrt{2\sqrt{5}+2}}\right)}{10\sqrt{2\sqrt{5}+2}}$

input
```
int(1/x^6/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
(-3*x^4-1/5)/x^5+1/4*sum(_R*ln(-55*_R^3-843*_R+34*x),_R=RootOf(25*_Z^4+380
*_Z^2-1))+1/4*sum(_R*ln(-55*_R^3+843*_R+34*x),_R=RootOf(25*_Z^4-380*_Z^2-1
))
```

3.403.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 324 vs. $2(121) = 242$.

Time = 0.26 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.87

$$\int \frac{1}{x^6(1-3x^4+x^8)}\,dx = -\frac{\sqrt{5}x^5\sqrt{17\sqrt{5}-38}\log\left(\sqrt{17\sqrt{5}-38}(5\sqrt{5}+11)+2x\right)-\sqrt{5}x^5\sqrt{17\sqrt{5}-38}\log\left(-\sqrt{17\sqrt{5}-38}(5\right.}{}$$

input
```
integrate(1/x^6/(x^8-3*x^4+1),x, algorithm="fricas")
```

output
```
-1/20*(sqrt(5)*x^5*sqrt(17*sqrt(5) - 38)*log(sqrt(17*sqrt(5) - 38)*(5*sqrt
(5) + 11) + 2*x) - sqrt(5)*x^5*sqrt(17*sqrt(5) - 38)*log(-sqrt(17*sqrt(5)
- 38)*(5*sqrt(5) + 11) + 2*x) - sqrt(5)*x^5*sqrt(17*sqrt(5) + 38)*log(sqrt
(17*sqrt(5) + 38)*(5*sqrt(5) - 11) + 2*x) + sqrt(5)*x^5*sqrt(17*sqrt(5) +
38)*log(-sqrt(17*sqrt(5) + 38)*(5*sqrt(5) - 11) + 2*x) - sqrt(5)*x^5*sqrt(
-17*sqrt(5) + 38)*log((5*sqrt(5) + 11)*sqrt(-17*sqrt(5) + 38) + 2*x) + sqr
t(5)*x^5*sqrt(-17*sqrt(5) + 38)*log(-(5*sqrt(5) + 11)*sqrt(-17*sqrt(5) + 3
8) + 2*x) + sqrt(5)*x^5*sqrt(-17*sqrt(5) - 38)*log((5*sqrt(5) - 11)*sqrt(-
17*sqrt(5) - 38) + 2*x) - sqrt(5)*x^5*sqrt(-17*sqrt(5) - 38)*log(-(5*sqrt(
5) - 11)*sqrt(-17*sqrt(5) - 38) + 2*x) + 60*x^4 + 4)/x^5
```

3.403.6 Sympy [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.42

$$\int \frac{1}{x^6 (1-3x^4+x^8)} \, dx = \text{RootSum}\left(6400t^4-6080t^2-1, \left(t\mapsto t\log\left(\frac{215808000t^7}{323}-\frac{194833880t^3}{323}+x\right)\right)\right) + \text{RootSum}\left(6400t^4+6080t^2-1, \left(t\mapsto t\log\left(\frac{215808000t^7}{323}-\frac{194833880t^3}{323}+x\right)\right)\right) + \frac{-15x^4-1}{5x^5}$$

input
```
integrate(1/x**6/(x**8-3*x**4+1),x)
```

output
```
RootSum(6400*_t**4 - 6080*_t**2 - 1, Lambda(_t, _t*log(215808000*_t**7/323
 - 194833880*_t**3/323 + x))) + RootSum(6400*_t**4 + 6080*_t**2 - 1, Lambd
a(_t, _t*log(215808000*_t**7/323 - 194833880*_t**3/323 + x))) + (-15*x**4
- 1)/(5*x**5)
```

3.403.7 Maxima [F]

$$\int \frac{1}{x^6 (1-3x^4+x^8)} \, dx = \int \frac{1}{(x^8-3\,x^4+1)x^6} \, dx$$

input
```
integrate(1/x^6/(x^8-3*x^4+1),x, algorithm="maxima")
```

output
```
-1/5*(15*x^4 + 1)/x^5 - 1/2*integrate((3*x^2 + 5)/(x^4 + x^2 - 1), x) - 1/
2*integrate((3*x^2 - 5)/(x^4 - x^2 - 1), x)
```

3.403.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^6 (1-3x^4+x^8)} \, dx = \frac{1}{10} \sqrt{85 \sqrt{5}-190} \arctan\left(\frac{x}{\sqrt{\frac{1}{2} \sqrt{5}+\frac{1}{2}}}\right) - \frac{1}{10} \sqrt{85 \sqrt{5}+190} \arctan\left(\frac{x}{\sqrt{\frac{1}{2} \sqrt{5}-\frac{1}{2}}}\right) - \frac{1}{20} \sqrt{85 \sqrt{5}-190} \log\left(\left|x+\sqrt{\frac{1}{2} \sqrt{5}+\frac{1}{2}}\right|\right) + \frac{1}{20} \sqrt{85 \sqrt{5}-190} \log\left(\left|x-\sqrt{\frac{1}{2} \sqrt{5}+\frac{1}{2}}\right|\right) + \frac{1}{20} \sqrt{85 \sqrt{5}+190} \log\left(\left|x+\sqrt{\frac{1}{2} \sqrt{5}-\frac{1}{2}}\right|\right) - \frac{1}{20} \sqrt{85 \sqrt{5}+190} \log\left(\left|x-\sqrt{\frac{1}{2} \sqrt{5}-\frac{1}{2}}\right|\right) - \frac{15 x^4+1}{5 x^5}$$

input
```
integrate(1/x^6/(x^8-3*x^4+1),x, algorithm="giac")
```

output
```
1/10*sqrt(85*sqrt(5) - 190)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) - 1/10*sqrt(
85*sqrt(5) + 190)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/20*sqrt(85*sqrt(5)
 - 190)*log(abs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/20*sqrt(85*sqrt(5) - 190
)*log(abs(x - sqrt(1/2*sqrt(5) + 1/2))) + 1/20*sqrt(85*sqrt(5) + 190)*log(
abs(x + sqrt(1/2*sqrt(5) - 1/2))) - 1/20*sqrt(85*sqrt(5) + 190)*log(abs(x
- sqrt(1/2*sqrt(5) - 1/2))) - 1/5*(15*x^4 + 1)/x^5
```

3.403.9 Mupad [B] (verification not implemented)

Time = 8.33 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.49

$$\int \frac{1}{x^6 (1 - 3x^4 + x^8)} \, dx$$

$$= -\frac{3x^4 + \frac{1}{5}}{x^5} - \frac{\operatorname{atan}\left(\frac{x\sqrt{-85\sqrt{5}-190}\,372096\mathrm{i}}{2550408\sqrt{5}+5702888} + \frac{\sqrt{5}\,x\sqrt{-85\sqrt{5}-190}\,832048\mathrm{i}}{5\left(2550408\sqrt{5}+5702888\right)}\right)\sqrt{-85\sqrt{5}-190}\,1\mathrm{i}}{10}$$
$$+ \frac{\operatorname{atan}\left(\frac{x\sqrt{190-85\sqrt{5}}\,372096\mathrm{i}}{2550408\sqrt{5}-5702888} - \frac{\sqrt{5}\,x\sqrt{190-85\sqrt{5}}\,832048\mathrm{i}}{5\left(2550408\sqrt{5}-5702888\right)}\right)\sqrt{190-85\sqrt{5}}\,1\mathrm{i}}{10}$$
$$+ \frac{\operatorname{atan}\left(\frac{x\sqrt{85\sqrt{5}-190}\,372096\mathrm{i}}{2550408\sqrt{5}-5702888} - \frac{\sqrt{5}\,x\sqrt{85\sqrt{5}-190}\,832048\mathrm{i}}{5\left(2550408\sqrt{5}-5702888\right)}\right)\sqrt{85\sqrt{5}-190}\,1\mathrm{i}}{10}$$
$$- \frac{\operatorname{atan}\left(\frac{x\sqrt{85\sqrt{5}+190}\,372096\mathrm{i}}{2550408\sqrt{5}+5702888} + \frac{\sqrt{5}\,x\sqrt{85\sqrt{5}+190}\,832048\mathrm{i}}{5\left(2550408\sqrt{5}+5702888\right)}\right)\sqrt{85\sqrt{5}+190}\,1\mathrm{i}}{10}$$

input
```
int(1/(x^6*(x^8 - 3*x^4 + 1)),x)
```

output
```
(atan((x*(190 - 85*5^(1/2))^(1/2)*372096i)/(2550408*5^(1/2) - 5702888) - (
5^(1/2)*x*(190 - 85*5^(1/2))^(1/2)*832048i)/(5*(2550408*5^(1/2) - 5702888)
))*(190 - 85*5^(1/2))^(1/2)*1i)/10 - (atan((x*(- 85*5^(1/2) - 190)^(1/2)*3
72096i)/(2550408*5^(1/2) + 5702888) + (5^(1/2)*x*(- 85*5^(1/2) - 190)^(1/2
)*832048i)/(5*(2550408*5^(1/2) + 5702888)))*(- 85*5^(1/2) - 190)^(1/2)*1i)
/10 + (atan((x*(85*5^(1/2) - 190)^(1/2)*372096i)/(2550408*5^(1/2) - 570288
8) - (5^(1/2)*x*(85*5^(1/2) - 190)^(1/2)*832048i)/(5*(2550408*5^(1/2) - 57
02888)))*(85*5^(1/2) - 190)^(1/2)*1i)/10 - (atan((x*(85*5^(1/2) + 190)^(1/
2)*372096i)/(2550408*5^(1/2) + 5702888) + (5^(1/2)*x*(85*5^(1/2) + 190)^(1
/2)*832048i)/(5*(2550408*5^(1/2) + 5702888)))*(85*5^(1/2) + 190)^(1/2)*1i)
/10 - (3*x^4 + 1/5)/x^5
```

3.404 $\int \frac{1}{x^8(1-3x^4+x^8)}\,dx$

3.404.1 Optimal result

Integrand size = 16, antiderivative size = 189

$$\int \frac{1}{x^8\left(1-3x^4+x^8\right)}\,dx = -\frac{1}{7x^7}-\frac{1}{x^3}-\frac{\sqrt[4]{\frac{1}{2}\left(39603-17711\sqrt{5}\right)}\arctan\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}}+\frac{\sqrt[4]{\frac{1}{2}\left(39603+17711\sqrt{5}\right)}\arctan\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{5}}-\frac{\sqrt[4]{\frac{1}{2}\left(39603-17711\sqrt{5}\right)}\operatorname{arctanh}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}}+\frac{\sqrt[4]{\frac{1}{2}\left(39603+17711\sqrt{5}\right)}\operatorname{arctanh}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{5}}$$

output
```
-1/7/x^7-1/x^3-1/20*arctan(2^(1/4)*x*(1/(3+5^(1/2)))^(1/4))*(39603-17711*5
^(1/2))^(1/4)*2^(3/4)*5^(1/2)-1/20*arctanh(2^(1/4)*x*(1/(3+5^(1/2)))^(1/4)
)*(39603-17711*5^(1/2))^(1/4)*2^(3/4)*5^(1/2)+1/20*arctan(1/2*x*(3+5^(1/2)
)^(1/4)*2^(3/4))*(39603+17711*5^(1/2))^(1/4)*2^(3/4)*5^(1/2)+1/20*arctanh(
1/2*x*(3+5^(1/2))^(1/4)*2^(3/4))*(39603+17711*5^(1/2))^(1/4)*2^(3/4)*5^(1/
2)
```

3.404.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^8(1-3x^4+x^8)}\,dx = -\frac{1}{7x^7} - \frac{1}{x^3} + \frac{(11+5\sqrt{5})\arctan\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)}{2\sqrt{10(-1+\sqrt{5})}} + \frac{(11-5\sqrt{5})\arctan\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{2\sqrt{10(1+\sqrt{5})}} - \frac{(-11-5\sqrt{5})\,\text{arctanh}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)}{2\sqrt{10(-1+\sqrt{5})}} - \frac{(-11+5\sqrt{5})\,\text{arctanh}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{2\sqrt{10(1+\sqrt{5})}}$$

input
```
Integrate[1/(x^8*(1 - 3*x^4 + x^8)),x]
```

output
```
-1/7*1/x^7 - x^(-3) + ((11 + 5*Sqrt[5])*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x])/
(2*Sqrt[10*(-1 + Sqrt[5])]) + ((11 - 5*Sqrt[5])*ArcTan[Sqrt[2/(1 + Sqrt[5]
)]*x])/(2*Sqrt[10*(1 + Sqrt[5])]) - ((-11 - 5*Sqrt[5])*ArcTanh[Sqrt[2/(-1
+ Sqrt[5])]*x])/(2*Sqrt[10*(-1 + Sqrt[5])]) - ((-11 + 5*Sqrt[5])*ArcTanh[S
qrt[2/(1 + Sqrt[5])]*x])/(2*Sqrt[10*(1 + Sqrt[5])])
```

3.404.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1704, 27, 1828, 27, 1752, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^8(x^8-3x^4+1)}dx$$

$$\Big\downarrow \text{1704}$$

$$\frac{1}{7}\int \frac{7(3-x^4)}{x^4(x^8-3x^4+1)}dx - \frac{1}{7x^7}$$

$$\Big\downarrow \; 27$$

$$\int \frac{3-x^4}{x^4\left(x^8-3x^4+1\right)}dx-\frac{1}{7x^7}$$

$$\Big\downarrow \; 1828$$

$$-\frac{1}{3}\int -\frac{3\left(8-3x^4\right)}{x^8-3x^4+1}dx-\frac{1}{7x^7}-\frac{1}{x^3}$$

$$\Big\downarrow \; 27$$

$$\int \frac{8-3x^4}{x^8-3x^4+1}dx-\frac{1}{7x^7}-\frac{1}{x^3}$$

$$\Big\downarrow \; 1752$$

$$-\frac{1}{10}\left(15-7\sqrt{5}\right)\int \frac{1}{x^4+\frac{1}{2}\left(-3-\sqrt{5}\right)}dx-\frac{1}{10}\left(15+7\sqrt{5}\right)\int \frac{1}{x^4+\frac{1}{2}\left(-3+\sqrt{5}\right)}dx-\frac{1}{7x^7}-\frac{1}{x^3}$$

$$\Big\downarrow \; 756$$

$$-\frac{1}{10}\left(15+7\sqrt{5}\right)\left(-\frac{\int \frac{1}{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}dx}{\sqrt{3-\sqrt{5}}}-\frac{\int \frac{1}{\sqrt{2}x^2+\sqrt{3-\sqrt{5}}}dx}{\sqrt{3-\sqrt{5}}}\right)-\frac{1}{10}\left(15-7\sqrt{5}\right)\left(-\frac{\int \frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}dx}{\sqrt{3+\sqrt{5}}}-\frac{\int \frac{1}{\sqrt{2}x^2+\sqrt{3+\sqrt{5}}}dx}{\sqrt{3+\sqrt{5}}}\right)-\frac{1}{7x^7}-\frac{1}{x^3}$$

$$\Big\downarrow \; 216$$

$$-\frac{1}{10}\left(15+7\sqrt{5}\right)\left(-\frac{\int \frac{1}{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}dx}{\sqrt{3-\sqrt{5}}}-\frac{\arctan\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{\sqrt[4]{2}\left(3-\sqrt{5}\right)^{3/4}}\right)-\frac{1}{10}\left(15-7\sqrt{5}\right)\left(-\frac{\int \frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}dx}{\sqrt{3+\sqrt{5}}}-\frac{\arctan\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt[4]{2}\left(3+\sqrt{5}\right)^{3/4}}\right)-\frac{1}{7x^7}-\frac{1}{x^3}$$

$$\Big\downarrow \; 219$$

3.404. $\int \frac{1}{x^8\left(1-3x^4+x^8\right)}\,dx$

$$-\frac{1}{10}\left(15-7\sqrt{5}\right)\left(-\frac{\arctan\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt[4]{2}\left(3+\sqrt{5}\right)^{3/4}}-\frac{\operatorname{arctanh}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt[4]{2}\left(3+\sqrt{5}\right)^{3/4}}\right)-\frac{1}{10}\left(15+7\sqrt{5}\right)\left(-\frac{\arctan\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{\sqrt[4]{2}\left(3-\sqrt{5}\right)^{3/4}}-\frac{\operatorname{arctanh}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{\sqrt[4]{2}\left(3-\sqrt{5}\right)^{3/4}}\right)-\frac{1}{7x^7}-\frac{1}{x^3}$$

input
```
Int[1/(x^8*(1 - 3*x^4 + x^8)),x]
```

output
```
-1/7*1/x^7 - x^(-3) - ((15 - 7*Sqrt[5])*(-(ArcTan[(2/(3 + Sqrt[5]))^(1/4)*
x]/(2^(1/4)*(3 + Sqrt[5])^(3/4))) - ArcTanh[(2/(3 + Sqrt[5]))^(1/4)*x]/(2^
(1/4)*(3 + Sqrt[5])^(3/4))))/10 - ((15 + 7*Sqrt[5])*(-(ArcTan[((3 + Sqrt[5
])/2)^(1/4)*x]/(2^(1/4)*(3 - Sqrt[5])^(3/4))) - ArcTanh[((3 + Sqrt[5])/2)^
(1/4)*x]/(2^(1/4)*(3 - Sqrt[5])^(3/4))))/10
```

3.404.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 1704
```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Simp[1/(a*d^n*(m + 1))   Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1)
+ c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{
a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] &&
LtQ[m, -1] && IntegerQ[p]
```

rule 1752
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

rule 1828
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Simp[1/(a*f^n*(m + 1))   Int[(f*x)^(m +
 n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) -
c*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x
] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Int
egerQ[p]
```

3.404.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.42

method	result
risch	$\frac{-x^4-\frac{1}{7}}{x^7}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(25\_Z^4-995\_Z^2-1\right)}\_R\ln\left(90\_R^3-3571\_R+89x\right)\right)}{4}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(25\_Z^4+995\_Z^2-1\right)}\_R\ln(\right.}{4}$
default	$-\frac{1}{7x^7}-\frac{1}{x^3}-\frac{\left(-11+5\sqrt{5}\right)\sqrt{5}\,\operatorname{arctanh}\left(\frac{2x}{\sqrt{2\sqrt{5}+2}}\right)}{10\sqrt{2\sqrt{5}+2}}+\frac{\sqrt{5}\left(11+5\sqrt{5}\right)\arctan\left(\frac{2x}{\sqrt{2\sqrt{5}-2}}\right)}{10\sqrt{2\sqrt{5}-2}}-\frac{\left(-11+5\sqrt{5}\right)\sqrt{5}\arctan\left(\frac{2x}{\sqrt{2\sqrt{5}+}}\right.}{10\sqrt{2\sqrt{5}+2}}$

input
```
int(1/x^8/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
(-x^4-1/7)/x^7+1/4*sum(_R*ln(90*_R^3-3571*_R+89*x),_R=RootOf(25*_Z^4-995*_
Z^2-1))+1/4*sum(_R*ln(-90*_R^3-3571*_R+89*x),_R=RootOf(25*_Z^4+995*_Z^2-1)
)
```

3.404.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 352 vs. $2(133) = 266$.

Time = 0.28 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.86

$$\int \frac{1}{x^8(1-3x^4+x^8)}\,dx = -\frac{7\sqrt{10}x^7\sqrt{89\sqrt{5}-199}\log\left(\sqrt{10}\sqrt{89\sqrt{5}-199}(9\sqrt{5}+20)+10x\right)-7\sqrt{10}x^7\sqrt{89\sqrt{5}-199}\log\left(-\sqrt{}\right.}{}$$

input
```
integrate(1/x^8/(x^8-3*x^4+1),x, algorithm="fricas")
```

output
```
-1/280*(7*sqrt(10)*x^7*sqrt(89*sqrt(5) - 199)*log(sqrt(10)*sqrt(89*sqrt(5)
 - 199)*(9*sqrt(5) + 20) + 10*x) - 7*sqrt(10)*x^7*sqrt(89*sqrt(5) - 199)*l
og(-sqrt(10)*sqrt(89*sqrt(5) - 199)*(9*sqrt(5) + 20) + 10*x) - 7*sqrt(10)*
x^7*sqrt(89*sqrt(5) + 199)*log(sqrt(10)*sqrt(89*sqrt(5) + 199)*(9*sqrt(5)
- 20) + 10*x) + 7*sqrt(10)*x^7*sqrt(89*sqrt(5) + 199)*log(-sqrt(10)*sqrt(8
9*sqrt(5) + 199)*(9*sqrt(5) - 20) + 10*x) + 7*sqrt(10)*x^7*sqrt(-89*sqrt(5
) + 199)*log(sqrt(10)*(9*sqrt(5) + 20)*sqrt(-89*sqrt(5) + 199) + 10*x) - 7
*sqrt(10)*x^7*sqrt(-89*sqrt(5) + 199)*log(-sqrt(10)*(9*sqrt(5) + 20)*sqrt(
-89*sqrt(5) + 199) + 10*x) - 7*sqrt(10)*x^7*sqrt(-89*sqrt(5) - 199)*log(sq
rt(10)*(9*sqrt(5) - 20)*sqrt(-89*sqrt(5) - 199) + 10*x) + 7*sqrt(10)*x^7*s
qrt(-89*sqrt(5) - 199)*log(-sqrt(10)*(9*sqrt(5) - 20)*sqrt(-89*sqrt(5) - 1
99) + 10*x) + 280*x^4 + 40)/x^7
```

3.404.6 Sympy [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.37

$$\int \frac{1}{x^8(1-3x^4+x^8)}\,dx = \text{RootSum}\left(6400t^4-15920t^2-1, \left(t\mapsto t\log\left(\frac{460800t^5}{17711}-\frac{2842588t}{17711}+x\right)\right)\right) + \text{RootSum}\left(6400t^4+15920t^2-1, \left(t\mapsto t\log\left(\frac{460800t^5}{17711}-\frac{2842588t}{17711}+x\right)\right)\right) + \frac{-7x^4-1}{7x^7}$$

input
```
integrate(1/x**8/(x**8-3*x**4+1),x)
```

output
```
RootSum(6400*_t**4 - 15920*_t**2 - 1, Lambda(_t, _t*log(460800*_t**5/17711
 - 2842588*_t/17711 + x))) + RootSum(6400*_t**4 + 15920*_t**2 - 1, Lambda(
_t, _t*log(460800*_t**5/17711 - 2842588*_t/17711 + x))) + (-7*x**4 - 1)/(7
*x**7)
```

3.404.7 Maxima [F]

$$\int \frac{1}{x^8(1-3x^4+x^8)}\,dx = \int \frac{1}{(x^8-3\,x^4+1)x^8}\,dx$$

input
```
integrate(1/x^8/(x^8-3*x^4+1),x, algorithm="maxima")
```

output
```
-1/7*(7*x^4 + 1)/x^7 - 1/2*integrate((5*x^2 + 8)/(x^4 + x^2 - 1), x) + 1/2
*integrate((5*x^2 - 8)/(x^4 - x^2 - 1), x)
```

3.404.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^8 (1-3x^4+x^8)}\,dx = -\frac{1}{20}\sqrt{890\sqrt{5}-1990}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right) + \frac{1}{20}\sqrt{890\sqrt{5}+1990}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right) - \frac{1}{40}\sqrt{890\sqrt{5}-1990}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right) + \frac{1}{40}\sqrt{890\sqrt{5}-1990}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right) + \frac{1}{40}\sqrt{890\sqrt{5}+1990}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right) - \frac{1}{40}\sqrt{890\sqrt{5}+1990}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right) - \frac{7\,x^4+1}{7\,x^7}$$

input
```
integrate(1/x^8/(x^8-3*x^4+1),x, algorithm="giac")
```

output
```
-1/20*sqrt(890*sqrt(5) - 1990)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) + 1/20*sq
rt(890*sqrt(5) + 1990)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/40*sqrt(890*s
qrt(5) - 1990)*log(abs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(890*sqrt(
5) - 1990)*log(abs(x - sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(890*sqrt(5) +
 1990)*log(abs(x + sqrt(1/2*sqrt(5) - 1/2))) - 1/40*sqrt(890*sqrt(5) + 199
0)*log(abs(x - sqrt(1/2*sqrt(5) - 1/2))) - 1/7*(7*x^4 + 1)/x^7
```

3.404.9 Mupad [B] (verification not implemented)

Time = 8.38 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.54

$$\int \frac{1}{x^8(1-3x^4+x^8)}\,dx$$
$$= -\frac{x^4+\frac{1}{7}}{x^7} + \frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{-89\sqrt{5}-199}\,6677047\mathrm{i}}{2\left(74049691\sqrt{5}+165580139\right)}+\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{-89\sqrt{5}-199}\,14930373\mathrm{i}}{10\left(74049691\sqrt{5}+165580139\right)}\right)\sqrt{-89\sqrt{5}-199}\,1\mathrm{i}}{20}$$
$$+ \frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{199-89\sqrt{5}}\,6677047\mathrm{i}}{2\left(74049691\sqrt{5}-165580139\right)}-\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{199-89\sqrt{5}}\,14930373\mathrm{i}}{10\left(74049691\sqrt{5}-165580139\right)}\right)\sqrt{199-89\sqrt{5}}\,1\mathrm{i}}{20}$$
$$- \frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{89\sqrt{5}-199}\,6677047\mathrm{i}}{2\left(74049691\sqrt{5}-165580139\right)}-\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{89\sqrt{5}-199}\,14930373\mathrm{i}}{10\left(74049691\sqrt{5}-165580139\right)}\right)\sqrt{89\sqrt{5}-199}\,1\mathrm{i}}{20}$$
$$- \frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{89\sqrt{5}+199}\,6677047\mathrm{i}}{2\left(74049691\sqrt{5}+165580139\right)}+\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{89\sqrt{5}+199}\,14930373\mathrm{i}}{10\left(74049691\sqrt{5}+165580139\right)}\right)\sqrt{89\sqrt{5}+199}\,1\mathrm{i}}{20}$$

input
```
int(1/(x^8*(x^8 - 3*x^4 + 1)),x)
```

output
```
(10^(1/2)*atan((10^(1/2)*x*(- 89*5^(1/2) - 199)^(1/2)*6677047i)/(2*(740496
91*5^(1/2) + 165580139)) + (5^(1/2)*10^(1/2)*x*(- 89*5^(1/2) - 199)^(1/2)*
14930373i)/(10*(74049691*5^(1/2) + 165580139)))*(- 89*5^(1/2) - 199)^(1/2)
*1i)/20 - (x^4 + 1/7)/x^7 + (10^(1/2)*atan((10^(1/2)*x*(199 - 89*5^(1/2))^
(1/2)*6677047i)/(2*(74049691*5^(1/2) - 165580139)) - (5^(1/2)*10^(1/2)*x*(
199 - 89*5^(1/2))^(1/2)*14930373i)/(10*(74049691*5^(1/2) - 165580139)))*(1
99 - 89*5^(1/2))^(1/2)*1i)/20 - (10^(1/2)*atan((10^(1/2)*x*(89*5^(1/2) - 1
99)^(1/2)*6677047i)/(2*(74049691*5^(1/2) - 165580139)) - (5^(1/2)*10^(1/2)
*x*(89*5^(1/2) - 199)^(1/2)*14930373i)/(10*(74049691*5^(1/2) - 165580139))
)*(89*5^(1/2) - 199)^(1/2)*1i)/20 - (10^(1/2)*atan((10^(1/2)*x*(89*5^(1/2)
 + 199)^(1/2)*6677047i)/(2*(74049691*5^(1/2) + 165580139)) + (5^(1/2)*10^(
1/2)*x*(89*5^(1/2) + 199)^(1/2)*14930373i)/(10*(74049691*5^(1/2) + 1655801
39)))*(89*5^(1/2) + 199)^(1/2)*1i)/20
```

3.405 $\int \frac{x^3}{2+3x^4+x^8} \, dx$

3.405.1 Optimal result

Integrand size = 16, antiderivative size = 21

$$\int \frac{x^3}{2+3x^4+x^8} \, dx = \frac{1}{4}\log\left(1+x^4\right)-\frac{1}{4}\log\left(2+x^4\right)$$

output
```
1/4*ln(x^4+1)-1/4*ln(x^4+2)
```

3.405.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{2+3x^4+x^8} \, dx = \frac{1}{4}\log\left(1+x^4\right)-\frac{1}{4}\log\left(2+x^4\right)$$

input
```
Integrate[x^3/(2 + 3*x^4 + x^8),x]
```

output
```
Log[1 + x^4]/4 - Log[2 + x^4]/4
```

3.405.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1690, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{x^8+3x^4+2} \, dx$$
$$\Big\downarrow \text{ 1690}$$
$$\frac{1}{4}\int \frac{1}{x^8+3x^4+2}dx^4$$
$$\Big\downarrow \text{ 1081}$$
$$\frac{1}{4}\int \left(\frac{1}{x^4+1}+\frac{1}{-x^4-2}\right)dx^4$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{1}{4}\left(\log\left(x^4+1\right)-\log\left(x^4+2\right)\right)$$

input
```
Int[x^3/(2 + 3*x^4 + x^8),x]
```

output
```
(Log[1 + x^4] - Log[2 + x^4])/4
```

3.405.3.1 Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
 + c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.405.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{\ln(x^4+1)}{4}-\frac{\ln(x^4+2)}{4}$	18
norman	$\frac{\ln(x^4+1)}{4}-\frac{\ln(x^4+2)}{4}$	18
risch	$\frac{\ln(x^4+1)}{4}-\frac{\ln(x^4+2)}{4}$	18
parallelrisch	$\frac{\ln(x^4+1)}{4}-\frac{\ln(x^4+2)}{4}$	18

input
```
int(x^3/(x^8+3*x^4+2),x,method=_RETURNVERBOSE)
```

output
```
1/4*ln(x^4+1)-1/4*ln(x^4+2)
```

3.405.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{x^3}{2+3x^4+x^8}\,dx = -\frac{1}{4}\log\left(x^4+2\right)+\frac{1}{4}\log\left(x^4+1\right)$$

input
```
integrate(x^3/(x^8+3*x^4+2),x, algorithm="fricas")
```

output
```
-1/4*log(x^4 + 2) + 1/4*log(x^4 + 1)
```

3.405.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{x^3}{2+3x^4+x^8}\,dx = \frac{\log\left(x^4+1\right)}{4}-\frac{\log\left(x^4+2\right)}{4}$$

input
```
integrate(x**3/(x**8+3*x**4+2),x)
```

output
```
log(x**4 + 1)/4 - log(x**4 + 2)/4
```

3.405.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{x^3}{2+3x^4+x^8}\,dx = -\frac{1}{4}\log\left(x^4+2\right)+\frac{1}{4}\log\left(x^4+1\right)$$

input
```
integrate(x^3/(x^8+3*x^4+2),x, algorithm="maxima")
```

output
```
-1/4*log(x^4 + 2) + 1/4*log(x^4 + 1)
```

3.405.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{x^3}{2+3x^4+x^8}\,dx = -\frac{1}{4}\log\left(x^4+2\right)+\frac{1}{4}\log\left(x^4+1\right)$$

input
```
integrate(x^3/(x^8+3*x^4+2),x, algorithm="giac")
```

output
```
-1/4*log(x^4 + 2) + 1/4*log(x^4 + 1)
```

3.405.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int \frac{x^3}{2+3x^4+x^8}\,dx = -\frac{\operatorname{atanh}\left(\frac{256}{9\,(144\,x^4+160)}-\frac{7}{9}\right)}{2}$$

input
```
int(x^3/(3*x^4 + x^8 + 2),x)
```

output
```
-atanh(256/(9*(144*x^4 + 160)) - 7/9)/2
```

3.406 $\int \frac{x^{11}}{2+3x^4+x^8}\,dx$

3.406.1 Optimal result

Integrand size = 16, antiderivative size = 26

$$\int \frac{x^{11}}{2+3x^4+x^8}\,dx=\frac{x^4}{4}+\frac{1}{4}\log\left(1+x^4\right)-\log\left(2+x^4\right)$$

output
```
1/4*x^4+1/4*ln(x^4+1)-ln(x^4+2)
```

3.406.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x^{11}}{2+3x^4+x^8}\,dx=\frac{x^4}{4}+\frac{1}{4}\log\left(1+x^4\right)-\log\left(2+x^4\right)$$

input
```
Integrate[x^11/(2 + 3*x^4 + x^8),x]
```

output
```
x^4/4 + Log[1 + x^4]/4 - Log[2 + x^4]
```

3.406.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1693, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{x^8+3x^4+2}dx$$
$$\Big\downarrow \text{1693}$$
$$\frac{1}{4}\int \frac{x^8}{x^8+3x^4+2}dx^4$$
$$\Big\downarrow \text{1141}$$
$$\frac{1}{4}\int \left(-\frac{4}{x^4+2}+1+\frac{1}{x^4+1}\right)dx^4$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{4}\left(x^4+\log\left(x^4+1\right)-4\log\left(x^4+2\right)\right)$$

input
```
Int[x^11/(2 + 3*x^4 + x^8),x]
```

output
```
(x^4 + Log[1 + x^4] - 4*Log[2 + x^4])/4
```

3.406.3.1 Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.406.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{x^4}{4}+\frac{\ln(x^4+1)}{4}-\ln\left(x^4+2\right)$	23
norman	$\frac{x^4}{4}+\frac{\ln(x^4+1)}{4}-\ln\left(x^4+2\right)$	23
risch	$\frac{x^4}{4}+\frac{\ln(x^4+1)}{4}-\ln\left(x^4+2\right)$	23
parallelrisch	$\frac{x^4}{4}+\frac{\ln(x^4+1)}{4}-\ln\left(x^4+2\right)$	23

input
```
int(x^11/(x^8+3*x^4+2),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4+1/4*ln(x^4+1)-ln(x^4+2)
```

3.406.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^{11}}{2+3x^4+x^8}\,dx=\frac{1}{4}\,x^4-\log\left(x^4+2\right)+\frac{1}{4}\,\log\left(x^4+1\right)$$

input
```
integrate(x^11/(x^8+3*x^4+2),x, algorithm="fricas")
```

output
```
1/4*x^4 - log(x^4 + 2) + 1/4*log(x^4 + 1)
```

3.406.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.73

$$\int \frac{x^{11}}{2+3x^4+x^8}\,dx = \frac{x^4}{4} + \frac{\log\left(x^4+1\right)}{4} - \log\left(x^4+2\right)$$

input
```
integrate(x**11/(x**8+3*x**4+2),x)
```

output
```
x**4/4 + log(x**4 + 1)/4 - log(x**4 + 2)
```

3.406.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^{11}}{2+3x^4+x^8}\,dx = \frac{1}{4}\,x^4 - \log\left(x^4+2\right) + \frac{1}{4}\,\log\left(x^4+1\right)$$

input
```
integrate(x^11/(x^8+3*x^4+2),x, algorithm="maxima")
```

output
```
1/4*x^4 - log(x^4 + 2) + 1/4*log(x^4 + 1)
```

3.406.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^{11}}{2+3x^4+x^8}\,dx = \frac{1}{4}\,x^4 - \log\left(x^4+2\right) + \frac{1}{4}\,\log\left(x^4+1\right)$$

input
```
integrate(x^11/(x^8+3*x^4+2),x, algorithm="giac")
```

output
```
1/4*x^4 - log(x^4 + 2) + 1/4*log(x^4 + 1)
```

3.406.9 Mupad [B] (verification not implemented)

Time = 8.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^{11}}{2+3x^4+x^8}\,dx = \frac{\ln(x^4+1)}{4} - \ln(x^4+2) + \frac{x^4}{4}$$

input
```
int(x^11/(3*x^4 + x^8 + 2),x)
```

output
```
log(x^4 + 1)/4 - log(x^4 + 2) + x^4/4
```

3.407 $\int \frac{x^9}{2+x^5+x^{10}}\,dx$

3.407.1 Optimal result

Integrand size = 14, antiderivative size = 37

$$\int \frac{x^9}{2+x^5+x^{10}}\,dx=-\frac{\arctan\left(\frac{1+2x^5}{\sqrt{7}}\right)}{5\sqrt{7}}+\frac{1}{10}\log\left(2+x^5+x^{10}\right)$$

output
```
1/10*ln(x^10+x^5+2)-1/35*arctan(1/7*(2*x^5+1)*7^(1/2))*7^(1/2)
```

3.407.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{x^9}{2+x^5+x^{10}}\,dx=-\frac{\arctan\left(\frac{1+2x^5}{\sqrt{7}}\right)}{5\sqrt{7}}+\frac{1}{10}\log\left(2+x^5+x^{10}\right)$$

input
```
Integrate[x^9/(2 + x^5 + x^10),x]
```

output
```
-1/5*ArcTan[(1 + 2*x^5)/Sqrt[7]]/Sqrt[7] + Log[2 + x^5 + x^10]/10
```

3.407.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {1693, 1142, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{x^{10}+x^5+2}\,dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{5}\int \frac{x^5}{x^{10}+x^5+2}dx^5$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{5}\left(\frac{1}{2}\int \frac{2x^5+1}{x^{10}+x^5+2}dx^5-\frac{1}{2}\int \frac{1}{x^{10}+x^5+2}dx^5\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{5}\left(\int \frac{1}{-x^{10}-7}d(2x^5+1)+\frac{1}{2}\int \frac{2x^5+1}{x^{10}+x^5+2}dx^5\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{5}\left(\frac{1}{2}\int \frac{2x^5+1}{x^{10}+x^5+2}dx^5-\frac{\arctan\left(\frac{2x^5+1}{\sqrt{7}}\right)}{\sqrt{7}}\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{5}\left(\frac{1}{2}\log\left(x^{10}+x^5+2\right)-\frac{\arctan\left(\frac{2x^5+1}{\sqrt{7}}\right)}{\sqrt{7}}\right)$$

input
```
Int[x^9/(2 + x^5 + x^10),x]
```

output
```
(-(ArcTan[(1 + 2*x^5)/Sqrt[7]]/Sqrt[7]) + Log[2 + x^5 + x^10]/2)/5
```

3.407.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.407.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{\ln\left(x^{10}+x^{5}+2\right)}{10}-\frac{\arctan\left(\frac{\left(2x^{5}+1\right)\sqrt{7}}{7}\right)\sqrt{7}}{35}$	31
risch	$\frac{\ln\left(4x^{10}+4x^{5}+8\right)}{10}-\frac{\arctan\left(\frac{\left(2x^{5}+1\right)\sqrt{7}}{7}\right)\sqrt{7}}{35}$	35

input
```
int(x^9/(x^10+x^5+2),x,method=_RETURNVERBOSE)
```

output
```
1/10*ln(x^10+x^5+2)-1/35*arctan(1/7*(2*x^5+1)*7^(1/2))*7^(1/2)
```

3.407.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{x^9}{2+x^5+x^{10}}\,dx = -\frac{1}{35}\,\sqrt{7}\arctan\left(\frac{1}{7}\,\sqrt{7}(2\,x^5+1)\right)+\frac{1}{10}\,\log\left(x^{10}+x^5+2\right)$$

input
```
integrate(x^9/(x^10+x^5+2),x, algorithm="fricas")
```

output
```
-1/35*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^5 + 1)) + 1/10*log(x^10 + x^5 + 2)
```

3.407.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{x^9}{2+x^5+x^{10}}\,dx = \frac{\log\left(x^{10}+x^5+2\right)}{10}-\frac{\sqrt{7}\,\mathrm{atan}\left(\frac{2\sqrt{7}x^5}{7}+\frac{\sqrt{7}}{7}\right)}{35}$$

input
```
integrate(x**9/(x**10+x**5+2),x)
```

output
```
log(x**10 + x**5 + 2)/10 - sqrt(7)*atan(2*sqrt(7)*x**5/7 + sqrt(7)/7)/35
```

3.407.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{x^9}{2+x^5+x^{10}}\,dx = -\frac{1}{35}\,\sqrt{7}\arctan\left(\frac{1}{7}\,\sqrt{7}(2\,x^5+1)\right)+\frac{1}{10}\,\log\left(x^{10}+x^5+2\right)$$

input
```
integrate(x^9/(x^10+x^5+2),x, algorithm="maxima")
```

output
```
-1/35*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^5 + 1)) + 1/10*log(x^10 + x^5 + 2)
```

3.407.8 Giac [A] (verification not implemented)

Time = 1.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{x^9}{2+x^5+x^{10}}\,dx = -\frac{1}{35}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2\,x^5+1)\right)+\frac{1}{10}\log\left(x^{10}+x^5+2\right)$$

input
```
integrate(x^9/(x^10+x^5+2),x, algorithm="giac")
```

output
```
-1/35*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^5 + 1)) + 1/10*log(x^10 + x^5 + 2)
```

3.407.9 Mupad [B] (verification not implemented)

Time = 8.40 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

$$\int \frac{x^9}{2+x^5+x^{10}}\,dx = \frac{\ln\left(x^{10}+x^5+2\right)}{10}-\frac{\sqrt{7}\,\mathrm{atan}\left(\frac{2\sqrt{7}\,x^5}{7}+\frac{\sqrt{7}}{7}\right)}{35}$$

input
```
int(x^9/(x^5 + x^10 + 2),x)
```

output
```
log(x^5 + x^10 + 2)/10 - (7^(1/2)*atan(7^(1/2)/7 + (2*7^(1/2)*x^5)/7))/35
```

3.408 $\int \frac{x^4}{2+x^5+x^{10}} \, dx$

3.408.1 Optimal result

Integrand size = 14, antiderivative size = 23

$$\int \frac{x^4}{2+x^5+x^{10}} \, dx = \frac{2\arctan\left(\frac{1+2x^5}{\sqrt{7}}\right)}{5\sqrt{7}}$$

output
```
2/35*arctan(1/7*(2*x^5+1)*7^(1/2))*7^(1/2)
```

3.408.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{2+x^5+x^{10}} \, dx = \frac{2\arctan\left(\frac{1+2x^5}{\sqrt{7}}\right)}{5\sqrt{7}}$$

input
```
Integrate[x^4/(2 + x^5 + x^10),x]
```

output
```
(2*ArcTan[(1 + 2*x^5)/Sqrt[7]])/(5*Sqrt[7])
```

3.408.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1690, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{x^{10}+x^5+2}dx$$
$$\Big\downarrow \text{1690}$$
$$\frac{1}{5}\int \frac{1}{x^{10}+x^5+2}dx^5$$
$$\Big\downarrow \text{1083}$$
$$-\frac{2}{5}\int \frac{1}{-x^{10}-7}d(2x^5+1)$$
$$\Big\downarrow \text{217}$$
$$\frac{2\arctan\left(\frac{2x^5+1}{\sqrt{7}}\right)}{5\sqrt{7}}$$

input
```
Int[x^4/(2 + x^5 + x^10),x]
```

output
```
(2*ArcTan[(1 + 2*x^5)/Sqrt[7]])/(5*Sqrt[7])
```

3.408.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

3.408.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{2 \arctan\left(\frac{\left(2x^5+1\right)\sqrt{7}}{7}\right)\sqrt{7}}{35}$	19
risch	$\frac{2 \arctan\left(\frac{\left(2x^5+1\right)\sqrt{7}}{7}\right)\sqrt{7}}{35}$	19

input
```
int(x^4/(x^10+x^5+2),x,method=_RETURNVERBOSE)
```

output
```
2/35*arctan(1/7*(2*x^5+1)*7^(1/2))*7^(1/2)
```

3.408.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{x^4}{2+x^5+x^{10}}\,dx = \frac{2}{35}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2\,x^5+1)\right)$$

input
```
integrate(x^4/(x^10+x^5+2),x, algorithm="fricas")
```

output
```
2/35*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^5 + 1))
```

3.408.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x^4}{2+x^5+x^{10}}\,dx = \frac{2\sqrt{7}\,\mathrm{atan}\left(\frac{2\sqrt{7}x^5}{7}+\frac{\sqrt{7}}{7}\right)}{35}$$

input
```
integrate(x**4/(x**10+x**5+2),x)
```

output
```
2*sqrt(7)*atan(2*sqrt(7)*x**5/7 + sqrt(7)/7)/35
```

3.408.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{x^4}{2+x^5+x^{10}}\,dx = \frac{2}{35}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2\,x^5+1)\right)$$

input
```
integrate(x^4/(x^10+x^5+2),x, algorithm="maxima")
```

output
```
2/35*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^5 + 1))
```

3.408.8 Giac [A] (verification not implemented)

Time = 1.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{x^4}{2+x^5+x^{10}}\,dx = \frac{2}{35}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2\,x^5+1)\right)$$

input
```
integrate(x^4/(x^10+x^5+2),x, algorithm="giac")
```

output
```
2/35*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^5 + 1))
```

3.408.9 Mupad [B] (verification not implemented)

Time = 8.42 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{x^4}{2+x^5+x^{10}}\,dx = \frac{2\sqrt{7}\,\mathrm{atan}\left(\frac{2\sqrt{7}\,x^5}{7}+\frac{\sqrt{7}}{7}\right)}{35}$$

input
```
int(x^4/(x^5 + x^10 + 2),x)
```

output
```
(2*7^(1/2)*atan(7^(1/2)/7 + (2*7^(1/2)*x^5)/7))/35
```

3.409 $\int \frac{1}{x(1+x^5+x^{10})}\,dx$

3.409.1 Optimal result

Integrand size = 14, antiderivative size = 39

$$\int \frac{1}{x(1+x^5+x^{10})}\,dx=-\frac{\arctan\left(\frac{1+2x^5}{\sqrt{3}}\right)}{5\sqrt{3}}+\log(x)-\frac{1}{10}\log\left(1+x^5+x^{10}\right)$$

output
```
ln(x)-1/10*ln(x^10+x^5+1)-1/15*arctan(1/3*(2*x^5+1)*3^(1/2))*3^(1/2)
```

3.409.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 197, normalized size of antiderivative = 5.05

$$\int \frac{1}{x(1+x^5+x^{10})}\,dx$$
$$=\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{5\sqrt{3}}+\log(x)-\frac{1}{10}\log\left(1+x+x^2\right)-\frac{1}{5}\text{RootSum}\Big[1-\#1+\#1^3-\#1^4+\#1^5-\#1^7$$
$$+\#1^8\&,\frac{-\log(x-\#1)\#1+2\log(x-\#1)\#1^2-\log(x-\#1)\#1^3+3\log(x-\#1)\#1^4-\log(x-\#1)\#}{-1+3\#1^2-4\#1^3+5\#1^4-7\#1^6+8\#1^7}$$

input
```
Integrate[1/(x*(1 + x^5 + x^10)),x]
```

output
```
ArcTan[(1 + 2*x)/Sqrt[3]]/(5*Sqrt[3]) + Log[x] - Log[1 + x + x^2]/10 - Roo
tSum[1 - #1 + #1^3 - #1^4 + #1^5 - #1^7 + #1^8 & , (-(Log[x - #1]*#1) + 2*
Log[x - #1]*#1^2 - Log[x - #1]*#1^3 + 3*Log[x - #1]*#1^4 - Log[x - #1]*#1^
5 - 3*Log[x - #1]*#1^6 + 4*Log[x - #1]*#1^7)/(-1 + 3*#1^2 - 4*#1^3 + 5*#1^
4 - 7*#1^6 + 8*#1^7) & ]/5
```

3.409.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1693, 1144, 25, 1142, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(x^{10}+x^5+1\right)}dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{5}\int \frac{1}{x^5\left(x^{10}+x^5+1\right)}dx^5$$

$$\Big\downarrow \text{1144}$$

$$\frac{1}{5}\left(\int -\frac{x^5+1}{x^{10}+x^5+1}dx^5+\log\left(x^5\right)\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{5}\left(\log\left(x^5\right)-\int \frac{x^5+1}{x^{10}+x^5+1}dx^5\right)$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{5}\left(-\frac{1}{2}\int \frac{1}{x^{10}+x^5+1}dx^5-\frac{1}{2}\int \frac{2x^5+1}{x^{10}+x^5+1}dx^5+\log\left(x^5\right)\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{5}\left(\int \frac{1}{-x^{10}-3}d\left(2x^5+1\right)-\frac{1}{2}\int \frac{2x^5+1}{x^{10}+x^5+1}dx^5+\log\left(x^5\right)\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{5}\left(-\frac{1}{2}\int \frac{2x^5+1}{x^{10}+x^5+1}dx^5-\frac{\arctan\left(\frac{2x^5+1}{\sqrt{3}}\right)}{\sqrt{3}}+\log\left(x^5\right)\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{5}\left(-\frac{\arctan\left(\frac{2x^5+1}{\sqrt{3}}\right)}{\sqrt{3}}+\log\left(x^5\right)-\frac{1}{2}\log\left(x^{10}+x^5+1\right)\right)$$

input
```
Int[1/(x*(1 + x^5 + x^10)),x]
```

output
```
(-(ArcTan[(1 + 2*x^5)/Sqrt[3]]/Sqrt[3]) + Log[x^5] - Log[1 + x^5 + x^10]/2
)/5
```

3.409.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1144
```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Simp[e*(Log[RemoveContent[d + e*x, x]]/(c*d^2 - b*d*e + a*e^2)), x] + S
imp[1/(c*d^2 - b*d*e + a*e^2)   Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2),
x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.409.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

method	result
risch	$\ln(x)-\frac{\ln(x^{10}+x^{5}+1)}{10}-\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x^{5}+\frac{1}{2}\right)\sqrt{3}}{3}\right)}{15}$
default	$\ln(x)-\frac{\left(\frac{1}{2}+\frac{i\sqrt{3}}{6}\right)\ln\left(2x^{4}+\left(-1+i\sqrt{3}\right)x^{3}+\left(-1-i\sqrt{3}\right)x^{2}+2x-1+i\sqrt{3}\right)}{5}-\frac{\left(\frac{1}{2}-\frac{i\sqrt{3}}{6}\right)\ln\left(2x^{4}+\left(-1-i\sqrt{3}\right)x^{3}+\left(-1+i\sqrt{3}\right)x^{2}+\right.}{5}$

input
```
int(1/x/(x^10+x^5+1),x,method=_RETURNVERBOSE)
```

output
```
ln(x)-1/10*ln(x^10+x^5+1)-1/15*3^(1/2)*arctan(2/3*(x^5+1/2)*3^(1/2))
```

3.409.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{1}{x\,(1+x^5+x^{10})}\,dx=-\frac{1}{15}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^5+1)\right)-\frac{1}{10}\,\log\left(x^{10}+x^5+1\right)+\log\left(x\right)$$

input
```
integrate(1/x/(x^10+x^5+1),x, algorithm="fricas")
```

output
```
-1/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^5 + 1)) - 1/10*log(x^10 + x^5 + 1) +
 log(x)
```

3.409.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{1}{x(1+x^5+x^{10})}\,dx = \log(x) - \frac{\log(x^{10}+x^5+1)}{10} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^5}{3}+\frac{\sqrt{3}}{3}\right)}{15}$$

input
```
integrate(1/x/(x**10+x**5+1),x)
```

output
```
log(x) - log(x**10 + x**5 + 1)/10 - sqrt(3)*atan(2*sqrt(3)*x**5/3 + sqrt(3
)/3)/15
```

3.409.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \frac{1}{x(1+x^5+x^{10})}\,dx = -\frac{1}{15}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^5+1)\right) - \frac{1}{10}\log(x^{10}+x^5+1) + \frac{1}{5}\log(x^5)$$

input
```
integrate(1/x/(x^10+x^5+1),x, algorithm="maxima")
```

output
```
-1/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^5 + 1)) - 1/10*log(x^10 + x^5 + 1) +
 1/5*log(x^5)
```

3.409.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int \frac{1}{x(1+x^5+x^{10})}\,dx = -\frac{1}{15}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^5+1)\right) - \frac{1}{10}\log(x^{10}+x^5+1) + \log(|x|)$$

input
```
integrate(1/x/(x^10+x^5+1),x, algorithm="giac")
```

output
```
-1/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^5 + 1)) - 1/10*log(x^10 + x^5 + 1) +
 log(abs(x))
```

3.409.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

$$\int \frac{1}{x\left(1+x^5+x^{10}\right)}\,dx = \ln\left(x\right)-\frac{\ln\left(x^{10}+x^5+1\right)}{10}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\,\sqrt{3}\,x^5}{3}+\frac{\sqrt{3}}{3}\right)}{15}$$

input
```
int(1/(x*(x^5 + x^10 + 1)),x)
```

output
```
log(x) - log(x^5 + x^10 + 1)/10 - (3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^5
)/3))/15
```

3.410 $\int \frac{1}{x^6(1+x^5+x^{10})}\,dx$

3.410.1 Optimal result

Integrand size = 14, antiderivative size = 48

$$\int \frac{1}{x^6(1+x^5+x^{10})}\,dx=-\frac{1}{5x^5}-\frac{\arctan\left(\frac{1+2x^5}{\sqrt{3}}\right)}{5\sqrt{3}}-\log(x)+\frac{1}{10}\log\left(1+x^5+x^{10}\right)$$

output
```
-1/5/x^5-ln(x)+1/10*ln(x^10+x^5+1)-1/15*arctan(1/3*(2*x^5+1)*3^(1/2))*3^(1
/2)
```

3.410.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 208, normalized size of antiderivative = 4.33

$$\int \frac{1}{x^6(1+x^5+x^{10})}\,dx=\frac{1}{30}\left(-\frac{6}{x^5}+2\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right)-30\log(x)+3\log\left(1+x+x^2\right)\right.$$
$$+6\text{RootSum}\Big[1-\#1+\#1^3-\#1^4+\#1^5-\#1^7$$
$$+\#1^8\&,\frac{-\log(x-\#1)+\log(x-\#1)\#1+\log(x-\#1)\#1^2-3\log(x-\#1)\#1^3+2\log(x-\#1)\#1^4 \ldots}{-1+3\#1^2-4\#1^3+5\#1^4-7\#1^6+8\ldots}$$

input
```
Integrate[1/(x^6*(1 + x^5 + x^10)),x]
```

output
```
(-6/x^5 + 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 30*Log[x] + 3*Log[1 + x +
x^2] + 6*RootSum[1 - #1 + #1^3 - #1^4 + #1^5 - #1^7 + #1^8 & , (-Log[x - #
1] + Log[x - #1]*#1 + Log[x - #1]*#1^2 - 3*Log[x - #1]*#1^3 + 2*Log[x - #1
]*#1^4 + Log[x - #1]*#1^5 - 4*Log[x - #1]*#1^6 + 4*Log[x - #1]*#1^7)/(-1 +
 3*#1^2 - 4*#1^3 + 5*#1^4 - 7*#1^6 + 8*#1^7) & ])/30
```

3.410.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {1693, 1145, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6\left(x^{10}+x^5+1\right)}dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{5}\int \frac{1}{x^{10}\left(x^{10}+x^5+1\right)}dx^5$$

$$\Big\downarrow \text{1145}$$

$$\frac{1}{5}\left(\int -\frac{x^5+1}{x^5\left(x^{10}+x^5+1\right)}dx^5-\frac{1}{x^5}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{5}\left(-\int \frac{x^5+1}{x^5\left(x^{10}+x^5+1\right)}dx^5-\frac{1}{x^5}\right)$$

$$\Big\downarrow \text{1200}$$

$$\frac{1}{5}\left(-\int \left(\frac{1}{x^5}-\frac{x^5}{x^{10}+x^5+1}\right)dx^5-\frac{1}{x^5}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}\left(-\frac{\arctan\left(\frac{2x^5+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{x^5}-\log\left(x^5\right)+\frac{1}{2}\log\left(x^{10}+x^5+1\right)\right)$$

input
```
Int[1/(x^6*(1 + x^5 + x^10)),x]
```

output
```
(-x^(-5) - ArcTan[(1 + 2*x^5)/Sqrt[3]]/Sqrt[3] - Log[x^5] + Log[1 + x^5 +
x^10]/2)/5
```

3.410.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1145
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp
[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[m, -1]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.410.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.79

method	result
risch	$-\frac{1}{5x^5}-\ln\left(x\right)+\frac{\ln\left(x^{10}+x^5+1\right)}{10}-\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x^5+\frac{1}{2}\right)\sqrt{3}}{3}\right)}{15}$
default	$-\frac{1}{5x^5}-\ln\left(x\right)+\frac{\left(\frac{1}{2}+\frac{i\sqrt{3}}{6}\right)\ln\left(2x^4+\left(-1-i\sqrt{3}\right)x^3+\left(-1+i\sqrt{3}\right)x^2+2x-1-i\sqrt{3}\right)}{5}+\frac{\left(\frac{1}{2}-\frac{i\sqrt{3}}{6}\right)\ln\left(2x^4+\left(-1+i\sqrt{3}\right)x^3+\left(-1\right.\right.}{5}$

input
```
int(1/x^6/(x^10+x^5+1),x,method=_RETURNVERBOSE)
```

output
```
-1/5/x^5-ln(x)+1/10*ln(x^10+x^5+1)-1/15*3^(1/2)*arctan(2/3*(x^5+1/2)*3^(1/
2))
```

3.410.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.02

$$\int \frac{1}{x^6 (1+x^5+x^{10})} \, dx = -\frac{2 \sqrt{3} x^5 \arctan\left(\frac{1}{3} \sqrt{3}(2 x^5+1)\right) - 3 x^5 \log\left(x^{10}+x^5+1\right) + 30 x^5 \log(x) + 6}{30 x^5}$$

input
```
integrate(1/x^6/(x^10+x^5+1),x, algorithm="fricas")
```

output
```
-1/30*(2*sqrt(3)*x^5*arctan(1/3*sqrt(3)*(2*x^5 + 1)) - 3*x^5*log(x^10 + x^
5 + 1) + 30*x^5*log(x) + 6)/x^5
```

3.410.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^6 (1+x^5+x^{10})} \, dx = -\log(x) + \frac{\log\left(x^{10}+x^5+1\right)}{10} - \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x^5}{3} + \frac{\sqrt{3}}{3}\right)}{15} - \frac{1}{5x^5}$$

input
```
integrate(1/x**6/(x**10+x**5+1),x)
```

output
```
-log(x) + log(x**10 + x**5 + 1)/10 - sqrt(3)*atan(2*sqrt(3)*x**5/3 + sqrt(
3)/3)/15 - 1/(5*x**5)
```

3.410.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^6 (1+x^5+x^{10})} \, dx = -\frac{1}{15} \sqrt{3} \arctan \left(\frac{1}{3} \sqrt{3}(2 x^5+1)\right) - \frac{1}{5 x^5} + \frac{1}{10} \log \left(x^{10}+x^5+1\right) - \frac{1}{5} \log \left(x^5\right)$$

input
```
integrate(1/x^6/(x^10+x^5+1),x, algorithm="maxima")
```

output
```
-1/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^5 + 1)) - 1/5/x^5 + 1/10*log(x^10 +
x^5 + 1) - 1/5*log(x^5)
```

3.410.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^6 (1+x^5+x^{10})} \, dx = -\frac{1}{15} \sqrt{3} \arctan \left(\frac{1}{3} \sqrt{3}(2 x^5+1)\right) + \frac{x^5-1}{5 x^5} + \frac{1}{10} \log \left(x^{10}+x^5+1\right) - \log \left(|x|\right)$$

input
```
integrate(1/x^6/(x^10+x^5+1),x, algorithm="giac")
```

output
```
-1/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^5 + 1)) + 1/5*(x^5 - 1)/x^5 + 1/10*l
og(x^10 + x^5 + 1) - log(abs(x))
```

3.410.9 Mupad [B] (verification not implemented)

Time = 8.30 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^6 (1+x^5+x^{10})} \, dx = \frac{\ln \left(x^{10}+x^5+1\right)}{10} - \ln \left(x\right) - \frac{\sqrt{3} \operatorname{atan}\left(\frac{2 \sqrt{3} x^5}{3}+\frac{\sqrt{3}}{3}\right)}{15} - \frac{1}{5 x^5}$$

input
```
int(1/(x^6*(x^5 + x^10 + 1)),x)
```

output
```
log(x^5 + x^10 + 1)/10 - log(x) - (3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^5
)/3))/15 - 1/(5*x^5)
```

3.411 $\int \frac{1}{x+x^6+x^{11}}\,dx$

3.411.1 Optimal result

Integrand size = 10, antiderivative size = 39

$$\int \frac{1}{x+x^6+x^{11}}\,dx=-\frac{\arctan\left(\frac{1+2x^5}{\sqrt{3}}\right)}{5\sqrt{3}}+\log(x)-\frac{1}{10}\log\left(1+x^5+x^{10}\right)$$

output
```
ln(x)-1/10*ln(x^10+x^5+1)-1/15*arctan(1/3*(2*x^5+1)*3^(1/2))*3^(1/2)
```

3.411.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 197, normalized size of antiderivative = 5.05

$$\int \frac{1}{x+x^6+x^{11}}\,dx$$
$$=\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{5\sqrt{3}}+\log(x)-\frac{1}{10}\log\left(1+x+x^2\right)-\frac{1}{5}\text{RootSum}\Big[1-\#1+\#1^3-\#1^4+\#1^5-\#1^7$$
$$+\#1^8\&,\frac{-\log(x-\#1)\#1+2\log(x-\#1)\#1^2-\log(x-\#1)\#1^3+3\log(x-\#1)\#1^4-\log(x-\#1)\#}{-1+3\#1^2-4\#1^3+5\#1^4-7\#1^6+8\#1^7}$$

input
```
Integrate[(x + x^6 + x^11)^(-1),x]
```

output
```
ArcTan[(1 + 2*x)/Sqrt[3]]/(5*Sqrt[3]) + Log[x] - Log[1 + x + x^2]/10 - Roo
tSum[1 - #1 + #1^3 - #1^4 + #1^5 - #1^7 + #1^8 & , (-(Log[x - #1]*#1) + 2*
Log[x - #1]*#1^2 - Log[x - #1]*#1^3 + 3*Log[x - #1]*#1^4 - Log[x - #1]*#1^
5 - 3*Log[x - #1]*#1^6 + 4*Log[x - #1]*#1^7)/(-1 + 3*#1^2 - 4*#1^3 + 5*#1^
4 - 7*#1^6 + 8*#1^7) & ]/5
```

3.411.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {1949, 1693, 1144, 25, 1142, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{11}+x^6+x} \, dx$$

$$\Big\downarrow \text{1949}$$

$$\int \frac{1}{x\left(x^{10}+x^5+1\right)}dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{5}\int \frac{1}{x^5\left(x^{10}+x^5+1\right)}dx^5$$

$$\Big\downarrow \text{1144}$$

$$\frac{1}{5}\left(\int -\frac{x^5+1}{x^{10}+x^5+1}dx^5+\log\left(x^5\right)\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{5}\left(\log\left(x^5\right)-\int \frac{x^5+1}{x^{10}+x^5+1}dx^5\right)$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{5}\left(-\frac{1}{2}\int \frac{1}{x^{10}+x^5+1}dx^5-\frac{1}{2}\int \frac{2x^5+1}{x^{10}+x^5+1}dx^5+\log\left(x^5\right)\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{5}\left(\int \frac{1}{-x^{10}-3}d\left(2x^5+1\right)-\frac{1}{2}\int \frac{2x^5+1}{x^{10}+x^5+1}dx^5+\log\left(x^5\right)\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{5}\left(-\frac{1}{2}\int \frac{2x^5+1}{x^{10}+x^5+1}dx^5 - \frac{\arctan\left(\frac{2x^5+1}{\sqrt{3}}\right)}{\sqrt{3}} + \log\left(x^5\right)\right)$$

$$\Big\downarrow \ 1103$$

$$\frac{1}{5}\left(-\frac{\arctan\left(\frac{2x^5+1}{\sqrt{3}}\right)}{\sqrt{3}} + \log\left(x^5\right) - \frac{1}{2}\log\left(x^{10}+x^5+1\right)\right)$$

input
```
Int[(x + x^6 + x^11)^(-1),x]
```

output
```
(-(ArcTan[(1 + 2*x^5)/Sqrt[3]]/Sqrt[3]) + Log[x^5] - Log[1 + x^5 + x^10]/2
)/5
```

3.411.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.411. $\int \frac{1}{x+x^6+x^{11}}\,dx$

rule 1144
```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Simp[e*(Log[RemoveContent[d + e*x, x]]/(c*d^2 - b*d*e + a*e^2)), x] + S
imp[1/(c*d^2 - b*d*e + a*e^2)   Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2),
x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 1949
```
Int[((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_), x_Symbol
] :> Int[x^(p*q)*(a + b*x^(n - q) + c*x^(2*(n - q)))^p, x] /; FreeQ[{a, b,
c, n, q}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && IntegerQ[p]
```

3.411.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

method	result
risch	$\ln\left(x\right)-\frac{\ln\left(x^{10}+x^{5}+1\right)}{10}-\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x^{5}+\frac{1}{2}\right)\sqrt{3}}{3}\right)}{15}$
default	$\ln\left(x\right)-\frac{\left(\frac{1}{2}+\frac{i\sqrt{3}}{6}\right)\ln\left(2x^{4}+\left(-1+i\sqrt{3}\right)x^{3}+\left(-1-i\sqrt{3}\right)x^{2}+2x-1+i\sqrt{3}\right)}{5}-\frac{\left(\frac{1}{2}-\frac{i\sqrt{3}}{6}\right)\ln\left(2x^{4}+\left(-1-i\sqrt{3}\right)x^{3}+\left(-1+i\sqrt{3}\right)x^{2}+\right.}{5}$

input
```
int(1/(x^11+x^6+x),x,method=_RETURNVERBOSE)
```

output
```
ln(x)-1/10*ln(x^10+x^5+1)-1/15*3^(1/2)*arctan(2/3*(x^5+1/2)*3^(1/2))
```

3.411.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{1}{x+x^6+x^{11}}\,dx = -\frac{1}{15}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x^5+1)\right) - \frac{1}{10}\log\left(x^{10}+x^5+1\right) + \log\left(x\right)$$

input
```
integrate(1/(x^11+x^6+x),x, algorithm="fricas")
```

output
```
-1/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^5 + 1)) - 1/10*log(x^10 + x^5 + 1) +
 log(x)
```

3.411.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{1}{x+x^6+x^{11}}\,dx = \log\left(x\right) - \frac{\log\left(x^{10}+x^5+1\right)}{10} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^5}{3}+\frac{\sqrt{3}}{3}\right)}{15}$$

input
```
integrate(1/(x**11+x**6+x),x)
```

output
```
log(x) - log(x**10 + x**5 + 1)/10 - sqrt(3)*atan(2*sqrt(3)*x**5/3 + sqrt(3
)/3)/15
```

3.411.7 Maxima [F]

$$\int \frac{1}{x+x^6+x^{11}}\,dx = \int \frac{1}{x^{11}+x^6+x}\,dx$$

input
```
integrate(1/(x^11+x^6+x),x, algorithm="maxima")
```

output
```
1/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/5*integrate((4*x^7 - 3*x^6
- x^5 + 3*x^4 - x^3 + 2*x^2 - x)/(x^8 - x^7 + x^5 - x^4 + x^3 - x + 1), x)
 - 1/10*log(x^2 + x + 1) + log(x)
```

3.411.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int \frac{1}{x+x^6+x^{11}}\,dx = -\frac{1}{15}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^5+1)\right) - \frac{1}{10}\log\left(x^{10}+x^5+1\right) + \log\left(|x|\right)$$

input
```
integrate(1/(x^11+x^6+x),x, algorithm="giac")
```

output
```
-1/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^5 + 1)) - 1/10*log(x^10 + x^5 + 1) +
 log(abs(x))
```

3.411.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

$$\int \frac{1}{x+x^6+x^{11}}\,dx = \ln(x) - \frac{\ln\left(x^{10}+x^5+1\right)}{10} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x^5}{3}+\frac{\sqrt{3}}{3}\right)}{15}$$

input
```
int(1/(x + x^6 + x^11),x)
```

output
```
log(x) - log(x^5 + x^10 + 1)/10 - (3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^5
)/3))/15
```

3.412 $\int \frac{x^3}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx$

3.412.1 Optimal result

Integrand size = 18, antiderivative size = 147

$$\int \frac{x^3}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx = -\frac{b(b^2-2ac)\,x}{c^4}+\frac{(b^2-ac)\,x^2}{2c^3}-\frac{bx^3}{3c^2}+\frac{x^4}{4c} + \frac{b(b^4-5ab^2c+5a^2c^2)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^5\sqrt{b^2-4ac}} + \frac{(b^4-3ab^2c+a^2c^2)\log\left(a+bx+cx^2\right)}{2c^5}$$

output
```
-b*(-2*a*c+b^2)*x/c^4+1/2*(-a*c+b^2)*x^2/c^3-1/3*b*x^3/c^2+1/4*x^4/c+1/2*(
a^2*c^2-3*a*b^2*c+b^4)*ln(c*x^2+b*x+a)/c^5+b*(5*a^2*c^2-5*a*b^2*c+b^4)*arc
tanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^5/(-4*a*c+b^2)^(1/2)
```

3.412.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.95

$$\int \frac{x^3}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx$$

$$= \frac{cx(-12b^3+6b^2cx-4bc(-6a+cx^2)+3c^2x(-2a+cx^2)) - \frac{12b(b^4-5ab^2c+5a^2c^2)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}} + 6(b^4-3ab}{12c^5}$$

input
```
Integrate[x^3/(c + a/x^2 + b/x),x]
```

output
```
(c*x*(-12*b^3 + 6*b^2*c*x - 4*b*c*(-6*a + c*x^2) + 3*c^2*x*(-2*a + c*x^2))
 - (12*b*(b^4 - 5*a*b^2*c + 5*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*
c]])/Sqrt[-b^2 + 4*a*c] + 6*(b^4 - 3*a*b^2*c + a^2*c^2)*Log[a + x*(b + c*x
)])/(12*c^5)
```

3.412.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1692, 1143, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\frac{a}{x^2}+\frac{b}{x}+c}dx$$

$$\Big\downarrow \text{1692}$$

$$\int \frac{x^5}{a+bx+cx^2}dx$$

$$\Big\downarrow \text{1143}$$

$$\int \left(\frac{x\left(a^2c^2-3ab^2c+b^4\right)+ab\left(b^2-2ac\right)}{c^4\left(a+bx+cx^2\right)}-\frac{b\left(b^2-2ac\right)}{c^4}+\frac{x\left(b^2-ac\right)}{c^3}-\frac{bx^2}{c^2}+\frac{x^3}{c}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\left(5a^2c^2-5ab^2c+b^4\right)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^5\sqrt{b^2-4ac}}+\frac{\left(a^2c^2-3ab^2c+b^4\right)\log\left(a+bx+cx^2\right)}{2c^5}-\frac{bx\left(b^2-2ac\right)}{c^4}+\frac{x^2\left(b^2-ac\right)}{2c^3}-\frac{bx^3}{3c^2}+\frac{x^4}{4c}$$

input
```
Int[x^3/(c + a/x^2 + b/x),x]
```

output
```
-((b*(b^2 - 2*a*c)*x)/c^4) + ((b^2 - a*c)*x^2)/(2*c^3) - (b*x^3)/(3*c^2) +
 x^4/(4*c) + (b*(b^4 - 5*a*b^2*c + 5*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2
 - 4*a*c]])/(c^5*Sqrt[b^2 - 4*a*c]) + ((b^4 - 3*a*b^2*c + a^2*c^2)*Log[a +
 b*x + c*x^2])/(2*c^5)
```

3.412.3.1 Defintions of rubi rules used

rule 1143
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x)^m/(a + b*x + c*x^2), x], x] /; FreeQ[{a,
b, c, d, e}, x] && IGtQ[m, 1]
```

rule 1692
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.412.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.12

method	result
default	$\frac{\frac{1}{4}c^3x^4-\frac{1}{3}b\,c^2x^3-\frac{1}{2}a\,c^2x^2+\frac{1}{2}b^2c\,x^2+2abcx-b^3x}{c^4}+\frac{\frac{\left(a^2c^2-3a\,b^2c+b^4\right)\ln\left(c\,x^2+bx+a\right)}{2c}+\frac{2\left(-2cb\,a^2+a\,b^3-\frac{\left(a^2c^2-3a\,b^2c+b^4\right)b}{2c}\right)\arctan(}{\sqrt{4ac-b^2}}}{c^4}$
risch	Expression too large to display

input
```
int(x^3/(c+a/x^2+b/x),x,method=_RETURNVERBOSE)
```

output
```
1/c^4*(1/4*c^3*x^4-1/3*b*c^2*x^3-1/2*a*c^2*x^2+1/2*b^2*c*x^2+2*a*b*c*x-b^3
*x)+1/c^4*(1/2*(a^2*c^2-3*a*b^2*c+b^4)/c*ln(c*x^2+b*x+a)+2*(-2*c*b*a^2+a*b
^3-1/2*(a^2*c^2-3*a*b^2*c+b^4)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*
a*c-b^2)^(1/2)))
```

3.412.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 466, normalized size of antiderivative = 3.17

$$\int \frac{x^3}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx$$
$$= \left[\frac{3\,(b^2c^4-4\,ac^5)x^4-4\,(b^3c^3-4\,abc^4)x^3+6\,(b^4c^2-5\,ab^2c^3+4\,a^2c^4)x^2+6\,(b^5-5\,ab^3c+5\,a^2bc^2)\sqrt{b^2-\cdots}}{\cdots}\right.$$

input
```
integrate(x^3/(c+a/x^2+b/x),x, algorithm="fricas")
```

output
```
[1/12*(3*(b^2*c^4 - 4*a*c^5)*x^4 - 4*(b^3*c^3 - 4*a*b*c^4)*x^3 + 6*(b^4*c^
2 - 5*a*b^2*c^3 + 4*a^2*c^4)*x^2 + 6*(b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*sqrt(
b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2
*c*x + b))/(c*x^2 + b*x + a)) - 12*(b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*x +
 6*(b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*log(c*x^2 + b*x + a))/(b
^2*c^5 - 4*a*c^6), 1/12*(3*(b^2*c^4 - 4*a*c^5)*x^4 - 4*(b^3*c^3 - 4*a*b*c^
4)*x^3 + 6*(b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*x^2 + 12*(b^5 - 5*a*b^3*c +
 5*a^2*b*c^2)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b
^2 - 4*a*c)) - 12*(b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*x + 6*(b^6 - 7*a*b^4
*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*log(c*x^2 + b*x + a))/(b^2*c^5 - 4*a*c^6)
]
```

3.412.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 605 vs. $2(144) = 288$.

Time = 0.73 (sec) , antiderivative size = 605, normalized size of antiderivative = 4.12

$$\int \frac{x^3}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx$$
$$= -\frac{bx^3}{3c^2} + x^2\left(-\frac{a}{2c^2}+\frac{b^2}{2c^3}\right) + x\left(\frac{2ab}{c^3}-\frac{b^3}{c^4}\right) + \left(-\frac{b\sqrt{-4ac+b^2}\cdot(5a^2c^2-5ab^2c+b^4)}{2c^5\cdot(4ac-b^2)}\right.$$
$$\left.+\frac{a^2c^2-3ab^2c+b^4}{2c^5}\right)\log\left(x+\frac{2a^3c^2-4a^2b^2c+ab^4-4ac^5\left(-\frac{b\sqrt{-4ac+b^2}\cdot(5a^2c^2-5ab^2c+b^4)}{2c^5\cdot(4ac-b^2)}+\frac{a^2c^2-3ab^2c+b^4}{2c^5}\right)}{5a^2bc^2-5ab^3c+b^5}\right.$$
$$+\left(\frac{b\sqrt{-4ac+b^2}\cdot(5a^2c^2-5ab^2c+b^4)}{2c^5\cdot(4ac-b^2)}\right.$$
$$\left.+\frac{a^2c^2-3ab^2c+b^4}{2c^5}\right)\log\left(x+\frac{2a^3c^2-4a^2b^2c+ab^4-4ac^5\left(\frac{b\sqrt{-4ac+b^2}\cdot(5a^2c^2-5ab^2c+b^4)}{2c^5\cdot(4ac-b^2)}+\frac{a^2c^2-3ab^2c+b^4}{2c^5}\right)+}{5a^2bc^2-5ab^3c+b^5}\right.$$
$$+\frac{x^4}{4c}$$

input
```
integrate(x**3/(c+a/x**2+b/x),x)
```

output
```
-b*x**3/(3*c**2) + x**2*(-a/(2*c**2) + b**2/(2*c**3)) + x*(2*a*b/c**3 - b*
*3/c**4) + (-b*sqrt(-4*a*c + b**2)*(5*a**2*c**2 - 5*a*b**2*c + b**4)/(2*c*
*5*(4*a*c - b**2)) + (a**2*c**2 - 3*a*b**2*c + b**4)/(2*c**5))*log(x + (2*
a**3*c**2 - 4*a**2*b**2*c + a*b**4 - 4*a*c**5*(-b*sqrt(-4*a*c + b**2)*(5*a
**2*c**2 - 5*a*b**2*c + b**4)/(2*c**5*(4*a*c - b**2)) + (a**2*c**2 - 3*a*b
**2*c + b**4)/(2*c**5)) + b**2*c**4*(-b*sqrt(-4*a*c + b**2)*(5*a**2*c**2 -
 5*a*b**2*c + b**4)/(2*c**5*(4*a*c - b**2)) + (a**2*c**2 - 3*a*b**2*c + b*
*4)/(2*c**5)))/(5*a**2*b*c**2 - 5*a*b**3*c + b**5)) + (b*sqrt(-4*a*c + b**
2)*(5*a**2*c**2 - 5*a*b**2*c + b**4)/(2*c**5*(4*a*c - b**2)) + (a**2*c**2
- 3*a*b**2*c + b**4)/(2*c**5))*log(x + (2*a**3*c**2 - 4*a**2*b**2*c + a*b*
*4 - 4*a*c**5*(b*sqrt(-4*a*c + b**2)*(5*a**2*c**2 - 5*a*b**2*c + b**4)/(2*
c**5*(4*a*c - b**2)) + (a**2*c**2 - 3*a*b**2*c + b**4)/(2*c**5)) + b**2*c*
*4*(b*sqrt(-4*a*c + b**2)*(5*a**2*c**2 - 5*a*b**2*c + b**4)/(2*c**5*(4*a*c
 - b**2)) + (a**2*c**2 - 3*a*b**2*c + b**4)/(2*c**5)))/(5*a**2*b*c**2 - 5*
a*b**3*c + b**5)) + x**4/(4*c)
```

3.412. $\int \frac{x^3}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx$

3.412.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{c + \frac{a}{x^2} + \frac{b}{x}} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3/(c+a/x^2+b/x),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.412.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.99

$$\int \frac{x^3}{c + \frac{a}{x^2} + \frac{b}{x}} \, dx = \frac{3\, c^3 x^4 - 4\, b c^2 x^3 + 6\, b^2 c x^2 - 6\, a c^2 x^2 - 12\, b^3 x + 24\, abcx}{12\, c^4} + \frac{(b^4 - 3\, a b^2 c + a^2 c^2) \log(cx^2 + bx + a)}{2\, c^5} - \frac{(b^5 - 5\, a b^3 c + 5\, a^2 b c^2) \arctan\left(\frac{2\, cx + b}{\sqrt{-b^2 + 4\, ac}}\right)}{\sqrt{-b^2 + 4\, ac} c^5}$$

input
```
integrate(x^3/(c+a/x^2+b/x),x, algorithm="giac")
```

output
```
1/12*(3*c^3*x^4 - 4*b*c^2*x^3 + 6*b^2*c*x^2 - 6*a*c^2*x^2 - 12*b^3*x + 24*
a*b*c*x)/c^4 + 1/2*(b^4 - 3*a*b^2*c + a^2*c^2)*log(c*x^2 + b*x + a)/c^5 -
(b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sq
rt(-b^2 + 4*a*c)*c^5)
```

3.412.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.24

$$\int \frac{x^3}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx = x\left(\frac{b\left(\frac{a}{c^2}-\frac{b^2}{c^3}\right)}{c}+\frac{a\,b}{c^3}\right)+\frac{x^4}{4\,c}-x^2\left(\frac{a}{2\,c^2}-\frac{b^2}{2\,c^3}\right) - \frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(-4\,a^3\,c^3+13\,a^2\,b^2\,c^2-7\,a\,b^4\,c+b^6\right)}{2\,\left(4\,a\,c^6-b^2\,c^5\right)} - \frac{b\,x^3}{3\,c^2}-\frac{b\,\mathrm{atan}\left(\frac{b+2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)\,\left(5\,a^2\,c^2-5\,a\,b^2\,c+b^4\right)}{c^5\,\sqrt{4\,a\,c-b^2}}$$

input
```
int(x^3/(c + a/x^2 + b/x),x)
```

output
```
x*((b*(a/c^2 - b^2/c^3))/c + (a*b)/c^3) + x^4/(4*c) - x^2*(a/(2*c^2) - b^2
/(2*c^3)) - (log(a + b*x + c*x^2)*(b^6 - 4*a^3*c^3 + 13*a^2*b^2*c^2 - 7*a*
b^4*c))/(2*(4*a*c^6 - b^2*c^5)) - (b*x^3)/(3*c^2) - (b*atan((b + 2*c*x)/(4
*a*c - b^2)^(1/2))*(b^4 + 5*a^2*c^2 - 5*a*b^2*c))/(c^5*(4*a*c - b^2)^(1/2)
)
```

3.413 $\int \frac{x^2}{c+\frac{a}{x^2}+\frac{b}{x}} \, dx$

3.413.1 Optimal result

Integrand size = 18, antiderivative size = 118

$$\int \frac{x^2}{c+\frac{a}{x^2}+\frac{b}{x}} \, dx = \frac{(b^2-ac)\,x}{c^3} - \frac{bx^2}{2c^2} + \frac{x^3}{3c} - \frac{(b^4-4ab^2c+2a^2c^2)\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^4\sqrt{b^2-4ac}} - \frac{b(b^2-2ac)\log\left(a+bx+cx^2\right)}{2c^4}$$

output
```
(-a*c+b^2)*x/c^3-1/2*b*x^2/c^2+1/3*x^3/c-1/2*b*(-2*a*c+b^2)*ln(c*x^2+b*x+a
)/c^4-(2*a^2*c^2-4*a*b^2*c+b^4)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^4/
(-4*a*c+b^2)^(1/2)
```

3.413.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.95

$$\int \frac{x^2}{c+\frac{a}{x^2}+\frac{b}{x}} \, dx = \frac{cx(6b^2-6ac-3bcx+2c^2x^2) + \frac{6(b^4-4ab^2c+2a^2c^2)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}} - 3(b^3-2abc)\log(a+x(b+cx))}{6c^4}$$

input
```
Integrate[x^2/(c + a/x^2 + b/x),x]
```

output
```
(c*x*(6*b^2 - 6*a*c - 3*b*c*x + 2*c^2*x^2) + (6*(b^4 - 4*a*b^2*c + 2*a^2*c
^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] - 3*(b^3 -
2*a*b*c)*Log[a + x*(b + c*x)])/(6*c^4)
```

3.413.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1692, 1143, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\frac{a}{x^2}+\frac{b}{x}+c}dx$$

$$\Big\downarrow \text{ 1692}$$

$$\int \frac{x^4}{a+bx+cx^2}dx$$

$$\Big\downarrow \text{ 1143}$$

$$\int \left(-\frac{bx\left(b^2-2ac\right)+a\left(b^2-ac\right)}{c^3\left(a+bx+cx^2\right)}+\frac{b^2-ac}{c^3}-\frac{bx}{c^2}+\frac{x^2}{c}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\left(2a^2c^2-4ab^2c+b^4\right)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^4\sqrt{b^2-4ac}}-\frac{b\left(b^2-2ac\right)\log\left(a+bx+cx^2\right)}{2c^4}+\frac{x\left(b^2-ac\right)}{c^3}-\frac{bx^2}{2c^2}+\frac{x^3}{3c}$$

input
```
Int[x^2/(c + a/x^2 + b/x),x]
```

output
```
((b^2 - a*c)*x)/c^3 - (b*x^2)/(2*c^2) + x^3/(3*c) - ((b^4 - 4*a*b^2*c + 2*
a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^4*Sqrt[b^2 - 4*a*c]) -
 (b*(b^2 - 2*a*c)*Log[a + b*x + c*x^2])/(2*c^4)
```

3.413.3.1 Defintions of rubi rules used

rule 1143
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x)^m/(a + b*x + c*x^2), x], x] /; FreeQ[{a,
b, c, d, e}, x] && IGtQ[m, 1]
```

rule 1692
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.413.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.08

method	result	size
default	$-\frac{-\frac{1}{3}c^2x^3+\frac{1}{2}bc\,x^2+acx-b^2x}{c^3}+\frac{\frac{\left(2abc-b^3\right)\ln\left(c\,x^2+bx+a\right)}{2c}+\frac{2\left(c\,a^2-b^2a-\frac{\left(2abc-b^3\right)b}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{c^3}$	128
risch	Expression too large to display	1138

input
```
int(x^2/(c+a/x^2+b/x),x,method=_RETURNVERBOSE)
```

output
```
-1/c^3*(-1/3*c^2*x^3+1/2*b*c*x^2+a*c*x-b^2*x)+1/c^3*(1/2*(2*a*b*c-b^3)/c*l
n(c*x^2+b*x+a)+2*(c*a^2-b^2*a-1/2*(2*a*b*c-b^3)*b/c)/(4*a*c-b^2)^(1/2)*arc
tan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

3.413.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 383, normalized size of antiderivative = 3.25

$$\int \frac{x^2}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx$$

$$= \left[\frac{2\,(b^2c^3-4\,ac^4)x^3-3\,(b^3c^2-4\,abc^3)x^2+3\,(b^4-4\,ab^2c+2\,a^2c^2)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4}}{cx^2+bx+a}\right.}{6\,(b^2c^4-4\,ac^5)}\right.$$

input
```
integrate(x^2/(c+a/x^2+b/x),x, algorithm="fricas")
```

output
```
[1/6*(2*(b^2*c^3 - 4*a*c^4)*x^3 - 3*(b^3*c^2 - 4*a*b*c^3)*x^2 + 3*(b^4 - 4
*a*b^2*c + 2*a^2*c^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2
*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 6*(b^4*c - 5*a*
b^2*c^2 + 4*a^2*c^3)*x - 3*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*log(c*x^2 + b*x
 + a))/(b^2*c^4 - 4*a*c^5), 1/6*(2*(b^2*c^3 - 4*a*c^4)*x^3 - 3*(b^3*c^2 -
4*a*b*c^3)*x^2 - 6*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*sqrt(-b^2 + 4*a*c)*arctan
(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 6*(b^4*c - 5*a*b^2*c^2 +
 4*a^2*c^3)*x - 3*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*log(c*x^2 + b*x + a))/(b
^2*c^4 - 4*a*c^5)]
```

3.413.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 498 vs. $2(110) = 220$.

Time = 0.60 (sec) , antiderivative size = 498, normalized size of antiderivative = 4.22

$$\int \frac{x^2}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx = -\frac{bx^2}{2c^2} + x\left(-\frac{a}{c^2}+\frac{b^2}{c^3}\right) + \left(\frac{b(2ac-b^2)}{2c^4} - \frac{\sqrt{-4ac+b^2}\cdot(2a^2c^2-4ab^2c+b^4)}{2c^4\cdot(4ac-b^2)}\right)\log\left(x + \frac{-3a^2bc+ab^3+4ac^4\left(\frac{b(2ac-b^2)}{2c^4} - \frac{\sqrt{-4ac+b^2}\cdot(2a^2c^2-4ab^2c+b^4)}{2c^4\cdot(4ac-b^2)}\right)}{2a^2c^2-4ab^2c \cdots}\right) + \left(\frac{b(2ac-b^2)}{2c^4} + \frac{\sqrt{-4ac+b^2}\cdot(2a^2c^2-4ab^2c+b^4)}{2c^4\cdot(4ac-b^2)}\right)\log\left(x + \frac{-3a^2bc+ab^3+4ac^4\left(\frac{b(2ac-b^2)}{2c^4} + \frac{\sqrt{-4ac+b^2}\cdot(2a^2c^2-4ab^2c+b^4)}{2c^4\cdot(4ac-b^2)}\right)}{2a^2c^2-4ab^2c \cdots}\right) + \frac{x^3}{3c}$$

input
```
integrate(x**2/(c+a/x**2+b/x),x)
```

output
```
-b*x**2/(2*c**2) + x*(-a/c**2 + b**2/c**3) + (b*(2*a*c - b**2)/(2*c**4) -
sqrt(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*c**4*(4*a*c - b**
2)))*log(x + (-3*a**2*b*c + a*b**3 + 4*a*c**4*(b*(2*a*c - b**2)/(2*c**4) -
 sqrt(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*c**4*(4*a*c - b*
*2))) - b**2*c**3*(b*(2*a*c - b**2)/(2*c**4) - sqrt(-4*a*c + b**2)*(2*a**2
*c**2 - 4*a*b**2*c + b**4)/(2*c**4*(4*a*c - b**2))))/(2*a**2*c**2 - 4*a*b*
*2*c + b**4)) + (b*(2*a*c - b**2)/(2*c**4) + sqrt(-4*a*c + b**2)*(2*a**2*c
**2 - 4*a*b**2*c + b**4)/(2*c**4*(4*a*c - b**2)))*log(x + (-3*a**2*b*c + a
*b**3 + 4*a*c**4*(b*(2*a*c - b**2)/(2*c**4) + sqrt(-4*a*c + b**2)*(2*a**2*
c**2 - 4*a*b**2*c + b**4)/(2*c**4*(4*a*c - b**2))) - b**2*c**3*(b*(2*a*c -
 b**2)/(2*c**4) + sqrt(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2
*c**4*(4*a*c - b**2))))/(2*a**2*c**2 - 4*a*b**2*c + b**4)) + x**3/(3*c)
```

3.413.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2/(c+a/x^2+b/x),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.413.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.96

$$\int \frac{x^2}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx = \frac{2\,c^2x^3-3\,bcx^2+6\,b^2x-6\,acx}{6\,c^3} - \frac{(b^3-2\,abc)\log\left(cx^2+bx+a\right)}{2\,c^4} + \frac{(b^4-4\,ab^2c+2\,a^2c^2)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}c^4}$$

input
```
integrate(x^2/(c+a/x^2+b/x),x, algorithm="giac")
```

output
```
1/6*(2*c^2*x^3 - 3*b*c*x^2 + 6*b^2*x - 6*a*c*x)/c^3 - 1/2*(b^3 - 2*a*b*c)*
log(c*x^2 + b*x + a)/c^4 + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*arctan((2*c*x + b
)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^4)
```

3.413.9 Mupad [B] (verification not implemented)

Time = 8.29 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.28

$$\int \frac{x^2}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx = \frac{x^3}{3\,c}-x\left(\frac{a}{c^2}-\frac{b^2}{c^3}\right)-\frac{b\,x^2}{2\,c^2}+\frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(8\,a^2\,b\,c^2-6\,a\,b^3\,c+b^5\right)}{2\,\left(4\,a\,c^5-b^2\,c^4\right)} + \frac{\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)\,\left(2\,a^2\,c^2-4\,a\,b^2\,c+b^4\right)}{c^4\,\sqrt{4\,a\,c-b^2}}$$

input
```
int(x^2/(c + a/x^2 + b/x),x)
```

output
```
x^3/(3*c) - x*(a/c^2 - b^2/c^3) - (b*x^2)/(2*c^2) + (log(a + b*x + c*x^2)*
(b^5 + 8*a^2*b*c^2 - 6*a*b^3*c))/(2*(4*a*c^5 - b^2*c^4)) + (atan(b/(4*a*c
- b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2))*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))
/(c^4*(4*a*c - b^2)^(1/2))
```

3.414 $\int \frac{x}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx$

3.414.1 Optimal result

Integrand size = 16, antiderivative size = 89

$$\int \frac{x}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx=-\frac{bx}{c^2}+\frac{x^2}{2c}+\frac{b(b^2-3ac)\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}}+\frac{(b^2-ac)\log\left(a+bx+cx^2\right)}{2c^3}$$

output
```
-b*x/c^2+1/2*x^2/c+1/2*(-a*c+b^2)*ln(c*x^2+b*x+a)/c^3+b*(-3*a*c+b^2)*arcta
nh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^3/(-4*a*c+b^2)^(1/2)
```

3.414.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.94

$$\int \frac{x}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx=\frac{cx(-2b+cx)-\frac{2b(b^2-3ac)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+(b^2-ac)\log(a+x(b+cx))}{2c^3}$$

input
```
Integrate[x/(c + a/x^2 + b/x),x]
```

output
```
(c*x*(-2*b + c*x) - (2*b*(b^2 - 3*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*
c]])/Sqrt[-b^2 + 4*a*c] + (b^2 - a*c)*Log[a + x*(b + c*x)])/(2*c^3)
```

3.414.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1692, 1143, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\frac{a}{x^2}+\frac{b}{x}+c}dx$$

$$\Big\downarrow \text{1692}$$

$$\int \frac{x^3}{a+bx+cx^2}dx$$

$$\Big\downarrow \text{1143}$$

$$\int \left(\frac{x(b^2-ac)+ab}{c^2(a+bx+cx^2)}-\frac{b}{c^2}+\frac{x}{c}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{b(b^2-3ac)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}}+\frac{(b^2-ac)\log(a+bx+cx^2)}{2c^3}-\frac{bx}{c^2}+\frac{x^2}{2c}$$

input
```
Int[x/(c + a/x^2 + b/x),x]
```

output
```
-((b*x)/c^2) + x^2/(2*c) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 -
 4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - a*c)*Log[a + b*x + c*x^2])/(2*
c^3)
```

3.414.3.1 Defintions of rubi rules used

rule 1143
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x)^m/(a + b*x + c*x^2), x], x] /; FreeQ[{a,
b, c, d, e}, x] && IGtQ[m, 1]
```

rule 1692
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.414.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.10

method	result
default	$-\frac{-\frac{1}{2}c\,x^2+bx}{c^2}+\frac{\frac{(-ac+b^2)\ln(c\,x^2+bx+a)}{2c}+\frac{2\left(ab-\frac{(-ac+b^2)b}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{c^2}$
risch	$\frac{x^2}{2c}-\frac{bx}{c^2}-\frac{2\ln\left(12a^2b\,c^2-7a\,b^3c+b^5-2\sqrt{-b^2(4ac-b^2)(3ac-b^2)^2}\,cx-\sqrt{-b^2(4ac-b^2)(3ac-b^2)^2}\,b\right)a^2}{c(4ac-b^2)}+\frac{5\ln\left(12a^2b\,c^2-7a\,b^3c-\right.}{}$

input
```
int(x/(c+a/x^2+b/x),x,method=_RETURNVERBOSE)
```

output
```
-1/c^2*(-1/2*c*x^2+b*x)+1/c^2*(1/2*(-a*c+b^2)/c*ln(c*x^2+b*x+a)+2*(a*b-1/2
*(-a*c+b^2)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

3.414.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 297, normalized size of antiderivative = 3.34

$$\int \frac{x}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx$$
$$=\left[\frac{(b^2c^2-4\,ac^3)x^2-(b^3-3\,abc)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)-2\,(b^3c-4\,abc^2)x+(b}{2\,(b^2c^3-4\,ac^4)}\right.$$

input
```
integrate(x/(c+a/x^2+b/x),x, algorithm="fricas")
```

output
```
[1/2*((b^2*c^2 - 4*a*c^3)*x^2 - (b^3 - 3*a*b*c)*sqrt(b^2 - 4*a*c)*log((2*c
^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b
*x + a)) - 2*(b^3*c - 4*a*b*c^2)*x + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*log(c*x
^2 + b*x + a))/(b^2*c^3 - 4*a*c^4), 1/2*((b^2*c^2 - 4*a*c^3)*x^2 + 2*(b^3
- 3*a*b*c)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2
- 4*a*c)) - 2*(b^3*c - 4*a*b*c^2)*x + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*log(c*
x^2 + b*x + a))/(b^2*c^3 - 4*a*c^4)]
```

3.414.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 381 vs. $2(83) = 166$.

Time = 0.47 (sec) , antiderivative size = 381, normalized size of antiderivative = 4.28

$$
\begin{aligned}
\int \frac{x}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx = & -\frac{bx}{c^2} + \left(-\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{2c^3\cdot(4ac-b^2)}\right. \\
& \left.-\frac{ac-b^2}{2c^3}\right)\log\left(x+\frac{2a^2c-ab^2+4ac^3\left(-\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{2c^3\cdot(4ac-b^2)}-\frac{ac-b^2}{2c^3}\right)-b^2c^2\left(-\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{2c^3\cdot(4ac-b^2)}-\frac{ac-b^2}{2c^3}\right.}{3abc-b^3}\right. \\
& + \left(\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{2c^3\cdot(4ac-b^2)}\right. \\
& \left.-\frac{ac-b^2}{2c^3}\right)\log\left(x+\frac{2a^2c-ab^2+4ac^3\left(\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{2c^3\cdot(4ac-b^2)}-\frac{ac-b^2}{2c^3}\right)-b^2c^2\left(\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{2c^3\cdot(4ac-b^2)}-\frac{ac-b^2}{2c^3}\right)}{3abc-b^3}\right) \\
& + \frac{x^2}{2c}
\end{aligned}
$$

input
```
integrate(x/(c+a/x**2+b/x),x)
```

output
```
-b*x/c**2 + (-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*c**3*(4*a*c - b**2))
 - (a*c - b**2)/(2*c**3))*log(x + (2*a**2*c - a*b**2 + 4*a*c**3*(-b*sqrt(-
4*a*c + b**2)*(3*a*c - b**2)/(2*c**3*(4*a*c - b**2)) - (a*c - b**2)/(2*c**
3)) - b**2*c**2*(-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*c**3*(4*a*c - b*
*2)) - (a*c - b**2)/(2*c**3)))/(3*a*b*c - b**3)) + (b*sqrt(-4*a*c + b**2)*
(3*a*c - b**2)/(2*c**3*(4*a*c - b**2)) - (a*c - b**2)/(2*c**3))*log(x + (2
*a**2*c - a*b**2 + 4*a*c**3*(b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*c**3*
(4*a*c - b**2)) - (a*c - b**2)/(2*c**3)) - b**2*c**2*(b*sqrt(-4*a*c + b**2
)*(3*a*c - b**2)/(2*c**3*(4*a*c - b**2)) - (a*c - b**2)/(2*c**3)))/(3*a*b*
c - b**3)) + x**2/(2*c)
```

3.414.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x/(c+a/x^2+b/x),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.414.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.97

$$\int \frac{x}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx = \frac{cx^2-2\,bx}{2\,c^2}+\frac{(b^2-ac)\log\left(cx^2+bx+a\right)}{2\,c^3}-\frac{(b^3-3\,abc)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}c^3}$$

input
```
integrate(x/(c+a/x^2+b/x),x, algorithm="giac")
```

output
```
1/2*(c*x^2 - 2*b*x)/c^2 + 1/2*(b^2 - a*c)*log(c*x^2 + b*x + a)/c^3 - (b^3
- 3*a*b*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^3)
```

3.414.9 Mupad [B] (verification not implemented)

Time = 8.32 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.26

$$\int \frac{x}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx = \frac{x^2}{2\,c}-\frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(4\,a^2\,c^2-5\,a\,b^2\,c+b^4\right)}{2\,\left(4\,a\,c^4-b^2\,c^3\right)} - \frac{b\,x}{c^2}+\frac{b\,\mathrm{atan}\left(\frac{b+2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)\,\left(3\,a\,c-b^2\right)}{c^3\,\sqrt{4\,a\,c-b^2}}$$

input
```
int(x/(c + a/x^2 + b/x),x)
```

output
```
x^2/(2*c) - (log(a + b*x + c*x^2)*(b^4 + 4*a^2*c^2 - 5*a*b^2*c))/(2*(4*a*c
^4 - b^2*c^3)) - (b*x)/c^2 + (b*atan((b + 2*c*x)/(4*a*c - b^2)^(1/2))*(3*a
*c - b^2))/(c^3*(4*a*c - b^2)^(1/2))
```

3.414. $\int \frac{x}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx$

3.415 $\int \frac{1}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx$

3.415.1 Optimal result

Integrand size = 14, antiderivative size = 70

$$\int \frac{1}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx=\frac{x}{c}-\frac{(b^2-2ac)\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx+cx^2\right)}{2c^2}$$

output
```
x/c-1/2*b*ln(c*x^2+b*x+a)/c^2-(-2*a*c+b^2)*arctanh((2*c*x+b)/(-4*a*c+b^2)^
(1/2))/c^2/(-4*a*c+b^2)^(1/2)
```

3.415.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.04

$$\int \frac{1}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx=\frac{x}{c}+\frac{(b^2-2ac)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{c^2\sqrt{-b^2+4ac}}-\frac{b\log\left(a+bx+cx^2\right)}{2c^2}$$

input
```
Integrate[(c + a/x^2 + b/x)^(-1),x]
```

output
```
x/c + ((b^2 - 2*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(c^2*Sqrt[-b^
2 + 4*a*c]) - (b*Log[a + b*x + c*x^2])/(2*c^2)
```

3.415.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1679, 1143, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\frac{a}{x^2}+\frac{b}{x}+c}dx$$

$$\Big\downarrow \text{1679}$$

$$\int \frac{x^2}{a+bx+cx^2}dx$$

$$\Big\downarrow \text{1143}$$

$$\int \left(\frac{1}{c}-\frac{a+bx}{c\left(a+bx+cx^2\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(b^2-2ac\right)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx+cx^2\right)}{2c^2}+\frac{x}{c}$$

input
```
Int[(c + a/x^2 + b/x)^(-1),x]
```

output
```
x/c - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^2*Sqrt[b^2
 - 4*a*c]) - (b*Log[a + b*x + c*x^2])/(2*c^2)
```

3.415.3.1 Defintions of rubi rules used

rule 1143
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x)^m/(a + b*x + c*x^2), x], x] /; FreeQ[{a,
b, c, d, e}, x] && IGtQ[m, 1]
```

rule 1679
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(
2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n]
 && LtQ[n, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.415.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.07

method	result
default	$\frac{x}{c}+\frac{-\frac{b\ln\left(c\,x^{2}+bx+a\right)}{2c}+\frac{2\left(-a+\frac{b^{2}}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{c}$
risch	$\frac{x}{c}-\frac{2\ln\left(-8a^{2}c^{2}+6a\,b^{2}c-b^{4}-2\sqrt{-(4ac-b^{2})(2ac-b^{2})^{2}}\,cx-\sqrt{-(4ac-b^{2})(2ac-b^{2})^{2}}\,b\right)ab}{c(4ac-b^{2})}+\frac{\ln\left(-8a^{2}c^{2}+6a\,b^{2}c-b^{4}-2\sqrt{-(4ac-\right.}{}$

input
```
int(1/(c+a/x^2+b/x),x,method=_RETURNVERBOSE)
```

output
```
x/c+1/c*(-1/2*b/c*ln(c*x^2+b*x+a)+2*(-a+1/2/c*b^2)/(4*a*c-b^2)^(1/2)*arcta
n((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

3.415.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 235, normalized size of antiderivative = 3.36

$$\int \frac{1}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx$$
$$=\left[-\frac{(b^2-2\,ac)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)-2\,(b^2c-4\,ac^2)x+(b^3-4\,abc)\log\left(cx^2\right.}{2\,(b^2c^2-4\,ac^3)}\right.$$
$$-\frac{2\,(b^2-2\,ac)\sqrt{-b^2+4\,ac}\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx+b)}{b^2-4\,ac}\right)-2\,(b^2c-4\,ac^2)x+(b^3-4\,abc)\log\left(cx^2+bx+a\right.}{2\,(b^2c^2-4\,ac^3)}$$

input
```
integrate(1/(c+a/x^2+b/x),x, algorithm="fricas")
```

output
```
[-1/2*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*
a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 2*(b^2*c - 4*a*c
^2)*x + (b^3 - 4*a*b*c)*log(c*x^2 + b*x + a))/(b^2*c^2 - 4*a*c^3), -1/2*(2
*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(
b^2 - 4*a*c)) - 2*(b^2*c - 4*a*c^2)*x + (b^3 - 4*a*b*c)*log(c*x^2 + b*x +
a))/(b^2*c^2 - 4*a*c^3)]
```

3.415.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(65) = 130$.

Time = 0.34 (sec) , antiderivative size = 306, normalized size of antiderivative = 4.37

$$\int \frac{1}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx = \left(-\frac{b}{2c^2} - \frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{2c^2\cdot(4ac-b^2)}\right)\log\left(x + \frac{-ab-4ac^2\left(-\frac{b}{2c^2}-\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{2c^2\cdot(4ac-b^2)}\right)+b^2c\left(-\frac{b}{2c^2}-\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{2c^2\cdot(4ac-b^2)}\right)}{2ac-b^2}\right) + \left(-\frac{b}{2c^2} + \frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{2c^2\cdot(4ac-b^2)}\right)\log\left(x + \frac{-ab-4ac^2\left(-\frac{b}{2c^2}+\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{2c^2\cdot(4ac-b^2)}\right)+b^2c\left(-\frac{b}{2c^2}+\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{2c^2\cdot(4ac-b^2)}\right)}{2ac-b^2}\right) + \frac{x}{c}$$

input
```
integrate(1/(c+a/x**2+b/x),x)
```

output
```
(-b/(2*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*c**2*(4*a*c - b**2)))
*log(x + (-a*b - 4*a*c**2*(-b/(2*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2
)/(2*c**2*(4*a*c - b**2))) + b**2*c*(-b/(2*c**2) - sqrt(-4*a*c + b**2)*(2*
a*c - b**2)/(2*c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + (-b/(2*c**2) + sqr
t(-4*a*c + b**2)*(2*a*c - b**2)/(2*c**2*(4*a*c - b**2)))*log(x + (-a*b - 4
*a*c**2*(-b/(2*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*c**2*(4*a*c -
 b**2))) + b**2*c*(-b/(2*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*c**
2*(4*a*c - b**2))))/(2*a*c - b**2)) + x/c
```

3.415.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(c+a/x^2+b/x),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.415.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.96

$$\int \frac{1}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx = \frac{x}{c} - \frac{b\log\left(cx^2+bx+a\right)}{2\,c^2} + \frac{\left(b^2-2\,ac\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}c^2}$$

input
```
integrate(1/(c+a/x^2+b/x),x, algorithm="giac")
```

output
```
x/c - 1/2*b*log(c*x^2 + b*x + a)/c^2 + (b^2 - 2*a*c)*arctan((2*c*x + b)/sq
rt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

3.415.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.46

$$\int \frac{1}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx = \frac{x}{c} + \frac{b^3\,\ln\left(c\,x^2+b\,x+a\right)}{2\,\left(4\,a\,c^3-b^2\,c^2\right)} - \frac{2\,a\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)}{c\,\sqrt{4\,a\,c-b^2}} + \frac{b^2\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)}{c^2\,\sqrt{4\,a\,c-b^2}} - \frac{2\,a\,b\,c\,\ln\left(c\,x^2+b\,x+a\right)}{4\,a\,c^3-b^2\,c^2}$$

input
```
int(1/(c + a/x^2 + b/x),x)
```

output
```
x/c + (b^3*log(a + b*x + c*x^2))/(2*(4*a*c^3 - b^2*c^2)) - (2*a*atan(b/(4*
a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2)))/(c*(4*a*c - b^2)^(1/2)) +
 (b^2*atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2)))/(c^2*(4*a
*c - b^2)^(1/2)) - (2*a*b*c*log(a + b*x + c*x^2))/(4*a*c^3 - b^2*c^2)
```

3.415. $\int \frac{1}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx$

3.416 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x}\,dx$

3.416.1 Optimal result

Integrand size = 18, antiderivative size = 56

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x}\,dx=\frac{b\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx+cx^2\right)}{2c}$$

output
```
1/2*ln(c*x^2+b*x+a)/c+b*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c/(-4*a*c+b^
2)^(1/2)
```

3.416.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.02

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x}\,dx=\frac{-\frac{2b\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+\log(a+x(b+cx))}{2c}$$

input
```
Integrate[1/((c + a/x^2 + b/x)*x),x]
```

output
```
((-2*b*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + Log[a
+ x*(b + c*x)])/(2*c)
```

3.416.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1692, 1142, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(\frac{a}{x^2}+\frac{b}{x}+c\right)}dx$$

$$\Big\downarrow \text{ 1692}$$

$$\int \frac{x}{a+bx+cx^2}dx$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{\int \frac{b+2cx}{cx^2+bx+a}dx}{2c}-\frac{b\int \frac{1}{cx^2+bx+a}dx}{2c}$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{b\int \frac{1}{b^2-(b+2cx)^2-4ac}d(b+2cx)}{c}+\frac{\int \frac{b+2cx}{cx^2+bx+a}dx}{2c}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\int \frac{b+2cx}{cx^2+bx+a}dx}{2c}+\frac{b\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{b\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx+cx^2\right)}{2c}$$

input
```
Int[1/((c + a/x^2 + b/x)*x),x]
```

output
```
(b*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 - 4*a*c]) + Log[a +
 b*x + c*x^2]/(2*c)
```

3.416.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1692
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

3.416.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

method	result
default	$\frac{\ln(c x^2+bx+a)}{2c}-\frac{b\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{c\sqrt{4ac-b^2}}$
risch	$\frac{2\ln\left(-2\sqrt{-b^2(4ac-b^2)}\,cx-4abc+b^3-\sqrt{-b^2(4ac-b^2)}\,b\right)a}{4ac-b^2}-\frac{\ln\left(-2\sqrt{-b^2(4ac-b^2)}\,cx-4abc+b^3-\sqrt{-b^2(4ac-b^2)}\,b\right)b^2}{2c(4ac-b^2)}+$ ln(−2

input
```
int(1/(c+a/x^2+b/x)/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(c*x^2+b*x+a)/c-b/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(
1/2))
```

3.416.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 185, normalized size of antiderivative = 3.30

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x}\,dx = \left[\frac{\sqrt{b^2-4\,ac}b\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)+(b^2-4\,ac)\log\left(cx^2+bx+a\right)}{2\,(b^2c-4\,ac^2)}, \frac{2\,\sqrt{-b^2+4\,ac}b\,\mathrm{arcta}}{}\right.$$

input
```
integrate(1/(c+a/x^2+b/x)/x,x, algorithm="fricas")
```

output
```
[1/2*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^
2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + (b^2 - 4*a*c)*log(c*x^2 + b*x
 + a))/(b^2*c - 4*a*c^2), 1/2*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-sqrt(-b^2 +
4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + (b^2 - 4*a*c)*log(c*x^2 + b*x + a))/(b
^2*c - 4*a*c^2)]
```

3.416.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. 2(49) = 98.

Time = 0.17 (sec) , antiderivative size = 216, normalized size of antiderivative = 3.86

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x}\,dx = \left(-\frac{b\sqrt{-4ac+b^2}}{2c\,(4ac-b^2)}+\frac{1}{2c}\right)\log\left(x+\frac{-4ac\left(-\frac{b\sqrt{-4ac+b^2}}{2c(4ac-b^2)}+\frac{1}{2c}\right)+2a+b^2\left(-\frac{b\sqrt{-4ac+b^2}}{2c(4ac-b^2)}+\frac{1}{2c}\right)}{b}\right) + \left(\frac{b\sqrt{-4ac+b^2}}{2c\,(4ac-b^2)}+\frac{1}{2c}\right)\log\left(x+\frac{-4ac\left(\frac{b\sqrt{-4ac+b^2}}{2c(4ac-b^2)}+\frac{1}{2c}\right)+2a+b^2\left(\frac{b\sqrt{-4ac+b^2}}{2c(4ac-b^2)}+\frac{1}{2c}\right)}{b}\right)$$

input
```
integrate(1/(c+a/x**2+b/x)/x,x)
```

output
```
(-b*sqrt(-4*a*c + b**2)/(2*c*(4*a*c - b**2)) + 1/(2*c))*log(x + (-4*a*c*(-
b*sqrt(-4*a*c + b**2)/(2*c*(4*a*c - b**2)) + 1/(2*c)) + 2*a + b**2*(-b*sqr
t(-4*a*c + b**2)/(2*c*(4*a*c - b**2)) + 1/(2*c)))/b) + (b*sqrt(-4*a*c + b*
*2)/(2*c*(4*a*c - b**2)) + 1/(2*c))*log(x + (-4*a*c*(b*sqrt(-4*a*c + b**2)
/(2*c*(4*a*c - b**2)) + 1/(2*c)) + 2*a + b**2*(b*sqrt(-4*a*c + b**2)/(2*c*
(4*a*c - b**2)) + 1/(2*c)))/b)
```

3.416.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(c+a/x^2+b/x)/x,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.416.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.98

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x}\,dx = -\frac{b\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}c} + \frac{\log\left(cx^2+bx+a\right)}{2\,c}$$

input
```
integrate(1/(c+a/x^2+b/x)/x,x, algorithm="giac")
```

output
```
-b*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c) + 1/2*log
(c*x^2 + b*x + a)/c
```

3.416.9 Mupad [B] (verification not implemented)

Time = 8.39 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.00

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x}\,dx = \frac{2\,a\,c\,\ln\left(c\,x^2+b\,x+a\right)}{4\,a\,c^2-b^2\,c} - \frac{b\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)}{c\,\sqrt{4\,a\,c-b^2}} - \frac{b^2\,\ln\left(c\,x^2+b\,x+a\right)}{2\,\left(4\,a\,c^2-b^2\,c\right)}$$

input
```
int(1/(x*(c + a/x^2 + b/x)),x)
```

output
```
(2*a*c*log(a + b*x + c*x^2))/(4*a*c^2 - b^2*c) - (b*atan(b/(4*a*c - b^2)^(
1/2) + (2*c*x)/(4*a*c - b^2)^(1/2)))/(c*(4*a*c - b^2)^(1/2)) - (b^2*log(a
+ b*x + c*x^2))/(2*(4*a*c^2 - b^2*c))
```

3.417 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^2}\,dx$

3.417.1 Optimal result

Integrand size = 18, antiderivative size = 36

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^2}\,dx=\frac{2\text{arctanh}\left(\frac{b+\frac{2a}{x}}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

output
```
2*arctanh((b+2*a/x)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)
```

3.417.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^2}\,dx=\frac{2\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}$$

input
```
Integrate[1/((c + a/x^2 + b/x)*x^2),x]
```

output
```
(2*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c]
```

3.417.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1690, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(\frac{a}{x^2}+\frac{b}{x}+c\right)}dx$$

$$\Big\downarrow \text{ 1690}$$

$$-\int \frac{1}{\frac{a}{x^2}+c+\frac{b}{x}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 1083}$$

$$2\int \frac{1}{b^2-4ac-\frac{1}{x^2}}d\left(\frac{2a}{x}+b\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{2\text{arctanh}\left(\frac{\frac{2a}{x}+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

input
```
Int[1/((c + a/x^2 + b/x)*x^2),x]
```

output
```
(2*ArcTanh[(b + (2*a)/x)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]
```

3.417.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

3.417.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{2\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}$	35
risch	$-\frac{\ln\left(2cx+\sqrt{-4ac+b^2}+b\right)}{\sqrt{-4ac+b^2}}+\frac{\ln\left(-2cx+\sqrt{-4ac+b^2}-b\right)}{\sqrt{-4ac+b^2}}$	61

input
```
int(1/(c+a/x^2+b/x)/x^2,x,method=_RETURNVERBOSE)
```

output
```
2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
```

3.417.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 120, normalized size of antiderivative = 3.33

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^2}\,dx = \left[\frac{\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)}{\sqrt{b^2-4\,ac}},\ -\frac{2\,\sqrt{-b^2+4\,ac}\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx+b)}{b^2-4\,ac}\right)}{b^2-4\,ac}\right]$$

input
```
integrate(1/(c+a/x^2+b/x)/x^2,x, algorithm="fricas")
```

output
```
[log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(
c*x^2 + b*x + a))/sqrt(b^2 - 4*a*c), -2*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b
^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c))/(b^2 - 4*a*c)]
```

3.417.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(32) = 64$.

Time = 0.11 (sec) , antiderivative size = 124, normalized size of antiderivative = 3.44

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^2}\,dx = -\sqrt{-\frac{1}{4ac-b^2}}\log\left(x+\frac{-4ac\sqrt{-\frac{1}{4ac-b^2}}+b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right) + \sqrt{-\frac{1}{4ac-b^2}}\log\left(x+\frac{4ac\sqrt{-\frac{1}{4ac-b^2}}-b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)$$

input
```
integrate(1/(c+a/x**2+b/x)/x**2,x)
```

output
```
-sqrt(-1/(4*a*c - b**2))*log(x + (-4*a*c*sqrt(-1/(4*a*c - b**2)) + b**2*sq
rt(-1/(4*a*c - b**2)) + b)/(2*c)) + sqrt(-1/(4*a*c - b**2))*log(x + (4*a*c
*sqrt(-1/(4*a*c - b**2)) - b**2*sqrt(-1/(4*a*c - b**2)) + b)/(2*c))
```

3.417.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(c+a/x^2+b/x)/x^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.417.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^2}\,dx = \frac{2\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}}$$

input
```
integrate(1/(c+a/x^2+b/x)/x^2,x, algorithm="giac")
```

output
```
2*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/sqrt(-b^2 + 4*a*c)
```

3.417.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.28

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^2}\,dx = \frac{2\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)}{\sqrt{4\,a\,c-b^2}}$$

input
```
int(1/(x^2*(c + a/x^2 + b/x)),x)
```

output
```
(2*atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2)))/(4*a*c - b^2
)^(1/2)
```

3.418 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^3}\,dx$

3.418.1 Optimal result

Integrand size = 18, antiderivative size = 62

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^3}\,dx=\frac{b\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}+\frac{\log(x)}{a}-\frac{\log\left(a+bx+cx^2\right)}{2a}$$

output
```
ln(x)/a-1/2*ln(c*x^2+b*x+a)/a+b*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/a/(-
4*a*c+b^2)^(1/2)
```

3.418.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.98

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^3}\,dx=-\frac{\frac{2b\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}-2\log(x)+\log(a+x(b+cx))}{2a}$$

input
```
Integrate[1/((c + a/x^2 + b/x)*x^3),x]
```

output
```
-1/2*((2*b*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] - 2*
Log[x] + Log[a + x*(b + c*x)])/a
```

3.418.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1692, 1144, 25, 1142, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\left(\frac{a}{x^2}+\frac{b}{x}+c\right)}dx$$

$$\Big\downarrow \ 1692$$

$$\int \frac{1}{x\left(a+bx+cx^2\right)}dx$$

$$\Big\downarrow \ 1144$$

$$\frac{\int -\frac{b+cx}{cx^2+bx+a}dx}{a}+\frac{\log(x)}{a}$$

$$\Big\downarrow \ 25$$

$$\frac{\log(x)}{a}-\frac{\int \frac{b+cx}{cx^2+bx+a}dx}{a}$$

$$\Big\downarrow \ 1142$$

$$\frac{\log(x)}{a}-\frac{\frac{1}{2}b\int \frac{1}{cx^2+bx+a}dx+\frac{1}{2}\int \frac{b+2cx}{cx^2+bx+a}dx}{a}$$

$$\Big\downarrow \ 1083$$

$$\frac{\log(x)}{a}-\frac{\frac{1}{2}\int \frac{b+2cx}{cx^2+bx+a}dx-b\int \frac{1}{b^2-(b+2cx)^2-4ac}d(b+2cx)}{a}$$

$$\Big\downarrow \ 219$$

$$\frac{\log(x)}{a}-\frac{\frac{1}{2}\int \frac{b+2cx}{cx^2+bx+a}dx-\frac{b\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}}{a}$$

$$\Big\downarrow \ 1103$$

$$\frac{\log(x)}{a}-\frac{\frac{1}{2}\log\left(a+bx+cx^2\right)-\frac{b\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}}{a}$$

input
```
Int[1/((c + a/x^2 + b/x)*x^3),x]
```

3.418. $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^3}\,dx$

output
```
Log[x]/a - (-((b*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]
) + Log[a + b*x + c*x^2]/2)/a
```

3.418.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1144
```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Simp[e*(Log[RemoveContent[d + e*x, x]]/(c*d^2 - b*d*e + a*e^2)), x] + S
imp[1/(c*d^2 - b*d*e + a*e^2)   Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2),
x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1692
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

3.418. $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^3}\,dx$

3.418.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.98

method	result
default	$\frac{\ln(x)}{a}+\frac{-\frac{\ln\left(c\,x^2+bx+a\right)}{2}-\frac{b\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{a}$
risch	$-\frac{2\ln\left(\left(\left(8a\,b^2c-2b^4+6\sqrt{-b^2(4ac-b^2)}\,ac-2\sqrt{-b^2(4ac-b^2)}\,b^2\right)x+12cb\,a^2-3a\,b^3-\sqrt{-b^2(4ac-b^2)}\,ab\right)c}{4ac-b^2}+\frac{\ln\left(\left(8a\,b^2c-2b^4+6\sqrt{-b\right.\right.}}{}$

input
```
int(1/(c+a/x^2+b/x)/x^3,x,method=_RETURNVERBOSE)
```

output
```
ln(x)/a+1/a*(-1/2*ln(c*x^2+b*x+a)-b/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*
a*c-b^2)^(1/2)))
```

3.418.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 211, normalized size of antiderivative = 3.40

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^3}\,dx$$
$$=\left[\frac{\sqrt{b^2-4\,ac}b\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)-(b^2-4\,ac)\log\left(cx^2+bx+a\right)+2\,(b^2-4\,ac)\log\left(x\right)}{2\,(ab^2-4\,a^2c)}\right.$$

input
```
integrate(1/(c+a/x^2+b/x)/x^3,x, algorithm="fricas")
```

output
```
[1/2*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^
2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - (b^2 - 4*a*c)*log(c*x^2 + b*x
 + a) + 2*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c), 1/2*(2*sqrt(-b^2 + 4*a*
c)*b*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - (b^2 - 4*a*c)
*log(c*x^2 + b*x + a) + 2*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c)]
```

3.418.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 564 vs. $2(54) = 108$.

Time = 4.46 (sec) , antiderivative size = 564, normalized size of antiderivative = 9.10

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^3}\,dx = \left(-\frac{b\sqrt{-4ac+b^2}}{2a\,(4ac-b^2)} - \frac{1}{2a}\right)\log\left(x + \frac{24a^4c^2\left(-\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)}-\frac{1}{2a}\right)^2 - 14a^3b^2c\left(-\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)}-\frac{1}{2a}\right)^2 - 12a^3c^2\left(-\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)}-\frac{1}{2a}\right) \cdots}{9abc^2 - 2 \cdots}\right.$$

$$+\left(\frac{b\sqrt{-4ac+b^2}}{2a\,(4ac-b^2)} - \frac{1}{2a}\right)\log\left(x + \frac{24a^4c^2\left(\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)}-\frac{1}{2a}\right)^2 - 14a^3b^2c\left(\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)}-\frac{1}{2a}\right)^2 - 12a^3c^2\left(\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)}-\frac{1}{2a}\right) + 2a^2 \cdots}{9abc^2 - 2b^3c \cdots}\right.$$

$$+\frac{\log(x)}{a}$$

input
```
integrate(1/(c+a/x**2+b/x)/x**3,x)
```

output
```
(-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))*log(x + (24*a**4*c
**2*(-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))**2 - 14*a**3*b
**2*c*(-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))**2 - 12*a**3
*c**2*(-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a)) + 2*a**2*b**
4*(-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))**2 + 3*a**2*b**2
*c*(-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a)) - 12*a**2*c**2
+ 11*a*b**2*c - 2*b**4)/(9*a*b*c**2 - 2*b**3*c)) + (b*sqrt(-4*a*c + b**2)/
(2*a*(4*a*c - b**2)) - 1/(2*a))*log(x + (24*a**4*c**2*(b*sqrt(-4*a*c + b**
2)/(2*a*(4*a*c - b**2)) - 1/(2*a))**2 - 14*a**3*b**2*c*(b*sqrt(-4*a*c + b*
*2)/(2*a*(4*a*c - b**2)) - 1/(2*a))**2 - 12*a**3*c**2*(b*sqrt(-4*a*c + b**
2)/(2*a*(4*a*c - b**2)) - 1/(2*a)) + 2*a**2*b**4*(b*sqrt(-4*a*c + b**2)/(2
*a*(4*a*c - b**2)) - 1/(2*a))**2 + 3*a**2*b**2*c*(b*sqrt(-4*a*c + b**2)/(2
*a*(4*a*c - b**2)) - 1/(2*a)) - 12*a**2*c**2 + 11*a*b**2*c - 2*b**4)/(9*a*
b*c**2 - 2*b**3*c)) + log(x)/a
```

3.418.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(c+a/x^2+b/x)/x^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.418.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^3}\,dx = -\frac{b\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}a} - \frac{\log\left(cx^2+bx+a\right)}{2\,a} + \frac{\log\left(|x|\right)}{a}$$

input
```
integrate(1/(c+a/x^2+b/x)/x^3,x, algorithm="giac")
```

output
```
-b*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a) - 1/2*log
(c*x^2 + b*x + a)/a + log(abs(x))/a
```

3.418.9 Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 213, normalized size of antiderivative = 3.44

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^3}\,dx = \frac{\ln(x)}{a} - \ln\left(b\,c - \left(x\left(6\,a\,c^2-2\,b^2\,c\right)-a\,b\,c\right)\left(\frac{1}{2\,a}-\frac{b\sqrt{b^2-4\,a\,c}}{2\left(a\,b^2-4\,a^2\,c\right)}\right) + 3\,c^2\,x\right)\left(\frac{1}{2\,a}-\frac{b\sqrt{b^2-4\,a\,c}}{2\left(a\,b^2-4\,a^2\,c\right)}\right) - \ln\left(\left(x\left(6\,a\,c^2-2\,b^2\,c\right)-a\,b\,c\right)\left(\frac{1}{2\,a}+\frac{b\sqrt{b^2-4\,a\,c}}{2\left(a\,b^2-4\,a^2\,c\right)}\right) - b\,c - 3\,c^2\,x\right)\left(\frac{1}{2\,a}+\frac{b\sqrt{b^2-4\,a\,c}}{2\left(a\,b^2-4\,a^2\,c\right)}\right)$$

input
```
int(1/(x^3*(c + a/x^2 + b/x)),x)
```

output
```
log(x)/a - log(b*c - (x*(6*a*c^2 - 2*b^2*c) - a*b*c)*(1/(2*a) - (b*(b^2 -
4*a*c)^(1/2))/(2*(a*b^2 - 4*a^2*c))) + 3*c^2*x)*(1/(2*a) - (b*(b^2 - 4*a*c
)^(1/2))/(2*(a*b^2 - 4*a^2*c))) - log((x*(6*a*c^2 - 2*b^2*c) - a*b*c)*(1/(
2*a) + (b*(b^2 - 4*a*c)^(1/2))/(2*(a*b^2 - 4*a^2*c))) - b*c - 3*c^2*x)*(1/
(2*a) + (b*(b^2 - 4*a*c)^(1/2))/(2*(a*b^2 - 4*a^2*c)))
```

3.418. $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^3}\,dx$

3.419 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^4}\,dx$

3.419.1 Optimal result

Integrand size = 18, antiderivative size = 81

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^4}\,dx=-\frac{1}{ax}-\frac{(b^2-2ac)\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx+cx^2\right)}{2a^2}$$

output
```
-1/a/x-b*ln(x)/a^2+1/2*b*ln(c*x^2+b*x+a)/a^2-(-2*a*c+b^2)*arctanh((2*c*x+b
)/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(1/2)
```

3.419.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.95

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^4}\,dx=\frac{-\frac{2a}{x}+\frac{2(b^2-2ac)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}-2b\log(x)+b\log(a+x(b+cx))}{2a^2}$$

input
```
Integrate[1/((c + a/x^2 + b/x)*x^4),x]
```

output
```
((-2*a)/x + (2*(b^2 - 2*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[
-b^2 + 4*a*c] - 2*b*Log[x] + b*Log[a + x*(b + c*x)])/(2*a^2)
```

3.419.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1692, 1145, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\left(\frac{a}{x^2}+\frac{b}{x}+c\right)}dx$$

$$\Big\downarrow \ 1692$$

$$\int \frac{1}{x^2\left(a+bx+cx^2\right)}dx$$

$$\Big\downarrow \ 1145$$

$$\frac{\int -\frac{b+cx}{x(cx^2+bx+a)}dx}{a}-\frac{1}{ax}$$

$$\Big\downarrow \ 25$$

$$-\frac{\int \frac{b+cx}{x(cx^2+bx+a)}dx}{a}-\frac{1}{ax}$$

$$\Big\downarrow \ 1200$$

$$-\frac{\int \left(\frac{b}{ax}+\frac{-b^2-cxb+ac}{a(cx^2+bx+a)}\right)dx}{a}-\frac{1}{ax}$$

$$\Big\downarrow \ 2009$$

$$-\frac{\frac{(b^2-2ac)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}-\frac{b\log(a+bx+cx^2)}{2a}+\frac{b\log(x)}{a}}{a}-\frac{1}{ax}$$

input
```
Int[1/((c + a/x^2 + b/x)*x^4),x]
```

output
```
-(1/(a*x)) - (((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a*Sq
rt[b^2 - 4*a*c]) + (b*Log[x])/a - (b*Log[a + b*x + c*x^2])/(2*a))/a
```

3.419.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1145
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp
[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[m, -1]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1692
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.419.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00

method	result
default	$-\frac{1}{ax}-\frac{b\ln(x)}{a^2}+\frac{\frac{b\ln\left(c\,x^2+bx+a\right)}{2}+\frac{2\left(-ac+\frac{b^2}{2}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{a^2}$
risch	$-\frac{1}{ax}-\frac{b\ln(x)}{a^2}+\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4a^3c-a^2b^2\right)\_Z^2+\left(-4abc+b^3\right)\_Z+c^2\right)}\_R\ln\left(\left(\left(6a^3c-2a^2b^2\right)\_R^2-2\_Rabc\right.\right.\right.$

input
```
int(1/(c+a/x^2+b/x)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/a/x-b*ln(x)/a^2+1/a^2*(1/2*b*ln(c*x^2+b*x+a)+2*(-a*c+1/2*b^2)/(4*a*c-b^
2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

3.419.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 269, normalized size of antiderivative = 3.32

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^4}\,dx$$
$$= \left[-\frac{(b^2-2\,ac)\sqrt{b^2-4\,ac}x\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)+2\,ab^2-8\,a^2c-(b^3-4\,abc)x\log\left(cx^2\right.}{2\,(a^2b^2-4\,a^3c)x}\right.$$
$$-\frac{2\,(b^2-2\,ac)\sqrt{-b^2+4\,ac}x\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx+b)}{b^2-4\,ac}\right)+2\,ab^2-8\,a^2c-(b^3-4\,abc)x\log\left(cx^2+bx+a\right.}{2\,(a^2b^2-4\,a^3c)x}$$

input
```
integrate(1/(c+a/x^2+b/x)/x^4,x, algorithm="fricas")
```

output
```
[-1/2*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*x*log((2*c^2*x^2 + 2*b*c*x + b^2 -
2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*a*b^2 - 8*a^
2*c - (b^3 - 4*a*b*c)*x*log(c*x^2 + b*x + a) + 2*(b^3 - 4*a*b*c)*x*log(x))
/((a^2*b^2 - 4*a^3*c)*x), -1/2*(2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*x*arcta
n(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*a*b^2 - 8*a^2*c - (b^
3 - 4*a*b*c)*x*log(c*x^2 + b*x + a) + 2*(b^3 - 4*a*b*c)*x*log(x))/((a^2*b^
2 - 4*a^3*c)*x)]
```

3.419.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^4}\,dx = \text{Timed out}$$

input
```
integrate(1/(c+a/x**2+b/x)/x**4,x)
```

output
```
Timed out
```

3.419.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(c+a/x^2+b/x)/x^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.419.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.98

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^4}\,dx = \frac{b\log\left(cx^2+bx+a\right)}{2\,a^2}-\frac{b\log\left(|x|\right)}{a^2}+\frac{(b^2-2\,ac)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}a^2}-\frac{1}{ax}$$

input
```
integrate(1/(c+a/x^2+b/x)/x^4,x, algorithm="giac")
```

output
```
1/2*b*log(c*x^2 + b*x + a)/a^2 - b*log(abs(x))/a^2 + (b^2 - 2*a*c)*arctan(
(2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^2) - 1/(a*x)
```

3.419.9 Mupad [B] (verification not implemented)

Time = 8.62 (sec) , antiderivative size = 339, normalized size of antiderivative = 4.19

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^4}\,dx$$

$$= \frac{\ln\left(2\,a\,b^3+2\,b^4\,x-2\,a\,b^2\,\sqrt{b^2-4\,a\,c}+a^2\,c\,\sqrt{b^2-4\,a\,c}-2\,b^3\,x\,\sqrt{b^2-4\,a\,c}+2\,a^2\,c^2\,x-7\,a^2\,b\,c-8\,a\,b^2\,c\right.}{4\,a^3\,c-a^2\,b^2}$$

$$-\frac{1}{a\,x}$$

$$-\frac{\ln\left(2\,a\,b^3+2\,b^4\,x+2\,a\,b^2\,\sqrt{b^2-4\,a\,c}-a^2\,c\,\sqrt{b^2-4\,a\,c}+2\,b^3\,x\,\sqrt{b^2-4\,a\,c}+2\,a^2\,c^2\,x-7\,a^2\,b\,c-8\,a\,b^2\right.}{4\,a^3\,c-a^2\,b^2}$$

$$-\frac{b\,\ln\left(x\right)}{a^2}$$

input
```
int(1/(x^4*(c + a/x^2 + b/x)),x)
```

output
```
(log(2*a*b^3 + 2*b^4*x - 2*a*b^2*(b^2 - 4*a*c)^(1/2) + a^2*c*(b^2 - 4*a*c)
^(1/2) - 2*b^3*x*(b^2 - 4*a*c)^(1/2) + 2*a^2*c^2*x - 7*a^2*b*c - 8*a*b^2*c
*x + 4*a*b*c*x*(b^2 - 4*a*c)^(1/2))*(a*(2*b*c - c*(b^2 - 4*a*c)^(1/2)) - b
^3/2 + (b^2*(b^2 - 4*a*c)^(1/2))/2))/(4*a^3*c - a^2*b^2) - 1/(a*x) - (log(
2*a*b^3 + 2*b^4*x + 2*a*b^2*(b^2 - 4*a*c)^(1/2) - a^2*c*(b^2 - 4*a*c)^(1/2
) + 2*b^3*x*(b^2 - 4*a*c)^(1/2) + 2*a^2*c^2*x - 7*a^2*b*c - 8*a*b^2*c*x -
4*a*b*c*x*(b^2 - 4*a*c)^(1/2))*(b^3/2 - a*(2*b*c + c*(b^2 - 4*a*c)^(1/2))
+ (b^2*(b^2 - 4*a*c)^(1/2))/2))/(4*a^3*c - a^2*b^2) - (b*log(x))/a^2
```

3.419. $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^4}\,dx$

3.420 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^5}\,dx$

3.420.1 Optimal result

Integrand size = 18, antiderivative size = 104

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^5}\,dx=-\frac{1}{2ax^2}+\frac{b}{a^2x}+\frac{b(b^2-3ac)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}}+\frac{(b^2-ac)\log(x)}{a^3}-\frac{(b^2-ac)\log(a+bx+cx^2)}{2a^3}$$

output
```
-1/2/a/x^2+b/a^2/x+(-a*c+b^2)*ln(x)/a^3-1/2*(-a*c+b^2)*ln(c*x^2+b*x+a)/a^3
+b*(-3*a*c+b^2)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)^(1/
2)
```

3.420.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.98

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^5}\,dx=\frac{-\frac{a^2}{x^2}+\frac{2ab}{x}-\frac{2b(b^2-3ac)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+2(b^2-ac)\log(x)+(-b^2+ac)\log(a+x(b+cx))}{2a^3}$$

input
```
Integrate[1/((c + a/x^2 + b/x)*x^5),x]
```

output
```
(-(a^2/x^2) + (2*a*b)/x - (2*b*(b^2 - 3*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2
+ 4*a*c]])/Sqrt[-b^2 + 4*a*c] + 2*(b^2 - a*c)*Log[x] + (-b^2 + a*c)*Log[a
+ x*(b + c*x)])/(2*a^3)
```

3.420.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1692, 1145, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5\left(\frac{a}{x^2}+\frac{b}{x}+c\right)}dx$$

$$\Big\downarrow \text{1692}$$

$$\int \frac{1}{x^3\left(a+bx+cx^2\right)}dx$$

$$\Big\downarrow \text{1145}$$

$$\frac{\int -\frac{b+cx}{x^2(cx^2+bx+a)}dx}{a}-\frac{1}{2ax^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{b+cx}{x^2(cx^2+bx+a)}dx}{a}-\frac{1}{2ax^2}$$

$$\Big\downarrow \text{1200}$$

$$-\frac{\int \left(\frac{b}{ax^2}+\frac{ac-b^2}{a^2x}+\frac{b(b^2-2ac)+c(b^2-ac)x}{a^2(cx^2+bx+a)}\right)dx}{a}-\frac{1}{2ax^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{b(b^2-3ac)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}}+\frac{(b^2-ac)\log(a+bx+cx^2)}{2a^2}-\frac{\log(x)(b^2-ac)}{a^2}-\frac{b}{ax}}{a}-\frac{1}{2ax^2}$$

input
```
Int[1/((c + a/x^2 + b/x)*x^5),x]
```

output
```
-1/2*1/(a*x^2) - (-(b/(a*x)) - (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b
^2 - 4*a*c]])/(a^2*Sqrt[b^2 - 4*a*c]) - ((b^2 - a*c)*Log[x])/a^2 + ((b^2 -
 a*c)*Log[a + b*x + c*x^2])/(2*a^2))/a
```

3.420.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1145
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp
[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[m, -1]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1692
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.420.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.23

method	result	size
default	$-\frac{1}{2 a \,x^{2}}+\frac{\left(-a c +b^{2}\right) \ln \left(x \right)}{a^{3}}+\frac{b}{a^{2} x}+\frac{\frac{\left(a \,c^{2}-b^{2} c \right) \ln \left(c \,x^{2}+b x +a \right)}{2 c}+\frac{2 \left(2 a b c -b^{3}-\frac{\left(a \,c^{2}-b^{2} c \right) b}{2 c}\right) \arctan \left(\frac{2 c x +b}{\sqrt{4 a c -b^{2}}}\right)}{\sqrt{4 a c -b^{2}}}}{a^{3}}$	128
risch	Expression too large to display	2265

input
```
int(1/(c+a/x^2+b/x)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/2/a/x^2+(-a*c+b^2)*ln(x)/a^3+b/a^2/x+1/a^3*(1/2*(a*c^2-b^2*c)/c*ln(c*x^
2+b*x+a)+2*(2*a*b*c-b^3-1/2*(a*c^2-b^2*c)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2
*c*x+b)/(4*a*c-b^2)^(1/2)))
```

3.420.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 358, normalized size of antiderivative = 3.44

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^5}\,dx = \left[-\frac{(b^3-3\,abc)\sqrt{b^2-4\,ac}x^2\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)+a^2b^2-4\,a^3c+(b^4-5\,ab^2c+4\,a^2c^2)}{2\,(a^3b^2-4\,a^4c)x^2}\right.$$

input
```
integrate(1/(c+a/x^2+b/x)/x^5,x, algorithm="fricas")
```

output
```
[-1/2*((b^3 - 3*a*b*c)*sqrt(b^2 - 4*a*c)*x^2*log((2*c^2*x^2 + 2*b*c*x + b^
2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + a^2*b^2 -
4*a^3*c + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*x^2*log(c*x^2 + b*x + a) - 2*(b^4
- 5*a*b^2*c + 4*a^2*c^2)*x^2*log(x) - 2*(a*b^3 - 4*a^2*b*c)*x)/((a^3*b^2 -
 4*a^4*c)*x^2), 1/2*(2*(b^3 - 3*a*b*c)*sqrt(-b^2 + 4*a*c)*x^2*arctan(-sqrt
(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - a^2*b^2 + 4*a^3*c - (b^4 - 5*a
*b^2*c + 4*a^2*c^2)*x^2*log(c*x^2 + b*x + a) + 2*(b^4 - 5*a*b^2*c + 4*a^2*
c^2)*x^2*log(x) + 2*(a*b^3 - 4*a^2*b*c)*x)/((a^3*b^2 - 4*a^4*c)*x^2)]
```

3.420.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^5}\,dx = \text{Timed out}$$

input
```
integrate(1/(c+a/x**2+b/x)/x**5,x)
```

output
```
Timed out
```

3.420.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^5}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(c+a/x^2+b/x)/x^5,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.420.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.01

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^5}\,dx = -\frac{(b^2-ac)\log(cx^2+bx+a)}{2\,a^3}+\frac{(b^2-ac)\log(|x|)}{a^3} - \frac{(b^3-3\,abc)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}a^3}+\frac{2\,abx-a^2}{2\,a^3x^2}$$

input
```
integrate(1/(c+a/x^2+b/x)/x^5,x, algorithm="giac")
```

output
```
-1/2*(b^2 - a*c)*log(c*x^2 + b*x + a)/a^3 + (b^2 - a*c)*log(abs(x))/a^3 -
(b^3 - 3*a*b*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)
*a^3) + 1/2*(2*a*b*x - a^2)/(a^3*x^2)
```

3.420.9 Mupad [B] (verification not implemented)

Time = 8.59 (sec) , antiderivative size = 447, normalized size of antiderivative = 4.30

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^5}\,dx$$

$$= \frac{\ln\left(2\,a\,b^4+2\,b^5\,x+6\,a^3\,c^2+2\,a\,b^3\,\sqrt{b^2-4\,a\,c}+2\,b^4\,x\,\sqrt{b^2-4\,a\,c}-9\,a^2\,b^2\,c-10\,a\,b^3\,c\,x-3\,a^2\,b\,c\,\sqrt{b^2-\right.}{4}$$

$$-\frac{\ln\left(2\,a\,b^4+2\,b^5\,x+6\,a^3\,c^2-2\,a\,b^3\,\sqrt{b^2-4\,a\,c}-2\,b^4\,x\,\sqrt{b^2-4\,a\,c}-9\,a^2\,b^2\,c-10\,a\,b^3\,c\,x+3\,a^2\,b\,c\,\sqrt{b^2\right.}{}$$

$$-\frac{\frac{1}{2\,a}-\frac{b\,x}{a^2}}{x^2}-\frac{\ln\left(x\right)\,\left(a\,c-b^2\right)}{a^3}$$

input
```
int(1/(x^5*(c + a/x^2 + b/x)),x)
```

output
```
(log(2*a*b^4 + 2*b^5*x + 6*a^3*c^2 + 2*a*b^3*(b^2 - 4*a*c)^(1/2) + 2*b^4*x
*(b^2 - 4*a*c)^(1/2) - 9*a^2*b^2*c - 10*a*b^3*c*x - 3*a^2*b*c*(b^2 - 4*a*c
)^(1/2) + 9*a^2*b*c^2*x + 3*a^2*c^2*x*(b^2 - 4*a*c)^(1/2) - 6*a*b^2*c*x*(b
^2 - 4*a*c)^(1/2))*(b^4/2 - a*((5*b^2*c)/2 + (3*b*c*(b^2 - 4*a*c)^(1/2))/2
) + (b^3*(b^2 - 4*a*c)^(1/2))/2 + 2*a^2*c^2))/(4*a^4*c - a^3*b^2) - (log(2
*a*b^4 + 2*b^5*x + 6*a^3*c^2 - 2*a*b^3*(b^2 - 4*a*c)^(1/2) - 2*b^4*x*(b^2
- 4*a*c)^(1/2) - 9*a^2*b^2*c - 10*a*b^3*c*x + 3*a^2*b*c*(b^2 - 4*a*c)^(1/2
) + 9*a^2*b*c^2*x - 3*a^2*c^2*x*(b^2 - 4*a*c)^(1/2) + 6*a*b^2*c*x*(b^2 - 4
*a*c)^(1/2))*(a*((5*b^2*c)/2 - (3*b*c*(b^2 - 4*a*c)^(1/2))/2) - b^4/2 + (b
^3*(b^2 - 4*a*c)^(1/2))/2 - 2*a^2*c^2))/(4*a^4*c - a^3*b^2) - (1/(2*a) - (
b*x)/a^2)/x^2 - (log(x)*(a*c - b^2))/a^3
```

3.421 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^6}\,dx$

3.421.1 Optimal result

Integrand size = 18, antiderivative size = 137

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^6}\,dx = -\frac{1}{3ax^3}+\frac{b}{2a^2x^2}-\frac{b^2-ac}{a^3x}-\frac{(b^4-4ab^2c+2a^2c^2)\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4\sqrt{b^2-4ac}} - \frac{b(b^2-2ac)\log(x)}{a^4}+\frac{b(b^2-2ac)\log\left(a+bx+cx^2\right)}{2a^4}$$

output
```
-1/3/a/x^3+1/2*b/a^2/x^2+(a*c-b^2)/a^3/x-b*(-2*a*c+b^2)*ln(x)/a^4+1/2*b*(-
2*a*c+b^2)*ln(c*x^2+b*x+a)/a^4-(2*a^2*c^2-4*a*b^2*c+b^4)*arctanh((2*c*x+b)
/(-4*a*c+b^2)^(1/2))/a^4/(-4*a*c+b^2)^(1/2)
```

3.421.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.96

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^6}\,dx = \frac{-\frac{2a^3}{x^3}+\frac{3a^2b}{x^2}+\frac{6a(-b^2+ac)}{x}+\frac{6(b^4-4ab^2c+2a^2c^2)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}-6(b^3-2abc)\log(x)+3(b^3-2abc)\log(a+x}{6a^4}$$

input
```
Integrate[1/((c + a/x^2 + b/x)*x^6),x]
```

output
```
((-2*a^3)/x^3 + (3*a^2*b)/x^2 + (6*a*(-b^2 + a*c))/x + (6*(b^4 - 4*a*b^2*c
 + 2*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] -
 6*(b^3 - 2*a*b*c)*Log[x] + 3*(b^3 - 2*a*b*c)*Log[a + x*(b + c*x)])/(6*a^4
)
```

3.421.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1692, 1145, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6\left(\frac{a}{x^2}+\frac{b}{x}+c\right)}dx$$

$$\Big\downarrow \text{1692}$$

$$\int \frac{1}{x^4\left(a+bx+cx^2\right)}dx$$

$$\Big\downarrow \text{1145}$$

$$\frac{\int -\frac{b+cx}{x^3(cx^2+bx+a)}dx}{a}-\frac{1}{3ax^3}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{b+cx}{x^3(cx^2+bx+a)}dx}{a}-\frac{1}{3ax^3}$$

$$\Big\downarrow \text{1200}$$

$$-\frac{\int \left(\frac{b}{ax^3}+\frac{b^3-2abc}{a^3x}+\frac{-b^4+3acb^2-c(b^2-2ac)xb-a^2c^2}{a^3(cx^2+bx+a)}+\frac{ac-b^2}{a^2x^2}\right)dx}{a}-\frac{1}{3ax^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{b(b^2-2ac)\log(a+bx+cx^2)}{2a^3}+\frac{b\log(x)(b^2-2ac)}{a^3}+\frac{b^2-ac}{a^2x}+\frac{(2a^2c^2-4ab^2c+b^4)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}}-\frac{b}{2ax^2}}{a}-\frac{1}{3ax^3}$$

input
```
Int[1/((c + a/x^2 + b/x)*x^6),x]
```

output
```
-1/3*1/(a*x^3) - (-1/2*b/(a*x^2) + (b^2 - a*c)/(a^2*x) + ((b^4 - 4*a*b^2*c
 + 2*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^3*Sqrt[b^2 - 4*a*
c]) + (b*(b^2 - 2*a*c)*Log[x])/a^3 - (b*(b^2 - 2*a*c)*Log[a + b*x + c*x^2]
)/(2*a^3))/a
```

3.421.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1145
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp
[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[m, -1]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1692
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.421.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.15

method	result
default	$-\frac{1}{3a\,x^3}-\frac{-ac+b^2}{x\,a^3}+\frac{b(2ac-b^2)\ln(x)}{a^4}+\frac{b}{2a^2x^2}+\frac{\frac{\left(-2ab\,c^2+b^3c\right)\ln\left(c\,x^2+bx+a\right)}{2c}+\frac{2\left(a^2c^2-3a\,b^2c+b^4-\frac{\left(-2ab\,c^2+b^3c\right)b}{2c}\right)\arctan\left(\right.}{\sqrt{4ac-b^2}}}{a^4}$
risch	Expression too large to display

input
```
int(1/(c+a/x^2+b/x)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/3/a/x^3-(-a*c+b^2)/x/a^3+b*(2*a*c-b^2)/a^4*ln(x)+1/2*b/a^2/x^2+1/a^4*(1
/2*(-2*a*b*c^2+b^3*c)/c*ln(c*x^2+b*x+a)+2*(a^2*c^2-3*a*b^2*c+b^4-1/2*(-2*a
*b*c^2+b^3*c)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

3.421.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 445, normalized size of antiderivative = 3.25

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^6}\,dx$$
$$= \left[\frac{3\,(b^4-4\,ab^2c+2\,a^2c^2)\sqrt{b^2-4\,ac}x^3\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)-2\,a^3b^2+8\,a^4c+3\,(b^5-6\,a\cdots}{\cdots}\right.$$
$$-\frac{6\,(b^4-4\,ab^2c+2\,a^2c^2)\sqrt{-b^2+4\,ac}x^3\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx+b)}{b^2-4\,ac}\right)+2\,a^3b^2-8\,a^4c-3\,(b^5-6\,ab^3c+8\cdots}{6\,(a\cdots}$$

input
```
integrate(1/(c+a/x^2+b/x)/x^6,x, algorithm="fricas")
```

output
```
[1/6*(3*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*sqrt(b^2 - 4*a*c)*x^3*log((2*c^2*x^2
 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a
)) - 2*a^3*b^2 + 8*a^4*c + 3*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*x^3*log(c*x^2
 + b*x + a) - 6*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*x^3*log(x) - 6*(a*b^4 - 5*
a^2*b^2*c + 4*a^3*c^2)*x^2 + 3*(a^2*b^3 - 4*a^3*b*c)*x)/((a^4*b^2 - 4*a^5*
c)*x^3), -1/6*(6*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*sqrt(-b^2 + 4*a*c)*x^3*arct
an(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*a^3*b^2 - 8*a^4*c -
3*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*x^3*log(c*x^2 + b*x + a) + 6*(b^5 - 6*a*
b^3*c + 8*a^2*b*c^2)*x^3*log(x) + 6*(a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*x^2
- 3*(a^2*b^3 - 4*a^3*b*c)*x)/((a^4*b^2 - 4*a^5*c)*x^3)]
```

3.421.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^6}\,dx = \text{Timed out}$$

input
```
integrate(1/(c+a/x**2+b/x)/x**6,x)
```

output
```
Timed out
```

3.421.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^6}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(c+a/x^2+b/x)/x^6,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.421.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.99

$$\begin{aligned}\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^6}\,dx &= \frac{(b^3-2\,abc)\log\left(cx^2+bx+a\right)}{2\,a^4} - \frac{(b^3-2\,abc)\log\left(|x|\right)}{a^4} \\ &\quad + \frac{(b^4-4\,ab^2c+2\,a^2c^2)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}a^4} \\ &\quad + \frac{3\,a^2bx-2\,a^3-6\,(ab^2-a^2c)x^2}{6\,a^4x^3}\end{aligned}$$

input
```
integrate(1/(c+a/x^2+b/x)/x^6,x, algorithm="giac")
```

output
```
1/2*(b^3 - 2*a*b*c)*log(c*x^2 + b*x + a)/a^4 - (b^3 - 2*a*b*c)*log(abs(x))
/a^4 + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c)
)/(sqrt(-b^2 + 4*a*c)*a^4) + 1/6*(3*a^2*b*x - 2*a^3 - 6*(a*b^2 - a^2*c)*x^
2)/(a^4*x^3)
```

3.421.9 Mupad [B] (verification not implemented)

Time = 8.60 (sec) , antiderivative size = 524, normalized size of antiderivative = 3.82

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^6}\,dx = \ln\left(2\,a\,b^4\,\sqrt{b^2-4\,a\,c}-2\,b^6\,x-2\,a\,b^5+2\,b^5\,x\,\sqrt{b^2-4\,a\,c}+11\,a^2\,b^3\,c-13\,a^3\,b\,c^2+2\,a^3\,c^3\,x+a^3\,c^2\,\sqrt{b^2-4\,a\,c}-17\,a^2\,b^2\,c^2\,x+12\,a\,b^4\,c\,x-5\,a^2\,b^2\,c\,\sqrt{b^2-4\,a\,c}-8\,a\,b^3\,c\,x\,\sqrt{b^2-4\,a\,c}+7\,a^2\,b\,c^2\,x\,\sqrt{b^2-4\,a\,c}\right)\left(\frac{b^3}{2\,a^4}-\frac{b^2\,\sqrt{b^2-4\,a\,c}}{2\,a^4}-\frac{b\,c}{a^3}+\frac{a^2\,c^2\,\sqrt{b^2-4\,a\,c}}{4\,a^5\,c-a^4\,b^2}\right)+\ln\left(2\,a\,b^5+2\,b^6\,x+2\,a\,b^4\,\sqrt{b^2-4\,a\,c}+2\,b^5\,x\,\sqrt{b^2-4\,a\,c}-11\,a^2\,b^3\,c+13\,a^3\,b\,c^2-2\,a^3\,c^3\,x+a^3\,c^2\,\sqrt{b^2-4\,a\,c}+17\,a^2\,b^2\,c^2\,x-12\,a\,b^4\,c\,x-5\,a^2\,b^2\,c\,\sqrt{b^2-4\,a\,c}-8\,a\,b^3\,c\,x\,\sqrt{b^2-4\,a\,c}+7\,a^2\,b\,c^2\,x\,\sqrt{b^2-4\,a\,c}\right)\left(\frac{b^3}{2\,a^4}+\frac{b^2\,\sqrt{b^2-4\,a\,c}}{2\,a^4}-\frac{b\,c}{a^3}-\frac{a^2\,c^2\,\sqrt{b^2-4\,a\,c}}{4\,a^5\,c-a^4\,b^2}\right)+\frac{\frac{x^2\,\left(a\,c-b^2\right)}{a^3}-\frac{1}{3\,a}+\frac{b\,x}{2\,a^2}}{x^3}+\frac{b\,\ln\left(x\right)\,\left(2\,a\,c-b^2\right)}{a^4}$$

input
```
int(1/(x^6*(c + a/x^2 + b/x)),x)
```

output
```
log(2*a*b^4*(b^2 - 4*a*c)^(1/2) - 2*b^6*x - 2*a*b^5 + 2*b^5*x*(b^2 - 4*a*c
)^(1/2) + 11*a^2*b^3*c - 13*a^3*b*c^2 + 2*a^3*c^3*x + a^3*c^2*(b^2 - 4*a*c
)^(1/2) - 17*a^2*b^2*c^2*x + 12*a*b^4*c*x - 5*a^2*b^2*c*(b^2 - 4*a*c)^(1/2
) - 8*a*b^3*c*x*(b^2 - 4*a*c)^(1/2) + 7*a^2*b*c^2*x*(b^2 - 4*a*c)^(1/2))*(
b^3/(2*a^4) - (b^2*(b^2 - 4*a*c)^(1/2))/(2*a^4) - (b*c)/a^3 + (a^2*c^2*(b^
2 - 4*a*c)^(1/2))/(4*a^5*c - a^4*b^2)) + log(2*a*b^5 + 2*b^6*x + 2*a*b^4*(
b^2 - 4*a*c)^(1/2) + 2*b^5*x*(b^2 - 4*a*c)^(1/2) - 11*a^2*b^3*c + 13*a^3*b
*c^2 - 2*a^3*c^3*x + a^3*c^2*(b^2 - 4*a*c)^(1/2) + 17*a^2*b^2*c^2*x - 12*a
*b^4*c*x - 5*a^2*b^2*c*(b^2 - 4*a*c)^(1/2) - 8*a*b^3*c*x*(b^2 - 4*a*c)^(1/
2) + 7*a^2*b*c^2*x*(b^2 - 4*a*c)^(1/2))*(b^3/(2*a^4) + (b^2*(b^2 - 4*a*c)^
(1/2))/(2*a^4) - (b*c)/a^3 - (a^2*c^2*(b^2 - 4*a*c)^(1/2))/(4*a^5*c - a^4*
b^2)) + ((x^2*(a*c - b^2))/a^3 - 1/(3*a) + (b*x)/(2*a^2))/x^3 + (b*log(x)*
(2*a*c - b^2))/a^4
```

3.421. $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^6}\,dx$

3.422 $\int \frac{x}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2}\,dx$

3.422.1 Optimal result

Integrand size = 16, antiderivative size = 196

$$\int \frac{x}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2}\,dx = -\frac{b(3b^2-11ac)\,x}{c^3\,(b^2-4ac)}+\frac{(3b^2-8ac)\,x^2}{2c^2\,(b^2-4ac)} - \frac{bx^3}{c\,(b^2-4ac)}+\frac{x^4(2a+bx)}{(b^2-4ac)\,(a+bx+cx^2)} + \frac{b(3b^4-20ab^2c+30a^2c^2)\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^4\,(b^2-4ac)^{3/2}} + \frac{(3b^2-2ac)\log\left(a+bx+cx^2\right)}{2c^4}$$

output
```
-b*(-11*a*c+3*b^2)*x/c^3/(-4*a*c+b^2)+1/2*(-8*a*c+3*b^2)*x^2/c^2/(-4*a*c+b
^2)-b*x^3/c/(-4*a*c+b^2)+x^4*(b*x+2*a)/(-4*a*c+b^2)/(c*x^2+b*x+a)+b*(30*a^
2*c^2-20*a*b^2*c+3*b^4)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^4/(-4*a*c+
b^2)^(3/2)+1/2*(-2*a*c+3*b^2)*ln(c*x^2+b*x+a)/c^4
```

3.422.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.83

$$\int \frac{x}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2}\,dx$$
$$= \frac{-4bcx+c^2x^2+\frac{2(2a^3c^2+b^5x+ab^3(b-5cx)+a^2bc(-4b+5cx))}{(b^2-4ac)(a+x(b+cx))}+\frac{2b(3b^4-20ab^2c+30a^2c^2)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}+(3b^2-2ac)\log(}{2c^4}$$

input
```
Integrate[x/(c + a/x^2 + b/x)^2,x]
```

output
```
(-4*b*c*x + c^2*x^2 + (2*(2*a^3*c^2 + b^5*x + a*b^3*(b - 5*c*x) + a^2*b*c*
(-4*b + 5*c*x)))/((b^2 - 4*a*c)*(a + x*(b + c*x))) + (2*b*(3*b^4 - 20*a*b^
2*c + 30*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(
3/2) + (3*b^2 - 2*a*c)*Log[a + x*(b + c*x)])/(2*c^4)
```

3.422.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1692, 1164, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(\frac{a}{x^2}+\frac{b}{x}+c\right)^2}dx$$

$$\Big\downarrow \; 1692$$

$$\int \frac{x^5}{(a+bx+cx^2)^2}dx$$

$$\Big\downarrow \; 1164$$

$$\frac{x^4(2a+bx)}{(b^2-4ac)\,(a+bx+cx^2)}-\frac{\int \frac{x^3(8a+3bx)}{cx^2+bx+a}dx}{b^2-4ac}$$

$$\Big\downarrow \; 1200$$

$$\frac{x^4(2a+bx)}{(b^2-4ac)\,(a+bx+cx^2)}-\frac{\int \left(\frac{3bx^2}{c}-\frac{(3b^2-8ac)x}{c^2}+\frac{b(3b^2-11ac)}{c^3}-\frac{ab(3b^2-11ac)+(b^2-4ac)\,(3b^2-2ac)x}{c^3(cx^2+bx+a)}\right)dx}{b^2-4ac}$$

$$\downarrow \text{2009}$$

$$\frac{x^4(2a+bx)}{(b^2-4ac)\,(a+bx+cx^2)} - \frac{-\frac{b(30a^2c^2-20ab^2c+3b^4)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^4\sqrt{b^2-4ac}} - \frac{(b^2-4ac)\,(3b^2-2ac)\log(a+bx+cx^2)}{2c^4} + \frac{bx(3b^2-11ac)}{c^3} - \frac{x^2(3b^2-8ac)}{2c^2} + \frac{bx^3}{c}}{b^2-4ac}$$

input
```
Int[x/(c + a/x^2 + b/x)^2,x]
```

output
```
(x^4*(2*a + b*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)) - ((b*(3*b^2 - 11*a*c)
*x)/c^3 - ((3*b^2 - 8*a*c)*x^2)/(2*c^2) + (b*x^3)/c - (b*(3*b^4 - 20*a*b^2
*c + 30*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^4*Sqrt[b^2 - 4
*a*c]) - ((b^2 - 4*a*c)*(3*b^2 - 2*a*c)*Log[a + b*x + c*x^2])/(2*c^4))/(b^
2 - 4*a*c)
```

3.422.3.1 Defintions of rubi rules used

rule 1164
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
 + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
 + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1692
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.422. $\int \frac{x}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2}\,dx$

3.422.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.21

method	result
default	$-\frac{-\frac{1}{2}c\,x^{2}+2bx}{c^{3}}+\frac{\frac{-\frac{b\left(5a^{2}c^{2}-5a\,b^{2}c+b^{4}\right)x}{c\left(4ac-b^{2}\right)}-\frac{a\left(2a^{2}c^{2}-4a\,b^{2}c+b^{4}\right)}{c\left(4ac-b^{2}\right)}}{c\,x^{2}+bx+a}+\frac{\frac{\left(-8a^{2}c^{2}+14a\,b^{2}c-3b^{4}\right)\ln\left(c\,x^{2}+bx+a\right)}{2c}+\frac{2\left(11cb\,a^{2}-3a\,b^{3}-\frac{\left(-8a^{2}c^{2}\cdots\right.}{\cdots}\right.}{4ac-b^{2}}}{c^{3}}$
risch	Expression too large to display

input
```
int(x/(c+a/x^2+b/x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/c^3*(-1/2*c*x^2+2*b*x)+1/c^3*((-b*(5*a^2*c^2-5*a*b^2*c+b^4)/c/(4*a*c-b^
2)*x-a/c*(2*a^2*c^2-4*a*b^2*c+b^4)/(4*a*c-b^2))/(c*x^2+b*x+a)+1/(4*a*c-b^2
)*(1/2*(-8*a^2*c^2+14*a*b^2*c-3*b^4)/c*ln(c*x^2+b*x+a)+2*(11*c*b*a^2-3*a*b
^3-1/2*(-8*a^2*c^2+14*a*b^2*c-3*b^4)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+
b)/(4*a*c-b^2)^(1/2))))
```

3.422.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 504 vs. 2(188) = 376.

Time = 0.29 (sec) , antiderivative size = 1029, normalized size of antiderivative = 5.25

$$\int \frac{x}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2}\,dx$$
$$= \left[\frac{2\,ab^6-16\,a^2b^4c+36\,a^3b^2c^2-16\,a^4c^3+(b^4c^3-8\,ab^2c^4+16\,a^2c^5)x^4-3\,(b^5c^2-8\,ab^3c^3+16\,a^2bc^4)x^3-\cdots}{\cdots}\right.$$

input
```
integrate(x/(c+a/x^2+b/x)^2,x, algorithm="fricas")
```

output
```
[1/2*(2*a*b^6 - 16*a^2*b^4*c + 36*a^3*b^2*c^2 - 16*a^4*c^3 + (b^4*c^3 - 8*
a*b^2*c^4 + 16*a^2*c^5)*x^4 - 3*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^3
 - (4*b^6*c - 33*a*b^4*c^2 + 72*a^2*b^2*c^3 - 16*a^3*c^4)*x^2 - (3*a*b^5 -
 20*a^2*b^3*c + 30*a^3*b*c^2 + (3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*x^2
 + (3*b^6 - 20*a*b^4*c + 30*a^2*b^2*c^2)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x
^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x +
 a)) + 2*(b^7 - 11*a*b^5*c + 41*a^2*b^3*c^2 - 52*a^3*b*c^3)*x + (3*a*b^6 -
 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c^3 + (3*b^6*c - 26*a*b^4*c^2 + 64
*a^2*b^2*c^3 - 32*a^3*c^4)*x^2 + (3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32
*a^3*b*c^3)*x)*log(c*x^2 + b*x + a))/(a*b^4*c^4 - 8*a^2*b^2*c^5 + 16*a^3*c
^6 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*x^2 + (b^5*c^4 - 8*a*b^3*c^5 + 1
6*a^2*b*c^6)*x), 1/2*(2*a*b^6 - 16*a^2*b^4*c + 36*a^3*b^2*c^2 - 16*a^4*c^3
 + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^4 - 3*(b^5*c^2 - 8*a*b^3*c^3 + 1
6*a^2*b*c^4)*x^3 - (4*b^6*c - 33*a*b^4*c^2 + 72*a^2*b^2*c^3 - 16*a^3*c^4)*
x^2 + 2*(3*a*b^5 - 20*a^2*b^3*c + 30*a^3*b*c^2 + (3*b^5*c - 20*a*b^3*c^2 +
 30*a^2*b*c^3)*x^2 + (3*b^6 - 20*a*b^4*c + 30*a^2*b^2*c^2)*x)*sqrt(-b^2 +
4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(b^7 - 11
*a*b^5*c + 41*a^2*b^3*c^2 - 52*a^3*b*c^3)*x + (3*a*b^6 - 26*a^2*b^4*c + 64
*a^3*b^2*c^2 - 32*a^4*c^3 + (3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*
a^3*c^4)*x^2 + (3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*x)*...
```

3.422.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1012 vs. $2(180) = 360$.

Time = 1.35 (sec) , antiderivative size = 1012, normalized size of antiderivative = 5.16

$$\int \frac{x}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2}\,dx = -\frac{2bx}{c^3} + \left(-\frac{b\sqrt{-\left(4ac-b^2\right)^3}\cdot\left(30a^2c^2-20ab^2c+3b^4\right)}{2c^4\cdot\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)} - \frac{2ac-3b^2}{2c^4}\right)\log\left(x+\frac{16a^3c^2-17a^2b^2c+16a^2c^5\left(-\frac{b\sqrt{-(4ac-b^2)^3}\cdot(30a^2c^2-20ab^2c+3b^4)}{2c^4\cdot(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}-\frac{2ac-3b^2}{2c^4}\right)+3ab^4-}{3}\right.$$

$$+\left(\frac{b\sqrt{-\left(4ac-b^2\right)^3}\cdot\left(30a^2c^2-20ab^2c+3b^4\right)}{2c^4\cdot\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)} - \frac{2ac-3b^2}{2c^4}\right)\log\left(x+\frac{16a^3c^2-17a^2b^2c+16a^2c^5\left(\frac{b\sqrt{-(4ac-b^2)^3}\cdot(30a^2c^2-20ab^2c+3b^4)}{2c^4\cdot(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}-\frac{2ac-3b^2}{2c^4}\right)+3ab^4-8}{30a}\right.$$

$$+\frac{-2a^3c^2+4a^2b^2c-ab^4+x(-5a^2bc^2+5ab^3c-b^5)}{4a^2c^5-ab^2c^4+x^2\cdot(4ac^6-b^2c^5)+x\left(4abc^5-b^3c^4\right)}+\frac{x^2}{2c^2}$$

input
```
integrate(x/(c+a/x**2+b/x)**2,x)
```

3.422. $\int \frac{x}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2}\,dx$

output
```
-2*b*x/c**3 + (-b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3
*b**4)/(2*c**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) -
(2*a*c - 3*b**2)/(2*c**4))*log(x + (16*a**3*c**2 - 17*a**2*b**2*c + 16*a**
2*c**5*(-b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/
(2*c**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - (2*a*c
- 3*b**2)/(2*c**4)) + 3*a*b**4 - 8*a*b**2*c**4*(-b*sqrt(-(4*a*c - b**2)**3
)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*c**4*(64*a**3*c**3 - 48*a**2*b*
*2*c**2 + 12*a*b**4*c - b**6)) - (2*a*c - 3*b**2)/(2*c**4)) + b**4*c**3*(-
b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*c**4*(
64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - (2*a*c - 3*b**2)
/(2*c**4)))/(30*a**2*b*c**2 - 20*a*b**3*c + 3*b**5)) + (b*sqrt(-(4*a*c - b
**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*c**4*(64*a**3*c**3 - 48*
a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - (2*a*c - 3*b**2)/(2*c**4))*log(x +
 (16*a**3*c**2 - 17*a**2*b**2*c + 16*a**2*c**5*(b*sqrt(-(4*a*c - b**2)**3)
*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*c**4*(64*a**3*c**3 - 48*a**2*b**
2*c**2 + 12*a*b**4*c - b**6)) - (2*a*c - 3*b**2)/(2*c**4)) + 3*a*b**4 - 8*
a*b**2*c**4*(b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b*
*4)/(2*c**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - (2*
a*c - 3*b**2)/(2*c**4)) + b**4*c**3*(b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c
**2 - 20*a*b**2*c + 3*b**4)/(2*c**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 +...
```

3.422.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x/(c+a/x^2+b/x)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.422.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.96

$$\int \frac{x}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2}\,dx = -\frac{(3\,b^5-20\,ab^3c+30\,a^2bc^2)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2c^4-4\,ac^5)\sqrt{-b^2+4\,ac}} + \frac{(3\,b^2-2\,ac)\log\left(cx^2+bx+a\right)}{2\,c^4} + \frac{c^2x^2-4\,bcx}{2\,c^4} + \frac{ab^4-4\,a^2b^2c+2\,a^3c^2+(b^5-5\,ab^3c+5\,a^2bc^2)x}{(cx^2+bx+a)(b^2-4\,ac)c^4}$$

input
```
integrate(x/(c+a/x^2+b/x)^2,x, algorithm="giac")
```

output
```
-(3*b^5 - 20*a*b^3*c + 30*a^2*b*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c)
)/((b^2*c^4 - 4*a*c^5)*sqrt(-b^2 + 4*a*c)) + 1/2*(3*b^2 - 2*a*c)*log(c*x^2
 + b*x + a)/c^4 + 1/2*(c^2*x^2 - 4*b*c*x)/c^4 + (a*b^4 - 4*a^2*b^2*c + 2*a
^3*c^2 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*x)/((c*x^2 + b*x + a)*(b^2 - 4*a*
c)*c^4)
```

3.422.9 Mupad [B] (verification not implemented)

Time = 8.69 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.95

$$\int \frac{x}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2}\,dx = \frac{x^2}{2\,c^2} - \frac{\frac{a\,\left(2\,a^2\,c^2-4\,a\,b^2\,c+b^4\right)}{c\,\left(4\,a\,c-b^2\right)}+\frac{b\,x\,\left(5\,a^2\,c^2-5\,a\,b^2\,c+b^4\right)}{c\,\left(4\,a\,c-b^2\right)}}{c^4\,x^2+b\,c^3\,x+a\,c^3} - \frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(128\,a^4\,c^4-288\,a^3\,b^2\,c^3+168\,a^2\,b^4\,c^2-38\,a\,b^6\,c+3\,b^8\right)}{2\,\left(64\,a^3\,c^7-48\,a^2\,b^2\,c^6+12\,a\,b^4\,c^5-b^6\,c^4\right)} - \frac{2\,b\,x}{c^3} + \frac{b\,\mathrm{atan}\left(\frac{c^4\,\left(\frac{2\,b\,x\,\left(30\,a^2\,c^2-20\,a\,b^2\,c+3\,b^4\right)}{c^3\,\left(4\,a\,c-b^2\right)^3}-\frac{b\,\left(b^3\,c^3-4\,a\,b\,c^4\right)\,\left(30\,a^2\,c^2-20\,a\,b^2\,c+3\,b^4\right)}{c^7\,\left(4\,a\,c-b^2\right)^4}\right)\,{\left(4\,a\,c-b^2\right)}^{5/2}}{30\,a^2\,b\,c^2-20\,a\,b^3\,c+3\,b^5}\right)\,\left(30\,a^2\,c^2-20\,a\,b^2\,c+3\,b^4\right)}{c^4\,{\left(4\,a\,c-b^2\right)}^{3/2}}$$

input
```
int(x/(c + a/x^2 + b/x)^2,x)
```

output
```
x^2/(2*c^2) - ((a*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/(c*(4*a*c - b^2)) + (b*x*
(b^4 + 5*a^2*c^2 - 5*a*b^2*c))/(c*(4*a*c - b^2)))/(a*c^3 + c^4*x^2 + b*c^3
*x) - (log(a + b*x + c*x^2)*(3*b^8 + 128*a^4*c^4 + 168*a^2*b^4*c^2 - 288*a
^3*b^2*c^3 - 38*a*b^6*c))/(2*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2
*b^2*c^6)) - (2*b*x)/c^3 + (b*atan((c^4*((2*b*x*(3*b^4 + 30*a^2*c^2 - 20*a
*b^2*c))/(c^3*(4*a*c - b^2)^3) - (b*(b^3*c^3 - 4*a*b*c^4)*(3*b^4 + 30*a^2*
c^2 - 20*a*b^2*c))/(c^7*(4*a*c - b^2)^4))*(4*a*c - b^2)^(5/2))/(3*b^5 + 30
*a^2*b*c^2 - 20*a*b^3*c))*(3*b^4 + 30*a^2*c^2 - 20*a*b^2*c))/(c^4*(4*a*c -
 b^2)^(3/2))
```

3.422. $\int \frac{x}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2}\,dx$

3.423 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2}\,dx$

3.423.1 Optimal result

Integrand size = 14, antiderivative size = 150

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2}\,dx = \frac{2(b^2-3ac)\,x}{c^2\,(b^2-4ac)}-\frac{bx^2}{c\,(b^2-4ac)}+\frac{x^3(2a+bx)}{(b^2-4ac)\,(a+bx+cx^2)} -\frac{2(b^4-6ab^2c+6a^2c^2)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\,(b^2-4ac)^{3/2}}-\frac{b\log\left(a+bx+cx^2\right)}{c^3}$$

output
```
2*(-3*a*c+b^2)*x/c^2/(-4*a*c+b^2)-b*x^2/c/(-4*a*c+b^2)+x^3*(b*x+2*a)/(-4*a
*c+b^2)/(c*x^2+b*x+a)-2*(6*a^2*c^2-6*a*b^2*c+b^4)*arctanh((2*c*x+b)/(-4*a*
c+b^2)^(1/2))/c^3/(-4*a*c+b^2)^(3/2)-b*ln(c*x^2+b*x+a)/c^3
```

3.423.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2}\,dx = \frac{cx+\frac{-b^4x-ab^2(b-4cx)+a^2c(3b-2cx)}{(b^2-4ac)(a+x(b+cx))}-\frac{2(b^4-6ab^2c+6a^2c^2)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}-b\log(a+x(b+cx))}{c^3}$$

input
```
Integrate[(c + a/x^2 + b/x)^(-2),x]
```

output
```
(c*x + (-(b^4*x) - a*b^2*(b - 4*c*x) + a^2*c*(3*b - 2*c*x))/((b^2 - 4*a*c)
*(a + x*(b + c*x))) - (2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTan[(b + 2*c*x)/
Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) - b*Log[a + x*(b + c*x)])/c^3
```

3.423.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {1679, 1164, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\frac{a}{x^2}+\frac{b}{x}+c\right)^2}dx$$

$$\Big\downarrow \text{ 1679}$$

$$\int \frac{x^4}{(a+bx+cx^2)^2}dx$$

$$\Big\downarrow \text{ 1164}$$

$$\frac{x^3(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}-\frac{\int \frac{2x^2(3a+bx)}{cx^2+bx+a}dx}{b^2-4ac}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{x^3(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}-\frac{2\int \frac{x^2(3a+bx)}{cx^2+bx+a}dx}{b^2-4ac}$$

$$\Big\downarrow \text{ 1200}$$

$$\frac{x^3(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}-\frac{2\int \left(-\frac{b^2-3ac}{c^2}+\frac{bx}{c}+\frac{a(b^2-3ac)+b(b^2-4ac)x}{c^2(cx^2+bx+a)}\right)dx}{b^2-4ac}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x^3(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}-\frac{2\left(\frac{(6a^2c^2-6ab^2c+b^4)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}}+\frac{b(b^2-4ac)\log(a+bx+cx^2)}{2c^3}-\frac{x(b^2-3ac)}{c^2}+\frac{bx^2}{2c}\right)}{b^2-4ac}$$

input
```
Int[(c + a/x^2 + b/x)^(-2),x]
```

output
```
(x^3*(2*a + b*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)) - (2*(-((((b^2 - 3*a*c)
*x)/c^2) + (b*x^2)/(2*c) + ((b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTanh[(b + 2*c
*x)/Sqrt[b^2 - 4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]) + (b*(b^2 - 4*a*c)*Log[a +
 b*x + c*x^2])/(2*c^3)))/(b^2 - 4*a*c)
```

3.423.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1164
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1679
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(
2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n]
 && LtQ[n, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.423.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.32

method	result	size
default	$\frac{x}{c^2}-\frac{\frac{-\frac{\left(2a^2c^2-4a\,b^2c+b^4\right)x}{c\left(4ac-b^2\right)}+\frac{ba\left(3ac-b^2\right)}{c\left(4ac-b^2\right)}}{c\,x^2+bx+a}+\frac{\frac{\left(4abc-b^3\right)\ln\left(c\,x^2+bx+a\right)}{c}+\frac{4\left(3c\,a^2-b^2a-\frac{\left(4abc-b^3\right)b}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{4ac-b^2}}{c^2}$	198
risch	Expression too large to display	1176

input
```
int(1/(c+a/x^2+b/x)^2,x,method=_RETURNVERBOSE)
```

output
```
x/c^2-1/c^2*((-(2*a^2*c^2-4*a*b^2*c+b^4)/c/(4*a*c-b^2)*x+b*a/c*(3*a*c-b^2)
/(4*a*c-b^2))/(c*x^2+b*x+a)+2/(4*a*c-b^2)*(1/2*(4*a*b*c-b^3)/c*ln(c*x^2+b*
x+a)+2*(3*c*a^2-b^2*a-1/2*(4*a*b*c-b^3)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c
*x+b)/(4*a*c-b^2)^(1/2))))
```

3.423.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 409 vs. 2(146) = 292.

Time = 0.30 (sec) , antiderivative size = 837, normalized size of antiderivative = 5.58

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2}\,dx$$

$$= \left[-\frac{ab^5-7\,a^2b^3c+12\,a^3bc^2-(b^4c^2-8\,ab^2c^3+16\,a^2c^4)x^3-(b^5c-8\,ab^3c^2+16\,a^2bc^3)x^2+(ab^4-6\,a^2b^2c\right.$$

$$-\frac{ab^5-7\,a^2b^3c+12\,a^3bc^2-(b^4c^2-8\,ab^2c^3+16\,a^2c^4)x^3-(b^5c-8\,ab^3c^2+16\,a^2bc^3)x^2+2\,(ab^4-6\,a^2b^2}{}$$

input
```
integrate(1/(c+a/x^2+b/x)^2,x, algorithm="fricas")
```

output
```
[-(a*b^5 - 7*a^2*b^3*c + 12*a^3*b*c^2 - (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^
4)*x^3 - (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2 + (a*b^4 - 6*a^2*b^2*c +
 6*a^3*c^2 + (b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^2 + (b^5 - 6*a*b^3*c + 6*
a^2*b*c^2)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + s
qrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + (b^6 - 9*a*b^4*c + 26*a
^2*b^2*c^2 - 24*a^3*c^3)*x + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + (b^5*c
- 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x)*
log(c*x^2 + b*x + a))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5 + (b^4*c^4 -
 8*a*b^2*c^5 + 16*a^2*c^6)*x^2 + (b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x)
, -(a*b^5 - 7*a^2*b^3*c + 12*a^3*b*c^2 - (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c
^4)*x^3 - (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2 + 2*(a*b^4 - 6*a^2*b^2*
c + 6*a^3*c^2 + (b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^2 + (b^5 - 6*a*b^3*c +
 6*a^2*b*c^2)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)
/(b^2 - 4*a*c)) + (b^6 - 9*a*b^4*c + 26*a^2*b^2*c^2 - 24*a^3*c^3)*x + (a*b
^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2
 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x)*log(c*x^2 + b*x + a))/(a*b^4*c^3
- 8*a^2*b^2*c^4 + 16*a^3*c^5 + (b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^2 +
(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x)]
```

3.423.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 842 vs. $2(141) = 282$.

Time = 1.07 (sec) , antiderivative size = 842, normalized size of antiderivative = 5.61

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2}\,dx = \left(-\frac{b}{c^3} - \frac{\sqrt{-\left(4ac-b^2\right)^3}\cdot\left(6a^2c^2-6ab^2c+b^4\right)}{c^3\cdot\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)}\right)\log\left(x+\frac{-10a^2bc-16a^2c^4\left(-\frac{b}{c^3}-\frac{\sqrt{-(4ac-b^2)^3}\cdot(6a^2c^2-6ab^2c+b^4)}{c^3\cdot(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}\right.}{}\right.$$

$$+\left(-\frac{b}{c^3} + \frac{\sqrt{-\left(4ac-b^2\right)^3}\cdot\left(6a^2c^2-6ab^2c+b^4\right)}{c^3\cdot\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)}\right)\log\left(x+\frac{-10a^2bc-16a^2c^4\left(-\frac{b}{c^3}+\frac{\sqrt{-(4ac-b^2)^3}\cdot(6a^2c^2-6ab^2c+b^4)}{c^3\cdot(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}\right.}{}\right.$$

$$+\frac{-3a^2bc+ab^3+x\left(2a^2c^2-4ab^2c+b^4\right)}{4a^2c^4-ab^2c^3+x^2\cdot\left(4ac^5-b^2c^4\right)+x\left(4abc^4-b^3c^3\right)}+\frac{x}{c^2}$$

input
```
integrate(1/(c+a/x**2+b/x)**2,x)
```

output
```
(-b/c**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(c**
3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))*log(x + (-10*a
**2*b*c - 16*a**2*c**4*(-b/c**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 -
6*a*b**2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c -
 b**6))) + 2*a*b**3 + 8*a*b**2*c**3*(-b/c**3 - sqrt(-(4*a*c - b**2)**3)*(6
*a**2*c**2 - 6*a*b**2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 +
12*a*b**4*c - b**6))) - b**4*c**2*(-b/c**3 - sqrt(-(4*a*c - b**2)**3)*(6*a
**2*c**2 - 6*a*b**2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12
*a*b**4*c - b**6))))/(12*a**2*c**2 - 12*a*b**2*c + 2*b**4)) + (-b/c**3 + s
qrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(c**3*(64*a**3*c
**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))*log(x + (-10*a**2*b*c - 16
*a**2*c**4*(-b/c**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c +
 b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) + 2
*a*b**3 + 8*a*b**2*c**3*(-b/c**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 -
 6*a*b**2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c
- b**6))) - b**4*c**2*(-b/c**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6
*a*b**2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c -
b**6))))/(12*a**2*c**2 - 12*a*b**2*c + 2*b**4)) + (-3*a**2*b*c + a*b**3 +
x*(2*a**2*c**2 - 4*a*b**2*c + b**4))/(4*a**2*c**4 - a*b**2*c**3 + x**2*(4*
a*c**5 - b**2*c**4) + x*(4*a*b*c**4 - b**3*c**3)) + x/c**2
```

3.423.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(c+a/x^2+b/x)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.423.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.07

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2}\,dx = \frac{2\left(b^4-6\,ab^2c+6\,a^2c^2\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^2c^3-4\,ac^4\right)\sqrt{-b^2+4\,ac}}+\frac{x}{c^2} - \frac{b\log\left(cx^2+bx+a\right)}{c^3}-\frac{\frac{\left(b^4-4\,ab^2c+2\,a^2c^2\right)x}{c}+\frac{ab^3-3\,a^2bc}{c}}{\left(cx^2+bx+a\right)\left(b^2-4\,ac\right)c^2}$$

input
```
integrate(1/(c+a/x^2+b/x)^2,x, algorithm="giac")
```

output
```
2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b
^2*c^3 - 4*a*c^4)*sqrt(-b^2 + 4*a*c)) + x/c^2 - b*log(c*x^2 + b*x + a)/c^3
 - ((b^4 - 4*a*b^2*c + 2*a^2*c^2)*x/c + (a*b^3 - 3*a^2*b*c)/c)/((c*x^2 + b
*x + a)*(b^2 - 4*a*c)*c^2)
```

3.423.9 Mupad [B] (verification not implemented)

Time = 8.47 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.74

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2}\,dx = \frac{x}{c^2}+\frac{\frac{a\,\left(b^3-3\,a\,b\,c\right)}{c\,\left(4\,a\,c-b^2\right)}+\frac{x\,\left(2\,a^2\,c^2-4\,a\,b^2\,c+b^4\right)}{c\,\left(4\,a\,c-b^2\right)}}{c^3\,x^2+b\,c^2\,x+a\,c^2} + \frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(-128\,a^3\,b\,c^3+96\,a^2\,b^3\,c^2-24\,a\,b^5\,c+2\,b^7\right)}{2\,\left(64\,a^3\,c^6-48\,a^2\,b^2\,c^5+12\,a\,b^4\,c^4-b^6\,c^3\right)} - \frac{2\,\mathrm{atan}\left(\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}-\frac{b^3\,c^2-4\,a\,b\,c^3}{c^2\,{\left(4\,a\,c-b^2\right)}^{3/2}}\right)\,\left(6\,a^2\,c^2-6\,a\,b^2\,c+b^4\right)}{c^3\,{\left(4\,a\,c-b^2\right)}^{3/2}}$$

input
```
int(1/(c + a/x^2 + b/x)^2,x)
```

output
```
x/c^2 + ((a*(b^3 - 3*a*b*c))/(c*(4*a*c - b^2)) + (x*(b^4 + 2*a^2*c^2 - 4*a
*b^2*c))/(c*(4*a*c - b^2)))/(a*c^2 + c^3*x^2 + b*c^2*x) + (log(a + b*x + c
*x^2)*(2*b^7 - 128*a^3*b*c^3 + 96*a^2*b^3*c^2 - 24*a*b^5*c))/(2*(64*a^3*c^
6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) - (2*atan((2*c*x)/(4*a*c - b
^2)^(1/2) - (b^3*c^2 - 4*a*b*c^3)/(c^2*(4*a*c - b^2)^(3/2)))*(b^4 + 6*a^2*
c^2 - 6*a*b^2*c))/(c^3*(4*a*c - b^2)^(3/2))
```

3.424 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x}\,dx$

3.424.1 Optimal result

Integrand size = 18, antiderivative size = 114

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x}\,dx = -\frac{bx}{c\left(b^2-4ac\right)}+\frac{x^2(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{b(b^2-6ac)\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\left(b^2-4ac\right)^{3/2}}+\frac{\log\left(a+bx+cx^2\right)}{2c^2}$$

output
```
-b*x/c/(-4*a*c+b^2)+x^2*(b*x+2*a)/(-4*a*c+b^2)/(c*x^2+b*x+a)+b*(-6*a*c+b^2
)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(3/2)+1/2*ln(c*x^
2+b*x+a)/c^2
```

3.424.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.96

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x}\,dx = \frac{\frac{2\left(-2a^2c+b^3x+ab(b-3cx)\right)}{(b^2-4ac)(a+x(b+cx))}+\frac{2b(b^2-6ac)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}+\log(a+x(b+cx))}{2c^2}$$

input
```
Integrate[1/((c + a/x^2 + b/x)^2*x),x]
```

output
```
((2*(-2*a^2*c + b^3*x + a*b*(b - 3*c*x)))/((b^2 - 4*a*c)*(a + x*(b + c*x))
) + (2*b*(b^2 - 6*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a
*c)^(3/2) + Log[a + x*(b + c*x)])/(2*c^2)
```

3.424.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1692, 1164, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(\frac{a}{x^2}+\frac{b}{x}+c\right)^2}dx$$

$$\Big\downarrow \text{1692}$$

$$\int \frac{x^3}{(a+bx+cx^2)^2}dx$$

$$\Big\downarrow \text{1164}$$

$$\frac{x^2(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}-\frac{\int \frac{x(4a+bx)}{cx^2+bx+a}dx}{b^2-4ac}$$

$$\Big\downarrow \text{1200}$$

$$\frac{x^2(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}-\frac{\int \left(\frac{b}{c}-\frac{ab+(b^2-4ac)x}{c(cx^2+bx+a)}\right)dx}{b^2-4ac}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^2(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}-\frac{-\frac{b(b^2-6ac)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{(b^2-4ac)\log(a+bx+cx^2)}{2c^2}+\frac{bx}{c}}{b^2-4ac}$$

input
```
Int[1/((c + a/x^2 + b/x)^2*x),x]
```

output
```
(x^2*(2*a + b*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)) - ((b*x)/c - (b*(b^2 -
 6*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^2*Sqrt[b^2 - 4*a*c]) -
((b^2 - 4*a*c)*Log[a + b*x + c*x^2])/(2*c^2))/(b^2 - 4*a*c)
```

3.424.3.1 Defintions of rubi rules used

rule 1164
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1692
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.424.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.48

method	result
default	$\frac{\frac{b(3ac-b^2)x}{c^2(4ac-b^2)}+\frac{a(2ac-b^2)}{(4ac-b^2)c^2}}{cx^2+bx+a}+\frac{\frac{(4ac-b^2)\ln(cx^2+bx+a)}{2c}+\frac{2\left(-ab-\frac{(4ac-b^2)b}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{c(4ac-b^2)}$
risch	$\frac{\frac{b(3ac-b^2)x}{c^2(4ac-b^2)}+\frac{a(2ac-b^2)}{(4ac-b^2)c^2}}{cx^2+bx+a}+\frac{8\ln\left(-24a^2bc^2+10ab^3c-b^5-2\sqrt{-b^2(4ac-b^2)(6ac-b^2)^2}\,cx-\sqrt{-b^2(4ac-b^2)(6ac-b^2)^2}\,b\right)a^2}{(4ac-b^2)^2}-\frac{4\ln}{}$

input
```
int(1/(c+a/x^2+b/x)^2/x,x,method=_RETURNVERBOSE)
```

output
```
(b/c^2*(3*a*c-b^2)/(4*a*c-b^2)*x+a*(2*a*c-b^2)/(4*a*c-b^2)/c^2)/(c*x^2+b*x
+a)+1/c/(4*a*c-b^2)*(1/2*(4*a*c-b^2)/c*ln(c*x^2+b*x+a)+2*(-a*b-1/2*(4*a*c-
b^2)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

3.424.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(108) = 216$.

Time = 0.32 (sec) , antiderivative size = 635, normalized size of antiderivative = 5.57

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x}\,dx$$
$$= \left[\frac{2\,ab^4 - 12\,a^2b^2c + 16\,a^3c^2 + (ab^3 - 6\,a^2bc + (b^3c - 6\,abc^2)x^2 + (b^4 - 6\,ab^2c)x)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx}{}\right.}{2\,(ab^4c^2 - 8\,a^2b^2c^3}\right.$$

input
```
integrate(1/(c+a/x^2+b/x)^2/x,x, algorithm="fricas")
```

output
```
[1/2*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + (a*b^3 - 6*a^2*b*c + (b^3*c -
6*a*b*c^2)*x^2 + (b^4 - 6*a*b^2*c)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2
*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) +
 2*(b^5 - 7*a*b^3*c + 12*a^2*b*c^2)*x + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2
+ (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2
)*x)*log(c*x^2 + b*x + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^4*
c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^2 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^
4)*x), 1/2*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(a*b^3 - 6*a^2*b*c + (
b^3*c - 6*a*b*c^2)*x^2 + (b^4 - 6*a*b^2*c)*x)*sqrt(-b^2 + 4*a*c)*arctan(-s
qrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(b^5 - 7*a*b^3*c + 12*a^2
*b*c^2)*x + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*
a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*log(c*x^2 + b*x + a))/(
a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c
^5)*x^2 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x)]
```

3.424.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 729 vs. $2(104) = 208$.

Time = 0.76 (sec) , antiderivative size = 729, normalized size of antiderivative = 6.39

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x}\,dx = \left(-\frac{b\sqrt{-\left(4ac-b^2\right)^3}\cdot\left(6ac-b^2\right)}{2c^2\cdot\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)} + \frac{1}{2c^2}\right)\log\left(x+\frac{-16a^2c^3\left(-\frac{b\sqrt{-(4ac-b^2)^3}\cdot(6ac-b^2)}{2c^2\cdot(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}+\frac{1}{2c^2}\right)+8a^2c+8ab^2c^2\left(-\frac{b\sqrt{-(4ac-b^2)^3}\cdot(6ac-}{2c^2\cdot(64a^3c^3-48a^2b^2c^2+12}\right.}{6abc-b^3}\right.$$
$$+\left(\frac{b\sqrt{-\left(4ac-b^2\right)^3}\cdot\left(6ac-b^2\right)}{2c^2\cdot\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)} + \frac{1}{2c^2}\right)\log\left(x+\frac{-16a^2c^3\left(\frac{b\sqrt{-(4ac-b^2)^3}\cdot(6ac-b^2)}{2c^2\cdot(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}+\frac{1}{2c^2}\right)+8a^2c+8ab^2c^2\left(\frac{b\sqrt{-(4ac-b^2)^3}\cdot(6ac-b^2)}{2c^2\cdot(64a^3c^3-48a^2b^2c^2+12ab^4c-}\right.}{6abc-b^3}\right.$$
$$+\frac{2a^2c-ab^2+x(3abc-b^3)}{4a^2c^3-ab^2c^2+x^2\cdot\left(4ac^4-b^2c^3\right)+x\left(4abc^3-b^3c^2\right)}$$

input
```
integrate(1/(c+a/x**2+b/x)**2/x,x)
```

output
```
(-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**
2*b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(2*c**2))*log(x + (-16*a**2*c**3*(-
b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*
b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(2*c**2)) + 8*a**2*c + 8*a*b**2*c**2*
(-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**
2*b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(2*c**2)) - a*b**2 - b**4*c*(-b*sqr
t(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*
c**2 + 12*a*b**4*c - b**6)) + 1/(2*c**2)))/(6*a*b*c - b**3)) + (b*sqrt(-(4
*a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2
+ 12*a*b**4*c - b**6)) + 1/(2*c**2))*log(x + (-16*a**2*c**3*(b*sqrt(-(4*a*
c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 1
2*a*b**4*c - b**6)) + 1/(2*c**2)) + 8*a**2*c + 8*a*b**2*c**2*(b*sqrt(-(4*a
*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 +
12*a*b**4*c - b**6)) + 1/(2*c**2)) - a*b**2 - b**4*c*(b*sqrt(-(4*a*c - b**
2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**
4*c - b**6)) + 1/(2*c**2)))/(6*a*b*c - b**3)) + (2*a**2*c - a*b**2 + x*(3*
a*b*c - b**3))/(4*a**2*c**3 - a*b**2*c**2 + x**2*(4*a*c**4 - b**2*c**3) +
x*(4*a*b*c**3 - b**3*c**2))
```

3.424.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(c+a/x^2+b/x)^2/x,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.424.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.10

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x}\,dx = -\frac{(b^3-6\,abc)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2c^2-4\,ac^3)\sqrt{-b^2+4\,ac}} + \frac{\log\left(cx^2+bx+a\right)}{2\,c^2} + \frac{ab^2-2\,a^2c+(b^3-3\,abc)x}{(cx^2+bx+a)(b^2-4\,ac)c^2}$$

input
```
integrate(1/(c+a/x^2+b/x)^2/x,x, algorithm="giac")
```

output
```
-(b^3 - 6*a*b*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2*c^2 - 4*a*c^
3)*sqrt(-b^2 + 4*a*c)) + 1/2*log(c*x^2 + b*x + a)/c^2 + (a*b^2 - 2*a^2*c +
 (b^3 - 3*a*b*c)*x)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*c^2)
```

3.424.9 Mupad [B] (verification not implemented)

Time = 8.52 (sec) , antiderivative size = 279, normalized size of antiderivative = 2.45

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x}\,dx = \frac{\frac{a\,(2\,a\,c-b^2)}{c^2\,(4\,a\,c-b^2)}+\frac{b\,x\,(3\,a\,c-b^2)}{c^2\,(4\,a\,c-b^2)}}{c\,x^2+b\,x+a} - \frac{\ln\left(c\,x^2+b\,x+a\right)\,(-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6)}{2\,(64\,a^3\,c^5-48\,a^2\,b^2\,c^4+12\,a\,b^4\,c^3-b^6\,c^2)} + \frac{b\,\mathrm{atan}\left(\frac{c^2\,(4\,a\,c-b^2)^{5/2}\left(\frac{2\,b\,x\,(6\,a\,c-b^2)}{c\,(4\,a\,c-b^2)^3}+\frac{b^2\,(4\,a\,c^2-b^2\,c)\,(6\,a\,c-b^2)}{c^3\,(4\,a\,c-b^2)^4}\right)}{b^3-6\,a\,b\,c}\right)(6\,a\,c-b^2)}{c^2\,(4\,a\,c-b^2)^{3/2}}$$

input
```
int(1/(x*(c + a/x^2 + b/x)^2),x)
```

output
```
((a*(2*a*c - b^2))/(c^2*(4*a*c - b^2)) + (b*x*(3*a*c - b^2))/(c^2*(4*a*c -
 b^2)))/(a + b*x + c*x^2) - (log(a + b*x + c*x^2)*(b^6 - 64*a^3*c^3 + 48*a
^2*b^2*c^2 - 12*a*b^4*c))/(2*(64*a^3*c^5 - b^6*c^2 + 12*a*b^4*c^3 - 48*a^2
*b^2*c^4)) + (b*atan((c^2*(4*a*c - b^2)^(5/2)*((2*b*x*(6*a*c - b^2))/(c*(4
*a*c - b^2)^3) + (b^2*(4*a*c^2 - b^2*c)*(6*a*c - b^2))/(c^3*(4*a*c - b^2)^
4)))/(b^3 - 6*a*b*c))*(6*a*c - b^2))/(c^2*(4*a*c - b^2)^(3/2))
```

3.425 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^2} \, dx$

3.425.1 Optimal result

Integrand size = 18, antiderivative size = 71

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^2} \, dx = \frac{b+\frac{2a}{x}}{(b^2-4ac)\left(c+\frac{a}{x^2}+\frac{b}{x}\right)} - \frac{4a\text{arctanh}\left(\frac{b+\frac{2a}{x}}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

output
```
(b+2*a/x)/(-4*a*c+b^2)/(c+a/x^2+b/x)-4*a*arctanh((b+2*a/x)/(-4*a*c+b^2)^(1
/2))/(-4*a*c+b^2)^(3/2)
```

3.425.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.14

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^2} \, dx = \frac{b^2x+a(b-2cx)}{c(-b^2+4ac)(a+x(b+cx))} + \frac{4a\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}$$

input
```
Integrate[1/((c + a/x^2 + b/x)^2*x^2),x]
```

output
```
(b^2*x + a*(b - 2*c*x))/(c*(-b^2 + 4*a*c)*(a + x*(b + c*x))) + (4*a*ArcTan
[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
```

3.425.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1690, 1086, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(\frac{a}{x^2}+\frac{b}{x}+c\right)^2}dx$$

$$\Big\downarrow \text{1690}$$

$$-\int \frac{1}{\left(\frac{a}{x^2}+c+\frac{b}{x}\right)^2}d\frac{1}{x}$$

$$\Big\downarrow \text{1086}$$

$$\frac{2a\int \frac{1}{\frac{a}{x^2}+c+\frac{b}{x}}d\frac{1}{x}}{b^2-4ac}+\frac{\frac{2a}{x}+b}{(b^2-4ac)\left(\frac{a}{x^2}+\frac{b}{x}+c\right)}$$

$$\Big\downarrow \text{1083}$$

$$\frac{\frac{2a}{x}+b}{(b^2-4ac)\left(\frac{a}{x^2}+\frac{b}{x}+c\right)}-\frac{4a\int \frac{1}{b^2-4ac-\frac{1}{x^2}}d\left(\frac{2a}{x}+b\right)}{b^2-4ac}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{2a}{x}+b}{(b^2-4ac)\left(\frac{a}{x^2}+\frac{b}{x}+c\right)}-\frac{4a\text{arctanh}\left(\frac{\frac{2a}{x}+b}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

input
```
Int[1/((c + a/x^2 + b/x)^2*x^2),x]
```

output
```
(b + (2*a)/x)/((b^2 - 4*a*c)*(c + a/x^2 + b/x)) - (4*a*ArcTanh[(b + (2*a)/
x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

3.425.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

3.425.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.37

method	result
default	$\frac{-\frac{\left(2ac-b^2\right)x}{c\left(4ac-b^2\right)}+\frac{ab}{c\left(4ac-b^2\right)}}{c\,x^2+bx+a}+\frac{4a\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}}$
risch	$\frac{-\frac{\left(2ac-b^2\right)x}{c\left(4ac-b^2\right)}+\frac{ab}{c\left(4ac-b^2\right)}}{c\,x^2+bx+a}+\frac{2a\ln\left(\left(-8a\,c^2+2b^2c\right)x+\left(-4ac+b^2\right)^{\frac{3}{2}}-4abc+b^3\right)}{\left(-4ac+b^2\right)^{\frac{3}{2}}}-\frac{2a\ln\left(\left(8a\,c^2-2b^2c\right)x+\left(-4ac+b^2\right)^{\frac{3}{2}}+4abc-b^3\right)}{\left(-4ac+b^2\right)^{\frac{3}{2}}}$

input
```
int(1/(c+a/x^2+b/x)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
(-(2*a*c-b^2)/c/(4*a*c-b^2)*x+a*b/c/(4*a*c-b^2))/(c*x^2+b*x+a)+4*a/(4*a*c-
b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
```

3.425.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. $2(67) = 134$.

Time = 0.28 (sec) , antiderivative size = 387, normalized size of antiderivative = 5.45

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^2}\,dx$$
$$= \left[-\frac{ab^3 - 4\,a^2bc + 2\,(ac^2x^2 + abcx + a^2c)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right) + (b^4 - 6\,ab^2c}{ab^4c - 8\,a^2b^2c^2 + 16\,a^3c^3 + (b^4c^2 - 8\,ab^2c^3 + 16\,a^2c^4)x^2 + (b^5c - 8\,ab^3c^2 + 16\,a^2bc^3)x}\right.$$
$$-\frac{ab^3 - 4\,a^2bc - 4\,(ac^2x^2 + abcx + a^2c)\sqrt{-b^2+4\,ac}\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx+b)}{b^2-4\,ac}\right) + (b^4 - 6\,ab^2c + 8\,a^2c^2)x}{ab^4c - 8\,a^2b^2c^2 + 16\,a^3c^3 + (b^4c^2 - 8\,ab^2c^3 + 16\,a^2c^4)x^2 + (b^5c - 8\,ab^3c^2 + 16\,a^2bc^3)x}$$

input
```
integrate(1/(c+a/x^2+b/x)^2/x^2,x, algorithm="fricas")
```

output
```
[-(a*b^3 - 4*a^2*b*c + 2*(a*c^2*x^2 + a*b*c*x + a^2*c)*sqrt(b^2 - 4*a*c)*l
og((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*
x^2 + b*x + a)) + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*x)/(a*b^4*c - 8*a^2*b^2*c^
2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^2 + (b^5*c - 8*a*b
^3*c^2 + 16*a^2*b*c^3)*x), -(a*b^3 - 4*a^2*b*c - 4*(a*c^2*x^2 + a*b*c*x +
a^2*c)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*
a*c)) + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*x)/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3
*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^2 + (b^5*c - 8*a*b^3*c^2 + 1
6*a^2*b*c^3)*x)]
```

3.425.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(60) = 120$.

Time = 0.32 (sec) , antiderivative size = 280, normalized size of antiderivative = 3.94

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^2}\,dx =$$
$$-2a\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x+\frac{-32a^3c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}+16a^2b^2c\sqrt{-\frac{1}{(4ac-b^2)^3}}-2ab^4\sqrt{-\frac{1}{(4ac-b^2)^3}}+2ab}{4ac}\right)$$
$$+2a\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x+\frac{32a^3c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}-16a^2b^2c\sqrt{-\frac{1}{(4ac-b^2)^3}}+2ab^4\sqrt{-\frac{1}{(4ac-b^2)^3}}+2ab}{4ac}\right)$$
$$+\frac{ab+x(-2ac+b^2)}{4a^2c^2-ab^2c+x^2\cdot(4ac^3-b^2c^2)+x\,(4abc^2-b^3c)}$$

input
```
integrate(1/(c+a/x**2+b/x)**2/x**2,x)
```

output
```
-2*a*sqrt(-1/(4*a*c - b**2)**3)*log(x + (-32*a**3*c**2*sqrt(-1/(4*a*c - b*
*2)**3) + 16*a**2*b**2*c*sqrt(-1/(4*a*c - b**2)**3) - 2*a*b**4*sqrt(-1/(4*
a*c - b**2)**3) + 2*a*b)/(4*a*c)) + 2*a*sqrt(-1/(4*a*c - b**2)**3)*log(x +
 (32*a**3*c**2*sqrt(-1/(4*a*c - b**2)**3) - 16*a**2*b**2*c*sqrt(-1/(4*a*c
- b**2)**3) + 2*a*b**4*sqrt(-1/(4*a*c - b**2)**3) + 2*a*b)/(4*a*c)) + (a*b
 + x*(-2*a*c + b**2))/(4*a**2*c**2 - a*b**2*c + x**2*(4*a*c**3 - b**2*c**2
) + x*(4*a*b*c**2 - b**3*c))
```

3.425.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(c+a/x^2+b/x)^2/x^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.425.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.24

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^2}\,dx = -\frac{4\,a\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}} - \frac{b^2x-2\,acx+ab}{(b^2c-4\,ac^2)(cx^2+bx+a)}$$

input
```
integrate(1/(c+a/x^2+b/x)^2/x^2,x, algorithm="giac")
```

output
```
-4*a*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a
*c)) - (b^2*x - 2*a*c*x + a*b)/((b^2*c - 4*a*c^2)*(c*x^2 + b*x + a))
```

3.425.9 Mupad [B] (verification not implemented)

Time = 8.31 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.90

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^2}\,dx = -\frac{\frac{x\,(2\,a\,c-b^2)}{c\,(4\,a\,c-b^2)}-\frac{a\,b}{c\,(4\,a\,c-b^2)}}{c\,x^2+b\,x+a} - \frac{4\,a\,\mathrm{atan}\left(\frac{\left(\frac{2\,a\,\left(b^3-4\,a\,b\,c\right)}{\left(4\,a\,c-b^2\right)^{5/2}}-\frac{4\,a\,c\,x}{\left(4\,a\,c-b^2\right)^{3/2}}\right)(4\,a\,c-b^2)}{2\,a}\right)}{\left(4\,a\,c-b^2\right)^{3/2}}$$

input
```
int(1/(x^2*(c + a/x^2 + b/x)^2),x)
```

output
```
- ((x*(2*a*c - b^2))/(c*(4*a*c - b^2)) - (a*b)/(c*(4*a*c - b^2)))/(a + b*x
 + c*x^2) - (4*a*atan((((2*a*(b^3 - 4*a*b*c))/(4*a*c - b^2)^(5/2) - (4*a*c
*x)/(4*a*c - b^2)^(3/2))*(4*a*c - b^2))/(2*a)))/(4*a*c - b^2)^(3/2)
```

3.426 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^3} \, dx$

3.426.1 Optimal result

Integrand size = 18, antiderivative size = 66

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^3} \, dx = \frac{2a+bx}{(b^2-4ac)\,(a+bx+cx^2)} - \frac{2b\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

output
```
(b*x+2*a)/(-4*a*c+b^2)/(c*x^2+b*x+a)-2*b*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1
/2))/(-4*a*c+b^2)^(3/2)
```

3.426.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.05

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^3} \, dx = \frac{2a+bx}{(b^2-4ac)\,(a+x(b+cx))} - \frac{2b\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}$$

input
```
Integrate[1/((c + a/x^2 + b/x)^2*x^3),x]
```

output
```
(2*a + b*x)/((b^2 - 4*a*c)*(a + x*(b + c*x))) - (2*b*ArcTan[(b + 2*c*x)/Sq
rt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
```

3.426.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1692, 1159, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\left(\frac{a}{x^2}+\frac{b}{x}+c\right)^2}dx$$

$$\Big\downarrow \text{1692}$$

$$\int \frac{x}{(a+bx+cx^2)^2}dx$$

$$\Big\downarrow \text{1159}$$

$$\frac{b\int \frac{1}{cx^2+bx+a}dx}{b^2-4ac}+\frac{2a+bx}{(b^2-4ac)(a+bx+cx^2)}$$

$$\Big\downarrow \text{1083}$$

$$\frac{2a+bx}{(b^2-4ac)(a+bx+cx^2)}-\frac{2b\int \frac{1}{b^2-(b+2cx)^2-4ac}d(b+2cx)}{b^2-4ac}$$

$$\Big\downarrow \text{219}$$

$$\frac{2a+bx}{(b^2-4ac)(a+bx+cx^2)}-\frac{2b\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

input
```
Int[1/((c + a/x^2 + b/x)^2*x^3),x]
```

output
```
(2*a + b*x)/((b^2 - 4*a*c)*(a + b*x + c*x^2)) - (2*b*ArcTanh[(b + 2*c*x)/S
qrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

3.426.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

rule 1692
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

3.426.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.06

method	result	size
default	$\frac{-bx-2a}{(4ac-b^2)(c\,x^2+bx+a)}-\frac{2b\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}}$	70
risch	$\frac{-\frac{bx}{4ac-b^2}-\frac{2a}{4ac-b^2}}{c\,x^2+bx+a}+\frac{b\ln\left(\left(-8a\,c^2+2b^2c\right)x-\left(-4ac+b^2\right)^{\frac{3}{2}}-4abc+b^3\right)}{(-4ac+b^2)^{\frac{3}{2}}}-\frac{b\ln\left(\left(8a\,c^2-2b^2c\right)x-\left(-4ac+b^2\right)^{\frac{3}{2}}+4abc-b^3\right)}{(-4ac+b^2)^{\frac{3}{2}}}$	148

input
```
int(1/(c+a/x^2+b/x)^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
(-b*x-2*a)/(4*a*c-b^2)/(c*x^2+b*x+a)-2*b/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b
)/(4*a*c-b^2)^(1/2))
```

3.426.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(62) = 124$.

Time = 0.30 (sec) , antiderivative size = 338, normalized size of antiderivative = 5.12

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^3}\,dx$$
$$= \left[\frac{2\,ab^2 - 8\,a^2c - (bcx^2 + b^2x + ab)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right) + (b^3-4\,abc)x}{ab^4 - 8\,a^2b^2c + 16\,a^3c^2 + (b^4c - 8\,ab^2c^2 + 16\,a^2c^3)x^2 + (b^5 - 8\,ab^3c + 16\,a^2bc^2)x}, \frac{2\,ab^2\cdots}{\cdots}\right.$$

input
```
integrate(1/(c+a/x^2+b/x)^2/x^3,x, algorithm="fricas")
```

output
```
[(2*a*b^2 - 8*a^2*c - (b*c*x^2 + b^2*x + a*b)*sqrt(b^2 - 4*a*c)*log((2*c^2
*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x
 + a)) + (b^3 - 4*a*b*c)*x)/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8
*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x), (2*a*b
^2 - 8*a^2*c - 2*(b*c*x^2 + b^2*x + a*b)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-
b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + (b^3 - 4*a*b*c)*x)/(a*b^4 - 8*a^
2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a
*b^3*c + 16*a^2*b*c^2)*x)]
```

3.426.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(60) = 120$.

Time = 0.29 (sec) , antiderivative size = 253, normalized size of antiderivative = 3.83

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^3}\,dx$$
$$= b\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x + \frac{-16a^2bc^2\sqrt{-\frac{1}{(4ac-b^2)^3}} + 8ab^3c\sqrt{-\frac{1}{(4ac-b^2)^3}} - b^5\sqrt{-\frac{1}{(4ac-b^2)^3}} + b^2}{2bc}\right)$$
$$- b\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x + \frac{16a^2bc^2\sqrt{-\frac{1}{(4ac-b^2)^3}} - 8ab^3c\sqrt{-\frac{1}{(4ac-b^2)^3}} + b^5\sqrt{-\frac{1}{(4ac-b^2)^3}} + b^2}{2bc}\right)$$
$$+ \frac{-2a-bx}{4a^2c - ab^2 + x^2\cdot(4ac^2-b^2c) + x\,(4abc-b^3)}$$

input
```
integrate(1/(c+a/x**2+b/x)**2/x**3,x)
```

output
```
b*sqrt(-1/(4*a*c - b**2)**3)*log(x + (-16*a**2*b*c**2*sqrt(-1/(4*a*c - b**
2)**3) + 8*a*b**3*c*sqrt(-1/(4*a*c - b**2)**3) - b**5*sqrt(-1/(4*a*c - b**
2)**3) + b**2)/(2*b*c)) - b*sqrt(-1/(4*a*c - b**2)**3)*log(x + (16*a**2*b*
c**2*sqrt(-1/(4*a*c - b**2)**3) - 8*a*b**3*c*sqrt(-1/(4*a*c - b**2)**3) +
b**5*sqrt(-1/(4*a*c - b**2)**3) + b**2)/(2*b*c)) + (-2*a - b*x)/(4*a**2*c
- a*b**2 + x**2*(4*a*c**2 - b**2*c) + x*(4*a*b*c - b**3))
```

3.426.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(c+a/x^2+b/x)^2/x^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.426.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.15

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^3}\,dx = \frac{2\,b\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}} + \frac{bx+2\,a}{(cx^2+bx+a)(b^2-4\,ac)}$$

input
```
integrate(1/(c+a/x^2+b/x)^2/x^3,x, algorithm="giac")
```

output
```
2*b*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a*
c)) + (b*x + 2*a)/((c*x^2 + b*x + a)*(b^2 - 4*a*c))
```

3.426.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.67

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^3}\,dx = -\frac{\frac{2a}{4ac-b^2}+\frac{bx}{4ac-b^2}}{cx^2+bx+a}-\frac{2b\,\mathrm{atan}\left(\frac{\left(\frac{b^2}{\left(4ac-b^2\right)^{3/2}}+\frac{2bcx}{\left(4ac-b^2\right)^{3/2}}\right)\left(4ac-b^2\right)}{b}\right)}{\left(4ac-b^2\right)^{3/2}}$$

input
```
int(1/(x^3*(c + a/x^2 + b/x)^2),x)
```

output
```
- ((2*a)/(4*a*c - b^2) + (b*x)/(4*a*c - b^2))/(a + b*x + c*x^2) - (2*b*ata
n(((b^2/(4*a*c - b^2)^(3/2) + (2*b*c*x)/(4*a*c - b^2)^(3/2))*(4*a*c - b^2)
)/b))/(4*a*c - b^2)^(3/2)
```

3.427 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^4} dx$

3.427.1 Optimal result

Integrand size = 18, antiderivative size = 66

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^4} dx = -\frac{b+2cx}{(b^2-4ac)(a+bx+cx^2)}+\frac{4c\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

output
```
(-2*c*x-b)/(-4*a*c+b^2)/(c*x^2+b*x+a)+4*c*arctanh((2*c*x+b)/(-4*a*c+b^2)^(
1/2))/(-4*a*c+b^2)^(3/2)
```

3.427.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.06

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^4} dx = -\frac{\frac{b+2cx}{a+x(b+cx)}+\frac{4c\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}}{b^2-4ac}$$

input
```
Integrate[1/((c + a/x^2 + b/x)^2*x^4),x]
```

output
```
-(((b + 2*c*x)/(a + x*(b + c*x)) + (4*c*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a
*c]])/Sqrt[-b^2 + 4*a*c])/(b^2 - 4*a*c))
```

3.427.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1692, 1086, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \left(\frac{a}{x^2}+\frac{b}{x}+c\right)^2} \, dx$$

$$\Big\downarrow \text{ 1692}$$

$$\int \frac{1}{(a+bx+cx^2)^2}dx$$

$$\Big\downarrow \text{ 1086}$$

$$-\frac{2c\int \frac{1}{cx^2+bx+a}dx}{b^2-4ac}-\frac{b+2cx}{(b^2-4ac)\,(a+bx+cx^2)}$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{4c\int \frac{1}{b^2-(b+2cx)^2-4ac}d(b+2cx)}{b^2-4ac}-\frac{b+2cx}{(b^2-4ac)\,(a+bx+cx^2)}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{4c\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx}{(b^2-4ac)\,(a+bx+cx^2)}$$

input
```
Int[1/((c + a/x^2 + b/x)^2*x^4),x]
```

output
```
-((b + 2*c*x)/((b^2 - 4*a*c)*(a + b*x + c*x^2))) + (4*c*ArcTanh[(b + 2*c*x
)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

3.427.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

rule 1692
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

3.427.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.03

method	result	size
default	$\frac{2cx+b}{\left(4ac-b^{2}\right)\left(c\,x^{2}+bx+a\right)}+\frac{4c\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^{2}}}\right)}{\left(4ac-b^{2}\right)^{\frac{3}{2}}}$	68
risch	$\frac{\frac{2cx}{4ac-b^{2}}+\frac{b}{4ac-b^{2}}}{c\,x^{2}+bx+a}+\frac{2c\ln\left(\left(-8a\,c^{2}+2b^{2}c\right)x+\left(-4ac+b^{2}\right)^{\frac{3}{2}}-4abc+b^{3}\right)}{\left(-4ac+b^{2}\right)^{\frac{3}{2}}}-\frac{2c\ln\left(\left(8a\,c^{2}-2b^{2}c\right)x+\left(-4ac+b^{2}\right)^{\frac{3}{2}}+4abc-b^{3}\right)}{\left(-4ac+b^{2}\right)^{\frac{3}{2}}}$	144

input
```
int(1/(c+a/x^2+b/x)^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)+4*c/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)
/(4*a*c-b^2)^(1/2))
```

3.427.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(62) = 124$.

Time = 0.27 (sec) , antiderivative size = 341, normalized size of antiderivative = 5.17

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^4}\,dx$$
$$=\left[-\frac{b^3-4\,abc+2\,(c^2x^2+bcx+ac)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)+2\,(b^2c-4\,ac^2)x}{ab^4-8\,a^2b^2c+16\,a^3c^2+(b^4c-8\,ab^2c^2+16\,a^2c^3)x^2+(b^5-8\,ab^3c+16\,a^2bc^2)x},\right.$$
$$\left.-\frac{b^3-4\,abc-4\,(c^2x^2+bcx+ac)\sqrt{-b^2+4\,ac}\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx+b)}{b^2-4\,ac}\right)+2\,(b^2c-4\,ac^2)x}{ab^4-8\,a^2b^2c+16\,a^3c^2+(b^4c-8\,ab^2c^2+16\,a^2c^3)x^2+(b^5-8\,ab^3c+16\,a^2bc^2)x}\right]$$

input
```
integrate(1/(c+a/x^2+b/x)^2/x^4,x, algorithm="fricas")
```

output
```
[-(b^3 - 4*a*b*c + 2*(c^2*x^2 + b*c*x + a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*
x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x
+ a)) + 2*(b^2*c - 4*a*c^2)*x)/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c
- 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x), -(b
^3 - 4*a*b*c - 4*(c^2*x^2 + b*c*x + a*c)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-
b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(b^2*c - 4*a*c^2)*x)/(a*b^4 -
8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 -
 8*a*b^3*c + 16*a^2*b*c^2)*x)]
```

3.427.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(61) = 122$.

Time = 0.31 (sec) , antiderivative size = 265, normalized size of antiderivative = 4.02

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^4}\,dx =$$
$$-2c\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x+\frac{-32a^2c^3\sqrt{-\frac{1}{(4ac-b^2)^3}}+16ab^2c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}-2b^4c\sqrt{-\frac{1}{(4ac-b^2)^3}}+2bc}{4c^2}\right)$$
$$+2c\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x+\frac{32a^2c^3\sqrt{-\frac{1}{(4ac-b^2)^3}}-16ab^2c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}+2b^4c\sqrt{-\frac{1}{(4ac-b^2)^3}}+2bc}{4c^2}\right)$$
$$+\frac{b+2cx}{4a^2c-ab^2+x^2\cdot(4ac^2-b^2c)+x\,(4abc-b^3)}$$

input
```
integrate(1/(c+a/x**2+b/x)**2/x**4,x)
```

output
```
-2*c*sqrt(-1/(4*a*c - b**2)**3)*log(x + (-32*a**2*c**3*sqrt(-1/(4*a*c - b*
*2)**3) + 16*a*b**2*c**2*sqrt(-1/(4*a*c - b**2)**3) - 2*b**4*c*sqrt(-1/(4*
a*c - b**2)**3) + 2*b*c)/(4*c**2)) + 2*c*sqrt(-1/(4*a*c - b**2)**3)*log(x
+ (32*a**2*c**3*sqrt(-1/(4*a*c - b**2)**3) - 16*a*b**2*c**2*sqrt(-1/(4*a*c
 - b**2)**3) + 2*b**4*c*sqrt(-1/(4*a*c - b**2)**3) + 2*b*c)/(4*c**2)) + (b
 + 2*c*x)/(4*a**2*c - a*b**2 + x**2*(4*a*c**2 - b**2*c) + x*(4*a*b*c - b**
3))
```

3.427.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(c+a/x^2+b/x)^2/x^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.427.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.15

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^4}\,dx = -\frac{4\,c\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}} - \frac{2\,cx+b}{(cx^2+bx+a)(b^2-4\,ac)}$$

input
```
integrate(1/(c+a/x^2+b/x)^2/x^4,x, algorithm="giac")
```

output
```
-4*c*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a
*c)) - (2*c*x + b)/((c*x^2 + b*x + a)*(b^2 - 4*a*c))
```

3.427.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.80

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^4}\,dx=\frac{\frac{b}{4\,a\,c-b^2}+\frac{2\,c\,x}{4\,a\,c-b^2}}{c\,x^2+b\,x+a}-\frac{4\,c\,\mathrm{atan}\left(\frac{\left(\frac{2\,c\,\left(b^3-4\,a\,b\,c\right)}{{\left(4\,a\,c-b^2\right)}^{5/2}}-\frac{4\,c^2\,x}{{\left(4\,a\,c-b^2\right)}^{3/2}}\right)\,\left(4\,a\,c-b^2\right)}{2\,c}\right)}{{\left(4\,a\,c-b^2\right)}^{3/2}}$$

input
```
int(1/(x^4*(c + a/x^2 + b/x)^2),x)
```

output
```
(b/(4*a*c - b^2) + (2*c*x)/(4*a*c - b^2))/(a + b*x + c*x^2) - (4*c*atan(((
(2*c*(b^3 - 4*a*b*c))/(4*a*c - b^2)^(5/2) - (4*c^2*x)/(4*a*c - b^2)^(3/2))
*(4*a*c - b^2))/(2*c)))/(4*a*c - b^2)^(3/2)
```

3.428 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^5} dx$

3.428.1 Optimal result

Integrand size = 18, antiderivative size = 108

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^5} dx = \frac{b^2-2ac+bcx}{a(b^2-4ac)(a+bx+cx^2)}+\frac{b(b^2-6ac)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2(b^2-4ac)^{3/2}}+\frac{\log(x)}{a^2}-\frac{\log(a+bx+cx^2)}{2a^2}$$

output
```
(b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^2+b*x+a)+b*(-6*a*c+b^2)*arctanh((2*c
*x+b)/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(3/2)+ln(x)/a^2-1/2*ln(c*x^2+b*
x+a)/a^2
```

3.428.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.99

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^5} dx = \frac{\frac{2a(b^2-2ac+bcx)}{(b^2-4ac)(a+x(b+cx))}+\frac{2b(b^2-6ac)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}+2\log(x)-\log(a+x(b+cx))}{2a^2}$$

input
```
Integrate[1/((c + a/x^2 + b/x)^2*x^5),x]
```

output
```
((2*a*(b^2 - 2*a*c + b*c*x))/((b^2 - 4*a*c)*(a + x*(b + c*x))) + (2*b*(b^2
 - 6*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + 2
*Log[x] - Log[a + x*(b + c*x)])/(2*a^2)
```

3.428.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.29, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1692, 1165, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5\left(\frac{a}{x^2}+\frac{b}{x}+c\right)^2}dx$$

$$\Big\downarrow \text{1692}$$

$$\int \frac{1}{x\left(a+bx+cx^2\right)^2}dx$$

$$\Big\downarrow \text{1165}$$

$$\frac{-2ac+b^2+bcx}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{\int -\frac{b^2+cxb-4ac}{x\left(cx^2+bx+a\right)}dx}{a\left(b^2-4ac\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{b^2+cxb-4ac}{x\left(cx^2+bx+a\right)}dx}{a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

$$\Big\downarrow \text{1200}$$

$$\frac{\int \left(\frac{b^2-4ac}{ax}+\frac{-b\left(b^2-5ac\right)-c\left(b^2-4ac\right)x}{a\left(cx^2+bx+a\right)}\right)dx}{a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{b\left(b^2-6ac\right)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}-\frac{\left(b^2-4ac\right)\log\left(a+bx+cx^2\right)}{2a}+\frac{\log(x)\left(b^2-4ac\right)}{a}}{a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

input
```
Int[1/((c + a/x^2 + b/x)^2*x^5),x]
```

output
```
(b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*(a + b*x + c*x^2)) + ((b*(b^2 - 6*a
*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a*Sqrt[b^2 - 4*a*c]) + ((b^2
- 4*a*c)*Log[x])/a - ((b^2 - 4*a*c)*Log[a + b*x + c*x^2])/(2*a))/(a*(b^2 -
 4*a*c))
```

3.428.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1692
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.428. $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^5}\,dx$

3.428.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.64

method	result	size
default	$\frac{\ln(x)}{a^2}-\frac{\frac{\frac{abcx}{4ac-b^2}-\frac{a\left(2ac-b^2\right)}{4ac-b^2}}{c\,x^2+bx+a}+\frac{\frac{\left(4a\,c^2-b^2c\right)\ln\left(c\,x^2+bx+a\right)}{2c}+\frac{2\left(5abc-b^3-\frac{\left(4a\,c^2-b^2c\right)b}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{4ac-b^2}}{a^2}$	177
risch	Expression too large to display	2292

input
```
int(1/(c+a/x^2+b/x)^2/x^5,x,method=_RETURNVERBOSE)
```

output
```
ln(x)/a^2-1/a^2*((a*b*c/(4*a*c-b^2)*x-a*(2*a*c-b^2)/(4*a*c-b^2))/(c*x^2+b*
x+a)+1/(4*a*c-b^2)*(1/2*(4*a*c^2-b^2*c)/c*ln(c*x^2+b*x+a)+2*(5*a*b*c-b^3-1
/2*(4*a*c^2-b^2*c)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/
2))))
```

3.428.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 381 vs. 2(102) = 204.

Time = 0.34 (sec) , antiderivative size = 781, normalized size of antiderivative = 7.23

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^5}\,dx$$

$$= \left[\frac{2\,ab^4-12\,a^2b^2c+16\,a^3c^2+(ab^3-6\,a^2bc+(b^3c-6\,abc^2)x^2+(b^4-6\,ab^2c)x)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx}{}\right.}{}\right.$$

input
```
integrate(1/(c+a/x^2+b/x)^2/x^5,x, algorithm="fricas")
```

output
```
[1/2*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + (a*b^3 - 6*a^2*b*c + (b^3*c -
6*a*b*c^2)*x^2 + (b^4 - 6*a*b^2*c)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2
*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) +
 2*(a*b^3*c - 4*a^2*b*c^2)*x - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c
- 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*log(
c*x^2 + b*x + a) + 2*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*
c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*log(x))/(a^3*b
^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*x
^2 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x), 1/2*(2*a*b^4 - 12*a^2*b^2*
c + 16*a^3*c^2 + 2*(a*b^3 - 6*a^2*b*c + (b^3*c - 6*a*b*c^2)*x^2 + (b^4 - 6
*a*b^2*c)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^
2 - 4*a*c)) + 2*(a*b^3*c - 4*a^2*b*c^2)*x - (a*b^4 - 8*a^2*b^2*c + 16*a^3*
c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b
*c^2)*x)*log(c*x^2 + b*x + a) + 2*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4
*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*l
og(x))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 +
16*a^4*c^3)*x^2 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x)]
```

3.428.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^5} \, dx = \text{Timed out}$$

input
```
integrate(1/(c+a/x**2+b/x)**2/x**5,x)
```

output
```
Timed out
```

3.428.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^5} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(c+a/x^2+b/x)^2/x^5,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.428.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.17

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^5}\,dx = -\frac{(b^3-6\,abc)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(a^2b^2-4\,a^3c)\sqrt{-b^2+4\,ac}} - \frac{\log\left(cx^2+bx+a\right)}{2\,a^2} + \frac{\log\left(|x|\right)}{a^2} + \frac{abcx+ab^2-2\,a^2c}{(cx^2+bx+a)(b^2-4\,ac)a^2}$$

input
```
integrate(1/(c+a/x^2+b/x)^2/x^5,x, algorithm="giac")
```

output
```
-(b^3 - 6*a*b*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((a^2*b^2 - 4*a^3*
c)*sqrt(-b^2 + 4*a*c)) - 1/2*log(c*x^2 + b*x + a)/a^2 + log(abs(x))/a^2 +
(a*b*c*x + a*b^2 - 2*a^2*c)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*a^2)
```

3.428.9 Mupad [B] (verification not implemented)

Time = 8.70 (sec) , antiderivative size = 620, normalized size of antiderivative = 5.74

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^5}\,dx = \frac{\ln(x)}{a^2} + \frac{\frac{2\,a\,c-b^2}{a\,(4\,a\,c-b^2)} - \frac{b\,c\,x}{a\,(4\,a\,c-b^2)}}{c\,x^2+b\,x+a} + \frac{\ln\left(2\,a\,b^6+2\,b^7\,x-96\,a^4\,c^3+2\,a\,b^3\,\sqrt{-(4\,a\,c-b^2)^3}-23\,a^2\,b^4\,c+2\,b^4\,x\,\sqrt{-(4\,a\,c-b^2)^3}+84\,a^3\,b^2\,c^2 \cdots\right.}{\cdots} + \frac{\ln\left(96\,a^4\,c^3-2\,b^7\,x-2\,a\,b^6+2\,a\,b^3\,\sqrt{-(4\,a\,c-b^2)^3}+23\,a^2\,b^4\,c+2\,b^4\,x\,\sqrt{-(4\,a\,c-b^2)^3}-84\,a^3\,b^2\,c^2 \cdots\right.}{\cdots}$$

input
```
int(1/(x^5*(c + a/x^2 + b/x)^2),x)
```

output
```
log(x)/a^2 + ((2*a*c - b^2)/(a*(4*a*c - b^2)) - (b*c*x)/(a*(4*a*c - b^2)))
/(a + b*x + c*x^2) + (log(2*a*b^6 + 2*b^7*x - 96*a^4*c^3 + 2*a*b^3*(-(4*a*
c - b^2)^3)^(1/2) - 23*a^2*b^4*c + 2*b^4*x*(-(4*a*c - b^2)^3)^(1/2) + 84*a
^3*b^2*c^2 + 94*a^2*b^3*c^2*x + 12*a^2*c^2*x*(-(4*a*c - b^2)^3)^(1/2) - 24
*a*b^5*c*x - 9*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2) - 120*a^3*b*c^3*x - 12*a*b
^2*c*x*(-(4*a*c - b^2)^3)^(1/2))*(b^6 - 64*a^3*c^3 + b^3*(-(4*a*c - b^2)^3
)^(1/2) + 48*a^2*b^2*c^2 - 12*a*b^4*c - 6*a*b*c*(-(4*a*c - b^2)^3)^(1/2)))
/(2*a^2*(4*a*c - b^2)^3) + (log(96*a^4*c^3 - 2*b^7*x - 2*a*b^6 + 2*a*b^3*(
-(4*a*c - b^2)^3)^(1/2) + 23*a^2*b^4*c + 2*b^4*x*(-(4*a*c - b^2)^3)^(1/2)
- 84*a^3*b^2*c^2 - 94*a^2*b^3*c^2*x + 12*a^2*c^2*x*(-(4*a*c - b^2)^3)^(1/2
) + 24*a*b^5*c*x - 9*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2) + 120*a^3*b*c^3*x -
12*a*b^2*c*x*(-(4*a*c - b^2)^3)^(1/2))*(b^6 - 64*a^3*c^3 - b^3*(-(4*a*c -
b^2)^3)^(1/2) + 48*a^2*b^2*c^2 - 12*a*b^4*c + 6*a*b*c*(-(4*a*c - b^2)^3)^(
1/2)))/(2*a^2*(4*a*c - b^2)^3)
```

3.428. $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^5}\,dx$

3.429 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^6}\,dx$

3.429.1 Optimal result

Integrand size = 18, antiderivative size = 148

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^6}\,dx = -\frac{2(b^2-3ac)}{a^2(b^2-4ac)\,x}+\frac{b^2-2ac+bcx}{a(b^2-4ac)\,x(a+bx+cx^2)} - \frac{2(b^4-6ab^2c+6a^2c^2)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3(b^2-4ac)^{3/2}} - \frac{2b\log(x)}{a^3}+\frac{b\log(a+bx+cx^2)}{a^3}$$

output
```
-2*(-3*a*c+b^2)/a^2/(-4*a*c+b^2)/x+(b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/x/(c*x
^2+b*x+a)-2*(6*a^2*c^2-6*a*b^2*c+b^4)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2)
)/a^3/(-4*a*c+b^2)^(3/2)-2*b*ln(x)/a^3+b*ln(c*x^2+b*x+a)/a^3
```

3.429.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.89

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^6}\,dx = -\frac{\frac{a}{x}+\frac{a(b^3-3abc+b^2cx-2ac^2x)}{(b^2-4ac)(a+x(b+cx))}+\frac{2(b^4-6ab^2c+6a^2c^2)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}+2b\log(x)-b\log(a+x(b+cx))}{a^3}$$

input
```
Integrate[1/((c + a/x^2 + b/x)^2*x^6),x]
```

output
```
-((a/x + (a*(b^3 - 3*a*b*c + b^2*c*x - 2*a*c^2*x))/((b^2 - 4*a*c)*(a + x*(
b + c*x))) + (2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2
 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + 2*b*Log[x] - b*Log[a + x*(b + c*x)])/a^
3)
```

3.429.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.17, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1692, 1165, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 \left(\frac{a}{x^2}+\frac{b}{x}+c\right)^2} \, dx$$

$$\Big\downarrow \text{1692}$$

$$\int \frac{1}{x^2 \left(a+bx+cx^2\right)^2}dx$$

$$\Big\downarrow \text{1165}$$

$$\frac{-2ac+b^2+bcx}{ax\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{\int -\frac{2\left(b^2+cxb-3ac\right)}{x^2\left(cx^2+bx+a\right)}dx}{a\left(b^2-4ac\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\int \frac{b^2+cxb-3ac}{x^2\left(cx^2+bx+a\right)}dx}{a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx}{ax\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

$$\Big\downarrow \text{1200}$$

$$\frac{2\int \left(\frac{b^2-3ac}{ax^2}+\frac{4abc-b^3}{a^2x}+\frac{b^4-5acb^2+c\left(b^2-4ac\right)xb+3a^2c^2}{a^2\left(cx^2+bx+a\right)}\right)dx}{a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx}{ax\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\left(-\frac{(6a^2c^2-6ab^2c+b^4)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}}+\frac{b(b^2-4ac)\log(a+bx+cx^2)}{2a^2}-\frac{b\log(x)(b^2-4ac)}{a^2}-\frac{b^2-3ac}{ax}\right)}{a(b^2-4ac)}+\frac{-2ac+b^2+bcx}{ax(b^2-4ac)(a+bx+cx^2)}$$

input
```
Int[1/((c + a/x^2 + b/x)^2*x^6),x]
```

output
```
(b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*x*(a + b*x + c*x^2)) + (2*(-((b^2 -
 3*a*c)/(a*x)) - ((b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b
^2 - 4*a*c]])/(a^2*Sqrt[b^2 - 4*a*c]) - (b*(b^2 - 4*a*c)*Log[x])/a^2 + (b*
(b^2 - 4*a*c)*Log[a + b*x + c*x^2])/(2*a^2)))/(a*(b^2 - 4*a*c))
```

3.429.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1692
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

3.429. $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2x^6}\,dx$

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.429.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.39

method	result
default	$-\frac{1}{a^{2} x}-\frac{2 b \ln \left(x\right)}{a^{3}}-\frac{\frac{\frac{a c \left(2 a c -b^{2}\right) x}{4 a c -b^{2}}+\frac{a b \left(3 a c -b^{2}\right)}{4 a c -b^{2}}}{c \,x^{2}+b x +a}+\frac{\frac{\left(-4 a b \,c^{2}+b^{3} c \right) \ln \left(c \,x^{2}+b x +a \right)}{c}+\frac{4 \left(3 a^{2} c^{2}-5 a \,b^{2} c +b^{4}-\frac{\left(-4 a b \,c^{2}+b^{3} c \right) b}{2 c}\right) \arctan \left(\frac{2 c \cdots}{\sqrt{4 a c \cdots}}\right)}{\sqrt{4 a c -b^{2}}}}{4 a c -b^{2}}}{a^{3}}$
risch	$\frac{-\frac{2 c \left(3 a c -b^{2}\right) x^{2}}{a^{2} \left(4 a c -b^{2}\right)}-\frac{b \left(7 a c -2 b^{2}\right) x}{a^{2} \left(4 a c -b^{2}\right)}-\frac{1}{a}}{x \left(c \,x^{2}+b x +a \right)}-\frac{2 b \ln \left(x \right)}{a^{3}}+2 \left(\sum_{\textit{\_R} =\operatorname{RootOf}\left(\left(64 a^{6} c^{3}-48 a^{5} b^{2} c^{2}+12 a^{4} b^{4} c -a^{3} b^{6}\right) \textit{\_Z}^{2}+\left(-64 b \,c^{3} a^{3}+48 b^{3} c^{2} a \cdots\right.\right.}\cdots\right.$

input
```
int(1/(c+a/x^2+b/x)^2/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/a^2/x-2*b*ln(x)/a^3-1/a^3*((a*c*(2*a*c-b^2)/(4*a*c-b^2)*x+a*b*(3*a*c-b^
2)/(4*a*c-b^2))/(c*x^2+b*x+a)+2/(4*a*c-b^2)*(1/2*(-4*a*b*c^2+b^3*c)/c*ln(c
*x^2+b*x+a)+2*(3*a^2*c^2-5*a*b^2*c+b^4-1/2*(-4*a*b*c^2+b^3*c)*b/c)/(4*a*c-
b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))))
```

3.429.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 478 vs. $2(144) = 288$.

Time = 0.40 (sec) , antiderivative size = 975, normalized size of antiderivative = 6.59

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^6}\,dx$$

$$= \left[-\frac{a^{2} b^{4} - 8 \, a^{3} b^{2} c + 16 \, a^{4} c^{2} + 2 \, {\left(a b^{4} c - 7 \, a^{2} b^{2} c^{2} + 12 \, a^{3} c^{3}\right)} x^{2} + {\left({\left(b^{4} c - 6 \, a b^{2} c^{2} + 6 \, a^{2} c^{3}\right)} x^{3} + {\left(b^{5} - 6 \, a b^{3} c \cdots\right.}\right.}}{\cdots}\right.$$

$$-\frac{a^{2} b^{4} - 8 \, a^{3} b^{2} c + 16 \, a^{4} c^{2} + 2 \, {\left(a b^{4} c - 7 \, a^{2} b^{2} c^{2} + 12 \, a^{3} c^{3}\right)} x^{2} + 2 \, {\left({\left(b^{4} c - 6 \, a b^{2} c^{2} + 6 \, a^{2} c^{3}\right)} x^{3} + {\left(b^{5} - 6 \, a b^{3} c \cdots\right.}\right.}}{\cdots}$$

input
```
integrate(1/(c+a/x^2+b/x)^2/x^6,x, algorithm="fricas")
```

output
```
[-(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + 2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^
3*c^3)*x^2 + ((b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^3 + (b^5 - 6*a*b^3*c + 6
*a^2*b*c^2)*x^2 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*x)*sqrt(b^2 - 4*a*c)*l
og((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*
x^2 + b*x + a)) + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*x - ((b^5*c - 8*
a*b^3*c^2 + 16*a^2*b*c^3)*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x^2 + (
a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x)*log(c*x^2 + b*x + a) + 2*((b^5*c -
8*a*b^3*c^2 + 16*a^2*b*c^3)*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x^2 +
 (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x)*log(x))/((a^3*b^4*c - 8*a^4*b^2*c
^2 + 16*a^5*c^3)*x^3 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2 + (a^4*b
^4 - 8*a^5*b^2*c + 16*a^6*c^2)*x), -(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 +
2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*x^2 + 2*((b^4*c - 6*a*b^2*c^2 + 6
*a^2*c^3)*x^3 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*x^2 + (a*b^4 - 6*a^2*b^2*c
 + 6*a^3*c^2)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)
/(b^2 - 4*a*c)) + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*x - ((b^5*c - 8*
a*b^3*c^2 + 16*a^2*b*c^3)*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x^2 + (
a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x)*log(c*x^2 + b*x + a) + 2*((b^5*c -
8*a*b^3*c^2 + 16*a^2*b*c^3)*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x^2 +
 (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x)*log(x))/((a^3*b^4*c - 8*a^4*b^2*c
^2 + 16*a^5*c^3)*x^3 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2 + (a^...
```

3.429.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^6}\,dx = \text{Timed out}$$

input
```
integrate(1/(c+a/x**2+b/x)**2/x**6,x)
```

output
```
Timed out
```

3.429.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^6}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(c+a/x^2+b/x)^2/x^6,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.429.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.16

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^6}\,dx = \frac{2\,(b^4-6\,ab^2c+6\,a^2c^2)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(a^3b^2-4\,a^4c)\sqrt{-b^2+4\,ac}} - \frac{2\,b^2cx^2-6\,ac^2x^2+2\,b^3x-7\,abcx+ab^2-4\,a^2c}{(a^2b^2-4\,a^3c)(cx^3+bx^2+ax)} + \frac{b\log\left(cx^2+bx+a\right)}{a^3} - \frac{2\,b\log\left(|x|\right)}{a^3}$$

input
```
integrate(1/(c+a/x^2+b/x)^2/x^6,x, algorithm="giac")
```

output
```
2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((a
^3*b^2 - 4*a^4*c)*sqrt(-b^2 + 4*a*c)) - (2*b^2*c*x^2 - 6*a*c^2*x^2 + 2*b^3
*x - 7*a*b*c*x + a*b^2 - 4*a^2*c)/((a^2*b^2 - 4*a^3*c)*(c*x^3 + b*x^2 + a*
x)) + b*log(c*x^2 + b*x + a)/a^3 - 2*b*log(abs(x))/a^3
```

3.429.9 Mupad [B] (verification not implemented)

Time = 8.85 (sec) , antiderivative size = 775, normalized size of antiderivative = 5.24

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^6}\,dx$$
$$= \ln\left(2\,a\,b^7+2\,b^8\,x+2\,a\,b^4\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-23\,a^2\,b^5\,c-108\,a^4\,b\,c^3+24\,a^4\,c^4\,x+2\,b^5\,x\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}+87\,a^3\,b^3\,c^2+3\,a^3\,c^2\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-9\,a^2\,b^2\,c\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}+97\,a^2\,b^4\,c^2\,x-138\,a^3\,b^2\,c^3\,x-24\,a\,b^6\,c\,x-12\,a\,b^3\,c\,x\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}+15\,a^2\,b\,c^2\,x\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}\right)\left(\frac{b^4\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}+6\,a^2\,c^2\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-6\,a\,b^2\,c\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}}{-64\,a^6\,c^3+48\,a^5\,b^2\,c^2-12\,a^4\,b^4\,c+a^3\,b^6}+\frac{b}{a^3}\right)-\frac{\frac{1}{a}-\frac{x\,\left(2\,b^3-7\,a\,b\,c\right)}{a^2\,\left(4\,a\,c-b^2\right)}+\frac{2\,c\,x^2\,\left(3\,a\,c-b^2\right)}{a^2\,\left(4\,a\,c-b^2\right)}}{c\,x^3+b\,x^2+a\,x}-\ln\left(2\,a\,b^4\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-2\,b^8\,x-2\,a\,b^7+23\,a^2\,b^5\,c+108\,a^4\,b\,c^3-24\,a^4\,c^4\,x+2\,b^5\,x\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-87\,a^3\,b^3\,c^2+3\,a^3\,c^2\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-9\,a^2\,b^2\,c\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-97\,a^2\,b^4\,c^2\,x+138\,a^3\,b^2\,c^3\,x+24\,a\,b^6\,c\,x-12\,a\,b^3\,c\,x\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}+15\,a^2\,b\,c^2\,x\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}\right)\left(\frac{b^4\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}+6\,a^2\,c^2\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-6\,a\,b^2\,c\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}}{-64\,a^6\,c^3+48\,a^5\,b^2\,c^2-12\,a^4\,b^4\,c+a^3\,b^6}-\frac{b}{a^3}\right)-\frac{2\,b\,\ln\left(x\right)}{a^3}$$

input
```
int(1/(x^6*(c + a/x^2 + b/x)^2),x)
```

output
```
log(2*a*b^7 + 2*b^8*x + 2*a*b^4*(-(4*a*c - b^2)^3)^(1/2) - 23*a^2*b^5*c -
108*a^4*b*c^3 + 24*a^4*c^4*x + 2*b^5*x*(-(4*a*c - b^2)^3)^(1/2) + 87*a^3*b
^3*c^2 + 3*a^3*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*(-(4*a*c - b^2)^
3)^(1/2) + 97*a^2*b^4*c^2*x - 138*a^3*b^2*c^3*x - 24*a*b^6*c*x - 12*a*b^3*
c*x*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b*c^2*x*(-(4*a*c - b^2)^3)^(1/2))*((
b^4*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^
2*c*(-(4*a*c - b^2)^3)^(1/2))/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^
5*b^2*c^2) + b/a^3) - (1/a - (x*(2*b^3 - 7*a*b*c))/(a^2*(4*a*c - b^2)) + (
2*c*x^2*(3*a*c - b^2))/(a^2*(4*a*c - b^2)))/(a*x + b*x^2 + c*x^3) - log(2*
a*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*b^8*x - 2*a*b^7 + 23*a^2*b^5*c + 108*a^
4*b*c^3 - 24*a^4*c^4*x + 2*b^5*x*(-(4*a*c - b^2)^3)^(1/2) - 87*a^3*b^3*c^2
 + 3*a^3*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*(-(4*a*c - b^2)^3)^(1/
2) - 97*a^2*b^4*c^2*x + 138*a^3*b^2*c^3*x + 24*a*b^6*c*x - 12*a*b^3*c*x*(-
(4*a*c - b^2)^3)^(1/2) + 15*a^2*b*c^2*x*(-(4*a*c - b^2)^3)^(1/2))*((b^4*(-
(4*a*c - b^2)^3)^(1/2) + 6*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*(-
(4*a*c - b^2)^3)^(1/2))/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*
c^2) - b/a^3) - (2*b*log(x))/a^3
```

3.429. $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^6}\,dx$

3.430 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^7}\,dx$

3.430.1 Optimal result

Integrand size = 18, antiderivative size = 202

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^7}\,dx = -\frac{3b^2-8ac}{2a^2(b^2-4ac)x^2}+\frac{b(3b^2-11ac)}{a^3(b^2-4ac)x} + \frac{b^2-2ac+bcx}{a(b^2-4ac)x^2(a+bx+cx^2)} + \frac{b(3b^4-20ab^2c+30a^2c^2)\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4(b^2-4ac)^{3/2}} + \frac{(3b^2-2ac)\log(x)}{a^4}-\frac{(3b^2-2ac)\log(a+bx+cx^2)}{2a^4}$$

output
```
1/2*(8*a*c-3*b^2)/a^2/(-4*a*c+b^2)/x^2+b*(-11*a*c+3*b^2)/a^3/(-4*a*c+b^2)/
x+(b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/x^2/(c*x^2+b*x+a)+b*(30*a^2*c^2-20*a*b^
2*c+3*b^4)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/a^4/(-4*a*c+b^2)^(3/2)+(-
2*a*c+3*b^2)*ln(x)/a^4-1/2*(-2*a*c+3*b^2)*ln(c*x^2+b*x+a)/a^4
```

3.430.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.87

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^7}\,dx = \frac{-\frac{a^2}{x^2}+\frac{4ab}{x}+\frac{2a\left(b^4-4ab^2c+2a^2c^2+b^3cx-3abc^2x\right)}{(b^2-4ac)(a+x(b+cx))}+\frac{2b\left(3b^4-20ab^2c+30a^2c^2\right)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}+2(3b^2-2ac)\log(x)+(-}{2a^4}$$

input
```
Integrate[1/((c + a/x^2 + b/x)^2*x^7),x]
```

output
```
(-(a^2/x^2) + (4*a*b)/x + (2*a*(b^4 - 4*a*b^2*c + 2*a^2*c^2 + b^3*c*x - 3*
a*b*c^2*x))/((b^2 - 4*a*c)*(a + x*(b + c*x))) + (2*b*(3*b^4 - 20*a*b^2*c +
 30*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
+ 2*(3*b^2 - 2*a*c)*Log[x] + (-3*b^2 + 2*a*c)*Log[a + x*(b + c*x)])/(2*a^4
)
```

3.430.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1692, 1165, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7\left(\frac{a}{x^2}+\frac{b}{x}+c\right)^2}dx$$

$$\Big\downarrow \text{ 1692}$$

$$\int \frac{1}{x^3\left(a+bx+cx^2\right)^2}dx$$

$$\Big\downarrow \text{ 1165}$$

$$\frac{-2ac+b^2+bcx}{ax^2\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{\int -\frac{3b^2+3cxb-8ac}{x^3\left(cx^2+bx+a\right)}dx}{a\left(b^2-4ac\right)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{3b^2+3cxb-8ac}{x^3(cx^2+bx+a)}dx}{a(b^2-4ac)}+\frac{-2ac+b^2+bcx}{ax^2(b^2-4ac)(a+bx+cx^2)}$$

$$\Big\downarrow \text{ 1200}$$

$$\frac{\int \left(\frac{3b^2-8ac}{ax^3}+\frac{(b^2-4ac)(3b^2-2ac)}{a^3x}+\frac{-b(3b^4-17acb^2+19a^2c^2)-c(3b^4-14acb^2+8a^2c^2)x}{a^3(cx^2+bx+a)}+\frac{b(11ac-3b^2)}{a^2x^2}\right)dx}{a(b^2-4ac)}+\frac{-2ac+b^2+bcx}{ax^2(b^2-4ac)(a+bx+cx^2)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{(b^2-4ac)(3b^2-2ac)\log(a+bx+cx^2)}{2a^3}+\frac{\log(x)(b^2-4ac)(3b^2-2ac)}{a^3}+\frac{b(3b^2-11ac)}{a^2x}+\frac{b(30a^2c^2-20ab^2c+3b^4)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}}-\frac{3b^2}{2}}{a(b^2-4ac)}+\frac{-2ac+b^2+bcx}{ax^2(b^2-4ac)(a+bx+cx^2)}$$

input
```
Int[1/((c + a/x^2 + b/x)^2*x^7),x]
```

output
```
(b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*x^2*(a + b*x + c*x^2)) + (-1/2*(3*b
^2 - 8*a*c)/(a*x^2) + (b*(3*b^2 - 11*a*c))/(a^2*x) + (b*(3*b^4 - 20*a*b^2*
c + 30*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^3*Sqrt[b^2 - 4*
a*c]) + ((b^2 - 4*a*c)*(3*b^2 - 2*a*c)*Log[x])/a^3 - ((b^2 - 4*a*c)*(3*b^2
 - 2*a*c)*Log[a + b*x + c*x^2])/(2*a^3))/(a*(b^2 - 4*a*c))
```

3.430.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

3.430. $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2x^7}\,dx$

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1692
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.430.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.26

method	result
default	$-\frac{1}{2a^2x^2}+\frac{(-2ac+3b^2)\ln(x)}{a^4}+\frac{2b}{a^3x}+\frac{\frac{\frac{acb(3ac-b^2)x}{4ac-b^2}-\frac{a(2a^2c^2-4ab^2c+b^4)}{4ac-b^2}}{cx^2+bx+a}+\frac{\frac{(8a^2c^3-14b^2ac^2+3b^4c)\ln(cx^2+bx+a)}{2c}+\frac{2\left(19a^2bc^2 \cdots\right.}{\cdots}}{\cdots}}{a^4}$
risch	Expression too large to display

input
```
int(1/(c+a/x^2+b/x)^2/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/2/a^2/x^2+(-2*a*c+3*b^2)*ln(x)/a^4+2/a^3*b/x+1/a^4*((a*c*b*(3*a*c-b^2)/
(4*a*c-b^2)*x-a*(2*a^2*c^2-4*a*b^2*c+b^4)/(4*a*c-b^2))/(c*x^2+b*x+a)+1/(4*
a*c-b^2)*(1/2*(8*a^2*c^3-14*a*b^2*c^2+3*b^4*c)/c*ln(c*x^2+b*x+a)+2*(19*a^2
*b*c^2-17*a*b^3*c+3*b^5-1/2*(8*a^2*c^3-14*a*b^2*c^2+3*b^4*c)*b/c)/(4*a*c-b
^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))))
```

3.430.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 603 vs. 2(194) = 388.

Time = 0.46 (sec) , antiderivative size = 1226, normalized size of antiderivative = 6.07

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^7} \, dx = \text{Too large to display}$$

input
```
integrate(1/(c+a/x^2+b/x)^2/x^7,x, algorithm="fricas")
```

output
```
[-1/2*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 - 2*(3*a*b^5*c - 23*a^2*b^3*c^2
+ 44*a^3*b*c^3)*x^3 - (6*a*b^6 - 49*a^2*b^4*c + 108*a^3*b^2*c^2 - 32*a^4*c
^3)*x^2 + ((3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*x^4 + (3*b^6 - 20*a*b^4
*c + 30*a^2*b^2*c^2)*x^3 + (3*a*b^5 - 20*a^2*b^3*c + 30*a^3*b*c^2)*x^2)*sq
rt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)
*(2*c*x + b))/(c*x^2 + b*x + a)) - 3*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2
)*x + ((3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*x^4 + (3*b^7
 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*x^3 + (3*a*b^6 - 26*a^2*b^4
*c + 64*a^3*b^2*c^2 - 32*a^4*c^3)*x^2)*log(c*x^2 + b*x + a) - 2*((3*b^6*c
- 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*x^4 + (3*b^7 - 26*a*b^5*c +
64*a^2*b^3*c^2 - 32*a^3*b*c^3)*x^3 + (3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*
c^2 - 32*a^4*c^3)*x^2)*log(x))/((a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*x
^4 + (a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x^3 + (a^5*b^4 - 8*a^6*b^2*c +
 16*a^7*c^2)*x^2), -1/2*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 - 2*(3*a*b^5*c
 - 23*a^2*b^3*c^2 + 44*a^3*b*c^3)*x^3 - (6*a*b^6 - 49*a^2*b^4*c + 108*a^3*
b^2*c^2 - 32*a^4*c^3)*x^2 - 2*((3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*x^4
 + (3*b^6 - 20*a*b^4*c + 30*a^2*b^2*c^2)*x^3 + (3*a*b^5 - 20*a^2*b^3*c + 3
0*a^3*b*c^2)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b
)/(b^2 - 4*a*c)) - 3*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x + ((3*b^6*c
- 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*x^4 + (3*b^7 - 26*a*b^5*c...
```

3.430.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^7} \, dx = \text{Timed out}$$

input
```
integrate(1/(c+a/x**2+b/x)**2/x**7,x)
```

output
```
Timed out
```

3.430.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^7}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(c+a/x^2+b/x)^2/x^7,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.430.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.13

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^7}\,dx = -\frac{(3\,b^5-20\,ab^3c+30\,a^2bc^2)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(a^4b^2-4\,a^5c)\sqrt{-b^2+4\,ac}} - \frac{(3\,b^2-2\,ac)\log\left(cx^2+bx+a\right)}{2\,a^4} + \frac{(3\,b^2-2\,ac)\log\left(|x|\right)}{a^4} - \frac{a^3b^2-4\,a^4c-2\,(3\,ab^3c-11\,a^2bc^2)x^3-(6\,ab^4-25\,a^2b^2c+8\,a^3c^2)x^2-3\,(a^2b^3-4\,a^3bc)x}{2\,(cx^2+bx+a)(b^2-4\,ac)a^4x^2}$$

input
```
integrate(1/(c+a/x^2+b/x)^2/x^7,x, algorithm="giac")
```

output
```
-(3*b^5 - 20*a*b^3*c + 30*a^2*b*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c)
)/((a^4*b^2 - 4*a^5*c)*sqrt(-b^2 + 4*a*c)) - 1/2*(3*b^2 - 2*a*c)*log(c*x^2
 + b*x + a)/a^4 + (3*b^2 - 2*a*c)*log(abs(x))/a^4 - 1/2*(a^3*b^2 - 4*a^4*c
 - 2*(3*a*b^3*c - 11*a^2*b*c^2)*x^3 - (6*a*b^4 - 25*a^2*b^2*c + 8*a^3*c^2)
*x^2 - 3*(a^2*b^3 - 4*a^3*b*c)*x)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*a^4*x^2
)
```

3.430.9 Mupad [B] (verification not implemented)

Time = 9.12 (sec) , antiderivative size = 914, normalized size of antiderivative = 4.52

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^7}\,dx$$

$$= \frac{\ln\left(6\,a\,b^8+6\,b^9\,x+192\,a^5\,c^4-6\,a\,b^5\,\sqrt{-\left(4\,a\,c-b^2\right)^3}-73\,a^2\,b^6\,c-6\,b^6\,x\,\sqrt{-\left(4\,a\,c-b^2\right)^3}+307\,a^3\,b^4\,c^2\right.}{}$$

$$-\frac{\ln\left(x\right)\,\left(2\,a\,c-3\,b^2\right)}{a^4}-\frac{\frac{1}{2\,a}-\frac{3\,b\,x}{2\,a^2}+\frac{x^2\,\left(8\,a^2\,c^2-25\,a\,b^2\,c+6\,b^4\right)}{2\,a^3\,\left(4\,a\,c-b^2\right)}-\frac{b\,c\,x^3\,\left(11\,a\,c-3\,b^2\right)}{a^3\,\left(4\,a\,c-b^2\right)}}{c\,x^4+b\,x^3+a\,x^2}$$

$$+\frac{\ln\left(6\,a\,b^8+6\,b^9\,x+192\,a^5\,c^4+6\,a\,b^5\,\sqrt{-\left(4\,a\,c-b^2\right)^3}-73\,a^2\,b^6\,c+6\,b^6\,x\,\sqrt{-\left(4\,a\,c-b^2\right)^3}+307\,a^3\,b^4\,c\right.}{}$$

input
```
int(1/(x^7*(c + a/x^2 + b/x)^2),x)
```

output
```
(log(6*a*b^8 + 6*b^9*x + 192*a^5*c^4 - 6*a*b^5*(-(4*a*c - b^2)^3)^(1/2) -
73*a^2*b^6*c - 6*b^6*x*(-(4*a*c - b^2)^3)^(1/2) + 307*a^3*b^4*c^2 - 492*a^
4*b^2*c^3 + 31*a^2*b^3*c*(-(4*a*c - b^2)^3)^(1/2) - 27*a^3*b*c^2*(-(4*a*c
- b^2)^3)^(1/2) + 339*a^2*b^5*c^2*x - 602*a^3*b^3*c^3*x + 24*a^3*c^3*x*(-(
4*a*c - b^2)^3)^(1/2) - 76*a*b^7*c*x + 312*a^4*b*c^4*x + 40*a*b^4*c*x*(-(4
*a*c - b^2)^3)^(1/2) - 69*a^2*b^2*c^2*x*(-(4*a*c - b^2)^3)^(1/2))*(3*b^8 +
 128*a^4*c^4 - 3*b^5*(-(4*a*c - b^2)^3)^(1/2) + 168*a^2*b^4*c^2 - 288*a^3*
b^2*c^3 - 38*a*b^6*c - 30*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) + 20*a*b^3*c*
(-(4*a*c - b^2)^3)^(1/2)))/(2*a^4*(4*a*c - b^2)^3) - (log(x)*(2*a*c - 3*b^
2))/a^4 - (1/(2*a) - (3*b*x)/(2*a^2) + (x^2*(6*b^4 + 8*a^2*c^2 - 25*a*b^2*
c))/(2*a^3*(4*a*c - b^2)) - (b*c*x^3*(11*a*c - 3*b^2))/(a^3*(4*a*c - b^2))
)/(a*x^2 + b*x^3 + c*x^4) + (log(6*a*b^8 + 6*b^9*x + 192*a^5*c^4 + 6*a*b^5
*(-(4*a*c - b^2)^3)^(1/2) - 73*a^2*b^6*c + 6*b^6*x*(-(4*a*c - b^2)^3)^(1/2
) + 307*a^3*b^4*c^2 - 492*a^4*b^2*c^3 - 31*a^2*b^3*c*(-(4*a*c - b^2)^3)^(1
/2) + 27*a^3*b*c^2*(-(4*a*c - b^2)^3)^(1/2) + 339*a^2*b^5*c^2*x - 602*a^3*
b^3*c^3*x - 24*a^3*c^3*x*(-(4*a*c - b^2)^3)^(1/2) - 76*a*b^7*c*x + 312*a^4
*b*c^4*x - 40*a*b^4*c*x*(-(4*a*c - b^2)^3)^(1/2) + 69*a^2*b^2*c^2*x*(-(4*a
*c - b^2)^3)^(1/2))*(3*b^8 + 128*a^4*c^4 + 3*b^5*(-(4*a*c - b^2)^3)^(1/2)
+ 168*a^2*b^4*c^2 - 288*a^3*b^2*c^3 - 38*a*b^6*c + 30*a^2*b*c^2*(-(4*a*c -
 b^2)^3)^(1/2) - 20*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2)))/(2*a^4*(4*a*c - ...
```

3.431 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3}\,dx$

3.431.1 Optimal result

Integrand size = 14, antiderivative size = 238

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3}\,dx = \frac{3(b^4-7ab^2c+10a^2c^2)\,x}{c^3\left(b^2-4ac\right)^2}-\frac{3b(b^2-6ac)\,x^2}{2c^2\left(b^2-4ac\right)^2} + \frac{x^5(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{x^3(a(b^2-10ac)+b(b^2-7ac)\,x)}{c\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} - \frac{3(b^6-10ab^4c+30a^2b^2c^2-20a^3c^3)\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^4\left(b^2-4ac\right)^{5/2}} - \frac{3b\log\left(a+bx+cx^2\right)}{2c^4}$$

output

```
3*(10*a^2*c^2-7*a*b^2*c+b^4)*x/c^3/(-4*a*c+b^2)^2-3/2*b*(-6*a*c+b^2)*x^2/c
^2/(-4*a*c+b^2)^2+1/2*x^5*(b*x+2*a)/(-4*a*c+b^2)/(c*x^2+b*x+a)^2+x^3*(a*(-
10*a*c+b^2)+b*(-7*a*c+b^2)*x)/c/(-4*a*c+b^2)^2/(c*x^2+b*x+a)-3*(-20*a^3*c^
3+30*a^2*b^2*c^2-10*a*b^4*c+b^6)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^4
/(-4*a*c+b^2)^(5/2)-3/2*b*ln(c*x^2+b*x+a)/c^4
```

3.431.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.09

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3}\,dx$$

$$= \frac{2c^2x+\frac{b^7-14ab^5c+61a^2b^3c^2-78a^3bc^3-6b^6cx+48ab^4c^2x-102a^2b^2c^3x+36a^3c^4x}{(b^2-4ac)^2(a+x(b+cx))}+\frac{-b^6x+a^2b^2c(5b-9cx)-ab^4(b-6cx)+a^3c^2(-5b+2cx)}{(b^2-4ac)(a+x(b+cx))^2}+}{2c^5}$$

input
```
Integrate[(c + a/x^2 + b/x)^(-3),x]
```

output
```
(2*c^2*x + (b^7 - 14*a*b^5*c + 61*a^2*b^3*c^2 - 78*a^3*b*c^3 - 6*b^6*c*x +
 48*a*b^4*c^2*x - 102*a^2*b^2*c^3*x + 36*a^3*c^4*x)/((b^2 - 4*a*c)^2*(a +
x*(b + c*x))) + (-(b^6*x) + a^2*b^2*c*(5*b - 9*c*x) - a*b^4*(b - 6*c*x) +
a^3*c^2*(-5*b + 2*c*x))/((b^2 - 4*a*c)*(a + x*(b + c*x))^2) + (6*c*(b^6 -
10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4
*a*c]])/(-b^2 + 4*a*c)^(5/2) - 3*b*c*Log[a + x*(b + c*x)])/(2*c^5)
```

3.431.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1679, 1164, 27, 1233, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\frac{a}{x^2}+\frac{b}{x}+c\right)^3}dx$$

$$\Big\downarrow \text{1679}$$

$$\int \frac{x^6}{(a+bx+cx^2)^3}dx$$

$$\Big\downarrow \text{1164}$$

$$\frac{x^5(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2}-\frac{\int \frac{2x^4(5a+bx)}{(cx^2+bx+a)^2}dx}{2(b^2-4ac)}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^5(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2}-\frac{\int \frac{x^4(5a+bx)}{(cx^2+bx+a)^2}dx}{b^2-4ac}$$

$\Big\downarrow$ 1233

$$\frac{x^5(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2}-\frac{\frac{\int \frac{3x^2\left(a\left(b^2-10ac\right)+b\left(b^2-6ac\right)x\right)}{cx^2+bx+a}dx}{c(b^2-4ac)}-\frac{x^3(bx(b^2-7ac)+a(b^2-10ac))}{c(b^2-4ac)(a+bx+cx^2)}}{b^2-4ac}$$

$\Big\downarrow$ 27

$$\frac{x^5(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2}-\frac{\frac{3\int \frac{x^2\left(a\left(b^2-10ac\right)+b\left(b^2-6ac\right)x\right)}{cx^2+bx+a}dx}{c(b^2-4ac)}-\frac{x^3(bx(b^2-7ac)+a(b^2-10ac))}{c(b^2-4ac)(a+bx+cx^2)}}{b^2-4ac}$$

$\Big\downarrow$ 1200

$$\frac{x^5(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2}-\frac{\frac{3\int \left(-\frac{b^4-7acb^2+10a^2c^2}{c^2}+\frac{b\left(b^2-6ac\right)x}{c}+\frac{bx\left(b^2-4ac\right)^2+a\left(b^4-7acb^2+10a^2c^2\right)}{c^2\left(cx^2+bx+a\right)}\right)dx}{c(b^2-4ac)}-\frac{x^3(bx(b^2-7ac)+a(b^2-10ac))}{c(b^2-4ac)(a+bx+cx^2)}}{b^2-4ac}$$

$\Big\downarrow$ 2009

$$\frac{x^5(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2}-\frac{\frac{3\left(-\frac{x\left(10a^2c^2-7ab^2c+b^4\right)}{c^2}+\frac{\left(-20a^3c^3+30a^2b^2c^2-10ab^4c+b^6\right)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}}+\frac{b\left(b^2-4ac\right)^2\log\left(a+bx+cx^2\right)}{2c^3}+\frac{bx^2\left(b^2-6ac\right)}{2c}\right)}{c(b^2-4ac)}-\frac{x^3(bx(b^2-7ac}{c(b^2-4ac)}}{b^2-4ac}$$

input
```
Int[(c + a/x^2 + b/x)^(-3),x]
```

output
```
(x^5*(2*a + b*x))/(2*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) - (-((x^3*(a*(b^2
- 10*a*c) + b*(b^2 - 7*a*c)*x))/(c*(b^2 - 4*a*c)*(a + b*x + c*x^2))) + (3*
(-(((b^4 - 7*a*b^2*c + 10*a^2*c^2)*x)/c^2) + (b*(b^2 - 6*a*c)*x^2)/(2*c) +
 ((b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*ArcTanh[(b + 2*c*x)/Sqr
t[b^2 - 4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]) + (b*(b^2 - 4*a*c)^2*Log[a + b*x
+ c*x^2])/(2*c^3)))/(c*(b^2 - 4*a*c)))/(b^2 - 4*a*c)
```

3.431. $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3}\,dx$

3.431.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1164
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1233
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Simp[1/(c*(
p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Sim
p[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f
*(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(
m + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*
p + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -1] &&
GtQ[m, 1] && ((EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) |
|  !ILtQ[m + 2*p + 3, 0])
```

rule 1679
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(
2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n]
 && LtQ[n, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.431. $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3} dx$

3.431.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.68

method	result
default	$\frac{x}{c^3}-\frac{\frac{-\frac{3\left(6c^3a^3-17a^2b^2c^2+8a\,b^4c-b^6\right)x^3}{16a^2c^2-8a\,b^2c+b^4}+\frac{b\left(42c^3a^3+41a^2b^2c^2-34a\,b^4c+5b^6\right)x^2}{2\left(16a^2c^2-8a\,b^2c+b^4\right)c}-\frac{a\left(14c^3a^3-71a^2b^2c^2+38a\,b^4c-5b^6\right)x}{c\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{b\,a^2\left(58a^2c^2-36a \cdots\right.}{2c\left(16a^2c^2-8a \cdots\right.}}{\left(c\,x^2+bx+a\right)^2} \cdots}{c^3}$
risch	Expression too large to display

input
```
int(1/(c+a/x^2+b/x)^3,x,method=_RETURNVERBOSE)
```

output
```
x/c^3-1/c^3*((-3*(6*a^3*c^3-17*a^2*b^2*c^2+8*a*b^4*c-b^6)/(16*a^2*c^2-8*a*
b^2*c+b^4)*x^3+1/2*b*(42*a^3*c^3+41*a^2*b^2*c^2-34*a*b^4*c+5*b^6)/(16*a^2*
c^2-8*a*b^2*c+b^4)/c*x^2-a/c*(14*a^3*c^3-71*a^2*b^2*c^2+38*a*b^4*c-5*b^6)/
(16*a^2*c^2-8*a*b^2*c+b^4)*x+1/2*b*a^2/c*(58*a^2*c^2-36*a*b^2*c+5*b^4)/(16
*a^2*c^2-8*a*b^2*c+b^4))/(c*x^2+b*x+a)^2+3/(16*a^2*c^2-8*a*b^2*c+b^4)*(1/2
*(16*a^2*b*c^2-8*a*b^3*c+b^5)/c*ln(c*x^2+b*x+a)+2*(10*a^3*c^2-7*a^2*b^2*c+
b^4*a-1/2*(16*a^2*b*c^2-8*a*b^3*c+b^5)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*
x+b)/(4*a*c-b^2)^(1/2))))
```

3.431.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 953 vs. $2(228) = 456$.

Time = 0.29 (sec) , antiderivative size = 1926, normalized size of antiderivative = 8.09

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c+a/x^2+b/x)^3,x, algorithm="fricas")
```

output
```
[-1/2*(5*a^2*b^7 - 56*a^3*b^5*c + 202*a^4*b^3*c^2 - 232*a^5*b*c^3 - 2*(b^6
*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*x^5 - 4*(b^7*c^2 - 12*a
*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*x^4 + 2*(2*b^8*c - 26*a*b^6*c^2
+ 123*a^2*b^4*c^3 - 254*a^3*b^2*c^4 + 200*a^4*c^5)*x^3 + (5*b^9 - 58*a*b^7
*c + 225*a^2*b^5*c^2 - 314*a^3*b^3*c^3 + 88*a^4*b*c^4)*x^2 + 3*(a^2*b^6 -
10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^3 + (b^6*c^2 - 10*a*b^4*c^3 + 30*
a^2*b^2*c^4 - 20*a^3*c^5)*x^4 + 2*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 -
 20*a^3*b*c^4)*x^3 + (b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 -
40*a^4*c^4)*x^2 + 2*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b*c^3)
*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 -
4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*(5*a*b^8 - 59*a^2*b^6*c + 235*a
^3*b^4*c^2 - 346*a^4*b^2*c^3 + 120*a^5*c^4)*x + 3*(a^2*b^7 - 12*a^3*b^5*c
+ 48*a^4*b^3*c^2 - 64*a^5*b*c^3 + (b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4
 - 64*a^3*b*c^5)*x^4 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b
^2*c^4)*x^3 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^
4*b*c^4)*x^2 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*
x)*log(c*x^2 + b*x + a))/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 -
64*a^5*c^7 + (b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)*x^4 +
2*(b^7*c^5 - 12*a*b^5*c^6 + 48*a^2*b^3*c^7 - 64*a^3*b*c^8)*x^3 + (b^8*c^4
- 10*a*b^6*c^5 + 24*a^2*b^4*c^6 + 32*a^3*b^2*c^7 - 128*a^4*c^8)*x^2 + 2...
```

3.431.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1714 vs. $2(236) = 472$.

Time = 3.04 (sec) , antiderivative size = 1714, normalized size of antiderivative = 7.20

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c+a/x**2+b/x)**3,x)
```

output
```
(-3*b/(2*c**4) - 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c
**2 + 10*a*b**4*c - b**6)/(2*c**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 +
640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)))*log(x + (
-66*a**3*b*c**2 - 64*a**3*c**6*(-3*b/(2*c**4) - 3*sqrt(-(4*a*c - b**2)**5)
*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*c**4*(1024*a**
5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 2
0*a*b**8*c - b**10))) + 27*a**2*b**3*c + 48*a**2*b**2*c**5*(-3*b/(2*c**4)
- 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4
*c - b**6)/(2*c**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c
**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10))) - 3*a*b**5 - 12*a*b**4*c
**4*(-3*b/(2*c**4) - 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b*
*2*c**2 + 10*a*b**4*c - b**6)/(2*c**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**
4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10))) + b**
6*c**3*(-3*b/(2*c**4) - 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2
*b**2*c**2 + 10*a*b**4*c - b**6)/(2*c**4*(1024*a**5*c**5 - 1280*a**4*b**2*
c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10))))/(
60*a**3*c**3 - 90*a**2*b**2*c**2 + 30*a*b**4*c - 3*b**6)) + (-3*b/(2*c**4)
 + 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**
4*c - b**6)/(2*c**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*
c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)))*log(x + (-66*a**3*b*...
```

3.431.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(c+a/x^2+b/x)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.431.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.18

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3}\,dx = \frac{3\,(b^6-10\,ab^4c+30\,a^2b^2c^2-20\,a^3c^3)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4c^4-8\,ab^2c^5+16\,a^2c^6)\sqrt{-b^2+4\,ac}}+\frac{x}{c^3}-\frac{3\,b\log\left(cx^2+bx+a\right)}{2\,c^4} - \frac{5\,a^2b^5-36\,a^3b^3c+58\,a^4bc^2+6\,(b^6c-8\,ab^4c^2+17\,a^2b^2c^3-6\,a^3c^4)x^3+(5\,b^7-34\,ab^5c+41\,a^2b^3c^2+42\cdots}{2\,(cx^2+bx+a)^2(b^2-4\,ac)^2c^4}$$

input
```
integrate(1/(c+a/x^2+b/x)^3,x, algorithm="giac")
```

output
```
3*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*arctan((2*c*x + b)/sqrt
(-b^2 + 4*a*c))/((b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*sqrt(-b^2 + 4*a*c))
+ x/c^3 - 3/2*b*log(c*x^2 + b*x + a)/c^4 - 1/2*(5*a^2*b^5 - 36*a^3*b^3*c +
 58*a^4*b*c^2 + 6*(b^6*c - 8*a*b^4*c^2 + 17*a^2*b^2*c^3 - 6*a^3*c^4)*x^3 +
 (5*b^7 - 34*a*b^5*c + 41*a^2*b^3*c^2 + 42*a^3*b*c^3)*x^2 + 2*(5*a*b^6 - 3
8*a^2*b^4*c + 71*a^3*b^2*c^2 - 14*a^4*c^3)*x)/((c*x^2 + b*x + a)^2*(b^2 -
4*a*c)^2*c^4)
```

3.431.9 Mupad [B] (verification not implemented)

Time = 8.59 (sec) , antiderivative size = 705, normalized size of antiderivative = 2.96

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3}\,dx = \frac{x}{c^3} - \frac{\frac{3\,x^3\,\left(-6\,a^3\,c^3+17\,a^2\,b^2\,c^2-8\,a\,b^4\,c+b^6\right)}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}+\frac{x^2\,\left(42\,a^3\,b\,c^3+41\,a^2\,b^3\,c^2-34\,a\,b^5\,c+5\,b^7\right)}{2\,c\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}+\frac{a^2\,\left(58\,a^2\,b\,c^2-36\,a\,b^3\,c+5\,b^5\right)}{2\,c\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}+\frac{a\,x\,\left(-14\,a^3\,c^3+7\cdots\right)}{c\,(16\,a^2\cdots)}}{a^2\,c^3+c^5\,x^4+x^2\,\left(b^2\,c^3+2\,a\,c^4\right)+2\,b\,c^4\,x^3+2\,a\,b\,c^3\,x} + \frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(-3072\,a^5\,b\,c^5+3840\,a^4\,b^3\,c^4-1920\,a^3\,b^5\,c^3+480\,a^2\,b^7\,c^2-60\,a\,b^9\,c+3\,b^{11}\right)}{2\,\left(1024\,a^5\,c^9-1280\,a^4\,b^2\,c^8+640\,a^3\,b^4\,c^7-160\,a^2\,b^6\,c^6+20\,a\,b^8\,c^5-b^{10}\,c^4\right)} + \frac{3\,\mathrm{atan}\left(\frac{\left(\frac{3\,x\,\left(-20\,a^3\,c^3+30\,a^2\,b^2\,c^2-10\,a\,b^4\,c+b^6\right)}{c^3\,\left(4\,a\,c-b^2\right)^5}+\frac{3\,\left(16\,a^2\,b\,c^5-8\,a\,b^3\,c^4+b^5\,c^3\right)\,\left(-20\,a^3\,c^3+30\,a^2\,b^2\,c^2-10\,a\,b^4\,c+b^6\right)}{2\,c^7\,\left(4\,a\,c-b^2\right)^5\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}\right)\,\left(32\,a^2\,c^6\,\left(4\,a\,c-b^2\right)^{5/2}\cdots\right)}{-60\,a^3\,c^3+90\,a^2\,b^2\,c^2-30\,a\,b^4\,c+3\,b^6}\cdots\right)\cdots}{c^4\,\left(4\,a\,c-b^2\right)^{5/2}}$$

input
```
int(1/(c + a/x^2 + b/x)^3,x)
```

output

```
x/c^3 - ((3*x^3*(b^6 - 6*a^3*c^3 + 17*a^2*b^2*c^2 - 8*a*b^4*c))/(b^4 + 16*
a^2*c^2 - 8*a*b^2*c) + (x^2*(5*b^7 + 42*a^3*b*c^3 + 41*a^2*b^3*c^2 - 34*a*
b^5*c))/(2*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (a^2*(5*b^5 + 58*a^2*b*c^2
- 36*a*b^3*c))/(2*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (a*x*(5*b^6 - 14*a^3
*c^3 + 71*a^2*b^2*c^2 - 38*a*b^4*c))/(c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(
a^2*c^3 + c^5*x^4 + x^2*(2*a*c^4 + b^2*c^3) + 2*b*c^4*x^3 + 2*a*b*c^3*x) +
 (log(a + b*x + c*x^2)*(3*b^11 - 3072*a^5*b*c^5 + 480*a^2*b^7*c^2 - 1920*a
^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 60*a*b^9*c))/(2*(1024*a^5*c^9 - b^10*c^4 +
 20*a*b^8*c^5 - 160*a^2*b^6*c^6 + 640*a^3*b^4*c^7 - 1280*a^4*b^2*c^8)) + (
3*atan((((3*x*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(c^3*(4*a*
c - b^2)^5) + (3*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*(b^6 - 20*a^3*c^3
+ 30*a^2*b^2*c^2 - 10*a*b^4*c))/(2*c^7*(4*a*c - b^2)^5*(b^4 + 16*a^2*c^2 -
 8*a*b^2*c)))*(32*a^2*c^6*(4*a*c - b^2)^(5/2) + 2*b^4*c^4*(4*a*c - b^2)^(5
/2) - 16*a*b^2*c^5*(4*a*c - b^2)^(5/2)))/(3*b^6 - 60*a^3*c^3 + 90*a^2*b^2*
c^2 - 30*a*b^4*c))*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(c^4*
(4*a*c - b^2)^(5/2))
```

3.431. $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3}\,dx$

3.432 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x}\,dx$

3.432.1 Optimal result

Integrand size = 18, antiderivative size = 190

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x}\,dx = -\frac{b(b^2-7ac)\,x}{c^2\,(b^2-4ac)^2}+\frac{x^4(2a+bx)}{2\,(b^2-4ac)\,(a+bx+cx^2)^2} + \frac{x^2(a(b^2-16ac)+b(b^2-10ac)\,x)}{2c\,(b^2-4ac)^2\,(a+bx+cx^2)} + \frac{b(b^4-10ab^2c+30a^2c^2)\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\,(b^2-4ac)^{5/2}}+\frac{\log\left(a+bx+cx^2\right)}{2c^3}$$

output
```
-b*(-7*a*c+b^2)*x/c^2/(-4*a*c+b^2)^2+1/2*x^4*(b*x+2*a)/(-4*a*c+b^2)/(c*x^2
+b*x+a)^2+1/2*x^2*(a*(-16*a*c+b^2)+b*(-10*a*c+b^2)*x)/c/(-4*a*c+b^2)^2/(c*
x^2+b*x+a)+b*(30*a^2*c^2-10*a*b^2*c+b^4)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1
/2))/c^3/(-4*a*c+b^2)^(5/2)+1/2*ln(c*x^2+b*x+a)/c^3
```

3.432.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.16

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x}\,dx$$

$$= \frac{\frac{-b^6+11ab^4c-39a^2b^2c^2+32a^3c^3+4b^5cx-30ab^3c^2x+50a^2bc^3x}{(b^2-4ac)^2(a+x(b+cx))}+\frac{2a^3c^2+b^5x+ab^3(b-5cx)+a^2bc(-4b+5cx)}{(b^2-4ac)(a+x(b+cx))^2}-\frac{2bc(b^4-10ab^2c+30a^2c^2)\,\text{arctan}\cdots}{(-b^2+4ac)^{5/2}}\cdots}{2c^4}$$

input
```
Integrate[1/((c + a/x^2 + b/x)^3*x),x]
```

output
```
((-b^6 + 11*a*b^4*c - 39*a^2*b^2*c^2 + 32*a^3*c^3 + 4*b^5*c*x - 30*a*b^3*c
^2*x + 50*a^2*b*c^3*x)/((b^2 - 4*a*c)^2*(a + x*(b + c*x))) + (2*a^3*c^2 +
b^5*x + a*b^3*(b - 5*c*x) + a^2*b*c*(-4*b + 5*c*x))/((b^2 - 4*a*c)*(a + x*
(b + c*x))^2) - (2*b*c*(b^4 - 10*a*b^2*c + 30*a^2*c^2)*ArcTan[(b + 2*c*x)/
Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2) + c*Log[a + x*(b + c*x)])/(2*c^4
)
```

3.432.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1692, 1164, 1233, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(\frac{a}{x^2}+\frac{b}{x}+c\right)^3}dx$$

$$\Big\downarrow \text{ 1692}$$

$$\int \frac{x^5}{(a+bx+cx^2)^3}dx$$

$$\Big\downarrow \text{ 1164}$$

$$\frac{x^4(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2}-\frac{\int \frac{x^3(8a+bx)}{(cx^2+bx+a)^2}dx}{2(b^2-4ac)}$$

$$\Big\downarrow \text{ 1233}$$

$$\frac{x^4(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2}-\frac{\frac{\int \frac{2x\left(a\left(b^2-16ac\right)+b\left(b^2-7ac\right)x\right)}{cx^2+bx+a}dx}{c(b^2-4ac)}-\frac{x^2\left(bx\left(b^2-10ac\right)+a\left(b^2-16ac\right)\right)}{c(b^2-4ac)(a+bx+cx^2)}}{2(b^2-4ac)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{x^4(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2}-\frac{\frac{2\int \frac{x\left(a\left(b^2-16ac\right)+b\left(b^2-7ac\right)x\right)}{cx^2+bx+a}dx}{c(b^2-4ac)}-\frac{x^2\left(bx\left(b^2-10ac\right)+a\left(b^2-16ac\right)\right)}{c(b^2-4ac)(a+bx+cx^2)}}{2(b^2-4ac)}$$

$$\Big\downarrow \text{ 1200}$$

$$\frac{x^4(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}-\frac{\frac{2\int\left(-b\left(7a-\frac{b^2}{c}\right)-\frac{x\left(b^2-4ac\right)^2+ab\left(b^2-7ac\right)}{c\left(cx^2+bx+a\right)}\right)dx}{c\left(b^2-4ac\right)}-\frac{x^2\left(bx\left(b^2-10ac\right)+a\left(b^2-16ac\right)\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}}{2\left(b^2-4ac\right)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x^4(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}-\frac{\frac{2\left(-\frac{b\left(30a^2c^2-10ab^2c+b^4\right)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{\left(b^2-4ac\right)^2\log\left(a+bx+cx^2\right)}{2c^2}-bx\left(7a-\frac{b^2}{c}\right)\right)}{c\left(b^2-4ac\right)}-\frac{x^2\left(bx\left(b^2-10ac\right)+a\left(b^2-16ac\right)\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}}{2\left(b^2-4ac\right)}$$

input
```
Int[1/((c + a/x^2 + b/x)^3*x),x]
```

output
```
(x^4*(2*a + b*x))/(2*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) - (-((x^2*(a*(b^2
- 16*a*c) + b*(b^2 - 10*a*c)*x))/(c*(b^2 - 4*a*c)*(a + b*x + c*x^2))) + (2
*(-(b*(7*a - b^2/c)*x) - (b*(b^4 - 10*a*b^2*c + 30*a^2*c^2)*ArcTanh[(b + 2
*c*x)/Sqrt[b^2 - 4*a*c]])/(c^2*Sqrt[b^2 - 4*a*c]) - ((b^2 - 4*a*c)^2*Log[a
 + b*x + c*x^2])/(2*c^2)))/(c*(b^2 - 4*a*c)))/(2*(b^2 - 4*a*c))
```

3.432.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1164
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1233
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Simp[1/(c*(
p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Sim
p[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f
*(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(
m + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*
p + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -1] &&
GtQ[m, 1] && ((EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) |
|  !ILtQ[m + 2*p + 3, 0])
```

rule 1692
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.432.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.88

method	result
default	$\frac{\frac{b\left(25a^2c^2-15a\,b^2c+2b^4\right)x^3}{c^2\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{\left(32c^3a^3+11a^2b^2c^2-19a\,b^4c+3b^6\right)x^2}{2c^3\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{ab\left(31a^2c^2-22a\,b^2c+3b^4\right)x}{\left(16a^2c^2-8a\,b^2c+b^4\right)c^3}+\frac{3a^2\left(8a^2c^2-7a\,b^2c+b^4\right)}{2c^3\left(16a^2c^2-8a\,b^2c+b^4\right)}}{\left(c\,x^2+bx+a\right)^2}+\frac{\left(16a^2c^2-8a\,b^2 \right.}{}$
risch	Expression too large to display

input
```
int(1/(c+a/x^2+b/x)^3/x,x,method=_RETURNVERBOSE)
```

output
```
(1/c^2*b*(25*a^2*c^2-15*a*b^2*c+2*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+1/2*
(32*a^3*c^3+11*a^2*b^2*c^2-19*a*b^4*c+3*b^6)/c^3/(16*a^2*c^2-8*a*b^2*c+b^4
)*x^2+a*b*(31*a^2*c^2-22*a*b^2*c+3*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/c^3*x+3
/2*a^2*(8*a^2*c^2-7*a*b^2*c+b^4)/c^3/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^2+b*
x+a)^2+1/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*(1/2*(16*a^2*c^2-8*a*b^2*c+b^4)/c*
ln(c*x^2+b*x+a)+2*(-7*c*b*a^2+a*b^3-1/2*(16*a^2*c^2-8*a*b^2*c+b^4)*b/c)/(4
*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

3.432.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 792 vs. $2(180) = 360$.

Time = 0.29 (sec) , antiderivative size = 1603, normalized size of antiderivative = 8.44

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c+a/x^2+b/x)^3/x,x, algorithm="fricas")
```

output
```
[1/2*(3*a^2*b^6 - 33*a^3*b^4*c + 108*a^4*b^2*c^2 - 96*a^5*c^3 + 2*(2*b^7*c
 - 23*a*b^5*c^2 + 85*a^2*b^3*c^3 - 100*a^3*b*c^4)*x^3 + (3*b^8 - 31*a*b^6*
c + 87*a^2*b^4*c^2 - 12*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + (a^2*b^5 - 10*a^3
*b^3*c + 30*a^4*b*c^2 + (b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*x^4 + 2*(b
^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*x^3 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*
c^2 + 60*a^3*b*c^3)*x^2 + 2*(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*x)*sqr
t(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*
(2*c*x + b))/(c*x^2 + b*x + a)) + 2*(3*a*b^7 - 34*a^2*b^5*c + 119*a^3*b^3*
c^2 - 124*a^4*b*c^3)*x + (a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5
*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*(b^7
*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^6*c
 + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^2*
b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x)*log(c*x^2 + b*x + a))/(a^2*b^6*c
^3 - 12*a^3*b^4*c^4 + 48*a^4*b^2*c^5 - 64*a^5*c^6 + (b^6*c^5 - 12*a*b^4*c^
6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*x^4 + 2*(b^7*c^4 - 12*a*b^5*c^5 + 48*a^2*
b^3*c^6 - 64*a^3*b*c^7)*x^3 + (b^8*c^3 - 10*a*b^6*c^4 + 24*a^2*b^4*c^5 + 3
2*a^3*b^2*c^6 - 128*a^4*c^7)*x^2 + 2*(a*b^7*c^3 - 12*a^2*b^5*c^4 + 48*a^3*
b^3*c^5 - 64*a^4*b*c^6)*x), 1/2*(3*a^2*b^6 - 33*a^3*b^4*c + 108*a^4*b^2*c^
2 - 96*a^5*c^3 + 2*(2*b^7*c - 23*a*b^5*c^2 + 85*a^2*b^3*c^3 - 100*a^3*b*c^
4)*x^3 + (3*b^8 - 31*a*b^6*c + 87*a^2*b^4*c^2 - 12*a^3*b^2*c^3 - 128*a^...
```

3.432.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1510 vs. $2(180) = 360$.

Time = 1.87 (sec) , antiderivative size = 1510, normalized size of antiderivative = 7.95

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c+a/x**2+b/x)**3/x,x)
```

output
```
(-b*sqrt(-(4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(
1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*
c**2 + 20*a*b**8*c - b**10)) + 1/(2*c**3))*log(x + (-64*a**3*c**5*(-b*sqrt
(-(4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(1024*a**
5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 2
0*a*b**8*c - b**10)) + 1/(2*c**3)) + 32*a**3*c**2 + 48*a**2*b**2*c**4*(-b*
sqrt(-(4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(1024
*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2
 + 20*a*b**8*c - b**10)) + 1/(2*c**3)) - 9*a**2*b**2*c - 12*a*b**4*c**3*(-
b*sqrt(-(4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(10
24*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c*
*2 + 20*a*b**8*c - b**10)) + 1/(2*c**3)) + a*b**4 + b**6*c**2*(-b*sqrt(-(4
*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(1024*a**5*c*
*5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*
b**8*c - b**10)) + 1/(2*c**3)))/(30*a**2*b*c**2 - 10*a*b**3*c + b**5)) + (
b*sqrt(-(4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(10
24*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c*
*2 + 20*a*b**8*c - b**10)) + 1/(2*c**3))*log(x + (-64*a**3*c**5*(b*sqrt(-(
4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(1024*a**5*c
**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 2...
```

3.432.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(c+a/x^2+b/x)^3/x,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.432.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.29

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x}\,dx = -\frac{(b^5-10\,ab^3c+30\,a^2bc^2)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4c^3-8\,ab^2c^4+16\,a^2c^5)\sqrt{-b^2+4\,ac}}+\frac{\log\left(cx^2+bx+a\right)}{2\,c^3} + \frac{3\,a^2b^4-21\,a^3b^2c+24\,a^4c^2+2\,(2\,b^5c-15\,ab^3c^2+25\,a^2bc^3)x^3+(3\,b^6-19\,ab^4c+11\,a^2b^2c^2+32\,a^3c^3)x^2}{2\,(cx^2+bx+a)^2(b^2-4\,ac)^2c^3}$$

input
```
integrate(1/(c+a/x^2+b/x)^3/x,x, algorithm="giac")
```

output
```
-(b^5 - 10*a*b^3*c + 30*a^2*b*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/
((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt(-b^2 + 4*a*c)) + 1/2*log(c*x^2
+ b*x + a)/c^3 + 1/2*(3*a^2*b^4 - 21*a^3*b^2*c + 24*a^4*c^2 + 2*(2*b^5*c -
 15*a*b^3*c^2 + 25*a^2*b*c^3)*x^3 + (3*b^6 - 19*a*b^4*c + 11*a^2*b^2*c^2 +
 32*a^3*c^3)*x^2 + 2*(3*a*b^5 - 22*a^2*b^3*c + 31*a^3*b*c^2)*x)/((c*x^2 +
b*x + a)^2*(b^2 - 4*a*c)^2*c^3)
```

3.432.9 Mupad [B] (verification not implemented)

Time = 8.82 (sec) , antiderivative size = 620, normalized size of antiderivative = 3.26

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x}\,dx = \frac{\frac{3\,a^2\,\left(8\,a^2\,c^2-7\,a\,b^2\,c+b^4\right)}{2\,c^3\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{x^2\,\left(32\,a^3\,c^3+11\,a^2\,b^2\,c^2-19\,a\,b^4\,c+3\,b^6\right)}{2\,c^3\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{b\,x^3\,\left(25\,a^2\,c^2-15\,a\,b^2\,c+2\,b^4\right)}{c^2\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{a\,b\,x\,\left(31\,a^2\,c^2-22\,a\,b^2\,c+3\,b^4\right)}{c^3\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}}{x^2\,\left(b^2+2\,a\,c\right)+a^2+c^2\,x^4+2\,a\,b\,x+2\,b\,c\,x^3} - \frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(-1024\,a^5\,c^5+1280\,a^4\,b^2\,c^4-640\,a^3\,b^4\,c^3+160\,a^2\,b^6\,c^2-20\,a\,b^8\,c+b^{10}\right)}{2\,\left(1024\,a^5\,c^8-1280\,a^4\,b^2\,c^7+640\,a^3\,b^4\,c^6-160\,a^2\,b^6\,c^5+20\,a\,b^8\,c^4-b^{10}\,c^3\right)} - \frac{b\,\mathrm{atan}\left(\frac{\left(\frac{b\,x\,\left(30\,a^2\,c^2-10\,a\,b^2\,c+b^4\right)}{c^2\,\left(4\,a\,c-b^2\right)^5}+\frac{b^2\,\left(16\,a^2\,c^4-8\,a\,b^2\,c^3+b^4\,c^2\right)\,\left(30\,a^2\,c^2-10\,a\,b^2\,c+b^4\right)}{2\,c^5\,\left(4\,a\,c-b^2\right)^5\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}\right)\,\left(32\,a^2\,c^5\,{\left(4\,a\,c-b^2\right)}^{5/2}+2\,b^4\,c^3\,{\left(4\,a\,c-b^2\right)}^{5/2}-16\right.}{30\,a^2\,b\,c^2-10\,a\,b^3\,c+b^5}\right.}{c^3\,{\left(4\,a\,c-b^2\right)}^{5/2}}$$

input
```
int(1/(x*(c + a/x^2 + b/x)^3),x)
```

output
```
((3*a^2*(b^4 + 8*a^2*c^2 - 7*a*b^2*c))/(2*c^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*
c)) + (x^2*(3*b^6 + 32*a^3*c^3 + 11*a^2*b^2*c^2 - 19*a*b^4*c))/(2*c^3*(b^4
 + 16*a^2*c^2 - 8*a*b^2*c)) + (b*x^3*(2*b^4 + 25*a^2*c^2 - 15*a*b^2*c))/(c
^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (a*b*x*(3*b^4 + 31*a^2*c^2 - 22*a*b^2
*c))/(c^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^2*(2*a*c + b^2) + a^2 + c^2*
x^4 + 2*a*b*x + 2*b*c*x^3) - (log(a + b*x + c*x^2)*(b^10 - 1024*a^5*c^5 +
160*a^2*b^6*c^2 - 640*a^3*b^4*c^3 + 1280*a^4*b^2*c^4 - 20*a*b^8*c))/(2*(10
24*a^5*c^8 - b^10*c^3 + 20*a*b^8*c^4 - 160*a^2*b^6*c^5 + 640*a^3*b^4*c^6 -
 1280*a^4*b^2*c^7)) - (b*atan((((b*x*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(c^2
*(4*a*c - b^2)^5) + (b^2*(16*a^2*c^4 + b^4*c^2 - 8*a*b^2*c^3)*(b^4 + 30*a^
2*c^2 - 10*a*b^2*c))/(2*c^5*(4*a*c - b^2)^5*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)
))*(32*a^2*c^5*(4*a*c - b^2)^(5/2) + 2*b^4*c^3*(4*a*c - b^2)^(5/2) - 16*a*
b^2*c^4*(4*a*c - b^2)^(5/2)))/(b^5 + 30*a^2*b*c^2 - 10*a*b^3*c))*(b^4 + 30
*a^2*c^2 - 10*a*b^2*c))/(c^3*(4*a*c - b^2)^(5/2))
```

3.432. $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x}\,dx$

3.433 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^2} \, dx$

3.433.1 Optimal result

Integrand size = 18, antiderivative size = 111

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^2} \, dx = \frac{b+\frac{2a}{x}}{2\left(b^2-4ac\right)\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2} - \frac{3a\left(b+\frac{2a}{x}\right)}{\left(b^2-4ac\right)^2\left(c+\frac{a}{x^2}+\frac{b}{x}\right)} + \frac{12a^2 \operatorname{arctanh}\left(\frac{b+\frac{2a}{x}}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

output
```
1/2*(b+2*a/x)/(-4*a*c+b^2)/(c+a/x^2+b/x)^2-3*a*(b+2*a/x)/(-4*a*c+b^2)^2/(c
+a/x^2+b/x)+12*a^2*arctanh((b+2*a/x)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(5/2
)
```

3.433.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.57

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^2} \, dx = \frac{1}{2}\left(\frac{b^5-8ab^3c+22a^2bc^2-2b^4cx+16ab^2c^2x-20a^2c^3x}{c^3\left(b^2-4ac\right)^2(a+x(b+cx))} + \frac{b^4x+ab^2(b-4cx)+a^2c(-3b+2cx)}{c^3\left(-b^2+4ac\right)(a+x(b+cx))^2} + \frac{24a^2\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\left(-b^2+4ac\right)^{5/2}}\right)$$

input
```
Integrate[1/((c + a/x^2 + b/x)^3*x^2),x]
```

output
```
((b^5 - 8*a*b^3*c + 22*a^2*b*c^2 - 2*b^4*c*x + 16*a*b^2*c^2*x - 20*a^2*c^3
*x)/(c^3*(b^2 - 4*a*c)^2*(a + x*(b + c*x))) + (b^4*x + a*b^2*(b - 4*c*x) +
 a^2*c*(-3*b + 2*c*x))/(c^3*(-b^2 + 4*a*c)*(a + x*(b + c*x))^2) + (24*a^2*
ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2))/2
```

3.433.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1690, 1086, 1086, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(\frac{a}{x^2}+\frac{b}{x}+c\right)^3}dx$$

$$\Big\downarrow \text{1690}$$

$$-\int \frac{1}{\left(\frac{a}{x^2}+c+\frac{b}{x}\right)^3}d\frac{1}{x}$$

$$\Big\downarrow \text{1086}$$

$$\frac{3a\int \frac{1}{\left(\frac{a}{x^2}+c+\frac{b}{x}\right)^2}d\frac{1}{x}}{b^2-4ac}+\frac{\frac{2a}{x}+b}{2\left(b^2-4ac\right)\left(\frac{a}{x^2}+\frac{b}{x}+c\right)^2}$$

$$\Big\downarrow \text{1086}$$

$$\frac{3a\left(-\frac{2a\int \frac{1}{\frac{a}{x^2}+c+\frac{b}{x}}d\frac{1}{x}}{b^2-4ac}-\frac{\frac{2a}{x}+b}{\left(b^2-4ac\right)\left(\frac{a}{x^2}+\frac{b}{x}+c\right)}\right)}{b^2-4ac}+\frac{\frac{2a}{x}+b}{2\left(b^2-4ac\right)\left(\frac{a}{x^2}+\frac{b}{x}+c\right)^2}$$

$$\Big\downarrow \text{1083}$$

$$\frac{3a\left(\frac{4a\int \frac{1}{b^2-4ac-\frac{1}{x^2}}d\left(\frac{2a}{x}+b\right)}{b^2-4ac}-\frac{\frac{2a}{x}+b}{\left(b^2-4ac\right)\left(\frac{a}{x^2}+\frac{b}{x}+c\right)}\right)}{b^2-4ac}+\frac{\frac{2a}{x}+b}{2\left(b^2-4ac\right)\left(\frac{a}{x^2}+\frac{b}{x}+c\right)^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{3a\left(\frac{4a\operatorname{arctanh}\left(\frac{\frac{2a}{x}+b}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{\frac{2a}{x}+b}{(b^2-4ac)\left(\frac{a}{x^2}+\frac{b}{x}+c\right)}\right)}{b^2-4ac}+\frac{\frac{2a}{x}+b}{2\left(b^2-4ac\right)\left(\frac{a}{x^2}+\frac{b}{x}+c\right)^2}$$

input
```
Int[1/((c + a/x^2 + b/x)^3*x^2),x]
```

output
```
(b + (2*a)/x)/(2*(b^2 - 4*a*c)*(c + a/x^2 + b/x)^2) + (3*a*(-((b + (2*a)/x
)/((b^2 - 4*a*c)*(c + a/x^2 + b/x))) + (4*a*ArcTanh[(b + (2*a)/x)/Sqrt[b^2
 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)))/(b^2 - 4*a*c)
```

3.433.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

3.433.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(105) = 210$.

Time = 0.10 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.34

method	result
default	$\frac{-\frac{\left(10a^2c^2-8a\,b^2c+b^4\right)x^3}{c\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{b\left(2a^2c^2+8a\,b^2c-b^4\right)x^2}{2c^2\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{a\left(6a^2c^2-10a\,b^2c+b^4\right)x}{\left(16a^2c^2-8a\,b^2c+b^4\right)c^2}+\frac{a^2b\left(10ac-b^2\right)}{2c^2\left(16a^2c^2-8a\,b^2c+b^4\right)}}{\left(c\,x^2+bx+a\right)^2}+\frac{12a^2\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(16a^2c^2-8a\,b^2c+b^4)\sqrt{4ac-b}}$
risch	$\frac{-\frac{\left(10a^2c^2-8a\,b^2c+b^4\right)x^3}{c\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{b\left(2a^2c^2+8a\,b^2c-b^4\right)x^2}{2c^2\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{a\left(6a^2c^2-10a\,b^2c+b^4\right)x}{\left(16a^2c^2-8a\,b^2c+b^4\right)c^2}+\frac{a^2b\left(10ac-b^2\right)}{2c^2\left(16a^2c^2-8a\,b^2c+b^4\right)}}{\left(c\,x^2+bx+a\right)^2}-\frac{6a^2\ln\left(\left(32a^2c^3-16b^2a\,c^2+2\right.\right.}{}$

input
```
int(1/(c+a/x^2+b/x)^3/x^2,x,method=_RETURNVERBOSE)
```

output
```
(-1/c*(10*a^2*c^2-8*a*b^2*c+b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+1/2*b*(2*a
^2*c^2+8*a*b^2*c-b^4)/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2-a*(6*a^2*c^2-10*a
*b^2*c+b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/c^2*x+1/2*a^2*b*(10*a*c-b^2)/c^2/(1
6*a^2*c^2-8*a*b^2*c+b^4))/(c*x^2+b*x+a)^2+12*a^2/(16*a^2*c^2-8*a*b^2*c+b^4
)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
```

3.433.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 466 vs. $2(105) = 210$.

Time = 0.29 (sec) , antiderivative size = 953, normalized size of antiderivative = 8.59

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^2}\,dx$$

$$= \left[-\frac{a^2b^5 - 14\,a^3b^3c + 40\,a^4bc^2 + 2\,(b^6c - 12\,ab^4c^2 + 42\,a^2b^2c^3 - 40\,a^3c^4)x^3 + (b^7 - 12\,ab^5c + 30\,a^2b^3c^2 + 8}{2\,(a^2b^6c^2 - 12\,a^3b^4c^3 + 48\,a^4b^2c^4 - 64\,a^5c^5 + (b^6c^4 - 12\,ab^4c^5 + 48\,a^2b^2c^6 - 64\,a^3c^7)x^4}\right.$$

$$-\frac{a^2b^5 - 14\,a^3b^3c + 40\,a^4bc^2 + 2\,(b^6c - 12\,ab^4c^2 + 42\,a^2b^2c^3 - 40\,a^3c^4)x^3 + (b^7 - 12\,ab^5c + 30\,a^2b^3c^2 + 8}{2\,(a^2b^6c^2 - 12\,a^3b^4c^3 + 48\,a^4b^2c^4 - 64\,a^5c^5 + (b^6c^4 - 12\,ab^4c^5 + 48\,a^2b^2c^6 - 64\,a^3c^7)x^4 + 2\,(}$$

input
```
integrate(1/(c+a/x^2+b/x)^3/x^2,x, algorithm="fricas")
```

output
```
[-1/2*(a^2*b^5 - 14*a^3*b^3*c + 40*a^4*b*c^2 + 2*(b^6*c - 12*a*b^4*c^2 + 4
2*a^2*b^2*c^3 - 40*a^3*c^4)*x^3 + (b^7 - 12*a*b^5*c + 30*a^2*b^3*c^2 + 8*a
^3*b*c^3)*x^2 - 12*(a^2*c^4*x^4 + 2*a^2*b*c^3*x^3 + 2*a^3*b*c^2*x + a^4*c^
2 + (a^2*b^2*c^2 + 2*a^3*c^3)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*
c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*
(a*b^6 - 14*a^2*b^4*c + 46*a^3*b^2*c^2 - 24*a^4*c^3)*x)/(a^2*b^6*c^2 - 12*
a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5 + (b^6*c^4 - 12*a*b^4*c^5 + 48*a
^2*b^2*c^6 - 64*a^3*c^7)*x^4 + 2*(b^7*c^3 - 12*a*b^5*c^4 + 48*a^2*b^3*c^5
- 64*a^3*b*c^6)*x^3 + (b^8*c^2 - 10*a*b^6*c^3 + 24*a^2*b^4*c^4 + 32*a^3*b^
2*c^5 - 128*a^4*c^6)*x^2 + 2*(a*b^7*c^2 - 12*a^2*b^5*c^3 + 48*a^3*b^3*c^4
- 64*a^4*b*c^5)*x), -1/2*(a^2*b^5 - 14*a^3*b^3*c + 40*a^4*b*c^2 + 2*(b^6*c
 - 12*a*b^4*c^2 + 42*a^2*b^2*c^3 - 40*a^3*c^4)*x^3 + (b^7 - 12*a*b^5*c + 3
0*a^2*b^3*c^2 + 8*a^3*b*c^3)*x^2 + 24*(a^2*c^4*x^4 + 2*a^2*b*c^3*x^3 + 2*a
^3*b*c^2*x + a^4*c^2 + (a^2*b^2*c^2 + 2*a^3*c^3)*x^2)*sqrt(-b^2 + 4*a*c)*a
rctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(a*b^6 - 14*a^2*b
^4*c + 46*a^3*b^2*c^2 - 24*a^4*c^3)*x)/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*
a^4*b^2*c^4 - 64*a^5*c^5 + (b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a
^3*c^7)*x^4 + 2*(b^7*c^3 - 12*a*b^5*c^4 + 48*a^2*b^3*c^5 - 64*a^3*b*c^6)*x
^3 + (b^8*c^2 - 10*a*b^6*c^3 + 24*a^2*b^4*c^4 + 32*a^3*b^2*c^5 - 128*a^4*c
^6)*x^2 + 2*(a*b^7*c^2 - 12*a^2*b^5*c^3 + 48*a^3*b^3*c^4 - 64*a^4*b*c^5...
```

3.433.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 547 vs. 2(94) = 188.

Time = 0.82 (sec) , antiderivative size = 547, normalized size of antiderivative = 4.93

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^2}\,dx =$$

$$-6a^2\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x+\frac{-384a^5c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}+288a^4b^2c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}-72a^3b^4c\sqrt{-\frac{1}{(4ac-b^2)^5}}+6}{12a^2c}\right.$$

$$+6a^2\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x+\frac{384a^5c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}-288a^4b^2c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}+72a^3b^4c\sqrt{-\frac{1}{(4ac-b^2)^5}}-6a}{12a^2c}\right.$$

$$+\frac{10a^3bc-a^2b^3+x^3(-20a^2c^3+16ab^2c^2-2b^4c)+x^2\cdot(2a^2bc^2+8ab^3c-b^5}{32a^4c^4-16a^3b^2c^3+2a^2b^4c^2+x^4\cdot(32a^2c^6-16ab^2c^5+2b^4c^4)+x^3\cdot(64a^2bc^5-32ab^3c^4+4b^5c^3)+x^2}$$

input
```
integrate(1/(c+a/x**2+b/x)**3/x**2,x)
```

output
```
-6*a**2*sqrt(-1/(4*a*c - b**2)**5)*log(x + (-384*a**5*c**3*sqrt(-1/(4*a*c
- b**2)**5) + 288*a**4*b**2*c**2*sqrt(-1/(4*a*c - b**2)**5) - 72*a**3*b**4
*c*sqrt(-1/(4*a*c - b**2)**5) + 6*a**2*b**6*sqrt(-1/(4*a*c - b**2)**5) + 6
*a**2*b)/(12*a**2*c)) + 6*a**2*sqrt(-1/(4*a*c - b**2)**5)*log(x + (384*a**
5*c**3*sqrt(-1/(4*a*c - b**2)**5) - 288*a**4*b**2*c**2*sqrt(-1/(4*a*c - b*
*2)**5) + 72*a**3*b**4*c*sqrt(-1/(4*a*c - b**2)**5) - 6*a**2*b**6*sqrt(-1/
(4*a*c - b**2)**5) + 6*a**2*b)/(12*a**2*c)) + (10*a**3*b*c - a**2*b**3 + x
**3*(-20*a**2*c**3 + 16*a*b**2*c**2 - 2*b**4*c) + x**2*(2*a**2*b*c**2 + 8*
a*b**3*c - b**5) + x*(-12*a**3*c**2 + 20*a**2*b**2*c - 2*a*b**4))/(32*a**4
*c**4 - 16*a**3*b**2*c**3 + 2*a**2*b**4*c**2 + x**4*(32*a**2*c**6 - 16*a*b
**2*c**5 + 2*b**4*c**4) + x**3*(64*a**2*b*c**5 - 32*a*b**3*c**4 + 4*b**5*c
**3) + x**2*(64*a**3*c**5 - 12*a*b**4*c**3 + 2*b**6*c**2) + x*(64*a**3*b*c
**4 - 32*a**2*b**3*c**3 + 4*a*b**5*c**2))
```

3.433.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(c+a/x^2+b/x)^3/x^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.433.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.82

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^2}\,dx = \frac{12\,a^2\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-b^2+4\,ac}} - \frac{2\,b^4cx^3-16\,ab^2c^2x^3+20\,a^2c^3x^3+b^5x^2-8\,ab^3cx^2-2\,a^2bc^2x^2+2\,ab^4x-20\,a^2b^2cx+12\,a^3c^2x+a^2b^3}{2\,(b^4c^2-8\,ab^2c^3+16\,a^2c^4)(cx^2+bx+a)^2}$$

input
```
integrate(1/(c+a/x^2+b/x)^3/x^2,x, algorithm="giac")
```

output
```
12*a^2*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c + 16*a^2*c
^2)*sqrt(-b^2 + 4*a*c)) - 1/2*(2*b^4*c*x^3 - 16*a*b^2*c^2*x^3 + 20*a^2*c^3
*x^3 + b^5*x^2 - 8*a*b^3*c*x^2 - 2*a^2*b*c^2*x^2 + 2*a*b^4*x - 20*a^2*b^2*
c*x + 12*a^3*c^2*x + a^2*b^3 - 10*a^3*b*c)/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^
2*c^4)*(c*x^2 + b*x + a)^2)
```

3.433.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 343, normalized size of antiderivative = 3.09

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^2}\,dx = \frac{12\,a^2\,\mathrm{atan}\left(\frac{\left(\frac{6\,a^2\,\left(16\,a^2\,b\,c^2-8\,a\,b^3\,c+b^5\right)}{{\left(4\,a\,c-b^2\right)}^{5/2}\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{12\,a^2\,c\,x}{{\left(4\,a\,c-b^2\right)}^{5/2}}\right)\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}{6\,a^2}\right)}{{\left(4\,a\,c-b^2\right)}^{5/2}} - \frac{\frac{x^3\,\left(10\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}{c\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{a^2\,\left(b^3-10\,a\,b\,c\right)}{2\,c^2\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}-\frac{x^2\,\left(2\,a^2\,b\,c^2+8\,a\,b^3\,c-b^5\right)}{2\,c^2\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{a\,x\,\left(6\,a^2\,c^2-10\,a\,b^2\,c+b^4\right)}{c^2\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}}{x^2\,\left(b^2+2\,a\,c\right)+a^2+c^2\,x^4+2\,a\,b\,x+2\,b\,c\,x^3}$$

input
```
int(1/(x^2*(c + a/x^2 + b/x)^3),x)
```

output
```
(12*a^2*atan((((6*a^2*(b^5 + 16*a^2*b*c^2 - 8*a*b^3*c))/((4*a*c - b^2)^(5/
2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (12*a^2*c*x)/(4*a*c - b^2)^(5/2))*(b^
4 + 16*a^2*c^2 - 8*a*b^2*c))/(6*a^2)))/(4*a*c - b^2)^(5/2) - ((x^3*(b^4 +
10*a^2*c^2 - 8*a*b^2*c))/(c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (a^2*(b^3 -
10*a*b*c))/(2*c^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (x^2*(2*a^2*b*c^2 - b^
5 + 8*a*b^3*c))/(2*c^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (a*x*(b^4 + 6*a^2
*c^2 - 10*a*b^2*c))/(c^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^2*(2*a*c + b^
2) + a^2 + c^2*x^4 + 2*a*b*x + 2*b*c*x^3)
```

3.434 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^3} \, dx$

3.434.1 Optimal result

Integrand size = 18, antiderivative size = 107

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^3} \, dx = -\frac{x^3(b+2cx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{3bx(2a+bx)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} + \frac{6ab\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

output
```
-1/2*x^3*(2*c*x+b)/(-4*a*c+b^2)/(c*x^2+b*x+a)^2+3/2*b*x*(b*x+2*a)/(-4*a*c+
b^2)^2/(c*x^2+b*x+a)+6*a*b*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b
^2)^(5/2)
```

3.434.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.18

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^3} \, dx = -\frac{8a^3c+b^4x^2+abx(2b^2+bcx+6c^2x^2)+a^2(b^2+10bcx+16c^2x^2)}{2c\left(b^2-4ac\right)^2\left(a+x(b+cx)\right)^2} - \frac{6ab\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\left(-b^2+4ac\right)^{5/2}}$$

input
```
Integrate[1/((c + a/x^2 + b/x)^3*x^3),x]
```

output
```
-1/2*(8*a^3*c + b^4*x^2 + a*b*x*(2*b^2 + b*c*x + 6*c^2*x^2) + a^2*(b^2 + 1
0*b*c*x + 16*c^2*x^2))/(c*(b^2 - 4*a*c)^2*(a + x*(b + c*x))^2) - (6*a*b*Ar
cTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2)
```

3.434.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1692, 1156, 1153, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \left(\frac{a}{x^2}+\frac{b}{x}+c\right)^3} \, dx$$

$$\Big\downarrow \text{1692}$$

$$\int \frac{x^3}{(a+bx+cx^2)^3}dx$$

$$\Big\downarrow \text{1156}$$

$$\frac{3b\int \frac{x^2}{(cx^2+bx+a)^2}dx}{2(b^2-4ac)}-\frac{x^3(b+2cx)}{2(b^2-4ac)(a+bx+cx^2)^2}$$

$$\Big\downarrow \text{1153}$$

$$\frac{3b\left(\frac{x(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}-\frac{2a\int \frac{1}{cx^2+bx+a}dx}{b^2-4ac}\right)}{2(b^2-4ac)}-\frac{x^3(b+2cx)}{2(b^2-4ac)(a+bx+cx^2)^2}$$

$$\Big\downarrow \text{1083}$$

$$\frac{3b\left(\frac{4a\int \frac{1}{b^2-(b+2cx)^2-4ac}d(b+2cx)}{b^2-4ac}+\frac{x(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}\right)}{2(b^2-4ac)}-\frac{x^3(b+2cx)}{2(b^2-4ac)(a+bx+cx^2)^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{3b\left(\frac{4a\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}+\frac{x(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}\right)}{2(b^2-4ac)}-\frac{x^3(b+2cx)}{2(b^2-4ac)(a+bx+cx^2)^2}$$

input
```
Int[1/((c + a/x^2 + b/x)^3*x^3),x]
```

output
```
-1/2*(x^3*(b + 2*c*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)^2) + (3*b*((x*(2*a
 + b*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)) + (4*a*ArcTanh[(b + 2*c*x)/Sqrt
[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)))/(2*(b^2 - 4*a*c))
```

3.434.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1153
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*(2*p + 3)*((c*d^2 -
b*d*e + a*e^2)/((p + 1)*(b^2 - 4*a*c)))   Int[(d + e*x)^(m - 2)*(a + b*x +
c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
&& LtQ[p, -1]
```

rule 1156
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^m*(b + 2*c*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*
(b^2 - 4*a*c))), x] + Simp[m*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*c)))   Int[
(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && EqQ[m + 2*p + 3, 0] && LtQ[p, -1]
```

rule 1692
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

3.434. $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^3}\,dx$

3.434.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 222 vs. $2(99) = 198$.

Time = 0.10 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.08

method	result
default	$\frac{-\frac{3abc\,x^3}{16a^2c^2-8a\,b^2c+b^4}-\frac{\left(16a^2c^2+a\,b^2c+b^4\right)x^2}{2c\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{\left(5ac+b^2\right)abx}{c\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{a^2\left(8ac+b^2\right)}{2c\left(16a^2c^2-8a\,b^2c+b^4\right)}}{\left(c\,x^2+bx+a\right)^2}-\frac{6ab\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(16a^2c^2-8a\,b^2c+b^4\right)\sqrt{4ac-b^2}}$
risch	$\frac{-\frac{3abc\,x^3}{16a^2c^2-8a\,b^2c+b^4}-\frac{\left(16a^2c^2+a\,b^2c+b^4\right)x^2}{2c\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{\left(5ac+b^2\right)abx}{c\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{a^2\left(8ac+b^2\right)}{2c\left(16a^2c^2-8a\,b^2c+b^4\right)}}{\left(c\,x^2+bx+a\right)^2}-\frac{3ab\ln\left(\left(32a^2c^3-16b^2a\,c^2+2b^4c\right)x-(\right.}{(-4ac-}$

input
```
int(1/(c+a/x^2+b/x)^3/x^3,x,method=_RETURNVERBOSE)
```

output
```
(-3*a*b*c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3-1/2*(16*a^2*c^2+a*b^2*c+b^4)/c/(1
6*a^2*c^2-8*a*b^2*c+b^4)*x^2-(5*a*c+b^2)*a*b/c/(16*a^2*c^2-8*a*b^2*c+b^4)*
x-1/2*a^2*(8*a*c+b^2)/c/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^2+b*x+a)^2-6*a*b/
(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^
(1/2))
```

3.434.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 426 vs. $2(99) = 198$.

Time = 0.29 (sec) , antiderivative size = 872, normalized size of antiderivative = 8.15

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^3}\,dx$$

$$= \left[-\frac{a^2b^4+4\,a^3b^2c-32\,a^4c^2+6\,(ab^3c^2-4\,a^2bc^3)x^3+(b^6-3\,ab^4c+12\,a^2b^2c^2-64\,a^3c^3)x^2-6\,(abc^3x^4}{2\,(a^2b^6c-12\,a^3b^4c^2+48\,a^4b^2c^3-64\,a^5c^4+(b^6c^3-12\,ab^4c^4+48\,a^2b^2c^5-64\,a^3c^6)x^4+2\,(b^7c^2-12\,ab}\right.$$

$$-\frac{a^2b^4+4\,a^3b^2c-32\,a^4c^2+6\,(ab^3c^2-4\,a^2bc^3)x^3+(b^6-3\,ab^4c+12\,a^2b^2c^2-64\,a^3c^3)x^2-12\,(abc}{2\,(a^2b^6c-12\,a^3b^4c^2+48\,a^4b^2c^3-64\,a^5c^4+(b^6c^3-12\,ab^4c^4+48\,a^2b^2c^5-64\,a^3c^6)x^4+2\,(b^7c^2-12\,ab}$$

input
```
integrate(1/(c+a/x^2+b/x)^3/x^3,x, algorithm="fricas")
```

output
```
[-1/2*(a^2*b^4 + 4*a^3*b^2*c - 32*a^4*c^2 + 6*(a*b^3*c^2 - 4*a^2*b*c^3)*x^
3 + (b^6 - 3*a*b^4*c + 12*a^2*b^2*c^2 - 64*a^3*c^3)*x^2 - 6*(a*b*c^3*x^4 +
 2*a*b^2*c^2*x^3 + 2*a^2*b^2*c*x + a^3*b*c + (a*b^3*c + 2*a^2*b*c^2)*x^2)*
sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*
c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*(a*b^5 + a^2*b^3*c - 20*a^3*b*c^2)*
x)/(a^2*b^6*c - 12*a^3*b^4*c^2 + 48*a^4*b^2*c^3 - 64*a^5*c^4 + (b^6*c^3 -
12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*x^4 + 2*(b^7*c^2 - 12*a*b^5*c^
3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*x^3 + (b^8*c - 10*a*b^6*c^2 + 24*a^2*b^
4*c^3 + 32*a^3*b^2*c^4 - 128*a^4*c^5)*x^2 + 2*(a*b^7*c - 12*a^2*b^5*c^2 +
48*a^3*b^3*c^3 - 64*a^4*b*c^4)*x), -1/2*(a^2*b^4 + 4*a^3*b^2*c - 32*a^4*c^
2 + 6*(a*b^3*c^2 - 4*a^2*b*c^3)*x^3 + (b^6 - 3*a*b^4*c + 12*a^2*b^2*c^2 -
64*a^3*c^3)*x^2 - 12*(a*b*c^3*x^4 + 2*a*b^2*c^2*x^3 + 2*a^2*b^2*c*x + a^3*
b*c + (a*b^3*c + 2*a^2*b*c^2)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 +
4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(a*b^5 + a^2*b^3*c - 20*a^3*b*c^2)*x
)/(a^2*b^6*c - 12*a^3*b^4*c^2 + 48*a^4*b^2*c^3 - 64*a^5*c^4 + (b^6*c^3 - 1
2*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*x^4 + 2*(b^7*c^2 - 12*a*b^5*c^3
 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*x^3 + (b^8*c - 10*a*b^6*c^2 + 24*a^2*b^4
*c^3 + 32*a^3*b^2*c^4 - 128*a^4*c^5)*x^2 + 2*(a*b^7*c - 12*a^2*b^5*c^2 + 4
8*a^3*b^3*c^3 - 64*a^4*b*c^4)*x)]
```

3.434.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 513 vs. $2(102) = 204$.

Time = 0.66 (sec) , antiderivative size = 513, normalized size of antiderivative = 4.79

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^3}\,dx$$

$$= 3ab\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x + \frac{-192a^4bc^3\sqrt{-\frac{1}{(4ac-b^2)^5}} + 144a^3b^3c^2\sqrt{-\frac{1}{(4ac-b^2)^5}} - 36a^2b^5c\sqrt{-\frac{1}{(4ac-b^2)^5}} + 3a}{6abc}\right.$$

$$-3ab\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x + \frac{192a^4bc^3\sqrt{-\frac{1}{(4ac-b^2)^5}} - 144a^3b^3c^2\sqrt{-\frac{1}{(4ac-b^2)^5}} + 36a^2b^5c\sqrt{-\frac{1}{(4ac-b^2)^5}} - 3a}{6abc}\right.$$

$$+\frac{-8a^3c - a^2b^2 - 6abc^2x^3 + x^2(-16a^2c^2 - ab^2c - b^4) + x(-}{32a^4c^3 - 16a^3b^2c^2 + 2a^2b^4c + x^4\cdot(32a^2c^5 - 16ab^2c^4 + 2b^4c^3) + x^3\cdot(64a^2bc^4 - 32ab^3c^3 + 4b^5c^2) + x^2\cdot}$$

input
```
integrate(1/(c+a/x**2+b/x)**3/x**3,x)
```

output
```
3*a*b*sqrt(-1/(4*a*c - b**2)**5)*log(x + (-192*a**4*b*c**3*sqrt(-1/(4*a*c
- b**2)**5) + 144*a**3*b**3*c**2*sqrt(-1/(4*a*c - b**2)**5) - 36*a**2*b**5
*c*sqrt(-1/(4*a*c - b**2)**5) + 3*a*b**7*sqrt(-1/(4*a*c - b**2)**5) + 3*a*
b**2)/(6*a*b*c)) - 3*a*b*sqrt(-1/(4*a*c - b**2)**5)*log(x + (192*a**4*b*c*
*3*sqrt(-1/(4*a*c - b**2)**5) - 144*a**3*b**3*c**2*sqrt(-1/(4*a*c - b**2)*
*5) + 36*a**2*b**5*c*sqrt(-1/(4*a*c - b**2)**5) - 3*a*b**7*sqrt(-1/(4*a*c
- b**2)**5) + 3*a*b**2)/(6*a*b*c)) + (-8*a**3*c - a**2*b**2 - 6*a*b*c**2*x
**3 + x**2*(-16*a**2*c**2 - a*b**2*c - b**4) + x*(-10*a**2*b*c - 2*a*b**3)
)/(32*a**4*c**3 - 16*a**3*b**2*c**2 + 2*a**2*b**4*c + x**4*(32*a**2*c**5 -
 16*a*b**2*c**4 + 2*b**4*c**3) + x**3*(64*a**2*b*c**4 - 32*a*b**3*c**3 + 4
*b**5*c**2) + x**2*(64*a**3*c**4 - 12*a*b**4*c**2 + 2*b**6*c) + x*(64*a**3
*b*c**3 - 32*a**2*b**3*c**2 + 4*a*b**5*c))
```

3.434.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(c+a/x^2+b/x)^3/x^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.434.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.52

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^3}\,dx = -\frac{6\,ab\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^4-8\,ab^2c+16\,a^2c^2\right)\sqrt{-b^2+4\,ac}} - \frac{6\,abc^2x^3+b^4x^2+ab^2cx^2+16\,a^2c^2x^2+2\,ab^3x+10\,a^2bcx+a^2b^2+8\,a^3c}{2\,\left(b^4c-8\,ab^2c^2+16\,a^2c^3\right)\left(cx^2+bx+a\right)^2}$$

input
```
integrate(1/(c+a/x^2+b/x)^3/x^3,x, algorithm="giac")
```

output
```
-6*a*b*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c + 16*a^2*c
^2)*sqrt(-b^2 + 4*a*c)) - 1/2*(6*a*b*c^2*x^3 + b^4*x^2 + a*b^2*c*x^2 + 16*
a^2*c^2*x^2 + 2*a*b^3*x + 10*a^2*b*c*x + a^2*b^2 + 8*a^3*c)/((b^4*c - 8*a*
b^2*c^2 + 16*a^2*c^3)*(c*x^2 + b*x + a)^2)
```

3.434.9 Mupad [B] (verification not implemented)

Time = 8.20 (sec) , antiderivative size = 271, normalized size of antiderivative = 2.53

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^3}\,dx = -\frac{\frac{x^2\,\left(16\,a^2\,c^2+a\,b^2\,c+b^4\right)}{2\,c\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{a^2\,\left(b^2+8\,a\,c\right)}{2\,c\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{3\,a\,b\,c\,x^3}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}+\frac{a\,b\,x\,\left(b^2+5\,a\,c\right)}{c\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}}{x^2\,\left(b^2+2\,a\,c\right)+a^2+c^2\,x^4+2\,a\,b\,x+2\,b\,c\,x^3} - \frac{6\,a\,b\,\mathrm{atan}\left(\frac{\left(\frac{3\,a\,b^2}{{\left(4\,a\,c-b^2\right)}^{5/2}}+\frac{6\,a\,b\,c\,x}{{\left(4\,a\,c-b^2\right)}^{5/2}}\right)\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}{3\,a\,b}\right)}{{\left(4\,a\,c-b^2\right)}^{5/2}}$$

input
```
int(1/(x^3*(c + a/x^2 + b/x)^3),x)
```

output
```
- ((x^2*(b^4 + 16*a^2*c^2 + a*b^2*c))/(2*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))
 + (a^2*(8*a*c + b^2))/(2*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*a*b*c*x^3
)/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (a*b*x*(5*a*c + b^2))/(c*(b^4 + 16*a^2*
c^2 - 8*a*b^2*c)))/(x^2*(2*a*c + b^2) + a^2 + c^2*x^4 + 2*a*b*x + 2*b*c*x^
3) - (6*a*b*atan((((3*a*b^2)/(4*a*c - b^2)^(5/2) + (6*a*b*c*x)/(4*a*c - b^
2)^(5/2))*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(3*a*b)))/(4*a*c - b^2)^(5/2)
```

3.435 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^4} \, dx$

3.435.1 Optimal result

Integrand size = 18, antiderivative size = 115

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^4} \, dx=\frac{x(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{3ab+\left(b^2+2ac\right)x}{\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}-\frac{2(b^2+2ac)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

output
```
1/2*x*(b*x+2*a)/(-4*a*c+b^2)/(c*x^2+b*x+a)^2+(3*a*b+(2*a*c+b^2)*x)/(-4*a*c
+b^2)^2/(c*x^2+b*x+a)-2*(2*a*c+b^2)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/
(-4*a*c+b^2)^(5/2)
```

3.435.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.14

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^4} \, dx=\frac{1}{2}\left(\frac{b^2x+a(b-2cx)}{c\left(-b^2+4ac\right)\left(a+x(b+cx)\right)^2}+\frac{\left(b^2+2ac\right)\left(b+2cx\right)}{c\left(b^2-4ac\right)^2\left(a+x(b+cx)\right)}+\frac{4(b^2+2ac)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\left(-b^2+4ac\right)^{5/2}}\right)$$

input
```
Integrate[1/((c + a/x^2 + b/x)^3*x^4),x]
```

output
```
((b^2*x + a*(b - 2*c*x))/(c*(-b^2 + 4*a*c)*(a + x*(b + c*x))^2) + ((b^2 +
2*a*c)*(b + 2*c*x))/(c*(b^2 - 4*a*c)^2*(a + x*(b + c*x))) + (4*(b^2 + 2*a*
c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2))/2
```

3.435.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1692, 1164, 27, 1159, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \left(\frac{a}{x^2}+\frac{b}{x}+c\right)^3}dx$$

$$\Big\downarrow \text{1692}$$

$$\int \frac{x^2}{(a+bx+cx^2)^3}dx$$

$$\Big\downarrow \text{1164}$$

$$\frac{x(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2}-\frac{\int \frac{2(a-bx)}{(cx^2+bx+a)^2}dx}{2(b^2-4ac)}$$

$$\Big\downarrow \text{27}$$

$$\frac{x(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2}-\frac{\int \frac{a-bx}{(cx^2+bx+a)^2}dx}{b^2-4ac}$$

$$\Big\downarrow \text{1159}$$

$$\frac{x(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2}-\frac{-\frac{(2ac+b^2)\int \frac{1}{cx^2+bx+a}dx}{b^2-4ac}-\frac{x(2ac+b^2)+3ab}{(b^2-4ac)(a+bx+cx^2)}}{b^2-4ac}$$

$$\Big\downarrow \text{1083}$$

$$\frac{x(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2}-\frac{\frac{2(2ac+b^2)\int \frac{1}{b^2-(b+2cx)^2-4ac}d(b+2cx)}{b^2-4ac}-\frac{x(2ac+b^2)+3ab}{(b^2-4ac)(a+bx+cx^2)}}{b^2-4ac}$$

$$\Big\downarrow \text{219}$$

$$\frac{x(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}-\frac{\frac{2\left(2ac+b^2\right)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{x\left(2ac+b^2\right)+3ab}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}}{b^2-4ac}$$

input
```
Int[1/((c + a/x^2 + b/x)^3*x^4),x]
```

output
```
(x*(2*a + b*x))/(2*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) - (-((3*a*b + (b^2 +
 2*a*c)*x)/((b^2 - 4*a*c)*(a + b*x + c*x^2))) + (2*(b^2 + 2*a*c)*ArcTanh[(
b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2))/(b^2 - 4*a*c)
```

3.435.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

rule 1164
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

3.435. $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3x^4}\,dx$

rule 1692
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

3.435.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.83

method	result
default	$\frac{\frac{c(2ac+b^2)x^3}{16a^2c^2-8a\,b^2c+b^4}+\frac{3b(2ac+b^2)x^2}{2(16a^2c^2-8a\,b^2c+b^4)}-\frac{a(2ac-5b^2)x}{16a^2c^2-8a\,b^2c+b^4}+\frac{3a^2b}{16a^2c^2-8a\,b^2c+b^4}}{(c\,x^2+bx+a)^2}+\frac{2(2ac+b^2)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(16a^2c^2-8a\,b^2c+b^4)\sqrt{4ac-b^2}}$
risch	$\frac{\frac{c(2ac+b^2)x^3}{16a^2c^2-8a\,b^2c+b^4}+\frac{3b(2ac+b^2)x^2}{2(16a^2c^2-8a\,b^2c+b^4)}-\frac{a(2ac-5b^2)x}{16a^2c^2-8a\,b^2c+b^4}+\frac{3a^2b}{16a^2c^2-8a\,b^2c+b^4}}{(c\,x^2+bx+a)^2}-\frac{2\ln\left((32a^2c^3-16b^2a\,c^2+2b^4c)x+(-4ac+b^2)^{\frac{5}{2}}\right.}{(-4ac+b^2)^{\frac{5}{2}}}$

input
```
int(1/(c+a/x^2+b/x)^3/x^4,x,method=_RETURNVERBOSE)
```

output
```
(c*(2*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+3/2*b*(2*a*c+b^2)/(16*a^2*c^
2-8*a*b^2*c+b^4)*x^2-a*(2*a*c-5*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x+3*a^2*b/
(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^2+b*x+a)^2+2*(2*a*c+b^2)/(16*a^2*c^2-8*a*
b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
```

3.435.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 433 vs. $2(109) = 218$.

Time = 0.28 (sec) , antiderivative size = 887, normalized size of antiderivative = 7.71

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^4}\,dx$$
$$= \left[\frac{6\,a^2b^3-24\,a^3bc+2\,(b^4c-2\,ab^2c^2-8\,a^2c^3)x^3+3\,(b^5-2\,ab^3c-8\,a^2bc^2)x^2+2\,((b^2c^2+2\,ac^3)x^4+a^2b^2}{2\,(a^2b^6-12\,a^3b^4c+48\,a^4b^2c^2-64\,a^5c^3+(b^6c^2-12\,ab^4c^3+48\,a^2b^2c^4-64\,a^3c^5)x^4+2}\right.$$

input
```
integrate(1/(c+a/x^2+b/x)^3/x^4,x, algorithm="fricas")
```

output
```
[1/2*(6*a^2*b^3 - 24*a^3*b*c + 2*(b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*x^3 + 3
*(b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*x^2 + 2*((b^2*c^2 + 2*a*c^3)*x^4 + a^2*b^
2 + 2*a^3*c + 2*(b^3*c + 2*a*b*c^2)*x^3 + (b^4 + 4*a*b^2*c + 4*a^2*c^2)*x^
2 + 2*(a*b^3 + 2*a^2*b*c)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x +
b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*(5*a*b
^4 - 22*a^2*b^2*c + 8*a^3*c^2)*x)/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2
 - 64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4
 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 1
0*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7
- 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x), 1/2*(6*a^2*b^3 - 24*a^
3*b*c + 2*(b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*x^3 + 3*(b^5 - 2*a*b^3*c - 8*a
^2*b*c^2)*x^2 - 4*((b^2*c^2 + 2*a*c^3)*x^4 + a^2*b^2 + 2*a^3*c + 2*(b^3*c
+ 2*a*b*c^2)*x^3 + (b^4 + 4*a*b^2*c + 4*a^2*c^2)*x^2 + 2*(a*b^3 + 2*a^2*b*
c)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a
*c)) + 2*(5*a*b^4 - 22*a^2*b^2*c + 8*a^3*c^2)*x)/(a^2*b^6 - 12*a^3*b^4*c +
 48*a^4*b^2*c^2 - 64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 -
64*a^3*c^5)*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)
*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*
x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x)]
```

3.435.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 570 vs. $2(107) = 214$.

Time = 0.75 (sec) , antiderivative size = 570, normalized size of antiderivative = 4.96

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^4}\,dx = -\sqrt{-\frac{1}{(4ac-b^2)^5}}\cdot(2ac$$

$$+b^2)\log\left(x+\frac{-64a^3c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}\cdot(2ac+b^2)+48a^2b^2c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}\cdot(2ac+b^2)-12ab^4c\sqrt{-\frac{1}{(4ac-b^2)^5}}}{4ac^2+2b^2c}\right.$$

$$+\sqrt{-\frac{1}{(4ac-b^2)^5}}\cdot(2ac$$

$$+b^2)\log\left(x+\frac{64a^3c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}\cdot(2ac+b^2)-48a^2b^2c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}\cdot(2ac+b^2)+12ab^4c\sqrt{-\frac{1}{(4ac-b^2)^5}}\cdot}{4ac^2+2b^2c}\right.$$

$$+\frac{6a^2b+x^3\cdot(4ac^2+2b^2c)+x^2\cdot(6abc+3b^3)+x(-4a^2c+}{32a^4c^2-16a^3b^2c+2a^2b^4+x^4\cdot(32a^2c^4-16ab^2c^3+2b^4c^2)+x^3\cdot(64a^2bc^3-32ab^3c^2+4b^5c)+x^2\cdot(64}$$

input
```
integrate(1/(c+a/x**2+b/x)**3/x**4,x)
```

output
```
-sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2)*log(x + (-64*a**3*c**3*sqrt(-1/
(4*a*c - b**2)**5)*(2*a*c + b**2) + 48*a**2*b**2*c**2*sqrt(-1/(4*a*c - b**
2)**5)*(2*a*c + b**2) - 12*a*b**4*c*sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b*
*2) + 2*a*b*c + b**6*sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2) + b**3)/(4*
a*c**2 + 2*b**2*c)) + sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2)*log(x + (6
4*a**3*c**3*sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2) - 48*a**2*b**2*c**2*
sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2) + 12*a*b**4*c*sqrt(-1/(4*a*c - b
**2)**5)*(2*a*c + b**2) + 2*a*b*c - b**6*sqrt(-1/(4*a*c - b**2)**5)*(2*a*c
 + b**2) + b**3)/(4*a*c**2 + 2*b**2*c)) + (6*a**2*b + x**3*(4*a*c**2 + 2*b
**2*c) + x**2*(6*a*b*c + 3*b**3) + x*(-4*a**2*c + 10*a*b**2))/(32*a**4*c**
2 - 16*a**3*b**2*c + 2*a**2*b**4 + x**4*(32*a**2*c**4 - 16*a*b**2*c**3 + 2
*b**4*c**2) + x**3*(64*a**2*b*c**3 - 32*a*b**3*c**2 + 4*b**5*c) + x**2*(64
*a**3*c**3 - 12*a*b**4*c + 2*b**6) + x*(64*a**3*b*c**2 - 32*a**2*b**3*c +
4*a*b**5))
```

3.435.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(c+a/x^2+b/x)^3/x^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.435.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.34

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^4}\,dx = \frac{2\,(b^2+2\,ac)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-b^2+4\,ac}} + \frac{2\,b^2cx^3+4\,ac^2x^3+3\,b^3x^2+6\,abcx^2+10\,ab^2x-4\,a^2cx+6\,a^2b}{2\,(b^4-8\,ab^2c+16\,a^2c^2)(cx^2+bx+a)^2}$$

input
```
integrate(1/(c+a/x^2+b/x)^3/x^4,x, algorithm="giac")
```

output
```
2*(b^2 + 2*a*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c +
 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) + 1/2*(2*b^2*c*x^3 + 4*a*c^2*x^3 + 3*b^3*
x^2 + 6*a*b*c*x^2 + 10*a*b^2*x - 4*a^2*c*x + 6*a^2*b)/((b^4 - 8*a*b^2*c +
16*a^2*c^2)*(c*x^2 + b*x + a)^2)
```

3.435.9 Mupad [B] (verification not implemented)

Time = 8.41 (sec) , antiderivative size = 313, normalized size of antiderivative = 2.72

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^4}\,dx = \frac{\frac{3\,a^2\,b}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}-\frac{a\,x\,\left(2\,a\,c-5\,b^2\right)}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}+\frac{3\,b\,x^2\,\left(b^2+2\,a\,c\right)}{2\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{c\,x^3\,\left(b^2+2\,a\,c\right)}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}}{x^2\,\left(b^2+2\,a\,c\right)+a^2+c^2\,x^4+2\,a\,b\,x+2\,b\,c\,x^3} + \frac{2\,\mathrm{atan}\left(\frac{\left(\frac{\left(b^2+2\,a\,c\right)\,\left(16\,a^2\,b\,c^2-8\,a\,b^3\,c+b^5\right)}{{\left(4\,a\,c-b^2\right)}^{5/2}\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{2\,c\,x\,\left(b^2+2\,a\,c\right)}{{\left(4\,a\,c-b^2\right)}^{5/2}}\right)\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}{b^2+2\,a\,c}\right)\,\left(b^2+2\,a\,c\right)}{{\left(4\,a\,c-b^2\right)}^{5/2}}$$

input
```
int(1/(x^4*(c + a/x^2 + b/x)^3),x)
```

output
```
((3*a^2*b)/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) - (a*x*(2*a*c - 5*b^2))/(b^4 + 1
6*a^2*c^2 - 8*a*b^2*c) + (3*b*x^2*(2*a*c + b^2))/(2*(b^4 + 16*a^2*c^2 - 8*
a*b^2*c)) + (c*x^3*(2*a*c + b^2))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(x^2*(2*
a*c + b^2) + a^2 + c^2*x^4 + 2*a*b*x + 2*b*c*x^3) + (2*atan(((((2*a*c + b^
2)*(b^5 + 16*a^2*b*c^2 - 8*a*b^3*c))/((4*a*c - b^2)^(5/2)*(b^4 + 16*a^2*c^
2 - 8*a*b^2*c)) + (2*c*x*(2*a*c + b^2))/(4*a*c - b^2)^(5/2))*(b^4 + 16*a^2
*c^2 - 8*a*b^2*c))/(2*a*c + b^2))*(2*a*c + b^2))/(4*a*c - b^2)^(5/2)
```

3.436 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^5} dx$

3.436.1 Optimal result

Integrand size = 18, antiderivative size = 103

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^5} dx = \frac{2a+bx}{2(b^2-4ac)(a+bx+cx^2)^2} - \frac{3b(b+2cx)}{2(b^2-4ac)^2(a+bx+cx^2)} + \frac{6bc\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}$$

output
```
1/2*(b*x+2*a)/(-4*a*c+b^2)/(c*x^2+b*x+a)^2-3/2*b*(2*c*x+b)/(-4*a*c+b^2)^2/
(c*x^2+b*x+a)+6*b*c*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(5/
2)
```

3.436.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.99

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^5} dx = \frac{\frac{(b^2-4ac)(2a+bx)}{(a+x(b+cx))^2} - \frac{3b(b+2cx)}{a+x(b+cx)} - \frac{12bc\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}}{2(b^2-4ac)^2}$$

input
```
Integrate[1/((c + a/x^2 + b/x)^3*x^5),x]
```

output
```
(((b^2 - 4*a*c)*(2*a + b*x))/(a + x*(b + c*x))^2 - (3*b*(b + 2*c*x))/(a +
x*(b + c*x)) - (12*b*c*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 +
 4*a*c])/(2*(b^2 - 4*a*c)^2)
```

3.436.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1692, 1159, 1086, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 \left(\frac{a}{x^2}+\frac{b}{x}+c\right)^3}dx$$

$$\Big\downarrow \text{ 1692}$$

$$\int \frac{x}{(a+bx+cx^2)^3}dx$$

$$\Big\downarrow \text{ 1159}$$

$$\frac{3b\int \frac{1}{(cx^2+bx+a)^2}dx}{2(b^2-4ac)}+\frac{2a+bx}{2(b^2-4ac)(a+bx+cx^2)^2}$$

$$\Big\downarrow \text{ 1086}$$

$$\frac{3b\left(-\frac{2c\int \frac{1}{cx^2+bx+a}dx}{b^2-4ac}-\frac{b+2cx}{(b^2-4ac)(a+bx+cx^2)}\right)}{2(b^2-4ac)}+\frac{2a+bx}{2(b^2-4ac)(a+bx+cx^2)^2}$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{3b\left(\frac{4c\int \frac{1}{b^2-(b+2cx)^2-4ac}d(b+2cx)}{b^2-4ac}-\frac{b+2cx}{(b^2-4ac)(a+bx+cx^2)}\right)}{2(b^2-4ac)}+\frac{2a+bx}{2(b^2-4ac)(a+bx+cx^2)^2}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{3b\left(\frac{4c\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx}{(b^2-4ac)(a+bx+cx^2)}\right)}{2(b^2-4ac)}+\frac{2a+bx}{2(b^2-4ac)(a+bx+cx^2)^2}$$

input
```
Int[1/((c + a/x^2 + b/x)^3*x^5),x]
```

output
```
(2*a + b*x)/(2*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) + (3*b*(-((b + 2*c*x)/((
b^2 - 4*a*c)*(a + b*x + c*x^2))) + (4*c*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a
*c]])/(b^2 - 4*a*c)^(3/2)))/(2*(b^2 - 4*a*c))
```

3.436.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

rule 1692
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

3.436.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.15

method	result
default	$\frac{-bx-2a}{2(4ac-b^2)(cx^2+bx+a)^2}-\frac{3b\left(\frac{2cx+b}{(4ac-b^2)(cx^2+bx+a)}+\frac{4c\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}}\right)}{2(4ac-b^2)}$
risch	$\frac{-\frac{3bc^2x^3}{16a^2c^2-8ab^2c+b^4}-\frac{9b^2cx^2}{2(16a^2c^2-8ab^2c+b^4)}-\frac{(5ac+b^2)bx}{16a^2c^2-8ab^2c+b^4}-\frac{a(8ac+b^2)}{2(16a^2c^2-8ab^2c+b^4)}}{(cx^2+bx+a)^2}-\frac{3bc\ln\left((32a^2c^3-16b^2ac^2+2b^4c)x-(-4ac\right.}{(-4ac+b^2)^{\frac{5}{2}}}$

input
```
int(1/(c+a/x^2+b/x)^3/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/2*(-b*x-2*a)/(4*a*c-b^2)/(c*x^2+b*x+a)^2-3/2*b/(4*a*c-b^2)*((2*c*x+b)/(4
*a*c-b^2)/(c*x^2+b*x+a)+4*c/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)
^(1/2)))
```

3.436.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 384 vs. $2(95) = 190$.

Time = 0.29 (sec) , antiderivative size = 788, normalized size of antiderivative = 7.65

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^5}\,dx$$

$$= \left[-\frac{ab^4+4a^2b^2c-32a^3c^2+6(b^3c^2-4abc^3)x^3+9(b^4c-4ab^2c^2)x^2-6(bc^3x^4+2b^2c^2x^3+}{2(a^2b^6-12a^3b^4c+48a^4b^2c^2-64a^5c^3+(b^6c^2-12ab^4c^3+48a^2b^2c^4-64a^3c^5)x^4+2(b^7c-12ab^5c^2}\right.$$

$$-\frac{ab^4+4a^2b^2c-32a^3c^2+6(b^3c^2-4abc^3)x^3+9(b^4c-4ab^2c^2)x^2-12(bc^3x^4+2b^2c^2}{2(a^2b^6-12a^3b^4c+48a^4b^2c^2-64a^5c^3+(b^6c^2-12ab^4c^3+48a^2b^2c^4-64a^3c^5)x^4+2(b^7c-12ab^5c^2}$$

input
```
integrate(1/(c+a/x^2+b/x)^3/x^5,x, algorithm="fricas")
```

output
```
[-1/2*(a*b^4 + 4*a^2*b^2*c - 32*a^3*c^2 + 6*(b^3*c^2 - 4*a*b*c^3)*x^3 + 9*
(b^4*c - 4*a*b^2*c^2)*x^2 - 6*(b*c^3*x^4 + 2*b^2*c^2*x^3 + 2*a*b^2*c*x + a
^2*b*c + (b^3*c + 2*a*b*c^2)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c
*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*(
b^5 + a*b^3*c - 20*a^2*b*c^2)*x)/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2
- 64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4
+ 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10
*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 -
 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x), -1/2*(a*b^4 + 4*a^2*b^2
*c - 32*a^3*c^2 + 6*(b^3*c^2 - 4*a*b*c^3)*x^3 + 9*(b^4*c - 4*a*b^2*c^2)*x^
2 - 12*(b*c^3*x^4 + 2*b^2*c^2*x^3 + 2*a*b^2*c*x + a^2*b*c + (b^3*c + 2*a*b
*c^2)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2
- 4*a*c)) + 2*(b^5 + a*b^3*c - 20*a^2*b*c^2)*x)/(a^2*b^6 - 12*a^3*b^4*c +
48*a^4*b^2*c^2 - 64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 6
4*a^3*c^5)*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*
x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x
^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x)]
```

3.436.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 481 vs. $2(95) = 190$.

Time = 0.60 (sec) , antiderivative size = 481, normalized size of antiderivative = 4.67

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^5}\,dx$$

$$= 3bc\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x+\frac{-192a^3bc^4\sqrt{-\frac{1}{(4ac-b^2)^5}}+144a^2b^3c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}-36ab^5c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}+3b}{6bc^2}\right.$$

$$-3bc\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x+\frac{192a^3bc^4\sqrt{-\frac{1}{(4ac-b^2)^5}}-144a^2b^3c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}+36ab^5c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}-3b}{6bc^2}\right.$$

$$+\frac{-8a^2c-ab^2-9b^2cx^2-6bc^2x^3+x(-10abc-2b^3)}{32a^4c^2-16a^3b^2c+2a^2b^4+x^4\cdot(32a^2c^4-16ab^2c^3+2b^4c^2)+x^3\cdot(64a^2bc^3-32ab^3c^2+4b^5c)+x^2\cdot(64}$$

input
```
integrate(1/(c+a/x**2+b/x)**3/x**5,x)
```

output
```
3*b*c*sqrt(-1/(4*a*c - b**2)**5)*log(x + (-192*a**3*b*c**4*sqrt(-1/(4*a*c
- b**2)**5) + 144*a**2*b**3*c**3*sqrt(-1/(4*a*c - b**2)**5) - 36*a*b**5*c*
*2*sqrt(-1/(4*a*c - b**2)**5) + 3*b**7*c*sqrt(-1/(4*a*c - b**2)**5) + 3*b*
*2*c)/(6*b*c**2)) - 3*b*c*sqrt(-1/(4*a*c - b**2)**5)*log(x + (192*a**3*b*c
**4*sqrt(-1/(4*a*c - b**2)**5) - 144*a**2*b**3*c**3*sqrt(-1/(4*a*c - b**2)
**5) + 36*a*b**5*c**2*sqrt(-1/(4*a*c - b**2)**5) - 3*b**7*c*sqrt(-1/(4*a*c
 - b**2)**5) + 3*b**2*c)/(6*b*c**2)) + (-8*a**2*c - a*b**2 - 9*b**2*c*x**2
 - 6*b*c**2*x**3 + x*(-10*a*b*c - 2*b**3))/(32*a**4*c**2 - 16*a**3*b**2*c
+ 2*a**2*b**4 + x**4*(32*a**2*c**4 - 16*a*b**2*c**3 + 2*b**4*c**2) + x**3*
(64*a**2*b*c**3 - 32*a*b**3*c**2 + 4*b**5*c) + x**2*(64*a**3*c**3 - 12*a*b
**4*c + 2*b**6) + x*(64*a**3*b*c**2 - 32*a**2*b**3*c + 4*a*b**5))
```

3.436.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^5}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(c+a/x^2+b/x)^3/x^5,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.436.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.31

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^5}\,dx = -\frac{6\,bc\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-b^2+4\,ac}} - \frac{6\,bc^2x^3+9\,b^2cx^2+2\,b^3x+10\,abcx+ab^2+8\,a^2c}{2\,(b^4-8\,ab^2c+16\,a^2c^2)(cx^2+bx+a)^2}$$

input
```
integrate(1/(c+a/x^2+b/x)^3/x^5,x, algorithm="giac")
```

output
```
-6*b*c*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c + 16*a^2*c
^2)*sqrt(-b^2 + 4*a*c)) - 1/2*(6*b*c^2*x^3 + 9*b^2*c*x^2 + 2*b^3*x + 10*a*
b*c*x + a*b^2 + 8*a^2*c)/((b^4 - 8*a*b^2*c + 16*a^2*c^2)*(c*x^2 + b*x + a)
^2)
```

3.436.9 Mupad [B] (verification not implemented)

Time = 8.39 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.46

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^5}\,dx = -\frac{\frac{8\,c\,a^2+a\,b^2}{2\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}+\frac{9\,b^2\,c\,x^2}{2\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}+\frac{3\,b\,c^2\,x^3}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}+\frac{b\,x\,(b^2+5\,a\,c)}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}}{x^2\,(b^2+2\,a\,c)+a^2+c^2\,x^4+2\,a\,b\,x+2\,b\,c\,x^3} - \frac{6\,b\,c\,\mathrm{atan}\left(\frac{\left(\frac{3\,b^2\,c}{\left(4\,a\,c-b^2\right)^{5/2}}+\frac{6\,b\,c^2\,x}{\left(4\,a\,c-b^2\right)^{5/2}}\right)\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}{3\,b\,c}\right)}{\left(4\,a\,c-b^2\right)^{5/2}}$$

input
```
int(1/(x^5*(c + a/x^2 + b/x)^3),x)
```

output
```
- ((a*b^2 + 8*a^2*c)/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (9*b^2*c*x^2)/(2
*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*b*c^2*x^3)/(b^4 + 16*a^2*c^2 - 8*a*b
^2*c) + (b*x*(5*a*c + b^2))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(x^2*(2*a*c +
b^2) + a^2 + c^2*x^4 + 2*a*b*x + 2*b*c*x^3) - (6*b*c*atan((((3*b^2*c)/(4*a
*c - b^2)^(5/2) + (6*b*c^2*x)/(4*a*c - b^2)^(5/2))*(b^4 + 16*a^2*c^2 - 8*a
*b^2*c))/(3*b*c)))/(4*a*c - b^2)^(5/2)
```

3.437 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^6} \, dx$

3.437.1 Optimal result

Integrand size = 18, antiderivative size = 103

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^6} \, dx = \frac{-b-2cx}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{3c(b+2cx)}{\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} - \frac{12c^2\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

output
```
1/2*(-2*c*x-b)/(-4*a*c+b^2)/(c*x^2+b*x+a)^2+3*c*(2*c*x+b)/(-4*a*c+b^2)^2/(
c*x^2+b*x+a)-12*c^2*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(5/
2)
```

3.437.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.94

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^6} \, dx = \frac{-\frac{(b+2cx)\left(b^2-6bcx-2c\left(5a+3cx^2\right)\right)}{(a+x(b+cx))^2} + \frac{24c^2\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}}{2\left(b^2-4ac\right)^2}$$

input
```
Integrate[1/((c + a/x^2 + b/x)^3*x^6),x]
```

output
```
(-(((b + 2*c*x)*(b^2 - 6*b*c*x - 2*c*(5*a + 3*c*x^2)))/(a + x*(b + c*x))^2
) + (24*c^2*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(2
*(b^2 - 4*a*c)^2)
```

3.437.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1692, 1086, 1086, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 \left(\frac{a}{x^2}+\frac{b}{x}+c\right)^3} \, dx$$

$$\Big\downarrow \text{1692}$$

$$\int \frac{1}{(a+bx+cx^2)^3}dx$$

$$\Big\downarrow \text{1086}$$

$$-\frac{3c\int \frac{1}{(cx^2+bx+a)^2}dx}{b^2-4ac}-\frac{b+2cx}{2(b^2-4ac)(a+bx+cx^2)^2}$$

$$\Big\downarrow \text{1086}$$

$$-\frac{3c\left(-\frac{2c\int \frac{1}{cx^2+bx+a}dx}{b^2-4ac}-\frac{b+2cx}{(b^2-4ac)(a+bx+cx^2)}\right)}{b^2-4ac}-\frac{b+2cx}{2(b^2-4ac)(a+bx+cx^2)^2}$$

$$\Big\downarrow \text{1083}$$

$$-\frac{3c\left(\frac{4c\int \frac{1}{b^2-(b+2cx)^2-4ac}d(b+2cx)}{b^2-4ac}-\frac{b+2cx}{(b^2-4ac)(a+bx+cx^2)}\right)}{b^2-4ac}-\frac{b+2cx}{2(b^2-4ac)(a+bx+cx^2)^2}$$

$$\Big\downarrow \text{219}$$

$$-\frac{3c\left(\frac{4c\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx}{(b^2-4ac)(a+bx+cx^2)}\right)}{b^2-4ac}-\frac{b+2cx}{2(b^2-4ac)(a+bx+cx^2)^2}$$

input
```
Int[1/((c + a/x^2 + b/x)^3*x^6),x]
```

output
```
-1/2*(b + 2*c*x)/((b^2 - 4*a*c)*(a + b*x + c*x^2)^2) - (3*c*(-((b + 2*c*x)
/((b^2 - 4*a*c)*(a + b*x + c*x^2))) + (4*c*ArcTanh[(b + 2*c*x)/Sqrt[b^2 -
4*a*c]])/(b^2 - 4*a*c)^(3/2)))/(b^2 - 4*a*c)
```

3.437.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

rule 1692
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

3.437.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.13

method	result
default	$\frac{2cx+b}{2(4ac-b^2)(c\,x^2+bx+a)^2}+\frac{3c\left(\frac{2cx+b}{\left(4ac-b^2\right)\left(c\,x^2+bx+a\right)}+\frac{4c\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}}\right)}{4ac-b^2}$
risch	$\frac{\frac{6c^3x^3}{16a^2c^2-8a\,b^2c+b^4}+\frac{9b\,c^2x^2}{16a^2c^2-8a\,b^2c+b^4}+\frac{2\left(5ac+b^2\right)cx}{16a^2c^2-8a\,b^2c+b^4}+\frac{b\left(10ac-b^2\right)}{32a^2c^2-16a\,b^2c+2b^4}}{(c\,x^2+bx+a)^2}-\frac{6c^2\ln\left(\left(32a^2c^3-16b^2a\,c^2+2b^4c\right)x+\left(-4ac+b^2\right)^{\frac{5}{2}}\right.}{\left(-4ac+b^2\right)^{\frac{5}{2}}}$

input
```
int(1/(c+a/x^2+b/x)^3/x^6,x,method=_RETURNVERBOSE)
```

output
```
1/2*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^2+3*c/(4*a*c-b^2)*((2*c*x+b)/(4*a*
c-b^2)/(c*x^2+b*x+a)+4*c/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1
/2)))
```

3.437.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 382 vs. $2(95) = 190$.

Time = 0.26 (sec) , antiderivative size = 785, normalized size of antiderivative = 7.62

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^6}\,dx$$
$$= \left[-\frac{b^5-14\,ab^3c+40\,a^2bc^2-12\,(b^2c^3-4\,ac^4)x^3-18\,(b^3c^2-4\,abc^3)x^2-12\,(c^4x^4+2\,bc^3x^3+}{2\,(a^2b^6-12\,a^3b^4c+48\,a^4b^2c^2-64\,a^5c^3+(b^6c^2-12\,ab^4c^3+48\,a^2b^2c^4-64\,a^3c^5)x^4+2\,(b^7c-12\,ab^5c^2}\right.$$
$$-\frac{b^5-14\,ab^3c+40\,a^2bc^2-12\,(b^2c^3-4\,ac^4)x^3-18\,(b^3c^2-4\,abc^3)x^2+24\,(c^4x^4+2\,bc^3}{2\,(a^2b^6-12\,a^3b^4c+48\,a^4b^2c^2-64\,a^5c^3+(b^6c^2-12\,ab^4c^3+48\,a^2b^2c^4-64\,a^3c^5)x^4+2\,(b^7c-12\,ab^5c^2}$$

input
```
integrate(1/(c+a/x^2+b/x)^3/x^6,x, algorithm="fricas")
```

output
```
[-1/2*(b^5 - 14*a*b^3*c + 40*a^2*b*c^2 - 12*(b^2*c^3 - 4*a*c^4)*x^3 - 18*(
b^3*c^2 - 4*a*b*c^3)*x^2 - 12*(c^4*x^4 + 2*b*c^3*x^3 + 2*a*b*c^2*x + a^2*c
^2 + (b^2*c^2 + 2*a*c^3)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x +
 b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 4*(b^4*
c + a*b^2*c^2 - 20*a^2*c^3)*x)/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 -
64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4 +
2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*a
*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 - 1
2*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x), -1/2*(b^5 - 14*a*b^3*c +
40*a^2*b*c^2 - 12*(b^2*c^3 - 4*a*c^4)*x^3 - 18*(b^3*c^2 - 4*a*b*c^3)*x^2 +
 24*(c^4*x^4 + 2*b*c^3*x^3 + 2*a*b*c^2*x + a^2*c^2 + (b^2*c^2 + 2*a*c^3)*x
^2)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c
)) - 4*(b^4*c + a*b^2*c^2 - 20*a^2*c^3)*x)/(a^2*b^6 - 12*a^3*b^4*c + 48*a^
4*b^2*c^2 - 64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3
*c^5)*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 +
 (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 +
2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x)]
```

3.437.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 474 vs. $2(95) = 190$.

Time = 0.70 (sec) , antiderivative size = 474, normalized size of antiderivative = 4.60

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^6}\,dx =$$

$$-6c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x+\frac{-384a^3c^5\sqrt{-\frac{1}{(4ac-b^2)^5}}+288a^2b^2c^4\sqrt{-\frac{1}{(4ac-b^2)^5}}-72ab^4c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}+6}{12c^3}\right.$$

$$+6c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x+\frac{384a^3c^5\sqrt{-\frac{1}{(4ac-b^2)^5}}-288a^2b^2c^4\sqrt{-\frac{1}{(4ac-b^2)^5}}+72ab^4c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}-6b^6}{12c^3}\right.$$

$$+\frac{10abc-b^3+18bc^2x^2+12c^3x^3+x(20ac^2+4b^2c)}{32a^4c^2-16a^3b^2c+2a^2b^4+x^4\cdot(32a^2c^4-16ab^2c^3+2b^4c^2)+x^3\cdot(64a^2bc^3-32ab^3c^2+4b^5c)+x^2\cdot(64}$$

input
```
integrate(1/(c+a/x**2+b/x)**3/x**6,x)
```

output
```
-6*c**2*sqrt(-1/(4*a*c - b**2)**5)*log(x + (-384*a**3*c**5*sqrt(-1/(4*a*c
- b**2)**5) + 288*a**2*b**2*c**4*sqrt(-1/(4*a*c - b**2)**5) - 72*a*b**4*c*
*3*sqrt(-1/(4*a*c - b**2)**5) + 6*b**6*c**2*sqrt(-1/(4*a*c - b**2)**5) + 6
*b*c**2)/(12*c**3)) + 6*c**2*sqrt(-1/(4*a*c - b**2)**5)*log(x + (384*a**3*
c**5*sqrt(-1/(4*a*c - b**2)**5) - 288*a**2*b**2*c**4*sqrt(-1/(4*a*c - b**2
)**5) + 72*a*b**4*c**3*sqrt(-1/(4*a*c - b**2)**5) - 6*b**6*c**2*sqrt(-1/(4
*a*c - b**2)**5) + 6*b*c**2)/(12*c**3)) + (10*a*b*c - b**3 + 18*b*c**2*x**
2 + 12*c**3*x**3 + x*(20*a*c**2 + 4*b**2*c))/(32*a**4*c**2 - 16*a**3*b**2*
c + 2*a**2*b**4 + x**4*(32*a**2*c**4 - 16*a*b**2*c**3 + 2*b**4*c**2) + x**
3*(64*a**2*b*c**3 - 32*a*b**3*c**2 + 4*b**5*c) + x**2*(64*a**3*c**3 - 12*a
*b**4*c + 2*b**6) + x*(64*a**3*b*c**2 - 32*a**2*b**3*c + 4*a*b**5))
```

3.437.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^6}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(c+a/x^2+b/x)^3/x^6,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.437.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.32

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^6}\,dx = \frac{12\,c^2 \arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-b^2+4\,ac}} + \frac{12\,c^3x^3+18\,bc^2x^2+4\,b^2cx+20\,ac^2x-b^3+10\,abc}{2\,(b^4-8\,ab^2c+16\,a^2c^2)(cx^2+bx+a)^2}$$

input
```
integrate(1/(c+a/x^2+b/x)^3/x^6,x, algorithm="giac")
```

output
```
12*c^2*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c + 16*a^2*c
^2)*sqrt(-b^2 + 4*a*c)) + 1/2*(12*c^3*x^3 + 18*b*c^2*x^2 + 4*b^2*c*x + 20*
a*c^2*x - b^3 + 10*a*b*c)/((b^4 - 8*a*b^2*c + 16*a^2*c^2)*(c*x^2 + b*x + a
)^2)
```

3.437.9 Mupad [B] (verification not implemented)

Time = 8.49 (sec) , antiderivative size = 285, normalized size of antiderivative = 2.77

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^6}\,dx = \frac{\frac{6\,c^3\,x^3}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}-\frac{b^3-10\,a\,b\,c}{2\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}+\frac{9\,b\,c^2\,x^2}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}+\frac{2\,c\,x\,(b^2+5\,a\,c)}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}}{x^2\,(b^2+2\,a\,c)+a^2+c^2\,x^4+2\,a\,b\,x+2\,b\,c\,x^3} + \frac{12\,c^2\,\mathrm{atan}\left(\frac{\left(\frac{12\,c^3\,x}{(4\,a\,c-b^2)^{5/2}}+\frac{6\,c^2\,(16\,a^2\,b\,c^2-8\,a\,b^3\,c+b^5)}{(4\,a\,c-b^2)^{5/2}\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}\right)(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}{6\,c^2}\right)}{(4\,a\,c-b^2)^{5/2}}$$

input
```
int(1/(x^6*(c + a/x^2 + b/x)^3),x)
```

output
```
((6*c^3*x^3)/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) - (b^3 - 10*a*b*c)/(2*(b^4 + 1
6*a^2*c^2 - 8*a*b^2*c)) + (9*b*c^2*x^2)/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (
2*c*x*(5*a*c + b^2))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(x^2*(2*a*c + b^2) +
a^2 + c^2*x^4 + 2*a*b*x + 2*b*c*x^3) + (12*c^2*atan((((12*c^3*x)/(4*a*c -
b^2)^(5/2) + (6*c^2*(b^5 + 16*a^2*b*c^2 - 8*a*b^3*c))/((4*a*c - b^2)^(5/2)
*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(6*c^2))
)/(4*a*c - b^2)^(5/2)
```

3.437. $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^6}\,dx$

3.438 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^7} \, dx$

3.438.1 Optimal result

Integrand size = 18, antiderivative size = 185

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^7} \, dx = \frac{b^2-2ac+bcx}{2a(b^2-4ac)(a+bx+cx^2)^2} + \frac{2b^4-15ab^2c+16a^2c^2+2bc(b^2-7ac)x}{2a^2(b^2-4ac)^2(a+bx+cx^2)} + \frac{b(b^4-10ab^2c+30a^2c^2)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3(b^2-4ac)^{5/2}} + \frac{\log(x)}{a^3} - \frac{\log(a+bx+cx^2)}{2a^3}$$

output
```
1/2*(b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^2+b*x+a)^2+1/2*(2*b^4-15*a*b^2*c
+16*a^2*c^2+2*b*c*(-7*a*c+b^2)*x)/a^2/(-4*a*c+b^2)^2/(c*x^2+b*x+a)+b*(30*a
^2*c^2-10*a*b^2*c+b^4)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b
^2)^(5/2)+ln(x)/a^3-1/2*ln(c*x^2+b*x+a)/a^3
```

3.438.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.96

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^7}\,dx = \frac{\frac{a^2(b^2-2ac+bcx)}{(b^2-4ac)(a+x(b+cx))^2}+\frac{a(2b^4-15ab^2c+16a^2c^2+2b^3cx-14abc^2x)}{(b^2-4ac)^2(a+x(b+cx))}-\frac{2b(b^4-10ab^2c+30a^2c^2)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{5/2}}+2\log(x)-\log(}{2a^3}$$

input
```
Integrate[1/((c + a/x^2 + b/x)^3*x^7),x]
```

output
```
((a^2*(b^2 - 2*a*c + b*c*x))/((b^2 - 4*a*c)*(a + x*(b + c*x))^2) + (a*(2*b
^4 - 15*a*b^2*c + 16*a^2*c^2 + 2*b^3*c*x - 14*a*b*c^2*x))/((b^2 - 4*a*c)^2
*(a + x*(b + c*x))) - (2*b*(b^4 - 10*a*b^2*c + 30*a^2*c^2)*ArcTan[(b + 2*c
*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2) + 2*Log[x] - Log[a + x*(b +
c*x)])/(2*a^3)
```

3.438.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.28, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1692, 1165, 25, 1235, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7\left(\frac{a}{x^2}+\frac{b}{x}+c\right)^3}dx$$

$$\Big\downarrow \text{ 1692}$$

$$\int \frac{1}{x\left(a+bx+cx^2\right)^3}dx$$

$$\Big\downarrow \text{ 1165}$$

$$\frac{-2ac+b^2+bcx}{2a\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}-\frac{\int -\frac{2(b^2-4ac)+3bcx}{x(cx^2+bx+a)^2}dx}{2a\left(b^2-4ac\right)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{2(b^2-4ac)+3bcx}{x(cx^2+bx+a)^2}dx}{2a\,(b^2-4ac)}+\frac{-2ac+b^2+bcx}{2a\,(b^2-4ac)\,(a+bx+cx^2)^2}$$

$\Big\downarrow$ 1235

$$\frac{\frac{16a^2c^2+2bcx(b^2-7ac)-15ab^2c+2b^4}{a(b^2-4ac)(a+bx+cx^2)}-\frac{\int -\frac{2\left(\left(b^2-4ac\right)^2+bc\left(b^2-7ac\right)x\right)}{x\left(cx^2+bx+a\right)}dx}{a(b^2-4ac)}}{2a\,(b^2-4ac)}+\frac{-2ac+b^2+bcx}{2a\,(b^2-4ac)\,(a+bx+cx^2)^2}$$

$\Big\downarrow$ 27

$$\frac{\frac{2\int \frac{\left(b^2-4ac\right)^2+bc\left(b^2-7ac\right)x}{x\left(cx^2+bx+a\right)}dx}{a(b^2-4ac)}+\frac{16a^2c^2+2bcx(b^2-7ac)-15ab^2c+2b^4}{a(b^2-4ac)(a+bx+cx^2)}}{2a\,(b^2-4ac)}+\frac{-2ac+b^2+bcx}{2a\,(b^2-4ac)\,(a+bx+cx^2)^2}$$

$\Big\downarrow$ 1200

$$\frac{\frac{2\int \left(\frac{\left(4ac-b^2\right)^2}{ax}+\frac{-cx\left(b^2-4ac\right)^2-b\left(b^4-9acb^2+23a^2c^2\right)}{a\left(cx^2+bx+a\right)}\right)dx}{a(b^2-4ac)}+\frac{16a^2c^2+2bcx(b^2-7ac)-15ab^2c+2b^4}{a(b^2-4ac)(a+bx+cx^2)}}{2a\,(b^2-4ac)}+\frac{-2ac+b^2+bcx}{2a\,(b^2-4ac)\,(a+bx+cx^2)^2}$$

$\Big\downarrow$ 2009

$$\frac{\frac{2\left(\frac{b\left(30a^2c^2-10ab^2c+b^4\right)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}-\frac{\left(b^2-4ac\right)^2\log\left(a+bx+cx^2\right)}{2a}+\frac{\log(x)\left(b^2-4ac\right)^2}{a}\right)}{a(b^2-4ac)}+\frac{16a^2c^2+2bcx(b^2-7ac)-15ab^2c+2b^4}{a(b^2-4ac)(a+bx+cx^2)}}{2a\,(b^2-4ac)}+\frac{-2ac+b^2+bcx}{2a\,(b^2-4ac)\,(a+bx+cx^2)^2}$$

input
```
Int[1/((c + a/x^2 + b/x)^3*x^7),x]
```

output
```
(b^2 - 2*a*c + b*c*x)/(2*a*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) + ((2*b^4 -
15*a*b^2*c + 16*a^2*c^2 + 2*b*c*(b^2 - 7*a*c)*x)/(a*(b^2 - 4*a*c)*(a + b*x
 + c*x^2)) + (2*((b*(b^4 - 10*a*b^2*c + 30*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sq
rt[b^2 - 4*a*c]])/(a*Sqrt[b^2 - 4*a*c]) + ((b^2 - 4*a*c)^2*Log[x])/a - ((b
^2 - 4*a*c)^2*Log[a + b*x + c*x^2])/(2*a)))/(a*(b^2 - 4*a*c)))/(2*a*(b^2 -
 4*a*c))
```

3.438. $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^7}\,dx$

3.438.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1692
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

3.438. $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^7}\,dx$

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.438.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 351 vs. $2(175) = 350$.

Time = 0.18 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.90

method	result
default	$\frac{\ln(x)}{a^3} - \frac{\frac{\frac{ab\,c^2(7ac-b^2)x^3}{16a^2c^2-8a\,b^2c+b^4} - \frac{ac(16a^2c^2-29a\,b^2c+4b^4)x^2}{2(16a^2c^2-8a\,b^2c+b^4)} + \frac{ab(a^2c^2+6a\,b^2c-b^4)x}{16a^2c^2-8a\,b^2c+b^4} - \frac{3a^2(8a^2c^2-7a\,b^2c+b^4)}{2(16a^2c^2-8a\,b^2c+b^4)}}{(c\,x^2+bx+a)^2} + \frac{(16a^2c^3-8b^2a\,c^2+b^4c)\ln(c}{2c}}{a^3}$
risch	Expression too large to display

input
```
int(1/(c+a/x^2+b/x)^3/x^7,x,method=_RETURNVERBOSE)
```

output
```
ln(x)/a^3-1/a^3*((a*b*c^2*(7*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3-1/2*a
*c*(16*a^2*c^2-29*a*b^2*c+4*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+a*b*(a^2*c
^2+6*a*b^2*c-b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x-3/2*a^2*(8*a^2*c^2-7*a*b^2*
c+b^4)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^2+b*x+a)^2+1/(16*a^2*c^2-8*a*b^2*c
+b^4)*(1/2*(16*a^2*c^3-8*a*b^2*c^2+b^4*c)/c*ln(c*x^2+b*x+a)+2*(23*a^2*b*c^
2-9*a*b^3*c+b^5-1/2*(16*a^2*c^3-8*a*b^2*c^2+b^4*c)*b/c)/(4*a*c-b^2)^(1/2)*
arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))))
```

3.438.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 983 vs. $2(175) = 350$.

Time = 0.52 (sec) , antiderivative size = 1985, normalized size of antiderivative = 10.73

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^7}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c+a/x^2+b/x)^3/x^7,x, algorithm="fricas")
```

output
```
[1/2*(3*a^2*b^6 - 33*a^3*b^4*c + 108*a^4*b^2*c^2 - 96*a^5*c^3 + 2*(a*b^5*c
^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*x^3 + (4*a*b^6*c - 45*a^2*b^4*c^2 + 13
2*a^3*b^2*c^3 - 64*a^4*c^4)*x^2 + (a^2*b^5 - 10*a^3*b^3*c + 30*a^4*b*c^2 +
 (b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*x^4 + 2*(b^6*c - 10*a*b^4*c^2 + 3
0*a^2*b^2*c^3)*x^3 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*x^2
 + 2*(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*x)*sqrt(b^2 - 4*a*c)*log((2*c
^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b
*x + a)) + 2*(a*b^7 - 10*a^2*b^5*c + 23*a^3*b^3*c^2 + 4*a^4*b*c^3)*x - (a^
2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c
^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b
^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b
^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*
a^4*b*c^3)*x)*log(c*x^2 + b*x + a) + 2*(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^
2*c^2 - 64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5
)*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^
8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a
*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x)*log(x))/(a^5*b^6 -
 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3 + (a^3*b^6*c^2 - 12*a^4*b^4*c^
3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*x^4 + 2*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*
a^5*b^3*c^3 - 64*a^6*b*c^4)*x^3 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*...
```

3.438.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^7}\,dx = \text{Timed out}$$

input
```
integrate(1/(c+a/x**2+b/x)**3/x**7,x)
```

output
```
Timed out
```

3.438.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^7}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(c+a/x^2+b/x)^3/x^7,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.438.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.29

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^7}\,dx$$
$$= -\frac{(b^5-10\,ab^3c+30\,a^2bc^2)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(a^3b^4-8\,a^4b^2c+16\,a^5c^2)\sqrt{-b^2+4\,ac}} - \frac{\log\left(cx^2+bx+a\right)}{2\,a^3} + \frac{\log\left(|x|\right)}{a^3}$$
$$+ \frac{3\,a^2b^4-21\,a^3b^2c+24\,a^4c^2+2\,(ab^3c^2-7\,a^2bc^3)x^3+(4\,ab^4c-29\,a^2b^2c^2+16\,a^3c^3)x^2+2\,(ab^5-6\,a^2b^3c}{2\,(cx^2+bx+a)^2(b^2-4\,ac)^2a^3}$$

input
```
integrate(1/(c+a/x^2+b/x)^3/x^7,x, algorithm="giac")
```

output
```
-(b^5 - 10*a*b^3*c + 30*a^2*b*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/
((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt(-b^2 + 4*a*c)) - 1/2*log(c*x^2
+ b*x + a)/a^3 + log(abs(x))/a^3 + 1/2*(3*a^2*b^4 - 21*a^3*b^2*c + 24*a^4*
c^2 + 2*(a*b^3*c^2 - 7*a^2*b*c^3)*x^3 + (4*a*b^4*c - 29*a^2*b^2*c^2 + 16*a
^3*c^3)*x^2 + 2*(a*b^5 - 6*a^2*b^3*c - a^3*b*c^2)*x)/((c*x^2 + b*x + a)^2*
(b^2 - 4*a*c)^2*a^3)
```

3.438.9 Mupad [B] (verification not implemented)

Time = 8.99 (sec) , antiderivative size = 1089, normalized size of antiderivative = 5.89

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^7}\,dx = \frac{\ln(x)}{a^3} + \frac{\frac{3\,(8\,a^2\,c^2-7\,a\,b^2\,c+b^4)}{2\,a\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}+\frac{x^2\,(16\,a^2\,c^3-29\,a\,b^2\,c^2+4\,b^4\,c)}{2\,a^2\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}-\frac{b\,x\,(a^2\,c^2+6\,a\,b^2\,c-b^4)}{a^2\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}-\frac{b\,c^2\,x^3\,(7\,a\,c-b^2)}{a^2\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}}{x^2\,(b^2+2\,a\,c)+a^2+c^2\,x^4+2\,a\,b\,x+2\,b\,c\,x^3} - \frac{\ln\left(1536\,a^6\,c^5-2\,b^{11}\,x-2\,a\,b^{10}+2\,a\,b^5\,\sqrt{-(4\,a\,c-b^2)^5}+39\,a^2\,b^8\,c+2\,b^6\,x\,\sqrt{-(4\,a\,c-b^2)^5}-303\,a^3 \right.}{} + \frac{\ln\left(2\,a\,b^{10}+2\,b^{11}\,x-1536\,a^6\,c^5+2\,a\,b^5\,\sqrt{-(4\,a\,c-b^2)^5}-39\,a^2\,b^8\,c+2\,b^6\,x\,\sqrt{-(4\,a\,c-b^2)^5}+303\,a^3 \right.}{}$$

input
```
int(1/(x^7*(c + a/x^2 + b/x)^3),x)
```

output
```
log(x)/a^3 + ((3*(b^4 + 8*a^2*c^2 - 7*a*b^2*c))/(2*a*(b^4 + 16*a^2*c^2 - 8
*a*b^2*c)) + (x^2*(4*b^4*c + 16*a^2*c^3 - 29*a*b^2*c^2))/(2*a^2*(b^4 + 16*
a^2*c^2 - 8*a*b^2*c)) - (b*x*(a^2*c^2 - b^4 + 6*a*b^2*c))/(a^2*(b^4 + 16*a
^2*c^2 - 8*a*b^2*c)) - (b*c^2*x^3*(7*a*c - b^2))/(a^2*(b^4 + 16*a^2*c^2 -
8*a*b^2*c)))/(x^2*(2*a*c + b^2) + a^2 + c^2*x^4 + 2*a*b*x + 2*b*c*x^3) - (
log(1536*a^6*c^5 - 2*b^11*x - 2*a*b^10 + 2*a*b^5*(-(4*a*c - b^2)^5)^(1/2)
+ 39*a^2*b^8*c + 2*b^6*x*(-(4*a*c - b^2)^5)^(1/2) - 303*a^3*b^6*c^2 + 1160
*a^4*b^4*c^3 - 2160*a^5*b^2*c^4 - 17*a^2*b^3*c*(-(4*a*c - b^2)^5)^(1/2) +
39*a^3*b*c^2*(-(4*a*c - b^2)^5)^(1/2) - 321*a^2*b^7*c^2*x + 1286*a^3*b^5*c
^3*x - 2560*a^4*b^3*c^4*x - 48*a^3*c^3*x*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b
^9*c*x + 2016*a^5*b*c^5*x - 20*a*b^4*c*x*(-(4*a*c - b^2)^5)^(1/2) + 63*a^2
*b^2*c^2*x*(-(4*a*c - b^2)^5)^(1/2))*(1024*a^5*c^5 - b^10 + b^5*(-(4*a*c -
 b^2)^5)^(1/2) - 160*a^2*b^6*c^2 + 640*a^3*b^4*c^3 - 1280*a^4*b^2*c^4 + 20
*a*b^8*c + 30*a^2*b*c^2*(-(4*a*c - b^2)^5)^(1/2) - 10*a*b^3*c*(-(4*a*c - b
^2)^5)^(1/2)))/(2*a^3*(4*a*c - b^2)^5) + (log(2*a*b^10 + 2*b^11*x - 1536*a
^6*c^5 + 2*a*b^5*(-(4*a*c - b^2)^5)^(1/2) - 39*a^2*b^8*c + 2*b^6*x*(-(4*a*
c - b^2)^5)^(1/2) + 303*a^3*b^6*c^2 - 1160*a^4*b^4*c^3 + 2160*a^5*b^2*c^4
- 17*a^2*b^3*c*(-(4*a*c - b^2)^5)^(1/2) + 39*a^3*b*c^2*(-(4*a*c - b^2)^5)^
(1/2) + 321*a^2*b^7*c^2*x - 1286*a^3*b^5*c^3*x + 2560*a^4*b^3*c^4*x - 48*a
^3*c^3*x*(-(4*a*c - b^2)^5)^(1/2) - 40*a*b^9*c*x - 2016*a^5*b*c^5*x - 2...
```

3.439 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^8}\,dx$

3.439.1 Optimal result

Integrand size = 18, antiderivative size = 239

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^8}\,dx = -\frac{3(b^2-5ac)\,(b^2-2ac)}{a^3\,(b^2-4ac)^2\,x}+\frac{b^2-2ac+bcx}{2a\,(b^2-4ac)\,x\,(a+bx+cx^2)^2} + \frac{3b^4-20ab^2c+20a^2c^2+3bc(b^2-6ac)\,x}{2a^2\,(b^2-4ac)^2\,x\,(a+bx+cx^2)} - \frac{3(b^6-10ab^4c+30a^2b^2c^2-20a^3c^3)\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4\,(b^2-4ac)^{5/2}} - \frac{3b\log(x)}{a^4}+\frac{3b\log\left(a+bx+cx^2\right)}{2a^4}$$

output

```
-3*(-5*a*c+b^2)*(-2*a*c+b^2)/a^3/(-4*a*c+b^2)^2/x+1/2*(b*c*x-2*a*c+b^2)/a/
(-4*a*c+b^2)/x/(c*x^2+b*x+a)^2+1/2*(3*b^4-20*a*b^2*c+20*a^2*c^2+3*b*c*(-6*
a*c+b^2)*x)/a^2/(-4*a*c+b^2)^2/x/(c*x^2+b*x+a)-3*(-20*a^3*c^3+30*a^2*b^2*c
^2-10*a*b^4*c+b^6)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/a^4/(-4*a*c+b^2)^
(5/2)-3*b*ln(x)/a^4+3/2*b*ln(c*x^2+b*x+a)/a^4
```

3.439.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.92

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^8}\,dx$$
$$= \frac{-\frac{2a}{x}+\frac{a^2(b^3-3abc+b^2cx-2ac^2x)}{(-b^2+4ac)(a+x(b+cx))^2}-\frac{a(4b^5-29ab^3c+46a^2bc^2+4b^4cx-26ab^2c^2x+28a^2c^3x)}{(b^2-4ac)^2(a+x(b+cx))}+\frac{6(b^6-10ab^4c+30a^2b^2c^2-20a^3c^3)\arctan\left(\frac{-}{\sqrt{}}\right.}{(-b^2+4ac)^{5/2}}}{2a^4}$$

input
```
Integrate[1/((c + a/x^2 + b/x)^3*x^8),x]
```

output
```
((-2*a)/x + (a^2*(b^3 - 3*a*b*c + b^2*c*x - 2*a*c^2*x))/((-b^2 + 4*a*c)*(a
 + x*(b + c*x))^2) - (a*(4*b^5 - 29*a*b^3*c + 46*a^2*b*c^2 + 4*b^4*c*x - 2
6*a*b^2*c^2*x + 28*a^2*c^3*x))/((b^2 - 4*a*c)^2*(a + x*(b + c*x))) + (6*(b
^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*ArcTan[(b + 2*c*x)/Sqrt[-b^
2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2) - 6*b*Log[x] + 3*b*Log[a + x*(b + c*x)])
/(2*a^4)
```

3.439.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {1692, 1165, 25, 1235, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^8\left(\frac{a}{x^2}+\frac{b}{x}+c\right)^3}dx$$

$$\Big\downarrow \text{1692}$$

$$\int \frac{1}{x^2(a+bx+cx^2)^3}dx$$

$$\Big\downarrow \text{1165}$$

$$\frac{-2ac+b^2+bcx}{2ax(b^2-4ac)(a+bx+cx^2)^2}-\frac{\int -\frac{3b^2+4cxb-10ac}{x^2(cx^2+bx+a)^2}dx}{2a(b^2-4ac)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{3b^2+4cxb-10ac}{x^2(cx^2+bx+a)^2}dx}{2a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx}{2ax\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

$\Big\downarrow$ 1235

$$\frac{\frac{20a^2c^2+3bcx\left(b^2-6ac\right)-20ab^2c+3b^4}{ax(b^2-4ac)(a+bx+cx^2)}-\frac{\int -\frac{6\left(\left(b^2-5ac\right)\left(b^2-2ac\right)+bc\left(b^2-6ac\right)x\right)}{x^2\left(cx^2+bx+a\right)}dx}{a(b^2-4ac)}}{2a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx}{2ax\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

$\Big\downarrow$ 27

$$\frac{\frac{6\int \frac{\left(b^2-5ac\right)\left(b^2-2ac\right)+bc\left(b^2-6ac\right)x}{x^2\left(cx^2+bx+a\right)}dx}{a(b^2-4ac)}+\frac{20a^2c^2+3bcx\left(b^2-6ac\right)-20ab^2c+3b^4}{ax(b^2-4ac)(a+bx+cx^2)}}{2a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx}{2ax\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

$\Big\downarrow$ 1200

$$\frac{\frac{6\int \left(-\frac{b\left(4ac-b^2\right)^2}{a^2x}+\frac{b^6-9acb^4+23a^2c^2b^2+c\left(b^2-4ac\right)^2xb-10a^3c^3}{a^2\left(cx^2+bx+a\right)}+\frac{\left(b^2-5ac\right)\left(b^2-2ac\right)}{ax^2}\right)dx}{a(b^2-4ac)}+\frac{20a^2c^2+3bcx\left(b^2-6ac\right)-20ab^2c+3b^4}{ax(b^2-4ac)(a+bx+cx^2)}}{2a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx}{2ax\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

$\Big\downarrow$ 2009

$$\frac{\frac{20a^2c^2+3bcx\left(b^2-6ac\right)-20ab^2c+3b^4}{ax(b^2-4ac)(a+bx+cx^2)}+\frac{6\left(\frac{b\left(b^2-4ac\right)^2\log\left(a+bx+cx^2\right)}{2a^2}-\frac{b\log(x)\left(b^2-4ac\right)^2}{a^2}-\frac{\left(-20a^3c^3+30a^2b^2c^2-10ab^4c+b^6\right)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}}\right.}{a(b^2-4ac)}}{2a\left(b^2-4ac\right)}$$

$$\frac{-2ac+b^2+bcx}{2ax\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

input
```
Int[1/((c + a/x^2 + b/x)^3*x^8),x]
```

output
```
(b^2 - 2*a*c + b*c*x)/(2*a*(b^2 - 4*a*c)*x*(a + b*x + c*x^2)^2) + ((3*b^4
- 20*a*b^2*c + 20*a^2*c^2 + 3*b*c*(b^2 - 6*a*c)*x)/(a*(b^2 - 4*a*c)*x*(a +
 b*x + c*x^2)) + (6*(-(((b^2 - 5*a*c)*(b^2 - 2*a*c))/(a*x)) - ((b^6 - 10*a
*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c
]])/(a^2*Sqrt[b^2 - 4*a*c]) - (b*(b^2 - 4*a*c)^2*Log[x])/a^2 + (b*(b^2 - 4
*a*c)^2*Log[a + b*x + c*x^2])/(2*a^2)))/(a*(b^2 - 4*a*c)))/(2*a*(b^2 - 4*a
*c))
```

3.439. $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3x^8}\,dx$

3.439.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1692
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

3.439. $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^8}\,dx$

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.439.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.69

method	result
default	$-\frac{1}{a^{3}x}-\frac{3b\ln(x)}{a^{4}}-\frac{\frac{a\,c^{2}\left(14a^{2}c^{2}-13a\,b^{2}c+2b^{4}\right)x^{3}}{16a^{2}c^{2}-8a\,b^{2}c+b^{4}}+\frac{abc\left(74a^{2}c^{2}-55a\,b^{2}c+8b^{4}\right)x^{2}}{32a^{2}c^{2}-16a\,b^{2}c+2b^{4}}+\frac{a\left(18c^{3}a^{3}+7a^{2}b^{2}c^{2}-12a\,b^{4}c+2b^{6}\right)x}{16a^{2}c^{2}-8a\,b^{2}c+b^{4}}+\frac{a^{2}b\left(58a^{2}c^{2}-36\cdots\right.}{32a^{2}c^{2}-16a\cdots}}{\left(c\,x^{2}+bx+a\right)^{2}\cdots}$
risch	$\frac{-\frac{3c^{2}\left(10a^{2}c^{2}-7a\,b^{2}c+b^{4}\right)x^{4}}{a^{3}\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{3bc\left(46a^{2}c^{2}-29a\,b^{2}c+4b^{4}\right)x^{3}}{2\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)a^{3}}-\frac{\left(50c^{3}a^{3}+7a^{2}b^{2}c^{2}-18a\,b^{4}c+3b^{6}\right)x^{2}}{a^{3}\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{b\left(122a^{2}c^{2}-68a\,b^{2}c+9b^{4}\right)x}{2a^{2}\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{1}{a}}{x\left(c\,x^{2}+bx+a\right)^{2}}-\frac{3b\ln\cdots}{a^{4}}$

input
```
int(1/(c+a/x^2+b/x)^3/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/a^3/x-3*b*ln(x)/a^4-1/a^4*((a*c^2*(14*a^2*c^2-13*a*b^2*c+2*b^4)/(16*a^2
*c^2-8*a*b^2*c+b^4)*x^3+1/2*a*b*c*(74*a^2*c^2-55*a*b^2*c+8*b^4)/(16*a^2*c^
2-8*a*b^2*c+b^4)*x^2+a*(18*a^3*c^3+7*a^2*b^2*c^2-12*a*b^4*c+2*b^6)/(16*a^2
*c^2-8*a*b^2*c+b^4)*x+1/2*a^2*b*(58*a^2*c^2-36*a*b^2*c+5*b^4)/(16*a^2*c^2-
8*a*b^2*c+b^4))/(c*x^2+b*x+a)^2+3/(16*a^2*c^2-8*a*b^2*c+b^4)*(1/2*(-16*a^2
*b*c^3+8*a*b^3*c^2-b^5*c)/c*ln(c*x^2+b*x+a)+2*(10*c^3*a^3-23*a^2*b^2*c^2+9
*a*b^4*c-b^6-1/2*(-16*a^2*b*c^3+8*a*b^3*c^2-b^5*c)*b/c)/(4*a*c-b^2)^(1/2)*
arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))))
```

3.439.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1130 vs. 2(229) = 458.

Time = 0.67 (sec) , antiderivative size = 2280, normalized size of antiderivative = 9.54

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^8}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c+a/x^2+b/x)^3/x^8,x, algorithm="fricas")
```

output
```
[-1/2*(2*a^3*b^6 - 24*a^4*b^4*c + 96*a^5*b^2*c^2 - 128*a^6*c^3 + 6*(a*b^6*
c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*x^4 + 3*(4*a*b^7*c - 4
5*a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*x^3 + 2*(3*a*b^8 - 30*a^2
*b^6*c + 79*a^3*b^4*c^2 + 22*a^4*b^2*c^3 - 200*a^5*c^4)*x^2 + 3*((b^6*c^2
- 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*x^5 + 2*(b^7*c - 10*a*b^5*c^
2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*x^4 + (b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2
 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*x^3 + 2*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3
*c^2 - 20*a^4*b*c^3)*x^2 + (a^2*b^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a
^5*c^3)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt
(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + (9*a^2*b^7 - 104*a^3*b^5*c
 + 394*a^4*b^3*c^2 - 488*a^5*b*c^3)*x - 3*((b^7*c^2 - 12*a*b^5*c^3 + 48*a^
2*b^3*c^4 - 64*a^3*b*c^5)*x^5 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 -
 64*a^3*b^2*c^4)*x^4 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3
 - 128*a^4*b*c^4)*x^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*
b^2*c^3)*x^2 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*x)
*log(c*x^2 + b*x + a) + 6*((b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a
^3*b*c^5)*x^5 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)
*x^4 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4
)*x^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*x^2 + (
a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*x)*log(x))/((a^...
```

3.439.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^8}\,dx = \text{Timed out}$$

input
```
integrate(1/(c+a/x**2+b/x)**3/x**8,x)
```

output
```
Timed out
```

3.439.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^8}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(c+a/x^2+b/x)^3/x^8,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.439.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.29

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^8}\,dx = \frac{3\,(b^6-10\,ab^4c+30\,a^2b^2c^2-20\,a^3c^3)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(a^4b^4-8\,a^5b^2c+16\,a^6c^2)\sqrt{-b^2+4\,ac}} + \frac{3\,b\log\left(cx^2+bx+a\right)}{2\,a^4} - \frac{3\,b\log\left(|x|\right)}{a^4} - \frac{2\,a^3b^4-16\,a^4b^2c+32\,a^5c^2+6\,(ab^4c^2-7\,a^2b^2c^3+10\,a^3c^4)x^4+3\,(4\,ab^5c-29\,a^2b^3c^2+46\,a^3bc^3)x^3+2\,(}{2\,(cx^2+bx+a)^2(b^2-4\,ac)^2a}$$

input
```
integrate(1/(c+a/x^2+b/x)^3/x^8,x, algorithm="giac")
```

output
```
3*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*arctan((2*c*x + b)/sqrt
(-b^2 + 4*a*c))/((a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*sqrt(-b^2 + 4*a*c))
+ 3/2*b*log(c*x^2 + b*x + a)/a^4 - 3*b*log(abs(x))/a^4 - 1/2*(2*a^3*b^4 -
16*a^4*b^2*c + 32*a^5*c^2 + 6*(a*b^4*c^2 - 7*a^2*b^2*c^3 + 10*a^3*c^4)*x^4
 + 3*(4*a*b^5*c - 29*a^2*b^3*c^2 + 46*a^3*b*c^3)*x^3 + 2*(3*a*b^6 - 18*a^2
*b^4*c + 7*a^3*b^2*c^2 + 50*a^4*c^3)*x^2 + (9*a^2*b^5 - 68*a^3*b^3*c + 122
*a^4*b*c^2)*x)/((c*x^2 + b*x + a)^2*(b^2 - 4*a*c)^2*a^4*x)
```

3.439.9 Mupad [B] (verification not implemented)

Time = 9.08 (sec) , antiderivative size = 1255, normalized size of antiderivative = 5.25

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^8}\,dx =$$

$$-\frac{\frac{1}{a}+\frac{x^2\,(50\,a^3\,c^3+7\,a^2\,b^2\,c^2-18\,a\,b^4\,c+3\,b^6)}{a^3\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}+\frac{x\,(122\,a^2\,b\,c^2-68\,a\,b^3\,c+9\,b^5)}{2\,a^2\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}+\frac{3\,x^3\,(46\,a^2\,b\,c^3-29\,a\,b^3\,c^2+4\,b^5\,c)}{2\,a^3\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}+\frac{3\,c^2\,x^4\,(10\,a^2\,c^2-7}{a^3\,(16\,a^2\,c^2-8\,a}}{x^3\,(b^2+2\,a\,c)+a^2\,x+c^2\,x^5+2\,a\,b\,x^2+2\,b\,c\,x^4}$$

$$-\frac{3\,b\,\ln(x)}{a^4}$$

$$-\frac{3\,\ln\left(2\,a\,b^{11}+2\,b^{12}\,x+2\,a\,b^6\,\sqrt{-(4\,a\,c-b^2)^5}-39\,a^2\,b^9\,c-1696\,a^6\,b\,c^5+320\,a^6\,c^6\,x+2\,b^7\,x\,\sqrt{-(4\,a\,c\right.}{}$$

$$-\frac{3\,\ln\left(2\,a\,b^{11}+2\,b^{12}\,x-2\,a\,b^6\,\sqrt{-(4\,a\,c-b^2)^5}-39\,a^2\,b^9\,c-1696\,a^6\,b\,c^5+320\,a^6\,c^6\,x-2\,b^7\,x\,\sqrt{-(4\,a\,c\right.}{}$$

input
```
int(1/(x^8*(c + a/x^2 + b/x)^3),x)
```

output
```
- (1/a + (x^2*(3*b^6 + 50*a^3*c^3 + 7*a^2*b^2*c^2 - 18*a*b^4*c))/(a^3*(b^4
 + 16*a^2*c^2 - 8*a*b^2*c)) + (x*(9*b^5 + 122*a^2*b*c^2 - 68*a*b^3*c))/(2*
a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*x^3*(4*b^5*c - 29*a*b^3*c^2 + 46*
a^2*b*c^3))/(2*a^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*c^2*x^4*(b^4 + 10*
a^2*c^2 - 7*a*b^2*c))/(a^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^3*(2*a*c +
b^2) + a^2*x + c^2*x^5 + 2*a*b*x^2 + 2*b*c*x^4) - (3*b*log(x))/a^4 - (3*lo
g(2*a*b^11 + 2*b^12*x + 2*a*b^6*(-(4*a*c - b^2)^5)^(1/2) - 39*a^2*b^9*c -
1696*a^6*b*c^5 + 320*a^6*c^6*x + 2*b^7*x*(-(4*a*c - b^2)^5)^(1/2) + 303*a^
3*b^7*c^2 - 1170*a^4*b^5*c^3 + 2240*a^5*b^3*c^4 - 10*a^4*c^3*(-(4*a*c - b^
2)^5)^(1/2) - 17*a^2*b^4*c*(-(4*a*c - b^2)^5)^(1/2) + 321*a^2*b^8*c^2*x -
1296*a^3*b^6*c^3*x + 2660*a^4*b^4*c^4*x - 2336*a^5*b^2*c^5*x - 40*a*b^10*c
*x + 39*a^3*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 20*a*b^5*c*x*(-(4*a*c - b^2
)^5)^(1/2) - 58*a^3*b*c^3*x*(-(4*a*c - b^2)^5)^(1/2) + 63*a^2*b^3*c^2*x*(-
(4*a*c - b^2)^5)^(1/2))*(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 1024*a^5*b*
c^5 + 160*a^2*b^7*c^2 - 640*a^3*b^5*c^3 + 1280*a^4*b^3*c^4 - 20*a^3*c^3*(-
(4*a*c - b^2)^5)^(1/2) - 20*a*b^9*c + 30*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1
/2) - 10*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2)))/(2*a^4*(4*a*c - b^2)^5) - (3*l
og(2*a*b^11 + 2*b^12*x - 2*a*b^6*(-(4*a*c - b^2)^5)^(1/2) - 39*a^2*b^9*c -
 1696*a^6*b*c^5 + 320*a^6*c^6*x - 2*b^7*x*(-(4*a*c - b^2)^5)^(1/2) + 303*a
^3*b^7*c^2 - 1170*a^4*b^5*c^3 + 2240*a^5*b^3*c^4 + 10*a^4*c^3*(-(4*a*c ...
```

3.440 $\int \frac{x^2}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx$

3.440.1 Optimal result

Integrand size = 18, antiderivative size = 40

$$\int \frac{x^2}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx=\frac{139x}{3375}-\frac{13x^2}{450}+\frac{x^3}{45}-\frac{16}{567}\log(2+3x)+\frac{\log(1+5x)}{4375}$$

output
```
139/3375*x-13/450*x^2+1/45*x^3-16/567*ln(2+3*x)+1/4375*ln(1+5*x)
```

3.440.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx=\frac{139x}{3375}-\frac{13x^2}{450}+\frac{x^3}{45}-\frac{16}{567}\log(2+3x)+\frac{\log(1+5x)}{4375}$$

input
```
Integrate[x^2/(15 + 2/x^2 + 13/x),x]
```

output
```
(139*x)/3375 - (13*x^2)/450 + x^3/45 - (16*Log[2 + 3*x])/567 + Log[1 + 5*x
]/4375
```

3.440.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1692, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\frac{2}{x^2}+\frac{13}{x}+15} \, dx$$
$$\Big\downarrow \text{ 1692}$$
$$\int \frac{x^4}{15x^2+13x+2}dx$$
$$\Big\downarrow \text{ 1141}$$
$$15\int \left(\frac{x^2}{225}-\frac{13x}{3375}-\frac{16}{2835(3x+2)}+\frac{1}{13125(5x+1)}+\frac{139}{50625}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$15\left(\frac{x^3}{675}-\frac{13x^2}{6750}+\frac{139x}{50625}-\frac{16\log(3x+2)}{8505}+\frac{\log(5x+1)}{65625}\right)$$

input
```
Int[x^2/(15 + 2/x^2 + 13/x),x]
```

output
```
15*((139*x)/50625 - (13*x^2)/6750 + x^3/675 - (16*Log[2 + 3*x])/8505 + Log
[1 + 5*x]/65625)
```

3.440.3.1 Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1692
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.440.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.68

method	result	size
parallelrisch	$\frac{x^3}{45}-\frac{13x^2}{450}+\frac{139x}{3375}+\frac{\ln\left(x+\frac{1}{5}\right)}{4375}-\frac{16\ln\left(x+\frac{2}{3}\right)}{567}$	27
default	$\frac{139x}{3375}-\frac{13x^2}{450}+\frac{x^3}{45}-\frac{16\ln(3x+2)}{567}+\frac{\ln(1+5x)}{4375}$	31
norman	$\frac{139x}{3375}-\frac{13x^2}{450}+\frac{x^3}{45}-\frac{16\ln(3x+2)}{567}+\frac{\ln(1+5x)}{4375}$	31
risch	$\frac{139x}{3375}-\frac{13x^2}{450}+\frac{x^3}{45}-\frac{16\ln(3x+2)}{567}+\frac{\ln(1+5x)}{4375}$	31

input
```
int(x^2/(15+2/x^2+13/x),x,method=_RETURNVERBOSE)
```

output
```
1/45*x^3-13/450*x^2+139/3375*x+1/4375*ln(x+1/5)-16/567*ln(x+2/3)
```

3.440.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.75

$$\int \frac{x^2}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx=\frac{1}{45}\,x^3-\frac{13}{450}\,x^2+\frac{139}{3375}\,x+\frac{1}{4375}\,\log\left(5\,x+1\right)-\frac{16}{567}\,\log\left(3\,x+2\right)$$

input
```
integrate(x^2/(15+2/x^2+13/x),x, algorithm="fricas")
```

output
```
1/45*x^3 - 13/450*x^2 + 139/3375*x + 1/4375*log(5*x + 1) - 16/567*log(3*x
+ 2)
```

3.440.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \frac{x^2}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx=\frac{x^3}{45}-\frac{13x^2}{450}+\frac{139x}{3375}+\frac{\log\left(x+\frac{1}{5}\right)}{4375}-\frac{16\log\left(x+\frac{2}{3}\right)}{567}$$

input
```
integrate(x**2/(15+2/x**2+13/x),x)
```

output
```
x**3/45 - 13*x**2/450 + 139*x/3375 + log(x + 1/5)/4375 - 16*log(x + 2/3)/5
67
```

3.440.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.75

$$\int \frac{x^2}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx=\frac{1}{45}\,x^3-\frac{13}{450}\,x^2+\frac{139}{3375}\,x+\frac{1}{4375}\,\log\left(5\,x+1\right)-\frac{16}{567}\,\log\left(3\,x+2\right)$$

input
```
integrate(x^2/(15+2/x^2+13/x),x, algorithm="maxima")
```

output
```
1/45*x^3 - 13/450*x^2 + 139/3375*x + 1/4375*log(5*x + 1) - 16/567*log(3*x
+ 2)
```

3.440.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int \frac{x^2}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx=\frac{1}{45}\,x^3-\frac{13}{450}\,x^2+\frac{139}{3375}\,x+\frac{1}{4375}\,\log\left(|5\,x+1|\right)-\frac{16}{567}\,\log\left(|3\,x+2|\right)$$

input
```
integrate(x^2/(15+2/x^2+13/x),x, algorithm="giac")
```

output
```
1/45*x^3 - 13/450*x^2 + 139/3375*x + 1/4375*log(abs(5*x + 1)) - 16/567*log
(abs(3*x + 2))
```

3.440.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.65

$$\int \frac{x^2}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx = \frac{139\,x}{3375}-\frac{16\,\ln\left(x+\frac{2}{3}\right)}{567}+\frac{\ln\left(x+\frac{1}{5}\right)}{4375}-\frac{13\,x^2}{450}+\frac{x^3}{45}$$

input
```
int(x^2/(13/x + 2/x^2 + 15),x)
```

output
```
(139*x)/3375 - (16*log(x + 2/3))/567 + log(x + 1/5)/4375 - (13*x^2)/450 +
x^3/45
```

3.441 $\int \frac{x}{15+\frac{2}{x^2}+\frac{13}{x}} \, dx$

3.441.1 Optimal result

Integrand size = 16, antiderivative size = 33

$$\int \frac{x}{15+\frac{2}{x^2}+\frac{13}{x}} \, dx = -\frac{13x}{225}+\frac{x^2}{30}+\frac{8}{189}\log(2+3x)-\frac{1}{875}\log(1+5x)$$

output
```
-13/225*x+1/30*x^2+8/189*ln(2+3*x)-1/875*ln(1+5*x)
```

3.441.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{x}{15+\frac{2}{x^2}+\frac{13}{x}} \, dx = -\frac{13x}{225}+\frac{x^2}{30}+\frac{8}{189}\log(2+3x)-\frac{1}{875}\log(1+5x)$$

input
```
Integrate[x/(15 + 2/x^2 + 13/x),x]
```

output
```
(-13*x)/225 + x^2/30 + (8*Log[2 + 3*x])/189 - Log[1 + 5*x]/875
```

3.441.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1692, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\frac{2}{x^2}+\frac{13}{x}+15}dx$$
$$\Big\downarrow \text{1692}$$
$$\int \frac{x^3}{15x^2+13x+2}dx$$
$$\Big\downarrow \text{1141}$$
$$15\int \left(\frac{x}{225}+\frac{8}{945(3x+2)}-\frac{1}{2625(5x+1)}-\frac{13}{3375}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$15\left(\frac{x^2}{450}-\frac{13x}{3375}+\frac{8\log(3x+2)}{2835}-\frac{\log(5x+1)}{13125}\right)$$

input
```
Int[x/(15 + 2/x^2 + 13/x),x]
```

output
```
15*((-13*x)/3375 + x^2/450 + (8*Log[2 + 3*x])/2835 - Log[1 + 5*x]/13125)
```

3.441.3.1 Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1692
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.441.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.67

method	result	size
parallelrisch	$\frac{x^{2}}{30}-\frac{13x}{225}-\frac{\ln\left(x+\frac{1}{5}\right)}{875}+\frac{8\ln\left(x+\frac{2}{3}\right)}{189}$	22
default	$-\frac{13x}{225}+\frac{x^{2}}{30}+\frac{8\ln(3x+2)}{189}-\frac{\ln(1+5x)}{875}$	26
norman	$-\frac{13x}{225}+\frac{x^{2}}{30}+\frac{8\ln(3x+2)}{189}-\frac{\ln(1+5x)}{875}$	26
risch	$-\frac{13x}{225}+\frac{x^{2}}{30}+\frac{8\ln(3x+2)}{189}-\frac{\ln(1+5x)}{875}$	26

input
```
int(x/(15+2/x^2+13/x),x,method=_RETURNVERBOSE)
```

output
```
1/30*x^2-13/225*x-1/875*ln(x+1/5)+8/189*ln(x+2/3)
```

3.441.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{x}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx=\frac{1}{30}\,x^{2}-\frac{13}{225}\,x-\frac{1}{875}\,\log\left(5\,x+1\right)+\frac{8}{189}\,\log\left(3\,x+2\right)$$

input
```
integrate(x/(15+2/x^2+13/x),x, algorithm="fricas")
```

output
```
1/30*x^2 - 13/225*x - 1/875*log(5*x + 1) + 8/189*log(3*x + 2)
```

3.441.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \frac{x}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx=\frac{x^2}{30}-\frac{13x}{225}-\frac{\log\left(x+\frac{1}{5}\right)}{875}+\frac{8\log\left(x+\frac{2}{3}\right)}{189}$$

input
```
integrate(x/(15+2/x**2+13/x),x)
```

output
```
x**2/30 - 13*x/225 - log(x + 1/5)/875 + 8*log(x + 2/3)/189
```

3.441.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{x}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx=\frac{1}{30}\,x^2-\frac{13}{225}\,x-\frac{1}{875}\,\log\left(5\,x+1\right)+\frac{8}{189}\,\log\left(3\,x+2\right)$$

input
```
integrate(x/(15+2/x^2+13/x),x, algorithm="maxima")
```

output
```
1/30*x^2 - 13/225*x - 1/875*log(5*x + 1) + 8/189*log(3*x + 2)
```

3.441.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \frac{x}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx=\frac{1}{30}\,x^2-\frac{13}{225}\,x-\frac{1}{875}\,\log\left(|5\,x+1|\right)+\frac{8}{189}\,\log\left(|3\,x+2|\right)$$

input
```
integrate(x/(15+2/x^2+13/x),x, algorithm="giac")
```

output
```
1/30*x^2 - 13/225*x - 1/875*log(abs(5*x + 1)) + 8/189*log(abs(3*x + 2))
```

3.441.9 Mupad [B] (verification not implemented)

Time = 8.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.64

$$\int \frac{x}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx = \frac{8\,\ln\left(x+\frac{2}{3}\right)}{189}-\frac{13\,x}{225}-\frac{\ln\left(x+\frac{1}{5}\right)}{875}+\frac{x^2}{30}$$

input
```
int(x/(13/x + 2/x^2 + 15),x)
```

output
```
(8*log(x + 2/3))/189 - (13*x)/225 - log(x + 1/5)/875 + x^2/30
```

3.442 $\int \frac{1}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx$

3.442.1 Optimal result

Integrand size = 14, antiderivative size = 26

$$\int \frac{1}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx=\frac{x}{15}-\frac{4}{63}\log(2+3x)+\frac{1}{175}\log(1+5x)$$

output
```
1/15*x-4/63*ln(2+3*x)+1/175*ln(1+5*x)
```

3.442.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx=\frac{x}{15}-\frac{4}{63}\log(2+3x)+\frac{1}{175}\log(1+5x)$$

input
```
Integrate[(15 + 2/x^2 + 13/x)^(-1),x]
```

output
```
x/15 - (4*Log[2 + 3*x])/63 + Log[1 + 5*x]/175
```

3.442.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1679, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\frac{2}{x^2}+\frac{13}{x}+15}dx$$

$$\Big\downarrow \text{ 1679}$$

$$\int \frac{x^2}{15x^2+13x+2}dx$$

$$\Big\downarrow \text{ 1141}$$

$$15\int \left(\frac{1}{525(5x+1)}+\frac{1}{225}-\frac{4}{315(3x+2)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$15\left(\frac{x}{225}-\frac{4}{945}\log(3x+2)+\frac{\log(5x+1)}{2625}\right)$$

input
```
Int[(15 + 2/x^2 + 13/x)^(-1),x]
```

output
```
15*(x/225 - (4*Log[2 + 3*x])/945 + Log[1 + 5*x]/2625)
```

3.442.3.1 Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1679
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(
2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n]
 && LtQ[n, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.442.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

method	result	size
parallelrisch	$\frac{x}{15}+\frac{\ln\left(x+\frac{1}{5}\right)}{175}-\frac{4\ln\left(x+\frac{2}{3}\right)}{63}$	17
default	$\frac{x}{15}-\frac{4\ln(3x+2)}{63}+\frac{\ln(1+5x)}{175}$	21
norman	$\frac{x}{15}-\frac{4\ln(3x+2)}{63}+\frac{\ln(1+5x)}{175}$	21
risch	$\frac{x}{15}-\frac{4\ln(3x+2)}{63}+\frac{\ln(1+5x)}{175}$	21

input
```
int(1/(15+2/x^2+13/x),x,method=_RETURNVERBOSE)
```

output
```
1/15*x+1/175*ln(x+1/5)-4/63*ln(x+2/3)
```

3.442.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{1}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx=\frac{1}{15}\,x+\frac{1}{175}\,\log\left(5\,x+1\right)-\frac{4}{63}\,\log\left(3\,x+2\right)$$

input
```
integrate(1/(15+2/x^2+13/x),x, algorithm="fricas")
```

output
```
1/15*x + 1/175*log(5*x + 1) - 4/63*log(3*x + 2)
```

3.442.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{1}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx=\frac{x}{15}+\frac{\log\left(x+\frac{1}{5}\right)}{175}-\frac{4\log\left(x+\frac{2}{3}\right)}{63}$$

input
```
integrate(1/(15+2/x**2+13/x),x)
```

output
```
x/15 + log(x + 1/5)/175 - 4*log(x + 2/3)/63
```

3.442.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{1}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx=\frac{1}{15}\,x+\frac{1}{175}\,\log\left(5\,x+1\right)-\frac{4}{63}\,\log\left(3\,x+2\right)$$

input
```
integrate(1/(15+2/x^2+13/x),x, algorithm="maxima")
```

output
```
1/15*x + 1/175*log(5*x + 1) - 4/63*log(3*x + 2)
```

3.442.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx=\frac{1}{15}\,x+\frac{1}{175}\,\log\left(|5\,x+1|\right)-\frac{4}{63}\,\log\left(|3\,x+2|\right)$$

input
```
integrate(1/(15+2/x^2+13/x),x, algorithm="giac")
```

output
```
1/15*x + 1/175*log(abs(5*x + 1)) - 4/63*log(abs(3*x + 2))
```

3.442.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.62

$$\int \frac{1}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx=\frac{x}{15}-\frac{4\ln\left(x+\frac{2}{3}\right)}{63}+\frac{\ln\left(x+\frac{1}{5}\right)}{175}$$

input
```
int(1/(13/x + 2/x^2 + 15),x)
```

output
```
x/15 - (4*log(x + 2/3))/63 + log(x + 1/5)/175
```

3.443 $\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x}\,dx$

3.443.1 Optimal result

Integrand size = 18, antiderivative size = 21

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x}\,dx=\frac{2}{21}\log(2+3x)-\frac{1}{35}\log(1+5x)$$

output
```
2/21*ln(2+3*x)-1/35*ln(1+5*x)
```

3.443.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x}\,dx=\frac{2}{21}\log(2+3x)-\frac{1}{35}\log(1+5x)$$

input
```
Integrate[1/((15 + 2/x^2 + 13/x)*x),x]
```

output
```
(2*Log[2 + 3*x])/21 - Log[1 + 5*x]/35
```

3.443.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1692, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\frac{2}{x^2}+\frac{13}{x}+15\right)x}dx$$

$$\Big\downarrow \text{1692}$$

$$\int \frac{x}{15x^2+13x+2}dx$$

$$\Big\downarrow \text{1141}$$

$$15\int \left(\frac{2}{105(3x+2)}-\frac{1}{105(5x+1)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$15\left(\frac{2}{315}\log(3x+2)-\frac{1}{525}\log(5x+1)\right)$$

input
```
Int[1/((15 + 2/x^2 + 13/x)*x),x]
```

output
```
15*((2*Log[2 + 3*x])/315 - Log[1 + 5*x]/525)
```

3.443.3.1 Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1692
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.443.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
parallelrisch	$-\frac{\ln\left(x+\frac{1}{5}\right)}{35}+\frac{2\ln\left(x+\frac{2}{3}\right)}{21}$	14
default	$\frac{2\ln(3x+2)}{21}-\frac{\ln(1+5x)}{35}$	18
norman	$\frac{2\ln(3x+2)}{21}-\frac{\ln(1+5x)}{35}$	18
risch	$\frac{2\ln(3x+2)}{21}-\frac{\ln(1+5x)}{35}$	18

input
```
int(1/(15+2/x^2+13/x)/x,x,method=_RETURNVERBOSE)
```

output
```
-1/35*ln(x+1/5)+2/21*ln(x+2/3)
```

3.443.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x}\,dx=-\frac{1}{35}\,\log\left(5\,x+1\right)+\frac{2}{21}\,\log\left(3\,x+2\right)$$

input
```
integrate(1/(15+2/x^2+13/x)/x,x, algorithm="fricas")
```

output
```
-1/35*log(5*x + 1) + 2/21*log(3*x + 2)
```

3.443.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x}\,dx = -\frac{\log\left(x+\frac{1}{5}\right)}{35}+\frac{2\log\left(x+\frac{2}{3}\right)}{21}$$

input
```
integrate(1/(15+2/x**2+13/x)/x,x)
```

output
```
-log(x + 1/5)/35 + 2*log(x + 2/3)/21
```

3.443.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x}\,dx = -\frac{1}{35}\log(5x+1)+\frac{2}{21}\log(3x+2)$$

input
```
integrate(1/(15+2/x^2+13/x)/x,x, algorithm="maxima")
```

output
```
-1/35*log(5*x + 1) + 2/21*log(3*x + 2)
```

3.443.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x}\,dx = -\frac{1}{35}\log(|5x+1|)+\frac{2}{21}\log(|3x+2|)$$

input
```
integrate(1/(15+2/x^2+13/x)/x,x, algorithm="giac")
```

output
```
-1/35*log(abs(5*x + 1)) + 2/21*log(abs(3*x + 2))
```

3.443.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x}\,dx=\frac{2\ln\left(x+\frac{2}{3}\right)}{21}-\frac{\ln\left(x+\frac{1}{5}\right)}{35}$$

input
```
int(1/(x*(13/x + 2/x^2 + 15)),x)
```

output
```
(2*log(x + 2/3))/21 - log(x + 1/5)/35
```

3.444 $\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^2}\,dx$

3.444.1 Optimal result

Integrand size = 18, antiderivative size = 23

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^2}\,dx=\frac{1}{7}\log\left(5+\frac{1}{x}\right)-\frac{1}{7}\log\left(3+\frac{2}{x}\right)$$

output
```
1/7*ln(5+1/x)-1/7*ln(3+2/x)
```

3.444.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^2}\,dx=-\frac{1}{7}\log(2+3x)+\frac{1}{7}\log(1+5x)$$

input
```
Integrate[1/((15 + 2/x^2 + 13/x)*x^2),x]
```

output
```
-1/7*Log[2 + 3*x] + Log[1 + 5*x]/7
```

3.444.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1690, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\frac{2}{x^2}+\frac{13}{x}+15\right)x^2}dx$$

$$\Big\downarrow \text{1690}$$

$$-\int \frac{1}{15+\frac{13}{x}+\frac{2}{x^2}}d\frac{1}{x}$$

$$\Big\downarrow \text{1081}$$

$$-2\int \left(\frac{1}{7\left(3+\frac{2}{x}\right)}-\frac{1}{14\left(5+\frac{1}{x}\right)}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$-2\left(\frac{1}{14}\log\left(\frac{2}{x}+3\right)-\frac{1}{14}\log\left(\frac{1}{x}+5\right)\right)$$

input
```
Int[1/((15 + 2/x^2 + 13/x)*x^2),x]
```

output
```
-2*(-1/14*Log[5 + x^(-1)] + Log[3 + 2/x]/14)
```

3.444.3.1 Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.444.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.61

method	result	size
parallelrisch	$\frac{\ln\left(x+\frac{1}{5}\right)}{7}-\frac{\ln\left(x+\frac{2}{3}\right)}{7}$	14
default	$\frac{\ln(1+5x)}{7}-\frac{\ln(3x+2)}{7}$	18
norman	$\frac{\ln(1+5x)}{7}-\frac{\ln(3x+2)}{7}$	18
risch	$\frac{\ln(1+5x)}{7}-\frac{\ln(3x+2)}{7}$	18

input
```
int(1/(15+2/x^2+13/x)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/7*ln(x+1/5)-1/7*ln(x+2/3)
```

3.444.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^2}\,dx=\frac{1}{7}\,\log\left(5\,x+1\right)-\frac{1}{7}\,\log\left(3\,x+2\right)$$

input
```
integrate(1/(15+2/x^2+13/x)/x^2,x, algorithm="fricas")
```

output
```
1/7*log(5*x + 1) - 1/7*log(3*x + 2)
```

3.444.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^2}\,dx=\frac{\log\left(x+\frac{1}{5}\right)}{7}-\frac{\log\left(x+\frac{2}{3}\right)}{7}$$

input
```
integrate(1/(15+2/x**2+13/x)/x**2,x)
```

output
```
log(x + 1/5)/7 - log(x + 2/3)/7
```

3.444.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^2}\,dx = \frac{1}{7}\,\log\left(5\,x+1\right)-\frac{1}{7}\,\log\left(3\,x+2\right)$$

input
```
integrate(1/(15+2/x^2+13/x)/x^2,x, algorithm="maxima")
```

output
```
1/7*log(5*x + 1) - 1/7*log(3*x + 2)
```

3.444.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^2}\,dx = \frac{1}{7}\,\log\left(|5\,x+1|\right)-\frac{1}{7}\,\log\left(|3\,x+2|\right)$$

input
```
integrate(1/(15+2/x^2+13/x)/x^2,x, algorithm="giac")
```

output
```
1/7*log(abs(5*x + 1)) - 1/7*log(abs(3*x + 2))
```

3.444.9 Mupad [B] (verification not implemented)

Time = 8.49 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.35

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^2}\,dx = -\frac{2\,\mathrm{atanh}\left(\frac{30\,x}{7}+\frac{13}{7}\right)}{7}$$

input
```
int(1/(x^2*(13/x + 2/x^2 + 15)),x)
```

output
```
-(2*atanh((30*x)/7 + 13/7))/7
```

3.445 $\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^3}\,dx$

3.445.1 Optimal result

Integrand size = 18, antiderivative size = 27

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^3}\,dx=\frac{\log(x)}{2}+\frac{3}{14}\log(2+3x)-\frac{5}{7}\log(1+5x)$$

output
```
1/2*ln(x)+3/14*ln(2+3*x)-5/7*ln(1+5*x)
```

3.445.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^3}\,dx=\frac{\log(x)}{2}+\frac{3}{14}\log(2+3x)-\frac{5}{7}\log(1+5x)$$

input
```
Integrate[1/((15 + 2/x^2 + 13/x)*x^3),x]
```

output
```
Log[x]/2 + (3*Log[2 + 3*x])/14 - (5*Log[1 + 5*x])/7
```

3.445.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1692, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\frac{2}{x^2}+\frac{13}{x}+15\right)x^3}dx$$

$$\Big\downarrow \text{1692}$$

$$\int \frac{1}{x\left(15x^2+13x+2\right)}dx$$

$$\Big\downarrow \text{1141}$$

$$15\int \left(\frac{3}{70(3x+2)}-\frac{5}{21(5x+1)}+\frac{1}{30x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$15\left(\frac{\log(x)}{30}+\frac{1}{70}\log(3x+2)-\frac{1}{21}\log(5x+1)\right)$$

input
```
Int[1/((15 + 2/x^2 + 13/x)*x^3),x]
```

output
```
15*(Log[x]/30 + Log[2 + 3*x]/70 - Log[1 + 5*x]/21)
```

3.445.3.1 Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1692
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.445.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

method	result	size
parallelrisch	$\frac{\ln(x)}{2}-\frac{5\ln(x+\frac{1}{5})}{7}+\frac{3\ln(x+\frac{2}{3})}{14}$	18
default	$\frac{\ln(x)}{2}+\frac{3\ln(3x+2)}{14}-\frac{5\ln(1+5x)}{7}$	22
norman	$\frac{\ln(x)}{2}+\frac{3\ln(3x+2)}{14}-\frac{5\ln(1+5x)}{7}$	22
risch	$\frac{\ln(x)}{2}+\frac{3\ln(3x+2)}{14}-\frac{5\ln(1+5x)}{7}$	22

input
```
int(1/(15+2/x^2+13/x)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(x)-5/7*ln(x+1/5)+3/14*ln(x+2/3)
```

3.445.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^3}\,dx=-\frac{5}{7}\log\left(5x+1\right)+\frac{3}{14}\log\left(3x+2\right)+\frac{1}{2}\log\left(x\right)$$

input
```
integrate(1/(15+2/x^2+13/x)/x^3,x, algorithm="fricas")
```

output
```
-5/7*log(5*x + 1) + 3/14*log(3*x + 2) + 1/2*log(x)
```

3.445.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^3}\,dx=\frac{\log(x)}{2}-\frac{5\log\left(x+\frac{1}{5}\right)}{7}+\frac{3\log\left(x+\frac{2}{3}\right)}{14}$$

input
```
integrate(1/(15+2/x**2+13/x)/x**3,x)
```

output
```
log(x)/2 - 5*log(x + 1/5)/7 + 3*log(x + 2/3)/14
```

3.445.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^3}\,dx=-\frac{5}{7}\log(5x+1)+\frac{3}{14}\log(3x+2)+\frac{1}{2}\log(x)$$

input
```
integrate(1/(15+2/x^2+13/x)/x^3,x, algorithm="maxima")
```

output
```
-5/7*log(5*x + 1) + 3/14*log(3*x + 2) + 1/2*log(x)
```

3.445.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^3}\,dx=-\frac{5}{7}\log(|5x+1|)+\frac{3}{14}\log(|3x+2|)+\frac{1}{2}\log(|x|)$$

input
```
integrate(1/(15+2/x^2+13/x)/x^3,x, algorithm="giac")
```

output
```
-5/7*log(abs(5*x + 1)) + 3/14*log(abs(3*x + 2)) + 1/2*log(abs(x))
```

3.445.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^3}\,dx = \frac{3\,\ln\left(x+\frac{2}{3}\right)}{14}-\frac{5\,\ln\left(x+\frac{1}{5}\right)}{7}+\frac{\ln\left(x\right)}{2}$$

input
```
int(1/(x^3*(13/x + 2/x^2 + 15)),x)
```

output
```
(3*log(x + 2/3))/14 - (5*log(x + 1/5))/7 + log(x)/2
```

3.446 $\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^4}\,dx$

3.446.1 Optimal result

Integrand size = 18, antiderivative size = 34

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^4}\,dx=-\frac{1}{2x}-\frac{13\log(x)}{4}-\frac{9}{28}\log(2+3x)+\frac{25}{7}\log(1+5x)$$

output
```
-1/2/x-13/4*ln(x)-9/28*ln(2+3*x)+25/7*ln(1+5*x)
```

3.446.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^4}\,dx=-\frac{1}{2x}-\frac{13\log(x)}{4}-\frac{9}{28}\log(2+3x)+\frac{25}{7}\log(1+5x)$$

input
```
Integrate[1/((15 + 2/x^2 + 13/x)*x^4),x]
```

output
```
-1/2*1/x - (13*Log[x])/4 - (9*Log[2 + 3*x])/28 + (25*Log[1 + 5*x])/7
```

3.446.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1692, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\frac{2}{x^2}+\frac{13}{x}+15\right)x^4}dx$$

$$\Big\downarrow \text{1692}$$

$$\int \frac{1}{x^2\left(15x^2+13x+2\right)}dx$$

$$\Big\downarrow \text{1141}$$

$$15\int \left(-\frac{9}{140(3x+2)}+\frac{25}{21(5x+1)}-\frac{13}{60x}+\frac{1}{30x^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$15\left(-\frac{1}{30x}-\frac{13\log(x)}{60}-\frac{3}{140}\log(3x+2)+\frac{5}{21}\log(5x+1)\right)$$

input
```
Int[1/((15 + 2/x^2 + 13/x)*x^4),x]
```

output
```
15*(-1/30*1/x - (13*Log[x])/60 - (3*Log[2 + 3*x])/140 + (5*Log[1 + 5*x])/2
1)
```

3.446.3.1 Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1692
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.446.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

method	result	size
default	$-\frac{1}{2x}-\frac{13\ln(x)}{4}-\frac{9\ln(3x+2)}{28}+\frac{25\ln(1+5x)}{7}$	27
norman	$-\frac{1}{2x}-\frac{13\ln(x)}{4}-\frac{9\ln(3x+2)}{28}+\frac{25\ln(1+5x)}{7}$	27
risch	$-\frac{1}{2x}-\frac{13\ln(x)}{4}-\frac{9\ln(3x+2)}{28}+\frac{25\ln(1+5x)}{7}$	27
parallelrisch	$-\frac{91\ln(x)x-100\ln\left(x+\frac{1}{5}\right)x+9\ln\left(x+\frac{2}{3}\right)x+14}{28x}$	27

input
```
int(1/(15+2/x^2+13/x)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/2/x-13/4*ln(x)-9/28*ln(3*x+2)+25/7*ln(1+5*x)
```

3.446.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^4}\,dx=\frac{100\,x\log\left(5\,x+1\right)-9\,x\log\left(3\,x+2\right)-91\,x\log\left(x\right)-14}{28\,x}$$

input
```
integrate(1/(15+2/x^2+13/x)/x^4,x, algorithm="fricas")
```

output
```
1/28*(100*x*log(5*x + 1) - 9*x*log(3*x + 2) - 91*x*log(x) - 14)/x
```

3.446.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^4}\,dx = -\frac{13\log{(x)}}{4} + \frac{25\log{\left(x+\frac{1}{5}\right)}}{7} - \frac{9\log{\left(x+\frac{2}{3}\right)}}{28} - \frac{1}{2x}$$

input
```
integrate(1/(15+2/x**2+13/x)/x**4,x)
```

output
```
-13*log(x)/4 + 25*log(x + 1/5)/7 - 9*log(x + 2/3)/28 - 1/(2*x)
```

3.446.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^4}\,dx = -\frac{1}{2\,x} + \frac{25}{7}\,\log\left(5\,x+1\right) - \frac{9}{28}\,\log\left(3\,x+2\right) - \frac{13}{4}\,\log\left(x\right)$$

input
```
integrate(1/(15+2/x^2+13/x)/x^4,x, algorithm="maxima")
```

output
```
-1/2/x + 25/7*log(5*x + 1) - 9/28*log(3*x + 2) - 13/4*log(x)
```

3.446.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^4}\,dx = -\frac{1}{2\,x} + \frac{25}{7}\,\log\left(|5\,x+1|\right) - \frac{9}{28}\,\log\left(|3\,x+2|\right) - \frac{13}{4}\,\log\left(|x|\right)$$

input
```
integrate(1/(15+2/x^2+13/x)/x^4,x, algorithm="giac")
```

output
```
-1/2/x + 25/7*log(abs(5*x + 1)) - 9/28*log(abs(3*x + 2)) - 13/4*log(abs(x)
)
```

3.446.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.65

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^4}\,dx = \frac{25\ln\left(x+\frac{1}{5}\right)}{7}-\frac{9\ln\left(x+\frac{2}{3}\right)}{28}-\frac{13\ln(x)}{4}-\frac{1}{2x}$$

input
```
int(1/(x^4*(13/x + 2/x^2 + 15)),x)
```

output
```
(25*log(x + 1/5))/7 - (9*log(x + 2/3))/28 - (13*log(x))/4 - 1/(2*x)
```

3.447 $\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^5}\,dx$

3.447.1 Optimal result

Integrand size = 18, antiderivative size = 41

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^5}\,dx=-\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\log(x)}{8}+\frac{27}{56}\log(2+3x)-\frac{125}{7}\log(1+5x)$$

output
```
-1/4/x^2+13/4/x+139/8*ln(x)+27/56*ln(2+3*x)-125/7*ln(1+5*x)
```

3.447.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^5}\,dx=-\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\log(x)}{8}+\frac{27}{56}\log(2+3x)-\frac{125}{7}\log(1+5x)$$

input
```
Integrate[1/((15 + 2/x^2 + 13/x)*x^5),x]
```

output
```
-1/4*1/x^2 + 13/(4*x) + (139*Log[x])/8 + (27*Log[2 + 3*x])/56 - (125*Log[1
 + 5*x])/7
```

3.447.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1692, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\frac{2}{x^2}+\frac{13}{x}+15\right)x^5}dx$$

$$\Big\downarrow \text{1692}$$

$$\int \frac{1}{x^3\left(15x^2+13x+2\right)}dx$$

$$\Big\downarrow \text{1141}$$

$$15\int \left(\frac{27}{280(3x+2)}-\frac{125}{21(5x+1)}+\frac{139}{120x}-\frac{13}{60x^2}+\frac{1}{30x^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$15\left(-\frac{1}{60x^2}+\frac{13}{60x}+\frac{139\log(x)}{120}+\frac{9}{280}\log(3x+2)-\frac{25}{21}\log(5x+1)\right)$$

input
```
Int[1/((15 + 2/x^2 + 13/x)*x^5),x]
```

output
```
15*(-1/60*1/x^2 + 13/(60*x) + (139*Log[x])/120 + (9*Log[2 + 3*x])/280 - (2
5*Log[1 + 5*x])/21)
```

3.447.3.1 Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1692
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.447.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

method	result	size
risch	$\frac{\frac{13x}{4}-\frac{1}{4}}{x^2}+\frac{139\ln(x)}{8}+\frac{27\ln(3x+2)}{56}-\frac{125\ln(1+5x)}{7}$	31
default	$-\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\ln(x)}{8}+\frac{27\ln(3x+2)}{56}-\frac{125\ln(1+5x)}{7}$	32
parallelrisch	$\frac{973\ln(x)x^2-1000\ln(x+\frac{1}{5})x^2+27\ln(x+\frac{2}{3})x^2-14+182x}{56x^2}$	36
norman	$\frac{-\frac{1}{4}x^2+\frac{13}{4}x^3}{x^4}+\frac{139\ln(x)}{8}+\frac{27\ln(3x+2)}{56}-\frac{125\ln(1+5x)}{7}$	37

input
```
int(1/(15+2/x^2+13/x)/x^5,x,method=_RETURNVERBOSE)
```

output
```
(13/4*x-1/4)/x^2+139/8*ln(x)+27/56*ln(3*x+2)-125/7*ln(1+5*x)
```

3.447.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^5}\,dx = -\frac{1000\,x^2\log\left(5\,x+1\right)-27\,x^2\log\left(3\,x+2\right)-973\,x^2\log\left(x\right)-182\,x+14}{56\,x^2}$$

input
```
integrate(1/(15+2/x^2+13/x)/x^5,x, algorithm="fricas")
```

output
```
-1/56*(1000*x^2*log(5*x + 1) - 27*x^2*log(3*x + 2) - 973*x^2*log(x) - 182*
x + 14)/x^2
```

3.447.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^5}\,dx = \frac{139\log(x)}{8} - \frac{125\log\left(x+\frac{1}{5}\right)}{7} + \frac{27\log\left(x+\frac{2}{3}\right)}{56} + \frac{13x-1}{4x^2}$$

input
```
integrate(1/(15+2/x**2+13/x)/x**5,x)
```

output
```
139*log(x)/8 - 125*log(x + 1/5)/7 + 27*log(x + 2/3)/56 + (13*x - 1)/(4*x**
2)
```

3.447.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^5}\,dx = \frac{13\,x-1}{4\,x^2} - \frac{125}{7}\log(5\,x+1) + \frac{27}{56}\log(3\,x+2) + \frac{139}{8}\log(x)$$

input
```
integrate(1/(15+2/x^2+13/x)/x^5,x, algorithm="maxima")
```

output
```
1/4*(13*x - 1)/x^2 - 125/7*log(5*x + 1) + 27/56*log(3*x + 2) + 139/8*log(x
)
```

3.447.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^5}\,dx = \frac{13\,x-1}{4\,x^2} - \frac{125}{7}\log(|5\,x+1|) + \frac{27}{56}\log(|3\,x+2|) + \frac{139}{8}\log(|x|)$$

input
```
integrate(1/(15+2/x^2+13/x)/x^5,x, algorithm="giac")
```

output
```
1/4*(13*x - 1)/x^2 - 125/7*log(abs(5*x + 1)) + 27/56*log(abs(3*x + 2)) + 1
39/8*log(abs(x))
```

3.447.9 Mupad [B] (verification not implemented)

Time = 8.55 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.63

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^5}\,dx=\frac{27\ln\left(x+\frac{2}{3}\right)}{56}-\frac{125\ln\left(x+\frac{1}{5}\right)}{7}+\frac{139\ln(x)}{8}+\frac{\frac{13x}{4}-\frac{1}{4}}{x^2}$$

input
```
int(1/(x^5*(13/x + 2/x^2 + 15)),x)
```

output
```
(27*log(x + 2/3))/56 - (125*log(x + 1/5))/7 + (139*log(x))/8 + ((13*x)/4 -
1/4)/x^2
```

3.448 $\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^6}\,dx$

3.448.1 Optimal result

Integrand size = 18, antiderivative size = 48

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^6}\,dx=-\frac{1}{6x^3}+\frac{13}{8x^2}-\frac{139}{8x}-\frac{1417\log(x)}{16}-\frac{81}{112}\log(2+3x)+\frac{625}{7}\log(1+5x)$$

output
```
-1/6/x^3+13/8/x^2-139/8/x-1417/16*ln(x)-81/112*ln(2+3*x)+625/7*ln(1+5*x)
```

3.448.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^6}\,dx=-\frac{1}{6x^3}+\frac{13}{8x^2}-\frac{139}{8x}-\frac{1417\log(x)}{16}-\frac{81}{112}\log(2+3x)+\frac{625}{7}\log(1+5x)$$

input
```
Integrate[1/((15 + 2/x^2 + 13/x)*x^6),x]
```

output
```
-1/6*1/x^3 + 13/(8*x^2) - 139/(8*x) - (1417*Log[x])/16 - (81*Log[2 + 3*x])
/112 + (625*Log[1 + 5*x])/7
```

3.448.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1692, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\frac{2}{x^2}+\frac{13}{x}+15\right)x^6}dx$$

$$\Big\downarrow \text{ 1692}$$

$$\int \frac{1}{x^4\left(15x^2+13x+2\right)}dx$$

$$\Big\downarrow \text{ 1141}$$

$$15\int \left(-\frac{81}{560(3x+2)}+\frac{625}{21(5x+1)}-\frac{1417}{240x}+\frac{139}{120x^2}-\frac{13}{60x^3}+\frac{1}{30x^4}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$15\left(-\frac{1}{90x^3}+\frac{13}{120x^2}-\frac{139}{120x}-\frac{1417\log(x)}{240}-\frac{27}{560}\log(3x+2)+\frac{125}{21}\log(5x+1)\right)$$

input
```
Int[1/((15 + 2/x^2 + 13/x)*x^6),x]
```

output
```
15*(-1/90*1/x^3 + 13/(120*x^2) - 139/(120*x) - (1417*Log[x])/240 - (27*Log
[2 + 3*x])/560 + (125*Log[1 + 5*x])/21)
```

3.448.3.1 Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1692
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.448.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.75

method	result	size
risch	$\frac{-\frac{139}{8}x^2+\frac{13}{8}x-\frac{1}{6}}{x^3}-\frac{1417\ln(x)}{16}-\frac{81\ln(3x+2)}{112}+\frac{625\ln(1+5x)}{7}$	36
default	$-\frac{1}{6x^3}+\frac{13}{8x^2}-\frac{139}{8x}-\frac{1417\ln(x)}{16}-\frac{81\ln(3x+2)}{112}+\frac{625\ln(1+5x)}{7}$	37
parallelrisch	$-\frac{29757\ln(x)x^3-30000\ln(x+\frac{1}{5})x^3+243\ln(x+\frac{2}{3})x^3+56+5838x^2-546x}{336x^3}$	41
norman	$\frac{-\frac{1}{6}x^2+\frac{13}{8}x^3-\frac{139}{8}x^4}{x^5}-\frac{1417\ln(x)}{16}-\frac{81\ln(3x+2)}{112}+\frac{625\ln(1+5x)}{7}$	42

input
```
int(1/(15+2/x^2+13/x)/x^6,x,method=_RETURNVERBOSE)
```

output
```
(-139/8*x^2+13/8*x-1/6)/x^3-1417/16*ln(x)-81/112*ln(3*x+2)+625/7*ln(1+5*x)
```

3.448.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.92

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^6}\,dx$$
$$=\frac{30000\,x^3\log\left(5\,x+1\right)-243\,x^3\log\left(3\,x+2\right)-29757\,x^3\log\left(x\right)-5838\,x^2+546\,x-56}{336\,x^3}$$

input
```
integrate(1/(15+2/x^2+13/x)/x^6,x, algorithm="fricas")
```

output
```
1/336*(30000*x^3*log(5*x + 1) - 243*x^3*log(3*x + 2) - 29757*x^3*log(x) -
5838*x^2 + 546*x - 56)/x^3
```

3.448.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.85

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^6}\,dx = -\frac{1417\log{(x)}}{16}+\frac{625\log{\left(x+\frac{1}{5}\right)}}{7} - \frac{81\log{\left(x+\frac{2}{3}\right)}}{112}+\frac{-417x^2+39x-4}{24x^3}$$

input
```
integrate(1/(15+2/x**2+13/x)/x**6,x)
```

output
```
-1417*log(x)/16 + 625*log(x + 1/5)/7 - 81*log(x + 2/3)/112 + (-417*x**2 +
39*x - 4)/(24*x**3)
```

3.448.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.75

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^6}\,dx = -\frac{417\,x^2-39\,x+4}{24\,x^3}+\frac{625}{7}\log\left(5\,x+1\right) - \frac{81}{112}\log\left(3\,x+2\right)-\frac{1417}{16}\log\left(x\right)$$

input
```
integrate(1/(15+2/x^2+13/x)/x^6,x, algorithm="maxima")
```

output
```
-1/24*(417*x^2 - 39*x + 4)/x^3 + 625/7*log(5*x + 1) - 81/112*log(3*x + 2)
- 1417/16*log(x)
```

3.448.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.81

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^6}\,dx = -\frac{417\,x^2-39\,x+4}{24\,x^3}+\frac{625}{7}\log\left(|5\,x+1|\right) - \frac{81}{112}\log\left(|3\,x+2|\right)-\frac{1417}{16}\log\left(|x|\right)$$

input
```
integrate(1/(15+2/x^2+13/x)/x^6,x, algorithm="giac")
```

output
```
-1/24*(417*x^2 - 39*x + 4)/x^3 + 625/7*log(abs(5*x + 1)) - 81/112*log(abs(
3*x + 2)) - 1417/16*log(abs(x))
```

3.448.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.67

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^6}\,dx = \frac{625\,\ln\left(x+\frac{1}{5}\right)}{7}-\frac{81\,\ln\left(x+\frac{2}{3}\right)}{112}-\frac{1417\,\ln\left(x\right)}{16}-\frac{\frac{139\,x^2}{8}-\frac{13\,x}{8}+\frac{1}{6}}{x^3}$$

input
```
int(1/(x^6*(13/x + 2/x^2 + 15)),x)
```

output
```
(625*log(x + 1/5))/7 - (81*log(x + 2/3))/112 - (1417*log(x))/16 - ((139*x^
2)/8 - (13*x)/8 + 1/6)/x^3
```

3.449 $\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2} dx$

3.449.1 Optimal result

Integrand size = 16, antiderivative size = 204

$$\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2} dx = -\frac{5}{24}\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}\left(7b+\frac{6c}{x}\right) - \frac{5\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\left(b(b^2+44ac)+\frac{2c(b^2+12ac)}{x}\right)}{64c} + \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2} x + \frac{5}{2}a^{3/2}b\operatorname{arctanh}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right) + \frac{5(b^4-24ab^2c-48a^2c^2)\operatorname{arctanh}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{128c^{3/2}}$$

output
```
-5/24*(a+c/x^2+b/x)^(3/2)*(7*b+6*c/x)+(a+c/x^2+b/x)^(5/2)*x+5/2*a^(3/2)*b*
arctanh(1/2*(2*a+b/x)/a^(1/2)/(a+c/x^2+b/x)^(1/2))+5/128*(-48*a^2*c^2-24*a
*b^2*c+b^4)*arctanh(1/2*(b+2*c/x)/c^(1/2)/(a+c/x^2+b/x)^(1/2))/c^(3/2)-5/6
4*(b*(44*a*c+b^2)+2*c*(12*a*c+b^2)/x)*(a+c/x^2+b/x)^(1/2)/c
```

3.449.2 Mathematica [A] (verified)

Time = 1.37 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.02

$$\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2} dx =$$
$$-\frac{\sqrt{a+\frac{c+bx}{x^2}}\left(15(b^4-24ab^2c-48a^2c^2)\,x^4\text{arctanh}\left(\frac{\sqrt{a}x-\sqrt{c+x(b+ax)}}{\sqrt{c}}\right)+\sqrt{c}\left(\sqrt{c+x(b+ax)}(48c^3+15b^3x^3\right.\right.}{192c^{3/2}x^3\sqrt{}}$$

input
```
Integrate[(a + c/x^2 + b/x)^(5/2),x]
```

output
```
-1/192*(Sqrt[a + (c + b*x)/x^2]*(15*(b^4 - 24*a*b^2*c - 48*a^2*c^2)*x^4*Ar
cTanh[(Sqrt[a]*x - Sqrt[c + x*(b + a*x)])/Sqrt[c]] + Sqrt[c]*(Sqrt[c + x*(
b + a*x)]*(48*c^3 + 15*b^3*x^3 + 8*c^2*x*(17*b + 27*a*x) + 2*c*x^2*(59*b^2
 + 278*a*b*x - 96*a^2*x^2)) + 480*a^(3/2)*b*c*x^4*Log[b + 2*a*x - 2*Sqrt[a
]*Sqrt[c + x*(b + a*x)]])))/(c^(3/2)*x^3*Sqrt[c + x*(b + a*x)])
```

3.449.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.07, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {1681, 1161, 1231, 25, 27, 1231, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{5/2} dx$$
$$\Big\downarrow \text{1681}$$
$$-\int \left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{5/2} x^2 d\frac{1}{x}$$
$$\Big\downarrow \text{1161}$$
$$x\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{5/2}-\frac{5}{2}\int \left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}\left(b+\frac{2c}{x}\right) x d\frac{1}{x}$$
$$\Big\downarrow \text{1231}$$

$$x\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{5/2}-$$
$$\frac{5}{2}\left(\frac{1}{12}\left(7b+\frac{6c}{x}\right)\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}-\frac{\int -c\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(8ab+\frac{b^2+12ac}{x}\right)xd\frac{1}{x}}{8c}\right)$$

$\downarrow$ 25

$$x\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{5/2}-$$
$$\frac{5}{2}\left(\frac{\int c\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(8ab+\frac{b^2+12ac}{x}\right)xd\frac{1}{x}}{8c}+\frac{1}{12}\left(7b+\frac{6c}{x}\right)\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}\right)$$

$\downarrow$ 27

$$x\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{5/2}-$$
$$\frac{5}{2}\left(\frac{1}{8}\int\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(8ab+\frac{b^2+12ac}{x}\right)xd\frac{1}{x}+\frac{1}{12}\left(7b+\frac{6c}{x}\right)\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}\right)$$

$\downarrow$ 1231

$$x\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{5/2}-$$
$$\frac{5}{2}\left(\frac{1}{8}\left(\frac{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{2c(12ac+b^2)}{x}+b(44ac+b^2)\right)}{4c}-\frac{\int -\frac{\left(64a^2bc-\frac{b^4-24acb^2-48a^2c^2}{x}\right)x}{2\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}d\frac{1}{x}}{4c}\right)+\frac{1}{12}\left(7b+\frac{6c}{x}\right)\left(a+\frac{b}{x}+\right.\right.$$

$\downarrow$ 27

$$x\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{5/2}-$$
$$\frac{5}{2}\left(\frac{1}{8}\left(\frac{\int\frac{\left(64a^2bc-\frac{b^4-24acb^2-48a^2c^2}{x}\right)x}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}d\frac{1}{x}}{8c}+\frac{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{2c(12ac+b^2)}{x}+b(44ac+b^2)\right)}{4c}\right)+\frac{1}{12}\left(7b+\frac{6c}{x}\right)\left(a+\frac{b}{x}+\right.\right.$$

$\downarrow$ 1269

$$x\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{5/2}-$$
$$\frac{5}{2}\left(\frac{1}{8}\left(\frac{64a^2bc\int\frac{x}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}d\frac{1}{x}-(-48a^2c^2-24ab^2c+b^4)\int\frac{1}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}d\frac{1}{x}}{8c}+\frac{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{2c(12ac+b^2)}{x}+b(44ac+\right.}{4c}\right.\right.$$

3.449. $\int\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2}dx$

$$\Big\downarrow \text{1092}$$

$$x\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{5/2}-$$

$$\frac{5}{2}\left(\frac{1}{8}\left(\frac{64a^2bc\int \frac{x}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}d\frac{1}{x}-2\left(-48a^2c^2-24ab^2c+b^4\right)\int \frac{1}{4c-\frac{1}{x^2}}d\frac{b+\frac{2c}{x}}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}}{8c}+\frac{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{2c(12ac+b^2)}{x}+b(44\right.}{4c}\right.\right.$$

$$\Big\downarrow \text{219}$$

$$x\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{5/2}-$$

$$\frac{5}{2}\left(\frac{1}{8}\left(\frac{64a^2bc\int \frac{x}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}d\frac{1}{x}-\frac{(-48a^2c^2-24ab^2c+b^4)\text{arctanh}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{\sqrt{c}}}{8c}+\frac{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{2c(12ac+b^2)}{x}+b(44ac+\right.}{4c}\right.\right.$$

$$\Big\downarrow \text{1154}$$

$$x\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{5/2}-$$

$$\frac{5}{2}\left(\frac{1}{8}\left(\frac{-128a^2bc\int \frac{1}{4a-\frac{1}{x^2}}d\frac{2a+\frac{b}{x}}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}-\frac{(-48a^2c^2-24ab^2c+b^4)\text{arctanh}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{\sqrt{c}}}{8c}+\frac{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{2c(12ac+b^2)}{x}+b\right.}{4c}\right.\right.$$

$$\Big\downarrow \text{219}$$

$$x\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{5/2}-$$

$$\frac{5}{2}\left(\frac{1}{8}\left(\frac{-64a^{3/2}bc\,\text{arctanh}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)-\frac{(-48a^2c^2-24ab^2c+b^4)\text{arctanh}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{\sqrt{c}}}{8c}+\frac{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{2c(12ac+\right.}{x}}{4c}\right.\right.$$

input
```
Int[(a + c/x^2 + b/x)^(5/2),x]
```

3.449. $\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2}\,dx$

output
```
(a + c/x^2 + b/x)^(5/2)*x - (5*(((a + c/x^2 + b/x)^(3/2)*(7*b + (6*c)/x))/
12 + ((Sqrt[a + c/x^2 + b/x]*(b*(b^2 + 44*a*c) + (2*c*(b^2 + 12*a*c))/x))/
(4*c) + (-64*a^(3/2)*b*c*ArcTanh[(2*a + b/x)/(2*Sqrt[a]*Sqrt[a + c/x^2 + b
/x])] - ((b^4 - 24*a*b^2*c - 48*a^2*c^2)*ArcTanh[(b + (2*c)/x)/(2*Sqrt[c]*
Sqrt[a + c/x^2 + b/x])])/Sqrt[c])/(8*c))/8))/2
```

3.449.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1161
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Si
mp[p/(e*(m + 1))   Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||
 LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b,
 c, d, e, m, p, x]
```

3.449. $\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2} dx$

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1681
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[
Int[(a + b/x^n + c/x^(2*n))^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, p}, x] &&
 EqQ[n2, 2*n] && ILtQ[n, 0]
```

3.449.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.22

method	result
risch	$-\frac{\left(556 a b c\,x^{3}+15 b^{3} x^{3}+216 a\,c^{2} x^{2}+118 b^{2} c\,x^{2}+136 b\,c^{2} x+48 c^{3}\right) \sqrt{\frac{a\,x^{2}+b x+c}{x^{2}}}}{192 x^{3} c}+\frac{\left(384 c b\,a^{\frac{3}{2}} \ln\left(\frac{\frac{b}{2}+a x}{\sqrt{a}}+\sqrt{a\,x^{2}+b x+c}\right)+128 a^{3} c \left(\sqrt{\ldots\right.\right.}{\ldots}$
default	$\frac{\left(\frac{a\,x^{2}+b x+c}{x^{2}}\right)^{\frac{5}{2}} x \left(6 a^{\frac{3}{2}} \left(a\,x^{2}+b x+c\right)^{\frac{7}{2}} b^{3} x^{3}-6 a^{\frac{3}{2}} \left(a\,x^{2}+b x+c\right)^{\frac{5}{2}} b^{4} x^{4}+600 a^{\frac{7}{2}} \sqrt{a\,x^{2}+b x+c}\, b\,c^{3} x^{5}-30 a^{\frac{5}{2}} \sqrt{a\,x^{2}+b x+c}\, b^{3} c^{2} x^{5}+960 \ln\ldots\right.}{\ldots}$

input
```
int((a+c/x^2+b/x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/192*(556*a*b*c*x^3+15*b^3*x^3+216*a*c^2*x^2+118*b^2*c*x^2+136*b*c^2*x+4
8*c^3)/x^3/c*((a*x^2+b*x+c)/x^2)^(1/2)+1/128/c*(384*c*b*a^(3/2)*ln((1/2*b+
a*x)/a^(1/2)+(a*x^2+b*x+c)^(1/2))+128*a^3*c*(1/a*(a*x^2+b*x+c)^(1/2)-1/2*b
/a^(3/2)*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*x+c)^(1/2)))-(240*a^2*c^2+120*a*b
^2*c-5*b^4)/c^(1/2)*ln((2*c+b*x+2*c^(1/2)*(a*x^2+b*x+c)^(1/2))/x))*((a*x^2
+b*x+c)/x^2)^(1/2)*x/(a*x^2+b*x+c)^(1/2)
```

3.449.5 Fricas [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 959, normalized size of antiderivative = 4.70

$$\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2} dx = \left[\frac{960\, a^{\frac{3}{2}} b c^2 x^3 \log\left(-8\, a^2 x^2 - 8\, abx - b^2 - 4\, ac - 4\, (2\, ax^2 + bx)\sqrt{a}\sqrt{\frac{ax^2+bx+c}{x^2}}\right) - 15\, (b^4 - 24 \cdots}{\cdots}\right.$$

$$-\frac{1920\,\sqrt{-a} abc^2 x^3 \arctan\left(\frac{(2\,ax^2+bx)\sqrt{-a}\sqrt{\frac{ax^2+bx+c}{x^2}}}{2\,(a^2x^2+abx+ac)}\right) + 15\,(b^4 - 24\,ab^2c - 48\,a^2c^2)\sqrt{c}x^3 \log\left(-\frac{8\,bcx+(b^2+4\,ac)x^2 \cdots}{\cdots}\right.}{\cdots}$$

$$-\frac{960\,\sqrt{-a} abc^2 x^3 \arctan\left(\frac{(2\,ax^2+bx)\sqrt{-a}\sqrt{\frac{ax^2+bx+c}{x^2}}}{2\,(a^2x^2+abx+ac)}\right) + 15\,(b^4 - 24\,ab^2c - 48\,a^2c^2)\sqrt{-c}x^3 \arctan\left(\frac{(bx^2+2\,cx)\sqrt{-c \cdots}}{2\,(acx^2+b \cdots}\right.}{384 \cdots}$$

input
```
integrate((a+c/x^2+b/x)^(5/2),x, algorithm="fricas")
```

output
```
[1/768*(960*a^(3/2)*b*c^2*x^3*log(-8*a^2*x^2 - 8*a*b*x - b^2 - 4*a*c - 4*(
2*a*x^2 + b*x)*sqrt(a)*sqrt((a*x^2 + b*x + c)/x^2)) - 15*(b^4 - 24*a*b^2*c
 - 48*a^2*c^2)*sqrt(c)*x^3*log(-(8*b*c*x + (b^2 + 4*a*c)*x^2 + 8*c^2 - 4*(
b*x^2 + 2*c*x)*sqrt(c)*sqrt((a*x^2 + b*x + c)/x^2))/x^2) + 4*(192*a^2*c^2*
x^4 - 136*b*c^3*x - 48*c^4 - (15*b^3*c + 556*a*b*c^2)*x^3 - 2*(59*b^2*c^2
+ 108*a*c^3)*x^2)*sqrt((a*x^2 + b*x + c)/x^2))/(c^2*x^3), -1/768*(1920*sqr
t(-a)*a*b*c^2*x^3*arctan(1/2*(2*a*x^2 + b*x)*sqrt(-a)*sqrt((a*x^2 + b*x +
c)/x^2)/(a^2*x^2 + a*b*x + a*c)) + 15*(b^4 - 24*a*b^2*c - 48*a^2*c^2)*sqrt
(c)*x^3*log(-(8*b*c*x + (b^2 + 4*a*c)*x^2 + 8*c^2 - 4*(b*x^2 + 2*c*x)*sqrt
(c)*sqrt((a*x^2 + b*x + c)/x^2))/x^2) - 4*(192*a^2*c^2*x^4 - 136*b*c^3*x -
 48*c^4 - (15*b^3*c + 556*a*b*c^2)*x^3 - 2*(59*b^2*c^2 + 108*a*c^3)*x^2)*s
qrt((a*x^2 + b*x + c)/x^2))/(c^2*x^3), 1/384*(480*a^(3/2)*b*c^2*x^3*log(-8
*a^2*x^2 - 8*a*b*x - b^2 - 4*a*c - 4*(2*a*x^2 + b*x)*sqrt(a)*sqrt((a*x^2 +
 b*x + c)/x^2)) - 15*(b^4 - 24*a*b^2*c - 48*a^2*c^2)*sqrt(-c)*x^3*arctan(1
/2*(b*x^2 + 2*c*x)*sqrt(-c)*sqrt((a*x^2 + b*x + c)/x^2)/(a*c*x^2 + b*c*x +
 c^2)) + 2*(192*a^2*c^2*x^4 - 136*b*c^3*x - 48*c^4 - (15*b^3*c + 556*a*b*c
^2)*x^3 - 2*(59*b^2*c^2 + 108*a*c^3)*x^2)*sqrt((a*x^2 + b*x + c)/x^2))/(c^
2*x^3), -1/384*(960*sqrt(-a)*a*b*c^2*x^3*arctan(1/2*(2*a*x^2 + b*x)*sqrt(-
a)*sqrt((a*x^2 + b*x + c)/x^2)/(a^2*x^2 + a*b*x + a*c)) + 15*(b^4 - 24*a*b
^2*c - 48*a^2*c^2)*sqrt(-c)*x^3*arctan(1/2*(b*x^2 + 2*c*x)*sqrt(-c)*sqr...
```

3.449.6 Sympy [F]

$$\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2} dx = \int \left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{\frac{5}{2}} dx$$

input
```
integrate((a+c/x**2+b/x)**(5/2),x)
```

output
```
Integral((a + b/x + c/x**2)**(5/2), x)
```

3.449.7 Maxima [F]

$$\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2} dx = \int \left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{\frac{5}{2}} dx$$

input
```
integrate((a+c/x^2+b/x)^(5/2),x, algorithm="maxima")
```

output
```
integrate((a + b/x + c/x^2)^(5/2), x)
```

3.449.8 Giac [F(-1)]

Timed out.

$$\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2} dx = \text{Timed out}$$

input
```
integrate((a+c/x^2+b/x)^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

3.449.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2} dx = \int \left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{5/2} dx$$

input
```
int((a + b/x + c/x^2)^(5/2),x)
```

output
```
int((a + b/x + c/x^2)^(5/2), x)
```

3.450 $\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2} dx$

3.450.1 Optimal result

Integrand size = 16, antiderivative size = 145

$$\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2} dx = -\frac{3}{4}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\left(3b+\frac{2c}{x}\right)+\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2} x + \frac{3}{2}\sqrt{a}b\,\text{arctanh}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right) - \frac{3(b^2+4ac)\,\text{arctanh}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{8\sqrt{c}}$$

output
```
(a+c/x^2+b/x)^(3/2)*x+3/2*b*arctanh(1/2*(2*a+b/x)/a^(1/2)/(a+c/x^2+b/x)^(1
/2))*a^(1/2)-3/8*(4*a*c+b^2)*arctanh(1/2*(b+2*c/x)/c^(1/2)/(a+c/x^2+b/x)^(
1/2))/c^(1/2)-3/4*(3*b+2*c/x)*(a+c/x^2+b/x)^(1/2)
```

3.450.2 Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.09

$$\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2} dx = \frac{\sqrt{a+\frac{c+bx}{x^2}}\left(3(b^2+4ac)\,x^2\text{arctanh}\left(\frac{\sqrt{a}x-\sqrt{c+x(b+ax)}}{\sqrt{c}}\right)-\sqrt{c}\Big((2c+x(5b-4ax))\sqrt{c+x(b+ax}\right.}{4\sqrt{c}x\sqrt{c+x(b+ax)}}$$

input
```
Integrate[(a + c/x^2 + b/x)^(3/2),x]
```

output
```
(Sqrt[a + (c + b*x)/x^2]*(3*(b^2 + 4*a*c)*x^2*ArcTanh[(Sqrt[a]*x - Sqrt[c
+ x*(b + a*x)])/Sqrt[c]] - Sqrt[c]*((2*c + x*(5*b - 4*a*x))*Sqrt[c + x*(b
+ a*x)] + 6*Sqrt[a]*b*x^2*Log[b + 2*a*x - 2*Sqrt[a]*Sqrt[c + x*(b + a*x)]]
)))/(4*Sqrt[c]*x*Sqrt[c + x*(b + a*x)])
```

3.450.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {1681, 1161, 1231, 25, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}dx$$

$$\Big\downarrow \text{1681}$$

$$-\int \left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}x^2d\frac{1}{x}$$

$$\Big\downarrow \text{1161}$$

$$x\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}-\frac{3}{2}\int \sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(b+\frac{2c}{x}\right)xd\frac{1}{x}$$

$$\Big\downarrow \text{1231}$$

$$x\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}-\frac{3}{2}\left(\frac{1}{2}\left(3b+\frac{2c}{x}\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}-\frac{\int -\frac{c\left(4ab+\frac{b^2+4ac}{x}\right)x}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}d\frac{1}{x}}{4c}\right)$$

$$\Big\downarrow \text{25}$$

$$x\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}-\frac{3}{2}\left(\frac{\int \frac{c\left(4ab+\frac{b^2+4ac}{x}\right)x}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}d\frac{1}{x}}{4c}+\frac{1}{2}\left(3b+\frac{2c}{x}\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\right)$$

$$\Big\downarrow \text{27}$$

$$x\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}-\frac{3}{2}\left(\frac{1}{4}\int \frac{\left(4ab+\frac{b^2+4ac}{x}\right)x}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}d\frac{1}{x}+\frac{1}{2}\left(3b+\frac{2c}{x}\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\right)$$

$\Big\downarrow$ 1269

$$x\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}-$$
$$\frac{3}{2}\left(\frac{1}{4}\left(\left(4ac+b^2\right)\int \frac{1}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}d\frac{1}{x}+4ab\int \frac{x}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}d\frac{1}{x}\right)+\frac{1}{2}\left(3b+\frac{2c}{x}\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\right)$$

$\Big\downarrow$ 1092

$$x\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}-$$
$$\frac{3}{2}\left(\frac{1}{4}\left(2\left(4ac+b^2\right)\int \frac{1}{4c-\frac{1}{x^2}}d\frac{b+\frac{2c}{x}}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}+4ab\int \frac{x}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}d\frac{1}{x}\right)+\frac{1}{2}\left(3b+\frac{2c}{x}\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\right)$$

$\Big\downarrow$ 219

$$x\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}-$$
$$\frac{3}{2}\left(\frac{1}{4}\left(4ab\int \frac{x}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}d\frac{1}{x}+\frac{\left(4ac+b^2\right)\text{arctanh}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{\sqrt{c}}\right)+\frac{1}{2}\left(3b+\frac{2c}{x}\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\right)$$

$\Big\downarrow$ 1154

$$x\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}-$$
$$\frac{3}{2}\left(\frac{1}{4}\left(\frac{\left(4ac+b^2\right)\text{arctanh}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{\sqrt{c}}-8ab\int \frac{1}{4a-\frac{1}{x^2}}d\frac{2a+\frac{b}{x}}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)+\frac{1}{2}\left(3b+\frac{2c}{x}\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\right)$$

$\Big\downarrow$ 219

$$x\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}-$$
$$\frac{3}{2}\left(\frac{1}{4}\left(\frac{\left(4ac+b^2\right)\text{arctanh}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{\sqrt{c}}-4\sqrt{a}b\,\text{arctanh}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)\right)+\frac{1}{2}\left(3b+\frac{2c}{x}\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\right)$$

3.450. $\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}\,dx$

input
```
Int[(a + c/x^2 + b/x)^(3/2),x]
```

output
```
(a + c/x^2 + b/x)^(3/2)*x - (3*((Sqrt[a + c/x^2 + b/x]*(3*b + (2*c)/x))/2
+ (-4*Sqrt[a]*b*ArcTanh[(2*a + b/x)/(2*Sqrt[a]*Sqrt[a + c/x^2 + b/x])] + (
(b^2 + 4*a*c)*ArcTanh[(b + (2*c)/x)/(2*Sqrt[c]*Sqrt[a + c/x^2 + b/x])])/Sq
rt[c])/4))/2
```

3.450.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1161
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Si
mp[p/(e*(m + 1))   Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||
 LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b,
 c, d, e, m, p, x]
```

3.450. $\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2} dx$

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1681
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[
Int[(a + b/x^n + c/x^(2*n))^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, p}, x] &&
 EqQ[n2, 2*n] && ILtQ[n, 0]
```

3.450.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.23

method	result
risch	$-\frac{(5bx+2c)\sqrt{\frac{a\,x^2+bx+c}{x^2}}}{4x}+\frac{\left(a\sqrt{a\,x^2+bx+c}+\frac{3\sqrt{a}\,b\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx+c}\right)}{2}-\frac{3\sqrt{c}\,\ln\left(\frac{2c+bx+2\sqrt{c}\,\sqrt{a\,x^2+bx+c}}{x}\right)a}{2}-\frac{3\ln\left(\frac{2c+bx+2\sqrt{c}}{\ldots}\right.}{8\ldots}\right.}{\sqrt{a\,x^2+bx+c}}$
default	$-\frac{\left(\frac{a\,x^2+bx+c}{x^2}\right)^{\frac{3}{2}}x\left(12a^{\frac{5}{2}}c^{\frac{5}{2}}\ln\left(\frac{2c+bx+2\sqrt{c}\,\sqrt{a\,x^2+bx+c}}{x}\right)x^2-2a^{\frac{5}{2}}\left(a\,x^2+bx+c\right)^{\frac{3}{2}}b\,x^3-4a^{\frac{5}{2}}\left(a\,x^2+bx+c\right)^{\frac{3}{2}}c\,x^2-6a^{\frac{5}{2}}\sqrt{a\,x^2+bx+c}\,b\ldots\right.}{}$

input
```
int((a+c/x^2+b/x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(5*b*x+2*c)/x*((a*x^2+b*x+c)/x^2)^(1/2)+(a*(a*x^2+b*x+c)^(1/2)+3/2*a^
(1/2)*b*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*x+c)^(1/2))-3/2*c^(1/2)*ln((2*c+b*
x+2*c^(1/2)*(a*x^2+b*x+c)^(1/2))/x)*a-3/8/c^(1/2)*ln((2*c+b*x+2*c^(1/2)*(a
*x^2+b*x+c)^(1/2))/x)*b^2)*((a*x^2+b*x+c)/x^2)^(1/2)*x/(a*x^2+b*x+c)^(1/2)
```

3.450.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 709, normalized size of antiderivative = 4.89

$$\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2} dx = \left[\frac{12\sqrt{a}bcx\log\left(-8a^2x^2-8abx-b^2-4ac-4(2ax^2+bx)\sqrt{a}\sqrt{\frac{ax^2+bx+c}{x^2}}\right)+3(b^2+4ac)\sqrt{}}{16c}\right.$$

$$-\frac{24\sqrt{-a}bcx\arctan\left(\frac{(2ax^2+bx)\sqrt{-a}\sqrt{\frac{ax^2+bx+c}{x^2}}}{2(a^2x^2+abx+ac)}\right)-3(b^2+4ac)\sqrt{c}x\log\left(-\frac{8bcx+(b^2+4ac)x^2+8c^2-4(bx^2+2cx)\sqrt{c}\sqrt{}}{x^2}\right.}{16cx}$$

$$-\frac{12\sqrt{-a}bcx\arctan\left(\frac{(2ax^2+bx)\sqrt{-a}\sqrt{\frac{ax^2+bx+c}{x^2}}}{2(a^2x^2+abx+ac)}\right)-3(b^2+4ac)\sqrt{-c}x\arctan\left(\frac{(bx^2+2cx)\sqrt{-c}\sqrt{\frac{ax^2+bx+c}{x^2}}}{2(acx^2+bcx+c^2)}\right)-2(4a}{8cx}$$

input
```
integrate((a+c/x^2+b/x)^(3/2),x, algorithm="fricas")
```

output
```
[1/16*(12*sqrt(a)*b*c*x*log(-8*a^2*x^2 - 8*a*b*x - b^2 - 4*a*c - 4*(2*a*x^
2 + b*x)*sqrt(a)*sqrt((a*x^2 + b*x + c)/x^2)) + 3*(b^2 + 4*a*c)*sqrt(c)*x*
log(-(8*b*c*x + (b^2 + 4*a*c)*x^2 + 8*c^2 - 4*(b*x^2 + 2*c*x)*sqrt(c)*sqrt
((a*x^2 + b*x + c)/x^2))/x^2) + 4*(4*a*c*x^2 - 5*b*c*x - 2*c^2)*sqrt((a*x^
2 + b*x + c)/x^2))/(c*x), -1/16*(24*sqrt(-a)*b*c*x*arctan(1/2*(2*a*x^2 + b
*x)*sqrt(-a)*sqrt((a*x^2 + b*x + c)/x^2)/(a^2*x^2 + a*b*x + a*c)) - 3*(b^2
 + 4*a*c)*sqrt(c)*x*log(-(8*b*c*x + (b^2 + 4*a*c)*x^2 + 8*c^2 - 4*(b*x^2 +
 2*c*x)*sqrt(c)*sqrt((a*x^2 + b*x + c)/x^2))/x^2) - 4*(4*a*c*x^2 - 5*b*c*x
 - 2*c^2)*sqrt((a*x^2 + b*x + c)/x^2))/(c*x), 1/8*(6*sqrt(a)*b*c*x*log(-8*
a^2*x^2 - 8*a*b*x - b^2 - 4*a*c - 4*(2*a*x^2 + b*x)*sqrt(a)*sqrt((a*x^2 +
b*x + c)/x^2)) + 3*(b^2 + 4*a*c)*sqrt(-c)*x*arctan(1/2*(b*x^2 + 2*c*x)*sqr
t(-c)*sqrt((a*x^2 + b*x + c)/x^2)/(a*c*x^2 + b*c*x + c^2)) + 2*(4*a*c*x^2
- 5*b*c*x - 2*c^2)*sqrt((a*x^2 + b*x + c)/x^2))/(c*x), -1/8*(12*sqrt(-a)*b
*c*x*arctan(1/2*(2*a*x^2 + b*x)*sqrt(-a)*sqrt((a*x^2 + b*x + c)/x^2)/(a^2*
x^2 + a*b*x + a*c)) - 3*(b^2 + 4*a*c)*sqrt(-c)*x*arctan(1/2*(b*x^2 + 2*c*x
)*sqrt(-c)*sqrt((a*x^2 + b*x + c)/x^2)/(a*c*x^2 + b*c*x + c^2)) - 2*(4*a*c
*x^2 - 5*b*c*x - 2*c^2)*sqrt((a*x^2 + b*x + c)/x^2))/(c*x)]
```

3.450.6 Sympy [F]

$$\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2} dx = \int \left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{\frac{3}{2}} dx$$

input
```
integrate((a+c/x**2+b/x)**(3/2),x)
```

output
```
Integral((a + b/x + c/x**2)**(3/2), x)
```

3.450.7 Maxima [F]

$$\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2} dx = \int \left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{\frac{3}{2}} dx$$

input
```
integrate((a+c/x^2+b/x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a + b/x + c/x^2)^(3/2), x)
```

3.450.8 Giac [F(-1)]

Timed out.

$$\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2} dx = \text{Timed out}$$

input
```
integrate((a+c/x^2+b/x)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.450.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2} dx = \int \left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2} dx$$

input
```
int((a + b/x + c/x^2)^(3/2),x)
```

output
```
int((a + b/x + c/x^2)^(3/2), x)
```

3.451 $\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,dx$

3.451.1 Optimal result

Integrand size = 16, antiderivative size = 105

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,dx=\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x+\frac{b\,\text{arctanh}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{2\sqrt{a}}-\sqrt{c}\,\text{arctanh}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)$$

output
```
1/2*b*arctanh(1/2*(2*a+b/x)/a^(1/2)/(a+c/x^2+b/x)^(1/2))/a^(1/2)-arctanh(1
/2*(b+2*c/x)/c^(1/2)/(a+c/x^2+b/x)^(1/2))*c^(1/2)+x*(a+c/x^2+b/x)^(1/2)
```

3.451.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.25

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,dx =\frac{x\sqrt{a+\frac{c+bx}{x^2}}\left(2\sqrt{a}\sqrt{c+x(b+ax)}+4\sqrt{a}\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{a}x-\sqrt{c+x(b+ax)}}{\sqrt{c}}\right)-b\log\left(b+2ax-2\sqrt{a}\sqrt{c+x(b\right.\right.}{2\sqrt{a}\sqrt{c+x(b+ax)}}$$

input
```
Integrate[Sqrt[a + c/x^2 + b/x],x]
```

output
```
(x*Sqrt[a + (c + b*x)/x^2]*(2*Sqrt[a]*Sqrt[c + x*(b + a*x)] + 4*Sqrt[a]*Sq
rt[c]*ArcTanh[(Sqrt[a]*x - Sqrt[c + x*(b + a*x)])/Sqrt[c]] - b*Log[b + 2*a
*x - 2*Sqrt[a]*Sqrt[c + x*(b + a*x)]]))/(2*Sqrt[a]*Sqrt[c + x*(b + a*x)])
```

3.451.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1681, 1161, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+\frac{b}{x}+\frac{c}{x^2}}dx$$

$$\Big\downarrow \text{1681}$$

$$-\int \sqrt{a+\frac{b}{x}+\frac{c}{x^2}}x^2d\frac{1}{x}$$

$$\Big\downarrow \text{1161}$$

$$x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}-\frac{1}{2}\int \frac{\left(b+\frac{2c}{x}\right)x}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}d\frac{1}{x}$$

$$\Big\downarrow \text{1269}$$

$$\frac{1}{2}\left(-2c\int \frac{1}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}d\frac{1}{x}-b\int \frac{x}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}d\frac{1}{x}\right)+x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{2}\left(-4c\int \frac{1}{4c-\frac{1}{x^2}}d\frac{b+\frac{2c}{x}}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}-b\int \frac{x}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}d\frac{1}{x}\right)+x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(-b\int \frac{x}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}d\frac{1}{x}-2\sqrt{c}\text{arctanh}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)\right)+x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{2}\left(2b\int \frac{1}{4a-\frac{1}{x^2}}d\frac{2a+\frac{b}{x}}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}-2\sqrt{c}\text{arctanh}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)\right)+x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{b\text{arctanh}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{\sqrt{a}}-2\sqrt{c}\text{arctanh}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)\right)+x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}$$

input
```
Int[Sqrt[a + c/x^2 + b/x],x]
```

output
```
Sqrt[a + c/x^2 + b/x]*x + ((b*ArcTanh[(2*a + b/x)/(2*Sqrt[a]*Sqrt[a + c/x^
2 + b/x])])/Sqrt[a] - 2*Sqrt[c]*ArcTanh[(b + (2*c)/x)/(2*Sqrt[c]*Sqrt[a +
c/x^2 + b/x])])/2
```

3.451.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

3.451. $\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,dx$

rule 1161
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Si
mp[p/(e*(m + 1))   Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||
 LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b,
 c, d, e, m, p, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1681
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[
Int[(a + b/x^n + c/x^(2*n))^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, p}, x] &&
 EqQ[n2, 2*n] && ILtQ[n, 0]
```

3.451.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.15

method	result	size
default	$\frac{\sqrt{\frac{a\,x^{2}+b x +c}{x^{2}}}\, x \left(-2 \sqrt{c}\, \ln \left(\frac{2 c +b x +2 \sqrt{c}\, \sqrt{a\,x^{2}+b x +c}}{x}\right) \sqrt{a}+b \ln \left(\frac{2 \sqrt{a\,x^{2}+b x +c}\, \sqrt{a}+2 a x +b}{2 \sqrt{a}}\right)+2 \sqrt{a\,x^{2}+b x +c}\, \sqrt{a}\right)}{2 \sqrt{a\,x^{2}+b x +c}\, \sqrt{a}}$	121

input
```
int((a+c/x^2+b/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*((a*x^2+b*x+c)/x^2)^(1/2)*x*(-2*c^(1/2)*ln((2*c+b*x+2*c^(1/2)*(a*x^2+b
*x+c)^(1/2))/x)*a^(1/2)+b*ln(1/2*(2*(a*x^2+b*x+c)^(1/2)*a^(1/2)+2*a*x+b)/a
^(1/2))+2*(a*x^2+b*x+c)^(1/2)*a^(1/2))/(a*x^2+b*x+c)^(1/2)/a^(1/2)
```

3.451.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 590, normalized size of antiderivative = 5.62

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,dx$$
$$=\left[\frac{4\,ax\sqrt{\frac{ax^2+bx+c}{x^2}}+\sqrt{a}b\log\left(-8\,a^2x^2-8\,abx-b^2-4\,ac-4\,(2\,ax^2+bx)\sqrt{a}\sqrt{\frac{ax^2+bx+c}{x^2}}\right)+2\,a\sqrt{c}\log\left(-\right.}{4\,a}\right.$$

input
```
integrate((a+c/x^2+b/x)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(4*a*x*sqrt((a*x^2 + b*x + c)/x^2) + sqrt(a)*b*log(-8*a^2*x^2 - 8*a*b
*x - b^2 - 4*a*c - 4*(2*a*x^2 + b*x)*sqrt(a)*sqrt((a*x^2 + b*x + c)/x^2))
+ 2*a*sqrt(c)*log(-(8*b*c*x + (b^2 + 4*a*c)*x^2 + 8*c^2 - 4*(b*x^2 + 2*c*x
)*sqrt(c)*sqrt((a*x^2 + b*x + c)/x^2))/x^2))/a, 1/2*(2*a*x*sqrt((a*x^2 + b
*x + c)/x^2) - sqrt(-a)*b*arctan(1/2*(2*a*x^2 + b*x)*sqrt(-a)*sqrt((a*x^2
+ b*x + c)/x^2)/(a^2*x^2 + a*b*x + a*c)) + a*sqrt(c)*log(-(8*b*c*x + (b^2
+ 4*a*c)*x^2 + 8*c^2 - 4*(b*x^2 + 2*c*x)*sqrt(c)*sqrt((a*x^2 + b*x + c)/x^
2))/x^2))/a, 1/4*(4*a*x*sqrt((a*x^2 + b*x + c)/x^2) + 4*a*sqrt(-c)*arctan(
1/2*(b*x^2 + 2*c*x)*sqrt(-c)*sqrt((a*x^2 + b*x + c)/x^2)/(a*c*x^2 + b*c*x
+ c^2)) + sqrt(a)*b*log(-8*a^2*x^2 - 8*a*b*x - b^2 - 4*a*c - 4*(2*a*x^2 +
b*x)*sqrt(a)*sqrt((a*x^2 + b*x + c)/x^2)))/a, 1/2*(2*a*x*sqrt((a*x^2 + b*x
 + c)/x^2) - sqrt(-a)*b*arctan(1/2*(2*a*x^2 + b*x)*sqrt(-a)*sqrt((a*x^2 +
b*x + c)/x^2)/(a^2*x^2 + a*b*x + a*c)) + 2*a*sqrt(-c)*arctan(1/2*(b*x^2 +
2*c*x)*sqrt(-c)*sqrt((a*x^2 + b*x + c)/x^2)/(a*c*x^2 + b*c*x + c^2)))/a]
```

3.451.6 Sympy [F]

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,dx=\int \sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,dx$$

input
```
integrate((a+c/x**2+b/x)**(1/2),x)
```

output
```
Integral(sqrt(a + b/x + c/x**2), x)
```

3.451.7 Maxima [F]

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,dx=\int \sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,dx$$

input
```
integrate((a+c/x^2+b/x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a + b/x + c/x^2), x)
```

3.451.8 Giac [F(-2)]

Exception generated.

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+c/x^2+b/x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.451.9 Mupad [B] (verification not implemented)

Time = 8.52 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.95

$$\begin{aligned}\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,dx&=x\sqrt{\frac{1}{x^2}}\sqrt{a\,x^2+b\,x+c}\\&\quad-\sqrt{c}\,x\,\ln\left(\frac{2\,c+2\,\sqrt{c}\,\sqrt{a\,x^2+b\,x+c}+b\,x}{x}\right)\sqrt{\frac{1}{x^2}}\\&\quad+\frac{b\,x\,\ln\left(\frac{\frac{b}{2}+\sqrt{a}\,\sqrt{a\,x^2+b\,x+c}+a\,x}{\sqrt{a}}\right)\sqrt{\frac{1}{x^2}}}{2\,\sqrt{a}}\end{aligned}$$

input
```
int((a + b/x + c/x^2)^(1/2),x)
```

output
```
x*(1/x^2)^(1/2)*(c + b*x + a*x^2)^(1/2) - c^(1/2)*x*log((2*c + 2*c^(1/2)*(
c + b*x + a*x^2)^(1/2) + b*x)/x)*(1/x^2)^(1/2) + (b*x*log((b/2 + a^(1/2)*(
c + b*x + a*x^2)^(1/2) + a*x)/a^(1/2))*(1/x^2)^(1/2))/(2*a^(1/2))
```

3.452 $\int \frac{1}{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\,dx$

3.452.1 Optimal result

Integrand size = 16, antiderivative size = 67

$$\int \frac{1}{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\,dx=\frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x}{a}-\frac{b\,\text{arctanh}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{2a^{3/2}}$$

output
```
-1/2*b*arctanh(1/2*(2*a+b/x)/a^(1/2)/(a+c/x^2+b/x)^(1/2))/a^(3/2)+x*(a+c/x
^2+b/x)^(1/2)/a
```

3.452.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.33

$$\int \frac{1}{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\,dx=\frac{2\sqrt{a}(c+x(b+ax))-b\sqrt{c+x(b+ax)}\,\text{arctanh}\left(\frac{b+2ax}{2\sqrt{a}\sqrt{c+x(b+ax)}}\right)}{2a^{3/2}x\sqrt{a+\frac{c+bx}{x^2}}}$$

input
```
Integrate[1/Sqrt[a + c/x^2 + b/x],x]
```

output
```
(2*Sqrt[a]*(c + x*(b + a*x)) - b*Sqrt[c + x*(b + a*x)]*ArcTanh[(b + 2*a*x)
/(2*Sqrt[a]*Sqrt[c + x*(b + a*x)])])/(2*a^(3/2)*x*Sqrt[a + (c + b*x)/x^2])
```

3.452.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1681, 1157, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\,dx$$

$$\Big\downarrow \ 1681$$

$$-\int \frac{x^2}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}d\frac{1}{x}$$

$$\Big\downarrow \ 1157$$

$$\frac{b\int \frac{x}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}d\frac{1}{x}}{2a}+\frac{x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{a}$$

$$\Big\downarrow \ 1154$$

$$\frac{x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{a}-\frac{b\int \frac{1}{4a-\frac{1}{x^2}}d\frac{2a+\frac{b}{x}}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}}{a}$$

$$\Big\downarrow \ 219$$

$$\frac{x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{a}-\frac{b\operatorname{arctanh}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{2a^{3/2}}$$

input
```
Int[1/Sqrt[a + c/x^2 + b/x],x]
```

output
```
(Sqrt[a + c/x^2 + b/x]*x)/a - (b*ArcTanh[(2*a + b/x)/(2*Sqrt[a]*Sqrt[a + c
/x^2 + b/x])])/(2*a^(3/2))
```

3.452.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1157
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Simp[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2))
  Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && EqQ[m + 2*p + 3, 0]
```

rule 1681
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[
Int[(a + b/x^n + c/x^(2*n))^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, p}, x] &&
 EqQ[n2, 2*n] && ILtQ[n, 0]
```

3.452.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.31

method	result	size
default	$\frac{\sqrt{a\,x^{2}+bx+c}\,\left(2\sqrt{a\,x^{2}+bx+c}\,a^{\frac{3}{2}}-b\ln\left(\frac{2\sqrt{a\,x^{2}+bx+c}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a\right)}{2\sqrt{\frac{a\,x^{2}+bx+c}{x^{2}}}\,x\,a^{\frac{5}{2}}}$	88
risch	$\frac{a\,x^{2}+bx+c}{a\sqrt{\frac{a\,x^{2}+bx+c}{x^{2}}}\,x}-\frac{b\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^{2}+bx+c}\right)\sqrt{a\,x^{2}+bx+c}}{2a^{\frac{3}{2}}\sqrt{\frac{a\,x^{2}+bx+c}{x^{2}}}\,x}$	97

input
```
int(1/(a+c/x^2+b/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(a*x^2+b*x+c)^(1/2)*(2*(a*x^2+b*x+c)^(1/2)*a^(3/2)-b*ln(1/2*(2*(a*x^2+
b*x+c)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a)/((a*x^2+b*x+c)/x^2)^(1/2)/x/a^(5
/2)
```

3.452.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.55

$$\int \frac{1}{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\,dx$$
$$= \left[\frac{4\,ax\sqrt{\frac{ax^2+bx+c}{x^2}}+\sqrt{a}b\log\left(-8\,a^2x^2-8\,abx-b^2-4\,ac+4\,(2\,ax^2+bx)\sqrt{a}\sqrt{\frac{ax^2+bx+c}{x^2}}\right)}{4\,a^2}, \frac{2\,ax\sqrt{\frac{ax^2+bx+c}{x^2}}}{}\right.$$

input
```
integrate(1/(a+c/x^2+b/x)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(4*a*x*sqrt((a*x^2 + b*x + c)/x^2) + sqrt(a)*b*log(-8*a^2*x^2 - 8*a*b
*x - b^2 - 4*a*c + 4*(2*a*x^2 + b*x)*sqrt(a)*sqrt((a*x^2 + b*x + c)/x^2)))
/a^2, 1/2*(2*a*x*sqrt((a*x^2 + b*x + c)/x^2) + sqrt(-a)*b*arctan(1/2*(2*a*
x^2 + b*x)*sqrt(-a)*sqrt((a*x^2 + b*x + c)/x^2)/(a^2*x^2 + a*b*x + a*c)))/
a^2]
```

3.452.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\,dx$$

input
```
integrate(1/(a+c/x**2+b/x)**(1/2),x)
```

output
```
Integral(1/sqrt(a + b/x + c/x**2), x)
```

3.452.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\,dx$$

input
```
integrate(1/(a+c/x^2+b/x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(a + b/x + c/x^2), x)
```

3.452.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+c/x^2+b/x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.452.9 Mupad [B] (verification not implemented)

Time = 8.58 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.79

$$\int \frac{1}{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\,dx = \frac{x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{a} - \frac{b\,\mathrm{atanh}\left(\frac{a+\frac{b}{2x}}{\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{2\,a^{3/2}}$$

input
```
int(1/(a + b/x + c/x^2)^(1/2),x)
```

output
```
(x*(a + b/x + c/x^2)^(1/2))/a - (b*atanh((a + b/(2*x))/(a^(1/2)*(a + b/x +
 c/x^2)^(1/2))))/(2*a^(3/2))
```

3.453 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}}\,dx$

3.453.1 Optimal result

Integrand size = 16, antiderivative size = 133

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}}\,dx=\frac{(3b^2-8ac)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x}{a^2(b^2-4ac)}-\frac{2\left(b^2-2ac+\frac{bc}{x}\right)x}{a(b^2-4ac)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}-\frac{3b\operatorname{arctanh}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{2a^{5/2}}$$

output
```
-3/2*b*arctanh(1/2*(2*a+b/x)/a^(1/2)/(a+c/x^2+b/x)^(1/2))/a^(5/2)-2*(b^2-2
*a*c+b*c/x)*x/a/(-4*a*c+b^2)/(a+c/x^2+b/x)^(1/2)+(-8*a*c+3*b^2)*x*(a+c/x^2
+b/x)^(1/2)/a^2/(-4*a*c+b^2)
```

3.453.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.05

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}}\,dx=-\frac{2\sqrt{a}(-3b^3x+10abcx+4ac(2c+ax^2)-b^2(3c+ax^2))-3b(b^2-4ac)\sqrt{c+x(b+ax)}\log\left(a^2\left(b+2ax-\right.\right.}{2a^{5/2}(b^2-4ac)x\sqrt{a+\frac{c+bx}{x^2}}}$$

input
```
Integrate[(a + c/x^2 + b/x)^(-3/2),x]
```

output
```
-1/2*(2*Sqrt[a]*(-3*b^3*x + 10*a*b*c*x + 4*a*c*(2*c + a*x^2) - b^2*(3*c +
a*x^2)) - 3*b*(b^2 - 4*a*c)*Sqrt[c + x*(b + a*x)]*Log[a^2*(b + 2*a*x - 2*S
qrt[a]*Sqrt[c + x*(b + a*x)]])])/(a^(5/2)*(b^2 - 4*a*c)*x*Sqrt[a + (c + b*x
)/x^2])
```

3.453.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1681, 1165, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{1681}$$

$$-\int \frac{x^2}{\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{1165}$$

$$\frac{2\int -\frac{\left(3b^2+\frac{2cb}{x}-8ac\right)x^2}{2\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}d\frac{1}{x}}{a\left(b^2-4ac\right)}-\frac{2x\left(-2ac+b^2+\frac{bc}{x}\right)}{a\left(b^2-4ac\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\left(3b^2+\frac{2cb}{x}-8ac\right)x^2}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}d\frac{1}{x}}{a\left(b^2-4ac\right)}-\frac{2x\left(-2ac+b^2+\frac{bc}{x}\right)}{a\left(b^2-4ac\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}$$

$$\Big\downarrow \text{1228}$$

$$-\frac{-\frac{3b\left(b^2-4ac\right)\int \frac{x}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}d\frac{1}{x}}{2a}-\frac{x\left(3b^2-8ac\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{a}}{a\left(b^2-4ac\right)}-\frac{2x\left(-2ac+b^2+\frac{bc}{x}\right)}{a\left(b^2-4ac\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}$$

$$\Big\downarrow \text{1154}$$

$$-\frac{\frac{3b(b^2-4ac)\int \frac{1}{4a-\frac{1}{x^2}}d\frac{2a+\frac{b}{x}}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}}{a}-\frac{x(3b^2-8ac)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{a}}{a(b^2-4ac)}-\frac{2x\left(-2ac+b^2+\frac{bc}{x}\right)}{a(b^2-4ac)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}$$

$$\Big\downarrow \; 219$$

$$-\frac{\frac{3b(b^2-4ac)\text{arctanh}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{2a^{3/2}}-\frac{x(3b^2-8ac)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{a}}{a(b^2-4ac)}-\frac{2x\left(-2ac+b^2+\frac{bc}{x}\right)}{a(b^2-4ac)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}$$

input
```
Int[(a + c/x^2 + b/x)^(-3/2),x]
```

output
```
(-2*(b^2 - 2*a*c + (b*c)/x)*x)/(a*(b^2 - 4*a*c)*Sqrt[a + c/x^2 + b/x]) - (
-(((3*b^2 - 8*a*c)*Sqrt[a + c/x^2 + b/x]*x)/a) + (3*b*(b^2 - 4*a*c)*ArcTan
h[(2*a + b/x)/(2*Sqrt[a]*Sqrt[a + c/x^2 + b/x])])/(2*a^(3/2)))/(a*(b^2 - 4
*a*c))
```

3.453.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

3.453. $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}}\,dx$

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1681
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[
Int[(a + b/x^n + c/x^(2*n))^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, p}, x] &&
 EqQ[n2, 2*n] && ILtQ[n, 0]
```

3.453.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.48

method	result
default	$\frac{\left(a\,x^{2}+bx+c\right)\left(8a^{\frac{7}{2}}c\,x^{2}-2a^{\frac{5}{2}}b^{2}x^{2}+20a^{\frac{5}{2}}bcx-6a^{\frac{3}{2}}b^{3}x+16a^{\frac{5}{2}}c^{2}-6a^{\frac{3}{2}}b^{2}c-12\ln\left(\frac{2\sqrt{a\,x^{2}+bx+c}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)\sqrt{a\,x^{2}+bx+c}\,a^{2}bc+3\ln\left(\cdots\right.\right.}{2a^{\frac{7}{2}}\left(\frac{a\,x^{2}+bx+c}{x^{2}}\right)^{\frac{3}{2}}x^{3}\left(4ac-b^{2}\right)}$
risch	$\frac{a\,x^{2}+bx+c}{a^{2}\sqrt{\frac{a\,x^{2}+bx+c}{x^{2}}}\,x}+\frac{\left(\frac{3bx}{2a^{2}\sqrt{a\,x^{2}+bx+c}}-\frac{b^{2}}{4a^{3}\sqrt{a\,x^{2}+bx+c}}-\frac{b^{3}x}{2a^{2}\left(4ac-b^{2}\right)\sqrt{a\,x^{2}+bx+c}}-\frac{b^{4}}{4a^{3}\left(4ac-b^{2}\right)\sqrt{a\,x^{2}+bx+c}}-\frac{3b\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^{2}\cdots}\right.}{2a^{\frac{5}{2}}}\cdots\right.}{\sqrt{\frac{a\,x^{2}+bx+c}{x^{2}}}\,x}$

input
```
int(1/(a+c/x^2+b/x)^(3/2),x,method=_RETURNVERBOSE)
```

3.453. $\int \frac{1}{\left(a+\frac{c}{x^{2}}+\frac{b}{x}\right)^{3/2}}\,dx$

output
```
1/2*(a*x^2+b*x+c)/a^(7/2)*(8*a^(7/2)*c*x^2-2*a^(5/2)*b^2*x^2+20*a^(5/2)*b*
c*x-6*a^(3/2)*b^3*x+16*a^(5/2)*c^2-6*a^(3/2)*b^2*c-12*ln(1/2*(2*(a*x^2+b*x
+c)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*(a*x^2+b*x+c)^(1/2)*a^2*b*c+3*ln(1/2*(
2*(a*x^2+b*x+c)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*(a*x^2+b*x+c)^(1/2)*a*b^3)
/((a*x^2+b*x+c)/x^2)^(3/2)/x^3/(4*a*c-b^2)
```

3.453.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 465, normalized size of antiderivative = 3.50

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}}\,dx = \left[\frac{3\,(b^3c-4\,abc^2+(ab^3-4\,a^2bc)x^2+(b^4-4\,ab^2c)x)\sqrt{a}\log\left(-8\,a^2x^2-8\,abx-b^2\right.}{4\,(a^3b^2c-4\right.}$$

input
```
integrate(1/(a+c/x^2+b/x)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(3*(b^3*c - 4*a*b*c^2 + (a*b^3 - 4*a^2*b*c)*x^2 + (b^4 - 4*a*b^2*c)*x
)*sqrt(a)*log(-8*a^2*x^2 - 8*a*b*x - b^2 - 4*a*c + 4*(2*a*x^2 + b*x)*sqrt(
a)*sqrt((a*x^2 + b*x + c)/x^2)) + 4*((a^2*b^2 - 4*a^3*c)*x^3 + (3*a*b^3 -
10*a^2*b*c)*x^2 + (3*a*b^2*c - 8*a^2*c^2)*x)*sqrt((a*x^2 + b*x + c)/x^2))/
(a^3*b^2*c - 4*a^4*c^2 + (a^4*b^2 - 4*a^5*c)*x^2 + (a^3*b^3 - 4*a^4*b*c)*x
), 1/2*(3*(b^3*c - 4*a*b*c^2 + (a*b^3 - 4*a^2*b*c)*x^2 + (b^4 - 4*a*b^2*c)
*x)*sqrt(-a)*arctan(1/2*(2*a*x^2 + b*x)*sqrt(-a)*sqrt((a*x^2 + b*x + c)/x^
2)/(a^2*x^2 + a*b*x + a*c)) + 2*((a^2*b^2 - 4*a^3*c)*x^3 + (3*a*b^3 - 10*a
^2*b*c)*x^2 + (3*a*b^2*c - 8*a^2*c^2)*x)*sqrt((a*x^2 + b*x + c)/x^2))/(a^3
*b^2*c - 4*a^4*c^2 + (a^4*b^2 - 4*a^5*c)*x^2 + (a^3*b^3 - 4*a^4*b*c)*x)]
```

3.453.6 Sympy [F]

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+c/x**2+b/x)**(3/2),x)
```

output
```
Integral((a + b/x + c/x**2)**(-3/2), x)
```

3.453.7 Maxima [F]

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}}\,dx=\int \frac{1}{\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+c/x^2+b/x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a + b/x + c/x^2)^(-3/2), x)
```

3.453.8 Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.74

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}}\,dx= -\frac{\left(3\,b^3\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)-12\,abc\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)+6\,\sqrt{a}b^2\sqrt{c}-16\,a^{\frac{3}{2}}c^{\frac{3}{2}}\right)\mathrm{sgn}(x)}{2\left(a^{\frac{5}{2}}b^2-4\,a^{\frac{7}{2}}c\right)} + \frac{\left(\frac{(ab^2-4\,a^2c)x}{a^2b^2\mathrm{sgn}(x)-4\,a^3c\mathrm{sgn}(x)}+\frac{3\,b^3-10\,abc}{a^2b^2\mathrm{sgn}(x)-4\,a^3c\mathrm{sgn}(x)}\right)x+\frac{3\,b^2c-8\,ac^2}{a^2b^2\mathrm{sgn}(x)-4\,a^3c\mathrm{sgn}(x)}}{\sqrt{ax^2+bx+c}} + \frac{3\,b\log\left(\left|2\left(\sqrt{a}x-\sqrt{ax^2+bx+c}\right)\sqrt{a}+b\right|\right)}{2\,a^{\frac{5}{2}}\mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/(a+c/x^2+b/x)^(3/2),x, algorithm="giac")
```

output
```
-1/2*(3*b^3*log(abs(b - 2*sqrt(a)*sqrt(c))) - 12*a*b*c*log(abs(b - 2*sqrt(
a)*sqrt(c))) + 6*sqrt(a)*b^2*sqrt(c) - 16*a^(3/2)*c^(3/2))*sgn(x)/(a^(5/2)
*b^2 - 4*a^(7/2)*c) + (((a*b^2 - 4*a^2*c)*x/(a^2*b^2*sgn(x) - 4*a^3*c*sgn(
x)) + (3*b^3 - 10*a*b*c)/(a^2*b^2*sgn(x) - 4*a^3*c*sgn(x)))*x + (3*b^2*c -
 8*a*c^2)/(a^2*b^2*sgn(x) - 4*a^3*c*sgn(x)))/sqrt(a*x^2 + b*x + c) + 3/2*b
*log(abs(2*(sqrt(a)*x - sqrt(a*x^2 + b*x + c))*sqrt(a) + b))/(a^(5/2)*sgn(
x))
```

3.453.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}}\,dx=\int \frac{1}{\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}}\,dx$$

input
```
int(1/(a + b/x + c/x^2)^(3/2),x)
```

output
```
int(1/(a + b/x + c/x^2)^(3/2), x)
```

3.454 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2}}\,dx$

3.454.1 Optimal result

Integrand size = 16, antiderivative size = 220

$$\begin{aligned}\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2}}\,dx &= \frac{(15b^4-100ab^2c+128a^2c^2)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x}{3a^3(b^2-4ac)^2} \\ &- \frac{2\left(b^2-2ac+\frac{bc}{x}\right)x}{3a(b^2-4ac)\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}} \\ &- \frac{2\left(5b^4-32ab^2c+32a^2c^2+\frac{bc(5b^2-28ac)}{x}\right)x}{3a^2(b^2-4ac)^2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}} - \frac{5b\,\text{arctanh}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{2a^{7/2}}\end{aligned}$$

output
```
-2/3*(b^2-2*a*c+b*c/x)*x/a/(-4*a*c+b^2)/(a+c/x^2+b/x)^(3/2)-5/2*b*arctanh(
1/2*(2*a+b/x)/a^(1/2)/(a+c/x^2+b/x)^(1/2))/a^(7/2)-2/3*(5*b^4-32*a*b^2*c+3
2*a^2*c^2+b*c*(-28*a*c+5*b^2)/x)*x/a^2/(-4*a*c+b^2)^2/(a+c/x^2+b/x)^(1/2)+
1/3*(128*a^2*c^2-100*a*b^2*c+15*b^4)*x*(a+c/x^2+b/x)^(1/2)/a^3/(-4*a*c+b^2
)^2
```

3.454.2 Mathematica [A] (verified)

Time = 1.20 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.12

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2}}\,dx = \frac{\frac{2\sqrt{a}(c+x(b+ax))\left(15b^6x^2+8a^2bc^2x\left(39c+32ax^2\right)-2ab^3cx\left(105c+74ax^2\right)+10b^5\left(3cx+2ax^3\right)+3b^4\left(5c^2-30acx^2+a^2\right.}{\left(b^2-4ac\right)^2}}{}$$

input

```
Integrate[(a + c/x^2 + b/x)^(-5/2),x]
```

output

```
((2*Sqrt[a]*(c + x*(b + a*x))*(15*b^6*x^2 + 8*a^2*b*c^2*x*(39*c + 32*a*x^2
) - 2*a*b^3*c*x*(105*c + 74*a*x^2) + 10*b^5*(3*c*x + 2*a*x^3) + 3*b^4*(5*c
^2 - 30*a*c*x^2 + a^2*x^4) + 16*a^2*c^2*(8*c^2 + 12*a*c*x^2 + 3*a^2*x^4) -
 4*a*b^2*c*(25*c^2 - 12*a*c*x^2 + 6*a^2*x^4)))/(b^2 - 4*a*c)^2 + 15*b*(c +
 x*(b + a*x))^(5/2)*Log[a^3*(b + 2*a*x - 2*Sqrt[a]*Sqrt[c + x*(b + a*x)])]
)/(6*a^(7/2)*x^5*(a + (c + b*x)/x^2)^(5/2))
```

3.454.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1681, 1165, 27, 1235, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{1681}$$

$$-\int \frac{x^2}{\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{5/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{1165}$$

$$\frac{2\int -\frac{\left(5b^2+\frac{6cb}{x}-16ac\right)x^2}{2\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}}d\frac{1}{x}}{3a\left(b^2-4ac\right)}-\frac{2x\left(-2ac+b^2+\frac{bc}{x}\right)}{3a\left(b^2-4ac\right)\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\left(5b^2+\frac{6cb}{x}-16ac\right)x^2}{\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}}d\frac{1}{x}}{3a\left(b^2-4ac\right)}-\frac{2x\left(-2ac+b^2+\frac{bc}{x}\right)}{3a\left(b^2-4ac\right)\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}}$$

$\Big\downarrow$ 1235

$$-\frac{\frac{2x\left(32a^2c^2+\frac{bc\left(5b^2-28ac\right)}{x}-32ab^2c+5b^4\right)}{a(b^2-4ac)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}-\frac{2\int -\frac{\left(15b^4-100acb^2+\frac{2c\left(5b^2-28ac\right)b}{x}+128a^2c^2\right)x^2}{2\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}d\frac{1}{x}}{a(b^2-4ac)}}{3a\left(b^2-4ac\right)}-\frac{2x\left(-2ac+b^2+\frac{bc}{x}\right)}{3a\left(b^2-4ac\right)\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}}$$

$\Big\downarrow$ 27

$$-\frac{\frac{\int \frac{\left(15b^4-100acb^2+\frac{2c\left(5b^2-28ac\right)b}{x}+128a^2c^2\right)x^2}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}d\frac{1}{x}}{a(b^2-4ac)}+\frac{2x\left(32a^2c^2+\frac{bc\left(5b^2-28ac\right)}{x}-32ab^2c+5b^4\right)}{a(b^2-4ac)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}}{3a\left(b^2-4ac\right)}-\frac{2x\left(-2ac+b^2+\frac{bc}{x}\right)}{3a\left(b^2-4ac\right)\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}}$$

$\Big\downarrow$ 1228

$$-\frac{\frac{-\frac{15b\left(b^2-4ac\right)^2\int \frac{x}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}d\frac{1}{x}}{2a}-\frac{x\left(128a^2c^2-100ab^2c+15b^4\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{a}}{a(b^2-4ac)}+\frac{2x\left(32a^2c^2+\frac{bc\left(5b^2-28ac\right)}{x}-32ab^2c+5b^4\right)}{a(b^2-4ac)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}}{3a\left(b^2-4ac\right)}-\frac{2x\left(-2ac+b^2+\frac{bc}{x}\right)}{3a\left(b^2-4ac\right)\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}}$$

$\Big\downarrow$ 1154

$$-\frac{\frac{\frac{15b\left(b^2-4ac\right)^2\int \frac{1}{4a-\frac{1}{x^2}}d\frac{2a+\frac{b}{x}}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}}{a}-\frac{x\left(128a^2c^2-100ab^2c+15b^4\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{a}}{a(b^2-4ac)}+\frac{2x\left(32a^2c^2+\frac{bc\left(5b^2-28ac\right)}{x}-32ab^2c+5b^4\right)}{a(b^2-4ac)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}}{3a\left(b^2-4ac\right)}-\frac{2x\left(-2ac+b^2+\frac{bc}{x}\right)}{3a\left(b^2-4ac\right)\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}}$$

$\Big\downarrow$ 219

3.454. $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2}}\,dx$

$$-\frac{\frac{2x\left(32a^2c^2+\frac{bc(5b^2-28ac)}{x}-32ab^2c+5b^4\right)}{a(b^2-4ac)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}+\frac{\frac{15b(b^2-4ac)^2\operatorname{arctanh}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{2a^{3/2}}-\frac{x(128a^2c^2-100ab^2c+15b^4)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{a}}{a(b^2-4ac)}}{3a(b^2-4ac)}-\frac{2x\left(-2ac+b^2+\frac{bc}{x}\right)}{3a(b^2-4ac)\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}}$$

input
```
Int[(a + c/x^2 + b/x)^(-5/2),x]
```

output
```
(-2*(b^2 - 2*a*c + (b*c)/x)*x)/(3*a*(b^2 - 4*a*c)*(a + c/x^2 + b/x)^(3/2))
 - ((2*(5*b^4 - 32*a*b^2*c + 32*a^2*c^2 + (b*c*(5*b^2 - 28*a*c))/x)*x)/(a*
(b^2 - 4*a*c)*Sqrt[a + c/x^2 + b/x]) + (-(((15*b^4 - 100*a*b^2*c + 128*a^2
*c^2)*Sqrt[a + c/x^2 + b/x]*x)/a) + (15*b*(b^2 - 4*a*c)^2*ArcTanh[(2*a + b
/x)/(2*Sqrt[a]*Sqrt[a + c/x^2 + b/x])])/(2*a^(3/2)))/(a*(b^2 - 4*a*c)))/(3
*a*(b^2 - 4*a*c))
```

3.454.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

3.454. $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2}}\,dx$

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1681
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[
Int[(a + b/x^n + c/x^(2*n))^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, p}, x] &&
 EqQ[n2, 2*n] && ILtQ[n, 0]
```

3.454. $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2}} dx$

3.454.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.71

method	result
default	$-\frac{(a\,x^2+bx+c)\left(-96a^{\frac{13}{2}}c^2x^4+48a^{\frac{11}{2}}b^2c\,x^4-512a^{\frac{11}{2}}b\,c^2x^3-6a^{\frac{9}{2}}b^4x^4-384a^{\frac{11}{2}}c^3x^2+296a^{\frac{9}{2}}b^3c\,x^3-96a^{\frac{9}{2}}b^2c^2x^2-40a^{\frac{7}{2}}b^5x^3-624a\right.}{}$
risch	Expression too large to display

input
```
int(1/(a+c/x^2+b/x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(a*x^2+b*x+c)*(-96*a^(13/2)*c^2*x^4+48*a^(11/2)*b^2*c*x^4-512*a^(11/2
)*b*c^2*x^3-6*a^(9/2)*b^4*x^4-384*a^(11/2)*c^3*x^2+296*a^(9/2)*b^3*c*x^3-9
6*a^(9/2)*b^2*c^2*x^2-40*a^(7/2)*b^5*x^3-624*a^(9/2)*b*c^3*x+180*a^(7/2)*b
^4*c*x^2-256*a^(9/2)*c^4+420*a^(7/2)*b^3*c^2*x-30*a^(5/2)*b^6*x^2+200*a^(7
/2)*b^2*c^3-60*a^(5/2)*b^5*c*x+240*(a*x^2+b*x+c)^(3/2)*ln(1/2*(2*(a*x^2+b*
x+c)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^4*b*c^2-120*(a*x^2+b*x+c)^(3/2)*ln(
1/2*(2*(a*x^2+b*x+c)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^3*b^3*c+15*(a*x^2+b
*x+c)^(3/2)*ln(1/2*(2*(a*x^2+b*x+c)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^2*b^
5-30*a^(5/2)*b^4*c^2)/a^(11/2)/((a*x^2+b*x+c)/x^2)^(5/2)/x^5/(4*a*c-b^2)^2
```

3.454.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 539 vs. 2(198) = 396.

Time = 0.41 (sec) , antiderivative size = 1081, normalized size of antiderivative = 4.91

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2}}\,dx = \left[\frac{15\,(b^5c^2-8\,ab^3c^3+16\,a^2bc^4+(a^2b^5-8\,a^3b^3c+16\,a^4bc^2)x^4+2\,(ab^6-8\,a^2b^4c+}{}\right.$$

input
```
integrate(1/(a+c/x^2+b/x)^(5/2),x, algorithm="fricas")
```

output
```
[1/12*(15*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4 + (a^2*b^5 - 8*a^3*b^3*c +
 16*a^4*b*c^2)*x^4 + 2*(a*b^6 - 8*a^2*b^4*c + 16*a^3*b^2*c^2)*x^3 + (b^7 -
 6*a*b^5*c + 32*a^3*b*c^3)*x^2 + 2*(b^6*c - 8*a*b^4*c^2 + 16*a^2*b^2*c^3)*
x)*sqrt(a)*log(-8*a^2*x^2 - 8*a*b*x - b^2 - 4*a*c + 4*(2*a*x^2 + b*x)*sqrt
(a)*sqrt((a*x^2 + b*x + c)/x^2)) + 4*(3*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^
2)*x^5 + 4*(5*a^2*b^5 - 37*a^3*b^3*c + 64*a^4*b*c^2)*x^4 + 3*(5*a*b^6 - 30
*a^2*b^4*c + 16*a^3*b^2*c^2 + 64*a^4*c^3)*x^3 + 6*(5*a*b^5*c - 35*a^2*b^3*
c^2 + 52*a^3*b*c^3)*x^2 + (15*a*b^4*c^2 - 100*a^2*b^2*c^3 + 128*a^3*c^4)*x
)*sqrt((a*x^2 + b*x + c)/x^2))/(a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^6*c^4 +
 (a^6*b^4 - 8*a^7*b^2*c + 16*a^8*c^2)*x^4 + 2*(a^5*b^5 - 8*a^6*b^3*c + 16*
a^7*b*c^2)*x^3 + (a^4*b^6 - 6*a^5*b^4*c + 32*a^7*c^3)*x^2 + 2*(a^4*b^5*c -
 8*a^5*b^3*c^2 + 16*a^6*b*c^3)*x), 1/6*(15*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2
*b*c^4 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^4 + 2*(a*b^6 - 8*a^2*b^4
*c + 16*a^3*b^2*c^2)*x^3 + (b^7 - 6*a*b^5*c + 32*a^3*b*c^3)*x^2 + 2*(b^6*c
 - 8*a*b^4*c^2 + 16*a^2*b^2*c^3)*x)*sqrt(-a)*arctan(1/2*(2*a*x^2 + b*x)*sq
rt(-a)*sqrt((a*x^2 + b*x + c)/x^2)/(a^2*x^2 + a*b*x + a*c)) + 2*(3*(a^3*b^
4 - 8*a^4*b^2*c + 16*a^5*c^2)*x^5 + 4*(5*a^2*b^5 - 37*a^3*b^3*c + 64*a^4*b
*c^2)*x^4 + 3*(5*a*b^6 - 30*a^2*b^4*c + 16*a^3*b^2*c^2 + 64*a^4*c^3)*x^3 +
 6*(5*a*b^5*c - 35*a^2*b^3*c^2 + 52*a^3*b*c^3)*x^2 + (15*a*b^4*c^2 - 100*a
^2*b^2*c^3 + 128*a^3*c^4)*x)*sqrt((a*x^2 + b*x + c)/x^2))/(a^4*b^4*c^2 ...
```

3.454.6 Sympy [F]

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+c/x**2+b/x)**(5/2),x)
```

output
```
Integral((a + b/x + c/x**2)**(-5/2), x)
```

3.454.7 Maxima [F]

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2}}\,dx=\int \frac{1}{\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+c/x^2+b/x)^(5/2),x, algorithm="maxima")
```

output
```
integrate((a + b/x + c/x^2)^(-5/2), x)
```

3.454.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 499 vs. $2(198) = 396$.

Time = 0.37 (sec) , antiderivative size = 499, normalized size of antiderivative = 2.27

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2}}\,dx=$$
$$-\frac{\left(15\,b^5\sqrt{c}\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)-120\,ab^3c^{\frac{3}{2}}\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)+240\,a^2bc^{\frac{5}{2}}\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)+30\,\sqrt{a}b^4c\right.}{6\left(a^{\frac{7}{2}}b^4\sqrt{c}-8\,a^{\frac{9}{2}}b^2c^{\frac{3}{2}}+16\,a^{\frac{11}{2}}c^{\frac{5}{2}}\right)}$$
$$+\frac{\left(\left(\left(\frac{3\left(a^2b^4-8\,a^3b^2c+16\,a^4c^2\right)x}{a^3b^4\mathrm{sgn}(x)-8\,a^4b^2c\mathrm{sgn}(x)+16\,a^5c^2\mathrm{sgn}(x)}+\frac{4\left(5\,ab^5-37\,a^2b^3c+64\,a^3bc^2\right)}{a^3b^4\mathrm{sgn}(x)-8\,a^4b^2c\mathrm{sgn}(x)+16\,a^5c^2\mathrm{sgn}(x)}\right)x+\frac{3\left(5\,b^6-30\,ab^4c+16\,a^2b^2c^2+64\,a^3c^3\right)}{a^3b^4\mathrm{sgn}(x)-8\,a^4b^2c\mathrm{sgn}(x)+16\,a^5c^2\mathrm{sgn}(x}\right.\right.}{3\left(ax^2+bx+c\right)^{\frac{3}{2}}}$$
$$+\frac{5\,b\log\left(\left|2\left(\sqrt{a}x-\sqrt{ax^2+bx+c}\right)\sqrt{a}+b\right|\right)}{2\,a^{\frac{7}{2}}\mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/(a+c/x^2+b/x)^(5/2),x, algorithm="giac")
```

output
```
-1/6*(15*b^5*sqrt(c)*log(abs(b - 2*sqrt(a)*sqrt(c))) - 120*a*b^3*c^(3/2)*l
og(abs(b - 2*sqrt(a)*sqrt(c))) + 240*a^2*b*c^(5/2)*log(abs(b - 2*sqrt(a)*s
qrt(c))) + 30*sqrt(a)*b^4*c - 200*a^(3/2)*b^2*c^2 + 256*a^(5/2)*c^3)*sgn(x
)/(a^(7/2)*b^4*sqrt(c) - 8*a^(9/2)*b^2*c^(3/2) + 16*a^(11/2)*c^(5/2)) + 1/
3*((((3*(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)*x/(a^3*b^4*sgn(x) - 8*a^4*b^2
*c*sgn(x) + 16*a^5*c^2*sgn(x)) + 4*(5*a*b^5 - 37*a^2*b^3*c + 64*a^3*b*c^2)
/(a^3*b^4*sgn(x) - 8*a^4*b^2*c*sgn(x) + 16*a^5*c^2*sgn(x)))*x + 3*(5*b^6 -
 30*a*b^4*c + 16*a^2*b^2*c^2 + 64*a^3*c^3)/(a^3*b^4*sgn(x) - 8*a^4*b^2*c*s
gn(x) + 16*a^5*c^2*sgn(x)))*x + 6*(5*b^5*c - 35*a*b^3*c^2 + 52*a^2*b*c^3)/
(a^3*b^4*sgn(x) - 8*a^4*b^2*c*sgn(x) + 16*a^5*c^2*sgn(x)))*x + (15*b^4*c^2
 - 100*a*b^2*c^3 + 128*a^2*c^4)/(a^3*b^4*sgn(x) - 8*a^4*b^2*c*sgn(x) + 16*
a^5*c^2*sgn(x)))/(a*x^2 + b*x + c)^(3/2) + 5/2*b*log(abs(2*(sqrt(a)*x - sq
rt(a*x^2 + b*x + c))*sqrt(a) + b))/(a^(7/2)*sgn(x))
```

3.454.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2}}\,dx=\int \frac{1}{\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{5/2}}\,dx$$

input
```
int(1/(a + b/x + c/x^2)^(5/2),x)
```

output
```
int(1/(a + b/x + c/x^2)^(5/2), x)
```

3.455 $\int \sqrt{a^2+\frac{b^2}{x^2}+\frac{2ab}{x}}\,dx$

3.455.1 Optimal result

Integrand size = 22, antiderivative size = 73

$$\int \sqrt{a^2+\frac{b^2}{x^2}+\frac{2ab}{x}}\,dx=\frac{a\sqrt{a^2+\frac{b^2}{x^2}+\frac{2ab}{x}}\,x}{a+\frac{b}{x}}-\frac{b\sqrt{a^2+\frac{b^2}{x^2}+\frac{2ab}{x}}\log\left(\frac{1}{x}\right)}{a+\frac{b}{x}}$$

output
```
a*x*(a^2+b^2/x^2+2*a*b/x)^(1/2)/(a+b/x)-b*ln(1/x)*(a^2+b^2/x^2+2*a*b/x)^(1
/2)/(a+b/x)
```

3.455.2 Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.44

$$\int \sqrt{a^2+\frac{b^2}{x^2}+\frac{2ab}{x}}\,dx=\frac{x\sqrt{\frac{(b+ax)^2}{x^2}}(ax+b\log(x))}{b+ax}$$

input
```
Integrate[Sqrt[a^2 + b^2/x^2 + (2*a*b)/x],x]
```

output
```
(x*Sqrt[(b + a*x)^2/x^2]*(a*x + b*Log[x]))/(b + a*x)
```

3.455.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.63, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1384, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a^2+\frac{2ab}{x}+\frac{b^2}{x^2}}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+\frac{2ab}{x}+\frac{b^2}{x^2}}\int \left(\frac{b^2}{x}+ab\right)dx}{b\left(a+\frac{b}{x}\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+\frac{2ab}{x}+\frac{b^2}{x^2}}\left(abx+b^2\log(x)\right)}{b\left(a+\frac{b}{x}\right)}$$

input
```
Int[Sqrt[a^2 + b^2/x^2 + (2*a*b)/x],x]
```

output
```
(Sqrt[a^2 + b^2/x^2 + (2*a*b)/x]*(a*b*x + b^2*Log[x]))/(b*(a + b/x))
```

3.455.3.1 Defintions of rubi rules used

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.455.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.55

method	result	size
default	$\frac{\sqrt{\frac{a^{2} x^{2}+2 a b x+b^{2}}{x^{2}}}\, x \left(a x+b \ln \left(x\right)\right)}{a x+b}$	40
risch	$\frac{\sqrt{\frac{\left(a x+b\right)^{2}}{x^{2}}}\, x^{2} a}{a x+b}+\frac{\sqrt{\frac{\left(a x+b\right)^{2}}{x^{2}}}\, x b \ln \left(x\right)}{a x+b}$	52

input
```
int((a^2+b^2/x^2+2*a*b/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((a^2*x^2+2*a*b*x+b^2)/x^2)^(1/2)/(a*x+b)*x*(a*x+b*ln(x))
```

3.455.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.11

$$\int \sqrt{a^2+\frac{b^2}{x^2}+\frac{2 a b}{x}}\, dx = a x + b \log \left(x\right)$$

input
```
integrate((a^2+b^2/x^2+2*a*b/x)^(1/2),x, algorithm="fricas")
```

output
```
a*x + b*log(x)
```

3.455.6 Sympy [F]

$$\int \sqrt{a^2+\frac{b^2}{x^2}+\frac{2 a b}{x}}\, dx = \int \sqrt{a^{2}+\frac{2 a b}{x}+\frac{b^{2}}{x^{2}}}\, dx$$

input
```
integrate((a**2+b**2/x**2+2*a*b/x)**(1/2),x)
```

output
```
Integral(sqrt(a**2 + 2*a*b/x + b**2/x**2), x)
```

3.455.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.11

$$\int \sqrt{a^2 + \frac{b^2}{x^2} + \frac{2ab}{x}}\,dx = ax + b\log(x)$$

input
```
integrate((a^2+b^2/x^2+2*a*b/x)^(1/2),x, algorithm="maxima")
```

output
```
a*x + b*log(x)
```

3.455.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.40

$$\int \sqrt{a^2 + \frac{b^2}{x^2} + \frac{2ab}{x}}\,dx = ax\mathrm{sgn}\left(ax^2 + bx\right) + b\log(|x|)\,\mathrm{sgn}\left(ax^2 + bx\right)$$

input
```
integrate((a^2+b^2/x^2+2*a*b/x)^(1/2),x, algorithm="giac")
```

output
```
a*x*sgn(a*x^2 + b*x) + b*log(abs(x))*sgn(a*x^2 + b*x)
```

3.455.9 Mupad [B] (verification not implemented)

Time = 8.39 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.84

$$\begin{aligned}\int \sqrt{a^2 + \frac{b^2}{x^2} + \frac{2ab}{x}}\,dx &= x\sqrt{\frac{1}{x^2}}\,\sqrt{a^2\,x^2 + 2\,a\,b\,x + b^2} \\ &\quad - x\,\ln\left(\frac{2\,\sqrt{b^2}\,\sqrt{a^2\,x^2 + 2\,a\,b\,x + b^2} + 2\,b^2 + 2\,a\,b\,x}{x}\right)\sqrt{b^2}\,\sqrt{\frac{1}{x^2}} \\ &\quad + \frac{a\,b\,x\,\ln\left(\frac{a\,b + \sqrt{a^2}\,\sqrt{a^2\,x^2 + 2\,a\,b\,x + b^2} + a^2\,x}{\sqrt{a^2}}\right)\sqrt{\frac{1}{x^2}}}{\sqrt{a^2}}\end{aligned}$$

input
```
int((a^2 + b^2/x^2 + (2*a*b)/x)^(1/2),x)
```

output
```
x*(1/x^2)^(1/2)*(b^2 + a^2*x^2 + 2*a*b*x)^(1/2) - x*log((2*(b^2)^(1/2)*(b^
2 + a^2*x^2 + 2*a*b*x)^(1/2) + 2*b^2 + 2*a*b*x)/x)*(b^2)^(1/2)*(1/x^2)^(1/
2) + (a*b*x*log((a*b + (a^2)^(1/2)*(b^2 + a^2*x^2 + 2*a*b*x)^(1/2) + a^2*x
)/(a^2)^(1/2))*(1/x^2)^(1/2))/(a^2)^(1/2)
```

3.455. $\int \sqrt{a^2 + \frac{b^2}{x^2} + \frac{2ab}{x}}\, dx$

3.456 $\int \frac{1}{c+\frac{a}{x^4}+\frac{b}{x^2}} \, dx$

3.456.1 Optimal result

Integrand size = 14, antiderivative size = 179

$$\int \frac{1}{c+\frac{a}{x^4}+\frac{b}{x^2}} \, dx = \frac{x}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

output

```
x/c-1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b+(2*a*c-b
^2)/(-4*a*c+b^2)^(1/2))/c^(3/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*a
rctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-2*a*c+b^2)/(-4*
a*c+b^2)^(1/2))/c^(3/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.456.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.13

$$\int \frac{1}{c+\frac{a}{x^4}+\frac{b}{x^2}} \, dx = \frac{x}{c} - \frac{\left(-b^2+2ac+b\sqrt{b^2-4ac}\right) \arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(b^2-2ac+b\sqrt{b^2-4ac}\right) \arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}$$

input
```
Integrate[(c + a/x^4 + b/x^2)^(-1),x]
```

output
```
x/c - ((-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqr
t[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqr
t[b^2 - 4*a*c]]) - ((b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sq
rt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*
Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

3.456.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1679, 1442, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\frac{a}{x^4}+\frac{b}{x^2}+c}dx$$

$$\Big\downarrow \text{ 1679}$$

$$\int \frac{x^4}{a+bx^2+cx^4}dx$$

$$\Big\downarrow \text{ 1442}$$

$$\frac{x}{c}-\frac{\int \frac{bx^2+a}{cx^4+bx^2+a}dx}{c}$$

$$\Big\downarrow \text{ 1480}$$

$$\frac{x}{c}-\frac{\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{c}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{x}{c}-\frac{\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{c}$$

input
```
Int[(c + a/x^4 + b/x^2)^(-1),x]
```

output
```
x/c - (((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/S
qrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]])
 + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[
b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/c
```

3.456.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1442
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
x] - Simp[d^4/(c*(m + 4*p + 1))   Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1679
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(
2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n]
 && LtQ[n, 0] && IntegerQ[p]
```

3.456.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.32

method	result
risch	$\frac{x}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(-\_R^2b-a\right)\ln\left(x-\_R\right)}{2c\_R^3+\_Rb}}{2c}$
default	$\frac{x}{c}+\frac{\left(-b\sqrt{-4ac+b^2}+2ac-b^2\right)\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,c\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\left(b^2-2ac-b\sqrt{-4ac+b^2}\right)\sqrt{2}\,\mathrm{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,c\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}$

input
```
int(1/(a/x^4+b/x^2+c),x,method=_RETURNVERBOSE)
```

output
```
x/c+1/2/c*sum((-_R^2*b-a)/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b
+a))
```

3.456.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1059 vs. 2(143) = 286.

Time = 0.28 (sec) , antiderivative size = 1059, normalized size of antiderivative = 5.92

$$\int \frac{1}{c+\frac{a}{x^4}+\frac{b}{x^2}}\,dx =$$

$$-\frac{\sqrt{\frac{1}{2}}c\sqrt{-\frac{b^3-3\,abc+(b^2c^3-4\,ac^4)\sqrt{\frac{b^4-2\,ab^2c+a^2c^2}{b^2c^6-4\,ac^7}}}{b^2c^3-4\,ac^4}}\log\left(-2\,(ab^2-a^2c)x+\sqrt{\frac{1}{2}}\left(b^4-5\,ab^2c+4\,a^2c^2-(b^3c^3-4\,a\right.\right.}{}$$

input
```
integrate(1/(c+a/x^4+b/x^2),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2
*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b
^2 - a^2*c)*x + sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 - (b^3*c^3 - 4*a*b*
c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3
*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4
*a*c^7)))/(b^2*c^3 - 4*a*c^4))) - sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c + (b^2*
c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2
*c^3 - 4*a*c^4))*log(-2*(a*b^2 - a^2*c)*x - sqrt(1/2)*(b^4 - 5*a*b^2*c + 4
*a^2*c^2 - (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6
 - 4*a*c^7)))*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b
^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) + sqrt(1/2)*c*
sqrt(-(b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2
)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b^2 - a^2*c)*x + sq
rt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 + (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2
*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3*a*b*c - (b^2*c^3
- 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3
 - 4*a*c^4))) - sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqr
t((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*l
og(-2*(a*b^2 - a^2*c)*x - sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 + (b^3*c^
3 - 4*a*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*s...
```

3.456.6 Sympy [A] (verification not implemented)

Time = 1.23 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.72

$$\int \frac{1}{c+\frac{a}{x^4}+\frac{b}{x^2}}\,dx$$
$$=\operatorname{RootSum}\left(t^4\cdot\left(256a^2c^5-128ab^2c^4+16b^4c^3\right)+t^2\cdot\left(48a^2bc^2-28ab^3c+4b^5\right)+a^3,\left(t\mapsto t\log\left(x+\frac{32t^3\right.\right.\right.$$
$$+\frac{x}{c}$$

input
```
integrate(1/(c+a/x**4+b/x**2),x)
```

output
```
RootSum(_t**4*(256*a**2*c**5 - 128*a*b**2*c**4 + 16*b**4*c**3) + _t**2*(48
*a**2*b*c**2 - 28*a*b**3*c + 4*b**5) + a**3, Lambda(_t, _t*log(x + (32*_t*
*3*a*b*c**4 - 8*_t**3*b**3*c**3 - 4*_t*a**2*c**2 + 8*_t*a*b**2*c - 2*_t*b*
*4)/(a**2*c - a*b**2)))) + x/c
```

3.456.7 Maxima [F]

$$\int \frac{1}{c + \frac{a}{x^4} + \frac{b}{x^2}}\,dx = \int \frac{1}{c + \frac{b}{x^2} + \frac{a}{x^4}}\,dx$$

input
```
integrate(1/(c+a/x^4+b/x^2),x, algorithm="maxima")
```

output
```
x/c - integrate((b*x^2 + a)/(c*x^4 + b*x^2 + a), x)/c
```

3.456.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2109 vs. $2(143) = 286$.

Time = 0.79 (sec) , antiderivative size = 2109, normalized size of antiderivative = 11.78

$$\int \frac{1}{c + \frac{a}{x^4} + \frac{b}{x^2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c+a/x^4+b/x^2),x, algorithm="giac")
```

output
```
x/c - 1/8*(2*b^5*c^4 - 12*a*b^3*c^5 + 16*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c
^5 - (2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*c^2 + 2*
(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 8*sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a*b^3*c^3 - 2*a*b^4*c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
3*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + sqrt(2)*s...
```

3.456.9 Mupad [B] (verification not implemented)

Time = 8.73 (sec) , antiderivative size = 3026, normalized size of antiderivative = 16.91

$$\int \frac{1}{c+\frac{a}{x^4}+\frac{b}{x^2}}\,dx = \text{Too large to display}$$

input
```
int(1/(c + a/x^4 + b/x^2),x)
```

output
```
x/c - atan(((((16*a^2*c^3 - 4*a*b^2*c^2)/c - (2*x*(4*b^3*c^3 - 16*a*b*c^4)
*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-
(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c
)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(
-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) -
 (2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1
/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*
c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i - (((16*a^2*c^3 - 4*a*b^2*c^2)/c +
 (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*
a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4
*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12
*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^
4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*(-(b
^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*
c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i)/(((
(16*a^2*c^3 - 4*a*b^2*c^2)/c - (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(b^5 + b^2*
(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^
3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(b^5 + b^2
*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)
^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*x*(b^4 ...
```

3.457 $\int \frac{1}{c+\frac{a}{x^6}+\frac{b}{x^3}}\,dx$

3.457.1 Optimal result

Integrand size = 14, antiderivative size = 631

$$\int \frac{1}{c+\frac{a}{x^6}+\frac{b}{x^3}}\,dx$$
$$=\frac{x}{c}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$
$$+\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$
$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$
$$-\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$
$$+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$
$$+\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

output
```
x/c-1/6*ln(2^(1/3)*c^(1/3)*x+(b-(-4*a*c+b^2)^(1/2))^(1/3))*(b+(2*a*c-b^2)/
(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)/(b-(-4*a*c+b^2)^(1/2))^(2/3)+1/12*ln(2
^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b-(-4*a*c+b^2)^(1/2))^(1/3)+(b-(-4*a
*c+b^2)^(1/2))^(2/3))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)/(
b-(-4*a*c+b^2)^(1/2))^(2/3)+1/6*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b-(-4*a
*c+b^2)^(1/2))^(1/3))*3^(1/2))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))*2^(2/3)/
c^(4/3)*3^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(2/3)-1/6*ln(2^(1/3)*c^(1/3)*x+(b+(
-4*a*c+b^2)^(1/2))^(1/3))*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4
/3)/(b+(-4*a*c+b^2)^(1/2))^(2/3)+1/12*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/
3)*x*(b+(-4*a*c+b^2)^(1/2))^(1/3)+(b+(-4*a*c+b^2)^(1/2))^(2/3))*(b+(-2*a*c
+b^2)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)/(b+(-4*a*c+b^2)^(1/2))^(2/3)+1/6
*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b+(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*
(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)*3^(1/2)/(b+(-4*a*c+b^2
)^(1/2))^(2/3)
```

3.457.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.11

$$\int \frac{1}{c+\frac{a}{x^6}+\frac{b}{x^3}}\,dx=\frac{x}{c}-\frac{\text{RootSum}\left[a+b\#1^3+c\#1^6\&,\frac{a\log(x-\#1)+b\log(x-\#1)\#1^3}{b\#1^2+2c\#1^5}\&\right]}{3c}$$

input
```
Integrate[(c + a/x^6 + b/x^3)^(-1),x]
```

output
```
x/c - RootSum[a + b*#1^3 + c*#1^6 & , (a*Log[x - #1] + b*Log[x - #1]*#1^3)
/(b*#1^2 + 2*c*#1^5) & ]/(3*c)
```

3.457.3 Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 521, normalized size of antiderivative = 0.83, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {1679, 1703, 1752, 750, 16, 27, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\frac{a}{x^6}+\frac{b}{x^3}+c}dx$$

$$\Big\downarrow \text{ 1679}$$

$$\int \frac{x^6}{a+bx^3+cx^6}dx$$

$$\Big\downarrow \text{ 1703}$$

$$\frac{x}{c}-\frac{\int \frac{bx^3+a}{cx^6+bx^3+a}dx}{c}$$

$$\Big\downarrow \text{ 1752}$$

$$\frac{x}{c}-\frac{\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^3+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\int \frac{1}{cx^3+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{c}$$

$$\Big\downarrow \text{ 750}$$

$$\frac{x}{c}-\frac{\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{2^{2/3}\int \frac{2\left(2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}x\right)}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\int \frac{1}{\sqrt[3]{c}x+\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}}dx}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right) \cdots}{\cdots}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{x}{c}-\frac{\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{2^{2/3}\int \frac{2\left(2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}x\right)}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x \cdots\right.}{3\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} \cdots\right)}{\cdots}$$

$$\Big\downarrow \text{ 27}$$

3.457. $\int \frac{1}{c+\frac{a}{x^6}+\frac{b}{x^3}}\,dx$

$$\frac{x}{c}-\frac{\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{2\ 2^{2/3}\int \frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}x}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\cdots\right.}{3\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\cdots\right.}{\cdots}$$

$\downarrow$ 1142

$$\frac{x}{c}-\frac{\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{2\ 2^{2/3}\left(\frac{3\sqrt[3]{b-\sqrt{b^2-4ac}}\int\frac{1}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{2\sqrt[3]{2}}-\int-\frac{\sqrt[3]{c}\cdots}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{\cdots}}\cdots\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\cdots\right.}{\cdots}$$

$\downarrow$ 25

3.457. $\int \frac{1}{c+\frac{a}{x^6}+\frac{b}{x^3}}\,dx$

$$\frac{\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(2\,2^{2/3}\left(\frac{3\sqrt[3]{b-\sqrt{b^2-4ac}}\int\frac{1}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{2\sqrt[3]{2}}+\frac{\int\frac{\sqrt[3]{c}\left(\cdots\right.}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\cdots}}{\cdots}\right.\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}\cdots}\quad \frac{x}{c}-$$

$\downarrow$ 27

$$\frac{\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(2\,2^{2/3}\left(\frac{3\sqrt[3]{b-\sqrt{b^2-4ac}}\int\frac{1}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{2\sqrt[3]{2}}+\frac{1}{4}\int\frac{\cdots}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{\cdots}}\right.\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}\cdots}\quad \frac{x}{c}-$$

$\downarrow$ 1082

3.457. $\int \frac{1}{c+\frac{a}{x^6}+\frac{b}{x^3}}\,dx$

$$\frac{x}{c}-\frac{\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{2\ 2^{2/3}\left(\frac{1}{4}\int\frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-4\sqrt[3]{c}x}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)^2-3}d}{2\sqrt[3]{c}}\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right.}{}$$

$\big\downarrow$ 217

$$\frac{x}{c}-\frac{\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{2\ 2^{2/3}\left(\frac{1}{4}\int\frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-4\sqrt[3]{c}x}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2\sqrt[3]{c}}\right)}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right.}{}$$

$\big\downarrow$ 1103

3.457. $\int\frac{1}{c+\frac{a}{x^6}+\frac{b}{x^3}}\,dx$

$$\frac{x}{c} - \frac{\frac{1}{2}\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{2\,2^{2/3}\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2\sqrt[3]{c}} - \frac{\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{4\sqrt[3]{c}}\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right.}{}$$

input
```
Int[(c + a/x^6 + b/x^3)^(-1),x]
```

output
```
x/c - (((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*((2^(2/3)*Log[(b - Sqrt[b^2
 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(
2/3)) + (2*2^(2/3)*(-1/2*(Sqrt[3]*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - S
qrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/c^(1/3) - Log[(b - Sqrt[b^2 - 4*a*c])^(
2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x
^2]/(4*c^(1/3))))/(3*(b - Sqrt[b^2 - 4*a*c])^(2/3))))/2 + ((b + (b^2 - 2*a
*c)/Sqrt[b^2 - 4*a*c])*((2^(2/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/
3)*c^(1/3)*x])/(3*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)) + (2*2^(2/3)*(-1/
2*(Sqrt[3]*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3)
)/Sqrt[3]])/c^(1/3) - Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*
(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2]/(4*c^(1/3))))/(3*(b
 + Sqrt[b^2 - 4*a*c])^(2/3))))/2)/c
```

3.457. $\int \frac{1}{c+\frac{a}{x^6}+\frac{b}{x^3}}\,dx$

3.457.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1679
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(
2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n]
 && LtQ[n, 0] && IntegerQ[p]
```

3.457. $\int \frac{1}{c+\frac{a}{x^6}+\frac{b}{x^3}}\,dx$

rule 1703
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(2*n - 1)*(d*x)^(m - 2*n + 1)*((a + b*x^n + c*x^(2*n))^(
p + 1)/(c*(m + 2*n*p + 1))), x] - Simp[d^(2*n)/(c*(m + 2*n*p + 1))   Int[(d
*x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x
^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && N
eQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0
] && IntegerQ[p]
```

rule 1752
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

3.457.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.09

method	result	size
default	$\frac{x}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}c+\_Z^{3}b+a\right)}\frac{\left(-\_R^{3}b-a\right)\ln\left(x-\_R\right)}{2\_R^{5}c+\_R^{2}b}}{3c}$	59
risch	$\frac{x}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}c+\_Z^{3}b+a\right)}\frac{\left(-\_R^{3}b-a\right)\ln\left(x-\_R\right)}{2\_R^{5}c+\_R^{2}b}}{3c}$	59

input
```
int(1/(c+a/x^6+b/x^3),x,method=_RETURNVERBOSE)
```

output
```
x/c+1/3/c*sum((-_R^3*b-a)/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^6*c+_Z^3
*b+a))
```

3.457.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2882 vs. 2(495) = 990.

Time = 0.39 (sec) , antiderivative size = 2882, normalized size of antiderivative = 4.57

$$\int \frac{1}{c+\frac{a}{x^6}+\frac{b}{x^3}}\,dx=\text{Too large to display}$$

input
```
integrate(1/(c+a/x^6+b/x^3),x, algorithm="fricas")
```

output
```
-1/6*((1/2)^(1/3)*(sqrt(-3)*c + c)*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*
sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*
c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))
^(1/3)*log(4*(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*x - (1/2)^(1/3)*(b^6 - 8*a*
b^4*c + 18*a^2*b^2*c^2 - 8*a^3*c^3 + sqrt(-3)*(b^6 - 8*a*b^4*c + 18*a^2*b^
2*c^2 - 8*a^3*c^3) - (b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6 + sqrt(-3)*(b^5
*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6))*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2
 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 -
 64*a^3*c^11)))*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6
*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9
+ 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)) - (1/2)^(1/
3)*(sqrt(-3)*c - c)*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a
*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*
c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)*log(4*(a
*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*x - (1/2)^(1/3)*(b^6 - 8*a*b^4*c + 18*a^2*
b^2*c^2 - 8*a^3*c^3 - sqrt(-3)*(b^6 - 8*a*b^4*c + 18*a^2*b^2*c^2 - 8*a^3*c
^3) - (b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6 - sqrt(-3)*(b^5*c^4 - 8*a*b^3*
c^5 + 16*a^2*b*c^6))*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c
^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))
*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*...
```

3.457.6 Sympy [A] (verification not implemented)

Time = 54.07 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.31

$$\int \frac{1}{c+\frac{a}{x^6}+\frac{b}{x^3}}\,dx$$
$$=\operatorname{RootSum}\left(t^6\cdot\left(46656a^3c^7-34992a^2b^2c^6+8748ab^4c^5-729b^6c^4\right)+t^3\cdot\left(864a^3bc^3-864a^2b^3c^2+270ab^5c\right.\right.$$
$$+\frac{x}{c}$$

input
```
integrate(1/(c+a/x**6+b/x**3),x)
```

output
```
RootSum(_t**6*(46656*a**3*c**7 - 34992*a**2*b**2*c**6 + 8748*a*b**4*c**5 -
 729*b**6*c**4) + _t**3*(864*a**3*b*c**3 - 864*a**2*b**3*c**2 + 270*a*b**5
*c - 27*b**7) + a**4, Lambda(_t, _t*log(x + (1296*_t**4*a**2*b*c**6 - 648*
_t**4*a*b**3*c**5 + 81*_t**4*b**5*c**4 - 12*_t*a**3*c**3 + 39*_t*a**2*b**2
*c**2 - 21*_t*a*b**4*c + 3*_t*b**6)/(2*a**3*c**2 - 4*a**2*b**2*c + a*b**4)
))) + x/c
```

3.457.7 Maxima [F]

$$\int \frac{1}{c+\frac{a}{x^6}+\frac{b}{x^3}}\,dx = \int \frac{1}{c+\frac{b}{x^3}+\frac{a}{x^6}}\,dx$$

input
```
integrate(1/(c+a/x^6+b/x^3),x, algorithm="maxima")
```

output
```
x/c - integrate((b*x^3 + a)/(c*x^6 + b*x^3 + a), x)/c
```

3.457.8 Giac [F]

$$\int \frac{1}{c+\frac{a}{x^6}+\frac{b}{x^3}}\,dx = \int \frac{1}{c+\frac{b}{x^3}+\frac{a}{x^6}}\,dx$$

input
```
integrate(1/(c+a/x^6+b/x^3),x, algorithm="giac")
```

output
```
integrate(1/(c + b/x^3 + a/x^6), x)
```

3.457.9 Mupad [B] (verification not implemented)

Time = 10.26 (sec) , antiderivative size = 2280, normalized size of antiderivative = 3.61

$$\int \frac{1}{c+\frac{a}{x^6}+\frac{b}{x^3}}\,dx = \text{Too large to display}$$

input
```
int(1/(c + a/x^6 + b/x^3),x)
```

output
```
log((3*a^2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c - (3*2^(2/3)*a*(-(b^4*(-(4*a
*c - b^2)^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*
a*c - b^2)^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(c^
4*(4*a*c - b^2)^3))^(1/3)*(b^4 + 2*a^2*c^2 - 4*a*b^2*c)*(b*(-(4*a*c - b^2)
^3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(4*c*(4*a*c - b^2)))*(-(b^4*(-(
4*a*c - b^2)^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-
(4*a*c - b^2)^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/
(54*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6)))^(1/3) + x/c +
 log((3*a^2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c + (3*2^(2/3)*a*((b^7 + b^4*
(-(4*a*c - b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*
a*c - b^2)^3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(c^
4*(4*a*c - b^2)^3))^(1/3)*(b^4 + 2*a^2*c^2 - 4*a*b^2*c)*(b*(-(4*a*c - b^2)
^3)^(1/2) - b^4 - 16*a^2*c^2 + 8*a*b^2*c))/(4*c*(4*a*c - b^2)))*((b^7 + b^
4*(-(4*a*c - b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(
4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(
54*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6)))^(1/3) + log((3
*a^2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c + (3*2^(2/3)*a*(3^(1/2)*1i - 1)*((
b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*a^2
*c^2*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^
(1/2))/(c^4*(4*a*c - b^2)^3))^(1/3)*(b^4 + 2*a^2*c^2 - 4*a*b^2*c)*(b*(-...
```

3.458 $\int \frac{1}{c+\frac{a}{x^8}+\frac{b}{x^4}}\,dx$

3.458.1 Optimal result

Integrand size = 14, antiderivative size = 376

$$\int \frac{1}{c+\frac{a}{x^8}+\frac{b}{x^4}}\,dx = \frac{x}{c} + \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}} + \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}$$

output
```
x/c+1/4*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(b+(-2*a*c
+b^2)/(-4*a*c+b^2)^(1/2))*2^(3/4)/c^(5/4)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)+1/
4*arctanh(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(b+(-2*a*c+b^2)
/(-4*a*c+b^2)^(1/2))*2^(3/4)/c^(5/4)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)+1/4*arc
tan(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(b+(2*a*c-b^2)/(-4*a*
c+b^2)^(1/2))*2^(3/4)/c^(5/4)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)+1/4*arctanh(2^
(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(b+(2*a*c-b^2)/(-4*a*c+b^2)
^(1/2))*2^(3/4)/c^(5/4)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)
```

3.458.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.19

$$\int \frac{1}{c+\frac{a}{x^8}+\frac{b}{x^4}}\,dx=\frac{x}{c}-\frac{\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{a\log(x-\#1)+b\log(x-\#1)\#1^4}{b\#1^3+2c\#1^7}\&\right]}{4c}$$

input
```
Integrate[(c + a/x^8 + b/x^4)^(-1),x]
```

output
```
x/c - RootSum[a + b*#1^4 + c*#1^8 & , (a*Log[x - #1] + b*Log[x - #1]*#1^4)
/(b*#1^3 + 2*c*#1^7) & ]/(4*c)
```

3.458.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 337, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1679, 1703, 1752, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\frac{a}{x^8}+\frac{b}{x^4}+c}\,dx$$
$$\Big\downarrow \ 1679$$
$$\int \frac{x^8}{a+bx^4+cx^8}dx$$

$$\Big\downarrow \ 1703$$

$$\frac{x}{c}-\frac{\int \frac{bx^4+a}{cx^8+bx^4+a}dx}{c}$$

$$\Big\downarrow \ 1752$$

$$\frac{x}{c}-\frac{\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^4+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\int \frac{1}{cx^4+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{c}$$

$$\Big\downarrow \ 756$$

$$\frac{x}{c}-\frac{\frac{1}{2}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\left(-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}dx}{\sqrt{-\sqrt{b^2-4ac}-b}}-\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x^2+\sqrt{-b-\sqrt{b^2-4ac}}}dx}{\sqrt{-\sqrt{b^2-4ac}-b}}\right)+\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(-\frac{\int \frac{1}{\sqrt{\sqrt{b^2-4ac}-b}-\sqrt{2}\sqrt{c}x^2}\ldots}{\sqrt{\sqrt{b^2-4ac}-b}}\ldots\right.}{c}$$

$$\Big\downarrow \ 218$$

$$\frac{x}{c}-\frac{\frac{1}{2}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\left(-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}dx}{\sqrt{-\sqrt{b^2-4ac}-b}}-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}\right)+\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(-\frac{\int \frac{1}{\sqrt{\sqrt{b^2-4ac}-b-\ldots}}\ldots}{\sqrt{\sqrt{b^2-4ac}\ldots}}\ldots\right.}{c}$$

$$\Big\downarrow \ 221$$

$$\frac{x}{c}-\frac{\frac{1}{2}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\left(-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}\right)+\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(-\frac{\arctan\ldots}{\sqrt[4]{2}\ldots}\ldots\right.}{c}$$

input
```
Int[(c + a/x^8 + b/x^4)^(-1),x]
```

3.458. $\int \frac{1}{c+\frac{a}{x^8}+\frac{b}{x^4}}\,dx$

output
```
x/c - (((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*(-(ArcTan[(2^(1/4)*c^(1/4)*x
)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b - Sqrt[b^2 - 4*a*c]
)^(3/4))) - ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2
^(1/4)*c^(1/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4))))/2 + ((b - (b^2 - 2*a*c)/S
qrt[b^2 - 4*a*c])*(-(ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(
1/4)]/(2^(1/4)*c^(1/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))) - ArcTanh[(2^(1/4)
*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b + Sqrt[b^
2 - 4*a*c])^(3/4))))/2)/c
```

3.458.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 1679
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(
2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n]
 && LtQ[n, 0] && IntegerQ[p]
```

rule 1703
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(2*n - 1)*(d*x)^(m - 2*n + 1)*((a + b*x^n + c*x^(2*n))^(
p + 1)/(c*(m + 2*n*p + 1))), x] - Simp[d^(2*n)/(c*(m + 2*n*p + 1))   Int[(d
*x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x
^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && N
eQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0
] && IntegerQ[p]
```

3.458. $\int \frac{1}{c+\frac{a}{x^8}+\frac{b}{x^4}}\,dx$

rule 1752
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

3.458.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.16

method	result	size
default	$\frac{x}{c}+\frac{\sum_{\_R=\operatorname{RootOf}\left(c\_Z^8+\_Z^4b+a\right)}\frac{\left(-\_R^4b-a\right)\ln\left(x-\_R\right)}{2\_R^7c+\_R^3b}}{4c}$	59
risch	$\frac{x}{c}+\frac{\sum_{\_R=\operatorname{RootOf}\left(c\_Z^8+\_Z^4b+a\right)}\frac{\left(-\_R^4b-a\right)\ln\left(x-\_R\right)}{2\_R^7c+\_R^3b}}{4c}$	59

input
```
int(1/(c+a/x^8+b/x^4),x,method=_RETURNVERBOSE)
```

output
```
x/c+1/4/c*sum((-_R^4*b-a)/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*c+_Z^4
*b+a))
```

3.458.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4001 vs. $2(296) = 592$.

Time = 0.46 (sec) , antiderivative size = 4001, normalized size of antiderivative = 10.64

$$\int \frac{1}{c+\frac{a}{x^8}+\frac{b}{x^4}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c+a/x^8+b/x^4),x, algorithm="fricas")
```

output
```
1/4*(c*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*
a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2
*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)
))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7))*log((a*b^4 - 3*a^2*b^2*c + a^3*c
^2)*x + 1/2*(b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3 - (b^5*c^5 - 8*a
*b^3*c^6 + 16*a^2*b*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^
2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13
)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b
^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^
3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/
(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))) - c*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*
a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6
*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*
c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^
7)))*log((a*b^4 - 3*a^2*b^2*c + a^3*c^2)*x - 1/2*(b^6 - 7*a*b^4*c + 13*a^2
*b^2*c^2 - 4*a^3*c^3 - (b^5*c^5 - 8*a*b^3*c^6 + 16*a^2*b*c^7)*sqrt((b^8 -
6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4
*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b
^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*
b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*...
```

3.458.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{c + \frac{a}{x^8} + \frac{b}{x^4}} \, dx = \text{Timed out}$$

input
```
integrate(1/(c+a/x**8+b/x**4),x)
```

output
```
Timed out
```

3.458.7 Maxima [F]

$$\int \frac{1}{c+\frac{a}{x^8}+\frac{b}{x^4}}\,dx = \int \frac{1}{c+\frac{b}{x^4}+\frac{a}{x^8}}\,dx$$

input
```
integrate(1/(c+a/x^8+b/x^4),x, algorithm="maxima")
```

output
```
x/c - integrate((b*x^4 + a)/(c*x^8 + b*x^4 + a), x)/c
```

3.458.8 Giac [F]

$$\int \frac{1}{c+\frac{a}{x^8}+\frac{b}{x^4}}\,dx = \int \frac{1}{c+\frac{b}{x^4}+\frac{a}{x^8}}\,dx$$

input
```
integrate(1/(c+a/x^8+b/x^4),x, algorithm="giac")
```

output
```
integrate(1/(c + b/x^4 + a/x^8), x)
```

3.458.9 Mupad [B] (verification not implemented)

Time = 9.60 (sec) , antiderivative size = 10382, normalized size of antiderivative = 27.61

$$\int \frac{1}{c+\frac{a}{x^8}+\frac{b}{x^4}}\,dx = \text{Too large to display}$$

input
```
int(1/(c + a/x^8 + b/x^4),x)
```

output
```
atan(((((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c - (4*x
*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 1
20*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c
*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96
*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4
- 2048*a^4*b^3*c^5))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c
^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) -
 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*
c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) - (4*x*(a^4
*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2)
+ 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2
)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^
4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)
*1i - (((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c + (4*x
*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 1
20*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c
*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96
*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4
- 2048*a^4*b^3*c^5))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c
^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2...
```

3.458. $\int \frac{1}{c+\frac{a}{x^8}+\frac{b}{x^4}}\,dx$

3.459 $\int \frac{\sqrt{a+b\sqrt{x}+cx}}{x}\,dx$

3.459.1 Optimal result

Integrand size = 20, antiderivative size = 106

$$\int \frac{\sqrt{a+b\sqrt{x}+cx}}{x}\,dx = 2\sqrt{a+b\sqrt{x}+cx}-2\sqrt{a}\text{arctanh}\left(\frac{2a+b\sqrt{x}}{2\sqrt{a}\sqrt{a+b\sqrt{x}+cx}}\right)+\frac{b\,\text{arctanh}\left(\frac{b+2c\sqrt{x}}{2\sqrt{c}\sqrt{a+b\sqrt{x}+cx}}\right)}{\sqrt{c}}$$

output
```
-2*arctanh(1/2*(2*a+b*x^(1/2))/a^(1/2)/(a+c*x+b*x^(1/2))^(1/2))*a^(1/2)+b*
arctanh(1/2*(b+2*c*x^(1/2))/c^(1/2)/(a+c*x+b*x^(1/2))^(1/2))/c^(1/2)+2*(a+
c*x+b*x^(1/2))^(1/2)
```

3.459.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.99

$$\int \frac{\sqrt{a+b\sqrt{x}+cx}}{x}\,dx = 2\sqrt{a+b\sqrt{x}+cx}+4\sqrt{a}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{x}-\sqrt{a+b\sqrt{x}+cx}}{\sqrt{a}}\right)-\frac{b\log\left(b+2c\sqrt{x}-2\sqrt{c}\sqrt{a+b\sqrt{x}+cx}\right)}{\sqrt{c}}$$

input
```
Integrate[Sqrt[a + b*Sqrt[x] + c*x]/x,x]
```

output
```
2*Sqrt[a + b*Sqrt[x] + c*x] + 4*Sqrt[a]*ArcTanh[(Sqrt[c]*Sqrt[x] - Sqrt[a
+ b*Sqrt[x] + c*x])/Sqrt[a]] - (b*Log[b + 2*c*Sqrt[x] - 2*Sqrt[c]*Sqrt[a +
 b*Sqrt[x] + c*x]])/Sqrt[c]
```

3.459.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1693, 1162, 25, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\sqrt{x}+cx}}{x}dx$$

$$\Big\downarrow \text{1693}$$

$$2\int \frac{\sqrt{a+cx+b\sqrt{x}}}{\sqrt{x}}d\sqrt{x}$$

$$\Big\downarrow \text{1162}$$

$$2\left(\sqrt{a+b\sqrt{x}+cx}-\frac{1}{2}\int -\frac{2a+b\sqrt{x}}{\sqrt{x}\sqrt{a+cx+b\sqrt{x}}}d\sqrt{x}\right)$$

$$\Big\downarrow \text{25}$$

$$2\left(\frac{1}{2}\int \frac{2a+b\sqrt{x}}{\sqrt{x}\sqrt{a+cx+b\sqrt{x}}}d\sqrt{x}+\sqrt{a+b\sqrt{x}+cx}\right)$$

$$\Big\downarrow \text{1269}$$

$$2\left(\frac{1}{2}\left(b\int \frac{1}{\sqrt{a+cx+b\sqrt{x}}}d\sqrt{x}+2a\int \frac{1}{\sqrt{x}\sqrt{a+cx+b\sqrt{x}}}d\sqrt{x}\right)+\sqrt{a+b\sqrt{x}+cx}\right)$$

$$\Big\downarrow \text{1092}$$

$$2\left(\frac{1}{2}\left(2b\int \frac{1}{4c-x}d\frac{b+2c\sqrt{x}}{\sqrt{a+cx+b\sqrt{x}}}+2a\int \frac{1}{\sqrt{x}\sqrt{a+cx+b\sqrt{x}}}d\sqrt{x}\right)+\sqrt{a+b\sqrt{x}+cx}\right)$$

$$\Big\downarrow \text{219}$$

$$2\left(\frac{1}{2}\left(2a\int \frac{1}{\sqrt{x}\sqrt{a+cx+b\sqrt{x}}}d\sqrt{x}+\frac{b\,\text{arctanh}\left(\frac{b+2c\sqrt{x}}{2\sqrt{c}\sqrt{a+b\sqrt{x}+cx}}\right)}{\sqrt{c}}\right)+\sqrt{a+b\sqrt{x}+cx}\right)$$

$$\Big\downarrow \text{1154}$$

$$2\left(\frac{1}{2}\left(\frac{b\text{arctanh}\left(\frac{b+2c\sqrt{x}}{2\sqrt{c}\sqrt{a+b\sqrt{x}+cx}}\right)}{\sqrt{c}}-4a\int\frac{1}{4a-x}d\frac{2a+b\sqrt{x}}{\sqrt{a+cx+b\sqrt{x}}}\right)+\sqrt{a+b\sqrt{x}+cx}\right)$$

$$\Big\downarrow \text{219}$$

$$2\left(\frac{1}{2}\left(\frac{b\text{arctanh}\left(\frac{b+2c\sqrt{x}}{2\sqrt{c}\sqrt{a+b\sqrt{x}+cx}}\right)}{\sqrt{c}}-2\sqrt{a}\text{arctanh}\left(\frac{2a+b\sqrt{x}}{2\sqrt{a}\sqrt{a+b\sqrt{x}+cx}}\right)\right)+\sqrt{a+b\sqrt{x}+cx}\right)$$

input
```
Int[Sqrt[a + b*Sqrt[x] + c*x]/x,x]
```

output
```
2*(Sqrt[a + b*Sqrt[x] + c*x] + (-2*Sqrt[a]*ArcTanh[(2*a + b*Sqrt[x])/(2*Sq
rt[a]*Sqrt[a + b*Sqrt[x] + c*x])] + (b*ArcTanh[(b + 2*c*Sqrt[x])/(2*Sqrt[c
]*Sqrt[a + b*Sqrt[x] + c*x])])/Sqrt[c])/2)
```

3.459.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

3.459. $\int \frac{\sqrt{a+b\sqrt{x}+cx}}{x}\,dx$

rule 1162
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p/(e*(m + 2*p + 1))   Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d -
b*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&
!ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.459.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$2\sqrt{a+cx+b\sqrt{x}}+\frac{b\ln\left(\frac{\frac{b}{2}+c\sqrt{x}}{\sqrt{c}}+\sqrt{a+cx+b\sqrt{x}}\right)}{\sqrt{c}}-2\sqrt{a}\,\ln\left(\frac{2a+b\sqrt{x}+2\sqrt{a}\,\sqrt{a+cx+b\sqrt{x}}}{\sqrt{x}}\right)$	84
default	$2\sqrt{a+cx+b\sqrt{x}}+\frac{b\ln\left(\frac{\frac{b}{2}+c\sqrt{x}}{\sqrt{c}}+\sqrt{a+cx+b\sqrt{x}}\right)}{\sqrt{c}}-2\sqrt{a}\,\ln\left(\frac{2a+b\sqrt{x}+2\sqrt{a}\,\sqrt{a+cx+b\sqrt{x}}}{\sqrt{x}}\right)$	84

input
```
int((a+c*x+b*x^(1/2))^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
2*(a+c*x+b*x^(1/2))^(1/2)+b*ln((1/2*b+c*x^(1/2))/c^(1/2)+(a+c*x+b*x^(1/2))
^(1/2))/c^(1/2)-2*a^(1/2)*ln((2*a+b*x^(1/2)+2*a^(1/2)*(a+c*x+b*x^(1/2))^(1
/2))/x^(1/2))
```

3.459.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sqrt{x}+cx}}{x}\,dx = \text{Timed out}$$

input
```
integrate((a+c*x+b*x^(1/2))^(1/2)/x,x, algorithm="fricas")
```

output
```
Timed out
```

3.459.6 Sympy [F]

$$\int \frac{\sqrt{a+b\sqrt{x}+cx}}{x}\,dx = \int \frac{\sqrt{a+b\sqrt{x}+cx}}{x}\,dx$$

input
```
integrate((a+c*x+b*x**(1/2))**(1/2)/x,x)
```

output
```
Integral(sqrt(a + b*sqrt(x) + c*x)/x, x)
```

3.459.7 Maxima [F]

$$\int \frac{\sqrt{a+b\sqrt{x}+cx}}{x}\,dx = \int \frac{\sqrt{cx+b\sqrt{x}+a}}{x}\,dx$$

input
```
integrate((a+c*x+b*x^(1/2))^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x + b*sqrt(x) + a)/x, x)
```

3.459.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+b\sqrt{x}+cx}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+c*x+b*x^(1/2))^(1/2)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.459.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sqrt{x}+cx}}{x}\,dx = \int \frac{\sqrt{a+c\,x+b\,\sqrt{x}}}{x}\,dx$$

input
```
int((a + c*x + b*x^(1/2))^(1/2)/x,x)
```

output
```
int((a + c*x + b*x^(1/2))^(1/2)/x, x)
```

3.460 $\int \left(\frac{b^2}{4c}+b\sqrt{x}+cx\right)^2 dx$

3.460.1 Optimal result

Integrand size = 23, antiderivative size = 40

$$\int \left(\frac{b^2}{4c}+b\sqrt{x}+cx\right)^2 dx = -\frac{b\left(b+2c\sqrt{x}\right)^5}{160c^4}+\frac{\left(b+2c\sqrt{x}\right)^6}{192c^4}$$

output
```
-1/160*b*(b+2*c*x^(1/2))^5/c^4+1/192*(b+2*c*x^(1/2))^6/c^4
```

3.460.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.38

$$\int \left(\frac{b^2}{4c}+b\sqrt{x}+cx\right)^2 dx = \frac{15b^4x+80b^3cx^{3/2}+180b^2c^2x^2+192bc^3x^{5/2}+80c^4x^3}{240c^2}$$

input
```
Integrate[(b^2/(4*c) + b*Sqrt[x] + c*x)^2,x]
```

output
```
(15*b^4*x + 80*b^3*c*x^(3/2) + 180*b^2*c^2*x^2 + 192*b*c^3*x^(5/2) + 80*c^
4*x^3)/(240*c^2)
```

3.460.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.18, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1379, 774, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{b^2}{4c}+b\sqrt{x}+cx\right)^2 dx$$

$$\Big\downarrow \text{ 1379}$$

$$\frac{\int \left(\frac{b}{2}+c\sqrt{x}\right)^4 dx}{c^2}$$

$$\Big\downarrow \text{ 774}$$

$$\frac{2\int \frac{1}{16}\left(b+2c\sqrt{x}\right)^4\sqrt{x}d\sqrt{x}}{c^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \left(b+2c\sqrt{x}\right)^4\sqrt{x}d\sqrt{x}}{8c^2}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{\int \left(\frac{\left(b+2c\sqrt{x}\right)^5}{2c}-\frac{b\left(b+2c\sqrt{x}\right)^4}{2c}\right)d\sqrt{x}}{8c^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{\left(b+2c\sqrt{x}\right)^6}{24c^2}-\frac{b\left(b+2c\sqrt{x}\right)^5}{20c^2}}{8c^2}$$

input
```
Int[(b^2/(4*c) + b*Sqrt[x] + c*x)^2,x]
```

output
```
(-1/20*(b*(b + 2*c*Sqrt[x])^5)/c^2 + (b + 2*c*Sqrt[x])^6/(24*c^2))/(8*c^2)
```

3.460.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 1379
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/
c^p   Int[(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n
2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p] && NeQ[p, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.460.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.25

method	result	size
derivativedivides	$\frac{\frac{8c^4x^3}{3}+\frac{32b\,c^3x^{\frac{5}{2}}}{5}+6b^2c^2x^2+\frac{8b^3c\,x^{\frac{3}{2}}}{3}+\frac{b^4x}{2}}{8c^2}$	50
default	$\frac{b^2x^2}{2}+\frac{b\left(\frac{8c^2x^{\frac{5}{2}}}{5}+\frac{2b^2x^{\frac{3}{2}}}{3}\right)}{2c}+\frac{\left(\frac{b^2}{c}+4cx\right)^3}{192c}$	52
trager	$\frac{\frac{\left(16c^4x^2+36b^2x\,c^2+16c^4x+3b^4+36b^2c^2+16c^4\right)(x-1)}{3}+\frac{16bc\,x^{\frac{3}{2}}\left(12c^2x+5b^2\right)}{15}}{16c^2}$	73

input
```
int((1/4/c*b^2+c*x+b*x^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
1/8/c^2*(8/3*c^4*x^3+32/5*b*c^3*x^(5/2)+6*b^2*c^2*x^2+8/3*b^3*c*x^(3/2)+1/
2*b^4*x)
```

3.460.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.32

$$\int \left(\frac{b^2}{4c}+b\sqrt{x}+cx\right)^2 dx = \frac{80\,c^4x^3+180\,b^2c^2x^2+15\,b^4x+16\,(12\,bc^3x^2+5\,b^3cx)\sqrt{x}}{240\,c^2}$$

input
```
integrate((1/4*b^2/c+c*x+b*x^(1/2))^2,x, algorithm="fricas")
```

output
```
1/240*(80*c^4*x^3 + 180*b^2*c^2*x^2 + 15*b^4*x + 16*(12*b*c^3*x^2 + 5*b^3*
c*x)*sqrt(x))/c^2
```

3.460.6 Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.28

$$\int \left(\frac{b^2}{4c}+b\sqrt{x}+cx\right)^2 dx = \frac{b^4x}{16c^2}+\frac{b^3x^{\frac{3}{2}}}{3c}+\frac{3b^2x^2}{4}+\frac{4bcx^{\frac{5}{2}}}{5}+\frac{c^2x^3}{3}$$

input
```
integrate((1/4*b**2/c+c*x+b*x**(1/2))**2,x)
```

output
```
b**4*x/(16*c**2) + b**3*x**(3/2)/(3*c) + 3*b**2*x**2/4 + 4*b*c*x**(5/2)/5
+ c**2*x**3/3
```

3.460.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.35

$$\int \left(\frac{b^2}{4c}+b\sqrt{x}+cx\right)^2 dx = \frac{1}{3}\,c^2x^3+\frac{4}{5}\,bcx^{\frac{5}{2}}+\frac{1}{2}\,b^2x^2+\frac{b^4x}{16\,c^2}+\frac{\left(3\,cx^2+4\,bx^{\frac{3}{2}}\right)b^2}{12\,c}$$

input
```
integrate((1/4*b^2/c+c*x+b*x^(1/2))^2,x, algorithm="maxima")
```

output
```
1/3*c^2*x^3 + 4/5*b*c*x^(5/2) + 1/2*b^2*x^2 + 1/16*b^4*x/c^2 + 1/12*(3*c*x
^2 + 4*b*x^(3/2))*b^2/c
```

3.460.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.22

$$\int \left(\frac{b^2}{4c}+b\sqrt{x}+cx\right)^2 dx = \frac{80\,c^4x^3+192\,bc^3x^{\frac{5}{2}}+180\,b^2c^2x^2+80\,b^3cx^{\frac{3}{2}}+15\,b^4x}{240\,c^2}$$

input
```
integrate((1/4*b^2/c+c*x+b*x^(1/2))^2,x, algorithm="giac")
```

output
```
1/240*(80*c^4*x^3 + 192*b*c^3*x^(5/2) + 180*b^2*c^2*x^2 + 80*b^3*c*x^(3/2)
 + 15*b^4*x)/c^2
```

3.460.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10

$$\int \left(\frac{b^2}{4c}+b\sqrt{x}+cx\right)^2 dx = \frac{3\,b^2\,x^2}{4}+\frac{c^2\,x^3}{3}+\frac{b^4\,x}{16\,c^2}+\frac{b^3\,x^{3/2}}{3\,c}+\frac{4\,b\,c\,x^{5/2}}{5}$$

input
```
int((c*x + b*x^(1/2) + b^2/(4*c))^2,x)
```

output
```
(3*b^2*x^2)/4 + (c^2*x^3)/3 + (b^4*x)/(16*c^2) + (b^3*x^(3/2))/(3*c) + (4*
b*c*x^(5/2))/5
```

3.461 $\int \frac{1}{\sqrt{a^2+2ab\sqrt{x}+b^2x}}\,dx$

3.461.1 Optimal result

Integrand size = 22, antiderivative size = 75

$$\int \frac{1}{\sqrt{a^2+2ab\sqrt{x}+b^2x}}\,dx=\frac{2\sqrt{a^2+2ab\sqrt{x}+b^2x}}{b^2}-\frac{2a\left(a+b\sqrt{x}\right)\log\left(a+b\sqrt{x}\right)}{b^2\sqrt{a^2+2ab\sqrt{x}+b^2x}}$$

output
```
-2*a*ln(a+b*x^(1/2))*(a+b*x^(1/2))/b^2/(a^2+b^2*x+2*a*b*x^(1/2))^(1/2)+2*(
a^2+b^2*x+2*a*b*x^(1/2))^(1/2)/b^2
```

3.461.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt{a^2+2ab\sqrt{x}+b^2x}}\,dx=\frac{2\left(a+b\sqrt{x}\right)\left(b\sqrt{x}-a\log\left(a+b\sqrt{x}\right)\right)}{b^2\sqrt{\left(a+b\sqrt{x}\right)^2}}$$

input
```
Integrate[1/Sqrt[a^2 + 2*a*b*Sqrt[x] + b^2*x],x]
```

output
```
(2*(a + b*Sqrt[x])*(b*Sqrt[x] - a*Log[a + b*Sqrt[x]]))/(b^2*Sqrt[(a + b*Sq
rt[x])^2])
```

3.461.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1680, 1100, 1079, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a^2+2ab\sqrt{x}+b^2x}}dx$$

$$\Big\downarrow \text{1680}$$

$$2\int \frac{\sqrt{x}}{\sqrt{a^2+2b\sqrt{x}a+b^2x}}d\sqrt{x}$$

$$\Big\downarrow \text{1100}$$

$$2\left(\frac{\sqrt{a^2+2ab\sqrt{x}+b^2x}}{b^2}-\frac{a\int \frac{1}{\sqrt{a^2+2b\sqrt{x}a+b^2x}}d\sqrt{x}}{b}\right)$$

$$\Big\downarrow \text{1079}$$

$$2\left(\frac{\sqrt{a^2+2ab\sqrt{x}+b^2x}}{b^2}-\frac{a\left(a+b\sqrt{x}\right)\int \frac{1}{\sqrt{x}b^2+ab}d\sqrt{x}}{\sqrt{a^2+2ab\sqrt{x}+b^2x}}\right)$$

$$\Big\downarrow \text{16}$$

$$2\left(\frac{\sqrt{a^2+2ab\sqrt{x}+b^2x}}{b^2}-\frac{a\left(a+b\sqrt{x}\right)\log\left(a+b\sqrt{x}\right)}{b^2\sqrt{a^2+2ab\sqrt{x}+b^2x}}\right)$$

input
```
Int[1/Sqrt[a^2 + 2*a*b*Sqrt[x] + b^2*x],x]
```

output
```
2*(Sqrt[a^2 + 2*a*b*Sqrt[x] + b^2*x]/b^2 - (a*(a + b*Sqrt[x])*Log[a + b*Sq
rt[x]])/(b^2*Sqrt[a^2 + 2*a*b*Sqrt[x] + b^2*x]))
```

3.461.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1100
```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

rule 1680
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*
n))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fr
actionQ[n]
```

3.461.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.55

method	result	size
derivativedivides	$-\frac{2(a+b\sqrt{x})\left(a\ln\left(a+b\sqrt{x}\right)-b\sqrt{x}\right)}{\sqrt{\left(a+b\sqrt{x}\right)^{2}}\,b^{2}}$	41
default	$\frac{(a+b\sqrt{x})\left(2b\sqrt{x}+a\ln\left(b\sqrt{x}-a\right)-a\ln\left(a+b\sqrt{x}\right)-a\ln\left(b^{2}x-a^{2}\right)\right)}{\sqrt{a^{2}+b^{2}x+2ab\sqrt{x}}\,b^{2}}$	75

input
```
int(1/(a^2+b^2*x+2*a*b*x^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(a+b*x^(1/2))*(a*ln(a+b*x^(1/2))-b*x^(1/2))/((a+b*x^(1/2))^2)^(1/2)/b^2
```

3.461.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a^2+2ab\sqrt{x}+b^2x}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a^2+b^2*x+2*a*b*x^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.461.6 Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.65

$$\int \frac{1}{\sqrt{a^2+2ab\sqrt{x}+b^2x}}\,dx = 2\left(\begin{cases} -\frac{a\left(\frac{a}{b}+\sqrt{x}\right)\log\left(\frac{a}{b}+\sqrt{x}\right)}{b\sqrt{b^2\left(\frac{a}{b}+\sqrt{x}\right)^2}}+\frac{\sqrt{a^2+2ab\sqrt{x}+b^2x}}{b^2} & \text{for } b^2\neq 0 \\ \frac{-a^2\sqrt{a^2+2ab\sqrt{x}}+\frac{\left(a^2+2ab\sqrt{x}\right)^{\frac{3}{2}}}{3}}{2a^2b^2} & \text{for } ab\neq 0 \\ \frac{x}{2\sqrt{a^2}} & \text{otherwise} \end{cases}\right)$$

input
```
integrate(1/(a**2+b**2*x+2*a*b*x**(1/2))**(1/2),x)
```

output
```
2*Piecewise((-a*(a/b + sqrt(x))*log(a/b + sqrt(x))/(b*sqrt(b**2*(a/b + sqr
t(x))**2)) + sqrt(a**2 + 2*a*b*sqrt(x) + b**2*x)/b**2, Ne(b**2, 0)), ((-a*
*2*sqrt(a**2 + 2*a*b*sqrt(x)) + (a**2 + 2*a*b*sqrt(x))**(3/2)/3)/(2*a**2*b
**2), Ne(a*b, 0)), (x/(2*sqrt(a**2)), True))
```

3.461.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.31

$$\int \frac{1}{\sqrt{a^2+2ab\sqrt{x}+b^2x}}\,dx = -\frac{2\,a\log\left(b\sqrt{x}+a\right)}{b^2}+\frac{2\,\sqrt{x}}{b}$$

input
```
integrate(1/(a^2+b^2*x+2*a*b*x^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
-2*a*log(b*sqrt(x) + a)/b^2 + 2*sqrt(x)/b
```

3.461.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.60

$$\int \frac{1}{\sqrt{a^2+2ab\sqrt{x}+b^2x}}\,dx = -\frac{2\,|a|\log\left(\left|\sqrt{b^2x}\mathrm{sgn}(a)\,\mathrm{sgn}(b)+|a|\right|\right)}{b^2}+\frac{2\,\sqrt{b^2x}}{b^2\mathrm{sgn}\,(a)\,\mathrm{sgn}\,(b)}$$

input
```
integrate(1/(a^2+b^2*x+2*a*b*x^(1/2))^(1/2),x, algorithm="giac")
```

output
```
-2*abs(a)*log(abs(sqrt(b^2*x)*sgn(a)*sgn(b) + abs(a)))/b^2 + 2*sqrt(b^2*x)
/(b^2*sgn(a)*sgn(b))
```

3.461.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a^2+2ab\sqrt{x}+b^2x}}\,dx = \int \frac{1}{\sqrt{b^2\,x+a^2+2\,a\,b\,\sqrt{x}}}\,dx$$

input
```
int(1/(b^2*x + a^2 + 2*a*b*x^(1/2))^(1/2),x)
```

output
```
int(1/(b^2*x + a^2 + 2*a*b*x^(1/2))^(1/2), x)
```

3.462 $\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{7/2} dx$

3.462.1 Optimal result

Integrand size = 26, antiderivative size = 137

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{7/2} dx = \frac{3a^2\left(a+b\sqrt[3]{x}\right)^7\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{8b^3} - \frac{2a\left(a+b\sqrt[3]{x}\right)^8\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{3b^3} + \frac{3\left(a+b\sqrt[3]{x}\right)^9\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{10b^3}$$

output
```
3/8*a^2*(a+b*x^(1/3))^7*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)/b^3-2/3*a*(a
+b*x^(1/3))^8*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)/b^3+3/10*(a+b*x^(1/3))
^9*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)/b^3
```

3.462.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.85

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{7/2} dx = \frac{\left(\left(a+b\sqrt[3]{x}\right)^2\right)^{7/2}\left(120a^7x+630a^6bx^{4/3}+1512a^5b^2x^{5/3}+2100a^4b^3x^2+1800a^3b^4x^{7/3}+94\right.}{120\left(a+b\sqrt[3]{x}\right)^7}$$

input
```
Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(7/2),x]
```

output
```
(((a + b*x^(1/3))^2)^(7/2)*(120*a^7*x + 630*a^6*b*x^(4/3) + 1512*a^5*b^2*x
^(5/3) + 2100*a^4*b^3*x^2 + 1800*a^3*b^4*x^(7/3) + 945*a^2*b^5*x^(8/3) + 2
80*a*b^6*x^3 + 36*b^7*x^(10/3)))/(120*(a + b*x^(1/3))^7)
```

3.462.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.78, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1384, 774, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{7/2} dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\int \left(\sqrt[3]{x}b^2+ab\right)^7dx}{ab^7+b^8\sqrt[3]{x}}$$

$$\Big\downarrow \text{774}$$

$$\frac{3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\int b^7\left(a+b\sqrt[3]{x}\right)^7x^{2/3}d\sqrt[3]{x}}{ab^7+b^8\sqrt[3]{x}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3b^7\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\int \left(a+b\sqrt[3]{x}\right)^7x^{2/3}d\sqrt[3]{x}}{ab^7+b^8\sqrt[3]{x}}$$

$$\Big\downarrow \text{49}$$

$$\frac{3b^7\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\int \left(\frac{\left(a+b\sqrt[3]{x}\right)^9}{b^2}-\frac{2a\left(a+b\sqrt[3]{x}\right)^8}{b^2}+\frac{a^2\left(a+b\sqrt[3]{x}\right)^7}{b^2}\right)d\sqrt[3]{x}}{ab^7+b^8\sqrt[3]{x}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3b^7\left(\frac{a^2\left(a+b\sqrt[3]{x}\right)^8}{8b^3}+\frac{\left(a+b\sqrt[3]{x}\right)^{10}}{10b^3}-\frac{2a\left(a+b\sqrt[3]{x}\right)^9}{9b^3}\right)\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{ab^7+b^8\sqrt[3]{x}}$$

input
```
Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(7/2),x]
```

3.462. $\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{7/2} dx$

output
```
(3*b^7*((a^2*(a + b*x^(1/3))^8)/(8*b^3) - (2*a*(a + b*x^(1/3))^9)/(9*b^3)
+ (a + b*x^(1/3))^10/(10*b^3))*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])/(a
*b^7 + b^8*x^(1/3))
```

3.462.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.462.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.72

method	result
derivativedivides	$\frac{\left(\left(a+b\,x^{\frac{1}{3}}\right)^{2}\right)^{\frac{7}{2}} x\left(36b^{7}x^{\frac{7}{3}}+280a\,b^{6}x^{2}+945a^{2}b^{5}x^{\frac{5}{3}}+1800b^{4}a^{3}x^{\frac{4}{3}}+2100a^{4}b^{3}x+1512b^{2}a^{5}x^{\frac{2}{3}}+630a^{6}b\,x^{\frac{1}{3}}+120a^{7}\right)}{120\left(a+b\,x^{\frac{1}{3}}\right)^{7}}$
default	$\frac{\left(a^{2}+2ab\,x^{\frac{1}{3}}+b^{2}x^{\frac{2}{3}}\right)^{\frac{7}{2}}\left(36b^{7}x^{\frac{10}{3}}+945a^{2}b^{5}x^{\frac{8}{3}}+1800b^{4}a^{3}x^{\frac{7}{3}}+1512b^{2}a^{5}x^{\frac{5}{3}}+630a^{6}b\,x^{\frac{4}{3}}+280a\,b^{6}x^{3}+2100a^{4}b^{3}x^{2}+120a\right.}{120\left(a+b\,x^{\frac{1}{3}}\right)^{7}}$

input
```
int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/120*((a+b*x^(1/3))^2)^(7/2)*x*(36*b^7*x^(7/3)+280*a*b^6*x^2+945*a^2*b^5*
x^(5/3)+1800*b^4*a^3*x^(4/3)+2100*a^4*b^3*x+1512*b^2*a^5*x^(2/3)+630*a^6*b
*x^(1/3)+120*a^7)/(a+b*x^(1/3))^7
```

3.462.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.61

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{7/2}\,dx=\frac{7}{3}\,ab^6x^3+\frac{35}{2}\,a^4b^3x^2+a^7x+\frac{63}{40}\left(5\,a^2b^5x^2+8\,a^5b^2x\right)x^{\frac{2}{3}}+\frac{3}{20}\left(2\,b^7x^3+100\,a^3b^4x^2+35\,a^6bx\right)x^{\frac{1}{3}}$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(7/2),x, algorithm="fricas")
```

output
```
7/3*a*b^6*x^3 + 35/2*a^4*b^3*x^2 + a^7*x + 63/40*(5*a^2*b^5*x^2 + 8*a^5*b^
2*x)*x^(2/3) + 3/20*(2*b^7*x^3 + 100*a^3*b^4*x^2 + 35*a^6*b*x)*x^(1/3)
```

3.462.6 Sympy [A] (verification not implemented)

Time = 5.62 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.70

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{7/2}\,dx=3\left(\begin{cases}\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}}\left(\frac{a^9}{360b^3}-\frac{a^8\sqrt[3]{x}}{360b^2}+\frac{a^7x^{\frac{2}{3}}}{360b}+\frac{119a^6x}{360}+\frac{511a^5bx^{\frac{4}{3}}}{360}+\frac{1001a^4b^2x^{\frac{5}{3}}}{360}+\frac{1099a^3 b}{360} \right. \\ \frac{\frac{a^4\left(a^2+2ab\sqrt[3]{x}\right)^{\frac{9}{2}}}{9}-\frac{2a^2\left(a^2+2ab\sqrt[3]{x}\right)^{\frac{11}{2}}}{11}+\frac{\left(a^2+2ab\sqrt[3]{x}\right)^{\frac{13}{2}}}{13}}{4a^3b^3} \\ \frac{x\left(a^2\right)^{\frac{7}{2}}}{3}\end{cases}\right.$$

input
```
integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(7/2),x)
```

output
```
3*Piecewise((sqrt(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))*(a**9/(360*b**3)
- a**8*x**(1/3)/(360*b**2) + a**7*x**(2/3)/(360*b) + 119*a**6*x/360 + 511*
a**5*b*x**(4/3)/360 + 1001*a**4*b**2*x**(5/3)/360 + 1099*a**3*b**3*x**2/36
0 + 701*a**2*b**4*x**(7/3)/360 + 61*a*b**5*x**(8/3)/90 + b**6*x**3/10), Ne
(b**2, 0)), ((a**4*(a**2 + 2*a*b*x**(1/3))**(9/2)/9 - 2*a**2*(a**2 + 2*a*b
*x**(1/3))**(11/2)/11 + (a**2 + 2*a*b*x**(1/3))**(13/2)/13)/(4*a**3*b**3),
 Ne(a*b, 0)), (x*(a**2)**(7/2)/3, True))
```

3.462.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.83

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{7/2}\,dx = \frac{3\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^{\frac{7}{2}}a^2x^{\frac{1}{3}}}{8\,b^2} + \frac{3\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^{\frac{7}{2}}a^3}{8\,b^3} + \frac{3\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^{\frac{9}{2}}x^{\frac{1}{3}}}{10\,b^2} - \frac{11\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^{\frac{9}{2}}a}{30\,b^3}$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(7/2),x, algorithm="maxima")
```

output
```
3/8*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^(7/2)*a^2*x^(1/3)/b^2 + 3/8*(b^2*x
^(2/3) + 2*a*b*x^(1/3) + a^2)^(7/2)*a^3/b^3 + 3/10*(b^2*x^(2/3) + 2*a*b*x^
(1/3) + a^2)^(9/2)*x^(1/3)/b^2 - 11/30*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)
^(9/2)*a/b^3
```

3.462.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.02

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{7/2}\,dx = \frac{3}{10}\,b^7x^{\frac{10}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right) + \frac{7}{3}\,ab^6x^3\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right) + \frac{63}{8}\,a^2b^5x^{\frac{8}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right) + 15\,a^3b^4x^{\frac{7}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right) + \frac{35}{2}\,a^4b^3x^2\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right) + \frac{63}{5}\,a^5b^2x^{\frac{5}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right) + \frac{21}{4}\,a^6bx^{\frac{4}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right) + a^7x\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(7/2),x, algorithm="giac")
```

output
```
3/10*b^7*x^(10/3)*sgn(b*x^(1/3) + a) + 7/3*a*b^6*x^3*sgn(b*x^(1/3) + a) +
63/8*a^2*b^5*x^(8/3)*sgn(b*x^(1/3) + a) + 15*a^3*b^4*x^(7/3)*sgn(b*x^(1/3)
 + a) + 35/2*a^4*b^3*x^2*sgn(b*x^(1/3) + a) + 63/5*a^5*b^2*x^(5/3)*sgn(b*x
^(1/3) + a) + 21/4*a^6*b*x^(4/3)*sgn(b*x^(1/3) + a) + a^7*x*sgn(b*x^(1/3)
+ a)
```

3.462.9 Mupad [F(-1)]

Timed out.

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{7/2}\,dx=\int \left(a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}\right)^{7/2}dx$$

input
```
int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(7/2),x)
```

output
```
int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(7/2), x)
```

3.463 $\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}\,dx$

3.463.1 Optimal result

Integrand size = 26, antiderivative size = 137

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}\,dx=\frac{a^2\left(a+b\sqrt[3]{x}\right)^5\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{2b^3}-\frac{6a\left(a+b\sqrt[3]{x}\right)^6\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{7b^3}+\frac{3\left(a+b\sqrt[3]{x}\right)^7\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{8b^3}$$

output
```
1/2*a^2*(a+b*x^(1/3))^5*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)/b^3-6/7*a*(a
+b*x^(1/3))^6*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)/b^3+3/8*(a+b*x^(1/3))^
7*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)/b^3
```

3.463.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.68

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}\,dx=\frac{\left(\left(a+b\sqrt[3]{x}\right)^2\right)^{5/2}\left(56a^5x+210a^4bx^{4/3}+336a^3b^2x^{5/3}+280a^2b^3x^2+120ab^4x^{7/3}+21b^5x^{8/3}\right.}{56\left(a+b\sqrt[3]{x}\right)^5}$$

input
```
Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(5/2),x]
```

output
```
(((a + b*x^(1/3))^2)^(5/2)*(56*a^5*x + 210*a^4*b*x^(4/3) + 336*a^3*b^2*x^(
5/3) + 280*a^2*b^3*x^2 + 120*a*b^4*x^(7/3) + 21*b^5*x^(8/3)))/(56*(a + b*x
^(1/3))^5)
```

3.463.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.78, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1384, 774, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\int \left(\sqrt[3]{x}b^2+ab\right)^5dx}{ab^5+b^6\sqrt[3]{x}}$$

$$\Big\downarrow \text{774}$$

$$\frac{3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\int b^5\left(a+b\sqrt[3]{x}\right)^5x^{2/3}d\sqrt[3]{x}}{ab^5+b^6\sqrt[3]{x}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3b^5\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\int \left(a+b\sqrt[3]{x}\right)^5x^{2/3}d\sqrt[3]{x}}{ab^5+b^6\sqrt[3]{x}}$$

$$\Big\downarrow \text{49}$$

$$\frac{3b^5\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\int \left(\frac{\left(a+b\sqrt[3]{x}\right)^7}{b^2}-\frac{2a\left(a+b\sqrt[3]{x}\right)^6}{b^2}+\frac{a^2\left(a+b\sqrt[3]{x}\right)^5}{b^2}\right)d\sqrt[3]{x}}{ab^5+b^6\sqrt[3]{x}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3b^5\left(\frac{a^2\left(a+b\sqrt[3]{x}\right)^6}{6b^3}+\frac{\left(a+b\sqrt[3]{x}\right)^8}{8b^3}-\frac{2a\left(a+b\sqrt[3]{x}\right)^7}{7b^3}\right)\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{ab^5+b^6\sqrt[3]{x}}$$

input
```
Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(5/2),x]
```

3.463. $\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}dx$

output
```
(3*b^5*((a^2*(a + b*x^(1/3))^6)/(6*b^3) - (2*a*(a + b*x^(1/3))^7)/(7*b^3)
+ (a + b*x^(1/3))^8/(8*b^3))*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])/(a*b
^5 + b^6*x^(1/3))
```

3.463.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.463.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.55

method	result	size
derivativedivides	$\frac{\left(\left(a+b\,x^{\frac{1}{3}}\right)^{2}\right)^{\frac{5}{2}} x\left(21 b^{5} x^{\frac{5}{3}}+120 b^{4} a\,x^{\frac{4}{3}}+280 a^{2} b^{3} x+336 a^{3} b^{2} x^{\frac{2}{3}}+210 b\,a^{4} x^{\frac{1}{3}}+56 a^{5}\right)}{56\left(a+b\,x^{\frac{1}{3}}\right)^{5}}$	76
default	$\frac{\left(a^{2}+2 a b\,x^{\frac{1}{3}}+b^{2} x^{\frac{2}{3}}\right)^{\frac{5}{2}}\left(21 b^{5} x^{\frac{8}{3}}+120 b^{4} a\,x^{\frac{7}{3}}+336 a^{3} b^{2} x^{\frac{5}{3}}+210 b\,a^{4} x^{\frac{4}{3}}+280 a^{2} b^{3} x^{2}+56 a^{5} x\right)}{56\left(a+b\,x^{\frac{1}{3}}\right)^{5}}$	87

3.463. $\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}\,dx$

input
```
int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/56*((a+b*x^(1/3))^2)^(5/2)*x*(21*b^5*x^(5/3)+120*b^4*a*x^(4/3)+280*a^2*b
^3*x+336*a^3*b^2*x^(2/3)+210*b*a^4*x^(1/3)+56*a^5)/(a+b*x^(1/3))^5
```

3.463.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.45

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}\,dx = 5\,a^2b^3x^2+a^5x + \frac{3}{8}\left(b^5x^2+16\,a^3b^2x\right)x^{\frac{2}{3}} + \frac{15}{28}\left(4\,ab^4x^2+7\,a^4bx\right)x^{\frac{1}{3}}$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(5/2),x, algorithm="fricas")
```

output
```
5*a^2*b^3*x^2 + a^5*x + 3/8*(b^5*x^2 + 16*a^3*b^2*x)*x^(2/3) + 15/28*(4*a*
b^4*x^2 + 7*a^4*b*x)*x^(1/3)
```

3.463.6 Sympy [A] (verification not implemented)

Time = 1.43 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.49

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}\,dx = 3\left(\begin{cases} \sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}}\left(\frac{a^7}{168b^3}-\frac{a^6\sqrt[3]{x}}{168b^2}+\frac{a^5x^{\frac{2}{3}}}{168b}+\frac{55a^4x}{168}+\frac{155a^3bx^{\frac{4}{3}}}{168}+\frac{181a^2b^2x^{\frac{5}{3}}}{168}+\frac{33ab^3x^2}{56} \right. \\ \frac{\frac{a^4\left(a^2+2ab\sqrt[3]{x}\right)^{\frac{7}{2}}}{7}-\frac{2a^2\left(a^2+2ab\sqrt[3]{x}\right)^{\frac{9}{2}}}{9}+\frac{\left(a^2+2ab\sqrt[3]{x}\right)^{\frac{11}{2}}}{11}}{4a^3b^3} \\ \frac{x\left(a^2\right)^{\frac{5}{2}}}{3} \end{cases}\right.$$

input
```
integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(5/2),x)
```

output
```
3*Piecewise((sqrt(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))*(a**7/(168*b**3)
- a**6*x**(1/3)/(168*b**2) + a**5*x**(2/3)/(168*b) + 55*a**4*x/168 + 155*a
**3*b*x**(4/3)/168 + 181*a**2*b**2*x**(5/3)/168 + 33*a*b**3*x**2/56 + b**4
*x**(7/3)/8), Ne(b**2, 0)), ((a**4*(a**2 + 2*a*b*x**(1/3))**(7/2)/7 - 2*a*
*2*(a**2 + 2*a*b*x**(1/3))**(9/2)/9 + (a**2 + 2*a*b*x**(1/3))**(11/2)/11)/
(4*a**3*b**3), Ne(a*b, 0)), (x*(a**2)**(5/2)/3, True))
```

3.463.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.83

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}\,dx=\frac{\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^{\frac{5}{2}}a^2x^{\frac{1}{3}}}{2\,b^2}+\frac{\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^{\frac{5}{2}}a^3}{2\,b^3}+\frac{3\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^{\frac{7}{2}}x^{\frac{1}{3}}}{8\,b^2}-\frac{27\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^{\frac{7}{2}}a}{56\,b^3}$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(5/2),x, algorithm="maxima")
```

output
```
1/2*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^(5/2)*a^2*x^(1/3)/b^2 + 1/2*(b^2*x
^(2/3) + 2*a*b*x^(1/3) + a^2)^(5/2)*a^3/b^3 + 3/8*(b^2*x^(2/3) + 2*a*b*x^(
1/3) + a^2)^(7/2)*x^(1/3)/b^2 - 27/56*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^
(7/2)*a/b^3
```

3.463.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.74

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}\,dx=\frac{3}{8}\,b^5x^{\frac{8}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)+\frac{15}{7}\,ab^4x^{\frac{7}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)+5\,a^2b^3x^2\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)+6\,a^3b^2x^{\frac{5}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)+\frac{15}{4}\,a^4bx^{\frac{4}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)+a^5x\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(5/2),x, algorithm="giac")
```

output
```
3/8*b^5*x^(8/3)*sgn(b*x^(1/3) + a) + 15/7*a*b^4*x^(7/3)*sgn(b*x^(1/3) + a)
 + 5*a^2*b^3*x^2*sgn(b*x^(1/3) + a) + 6*a^3*b^2*x^(5/3)*sgn(b*x^(1/3) + a)
 + 15/4*a^4*b*x^(4/3)*sgn(b*x^(1/3) + a) + a^5*x*sgn(b*x^(1/3) + a)
```

3.463.9 Mupad [F(-1)]

Timed out.

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}\,dx=\int \left(a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}\right)^{5/2}dx$$

input
```
int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(5/2),x)
```

output
```
int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(5/2), x)
```

3.464 $\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}\,dx$

3.464.1 Optimal result

Integrand size = 26, antiderivative size = 137

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}\,dx=\frac{3a^2\left(a+b\sqrt[3]{x}\right)^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{4b^3}-\frac{6a\left(a+b\sqrt[3]{x}\right)^4\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{5b^3}+\frac{\left(a+b\sqrt[3]{x}\right)^5\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{2b^3}$$

output
```
3/4*a^2*(a+b*x^(1/3))^3*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)/b^3-6/5*a*(a
+b*x^(1/3))^4*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)/b^3+1/2*(a+b*x^(1/3))^
5*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)/b^3
```

3.464.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.49

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}\,dx=\frac{\left(\left(a+b\sqrt[3]{x}\right)^2\right)^{3/2}\left(20a^3x+45a^2bx^{4/3}+36ab^2x^{5/3}+10b^3x^2\right)}{20\left(a+b\sqrt[3]{x}\right)^3}$$

input
```
Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(3/2),x]
```

output
```
(((a + b*x^(1/3))^2)^(3/2)*(20*a^3*x + 45*a^2*b*x^(4/3) + 36*a*b^2*x^(5/3)
 + 10*b^3*x^2))/(20*(a + b*x^(1/3))^3)
```

3.464.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.68, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1384, 774, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\int \left(\sqrt[3]{x}b^2+ab\right)^3dx}{ab^3+b^4\sqrt[3]{x}}$$

$$\Big\downarrow \text{774}$$

$$\frac{3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\int b^3\left(a+b\sqrt[3]{x}\right)^3x^{2/3}d\sqrt[3]{x}}{ab^3+b^4\sqrt[3]{x}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3b^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\int \left(a+b\sqrt[3]{x}\right)^3x^{2/3}d\sqrt[3]{x}}{ab^3+b^4\sqrt[3]{x}}$$

$$\Big\downarrow \text{49}$$

$$\frac{3b^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\int \left(x^{2/3}a^3+3bxa^2+3b^2x^{4/3}a+b^3x^{5/3}\right)d\sqrt[3]{x}}{ab^3+b^4\sqrt[3]{x}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3b^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(\frac{a^3x}{3}+\frac{3}{4}a^2bx^{4/3}+\frac{3}{5}ab^2x^{5/3}+\frac{b^3x^2}{6}\right)}{ab^3+b^4\sqrt[3]{x}}$$

input
```
Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(3/2),x]
```

output
```
(3*b^3*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]*((a^3*x)/3 + (3*a^2*b*x^(4/
3))/4 + (3*a*b^2*x^(5/3))/5 + (b^3*x^2)/6))/(a*b^3 + b^4*x^(1/3))
```

3.464.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.464.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.39

method	result	size
derivativedivides	$\frac{\left(\left(a+b\,x^{\frac{1}{3}}\right)^{2}\right)^{\frac{3}{2}} x \left(10 b^{3} x+36 b^{2} a\,x^{\frac{2}{3}}+45 a^{2} b\,x^{\frac{1}{3}}+20 a^{3}\right)}{20 \left(a+b\,x^{\frac{1}{3}}\right)^{3}}$	54
default	$\frac{\left(a^{2}+2 a b\,x^{\frac{1}{3}}+b^{2} x^{\frac{2}{3}}\right)^{\frac{3}{2}} \left(36 b^{2} a\,x^{\frac{5}{3}}+45 a^{2} b\,x^{\frac{4}{3}}+10 b^{3} x^{2}+20 a^{3} x\right)}{20 \left(a+b\,x^{\frac{1}{3}}\right)^{3}}$	65

input
```
int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/20*((a+b*x^(1/3))^2)^(3/2)*x*(10*b^3*x+36*b^2*a*x^(2/3)+45*a^2*b*x^(1/3)
+20*a^3)/(a+b*x^(1/3))^3
```

3.464.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.23

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}\,dx=\frac{1}{2}\,b^3x^2+\frac{9}{5}\,ab^2x^{\frac{5}{3}}+\frac{9}{4}\,a^2bx^{\frac{4}{3}}+a^3x$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(3/2),x, algorithm="fricas")
```

output
```
1/2*b^3*x^2 + 9/5*a*b^2*x^(5/3) + 9/4*a^2*b*x^(4/3) + a^3*x
```

3.464.6 Sympy [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.28

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}\,dx=3\left(\begin{cases}\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}}\left(\frac{a^5}{60b^3}-\frac{a^4\sqrt[3]{x}}{60b^2}+\frac{a^3x^{\frac{2}{3}}}{60b}+\frac{19a^2x}{60}+\frac{13abx^{\frac{4}{3}}}{30}+\frac{b^2x^{\frac{5}{3}}}{6}\right) & \text{for } b^2\neq 0\\ \frac{\frac{a^4\left(a^2+2ab\sqrt[3]{x}\right)^{\frac{5}{2}}}{5}-\frac{2a^2\left(a^2+2ab\sqrt[3]{x}\right)^{\frac{7}{2}}}{7}+\frac{\left(a^2+2ab\sqrt[3]{x}\right)^{\frac{9}{2}}}{9}}{4a^3b^3} & \text{for } ab\neq 0\\ \frac{x(a^2)^{\frac{3}{2}}}{3} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(3/2),x)
```

output
```
3*Piecewise((sqrt(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))*(a**5/(60*b**3) -
 a**4*x**(1/3)/(60*b**2) + a**3*x**(2/3)/(60*b) + 19*a**2*x/60 + 13*a*b*x*
*(4/3)/30 + b**2*x**(5/3)/6), Ne(b**2, 0)), ((a**4*(a**2 + 2*a*b*x**(1/3))
**(5/2)/5 - 2*a**2*(a**2 + 2*a*b*x**(1/3))**(7/2)/7 + (a**2 + 2*a*b*x**(1/
3))**(9/2)/9)/(4*a**3*b**3), Ne(a*b, 0)), (x*(a**2)**(3/2)/3, True))
```

3.464.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.83

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}\,dx=\frac{3\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^{\frac{3}{2}}a^2x^{\frac{1}{3}}}{4\,b^2}+\frac{3\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^{\frac{3}{2}}a^3}{4\,b^3}+\frac{\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^{\frac{5}{2}}x^{\frac{1}{3}}}{2\,b^2}-\frac{7\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^{\frac{5}{2}}a}{10\,b^3}$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(3/2),x, algorithm="maxima")
```

output
```
3/4*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^(3/2)*a^2*x^(1/3)/b^2 + 3/4*(b^2*x
^(2/3) + 2*a*b*x^(1/3) + a^2)^(3/2)*a^3/b^3 + 1/2*(b^2*x^(2/3) + 2*a*b*x^(
1/3) + a^2)^(5/2)*x^(1/3)/b^2 - 7/10*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^(
5/2)*a/b^3
```

3.464.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.47

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}\,dx=\frac{1}{2}\,b^3x^2\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)+\frac{9}{5}\,ab^2x^{\frac{5}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)+\frac{9}{4}\,a^2bx^{\frac{4}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)+a^3x\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(3/2),x, algorithm="giac")
```

output
```
1/2*b^3*x^2*sgn(b*x^(1/3) + a) + 9/5*a*b^2*x^(5/3)*sgn(b*x^(1/3) + a) + 9/
4*a^2*b*x^(4/3)*sgn(b*x^(1/3) + a) + a^3*x*sgn(b*x^(1/3) + a)
```

3.464.9 Mupad [F(-1)]

Timed out.

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}\,dx=\int \left(a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}\right)^{3/2}\,dx$$

input
```
int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(3/2),x)
```

output
```
int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(3/2), x)
```

3.465 $\int \sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\,dx$

3.465.1 Optimal result

Integrand size = 26, antiderivative size = 88

$$\int \sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\,dx=\frac{a\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\,x}{a+b\sqrt[3]{x}}+\frac{3b\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\,x^{4/3}}{4\left(a+b\sqrt[3]{x}\right)}$$

output
```
a*x*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)/(a+b*x^(1/3))+3/4*b*x^(4/3)*(a^2
+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)/(a+b*x^(1/3))
```

3.465.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.49

$$\int \sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\,dx=\frac{\sqrt{\left(a+b\sqrt[3]{x}\right)^2}\left(4ax+3bx^{4/3}\right)}{4\left(a+b\sqrt[3]{x}\right)}$$

input
```
Integrate[Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)],x]
```

output
```
(Sqrt[(a + b*x^(1/3))^2]*(4*a*x + 3*b*x^(4/3)))/(4*(a + b*x^(1/3)))
```

3.465.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.67, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1384, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\,dx$$

$$\Big\downarrow \ 1384$$

$$\frac{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\int \left(\sqrt[3]{x}b^2+ab\right)dx}{ab+b^2\sqrt[3]{x}}$$

$$\Big\downarrow \ 2009$$

$$\frac{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(abx+\frac{3}{4}b^2x^{4/3}\right)}{ab+b^2\sqrt[3]{x}}$$

input
```
Int[Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)],x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]*(a*b*x + (3*b^2*x^(4/3))/4))/(a*b
 + b^2*x^(1/3))
```

3.465.3.1 Defintions of rubi rules used

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.465.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.48

method	result	size
derivativedivides	$\frac{\operatorname{csgn}\left(a+b\,x^{\frac{1}{3}}\right)\left(a+b\,x^{\frac{1}{3}}\right)^{2}\left(3b^{2}x^{\frac{2}{3}}-2ab\,x^{\frac{1}{3}}+a^{2}\right)}{4b^{3}}$	42
default	$\frac{\sqrt{a^{2}+2ab\,x^{\frac{1}{3}}+b^{2}x^{\frac{2}{3}}}\left(3b\,x^{\frac{4}{3}}+4ax\right)}{4a+4b\,x^{\frac{1}{3}}}$	43

input
```
int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*csgn(a+b*x^(1/3))*(a+b*x^(1/3))^2*(3*b^2*x^(2/3)-2*a*b*x^(1/3)+a^2)/b^
3
```

3.465.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.11

$$\int \sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\,dx = \frac{3}{4}\,bx^{\frac{4}{3}} + ax$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2),x, algorithm="fricas")
```

output
```
3/4*b*x^(4/3) + a*x
```

3.465.6 Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.66

$$\int \sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\,dx = 3\left(\begin{cases} \sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}}\left(\frac{a^3}{12b^3}-\frac{a^2\sqrt[3]{x}}{12b^2}+\frac{ax^{\frac{2}{3}}}{12b}+\frac{x}{4}\right) & \text{for } b^2 \neq 0 \\ \frac{\frac{a^4\left(a^2+2ab\sqrt[3]{x}\right)^{\frac{3}{2}}}{3}-\frac{2a^2\left(a^2+2ab\sqrt[3]{x}\right)^{\frac{5}{2}}}{5}+\frac{\left(a^2+2ab\sqrt[3]{x}\right)^{\frac{7}{2}}}{7}}{4a^3b^3} & \text{for } ab \neq 0 \\ \frac{x\sqrt{a^2}}{3} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(1/2),x)
```

output
```
3*Piecewise((sqrt(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))*(a**3/(12*b**3) -
 a**2*x**(1/3)/(12*b**2) + a*x**(2/3)/(12*b) + x/4), Ne(b**2, 0)), ((a**4*
(a**2 + 2*a*b*x**(1/3))**(3/2)/3 - 2*a**2*(a**2 + 2*a*b*x**(1/3))**(5/2)/5
 + (a**2 + 2*a*b*x**(1/3))**(7/2)/7)/(4*a**3*b**3), Ne(a*b, 0)), (x*sqrt(a
**2)/3, True))
```

3.465.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.30

$$\int \sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}}\,dx = \frac{3\sqrt{b^2x^{\frac{2}{3}} + 2\,abx^{\frac{1}{3}} + a^2}a^2x^{\frac{1}{3}}}{2\,b^2} + \frac{3\sqrt{b^2x^{\frac{2}{3}} + 2\,abx^{\frac{1}{3}} + a^2}a^3}{2\,b^3} + \frac{3\left(b^2x^{\frac{2}{3}} + 2\,abx^{\frac{1}{3}} + a^2\right)^{\frac{3}{2}}x^{\frac{1}{3}}}{4\,b^2} - \frac{5\left(b^2x^{\frac{2}{3}} + 2\,abx^{\frac{1}{3}} + a^2\right)^{\frac{3}{2}}a}{4\,b^3}$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2),x, algorithm="maxima")
```

output
```
3/2*sqrt(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)*a^2*x^(1/3)/b^2 + 3/2*sqrt(b^2
*x^(2/3) + 2*a*b*x^(1/3) + a^2)*a^3/b^3 + 3/4*(b^2*x^(2/3) + 2*a*b*x^(1/3)
 + a^2)^(3/2)*x^(1/3)/b^2 - 5/4*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^(3/2)*
a/b^3
```

3.465.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.30

$$\int \sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}}\,dx = \frac{3}{4}\,bx^{\frac{4}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}} + a\right) + ax\mathrm{sgn}\left(bx^{\frac{1}{3}} + a\right)$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2),x, algorithm="giac")
```

output
```
3/4*b*x^(4/3)*sgn(b*x^(1/3) + a) + a*x*sgn(b*x^(1/3) + a)
```

3.465.9 Mupad [B] (verification not implemented)

Time = 8.74 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.81

$$\int \sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\,dx = \frac{\sqrt{a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}}\,\left(a^3-4\,a^2\,b\,x^{1/3}-5\,a\,b^2\,x^{2/3}+3\,b\,x^{1/3}\,\left(a^2+b^2\,x^{2/}\right.\right.}{4\,b^3}$$

input
```
int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(1/2),x)
```

output
```
((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(1/2)*(a^3 - 4*a^2*b*x^(1/3) - 5*a*b^
2*x^(2/3) + 3*b*x^(1/3)*(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))))/(4*b^3)
```

3.466 $\int \frac{1}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}\,dx$

3.466.1 Optimal result

Integrand size = 26, antiderivative size = 147

$$\int \frac{1}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}\,dx = -\frac{3a\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{b^2\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}} + \frac{3\left(a+b\sqrt[3]{x}\right)x^{2/3}}{2b\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}} + \frac{3a^2\left(a+b\sqrt[3]{x}\right)\log\left(a+b\sqrt[3]{x}\right)}{b^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

output
```
-3*a*(a+b*x^(1/3))*x^(1/3)/b^2/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)+3/2*(
a+b*x^(1/3))*x^(2/3)/b/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)+3*a^2*(a+b*x^
(1/3))*ln(a+b*x^(1/3))/b^3/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)
```

3.466.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.44

$$\int \frac{1}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}\,dx = \frac{3\left(a+b\sqrt[3]{x}\right)\left(b\left(-2a+b\sqrt[3]{x}\right)\sqrt[3]{x}+2a^2\log\left(a+b\sqrt[3]{x}\right)\right)}{2b^3\sqrt{\left(a+b\sqrt[3]{x}\right)^2}}$$

input
```
Integrate[1/Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)],x]
```

output
```
(3*(a + b*x^(1/3))*(b*(-2*a + b*x^(1/3))*x^(1/3) + 2*a^2*Log[a + b*x^(1/3)
]))/(2*b^3*Sqrt[(a + b*x^(1/3))^2])
```

3.466.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.58, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1384, 774, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\left(ab+b^2\sqrt[3]{x}\right)\int \frac{1}{\sqrt[3]{x}b^2+ab}dx}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

$$\Big\downarrow \text{774}$$

$$\frac{3\left(ab+b^2\sqrt[3]{x}\right)\int \frac{x^{2/3}}{b\left(a+b\sqrt[3]{x}\right)}d\sqrt[3]{x}}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\left(ab+b^2\sqrt[3]{x}\right)\int \frac{x^{2/3}}{a+b\sqrt[3]{x}}d\sqrt[3]{x}}{b\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

$$\Big\downarrow \text{49}$$

$$\frac{3\left(ab+b^2\sqrt[3]{x}\right)\int \left(\frac{a^2}{b^2\left(a+b\sqrt[3]{x}\right)}-\frac{a}{b^2}+\frac{\sqrt[3]{x}}{b}\right)d\sqrt[3]{x}}{b\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\left(ab+b^2\sqrt[3]{x}\right)\left(\frac{a^2\log\left(a+b\sqrt[3]{x}\right)}{b^3}-\frac{a\sqrt[3]{x}}{b^2}+\frac{x^{2/3}}{2b}\right)}{b\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

input
```
Int[1/Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)],x]
```

output
```
(3*(a*b + b^2*x^(1/3))*(-((a*x^(1/3))/b^2) + x^(2/3)/(2*b) + (a^2*Log[a +
b*x^(1/3)])/b^3))/(b*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])
```

3.466.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.466.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.35

method	result	size
derivativedivides	$\frac{3\left(a+b\,x^{\frac{1}{3}}\right)\left(b^{2} x^{\frac{2}{3}}+2 a^{2} \ln\left(a+b\,x^{\frac{1}{3}}\right)-2 a b\,x^{\frac{1}{3}}\right)}{2 \sqrt{\left(a+b\,x^{\frac{1}{3}}\right)^{2}}\, b^{3}}$	52
default	$\frac{\left(a+b\,x^{\frac{1}{3}}\right)\left(3 b^{2} x^{\frac{2}{3}}-6 a b\,x^{\frac{1}{3}}+2 a^{2} \ln\left(b^{3} x+a^{3}\right)-2 a^{2} \ln\left(b^{2} x^{\frac{2}{3}}-a b\,x^{\frac{1}{3}}+a^{2}\right)+4 a^{2} \ln\left(a+b\,x^{\frac{1}{3}}\right)\right)}{2 \sqrt{a^{2}+2 a b\,x^{\frac{1}{3}}+b^{2} x^{\frac{2}{3}}}\, b^{3}}$	101

input
```
int(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
3/2*(a+b*x^(1/3))*(b^2*x^(2/3)+2*a^2*ln(a+b*x^(1/3))-2*a*b*x^(1/3))/((a+b*
x^(1/3))^2)^(1/2)/b^3
```

3.466.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.22

$$\int \frac{1}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}\,dx=\frac{3\left(2\,a^2\log\left(bx^{\frac{1}{3}}+a\right)+b^2x^{\frac{2}{3}}-2\,abx^{\frac{1}{3}}\right)}{2\,b^3}$$

input
```
integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2),x, algorithm="fricas")
```

output
```
3/2*(2*a^2*log(b*x^(1/3) + a) + b^2*x^(2/3) - 2*a*b*x^(1/3))/b^3
```

3.466.6 Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.16

$$\int \frac{1}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}\,dx=3\left(\begin{cases}\frac{a^2\left(\frac{a}{b}+\sqrt[3]{x}\right)\log\left(\frac{a}{b}+\sqrt[3]{x}\right)}{b^2\sqrt{b^2\left(\frac{a}{b}+\sqrt[3]{x}\right)^2}}+\left(-\frac{3a}{2b^3}+\frac{\sqrt[3]{x}}{2b^2}\right)\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}} & \text{for } b^2 \\ \frac{a^4\sqrt{a^2+2ab\sqrt[3]{x}}-\frac{2a^2\left(a^2+2ab\sqrt[3]{x}\right)^{\frac{3}{2}}}{3}+\frac{\left(a^2+2ab\sqrt[3]{x}\right)^{\frac{5}{2}}}{5}}{4a^3b^3} & \text{for } ab \\ \frac{x}{3\sqrt{a^2}} & \text{other} \end{cases}\right.$$

input
```
integrate(1/(a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(1/2),x)
```

output
```
3*Piecewise((a**2*(a/b + x**(1/3))*log(a/b + x**(1/3))/(b**2*sqrt(b**2*(a/
b + x**(1/3))**2)) + (-3*a/(2*b**3) + x**(1/3)/(2*b**2))*sqrt(a**2 + 2*a*b
*x**(1/3) + b**2*x**(2/3)), Ne(b**2, 0)), ((a**4*sqrt(a**2 + 2*a*b*x**(1/3
)) - 2*a**2*(a**2 + 2*a*b*x**(1/3))**(3/2)/3 + (a**2 + 2*a*b*x**(1/3))**(5
/2)/5)/(4*a**3*b**3), Ne(a*b, 0)), (x/(3*sqrt(a**2)), True))
```

3.466.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.24

$$\int \frac{1}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}\,dx = \frac{3\,a^2\log\left(x^{\frac{1}{3}}+\frac{a}{b}\right)}{b^3}+\frac{3\,x^{\frac{2}{3}}}{2\,b}-\frac{3\,ax^{\frac{1}{3}}}{b^2}$$

input
```
integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2),x, algorithm="maxima")
```

output
```
3*a^2*log(x^(1/3) + a/b)/b^3 + 3/2*x^(2/3)/b - 3*a*x^(1/3)/b^2
```

3.466.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.41

$$\int \frac{1}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}\,dx = \frac{3\left(bx^{\frac{2}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)-2\,ax^{\frac{1}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)\right)}{2\,b^2} + \frac{3\,a^2\log\left(\left|bx^{\frac{1}{3}}+a\right|\right)}{b^3\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)}$$

input
```
integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2),x, algorithm="giac")
```

output
```
3/2*(b*x^(2/3)*sgn(b*x^(1/3) + a) - 2*a*x^(1/3)*sgn(b*x^(1/3) + a))/b^2 +
3*a^2*log(abs(b*x^(1/3) + a))/(b^3*sgn(b*x^(1/3) + a))
```

3.466.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}\,dx = \int \frac{1}{\sqrt{a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}}}\,dx$$

input
```
int(1/(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(1/2),x)
```

output
```
int(1/(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(1/2), x)
```

3.467 $\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}}\,dx$

3.467.1 Optimal result

Integrand size = 26, antiderivative size = 130

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}}\,dx = \frac{6a}{b^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}} - \frac{3a^2}{2b^3\left(a+b\sqrt[3]{x}\right)\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}} + \frac{3\left(a+b\sqrt[3]{x}\right)\log\left(a+b\sqrt[3]{x}\right)}{b^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

output
```
6*a/b^3/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)-3/2*a^2/b^3/(a+b*x^(1/3))/(a
^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)+3*(a+b*x^(1/3))*ln(a+b*x^(1/3))/b^3/(a
^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)
```

3.467.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.55

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}}\,dx = \frac{3a\left(3a+4b\sqrt[3]{x}\right)+6\left(a+b\sqrt[3]{x}\right)^2\log\left(a+b\sqrt[3]{x}\right)}{2b^3\left(a+b\sqrt[3]{x}\right)\sqrt{\left(a+b\sqrt[3]{x}\right)^2}}$$

input
```
Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-3/2),x]
```

output
```
(3*a*(3*a + 4*b*x^(1/3)) + 6*(a + b*x^(1/3))^2*Log[a + b*x^(1/3)])/(2*b^3*
(a + b*x^(1/3))*Sqrt[(a + b*x^(1/3))^2])
```

3.467.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.76, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1384, 774, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\left(ab^3+b^4\sqrt[3]{x}\right)\int \frac{1}{\left(\sqrt[3]{x}b^2+ab\right)^3}dx}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

$$\Big\downarrow \text{774}$$

$$\frac{3\left(ab^3+b^4\sqrt[3]{x}\right)\int \frac{x^{2/3}}{b^3\left(a+b\sqrt[3]{x}\right)^3}d\sqrt[3]{x}}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\left(ab^3+b^4\sqrt[3]{x}\right)\int \frac{x^{2/3}}{\left(a+b\sqrt[3]{x}\right)^3}d\sqrt[3]{x}}{b^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

$$\Big\downarrow \text{49}$$

$$\frac{3\left(ab^3+b^4\sqrt[3]{x}\right)\int \left(\frac{a^2}{b^2\left(a+b\sqrt[3]{x}\right)^3}-\frac{2a}{b^2\left(a+b\sqrt[3]{x}\right)^2}+\frac{1}{b^2\left(a+b\sqrt[3]{x}\right)}\right)d\sqrt[3]{x}}{b^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\left(ab^3+b^4\sqrt[3]{x}\right)\left(-\frac{a^2}{2b^3\left(a+b\sqrt[3]{x}\right)^2}+\frac{2a}{b^3\left(a+b\sqrt[3]{x}\right)}+\frac{\log\left(a+b\sqrt[3]{x}\right)}{b^3}\right)}{b^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

input
```
Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-3/2),x]
```

output
```
(3*(a*b^3 + b^4*x^(1/3))*(-1/2*a^2/(b^3*(a + b*x^(1/3))^2) + (2*a)/(b^3*(a
 + b*x^(1/3))) + Log[a + b*x^(1/3)]/b^3))/(b^3*Sqrt[a^2 + 2*a*b*x^(1/3) +
b^2*x^(2/3)])
```

3.467.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.467.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.62

method	result
derivativedivides	$\frac{3\left(2\ln\left(a+b\,x^{\frac{1}{3}}\right)b^{2}x^{\frac{2}{3}}+4\ln\left(a+b\,x^{\frac{1}{3}}\right)ab\,x^{\frac{1}{3}}+2a^{2}\ln\left(a+b\,x^{\frac{1}{3}}\right)+4ab\,x^{\frac{1}{3}}+3a^{2}\right)\left(a+b\,x^{\frac{1}{3}}\right)}{2b^{3}\left(\left(a+b\,x^{\frac{1}{3}}\right)^{2}\right)^{\frac{3}{2}}}$
default	$\frac{\left(8x^{3}\ln\left(b^{3}x+a^{3}\right)a^{3}b^{9}-8x^{3}\ln\left(b^{2}x^{\frac{2}{3}}-ab\,x^{\frac{1}{3}}+a^{2}\right)a^{3}b^{9}+16x^{3}\ln\left(a+b\,x^{\frac{1}{3}}\right)a^{3}b^{9}-27x^{\frac{4}{3}}a^{8}b^{4}+3x^{\frac{2}{3}}a^{10}b^{2}-6x^{\frac{1}{3}}a^{11}b+2\ln\left(\right.\right.}{}$

input
```
int(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
3/2*(2*ln(a+b*x^(1/3))*b^2*x^(2/3)+4*ln(a+b*x^(1/3))*a*b*x^(1/3)+2*a^2*ln(
a+b*x^(1/3))+4*a*b*x^(1/3)+3*a^2)*(a+b*x^(1/3))/b^3/((a+b*x^(1/3))^2)^(3/2
)
```

3.467.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.87

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}}\,dx = \frac{3\left(6\,a^3b^3x+3\,a^6+2\,(b^6x^2+2\,a^3b^3x+a^6)\log\left(bx^{\frac{1}{3}}+a\right)+(4\,ab^5x+a^4b\right.}{2\,(b^9x^2+2\,a^3b^6x+a^6b^3)}$$

input
```
integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(3/2),x, algorithm="fricas")
```

output
```
3/2*(6*a^3*b^3*x + 3*a^6 + 2*(b^6*x^2 + 2*a^3*b^3*x + a^6)*log(b*x^(1/3) +
 a) + (4*a*b^5*x + a^4*b^2)*x^(2/3) - (5*a^2*b^4*x + 2*a^5*b)*x^(1/3))/(b^
9*x^2 + 2*a^3*b^6*x + a^6*b^3)
```

3.467.6 Sympy [F]

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}}\,dx = \int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(3/2),x)
```

output
```
Integral((a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**(-3/2), x)
```

3.467.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.42

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}}\,dx = \frac{3\,\log\left(x^{\frac{1}{3}}+\frac{a}{b}\right)}{b^3}+\frac{6\,ax^{\frac{1}{3}}}{b^4\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^2}+\frac{9\,a^2}{2\,b^5\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^2}$$

input
```
integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(3/2),x, algorithm="maxima")
```

output
```
3*log(x^(1/3) + a/b)/b^3 + 6*a*x^(1/3)/(b^4*(x^(1/3) + a/b)^2) + 9/2*a^2/(
b^5*(x^(1/3) + a/b)^2)
```

3.467.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.49

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}}\,dx = \frac{3\,\log\left(\left|bx^{\frac{1}{3}}+a\right|\right)}{b^3\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)}+\frac{3\left(4\,ax^{\frac{1}{3}}+\frac{3\,a^2}{b}\right)}{2\left(bx^{\frac{1}{3}}+a\right)^2b^2\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)}$$

input
```
integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(3/2),x, algorithm="giac")
```

output
```
3*log(abs(b*x^(1/3) + a))/(b^3*sgn(b*x^(1/3) + a)) + 3/2*(4*a*x^(1/3) + 3*
a^2/b)/((b*x^(1/3) + a)^2*b^2*sgn(b*x^(1/3) + a))
```

3.467.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}}\,dx = \int \frac{1}{\left(a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}\right)^{3/2}}\,dx$$

input
```
int(1/(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(3/2),x)
```

output
```
int(1/(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(3/2), x)
```

3.468 $\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}}\,dx$

3.468.1 Optimal result

Integrand size = 26, antiderivative size = 135

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}}\,dx = -\frac{3a^2}{4b^3\left(a+b\sqrt[3]{x}\right)^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}} + \frac{2a}{b^3\left(a+b\sqrt[3]{x}\right)^2\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}} - \frac{3}{2b^3\left(a+b\sqrt[3]{x}\right)\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

output
```
-3/4*a^2/b^3/(a+b*x^(1/3))^3/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)+2*a/b^3
/(a+b*x^(1/3))^2/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)-3/2/b^3/(a+b*x^(1/3
))/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)
```

3.468.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.41

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}}\,dx = \frac{\left(a+b\sqrt[3]{x}\right)\left(-a^2-4ab\sqrt[3]{x}-6b^2x^{2/3}\right)}{4b^3\left(\left(a+b\sqrt[3]{x}\right)^2\right)^{5/2}}$$

input
```
Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-5/2),x]
```

output
```
((a + b*x^(1/3))*(-a^2 - 4*a*b*x^(1/3) - 6*b^2*x^(2/3)))/(4*b^3*((a + b*x^
(1/3))^2)^(5/2))
```

3.468.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.78, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1384, 774, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\left(ab^5+b^6\sqrt[3]{x}\right)\int \frac{1}{\left(\sqrt[3]{x}b^2+ab\right)^5}dx}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

$$\Big\downarrow \text{774}$$

$$\frac{3\left(ab^5+b^6\sqrt[3]{x}\right)\int \frac{x^{2/3}}{b^5\left(a+b\sqrt[3]{x}\right)^5}d\sqrt[3]{x}}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\left(ab^5+b^6\sqrt[3]{x}\right)\int \frac{x^{2/3}}{\left(a+b\sqrt[3]{x}\right)^5}d\sqrt[3]{x}}{b^5\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

$$\Big\downarrow \text{53}$$

$$\frac{3\left(ab^5+b^6\sqrt[3]{x}\right)\int \left(\frac{a^2}{b^2\left(a+b\sqrt[3]{x}\right)^5}-\frac{2a}{b^2\left(a+b\sqrt[3]{x}\right)^4}+\frac{1}{b^2\left(a+b\sqrt[3]{x}\right)^3}\right)d\sqrt[3]{x}}{b^5\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\left(-\frac{a^2}{4b^3\left(a+b\sqrt[3]{x}\right)^4}+\frac{2a}{3b^3\left(a+b\sqrt[3]{x}\right)^3}-\frac{1}{2b^3\left(a+b\sqrt[3]{x}\right)^2}\right)\left(ab^5+b^6\sqrt[3]{x}\right)}{b^5\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

input
```
Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-5/2),x]
```

output
```
(3*(-1/4*a^2/(b^3*(a + b*x^(1/3))^4) + (2*a)/(3*b^3*(a + b*x^(1/3))^3) - 1
/(2*b^3*(a + b*x^(1/3))^2))*(a*b^5 + b^6*x^(1/3)))/(b^5*Sqrt[a^2 + 2*a*b*x
^(1/3) + b^2*x^(2/3)])
```

3.468.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.468. $\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}}\,dx$

3.468.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.32

method	result
derivativedivides	$-\frac{\left(6b^{2}x^{\frac{2}{3}}+4ab\,x^{\frac{1}{3}}+a^{2}\right)\left(a+b\,x^{\frac{1}{3}}\right)}{4b^{3}\left(\left(a+b\,x^{\frac{1}{3}}\right)^{2}\right)^{\frac{5}{2}}}$
default	$-\frac{\left(6x^{\frac{22}{3}}b^{22}+45x^{\frac{20}{3}}a^{2}b^{20}-36x^{\frac{19}{3}}a^{3}b^{19}+144x^{\frac{17}{3}}a^{5}b^{17}-189x^{\frac{16}{3}}a^{6}b^{16}+126x^{\frac{14}{3}}a^{8}b^{14}-276x^{\frac{13}{3}}a^{9}b^{13}-36x^{\frac{11}{3}}a^{11}b^{11}-1\right.}{4b^{3}\left(b^{2}x^{\frac{2}{3}}\right.}$

input
```
int(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(6*b^2*x^(2/3)+4*a*b*x^(1/3)+a^2)*(a+b*x^(1/3))/b^3/((a+b*x^(1/3))^2)
^(5/2)
```

3.468.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.01

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}}\,dx = \frac{20\,ab^9x^3-60\,a^4b^6x^2-a^{10}-9\,(5\,a^2b^8x^2-4\,a^5b^5x)x^{\frac{2}{3}}-3\,(2\,b^{10}x^3-20\,a}{4\,(b^{15}x^4+4\,a^3b^{12}x^3+6\,a^6b^9x^2+4\,a^9b^6x+a^{12}b^3)}$$

input
```
integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(5/2),x, algorithm="fricas")
```

output
```
1/4*(20*a*b^9*x^3 - 60*a^4*b^6*x^2 - a^10 - 9*(5*a^2*b^8*x^2 - 4*a^5*b^5*x
)*x^(2/3) - 3*(2*b^10*x^3 - 20*a^3*b^7*x^2 + 5*a^6*b^4*x)*x^(1/3))/(b^15*x
^4 + 4*a^3*b^12*x^3 + 6*a^6*b^9*x^2 + 4*a^9*b^6*x + a^12*b^3)
```

3.468.6 Sympy [F]

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}}\,dx = \int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(5/2),x)
```

output
```
Integral((a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**(-5/2), x)
```

3.468.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.39

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}}\,dx=-\frac{3}{2\,b^5\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^2}+\frac{2\,a}{b^6\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^3}-\frac{3\,a^2}{4\,b^7\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^4}$$

input
```
integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(5/2),x, algorithm="maxima")
```

output
```
-3/2/(b^5*(x^(1/3) + a/b)^2) + 2*a/(b^6*(x^(1/3) + a/b)^3) - 3/4*a^2/(b^7*
(x^(1/3) + a/b)^4)
```

3.468.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.32

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}}\,dx=-\frac{6\,b^2x^{\frac{2}{3}}+4\,abx^{\frac{1}{3}}+a^2}{4\left(bx^{\frac{1}{3}}+a\right)^4b^3\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)}$$

input
```
integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(5/2),x, algorithm="giac")
```

output
```
-1/4*(6*b^2*x^(2/3) + 4*a*b*x^(1/3) + a^2)/((b*x^(1/3) + a)^4*b^3*sgn(b*x^
(1/3) + a))
```

3.468.9 Mupad [B] (verification not implemented)

Time = 9.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.39

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}}\,dx=-\frac{\sqrt{a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}}\,\left(a^2+6\,b^2\,x^{2/3}+4\,a\,b\,x^{1/3}\right)}{4\,b^3\,\left(a+b\,x^{1/3}\right)^5}$$

input
```
int(1/(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(5/2),x)
```

output
```
-((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(1/2)*(a^2 + 6*b^2*x^(2/3) + 4*a*b*x
^(1/3)))/(4*b^3*(a + b*x^(1/3))^5)
```

3.469 $\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{7/2}}\,dx$

3.469.1 Optimal result

Integrand size = 26, antiderivative size = 137

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{7/2}}\,dx = -\frac{a^2}{2b^3\left(a+b\sqrt[3]{x}\right)^5\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}} + \frac{6a}{5b^3\left(a+b\sqrt[3]{x}\right)^4\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}} - \frac{3}{4b^3\left(a+b\sqrt[3]{x}\right)^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

output
```
-1/2*a^2/b^3/(a+b*x^(1/3))^5/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)+6/5*a/b
^3/(a+b*x^(1/3))^4/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)-3/4/b^3/(a+b*x^(1
/3))^3/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)
```

3.469.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.41

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{7/2}}\,dx = \frac{\left(a+b\sqrt[3]{x}\right)\left(-a^2-6ab\sqrt[3]{x}-15b^2x^{2/3}\right)}{20b^3\left(\left(a+b\sqrt[3]{x}\right)^2\right)^{7/2}}$$

input
```
Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-7/2),x]
```

output
```
((a + b*x^(1/3))*(-a^2 - 6*a*b*x^(1/3) - 15*b^2*x^(2/3)))/(20*b^3*((a + b*
x^(1/3))^2)^(7/2))
```

3.469.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.77, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1384, 774, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{7/2}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\left(ab^7+b^8\sqrt[3]{x}\right)\int \frac{1}{\left(\sqrt[3]{x}b^2+ab\right)^7}dx}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

$$\Big\downarrow \text{774}$$

$$\frac{3\left(ab^7+b^8\sqrt[3]{x}\right)\int \frac{x^{2/3}}{b^7\left(a+b\sqrt[3]{x}\right)^7}d\sqrt[3]{x}}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\left(ab^7+b^8\sqrt[3]{x}\right)\int \frac{x^{2/3}}{\left(a+b\sqrt[3]{x}\right)^7}d\sqrt[3]{x}}{b^7\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

$$\Big\downarrow \text{53}$$

$$\frac{3\left(ab^7+b^8\sqrt[3]{x}\right)\int \left(\frac{a^2}{b^2\left(a+b\sqrt[3]{x}\right)^7}-\frac{2a}{b^2\left(a+b\sqrt[3]{x}\right)^6}+\frac{1}{b^2\left(a+b\sqrt[3]{x}\right)^5}\right)d\sqrt[3]{x}}{b^7\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\left(-\frac{a^2}{6b^3\left(a+b\sqrt[3]{x}\right)^6}+\frac{2a}{5b^3\left(a+b\sqrt[3]{x}\right)^5}-\frac{1}{4b^3\left(a+b\sqrt[3]{x}\right)^4}\right)\left(ab^7+b^8\sqrt[3]{x}\right)}{b^7\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

input
```
Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-7/2),x]
```

output
```
(3*(-1/6*a^2/(b^3*(a + b*x^(1/3))^6) + (2*a)/(5*b^3*(a + b*x^(1/3))^5) - 1
/(4*b^3*(a + b*x^(1/3))^4))*(a*b^7 + b^8*x^(1/3)))/(b^7*Sqrt[a^2 + 2*a*b*x
^(1/3) + b^2*x^(2/3)])
```

3.469.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.469. $\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{7/2}}\,dx$

3.469.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.31

method	result
derivativedivides	$-\frac{\left(15b^2x^{\frac{2}{3}}+6ab\,x^{\frac{1}{3}}+a^2\right)\left(a+b\,x^{\frac{1}{3}}\right)}{20b^3\left(\left(a+b\,x^{\frac{1}{3}}\right)^2\right)^{\frac{7}{2}}}$
default	$-\frac{\left(280x^9a^5b^{27}-540x^{\frac{29}{3}}a^3b^{29}-84x^{\frac{31}{3}}a\,b^{31}-2106x^{\frac{26}{3}}a^6b^{26}+567x^{\frac{28}{3}}a^4b^{28}-792x^{\frac{23}{3}}a^9b^{23}+3996x^{\frac{25}{3}}a^7b^{25}+7344x^{\frac{20}{3}}a\right.}{}$

input
```
int(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/20*(15*b^2*x^(2/3)+6*a*b*x^(1/3)+a^2)*(a+b*x^(1/3))/b^3/((a+b*x^(1/3))^
2)^(7/2)
```

3.469.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.53

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{7/2}}\,dx =$$
$$-\frac{280\,a^2b^{12}x^4-1400\,a^5b^9x^3+735\,a^8b^6x^2-14\,a^{11}b^3x+a^{14}+3\,(5\,b^{14}x^4-210\,a^3b^{11}x^3+483\,a^6b^8x^2-112\,a^9}{20\,(b^{21}x^6+6\,a^3b^{18}x^5+15\,a^6b^{15}x^4+20\,a^9b^{12}x^3+15\,a^{12}b^9x^2}$$

input
```
integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(7/2),x, algorithm="fricas")
```

output
```
-1/20*(280*a^2*b^12*x^4 - 1400*a^5*b^9*x^3 + 735*a^8*b^6*x^2 - 14*a^11*b^3
*x + a^14 + 3*(5*b^14*x^4 - 210*a^3*b^11*x^3 + 483*a^6*b^8*x^2 - 112*a^9*b
^5*x)*x^(2/3) - 3*(28*a*b^13*x^4 - 357*a^4*b^10*x^3 + 390*a^7*b^7*x^2 - 35
*a^10*b^4*x)*x^(1/3))/(b^21*x^6 + 6*a^3*b^18*x^5 + 15*a^6*b^15*x^4 + 20*a^
9*b^12*x^3 + 15*a^12*b^9*x^2 + 6*a^15*b^6*x + a^18*b^3)
```

3.469.6 Sympy [F]

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{7/2}}\,dx=\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(7/2),x)
```

output
```
Integral((a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**(-7/2), x)
```

3.469.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.39

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{7/2}}\,dx=-\frac{3}{4\,b^7\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^4}+\frac{6\,a}{5\,b^8\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^5}-\frac{a^2}{2\,b^9\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^6}$$

input
```
integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(7/2),x, algorithm="maxima")
```

output
```
-3/4/(b^7*(x^(1/3) + a/b)^4) + 6/5*a/(b^8*(x^(1/3) + a/b)^5) - 1/2*a^2/(b^
9*(x^(1/3) + a/b)^6)
```

3.469.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.31

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{7/2}}\,dx=-\frac{15\,b^2x^{\frac{2}{3}}+6\,abx^{\frac{1}{3}}+a^2}{20\left(bx^{\frac{1}{3}}+a\right)^6b^3\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)}$$

input
```
integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(7/2),x, algorithm="giac")
```

output
```
-1/20*(15*b^2*x^(2/3) + 6*a*b*x^(1/3) + a^2)/((b*x^(1/3) + a)^6*b^3*sgn(b*
x^(1/3) + a))
```

3.469.9 Mupad [B] (verification not implemented)

Time = 9.49 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.39

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{7/2}}\,dx=-\frac{\sqrt{a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}}\,\left(a^2+15\,b^2\,x^{2/3}+6\,a\,b\,x^{1/3}\right)}{20\,b^3\,{\left(a+b\,x^{1/3}\right)}^7}$$

input
```
int(1/(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(7/2),x)
```

output
```
-((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(1/2)*(a^2 + 15*b^2*x^(2/3) + 6*a*b*
x^(1/3)))/(20*b^3*(a + b*x^(1/3))^7)
```

3.470 $\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{9/2}}\,dx$

3.470.1 Optimal result

Integrand size = 26, antiderivative size = 137

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{9/2}}\,dx=-\frac{3a^2}{8b^3\left(a+b\sqrt[3]{x}\right)^7\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{6a}{7b^3\left(a+b\sqrt[3]{x}\right)^6\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}-\frac{1}{2b^3\left(a+b\sqrt[3]{x}\right)^5\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

output
```
-3/8*a^2/b^3/(a+b*x^(1/3))^7/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)+6/7*a/b
^3/(a+b*x^(1/3))^6/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)-1/2/b^3/(a+b*x^(1
/3))^5/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)
```

3.470.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.41

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{9/2}}\,dx=\frac{\left(a+b\sqrt[3]{x}\right)\left(-a^2-8ab\sqrt[3]{x}-28b^2x^{2/3}\right)}{56b^3\left(\left(a+b\sqrt[3]{x}\right)^2\right)^{9/2}}$$

input
```
Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-9/2),x]
```

output
```
((a + b*x^(1/3))*(-a^2 - 8*a*b*x^(1/3) - 28*b^2*x^(2/3)))/(56*b^3*((a + b*
x^(1/3))^2)^(9/2))
```

3.470.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.77, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1384, 774, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{9/2}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\left(ab^9+b^{10}\sqrt[3]{x}\right)\int \frac{1}{\left(\sqrt[3]{x}b^2+ab\right)^9}dx}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

$$\Big\downarrow \text{774}$$

$$\frac{3\left(ab^9+b^{10}\sqrt[3]{x}\right)\int \frac{x^{2/3}}{b^9\left(a+b\sqrt[3]{x}\right)^9}d\sqrt[3]{x}}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\left(ab^9+b^{10}\sqrt[3]{x}\right)\int \frac{x^{2/3}}{\left(a+b\sqrt[3]{x}\right)^9}d\sqrt[3]{x}}{b^9\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

$$\Big\downarrow \text{53}$$

$$\frac{3\left(ab^9+b^{10}\sqrt[3]{x}\right)\int \left(\frac{a^2}{b^2\left(a+b\sqrt[3]{x}\right)^9}-\frac{2a}{b^2\left(a+b\sqrt[3]{x}\right)^8}+\frac{1}{b^2\left(a+b\sqrt[3]{x}\right)^7}\right)d\sqrt[3]{x}}{b^9\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\left(-\frac{a^2}{8b^3\left(a+b\sqrt[3]{x}\right)^8}+\frac{2a}{7b^3\left(a+b\sqrt[3]{x}\right)^7}-\frac{1}{6b^3\left(a+b\sqrt[3]{x}\right)^6}\right)\left(ab^9+b^{10}\sqrt[3]{x}\right)}{b^9\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

input
```
Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-9/2),x]
```

output
```
(3*(-1/8*a^2/(b^3*(a + b*x^(1/3))^8) + (2*a)/(7*b^3*(a + b*x^(1/3))^7) - 1
/(6*b^3*(a + b*x^(1/3))^6))*(a*b^9 + b^10*x^(1/3)))/(b^9*Sqrt[a^2 + 2*a*b*
x^(1/3) + b^2*x^(2/3)])
```

3.470.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.470.4 Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.31

method	result
derivativedivides	$-\frac{\left(28b^{2}x^{\frac{2}{3}}+8ab\,x^{\frac{1}{3}}+a^{2}\right)\left(a+b\,x^{\frac{1}{3}}\right)}{56b^{3}\left(\left(a+b\,x^{\frac{1}{3}}\right)^{2}\right)^{\frac{9}{2}}}$
default	$\frac{\left(-a^{42}+202496a^{27}b^{15}x^{5}+31696a^{30}b^{12}x^{4}-11704a^{33}b^{9}x^{3}-3844a^{36}b^{6}x^{2}+40a^{39}b^{3}x-46480a^{9}b^{33}x^{11}-190568a^{12}b^{30}x^{10}\right.}{}$

input
```
int(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/56*(28*b^2*x^(2/3)+8*a*b*x^(1/3)+a^2)*(a+b*x^(1/3))/b^3/((a+b*x^(1/3))^
2)^(9/2)
```

3.470.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 275 vs. 2(107) = 214.

Time = 0.41 (sec) , antiderivative size = 275, normalized size of antiderivative = 2.01

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{9/2}}\,dx =$$

$$-\frac{28\,b^{18}x^{6}-2856\,a^{3}b^{15}x^{5}+18186\,a^{6}b^{12}x^{4}-20608\,a^{9}b^{9}x^{3}+4200\,a^{12}b^{6}x^{2}-48\,a^{15}b^{3}x+a^{18}-27\left(8\,ab^{17}x^{5}-\right.}{56\left(b^{27}x^{8}+8\,a^{3}b^{24}x^{7}+28\,a^{6}b^{21}x^{6}+56\,a\right.}$$

input
```
integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(9/2),x, algorithm="fricas")
```

output
```
-1/56*(28*b^18*x^6 - 2856*a^3*b^15*x^5 + 18186*a^6*b^12*x^4 - 20608*a^9*b^
9*x^3 + 4200*a^12*b^6*x^2 - 48*a^15*b^3*x + a^18 - 27*(8*a*b^17*x^5 - 244*
a^4*b^14*x^4 + 840*a^7*b^11*x^3 - 553*a^10*b^8*x^2 + 56*a^13*b^5*x)*x^(2/3
) + 27*(35*a^2*b^16*x^5 - 448*a^5*b^13*x^4 + 876*a^8*b^10*x^3 - 328*a^11*b
^7*x^2 + 14*a^14*b^4*x)*x^(1/3))/(b^27*x^8 + 8*a^3*b^24*x^7 + 28*a^6*b^21*
x^6 + 56*a^9*b^18*x^5 + 70*a^12*b^15*x^4 + 56*a^15*b^12*x^3 + 28*a^18*b^9*
x^2 + 8*a^21*b^6*x + a^24*b^3)
```

3.470.6 Sympy [F]

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{9/2}}\,dx=\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^{\frac{9}{2}}}\,dx$$

input
```
integrate(1/(a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(9/2),x)
```

output
```
Integral((a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**(-9/2), x)
```

3.470.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.39

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{9/2}}\,dx=-\frac{1}{2\,b^9\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^6}+\frac{6\,a}{7\,b^{10}\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^7}-\frac{3\,a^2}{8\,b^{11}\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^8}$$

input
```
integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(9/2),x, algorithm="maxima")
```

output
```
-1/2/(b^9*(x^(1/3) + a/b)^6) + 6/7*a/(b^10*(x^(1/3) + a/b)^7) - 3/8*a^2/(b
^11*(x^(1/3) + a/b)^8)
```

3.470.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.31

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{9/2}}\,dx=-\frac{28\,b^2x^{\frac{2}{3}}+8\,abx^{\frac{1}{3}}+a^2}{56\left(bx^{\frac{1}{3}}+a\right)^8b^3\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)}$$

input
```
integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(9/2),x, algorithm="giac")
```

output
```
-1/56*(28*b^2*x^(2/3) + 8*a*b*x^(1/3) + a^2)/((b*x^(1/3) + a)^8*b^3*sgn(b*
x^(1/3) + a))
```

3.470.9 Mupad [B] (verification not implemented)

Time = 9.60 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.39

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{9/2}}\,dx = -\frac{\sqrt{a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}}\,\left(a^2+28\,b^2\,x^{2/3}+8\,a\,b\,x^{1/3}\right)}{56\,b^3\,{\left(a+b\,x^{1/3}\right)}^9}$$

input
```
int(1/(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(9/2),x)
```

output
```
-((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(1/2)*(a^2 + 28*b^2*x^(2/3) + 8*a*b*
x^(1/3)))/(56*b^3*(a + b*x^(1/3))^9)
```

3.471 $\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{11/2}}\,dx$

3.471.1 Optimal result

Integrand size = 26, antiderivative size = 137

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{11/2}}\,dx = -\frac{3a^2}{10b^3\left(a+b\sqrt[3]{x}\right)^9\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}} + \frac{2a}{3b^3\left(a+b\sqrt[3]{x}\right)^8\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}} - \frac{3}{8b^3\left(a+b\sqrt[3]{x}\right)^7\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

output
```
-3/10*a^2/b^3/(a+b*x^(1/3))^9/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)+2/3*a/
b^3/(a+b*x^(1/3))^8/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)-3/8/b^3/(a+b*x^(
1/3))^7/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)
```

3.471.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.41

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{11/2}}\,dx = \frac{\left(a+b\sqrt[3]{x}\right)\left(-a^2-10ab\sqrt[3]{x}-45b^2x^{2/3}\right)}{120b^3\left(\left(a+b\sqrt[3]{x}\right)^2\right)^{11/2}}$$

input
```
Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-11/2),x]
```

output
```
((a + b*x^(1/3))*(-a^2 - 10*a*b*x^(1/3) - 45*b^2*x^(2/3)))/(120*b^3*((a +
b*x^(1/3))^2)^(11/2))
```

3.471.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.77, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1384, 774, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{11/2}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\left(ab^{11}+b^{12}\sqrt[3]{x}\right)\int \frac{1}{\left(\sqrt[3]{x}b^2+ab\right)^{11}}dx}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

$$\Big\downarrow \text{774}$$

$$\frac{3\left(ab^{11}+b^{12}\sqrt[3]{x}\right)\int \frac{x^{2/3}}{b^{11}\left(a+b\sqrt[3]{x}\right)^{11}}d\sqrt[3]{x}}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\left(ab^{11}+b^{12}\sqrt[3]{x}\right)\int \frac{x^{2/3}}{\left(a+b\sqrt[3]{x}\right)^{11}}d\sqrt[3]{x}}{b^{11}\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

$$\Big\downarrow \text{53}$$

$$\frac{3\left(ab^{11}+b^{12}\sqrt[3]{x}\right)\int \left(\frac{a^2}{b^2\left(a+b\sqrt[3]{x}\right)^{11}}-\frac{2a}{b^2\left(a+b\sqrt[3]{x}\right)^{10}}+\frac{1}{b^2\left(a+b\sqrt[3]{x}\right)^9}\right)d\sqrt[3]{x}}{b^{11}\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\left(-\frac{a^2}{10b^3\left(a+b\sqrt[3]{x}\right)^{10}}+\frac{2a}{9b^3\left(a+b\sqrt[3]{x}\right)^9}-\frac{1}{8b^3\left(a+b\sqrt[3]{x}\right)^8}\right)\left(ab^{11}+b^{12}\sqrt[3]{x}\right)}{b^{11}\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

input
```
Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-11/2),x]
```

output
```
(3*(-1/10*a^2/(b^3*(a + b*x^(1/3))^10) + (2*a)/(9*b^3*(a + b*x^(1/3))^9) -
 1/(8*b^3*(a + b*x^(1/3))^8))*(a*b^11 + b^12*x^(1/3)))/(b^11*Sqrt[a^2 + 2*
a*b*x^(1/3) + b^2*x^(2/3)])
```

3.471.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.471. $\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{11/2}}\,dx$

3.471.4 Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.31

method	result
derivativedivides	$-\frac{\left(45b^{2}x^{\frac{2}{3}}+10ab\,x^{\frac{1}{3}}+a^{2}\right)\left(a+b\,x^{\frac{1}{3}}\right)}{120b^{3}\left(\left(a+b\,x^{\frac{1}{3}}\right)^{2}\right)^{\frac{11}{2}}}$
default	$-\frac{\left(-5834520x^{\frac{34}{3}}a^{18}b^{34}+2207250x^{\frac{38}{3}}a^{14}b^{38}-13023000x^{\frac{31}{3}}a^{21}b^{31}+6925500x^{\frac{35}{3}}a^{17}b^{35}-12561075x^{\frac{28}{3}}a^{24}b^{28}+865066\right.}{}$

input
```
int(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(11/2),x,method=_RETURNVERBOSE)
```

output
```
-1/120*(45*b^2*x^(2/3)+10*a*b*x^(1/3)+a^2)*(a+b*x^(1/3))/b^3/((a+b*x^(1/3)
)^2)^(11/2)
```

3.471.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. 2(107) = 214.

Time = 0.58 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.50

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{11/2}}\,dx = \frac{440\,ab^{21}x^7-25630\,a^4b^{18}x^6+186252\,a^7b^{15}x^5-326150\,a^{10}b^{12}x^4+15400}{}$$

input
```
integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(11/2),x, algorithm="fricas")
```

output
```
1/120*(440*a*b^21*x^7 - 25630*a^4*b^18*x^6 + 186252*a^7*b^15*x^5 - 326150*
a^10*b^12*x^4 + 154000*a^13*b^9*x^3 - 16005*a^16*b^6*x^2 + 110*a^19*b^3*x
- a^22 - 27*(88*a^2*b^20*x^6 - 2200*a^5*b^17*x^5 + 9625*a^8*b^14*x^4 - 109
10*a^11*b^11*x^3 + 3245*a^14*b^8*x^2 - 176*a^17*b^5*x)*x^(2/3) - 9*(5*b^22
*x^7 - 990*a^3*b^19*x^6 + 12705*a^6*b^16*x^5 - 34760*a^9*b^13*x^4 + 25542*
a^12*b^10*x^3 - 4620*a^15*b^7*x^2 + 110*a^18*b^4*x)*x^(1/3))/(b^33*x^10 +
10*a^3*b^30*x^9 + 45*a^6*b^27*x^8 + 120*a^9*b^24*x^7 + 210*a^12*b^21*x^6 +
 252*a^15*b^18*x^5 + 210*a^18*b^15*x^4 + 120*a^21*b^12*x^3 + 45*a^24*b^9*x
^2 + 10*a^27*b^6*x + a^30*b^3)
```

3.471.6 Sympy [F]

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{11/2}}\,dx = \int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^{\frac{11}{2}}}\,dx$$

input
```
integrate(1/(a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(11/2),x)
```

output
```
Integral((a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**(-11/2), x)
```

3.471.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.39

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{11/2}}\,dx = -\frac{3}{8\,b^{11}\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^{8}}+\frac{2\,a}{3\,b^{12}\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^{9}}-\frac{3\,a^2}{10\,b^{13}\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^{10}}$$

input
```
integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(11/2),x, algorithm="maxima")
```

output
```
-3/8/(b^11*(x^(1/3) + a/b)^8) + 2/3*a/(b^12*(x^(1/3) + a/b)^9) - 3/10*a^2/
(b^13*(x^(1/3) + a/b)^10)
```

3.471.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.31

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{11/2}}\,dx = -\frac{45\,b^2x^{\frac{2}{3}}+10\,abx^{\frac{1}{3}}+a^2}{120\left(bx^{\frac{1}{3}}+a\right)^{10}b^3\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)}$$

input
```
integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(11/2),x, algorithm="giac")
```

output
```
-1/120*(45*b^2*x^(2/3) + 10*a*b*x^(1/3) + a^2)/((b*x^(1/3) + a)^10*b^3*sgn
(b*x^(1/3) + a))
```

3.471.9 Mupad [B] (verification not implemented)

Time = 10.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.39

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{11/2}}\,dx=-\frac{\sqrt{a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}}\,\left(a^2+45\,b^2\,x^{2/3}+10\,a\,b\,x^{1/3}\right)}{120\,b^3\,{\left(a+b\,x^{1/3}\right)}^{11}}$$

input
```
int(1/(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(11/2),x)
```

output
```
-((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(1/2)*(a^2 + 45*b^2*x^(2/3) + 10*a*b
*x^(1/3)))/(120*b^3*(a + b*x^(1/3))^11)
```

3.472 $\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p (dx)^m\,dx$

3.472.1 Optimal result

Integrand size = 30, antiderivative size = 77

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p (dx)^m\,dx=\frac{\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x(dx)^m \text{Hypergeometric2F1}\left(3(1+m),-2p,\right.}{1+m}$$

output
```
(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x*(d*x)^m*hypergeom([-2*p, 3+3*m],[4+3*m
],-b*x^(1/3)/a)/(1+m)/((1+b*x^(1/3)/a)^(2*p))
```

3.472.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.88

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p (dx)^m\,dx=\frac{\left(\left(a+b\sqrt[3]{x}\right)^2\right)^p\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p} x(dx)^m \text{Hypergeometric2F1}\left(3(1+m),-2p,1+3(1\right.}{1+m}$$

input
```
Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*(d*x)^m,x]
```

output
```
(((a + b*x^(1/3))^2)^p*x*(d*x)^m*Hypergeometric2F1[3*(1 + m), -2*p, 1 + 3*
(1 + m), -((b*x^(1/3))/a)])/((1 + m)*(1 + (b*x^(1/3))/a)^(2*p))
```

3.472.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1385, 866, 864, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p dx$$

$$\Big\downarrow \text{1385}$$

$$\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\int \left(\frac{\sqrt[3]{x}b}{a}+1\right)^{2p}(dx)^m dx$$

$$\Big\downarrow \text{866}$$

$$x^{-m}(dx)^m\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\int \left(\frac{\sqrt[3]{x}b}{a}+1\right)^{2p}x^m dx$$

$$\Big\downarrow \text{864}$$

$$3x^{-m}(dx)^m\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\int \left(\frac{\sqrt[3]{x}b}{a}+1\right)^{2p}x^{\frac{1}{3}(3m+2)}d\sqrt[3]{x}$$

$$\Big\downarrow \text{74}$$

$$\frac{x(dx)^m\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p \text{Hypergeometric2F1}\left(3(m+1),-2p,3m+4,-\frac{b\sqrt[3]{x}}{a}\right)}{m+1}$$

input
```
Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*(d*x)^m,x]
```

output
```
((a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*x*(d*x)^m*Hypergeometric2F1[3*(1 +
m), -2*p, 4 + 3*m, -((b*x^(1/3))/a)])/((1 + m)*(1 + (b*x^(1/3))/a)^(2*p))
```

3.472.3.1 Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 864
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x
^(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

rule 866
```
Int[((c_)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^Int
Part[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a + b*x^n)^p, x], x] /
; FreeQ[{a, b, c, m, p}, x] && FractionQ[n]
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

3.472.4 Maple [F]

$$\int \left(a^2+2ab\,x^{\frac{1}{3}}+b^2x^{\frac{2}{3}}\right)^p (dx)^m\,dx$$

input
```
int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*(d*x)^m,x)
```

output
```
int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*(d*x)^m,x)
```

3.472.5 Fricas [F(-2)]

Exception generated.

$$\int \left(a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}\right)^p (dx)^m \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*(d*x)^m,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   algl
ogextint: unimplemented
```

3.472.6 Sympy [F]

$$\int \left(a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}\right)^p (dx)^m \, dx = \int (dx)^m \left(\left(a + b\sqrt[3]{x}\right)^2\right)^p \, dx$$

input
```
integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p*(d*x)**m,x)
```

output
```
Integral((d*x)**m*((a + b*x**(1/3))**2)**p, x)
```

3.472.7 Maxima [F]

$$\int \left(a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}\right)^p (dx)^m \, dx = \int \left(b^2x^{\frac{2}{3}} + 2\,abx^{\frac{1}{3}} + a^2\right)^p (dx)^m \, dx$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*(d*x)^m,x, algorithm="maxima")
```

output
```
integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*(d*x)^m, x)
```

3.472.8 Giac [F]

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p (dx)^m\,dx=\int \left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^p (dx)^m\,dx$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*(d*x)^m,x, algorithm="giac")
```

output
```
integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*(d*x)^m, x)
```

3.472.9 Mupad [F(-1)]

Timed out.

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p (dx)^m\,dx=\int (d\,x)^m\,\left(a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}\right)^p\,dx$$

input
```
int((d*x)^m*(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p,x)
```

output
```
int((d*x)^m*(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p, x)
```

3.473 $\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x^2\,dx$

3.473.1 Optimal result

Integrand size = 28, antiderivative size = 468

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x^2\,dx = \frac{3a^9\left(1+\frac{b\sqrt[3]{x}}{a}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(1+2p)} - \frac{12a^9\left(1+\frac{b\sqrt[3]{x}}{a}\right)^2\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(1+p)} + \frac{84a^9\left(1+\frac{b\sqrt[3]{x}}{a}\right)^3\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(3+2p)} - \frac{84a^9\left(1+\frac{b\sqrt[3]{x}}{a}\right)^4\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(2+p)} + \frac{210a^9\left(1+\frac{b\sqrt[3]{x}}{a}\right)^5\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(5+2p)} - \frac{84a^9\left(1+\frac{b\sqrt[3]{x}}{a}\right)^6\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(3+p)} + \frac{84a^9\left(1+\frac{b\sqrt[3]{x}}{a}\right)^7\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(7+2p)} - \frac{12a^9\left(1+\frac{b\sqrt[3]{x}}{a}\right)^8\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(4+p)} + \frac{3a^9\left(1+\frac{b\sqrt[3]{x}}{a}\right)^9\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(9+2p)}$$

output
```
3*a^9*(1+b*x^(1/3)/a)*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^9/(1+2*p)-12*a^9
*(1+b*x^(1/3)/a)^2*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^9/(p+1)+84*a^9*(1+b
*x^(1/3)/a)^3*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^9/(3+2*p)-84*a^9*(1+b*x^
(1/3)/a)^4*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^9/(2+p)+210*a^9*(1+b*x^(1/3
)/a)^5*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^9/(5+2*p)-84*a^9*(1+b*x^(1/3)/a
)^6*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^9/(3+p)+84*a^9*(1+b*x^(1/3)/a)^7*(
a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^9/(7+2*p)-12*a^9*(1+b*x^(1/3)/a)^8*(a^2
+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^9/(4+p)+3*a^9*(1+b*x^(1/3)/a)^9*(a^2+2*a*b
*x^(1/3)+b^2*x^(2/3))^p/b^9/(9+2*p)
```

3.473.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.44

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x^2\,dx = \frac{3\left(\frac{a^8}{1+2p}-\frac{4a^7\left(a+b\sqrt[3]{x}\right)}{1+p}+\frac{28a^6\left(a+b\sqrt[3]{x}\right)^2}{3+2p}-\frac{28a^5\left(a+b\sqrt[3]{x}\right)^3}{2+p}+\frac{70a^4\left(a+b\sqrt[3]{x}\right)^4}{5+2p}-\frac{28a^3\left(a+b\sqrt[3]{x}\right)^5}{3+p}\right.}{b^9}$$

input
```
Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*x^2,x]
```

output
```
(3*(a^8/(1 + 2*p) - (4*a^7*(a + b*x^(1/3)))/(1 + p) + (28*a^6*(a + b*x^(1/
3))^2)/(3 + 2*p) - (28*a^5*(a + b*x^(1/3))^3)/(2 + p) + (70*a^4*(a + b*x^(
1/3))^4)/(5 + 2*p) - (28*a^3*(a + b*x^(1/3))^5)/(3 + p) + (28*a^2*(a + b*x
^(1/3))^6)/(7 + 2*p) - (4*a*(a + b*x^(1/3))^7)/(4 + p) + (a + b*x^(1/3))^8
/(9 + 2*p))*(a + b*x^(1/3))*((a + b*x^(1/3))^2)^p)/b^9
```

3.473.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 330, normalized size of antiderivative = 0.71, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1385, 798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\,dx$$

$$\Big\downarrow \text{1385}$$

$$\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\int\left(\frac{\sqrt[3]{x}b}{a}+1\right)^{2p}x^2dx$$

$$\Big\downarrow \text{798}$$

$$3\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\int\left(\frac{\sqrt[3]{x}b}{a}+1\right)^{2p}x^{8/3}d\sqrt[3]{x}$$

$$\Big\downarrow \text{53}$$

$$3\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\int\left(\frac{a^8\left(\frac{\sqrt[3]{x}b}{a}+1\right)^{2p}}{b^8}-\frac{8a^8\left(\frac{\sqrt[3]{x}b}{a}+1\right)^{2p+1}}{b^8}+\frac{28a^8\left(\frac{\sqrt[3]{x}b}{a}+1\right)^{2p+2}}{b^8}\right.$$

$$\Big\downarrow \text{2009}$$

$$3\left(-\frac{4a^9\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{2(p+1)}}{b^9(p+1)}-\frac{28a^9\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{2(p+2)}}{b^9(p+2)}-\frac{28a^9\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{2(p+3)}}{b^9(p+3)}-\frac{4a^9\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{2(p+4)}}{b^9(p+4)}+\frac{a^9\left(\frac{b\sqrt[3]{x}}{a}\right.}{b^9}\right.$$

input
```
Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*x^2,x]
```

output
```
(3*((-4*a^9*(1 + (b*x^(1/3))/a)^(2*(1 + p)))/(b^9*(1 + p)) - (28*a^9*(1 +
(b*x^(1/3))/a)^(2*(2 + p)))/(b^9*(2 + p)) - (28*a^9*(1 + (b*x^(1/3))/a)^(2
*(3 + p)))/(b^9*(3 + p)) - (4*a^9*(1 + (b*x^(1/3))/a)^(2*(4 + p)))/(b^9*(4
 + p)) + (a^9*(1 + (b*x^(1/3))/a)^(1 + 2*p))/(b^9*(1 + 2*p)) + (28*a^9*(1
+ (b*x^(1/3))/a)^(3 + 2*p))/(b^9*(3 + 2*p)) + (70*a^9*(1 + (b*x^(1/3))/a)^
(5 + 2*p))/(b^9*(5 + 2*p)) + (28*a^9*(1 + (b*x^(1/3))/a)^(7 + 2*p))/(b^9*(
7 + 2*p)) + (a^9*(1 + (b*x^(1/3))/a)^(9 + 2*p))/(b^9*(9 + 2*p)))*(a^2 + 2*
a*b*x^(1/3) + b^2*x^(2/3))^p)/(1 + (b*x^(1/3))/a)^(2*p)
```

3.473. $\int\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x^2\,dx$

3.473.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.473.4 Maple [F]

$$\int \left(a^{2}+2 a b \,x^{\frac{1}{3}}+b^{2} x^{\frac{2}{3}}\right)^{p} x^{2}d x$$

input
```
int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x^2,x)
```

output
```
int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x^2,x)
```

3.473.5 Fricas [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 579, normalized size of antiderivative = 1.24

$$\int \left(a^2+2ab\sqrt[3]{x} +b^2x^{2/3}\right)^p x^2\,dx = \frac{3\left(2520\,a^9+\left(16\,b^9p^8+288\,b^9p^7+2184\,b^9p^6+9072\,b^9p^5+22449\,b^9p^4+33642\,b^9p^3+295\right.\right.}{}$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x^2,x, algorithm="fricas")
```

output
```
3*(2520*a^9 + (16*b^9*p^8 + 288*b^9*p^7 + 2184*b^9*p^6 + 9072*b^9*p^5 + 22
449*b^9*p^4 + 33642*b^9*p^3 + 29531*b^9*p^2 + 13698*b^9*p + 2520*b^9)*x^3
+ 28*(8*a^3*b^6*p^6 + 60*a^3*b^6*p^5 + 170*a^3*b^6*p^4 + 225*a^3*b^6*p^3 +
 137*a^3*b^6*p^2 + 30*a^3*b^6*p)*x^2 - 1680*(2*a^6*b^3*p^3 + 3*a^6*b^3*p^2
 + a^6*b^3*p)*x + (5040*a^7*b^2*p^2 + 2520*a^7*b^2*p + (16*a*b^8*p^8 + 224
*a*b^8*p^7 + 1288*a*b^8*p^6 + 3920*a*b^8*p^5 + 6769*a*b^8*p^4 + 6566*a*b^8
*p^3 + 3267*a*b^8*p^2 + 630*a*b^8*p)*x^2 - 168*(4*a^4*b^5*p^5 + 20*a^4*b^5
*p^4 + 35*a^4*b^5*p^3 + 25*a^4*b^5*p^2 + 6*a^4*b^5*p)*x)*x^(2/3) - 4*(1260
*a^8*b*p + 2*(8*a^2*b^7*p^7 + 84*a^2*b^7*p^6 + 350*a^2*b^7*p^5 + 735*a^2*b
^7*p^4 + 812*a^2*b^7*p^3 + 441*a^2*b^7*p^2 + 90*a^2*b^7*p)*x^2 - 105*(4*a^
5*b^4*p^4 + 12*a^5*b^4*p^3 + 11*a^5*b^4*p^2 + 3*a^5*b^4*p)*x)*x^(1/3))*(b^
2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/(32*b^9*p^9 + 720*b^9*p^8 + 6960*b^9*p^
7 + 37800*b^9*p^6 + 126546*b^9*p^5 + 269325*b^9*p^4 + 361840*b^9*p^3 + 293
175*b^9*p^2 + 128322*b^9*p + 22680*b^9)
```

3.473.6 Sympy [F]

$$\int \left(a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}\right)^p x^2\,dx = \int x^2\left(\left(a + b\sqrt[3]{x}\right)^2\right)^p\,dx$$

input
```
integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p*x**2,x)
```

output
```
Integral(x**2*((a + b*x**(1/3))**2)**p, x)
```

3.473.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 362, normalized size of antiderivative = 0.77

$$\int \left(a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}\right)^p x^2\,dx = \frac{3\left(\left(16\,p^8 + 288\,p^7 + 2184\,p^6 + 9072\,p^5 + 22449\,p^4 + 33642\,p^3 + 29531\,p^2 + 13698\,p + 252\right.\right.}{}$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x^2,x, algorithm="maxima")
```

output
```
3*((16*p^8 + 288*p^7 + 2184*p^6 + 9072*p^5 + 22449*p^4 + 33642*p^3 + 29531
*p^2 + 13698*p + 2520)*b^9*x^3 + (16*p^8 + 224*p^7 + 1288*p^6 + 3920*p^5 +
 6769*p^4 + 6566*p^3 + 3267*p^2 + 630*p)*a*b^8*x^(8/3) - 8*(8*p^7 + 84*p^6
 + 350*p^5 + 735*p^4 + 812*p^3 + 441*p^2 + 90*p)*a^2*b^7*x^(7/3) + 28*(8*p
^6 + 60*p^5 + 170*p^4 + 225*p^3 + 137*p^2 + 30*p)*a^3*b^6*x^2 - 168*(4*p^5
 + 20*p^4 + 35*p^3 + 25*p^2 + 6*p)*a^4*b^5*x^(5/3) + 420*(4*p^4 + 12*p^3 +
 11*p^2 + 3*p)*a^5*b^4*x^(4/3) - 1680*(2*p^3 + 3*p^2 + p)*a^6*b^3*x + 2520
*(2*p^2 + p)*a^7*b^2*x^(2/3) - 5040*a^8*b*p*x^(1/3) + 2520*a^9)*(b*x^(1/3)
 + a)^(2*p)/((32*p^9 + 720*p^8 + 6960*p^7 + 37800*p^6 + 126546*p^5 + 26932
5*p^4 + 361840*p^3 + 293175*p^2 + 128322*p + 22680)*b^9)
```

3.473.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1564 vs. $2(414) = 828$.

Time = 0.32 (sec) , antiderivative size = 1564, normalized size of antiderivative = 3.34

$$\int \left(a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}\right)^p x^2\,dx = \text{Too large to display}$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x^2,x, algorithm="giac")
```

output
```
3*(16*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^9*p^8*x^3 + 16*(b^2*x^(2/3)
+ 2*a*b*x^(1/3) + a^2)^p*a*b^8*p^8*x^(8/3) + 288*(b^2*x^(2/3) + 2*a*b*x^(1
/3) + a^2)^p*b^9*p^7*x^3 + 224*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a*b^8
*p^7*x^(8/3) - 64*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^2*b^7*p^7*x^(7/3
) + 2184*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^9*p^6*x^3 + 1288*(b^2*x^(
2/3) + 2*a*b*x^(1/3) + a^2)^p*a*b^8*p^6*x^(8/3) - 672*(b^2*x^(2/3) + 2*a*b
*x^(1/3) + a^2)^p*a^2*b^7*p^6*x^(7/3) + 224*(b^2*x^(2/3) + 2*a*b*x^(1/3) +
 a^2)^p*a^3*b^6*p^6*x^2 + 9072*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^9*p
^5*x^3 + 3920*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a*b^8*p^5*x^(8/3) - 28
00*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^2*b^7*p^5*x^(7/3) + 1680*(b^2*x
^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^3*b^6*p^5*x^2 + 22449*(b^2*x^(2/3) + 2*a
*b*x^(1/3) + a^2)^p*b^9*p^4*x^3 - 672*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^
p*a^4*b^5*p^5*x^(5/3) + 6769*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a*b^8*p
^4*x^(8/3) - 5880*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^2*b^7*p^4*x^(7/3
) + 4760*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^3*b^6*p^4*x^2 + 33642*(b^
2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^9*p^3*x^3 - 3360*(b^2*x^(2/3) + 2*a*b
*x^(1/3) + a^2)^p*a^4*b^5*p^4*x^(5/3) + 6566*(b^2*x^(2/3) + 2*a*b*x^(1/3)
+ a^2)^p*a*b^8*p^3*x^(8/3) + 1680*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^
5*b^4*p^4*x^(4/3) - 6496*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^2*b^7*p^3
*x^(7/3) + 6300*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^3*b^6*p^3*x^2 +...
```

3.473.9 Mupad [B] (verification not implemented)

Time = 9.61 (sec) , antiderivative size = 777, normalized size of antiderivative = 1.66

$$\int \left(a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}\right)^p x^2\,dx = \left(a^2 + b^2\,x^{2/3} + 2\,a\,b\,x^{1/3}\right)^p \left(\frac{3\,x^3\,(16\,p^8 + 288\,p^7 + 2184\,p^6 + 9072\,p^5 + 22449\,p^4 + 3}{32\,p^9 + 720\,p^8 + 6960\,p^7 + 37800\,p^6 + 126546\,p^5 + 269325\,p^4}\right.$$

input
```
int(x^2*(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p,x)
```

output
```
(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p*((3*x^3*(13698*p + 29531*p^2 + 33642
*p^3 + 22449*p^4 + 9072*p^5 + 2184*p^6 + 288*p^7 + 16*p^8 + 2520))/(128322
*p + 293175*p^2 + 361840*p^3 + 269325*p^4 + 126546*p^5 + 37800*p^6 + 6960*
p^7 + 720*p^8 + 32*p^9 + 22680) + (7560*a^9)/(b^9*(128322*p + 293175*p^2 +
 361840*p^3 + 269325*p^4 + 126546*p^5 + 37800*p^6 + 6960*p^7 + 720*p^8 + 3
2*p^9 + 22680)) - (15120*a^8*p*x^(1/3))/(b^8*(128322*p + 293175*p^2 + 3618
40*p^3 + 269325*p^4 + 126546*p^5 + 37800*p^6 + 6960*p^7 + 720*p^8 + 32*p^9
 + 22680)) + (3*a*p*x^(8/3)*(3267*p + 6566*p^2 + 6769*p^3 + 3920*p^4 + 128
8*p^5 + 224*p^6 + 16*p^7 + 630))/(b*(128322*p + 293175*p^2 + 361840*p^3 +
269325*p^4 + 126546*p^5 + 37800*p^6 + 6960*p^7 + 720*p^8 + 32*p^9 + 22680)
) + (84*a^3*p*x^2*(137*p + 225*p^2 + 170*p^3 + 60*p^4 + 8*p^5 + 30))/(b^3*
(128322*p + 293175*p^2 + 361840*p^3 + 269325*p^4 + 126546*p^5 + 37800*p^6
+ 6960*p^7 + 720*p^8 + 32*p^9 + 22680)) - (5040*a^6*p*x*(3*p + 2*p^2 + 1))
/(b^6*(128322*p + 293175*p^2 + 361840*p^3 + 269325*p^4 + 126546*p^5 + 3780
0*p^6 + 6960*p^7 + 720*p^8 + 32*p^9 + 22680)) - (24*a^2*p*x^(7/3)*(441*p +
 812*p^2 + 735*p^3 + 350*p^4 + 84*p^5 + 8*p^6 + 90))/(b^2*(128322*p + 2931
75*p^2 + 361840*p^3 + 269325*p^4 + 126546*p^5 + 37800*p^6 + 6960*p^7 + 720
*p^8 + 32*p^9 + 22680)) + (7560*a^7*p*x^(2/3)*(2*p + 1))/(b^7*(128322*p +
293175*p^2 + 361840*p^3 + 269325*p^4 + 126546*p^5 + 37800*p^6 + 6960*p^7 +
 720*p^8 + 32*p^9 + 22680)) + (1260*a^5*p*x^(4/3)*(11*p + 12*p^2 + 4*p^...
```

3.473. $\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x^2\,dx$

3.474 $\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x\,dx$

3.474.1 Optimal result

Integrand size = 26, antiderivative size = 315

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x\,dx = -\frac{3a^6\left(1+\frac{b\sqrt[3]{x}}{a}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^6(1+2p)} + \frac{15a^6\left(1+\frac{b\sqrt[3]{x}}{a}\right)^2\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{2b^6(1+p)} - \frac{30a^6\left(1+\frac{b\sqrt[3]{x}}{a}\right)^3\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^6(3+2p)} + \frac{15a^6\left(1+\frac{b\sqrt[3]{x}}{a}\right)^4\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^6(2+p)} - \frac{15a^6\left(1+\frac{b\sqrt[3]{x}}{a}\right)^5\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^6(5+2p)} + \frac{3a^6\left(1+\frac{b\sqrt[3]{x}}{a}\right)^6\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{2b^6(3+p)}$$

output
```
-3*a^6*(1+b*x^(1/3)/a)*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^6/(1+2*p)+15/2*
a^6*(1+b*x^(1/3)/a)^2*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^6/(p+1)-30*a^6*(
1+b*x^(1/3)/a)^3*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^6/(3+2*p)+15*a^6*(1+b
*x^(1/3)/a)^4*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^6/(2+p)-15*a^6*(1+b*x^(1
/3)/a)^5*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^6/(5+2*p)+3/2*a^6*(1+b*x^(1/3
)/a)^6*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^6/(3+p)
```

3.474.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.45

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x\,dx = \frac{3\left(-\frac{2a^5}{1+2p}+\frac{5a^4\left(a+b\sqrt[3]{x}\right)}{1+p}-\frac{20a^3\left(a+b\sqrt[3]{x}\right)^2}{3+2p}+\frac{10a^2\left(a+b\sqrt[3]{x}\right)^3}{2+p}-\frac{10a\left(a+b\sqrt[3]{x}\right)^4}{5+2p}+\frac{\left(a+b\sqrt[3]{x}\right)^5}{3+p}\right)(}{2b^6}$$

input
```
Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*x,x]
```

output
```
(3*((-2*a^5)/(1 + 2*p) + (5*a^4*(a + b*x^(1/3)))/(1 + p) - (20*a^3*(a + b*
x^(1/3))^2)/(3 + 2*p) + (10*a^2*(a + b*x^(1/3))^3)/(2 + p) - (10*a*(a + b*
x^(1/3))^4)/(5 + 2*p) + (a + b*x^(1/3))^5/(3 + p))*(a + b*x^(1/3))*((a + b
*x^(1/3))^2)^p)/(2*b^6)
```

3.474.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.76, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1385, 798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p dx$$

$$\Big\downarrow \ 1385$$

$$\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\int\left(\frac{\sqrt[3]{x}b}{a}+1\right)^{2p}xdx$$

$\big\downarrow$ 798

$$3\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\int\left(\frac{\sqrt[3]{x}b}{a}+1\right)^{2p}x^{5/3}d\sqrt[3]{x}$$

$\big\downarrow$ 53

$$3\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\int\left(-\frac{a^5\left(\frac{\sqrt[3]{x}b}{a}+1\right)^{2p}}{b^5}+\frac{5a^5\left(\frac{\sqrt[3]{x}b}{a}+1\right)^{2p+1}}{b^5}-\frac{10a^5\left(\frac{\sqrt[3]{x}b}{a}+1\right)^{2p+2}}{b^5}\right.$$

$\big\downarrow$ 2009

$$3\left(\frac{5a^6\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{2(p+1)}}{2b^6(p+1)}+\frac{5a^6\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{2(p+2)}}{b^6(p+2)}+\frac{a^6\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{2(p+3)}}{2b^6(p+3)}-\frac{a^6\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{2p+1}}{b^6(2p+1)}-\frac{10a^6\left(\frac{b\sqrt[3]{x}}{a}+\right.}{b^6(2p+3}\right.$$

input
```
Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*x,x]
```

output
```
(3*((5*a^6*(1 + (b*x^(1/3))/a)^(2*(1 + p)))/(2*b^6*(1 + p)) + (5*a^6*(1 +
(b*x^(1/3))/a)^(2*(2 + p)))/(b^6*(2 + p)) + (a^6*(1 + (b*x^(1/3))/a)^(2*(3
 + p)))/(2*b^6*(3 + p)) - (a^6*(1 + (b*x^(1/3))/a)^(1 + 2*p))/(b^6*(1 + 2*
p)) - (10*a^6*(1 + (b*x^(1/3))/a)^(3 + 2*p))/(b^6*(3 + 2*p)) - (5*a^6*(1 +
 (b*x^(1/3))/a)^(5 + 2*p))/(b^6*(5 + 2*p)))*(a^2 + 2*a*b*x^(1/3) + b^2*x^(
2/3))^p)/(1 + (b*x^(1/3))/a)^(2*p)
```

3.474.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

3.474. $\int\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x\,dx$

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.474.4 Maple [F]

$$\int \left(a^2+2ab\,x^{\frac{1}{3}}+b^2x^{\frac{2}{3}}\right)^p x dx$$

input
```
int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x,x)
```

output
```
int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x,x)
```

3.474.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 297, normalized size of antiderivative = 0.94

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x\,dx =$$

$$-\frac{3\left(30\,a^6-(8\,b^6p^5+60\,b^6p^4+170\,b^6p^3+225\,b^6p^2+137\,b^6p+30\,b^6)x^2-20\,(2\,a^3b^3p^3+3\,a^3b^3p^2+a^3b^3p)x\right.}{2}$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x,x, algorithm="fricas")
```

output
```
-3/2*(30*a^6 - (8*b^6*p^5 + 60*b^6*p^4 + 170*b^6*p^3 + 225*b^6*p^2 + 137*b
^6*p + 30*b^6)*x^2 - 20*(2*a^3*b^3*p^3 + 3*a^3*b^3*p^2 + a^3*b^3*p)*x + 2*
(30*a^4*b^2*p^2 + 15*a^4*b^2*p - (4*a*b^5*p^5 + 20*a*b^5*p^4 + 35*a*b^5*p^
3 + 25*a*b^5*p^2 + 6*a*b^5*p)*x)*x^(2/3) - 5*(12*a^5*b*p - (4*a^2*b^4*p^4
+ 12*a^2*b^4*p^3 + 11*a^2*b^4*p^2 + 3*a^2*b^4*p)*x)*x^(1/3))*(b^2*x^(2/3)
+ 2*a*b*x^(1/3) + a^2)^p/(8*b^6*p^6 + 84*b^6*p^5 + 350*b^6*p^4 + 735*b^6*p
^3 + 812*b^6*p^2 + 441*b^6*p + 90*b^6)
```

3.474.6 Sympy [F]

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x\,dx=\int x\left(\left(a+b\sqrt[3]{x}\right)^2\right)^p\,dx$$

input
```
integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p*x,x)
```

output
```
Integral(x*((a + b*x**(1/3))**2)**p, x)
```

3.474.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.63

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x\,dx=\frac{3\left((8\,p^5+60\,p^4+170\,p^3+225\,p^2+137\,p+30)b^6x^2+2\,(4\,p^5+20\,p^4+35\,p^3+25\,p^2+6\,p\right.}{2\,(8\,p^6+}$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x,x, algorithm="maxima")
```

output
```
3/2*((8*p^5 + 60*p^4 + 170*p^3 + 225*p^2 + 137*p + 30)*b^6*x^2 + 2*(4*p^5
+ 20*p^4 + 35*p^3 + 25*p^2 + 6*p)*a*b^5*x^(5/3) - 5*(4*p^4 + 12*p^3 + 11*p
^2 + 3*p)*a^2*b^4*x^(4/3) + 20*(2*p^3 + 3*p^2 + p)*a^3*b^3*x - 30*(2*p^2 +
 p)*a^4*b^2*x^(2/3) + 60*a^5*b*p*x^(1/3) - 30*a^6)*(b*x^(1/3) + a)^(2*p)/(
(8*p^6 + 84*p^5 + 350*p^4 + 735*p^3 + 812*p^2 + 441*p + 90)*b^6)
```

3.474.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 745 vs. 2(275) = 550.

Time = 0.32 (sec) , antiderivative size = 745, normalized size of antiderivative = 2.37

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x\,dx=\frac{3\left(8\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^pb^6p^5x^2+8\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^pab^5p^5x^{\frac{5}{3}}+60\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}\right.\right.}{}$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x,x, algorithm="giac")
```

output
```
3/2*(8*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^6*p^5*x^2 + 8*(b^2*x^(2/3)
+ 2*a*b*x^(1/3) + a^2)^p*a*b^5*p^5*x^(5/3) + 60*(b^2*x^(2/3) + 2*a*b*x^(1/
3) + a^2)^p*b^6*p^4*x^2 + 40*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a*b^5*p
^4*x^(5/3) - 20*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^2*b^4*p^4*x^(4/3)
+ 170*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^6*p^3*x^2 + 70*(b^2*x^(2/3)
+ 2*a*b*x^(1/3) + a^2)^p*a*b^5*p^3*x^(5/3) - 60*(b^2*x^(2/3) + 2*a*b*x^(1/
3) + a^2)^p*a^2*b^4*p^3*x^(4/3) + 40*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p
*a^3*b^3*p^3*x + 225*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^6*p^2*x^2 + 5
0*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a*b^5*p^2*x^(5/3) - 55*(b^2*x^(2/3
) + 2*a*b*x^(1/3) + a^2)^p*a^2*b^4*p^2*x^(4/3) + 60*(b^2*x^(2/3) + 2*a*b*x
^(1/3) + a^2)^p*a^3*b^3*p^2*x + 137*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*
b^6*p*x^2 - 60*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^4*b^2*p^2*x^(2/3) +
 12*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a*b^5*p*x^(5/3) - 15*(b^2*x^(2/3
) + 2*a*b*x^(1/3) + a^2)^p*a^2*b^4*p*x^(4/3) + 20*(b^2*x^(2/3) + 2*a*b*x^(
1/3) + a^2)^p*a^3*b^3*p*x + 30*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^6*x
^2 - 30*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^4*b^2*p*x^(2/3) + 60*(b^2*
x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^5*b*p*x^(1/3) - 30*(b^2*x^(2/3) + 2*a*b
*x^(1/3) + a^2)^p*a^6)/(8*b^6*p^6 + 84*b^6*p^5 + 350*b^6*p^4 + 735*b^6*p^3
 + 812*b^6*p^2 + 441*b^6*p + 90*b^6)
```

3.474.9 Mupad [B] (verification not implemented)

Time = 8.76 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.24

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x\,dx = \left(a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}\right)^p\,\left(\frac{3\,x^2\,\left(8\,p^5+60\,p^4+170\,p^3+225\,p^2+137\,p+30\right)}{2\,\left(8\,p^6+84\,p^5+350\,p^4+735\,p^3+812\,p^2+441\,p+90\right)}-\frac{\ldots}{b^6\,(\ldots}$$

input
```
int(x*(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p,x)
```

output
```
(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p*((3*x^2*(137*p + 225*p^2 + 170*p^3 +
 60*p^4 + 8*p^5 + 30))/(2*(441*p + 812*p^2 + 735*p^3 + 350*p^4 + 84*p^5 +
8*p^6 + 90)) - (45*a^6)/(b^6*(441*p + 812*p^2 + 735*p^3 + 350*p^4 + 84*p^5
 + 8*p^6 + 90)) + (90*a^5*p*x^(1/3))/(b^5*(441*p + 812*p^2 + 735*p^3 + 350
*p^4 + 84*p^5 + 8*p^6 + 90)) - (15*a^2*p*x^(4/3)*(11*p + 12*p^2 + 4*p^3 +
3))/(2*b^2*(441*p + 812*p^2 + 735*p^3 + 350*p^4 + 84*p^5 + 8*p^6 + 90)) +
(30*a^3*p*x*(3*p + 2*p^2 + 1))/(b^3*(441*p + 812*p^2 + 735*p^3 + 350*p^4 +
 84*p^5 + 8*p^6 + 90)) - (45*a^4*p*x^(2/3)*(2*p + 1))/(b^4*(441*p + 812*p^
2 + 735*p^3 + 350*p^4 + 84*p^5 + 8*p^6 + 90)) + (3*a*p*x^(5/3)*(25*p + 35*
p^2 + 20*p^3 + 4*p^4 + 6))/(b*(441*p + 812*p^2 + 735*p^3 + 350*p^4 + 84*p^
5 + 8*p^6 + 90)))
```

3.474. $\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x\,dx$

3.475 $\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\,dx$

3.475.1 Optimal result

Integrand size = 24, antiderivative size = 142

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\,dx=\frac{3a^2\left(a+b\sqrt[3]{x}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^3(1+2p)}-\frac{3a\left(a+b\sqrt[3]{x}\right)^2\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^3(1+p)}+\frac{3\left(a+b\sqrt[3]{x}\right)^3\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^3(3+2p)}$$

output
```
3*a^2*(a+b*x^(1/3))*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^3/(1+2*p)-3*a*(a+b
*x^(1/3))^2*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^3/(p+1)+3*(a+b*x^(1/3))^3*
(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^3/(3+2*p)
```

3.475.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.58

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\,dx=\frac{3\left(a+b\sqrt[3]{x}\right)\left(\left(a+b\sqrt[3]{x}\right)^2\right)^p\left(a^2-ab(1+2p)\sqrt[3]{x}+b^2\left(1+3p+2p^2\right)x^{2/3}\right)}{b^3(1+p)(1+2p)(3+2p)}$$

input
```
Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p,x]
```

output
```
(3*(a + b*x^(1/3))*((a + b*x^(1/3))^2)^p*(a^2 - a*b*(1 + 2*p)*x^(1/3) + b^
2*(1 + 3*p + 2*p^2)*x^(2/3)))/(b^3*(1 + p)*(1 + 2*p)*(3 + 2*p))
```

3.475.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1385, 774, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p dx$$

$$\Big\downarrow \text{1385}$$

$$\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\int \left(\frac{\sqrt[3]{x}b}{a}+1\right)^{2p}dx$$

$$\Big\downarrow \text{774}$$

$$3\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\int \left(\frac{\sqrt[3]{x}b}{a}+1\right)^{2p}x^{2/3}d\sqrt[3]{x}$$

$$\Big\downarrow \text{53}$$

$$3\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\int \left(\frac{a^2\left(\frac{\sqrt[3]{x}b}{a}+1\right)^{2p}}{b^2}-\frac{2a^2\left(\frac{\sqrt[3]{x}b}{a}+1\right)^{2p+1}}{b^2}+\frac{a^2\left(\frac{\sqrt[3]{x}b}{a}+1\right)^{2p+2}}{b^2}\right)d$$

$$\Big\downarrow \text{2009}$$

$$3\left(-\frac{a^3\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{2(p+1)}}{b^3(p+1)}+\frac{a^3\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{2p+1}}{b^3(2p+1)}+\frac{a^3\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{2p+3}}{b^3(2p+3)}\right)\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p$$

input
```
Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p,x]
```

output
```
(3*(-((a^3*(1 + (b*x^(1/3))/a)^(2*(1 + p)))/(b^3*(1 + p))) + (a^3*(1 + (b*
x^(1/3))/a)^(1 + 2*p))/(b^3*(1 + 2*p)) + (a^3*(1 + (b*x^(1/3))/a)^(3 + 2*p
))/(b^3*(3 + 2*p)))*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(1 + (b*x^(1/3)
)/a)^(2*p)
```

3.475.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.475.4 Maple [F]

$$\int \left(a^2 + 2ab\, x^{\frac{1}{3}} + b^2 x^{\frac{2}{3}}\right)^p dx$$

input
```
int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p,x)
```

output
```
int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p,x)
```

3.475.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.77

$$\int \left(a^2 + 2ab\sqrt[3]{x} + b^2 x^{2/3}\right)^p dx = -\frac{3\left(2\,a^2bpx^{\frac{1}{3}} - a^3 - (2\,b^3p^2 + 3\,b^3p + b^3)x - (2\,ab^2p^2 + ab^2p)x^{\frac{2}{3}}\right)\left(b^2x^{\frac{2}{3}} + 2\,abx^{\frac{1}{3}} + a^2\right)^p}{4\,b^3p^3 + 12\,b^3p^2 + 11\,b^3p + 3\,b^3}$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p,x, algorithm="fricas")
```

output
```
-3*(2*a^2*b*p*x^(1/3) - a^3 - (2*b^3*p^2 + 3*b^3*p + b^3)*x - (2*a*b^2*p^2
 + a*b^2*p)*x^(2/3))*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/(4*b^3*p^3 + 12
*b^3*p^2 + 11*b^3*p + 3*b^3)
```

3.475.6 Sympy [F]

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\,dx=\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^p\,dx$$

input
```
integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p,x)
```

output
```
Integral((a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**p, x)
```

3.475.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.54

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\,dx=\frac{3\left((2\,p^2+3\,p+1)b^3x+(2\,p^2+p)ab^2x^{\frac{2}{3}}-2\,a^2bpx^{\frac{1}{3}}+a^3\right)\left(bx^{\frac{1}{3}}+a\right)^{2\,p}}{(4\,p^3+12\,p^2+11\,p+3)b^3}$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p,x, algorithm="maxima")
```

output
```
3*((2*p^2 + 3*p + 1)*b^3*x + (2*p^2 + p)*a*b^2*x^(2/3) - 2*a^2*b*p*x^(1/3)
 + a^3)*(b*x^(1/3) + a)^(2*p)/((4*p^3 + 12*p^2 + 11*p + 3)*b^3)
```

3.475.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.61

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\,dx = \frac{3\left(2\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^pb^3p^2x+2\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^pab^2p^2x^{\frac{2}{3}}+3\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+\right.\right.}{}$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p,x, algorithm="giac")
```

output
```
3*(2*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^3*p^2*x + 2*(b^2*x^(2/3) + 2*
a*b*x^(1/3) + a^2)^p*a*b^2*p^2*x^(2/3) + 3*(b^2*x^(2/3) + 2*a*b*x^(1/3) +
a^2)^p*b^3*p*x + (b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a*b^2*p*x^(2/3) - 2
*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^2*b*p*x^(1/3) + (b^2*x^(2/3) + 2*
a*b*x^(1/3) + a^2)^p*b^3*x + (b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^3)/(4
*b^3*p^3 + 12*b^3*p^2 + 11*b^3*p + 3*b^3)
```

3.475.9 Mupad [B] (verification not implemented)

Time = 8.44 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.97

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\,dx = \left(a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}\right)^p\left(\frac{3\,x\,(2\,p^2+3\,p+1)}{4\,p^3+12\,p^2+11\,p+3}+\frac{3\,a^3}{b^3\,(4\,p^3+12\,p^2+11\,p+3)}-\frac{}{b^2\,(4\,p^3}\right.$$

input
```
int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p,x)
```

output
```
(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p*((3*x*(3*p + 2*p^2 + 1))/(11*p + 12*
p^2 + 4*p^3 + 3) + (3*a^3)/(b^3*(11*p + 12*p^2 + 4*p^3 + 3)) - (6*a^2*p*x^
(1/3))/(b^2*(11*p + 12*p^2 + 4*p^3 + 3)) + (3*a*p*x^(2/3)*(2*p + 1))/(b*(1
1*p + 12*p^2 + 4*p^3 + 3)))
```

3.476 $\int \frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x}\,dx$

3.476.1 Optimal result

Integrand size = 28, antiderivative size = 69

$$\int \frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x}\,dx = -\frac{3\left(1+\frac{b\sqrt[3]{x}}{a}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p \text{Hypergeometric2F1}\left(1,1+2p,2(1+p),1+\frac{b\sqrt[3]{x}}{a}\right)}{1+2p}$$

output
```
-3*(1+b*x^(1/3)/a)*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*hypergeom([1, 1+2*p],
[2+2*p],1+b*x^(1/3)/a)/(1+2*p)
```

3.476.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.84

$$\int \frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x}\,dx = -\frac{3\left(a+b\sqrt[3]{x}\right)\left(\left(a+b\sqrt[3]{x}\right)^2\right)^p \text{Hypergeometric2F1}\left(1,1+2p,2+2p,1+\frac{b\sqrt[3]{x}}{a}\right)}{a(1+2p)}$$

input
```
Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p/x,x]
```

output
```
(-3*(a + b*x^(1/3))*((a + b*x^(1/3))^2)^p*Hypergeometric2F1[1, 1 + 2*p, 2
+ 2*p, 1 + (b*x^(1/3))/a])/(a*(1 + 2*p))
```

3.476.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {1385, 798, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x}dx$$

$$\Big\downarrow \text{1385}$$

$$\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\int \frac{\left(\frac{\sqrt[3]{x}b}{a}+1\right)^{2p}}{x}dx$$

$$\Big\downarrow \text{798}$$

$$3\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\int \frac{\left(\frac{\sqrt[3]{x}b}{a}+1\right)^{2p}}{\sqrt[3]{x}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{75}$$

$$-\frac{3\left(\frac{b\sqrt[3]{x}}{a}+1\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\text{Hypergeometric2F1}\left(1,2p+1,2(p+1),\frac{\sqrt[3]{x}b}{a}+1\right)}{2p+1}$$

input
```
Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p/x,x]
```

output
```
(-3*(1 + (b*x^(1/3))/a)*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*Hypergeometr
ic2F1[1, 1 + 2*p, 2*(1 + p), 1 + (b*x^(1/3))/a])/(1 + 2*p)
```

3.476.3.1 Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

3.476.4 Maple [F]

$$\int \frac{\left(a^{2}+2 a b \,x^{\frac{1}{3}}+b^{2} x^{\frac{2}{3}}\right)^{p}}{x}d x$$

input
```
int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x,x)
```

output
```
int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x,x)
```

3.476.5 Fricas [F]

$$\int \frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x}\,dx=\int \frac{{\left(b^{2} x^{\frac{2}{3}} + 2 \, a b x^{\frac{1}{3}} + a^{2}\right)}^{p}}{x}\,{d x}$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x,x, algorithm="fricas")
```

output
```
integral((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/x, x)
```

3.476.6 Sympy [F]

$$\int \frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x}\,dx=\int \frac{\left(\left(a+b\sqrt[3]{x}\right)^2\right)^p}{x}\,dx$$

input
```
integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p/x,x)
```

output
```
Integral(((a + b*x**(1/3))**2)**p/x, x)
```

3.476.7 Maxima [F]

$$\int \frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x}\,dx=\int \frac{\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^p}{x}\,dx$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x,x, algorithm="maxima")
```

output
```
integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/x, x)
```

3.476.8 Giac [F]

$$\int \frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x}\,dx=\int \frac{\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^p}{x}\,dx$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x,x, algorithm="giac")
```

output
```
integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/x, x)
```

3.476.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x}\,dx=\int \frac{\left(a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}\right)^p}{x}\,dx$$

input
```
int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p/x,x)
```

output
```
int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p/x, x)
```

3.477 $\int \frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x^2}\,dx$

3.477.1 Optimal result

Integrand size = 28, antiderivative size = 75

$$\int \frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x^2}\,dx=\frac{3b^3\left(1+\frac{b\sqrt[3]{x}}{a}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p \text{Hypergeometric2F1}\left(4,1+2p,2(\right.}{a^3(1+2p)}$$

output
```
3*b^3*(1+b*x^(1/3)/a)*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*hypergeom([4, 1+2*
p],[2+2*p],1+b*x^(1/3)/a)/a^3/(1+2*p)
```

3.477.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.81

$$\int \frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x^2}\,dx=\frac{3b^3\left(a+b\sqrt[3]{x}\right)\left(\left(a+b\sqrt[3]{x}\right)^2\right)^p \text{Hypergeometric2F1}\left(4,1+2p,2+2p,1+\right.}{a^4(1+2p)}$$

input
```
Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p/x^2,x]
```

output
```
(3*b^3*(a + b*x^(1/3))*((a + b*x^(1/3))^2)^p*Hypergeometric2F1[4, 1 + 2*p,
 2 + 2*p, 1 + (b*x^(1/3))/a])/(a^4*(1 + 2*p))
```

3.477.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {1385, 798, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x^2}dx$$

$$\Big\downarrow \text{1385}$$

$$\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\int \frac{\left(\frac{\sqrt[3]{x}b}{a}+1\right)^{2p}}{x^2}dx$$

$$\Big\downarrow \text{798}$$

$$3\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\int \frac{\left(\frac{\sqrt[3]{x}b}{a}+1\right)^{2p}}{x^{4/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{75}$$

$$\frac{3b^3\left(\frac{b\sqrt[3]{x}}{a}+1\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\text{Hypergeometric2F1}\left(4,2p+1,2(p+1),\frac{\sqrt[3]{x}b}{a}+1\right)}{a^3(2p+1)}$$

input
```
Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p/x^2,x]
```

output
```
(3*b^3*(1 + (b*x^(1/3))/a)*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*Hypergeom
etric2F1[4, 1 + 2*p, 2*(1 + p), 1 + (b*x^(1/3))/a])/(a^3*(1 + 2*p))
```

3.477.3.1 Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

3.477.4 Maple [F]

$$\int \frac{\left(a^{2}+2 a b \,x^{\frac{1}{3}}+b^{2} x^{\frac{2}{3}}\right)^{p}}{x^{2}}d x$$

input
```
int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x^2,x)
```

output
```
int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x^2,x)
```

3.477.5 Fricas [F]

$$\int \frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x^2}\,dx=\int \frac{\left(b^{2} x^{\frac{2}{3}}+2 \, a b x^{\frac{1}{3}}+a^{2}\right)^{p}}{x^{2}}\,dx$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x^2,x, algorithm="fricas")
```

output
```
integral((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/x^2, x)
```

3.477.6 Sympy [F]

$$\int \frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x^2}\,dx=\int \frac{\left(\left(a+b\sqrt[3]{x}\right)^2\right)^p}{x^2}\,dx$$

input
```
integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p/x**2,x)
```

output
```
Integral(((a + b*x**(1/3))**2)**p/x**2, x)
```

3.477.7 Maxima [F]

$$\int \frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x^2}\,dx=\int \frac{\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^p}{x^2}\,dx$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x^2,x, algorithm="maxima")
```

output
```
integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/x^2, x)
```

3.477.8 Giac [F]

$$\int \frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x^2}\,dx=\int \frac{\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^p}{x^2}\,dx$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x^2,x, algorithm="giac")
```

output
```
integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/x^2, x)
```

3.477.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x^2}\,dx=\int \frac{\left(a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}\right)^p}{x^2}\,dx$$

input
```
int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p/x^2,x)
```

output
```
int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p/x^2, x)
```

3.478 $\int \left(\frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x^2} - \frac{2b^3(1-2p)(1-p)p\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{3a^3x} \right.$

3.478.1 Optimal result

Integrand size = 77, antiderivative size = 146

$$\int \left(\frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x^2} - \frac{2b^3(1-2p)(1-p)p\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{3a^3x} \right) dx = -\frac{\left(a+b\sqrt[3]{x}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{ax} + \frac{b(1-p)\left(a+b\sqrt[3]{x}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{a^2x^{2/3}} - \frac{b^2(1-2p)(1-p)\left(a+b\sqrt[3]{x}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{a^3\sqrt[3]{x}}$$

output
```
-(a+b*x^(1/3))*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/a/x+b*(1-p)*(a+b*x^(1/3))
*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/a^2/x^(2/3)-b^2*(1-2*p)*(1-p)*(a+b*x^(1
/3))*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/a^3/x^(1/3)
```

3.478.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.33 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.69

$$\int \left(\frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x^2} - \frac{2b^3(1-2p)(1-p)p\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{3a^3x}\right)dx = \frac{b^3\left(a+b\sqrt[3]{x}\right)\left(\left(a+b\sqrt[3]{x}\right)^2\right)^p\left(2p(1-3p+2p^2)\,\text{Hype}\right.}{}$$

input
```
Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p/x^2 - (2*b^3*(1 - 2*p)*(1
- p)*p*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(3*a^3*x),x]
```

output
```
(b^3*(a + b*x^(1/3))*((a + b*x^(1/3))^2)^p*(2*p*(1 - 3*p + 2*p^2)*Hypergeo
metric2F1[1, 1 + 2*p, 2*(1 + p), 1 + (b*x^(1/3))/a] + 3*Hypergeometric2F1[
4, 1 + 2*p, 2*(1 + p), 1 + (b*x^(1/3))/a]))/(a^3*(a + 2*a*p))
```

3.478.3 Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.31 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.11, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.013, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x^2}-\frac{2b^3(1-2p)(1-p)p\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{3a^3x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2b^3(1-2p)(1-p)p\left(\frac{b\sqrt[3]{x}}{a}+1\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p \text{Hypergeometric2F1}\left(1,2p+1,2(p+1),\frac{\sqrt[3]{x}b}{a}+1\right)}{a^3(2p+1)}+\frac{3b^3\left(\frac{b\sqrt[3]{x}}{a}+1\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p \text{Hypergeometric2F1}\left(4,2p+1,2(p+1),\frac{\sqrt[3]{x}b}{a}+1\right)}{a^3(2p+1)}$$

input
```
Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p/x^2 - (2*b^3*(1 - 2*p)*(1 - p)*p
*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(3*a^3*x),x]
```

output
```
(2*b^3*(1 - 2*p)*(1 - p)*p*(1 + (b*x^(1/3))/a)*(a^2 + 2*a*b*x^(1/3) + b^2*
x^(2/3))^p*Hypergeometric2F1[1, 1 + 2*p, 2*(1 + p), 1 + (b*x^(1/3))/a])/(a
^3*(1 + 2*p)) + (3*b^3*(1 + (b*x^(1/3))/a)*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2
/3))^p*Hypergeometric2F1[4, 1 + 2*p, 2*(1 + p), 1 + (b*x^(1/3))/a])/(a^3*(
1 + 2*p))
```

3.478.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.478.4 Maple [F]

$$\int \left(\frac{\left(a^{2}+2ab\,x^{\frac{1}{3}}+b^{2}x^{\frac{2}{3}}\right)^{p}}{x^{2}}-\frac{2b^{3}(1-2p)\left(-p+1\right)p\left(a^{2}+2ab\,x^{\frac{1}{3}}+b^{2}x^{\frac{2}{3}}\right)^{p}}{3a^{3}x}\right)dx$$

input
```
int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x^2-2/3*b^3*(1-2*p)*(-p+1)*p*(a^2+2*
a*b*x^(1/3)+b^2*x^(2/3))^p/a^3/x,x)
```

output
```
int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x^2-2/3*b^3*(1-2*p)*(-p+1)*p*(a^2+2*
a*b*x^(1/3)+b^2*x^(2/3))^p/a^3/x,x)
```

3.478.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.56

$$\int \left(\frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x^2}-\frac{2b^3(1-2p)(1-p)p\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{3a^3x}\right)dx = -\frac{\left(a^{2}bpx^{\frac{1}{3}}+a^{3}+(2\,b^{3}p^{2}-3\,b^{3}p+b^{3})x+2\,(ab^{2}p^{2}-ab^{2}p)x^{\frac{2}{3}}\right)\left(b^{2}x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^{2}\right)^{p}}{a^{3}x}$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x^2-2/3*b^3*(1-2*p)*(1-p)*p*(a
^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/a^3/x,x, algorithm="fricas")
```

output
```
-(a^2*b*p*x^(1/3) + a^3 + (2*b^3*p^2 - 3*b^3*p + b^3)*x + 2*(a*b^2*p^2 - a
*b^2*p)*x^(2/3))*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/(a^3*x)
```

3.478.6 Sympy [F]

$$\int \left(\frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x^2}-\frac{2b^3(1-2p)(1-p)p\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{3a^3x}\right)dx=$$

$$-\frac{\int \left(-\frac{3a^3\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^p}{x^2}\right)dx+\int \frac{2b^3p\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^p}{x}dx+\int \left(-\frac{6b^3p^2\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^p}{x}\right)dx+\int \frac{4b^3}{}}{3a^3}$$

input
```
integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p/x**2-2/3*b**3*(1-2*p)*(1-
p)*p*(a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p/a**3/x,x)
```

output
```
-(Integral(-3*a**3*(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**p/x**2, x) + I
ntegral(2*b**3*p*(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**p/x, x) + Integr
al(-6*b**3*p**2*(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**p/x, x) + Integra
l(4*b**3*p**3*(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**p/x, x))/(3*a**3)
```

3.478.7 Maxima [F]

$$\int \left(\frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x^2}-\frac{2b^3(1-2p)(1-p)p\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{3a^3x}\right)dx=\int -\frac{2\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^pb^3(2\,p-1)(p-1)p}{3\,a^3x}+$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x^2-2/3*b^3*(1-2*p)*(1-p)*p*(a
^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/a^3/x,x, algorithm="maxima")
```

output
```
integrate(-2/3*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^3*(2*p - 1)*(p - 1)
*p/(a^3*x) + (b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/x^2, x)
```

3.478.8 Giac [F]

$$\int \left(\frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x^2} - \frac{2b^3(1-2p)(1-p)p\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{3a^3x}\right) dx = \int -\frac{2\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^p b^3(2\,p-1)(p-1)p}{3\,a^3x} + \frac{(}{}$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x^2-2/3*b^3*(1-2*p)*(1-p)*p*(a
^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/a^3/x,x, algorithm="giac")
```

output
```
integrate(-2/3*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^3*(2*p - 1)*(p - 1)
*p/(a^3*x) + (b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/x^2, x)
```

3.478.9 Mupad [B] (verification not implemented)

Time = 8.60 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.47

$$\int \left(\frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x^2} - \frac{2b^3(1-2p)(1-p)p\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{3a^3x}\right) dx = -\frac{\left(a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}\right)^p\left(\frac{b^3\,x\,\left(2\,p^2-3\,p+1\right)}{a^3}+\frac{b\,p\,x^{1/3}}{a}+\frac{2\,b^2\,p\,x^{2/3}\,\left(p-1\right)}{a^2}+1\right)}{x}$$

input
```
int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p/x^2 - (2*b^3*p*(2*p - 1)*(p - 1)
*(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p)/(3*a^3*x),x)
```

output
```
-((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p*((b^3*x*(2*p^2 - 3*p + 1))/a^3 + (
b*p*x^(1/3))/a + (2*b^2*p*x^(2/3)*(p - 1))/a^2 + 1))/x
```

3.479 $\int \frac{1}{\left(a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}\right)^{3/2}}\,dx$

3.479.1 Optimal result

Integrand size = 26, antiderivative size = 176

$$\int \frac{1}{\left(a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}\right)^{3/2}}\,dx=-\frac{12a^2}{b^4\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}+\frac{2a^3}{b^4\left(a+b\sqrt[4]{x}\right)\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}+\frac{4\left(a+b\sqrt[4]{x}\right)\sqrt[4]{x}}{b^3\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}-\frac{12a\left(a+b\sqrt[4]{x}\right)\log\left(a+b\sqrt[4]{x}\right)}{b^4\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}$$

output
```
-12*a^2/b^4/(a^2+2*a*b*x^(1/4)+b^2*x^(1/2))^(1/2)+2*a^3/b^4/(a+b*x^(1/4))/
(a^2+2*a*b*x^(1/4)+b^2*x^(1/2))^(1/2)+4*(a+b*x^(1/4))*x^(1/4)/b^3/(a^2+2*a
*b*x^(1/4)+b^2*x^(1/2))^(1/2)-12*a*(a+b*x^(1/4))*ln(a+b*x^(1/4))/b^4/(a^2+
2*a*b*x^(1/4)+b^2*x^(1/2))^(1/2)
```

3.479.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.53

$$\int \frac{1}{\left(a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}\right)^{3/2}}\,dx=\frac{2\left(-5a^3-4a^2b\sqrt[4]{x}+4ab^2\sqrt{x}+2b^3x^{3/4}-6a\left(a+b\sqrt[4]{x}\right)^2\log\left(a+b\sqrt[4]{x}\right)\right)}{b^4\left(a+b\sqrt[4]{x}\right)\sqrt{\left(a+b\sqrt[4]{x}\right)^2}}$$

input
```
Integrate[(a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x])^(-3/2),x]
```

output
```
(2*(-5*a^3 - 4*a^2*b*x^(1/4) + 4*a*b^2*Sqrt[x] + 2*b^3*x^(3/4) - 6*a*(a +
b*x^(1/4))^2*Log[a + b*x^(1/4)]))/(b^4*(a + b*x^(1/4))*Sqrt[(a + b*x^(1/4)
)^2])
```

3.479.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.64, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1384, 774, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\left(ab^3+b^4\sqrt[4]{x}\right)\int \frac{1}{\left(\sqrt[4]{x}b^2+ab\right)^3}dx}{\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}$$

$$\Big\downarrow \text{ 774}$$

$$\frac{4\left(ab^3+b^4\sqrt[4]{x}\right)\int \frac{x^{3/4}}{b^3\left(a+b\sqrt[4]{x}\right)^3}d\sqrt[4]{x}}{\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{4\left(ab^3+b^4\sqrt[4]{x}\right)\int \frac{x^{3/4}}{\left(a+b\sqrt[4]{x}\right)^3}d\sqrt[4]{x}}{b^3\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{4\left(ab^3+b^4\sqrt[4]{x}\right)\int \left(-\frac{a^3}{b^3\left(a+b\sqrt[4]{x}\right)^3}+\frac{3a^2}{b^3\left(a+b\sqrt[4]{x}\right)^2}-\frac{3a}{b^3\left(a+b\sqrt[4]{x}\right)}+\frac{1}{b^3}\right)d\sqrt[4]{x}}{b^3\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{4(ab^3+b^4\sqrt[4]{x})\left(\frac{a^3}{2b^4\left(a+b\sqrt[4]{x}\right)^2}-\frac{3a^2}{b^4\left(a+b\sqrt[4]{x}\right)}-\frac{3a\log\left(a+b\sqrt[4]{x}\right)}{b^4}+\frac{\sqrt[4]{x}}{b^3}\right)}{b^3\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}$$

input
```
Int[(a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x])^(-3/2),x]
```

output
```
(4*(a*b^3 + b^4*x^(1/4))*(a^3/(2*b^4*(a + b*x^(1/4))^2) - (3*a^2)/(b^4*(a
+ b*x^(1/4))) + x^(1/4)/b^3 - (3*a*Log[a + b*x^(1/4)])/b^4))/(b^3*Sqrt[a^2
 + 2*a*b*x^(1/4) + b^2*Sqrt[x]])
```

3.479.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.479. $\int \frac{1}{\left(a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}\right)^{3/2}}\,dx$

3.479.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.59

method	result	s
derivativedivides	$-\frac{2\left(6\ln\left(a+b\,x^{\frac{1}{4}}\right)a\,b^{2}\sqrt{x}-2b^{3}x^{\frac{3}{4}}+12\ln\left(a+b\,x^{\frac{1}{4}}\right)a^{2}b\,x^{\frac{1}{4}}-4a\,b^{2}\sqrt{x}+6\ln\left(a+b\,x^{\frac{1}{4}}\right)a^{3}+4a^{2}b\,x^{\frac{1}{4}}+5a^{3}\right)\left(a+b\,x^{\frac{1}{4}}\right)}{b^{4}\left(\left(a+b\,x^{\frac{1}{4}}\right)^{2}\right)^{\frac{3}{2}}}$	1
default	Expression too large to display	1

input
```
int(1/(a^2+2*a*b*x^(1/4)+b^2*x^(1/2))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(6*ln(a+b*x^(1/4))*a*b^2*x^(1/2)-2*b^3*x^(3/4)+12*ln(a+b*x^(1/4))*a^2*b
*x^(1/4)-4*a*b^2*x^(1/2)+6*ln(a+b*x^(1/4))*a^3+4*a^2*b*x^(1/4)+5*a^3)*(a+b
*x^(1/4))/b^4/((a+b*x^(1/4))^2)^(3/2)
```

3.479.5 Fricas [A] (verification not implemented)

Time = 1.99 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.84

$$\int \frac{1}{\left(a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}\right)^{3/2}}\,dx = \frac{2\left(9\,a^5b^4x-5\,a^9-6\left(ab^8x^2-2\,a^5b^4x+a^9\right)\log\left(bx^{\frac{1}{4}}+a\right)-2\left(3\,a^2b^7x-\right.\right.}{b^{12}x^2-2\,a^4b^8x}$$

input
```
integrate(1/(a^2+2*a*b*x^(1/4)+b^2*x^(1/2))^(3/2),x, algorithm="fricas")
```

output
```
2*(9*a^5*b^4*x - 5*a^9 - 6*(a*b^8*x^2 - 2*a^5*b^4*x + a^9)*log(b*x^(1/4) +
 a) - 2*(3*a^2*b^7*x - a^6*b^3)*x^(3/4) + (7*a^3*b^6*x - 3*a^7*b^2)*sqrt(x
) + 2*(b^9*x^2 - 6*a^4*b^5*x + 3*a^8*b)*x^(1/4))/(b^12*x^2 - 2*a^4*b^8*x +
 a^8*b^4)
```

3.479.6 Sympy [F]

$$\int \frac{1}{\left(a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}\right)^{3/2}}\,dx=\int \frac{1}{\left(a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a**2+2*a*b*x**(1/4)+b**2*x**(1/2))**(3/2),x)
```

output
```
Integral((a**2 + 2*a*b*x**(1/4) + b**2*sqrt(x))**(-3/2), x)
```

3.479.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.65

$$\int \frac{1}{\left(a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}\right)^{3/2}}\,dx=\frac{4\sqrt{x}}{\sqrt{b^2\sqrt{x}+2\,abx^{\frac{1}{4}}+a^2}b^2}-\frac{12\,a\log\left(x^{\frac{1}{4}}+\frac{a}{b}\right)}{b^4}+\frac{8\,a^2}{\sqrt{b^2\sqrt{x}+2\,abx^{\frac{1}{4}}+a^2}b^4}-\frac{24\,a^2x^{\frac{1}{4}}}{b^5\left(x^{\frac{1}{4}}+\frac{a}{b}\right)^2}-\frac{22\,a^3}{b^6\left(x^{\frac{1}{4}}+\frac{a}{b}\right)^2}$$

input
```
integrate(1/(a^2+2*a*b*x^(1/4)+b^2*x^(1/2))^(3/2),x, algorithm="maxima")
```

output
```
4*sqrt(x)/(sqrt(b^2*sqrt(x) + 2*a*b*x^(1/4) + a^2)*b^2) - 12*a*log(x^(1/4)
 + a/b)/b^4 + 8*a^2/(sqrt(b^2*sqrt(x) + 2*a*b*x^(1/4) + a^2)*b^4) - 24*a^2
*x^(1/4)/(b^5*(x^(1/4) + a/b)^2) - 22*a^3/(b^6*(x^(1/4) + a/b)^2)
```

3.479.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{\left(a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}\right)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(1/(a^2+2*a*b*x^(1/4)+b^2*x^(1/2))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.479.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}\right)^{3/2}}\,dx=\int \frac{1}{\left(a^2+b^2\,\sqrt{x}+2\,a\,b\,x^{1/4}\right)^{3/2}}\,dx$$

input
```
int(1/(a^2 + b^2*x^(1/2) + 2*a*b*x^(1/4))^(3/2),x)
```

output
```
int(1/(a^2 + b^2*x^(1/2) + 2*a*b*x^(1/4))^(3/2), x)
```

3.480 $\int \frac{1}{\left(a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}\right)^{5/2}}\,dx$

3.480.1 Optimal result

Integrand size = 26, antiderivative size = 268

$$\begin{aligned}\int \frac{1}{\left(a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}\right)^{5/2}}\,dx &= -\frac{60a^2}{b^6\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}} \\ &+ \frac{3a^5}{2b^6\left(a+b\sqrt[6]{x}\right)^3\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}} \\ &- \frac{10a^4}{b^6\left(a+b\sqrt[6]{x}\right)^2\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}}+\frac{30a^3}{b^6\left(a+b\sqrt[6]{x}\right)\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}} \\ &+ \frac{6\left(a+b\sqrt[6]{x}\right)\sqrt[6]{x}}{b^5\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}}-\frac{30a\left(a+b\sqrt[6]{x}\right)\log\left(a+b\sqrt[6]{x}\right)}{b^6\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}}\end{aligned}$$

output
```
-60*a^2/b^6/(a^2+2*a*b*x^(1/6)+b^2*x^(1/3))^(1/2)+3/2*a^5/b^6/(a+b*x^(1/6)
)^3/(a^2+2*a*b*x^(1/6)+b^2*x^(1/3))^(1/2)-10*a^4/b^6/(a+b*x^(1/6))^2/(a^2+
2*a*b*x^(1/6)+b^2*x^(1/3))^(1/2)+30*a^3/b^6/(a+b*x^(1/6))/(a^2+2*a*b*x^(1/
6)+b^2*x^(1/3))^(1/2)+6*(a+b*x^(1/6))*x^(1/6)/b^5/(a^2+2*a*b*x^(1/6)+b^2*x
^(1/3))^(1/2)-30*a*(a+b*x^(1/6))*ln(a+b*x^(1/6))/b^6/(a^2+2*a*b*x^(1/6)+b^
2*x^(1/3))^(1/2)
```

3.480.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.45

$$\int \frac{1}{\left(a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}\right)^{5/2}}\,dx = \frac{-77a^5-248a^4b\sqrt[6]{x}-252a^3b^2\sqrt[3]{x}-48a^2b^3\sqrt{x}+48ab^4x^{2/3}+12b^5x^{5/6}-}{2b^6\left(a+b\sqrt[6]{x}\right)^3\sqrt{\left(a+b\sqrt[6]{x}\right)^2}}$$

input
```
Integrate[(a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3))^(-5/2),x]
```

output
```
(-77*a^5 - 248*a^4*b*x^(1/6) - 252*a^3*b^2*x^(1/3) - 48*a^2*b^3*Sqrt[x] +
48*a*b^4*x^(2/3) + 12*b^5*x^(5/6) - 60*a*(a + b*x^(1/6))^4*Log[a + b*x^(1/
6)])/(2*b^6*(a + b*x^(1/6))^3*Sqrt[(a + b*x^(1/6))^2])
```

3.480.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.57, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1384, 774, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}\right)^{5/2}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\left(ab^5+b^6\sqrt[6]{x}\right)\int \frac{1}{\left(\sqrt[6]{x}b^2+ab\right)^5}dx}{\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}}$$

$$\Big\downarrow \text{774}$$

$$\frac{6\left(ab^5+b^6\sqrt[6]{x}\right)\int \frac{x^{5/6}}{b^5\left(a+b\sqrt[6]{x}\right)^5}d\sqrt[6]{x}}{\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}}$$

$$\Big\downarrow \text{27}$$

$$\frac{6\left(ab^5+b^6\sqrt[6]{x}\right)\int \frac{x^{5/6}}{\left(a+b\sqrt[6]{x}\right)^5}d\sqrt[6]{x}}{b^5\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}}$$

$$\Big\downarrow \text{49}$$

$$\frac{6\left(ab^5+b^6\sqrt[6]{x}\right)\int\left(-\frac{a^5}{b^5\left(a+b\sqrt[6]{x}\right)^5}+\frac{5a^4}{b^5\left(a+b\sqrt[6]{x}\right)^4}-\frac{10a^3}{b^5\left(a+b\sqrt[6]{x}\right)^3}+\frac{10a^2}{b^5\left(a+b\sqrt[6]{x}\right)^2}-\frac{5a}{b^5\left(a+b\sqrt[6]{x}\right)}+\frac{1}{b^5}\right)d\sqrt[6]{x}}{b^5\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{6\left(ab^5+b^6\sqrt[6]{x}\right)\left(\frac{a^5}{4b^6\left(a+b\sqrt[6]{x}\right)^4}-\frac{5a^4}{3b^6\left(a+b\sqrt[6]{x}\right)^3}+\frac{5a^3}{b^6\left(a+b\sqrt[6]{x}\right)^2}-\frac{10a^2}{b^6\left(a+b\sqrt[6]{x}\right)}-\frac{5a\log\left(a+b\sqrt[6]{x}\right)}{b^6}+\frac{\sqrt[6]{x}}{b^5}\right)}{b^5\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}}$$

input
```
Int[(a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3))^(-5/2),x]
```

output
```
(6*(a*b^5 + b^6*x^(1/6))*(a^5/(4*b^6*(a + b*x^(1/6))^4) - (5*a^4)/(3*b^6*(
a + b*x^(1/6))^3) + (5*a^3)/(b^6*(a + b*x^(1/6))^2) - (10*a^2)/(b^6*(a + b
*x^(1/6))) + x^(1/6)/b^5 - (5*a*Log[a + b*x^(1/6)])/b^6))/(b^5*Sqrt[a^2 +
2*a*b*x^(1/6) + b^2*x^(1/3)])
```

3.480.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

3.480. $\int \frac{1}{\left(a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}\right)^{5/2}}\,dx$

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.480.4 Maple [A] (verified)

Time = 5.45 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.61

method	result
derivativedivides	$-\frac{\left(60\ln\left(a+b\,x^{\frac{1}{6}}\right)a\,b^{4}x^{\frac{2}{3}}-12b^{5}x^{\frac{5}{6}}+240\ln\left(a+b\,x^{\frac{1}{6}}\right)a^{2}b^{3}\sqrt{x}-48a\,b^{4}x^{\frac{2}{3}}+360\ln\left(a+b\,x^{\frac{1}{6}}\right)a^{3}b^{2}x^{\frac{1}{3}}+48a^{2}b^{3}\sqrt{x}+240\ln(\right.}{2b^{6}\left(\left(a+b\,x^{\frac{1}{6}}\right)^{2}\right)^{\frac{5}{2}}}$
default	Expression too large to display

input
```
int(1/(a^2+2*a*b*x^(1/6)+b^2*x^(1/3))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(60*ln(a+b*x^(1/6))*a*b^4*x^(2/3)-12*b^5*x^(5/6)+240*ln(a+b*x^(1/6))*
a^2*b^3*x^(1/2)-48*a*b^4*x^(2/3)+360*ln(a+b*x^(1/6))*a^3*b^2*x^(1/3)+48*a^
2*b^3*x^(1/2)+240*ln(a+b*x^(1/6))*a^4*b*x^(1/6)+252*a^3*b^2*x^(1/3)+60*ln(
a+b*x^(1/6))*a^5+248*a^4*b*x^(1/6)+77*a^5)*(a+b*x^(1/6))/b^6/((a+b*x^(1/6)
)^2)^(5/2)
```

3.480.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}\right)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate(1/(a^2+2*a*b*x^(1/6)+b^2*x^(1/3))^(5/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.480.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a**2+2*a*b*x**(1/6)+b**2*x**(1/3))**(5/2),x)
```

output
```
Timed out
```

3.480.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.44

$$\int \frac{1}{\left(a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}\right)^{5/2}}\,dx = \frac{12\,b^5x^{\frac{5}{6}} + 48\,ab^4x^{\frac{2}{3}} - 48\,a^2b^3\sqrt{x} - 252\,a^3b^2x^{\frac{1}{3}} - 248\,a^4bx^{\frac{1}{6}} - 77\,a^5}{2\left(b^{10}x^{\frac{2}{3}} + 4\,ab^9\sqrt{x} + 6\,a^2b^8x^{\frac{1}{3}} + 4\,a^3b^7x^{\frac{1}{6}} + a^4b^6\right)} - \frac{30\,a\log\left(bx^{\frac{1}{6}}+a\right)}{b^6}$$

input
```
integrate(1/(a^2+2*a*b*x^(1/6)+b^2*x^(1/3))^(5/2),x, algorithm="maxima")
```

output
```
1/2*(12*b^5*x^(5/6) + 48*a*b^4*x^(2/3) - 48*a^2*b^3*sqrt(x) - 252*a^3*b^2*
x^(1/3) - 248*a^4*b*x^(1/6) - 77*a^5)/(b^10*x^(2/3) + 4*a*b^9*sqrt(x) + 6*
a^2*b^8*x^(1/3) + 4*a^3*b^7*x^(1/6) + a^4*b^6) - 30*a*log(b*x^(1/6) + a)/b
^6
```

3.480.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.39

$$\int \frac{1}{\left(a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}\right)^{5/2}}\,dx = -\frac{30\,a\log\left(\left|bx^{\frac{1}{6}}+a\right|\right)}{b^6\mathrm{sgn}\left(bx^{\frac{1}{6}}+a\right)} + \frac{6\,x^{\frac{1}{6}}}{b^5\mathrm{sgn}\left(bx^{\frac{1}{6}}+a\right)} - \frac{120\,a^2b^3\sqrt{x} + 300\,a^3b^2x^{\frac{1}{3}} + 260\,a^4bx^{\frac{1}{6}} + 77\,a^5}{2\left(bx^{\frac{1}{6}}+a\right)^4 b^6\mathrm{sgn}\left(bx^{\frac{1}{6}}+a\right)}$$

input
```
integrate(1/(a^2+2*a*b*x^(1/6)+b^2*x^(1/3))^(5/2),x, algorithm="giac")
```

output
```
-30*a*log(abs(b*x^(1/6) + a))/(b^6*sgn(b*x^(1/6) + a)) + 6*x^(1/6)/(b^5*sg
n(b*x^(1/6) + a)) - 1/2*(120*a^2*b^3*sqrt(x) + 300*a^3*b^2*x^(1/3) + 260*a
^4*b*x^(1/6) + 77*a^5)/((b*x^(1/6) + a)^4*b^6*sgn(b*x^(1/6) + a))
```

3.480.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}\right)^{5/2}}\,dx=\int \frac{1}{\left(a^2+b^2\,x^{1/3}+2\,a\,b\,x^{1/6}\right)^{5/2}}\,dx$$

input
```
int(1/(a^2 + b^2*x^(1/3) + 2*a*b*x^(1/6))^(5/2),x)
```

output
```
int(1/(a^2 + b^2*x^(1/3) + 2*a*b*x^(1/6))^(5/2), x)
```

3.481 $\int \left(a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}\right)^{3/2} dx$

3.481.1 Optimal result

Integrand size = 24, antiderivative size = 179

$$\int \left(a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}\right)^{3/2} dx = -\frac{2b^3\sqrt{a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}}}{\left(a+\frac{b}{\sqrt{x}}\right)\sqrt{x}}+\frac{6a^2b\sqrt{a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}}\sqrt{x}}{a+\frac{b}{\sqrt{x}}} + \frac{a^3\sqrt{a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}}x}{a+\frac{b}{\sqrt{x}}}+\frac{6ab^2\sqrt{a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}}\log\left(\sqrt{x}\right)}{a+\frac{b}{\sqrt{x}}}$$

output
```
a^3*x*(a^2+b^2/x+2*a*b/x^(1/2))^(1/2)/(a+b/x^(1/2))+3*a*b^2*ln(x)*(a^2+b^2
/x+2*a*b/x^(1/2))^(1/2)/(a+b/x^(1/2))-2*b^3*(a^2+b^2/x+2*a*b/x^(1/2))^(1/2
)/(a+b/x^(1/2))/x^(1/2)+6*a^2*b*x^(1/2)*(a^2+b^2/x+2*a*b/x^(1/2))^(1/2)/(a
+b/x^(1/2))
```

3.481.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.37

$$\int \left(a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}\right)^{3/2} dx = \frac{\sqrt{\frac{\left(b+a\sqrt{x}\right)^2}{x}}\left(-2b^3+6a^2bx+a^3x^{3/2}+3ab^2\sqrt{x}\log(x)\right)}{b+a\sqrt{x}}$$

input
```
Integrate[(a^2 + b^2/x + (2*a*b)/Sqrt[x])^(3/2),x]
```

output
```
(Sqrt[(b + a*Sqrt[x])^2/x]*(-2*b^3 + 6*a^2*b*x + a^3*x^(3/2) + 3*a*b^2*Sqr
t[x]*Log[x]))/(b + a*Sqrt[x])
```

3.481.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.49, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1384, 774, 27, 795, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+\frac{2ab}{\sqrt{x}}+\frac{b^2}{x}\right)^{3/2}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2+\frac{2ab}{\sqrt{x}}+\frac{b^2}{x}}\int \left(\frac{b^2}{\sqrt{x}}+ab\right)^3dx}{ab^3+\frac{b^4}{\sqrt{x}}}$$

$$\Big\downarrow \text{ 774}$$

$$\frac{2\sqrt{a^2+\frac{2ab}{\sqrt{x}}+\frac{b^2}{x}}\int b^3\left(a+\frac{b}{\sqrt{x}}\right)^3\sqrt{x}d\sqrt{x}}{ab^3+\frac{b^4}{\sqrt{x}}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2b^3\sqrt{a^2+\frac{2ab}{\sqrt{x}}+\frac{b^2}{x}}\int \left(a+\frac{b}{\sqrt{x}}\right)^3\sqrt{x}d\sqrt{x}}{ab^3+\frac{b^4}{\sqrt{x}}}$$

$$\Big\downarrow \text{ 795}$$

$$\frac{2b^3\sqrt{a^2+\frac{2ab}{\sqrt{x}}+\frac{b^2}{x}}\int \frac{(\sqrt{x}a+b)^3}{x}d\sqrt{x}}{ab^3+\frac{b^4}{\sqrt{x}}}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{2b^3\sqrt{a^2+\frac{2ab}{\sqrt{x}}+\frac{b^2}{x}}\int \left(\sqrt{x}a^3+3ba^2+\frac{3b^2a}{\sqrt{x}}+\frac{b^3}{x}\right)d\sqrt{x}}{ab^3+\frac{b^4}{\sqrt{x}}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2b^3\sqrt{a^2+\frac{2ab}{\sqrt{x}}+\frac{b^2}{x}}\left(\frac{a^3x}{2}+3a^2b\sqrt{x}+3ab^2\log\left(\sqrt{x}\right)-\frac{b^3}{\sqrt{x}}\right)}{ab^3+\frac{b^4}{\sqrt{x}}}$$

input
```
Int[(a^2 + b^2/x + (2*a*b)/Sqrt[x])^(3/2),x]
```

output
```
(2*b^3*Sqrt[a^2 + b^2/x + (2*a*b)/Sqrt[x]]*(-(b^3/Sqrt[x]) + 3*a^2*b*Sqrt[
x] + (a^3*x)/2 + 3*a*b^2*Log[Sqrt[x]]))/(a*b^3 + b^4/Sqrt[x])
```

3.481.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.481. $\int \left(a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}\right)^{3/2} dx$

3.481.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.36

method	result	size
derivativedivides	$\frac{\left(\frac{a^2x+b^2+2ab\sqrt{x}}{x}\right)^{\frac{3}{2}} x\left(a^3x^{\frac{3}{2}}+3b^2a\ln(x)\sqrt{x}+6a^2bx-2b^3\right)}{\left(a\sqrt{x}+b\right)^3}$	65
default	$\frac{\left(\frac{a^2x^{\frac{3}{2}}+b^2\sqrt{x}+2abx}{x^{\frac{3}{2}}}\right)^{\frac{3}{2}}\left(x^{\frac{5}{2}}a^3+3x^{\frac{3}{2}}\ln(x)ab^2+6a^2bx^2-2b^3x\right)}{\left(a\sqrt{x}+b\right)^3}$	71

input
```
int((a^2+b^2/x+2*a*b/x^(1/2))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((a^2*x+b^2+2*a*b*x^(1/2))/x)^(3/2)*x*(a^3*x^(3/2)+3*b^2*a*ln(x)*x^(1/2)+6
*a^2*b*x-2*b^3)/(a*x^(1/2)+b)^3
```

3.481.5 Fricas [F(-1)]

Timed out.

$$\int \left(a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}\right)^{3/2} dx = \text{Timed out}$$

input
```
integrate((a^2+b^2/x+2*a*b/x^(1/2))^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.481.6 Sympy [F]

$$\int \left(a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}\right)^{3/2} dx = \int \left(a^2+\frac{2ab}{\sqrt{x}}+\frac{b^2}{x}\right)^{\frac{3}{2}} dx$$

input
```
integrate((a**2+b**2/x+2*a*b/x**(1/2))**(3/2),x)
```

output
```
Integral((a**2 + 2*a*b/sqrt(x) + b**2/x)**(3/2), x)
```

3.481.7 Maxima [F]

$$\int \left(a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}\right)^{3/2} dx = \int \left(a^2+\frac{2\,ab}{\sqrt{x}}+\frac{b^2}{x}\right)^{\frac{3}{2}} dx$$

input
```
integrate((a^2+b^2/x+2*a*b/x^(1/2))^(3/2),x, algorithm="maxima")
```

output
```
a^3*x + 3*a*b^2*integrate(1/x, x) + 6*a^2*b*sqrt(x) - 2*b^3/sqrt(x)
```

3.481.8 Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.45

$$\int \left(a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}\right)^{3/2} dx = a^3x\mathrm{sgn}\left(ax+b\sqrt{x}\right)\mathrm{sgn}(x) + 3\,ab^2\log\left(|x|\right)\mathrm{sgn}\left(ax+b\sqrt{x}\right)\mathrm{sgn}(x) + 6\,a^2b\sqrt{x}\mathrm{sgn}\left(ax+b\sqrt{x}\right)\mathrm{sgn}(x) - \frac{2\,b^3\mathrm{sgn}\left(ax+b\sqrt{x}\right)\mathrm{sgn}(x)}{\sqrt{x}}$$

input
```
integrate((a^2+b^2/x+2*a*b/x^(1/2))^(3/2),x, algorithm="giac")
```

output
```
a^3*x*sgn(a*x + b*sqrt(x))*sgn(x) + 3*a*b^2*log(abs(x))*sgn(a*x + b*sqrt(x
))*sgn(x) + 6*a^2*b*sqrt(x)*sgn(a*x + b*sqrt(x))*sgn(x) - 2*b^3*sgn(a*x +
b*sqrt(x))*sgn(x)/sqrt(x)
```

3.481.9 Mupad [F(-1)]

Timed out.

$$\int \left(a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}\right)^{3/2} dx = \int \left(a^2+\frac{b^2}{x}+\frac{2\,a\,b}{\sqrt{x}}\right)^{3/2} dx$$

input
```
int((a^2 + b^2/x + (2*a*b)/x^(1/2))^(3/2),x)
```

output
```
int((a^2 + b^2/x + (2*a*b)/x^(1/2))^(3/2), x)
```

3.482 $\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{7/2} dx$

3.482.1 Optimal result

Integrand size = 26, antiderivative size = 391

$$\begin{aligned}\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{7/2} dx = & -\frac{3b^7\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}{4\left(a+\frac{b}{\sqrt[3]{x}}\right)x^{4/3}}-\frac{7ab^6\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}{\left(a+\frac{b}{\sqrt[3]{x}}\right)x} \\ & -\frac{63a^2b^5\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}{2\left(a+\frac{b}{\sqrt[3]{x}}\right)x^{2/3}}-\frac{105a^3b^4\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}{\left(a+\frac{b}{\sqrt[3]{x}}\right)\sqrt[3]{x}} \\ & +\frac{63a^5b^2\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\sqrt[3]{x}}{a+\frac{b}{\sqrt[3]{x}}}+\frac{21a^6b\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}x^{2/3}}{2\left(a+\frac{b}{\sqrt[3]{x}}\right)} \\ & +\frac{a^7\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}x}{a+\frac{b}{\sqrt[3]{x}}}+\frac{105a^4b^3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\log\left(\sqrt[3]{x}\right)}{a+\frac{b}{\sqrt[3]{x}}}\end{aligned}$$

output
```
-3/4*b^7*(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))/x^(4/3)-7*a*b
^6*(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))/x-63/2*a^2*b^5*(a^2
+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))/x^(2/3)-105*a^3*b^4*(a^2+b
^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))/x^(1/3)+63*a^5*b^2*x^(1/3)*(
a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))+21/2*a^6*b*x^(2/3)*(a^2
+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))+a^7*x*(a^2+b^2/x^(2/3)+2*a
*b/x^(1/3))^(1/2)/(a+b/x^(1/3))+35*a^4*b^3*ln(x)*(a^2+b^2/x^(2/3)+2*a*b/x^
(1/3))^(1/2)/(a+b/x^(1/3))
```

3.482.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.32

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{7/2} dx = \frac{\sqrt{\frac{\left(b+a\sqrt[3]{x}\right)^2}{x^{2/3}}}\left(-3b^7-28ab^6\sqrt[3]{x}-126a^2b^5x^{2/3}-420a^3b^4x+252a^5b^2x^{5/3}+42a^6bx^2+4a^7x^7\right.}{4\left(b+a\sqrt[3]{x}\right)x}$$

input
```
Integrate[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(7/2),x]
```

output
```
(Sqrt[(b + a*x^(1/3))^2/x^(2/3)]*(-3*b^7 - 28*a*b^6*x^(1/3) - 126*a^2*b^5*
x^(2/3) - 420*a^3*b^4*x + 252*a^5*b^2*x^(5/3) + 42*a^6*b*x^2 + 4*a^7*x^(7/
3) + 140*a^4*b^3*x^(4/3)*Log[x]))/(4*(b + a*x^(1/3))*x)
```

3.482.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.38, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1384, 774, 27, 795, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}\right)^{7/2} dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\int \left(\frac{b^2}{\sqrt[3]{x}}+ab\right)^7dx}{ab^7+\frac{b^8}{\sqrt[3]{x}}}$$

$$\Big\downarrow \; 774$$

$$\frac{3\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\int b^7\left(a+\frac{b}{\sqrt[3]{x}}\right)^7x^{2/3}d\sqrt[3]{x}}{ab^7+\frac{b^8}{\sqrt[3]{x}}}$$

$$\Big\downarrow \; 27$$

$$\frac{3b^7\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^7x^{2/3}d\sqrt[3]{x}}{ab^7+\frac{b^8}{\sqrt[3]{x}}}$$

$$\Big\downarrow \; 795$$

$$\frac{3b^7\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\int \frac{\left(\sqrt[3]{x}a+b\right)^7}{x^{5/3}}d\sqrt[3]{x}}{ab^7+\frac{b^8}{\sqrt[3]{x}}}$$

$$\Big\downarrow \; 49$$

$$\frac{3b^7\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\int \left(x^{2/3}a^7+7b\sqrt[3]{x}a^6+21b^2a^5+\frac{35b^3a^4}{\sqrt[3]{x}}+\frac{35b^4a^3}{x^{2/3}}+\frac{21b^5a^2}{x}+\frac{7b^6a}{x^{4/3}}+\frac{b^7}{x^{5/3}}\right)d\sqrt[3]{x}}{ab^7+\frac{b^8}{\sqrt[3]{x}}}$$

$$\Big\downarrow \; 2009$$

$$\frac{3b^7\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\left(\frac{a^7x}{3}+\frac{7}{2}a^6bx^{2/3}+21a^5b^2\sqrt[3]{x}+35a^4b^3\log\left(\sqrt[3]{x}\right)-\frac{35a^3b^4}{\sqrt[3]{x}}-\frac{21a^2b^5}{2x^{2/3}}-\frac{7ab^6}{3x}-\frac{b^7}{4x^{4/3}}\right)}{ab^7+\frac{b^8}{\sqrt[3]{x}}}$$

input
```
Int[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(7/2),x]
```

output
```
(3*b^7*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*(-1/4*b^7/x^(4/3) - (7*a*
b^6)/(3*x) - (21*a^2*b^5)/(2*x^(2/3)) - (35*a^3*b^4)/x^(1/3) + 21*a^5*b^2*
x^(1/3) + (7*a^6*b*x^(2/3))/2 + (a^7*x)/3 + 35*a^4*b^3*Log[x^(1/3)]))/(a*b
^7 + b^8/x^(1/3))
```

3.482. $\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{7/2}dx$

3.482.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.482.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.29

method	result
derivativedivides	$\frac{\left(\frac{x^{\frac{2}{3}} a^{2}+2 a b \,x^{\frac{1}{3}}+b^{2}}{x^{\frac{2}{3}}}\right)^{\frac{7}{2}} x \left(4 a^{7} x^{\frac{7}{3}}+42 x^{2} a^{6} b +140 a^{4} b^{3} \ln \left(x \right) x^{\frac{4}{3}}+252 b^{2} a^{5} x^{\frac{5}{3}}-420 b^{4} a^{3} x -126 a^{2} b^{5} x^{\frac{2}{3}}-28 a \,b^{6} x^{\frac{1}{3}}-3 b^{7}\right)}{4 \left(b +a \,x^{\frac{1}{3}}\right)^{7}}$
default	$\frac{\left(\frac{x^{\frac{2}{3}} a^{2}+2 a b \,x^{\frac{1}{3}}+b^{2}}{x^{\frac{2}{3}}}\right)^{\frac{7}{2}} \left(42 a^{6} b \,x^{3}+252 b^{2} a^{5} x^{\frac{8}{3}}+140 a^{4} b^{3} \ln \left(x \right) x^{\frac{7}{3}}+4 a^{7} x^{\frac{10}{3}}-28 a \,b^{6} x^{\frac{4}{3}}-420 b^{4} a^{3} x^{2}-126 a^{2} b^{5} x^{\frac{5}{3}}-3 b^{7} x \right)}{4 \left(b +a \,x^{\frac{1}{3}}\right)^{7}}$

3.482. $\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{7/2} dx$

input
```
int((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*((x^(2/3)*a^2+2*a*b*x^(1/3)+b^2)/x^(2/3))^(7/2)*x*(4*a^7*x^(7/3)+42*x^
2*a^6*b+140*a^4*b^3*ln(x)*x^(4/3)+252*b^2*a^5*x^(5/3)-420*b^4*a^3*x-126*a^
2*b^5*x^(2/3)-28*a*b^6*x^(1/3)-3*b^7)/(b+a*x^(1/3))^7
```

3.482.5 Fricas [F(-1)]

Timed out.

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{7/2} dx = \text{Timed out}$$

input
```
integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(7/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.482.6 Sympy [F]

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{7/2} dx = \int \left(a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{\frac{2}{3}}}\right)^{\frac{7}{2}} dx$$

input
```
integrate((a**2+b**2/x**(2/3)+2*a*b/x**(1/3))**(7/2),x)
```

output
```
Integral((a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3))**(7/2), x)
```

3.482.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.20

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{7/2} dx = 35\,a^4b^3\log\left(x\right) + \frac{4\,a^7x^{\frac{7}{3}}+42\,a^6bx^2+252\,a^5b^2x^{\frac{5}{3}}-420\,a^3b^4x-126\,a^2b^5x^{\frac{2}{3}}-28\,ab^6x^{\frac{1}{3}}-3\,b^7}{4\,x^{\frac{4}{3}}}$$

input
```
integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(7/2),x, algorithm="maxima")
```

output
```
35*a^4*b^3*log(x) + 1/4*(4*a^7*x^(7/3) + 42*a^6*b*x^2 + 252*a^5*b^2*x^(5/3
) - 420*a^3*b^4*x - 126*a^2*b^5*x^(2/3) - 28*a*b^6*x^(1/3) - 3*b^7)/x^(4/3
)
```

3.482.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.44

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{7/2} dx = a^7 x\operatorname{sgn}\left(ax+bx^{\frac{2}{3}}\right)\operatorname{sgn}(x)+35\,a^4b^3\log\left(|x|\right)\operatorname{sgn}\left(ax+bx^{\frac{2}{3}}\right)\operatorname{sgn}(x) + \frac{21}{2}\,a^6bx^{\frac{2}{3}}\operatorname{sgn}\left(ax+bx^{\frac{2}{3}}\right)\operatorname{sgn}(x)+63\,a^5b^2x^{\frac{1}{3}}\operatorname{sgn}\left(ax+bx^{\frac{2}{3}}\right)\operatorname{sgn}(x) - \frac{420\,a^3b^4x\operatorname{sgn}\left(ax+bx^{\frac{2}{3}}\right)\operatorname{sgn}(x)+126\,a^2b^5x^{\frac{2}{3}}\operatorname{sgn}\left(ax+bx^{\frac{2}{3}}\right)\operatorname{sgn}(x)+28\,ab^6x^{\frac{1}{3}}\operatorname{sgn}\left(ax+bx^{\frac{2}{3}}\right)\operatorname{sgn}(x)+}{4\,x^{\frac{4}{3}}}$$

input
```
integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(7/2),x, algorithm="giac")
```

output
```
a^7*x*sgn(a*x + b*x^(2/3))*sgn(x) + 35*a^4*b^3*log(abs(x))*sgn(a*x + b*x^(
2/3))*sgn(x) + 21/2*a^6*b*x^(2/3)*sgn(a*x + b*x^(2/3))*sgn(x) + 63*a^5*b^2
*x^(1/3)*sgn(a*x + b*x^(2/3))*sgn(x) - 1/4*(420*a^3*b^4*x*sgn(a*x + b*x^(2
/3))*sgn(x) + 126*a^2*b^5*x^(2/3)*sgn(a*x + b*x^(2/3))*sgn(x) + 28*a*b^6*x
^(1/3)*sgn(a*x + b*x^(2/3))*sgn(x) + 3*b^7*sgn(a*x + b*x^(2/3))*sgn(x))/x^
(4/3)
```

3.482.9 Mupad [F(-1)]

Timed out.

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{7/2} dx = \int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2\,a\,b}{x^{1/3}}\right)^{7/2} dx$$

input
```
int((a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(7/2),x)
```

output
```
int((a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(7/2), x)
```

3.483 $\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{5/2} dx$

3.483.1 Optimal result

Integrand size = 26, antiderivative size = 291

$$\begin{aligned}\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{5/2} dx =& -\frac{3b^5\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}{2\left(a+\frac{b}{\sqrt[3]{x}}\right)x^{2/3}}-\frac{15ab^4\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}{\left(a+\frac{b}{\sqrt[3]{x}}\right)\sqrt[3]{x}} \\ &+\frac{30a^3b^2\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\sqrt[3]{x}}{a+\frac{b}{\sqrt[3]{x}}}+\frac{15a^4b\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}x^{2/3}}{2\left(a+\frac{b}{\sqrt[3]{x}}\right)} \\ &+\frac{a^5\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}x}{a+\frac{b}{\sqrt[3]{x}}}+\frac{30a^2b^3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\log\left(\sqrt[3]{x}\right)}{a+\frac{b}{\sqrt[3]{x}}}\end{aligned}$$

output
```
-3/2*b^5*(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))/x^(2/3)-15*a*
b^4*(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))/x^(1/3)+30*a^3*b^2
*x^(1/3)*(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))+15/2*a^4*b*x^
(2/3)*(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))+a^5*x*(a^2+b^2/x
^(2/3)+2*a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))+10*a^2*b^3*ln(x)*(a^2+b^2/x^(2/3
)+2*a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))
```

3.483.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.34

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{5/2} dx = \frac{\left(b+a\sqrt[3]{x}\right)\left(-3b^5-30ab^4\sqrt[3]{x}+60a^3b^2x+15a^4bx^{4/3}+2a^5x^{5/3}+20a^2b^3x^{2/3}\log(x)\right)}{2\sqrt{\frac{\left(b+a\sqrt[3]{x}\right)^2}{x^{2/3}}}x}$$

input
```
Integrate[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(5/2),x]
```

output
```
((b + a*x^(1/3))*(-3*b^5 - 30*a*b^4*x^(1/3) + 60*a^3*b^2*x + 15*a^4*b*x^(4
/3) + 2*a^5*x^(5/3) + 20*a^2*b^3*x^(2/3)*Log[x]))/(2*Sqrt[(b + a*x^(1/3))^
2/x^(2/3)]*x)
```

3.483.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.41, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1384, 774, 27, 795, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}\right)^{5/2} dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\int \left(\frac{b^2}{\sqrt[3]{x}}+ab\right)^5 dx}{ab^5+\frac{b^6}{\sqrt[3]{x}}}$$

$$\Big\downarrow \text{774}$$

$$\frac{3\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\int b^5\left(a+\frac{b}{\sqrt[3]{x}}\right)^5 x^{2/3}d\sqrt[3]{x}}{ab^5+\frac{b^6}{\sqrt[3]{x}}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3b^5\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^5 x^{2/3}d\sqrt[3]{x}}{ab^5+\frac{b^6}{\sqrt[3]{x}}}$$

$$\Big\downarrow \text{795}$$

$$\frac{3b^5\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\int \frac{\left(\sqrt[3]{x}a+b\right)^5}{x}d\sqrt[3]{x}}{ab^5+\frac{b^6}{\sqrt[3]{x}}}$$

$$\Big\downarrow \text{49}$$

$$\frac{3b^5\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\int \left(x^{2/3}a^5+5b\sqrt[3]{x}a^4+10b^2a^3+\frac{10b^3a^2}{\sqrt[3]{x}}+\frac{5b^4a}{x^{2/3}}+\frac{b^5}{x}\right)d\sqrt[3]{x}}{ab^5+\frac{b^6}{\sqrt[3]{x}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3b^5\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\left(\frac{a^5x}{3}+\frac{5}{2}a^4bx^{2/3}+10a^3b^2\sqrt[3]{x}+10a^2b^3\log\left(\sqrt[3]{x}\right)-\frac{5ab^4}{\sqrt[3]{x}}-\frac{b^5}{2x^{2/3}}\right)}{ab^5+\frac{b^6}{\sqrt[3]{x}}}$$

input
```
Int[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(5/2),x]
```

output
```
(3*b^5*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*(-1/2*b^5/x^(2/3) - (5*a*
b^4)/x^(1/3) + 10*a^3*b^2*x^(1/3) + (5*a^4*b*x^(2/3))/2 + (a^5*x)/3 + 10*a
^2*b^3*Log[x^(1/3)]))/(a*b^5 + b^6/x^(1/3))
```

3.483.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

3.483. $\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{5/2}dx$

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.483.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.31

method	result	size
derivativedivides	$\frac{\left(\frac{x^{\frac{2}{3}} a^{2}+2 a b \,x^{\frac{1}{3}}+b^{2}}{x^{\frac{2}{3}}}\right)^{\frac{5}{2}} x \left(2 a^{5} x^{\frac{5}{3}}+15 b \,a^{4} x^{\frac{4}{3}}+20 a^{2} b^{3} \ln \left(x \right) x^{\frac{2}{3}}+60 a^{3} b^{2} x -30 b^{4} a \,x^{\frac{1}{3}}-3 b^{5}\right)}{2 \left(b +a \,x^{\frac{1}{3}}\right)^{5}}$	91
default	$\frac{\left(\frac{x^{\frac{2}{3}} a^{2}+2 a b \,x^{\frac{1}{3}}+b^{2}}{x^{\frac{2}{3}}}\right)^{\frac{5}{2}} x \left(2 a^{5} x^{\frac{5}{3}}+15 b \,a^{4} x^{\frac{4}{3}}+20 a^{2} b^{3} \ln \left(x \right) x^{\frac{2}{3}}+60 a^{3} b^{2} x -30 b^{4} a \,x^{\frac{1}{3}}-3 b^{5}\right)}{2 \left(b +a \,x^{\frac{1}{3}}\right)^{5}}$	91

input
```
int((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*((x^(2/3)*a^2+2*a*b*x^(1/3)+b^2)/x^(2/3))^(5/2)*x*(2*a^5*x^(5/3)+15*b*
a^4*x^(4/3)+20*a^2*b^3*ln(x)*x^(2/3)+60*a^3*b^2*x-30*b^4*a*x^(1/3)-3*b^5)/
(b+a*x^(1/3))^5
```

3.483.5 Fricas [F(-1)]

Timed out.

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{5/2}\,dx=\text{Timed out}$$

input
```
integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(5/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.483.6 Sympy [F]

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{5/2}\,dx=\int \left(a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{\frac{2}{3}}}\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((a**2+b**2/x**(2/3)+2*a*b/x**(1/3))**(5/2),x)
```

output
```
Integral((a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3))**(5/2), x)
```

3.483.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.20

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{5/2}\,dx=10\,a^2b^3\log\left(x\right)+\frac{2\,a^5x^{\frac{5}{3}}+15\,a^4bx^{\frac{4}{3}}+60\,a^3b^2x-30\,ab^4x^{\frac{1}{3}}-3\,b^5}{2\,x^{\frac{2}{3}}}$$

input
```
integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(5/2),x, algorithm="maxima")
```

output
```
10*a^2*b^3*log(x) + 1/2*(2*a^5*x^(5/3) + 15*a^4*b*x^(4/3) + 60*a^3*b^2*x -
 30*a*b^4*x^(1/3) - 3*b^5)/x^(2/3)
```

3.483.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.44

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{5/2} dx = a^5 x \operatorname{sgn}\left(ax+bx^{\frac{2}{3}}\right)\operatorname{sgn}(x) + 10\,a^2b^3\log\left(|x|\right)\operatorname{sgn}\left(ax+bx^{\frac{2}{3}}\right)\operatorname{sgn}(x) + \frac{15}{2}\,a^4 b x^{\frac{2}{3}}\operatorname{sgn}\left(ax+bx^{\frac{2}{3}}\right)\operatorname{sgn}(x) + 30\,a^3b^2x^{\frac{1}{3}}\operatorname{sgn}\left(ax+bx^{\frac{2}{3}}\right)\operatorname{sgn}(x) - \frac{3\left(10\,ab^4x^{\frac{1}{3}}\operatorname{sgn}\left(ax+bx^{\frac{2}{3}}\right)\operatorname{sgn}(x)+b^5\operatorname{sgn}\left(ax+bx^{\frac{2}{3}}\right)\operatorname{sgn}(x)\right)}{2\,x^{\frac{2}{3}}}$$

input
```
integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(5/2),x, algorithm="giac")
```

output
```
a^5*x*sgn(a*x + b*x^(2/3))*sgn(x) + 10*a^2*b^3*log(abs(x))*sgn(a*x + b*x^(
2/3))*sgn(x) + 15/2*a^4*b*x^(2/3)*sgn(a*x + b*x^(2/3))*sgn(x) + 30*a^3*b^2
*x^(1/3)*sgn(a*x + b*x^(2/3))*sgn(x) - 3/2*(10*a*b^4*x^(1/3)*sgn(a*x + b*x
^(2/3))*sgn(x) + b^5*sgn(a*x + b*x^(2/3))*sgn(x))/x^(2/3)
```

3.483.9 Mupad [F(-1)]

Timed out.

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{5/2} dx = \int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2\,a\,b}{x^{1/3}}\right)^{5/2} dx$$

input
```
int((a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(5/2),x)
```

output
```
int((a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(5/2), x)
```

3.484 $\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{3/2} dx$

3.484.1 Optimal result

Integrand size = 26, antiderivative size = 189

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{3/2} dx = \frac{9ab^2\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\sqrt[3]{x}}{a+\frac{b}{\sqrt[3]{x}}} + \frac{9a^2b\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}x^{2/3}}{2\left(a+\frac{b}{\sqrt[3]{x}}\right)} + \frac{a^3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}x}{a+\frac{b}{\sqrt[3]{x}}} + \frac{3b^3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\log\left(\sqrt[3]{x}\right)}{a+\frac{b}{\sqrt[3]{x}}}$$

output
```
9*a*b^2*x^(1/3)*(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))+9/2*a^
2*b*x^(2/3)*(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))+a^3*x*(a^2
+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))+b^3*ln(x)*(a^2+b^2/x^(2/3)
+2*a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))
```

3.484.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.40

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{3/2} dx=\frac{\left(b+a\sqrt[3]{x}\right)\left(18ab^2\sqrt[3]{x}+9a^2bx^{2/3}+2a^3x+2b^3\log(x)\right)}{2\sqrt{\frac{\left(b+a\sqrt[3]{x}\right)^2}{x^{2/3}}}\sqrt[3]{x}}$$

input
```
Integrate[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(3/2),x]
```

output
```
((b + a*x^(1/3))*(18*a*b^2*x^(1/3) + 9*a^2*b*x^(2/3) + 2*a^3*x + 2*b^3*Log
[x]))/(2*Sqrt[(b + a*x^(1/3))^2/x^(2/3)]*x^(1/3))
```

3.484.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.48, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1384, 774, 27, 795, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}\right)^{3/2} dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\int \left(\frac{b^2}{\sqrt[3]{x}}+ab\right)^3dx}{ab^3+\frac{b^4}{\sqrt[3]{x}}}$$

$$\Big\downarrow \text{774}$$

$$\frac{3\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\int b^3\left(a+\frac{b}{\sqrt[3]{x}}\right)^3x^{2/3}d\sqrt[3]{x}}{ab^3+\frac{b^4}{\sqrt[3]{x}}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3b^3\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^{2/3}d\sqrt[3]{x}}{ab^3+\frac{b^4}{\sqrt[3]{x}}}$$

$$\Big\downarrow \text{795}$$

$$\frac{3b^3\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\int \frac{\left(\sqrt[3]{x}a+b\right)^3}{\sqrt[3]{x}}d\sqrt[3]{x}}{ab^3+\frac{b^4}{\sqrt[3]{x}}}$$

$$\Big\downarrow \text{49}$$

$$\frac{3b^3\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\int \left(x^{2/3}a^3+3b\sqrt[3]{x}a^2+3b^2a+\frac{b^3}{\sqrt[3]{x}}\right)d\sqrt[3]{x}}{ab^3+\frac{b^4}{\sqrt[3]{x}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3b^3\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\left(\frac{a^3x}{3}+\frac{3}{2}a^2bx^{2/3}+3ab^2\sqrt[3]{x}+b^3\log\left(\sqrt[3]{x}\right)\right)}{ab^3+\frac{b^4}{\sqrt[3]{x}}}$$

input
```
Int[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(3/2),x]
```

output
```
(3*b^3*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*(3*a*b^2*x^(1/3) + (3*a^2
*b*x^(2/3))/2 + (a^3*x)/3 + b^3*Log[x^(1/3)]))/(a*b^3 + b^4/x^(1/3))
```

3.484.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

3.484. $\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{3/2} dx$

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.484.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.37

method	result	size
derivativedivides	$\frac{\left(\frac{x^{\frac{2}{3}} a^{2}+2 a b \,x^{\frac{1}{3}}+b^{2}}{x^{\frac{2}{3}}}\right)^{\frac{3}{2}} x \left(2 a^{3} x +9 a^{2} b \,x^{\frac{2}{3}}+2 b^{3} \ln \left(x \right)+18 b^{2} a \,x^{\frac{1}{3}}\right)}{2 \left(b +a \,x^{\frac{1}{3}}\right)^{3}}$	69
default	$\frac{\left(\frac{x^{\frac{2}{3}} a^{2}+2 a b \,x^{\frac{1}{3}}+b^{2}}{x^{\frac{2}{3}}}\right)^{\frac{3}{2}} x \left(2 a^{3} x +9 a^{2} b \,x^{\frac{2}{3}}+2 b^{3} \ln \left(x \right)+18 b^{2} a \,x^{\frac{1}{3}}\right)}{2 \left(b +a \,x^{\frac{1}{3}}\right)^{3}}$	69

input
```
int((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*((x^(2/3)*a^2+2*a*b*x^(1/3)+b^2)/x^(2/3))^(3/2)*x*(2*a^3*x+9*a^2*b*x^(
2/3)+2*b^3*ln(x)+18*b^2*a*x^(1/3))/(b+a*x^(1/3))^3
```

3.484.5 Fricas [F(-1)]

Timed out.

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{3/2} dx = \text{Timed out}$$

input
```
integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.484.6 Sympy [F]

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{3/2} dx = \int \left(a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{\frac{2}{3}}}\right)^{\frac{3}{2}} dx$$

input
```
integrate((a**2+b**2/x**(2/3)+2*a*b/x**(1/3))**(3/2),x)
```

output
```
Integral((a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3))**(3/2), x)
```

3.484.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.16

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{3/2} dx = a^3x + b^3\log(x) + \frac{9}{2}a^2bx^{\frac{2}{3}} + 9ab^2x^{\frac{1}{3}}$$

input
```
integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(3/2),x, algorithm="maxima")
```

output
```
a^3*x + b^3*log(x) + 9/2*a^2*b*x^(2/3) + 9*a*b^2*x^(1/3)
```

3.484.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.42

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{3/2} dx = a^3 x \operatorname{sgn}\left(ax+bx^{\frac{2}{3}}\right)\operatorname{sgn}(x)+b^3\log\left(|x|\right)\operatorname{sgn}\left(ax+bx^{\frac{2}{3}}\right)\operatorname{sgn}(x)+\frac{9}{2}a^2bx^{\frac{2}{3}}\operatorname{sgn}\left(ax+bx^{\frac{2}{3}}\right)\operatorname{sgn}(x)+9ab^2x^{\frac{1}{3}}\operatorname{sgn}\left(ax+bx^{\frac{2}{3}}\right)\operatorname{sgn}(x)$$

input
```
integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(3/2),x, algorithm="giac")
```

output
```
a^3*x*sgn(a*x + b*x^(2/3))*sgn(x) + b^3*log(abs(x))*sgn(a*x + b*x^(2/3))*s
gn(x) + 9/2*a^2*b*x^(2/3)*sgn(a*x + b*x^(2/3))*sgn(x) + 9*a*b^2*x^(1/3)*sg
n(a*x + b*x^(2/3))*sgn(x)
```

3.484.9 Mupad [F(-1)]

Timed out.

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{3/2} dx = \int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2\,a\,b}{x^{1/3}}\right)^{3/2} dx$$

input
```
int((a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(3/2),x)
```

output
```
int((a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(3/2), x)
```

3.485 $\int \sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\,dx$

3.485.1 Optimal result

Integrand size = 26, antiderivative size = 88

$$\int \sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\,dx=\frac{3b\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\,x^{2/3}}{2\left(a+\frac{b}{\sqrt[3]{x}}\right)}+\frac{a\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\,x}{a+\frac{b}{\sqrt[3]{x}}}$$

output
```
3/2*b*x^(2/3)*(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))+a*x*(a^2
+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))
```

3.485.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.56

$$\int \sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\,dx=\frac{\left(3b+2a\sqrt[3]{x}\right)\sqrt{\frac{\left(b+a\sqrt[3]{x}\right)^2}{x^{2/3}}}\,x}{2\left(b+a\sqrt[3]{x}\right)}$$

input
```
Integrate[Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)],x]
```

output
```
((3*b + 2*a*x^(1/3))*Sqrt[(b + a*x^(1/3))^2/x^(2/3)]*x)/(2*(b + a*x^(1/3))
)
```

3.485.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.67, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1384, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}dx$$
$$\Big\downarrow \text{1384}$$
$$\frac{\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\int \left(\frac{b^2}{\sqrt[3]{x}}+ab\right)dx}{ab+\frac{b^2}{\sqrt[3]{x}}}$$
$$\Big\downarrow \text{2009}$$
$$\frac{\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\left(abx+\frac{3}{2}b^2x^{2/3}\right)}{ab+\frac{b^2}{\sqrt[3]{x}}}$$

input
```
Int[Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)],x]
```

output
```
(Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*((3*b^2*x^(2/3))/2 + a*b*x))/(a
*b + b^2/x^(1/3))
```

3.485.3.1 Defintions of rubi rules used

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))    Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.485.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.53

method	result	size
derivativedivides	$\frac{\sqrt{\frac{x^{\frac{2}{3}} a^{2}+2 a b\, x^{\frac{1}{3}}+b^{2}}{x^{\frac{2}{3}}}}\, x \left(2 a\, x^{\frac{1}{3}}+3 b \right)}{2 b +2 a\, x^{\frac{1}{3}}}$	47
default	$\frac{\sqrt{\frac{x^{\frac{2}{3}} a^{2}+2 a b\, x^{\frac{1}{3}}+b^{2}}{x^{\frac{2}{3}}}}\, x^{\frac{1}{3}} \left(3 b\, x^{\frac{2}{3}}+2 a x \right)}{2 b +2 a\, x^{\frac{1}{3}}}$	50

input
```
int((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*((x^(2/3)*a^2+2*a*b*x^(1/3)+b^2)/x^(2/3))^(1/2)*x*(2*a*x^(1/3)+3*b)/(b
+a*x^(1/3))
```

3.485.5 Fricas [F(-1)]

Timed out.

$$\int \sqrt{a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}}\, dx = \text{Timed out}$$

input
```
integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.485.6 Sympy [F]

$$\int \sqrt{a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}}\, dx = \int \sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{\frac{2}{3}}}}\, dx$$

input
```
integrate((a**2+b**2/x**(2/3)+2*a*b/x**(1/3))**(1/2),x)
```

output
```
Integral(sqrt(a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3)), x)
```

3.485.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.11

$$\int \sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\,dx = ax+\frac{3}{2}\,bx^{\frac{2}{3}}$$

input
```
integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2),x, algorithm="maxima")
```

output
```
a*x + 3/2*b*x^(2/3)
```

3.485.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.39

$$\int \sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\,dx = ax\text{sgn}\left(ax+bx^{\frac{2}{3}}\right)\text{sgn}(x)+\frac{3}{2}\,bx^{\frac{2}{3}}\text{sgn}\left(ax+bx^{\frac{2}{3}}\right)\text{sgn}(x)$$

input
```
integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2),x, algorithm="giac")
```

output
```
a*x*sgn(a*x + b*x^(2/3))*sgn(x) + 3/2*b*x^(2/3)*sgn(a*x + b*x^(2/3))*sgn(x
)
```

3.485.9 Mupad [B] (verification not implemented)

Time = 8.48 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.44

$$\int \sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\,dx = \frac{x\left(a+\frac{3\,b}{2\,x^{1/3}}\right)\,\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2\,a\,b}{x^{1/3}}}}{a+\frac{b}{x^{1/3}}}$$

input
```
int((a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(1/2),x)
```

output
```
(x*(a + (3*b)/(2*x^(1/3)))*(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(1/2))/(a
 + b/x^(1/3))
```

3.486 $\int \frac{1}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}\,dx$

3.486.1 Optimal result

Integrand size = 26, antiderivative size = 190

$$\int \frac{1}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}\,dx=\frac{3b^2\left(a+\frac{b}{\sqrt[3]{x}}\right)\sqrt[3]{x}}{a^3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}-\frac{3b\left(a+\frac{b}{\sqrt[3]{x}}\right)x^{2/3}}{2a^2\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}+\frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)x}{a\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}-\frac{3b^3\left(a+\frac{b}{\sqrt[3]{x}}\right)\log\left(b+a\sqrt[3]{x}\right)}{a^4\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}$$

output
```
3*b^2*(a+b/x^(1/3))*x^(1/3)/a^3/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)-3/2*
b*(a+b/x^(1/3))*x^(2/3)/a^2/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)+(a+b/x^(
1/3))*x/a/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)-3*b^3*(a+b/x^(1/3))*ln(b+a
*x^(1/3))/a^4/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)
```

3.486.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.45

$$\int \frac{1}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}\,dx=\frac{\left(b+a\sqrt[3]{x}\right)\left(6ab^2\sqrt[3]{x}-3a^2bx^{2/3}+2a^3x-6b^3\log\left(b+a\sqrt[3]{x}\right)\right)}{2a^4\sqrt{\frac{\left(b+a\sqrt[3]{x}\right)^2}{x^{2/3}}}\sqrt[3]{x}}$$

input
```
Integrate[1/Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)],x]
```

output
```
((b + a*x^(1/3))*(6*a*b^2*x^(1/3) - 3*a^2*b*x^(2/3) + 2*a^3*x - 6*b^3*Log[
b + a*x^(1/3)]))/(2*a^4*Sqrt[(b + a*x^(1/3))^2/x^(2/3)]*x^(1/3))
```

3.486.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.51, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1384, 774, 27, 795, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)\int \frac{1}{\frac{b^2}{\sqrt[3]{x}}+ab}dx}{\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}$$

$$\Big\downarrow \text{774}$$

$$\frac{3\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)\int \frac{x^{2/3}}{b\left(a+\frac{b}{\sqrt[3]{x}}\right)}d\sqrt[3]{x}}{\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)\int \frac{x^{2/3}}{a+\frac{b}{\sqrt[3]{x}}}d\sqrt[3]{x}}{b\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}$$

$$\Big\downarrow \text{795}$$

$$\frac{3\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)\int \frac{x}{\sqrt[3]{x}a+b}d\sqrt[3]{x}}{b\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}$$

$$\Big\downarrow \text{49}$$

$$\frac{3\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)\int \left(-\frac{b^3}{a^3\left(\sqrt[3]{x}a+b\right)}+\frac{b^2}{a^3}-\frac{\sqrt[3]{x}b}{a^2}+\frac{x^{2/3}}{a}\right)d\sqrt[3]{x}}{b\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)\left(-\frac{b^3\log\left(a\sqrt[3]{x}+b\right)}{a^4}+\frac{b^2\sqrt[3]{x}}{a^3}-\frac{bx^{2/3}}{2a^2}+\frac{x}{3a}\right)}{b\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}$$

input
```
Int[1/Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)],x]
```

output
```
(3*(a*b + b^2/x^(1/3))*((b^2*x^(1/3))/a^3 - (b*x^(2/3))/(2*a^2) + x/(3*a)
- (b^3*Log[b + a*x^(1/3)])/a^4))/(b*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/
3)])
```

3.486.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

3.486. $\int \frac{1}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}dx$

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.486.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.41

method	result	size
derivativedivides	$-\frac{\left(b+a\,x^{\frac{1}{3}}\right)\left(-2a^{3}x+3a^{2}b\,x^{\frac{2}{3}}+6b^{3}\ln\left(b+a\,x^{\frac{1}{3}}\right)-6b^{2}a\,x^{\frac{1}{3}}\right)}{2\sqrt{\frac{x^{\frac{2}{3}}a^{2}+2ab\,x^{\frac{1}{3}}+b^{2}}{x^{\frac{2}{3}}}}\,x^{\frac{1}{3}}a^{4}}$	78
default	$-\frac{\left(b+a\,x^{\frac{1}{3}}\right)\left(-2a^{3}x+3a^{2}b\,x^{\frac{2}{3}}+6b^{3}\ln\left(b+a\,x^{\frac{1}{3}}\right)-6b^{2}a\,x^{\frac{1}{3}}\right)}{2\sqrt{\frac{x^{\frac{2}{3}}a^{2}+2ab\,x^{\frac{1}{3}}+b^{2}}{x^{\frac{2}{3}}}}\,x^{\frac{1}{3}}a^{4}}$	78

input
```
int(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(b+a*x^(1/3))*(-2*a^3*x+3*a^2*b*x^(2/3)+6*b^3*ln(b+a*x^(1/3))-6*b^2*a
*x^(1/3))/((x^(2/3)*a^2+2*a*b*x^(1/3)+b^2)/x^(2/3))^(1/2)/x^(1/3)/a^4
```

3.486.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.486.6 Sympy [F]

$$\int \frac{1}{\sqrt{a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}}}\,dx = \int \frac{1}{\sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{\frac{2}{3}}}}}\,dx$$

input
```
integrate(1/(a**2+b**2/x**(2/3)+2*a*b/x**(1/3))**(1/2),x)
```

output
```
Integral(1/sqrt(a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3)), x)
```

3.486.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.23

$$\int \frac{1}{\sqrt{a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}}}\,dx = -\frac{3\,b^3 \log\left(ax^{\frac{1}{3}} + b\right)}{a^4} + \frac{2\,a^2x - 3\,abx^{\frac{2}{3}} + 6\,b^2x^{\frac{1}{3}}}{2\,a^3}$$

input
```
integrate(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2),x, algorithm="maxima")
```

output
```
-3*b^3*log(a*x^(1/3) + b)/a^4 + 1/2*(2*a^2*x - 3*a*b*x^(2/3) + 6*b^2*x^(1/
3))/a^3
```

3.486.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.41

$$\int \frac{1}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}\,dx=-\frac{3\,b^3\log\left(\left|ax^{\frac{1}{3}}+b\right|\right)}{a^4\mathrm{sgn}\left(ax+bx^{\frac{2}{3}}\right)\mathrm{sgn}\left(x\right)}+\frac{2\,a^2x-3\,abx^{\frac{2}{3}}+6\,b^2x^{\frac{1}{3}}}{2\,a^3\mathrm{sgn}\left(ax+bx^{\frac{2}{3}}\right)\mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2),x, algorithm="giac")
```

output
```
-3*b^3*log(abs(a*x^(1/3) + b))/(a^4*sgn(a*x + b*x^(2/3))*sgn(x)) + 1/2*(2*
a^2*x - 3*a*b*x^(2/3) + 6*b^2*x^(1/3))/(a^3*sgn(a*x + b*x^(2/3))*sgn(x))
```

3.486.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}\,dx=\int \frac{1}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2\,a\,b}{x^{1/3}}}}\,dx$$

input
```
int(1/(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(1/2),x)
```

output
```
int(1/(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(1/2), x)
```

3.487 $\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{3/2}}\,dx$

3.487.1 Optimal result

Integrand size = 26, antiderivative size = 300

$$\begin{aligned}\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{3/2}}\,dx &= \frac{3b^5\left(a+\frac{b}{\sqrt[3]{x}}\right)}{2a^6\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\left(b+a\sqrt[3]{x}\right)^2} \\ &\quad - \frac{15b^4\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^6\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\left(b+a\sqrt[3]{x}\right)} + \frac{18b^2\left(a+\frac{b}{\sqrt[3]{x}}\right)\sqrt[3]{x}}{a^5\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}} \\ &\quad - \frac{9b\left(a+\frac{b}{\sqrt[3]{x}}\right)x^{2/3}}{2a^4\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}} + \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)x}{a^3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}} - \frac{30b^3\left(a+\frac{b}{\sqrt[3]{x}}\right)\log\left(b+a\sqrt[3]{x}\right)}{a^6\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}\end{aligned}$$

output
```
3/2*b^5*(a+b/x^(1/3))/a^6/(b+a*x^(1/3))^2/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^
(1/2)-15*b^4*(a+b/x^(1/3))/a^6/(b+a*x^(1/3))/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3
))^(1/2)+18*b^2*(a+b/x^(1/3))*x^(1/3)/a^5/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^
(1/2)-9/2*b*(a+b/x^(1/3))*x^(2/3)/a^4/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2
)+(a+b/x^(1/3))*x/a^3/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)-30*b^3*(a+b/x^
(1/3))*ln(b+a*x^(1/3))/a^6/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)
```

3.487.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.42

$$\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{3/2}}\,dx=\frac{\left(b+a\sqrt[3]{x}\right)\left(-27b^5+6ab^4\sqrt[3]{x}+63a^2b^3x^{2/3}+20a^3b^2x-5a^4bx^{4/3}+2a^5x^{5/3}\right.}{2a^6\left(\frac{\left(b+a\sqrt[3]{x}\right)^2}{x^{2/3}}\right)^{3/2}x}$$

input
```
Integrate[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(-3/2),x]
```

output
```
((b + a*x^(1/3))*(-27*b^5 + 6*a*b^4*x^(1/3) + 63*a^2*b^3*x^(2/3) + 20*a^3*
b^2*x - 5*a^4*b*x^(4/3) + 2*a^5*x^(5/3) - 60*b^3*(b + a*x^(1/3))^2*Log[b +
 a*x^(1/3)]))/(2*a^6*((b + a*x^(1/3))^2/x^(2/3))^(3/2)*x)
```

3.487.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.46, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1384, 774, 27, 795, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}\right)^{3/2}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\left(ab^3+\frac{b^4}{\sqrt[3]{x}}\right)\int \frac{1}{\left(\frac{b^2}{\sqrt[3]{x}}+ab\right)^3}dx}{\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}$$

$$\Big\downarrow \text{774}$$

$$\frac{3\left(ab^3+\frac{b^4}{\sqrt[3]{x}}\right)\int \frac{x^{2/3}}{b^3\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}d\sqrt[3]{x}}{\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}$$

$$\downarrow \text{27}$$

$$\frac{3\left(ab^3+\frac{b^4}{\sqrt[3]{x}}\right)\int \frac{x^{2/3}}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}d\sqrt[3]{x}}{b^3\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}$$

$$\downarrow \text{795}$$

$$\frac{3\left(ab^3+\frac{b^4}{\sqrt[3]{x}}\right)\int \frac{x^{5/3}}{\left(\sqrt[3]{x}a+b\right)^3}d\sqrt[3]{x}}{b^3\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}$$

$$\downarrow \text{49}$$

$$\frac{3\left(ab^3+\frac{b^4}{\sqrt[3]{x}}\right)\int \left(-\frac{b^5}{a^5\left(\sqrt[3]{x}a+b\right)^3}+\frac{5b^4}{a^5\left(\sqrt[3]{x}a+b\right)^2}-\frac{10b^3}{a^5\left(\sqrt[3]{x}a+b\right)}+\frac{6b^2}{a^5}-\frac{3\sqrt[3]{x}b}{a^4}+\frac{x^{2/3}}{a^3}\right)d\sqrt[3]{x}}{b^3\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}$$

$$\downarrow \text{2009}$$

$$\frac{3\left(ab^3+\frac{b^4}{\sqrt[3]{x}}\right)\left(\frac{b^5}{2a^6\left(a\sqrt[3]{x}+b\right)^2}-\frac{5b^4}{a^6\left(a\sqrt[3]{x}+b\right)}-\frac{10b^3\log\left(a\sqrt[3]{x}+b\right)}{a^6}+\frac{6b^2\sqrt[3]{x}}{a^5}-\frac{3bx^{2/3}}{2a^4}+\frac{x}{3a^3}\right)}{b^3\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}$$

input
```
Int[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(-3/2),x]
```

output
```
(3*(a*b^3 + b^4/x^(1/3))*(b^5/(2*a^6*(b + a*x^(1/3))^2) - (5*b^4)/(a^6*(b
+ a*x^(1/3))) + (6*b^2*x^(1/3))/a^5 - (3*b*x^(2/3))/(2*a^4) + x/(3*a^3) -
(10*b^3*Log[b + a*x^(1/3)])/a^6))/(b^3*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^
(1/3)])
```

3.487. $\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{3/2}}\,dx$

3.487.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.487.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.47

method	result
derivativedivides	$-\frac{\left(-2a^{5}x^{\frac{5}{3}}+5b\,a^{4}x^{\frac{4}{3}}+60\ln\left(b+a\,x^{\frac{1}{3}}\right)a^{2}b^{3}x^{\frac{2}{3}}-20a^{3}b^{2}x+120\ln\left(b+a\,x^{\frac{1}{3}}\right)a\,b^{4}x^{\frac{1}{3}}-63b^{3}x^{\frac{2}{3}}a^{2}+60\ln\left(b+a\,x^{\frac{1}{3}}\right)b^{5}-6b^{4}\right.}{2a^{6}x\left(\frac{x^{\frac{2}{3}}a^{2}+2ab\,x^{\frac{1}{3}}+b^{2}}{x^{\frac{2}{3}}}\right)^{\frac{3}{2}}}$
default	$\frac{\left(2a^{5}x^{\frac{5}{3}}-5b\,a^{4}x^{\frac{4}{3}}-60\ln\left(b+a\,x^{\frac{1}{3}}\right)a^{2}b^{3}x^{\frac{2}{3}}+63b^{3}x^{\frac{2}{3}}a^{2}-120\ln\left(b+a\,x^{\frac{1}{3}}\right)a\,b^{4}x^{\frac{1}{3}}+6b^{4}a\,x^{\frac{1}{3}}-60\ln\left(b+a\,x^{\frac{1}{3}}\right)b^{5}+20a^{3}b^{2}x\right.}{2a^{6}x\left(\frac{x^{\frac{2}{3}}a^{2}+2ab\,x^{\frac{1}{3}}+b^{2}}{x^{\frac{2}{3}}}\right)^{\frac{3}{2}}}$

3.487. $\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{3/2}}\,dx$

input
```
int(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-2*a^5*x^(5/3)+5*b*a^4*x^(4/3)+60*ln(b+a*x^(1/3))*a^2*b^3*x^(2/3)-20
*a^3*b^2*x+120*ln(b+a*x^(1/3))*a*b^4*x^(1/3)-63*b^3*x^(2/3)*a^2+60*ln(b+a*
x^(1/3))*b^5-6*b^4*a*x^(1/3)+27*b^5)*(b+a*x^(1/3))/a^6/x/((x^(2/3)*a^2+2*a
*b*x^(1/3)+b^2)/x^(2/3))^(3/2)
```

3.487.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.487.6 Sympy [F]

$$\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{3/2}}\,dx=\int \frac{1}{\left(a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{\frac{2}{3}}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a**2+b**2/x**(2/3)+2*a*b/x**(1/3))**(3/2),x)
```

output
```
Integral((a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3))**(-3/2), x)
```

3.487.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.32

$$\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{3/2}}\,dx = \frac{2\,a^5x^{\frac{5}{3}}-5\,a^4bx^{\frac{4}{3}}+20\,a^3b^2x+63\,a^2b^3x^{\frac{2}{3}}+6\,ab^4x^{\frac{1}{3}}-27\,b^5}{2\left(a^8x^{\frac{2}{3}}+2\,a^7bx^{\frac{1}{3}}+a^6b^2\right)} - \frac{30\,b^3\log\left(ax^{\frac{1}{3}}+b\right)}{a^6}$$

input
```
integrate(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(3/2),x, algorithm="maxima")
```

output
```
1/2*(2*a^5*x^(5/3) - 5*a^4*b*x^(4/3) + 20*a^3*b^2*x + 63*a^2*b^3*x^(2/3) +
 6*a*b^4*x^(1/3) - 27*b^5)/(a^8*x^(2/3) + 2*a^7*b*x^(1/3) + a^6*b^2) - 30*
b^3*log(a*x^(1/3) + b)/a^6
```

3.487.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.42

$$\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{3/2}}\,dx = -\frac{30\,b^3\log\left(\left|ax^{\frac{1}{3}}+b\right|\right)}{a^6\mathrm{sgn}\left(ax+bx^{\frac{2}{3}}\right)\mathrm{sgn}\left(x\right)} - \frac{3\left(10\,ab^4x^{\frac{1}{3}}+9\,b^5\right)}{2\left(ax^{\frac{1}{3}}+b\right)^2a^6\mathrm{sgn}\left(ax+bx^{\frac{2}{3}}\right)\mathrm{sgn}\left(x\right)} + \frac{2\,a^6x-9\,a^5bx^{\frac{2}{3}}+36\,a^4b^2x^{\frac{1}{3}}}{2\,a^9\mathrm{sgn}\left(ax+bx^{\frac{2}{3}}\right)\mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(3/2),x, algorithm="giac")
```

output
```
-30*b^3*log(abs(a*x^(1/3) + b))/(a^6*sgn(a*x + b*x^(2/3))*sgn(x)) - 3/2*(1
0*a*b^4*x^(1/3) + 9*b^5)/((a*x^(1/3) + b)^2*a^6*sgn(a*x + b*x^(2/3))*sgn(x
)) + 1/2*(2*a^6*x - 9*a^5*b*x^(2/3) + 36*a^4*b^2*x^(1/3))/(a^9*sgn(a*x + b
*x^(2/3))*sgn(x))
```

3.487.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{3/2}}\,dx=\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2\,a\,b}{x^{1/3}}\right)^{3/2}}\,dx$$

input
```
int(1/(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(3/2),x)
```

output
```
int(1/(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(3/2), x)
```

3.488 $\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{5/2}} dx$

3.488.1 Optimal result

Integrand size = 26, antiderivative size = 410

$$\begin{aligned}\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{5/2}} dx &= \frac{3b^7\left(a+\frac{b}{\sqrt[3]{x}}\right)}{4a^8\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\left(b+a\sqrt[3]{x}\right)^4} \\ &- \frac{7b^6\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^8\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\left(b+a\sqrt[3]{x}\right)^3} + \frac{63b^5\left(a+\frac{b}{\sqrt[3]{x}}\right)}{2a^8\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\left(b+a\sqrt[3]{x}\right)^2} \\ &- \frac{105b^4\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^8\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\left(b+a\sqrt[3]{x}\right)} + \frac{45b^2\left(a+\frac{b}{\sqrt[3]{x}}\right)\sqrt[3]{x}}{a^7\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}} \\ &- \frac{15b\left(a+\frac{b}{\sqrt[3]{x}}\right)x^{2/3}}{2a^6\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}} + \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)x}{a^5\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}} - \frac{105b^3\left(a+\frac{b}{\sqrt[3]{x}}\right)\log\left(b+a\sqrt[3]{x}\right)}{a^8\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}\end{aligned}$$

output
```
3/4*b^7*(a+b/x^(1/3))/a^8/(b+a*x^(1/3))^4/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^
(1/2)-7*b^6*(a+b/x^(1/3))/a^8/(b+a*x^(1/3))^3/(a^2+b^2/x^(2/3)+2*a*b/x^(1/
3))^(1/2)+63/2*b^5*(a+b/x^(1/3))/a^8/(b+a*x^(1/3))^2/(a^2+b^2/x^(2/3)+2*a*
b/x^(1/3))^(1/2)-105*b^4*(a+b/x^(1/3))/a^8/(b+a*x^(1/3))/(a^2+b^2/x^(2/3)+
2*a*b/x^(1/3))^(1/2)+45*b^2*(a+b/x^(1/3))*x^(1/3)/a^7/(a^2+b^2/x^(2/3)+2*a
*b/x^(1/3))^(1/2)-15/2*b*(a+b/x^(1/3))*x^(2/3)/a^6/(a^2+b^2/x^(2/3)+2*a*b/
x^(1/3))^(1/2)+(a+b/x^(1/3))*x/a^5/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)-1
05*b^3*(a+b/x^(1/3))*ln(b+a*x^(1/3))/a^8/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(
1/2)
```

3.488.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.37

$$\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{5/2}}\,dx=\frac{\left(b+a\sqrt[3]{x}\right)\left(-319b^7-856ab^6\sqrt[3]{x}-444a^2b^5x^{2/3}+544a^3b^4x+556a^4b^3x^{4/3}+\right.}{4a^8\left(\frac{\left(b+a\sqrt[3]{x}\right)^2}{x^{2/3}}\right.}$$

input
```
Integrate[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(-5/2),x]
```

output
```
((b + a*x^(1/3))*(-319*b^7 - 856*a*b^6*x^(1/3) - 444*a^2*b^5*x^(2/3) + 544
*a^3*b^4*x + 556*a^4*b^3*x^(4/3) + 84*a^5*b^2*x^(5/3) - 14*a^6*b*x^2 + 4*a
^7*x^(7/3) - 420*b^3*(b + a*x^(1/3))^4*Log[b + a*x^(1/3)]))/(4*a^8*((b + a
*x^(1/3))^2/x^(2/3))^(5/2)*x^(5/3))
```

3.488.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.44, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1384, 774, 27, 795, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}\right)^{5/2}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\left(ab^5+\frac{b^6}{\sqrt[3]{x}}\right)\int \frac{1}{\left(\frac{b^2}{\sqrt[3]{x}}+ab\right)^5}dx}{\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}$$

$$\Big\downarrow \text{774}$$

$$\frac{3\left(ab^5+\frac{b^6}{\sqrt[3]{x}}\right)\int \frac{x^{2/3}}{b^5\left(a+\frac{b}{\sqrt[3]{x}}\right)^5}d\sqrt[3]{x}}{\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\left(ab^5+\frac{b^6}{\sqrt[3]{x}}\right)\int \frac{x^{2/3}}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^5}d\sqrt[3]{x}}{b^5\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}$$

$$\Big\downarrow \text{795}$$

$$\frac{3\left(ab^5+\frac{b^6}{\sqrt[3]{x}}\right)\int \frac{x^{7/3}}{\left(\sqrt[3]{x}a+b\right)^5}d\sqrt[3]{x}}{b^5\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}$$

$$\Big\downarrow \text{49}$$

$$\frac{3\left(ab^5+\frac{b^6}{\sqrt[3]{x}}\right)\int \left(-\frac{b^7}{a^7\left(\sqrt[3]{x}a+b\right)^5}+\frac{7b^6}{a^7\left(\sqrt[3]{x}a+b\right)^4}-\frac{21b^5}{a^7\left(\sqrt[3]{x}a+b\right)^3}+\frac{35b^4}{a^7\left(\sqrt[3]{x}a+b\right)^2}-\frac{35b^3}{a^7\left(\sqrt[3]{x}a+b\right)}+\frac{15b^2}{a^7}-\frac{5\sqrt[3]{x}b}{a^6}+\frac{x^{2/}}{a}\right.}{b^5\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\left(ab^5+\frac{b^6}{\sqrt[3]{x}}\right)\left(\frac{b^7}{4a^8\left(a\sqrt[3]{x}+b\right)^4}-\frac{7b^6}{3a^8\left(a\sqrt[3]{x}+b\right)^3}+\frac{21b^5}{2a^8\left(a\sqrt[3]{x}+b\right)^2}-\frac{35b^4}{a^8\left(a\sqrt[3]{x}+b\right)}-\frac{35b^3\log\left(a\sqrt[3]{x}+b\right)}{a^8}+\frac{15b^2\sqrt[3]{x}}{a^7}-\frac{5bx^{2/3}}{2a^6}\right.}{b^5\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}$$

3.488. $\displaystyle \int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{5/2}}dx$

input
```
Int[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(-5/2),x]
```

output
```
(3*(a*b^5 + b^6/x^(1/3))*(b^7/(4*a^8*(b + a*x^(1/3))^4) - (7*b^6)/(3*a^8*(
b + a*x^(1/3))^3) + (21*b^5)/(2*a^8*(b + a*x^(1/3))^2) - (35*b^4)/(a^8*(b
+ a*x^(1/3))) + (15*b^2*x^(1/3))/a^7 - (5*b*x^(2/3))/(2*a^6) + x/(3*a^5) -
 (35*b^3*Log[b + a*x^(1/3)])/a^8))/(b^5*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x
^(1/3)])
```

3.488.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.488. $\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{5/2}}\,dx$

3.488.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.49

method	result
derivativedivides	$-\frac{\left(-4a^7x^{\frac{7}{3}}+14x^2a^6b+420\ln\left(b+a\,x^{\frac{1}{3}}\right)a^4b^3x^{\frac{4}{3}}-84b^2a^5x^{\frac{5}{3}}+1680\ln\left(b+a\,x^{\frac{1}{3}}\right)a^3b^4x-556a^4b^3x^{\frac{4}{3}}+2520\ln\left(b+a\,x^{\frac{1}{3}}\right)a\right.}{4a^8x^{\frac{5}{3}}\left(\frac{x^{\frac{2}{3}}a^2+2al}{x^{\frac{2}{3}}}\right.}$
default	$\frac{\left(4a^7x^{\frac{7}{3}}+84b^2a^5x^{\frac{5}{3}}-420\ln\left(b+a\,x^{\frac{1}{3}}\right)a^4b^3x^{\frac{4}{3}}+556a^4b^3x^{\frac{4}{3}}-2520\ln\left(b+a\,x^{\frac{1}{3}}\right)a^2b^5x^{\frac{2}{3}}-444a^2b^5x^{\frac{2}{3}}-1680\ln\left(b+a\,x^{\frac{1}{3}}\right)a\right.}{4a^8x^{\frac{5}{3}}\left(\frac{x^{\frac{2}{3}}a^2+2ab\,x^{\frac{1}{3}}}{x^{\frac{2}{3}}}\right.}$

input
```
int(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-4*a^7*x^(7/3)+14*x^2*a^6*b+420*ln(b+a*x^(1/3))*a^4*b^3*x^(4/3)-84*b
^2*a^5*x^(5/3)+1680*ln(b+a*x^(1/3))*a^3*b^4*x-556*a^4*b^3*x^(4/3)+2520*ln(
b+a*x^(1/3))*a^2*b^5*x^(2/3)-544*b^4*a^3*x+1680*ln(b+a*x^(1/3))*a*b^6*x^(1
/3)+444*a^2*b^5*x^(2/3)+420*ln(b+a*x^(1/3))*b^7+856*a*b^6*x^(1/3)+319*b^7)
*(b+a*x^(1/3))/a^8/x^(5/3)/((x^(2/3)*a^2+2*a*b*x^(1/3)+b^2)/x^(2/3))^(5/2)
```

3.488.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(5/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.488.6 Sympy [F]

$$\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{5/2}}\,dx=\int \frac{1}{\left(a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{\frac{2}{3}}}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a**2+b**2/x**(2/3)+2*a*b/x**(1/3))**(5/2),x)
```

output
```
Integral((a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3))**(-5/2), x)
```

3.488.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.34

$$\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{5/2}}\,dx=\frac{4\,a^7x^{\frac{7}{3}}-14\,a^6bx^2+84\,a^5b^2x^{\frac{5}{3}}+556\,a^4b^3x^{\frac{4}{3}}+544\,a^3b^4x-444\,a^2b^5x^{\frac{2}{3}}-856}{4\left(a^{12}x^{\frac{4}{3}}+4\,a^{11}bx+6\,a^{10}b^2x^{\frac{2}{3}}+4\,a^9b^3x^{\frac{1}{3}}+a^8b^4\right)}-\frac{105\,b^3\log\left(ax^{\frac{1}{3}}+b\right)}{a^8}$$

input
```
integrate(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(5/2),x, algorithm="maxima")
```

output
```
1/4*(4*a^7*x^(7/3) - 14*a^6*b*x^2 + 84*a^5*b^2*x^(5/3) + 556*a^4*b^3*x^(4/
3) + 544*a^3*b^4*x - 444*a^2*b^5*x^(2/3) - 856*a*b^6*x^(1/3) - 319*b^7)/(a
^12*x^(4/3) + 4*a^11*b*x + 6*a^10*b^2*x^(2/3) + 4*a^9*b^3*x^(1/3) + a^8*b^
4) - 105*b^3*log(a*x^(1/3) + b)/a^8
```

3.488.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.36

$$\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{5/2}}\,dx=-\frac{105\,b^3\log\left(\left|ax^{\frac{1}{3}}+b\right|\right)}{a^8\mathrm{sgn}\left(ax+bx^{\frac{2}{3}}\right)\mathrm{sgn}\left(x\right)}-\frac{420\,a^3b^4x+1134\,a^2b^5x^{\frac{2}{3}}+1036\,ab^6x^{\frac{1}{3}}+319\,b^7}{4\left(ax^{\frac{1}{3}}+b\right)^4a^8\mathrm{sgn}\left(ax+bx^{\frac{2}{3}}\right)\mathrm{sgn}\left(x\right)}+\frac{2\,a^{10}x-15\,a^9bx^{\frac{2}{3}}+90\,a^8b^2x^{\frac{1}{3}}}{2\,a^{15}\mathrm{sgn}\left(ax+bx^{\frac{2}{3}}\right)\mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(5/2),x, algorithm="giac")
```

output
```
-105*b^3*log(abs(a*x^(1/3) + b))/(a^8*sgn(a*x + b*x^(2/3))*sgn(x)) - 1/4*(
420*a^3*b^4*x + 1134*a^2*b^5*x^(2/3) + 1036*a*b^6*x^(1/3) + 319*b^7)/((a*x
^(1/3) + b)^4*a^8*sgn(a*x + b*x^(2/3))*sgn(x)) + 1/2*(2*a^10*x - 15*a^9*b*
x^(2/3) + 90*a^8*b^2*x^(1/3))/(a^15*sgn(a*x + b*x^(2/3))*sgn(x))
```

3.488.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{5/2}}\,dx=\int \frac{1}{{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2\,a\,b}{x^{1/3}}\right)}^{5/2}}\,dx$$

input
```
int(1/(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(5/2),x)
```

output
```
int(1/(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(5/2), x)
```

3.489 $\int \left(a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}\right)^{5/2}\,dx$

3.489.1 Optimal result

Integrand size = 26, antiderivative size = 289

$$\int \left(a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}\right)^{5/2}\,dx=-\frac{4b^5\sqrt{a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}}}{\left(a+\frac{b}{\sqrt[4]{x}}\right)\sqrt[4]{x}}+\frac{40a^2b^3\sqrt{a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}}\sqrt[4]{x}}{a+\frac{b}{\sqrt[4]{x}}}+\frac{20a^3b^2\sqrt{a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}}\sqrt{x}}{a+\frac{b}{\sqrt[4]{x}}}+\frac{20a^4b\sqrt{a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}}x^{3/4}}{3\left(a+\frac{b}{\sqrt[4]{x}}\right)}+\frac{a^5\sqrt{a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}}x}{a+\frac{b}{\sqrt[4]{x}}}+\frac{20ab^4\sqrt{a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}}\log\left(\sqrt[4]{x}\right)}{a+\frac{b}{\sqrt[4]{x}}}$$

output
```
-4*b^5*(a^2+2*a*b/x^(1/4)+b^2/x^(1/2))^(1/2)/(a+b/x^(1/4))/x^(1/4)+40*a^2*
b^3*x^(1/4)*(a^2+2*a*b/x^(1/4)+b^2/x^(1/2))^(1/2)/(a+b/x^(1/4))+20/3*a^4*b
*x^(3/4)*(a^2+2*a*b/x^(1/4)+b^2/x^(1/2))^(1/2)/(a+b/x^(1/4))+a^5*x*(a^2+2*
a*b/x^(1/4)+b^2/x^(1/2))^(1/2)/(a+b/x^(1/4))+5*a*b^4*ln(x)*(a^2+2*a*b/x^(1
/4)+b^2/x^(1/2))^(1/2)/(a+b/x^(1/4))+20*a^3*b^2*x^(1/2)*(a^2+2*a*b/x^(1/4)
+b^2/x^(1/2))^(1/2)/(a+b/x^(1/4))
```

3.489.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.34

$$\int \left(a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}\right)^{5/2} dx = \frac{\sqrt{\frac{\left(b+a\sqrt[4]{x}\right)^2}{\sqrt{x}}}\left(-12b^5+120a^2b^3\sqrt{x}+60a^3b^2x^{3/4}+20a^4bx+3a^5x^{5/4}+15ab^4\sqrt[4]{x}\log(x)\right)}{3\left(b+a\sqrt[4]{x}\right)}$$

input
```
Integrate[(a^2 + b^2/Sqrt[x] + (2*a*b)/x^(1/4))^(5/2),x]
```

output
```
(Sqrt[(b + a*x^(1/4))^2/Sqrt[x]]*(-12*b^5 + 120*a^2*b^3*Sqrt[x] + 60*a^3*b
^2*x^(3/4) + 20*a^4*b*x + 3*a^5*x^(5/4) + 15*a*b^4*x^(1/4)*Log[x]))/(3*(b
+ a*x^(1/4)))
```

3.489.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.41, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1384, 774, 27, 795, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+\frac{2ab}{\sqrt[4]{x}}+\frac{b^2}{\sqrt{x}}\right)^{5/2} dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+\frac{2ab}{\sqrt[4]{x}}+\frac{b^2}{\sqrt{x}}}\int \left(\frac{b^2}{\sqrt[4]{x}}+ab\right)^5 dx}{ab^5+\frac{b^6}{\sqrt[4]{x}}}$$

$$\Big\downarrow \text{774}$$

$$\frac{4\sqrt{a^2+\frac{2ab}{\sqrt[4]{x}}+\frac{b^2}{\sqrt{x}}}\int b^5\left(a+\frac{b}{\sqrt[4]{x}}\right)^5 x^{3/4}d\sqrt[4]{x}}{ab^5+\frac{b^6}{\sqrt[4]{x}}}$$

$$\Big\downarrow \text{27}$$

$$\frac{4b^5\sqrt{a^2+\frac{2ab}{\sqrt[4]{x}}+\frac{b^2}{\sqrt{x}}}\int \left(a+\frac{b}{\sqrt[4]{x}}\right)^5 x^{3/4}d\sqrt[4]{x}}{ab^5+\frac{b^6}{\sqrt[4]{x}}}$$

$$\Big\downarrow \text{795}$$

$$\frac{4b^5\sqrt{a^2+\frac{2ab}{\sqrt[4]{x}}+\frac{b^2}{\sqrt{x}}}\int \frac{\left(\sqrt[4]{x}a+b\right)^5}{\sqrt{x}}d\sqrt[4]{x}}{ab^5+\frac{b^6}{\sqrt[4]{x}}}$$

$$\Big\downarrow \text{49}$$

$$\frac{4b^5\sqrt{a^2+\frac{2ab}{\sqrt[4]{x}}+\frac{b^2}{\sqrt{x}}}\int \left(x^{3/4}a^5+5b\sqrt{x}a^4+10b^2\sqrt[4]{x}a^3+10b^3a^2+\frac{5b^4a}{\sqrt[4]{x}}+\frac{b^5}{\sqrt{x}}\right)d\sqrt[4]{x}}{ab^5+\frac{b^6}{\sqrt[4]{x}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{4b^5\sqrt{a^2+\frac{2ab}{\sqrt[4]{x}}+\frac{b^2}{\sqrt{x}}}\left(\frac{a^5x}{4}+\frac{5}{3}a^4bx^{3/4}+5a^3b^2\sqrt{x}+10a^2b^3\sqrt[4]{x}+5ab^4\log\left(\sqrt[4]{x}\right)-\frac{b^5}{\sqrt[4]{x}}\right)}{ab^5+\frac{b^6}{\sqrt[4]{x}}}$$

input
```
Int[(a^2 + b^2/Sqrt[x] + (2*a*b)/x^(1/4))^(5/2),x]
```

output
```
(4*b^5*Sqrt[a^2 + b^2/Sqrt[x] + (2*a*b)/x^(1/4)]*(-(b^5/x^(1/4)) + 10*a^2*
b^3*x^(1/4) + 5*a^3*b^2*Sqrt[x] + (5*a^4*b*x^(3/4))/3 + (a^5*x)/4 + 5*a*b^
4*Log[x^(1/4)]))/(a*b^5 + b^6/x^(1/4))
```

3.489.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

3.489. $\int \left(a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}\right)^{5/2} dx$

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.489.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.31

method	result	size
derivativedivides	$\frac{\left(\frac{a^{2} \sqrt{x}+b^{2}+2 a b \,x^{\frac{1}{4}}}{\sqrt{x}}\right)^{\frac{5}{2}} x \left(3 a^{5} x^{\frac{5}{4}}+20 x \,a^{4} b +60 a^{3} b^{2} x^{\frac{3}{4}}+15 b^{4} a \ln \left(x \right) x^{\frac{1}{4}}+120 a^{2} b^{3} \sqrt{x}-12 b^{5}\right)}{3 \left(a \,x^{\frac{1}{4}}+b \right)^{5}}$	91
default	$\frac{\left(\frac{a^{2} x^{\frac{3}{4}}+b^{2} x^{\frac{1}{4}}+2 a b \sqrt{x}}{x^{\frac{3}{4}}}\right)^{\frac{5}{2}} x \left(3 a^{5} x^{\frac{5}{4}}+20 x \,a^{4} b +60 a^{3} b^{2} x^{\frac{3}{4}}+15 b^{4} a \ln \left(x \right) x^{\frac{1}{4}}+120 a^{2} b^{3} \sqrt{x}-12 b^{5}\right)}{3 \left(a \,x^{\frac{1}{4}}+b \right)^{5}}$	95

input
```
int((a^2+2*a*b/x^(1/4)+b^2/x^(1/2))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*((a^2*x^(1/2)+b^2+2*a*b*x^(1/4))/x^(1/2))^(5/2)*x*(3*a^5*x^(5/4)+20*x*
a^4*b+60*a^3*b^2*x^(3/4)+15*b^4*a*ln(x)*x^(1/4)+120*a^2*b^3*x^(1/2)-12*b^5
)/(a*x^(1/4)+b)^5
```

3.489.5 Fricas [F(-1)]

Timed out.

$$\int \left(a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}\right)^{5/2}\,dx=\text{Timed out}$$

input
```
integrate((a^2+2*a*b/x^(1/4)+b^2/x^(1/2))^(5/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.489.6 Sympy [F(-1)]

Timed out.

$$\int \left(a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}\right)^{5/2}\,dx=\text{Timed out}$$

input
```
integrate((a**2+2*a*b/x**(1/4)+b**2/x**(1/2))**(5/2),x)
```

output
```
Timed out
```

3.489.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.20

$$\int \left(a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}\right)^{5/2}\,dx=5\,ab^4\log\left(x\right)+\frac{3\,a^5x^{\frac{5}{4}}+20\,a^4bx+60\,a^3b^2x^{\frac{3}{4}}+120\,a^2b^3\sqrt{x}-12\,b^5}{3\,x^{\frac{1}{4}}}$$

input
```
integrate((a^2+2*a*b/x^(1/4)+b^2/x^(1/2))^(5/2),x, algorithm="maxima")
```

output
```
5*a*b^4*log(x) + 1/3*(3*a^5*x^(5/4) + 20*a^4*b*x + 60*a^3*b^2*x^(3/4) + 12
0*a^2*b^3*sqrt(x) - 12*b^5)/x^(1/4)
```

3.489.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.44

$$\int \left(a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}\right)^{5/2} dx = a^5x\operatorname{sgn}\left(ax+bx^{\frac{3}{4}}\right)\operatorname{sgn}(x) + 5\,ab^4\log\left(|x|\right)\operatorname{sgn}\left(ax+bx^{\frac{3}{4}}\right)\operatorname{sgn}(x) + \frac{20}{3}\,a^4bx^{\frac{3}{4}}\operatorname{sgn}\left(ax+bx^{\frac{3}{4}}\right)\operatorname{sgn}(x) + 20\,a^3b^2\sqrt{x}\operatorname{sgn}\left(ax+bx^{\frac{3}{4}}\right)\operatorname{sgn}(x) + 40\,a^2b^3x^{\frac{1}{4}}\operatorname{sgn}\left(ax+bx^{\frac{3}{4}}\right)\operatorname{sgn}(x) - \frac{4\,b^5\operatorname{sgn}\left(ax+bx^{\frac{3}{4}}\right)\operatorname{sgn}(x)}{x^{\frac{1}{4}}}$$

input
```
integrate((a^2+2*a*b/x^(1/4)+b^2/x^(1/2))^(5/2),x, algorithm="giac")
```

output
```
a^5*x*sgn(a*x + b*x^(3/4))*sgn(x) + 5*a*b^4*log(abs(x))*sgn(a*x + b*x^(3/4
))*sgn(x) + 20/3*a^4*b*x^(3/4)*sgn(a*x + b*x^(3/4))*sgn(x) + 20*a^3*b^2*sq
rt(x)*sgn(a*x + b*x^(3/4))*sgn(x) + 40*a^2*b^3*x^(1/4)*sgn(a*x + b*x^(3/4)
)*sgn(x) - 4*b^5*sgn(a*x + b*x^(3/4))*sgn(x)/x^(1/4)
```

3.489.9 Mupad [F(-1)]

Timed out.

$$\int \left(a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}\right)^{5/2} dx = \int \left(a^2+\frac{b^2}{\sqrt{x}}+\frac{2\,a\,b}{x^{1/4}}\right)^{5/2} dx$$

input
```
int((a^2 + b^2/x^(1/2) + (2*a*b)/x^(1/4))^(5/2),x)
```

output
```
int((a^2 + b^2/x^(1/2) + (2*a*b)/x^(1/4))^(5/2), x)
```

3.490 $\int \left(a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}\right)^{5/2} dx$

3.490.1 Optimal result

Integrand size = 26, antiderivative size = 291

$$\begin{aligned}
\int \left(a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}\right)^{5/2} dx &= \frac{25ab^4\sqrt{a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}}\sqrt[5]{x}}{a+\frac{b}{\sqrt[5]{x}}} \\
&+ \frac{25a^2b^3\sqrt{a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}}x^{2/5}}{a+\frac{b}{\sqrt[5]{x}}} + \frac{50a^3b^2\sqrt{a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}}x^{3/5}}{3\left(a+\frac{b}{\sqrt[5]{x}}\right)} \\
&+ \frac{25a^4b\sqrt{a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}}x^{4/5}}{4\left(a+\frac{b}{\sqrt[5]{x}}\right)} + \frac{a^5\sqrt{a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}}x}{a+\frac{b}{\sqrt[5]{x}}} \\
&+ \frac{5b^5\sqrt{a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}}\log\left(\sqrt[5]{x}\right)}{a+\frac{b}{\sqrt[5]{x}}}
\end{aligned}$$

output
```
25*a*b^4*x^(1/5)*(a^2+b^2/x^(2/5)+2*a*b/x^(1/5))^(1/2)/(a+b/x^(1/5))+25*a^
2*b^3*x^(2/5)*(a^2+b^2/x^(2/5)+2*a*b/x^(1/5))^(1/2)/(a+b/x^(1/5))+50/3*a^3
*b^2*x^(3/5)*(a^2+b^2/x^(2/5)+2*a*b/x^(1/5))^(1/2)/(a+b/x^(1/5))+25/4*a^4*
b*x^(4/5)*(a^2+b^2/x^(2/5)+2*a*b/x^(1/5))^(1/2)/(a+b/x^(1/5))+a^5*x*(a^2+b
^2/x^(2/5)+2*a*b/x^(1/5))^(1/2)/(a+b/x^(1/5))+b^5*ln(x)*(a^2+b^2/x^(2/5)+2
*a*b/x^(1/5))^(1/2)/(a+b/x^(1/5))
```

3.490.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.35

$$\int \left(a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}\right)^{5/2} dx = \frac{\left(b+a\sqrt[5]{x}\right)\left(300ab^4\sqrt[5]{x}+300a^2b^3x^{2/5}+200a^3b^2x^{3/5}+75a^4bx^{4/5}+12a^5x+12b^5\log(x)\right)}{12\sqrt{\frac{\left(b+a\sqrt[5]{x}\right)^2}{x^{2/5}}}\sqrt[5]{x}}$$

input
```
Integrate[(a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5))^(5/2),x]
```

output
```
((b + a*x^(1/5))*(300*a*b^4*x^(1/5) + 300*a^2*b^3*x^(2/5) + 200*a^3*b^2*x^
(3/5) + 75*a^4*b*x^(4/5) + 12*a^5*x + 12*b^5*Log[x]))/(12*Sqrt[(b + a*x^(1
/5))^2/x^(2/5)]*x^(1/5))
```

3.490.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.41, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1384, 774, 27, 795, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{2/5}}\right)^{5/2} dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{2/5}}}\int\left(\frac{b^2}{\sqrt[5]{x}}+ab\right)^5dx}{ab^5+\frac{b^6}{\sqrt[5]{x}}}$$

$$\Big\downarrow \; 774$$

$$\frac{5\sqrt{a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{2/5}}}\int b^5\left(a+\frac{b}{\sqrt[5]{x}}\right)^5x^{4/5}d\sqrt[5]{x}}{ab^5+\frac{b^6}{\sqrt[5]{x}}}$$

$$\Big\downarrow \; 27$$

$$\frac{5b^5\sqrt{a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{2/5}}}\int\left(a+\frac{b}{\sqrt[5]{x}}\right)^5x^{4/5}d\sqrt[5]{x}}{ab^5+\frac{b^6}{\sqrt[5]{x}}}$$

$$\Big\downarrow \; 795$$

$$\frac{5b^5\sqrt{a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{2/5}}}\int\frac{\left(\sqrt[5]{x}a+b\right)^5}{\sqrt[5]{x}}d\sqrt[5]{x}}{ab^5+\frac{b^6}{\sqrt[5]{x}}}$$

$$\Big\downarrow \; 49$$

$$\frac{5b^5\sqrt{a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{2/5}}}\int\left(x^{4/5}a^5+5bx^{3/5}a^4+10b^2x^{2/5}a^3+10b^3\sqrt[5]{x}a^2+5b^4a+\frac{b^5}{\sqrt[5]{x}}\right)d\sqrt[5]{x}}{ab^5+\frac{b^6}{\sqrt[5]{x}}}$$

$$\Big\downarrow \; 2009$$

$$\frac{5b^5\sqrt{a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{2/5}}}\left(\frac{a^5x}{5}+\frac{5}{4}a^4bx^{4/5}+\frac{10}{3}a^3b^2x^{3/5}+5a^2b^3x^{2/5}+5ab^4\sqrt[5]{x}+b^5\log\left(\sqrt[5]{x}\right)\right)}{ab^5+\frac{b^6}{\sqrt[5]{x}}}$$

input
```
Int[(a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5))^(5/2),x]
```

output
```
(5*b^5*Sqrt[a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5)]*(5*a*b^4*x^(1/5) + 5*a^2*
b^3*x^(2/5) + (10*a^3*b^2*x^(3/5))/3 + (5*a^4*b*x^(4/5))/4 + (a^5*x)/5 + b
^5*Log[x^(1/5)]))/(a*b^5 + b^6/x^(1/5))
```

3.490. $\int\left(a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}\right)^{5/2}dx$

3.490.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.490.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.31

method	result	size
derivativedivides	$\frac{\left(\frac{a^{2} x^{\frac{2}{5}}+2 a b \,x^{\frac{1}{5}}+b^{2}}{x^{\frac{2}{5}}}\right)^{\frac{5}{2}} x \left(12 a^{5} x +75 b \,a^{4} x^{\frac{4}{5}}+200 a^{3} b^{2} x^{\frac{3}{5}}+300 a^{2} b^{3} x^{\frac{2}{5}}+12 b^{5} \ln \left(x \right)+300 b^{4} a \,x^{\frac{1}{5}}\right)}{12 \left(a \,x^{\frac{1}{5}}+b \right)^{5}}$	91
default	$\frac{\left(\frac{a^{2} x^{\frac{2}{5}}+2 a b \,x^{\frac{1}{5}}+b^{2}}{x^{\frac{2}{5}}}\right)^{\frac{5}{2}} x \left(12 a^{5} x +75 b \,a^{4} x^{\frac{4}{5}}+200 a^{3} b^{2} x^{\frac{3}{5}}+300 a^{2} b^{3} x^{\frac{2}{5}}+12 b^{5} \ln \left(x \right)+300 b^{4} a \,x^{\frac{1}{5}}\right)}{12 \left(a \,x^{\frac{1}{5}}+b \right)^{5}}$	91

3.490. $\int \left(a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}\right)^{5/2} dx$

input
```
int((a^2+b^2/x^(2/5)+2*a*b/x^(1/5))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/12*((a^2*x^(2/5)+2*a*b*x^(1/5)+b^2)/x^(2/5))^(5/2)*x*(12*a^5*x+75*b*a^4*
x^(4/5)+200*a^3*b^2*x^(3/5)+300*a^2*b^3*x^(2/5)+12*b^5*ln(x)+300*b^4*a*x^(
1/5))/(a*x^(1/5)+b)^5
```

3.490.5 Fricas [F(-1)]

Timed out.

$$\int \left(a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}\right)^{5/2} dx = \text{Timed out}$$

input
```
integrate((a^2+b^2/x^(2/5)+2*a*b/x^(1/5))^(5/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.490.6 Sympy [F]

$$\int \left(a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}\right)^{5/2} dx = \int \left(a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{\frac{2}{5}}}\right)^{\frac{5}{2}} dx$$

input
```
integrate((a**2+b**2/x**(2/5)+2*a*b/x**(1/5))**(5/2),x)
```

output
```
Integral((a**2 + 2*a*b/x**(1/5) + b**2/x**(2/5))**(5/2), x)
```

3.490.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.18

$$\int \left(a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}\right)^{5/2} dx = a^5x+b^5\log(x)+\frac{25}{4}a^4bx^{\frac{4}{5}}+\frac{50}{3}a^3b^2x^{\frac{3}{5}}+25a^2b^3x^{\frac{2}{5}}+25ab^4x^{\frac{1}{5}}$$

input
```
integrate((a^2+b^2/x^(2/5)+2*a*b/x^(1/5))^(5/2),x, algorithm="maxima")
```

output
```
a^5*x + b^5*log(x) + 25/4*a^4*b*x^(4/5) + 50/3*a^3*b^2*x^(3/5) + 25*a^2*b^
3*x^(2/5) + 25*a*b^4*x^(1/5)
```

3.490.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.43

$$\int \left(a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}\right)^{5/2} dx = a^5x\text{sgn}\left(ax+bx^{\frac{4}{5}}\right)\text{sgn}(x)+b^5\log\left(|x|\right)\text{sgn}\left(ax+bx^{\frac{4}{5}}\right)\text{sgn}(x)+\frac{25}{4}a^4bx^{\frac{4}{5}}\text{sgn}\left(ax+bx^{\frac{4}{5}}\right)\text{sgn}(x)+\frac{50}{3}a^3b^2x^{\frac{3}{5}}\text{sgn}\left(ax+bx^{\frac{4}{5}}\right)\text{sgn}(x)+25\,a^2b^3x^{\frac{2}{5}}\text{sgn}\left(ax+bx^{\frac{4}{5}}\right)\text{sgn}(x)+25\,ab^4x^{\frac{1}{5}}\text{sgn}\left(ax+bx^{\frac{4}{5}}\right)\text{sgn}(x)$$

input
```
integrate((a^2+b^2/x^(2/5)+2*a*b/x^(1/5))^(5/2),x, algorithm="giac")
```

output
```
a^5*x*sgn(a*x + b*x^(4/5))*sgn(x) + b^5*log(abs(x))*sgn(a*x + b*x^(4/5))*s
gn(x) + 25/4*a^4*b*x^(4/5)*sgn(a*x + b*x^(4/5))*sgn(x) + 50/3*a^3*b^2*x^(3
/5)*sgn(a*x + b*x^(4/5))*sgn(x) + 25*a^2*b^3*x^(2/5)*sgn(a*x + b*x^(4/5))*
sgn(x) + 25*a*b^4*x^(1/5)*sgn(a*x + b*x^(4/5))*sgn(x)
```

3.490.9 Mupad [F(-1)]

Timed out.

$$\int \left(a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}\right)^{5/2} dx = \int \left(a^2+\frac{b^2}{x^{2/5}}+\frac{2\,a\,b}{x^{1/5}}\right)^{5/2} dx$$

input
```
int((a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5))^(5/2),x)
```

output
```
int((a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5))^(5/2), x)
```

3.491 $\int \frac{1}{\left(a^2+2ab\sqrt[5]{x}+b^2x^{2/5}\right)^{5/2}}\,dx$

3.491.1 Optimal result

Integrand size = 26, antiderivative size = 222

$$\int \frac{1}{\left(a^2+2ab\sqrt[5]{x}+b^2x^{2/5}\right)^{5/2}}\,dx=\frac{20a}{b^5\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}-\frac{5a^4}{4b^5\left(a+b\sqrt[5]{x}\right)^3\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}+\frac{20a^3}{3b^5\left(a+b\sqrt[5]{x}\right)^2\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}-\frac{15a^2}{b^5\left(a+b\sqrt[5]{x}\right)\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}+\frac{5\left(a+b\sqrt[5]{x}\right)\log\left(a+b\sqrt[5]{x}\right)}{b^5\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}$$

output
```
20*a/b^5/(a^2+2*a*b*x^(1/5)+b^2*x^(2/5))^(1/2)-5/4*a^4/b^5/(a+b*x^(1/5))^3
/(a^2+2*a*b*x^(1/5)+b^2*x^(2/5))^(1/2)+20/3*a^3/b^5/(a+b*x^(1/5))^2/(a^2+2
*a*b*x^(1/5)+b^2*x^(2/5))^(1/2)-15*a^2/b^5/(a+b*x^(1/5))/(a^2+2*a*b*x^(1/5
)+b^2*x^(2/5))^(1/2)+5*(a+b*x^(1/5))*ln(a+b*x^(1/5))/b^5/(a^2+2*a*b*x^(1/5
)+b^2*x^(2/5))^(1/2)
```

3.491.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.44

$$\int \frac{1}{\left(a^2+2ab\sqrt[5]{x}+b^2x^{2/5}\right)^{5/2}}\,dx=\frac{5a\left(25a^3+88a^2b\sqrt[5]{x}+108ab^2x^{2/5}+48b^3x^{3/5}\right)+60\left(a+b\sqrt[5]{x}\right)^4\log\left(a+b\right.}{12b^5\left(a+b\sqrt[5]{x}\right)^3\sqrt{\left(a+b\sqrt[5]{x}\right)^2}}$$

input
```
Integrate[(a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5))^(-5/2),x]
```

output
```
(5*a*(25*a^3 + 88*a^2*b*x^(1/5) + 108*a*b^2*x^(2/5) + 48*b^3*x^(3/5)) + 60
*(a + b*x^(1/5))^4*Log[a + b*x^(1/5)])/(12*b^5*(a + b*x^(1/5))^3*Sqrt[(a +
 b*x^(1/5))^2])
```

3.491.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.63, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1384, 774, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a^2+2ab\sqrt[5]{x}+b^2x^{2/5}\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\left(ab^5+b^6\sqrt[5]{x}\right)\int \frac{1}{\left(\sqrt[5]{x}b^2+ab\right)^5}dx}{\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}$$

$$\Big\downarrow \text{ 774}$$

$$\frac{5\left(ab^5+b^6\sqrt[5]{x}\right)\int \frac{x^{4/5}}{b^5\left(a+b\sqrt[5]{x}\right)^5}d\sqrt[5]{x}}{\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{5\left(ab^5+b^6\sqrt[5]{x}\right)\int \frac{x^{4/5}}{\left(a+b\sqrt[5]{x}\right)^5}d\sqrt[5]{x}}{b^5\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}$$

$$\Big\downarrow 49$$

$$\frac{5\left(ab^5+b^6\sqrt[5]{x}\right)\int\left(\frac{a^4}{b^4\left(a+b\sqrt[5]{x}\right)^5}-\frac{4a^3}{b^4\left(a+b\sqrt[5]{x}\right)^4}+\frac{6a^2}{b^4\left(a+b\sqrt[5]{x}\right)^3}-\frac{4a}{b^4\left(a+b\sqrt[5]{x}\right)^2}+\frac{1}{b^4\left(a+b\sqrt[5]{x}\right)}\right)d\sqrt[5]{x}}{b^5\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}$$

$$\Big\downarrow 2009$$

$$\frac{5\left(ab^5+b^6\sqrt[5]{x}\right)\left(-\frac{a^4}{4b^5\left(a+b\sqrt[5]{x}\right)^4}+\frac{4a^3}{3b^5\left(a+b\sqrt[5]{x}\right)^3}-\frac{3a^2}{b^5\left(a+b\sqrt[5]{x}\right)^2}+\frac{4a}{b^5\left(a+b\sqrt[5]{x}\right)}+\frac{\log\left(a+b\sqrt[5]{x}\right)}{b^5}\right)}{b^5\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}$$

input
```
Int[(a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5))^(-5/2),x]
```

output
```
(5*(a*b^5 + b^6*x^(1/5))*(-1/4*a^4/(b^5*(a + b*x^(1/5))^4) + (4*a^3)/(3*b^
5*(a + b*x^(1/5))^3) - (3*a^2)/(b^5*(a + b*x^(1/5))^2) + (4*a)/(b^5*(a + b
*x^(1/5))) + Log[a + b*x^(1/5)]/b^5))/(b^5*Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*
x^(2/5)])
```

3.491.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

3.491. $\int \frac{1}{\left(a^2+2ab\sqrt[5]{x}+b^2x^{2/5}\right)^{5/2}}\,dx$

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.491.4 Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.64

method	result
derivativedivides	$\frac{5\left(12\ln\left(a+b\,x^{\frac{1}{5}}\right)b^{4}x^{\frac{4}{5}}+48\ln\left(a+b\,x^{\frac{1}{5}}\right)a\,b^{3}x^{\frac{3}{5}}+72\ln\left(a+b\,x^{\frac{1}{5}}\right)a^{2}b^{2}x^{\frac{2}{5}}+48a\,b^{3}x^{\frac{3}{5}}+48\ln\left(a+b\,x^{\frac{1}{5}}\right)a^{3}b\,x^{\frac{1}{5}}+108a^{2}b^{2}x^{\frac{2}{5}}\right.}{12b^{5}\left(\left(a+b\,x^{\frac{1}{5}}\right)^{2}\right)^{\frac{5}{2}}}$
default	Expression too large to display

input
```
int(1/(a^2+2*a*b*x^(1/5)+b^2*x^(2/5))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
5/12*(12*ln(a+b*x^(1/5))*b^4*x^(4/5)+48*ln(a+b*x^(1/5))*a*b^3*x^(3/5)+72*l
n(a+b*x^(1/5))*a^2*b^2*x^(2/5)+48*a*b^3*x^(3/5)+48*ln(a+b*x^(1/5))*a^3*b*x
^(1/5)+108*a^2*b^2*x^(2/5)+12*ln(a+b*x^(1/5))*a^4+88*a^3*b*x^(1/5)+25*a^4)
*(a+b*x^(1/5))/b^5/((a+b*x^(1/5))^2)^(5/2)
```

3.491.5 Fricas [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.36

$$\int \frac{1}{\left(a^2+2ab\sqrt[5]{x}+b^2x^{2/5}\right)^{5/2}}\,dx = \frac{5\left(300\,a^5b^{15}x^3+100\,a^{15}b^5x+25\,a^{20}+12\,(b^{20}x^4+4\,a^5b^{15}x^3+6\,a^{10}b^{10}x^2\right.}{}$$

input
```
integrate(1/(a^2+2*a*b*x^(1/5)+b^2*x^(2/5))^(5/2),x, algorithm="fricas")
```

output
```
5/12*(300*a^5*b^15*x^3 + 100*a^15*b^5*x + 25*a^20 + 12*(b^20*x^4 + 4*a^5*b
^15*x^3 + 6*a^10*b^10*x^2 + 4*a^15*b^5*x + a^20)*log(b*x^(1/5) + a) + (48*
a*b^19*x^3 - 226*a^6*b^14*x^2 + 104*a^11*b^9*x + 3*a^16*b^4)*x^(4/5) - (84
*a^2*b^18*x^3 - 228*a^7*b^13*x^2 + 67*a^12*b^8*x + 4*a^17*b^3)*x^(3/5) + (
136*a^3*b^17*x^3 - 197*a^8*b^12*x^2 + 48*a^13*b^7*x + 6*a^18*b^2)*x^(2/5)
- (207*a^4*b^16*x^3 - 124*a^9*b^11*x^2 + 56*a^14*b^6*x + 12*a^19*b)*x^(1/5
))/(b^25*x^4 + 4*a^5*b^20*x^3 + 6*a^10*b^15*x^2 + 4*a^15*b^10*x + a^20*b^5
)
```

3.491.6 Sympy [F]

$$\int \frac{1}{\left(a^2+2ab\sqrt[5]{x}+b^2x^{2/5}\right)^{5/2}}\,dx=\int \frac{1}{\left(a^2+2ab\sqrt[5]{x}+b^2x^{\frac{2}{5}}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a**2+2*a*b*x**(1/5)+b**2*x**(2/5))**(5/2),x)
```

output
```
Integral((a**2 + 2*a*b*x**(1/5) + b**2*x**(2/5))**(-5/2), x)
```

3.491.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.45

$$\int \frac{1}{\left(a^2+2ab\sqrt[5]{x}+b^2x^{2/5}\right)^{5/2}}\,dx=\frac{5\left(48\,ab^3x^{\frac{3}{5}}+108\,a^2b^2x^{\frac{2}{5}}+88\,a^3bx^{\frac{1}{5}}+25\,a^4\right)}{12\left(b^9x^{\frac{4}{5}}+4\,ab^8x^{\frac{3}{5}}+6\,a^2b^7x^{\frac{2}{5}}+4\,a^3b^6x^{\frac{1}{5}}+a^4b^5\right)}+\frac{5\,\log\left(bx^{\frac{1}{5}}+a\right)}{b^5}$$

input
```
integrate(1/(a^2+2*a*b*x^(1/5)+b^2*x^(2/5))^(5/2),x, algorithm="maxima")
```

output
```
5/12*(48*a*b^3*x^(3/5) + 108*a^2*b^2*x^(2/5) + 88*a^3*b*x^(1/5) + 25*a^4)/
(b^9*x^(4/5) + 4*a*b^8*x^(3/5) + 6*a^2*b^7*x^(2/5) + 4*a^3*b^6*x^(1/5) + a
^4*b^5) + 5*log(b*x^(1/5) + a)/b^5
```

3.491.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.38

$$\int \frac{1}{\left(a^2+2ab\sqrt[5]{x}+b^2x^{2/5}\right)^{5/2}}\,dx=\frac{5\,\log\left(\left|bx^{\frac{1}{5}}+a\right|\right)}{b^5\mathrm{sgn}\left(bx^{\frac{1}{5}}+a\right)}+\frac{5\left(48\,ab^2x^{\frac{3}{5}}+108\,a^2bx^{\frac{2}{5}}+88\,a^3x^{\frac{1}{5}}+\frac{25\,a^4}{b}\right)}{12\left(bx^{\frac{1}{5}}+a\right)^4b^4\mathrm{sgn}\left(bx^{\frac{1}{5}}+a\right)}$$

input
```
integrate(1/(a^2+2*a*b*x^(1/5)+b^2*x^(2/5))^(5/2),x, algorithm="giac")
```

output
```
5*log(abs(b*x^(1/5) + a))/(b^5*sgn(b*x^(1/5) + a)) + 5/12*(48*a*b^2*x^(3/5
) + 108*a^2*b*x^(2/5) + 88*a^3*x^(1/5) + 25*a^4/b)/((b*x^(1/5) + a)^4*b^4*
sgn(b*x^(1/5) + a))
```

3.491.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a^2+2ab\sqrt[5]{x}+b^2x^{2/5}\right)^{5/2}}\,dx=\int \frac{1}{{\left(a^2+b^2\,x^{2/5}+2\,a\,b\,x^{1/5}\right)}^{5/2}}\,dx$$

input
```
int(1/(a^2 + b^2*x^(2/5) + 2*a*b*x^(1/5))^(5/2),x)
```

output
```
int(1/(a^2 + b^2*x^(2/5) + 2*a*b*x^(1/5))^(5/2), x)
```

3.492 $\int \left(a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}\right)^{7/2} dx$

3.492.1 Optimal result

Integrand size = 26, antiderivative size = 391

$$\begin{aligned}\int \left(a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}\right)^{7/2} dx &= -\frac{6b^7\sqrt{a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}}}{\left(a+\frac{b}{\sqrt[6]{x}}\right)\sqrt[6]{x}} \\ &+ \frac{126a^2b^5\sqrt{a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}}\sqrt[6]{x}}{a+\frac{b}{\sqrt[6]{x}}}+\frac{105a^3b^4\sqrt{a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}}\sqrt[3]{x}}{a+\frac{b}{\sqrt[6]{x}}} \\ &+ \frac{70a^4b^3\sqrt{a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}}\sqrt{x}}{a+\frac{b}{\sqrt[6]{x}}}+\frac{63a^5b^2\sqrt{a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}}x^{2/3}}{2\left(a+\frac{b}{\sqrt[6]{x}}\right)} \\ &+ \frac{42a^6b\sqrt{a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}}x^{5/6}}{5\left(a+\frac{b}{\sqrt[6]{x}}\right)}+\frac{a^7\sqrt{a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}}x}{a+\frac{b}{\sqrt[6]{x}}} \\ &+ \frac{42ab^6\sqrt{a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}}\log\left(\sqrt[6]{x}\right)}{a+\frac{b}{\sqrt[6]{x}}}\end{aligned}$$

output
```
-6*b^7*(a^2+b^2/x^(1/3)+2*a*b/x^(1/6))^(1/2)/(a+b/x^(1/6))/x^(1/6)+126*a^2
*b^5*x^(1/6)*(a^2+b^2/x^(1/3)+2*a*b/x^(1/6))^(1/2)/(a+b/x^(1/6))+105*a^3*b
^4*x^(1/3)*(a^2+b^2/x^(1/3)+2*a*b/x^(1/6))^(1/2)/(a+b/x^(1/6))+63/2*a^5*b^
2*x^(2/3)*(a^2+b^2/x^(1/3)+2*a*b/x^(1/6))^(1/2)/(a+b/x^(1/6))+42/5*a^6*b*x
^(5/6)*(a^2+b^2/x^(1/3)+2*a*b/x^(1/6))^(1/2)/(a+b/x^(1/6))+a^7*x*(a^2+b^2/
x^(1/3)+2*a*b/x^(1/6))^(1/2)/(a+b/x^(1/6))+7*a*b^6*ln(x)*(a^2+b^2/x^(1/3)+
2*a*b/x^(1/6))^(1/2)/(a+b/x^(1/6))+70*a^4*b^3*(a^2+b^2/x^(1/3)+2*a*b/x^(1/
6))^(1/2)*x^(1/2)/(a+b/x^(1/6))
```

3.492.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.32

$$\int \left(a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}\right)^{7/2} dx = \frac{\sqrt{\frac{\left(b+a\sqrt[6]{x}\right)^2}{\sqrt[3]{x}}}\left(-60b^7+1260a^2b^5\sqrt[3]{x}+1050a^3b^4\sqrt{x}+700a^4b^3x^{2/3}+315a^5b^2x^{5/6}+84a^6bx+\right.}{10\left(b+a\sqrt[6]{x}\right)}$$

input
```
Integrate[(a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6))^(7/2),x]
```

output
```
(Sqrt[(b + a*x^(1/6))^2/x^(1/3)]*(-60*b^7 + 1260*a^2*b^5*x^(1/3) + 1050*a^
3*b^4*Sqrt[x] + 700*a^4*b^3*x^(2/3) + 315*a^5*b^2*x^(5/6) + 84*a^6*b*x + 1
0*a^7*x^(7/6) + 70*a*b^6*x^(1/6)*Log[x]))/(10*(b + a*x^(1/6)))
```

3.492.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.38, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1384, 774, 27, 795, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}\right)^{7/2} dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}}\int\left(\frac{b^2}{\sqrt[6]{x}}+ab\right)^7dx}{ab^7+\frac{b^8}{\sqrt[6]{x}}}$$

$$\Big\downarrow \text{774}$$

$$\frac{6\sqrt{a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}}\int b^7\left(a+\frac{b}{\sqrt[6]{x}}\right)^7x^{5/6}d\sqrt[6]{x}}{ab^7+\frac{b^8}{\sqrt[6]{x}}}$$

$$\Big\downarrow \text{27}$$

$$\frac{6b^7\sqrt{a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}}\int\left(a+\frac{b}{\sqrt[6]{x}}\right)^7x^{5/6}d\sqrt[6]{x}}{ab^7+\frac{b^8}{\sqrt[6]{x}}}$$

$$\Big\downarrow \text{795}$$

$$\frac{6b^7\sqrt{a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}}\int\frac{\left(\sqrt[6]{x}a+b\right)^7}{\sqrt[3]{x}}d\sqrt[6]{x}}{ab^7+\frac{b^8}{\sqrt[6]{x}}}$$

$$\Big\downarrow \text{49}$$

$$\frac{6b^7\sqrt{a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}}\int\left(x^{5/6}a^7+7bx^{2/3}a^6+21b^2\sqrt{x}a^5+35b^3\sqrt[3]{x}a^4+35b^4\sqrt[6]{x}a^3+21b^5a^2+\frac{7b^6a}{\sqrt[6]{x}}+\frac{b^7}{\sqrt[3]{x}}\right)d\sqrt[6]{x}}{ab^7+\frac{b^8}{\sqrt[6]{x}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{6b^7\sqrt{a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}}\left(\frac{a^7x}{6}+\frac{7}{5}a^6bx^{5/6}+\frac{21}{4}a^5b^2x^{2/3}+\frac{35}{3}a^4b^3\sqrt{x}+\frac{35}{2}a^3b^4\sqrt[3]{x}+21a^2b^5\sqrt[6]{x}+7ab^6\log\left(\sqrt[6]{x}\right)-\frac{b^7}{\sqrt[6]{x}}\right)}{ab^7+\frac{b^8}{\sqrt[6]{x}}}$$

input
```
Int[(a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6))^(7/2),x]
```

output
```
(6*b^7*Sqrt[a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6)]*(-(b^7/x^(1/6)) + 21*a^2*
b^5*x^(1/6) + (35*a^3*b^4*x^(1/3))/2 + (35*a^4*b^3*Sqrt[x])/3 + (21*a^5*b^
2*x^(2/3))/4 + (7*a^6*b*x^(5/6))/5 + (a^7*x)/6 + 7*a*b^6*Log[x^(1/6)]))/(a
*b^7 + b^8/x^(1/6))
```

3.492. $\int\left(a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}\right)^{7/2}dx$

3.492.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.492.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.30

$$\frac{\left(\frac{a^{2}\sqrt{x}+2ab\,x^{\frac{1}{3}}+x^{\frac{1}{6}}b^{2}}{\sqrt{x}}\right)^{\frac{7}{2}} x\left(10a^{7}x^{\frac{7}{6}}+84a^{6}bx+315b^{2}a^{5}x^{\frac{5}{6}}+700a^{4}b^{3}x^{\frac{2}{3}}+1050b^{4}a^{3}\sqrt{x}+70a\,b^{6}\ln\left(x\right)x^{\frac{1}{6}}+126\right.}{10\left(a\,x^{\frac{1}{6}}+b\right)^{7}}$$

input
```
int((a^2+b^2/x^(1/3)+2*a*b/x^(1/6))^(7/2),x)
```

3.492. $\int\left(a^{2}+\frac{b^{2}}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}\right)^{7/2}dx$

output
```
1/10*((a^2*x^(1/2)+2*a*b*x^(1/3)+x^(1/6)*b^2)/x^(1/2))^(7/2)*x*(10*a^7*x^(
7/6)+84*a^6*b*x+315*b^2*a^5*x^(5/6)+700*a^4*b^3*x^(2/3)+1050*b^4*a^3*x^(1/
2)+70*a*b^6*ln(x)*x^(1/6)+1260*a^2*b^5*x^(1/3)-60*b^7)/(a*x^(1/6)+b)^7
```

3.492.5 Fricas [F(-1)]

Timed out.

$$\int \left(a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}\right)^{7/2} dx = \text{Timed out}$$

input
```
integrate((a^2+b^2/x^(1/3)+2*a*b/x^(1/6))^(7/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.492.6 Sympy [F(-1)]

Timed out.

$$\int \left(a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}\right)^{7/2} dx = \text{Timed out}$$

input
```
integrate((a**2+b**2/x**(1/3)+2*a*b/x**(1/6))**(7/2),x)
```

output
```
Timed out
```

3.492.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.20

$$\int \left(a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}\right)^{7/2} dx = 7\,ab^6\log\left(x\right) + \frac{10\,a^7x^{\frac{7}{6}}+84\,a^6bx+315\,a^5b^2x^{\frac{5}{6}}+700\,a^4b^3x^{\frac{2}{3}}+1050\,a^3b^4\sqrt{x}+1260\,a^2b^5x^{\frac{1}{3}}-60\,b^7}{10\,x^{\frac{1}{6}}}$$

input
```
integrate((a^2+b^2/x^(1/3)+2*a*b/x^(1/6))^(7/2),x, algorithm="maxima")
```

output
```
7*a*b^6*log(x) + 1/10*(10*a^7*x^(7/6) + 84*a^6*b*x + 315*a^5*b^2*x^(5/6) +
 700*a^4*b^3*x^(2/3) + 1050*a^3*b^4*sqrt(x) + 1260*a^2*b^5*x^(1/3) - 60*b^
7)/x^(1/6)
```

3.492.8 Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.44

$$\int \left(a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}\right)^{7/2} dx = a^7x\text{sgn}\left(ax+bx^{\frac{5}{6}}\right)\text{sgn}(x) + 7\,ab^6\log\left(|x|\right)\text{sgn}\left(ax+bx^{\frac{5}{6}}\right)\text{sgn}(x) + \frac{42}{5}\,a^6bx^{\frac{5}{6}}\text{sgn}\left(ax+bx^{\frac{5}{6}}\right)\text{sgn}(x) + \frac{63}{2}\,a^5b^2x^{\frac{2}{3}}\text{sgn}\left(ax+bx^{\frac{5}{6}}\right)\text{sgn}(x) + 70\,a^4b^3\sqrt{x}\text{sgn}\left(ax+bx^{\frac{5}{6}}\right)\text{sgn}(x) + 105\,a^3b^4x^{\frac{1}{3}}\text{sgn}\left(ax+bx^{\frac{5}{6}}\right)\text{sgn}(x) + 126\,a^2b^5x^{\frac{1}{6}}\text{sgn}\left(ax+bx^{\frac{5}{6}}\right)\text{sgn}(x) - \frac{6\,b^7\text{sgn}\left(ax+bx^{\frac{5}{6}}\right)\text{sgn}(x)}{x^{\frac{1}{6}}}$$

input
```
integrate((a^2+b^2/x^(1/3)+2*a*b/x^(1/6))^(7/2),x, algorithm="giac")
```

output
```
a^7*x*sgn(a*x + b*x^(5/6))*sgn(x) + 7*a*b^6*log(abs(x))*sgn(a*x + b*x^(5/6
))*sgn(x) + 42/5*a^6*b*x^(5/6)*sgn(a*x + b*x^(5/6))*sgn(x) + 63/2*a^5*b^2*
x^(2/3)*sgn(a*x + b*x^(5/6))*sgn(x) + 70*a^4*b^3*sqrt(x)*sgn(a*x + b*x^(5/
6))*sgn(x) + 105*a^3*b^4*x^(1/3)*sgn(a*x + b*x^(5/6))*sgn(x) + 126*a^2*b^5
*x^(1/6)*sgn(a*x + b*x^(5/6))*sgn(x) - 6*b^7*sgn(a*x + b*x^(5/6))*sgn(x)/x
^(1/6)
```

3.492.9 Mupad [F(-1)]

Timed out.

$$\int \left(a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}\right)^{7/2}\,dx=\int \left(a^2+\frac{b^2}{x^{1/3}}+\frac{2\,a\,b}{x^{1/6}}\right)^{7/2}\,dx$$

input
```
int((a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6))^(7/2),x)
```

output
```
int((a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6))^(7/2), x)
```

3.493 $\int \frac{x^{-1+4n}}{bx^n+cx^{2n}}\,dx$

3.493.1 Optimal result

Integrand size = 23, antiderivative size = 46

$$\int \frac{x^{-1+4n}}{bx^n+cx^{2n}}\,dx=-\frac{bx^n}{c^2n}+\frac{x^{2n}}{2cn}+\frac{b^2\log(b+cx^n)}{c^3n}$$

output
```
-b*x^n/c^2/n+1/2*x^(2*n)/c/n+b^2*ln(b+c*x^n)/c^3/n
```

3.493.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \frac{x^{-1+4n}}{bx^n+cx^{2n}}\,dx=\frac{cx^n(-2b+cx^n)+2b^2\log(b+cx^n)}{2c^3n}$$

input
```
Integrate[x^(-1 + 4*n)/(b*x^n + c*x^(2*n)),x]
```

output
```
(c*x^n*(-2*b + c*x^n) + 2*b^2*Log[b + c*x^n])/(2*c^3*n)
```

3.493.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {10, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{4n-1}}{bx^n + cx^{2n}} \, dx$$

$$\Big\downarrow \text{10}$$

$$\int \frac{x^{3n-1}}{b + cx^n} dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int \frac{x^{2n}}{cx^n+b} dx^n}{n}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(\frac{x^n}{c} + \frac{b^2}{c^2(cx^n+b)} - \frac{b}{c^2}\right) dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{b^2 \log(b+cx^n)}{c^3} - \frac{bx^n}{c^2} + \frac{x^{2n}}{2c}}{n}$$

input
```
Int[x^(-1 + 4*n)/(b*x^n + c*x^(2*n)),x]
```

output
```
(-((b*x^n)/c^2) + x^(2*n)/(2*c) + (b^2*Log[b + c*x^n])/c^3)/n
```

3.493.3.1 Defintions of rubi rules used

rule 10
```
Int[(u_.)*((e_.)*(x_))^(m_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x
_Symbol] :> Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*(a + b*x^(s - r))^p, x],
 x] /; FreeQ[{a, b, e, m, r, s}, x] && IntegerQ[p] && (IntegerQ[p*r] || GtQ
[e, 0]) && PosQ[s - r]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.493.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02

method	result	size
risch	$\frac{x^{2n}}{2cn}-\frac{b\,x^{n}}{c^{2}n}+\frac{b^{2}\ln\left(x^{n}+\frac{b}{c}\right)}{c^{3}n}$	47
norman	$\left(\frac{\mathrm{e}^{3n\ln(x)}}{2cn}-\frac{b\,\mathrm{e}^{2n\ln(x)}}{c^{2}n}\right)\mathrm{e}^{-n\ln(x)}+\frac{b^{2}\ln\left(c\,\mathrm{e}^{n\ln(x)}+b\right)}{c^{3}n}$	62

input
```
int(x^(-1+4*n)/(b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

output
```
1/2/c/n*(x^n)^2-b*x^n/c^2/n+b^2/c^3/n*ln(x^n+b/c)
```

3.493.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \frac{x^{-1+4n}}{bx^n+cx^{2n}}\,dx=\frac{c^2x^{2\,n}-2\,bcx^n+2\,b^2\log\left(cx^n+b\right)}{2\,c^3n}$$

input
```
integrate(x^(-1+4*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
1/2*(c^2*x^(2*n) - 2*b*c*x^n + 2*b^2*log(c*x^n + b))/(c^3*n)
```

3.493.6 Sympy [A] (verification not implemented)

Time = 22.44 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.37

$$\int \frac{x^{-1+4n}}{bx^n + cx^{2n}}\,dx = \begin{cases} \frac{\log(x)}{b} & \text{for } c = 0 \wedge n = 0 \\ \frac{xx^{-n}x^{4n-1}}{3bn} & \text{for } c = 0 \\ \frac{\log(x)}{b+c} & \text{for } n = 0 \\ \frac{b^2\log\left(\frac{b}{c}+x^n\right)}{c^3n} - \frac{bx^n}{c^2n} + \frac{x^{2n}}{2cn} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1+4*n)/(b*x**n+c*x**(2*n)),x)
```

output
```
Piecewise((log(x)/b, Eq(c, 0) & Eq(n, 0)), (x*x**(4*n - 1)/(3*b*n*x**n), E
q(c, 0)), (log(x)/(b + c), Eq(n, 0)), (b**2*log(b/c + x**n)/(c**3*n) - b*x
**n/(c**2*n) + x**(2*n)/(2*c*n), True))
```

3.493.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int \frac{x^{-1+4n}}{bx^n + cx^{2n}}\,dx = \frac{b^2\log\left(\frac{cx^n+b}{c}\right)}{c^3n} + \frac{cx^{2n} - 2\,bx^n}{2\,c^2n}$$

input
```
integrate(x^(-1+4*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
b^2*log((c*x^n + b)/c)/(c^3*n) + 1/2*(c*x^(2*n) - 2*b*x^n)/(c^2*n)
```

3.493.8 Giac [F]

$$\int \frac{x^{-1+4n}}{bx^n + cx^{2n}}\,dx = \int \frac{x^{4n-1}}{cx^{2n} + bx^n}\,dx$$

input
```
integrate(x^(-1+4*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate(x^(4*n - 1)/(c*x^(2*n) + b*x^n), x)
```

3.493.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+4n}}{bx^n + cx^{2n}}\,dx = \int \frac{x^{4\,n-1}}{b\,x^n + c\,x^{2\,n}}\,dx$$

input
```
int(x^(4*n - 1)/(b*x^n + c*x^(2*n)),x)
```

output
```
int(x^(4*n - 1)/(b*x^n + c*x^(2*n)), x)
```

3.494 $\int \frac{x^{-1+3n}}{bx^n+cx^{2n}}\,dx$

3.494.1 Optimal result

Integrand size = 23, antiderivative size = 28

$$\int \frac{x^{-1+3n}}{bx^n+cx^{2n}}\,dx=\frac{x^n}{cn}-\frac{b\log\left(b+cx^n\right)}{c^2n}$$

output
```
x^n/c/n-b*ln(b+c*x^n)/c^2/n
```

3.494.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{x^{-1+3n}}{bx^n+cx^{2n}}\,dx=\frac{cx^n-b\log\left(cn(b+cx^n)\right)}{c^2n}$$

input
```
Integrate[x^(-1 + 3*n)/(b*x^n + c*x^(2*n)),x]
```

output
```
(c*x^n - b*Log[c*n*(b + c*x^n)])/(c^2*n)
```

3.494.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {10, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3n-1}}{bx^n+cx^{2n}}dx$$

$$\Big\downarrow \text{10}$$

$$\int \frac{x^{2n-1}}{b+cx^n}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int \frac{x^n}{cx^n+b}dx^n}{n}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(\frac{1}{c}-\frac{b}{c(cx^n+b)}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{x^n}{c}-\frac{b\log(b+cx^n)}{c^2}}{n}$$

input
```
Int[x^(-1 + 3*n)/(b*x^n + c*x^(2*n)),x]
```

output
```
(x^n/c - (b*Log[b + c*x^n])/c^2)/n
```

3.494.3.1 Defintions of rubi rules used

rule 10
```
Int[(u_.)*((e_.)*(x_))^(m_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x
_Symbol] :> Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*(a + b*x^(s - r))^p, x],
 x] /; FreeQ[{a, b, e, m, r, s}, x] && IntegerQ[p] && (IntegerQ[p*r] || GtQ
[e, 0]) && PosQ[s - r]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.494.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

method	result	size
risch	$\frac{x^n}{cn}-\frac{b\ln\left(x^n+\frac{b}{c}\right)}{c^2n}$	31
norman	$\frac{\mathrm{e}^{n\ln(x)}}{cn}-\frac{b\ln\left(c\,\mathrm{e}^{n\ln(x)}+b\right)}{c^2n}$	33

input
```
int(x^(-1+3*n)/(b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

output
```
x^n/c/n-b/c^2/n*ln(x^n+b/c)
```

3.494.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{x^{-1+3n}}{bx^n+cx^{2n}}\,dx=\frac{cx^n-b\log\left(cx^n+b\right)}{c^2n}$$

input
```
integrate(x^(-1+3*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
(c*x^n - b*log(c*x^n + b))/(c^2*n)
```

3.494.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. 2(20) = 40.

Time = 8.96 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.71

$$\int \frac{x^{-1+3n}}{bx^n + cx^{2n}}\,dx = \begin{cases} \frac{\log(x)}{b} & \text{for } c = 0 \wedge n = 0 \\ \frac{xx^{-n}x^{3n-1}}{2bn} & \text{for } c = 0 \\ \frac{\log(x)}{b+c} & \text{for } n = 0 \\ -\frac{b\log\left(\frac{b}{c}+x^n\right)}{c^2n} + \frac{x^n}{cn} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1+3*n)/(b*x**n+c*x**(2*n)),x)
```

output
```
Piecewise((log(x)/b, Eq(c, 0) & Eq(n, 0)), (x*x**(3*n - 1)/(2*b*n*x**n), E
q(c, 0)), (log(x)/(b + c), Eq(n, 0)), (-b*log(b/c + x**n)/(c**2*n) + x**n/
(c*n), True))
```

3.494.7 Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.14

$$\int \frac{x^{-1+3n}}{bx^n + cx^{2n}}\,dx = \frac{x^n}{cn} - \frac{b\log\left(\frac{cx^n+b}{c}\right)}{c^2n}$$

input
```
integrate(x^(-1+3*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
x^n/(c*n) - b*log((c*x^n + b)/c)/(c^2*n)
```

3.494.8 Giac [F]

$$\int \frac{x^{-1+3n}}{bx^n + cx^{2n}}\,dx = \int \frac{x^{3n-1}}{cx^{2n} + bx^n}\,dx$$

input
```
integrate(x^(-1+3*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate(x^(3*n - 1)/(c*x^(2*n) + b*x^n), x)
```

3.494.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+3n}}{bx^n + cx^{2n}}\,dx = \int \frac{x^{3\,n-1}}{b\,x^n + c\,x^{2\,n}}\,dx$$

input
```
int(x^(3*n - 1)/(b*x^n + c*x^(2*n)),x)
```

output
```
int(x^(3*n - 1)/(b*x^n + c*x^(2*n)), x)
```

3.495 $\int \frac{x^{-1+2n}}{bx^n+cx^{2n}}\,dx$

3.495.1 Optimal result

Integrand size = 23, antiderivative size = 15

$$\int \frac{x^{-1+2n}}{bx^n+cx^{2n}}\,dx=\frac{\log(b+cx^n)}{cn}$$

output
```
ln(b+c*x^n)/c/n
```

3.495.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1+2n}}{bx^n+cx^{2n}}\,dx=\frac{\log(b+cx^n)}{cn}$$

input
```
Integrate[x^(-1 + 2*n)/(b*x^n + c*x^(2*n)),x]
```

output
```
Log[b + c*x^n]/(c*n)
```

3.495.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {10, 792}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{2n-1}}{bx^n+cx^{2n}}dx$$
$$\Big\downarrow \; 10$$
$$\int \frac{x^{n-1}}{b+cx^n}dx$$
$$\Big\downarrow \; 792$$
$$\frac{\log(b+cx^n)}{cn}$$

input
```
Int[x^(-1 + 2*n)/(b*x^n + c*x^(2*n)),x]
```

output
```
Log[b + c*x^n]/(c*n)
```

3.495.3.1 Defintions of rubi rules used

rule 10
```
Int[(u_.)*((e_.)*(x_))^(m_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x
_Symbol] :> Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*(a + b*x^(s - r))^p, x],
 x] /; FreeQ[{a, b, e, m, r, s}, x] && IntegerQ[p] && (IntegerQ[p*r] || GtQ
[e, 0]) && PosQ[s - r]
```

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

3.495.4 Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.20

method	result	size
norman	$\frac{\ln\left(c\,{\mathrm e}^{n\ln\left(x\right)}+b\right)}{c n}$	18
risch	$\frac{\ln\left(x^{n}+\frac{b}{c}\right)}{c n}$	18

input
```
int(x^(-1+2*n)/(b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

output
```
1/c/n*ln(c*exp(n*ln(x))+b)
```

3.495.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1+2n}}{bx^n+cx^{2n}}\,dx=\frac{\log\left(cx^n+b\right)}{cn}$$

input
```
integrate(x^(-1+2*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
log(c*x^n + b)/(c*n)
```

3.495.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(10) = 20.

Time = 1.75 (sec) , antiderivative size = 46, normalized size of antiderivative = 3.07

$$\int \frac{x^{-1+2n}}{bx^n+cx^{2n}}\,dx=\begin{cases}\frac{\log\left(x\right)}{b} & \text{for}\: c=0\wedge n=0\\ \frac{xx^{-n}x^{2n-1}}{bn} & \text{for}\: c=0\\ \frac{\log\left(x\right)}{b+c} & \text{for}\: n=0\\ -\frac{\log\left(x\right)}{c}+\frac{\log\left(\frac{bx^n}{c}+x^{2n}\right)}{cn} & \text{otherwise}\end{cases}$$

input
```
integrate(x**(-1+2*n)/(b*x**n+c*x**(2*n)),x)
```

output
```
Piecewise((log(x)/b, Eq(c, 0) & Eq(n, 0)), (x*x**(2*n - 1)/(b*n*x**n), Eq(
c, 0)), (log(x)/(b + c), Eq(n, 0)), (-log(x)/c + log(b*x**n/c + x**(2*n))/
(c*n), True))
```

3.495.7 Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int \frac{x^{-1+2n}}{bx^n + cx^{2n}}\,dx = \frac{\log\left(\frac{cx^n+b}{c}\right)}{cn}$$

input
```
integrate(x^(-1+2*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
log((c*x^n + b)/c)/(c*n)
```

3.495.8 Giac [F]

$$\int \frac{x^{-1+2n}}{bx^n + cx^{2n}}\,dx = \int \frac{x^{2n-1}}{cx^{2n} + bx^n}\,dx$$

input
```
integrate(x^(-1+2*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate(x^(2*n - 1)/(c*x^(2*n) + b*x^n), x)
```

3.495.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}}{bx^n + cx^{2n}}\,dx = \int \frac{x^{2n-1}}{b\,x^n + c\,x^{2n}}\,dx$$

input
```
int(x^(2*n - 1)/(b*x^n + c*x^(2*n)),x)
```

output
```
int(x^(2*n - 1)/(b*x^n + c*x^(2*n)), x)
```

3.496 $\int \frac{x^{-1+n}}{bx^n+cx^{2n}}\,dx$

3.496.1 Optimal result

Integrand size = 21, antiderivative size = 23

$$\int \frac{x^{-1+n}}{bx^n+cx^{2n}}\,dx = \frac{\log(x)}{b} - \frac{\log(b+cx^n)}{bn}$$

output
```
ln(x)/b-ln(b+c*x^n)/b/n
```

3.496.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^{-1+n}}{bx^n+cx^{2n}}\,dx = \frac{\log(x^n)-\log(bn(b+cx^n))}{bn}$$

input
```
Integrate[x^(-1 + n)/(b*x^n + c*x^(2*n)),x]
```

output
```
(Log[x^n] - Log[b*n*(b + c*x^n)])/(b*n)
```

3.496.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {10, 798, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{n-1}}{bx^n+cx^{2n}}dx$$

$$\Big\downarrow \text{10}$$

$$\int \frac{1}{x(b+cx^n)}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int \frac{x^{-n}}{cx^n+b}dx^n}{n}$$

$$\Big\downarrow \text{47}$$

$$\frac{\frac{\int x^{-n}dx^n}{b}-\frac{c\int \frac{1}{cx^n+b}dx^n}{b}}{n}$$

$$\Big\downarrow \text{14}$$

$$\frac{\frac{\log(x^n)}{b}-\frac{c\int \frac{1}{cx^n+b}dx^n}{b}}{n}$$

$$\Big\downarrow \text{16}$$

$$\frac{\frac{\log(x^n)}{b}-\frac{\log(b+cx^n)}{b}}{n}$$

input
```
Int[x^(-1 + n)/(b*x^n + c*x^(2*n)),x]
```

output
```
(Log[x^n]/b - Log[b + c*x^n]/b)/n
```

3.496.3.1 Defintions of rubi rules used

rule 10
```
Int[(u_.)*((e_.)*(x_))^(m_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x
_Symbol] :> Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*(a + b*x^(s - r))^p, x],
 x] /; FreeQ[{a, b, e, m, r, s}, x] && IntegerQ[p] && (IntegerQ[p*r] || GtQ
[e, 0]) && PosQ[s - r]
```

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

3.496.4 Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

method	result	size
norman	$\frac{\ln(x)}{b}-\frac{\ln\left(c\,{\mathrm e}^{n\ln(x)}+b\right)}{bn}$	26
risch	$\frac{\ln(x)}{b}-\frac{\ln\left(x^{n}+\frac{b}{c}\right)}{bn}$	26

input
```
int(x^(-1+n)/(b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

output
```
ln(x)/b-1/b/n*ln(c*exp(n*ln(x))+b)
```

3.496.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{x^{-1+n}}{bx^n + cx^{2n}}\,dx = \frac{n\log(x) - \log(cx^n + b)}{bn}$$

input
```
integrate(x^(-1+n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
(n*log(x) - log(c*x^n + b))/(b*n)
```

3.496.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(15) = 30.

Time = 1.74 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.35

$$\int \frac{x^{-1+n}}{bx^n + cx^{2n}}\,dx = \begin{cases} \tilde{\infty}\log(x) & \text{for } b = 0 \wedge c = 0 \wedge n = 0 \\ -\frac{xx^{-2n}x^{n-1}}{cn} & \text{for } b = 0 \\ \frac{\log(x)}{b} & \text{for } c = 0 \\ \frac{\log(x)}{b+c} & \text{for } n = 0 \\ \frac{2\log(x)}{b} - \frac{\log\left(\frac{bx^n}{c} + x^{2n}\right)}{bn} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1+n)/(b*x**n+c*x**(2*n)),x)
```

output
```
Piecewise((zoo*log(x), Eq(b, 0) & Eq(c, 0) & Eq(n, 0)), (-x*x**(n - 1)/(c*
n*x**(2*n)), Eq(b, 0)), (log(x)/b, Eq(c, 0)), (log(x)/(b + c), Eq(n, 0)),
(2*log(x)/b - log(b*x**n/c + x**(2*n))/(b*n), True))
```

3.496.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x^{-1+n}}{bx^n + cx^{2n}}\,dx = \frac{\log(x)}{b} - \frac{\log\left(\frac{cx^n+b}{c}\right)}{bn}$$

input
```
integrate(x^(-1+n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
log(x)/b - log((c*x^n + b)/c)/(b*n)
```

3.496.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^{-1+n}}{bx^n + cx^{2n}}\,dx = \frac{\log(|x|)}{b} - \frac{\log(|cx^n + b|)}{bn}$$

input
```
integrate(x^(-1+n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
log(abs(x))/b - log(abs(c*x^n + b))/(b*n)
```

3.496.9 Mupad [B] (verification not implemented)

Time = 8.71 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{x^{-1+n}}{bx^n + cx^{2n}}\,dx = -\frac{2\,\mathrm{atanh}\left(\frac{2\,c\,x^n}{b} + 1\right)}{b\,n}$$

input
```
int(x^(n - 1)/(b*x^n + c*x^(2*n)),x)
```

output
```
-(2*atanh((2*c*x^n)/b + 1))/(b*n)
```

3.497 $\int \frac{x^{-1-n}}{bx^n+cx^{2n}}\,dx$

3.497.1 Optimal result

Integrand size = 23, antiderivative size = 57

$$\int \frac{x^{-1-n}}{bx^n+cx^{2n}}\,dx=-\frac{x^{-2n}}{2bn}+\frac{cx^{-n}}{b^2n}+\frac{c^2\log(x)}{b^3}-\frac{c^2\log(b+cx^n)}{b^3n}$$

output
```
-1/2/b/n/(x^(2*n))+c/b^2/n/(x^n)+c^2*ln(x)/b^3-c^2*ln(b+c*x^n)/b^3/n
```

3.497.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.84

$$\int \frac{x^{-1-n}}{bx^n+cx^{2n}}\,dx=-\frac{bx^{-2n}(b-2cx^n)-2c^2\log(x^n)+2c^2\log(b+cx^n)}{2b^3n}$$

input
```
Integrate[x^(-1 - n)/(b*x^n + c*x^(2*n)),x]
```

output
```
-1/2*((b*(b - 2*c*x^n))/x^(2*n) - 2*c^2*Log[x^n] + 2*c^2*Log[b + c*x^n])/(
b^3*n)
```

3.497.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {10, 798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-n-1}}{bx^n+cx^{2n}}dx$$
$$\Big\downarrow \ 10$$
$$\int \frac{x^{-2n-1}}{b+cx^n}dx$$
$$\Big\downarrow \ 798$$
$$\frac{\int \frac{x^{-3n}}{cx^n+b}dx^n}{n}$$
$$\Big\downarrow \ 54$$
$$\frac{\int \left(\frac{x^{-3n}}{b}-\frac{cx^{-2n}}{b^2}+\frac{c^2x^{-n}}{b^3}-\frac{c^3}{b^3(cx^n+b)}\right)dx^n}{n}$$
$$\Big\downarrow \ 2009$$
$$\frac{\frac{c^2\log(x^n)}{b^3}-\frac{c^2\log(b+cx^n)}{b^3}+\frac{cx^{-n}}{b^2}-\frac{x^{-2n}}{2b}}{n}$$

input
```
Int[x^(-1 - n)/(b*x^n + c*x^(2*n)),x]
```

output
```
(-1/2*1/(b*x^(2*n)) + c/(b^2*x^n) + (c^2*Log[x^n])/b^3 - (c^2*Log[b + c*x^
n])/b^3)/n
```

3.497.3.1 Defintions of rubi rules used

rule 10
```
Int[(u_.)*((e_.)*(x_))^(m_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x
_Symbol] :> Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*(a + b*x^(s - r))^p, x],
 x] /; FreeQ[{a, b, e, m, r, s}, x] && IntegerQ[p] && (IntegerQ[p*r] || GtQ
[e, 0]) && PosQ[s - r]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.497.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.02

method	result	size
risch	$\frac{c\,x^{-n}}{b^{2} n}-\frac{x^{-2 n}}{2 b n}+\frac{c^{2} \ln \left(x \right)}{b^{3}}-\frac{c^{2} \ln \left(x^{n}+\frac{b}{c}\right)}{b^{3} n}$	58
norman	$\left(\frac{c\,{\mathrm e}^{n \ln \left(x \right)}}{b^{2} n}-\frac{1}{2 b n}+\frac{c^{2} \ln \left(x \right) {\mathrm e}^{2 n \ln \left(x \right)}}{b^{3}}\right) {\mathrm e}^{-2 n \ln \left(x \right)}-\frac{c^{2} \ln \left(c\,{\mathrm e}^{n \ln \left(x \right)}+b \right)}{b^{3} n}$	69

input
```
int(x^(-1-n)/(b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

output
```
c/b^2/n/(x^n)-1/2/b/n/(x^n)^2+c^2*ln(x)/b^3-c^2/b^3/n*ln(x^n+b/c)
```

3.497.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.04

$$\int \frac{x^{-1-n}}{bx^n + cx^{2n}}\, dx = \frac{2\, c^2 n x^{2\, n} \log{(x)} - 2\, c^2 x^{2\, n} \log{(cx^n + b)} + 2\, bcx^n - b^2}{2\, b^3 n x^{2\, n}}$$

input
```
integrate(x^(-1-n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
1/2*(2*c^2*n*x^(2*n)*log(x) - 2*c^2*x^(2*n)*log(c*x^n + b) + 2*b*c*x^n - b
^2)/(b^3*n*x^(2*n))
```

3.497.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. 2(48) = 96.

Time = 29.32 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.75

$$\int \frac{x^{-1-n}}{bx^n + cx^{2n}}\, dx = \begin{cases} \tilde{\infty} \log{(x)} & \text{for } b = 0 \wedge c = 0 \wedge n = 0 \\ -\frac{x x^{-2n} x^{-n-1}}{3cn} & \text{for } b = 0 \\ -\frac{x x^{-n} x^{-n-1}}{2bn} & \text{for } c = 0 \\ \frac{\log{(x)}}{b+c} & \text{for } n = 0 \\ -\frac{x^{-2n}}{2bn} + \frac{cx^{-n}}{b^2 n} + \frac{c^2 \log{(x^n)}}{b^3 n} - \frac{c^2 \log{\left(\frac{b}{c} + x^n\right)}}{b^3 n} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1-n)/(b*x**n+c*x**(2*n)),x)
```

output
```
Piecewise((zoo*log(x), Eq(b, 0) & Eq(c, 0) & Eq(n, 0)), (-x*x**(-n - 1)/(3
*c*n*x**(2*n)), Eq(b, 0)), (-x*x**(-n - 1)/(2*b*n*x**n), Eq(c, 0)), (log(x
)/(b + c), Eq(n, 0)), (-1/(2*b*n*x**(2*n)) + c/(b**2*n*x**n) + c**2*log(x*
*n)/(b**3*n) - c**2*log(b/c + x**n)/(b**3*n), True))
```

3.497.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.02

$$\int \frac{x^{-1-n}}{bx^n+cx^{2n}}\,dx = \frac{c^2\log(x)}{b^3} - \frac{c^2\log\left(\frac{cx^n+b}{c}\right)}{b^3n} + \frac{2\,cx^n-b}{2\,b^2nx^{2\,n}}$$

input
```
integrate(x^(-1-n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
c^2*log(x)/b^3 - c^2*log((c*x^n + b)/c)/(b^3*n) + 1/2*(2*c*x^n - b)/(b^2*n
*x^(2*n))
```

3.497.8 Giac [F]

$$\int \frac{x^{-1-n}}{bx^n+cx^{2n}}\,dx = \int \frac{x^{-n-1}}{cx^{2\,n}+bx^n}\,dx$$

input
```
integrate(x^(-1-n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate(x^(-n - 1)/(c*x^(2*n) + b*x^n), x)
```

3.497.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-n}}{bx^n+cx^{2n}}\,dx = \int \frac{1}{x^{n+1}\,\left(b\,x^n+c\,x^{2\,n}\right)}\,dx$$

input
```
int(1/(x^(n + 1)*(b*x^n + c*x^(2*n))),x)
```

output
```
int(1/(x^(n + 1)*(b*x^n + c*x^(2*n))), x)
```

3.498 $\int \frac{x^{-1-2n}}{bx^n+cx^{2n}}\,dx$

3.498.1 Optimal result

Integrand size = 23, antiderivative size = 76

$$\int \frac{x^{-1-2n}}{bx^n+cx^{2n}}\,dx=-\frac{x^{-3n}}{3bn}+\frac{cx^{-2n}}{2b^2n}-\frac{c^2x^{-n}}{b^3n}-\frac{c^3\log(x)}{b^4}+\frac{c^3\log\left(b+cx^n\right)}{b^4n}$$

output
```
-1/3/b/n/(x^(3*n))+1/2*c/b^2/n/(x^(2*n))-c^2/b^3/n/(x^n)-c^3*ln(x)/b^4+c^3
*ln(b+c*x^n)/b^4/n
```

3.498.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.83

$$\int \frac{x^{-1-2n}}{bx^n+cx^{2n}}\,dx=-\frac{bx^{-3n}\left(2b^2-3bcx^n+6c^2x^{2n}\right)+6c^3\log\left(x^n\right)-6c^3\log\left(b+cx^n\right)}{6b^4n}$$

input
```
Integrate[x^(-1 - 2*n)/(b*x^n + c*x^(2*n)),x]
```

output
```
-1/6*((b*(2*b^2 - 3*b*c*x^n + 6*c^2*x^(2*n)))/x^(3*n) + 6*c^3*Log[x^n] - 6
*c^3*Log[b + c*x^n])/(b^4*n)
```

3.498.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {10, 798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-2n-1}}{bx^n+cx^{2n}}\,dx$$

$$\Big\downarrow \; 10$$

$$\int \frac{x^{-3n-1}}{b+cx^n}dx$$

$$\Big\downarrow \; 798$$

$$\frac{\int \frac{x^{-4n}}{cx^n+b}dx^n}{n}$$

$$\Big\downarrow \; 54$$

$$\frac{\int \left(\frac{x^{-4n}}{b}-\frac{cx^{-3n}}{b^2}+\frac{c^2x^{-2n}}{b^3}-\frac{c^3x^{-n}}{b^4}+\frac{c^4}{b^4(cx^n+b)}\right)dx^n}{n}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{c^3\log(x^n)}{b^4}+\frac{c^3\log(b+cx^n)}{b^4}-\frac{c^2x^{-n}}{b^3}+\frac{cx^{-2n}}{2b^2}-\frac{x^{-3n}}{3b}}{n}$$

input
```
Int[x^(-1 - 2*n)/(b*x^n + c*x^(2*n)),x]
```

output
```
(-1/3*1/(b*x^(3*n)) + c/(2*b^2*x^(2*n)) - c^2/(b^3*x^n) - (c^3*Log[x^n])/b
^4 + (c^3*Log[b + c*x^n])/b^4)/n
```

3.498.3.1 Defintions of rubi rules used

rule 10
```
Int[(u_.)*((e_.)*(x_))^(m_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x
_Symbol] :> Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*(a + b*x^(s - r))^p, x],
 x] /; FreeQ[{a, b, e, m, r, s}, x] && IntegerQ[p] && (IntegerQ[p*r] || GtQ
[e, 0]) && PosQ[s - r]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.498.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.99

method	result	size
risch	$-\frac{c^2 x^{-n}}{b^3 n}+\frac{c\, x^{-2n}}{2b^2 n}-\frac{x^{-3n}}{3bn}-\frac{c^3 \ln(x)}{b^4}+\frac{c^3 \ln\left(x^n+\frac{b}{c}\right)}{b^4 n}$	75
norman	$\left(-\frac{1}{3bn}+\frac{c\,\mathrm{e}^{n\ln(x)}}{2b^2 n}-\frac{c^2 \mathrm{e}^{2n\ln(x)}}{b^3 n}-\frac{c^3 \ln(x)\mathrm{e}^{3n\ln(x)}}{b^4}\right)\mathrm{e}^{-3n\ln(x)}+\frac{c^3 \ln\left(c\,\mathrm{e}^{n\ln(x)}+b\right)}{b^4 n}$	88

input
```
int(x^(-1-2*n)/(b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

output
```
-c^2/b^3/n/(x^n)+1/2*c/b^2/n/(x^n)^2-1/3/b/n/(x^n)^3-c^3*ln(x)/b^4+c^3/b^4
/n*ln(x^n+b/c)
```

3.498.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.95

$$\int \frac{x^{-1-2n}}{bx^n+cx^{2n}}\,dx = -\frac{6\,c^3nx^{3\,n}\log\left(x\right)-6\,c^3x^{3\,n}\log\left(cx^n+b\right)+6\,bc^2x^{2\,n}-3\,b^2cx^n+2\,b^3}{6\,b^4nx^{3\,n}}$$

input
```
integrate(x^(-1-2*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
-1/6*(6*c^3*n*x^(3*n)*log(x) - 6*c^3*x^(3*n)*log(c*x^n + b) + 6*b*c^2*x^(2
*n) - 3*b^2*c*x^n + 2*b^3)/(b^4*n*x^(3*n))
```

3.498.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{x^{-1-2n}}{bx^n+cx^{2n}}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(x**(-1-2*n)/(b*x**n+c*x**(2*n)),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.498.7 Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.93

$$\int \frac{x^{-1-2n}}{bx^n+cx^{2n}}\,dx = -\frac{c^3\log\left(x\right)}{b^4}+\frac{c^3\log\left(\frac{cx^n+b}{c}\right)}{b^4n}-\frac{6\,c^2x^{2\,n}-3\,bcx^n+2\,b^2}{6\,b^3nx^{3\,n}}$$

input
```
integrate(x^(-1-2*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
-c^3*log(x)/b^4 + c^3*log((c*x^n + b)/c)/(b^4*n) - 1/6*(6*c^2*x^(2*n) - 3*
b*c*x^n + 2*b^2)/(b^3*n*x^(3*n))
```

3.498.8 Giac [F]

$$\int \frac{x^{-1-2n}}{bx^n + cx^{2n}}\,dx = \int \frac{x^{-2n-1}}{cx^{2n} + bx^n}\,dx$$

input
```
integrate(x^(-1-2*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate(x^(-2*n - 1)/(c*x^(2*n) + b*x^n), x)
```

3.498.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-2n}}{bx^n + cx^{2n}}\,dx = \int \frac{1}{x^{2n+1}\,(b\,x^n + c\,x^{2n})}\,dx$$

input
```
int(1/(x^(2*n + 1)*(b*x^n + c*x^(2*n))),x)
```

output
```
int(1/(x^(2*n + 1)*(b*x^n + c*x^(2*n))), x)
```

3.499 $\int \frac{x^{-1-3n}}{bx^n+cx^{2n}}\,dx$

3.499.1 Optimal result

Integrand size = 23, antiderivative size = 93

$$\int \frac{x^{-1-3n}}{bx^n+cx^{2n}}\,dx=-\frac{x^{-4n}}{4bn}+\frac{cx^{-3n}}{3b^2n}-\frac{c^2x^{-2n}}{2b^3n}+\frac{c^3x^{-n}}{b^4n}+\frac{c^4\log(x)}{b^5}-\frac{c^4\log\left(b+cx^n\right)}{b^5n}$$

output
```
-1/4/b/n/(x^(4*n))+1/3*c/b^2/n/(x^(3*n))-1/2*c^2/b^3/n/(x^(2*n))+c^3/b^4/n
/(x^n)+c^4*ln(x)/b^5-c^4*ln(b+c*x^n)/b^5/n
```

3.499.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.82

$$\int \frac{x^{-1-3n}}{bx^n+cx^{2n}}\,dx=-\frac{bx^{-4n}\left(3b^3-4b^2cx^n+6bc^2x^{2n}-12c^3x^{3n}\right)-12c^4\log\left(x^n\right)+12c^4\log\left(b+cx^n\right)}{12b^5n}$$

input
```
Integrate[x^(-1 - 3*n)/(b*x^n + c*x^(2*n)),x]
```

output
```
-1/12*((b*(3*b^3 - 4*b^2*c*x^n + 6*b*c^2*x^(2*n) - 12*c^3*x^(3*n)))/x^(4*n
) - 12*c^4*Log[x^n] + 12*c^4*Log[b + c*x^n])/(b^5*n)
```

3.499.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {10, 798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-3n-1}}{bx^n+cx^{2n}}dx$$
$$\Big\downarrow \text{ 10}$$
$$\int \frac{x^{-4n-1}}{b+cx^n}dx$$
$$\Big\downarrow \text{ 798}$$
$$\frac{\int \frac{x^{-5n}}{cx^n+b}dx^n}{n}$$
$$\Big\downarrow \text{ 54}$$
$$\frac{\int \left(\frac{x^{-5n}}{b}-\frac{cx^{-4n}}{b^2}+\frac{c^2x^{-3n}}{b^3}-\frac{c^3x^{-2n}}{b^4}+\frac{c^4x^{-n}}{b^5}-\frac{c^5}{b^5(cx^n+b)}\right)dx^n}{n}$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{\frac{c^4\log(x^n)}{b^5}-\frac{c^4\log(b+cx^n)}{b^5}+\frac{c^3x^{-n}}{b^4}-\frac{c^2x^{-2n}}{2b^3}+\frac{cx^{-3n}}{3b^2}-\frac{x^{-4n}}{4b}}{n}$$

input
```
Int[x^(-1 - 3*n)/(b*x^n + c*x^(2*n)),x]
```

output
```
(-1/4*1/(b*x^(4*n)) + c/(3*b^2*x^(3*n)) - c^2/(2*b^3*x^(2*n)) + c^3/(b^4*x
^n) + (c^4*Log[x^n])/b^5 - (c^4*Log[b + c*x^n])/b^5)/n
```

3.499.3.1 Defintions of rubi rules used

rule 10
```
Int[(u_.)*((e_.)*(x_))^(m_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x
_Symbol] :> Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*(a + b*x^(s - r))^p, x],
 x] /; FreeQ[{a, b, e, m, r, s}, x] && IntegerQ[p] && (IntegerQ[p*r] || GtQ
[e, 0]) && PosQ[s - r]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.499.4 Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.97

method	result	size
risch	$\frac{c^{3} x^{-n}}{b^{4} n}-\frac{c^{2} x^{-2 n}}{2 b^{3} n}+\frac{c\, x^{-3 n}}{3 b^{2} n}-\frac{x^{-4 n}}{4 b n}+\frac{c^{4} \ln \left(x \right)}{b^{5}}-\frac{c^{4} \ln \left(x^{n}+\frac{b}{c}\right)}{b^{5} n}$	90
norman	$\left(\frac{c^{3} {\mathrm e}^{3 n \ln \left(x \right)}}{b^{4} n}-\frac{1}{4 b n}+\frac{c\, {\mathrm e}^{n \ln \left(x \right)}}{3 b^{2} n}-\frac{c^{2} {\mathrm e}^{2 n \ln \left(x \right)}}{2 b^{3} n}+\frac{c^{4} \ln \left(x \right) {\mathrm e}^{4 n \ln \left(x \right)}}{b^{5}}\right) {\mathrm e}^{-4 n \ln \left(x \right)}-\frac{c^{4} \ln \left(c\, {\mathrm e}^{n \ln \left(x \right)}+b \right)}{b^{5} n}$	105

input
```
int(x^(-1-3*n)/(b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

output
```
c^3/b^4/n/(x^n)-1/2*c^2/b^3/n/(x^n)^2+1/3*c/b^2/n/(x^n)^3-1/4/b/n/(x^n)^4+
c^4*ln(x)/b^5-c^4/b^5/n*ln(x^n+b/c)
```

3.499.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.91

$$\int \frac{x^{-1-3n}}{bx^n + cx^{2n}}\,dx = \frac{12\,c^4 n x^{4\,n}\log\left(x\right) - 12\,c^4 x^{4\,n}\log\left(cx^n + b\right) + 12\,bc^3x^{3\,n} - 6\,b^2c^2x^{2\,n} + 4\,b^3cx^n - 3\,b^4}{12\,b^5 n x^{4\,n}}$$

input
```
integrate(x^(-1-3*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
1/12*(12*c^4*n*x^(4*n)*log(x) - 12*c^4*x^(4*n)*log(c*x^n + b) + 12*b*c^3*x
^(3*n) - 6*b^2*c^2*x^(2*n) + 4*b^3*c*x^n - 3*b^4)/(b^5*n*x^(4*n))
```

3.499.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{x^{-1-3n}}{bx^n + cx^{2n}}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(x**(-1-3*n)/(b*x**n+c*x**(2*n)),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.499.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.90

$$\int \frac{x^{-1-3n}}{bx^n + cx^{2n}}\,dx = \frac{c^4\log\left(x\right)}{b^5} - \frac{c^4\log\left(\frac{cx^n+b}{c}\right)}{b^5 n} + \frac{12\,c^3x^{3\,n} - 6\,bc^2x^{2\,n} + 4\,b^2cx^n - 3\,b^3}{12\,b^4 n x^{4\,n}}$$

input
```
integrate(x^(-1-3*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
c^4*log(x)/b^5 - c^4*log((c*x^n + b)/c)/(b^5*n) + 1/12*(12*c^3*x^(3*n) - 6
*b*c^2*x^(2*n) + 4*b^2*c*x^n - 3*b^3)/(b^4*n*x^(4*n))
```

3.499.8 Giac [F]

$$\int \frac{x^{-1-3n}}{bx^n + cx^{2n}}\,dx = \int \frac{x^{-3n-1}}{cx^{2n} + bx^n}\,dx$$

input
```
integrate(x^(-1-3*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate(x^(-3*n - 1)/(c*x^(2*n) + b*x^n), x)
```

3.499.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-3n}}{bx^n + cx^{2n}}\,dx = \int \frac{1}{x^{3n+1}\,(b\,x^n + c\,x^{2n})}\,dx$$

input
```
int(1/(x^(3*n + 1)*(b*x^n + c*x^(2*n))),x)
```

output
```
int(1/(x^(3*n + 1)*(b*x^n + c*x^(2*n))), x)
```

3.500 $\int \frac{x^{-1+\frac{n}{4}}}{bx^n+cx^{2n}}\,dx$

3.500.1 Optimal result

Integrand size = 25, antiderivative size = 236

$$\int \frac{x^{-1+\frac{n}{4}}}{bx^n+cx^{2n}}\,dx = -\frac{4x^{-3n/4}}{3bn}+\frac{\sqrt{2}c^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{b}}\right)}{b^{7/4}n} - \frac{\sqrt{2}c^{3/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{b}}\right)}{b^{7/4}n} + \frac{c^{3/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}x^{n/4}+\sqrt{c}x^{n/2}\right)}{\sqrt{2}b^{7/4}n} - \frac{c^{3/4}\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}x^{n/4}+\sqrt{c}x^{n/2}\right)}{\sqrt{2}b^{7/4}n}$$

output
```
-4/3/b/n/(x^(3/4*n))+1/2*c^(3/4)*ln(-b^(1/4)*c^(1/4)*x^(1/4*n)*2^(1/2)+b^(
1/2)+x^(1/2*n)*c^(1/2))/b^(7/4)/n*2^(1/2)-1/2*c^(3/4)*ln(b^(1/4)*c^(1/4)*x
^(1/4*n)*2^(1/2)+b^(1/2)+x^(1/2*n)*c^(1/2))/b^(7/4)/n*2^(1/2)-c^(3/4)*arct
an(-1+c^(1/4)*x^(1/4*n)*2^(1/2)/b^(1/4))*2^(1/2)/b^(7/4)/n-c^(3/4)*arctan(
1+c^(1/4)*x^(1/4*n)*2^(1/2)/b^(1/4))*2^(1/2)/b^(7/4)/n
```

3.500.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.14

$$\int \frac{x^{-1+\frac{n}{4}}}{bx^n + cx^{2n}}\,dx = -\frac{4x^{-3n/4}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},1,\frac{1}{4},-\frac{cx^n}{b}\right)}{3bn}$$

input
```
Integrate[x^(-1 + n/4)/(b*x^n + c*x^(2*n)),x]
```

output
```
(-4*Hypergeometric2F1[-3/4, 1, 1/4, -((c*x^n)/b)])/(3*b*n*x^((3*n)/4))
```

3.500.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.10, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {10, 886, 868, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{\frac{n}{4}-1}}{bx^n + cx^{2n}}\,dx$$

$$\Big\downarrow \text{10}$$

$$\int \frac{x^{-\frac{3n}{4}-1}}{b + cx^n}dx$$

$$\Big\downarrow \text{886}$$

$$-\frac{c\int \frac{x^{\frac{n-4}{4}}}{cx^n+b}dx}{b} - \frac{4x^{-3n/4}}{3bn}$$

$$\Big\downarrow \text{868}$$

$$-\frac{4c\int \frac{1}{cx^n+b}dx^{n/4}}{bn} - \frac{4x^{-3n/4}}{3bn}$$

$$\Big\downarrow \text{755}$$

$$-\frac{4c\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x^{n/2}}{cx^n+b}dx^{n/4}}{2\sqrt{b}} + \frac{\int \frac{\sqrt{c}x^{n/2}+\sqrt{b}}{cx^n+b}dx^{n/4}}{2\sqrt{b}}\right)}{bn} - \frac{4x^{-3n/4}}{3bn}$$

$$\Big\downarrow \text{1476}$$

$$-\frac{4c\left(\frac{\frac{\int \frac{1}{-\frac{\sqrt{2}\sqrt[4]{b}x^{n/4}}{\sqrt[4]{c}}+x^{n/2}+\frac{\sqrt{b}}{\sqrt{c}}}dx^{n/4}}{2\sqrt{c}}+\frac{\int \frac{1}{\frac{\sqrt{2}\sqrt[4]{b}x^{n/4}}{\sqrt[4]{c}}+x^{n/2}+\frac{\sqrt{b}}{\sqrt{c}}}dx^{n/4}}{2\sqrt{c}}}{2\sqrt{b}}+\frac{\int \frac{\sqrt{b}-\sqrt{c}x^{n/2}}{cx^n+b}dx^{n/4}}{2\sqrt{b}}\right)}{bn}-\frac{4x^{-3n/4}}{3bn}$$

$$\Big\downarrow \text{1082}$$

$$-\frac{4c\left(\frac{\frac{\int \frac{1}{-x^{n/2}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int \frac{1}{-x^{n/2}-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\int \frac{\sqrt{b}-\sqrt{c}x^{n/2}}{cx^n+b}dx^{n/4}}{2\sqrt{b}}\right)}{bn}-\frac{4x^{-3n/4}}{3bn}$$

$$\Big\downarrow \text{217}$$

$$-\frac{4c\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x^{n/2}}{cx^n+b}dx^{n/4}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{bn}-\frac{4x^{-3n/4}}{3bn}$$

$$\Big\downarrow \text{1479}$$

$$-\frac{4c\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}x^{n/4}}{\sqrt[4]{c}\left(-\frac{\sqrt{2}\sqrt[4]{b}x^{n/4}}{\sqrt[4]{c}}+x^{n/2}+\frac{\sqrt{b}}{\sqrt{c}}\right)}dx^{n/4}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x^{n/4}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(\frac{\sqrt{2}\sqrt[4]{b}x^{n/4}}{\sqrt[4]{c}}+x^{n/2}+\frac{\sqrt{b}}{\sqrt{c}}\right)}dx^{n/4}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x^{n}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{bn}$$

$$\frac{4x^{-3n/4}}{3bn}$$

$$\Big\downarrow \text{25}$$

3.500. $\int \frac{x^{-1+\frac{n}{4}}}{bx^n+cx^{2n}}\,dx$

$$-\frac{4c\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}x^{n/4}}{\sqrt[4]{c}\left(-\frac{\sqrt{2}\sqrt[4]{b}x^{n/4}}{\sqrt[4]{c}}+x^{n/2}+\frac{\sqrt{b}}{\sqrt{c}}\right)}dx^{n/4}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x^{n/4}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(\frac{\sqrt{2}\sqrt[4]{b}x^{n/4}}{\sqrt[4]{c}}+x^{n/2}+\frac{\sqrt{b}}{\sqrt{c}}\right)}dx^{n/4}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{bn}$$

$$\frac{4x^{-3n/4}}{3bn}$$

$\downarrow$ 27

$$-\frac{4c\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}x^{n/4}}{-\frac{\sqrt{2}\sqrt[4]{b}x^{n/4}}{\sqrt[4]{c}}+x^{n/2}+\frac{\sqrt{b}}{\sqrt{c}}}dx^{n/4}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}x^{n/4}+\sqrt[4]{b}}{\frac{\sqrt{2}\sqrt[4]{b}x^{n/4}}{\sqrt[4]{c}}+x^{n/2}+\frac{\sqrt{b}}{\sqrt{c}}}dx^{n/4}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{bn}-$$

$$\frac{4x^{-3n/4}}{3bn}$$

$\downarrow$ 1103

$$-\frac{4c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}x^{n/4}+\sqrt{b}+\sqrt{c}x^{n/2}\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}x^{n/4}+\sqrt{b}+\sqrt{c}x^{n/2}\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{bn}-$$

$$\frac{4x^{-3n/4}}{3bn}$$

input
```
Int[x^(-1 + n/4)/(b*x^n + c*x^(2*n)),x]
```

3.500. $\int \frac{x^{-1+\frac{n}{4}}}{bx^n+cx^{2n}}\,dx$

output
```
-4/(3*b*n*x^((3*n)/4)) - (4*c*((-(ArcTan[1 - (Sqrt[2]*c^(1/4)*x^(n/4))/b^(
1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4))) + ArcTan[1 + (Sqrt[2]*c^(1/4)*x^(n/4))/b^
(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[b]) + (-1/2*Log[Sqrt[b] - Sqrt[2
]*b^(1/4)*c^(1/4)*x^(n/4) + Sqrt[c]*x^(n/2)]/(Sqrt[2]*b^(1/4)*c^(1/4)) + L
og[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*x^(n/4) + Sqrt[c]*x^(n/2)]/(2*Sqrt[2]
*b^(1/4)*c^(1/4)))/(2*Sqrt[b])))/(b*n)
```

3.500.3.1 Defintions of rubi rules used

rule 10
```
Int[(u_.)*((e_.)*(x_))^(m_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x
_Symbol] :> Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*(a + b*x^(s - r))^p, x],
 x] /; FreeQ[{a, b, e, m, r, s}, x] && IntegerQ[p] && (IntegerQ[p*r] || GtQ
[e, 0]) && PosQ[s - r]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 868
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(m + 1)
 Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[
{a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

3.500. $\int \frac{x^{-1+\frac{n}{4}}}{bx^n+cx^{2n}} dx$

rule 886
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[x^(m + 1)/(a*(m +
 1)), x] - Simp[b/a   Int[x^Simplify[m + n]/(a + b*x^n), x], x] /; FreeQ[{a
, b, m, n}, x] && FractionQ[Simplify[(m + 1)/n]] && SumSimplerQ[m, n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.500.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.69 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.23

method	result	size
risch	$-\frac{4x^{-\frac{3n}{4}}}{3bn}+\left(\sum_{\_R=\operatorname{RootOf}\left(b^{7}n^{4}\_Z^{4}+c^{3}\right)}\_R\ln\left(x^{\frac{n}{4}}-\frac{b^{2}n\_R}{c}\right)\right)$	54

input
```
int(x^(-1+1/4*n)/(b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

output
```
-4/3/b/n/(x^(1/4*n))^3+sum(_R*ln(x^(1/4*n)-b^2*n/c*_R),_R=RootOf(_Z^4*b^7*
n^4+c^3))
```

3.500.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.14

$$\int \frac{x^{-1+\frac{n}{4}}}{bx^n+cx^{2n}}\,dx =$$

$$-\frac{3\,bnx^3x^{\frac{3}{4}n-3}\left(-\frac{c^3}{b^7n^4}\right)^{\frac{1}{4}}\log\left(\frac{b^2n\left(-\frac{c^3}{b^7n^4}\right)^{\frac{1}{4}}+cxx^{\frac{1}{4}n-1}}{x}\right)-3\,bnx^3x^{\frac{3}{4}n-3}\left(-\frac{c^3}{b^7n^4}\right)^{\frac{1}{4}}\log\left(-\frac{b^2n\left(-\frac{c^3}{b^7n^4}\right)^{\frac{1}{4}}-cxx^{\frac{1}{4}n-1}}{x}\right.}{}$$

input
```
integrate(x^(-1+1/4*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
-1/3*(3*b*n*x^3*x^(3/4*n - 3)*(-c^3/(b^7*n^4))^(1/4)*log((b^2*n*(-c^3/(b^7
*n^4))^(1/4) + c*x*x^(1/4*n - 1))/x) - 3*b*n*x^3*x^(3/4*n - 3)*(-c^3/(b^7*
n^4))^(1/4)*log(-(b^2*n*(-c^3/(b^7*n^4))^(1/4) - c*x*x^(1/4*n - 1))/x) + 3
*I*b*n*x^3*x^(3/4*n - 3)*(-c^3/(b^7*n^4))^(1/4)*log((I*b^2*n*(-c^3/(b^7*n^
4))^(1/4) + c*x*x^(1/4*n - 1))/x) - 3*I*b*n*x^3*x^(3/4*n - 3)*(-c^3/(b^7*n
^4))^(1/4)*log((-I*b^2*n*(-c^3/(b^7*n^4))^(1/4) + c*x*x^(1/4*n - 1))/x) +
4)/(b*n*x^3*x^(3/4*n - 3))
```

3.500.6 Sympy [F]

$$\int \frac{x^{-1+\frac{n}{4}}}{bx^n+cx^{2n}}\,dx = \int \frac{x^{-n}x^{\frac{n}{4}-1}}{b+cx^n}\,dx$$

input
```
integrate(x**(-1+1/4*n)/(b*x**n+c*x**(2*n)),x)
```

output
```
Integral(x**(n/4 - 1)/(x**n*(b + c*x**n)), x)
```

3.500.7 Maxima [F]

$$\int \frac{x^{-1+\frac{n}{4}}}{bx^n+cx^{2n}}\,dx=\int \frac{x^{\frac{1}{4}\,n-1}}{cx^{2\,n}+bx^n}\,dx$$

input
```
integrate(x^(-1+1/4*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
-c*integrate(x^(1/4*n)/(b*c*x*x^n + b^2*x), x) - 4/3/(b*n*x^(3/4*n))
```

3.500.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.86

$$\int \frac{x^{-1+\frac{n}{4}}}{bx^n+cx^{2n}}\,dx = -\frac{\frac{6\sqrt{2}(bc^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,(x^n)^{\frac{1}{4}}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{b^2}+\frac{6\sqrt{2}(bc^3)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,(x^n)^{\frac{1}{4}}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{b^2}+\frac{3\sqrt{2}(bc^3)^{\frac{1}{4}}\log\left(x^{\frac{1}{2}\,n}+\sqrt{2}(x^n)^{\frac{1}{4}}\left(\frac{b}{c}\right.\right.}{b^2}}{6\,n}$$

input
```
integrate(x^(-1+1/4*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
-1/6*(6*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*
(x^n)^(1/4))/(b/c)^(1/4))/b^2 + 6*sqrt(2)*(b*c^3)^(1/4)*arctan(-1/2*sqrt(2
)*(sqrt(2)*(b/c)^(1/4) - 2*(x^n)^(1/4))/(b/c)^(1/4))/b^2 + 3*sqrt(2)*(b*c^
3)^(1/4)*log(x^(1/2*n) + sqrt(2)*(x^n)^(1/4)*(b/c)^(1/4) + sqrt(b/c))/b^2
- 3*sqrt(2)*(b*c^3)^(1/4)*log(x^(1/2*n) - sqrt(2)*(x^n)^(1/4)*(b/c)^(1/4)
+ sqrt(b/c))/b^2 + 8/(b*x^(3/4*n)))/n
```

3.500.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+\frac{n}{4}}}{bx^n + cx^{2n}}\,dx = \int \frac{x^{\frac{n}{4}-1}}{b\,x^n + c\,x^{2\,n}}\,dx$$

input
```
int(x^(n/4 - 1)/(b*x^n + c*x^(2*n)),x)
```

output
```
int(x^(n/4 - 1)/(b*x^n + c*x^(2*n)), x)
```

3.501 $\int \frac{x^{-1+\frac{n}{3}}}{bx^n+cx^{2n}}\,dx$

3.501.1 Optimal result

Integrand size = 25, antiderivative size = 160

$$\int \frac{x^{-1+\frac{n}{3}}}{bx^n+cx^{2n}}\,dx = -\frac{3x^{-2n/3}}{2bn}+\frac{\sqrt{3}c^{2/3}\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{c}x^{n/3}}{\sqrt{3}\sqrt[3]{b}}\right)}{b^{5/3}n} - \frac{c^{2/3}\log\left(\sqrt[3]{b}+\sqrt[3]{c}x^{n/3}\right)}{b^{5/3}n}+\frac{c^{2/3}\log\left(b^{2/3}-\sqrt[3]{b}\sqrt[3]{c}x^{n/3}+c^{2/3}x^{2n/3}\right)}{2b^{5/3}n}$$

output
```
-3/2/b/n/(x^(2/3*n))-c^(2/3)*ln(b^(1/3)+c^(1/3)*x^(1/3*n))/b^(5/3)/n+1/2*c
^(2/3)*ln(b^(2/3)-b^(1/3)*c^(1/3)*x^(1/3*n)+c^(2/3)*x^(2/3*n))/b^(5/3)/n+c
^(2/3)*arctan(1/3*(b^(1/3)-2*c^(1/3)*x^(1/3*n))/b^(1/3)*3^(1/2))*3^(1/2)/b
^(5/3)/n
```

3.501.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.21

$$\int \frac{x^{-1+\frac{n}{3}}}{bx^n+cx^{2n}}\,dx = -\frac{3x^{-2n/3}\,\text{Hypergeometric2F1}\left(-\frac{2}{3},1,\frac{1}{3},-\frac{cx^n}{b}\right)}{2bn}$$

input
```
Integrate[x^(-1 + n/3)/(b*x^n + c*x^(2*n)),x]
```

output
```
(-3*Hypergeometric2F1[-2/3, 1, 1/3, -((c*x^n)/b)])/(2*b*n*x^((2*n)/3))
```

3.501.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {10, 886, 868, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{\frac{n}{3}-1}}{bx^n+cx^{2n}}dx$$

$$\Big\downarrow \text{ 10}$$

$$\int \frac{x^{-\frac{2n}{3}-1}}{b+cx^n}dx$$

$$\Big\downarrow \text{ 886}$$

$$-\frac{c\int \frac{x^{\frac{n-3}{3}}}{cx^n+b}dx}{b}-\frac{3x^{-2n/3}}{2bn}$$

$$\Big\downarrow \text{ 868}$$

$$-\frac{3c\int \frac{1}{cx^n+b}dx^{n/3}}{bn}-\frac{3x^{-2n/3}}{2bn}$$

$$\Big\downarrow \text{ 750}$$

$$-\frac{3c\left(\frac{\int \frac{2\sqrt[3]{b}-\sqrt[3]{c}x^{n/3}}{-\sqrt[3]{b}\sqrt[3]{c}x^{n/3}+c^{2/3}x^{2n/3}+b^{2/3}}dx^{n/3}}{3b^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{c}x^{n/3}+\sqrt[3]{b}}dx^{n/3}}{3b^{2/3}}\right)}{bn}-\frac{3x^{-2n/3}}{2bn}$$

$$\Big\downarrow \text{ 16}$$

$$-\frac{3c\left(\frac{\int \frac{2\sqrt[3]{b}-\sqrt[3]{c}x^{n/3}}{-\sqrt[3]{b}\sqrt[3]{c}x^{n/3}+c^{2/3}x^{2n/3}+b^{2/3}}dx^{n/3}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{b}+\sqrt[3]{c}x^{n/3}\right)}{3b^{2/3}\sqrt[3]{c}}\right)}{bn}-\frac{3x^{-2n/3}}{2bn}$$

$$\Big\downarrow \text{ 1142}$$

$$-\frac{3c\left(\frac{\frac{3}{2}\sqrt[3]{b}\int \frac{1}{-\sqrt[3]{b}\sqrt[3]{c}x^{n/3}+c^{2/3}x^{2n/3}+b^{2/3}}dx^{n/3}-\frac{\int -\frac{\sqrt[3]{c}\left(\sqrt[3]{b}-2\sqrt[3]{c}x^{n/3}\right)}{-\sqrt[3]{b}\sqrt[3]{c}x^{n/3}+c^{2/3}x^{2n/3}+b^{2/3}}dx^{n/3}}{2\sqrt[3]{c}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{b}+\sqrt[3]{c}x^{n/3}\right)}{3b^{2/3}\sqrt[3]{c}}\right)}{bn}-\frac{3x^{-2n/3}}{2bn}$$

$\big\downarrow$ 25

$$-\frac{3c\left(\frac{\frac{3}{2}\sqrt[3]{b}\int \frac{1}{-\sqrt[3]{b}\sqrt[3]{c}x^{n/3}+c^{2/3}x^{2n/3}+b^{2/3}}dx^{n/3}+\frac{\int \frac{\sqrt[3]{c}\left(\sqrt[3]{b}-2\sqrt[3]{c}x^{n/3}\right)}{-\sqrt[3]{b}\sqrt[3]{c}x^{n/3}+c^{2/3}x^{2n/3}+b^{2/3}}dx^{n/3}}{2\sqrt[3]{c}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{b}+\sqrt[3]{c}x^{n/3}\right)}{3b^{2/3}\sqrt[3]{c}}\right)}{bn}-\frac{3x^{-2n/3}}{2bn}$$

$\big\downarrow$ 27

$$-\frac{3c\left(\frac{\frac{3}{2}\sqrt[3]{b}\int \frac{1}{-\sqrt[3]{b}\sqrt[3]{c}x^{n/3}+c^{2/3}x^{2n/3}+b^{2/3}}dx^{n/3}+\frac{1}{2}\int \frac{\sqrt[3]{b}-2\sqrt[3]{c}x^{n/3}}{-\sqrt[3]{b}\sqrt[3]{c}x^{n/3}+c^{2/3}x^{2n/3}+b^{2/3}}dx^{n/3}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{b}+\sqrt[3]{c}x^{n/3}\right)}{3b^{2/3}\sqrt[3]{c}}\right)}{bn}-\frac{3x^{-2n/3}}{2bn}$$

$\big\downarrow$ 1082

$$-\frac{3c\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{b}-2\sqrt[3]{c}x^{n/3}}{-\sqrt[3]{b}\sqrt[3]{c}x^{n/3}+c^{2/3}x^{2n/3}+b^{2/3}}dx^{n/3}+\frac{3\int \frac{1}{-x^{2n/3}-3}d\left(1-\frac{2\sqrt[3]{c}x^{n/3}}{\sqrt[3]{b}}\right)}{\sqrt[3]{c}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{b}+\sqrt[3]{c}x^{n/3}\right)}{3b^{2/3}\sqrt[3]{c}}\right)}{bn}-\frac{3x^{-2n/3}}{2bn}$$

$\big\downarrow$ 217

3.501. $\int \frac{x^{-1+\frac{n}{3}}}{bx^n+cx^{2n}}\,dx$

$$-\frac{3c\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{b}-2\sqrt[3]{c}x^{n/3}}{-\sqrt[3]{b}\sqrt[3]{c}x^{n/3}+c^{2/3}x^{2n/3}+b^{2/3}}dx^{n/3}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{c}x^{n/3}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt[3]{c}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{b}+\sqrt[3]{c}x^{n/3}\right)}{3b^{2/3}\sqrt[3]{c}}\right)}{bn}-\frac{3x^{-2n/3}}{2bn}$$

$$\Big\downarrow \; 1103$$

$$-\frac{3c\left(\frac{-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{c}x^{n/3}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt[3]{c}}-\frac{\log\left(b^{2/3}-\sqrt[3]{b}\sqrt[3]{c}x^{n/3}+c^{2/3}x^{2n/3}\right)}{2\sqrt[3]{c}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{b}+\sqrt[3]{c}x^{n/3}\right)}{3b^{2/3}\sqrt[3]{c}}\right)}{bn}-\frac{3x^{-2n/3}}{2bn}$$

input
```
Int[x^(-1 + n/3)/(b*x^n + c*x^(2*n)),x]
```

output
```
-3/(2*b*n*x^((2*n)/3)) - (3*c*(Log[b^(1/3) + c^(1/3)*x^(n/3)]/(3*b^(2/3)*c
^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*c^(1/3)*x^(n/3))/b^(1/3))/Sqrt[3]])/c
^(1/3)) - Log[b^(2/3) - b^(1/3)*c^(1/3)*x^(n/3) + c^(2/3)*x^((2*n)/3)]/(2*
c^(1/3)))/(3*b^(2/3))))/(b*n)
```

3.501.3.1 Defintions of rubi rules used

rule 10
```
Int[(u_.)*((e_.)*(x_))^(m_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x
_Symbol] :> Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*(a + b*x^(s - r))^p, x],
 x] /; FreeQ[{a, b, e, m, r, s}, x] && IntegerQ[p] && (IntegerQ[p*r] || GtQ
[e, 0]) && PosQ[s - r]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

3.501. $\int \frac{x^{-1+\frac{n}{3}}}{bx^n+cx^{2n}}\,dx$

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 868
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(m + 1)
 Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[
{a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

rule 886
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[x^(m + 1)/(a*(m +
 1)), x] - Simp[b/a   Int[x^Simplify[m + n]/(a + b*x^n), x], x] /; FreeQ[{a
, b, m, n}, x] && FractionQ[Simplify[(m + 1)/n]] && SumSimplerQ[m, n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.501. $\int \frac{x^{-1+\frac{n}{3}}}{bx^n+cx^{2n}}\,dx$

3.501.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.67 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.34

method	result	size
risch	$-\frac{3x^{-\frac{2n}{3}}}{2bn}+\left(\sum_{\_R=\mathrm{RootOf}\left(b^{5}n^{3}\_Z^{3}+c^{2}\right)}\_R\ln\left(x^{\frac{n}{3}}-\frac{b^{2}n\_R}{c}\right)\right)$	54

input
```
int(x^(-1+1/3*n)/(b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

output
```
-3/2/b/n/(x^(1/3*n))^2+sum(_R*ln(x^(1/3*n)-b^2*n/c*_R),_R=RootOf(_Z^3*b^5*
n^3+c^2))
```

3.501.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.32

$$\int \frac{x^{-1+\frac{n}{3}}}{bx^{n}+cx^{2n}}\,dx = \frac{2\sqrt{3}x^{2}x^{\frac{2}{3}n-2}\left(-\frac{c^{2}}{b^{2}}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bxx^{\frac{1}{3}n-1}\left(-\frac{c^{2}}{b^{2}}\right)^{\frac{2}{3}}-\sqrt{3}c}{3c}\right)+2x^{2}x^{\frac{2}{3}n-2}\left(-\frac{c^{2}}{b^{2}}\right)^{\frac{1}{3}}\log\left(\frac{cxx^{\frac{1}{3}n-1}-b\left(-\frac{c^{2}}{b^{2}}\right)^{\frac{1}{3}}}{x}\right)-x^{2}\cdots}{2bnx^{2}x^{\frac{2}{3}n-2}}$$

input
```
integrate(x^(-1+1/3*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
1/2*(2*sqrt(3)*x^2*x^(2/3*n - 2)*(-c^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*
x*x^(1/3*n - 1)*(-c^2/b^2)^(2/3) - sqrt(3)*c)/c) + 2*x^2*x^(2/3*n - 2)*(-c
^2/b^2)^(1/3)*log((c*x*x^(1/3*n - 1) - b*(-c^2/b^2)^(1/3))/x) - x^2*x^(2/3
*n - 2)*(-c^2/b^2)^(1/3)*log((c^2*x^2*x^(2/3*n - 2) + b*c*x*x^(1/3*n - 1)*
(-c^2/b^2)^(1/3) + b^2*(-c^2/b^2)^(2/3))/x^2) - 3)/(b*n*x^2*x^(2/3*n - 2))
```

3.501.6 Sympy [F]

$$\int \frac{x^{-1+\frac{n}{3}}}{bx^n + cx^{2n}}\,dx = \int \frac{x^{-n}x^{\frac{n}{3}-1}}{b + cx^n}\,dx$$

input
```
integrate(x**(-1+1/3*n)/(b*x**n+c*x**(2*n)),x)
```

output
```
Integral(x**(n/3 - 1)/(x**n*(b + c*x**n)), x)
```

3.501.7 Maxima [F]

$$\int \frac{x^{-1+\frac{n}{3}}}{bx^n + cx^{2n}}\,dx = \int \frac{x^{\frac{1}{3}\,n-1}}{cx^{2\,n} + bx^n}\,dx$$

input
```
integrate(x^(-1+1/3*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
-c*integrate(x^(1/3*n)/(b*c*x*x^n + b^2*x), x) - 3/2/(b*n*x^(2/3*n))
```

3.501.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.85

$$\int \frac{x^{-1+\frac{n}{3}}}{bx^n + cx^{2n}}\,dx$$
$$= \frac{\frac{2\,c\left(-\frac{b}{c}\right)^{\frac{1}{3}}\log\left(\left|x^{\frac{1}{3}\,n}-\left(-\frac{b}{c}\right)^{\frac{1}{3}}\right|\right)}{b^2} - \frac{2\,\sqrt{3}\left(-bc^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}\,n}+\left(-\frac{b}{c}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{b}{c}\right)^{\frac{1}{3}}}\right)}{b^2} - \frac{\left(-bc^2\right)^{\frac{1}{3}}\log\left(x^{\frac{1}{3}\,n}\left(-\frac{b}{c}\right)^{\frac{1}{3}}+\left(x^n\right)^{\frac{2}{3}}+\left(-\frac{b}{c}\right)^{\frac{2}{3}}\right)}{b^2} - \frac{\ }{b(x}}{2\,n}$$

input
```
integrate(x^(-1+1/3*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
1/2*(2*c*(-b/c)^(1/3)*log(abs(x^(1/3*n) - (-b/c)^(1/3)))/b^2 - 2*sqrt(3)*(
-b*c^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x^(1/3*n) + (-b/c)^(1/3))/(-b/c)^(1/3)
)/b^2 - (-b*c^2)^(1/3)*log(x^(1/3*n)*(-b/c)^(1/3) + (x^n)^(2/3) + (-b/c)^(
2/3))/b^2 - 3/(b*(x^n)^(2/3)))/n
```

3.501.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+\frac{n}{3}}}{bx^n + cx^{2n}}\,dx = \int \frac{x^{\frac{n}{3}-1}}{b\,x^n + c\,x^{2\,n}}\,dx$$

input
```
int(x^(n/3 - 1)/(b*x^n + c*x^(2*n)),x)
```

output
```
int(x^(n/3 - 1)/(b*x^n + c*x^(2*n)), x)
```

3.502 $\int \frac{x^{-1+\frac{n}{2}}}{bx^n+cx^{2n}}\,dx$

3.502.1 Optimal result

Integrand size = 25, antiderivative size = 50

$$\int \frac{x^{-1+\frac{n}{2}}}{bx^n+cx^{2n}}\,dx=-\frac{2x^{-n/2}}{bn}+\frac{2\sqrt{c}\arctan\left(\frac{\sqrt{b}x^{-n/2}}{\sqrt{c}}\right)}{b^{3/2}n}$$

output
```
-2/b/n/(x^(1/2*n))+2*arctan(b^(1/2)/(x^(1/2*n))/c^(1/2))*c^(1/2)/b^(3/2)/n
```

3.502.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.64

$$\int \frac{x^{-1+\frac{n}{2}}}{bx^n+cx^{2n}}\,dx=-\frac{2x^{-n/2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\frac{cx^n}{b}\right)}{bn}$$

input
```
Integrate[x^(-1 + n/2)/(b*x^n + c*x^(2*n)),x]
```

output
```
(-2*Hypergeometric2F1[-1/2, 1, 1/2, -((c*x^n)/b)])/(b*n*x^(n/2))
```

3.502.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {10, 868, 772, 262, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{\frac{n}{2}-1}}{bx^n+cx^{2n}}\,dx$$

$$\Big\downarrow \; 10$$

$$\int \frac{x^{-\frac{n}{2}-1}}{b+cx^n}dx$$

$$\Big\downarrow \; 868$$

$$-\frac{2\int \frac{1}{cx^n+b}dx^{-n/2}}{n}$$

$$\Big\downarrow \; 772$$

$$-\frac{2\int \frac{x^{-n}}{bx^{-n}+c}dx^{-n/2}}{n}$$

$$\Big\downarrow \; 262$$

$$-\frac{2\left(\frac{x^{-n/2}}{b}-\frac{c\int \frac{1}{bx^{-n}+c}dx^{-n/2}}{b}\right)}{n}$$

$$\Big\downarrow \; 218$$

$$-\frac{2\left(\frac{x^{-n/2}}{b}-\frac{\sqrt{c}\arctan\left(\frac{\sqrt{b}x^{-n/2}}{\sqrt{c}}\right)}{b^{3/2}}\right)}{n}$$

input
```
Int[x^(-1 + n/2)/(b*x^n + c*x^(2*n)),x]
```

output
```
(-2*(1/(b*x^(n/2)) - (Sqrt[c]*ArcTan[Sqrt[b]/(Sqrt[c]*x^(n/2))])/b^(3/2)))
/n
```

3.502.3.1 Defintions of rubi rules used

rule 10
```
Int[(u_.)*((e_.)*(x_))^(m_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x
_Symbol] :> Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*(a + b*x^(s - r))^p, x],
 x] /; FreeQ[{a, b, e, m, r, s}, x] && IntegerQ[p] && (IntegerQ[p*r] || GtQ
[e, 0]) && PosQ[s - r]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 772
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && ILtQ[n, 0] && IntegerQ[p]
```

rule 868
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(m + 1)
 Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[
{a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

3.502.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.58

method	result	size
risch	$-\frac{2x^{-\frac{n}{2}}}{bn}+\frac{\sqrt{-bc}\,\ln\left(x^{\frac{n}{2}}-\frac{\sqrt{-bc}}{c}\right)}{b^2n}-\frac{\sqrt{-bc}\,\ln\left(x^{\frac{n}{2}}+\frac{\sqrt{-bc}}{c}\right)}{b^2n}$	79

input
```
int(x^(-1+1/2*n)/(b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

output
```
-2/b/n/(x^(1/2*n))+1/b^2*(-b*c)^(1/2)/n*ln(x^(1/2*n)-1/c*(-b*c)^(1/2))-1/b
^2*(-b*c)^(1/2)/n*ln(x^(1/2*n)+1/c*(-b*c)^(1/2))
```

3.502.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 151, normalized size of antiderivative = 3.02

$$\int \frac{x^{-1+\frac{n}{2}}}{bx^n+cx^{2n}}\,dx = \left[\frac{xx^{\frac{1}{2}n-1}\sqrt{-\frac{c}{b}}\log\left(\frac{cx^2x^{n-2}-2\,bxx^{\frac{1}{2}n-1}\sqrt{-\frac{c}{b}}-b}{cx^2x^{n-2}+b}\right)-2}{bnxx^{\frac{1}{2}n-1}},\ \frac{2\left(xx^{\frac{1}{2}n-1}\sqrt{\frac{c}{b}}\arctan\left(\frac{b\sqrt{\frac{c}{b}}}{cxx^{\frac{1}{2}n-1}}\right)-1\right)}{bnxx^{\frac{1}{2}n-1}}\right]$$

input
```
integrate(x^(-1+1/2*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
[(x*x^(1/2*n - 1)*sqrt(-c/b)*log((c*x^2*x^(n - 2) - 2*b*x*x^(1/2*n - 1)*sq
rt(-c/b) - b)/(c*x^2*x^(n - 2) + b)) - 2)/(b*n*x*x^(1/2*n - 1)), 2*(x*x^(1
/2*n - 1)*sqrt(c/b)*arctan(b*sqrt(c/b)/(c*x*x^(1/2*n - 1))) - 1)/(b*n*x*x^
(1/2*n - 1))]
```

3.502.6 Sympy [F]

$$\int \frac{x^{-1+\frac{n}{2}}}{bx^n+cx^{2n}}\,dx = \int \frac{x^{-n}x^{\frac{n}{2}-1}}{b+cx^n}\,dx$$

input
```
integrate(x**(-1+1/2*n)/(b*x**n+c*x**(2*n)),x)
```

output
```
Integral(x**(n/2 - 1)/(x**n*(b + c*x**n)), x)
```

3.502.7 Maxima [F]

$$\int \frac{x^{-1+\frac{n}{2}}}{bx^n+cx^{2n}}\,dx = \int \frac{x^{\frac{1}{2}n-1}}{cx^{2n}+bx^n}\,dx$$

input
```
integrate(x^(-1+1/2*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
-c*integrate(x^(1/2*n)/(b*c*x*x^n + b^2*x), x) - 2/(b*n*x^(1/2*n))
```

3.502.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.76

$$\int \frac{x^{-1+\frac{n}{2}}}{bx^n + cx^{2n}}\,dx = -\frac{2\left(\frac{c\arctan\left(\frac{c\sqrt{x^n}}{\sqrt{bc}}\right)}{\sqrt{bc}b} + \frac{1}{b\sqrt{x^n}}\right)}{n}$$

input
```
integrate(x^(-1+1/2*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
-2*(c*arctan(c*sqrt(x^n)/sqrt(b*c))/(sqrt(b*c)*b) + 1/(b*sqrt(x^n)))/n
```

3.502.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+\frac{n}{2}}}{bx^n + cx^{2n}}\,dx = \int \frac{x^{\frac{n}{2}-1}}{b\,x^n + c\,x^{2\,n}}\,dx$$

input
```
int(x^(n/2 - 1)/(b*x^n + c*x^(2*n)),x)
```

output
```
int(x^(n/2 - 1)/(b*x^n + c*x^(2*n)), x)
```

3.503 $\int \frac{x^{-1-\frac{n}{2}}}{bx^n+cx^{2n}}\,dx$

3.503.1 Optimal result

Integrand size = 25, antiderivative size = 68

$$\int \frac{x^{-1-\frac{n}{2}}}{bx^n+cx^{2n}}\,dx=-\frac{2x^{-3n/2}}{3bn}+\frac{2cx^{-n/2}}{b^2n}-\frac{2c^{3/2}\arctan\left(\frac{\sqrt{b}x^{-n/2}}{\sqrt{c}}\right)}{b^{5/2}n}$$

output
```
-2/3/b/n/(x^(3/2*n))+2*c/b^2/n/(x^(1/2*n))-2*c^(3/2)*arctan(b^(1/2)/(x^(1/
2*n))/c^(1/2))/b^(5/2)/n
```

3.503.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.50

$$\int \frac{x^{-1-\frac{n}{2}}}{bx^n+cx^{2n}}\,dx=-\frac{2x^{-3n/2}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\frac{cx^n}{b}\right)}{3bn}$$

input
```
Integrate[x^(-1 - n/2)/(b*x^n + c*x^(2*n)),x]
```

output
```
(-2*Hypergeometric2F1[-3/2, 1, -1/2, -((c*x^n)/b)])/(3*b*n*x^((3*n)/2))
```

3.503.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {10, 886, 868, 772, 262, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-\frac{n}{2}-1}}{bx^n+cx^{2n}}dx$$

$$\Big\downarrow \text{10}$$

$$\int \frac{x^{-\frac{3n}{2}-1}}{b+cx^n}dx$$

$$\Big\downarrow \text{886}$$

$$-\frac{c\int \frac{x^{-\frac{n}{2}-1}}{cx^n+b}dx}{b}-\frac{2x^{-3n/2}}{3bn}$$

$$\Big\downarrow \text{868}$$

$$\frac{2c\int \frac{1}{cx^n+b}dx^{-n/2}}{bn}-\frac{2x^{-3n/2}}{3bn}$$

$$\Big\downarrow \text{772}$$

$$\frac{2c\int \frac{x^{-n}}{bx^{-n}+c}dx^{-n/2}}{bn}-\frac{2x^{-3n/2}}{3bn}$$

$$\Big\downarrow \text{262}$$

$$\frac{2c\left(\frac{x^{-n/2}}{b}-\frac{c\int \frac{1}{bx^{-n}+c}dx^{-n/2}}{b}\right)}{bn}-\frac{2x^{-3n/2}}{3bn}$$

$$\Big\downarrow \text{218}$$

$$\frac{2c\left(\frac{x^{-n/2}}{b}-\frac{\sqrt{c}\arctan\left(\frac{\sqrt{b}x^{-n/2}}{\sqrt{c}}\right)}{b^{3/2}}\right)}{bn}-\frac{2x^{-3n/2}}{3bn}$$

input
```
Int[x^(-1 - n/2)/(b*x^n + c*x^(2*n)),x]
```

output
```
-2/(3*b*n*x^((3*n)/2)) + (2*c*(1/(b*x^(n/2)) - (Sqrt[c]*ArcTan[Sqrt[b]/(Sq
rt[c]*x^(n/2))])/b^(3/2)))/(b*n)
```

3.503.3.1 Defintions of rubi rules used

rule 10
```
Int[(u_.)*((e_.)*(x_))^(m_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x
_Symbol] :> Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*(a + b*x^(s - r))^p, x],
 x] /; FreeQ[{a, b, e, m, r, s}, x] && IntegerQ[p] && (IntegerQ[p*r] || GtQ
[e, 0]) && PosQ[s - r]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 772
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && ILtQ[n, 0] && IntegerQ[p]
```

rule 868
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(m + 1)
 Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[
{a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

rule 886
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[x^(m + 1)/(a*(m +
 1)), x] - Simp[b/a   Int[x^Simplify[m + n]/(a + b*x^n), x], x] /; FreeQ[{a
, b, m, n}, x] && FractionQ[Simplify[(m + 1)/n]] && SumSimplerQ[m, n]
```

3.503.4 Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.43

method	result	size
risch	$\frac{2c\,x^{-\frac{n}{2}}}{b^{2} n}-\frac{2x^{-\frac{3n}{2}}}{3bn}+\frac{\sqrt{-bc}\,c\ln\left(x^{\frac{n}{2}}+\frac{\sqrt{-bc}}{c}\right)}{b^{3} n}-\frac{\sqrt{-bc}\,c\ln\left(x^{\frac{n}{2}}-\frac{\sqrt{-bc}}{c}\right)}{b^{3} n}$	97

input
```
int(x^(-1-1/2*n)/(b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

output
```
2*c/b^2/n/(x^(1/2*n))-2/3/b/n/(x^(1/2*n))^3+1/b^3*(-b*c)^(1/2)*c/n*ln(x^(1
/2*n)+1/c*(-b*c)^(1/2))-1/b^3*(-b*c)^(1/2)*c/n*ln(x^(1/2*n)-1/c*(-b*c)^(1/
2))
```

3.503.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.37

$$\int \frac{x^{-1-\frac{n}{2}}}{bx^n + cx^{2n}}\,dx = \left[-\frac{2\,bx^3x^{-\frac{3}{2}\,n-3} - 6\,cxx^{-\frac{1}{2}\,n-1} - 3\,c\sqrt{-\frac{c}{b}}\log\left(\frac{bx^2x^{-n-2}-2\,bxx^{-\frac{1}{2}\,n-1}\sqrt{-\frac{c}{b}}-c}{bx^2x^{-n-2}+c}\right)}{3\,b^2n},\ -\frac{2\left(bx^3x^{-\frac{3}{2}\,n-3} - 3\,cxx^{-\frac{1}{2}\,n-1} - 3\,c\sqrt{\frac{c}{b}}\arctan\left(\frac{\sqrt{\frac{c}{b}}}{xx^{-\frac{1}{2}\,n-1}}\right)\right)}{3\,b^2n}\right]$$

input
```
integrate(x^(-1-1/2*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
[-1/3*(2*b*x^3*x^(-3/2*n - 3) - 6*c*x*x^(-1/2*n - 1) - 3*c*sqrt(-c/b)*log(
(b*x^2*x^(-n - 2) - 2*b*x*x^(-1/2*n - 1)*sqrt(-c/b) - c)/(b*x^2*x^(-n - 2)
 + c)))/(b^2*n), -2/3*(b*x^3*x^(-3/2*n - 3) - 3*c*x*x^(-1/2*n - 1) - 3*c*s
qrt(c/b)*arctan(sqrt(c/b)/(x*x^(-1/2*n - 1))))/(b^2*n)]
```

3.503.6 Sympy [F]

$$\int \frac{x^{-1-\frac{n}{2}}}{bx^n + cx^{2n}}\,dx = \int \frac{x^{-n}x^{-\frac{n}{2}-1}}{b + cx^n}\,dx$$

input
```
integrate(x**(-1-1/2*n)/(b*x**n+c*x**(2*n)),x)
```

output
```
Integral(x**(-n/2 - 1)/(x**n*(b + c*x**n)), x)
```

3.503.7 Maxima [F]

$$\int \frac{x^{-1-\frac{n}{2}}}{bx^n + cx^{2n}}\,dx = \int \frac{x^{-\frac{1}{2}\,n-1}}{cx^{2\,n} + bx^n}\,dx$$

input
```
integrate(x^(-1-1/2*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
c^2*integrate(x^(1/2*n)/(b^2*c*x*x^n + b^3*x), x) + 2/3*(3*c*x^n - b)/(b^2
*n*x^(3/2*n))
```

3.503.8 Giac [F]

$$\int \frac{x^{-1-\frac{n}{2}}}{bx^n + cx^{2n}}\,dx = \int \frac{x^{-\frac{1}{2}\,n-1}}{cx^{2\,n} + bx^n}\,dx$$

input
```
integrate(x^(-1-1/2*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate(x^(-1/2*n - 1)/(c*x^(2*n) + b*x^n), x)
```

3.503.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-\frac{n}{2}}}{bx^n + cx^{2n}}\,dx = \int \frac{1}{x^{\frac{n}{2}+1}\,(b\,x^n + c\,x^{2\,n})}\,dx$$

input
```
int(1/(x^(n/2 + 1)*(b*x^n + c*x^(2*n))),x)
```

output
```
int(1/(x^(n/2 + 1)*(b*x^n + c*x^(2*n))), x)
```

3.504 $\int \frac{x^{-1-\frac{n}{3}}}{bx^n+cx^{2n}}\,dx$

3.504.1 Optimal result

Integrand size = 25, antiderivative size = 176

$$\int \frac{x^{-1-\frac{n}{3}}}{bx^n+cx^{2n}}\,dx=-\frac{3x^{-4n/3}}{4bn}+\frac{3cx^{-n/3}}{b^2n}+\frac{\sqrt{3}c^{4/3}\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{b}x^{-n/3}}{\sqrt{3}\sqrt[3]{c}}\right)}{b^{7/3}n}-\frac{c^{4/3}\log\left(\sqrt[3]{c}+\sqrt[3]{b}x^{-n/3}\right)}{b^{7/3}n}+\frac{c^{4/3}\log\left(c^{2/3}+b^{2/3}x^{-2n/3}-\sqrt[3]{b}\sqrt[3]{c}x^{-n/3}\right)}{2b^{7/3}n}$$

output
```
-3/4/b/n/(x^(4/3*n))+3*c/b^2/n/(x^(1/3*n))-c^(4/3)*ln(c^(1/3)+b^(1/3)/(x^(
1/3*n)))/b^(7/3)/n+1/2*c^(4/3)*ln(c^(2/3)+b^(2/3)/(x^(2/3*n))-b^(1/3)*c^(1
/3)/(x^(1/3*n)))/b^(7/3)/n+c^(4/3)*arctan(1/3*(1-2*b^(1/3)/c^(1/3)/(x^(1/3
*n)))*3^(1/2))*3^(1/2)/b^(7/3)/n
```

3.504.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.19

$$\int \frac{x^{-1-\frac{n}{3}}}{bx^n+cx^{2n}}\,dx=-\frac{3x^{-4n/3}\,\text{Hypergeometric2F1}\left(-\frac{4}{3},1,-\frac{1}{3},-\frac{cx^n}{b}\right)}{4bn}$$

input
```
Integrate[x^(-1 - n/3)/(b*x^n + c*x^(2*n)),x]
```

output
```
(-3*Hypergeometric2F1[-4/3, 1, -1/3, -((c*x^n)/b)])/(4*b*n*x^((4*n)/3))
```

3.504.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.01, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {10, 886, 868, 772, 843, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-\frac{n}{3}-1}}{bx^n+cx^{2n}}dx$$

$$\Big\downarrow \ 10$$

$$\int \frac{x^{-\frac{4n}{3}-1}}{b+cx^n}dx$$

$$\Big\downarrow \ 886$$

$$-\frac{c\int \frac{x^{-\frac{n}{3}-1}}{cx^n+b}dx}{b}-\frac{3x^{-4n/3}}{4bn}$$

$$\Big\downarrow \ 868$$

$$\frac{3c\int \frac{1}{cx^n+b}dx^{-n/3}}{bn}-\frac{3x^{-4n/3}}{4bn}$$

$$\Big\downarrow \ 772$$

$$\frac{3c\int \frac{x^{-n}}{bx^{-n}+c}dx^{-n/3}}{bn}-\frac{3x^{-4n/3}}{4bn}$$

$$\Big\downarrow \ 843$$

$$\frac{3c\left(\frac{x^{-n/3}}{b}-\frac{c\int \frac{1}{bx^{-n}+c}dx^{-n/3}}{b}\right)}{bn}-\frac{3x^{-4n/3}}{4bn}$$

$$\Big\downarrow \ 750$$

$$\frac{3c\left(\frac{x^{-n/3}}{b}-\frac{c\left(\frac{\int \frac{2\sqrt[3]{c}-\sqrt[3]{b}x^{-n/3}}{b^{2/3}x^{-2n/3}-\sqrt[3]{b}\sqrt[3]{c}x^{-n/3}+c^{2/3}}dx^{-n/3}}{3c^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x^{-n/3}+\sqrt[3]{c}}dx^{-n/3}}{3c^{2/3}}\right)}{b}\right)}{bn}-\frac{3x^{-4n/3}}{4bn}$$

$$\Big\downarrow \ 16$$

$$\frac{3c\left(\frac{x^{-n/3}}{b}-\frac{c\left(\frac{\int \frac{2\sqrt[3]{c}-\sqrt[3]{b}x^{-n/3}}{b^{2/3}x^{-2n/3}-\sqrt[3]{b}\sqrt[3]{c}x^{-n/3}+c^{2/3}}dx^{-n/3}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{b}x^{-n/3}+\sqrt[3]{c}\right)}{3\sqrt[3]{b}c^{2/3}}\right)}{b}\right)}{bn}-\frac{3x^{-4n/3}}{4bn}$$

$\Big\downarrow$ 1142

$$\frac{3c\left(\frac{x^{-n/3}}{b}-\frac{c\left(\frac{\frac{3}{2}\sqrt[3]{c}\int \frac{1}{b^{2/3}x^{-2n/3}-\sqrt[3]{b}\sqrt[3]{c}x^{-n/3}+c^{2/3}}dx^{-n/3}-\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{c}-2\sqrt[3]{b}x^{-n/3}\right)}{b^{2/3}x^{-2n/3}-\sqrt[3]{b}\sqrt[3]{c}x^{-n/3}+c^{2/3}}dx^{-n/3}}{2\sqrt[3]{b}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{b}x^{-n/3}+\sqrt[3]{c}\right)}{3\sqrt[3]{b}c^{2/3}}\right)}{b}\right)}{bn}$$

$$\frac{3x^{-4n/3}}{4bn}$$

$\Big\downarrow$ 25

3.504. $\int \frac{x^{-1-\frac{n}{3}}}{bx^n+cx^{2n}}\,dx$

$$\frac{3c\left(\frac{x^{-n/3}}{b}-\frac{c\left(\frac{\frac{3}{2}\sqrt[3]{c}\int\frac{1}{b^{2/3}x^{-2n/3}-\sqrt[3]{b}\sqrt[3]{c}x^{-n/3}+c^{2/3}}dx^{-n/3}+\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{c}-2\sqrt[3]{b}x^{-n/3}\right)}{b^{2/3}x^{-2n/3}-\sqrt[3]{b}\sqrt[3]{c}x^{-n/3}+c^{2/3}}dx^{-n/3}}{2\sqrt[3]{b}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{b}x^{-n/3}+\sqrt[3]{c}\right)}{3\sqrt[3]{b}c^{2/3}}\right)}{b}\right)}{bn}-\frac{3x^{-4n/3}}{4bn}$$

$$\Big\downarrow 27$$

$$\frac{3c\left(\frac{x^{-n/3}}{b}-\frac{c\left(\frac{\frac{3}{2}\sqrt[3]{c}\int\frac{1}{b^{2/3}x^{-2n/3}-\sqrt[3]{b}\sqrt[3]{c}x^{-n/3}+c^{2/3}}dx^{-n/3}+\frac{1}{2}\int\frac{\sqrt[3]{c}-2\sqrt[3]{b}x^{-n/3}}{b^{2/3}x^{-2n/3}-\sqrt[3]{b}\sqrt[3]{c}x^{-n/3}+c^{2/3}}dx^{-n/3}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{b}x^{-n/3}+\sqrt[3]{c}\right)}{3\sqrt[3]{b}c^{2/3}}\right)}{b}\right)}{bn}-\frac{3x^{-4n/3}}{4bn}$$

$$\Big\downarrow 1082$$

3.504. $\int \frac{x^{-1-\frac{n}{3}}}{bx^n+cx^{2n}}\,dx$

$$\frac{3c\left(\frac{x^{-n/3}}{b}-\frac{c\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{c}-2\sqrt[3]{b}x^{-n/3}}{b^{2/3}x^{-2n/3}-\sqrt[3]{b}\sqrt[3]{c}x^{-n/3}+c^{2/3}}dx^{-n/3}+\frac{3\int \frac{1}{-x^{-2n/3}-3}d\left(1-\frac{2\sqrt[3]{b}x^{-n/3}}{\sqrt[3]{c}}\right)}{\sqrt[3]{b}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{b}x^{-n/3}+\sqrt[3]{c}\right)}{3\sqrt[3]{b}c^{2/3}}\right)}{b}\right)}{bn}-\frac{3x^{-4n/3}}{4bn}$$

$$\Big\downarrow \text{217}$$

$$\frac{3c\left(\frac{x^{-n/3}}{b}-\frac{c\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{c}-2\sqrt[3]{b}x^{-n/3}}{b^{2/3}x^{-2n/3}-\sqrt[3]{b}\sqrt[3]{c}x^{-n/3}+c^{2/3}}dx^{-n/3}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x^{-n/3}}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{b}x^{-n/3}+\sqrt[3]{c}\right)}{3\sqrt[3]{b}c^{2/3}}\right)}{b}\right)}{bn}-\frac{3x^{-4n/3}}{4bn}$$

$$\Big\downarrow \text{1103}$$

3.504. $\int \frac{x^{-1-\frac{n}{3}}}{bx^n+cx^{2n}}\,dx$

$$\frac{3c\left(\frac{x^{-n/3}}{b}-\frac{c\left(\frac{-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x^{-n/3}}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}-\frac{\log\left(b^{2/3}x^{-2n/3}-\sqrt[3]{b}\sqrt[3]{c}x^{-n/3}+c^{2/3}\right)}{2\sqrt[3]{b}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{b}x^{-n/3}+\sqrt[3]{c}\right)}{3\sqrt[3]{b}c^{2/3}}\right)}{b}\right)}{bn}-\frac{3x^{-4n/3}}{4bn}$$

input
```
Int[x^(-1 - n/3)/(b*x^n + c*x^(2*n)),x]
```

output
```
-3/(4*b*n*x^((4*n)/3)) + (3*c*(1/(b*x^(n/3)) - (c*(Log[c^(1/3) + b^(1/3)/x
^(n/3)]/(3*b^(1/3)*c^(2/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3))/(c^(1/3)
*x^(n/3)))/Sqrt[3]])/b^(1/3)) - Log[c^(2/3) + b^(2/3)/x^((2*n)/3) - (b^(1/
3)*c^(1/3))/x^(n/3)]/(2*b^(1/3)))/(3*c^(2/3))))/b))/(b*n)
```

3.504.3.1 Defintions of rubi rules used

rule 10
```
Int[(u_.)*((e_.)*(x_))^(m_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x
_Symbol] :> Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*(a + b*x^(s - r))^p, x],
 x] /; FreeQ[{a, b, e, m, r, s}, x] && IntegerQ[p] && (IntegerQ[p*r] || GtQ
[e, 0]) && PosQ[s - r]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 772
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && ILtQ[n, 0] && IntegerQ[p]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 868
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(m + 1)
 Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[
{a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

rule 886
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[x^(m + 1)/(a*(m +
 1)), x] - Simp[b/a   Int[x^Simplify[m + n]/(a + b*x^n), x], x] /; FreeQ[{a
, b, m, n}, x] && FractionQ[Simplify[(m + 1)/n]] && SumSimplerQ[m, n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.504. $\int \frac{x^{-1-\frac{n}{3}}}{bx^n+cx^{2n}}\,dx$

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.504.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.67 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.41

method	result	size
risch	$\frac{3c\,x^{-\frac{n}{3}}}{b^{2}n}-\frac{3x^{-\frac{4n}{3}}}{4bn}+\left(\sum_{\_R=\operatorname{RootOf}\left(b^{7}n^{3}\_Z^{3}+c^{4}\right)}\_R\ln\left(x^{\frac{n}{3}}+\frac{b^{5}n^{2}\_R^{2}}{c^{3}}\right)\right)$	73

input
```
int(x^(-1-1/3*n)/(b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

output
```
3*c/b^2/n/(x^(1/3*n))-3/4/b/n/(x^(1/3*n))^4+sum(_R*ln(x^(1/3*n)+b^5*n^2/c^
3*_R^2),_R=RootOf(_Z^3*b^7*n^3+c^4))
```

3.504.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.97

$$\int \frac{x^{-1-\frac{n}{3}}}{bx^n+cx^{2n}}\,dx = -\frac{3\,bx^{4}x^{-\frac{4}{3}\,n-4}-12\,cxx^{-\frac{1}{3}\,n-1}-4\,\sqrt{3}c\left(-\frac{c}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}bxx^{-\frac{1}{3}\,n-1}\left(-\frac{c}{b}\right)^{\frac{2}{3}}-\sqrt{3}c}{3\,c}\right)-4\,c\left(-\frac{c}{b}\right)^{\frac{1}{3}}\log\left(\frac{xx^{-\frac{1}{3}\,n-1}-\cdots}{x}\right.}{4\,b^{2}n}$$

input
```
integrate(x^(-1-1/3*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
-1/4*(3*b*x^4*x^(-4/3*n - 4) - 12*c*x*x^(-1/3*n - 1) - 4*sqrt(3)*c*(-c/b)^
(1/3)*arctan(1/3*(2*sqrt(3)*b*x*x^(-1/3*n - 1)*(-c/b)^(2/3) - sqrt(3)*c)/c
) - 4*c*(-c/b)^(1/3)*log((x*x^(-1/3*n - 1) - (-c/b)^(1/3))/x) + 2*c*(-c/b)
^(1/3)*log((x^2*x^(-2/3*n - 2) + x*x^(-1/3*n - 1)*(-c/b)^(1/3) + (-c/b)^(2
/3))/x^2))/(b^2*n)
```

3.504.6 Sympy [F]

$$\int \frac{x^{-1-\frac{n}{3}}}{bx^n+cx^{2n}}\,dx=\int \frac{x^{-n}x^{-\frac{n}{3}-1}}{b+cx^n}\,dx$$

input
```
integrate(x**(-1-1/3*n)/(b*x**n+c*x**(2*n)),x)
```

output
```
Integral(x**(-n/3 - 1)/(x**n*(b + c*x**n)), x)
```

3.504.7 Maxima [F]

$$\int \frac{x^{-1-\frac{n}{3}}}{bx^n+cx^{2n}}\,dx=\int \frac{x^{-\frac{1}{3}\,n-1}}{cx^{2\,n}+bx^n}\,dx$$

input
```
integrate(x^(-1-1/3*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
c^2*integrate(x^(2/3*n)/(b^2*c*x*x^n + b^3*x), x) + 3/4*(4*c*x^n - b)/(b^2
*n*x^(4/3*n))
```

3.504.8 Giac [F]

$$\int \frac{x^{-1-\frac{n}{3}}}{bx^n+cx^{2n}}\,dx=\int \frac{x^{-\frac{1}{3}\,n-1}}{cx^{2\,n}+bx^n}\,dx$$

input
```
integrate(x^(-1-1/3*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate(x^(-1/3*n - 1)/(c*x^(2*n) + b*x^n), x)
```

3.504.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-\frac{n}{3}}}{bx^n + cx^{2n}}\,dx = \int \frac{1}{x^{\frac{n}{3}+1}\,\left(b\,x^n + c\,x^{2\,n}\right)}\,dx$$

input
```
int(1/(x^(n/3 + 1)*(b*x^n + c*x^(2*n))),x)
```

output
```
int(1/(x^(n/3 + 1)*(b*x^n + c*x^(2*n))), x)
```

3.505 $\int \frac{x^{-1-\frac{n}{4}}}{bx^n+cx^{2n}}\,dx$

3.505.1 Optimal result

Integrand size = 25, antiderivative size = 252

$$\int \frac{x^{-1-\frac{n}{4}}}{bx^n+cx^{2n}}\,dx=-\frac{4x^{-5n/4}}{5bn}+\frac{4cx^{-n/4}}{b^2n}+\frac{\sqrt{2}c^{5/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{c}}\right)}{b^{9/4}n}-\frac{\sqrt{2}c^{5/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{c}}\right)}{b^{9/4}n}+\frac{c^{5/4}\log\left(\sqrt{c}+\sqrt{b}x^{-n/2}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}x^{-n/4}\right)}{\sqrt{2}b^{9/4}n}-\frac{c^{5/4}\log\left(\sqrt{c}+\sqrt{b}x^{-n/2}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}x^{-n/4}\right)}{\sqrt{2}b^{9/4}n}$$

output
```
-4/5/b/n/(x^(5/4*n))+4*c/b^2/n/(x^(1/4*n))+1/2*c^(5/4)*ln(-b^(1/4)*c^(1/4)
*2^(1/2)/(x^(1/4*n))+b^(1/2)/(x^(1/2*n))+c^(1/2))/b^(9/4)/n*2^(1/2)-1/2*c^
(5/4)*ln(b^(1/4)*c^(1/4)*2^(1/2)/(x^(1/4*n))+b^(1/2)/(x^(1/2*n))+c^(1/2))/
b^(9/4)/n*2^(1/2)+c^(5/4)*arctan(1-b^(1/4)*2^(1/2)/c^(1/4)/(x^(1/4*n)))*2^
(1/2)/b^(9/4)/n-c^(5/4)*arctan(1+b^(1/4)*2^(1/2)/c^(1/4)/(x^(1/4*n)))*2^(1
/2)/b^(9/4)/n
```

3.505.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.13

$$\int \frac{x^{-1-\frac{n}{4}}}{bx^n + cx^{2n}} \, dx = -\frac{4x^{-5n/4} \text{Hypergeometric2F1}\left(-\frac{5}{4}, 1, -\frac{1}{4}, -\frac{cx^n}{b}\right)}{5bn}$$

input
```
Integrate[x^(-1 - n/4)/(b*x^n + c*x^(2*n)),x]
```

output
```
(-4*Hypergeometric2F1[-5/4, 1, -1/4, -((c*x^n)/b)])/(5*b*n*x^((5*n)/4))
```

3.505.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.10, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {10, 886, 868, 772, 843, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-\frac{n}{4}-1}}{bx^n + cx^{2n}} \, dx$$

$$\Big\downarrow \text{10}$$

$$\int \frac{x^{-\frac{5n}{4}-1}}{b + cx^n} dx$$

$$\Big\downarrow \text{886}$$

$$-\frac{c \int \frac{x^{-\frac{n}{4}-1}}{cx^n+b} dx}{b} - \frac{4x^{-5n/4}}{5bn}$$

$$\Big\downarrow \text{868}$$

$$\frac{4c \int \frac{1}{cx^n+b} dx^{-n/4}}{bn} - \frac{4x^{-5n/4}}{5bn}$$

$$\Big\downarrow \text{772}$$

$$\frac{4c \int \frac{x^{-n}}{bx^{-n}+c} dx^{-n/4}}{bn} - \frac{4x^{-5n/4}}{5bn}$$

$$\Big\downarrow \text{843}$$

$$\frac{4c\left(\frac{x^{-n/4}}{b}-\frac{c\int \frac{1}{bx^{-n}+c}dx^{-n/4}}{b}\right)}{bn}-\frac{4x^{-5n/4}}{5bn}$$

$\Big\downarrow$ 755

$$\frac{4c\left(\frac{x^{-n/4}}{b}-\frac{c\left(\frac{\int \frac{\sqrt{c}-\sqrt{b}x^{-n/2}}{bx^{-n}+c}dx^{-n/4}}{2\sqrt{c}}+\frac{\int \frac{\sqrt{b}x^{-n/2}+\sqrt{c}}{bx^{-n}+c}dx^{-n/4}}{2\sqrt{c}}\right)}{b}\right)}{bn}-\frac{4x^{-5n/4}}{5bn}$$

$\Big\downarrow$ 1476

$$\frac{4c\left(\frac{x^{-n/4}}{b}-\frac{c\left(\frac{\int \frac{\sqrt{c}-\sqrt{b}x^{-n/2}}{bx^{-n}+c}dx^{-n/4}}{2\sqrt{c}}+\frac{\frac{\int \frac{1}{x^{-n/2}-\frac{\sqrt{2}\sqrt[4]{c}x^{-n/4}}{\sqrt[4]{b}}+\frac{\sqrt{c}}{\sqrt{b}}}dx^{-n/4}}{2\sqrt{b}}+\frac{\int \frac{1}{x^{-n/2}+\frac{\sqrt{2}\sqrt[4]{c}x^{-n/4}}{\sqrt[4]{b}}+\frac{\sqrt{c}}{\sqrt{b}}}dx^{-n/4}}{2\sqrt{b}}}{2\sqrt{c}}\right)}{b}\right)}{bn}-\frac{4x^{-5n/4}}{5bn}$$

$\Big\downarrow$ 1082

$$\frac{4c\left(\frac{x^{-n/4}}{b}-\frac{c\left(\frac{\frac{\int \frac{1}{-x^{-n/2}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int \frac{1}{-x^{-n/2}-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}+\frac{\int \frac{\sqrt{c}-\sqrt{b}x^{-n/2}}{bx^{-n}+c}dx^{-n/4}}{2\sqrt{c}}\right)}{b}\right)}{bn}-\frac{4x^{-5n/4}}{5bn}$$

$\Big\downarrow$ 217

3.505. $\int \frac{x^{-1-\frac{n}{4}}}{bx^n+cx^{2n}}\,dx$

$$\frac{4c\left(\frac{x^{-n/4}}{b}-\frac{c\left(\frac{\int\frac{\sqrt{c}-\sqrt{b}x^{-n/2}}{bx^{-n}+c}\,dx^{-n/4}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{b}\right)}{bn}-\frac{4x^{-5n/4}}{5bn}$$

$\downarrow$ 1479

$$\frac{4c\left(\frac{x^{-n/4}}{b}-\frac{c\left(\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{b}\left(x^{-n/2}-\frac{\sqrt{2}\sqrt[4]{c}x^{-n/4}}{\sqrt[4]{b}}+\frac{\sqrt{c}}{\sqrt{b}}\right)}dx^{-n/4}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x^{-n/4}+\sqrt[4]{c}\right)}{\sqrt[4]{b}\left(x^{-n/2}+\frac{\sqrt{2}\sqrt[4]{c}x^{-n/4}}{\sqrt[4]{b}}+\frac{\sqrt{c}}{\sqrt{b}}\right)}dx^{-n/4}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\cdots}{2\sqrt{c}}\cdots\right)}{b}\cdots\right)}{bn}\cdots$$

$$\frac{4x^{-5n/4}}{5bn}$$

$\downarrow$ 25

3.505. $\int \frac{x^{-1-\frac{n}{4}}}{bx^n+cx^{2n}}\,dx$

$$\frac{4c\left(\frac{x^{-n/4}}{b}-\frac{c\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{b}\left(x^{-n/2}-\frac{\sqrt{2}\sqrt[4]{c}x^{-n/4}}{\sqrt[4]{b}}+\frac{\sqrt{c}}{\sqrt{b}}\right)}dx^{-n/4}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x^{-n/4}+\sqrt[4]{c}\right)}{\sqrt[4]{b}\left(x^{-n/2}+\frac{\sqrt{2}\sqrt[4]{c}x^{-n/4}}{\sqrt[4]{b}}+\frac{\sqrt{c}}{\sqrt{b}}\right)}dx^{-n/4}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan}{}}{2\sqrt{c}}\right)}{b}\right)}{bn}$$

$$\frac{4x^{-5n/4}}{5bn}$$

$\downarrow$ 27

$$\frac{4c\left(\frac{x^{-n/4}}{b}-\frac{c\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{b}x^{-n/4}}{x^{-n/2}-\frac{\sqrt{2}\sqrt[4]{c}x^{-n/4}}{\sqrt[4]{b}}+\frac{\sqrt{c}}{\sqrt{b}}}dx^{-n/4}}{2\sqrt{2}\sqrt{b}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}+\sqrt[4]{c}}{x^{-n/2}+\frac{\sqrt{2}\sqrt[4]{c}x^{-n/4}}{\sqrt[4]{b}}+\frac{\sqrt{c}}{\sqrt{b}}}dx^{-n/4}}{2\sqrt{b}\sqrt[4]{c}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{b}\right)}{bn}$$

$$\frac{4x^{-5n/4}}{5bn}$$

$\downarrow$ 1103

3.505. $\int \frac{x^{-1-\frac{n}{4}}}{bx^n+cx^{2n}}\,dx$

$$\frac{4c\left(\frac{x^{-n/4}}{b}-\frac{c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}x^{-n/4}+\sqrt{b}x^{-n/2}+\sqrt{c}\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}x^{-n/4}+\sqrt{b}x^{-}\cdots\right.}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}\cdots}{2\sqrt{c}}\cdots\right)}{b}\cdots\right)}{bn}$$

$$\frac{4x^{-5n/4}}{5bn}$$

input
```
Int[x^(-1 - n/4)/(b*x^n + c*x^(2*n)),x]
```

output
```
-4/(5*b*n*x^((5*n)/4)) + (4*c*(1/(b*x^(n/4)) - (c*((-(ArcTan[1 - (Sqrt[2]*
b^(1/4))/(c^(1/4)*x^(n/4))]/(Sqrt[2]*b^(1/4)*c^(1/4))) + ArcTan[1 + (Sqrt[
2]*b^(1/4))/(c^(1/4)*x^(n/4))]/(Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[c]) + (-
1/2*Log[Sqrt[c] + Sqrt[b]/x^(n/2) - (Sqrt[2]*b^(1/4)*c^(1/4))/x^(n/4)]/(Sq
rt[2]*b^(1/4)*c^(1/4)) + Log[Sqrt[c] + Sqrt[b]/x^(n/2) + (Sqrt[2]*b^(1/4)*
c^(1/4))/x^(n/4)]/(2*Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[c])))/b))/(b*n)
```

3.505.3.1 Defintions of rubi rules used

rule 10
```
Int[(u_.)*((e_.)*(x_))^(m_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x
_Symbol] :> Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*(a + b*x^(s - r))^p, x],
 x] /; FreeQ[{a, b, e, m, r, s}, x] && IntegerQ[p] && (IntegerQ[p*r] || GtQ
[e, 0]) && PosQ[s - r]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 772
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && ILtQ[n, 0] && IntegerQ[p]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 868
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(m + 1)
 Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[
{a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

rule 886
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[x^(m + 1)/(a*(m +
 1)), x] - Simp[b/a   Int[x^Simplify[m + n]/(a + b*x^n), x], x] /; FreeQ[{a
, b, m, n}, x] && FractionQ[Simplify[(m + 1)/n]] && SumSimplerQ[m, n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

3.505. $\int \frac{x^{-1-\frac{n}{4}}}{bx^n+cx^{2n}}\,dx$

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.505.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.69 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.29

method	result	size
risch	$\frac{4c\,x^{-\frac{n}{4}}}{b^2 n}-\frac{4x^{-\frac{5n}{4}}}{5bn}+\left(\sum_{\_R=\operatorname{RootOf}\left(b^9n^4\_Z^4+c^5\right)}\_R\ln\left(x^{\frac{n}{4}}+\frac{b^7n^3\_R^3}{c^4}\right)\right)$	73

input
```
int(x^(-1-1/4*n)/(b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

output
```
4*c/b^2/n/(x^(1/4*n))-4/5/b/n/(x^(1/4*n))^5+sum(_R*ln(x^(1/4*n)+b^7*n^3/c^
4*_R^3),_R=RootOf(_Z^4*b^9*n^4+c^5))
```

3.505.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.99

$$\int \frac{x^{-1-\frac{n}{4}}}{bx^n+cx^{2n}}\,dx =$$

$$-\frac{4\,bx^5x^{-\frac{5}{4}\,n-5}+5\,b^2n\left(-\frac{c^5}{b^9n^4}\right)^{\frac{1}{4}}\log\left(\frac{b^2n\left(-\frac{c^5}{b^9n^4}\right)^{\frac{1}{4}}+cxx^{-\frac{1}{4}\,n-1}}{x}\right)-5\,b^2n\left(-\frac{c^5}{b^9n^4}\right)^{\frac{1}{4}}\log\left(-\frac{b^2n\left(-\frac{c^5}{b^9n^4}\right)^{\frac{1}{4}}-cxx^{-\frac{1}{4}}}{x}\right.}{}$$

input
```
integrate(x^(-1-1/4*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
-1/5*(4*b*x^5*x^(-5/4*n - 5) + 5*b^2*n*(-c^5/(b^9*n^4))^(1/4)*log((b^2*n*(
-c^5/(b^9*n^4))^(1/4) + c*x*x^(-1/4*n - 1))/x) - 5*b^2*n*(-c^5/(b^9*n^4))^
(1/4)*log(-(b^2*n*(-c^5/(b^9*n^4))^(1/4) - c*x*x^(-1/4*n - 1))/x) + 5*I*b^
2*n*(-c^5/(b^9*n^4))^(1/4)*log((I*b^2*n*(-c^5/(b^9*n^4))^(1/4) + c*x*x^(-1
/4*n - 1))/x) - 5*I*b^2*n*(-c^5/(b^9*n^4))^(1/4)*log((-I*b^2*n*(-c^5/(b^9*
n^4))^(1/4) + c*x*x^(-1/4*n - 1))/x) - 20*c*x*x^(-1/4*n - 1))/(b^2*n)
```

3.505.6 Sympy [F]

$$\int \frac{x^{-1-\frac{n}{4}}}{bx^n+cx^{2n}}\,dx=\int \frac{x^{-n}x^{-\frac{n}{4}-1}}{b+cx^n}\,dx$$

input
```
integrate(x**(-1-1/4*n)/(b*x**n+c*x**(2*n)),x)
```

output
```
Integral(x**(-n/4 - 1)/(x**n*(b + c*x**n)), x)
```

3.505.7 Maxima [F]

$$\int \frac{x^{-1-\frac{n}{4}}}{bx^n+cx^{2n}}\,dx=\int \frac{x^{-\frac{1}{4}n-1}}{cx^{2n}+bx^n}\,dx$$

input
```
integrate(x^(-1-1/4*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
c^2*integrate(x^(3/4*n)/(b^2*c*x*x^n + b^3*x), x) + 4/5*(5*c*x^n - b)/(b^2
*n*x^(5/4*n))
```

3.505.8 Giac [F]

$$\int \frac{x^{-1-\frac{n}{4}}}{bx^n + cx^{2n}}\,dx = \int \frac{x^{-\frac{1}{4}n-1}}{cx^{2n} + bx^n}\,dx$$

input
```
integrate(x^(-1-1/4*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate(x^(-1/4*n - 1)/(c*x^(2*n) + b*x^n), x)
```

3.505.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-\frac{n}{4}}}{bx^n + cx^{2n}}\,dx = \int \frac{1}{x^{\frac{n}{4}+1}\,(b\,x^n + c\,x^{2\,n})}\,dx$$

input
```
int(1/(x^(n/4 + 1)*(b*x^n + c*x^(2*n))),x)
```

output
```
int(1/(x^(n/4 + 1)*(b*x^n + c*x^(2*n))), x)
```

3.506 $\int x^{-1-n(-1+p)}\left(bx^n+cx^{2n}\right)^p\,dx$

3.506.1 Optimal result

Integrand size = 26, antiderivative size = 37

$$\int x^{-1-n(-1+p)}\left(bx^n+cx^{2n}\right)^p\,dx=\frac{x^{-n(1+p)}\left(bx^n+cx^{2n}\right)^{1+p}}{cn(1+p)}$$

output
```
(b*x^n+c*x^(2*n))^(p+1)/c/n/(p+1)/(x^(n*(p+1)))
```

3.506.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03

$$\int x^{-1-n(-1+p)}\left(bx^n+cx^{2n}\right)^p\,dx=\frac{x^{-np}(b+cx^n)\left(x^n(b+cx^n)\right)^p}{cn(1+p)}$$

input
```
Integrate[x^(-1 - n*(-1 + p))*(b*x^n + c*x^(2*n))^p,x]
```

output
```
((b + c*x^n)*(x^n*(b + c*x^n))^p)/(c*n*(1 + p)*x^(n*p))
```

3.506.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-n(p-1)-1}\left(bx^n+cx^{2n}\right)^p\,dx$$

$$\Big\downarrow \; 1920$$

$$\frac{x^{-n(p+1)}\left(bx^n+cx^{2n}\right)^{p+1}}{cn(p+1)}$$

input
```
Int[x^(-1 - n*(-1 + p))*(b*x^n + c*x^(2*n))^p,x]
```

output
```
(b*x^n + c*x^(2*n))^(1 + p)/(c*n*(1 + p)*x^(n*(1 + p)))
```

3.506.3.1 Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

3.506.4 Maple [F]

$$\int x^{-1-n(-1+p)}\left(b\,x^{n}+c\,x^{2n}\right)^{p}dx$$

input
```
int(x^(-1-n*(-1+p))*(b*x^n+c*x^(2*n))^p,x)
```

output
```
int(x^(-1-n*(-1+p))*(b*x^n+c*x^(2*n))^p,x)
```

3.506.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.59

$$\int x^{-1-n(-1+p)}\left(bx^n+cx^{2n}\right)^p\,dx=\frac{\left(cxx^{-np+n-1}x^n+bxx^{-np+n-1}\right)\left(cx^{2\,n}+bx^n\right)^p}{(cnp+cn)x^n}$$

input
```
integrate(x^(-1-n*(-1+p))*(b*x^n+c*x^(2*n))^p,x, algorithm="fricas")
```

output
```
(c*x*x^(-n*p + n - 1)*x^n + b*x*x^(-n*p + n - 1))*(c*x^(2*n) + b*x^n)^p/((
c*n*p + c*n)*x^n)
```

3.506.6 Sympy [F]

$$\int x^{-1-n(-1+p)}\left(bx^n+cx^{2n}\right)^p\,dx=\int x^{-n(p-1)-1}(x^n(b+cx^n))^p\,dx$$

input
```
integrate(x**(-1-n*(-1+p))*(b*x**n+c*x**(2*n))**p,x)
```

output
```
Integral(x**(-n*(p - 1) - 1)*(x**n*(b + c*x**n))**p, x)
```

3.506.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.16

$$\int x^{-1-n(-1+p)}\left(bx^n+cx^{2n}\right)^p\,dx=\frac{(cx^n+b)e^{(-np\log(x)+p\log(cx^n+b)+p\log(x^n))}}{cn(p+1)}$$

input
```
integrate(x^(-1-n*(-1+p))*(b*x^n+c*x^(2*n))^p,x, algorithm="maxima")
```

output
```
(c*x^n + b)*e^(-n*p*log(x) + p*log(c*x^n + b) + p*log(x^n))/(c*n*(p + 1))
```

3.506.8 Giac [F]

$$\int x^{-1-n(-1+p)}\left(bx^n+cx^{2n}\right)^p\,dx=\int \left(cx^{2n}+bx^n\right)^p x^{-n(p-1)-1}\,dx$$

input
```
integrate(x^(-1-n*(-1+p))*(b*x^n+c*x^(2*n))^p,x, algorithm="giac")
```

output
```
integrate((c*x^(2*n) + b*x^n)^p*x^(-n*(p - 1) - 1), x)
```

3.506.9 Mupad [F(-1)]

Timed out.

$$\int x^{-1-n(-1+p)}\left(bx^n+cx^{2n}\right)^p\,dx=\int \frac{\left(b\,x^n+c\,x^{2\,n}\right)^p}{x^{n\,(p-1)+1}}\,dx$$

input
```
int((b*x^n + c*x^(2*n))^p/x^(n*(p - 1) + 1),x)
```

output
```
int((b*x^n + c*x^(2*n))^p/x^(n*(p - 1) + 1), x)
```

3.507 $\int x^{-1-n(1+2p)}(bx^n + cx^{2n})^p \, dx$

3.507.1 Optimal result

Integrand size = 28, antiderivative size = 38

$$\int x^{-1-n(1+2p)}(bx^n + cx^{2n})^p \, dx = -\frac{x^{-2n(1+p)}(bx^n + cx^{2n})^{1+p}}{bn(1+p)}$$

output
```
-(b*x^n+c*x^(2*n))^(p+1)/b/n/(p+1)/(x^(2*n*(p+1)))
```

3.507.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.13

$$\int x^{-1-n(1+2p)}(bx^n + cx^{2n})^p \, dx = -\frac{x^{-n(1+2p)}(b+cx^n)(x^n(b+cx^n))^p}{bn(1+p)}$$

input
```
Integrate[x^(-1 - n*(1 + 2*p))*(b*x^n + c*x^(2*n))^p,x]
```

output
```
-(((b + c*x^n)*(x^n*(b + c*x^n))^p)/(b*n*(1 + p)*x^(n*(1 + 2*p))))
```

3.507.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-n(2p+1)-1}\left(bx^n+cx^{2n}\right)^p\,dx$$

$$\Big\downarrow \ 1920$$

$$-\frac{x^{-2n(p+1)}\left(bx^n+cx^{2n}\right)^{p+1}}{bn(p+1)}$$

input
```
Int[x^(-1 - n*(1 + 2*p))*(b*x^n + c*x^(2*n))^p,x]
```

output
```
-((b*x^n + c*x^(2*n))^(1 + p)/(b*n*(1 + p)*x^(2*n*(1 + p))))
```

3.507.3.1 Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

3.507.4 Maple [F]

$$\int x^{-1-n(1+2p)}\left(b\,x^n+c\,x^{2n}\right)^p dx$$

input
```
int(x^(-1-n*(1+2*p))*(b*x^n+c*x^(2*n))^p,x)
```

output
```
int(x^(-1-n*(1+2*p))*(b*x^n+c*x^(2*n))^p,x)
```

3.507.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.55

$$\int x^{-1-n(1+2p)}\left(bx^n+cx^{2n}\right)^p\,dx=-\frac{\left(cxx^{-2\,np-n-1}x^n+bxx^{-2\,np-n-1}\right)\left(cx^{2\,n}+bx^n\right)^p}{bnp+bn}$$

input
```
integrate(x^(-1-n*(1+2*p))*(b*x^n+c*x^(2*n))^p,x, algorithm="fricas")
```

output
```
-(c*x*x^(-2*n*p - n - 1)*x^n + b*x*x^(-2*n*p - n - 1))*(c*x^(2*n) + b*x^n)
^p/(b*n*p + b*n)
```

3.507.6 Sympy [F]

$$\int x^{-1-n(1+2p)}\left(bx^n+cx^{2n}\right)^p\,dx=\int x^{-n(2p+1)-1}(x^n(b+cx^n))^p\,dx$$

input
```
integrate(x**(-1-n*(1+2*p))*(b*x**n+c*x**(2*n))**p,x)
```

output
```
Integral(x**(-n*(2*p + 1) - 1)*(x**n*(b + c*x**n))**p, x)
```

3.507.7 Maxima [F]

$$\int x^{-1-n(1+2p)}\left(bx^n+cx^{2n}\right)^p\,dx=\int\left(cx^{2\,n}+bx^n\right)^px^{-n(2\,p+1)-1}\,dx$$

input
```
integrate(x^(-1-n*(1+2*p))*(b*x^n+c*x^(2*n))^p,x, algorithm="maxima")
```

output
```
integrate((c*x^(2*n) + b*x^n)^p*x^(-n*(2*p + 1) - 1), x)
```

3.507.8 Giac [F]

$$\int x^{-1-n(1+2p)}\left(bx^n+cx^{2n}\right)^p\,dx=\int \left(cx^{2n}+bx^n\right)^p x^{-n(2p+1)-1}\,dx$$

input
```
integrate(x^(-1-n*(1+2*p))*(b*x^n+c*x^(2*n))^p,x, algorithm="giac")
```

output
```
integrate((c*x^(2*n) + b*x^n)^p*x^(-n*(2*p + 1) - 1), x)
```

3.507.9 Mupad [F(-1)]

Timed out.

$$\int x^{-1-n(1+2p)}\left(bx^n+cx^{2n}\right)^p\,dx=\int \frac{\left(b\,x^n+c\,x^{2\,n}\right)^p}{x^{n\,(2\,p+1)+1}}\,dx$$

input
```
int((b*x^n + c*x^(2*n))^p/x^(n*(2*p + 1) + 1),x)
```

output
```
int((b*x^n + c*x^(2*n))^p/x^(n*(2*p + 1) + 1), x)
```

3.508 $\int x^{-1+2n}(a^2+2abx^n+b^2x^{2n})^{5/2}\,dx$

3.508.1 Optimal result

Integrand size = 32, antiderivative size = 112

$$\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^{5/2}\,dx = -\frac{a(a+bx^n)^6\sqrt{a^2+2abx^n+b^2x^{2n}}}{6n\,(ab^2+b^3x^n)}+\frac{(a+bx^n)^7\sqrt{a^2+2abx^n+b^2x^{2n}}}{7n\,(ab^2+b^3x^n)}$$

output
```
-1/6*a*(a+b*x^n)^6*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/n/(a*b^2+b^3*x^n)+1/7
*(a+b*x^n)^7*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/n/(a*b^2+b^3*x^n)
```

3.508.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.86

$$\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^{5/2}\,dx = \frac{x^{2n}\left((a+bx^n)^2\right)^{5/2}\left(21a^5+70a^4bx^n+105a^3b^2x^{2n}+84a^2b^3x^{3n}+35ab^4x^{4n}+6b^5x^{5n}\right)}{42n\,(a+bx^n)^5}$$

input
```
Integrate[x^(-1 + 2*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(5/2),x]
```

output
```
(x^(2*n)*((a + b*x^n)^2)^(5/2)*(21*a^5 + 70*a^4*b*x^n + 105*a^3*b^2*x^(2*n
) + 84*a^2*b^3*x^(3*n) + 35*a*b^4*x^(4*n) + 6*b^5*x^(5*n)))/(42*n*(a + b*x
^n)^5)
```

3.508.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.69, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1693, 1100, 1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{2n-1} \left(a^2+2abx^n+b^2x^{2n}\right)^{5/2} \, dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{\int x^n \left(2abx^n+b^2x^{2n}+a^2\right)^{5/2}dx^n}{n}$$

$$\Big\downarrow \text{1100}$$

$$\frac{\frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{7/2}}{7b^2}-\frac{a\int \left(2abx^n+b^2x^{2n}+a^2\right)^{5/2}dx^n}{b}}{n}$$

$$\Big\downarrow \text{1079}$$

$$\frac{\frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{7/2}}{7b^2}-\frac{a\sqrt{a^2+2abx^n+b^2x^{2n}}\int \left(b^2x^n+ab\right)^5dx^n}{b^6(a+bx^n)}}{n}$$

$$\Big\downarrow \text{17}$$

$$\frac{\frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{7/2}}{7b^2}-\frac{a(a+bx^n)^5\sqrt{a^2+2abx^n+b^2x^{2n}}}{6b^2}}{n}$$

input
```
Int[x^(-1 + 2*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(5/2),x]
```

output
```
(-1/6*(a*(a + b*x^n)^5*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/b^2 + (a^2 + 2
*a*b*x^n + b^2*x^(2*n))^(7/2)/(7*b^2))/n
```

3.508.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1100
```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.508.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.86

method	result
risch	$\frac{\sqrt{(a+b\,x^{n})^{2}}\,b^{5}x^{7n}}{7(a+b\,x^{n})n}+\frac{5\sqrt{(a+b\,x^{n})^{2}}\,b^{4}a\,x^{6n}}{6(a+b\,x^{n})n}+\frac{2\sqrt{(a+b\,x^{n})^{2}}\,a^{2}b^{3}x^{5n}}{(a+b\,x^{n})n}+\frac{5\sqrt{(a+b\,x^{n})^{2}}\,a^{3}b^{2}x^{4n}}{2(a+b\,x^{n})n}+\frac{5\sqrt{(a+b\,x^{n})^{2}}\,b\,a^{4}x^{3n}}{3(a+b\,x^{n})n}+$

input
```
int(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/7*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b^5/n*(x^n)^7+5/6*((a+b*x^n)^2)^(1/2)/(a
+b*x^n)*b^4*a/n*(x^n)^6+2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^2*b^3/n*(x^n)^5+
5/2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^3*b^2/n*(x^n)^4+5/3*((a+b*x^n)^2)^(1/2
)/(a+b*x^n)*b*a^4/n*(x^n)^3+1/2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^5/n*(x^n)^
2
```

3.508.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.66

$$\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^{5/2}\,dx=\frac{6\,b^5x^{7\,n}+35\,ab^4x^{6\,n}+84\,a^2b^3x^{5\,n}+105\,a^3b^2x^{4\,n}+70\,a^4bx^{3\,n}+21\,a^5x^{2\,n}}{42\,n}$$

input
```
integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x, algorithm="frica
s")
```

output
```
1/42*(6*b^5*x^(7*n) + 35*a*b^4*x^(6*n) + 84*a^2*b^3*x^(5*n) + 105*a^3*b^2*
x^(4*n) + 70*a^4*b*x^(3*n) + 21*a^5*x^(2*n))/n
```

3.508.6 Sympy [F(-1)]

Timed out.

$$\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^{5/2}\,dx=\text{Timed out}$$

input
```
integrate(x**(-1+2*n)*(a**2+2*a*b*x**n+b**2*x**(2*n))**(5/2),x)
```

output
```
Timed out
```

3.508.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.66

$$\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^{5/2}\,dx=\frac{6\,b^5x^{7\,n}+35\,ab^4x^{6\,n}+84\,a^2b^3x^{5\,n}+105\,a^3b^2x^{4\,n}+70\,a^4bx^{3\,n}+21\,a^5x^{2\,n}}{42\,n}$$

input
```
integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x, algorithm="maxim
a")
```

output
```
1/42*(6*b^5*x^(7*n) + 35*a*b^4*x^(6*n) + 84*a^2*b^3*x^(5*n) + 105*a^3*b^2*
x^(4*n) + 70*a^4*b*x^(3*n) + 21*a^5*x^(2*n))/n
```

3.508.8 Giac [F]

$$\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^{5/2}\,dx=\int \left(b^2x^{2\,n}+2\,abx^n+a^2\right)^{\frac{5}{2}}x^{2\,n-1}\,dx$$

input
```
integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x, algorithm="giac"
)
```

output
```
integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(5/2)*x^(2*n - 1), x)
```

3.508.9 Mupad [F(-1)]

Timed out.

$$\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^{5/2}\,dx=\int x^{2\,n-1}\,{\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)}^{5/2}\,dx$$

input
```
int(x^(2*n - 1)*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(5/2),x)
```

output
```
int(x^(2*n - 1)*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(5/2), x)
```

3.509 $\int x^{-1+2n}(a^2+2abx^n+b^2x^{2n})^{3/2}\,dx$

3.509.1 Optimal result

Integrand size = 32, antiderivative size = 112

$$\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx = -\frac{a(a+bx^n)^4\sqrt{a^2+2abx^n+b^2x^{2n}}}{4n\,(ab^2+b^3x^n)}+\frac{(a+bx^n)^5\sqrt{a^2+2abx^n+b^2x^{2n}}}{5n\,(ab^2+b^3x^n)}$$

output
```
-1/4*a*(a+b*x^n)^4*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/n/(a*b^2+b^3*x^n)+1/5
*(a+b*x^n)^5*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/n/(a*b^2+b^3*x^n)
```

3.509.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.62

$$\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx = \frac{x^{2n}\left((a+bx^n)^2\right)^{3/2}\left(10a^3+20a^2bx^n+15ab^2x^{2n}+4b^3x^{3n}\right)}{20n\,(a+bx^n)^3}$$

input
```
Integrate[x^(-1 + 2*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

output
```
(x^(2*n)*((a + b*x^n)^2)^(3/2)*(10*a^3 + 20*a^2*b*x^n + 15*a*b^2*x^(2*n) +
 4*b^3*x^(3*n)))/(20*n*(a + b*x^n)^3)
```

3.509.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.69, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1693, 1100, 1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{2n-1}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{\int x^n\left(2abx^n+b^2x^{2n}+a^2\right)^{3/2}dx^n}{n}$$

$$\Big\downarrow \text{1100}$$

$$\frac{\frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{5/2}}{5b^2}-\frac{a\int\left(2abx^n+b^2x^{2n}+a^2\right)^{3/2}dx^n}{b}}{n}$$

$$\Big\downarrow \text{1079}$$

$$\frac{\frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{5/2}}{5b^2}-\frac{a\sqrt{a^2+2abx^n+b^2x^{2n}}\int\left(b^2x^n+ab\right)^3dx^n}{b^4(a+bx^n)}}{n}$$

$$\Big\downarrow \text{17}$$

$$\frac{\frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{5/2}}{5b^2}-\frac{a(a+bx^n)^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{4b^2}}{n}$$

input
```
Int[x^(-1 + 2*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

output
```
(-1/4*(a*(a + b*x^n)^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/b^2 + (a^2 + 2
*a*b*x^n + b^2*x^(2*n))^(5/2)/(5*b^2))/n
```

3.509.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1100
```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.509.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.21

method	result	size
risch	$\frac{\sqrt{(a+b\,x^{n})^{2}}\,b^{3}x^{5n}}{5(a+b\,x^{n})n}+\frac{3\sqrt{(a+b\,x^{n})^{2}}\,b^{2}a\,x^{4n}}{4(a+b\,x^{n})n}+\frac{\sqrt{(a+b\,x^{n})^{2}}\,a^{2}b\,x^{3n}}{(a+b\,x^{n})n}+\frac{\sqrt{(a+b\,x^{n})^{2}}\,a^{3}x^{2n}}{2(a+b\,x^{n})n}$	135

input
```
int(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b^3/n*(x^n)^5+3/4*((a+b*x^n)^2)^(1/2)/(a
+b*x^n)*b^2*a/n*(x^n)^4+((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^2*b/n*(x^n)^3+1/2*
((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^3/n*(x^n)^2
```

3.509.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.43

$$\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx=\frac{4\,b^3x^{5\,n}+15\,ab^2x^{4\,n}+20\,a^2bx^{3\,n}+10\,a^3x^{2\,n}}{20\,n}$$

input
```
integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="frica
s")
```

output
```
1/20*(4*b^3*x^(5*n) + 15*a*b^2*x^(4*n) + 20*a^2*b*x^(3*n) + 10*a^3*x^(2*n)
)/n
```

3.509.6 Sympy [F(-1)]

Timed out.

$$\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(x**(-1+2*n)*(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

output
```
Timed out
```

3.509.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.43

$$\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx=\frac{4\,b^3x^{5\,n}+15\,ab^2x^{4\,n}+20\,a^2bx^{3\,n}+10\,a^3x^{2\,n}}{20\,n}$$

input
```
integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxim
a")
```

output
```
1/20*(4*b^3*x^(5*n) + 15*a*b^2*x^(4*n) + 20*a^2*b*x^(3*n) + 10*a^3*x^(2*n)
)/n
```

3.509.8 Giac [F]

$$\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx=\int\left(b^2x^{2\,n}+2\,abx^n+a^2\right)^{\frac{3}{2}}x^{2\,n-1}\,dx$$

input
```
integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac"
)
```

output
```
integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)*x^(2*n - 1), x)
```

3.509.9 Mupad [F(-1)]

Timed out.

$$\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx=\int x^{2\,n-1}\,\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)^{3/2}\,dx$$

input
```
int(x^(2*n - 1)*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2),x)
```

output
```
int(x^(2*n - 1)*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2), x)
```

3.510 $\int x^{-1+2n}\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx$

3.510.1 Optimal result

Integrand size = 32, antiderivative size = 99

$$\int x^{-1+2n}\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\frac{ax^{2n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2n(a+bx^n)}+\frac{b^2x^{3n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{3n(ab+b^2x^n)}$$

output
```
1/2*a*x^(2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/n/(a+b*x^n)+1/3*b^2*x^(3*n
)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/n/(a*b+b^2*x^n)
```

3.510.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.44

$$\int x^{-1+2n}\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\frac{x^{2n}\sqrt{(a+bx^n)^2}(3a+2bx^n)}{6n(a+bx^n)}$$

input
```
Integrate[x^(-1 + 2*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

output
```
(x^(2*n)*Sqrt[(a + b*x^n)^2]*(3*a + 2*b*x^n))/(6*n*(a + b*x^n))
```

3.510.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.76, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1693, 1100, 1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{2n-1}\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{\int x^n\sqrt{2abx^n+b^2x^{2n}+a^2}dx^n}{n}$$

$$\Big\downarrow \text{1100}$$

$$\frac{\frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{3b^2}-\frac{a\int \sqrt{2abx^n+b^2x^{2n}+a^2}dx^n}{b}}{n}$$

$$\Big\downarrow \text{1079}$$

$$\frac{\frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{3b^2}-\frac{a\sqrt{a^2+2abx^n+b^2x^{2n}}\int (b^2x^n+ab)dx^n}{b^2(a+bx^n)}}{n}$$

$$\Big\downarrow \text{17}$$

$$\frac{\frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{3b^2}-\frac{a(a+bx^n)\sqrt{a^2+2abx^n+b^2x^{2n}}}{2b^2}}{n}$$

input
```
Int[x^(-1 + 2*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

output
```
(-1/2*(a*(a + b*x^n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/b^2 + (a^2 + 2*a
*b*x^n + b^2*x^(2*n))^(3/2)/(3*b^2))/n
```

3.510.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1100
```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.510.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.65

method	result	size
risch	$\frac{\sqrt{(a+b\,x^{n})^{2}}\,b\,x^{3n}}{3(a+b\,x^{n})n}+\frac{\sqrt{(a+b\,x^{n})^{2}}\,a\,x^{2n}}{2(a+b\,x^{n})n}$	64

input
```
int(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b/n*(x^n)^3+1/2*((a+b*x^n)^2)^(1/2)/(a+b
*x^n)*a/n*(x^n)^2
```

3.510.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.22

$$\int x^{-1+2n}\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\frac{2\,bx^{3\,n}+3\,ax^{2\,n}}{6\,n}$$

input
```
integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="frica
s")
```

output
```
1/6*(2*b*x^(3*n) + 3*a*x^(2*n))/n
```

3.510.6 Sympy [F]

$$\int x^{-1+2n}\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\int x^{2n-1}\sqrt{(a+bx^n)^2}\,dx$$

input
```
integrate(x**(-1+2*n)*(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

output
```
Integral(x**(2*n - 1)*sqrt((a + b*x**n)**2), x)
```

3.510.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.22

$$\int x^{-1+2n}\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\frac{2\,bx^{3\,n}+3\,ax^{2\,n}}{6\,n}$$

input
```
integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxim
a")
```

output
```
1/6*(2*b*x^(3*n) + 3*a*x^(2*n))/n
```

3.510.8 Giac [F]

$$\int x^{-1+2n}\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx = \int \sqrt{b^2x^{2n}+2\,abx^n+a^2}x^{2\,n-1}\,dx$$

input
```
integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac"
)
```

output
```
integrate(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x^(2*n - 1), x)
```

3.510.9 Mupad [F(-1)]

Timed out.

$$\int x^{-1+2n}\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx = \int x^{2\,n-1}\,\sqrt{a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n}\,dx$$

input
```
int(x^(2*n - 1)*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2),x)
```

output
```
int(x^(2*n - 1)*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2), x)
```

3.511 $\int \frac{x^{-1+2n}}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx$

3.511.1 Optimal result

Integrand size = 32, antiderivative size = 90

$$\int \frac{x^{-1+2n}}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx=\frac{x^n(a+bx^n)}{bn\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{a(a+bx^n)\log(a+bx^n)}{b^2n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

output
```
x^n*(a+b*x^n)/b/n/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)-a*(a+b*x^n)*ln(a+b*x^n
)/b^2/n/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)
```

3.511.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.52

$$\int \frac{x^{-1+2n}}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx=\frac{(a+bx^n)\,(bx^n-a\log(bn(a+bx^n)))}{b^2n\sqrt{(a+bx^n)^2}}$$

input
```
Integrate[x^(-1 + 2*n)/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

output
```
((a + b*x^n)*(b*x^n - a*Log[b*n*(a + b*x^n)]))/(b^2*n*Sqrt[(a + b*x^n)^2])
```

3.511.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1693, 1100, 1079, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{2n-1}}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx$$

$$\Big\downarrow \ 1693$$

$$\frac{\int \frac{x^n}{\sqrt{2abx^n+b^2x^{2n}+a^2}}dx^n}{n}$$

$$\Big\downarrow \ 1100$$

$$\frac{\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{b^2}-\frac{a\int \frac{1}{\sqrt{2abx^n+b^2x^{2n}+a^2}}dx^n}{b}}{n}$$

$$\Big\downarrow \ 1079$$

$$\frac{\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{b^2}-\frac{a(a+bx^n)\int \frac{1}{b^2x^n+ab}dx^n}{\sqrt{a^2+2abx^n+b^2x^{2n}}}}{n}$$

$$\Big\downarrow \ 16$$

$$\frac{\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{b^2}-\frac{a(a+bx^n)\log(a+bx^n)}{b^2\sqrt{a^2+2abx^n+b^2x^{2n}}}}{n}$$

input
```
Int[x^(-1 + 2*n)/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/b^2 - (a*(a + b*x^n)*Log[a + b*x^n])/
(b^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]))/n
```

3.511.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1100
```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.511.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.79

method	result	size
risch	$\frac{\sqrt{(a+b\,x^n)^2}\,x^n}{(a+b\,x^n)bn} - \frac{\sqrt{(a+b\,x^n)^2}\,a\ln\left(x^n+\frac{a}{b}\right)}{(a+b\,x^n)b^2n}$	71

input
```
int(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((a+b*x^n)^2)^(1/2)/(a+b*x^n)/b/n*x^n-((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a/b^2/
n*ln(x^n+a/b)
```

3.511.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.27

$$\int \frac{x^{-1+2n}}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx=\frac{bx^n-a\log(bx^n+a)}{b^2n}$$

input
```
integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="frica
s")
```

output
```
(b*x^n - a*log(b*x^n + a))/(b^2*n)
```

3.511.6 Sympy [F]

$$\int \frac{x^{-1+2n}}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx=\int \frac{x^{2n-1}}{\sqrt{(a+bx^n)^2}}\,dx$$

input
```
integrate(x**(-1+2*n)/(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

output
```
Integral(x**(2*n - 1)/sqrt((a + b*x**n)**2), x)
```

3.511.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.36

$$\int \frac{x^{-1+2n}}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx=\frac{x^n}{bn}-\frac{a\log\left(\frac{bx^n+a}{b}\right)}{b^2n}$$

input
```
integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxim
a")
```

output
```
x^n/(b*n) - a*log((b*x^n + a)/b)/(b^2*n)
```

3.511.8 Giac [F]

$$\int \frac{x^{-1+2n}}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \int \frac{x^{2\,n-1}}{\sqrt{b^2x^{2\,n}+2\,abx^n+a^2}}\,dx$$

input
```
integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac"
)
```

output
```
integrate(x^(2*n - 1)/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

3.511.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \int \frac{x^{2\,n-1}}{\sqrt{a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n}}\,dx$$

input
```
int(x^(2*n - 1)/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2),x)
```

output
```
int(x^(2*n - 1)/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2), x)
```

3.512 $\int \frac{x^{-1+2n}}{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx$

3.512.1 Optimal result

Integrand size = 32, antiderivative size = 48

$$\int \frac{x^{-1+2n}}{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx=\frac{x^{2n}}{2an\left(a+bx^n\right)\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

output
```
1/2*x^(2*n)/a/n/(a+b*x^n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)
```

3.512.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.83

$$\int \frac{x^{-1+2n}}{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx=\frac{\left(-a-2bx^n\right)\left(a+bx^n\right)}{2b^2n\left(\left(a+bx^n\right)^2\right)^{3/2}}$$

input
```
Integrate[x^(-1 + 2*n)/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

output
```
((-a - 2*b*x^n)*(a + b*x^n))/(2*b^2*n*((a + b*x^n)^2)^(3/2))
```

3.512.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.56, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {1693, 1100, 1078}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{2n-1}}{(a^2+2abx^n+b^2x^{2n})^{3/2}}dx$$

$$\Big\downarrow \text{ 1693}$$

$$\frac{\int \frac{x^n}{(2abx^n+b^2x^{2n}+a^2)^{3/2}}dx^n}{n}$$

$$\Big\downarrow \text{ 1100}$$

$$\frac{-\frac{a\int \frac{1}{\left(2abx^n+b^2x^{2n}+a^2\right)^{3/2}}dx^n}{b}-\frac{1}{b^2\sqrt{a^2+2abx^n+b^2x^{2n}}}}{n}$$

$$\Big\downarrow \text{ 1078}$$

$$\frac{\frac{a}{2b^2(a+bx^n)\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{1}{b^2\sqrt{a^2+2abx^n+b^2x^{2n}}}}{n}$$

input
```
Int[x^(-1 + 2*n)/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

output
```
(-(1/(b^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])) + a/(2*b^2*(a + b*x^n)*Sqr
t[a^2 + 2*a*b*x^n + b^2*x^(2*n)]))/n
```

3.512.3.1 Defintions of rubi rules used

rule 1078
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[2*((a + b*x
+ c*x^2)^(p + 1)/((2*p + 1)*(b + 2*c*x))), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 1100
```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.512.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.77

method	result	size
risch	$-\frac{\sqrt{(a+b\,x^{n})^{2}}\,(2b\,x^{n}+a)}{2(a+b\,x^{n})^{3}b^{2}n}$	37

input
```
int(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)^3*(2*b*x^n+a)/b^2/n
```

3.512.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.85

$$\int \frac{x^{-1+2n}}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx = -\frac{2\,bx^n+a}{2\,(b^4nx^{2\,n}+2\,ab^3nx^n+a^2b^2n)}$$

input
```
integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="frica
s")
```

output
```
-1/2*(2*b*x^n + a)/(b^4*n*x^(2*n) + 2*a*b^3*n*x^n + a^2*b^2*n)
```

3.512.6 Sympy [F]

$$\int \frac{x^{-1+2n}}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx = \int \frac{x^{2n-1}}{\left((a+bx^n)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**(-1+2*n)/(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

output
```
Integral(x**(2*n - 1)/((a + b*x**n)**2)**(3/2), x)
```

3.512.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.85

$$\int \frac{x^{-1+2n}}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx = -\frac{2\,bx^n+a}{2\,(b^4nx^{2\,n}+2\,ab^3nx^n+a^2b^2n)}$$

input
```
integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxim
a")
```

output
```
-1/2*(2*b*x^n + a)/(b^4*n*x^(2*n) + 2*a*b^3*n*x^n + a^2*b^2*n)
```

3.512.8 Giac [F]

$$\int \frac{x^{-1+2n}}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx = \int \frac{x^{2\,n-1}}{(b^2x^{2\,n}+2\,abx^n+a^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac"
)
```

output
```
integrate(x^(2*n - 1)/(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2), x)
```

3.512.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}}{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}\, dx = \int \frac{x^{2\,n-1}}{(a^2 + b^2\,x^{2\,n} + 2\,a\,b\,x^n)^{3/2}}\, dx$$

input
```
int(x^(2*n - 1)/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2),x)
```

output
```
int(x^(2*n - 1)/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2), x)
```

3.513 $\int \frac{x^{-1+2n}}{(a^2+2abx^n+b^2x^{2n})^{5/2}}\,dx$

3.513.1 Optimal result

Integrand size = 32, antiderivative size = 88

$$\int \frac{x^{-1+2n}}{(a^2+2abx^n+b^2x^{2n})^{5/2}}\,dx = \frac{a}{4b^2n(a+bx^n)^3\sqrt{a^2+2abx^n+b^2x^{2n}}} - \frac{1}{3b^2n(a+bx^n)^2\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

output
```
1/4*a/b^2/n/(a+b*x^n)^3/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)-1/3/b^2/n/(a+b*x
^n)^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)
```

3.513.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.45

$$\int \frac{x^{-1+2n}}{(a^2+2abx^n+b^2x^{2n})^{5/2}}\,dx = \frac{(-a-4bx^n)(a+bx^n)}{12b^2n\left((a+bx^n)^2\right)^{5/2}}$$

input
```
Integrate[x^(-1 + 2*n)/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(5/2),x]
```

output
```
((-a - 4*b*x^n)*(a + b*x^n))/(12*b^2*n*((a + b*x^n)^2)^(5/2))
```

3.513.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.88, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {1693, 1100, 1078}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{2n-1}}{(a^2+2abx^n+b^2x^{2n})^{5/2}}\,dx$$

$$\Big\downarrow \; 1693$$

$$\frac{\int \frac{x^n}{(2abx^n+b^2x^{2n}+a^2)^{5/2}}dx^n}{n}$$

$$\Big\downarrow \; 1100$$

$$\frac{-\frac{a\int \frac{1}{\left(2abx^n+b^2x^{2n}+a^2\right)^{5/2}}dx^n}{b}-\frac{1}{3b^2(a^2+2abx^n+b^2x^{2n})^{3/2}}}{n}$$

$$\Big\downarrow \; 1078$$

$$\frac{\frac{a}{4b^2(a+bx^n)(a^2+2abx^n+b^2x^{2n})^{3/2}}-\frac{1}{3b^2(a^2+2abx^n+b^2x^{2n})^{3/2}}}{n}$$

input
```
Int[x^(-1 + 2*n)/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(5/2),x]
```

output
```
(-1/3*1/(b^2*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)) + a/(4*b^2*(a + b*x^n)
*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)))/n
```

3.513.3.1 Defintions of rubi rules used

rule 1078
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[2*((a + b*x
+ c*x^2)^(p + 1)/((2*p + 1)*(b + 2*c*x))), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 1100
```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.513.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.42

method	result	size
risch	$-\frac{\sqrt{(a+b\,x^{n})^{2}}\,(4b\,x^{n}+a)}{12(a+b\,x^{n})^{5}b^{2}n}$	37

input
```
int(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12*((a+b*x^n)^2)^(1/2)/(a+b*x^n)^5*(4*b*x^n+a)/b^2/n
```

3.513.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.78

$$\int \frac{x^{-1+2n}}{(a^2+2abx^n+b^2x^{2n})^{5/2}}\,dx=-\frac{4\,bx^n+a}{12\,(b^6nx^{4\,n}+4\,ab^5nx^{3\,n}+6\,a^2b^4nx^{2\,n}+4\,a^3b^3nx^n+a^4b^2n)}$$

input
```
integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x, algorithm="frica
s")
```

output
```
-1/12*(4*b*x^n + a)/(b^6*n*x^(4*n) + 4*a*b^5*n*x^(3*n) + 6*a^2*b^4*n*x^(2*
n) + 4*a^3*b^3*n*x^n + a^4*b^2*n)
```

3.513.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}}{\left(a^2+2abx^n+b^2x^{2n}\right)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate(x**(-1+2*n)/(a**2+2*a*b*x**n+b**2*x**(2*n))**(5/2),x)
```

output
```
Timed out
```

3.513.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.78

$$\int \frac{x^{-1+2n}}{\left(a^2+2abx^n+b^2x^{2n}\right)^{5/2}}\,dx=-\frac{4\,bx^n+a}{12\,\left(b^6nx^{4\,n}+4\,ab^5nx^{3\,n}+6\,a^2b^4nx^{2\,n}+4\,a^3b^3nx^n+a^4b^2n\right)}$$

input
```
integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x, algorithm="maxim
a")
```

output
```
-1/12*(4*b*x^n + a)/(b^6*n*x^(4*n) + 4*a*b^5*n*x^(3*n) + 6*a^2*b^4*n*x^(2*
n) + 4*a^3*b^3*n*x^n + a^4*b^2*n)
```

3.513.8 Giac [F]

$$\int \frac{x^{-1+2n}}{\left(a^2+2abx^n+b^2x^{2n}\right)^{5/2}}\,dx=\int \frac{x^{2\,n-1}}{\left(b^2x^{2\,n}+2\,abx^n+a^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x, algorithm="giac"
)
```

output
```
integrate(x^(2*n - 1)/(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(5/2), x)
```

3.513.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}}{(a^2+2abx^n+b^2x^{2n})^{5/2}}\,dx = \int \frac{x^{2\,n-1}}{\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)^{5/2}}\,dx$$

input
```
int(x^(2*n - 1)/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(5/2),x)
```

output
```
int(x^(2*n - 1)/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(5/2), x)
```

3.514 $\int \frac{x^{-1+2n}}{(a^2+2abx^n+b^2x^{2n})^{7/2}}\,dx$

3.514.1 Optimal result

Integrand size = 32, antiderivative size = 88

$$\int \frac{x^{-1+2n}}{(a^2+2abx^n+b^2x^{2n})^{7/2}}\,dx = \frac{a}{6b^2n(a+bx^n)^5\sqrt{a^2+2abx^n+b^2x^{2n}}} - \frac{1}{5b^2n(a+bx^n)^4\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

output
```
1/6*a/b^2/n/(a+b*x^n)^5/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)-1/5/b^2/n/(a+b*x
^n)^4/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)
```

3.514.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.45

$$\int \frac{x^{-1+2n}}{(a^2+2abx^n+b^2x^{2n})^{7/2}}\,dx = \frac{(-a-6bx^n)(a+bx^n)}{30b^2n\left((a+bx^n)^2\right)^{7/2}}$$

input
```
Integrate[x^(-1 + 2*n)/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(7/2),x]
```

output
```
((-a - 6*b*x^n)*(a + b*x^n))/(30*b^2*n*((a + b*x^n)^2)^(7/2))
```

3.514.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.88, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {1693, 1100, 1078}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{2n-1}}{(a^2+2abx^n+b^2x^{2n})^{7/2}}dx$$

$$\Big\downarrow \ 1693$$

$$\frac{\int \frac{x^n}{(2abx^n+b^2x^{2n}+a^2)^{7/2}}dx^n}{n}$$

$$\Big\downarrow \ 1100$$

$$\frac{-\frac{a\int \frac{1}{\left(2abx^n+b^2x^{2n}+a^2\right)^{7/2}}dx^n}{b}-\frac{1}{5b^2(a^2+2abx^n+b^2x^{2n})^{5/2}}}{n}$$

$$\Big\downarrow \ 1078$$

$$\frac{\frac{a}{6b^2(a+bx^n)(a^2+2abx^n+b^2x^{2n})^{5/2}}-\frac{1}{5b^2(a^2+2abx^n+b^2x^{2n})^{5/2}}}{n}$$

input
```
Int[x^(-1 + 2*n)/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(7/2),x]
```

output
```
(-1/5*1/(b^2*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(5/2)) + a/(6*b^2*(a + b*x^n)
*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(5/2)))/n
```

3.514.3.1 Defintions of rubi rules used

rule 1078
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[2*((a + b*x
+ c*x^2)^(p + 1)/((2*p + 1)*(b + 2*c*x))), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 1100
```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.514.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.42

method	result	size
risch	$-\frac{\sqrt{(a+b\,x^n)^2}\,(6b\,x^n+a)}{30(a+b\,x^n)^7 b^2 n}$	37

input
```
int(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/30*((a+b*x^n)^2)^(1/2)/(a+b*x^n)^7*(6*b*x^n+a)/b^2/n
```

3.514.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.10

$$\int \frac{x^{-1+2n}}{(a^2+2abx^n+b^2x^{2n})^{7/2}}\,dx =$$
$$-\frac{6\,bx^n+a}{30\,(b^8nx^{6\,n}+6\,ab^7nx^{5\,n}+15\,a^2b^6nx^{4\,n}+20\,a^3b^5nx^{3\,n}+15\,a^4b^4nx^{2\,n}+6\,a^5b^3nx^n+a^6b^2n)}$$

input
```
integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(7/2),x, algorithm="frica
s")
```

output
```
-1/30*(6*b*x^n + a)/(b^8*n*x^(6*n) + 6*a*b^7*n*x^(5*n) + 15*a^2*b^6*n*x^(4
*n) + 20*a^3*b^5*n*x^(3*n) + 15*a^4*b^4*n*x^(2*n) + 6*a^5*b^3*n*x^n + a^6*
b^2*n)
```

3.514.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}}{(a^2+2abx^n+b^2x^{2n})^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**(-1+2*n)/(a**2+2*a*b*x**n+b**2*x**(2*n))**(7/2),x)
```

output
```
Timed out
```

3.514.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.10

$$\int \frac{x^{-1+2n}}{(a^2+2abx^n+b^2x^{2n})^{7/2}}\,dx = -\frac{6\,bx^n+a}{30\,(b^8nx^{6\,n}+6\,ab^7nx^{5\,n}+15\,a^2b^6nx^{4\,n}+20\,a^3b^5nx^{3\,n}+15\,a^4b^4nx^{2\,n}+6\,a^5b^3nx^n+a^6b^2n)}$$

input
```
integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(7/2),x, algorithm="maxim
a")
```

output
```
-1/30*(6*b*x^n + a)/(b^8*n*x^(6*n) + 6*a*b^7*n*x^(5*n) + 15*a^2*b^6*n*x^(4
*n) + 20*a^3*b^5*n*x^(3*n) + 15*a^4*b^4*n*x^(2*n) + 6*a^5*b^3*n*x^n + a^6*
b^2*n)
```

3.514.8 Giac [F]

$$\int \frac{x^{-1+2n}}{(a^2+2abx^n+b^2x^{2n})^{7/2}}\,dx = \int \frac{x^{2\,n-1}}{(b^2x^{2\,n}+2\,abx^n+a^2)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(7/2),x, algorithm="giac"
)
```

output
```
integrate(x^(2*n - 1)/(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(7/2), x)
```

3.514.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}}{(a^2+2abx^n+b^2x^{2n})^{7/2}}\,dx=\int \frac{x^{2\,n-1}}{{\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)}^{7/2}}\,dx$$

input
```
int(x^(2*n - 1)/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(7/2),x)
```

output
```
int(x^(2*n - 1)/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(7/2), x)
```

3.515 $\int (dx)^m \sqrt{a^2+2abx^n+b^2x^{2n}}\,dx$

3.515.1 Optimal result

Integrand size = 30, antiderivative size = 108

$$\int (dx)^m \sqrt{a^2+2abx^n+b^2x^{2n}}\,dx = \frac{a(dx)^{1+m}\sqrt{a^2+2abx^n+b^2x^{2n}}}{d(1+m)\,(a+bx^n)} + \frac{b^2x^{1+n}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1+m+n)\,(ab+b^2x^n)}$$

output
```
a*(d*x)^(1+m)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/d/(1+m)/(a+b*x^n)+b^2*x^(1
+n)*(d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(1+m+n)/(a*b+b^2*x^n)
```

3.515.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.51

$$\int (dx)^m \sqrt{a^2+2abx^n+b^2x^{2n}}\,dx = \frac{x(dx)^m\sqrt{(a+bx^n)^2}(a(1+m+n)+b(1+m)x^n)}{(1+m)(1+m+n)\,(a+bx^n)}$$

input
```
Integrate[(d*x)^m*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

output
```
(x*(d*x)^m*Sqrt[(a + b*x^n)^2]*(a*(1 + m + n) + b*(1 + m)*x^n))/((1 + m)*(
1 + m + n)*(a + b*x^n))
```

3.515.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.71, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1384, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \sqrt{a^2+2abx^n+b^2x^{2n}}\,dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int (dx)^m\left(b^2x^n+ab\right)dx}{ab+b^2x^n}$$

$$\Big\downarrow \text{ 802}$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \left(b^2x^n(dx)^m+ab(dx)^m\right)dx}{ab+b^2x^n}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\left(\frac{ab(dx)^{m+1}}{d(m+1)}+\frac{b^2x^{n+1}(dx)^m}{m+n+1}\right)}{ab+b^2x^n}$$

input
```
Int[(d*x)^m*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]*((b^2*x^(1 + n)*(d*x)^m)/(1 + m + n)
+ (a*b*(d*x)^(1 + m))/(d*(1 + m))))/(a*b + b^2*x^n)
```

3.515.3.1 Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.515.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.92

method	result	size
risch	$\frac{\sqrt{(a+b\,x^{n})^{2}}\,x(mb\,x^{n}+am+an+b\,x^{n}+a)d^{m}x^{m}\mathrm{e}^{\frac{i\pi\,\operatorname{csgn}(idx)m(\operatorname{csgn}(idx)-\operatorname{csgn}(ix))(-\operatorname{csgn}(idx)+\operatorname{csgn}(id))}{2}}}{(a+b\,x^{n})(1+m)(1+m+n)}$	99

input
```
int((d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((a+b*x^n)^2)^(1/2)/(a+b*x^n)*x*(m*b*x^n+a*m+a*n+b*x^n+a)/(1+m)/(1+m+n)*d^
m*x^m*exp(1/2*I*Pi*csgn(I*d*x)*m*(csgn(I*d*x)-csgn(I*x))*(-csgn(I*d*x)+csg
n(I*d)))
```

3.515.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.53

$$\int (dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx = \frac{(bm+b)xx^ne^{(m\log(d)+m\log(x))}+(am+an+a)xe^{(m\log(d)+m\log(x))}}{m^2+(m+1)n+2\,m+1}$$

input
```
integrate((d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")
```

output
```
((b*m + b)*x*x^n*e^(m*log(d) + m*log(x)) + (a*m + a*n + a)*x*e^(m*log(d) +
 m*log(x)))/(m^2 + (m + 1)*n + 2*m + 1)
```

3.515.6 Sympy [F]

$$\int (dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx = \int (dx)^m\sqrt{(a+bx^n)^2}\,dx$$

input
```
integrate((d*x)**m*(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

output
```
Integral((d*x)**m*sqrt((a + b*x**n)**2), x)
```

3.515.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.44

$$\int (dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx = \frac{ad^m(m+n+1)xx^m+bd^m(m+1)xe^{(m\log(x)+n\log(x))}}{m^2+m(n+2)+n+1}$$

input
```
integrate((d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")
```

output
```
(a*d^m*(m + n + 1)*x*x^m + b*d^m*(m + 1)*x*e^(m*log(x) + n*log(x)))/(m^2 +
 m*(n + 2) + n + 1)
```

3.515.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.60

$$\int (dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx = \frac{bmxx^ne^{(m\log(d)+m\log(x))}\mathrm{sgn}(bx^n+a)+amxe^{(m\log(d)+m\log(x))}\mathrm{sgn}(bx^n+a)+bmxe^{(m\log(d)+m\log(x))}\mathrm{sgn}(bx^n}{}$$

input
```
integrate((d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
```

output
```
(b*m*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + a*m*x*e^(m*log(d) + m*
log(x))*sgn(b*x^n + a) + b*m*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + a*
n*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + b*x*x^n*e^(m*log(d) + m*log(x
))*sgn(b*x^n + a) + a*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + b*x*e^(m*
log(d) + m*log(x))*sgn(b*x^n + a))/(m^2 + m*n + 2*m + n + 1)
```

3.515.9 Mupad [F(-1)]

Timed out.

$$\int (dx)^m \sqrt{a^2+2abx^n+b^2x^{2n}}\,dx = \int (d\,x)^m\,\sqrt{a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n}\,dx$$

input
```
int((d*x)^m*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2),x)
```

output
```
int((d*x)^m*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2), x)
```

3.516 $\int x^2\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx$

3.516.1 Optimal result

Integrand size = 28, antiderivative size = 93

$$\int x^2\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\frac{ax^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{3(a+bx^n)}+\frac{b^2x^{3+n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(3+n)(ab+b^2x^n)}$$

output
```
1/3*a*x^3*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(a+b*x^n)+b^2*x^(3+n)*(a^2+2*a
*b*x^n+b^2*x^(2*n))^(1/2)/(3+n)/(a*b+b^2*x^n)
```

3.516.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.49

$$\int x^2\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\frac{x^3\sqrt{(a+bx^n)^2}(a(3+n)+3bx^n)}{3(3+n)(a+bx^n)}$$

input
```
Integrate[x^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

output
```
(x^3*Sqrt[(a + b*x^n)^2]*(a*(3 + n) + 3*b*x^n))/(3*(3 + n)*(a + b*x^n))
```

3.516.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.67, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {1384, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sqrt{a^2+2abx^n+b^2x^{2n}}\,dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int x^2\left(b^2x^n+ab\right)dx}{ab+b^2x^n}$$

$$\Big\downarrow \text{ 802}$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \left(b^2x^{n+2}+abx^2\right)dx}{ab+b^2x^n}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\left(\frac{1}{3}abx^3+\frac{b^2x^{n+3}}{n+3}\right)}{ab+b^2x^n}$$

input
```
Int[x^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]*((a*b*x^3)/3 + (b^2*x^(3 + n))/(3 + n
)))/(a*b + b^2*x^n)
```

3.516.3.1 Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.516.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.66

method	result	size
risch	$\frac{\sqrt{(a+b\,x^n)^2}\,a\,x^3}{3a+3b\,x^n}+\frac{\sqrt{(a+b\,x^n)^2}\,b\,x^3x^n}{(a+b\,x^n)(3+n)}$	61

input
```
int(x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a*x^3+((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b/(3
+n)*x^3*x^n
```

3.516.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.30

$$\int x^2\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\frac{3\,bx^3x^n+(an+3\,a)x^3}{3\,(n+3)}$$

input
```
integrate(x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")
```

output
```
1/3*(3*b*x^3*x^n + (a*n + 3*a)*x^3)/(n + 3)
```

3.516.6 Sympy [F]

$$\int x^2\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\int x^2\sqrt{(a+bx^n)^2}\,dx$$

input
```
integrate(x**2*(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

output
```
Integral(x**2*sqrt((a + b*x**n)**2), x)
```

3.516.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.27

$$\int x^2\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\frac{3\,bx^3x^n+a(n+3)x^3}{3\,(n+3)}$$

input
```
integrate(x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")
```

output
```
1/3*(3*b*x^3*x^n + a*(n + 3)*x^3)/(n + 3)
```

3.516.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.57

$$\int x^2\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx$$
$$=\frac{3\,bx^3x^n\text{sgn}(bx^n+a)+anx^3\text{sgn}(bx^n+a)+3\,ax^3\text{sgn}(bx^n+a)}{3\,(n+3)}$$

input
```
integrate(x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
```

output
```
1/3*(3*b*x^3*x^n*sgn(b*x^n + a) + a*n*x^3*sgn(b*x^n + a) + 3*a*x^3*sgn(b*x
^n + a))/(n + 3)
```

3.516.9 Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\int x^2\,\sqrt{a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n}\,dx$$

input
```
int(x^2*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2),x)
```

output
```
int(x^2*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2), x)
```

3.517 $\int x\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx$

3.517.1 Optimal result

Integrand size = 26, antiderivative size = 93

$$\int x\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\frac{ax^2\sqrt{a^2+2abx^n+b^2x^{2n}}}{2\left(a+bx^n\right)}+\frac{b^2x^{2+n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2+n)\left(ab+b^2x^n\right)}$$

output
```
1/2*a*x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(a+b*x^n)+b^2*x^(2+n)*(a^2+2*a
*b*x^n+b^2*x^(2*n))^(1/2)/(2+n)/(a*b+b^2*x^n)
```

3.517.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.49

$$\int x\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\frac{x^2\sqrt{(a+bx^n)^2}(a(2+n)+2bx^n)}{2(2+n)\left(a+bx^n\right)}$$

input
```
Integrate[x*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

output
```
(x^2*Sqrt[(a + b*x^n)^2]*(a*(2 + n) + 2*b*x^n))/(2*(2 + n)*(a + b*x^n))
```

3.517.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.67, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1384, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx$$

$$\Big\downarrow \ 1384$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int x(b^2x^n+ab)\,dx}{ab+b^2x^n}$$

$$\Big\downarrow \ 802$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int (b^2x^{n+1}+abx)\,dx}{ab+b^2x^n}$$

$$\Big\downarrow \ 2009$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\left(\frac{1}{2}abx^2+\frac{b^2x^{n+2}}{n+2}\right)}{ab+b^2x^n}$$

input
```
Int[x*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]*((a*b*x^2)/2 + (b^2*x^(2 + n))/(2 + n
)))/(a*b + b^2*x^n)
```

3.517.3.1 Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.517.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.66

method	result	size
risch	$\frac{\sqrt{(a+b\,x^{n})^{2}}\,a\,x^{2}}{2a+2b\,x^{n}}+\frac{\sqrt{(a+b\,x^{n})^{2}}\,b\,x^{2}x^{n}}{(a+b\,x^{n})(2+n)}$	61

input
```
int(x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a*x^2+((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b/(2
+n)*x^2*x^n
```

3.517.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.30

$$\int x\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\frac{2\,bx^2x^n+(an+2\,a)x^2}{2\,(n+2)}$$

input
```
integrate(x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")
```

output
```
1/2*(2*b*x^2*x^n + (a*n + 2*a)*x^2)/(n + 2)
```

3.517.6 Sympy [F]

$$\int x\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\int x\sqrt{(a+bx^n)^2}\,dx$$

input
```
integrate(x*(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

output
```
Integral(x*sqrt((a + b*x**n)**2), x)
```

3.517.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.27

$$\int x\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\frac{2\,bx^2x^n+a(n+2)x^2}{2\,(n+2)}$$

input
```
integrate(x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")
```

output
```
1/2*(2*b*x^2*x^n + a*(n + 2)*x^2)/(n + 2)
```

3.517.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.57

$$\int x\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\frac{2\,bx^2x^n\mathrm{sgn}(bx^n+a)+anx^2\mathrm{sgn}(bx^n+a)+2\,ax^2\mathrm{sgn}(bx^n+a)}{2\,(n+2)}$$

input
```
integrate(x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
```

output
```
1/2*(2*b*x^2*x^n*sgn(b*x^n + a) + a*n*x^2*sgn(b*x^n + a) + 2*a*x^2*sgn(b*x
^n + a))/(n + 2)
```

3.517.9 Mupad [F(-1)]

Timed out.

$$\int x\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\int x\,\sqrt{a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n}\,dx$$

input
```
int(x*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2),x)
```

output
```
int(x*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2), x)
```

3.518 $\int \sqrt{a^2+2abx^n+b^2x^{2n}}\,dx$

3.518.1 Optimal result

Integrand size = 24, antiderivative size = 88

$$\int \sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\frac{ax\sqrt{a^2+2abx^n+b^2x^{2n}}}{a+bx^n}+\frac{b^2x^{1+n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1+n)\left(ab+b^2x^n\right)}$$

output
```
a*x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(a+b*x^n)+b^2*x^(1+n)*(a^2+2*a*b*x^n
+b^2*x^(2*n))^(1/2)/(1+n)/(a*b+b^2*x^n)
```

3.518.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.44

$$\int \sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\frac{x\sqrt{(a+bx^n)^2}(a+an+bx^n)}{(1+n)(a+bx^n)}$$

input
```
Integrate[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

output
```
(x*Sqrt[(a + b*x^n)^2]*(a + a*n + b*x^n))/((1 + n)*(a + b*x^n))
```

3.518.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.65, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1384, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a^2+2abx^n+b^2x^{2n}}\,dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \left(b^2x^n+ab\right)dx}{ab+b^2x^n}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\left(abx+\frac{b^2x^{n+1}}{n+1}\right)}{ab+b^2x^n}$$

input
```
Int[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]*(a*b*x + (b^2*x^(1 + n))/(1 + n)))/(a
*b + b^2*x^n)
```

3.518.3.1 Defintions of rubi rules used

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.518.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.64

method	result	size
risch	$\frac{\sqrt{(a+b\,x^n)^2}\,ax}{a+b\,x^n}+\frac{\sqrt{(a+b\,x^n)^2}\,bx\,x^n}{(a+b\,x^n)(1+n)}$	56

input
```
int((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a*x+((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b/(1+n)*x*
x^n
```

3.518.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.23

$$\int \sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\frac{bxx^n+(an+a)x}{n+1}$$

input
```
integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")
```

output
```
(b*x*x^n + (a*n + a)*x)/(n + 1)
```

3.518.6 Sympy [F]

$$\int \sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\int \sqrt{a^2+2abx^n+b^2x^{2n}}\,dx$$

input
```
integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

output
```
Integral(sqrt(a**2 + 2*a*b*x**n + b**2*x**(2*n)), x)
```

3.518.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.22

$$\int \sqrt{a^2 + 2abx^n + b^2x^{2n}}\,dx = \frac{a(n+1)x + bxx^n}{n+1}$$

input
```
integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")
```

output
```
(a*(n + 1)*x + b*x*x^n)/(n + 1)
```

3.518.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.28

$$\int \sqrt{a^2 + 2abx^n + b^2x^{2n}}\,dx = \left(ax + \frac{bx^{n+1}}{n+1}\right)\operatorname{sgn}(bx^n + a)$$

input
```
integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
```

output
```
(a*x + b*x^(n + 1)/(n + 1))*sgn(b*x^n + a)
```

3.518.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a^2 + 2abx^n + b^2x^{2n}}\,dx = \int \sqrt{a^2 + b^2\,x^{2\,n} + 2\,a\,b\,x^n}\,dx$$

input
```
int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2),x)
```

output
```
int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2), x)
```

3.519 $\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x}\,dx$

3.519.1 Optimal result

Integrand size = 28, antiderivative size = 85

$$\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x}\,dx=\frac{b^2x^n\sqrt{a^2+2abx^n+b^2x^{2n}}}{n\left(ab+b^2x^n\right)}+\frac{a\sqrt{a^2+2abx^n+b^2x^{2n}}\log(x)}{a+bx^n}$$

output
```
b^2*x^n*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/n/(a*b+b^2*x^n)+a*ln(x)*(a^2+2*a
*b*x^n+b^2*x^(2*n))^(1/2)/(a+b*x^n)
```

3.519.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.45

$$\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x}\,dx=\frac{\sqrt{(a+bx^n)^2}(bx^n+a\log(x^n))}{n(a+bx^n)}$$

input
```
Integrate[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/x,x]
```

output
```
(Sqrt[(a + b*x^n)^2]*(b*x^n + a*Log[x^n]))/(n*(a + b*x^n))
```

3.519.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.64, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {1384, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x}\,dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \frac{b^2x^n+ab}{x}dx}{ab+b^2x^n}$$

$$\Big\downarrow \text{ 802}$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \left(b^2x^{n-1}+\frac{ab}{x}\right)dx}{ab+b^2x^n}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\left(ab\log(x)+\frac{b^2x^n}{n}\right)}{ab+b^2x^n}$$

input
```
Int[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/x,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]*((b^2*x^n)/n + a*b*Log[x]))/(a*b + b^
2*x^n)
```

3.519.3.1 Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.519.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.64

method	result	size
risch	$\frac{\sqrt{(a+b\,x^n)^2}\,a\ln(x)}{a+b\,x^n}+\frac{\sqrt{(a+b\,x^n)^2}\,b\,x^n}{(a+b\,x^n)n}$	54

input
```
int((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a*ln(x)+((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b/n*x^
n
```

3.519.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.18

$$\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x}\,dx=\frac{an\log(x)+bx^n}{n}$$

input
```
integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x,x, algorithm="fricas")
```

output
```
(a*n*log(x) + b*x^n)/n
```

3.519.6 Sympy [F]

$$\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x}\,dx = \int \frac{\sqrt{(a+bx^n)^2}}{x}\,dx$$

input
```
integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2)/x,x)
```

output
```
Integral(sqrt((a + b*x**n)**2)/x, x)
```

3.519.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.15

$$\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x}\,dx = a\log(x) + \frac{bx^n}{n}$$

input
```
integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x,x, algorithm="maxima")
```

output
```
a*log(x) + b*x^n/n
```

3.519.8 Giac [F]

$$\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x}\,dx = \int \frac{\sqrt{b^2x^{2n}+2abx^n+a^2}}{x}\,dx$$

input
```
integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x,x, algorithm="giac")
```

output
```
integrate(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)/x, x)
```

3.519.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x}\,dx=\int \frac{\sqrt{a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n}}{x}\,dx$$

input
```
int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)/x,x)
```

output
```
int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)/x, x)
```

3.520 $\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x^2}\,dx$

3.520.1 Optimal result

Integrand size = 28, antiderivative size = 94

$$\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x^2}\,dx=-\frac{a\sqrt{a^2+2abx^n+b^2x^{2n}}}{x\,(a+bx^n)}-\frac{b^2x^{-1+n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1-n)\,(ab+b^2x^n)}$$

output
```
-a*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x/(a+b*x^n)-b^2*x^(-1+n)*(a^2+2*a*b*x
^n+b^2*x^(2*n))^(1/2)/(1-n)/(a*b+b^2*x^n)
```

3.520.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.45

$$\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x^2}\,dx=\frac{\sqrt{(a+bx^n)^2}(a-an+bx^n)}{(-1+n)x\,(a+bx^n)}$$

input
```
Integrate[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/x^2,x]
```

output
```
(Sqrt[(a + b*x^n)^2]*(a - a*n + b*x^n))/((-1 + n)*x*(a + b*x^n))
```

3.520.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.67, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {1384, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x^2}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \frac{b^2x^n+ab}{x^2}dx}{ab+b^2x^n}$$

$$\Big\downarrow \text{802}$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \left(b^2x^{n-2}+\frac{ab}{x^2}\right)dx}{ab+b^2x^n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(-\frac{ab}{x}-\frac{b^2x^{n-1}}{1-n}\right)\sqrt{a^2+2abx^n+b^2x^{2n}}}{ab+b^2x^n}$$

input
```
Int[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/x^2,x]
```

output
```
((-((a*b)/x) - (b^2*x^(-1 + n))/(1 - n))*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n
)])/(a*b + b^2*x^n)
```

3.520.3.1 Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.520.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.65

method	result	size
risch	$-\frac{\sqrt{(a+b\,x^{n})^{2}}\,a}{(a+b\,x^{n})x}+\frac{\sqrt{(a+b\,x^{n})^{2}}\,b\,x^{n}}{(a+b\,x^{n})(-1+n)x}$	61

input
```
int((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a/x+((a+b*x^n)^2)^(1/2)/(a+b*x^n)/(-1+n)*b/
x*x^n
```

3.520.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.24

$$\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x^2}\,dx=-\frac{an-bx^n-a}{(n-1)x}$$

input
```
integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^2,x, algorithm="fricas")
```

output
```
-(a*n - b*x^n - a)/((n - 1)*x)
```

3.520.6 Sympy [F]

$$\int \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{x^2}\, dx = \int \frac{\sqrt{(a + bx^n)^2}}{x^2}\, dx$$

input
```
integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2)/x**2,x)
```

output
```
Integral(sqrt((a + b*x**n)**2)/x**2, x)
```

3.520.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.23

$$\int \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{x^2}\, dx = -\frac{a(n-1) - bx^n}{(n-1)x}$$

input
```
integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^2,x, algorithm="maxima")
```

output
```
-(a*(n - 1) - b*x^n)/((n - 1)*x)
```

3.520.8 Giac [F]

$$\int \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{x^2}\, dx = \int \frac{\sqrt{b^2x^{2n} + 2\,abx^n + a^2}}{x^2}\, dx$$

input
```
integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^2,x, algorithm="giac")
```

output
```
integrate(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)/x^2, x)
```

3.520.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x^2}\,dx=\int \frac{\sqrt{a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n}}{x^2}\,dx$$

input
```
int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)/x^2,x)
```

output
```
int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)/x^2, x)
```

3.521 $\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x^3}\,dx$

3.521.1 Optimal result

Integrand size = 28, antiderivative size = 96

$$\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x^3}\,dx = -\frac{a\sqrt{a^2+2abx^n+b^2x^{2n}}}{2x^2\left(a+bx^n\right)}-\frac{b^2x^{-2+n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2-n)\left(ab+b^2x^n\right)}$$

output
```
-1/2*a*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^2/(a+b*x^n)-b^2*x^(-2+n)*(a^2+2
*a*b*x^n+b^2*x^(2*n))^(1/2)/(2-n)/(a*b+b^2*x^n)
```

3.521.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.49

$$\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x^3}\,dx = \frac{\sqrt{(a+bx^n)^2}(-a(-2+n)+2bx^n)}{2(-2+n)x^2\left(a+bx^n\right)}$$

input
```
Integrate[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/x^3,x]
```

output
```
(Sqrt[(a + b*x^n)^2]*(-(a*(-2 + n)) + 2*b*x^n))/(2*(-2 + n)*x^2*(a + b*x^n
))
```

3.521.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.68, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {1384, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x^3}\,dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \frac{b^2x^n+ab}{x^3}dx}{ab+b^2x^n}$$

$$\Big\downarrow \text{ 802}$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \left(b^2x^{n-3}+\frac{ab}{x^3}\right)dx}{ab+b^2x^n}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\left(-\frac{ab}{2x^2}-\frac{b^2x^{n-2}}{2-n}\right)\sqrt{a^2+2abx^n+b^2x^{2n}}}{ab+b^2x^n}$$

input
```
Int[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/x^3,x]
```

output
```
((-1/2*(a*b)/x^2 - (b^2*x^(-2 + n))/(2 - n))*Sqrt[a^2 + 2*a*b*x^n + b^2*x^
(2*n)])/(a*b + b^2*x^n)
```

3.521.3.1 Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.521.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.64

method	result	size
risch	$-\frac{\sqrt{\left(a+b\,x^{n}\right)^{2}}\,a}{2\left(a+b\,x^{n}\right)x^{2}}+\frac{\sqrt{\left(a+b\,x^{n}\right)^{2}}\,b\,x^{n}}{\left(a+b\,x^{n}\right)\left(-2+n\right)x^{2}}$	61

input
```
int((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a/x^2+((a+b*x^n)^2)^(1/2)/(a+b*x^n)/(-2
+n)*b/x^2*x^n
```

3.521.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.24

$$\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x^3}\,dx=-\frac{an-2\,bx^n-2\,a}{2\,(n-2)x^2}$$

input
```
integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^3,x, algorithm="fricas")
```

output
```
-1/2*(a*n - 2*b*x^n - 2*a)/((n - 2)*x^2)
```

3.521.6 Sympy [F]

$$\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x^3}\,dx = \int \frac{\sqrt{(a+bx^n)^2}}{x^3}\,dx$$

input
```
integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2)/x**3,x)
```

output
```
Integral(sqrt((a + b*x**n)**2)/x**3, x)
```

3.521.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.23

$$\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x^3}\,dx = -\frac{a(n-2)-2\,bx^n}{2\,(n-2)x^2}$$

input
```
integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^3,x, algorithm="maxima")
```

output
```
-1/2*(a*(n - 2) - 2*b*x^n)/((n - 2)*x^2)
```

3.521.8 Giac [F]

$$\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x^3}\,dx = \int \frac{\sqrt{b^2x^{2\,n}+2\,abx^n+a^2}}{x^3}\,dx$$

input
```
integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^3,x, algorithm="giac")
```

output
```
integrate(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)/x^3, x)
```

3.521.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x^3}\,dx=\int \frac{\sqrt{a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n}}{x^3}\,dx$$

input
```
int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)/x^3,x)
```

output
```
int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)/x^3, x)
```

3.522 $\int (dx)^m \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2} dx$

3.522.1 Optimal result

Integrand size = 30, antiderivative size = 238

$$\int (dx)^m \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2} dx = \frac{a^3(dx)^{1+m}\sqrt{a^2+2abx^n+b^2x^{2n}}}{d(1+m)\left(a+bx^n\right)} + \frac{3a^2b^2x^{1+n}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1+m+n)\left(ab+b^2x^n\right)} + \frac{3ab^3x^{1+2n}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1+m+2n)\left(ab+b^2x^n\right)} + \frac{b^4x^{1+3n}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1+m+3n)\left(ab+b^2x^n\right)}$$

output
```
a^3*(d*x)^(1+m)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/d/(1+m)/(a+b*x^n)+3*a^2*
b^2*x^(1+n)*(d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(1+m+n)/(a*b+b^2*x^n
)+3*a*b^3*x^(1+2*n)*(d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(1+m+2*n)/(a
*b+b^2*x^n)+b^4*x^(1+3*n)*(d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(1+m+3
*n)/(a*b+b^2*x^n)
```

3.522.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.38

$$\int (dx)^m \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2} dx = \frac{x(dx)^m\left(\left(a+bx^n\right)^2\right)^{3/2}\left(\frac{a^3}{1+m}+\frac{3a^2bx^n}{1+m+n}+\frac{3ab^2x^{2n}}{1+m+2n}+\frac{b^3x^{3n}}{1+m+3n}\right)}{\left(a+bx^n\right)^3}$$

input
```
Integrate[(d*x)^m*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

output
```
(x*(d*x)^m*((a + b*x^n)^2)^(3/2)*(a^3/(1 + m) + (3*a^2*b*x^n)/(1 + m + n)
+ (3*a*b^2*x^(2*n))/(1 + m + 2*n) + (b^3*x^(3*n))/(1 + m + 3*n)))/(a + b*x
^n)^3
```

3.522.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.58, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1384, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2} dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}} \int (dx)^m \left(b^2x^n+ab\right)^3 dx}{ab^3+b^4x^n}$$

$$\Big\downarrow \text{802}$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}} \int \left(3a^2b^4x^n(dx)^m+3ab^5x^{2n}(dx)^m+b^6x^{3n}(dx)^m+a^3b^3(dx)^m\right) dx}{ab^3+b^4x^n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}} \left(\frac{a^3b^3(dx)^{m+1}}{d(m+1)}+\frac{3a^2b^4x^{n+1}(dx)^m}{m+n+1}+\frac{3ab^5x^{2n+1}(dx)^m}{m+2n+1}+\frac{b^6x^{3n+1}(dx)^m}{m+3n+1}\right)}{ab^3+b^4x^n}$$

input
```
Int[(d*x)^m*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]*((3*a^2*b^4*x^(1 + n)*(d*x)^m)/(1 + m
 + n) + (3*a*b^5*x^(1 + 2*n)*(d*x)^m)/(1 + m + 2*n) + (b^6*x^(1 + 3*n)*(d*
x)^m)/(1 + m + 3*n) + (a^3*b^3*(d*x)^(1 + m))/(d*(1 + m))))/(a*b^3 + b^4*x
^n)
```

3.522.3.1 Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.522.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 499, normalized size of antiderivative = 2.10

method	result
risch	$\frac{\sqrt{(a+b\,x^n)^2}\,x(a^3+3m\,a^3+3a\,b^2m^3x^{2n}+6b^3mn\,x^{3n}+9a\,b^2m^2x^{2n}+9a\,b^2n^2x^{2n}+9m\,b^2a\,x^{2n}+3b^3m^2n\,x^{3n}+2b^3m\,n^2x^{3n}+12b^2a\,x^{2n}}{}$

input
```
int((d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((a+b*x^n)^2)^(1/2)/(a+b*x^n)*x*(a^3+3*m*a^3+3*a*b^2*m^3*(x^n)^2+6*b^3*m*n
*(x^n)^3+3*a^2*b*m^3*x^n+9*m*a^2*b*x^n+15*a^2*b*n*x^n+9*a*b^2*m^2*(x^n)^2+
9*a*b^2*n^2*(x^n)^2+9*a^2*b*m^2*x^n+18*a^2*b*n^2*x^n+9*m*b^2*a*(x^n)^2+3*b
^3*m^2*n*(x^n)^3+2*b^3*m*n^2*(x^n)^3+12*b^2*a*(x^n)^2*n+a^3*m^3+3*a^3*m^2+
(x^n)^3*b^3+9*a*b^2*m*n^2*(x^n)^2+15*a^2*b*m^2*n*x^n+18*a^2*b*m*n^2*x^n+24
*a*b^2*m*n*(x^n)^2+30*a^2*b*m*n*x^n+11*a^3*m*n^2+12*a^3*m*n+6*a^3*m^2*n+3*
b^3*m^2*(x^n)^3+b^3*m^3*(x^n)^3+2*b^3*n^2*(x^n)^3+3*m*b^3*(x^n)^3+3*b^3*(x
^n)^3*n+11*a^3*n^2+6*a^3*n+3*(x^n)^2*a*b^2+3*x^n*a^2*b+6*a^3*n^3+12*a*b^2*
m^2*n*(x^n)^2)/(1+m)/(1+m+n)/(1+m+2*n)/(1+m+3*n)*d^m*x^m*exp(1/2*I*Pi*csgn
(I*d*x)*m*(csgn(I*d*x)-csgn(I*x))*(-csgn(I*d*x)+csgn(I*d)))
```

3.522.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.64

$$\int (dx)^m \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2} dx = \frac{(b^3m^3+3\,b^3m^2+3\,b^3m+b^3+2\,(b^3m+b^3)n^2+3\,(b^3m^2+2\,b^3m+b^3)n)xx^{3\,n}e^{(m\log(d)+m\,\mathrm{lo}}}{}$$

input
```
integrate((d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")
```

output
```
((b^3*m^3 + 3*b^3*m^2 + 3*b^3*m + b^3 + 2*(b^3*m + b^3)*n^2 + 3*(b^3*m^2 +
 2*b^3*m + b^3)*n)*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 3*(a*b^2*m^3 + 3*a*
b^2*m^2 + 3*a*b^2*m + a*b^2 + 3*(a*b^2*m + a*b^2)*n^2 + 4*(a*b^2*m^2 + 2*a
*b^2*m + a*b^2)*n)*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 3*(a^2*b*m^3 + 3*a^
2*b*m^2 + 3*a^2*b*m + a^2*b + 6*(a^2*b*m + a^2*b)*n^2 + 5*(a^2*b*m^2 + 2*a
^2*b*m + a^2*b)*n)*x*x^n*e^(m*log(d) + m*log(x)) + (a^3*m^3 + 6*a^3*n^3 +
3*a^3*m^2 + 3*a^3*m + a^3 + 11*(a^3*m + a^3)*n^2 + 6*(a^3*m^2 + 2*a^3*m +
a^3)*n)*x*e^(m*log(d) + m*log(x)))/(m^4 + 6*(m + 1)*n^3 + 4*m^3 + 11*(m^2
+ 2*m + 1)*n^2 + 6*m^2 + 6*(m^3 + 3*m^2 + 3*m + 1)*n + 4*m + 1)
```

3.522.6 Sympy [F]

$$\int (dx)^m \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2} dx = \int (dx)^m \left((a+bx^n)^2\right)^{\frac{3}{2}} dx$$

input
```
integrate((d*x)**m*(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

output
```
Integral((d*x)**m*((a + b*x**n)**2)**(3/2), x)
```

3.522.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.16

$$\int (dx)^m \left(a^2 + 2abx^n + b^2x^{2n}\right)^{3/2} dx = \frac{(m^3 + 3m^2(2n+1) + 6n^3 + (11n^2 + 12n + 3)m + 11n^2 + 6n + 1)a^3d^mxx^m + (m^3 + 3r}{}$$

input
```
integrate((d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")
```

output
```
((m^3 + 3*m^2*(2*n + 1) + 6*n^3 + (11*n^2 + 12*n + 3)*m + 11*n^2 + 6*n + 1
)*a^3*d^m*x*x^m + (m^3 + 3*m^2*(n + 1) + (2*n^2 + 6*n + 3)*m + 2*n^2 + 3*n
 + 1)*b^3*d^m*x*e^(m*log(x) + 3*n*log(x)) + 3*(m^3 + m^2*(4*n + 3) + (3*n^
2 + 8*n + 3)*m + 3*n^2 + 4*n + 1)*a*b^2*d^m*x*e^(m*log(x) + 2*n*log(x)) +
3*(m^3 + m^2*(5*n + 3) + (6*n^2 + 10*n + 3)*m + 6*n^2 + 5*n + 1)*a^2*b*d^m
*x*e^(m*log(x) + n*log(x)))/(m^4 + 2*m^3*(3*n + 2) + (11*n^2 + 18*n + 6)*m
^2 + 6*n^3 + 2*(3*n^3 + 11*n^2 + 9*n + 2)*m + 11*n^2 + 6*n + 1)
```

3.522.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2719 vs. 2(230) = 460.

Time = 0.41 (sec) , antiderivative size = 2719, normalized size of antiderivative = 11.42

$$\int (dx)^m \left(a^2 + 2abx^n + b^2x^{2n}\right)^{3/2} dx = \text{Too large to display}$$

input
```
integrate((d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
```

output
```
(b^3*m^3*x*x^(3*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*b^3*m^2*n*x*
x^(3*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 2*b^3*m*n^2*x*x^(3*n)*e^(
m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*a*b^2*m^3*x*x^(2*n)*e^(m*log(d) +
m*log(x))*sgn(b*x^n + a) + b^3*m^3*x*x^(2*n)*e^(m*log(d) + m*log(x))*sgn(b
*x^n + a) + 12*a*b^2*m^2*n*x*x^(2*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a
) + 3*b^3*m^2*n*x*x^(2*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 9*a*b^2
*m*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 2*b^3*m*n^2*x*x^
(2*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*a^2*b*m^3*x*x^n*e^(m*log(
d) + m*log(x))*sgn(b*x^n + a) + 3*a*b^2*m^3*x*x^n*e^(m*log(d) + m*log(x))*
sgn(b*x^n + a) + b^3*m^3*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 15
*a^2*b*m^2*n*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 12*a*b^2*m^2*n
*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*b^3*m^2*n*x*x^n*e^(m*log
(d) + m*log(x))*sgn(b*x^n + a) + 18*a^2*b*m*n^2*x*x^n*e^(m*log(d) + m*log(
x))*sgn(b*x^n + a) + 9*a*b^2*m*n^2*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n
 + a) + 2*b^3*m*n^2*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + a^3*m^3
*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*a^2*b*m^3*x*e^(m*log(d) + m*
log(x))*sgn(b*x^n + a) + 3*a*b^2*m^3*x*e^(m*log(d) + m*log(x))*sgn(b*x^n +
 a) + b^3*m^3*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 6*a^3*m^2*n*x*e^(
m*log(d) + m*log(x))*sgn(b*x^n + a) + 15*a^2*b*m^2*n*x*e^(m*log(d) + m*log
(x))*sgn(b*x^n + a) + 12*a*b^2*m^2*n*x*e^(m*log(d) + m*log(x))*sgn(b*x^...
```

3.522.9 Mupad [F(-1)]

Timed out.

$$\int (dx)^m \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2} \, dx = \int (d\,x)^m\,\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)^{3/2} \,dx$$

input
```
int((d*x)^m*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2),x)
```

output
```
int((d*x)^m*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2), x)
```

3.523 $\int x^2(a^2+2abx^n+b^2x^{2n})^{3/2}\,dx$

3.523.1 Optimal result

Integrand size = 28, antiderivative size = 212

$$\int x^2\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx=\frac{a^3x^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{3\left(a+bx^n\right)}+\frac{b^4x^{3(1+n)}\sqrt{a^2+2abx^n+b^2x^{2n}}}{3(1+n)\left(ab+b^2x^n\right)}+\frac{3a^2b^2x^{3+n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(3+n)\left(ab+b^2x^n\right)}+\frac{3ab^3x^{3+2n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(3+2n)\left(ab+b^2x^n\right)}$$

output
```
1/3*a^3*x^3*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(a+b*x^n)+1/3*b^4*x^(3+3*n)*
(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(1+n)/(a*b+b^2*x^n)+3*a^2*b^2*x^(3+n)*(a
^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(3+n)/(a*b+b^2*x^n)+3*a*b^3*x^(3+2*n)*(a^2
+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(3+2*n)/(a*b+b^2*x^n)
```

3.523.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.58

$$\int x^2\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx=\frac{x^3\sqrt{\left(a+bx^n\right)^2}\left(a^3(9+18n+11n^2+2n^3)+9a^2b(3+5n+2n^2)\,x^n+9ab^2(3+4n+n^2)\,x^2\right.}{3(1+n)(3+n)(3+2n)\left(a+bx^n\right)}$$

input
```
Integrate[x^2*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

output
```
(x^3*Sqrt[(a + b*x^n)^2]*(a^3*(9 + 18*n + 11*n^2 + 2*n^3) + 9*a^2*b*(3 + 5
*n + 2*n^2)*x^n + 9*a*b^2*(3 + 4*n + n^2)*x^(2*n) + b^3*(9 + 9*n + 2*n^2)*
x^(3*n)))/(3*(1 + n)*(3 + n)*(3 + 2*n)*(a + b*x^n))
```

3.523.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.52, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {1384, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2} dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}} \int x^2 \left(b^2x^n+ab\right)^3 dx}{ab^3+b^4x^n}$$

$$\Big\downarrow \text{802}$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}} \int \left(3ab^5x^{2(n+1)}+3a^2b^4x^{n+2}+b^6x^{3n+2}+a^3b^3x^2\right) dx}{ab^3+b^4x^n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}} \left(\frac{1}{3}a^3b^3x^3+\frac{3a^2b^4x^{n+3}}{n+3}+\frac{3ab^5x^{2n+3}}{2n+3}+\frac{b^6x^{3(n+1)}}{3(n+1)}\right)}{ab^3+b^4x^n}$$

input
```
Int[x^2*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]*((a^3*b^3*x^3)/3 + (b^6*x^(3*(1 + n))
)/(3*(1 + n)) + (3*a^2*b^4*x^(3 + n))/(3 + n) + (3*a*b^5*x^(3 + 2*n))/(3 +
 2*n)))/(a*b^3 + b^4*x^n)
```

3.523.3.1 Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.523.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.69

method	result	size
risch	$\frac{\sqrt{(a+b x^n)^2}\, a^3 x^3}{3a+3b x^n}+\frac{\sqrt{(a+b x^n)^2}\, b^3 x^3 x^{3n}}{3(a+b x^n)(1+n)}+\frac{3\sqrt{(a+b x^n)^2}\, b^2 a x^3 x^{2n}}{(a+b x^n)(3+2n)}+\frac{3\sqrt{(a+b x^n)^2}\, a^2 b x^3 x^n}{(a+b x^n)(3+n)}$	146

input
```
int(x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^3*x^3+1/3*((a+b*x^n)^2)^(1/2)/(a+b*x^n
)*b^3*x^3/(1+n)*(x^n)^3+3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b^2*a/(3+2*n)*x^3*
(x^n)^2+3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^2*b/(3+n)*x^3*x^n
```

3.523.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.68

$$\int x^2(a^2+2abx^n+b^2x^{2n})^{3/2}\,dx = \frac{(2\,b^3n^2+9\,b^3n+9\,b^3)x^3x^{3\,n}+9\,(ab^2n^2+4\,ab^2n+3\,ab^2)x^3x^{2\,n}+9\,(2\,a^2bn^2+5\,a^2bn+3\,a^2}{3\,(2\,n^3+11\,n^2+18\,n+9)}$$

input
```
integrate(x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")
```

output
```
1/3*((2*b^3*n^2 + 9*b^3*n + 9*b^3)*x^3*x^(3*n) + 9*(a*b^2*n^2 + 4*a*b^2*n
+ 3*a*b^2)*x^3*x^(2*n) + 9*(2*a^2*b*n^2 + 5*a^2*b*n + 3*a^2*b)*x^3*x^n + (
2*a^3*n^3 + 11*a^3*n^2 + 18*a^3*n + 9*a^3)*x^3)/(2*n^3 + 11*n^2 + 18*n + 9
)
```

3.523.6 Sympy [F]

$$\int x^2 \left(a^2 + 2abx^n + b^2x^{2n}\right)^{3/2} dx = \int x^2 \left(\left(a + bx^n\right)^2\right)^{\frac{3}{2}} dx$$

input
```
integrate(x**2*(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

output
```
Integral(x**2*((a + b*x**n)**2)**(3/2), x)
```

3.523.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.51

$$\int x^2 \left(a^2 + 2abx^n + b^2x^{2n}\right)^{3/2} dx = \frac{(2\,n^2 + 9\,n + 9)b^3x^3x^{3\,n} + 9\,(n^2 + 4\,n + 3)ab^2x^3x^{2\,n} + 9\,(2\,n^2 + 5\,n + 3)a^2bx^3x^n + (2\,n^3 + 1}{3\,(2\,n^3 + 11\,n^2 + 18\,n + 9)}$$

input
```
integrate(x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")
```

output
```
1/3*((2*n^2 + 9*n + 9)*b^3*x^3*x^(3*n) + 9*(n^2 + 4*n + 3)*a*b^2*x^3*x^(2*
n) + 9*(2*n^2 + 5*n + 3)*a^2*b*x^3*x^n + (2*n^3 + 11*n^2 + 18*n + 9)*a^3*x
^3)/(2*n^3 + 11*n^2 + 18*n + 9)
```

3.523.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.38

$$\int x^2 (a^2 + 2abx^n + b^2x^{2n})^{3/2} \, dx = \frac{2\, b^3 n^2 x^3 x^{3\,n} \mathrm{sgn}(bx^n + a) + 9\, ab^2 n^2 x^3 x^{2\,n} \mathrm{sgn}(bx^n + a) + 18\, a^2 bn^2 x^3 x^n \mathrm{sgn}(bx^n + a) + 2\, a^3 n^3 \cdots}{\cdots}$$

input
```
integrate(x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
```

output
```
1/3*(2*b^3*n^2*x^3*x^(3*n)*sgn(b*x^n + a) + 9*a*b^2*n^2*x^3*x^(2*n)*sgn(b*
x^n + a) + 18*a^2*b*n^2*x^3*x^n*sgn(b*x^n + a) + 2*a^3*n^3*x^3*sgn(b*x^n +
 a) + 9*b^3*n*x^3*x^(3*n)*sgn(b*x^n + a) + 36*a*b^2*n*x^3*x^(2*n)*sgn(b*x^
n + a) + 45*a^2*b*n*x^3*x^n*sgn(b*x^n + a) + 11*a^3*n^2*x^3*sgn(b*x^n + a)
 + 9*b^3*x^3*x^(3*n)*sgn(b*x^n + a) + 27*a*b^2*x^3*x^(2*n)*sgn(b*x^n + a)
+ 27*a^2*b*x^3*x^n*sgn(b*x^n + a) + 18*a^3*n*x^3*sgn(b*x^n + a) + 9*a^3*x^
3*sgn(b*x^n + a))/(2*n^3 + 11*n^2 + 18*n + 9)
```

3.523.9 Mupad [F(-1)]

Timed out.

$$\int x^2 (a^2 + 2abx^n + b^2x^{2n})^{3/2} \, dx = \int x^2 \left(a^2 + b^2 x^{2\,n} + 2\,a\,b\,x^n\right)^{3/2} dx$$

input
```
int(x^2*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2),x)
```

output
```
int(x^2*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2), x)
```

3.524 $\int x(a^2 + 2abx^n + b^2x^{2n})^{3/2}\, dx$

3.524.1 Optimal result

Integrand size = 26, antiderivative size = 211

$$\int x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\, dx = \frac{a^3x^2\sqrt{a^2+2abx^n+b^2x^{2n}}}{2\left(a+bx^n\right)} + \frac{3ab^3x^{2(1+n)}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2(1+n)\left(ab+b^2x^n\right)} + \frac{3a^2b^2x^{2+n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2+n)\left(ab+b^2x^n\right)} + \frac{b^4x^{2+3n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2+3n)\left(ab+b^2x^n\right)}$$

output
```
1/2*a^3*x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(a+b*x^n)+3/2*a*b^3*x^(2+2*n
)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(1+n)/(a*b+b^2*x^n)+3*a^2*b^2*x^(2+n)*
(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(2+n)/(a*b+b^2*x^n)+b^4*x^(2+3*n)*(a^2+2
*a*b*x^n+b^2*x^(2*n))^(1/2)/(2+3*n)/(a*b+b^2*x^n)
```

3.524.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.59

$$\int x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\, dx = \frac{x^2\sqrt{\left(a+bx^n\right)^2}\left(a^3(4+12n+11n^2+3n^3)+6a^2b(2+5n+3n^2)\,x^n+3ab^2(4+8n+3n^2)\,x\right.}{2(1+n)(2+n)(2+3n)\left(a+bx^n\right)}$$

input
```
Integrate[x*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

output
```
(x^2*Sqrt[(a + b*x^n)^2]*(a^3*(4 + 12*n + 11*n^2 + 3*n^3) + 6*a^2*b*(2 + 5
*n + 3*n^2)*x^n + 3*a*b^2*(4 + 8*n + 3*n^2)*x^(2*n) + 2*b^3*(2 + 3*n + n^2
)*x^(3*n)))/(2*(1 + n)*(2 + n)*(2 + 3*n)*(a + b*x^n))
```

3.524.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.52, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1384, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}dx$$

$$\Big\downarrow \ 1384$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int x\left(b^2x^n+ab\right)^3dx}{ab^3+b^4x^n}$$

$$\Big\downarrow \ 802$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \left(3a^2b^4x^{n+1}+3ab^5x^{2n+1}+b^6x^{3n+1}+a^3b^3x\right)dx}{ab^3+b^4x^n}$$

$$\Big\downarrow \ 2009$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\left(\frac{1}{2}a^3b^3x^2+\frac{3a^2b^4x^{n+2}}{n+2}+\frac{3ab^5x^{2(n+1)}}{2(n+1)}+\frac{b^6x^{3n+2}}{3n+2}\right)}{ab^3+b^4x^n}$$

input
```
Int[x*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]*((a^3*b^3*x^2)/2 + (3*a*b^5*x^(2*(1 +
 n)))/(2*(1 + n)) + (3*a^2*b^4*x^(2 + n))/(2 + n) + (b^6*x^(2 + 3*n))/(2 +
 3*n)))/(a*b^3 + b^4*x^n)
```

3.524.3.1 Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.524.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.69

method	result	size
risch	$\frac{\sqrt{(a+b\,x^{n})^{2}}\,a^{3}x^{2}}{2a+2b\,x^{n}}+\frac{\sqrt{(a+b\,x^{n})^{2}}\,b^{3}x^{2}x^{3n}}{(a+b\,x^{n})(2+3n)}+\frac{3\sqrt{(a+b\,x^{n})^{2}}\,a\,b^{2}x^{2}x^{2n}}{2(a+b\,x^{n})(1+n)}+\frac{3\sqrt{(a+b\,x^{n})^{2}}\,a^{2}b\,x^{2}x^{n}}{(a+b\,x^{n})(2+n)}$	145

input
```
int(x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^3*x^2+((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b^
3/(2+3*n)*x^2*(x^n)^3+3/2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a*b^2*x^2/(1+n)*(x
^n)^2+3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^2*b/(2+n)*x^2*x^n
```

3.524.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.69

$$\int x\left(a^{2}+2abx^{n}+b^{2}x^{2n}\right)^{3/2}\,dx=\frac{2\,(b^{3}n^{2}+3\,b^{3}n+2\,b^{3})x^{2}x^{3\,n}+3\,(3\,ab^{2}n^{2}+8\,ab^{2}n+4\,ab^{2})x^{2}x^{2\,n}+6\,(3\,a^{2}bn^{2}+5\,a^{2}bn+2\,a}{2\,(3\,n^{3}+11\,n^{2}+12\,n+4)}$$

input
```
integrate(x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")
```

output
```
1/2*(2*(b^3*n^2 + 3*b^3*n + 2*b^3)*x^2*x^(3*n) + 3*(3*a*b^2*n^2 + 8*a*b^2*
n + 4*a*b^2)*x^2*x^(2*n) + 6*(3*a^2*b*n^2 + 5*a^2*b*n + 2*a^2*b)*x^2*x^n +
 (3*a^3*n^3 + 11*a^3*n^2 + 12*a^3*n + 4*a^3)*x^2)/(3*n^3 + 11*n^2 + 12*n +
 4)
```

3.524.6 Sympy [F]

$$\int x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx=\int x\left((a+bx^n)^2\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x*(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

output
```
Integral(x*((a + b*x**n)**2)**(3/2), x)
```

3.524.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.52

$$\int x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx=\frac{2\,(n^2+3\,n+2)b^3x^2x^{3\,n}+3\,(3\,n^2+8\,n+4)ab^2x^2x^{2\,n}+6\,(3\,n^2+5\,n+2)a^2bx^2x^n+(3\,n^3+}{2\,(3\,n^3+11\,n^2+12\,n+4)}$$

input
```
integrate(x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")
```

output
```
1/2*(2*(n^2 + 3*n + 2)*b^3*x^2*x^(3*n) + 3*(3*n^2 + 8*n + 4)*a*b^2*x^2*x^(
2*n) + 6*(3*n^2 + 5*n + 2)*a^2*b*x^2*x^n + (3*n^3 + 11*n^2 + 12*n + 4)*a^3
*x^2)/(3*n^3 + 11*n^2 + 12*n + 4)
```

3.524.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.38

$$\int x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx=\frac{2\,b^3n^2x^2x^{3\,n}\mathrm{sgn}(bx^n+a)+9\,ab^2n^2x^2x^{2\,n}\mathrm{sgn}(bx^n+a)+18\,a^2bn^2x^2x^n\mathrm{sgn}(bx^n+a)+3\,a^3n^3\cdots}{}$$

input
```
integrate(x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
```

output
```
1/2*(2*b^3*n^2*x^2*x^(3*n)*sgn(b*x^n + a) + 9*a*b^2*n^2*x^2*x^(2*n)*sgn(b*
x^n + a) + 18*a^2*b*n^2*x^2*x^n*sgn(b*x^n + a) + 3*a^3*n^3*x^2*sgn(b*x^n +
 a) + 6*b^3*n*x^2*x^(3*n)*sgn(b*x^n + a) + 24*a*b^2*n*x^2*x^(2*n)*sgn(b*x^
n + a) + 30*a^2*b*n*x^2*x^n*sgn(b*x^n + a) + 11*a^3*n^2*x^2*sgn(b*x^n + a)
 + 4*b^3*x^2*x^(3*n)*sgn(b*x^n + a) + 12*a*b^2*x^2*x^(2*n)*sgn(b*x^n + a)
+ 12*a^2*b*x^2*x^n*sgn(b*x^n + a) + 12*a^3*n*x^2*sgn(b*x^n + a) + 4*a^3*x^
2*sgn(b*x^n + a))/(3*n^3 + 11*n^2 + 12*n + 4)
```

3.524.9 Mupad [F(-1)]

Timed out.

$$\int x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx=\int x\,\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)^{3/2}\,dx$$

input
```
int(x*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2),x)
```

output
```
int(x*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2), x)
```

3.525 $\int (a^2+2abx^n+b^2x^{2n})^{3/2}\,dx$

3.525.1 Optimal result

Integrand size = 24, antiderivative size = 206

$$\int (a^2+2abx^n+b^2x^{2n})^{3/2}\,dx=\frac{a^3x(a^2+2abx^n+b^2x^{2n})^{3/2}}{(a+bx^n)^3}+\frac{3a^2b^4x^{1+n}(a^2+2abx^n+b^2x^{2n})^{3/2}}{(1+n)\,(ab+b^2x^n)^3}+\frac{3ab^5x^{1+2n}(a^2+2abx^n+b^2x^{2n})^{3/2}}{(1+2n)\,(ab+b^2x^n)^3}+\frac{b^6x^{1+3n}(a^2+2abx^n+b^2x^{2n})^{3/2}}{(1+3n)\,(ab+b^2x^n)^3}$$

output
```
a^3*x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/(a+b*x^n)^3+3*a^2*b^4*x^(1+n)*(a^2
+2*a*b*x^n+b^2*x^(2*n))^(3/2)/(1+n)/(a*b+b^2*x^n)^3+3*a*b^5*x^(1+2*n)*(a^2
+2*a*b*x^n+b^2*x^(2*n))^(3/2)/(1+2*n)/(a*b+b^2*x^n)^3+b^6*x^(1+3*n)*(a^2+2
*a*b*x^n+b^2*x^(2*n))^(3/2)/(1+3*n)/(a*b+b^2*x^n)^3
```

3.525.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.59

$$\int (a^2+2abx^n+b^2x^{2n})^{3/2}\,dx=\frac{x\sqrt{(a+bx^n)^2}(a^3(1+6n+11n^2+6n^3)+3a^2b(1+5n+6n^2)\,x^n+3ab^2(1+4n+3n^2)\,x^{2n}}{(1+n)(1+2n)(1+3n)\,(a+bx^n)}$$

input
```
Integrate[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

output
```
(x*Sqrt[(a + b*x^n)^2]*(a^3*(1 + 6*n + 11*n^2 + 6*n^3) + 3*a^2*b*(1 + 5*n
+ 6*n^2)*x^n + 3*a*b^2*(1 + 4*n + 3*n^2)*x^(2*n) + b^3*(1 + 3*n + 2*n^2)*x
^(3*n)))/((1 + n)*(1 + 2*n)*(1 + 3*n)*(a + b*x^n))
```

3.525.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.51, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1384, 775, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \left(b^2x^n+ab\right)^3dx}{ab^3+b^4x^n}$$

$$\Big\downarrow \text{775}$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \left(3a^2b^4x^n+3ab^5x^{2n}+b^6x^{3n}+a^3b^3\right)dx}{ab^3+b^4x^n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\left(a^3b^3x+\frac{3a^2b^4x^{n+1}}{n+1}+\frac{3ab^5x^{2n+1}}{2n+1}+\frac{b^6x^{3n+1}}{3n+1}\right)}{ab^3+b^4x^n}$$

input
```
Int[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]*(a^3*b^3*x + (3*a^2*b^4*x^(1 + n))/(1
 + n) + (3*a*b^5*x^(1 + 2*n))/(1 + 2*n) + (b^6*x^(1 + 3*n))/(1 + 3*n)))/(a
*b^3 + b^4*x^n)
```

3.525.3.1 Defintions of rubi rules used

rule 775
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.525.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.67

method	result	size
risch	$\frac{\sqrt{\left(a+b\,x^{n}\right)^{2}}\,a^{3}x}{a+b\,x^{n}}+\frac{\sqrt{\left(a+b\,x^{n}\right)^{2}}\,b^{3}x\,x^{3n}}{\left(a+b\,x^{n}\right)\left(1+3n\right)}+\frac{3\sqrt{\left(a+b\,x^{n}\right)^{2}}\,b^{2}ax\,x^{2n}}{\left(a+b\,x^{n}\right)\left(1+2n\right)}+\frac{3\sqrt{\left(a+b\,x^{n}\right)^{2}}\,a^{2}bx\,x^{n}}{\left(a+b\,x^{n}\right)\left(1+n\right)}$	138

input
```
int((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^3*x+((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b^3/(1+3
*n)*x*(x^n)^3+3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b^2*a/(1+2*n)*x*(x^n)^2+3*((
a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^2*b/(1+n)*x*x^n
```

3.525.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.63

$$\int \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx=\frac{(2\,b^3n^2+3\,b^3n+b^3)xx^{3\,n}+3\,(3\,ab^2n^2+4\,ab^2n+ab^2)xx^{2\,n}+3\,(6\,a^2bn^2+5\,a^2bn+a^2b)xx^{n}}{6\,n^3+11\,n^2+6\,n+1}$$

input
```
integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")
```

output
```
((2*b^3*n^2 + 3*b^3*n + b^3)*x*x^(3*n) + 3*(3*a*b^2*n^2 + 4*a*b^2*n + a*b^
2)*x*x^(2*n) + 3*(6*a^2*b*n^2 + 5*a^2*b*n + a^2*b)*x*x^n + (6*a^3*n^3 + 11
*a^3*n^2 + 6*a^3*n + a^3)*x)/(6*n^3 + 11*n^2 + 6*n + 1)
```

3.525.6 Sympy [F]

$$\int \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx=\int \left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

output
```
Integral((a**2 + 2*a*b*x**n + b**2*x**(2*n))**(3/2), x)
```

3.525.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.49

$$\int \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx=\frac{(2\,n^2+3\,n+1)b^3xx^{3\,n}+3\,(3\,n^2+4\,n+1)ab^2xx^{2\,n}+3\,(6\,n^2+5\,n+1)a^2bxx^n+(6\,n^3+1}{6\,n^3+11\,n^2+6\,n+1}$$

input
```
integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")
```

output
```
((2*n^2 + 3*n + 1)*b^3*x*x^(3*n) + 3*(3*n^2 + 4*n + 1)*a*b^2*x*x^(2*n) + 3
*(6*n^2 + 5*n + 1)*a^2*b*x*x^n + (6*n^3 + 11*n^2 + 6*n + 1)*a^3*x)/(6*n^3
+ 11*n^2 + 6*n + 1)
```

3.525.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.28

$$\int \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2} dx = \frac{6\,a^3n^3x\mathrm{sgn}(bx^n+a)+2\,b^3n^2xx^{3\,n}\mathrm{sgn}(bx^n+a)+9\,ab^2n^2xx^{2\,n}\mathrm{sgn}(bx^n+a)+18\,a^2bn^2xx^n\mathrm{s}}{}$$

input
```
integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
```

output
```
(6*a^3*n^3*x*sgn(b*x^n + a) + 2*b^3*n^2*x*x^(3*n)*sgn(b*x^n + a) + 9*a*b^2
*n^2*x*x^(2*n)*sgn(b*x^n + a) + 18*a^2*b*n^2*x*x^n*sgn(b*x^n + a) + 11*a^3
*n^2*x*sgn(b*x^n + a) + 3*b^3*n*x*x^(3*n)*sgn(b*x^n + a) + 12*a*b^2*n*x*x^
(2*n)*sgn(b*x^n + a) + 15*a^2*b*n*x*x^n*sgn(b*x^n + a) + 6*a^3*n*x*sgn(b*x
^n + a) + b^3*x*x^(3*n)*sgn(b*x^n + a) + 3*a*b^2*x*x^(2*n)*sgn(b*x^n + a)
+ 3*a^2*b*x*x^n*sgn(b*x^n + a) + a^3*x*sgn(b*x^n + a))/(6*n^3 + 11*n^2 + 6
*n + 1)
```

3.525.9 Mupad [F(-1)]

Timed out.

$$\int \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2} dx = \int \left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)^{3/2} dx$$

input
```
int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2),x)
```

output
```
int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2), x)
```

3.526 $\int \frac{(a^2+2abx^n+b^2x^{2n})^{3/2}}{x} dx$

3.526.1 Optimal result

Integrand size = 28, antiderivative size = 196

$$\int \frac{(a^2+2abx^n+b^2x^{2n})^{3/2}}{x} dx = \frac{3a^2b^2x^n\sqrt{a^2+2abx^n+b^2x^{2n}}}{n(ab+b^2x^n)} + \frac{3ab^3x^{2n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2n(ab+b^2x^n)} + \frac{b^4x^{3n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{3n(ab+b^2x^n)} + \frac{a^3\sqrt{a^2+2abx^n+b^2x^{2n}}\log(x)}{a+bx^n}$$

output
```
3*a^2*b^2*x^n*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/n/(a*b+b^2*x^n)+3/2*a*b^3*
x^(2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/n/(a*b+b^2*x^n)+1/3*b^4*x^(3*n)*
(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/n/(a*b+b^2*x^n)+a^3*ln(x)*(a^2+2*a*b*x^n
+b^2*x^(2*n))^(1/2)/(a+b*x^n)
```

3.526.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.34

$$\int \frac{(a^2+2abx^n+b^2x^{2n})^{3/2}}{x} dx = \frac{\sqrt{(a+bx^n)^2}(bx^n(18a^2+9abx^n+2b^2x^{2n})+6a^3\log(x^n))}{6n(a+bx^n)}$$

input
```
Integrate[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)/x,x]
```

output
```
(Sqrt[(a + b*x^n)^2]*(b*x^n*(18*a^2 + 9*a*b*x^n + 2*b^2*x^(2*n)) + 6*a^3*L
og[x^n]))/(6*n*(a + b*x^n))
```

3.526.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.45, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1384, 798, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{x}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \frac{\left(b^2x^n+ab\right)^3}{x}dx}{ab^3+b^4x^n}$$

$$\Big\downarrow \text{ 798}$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int b^3x^{-n}(bx^n+a)^3dx^n}{n\left(ab^3+b^4x^n\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{b^3\sqrt{a^2+2abx^n+b^2x^{2n}}\int x^{-n}(bx^n+a)^3dx^n}{n\left(ab^3+b^4x^n\right)}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{b^3\sqrt{a^2+2abx^n+b^2x^{2n}}\int \left(a^3x^{-n}+3ab^2x^n+b^3x^{2n}+3a^2b\right)dx^n}{n\left(ab^3+b^4x^n\right)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{b^3\sqrt{a^2+2abx^n+b^2x^{2n}}\left(a^3\log\left(x^n\right)+3a^2bx^n+\frac{3}{2}ab^2x^{2n}+\frac{1}{3}b^3x^{3n}\right)}{n\left(ab^3+b^4x^n\right)}$$

input
```
Int[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)/x,x]
```

output
```
(b^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]*(3*a^2*b*x^n + (3*a*b^2*x^(2*n))/
2 + (b^3*x^(3*n))/3 + a^3*Log[x^n]))/(n*(a*b^3 + b^4*x^n))
```

3.526.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.526.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.65

method	result	size
risch	$\frac{\sqrt{\left(a+b\,x^{n}\right)^{2}}\,a^{3}\ln\left(x\right)}{a+b\,x^{n}}+\frac{\sqrt{\left(a+b\,x^{n}\right)^{2}}\,b^{3}x^{3n}}{3\left(a+b\,x^{n}\right)n}+\frac{3\sqrt{\left(a+b\,x^{n}\right)^{2}}\,b^{2}a\,x^{2n}}{2\left(a+b\,x^{n}\right)n}+\frac{3\sqrt{\left(a+b\,x^{n}\right)^{2}}\,a^{2}b\,x^{n}}{\left(a+b\,x^{n}\right)n}$	127

input
```
int((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^3*ln(x)+1/3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*
b^3/n*(x^n)^3+3/2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b^2*a/n*(x^n)^2+3*((a+b*x^
n)^2)^(1/2)/(a+b*x^n)*a^2*b/n*x^n
```

3.526.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.22

$$\int \frac{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}{x} dx = \frac{6\,a^3n\log{(x)} + 2\,b^3x^{3\,n} + 9\,ab^2x^{2\,n} + 18\,a^2bx^n}{6\,n}$$

input
```
integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x,x, algorithm="fricas")
```

output
```
1/6*(6*a^3*n*log(x) + 2*b^3*x^(3*n) + 9*a*b^2*x^(2*n) + 18*a^2*b*x^n)/n
```

3.526.6 Sympy [F]

$$\int \frac{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}{x} dx = \int \frac{\left((a + bx^n)^2\right)^{\frac{3}{2}}}{x} dx$$

input
```
integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2)/x,x)
```

output
```
Integral(((a + b*x**n)**2)**(3/2)/x, x)
```

3.526.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.22

$$\int \frac{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}{x} dx = a^3\log{(x)} + \frac{2\,b^3x^{3\,n} + 9\,ab^2x^{2\,n} + 18\,a^2bx^n}{6\,n}$$

input
```
integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x,x, algorithm="maxima")
```

output
```
a^3*log(x) + 1/6*(2*b^3*x^(3*n) + 9*a*b^2*x^(2*n) + 18*a^2*b*x^n)/n
```

3.526.8 Giac [F]

$$\int \frac{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}{x}\,dx = \int \frac{(b^2x^{2n} + 2\,abx^n + a^2)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x,x, algorithm="giac")
```

output
```
integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)/x, x)
```

3.526.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}{x}\,dx = \int \frac{(a^2 + b^2\,x^{2\,n} + 2\,a\,b\,x^n)^{3/2}}{x}\,dx$$

input
```
int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)/x,x)
```

output
```
int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)/x, x)
```

3.527 $\int \frac{(a^2+2abx^n+b^2x^{2n})^{3/2}}{x^2}\,dx$

3.527.1 Optimal result

Integrand size = 28, antiderivative size = 212

$$\int \frac{(a^2+2abx^n+b^2x^{2n})^{3/2}}{x^2}\,dx=-\frac{a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{x(a+bx^n)}-\frac{3a^2b^2x^{-1+n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1-n)(ab+b^2x^n)}-\frac{3ab^3x^{-1+2n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1-2n)(ab+b^2x^n)}-\frac{b^4x^{-1+3n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1-3n)(ab+b^2x^n)}$$

output
```
-a^3*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x/(a+b*x^n)-3*a^2*b^2*x^(-1+n)*(a^2
+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(1-n)/(a*b+b^2*x^n)-3*a*b^3*x^(-1+2*n)*(a^2+
2*a*b*x^n+b^2*x^(2*n))^(1/2)/(1-2*n)/(a*b+b^2*x^n)-b^4*x^(-1+3*n)*(a^2+2*a
*b*x^n+b^2*x^(2*n))^(1/2)/(1-3*n)/(a*b+b^2*x^n)
```

3.527.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.58

$$\int \frac{(a^2+2abx^n+b^2x^{2n})^{3/2}}{x^2}\,dx=\frac{\sqrt{(a+bx^n)^2}\,(a^3(1-6n+11n^2-6n^3)+3a^2b(1-5n+6n^2)\,x^n+3ab^2(1-}{(-1+n)(-1+2n)(-1+3n)x\,(a+b}$$

input
```
Integrate[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)/x^2,x]
```

output
```
(Sqrt[(a + b*x^n)^2]*(a^3*(1 - 6*n + 11*n^2 - 6*n^3) + 3*a^2*b*(1 - 5*n +
6*n^2)*x^n + 3*a*b^2*(1 - 4*n + 3*n^2)*x^(2*n) + b^3*(1 - 3*n + 2*n^2)*x^(
3*n)))/((-1 + n)*(-1 + 2*n)*(-1 + 3*n)*x*(a + b*x^n))
```

3.527.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.52, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {1384, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{x^2}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \frac{\left(b^2x^n+ab\right)^3}{x^2}dx}{ab^3+b^4x^n}$$

$$\Big\downarrow \text{802}$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \left(3a^2b^4x^{n-2}+3ab^5x^{2(n-1)}+b^6x^{3n-2}+\frac{a^3b^3}{x^2}\right)dx}{ab^3+b^4x^n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\left(-\frac{a^3b^3}{x}-\frac{3a^2b^4x^{n-1}}{1-n}-\frac{3ab^5x^{2n-1}}{1-2n}-\frac{b^6x^{3n-1}}{1-3n}\right)}{ab^3+b^4x^n}$$

input
```
Int[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)/x^2,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]*(-((a^3*b^3)/x) - (3*a^2*b^4*x^(-1 +
n))/(1 - n) - (3*a*b^5*x^(-1 + 2*n))/(1 - 2*n) - (b^6*x^(-1 + 3*n))/(1 - 3
*n)))/(a*b^3 + b^4*x^n)
```

3.527.3.1 Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.527.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.69

method	result	size
risch	$-\frac{\sqrt{\left(a+b\,x^{n}\right)^{2}}\,a^{3}}{\left(a+b\,x^{n}\right)x}+\frac{\sqrt{\left(a+b\,x^{n}\right)^{2}}\,b^{3}x^{3n}}{\left(a+b\,x^{n}\right)\left(-1+3n\right)x}+\frac{3\sqrt{\left(a+b\,x^{n}\right)^{2}}\,b^{2}a\,x^{2n}}{\left(a+b\,x^{n}\right)\left(-1+2n\right)x}+\frac{3\sqrt{\left(a+b\,x^{n}\right)^{2}}\,a^{2}b\,x^{n}}{\left(a+b\,x^{n}\right)\left(-1+n\right)x}$	147

input
```
int((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^3/x+((a+b*x^n)^2)^(1/2)/(a+b*x^n)/(-1+3*n
)*b^3/x*(x^n)^3+3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)/(-1+2*n)*b^2*a/x*(x^n)^2+3
*((a+b*x^n)^2)^(1/2)/(a+b*x^n)/(-1+n)*a^2*b/x*x^n
```

3.527.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.62

$$\int \frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{x^2}\,dx =$$

$$-\frac{6\,a^3n^3-11\,a^3n^2+6\,a^3n-a^3-(2\,b^3n^2-3\,b^3n+b^3)x^{3\,n}-3\,(3\,ab^2n^2-4\,ab^2n+ab^2)x^{2\,n}-3\,(6\,a^2bn^2-5}{(6\,n^3-11\,n^2+6\,n-1)x}$$

input
```
integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x^2,x, algorithm="fricas")
```

output
```
-(6*a^3*n^3 - 11*a^3*n^2 + 6*a^3*n - a^3 - (2*b^3*n^2 - 3*b^3*n + b^3)*x^(
3*n) - 3*(3*a*b^2*n^2 - 4*a*b^2*n + a*b^2)*x^(2*n) - 3*(6*a^2*b*n^2 - 5*a^
2*b*n + a^2*b)*x^n)/((6*n^3 - 11*n^2 + 6*n - 1)*x)
```

3.527.6 Sympy [F]

$$\int \frac{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}{x^2}\,dx = \int \frac{\left((a + bx^n)^2\right)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2)/x**2,x)
```

output
```
Integral(((a + b*x**n)**2)**(3/2)/x**2, x)
```

3.527.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.48

$$\int \frac{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}{x^2}\,dx = \frac{(2\,n^2 - 3\,n + 1)b^3x^{3\,n} + 3\,(3\,n^2 - 4\,n + 1)ab^2x^{2\,n} + 3\,(6\,n^2 - 5\,n + 1)a^2bx^n}{(6\,n^3 - 11\,n^2 + 6\,n - 1)x}$$

input
```
integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x^2,x, algorithm="maxima")
```

output
```
((2*n^2 - 3*n + 1)*b^3*x^(3*n) + 3*(3*n^2 - 4*n + 1)*a*b^2*x^(2*n) + 3*(6*
n^2 - 5*n + 1)*a^2*b*x^n - (6*n^3 - 11*n^2 + 6*n - 1)*a^3)/((6*n^3 - 11*n^
2 + 6*n - 1)*x)
```

3.527.8 Giac [F]

$$\int \frac{(a^2+2abx^n+b^2x^{2n})^{3/2}}{x^2}\,dx=\int \frac{(b^2x^{2n}+2\,abx^n+a^2)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x^2,x, algorithm="giac")
```

output
```
integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)/x^2, x)
```

3.527.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2+2abx^n+b^2x^{2n})^{3/2}}{x^2}\,dx=\int \frac{(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n)^{3/2}}{x^2}\,dx$$

input
```
int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)/x^2,x)
```

output
```
int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)/x^2, x)
```

3.528 $\int \frac{(a^2+2abx^n+b^2x^{2n})^{3/2}}{x^3}\,dx$

3.528.1 Optimal result

Integrand size = 28, antiderivative size = 218

$$\int \frac{(a^2+2abx^n+b^2x^{2n})^{3/2}}{x^3}\,dx = -\frac{a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{2x^2(a+bx^n)} - \frac{3ab^3x^{-2(1-n)}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2(1-n)(ab+b^2x^n)} - \frac{3a^2b^2x^{-2+n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2-n)(ab+b^2x^n)} - \frac{b^4x^{-2+3n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2-3n)(ab+b^2x^n)}$$

output
```
-1/2*a^3*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^2/(a+b*x^n)-3/2*a*b^3*(a^2+2*
a*b*x^n+b^2*x^(2*n))^(1/2)/(1-n)/(x^(2-2*n))/(a*b+b^2*x^n)-3*a^2*b^2*x^(-2
+n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(2-n)/(a*b+b^2*x^n)-b^4*x^(-2+3*n)*(
a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(2-3*n)/(a*b+b^2*x^n)
```

3.528.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.57

$$\int \frac{(a^2+2abx^n+b^2x^{2n})^{3/2}}{x^3}\,dx = \frac{\sqrt{(a+bx^n)^2}(a^3(4-12n+11n^2-3n^3)+6a^2b(2-5n+3n^2)\,x^n+3ab^2(4}{2(-2+n)(-1+n)(-2+3n)x^2\,(a+}$$

input
```
Integrate[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)/x^3,x]
```

output
```
(Sqrt[(a + b*x^n)^2]*(a^3*(4 - 12*n + 11*n^2 - 3*n^3) + 6*a^2*b*(2 - 5*n +
 3*n^2)*x^n + 3*a*b^2*(4 - 8*n + 3*n^2)*x^(2*n) + 2*b^3*(2 - 3*n + n^2)*x^
(3*n)))/(2*(-2 + n)*(-1 + n)*(-2 + 3*n)*x^2*(a + b*x^n))
```

3.528.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.54, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {1384, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{x^3}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \frac{\left(b^2x^n+ab\right)^3}{x^3}dx}{ab^3+b^4x^n}$$

$$\Big\downarrow \text{ 802}$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \left(3a^2b^4x^{n-3}+b^6x^{3(n-1)}+3ab^5x^{2n-3}+\frac{a^3b^3}{x^3}\right)dx}{ab^3+b^4x^n}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\left(-\frac{a^3b^3}{2x^2}-\frac{3a^2b^4x^{n-2}}{2-n}-\frac{3ab^5x^{-2(1-n)}}{2(1-n)}-\frac{b^6x^{3n-2}}{2-3n}\right)}{ab^3+b^4x^n}$$

input
```
Int[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)/x^3,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]*(-1/2*(a^3*b^3)/x^2 - (3*a*b^5)/(2*(1
 - n)*x^(2*(1 - n))) - (3*a^2*b^4*x^(-2 + n))/(2 - n) - (b^6*x^(-2 + 3*n))
/(2 - 3*n)))/(a*b^3 + b^4*x^n)
```

3.528.3.1 Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.528.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.67

method	result	size
risch	$-\frac{\sqrt{(a+b\,x^n)^2}\,a^3}{2(a+b\,x^n)x^2}+\frac{\sqrt{(a+b\,x^n)^2}\,b^3x^{3n}}{(a+b\,x^n)(-2+3n)x^2}+\frac{3\sqrt{(a+b\,x^n)^2}\,b^2a\,x^{2n}}{2(a+b\,x^n)(-1+n)x^2}+\frac{3\sqrt{(a+b\,x^n)^2}\,a^2b\,x^n}{(a+b\,x^n)(-2+n)x^2}$	145

input
```
int((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^3/x^2+((a+b*x^n)^2)^(1/2)/(a+b*x^n)/(
-2+3*n)*b^3/x^2*(x^n)^3+3/2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)/(-1+n)*b^2*a/x^2
*(x^n)^2+3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)/(-2+n)*a^2*b/x^2*x^n
```

3.528.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.61

$$\int \frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{x^3}\,dx =$$
$$-\frac{3\,a^3n^3-11\,a^3n^2+12\,a^3n-4\,a^3-2\,(b^3n^2-3\,b^3n+2\,b^3)x^{3\,n}-3\,(3\,ab^2n^2-8\,ab^2n+4\,ab^2)x^{2\,n}-6\,(3\,a^2b}{2\,(3\,n^3-11\,n^2+12\,n-4)x^2}$$

input
```
integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x^3,x, algorithm="fricas")
```

output
```
-1/2*(3*a^3*n^3 - 11*a^3*n^2 + 12*a^3*n - 4*a^3 - 2*(b^3*n^2 - 3*b^3*n + 2
*b^3)*x^(3*n) - 3*(3*a*b^2*n^2 - 8*a*b^2*n + 4*a*b^2)*x^(2*n) - 6*(3*a^2*b
*n^2 - 5*a^2*b*n + 2*a^2*b)*x^n)/((3*n^3 - 11*n^2 + 12*n - 4)*x^2)
```

3.528.6 Sympy [F]

$$\int \frac{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}{x^3}\,dx = \int \frac{\left((a + bx^n)^2\right)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2)/x**3,x)
```

output
```
Integral(((a + b*x**n)**2)**(3/2)/x**3, x)
```

3.528.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.46

$$\int \frac{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}{x^3}\,dx = \frac{2\,(n^2 - 3\,n + 2)b^3x^{3\,n} + 3\,(3\,n^2 - 8\,n + 4)ab^2x^{2\,n} + 6\,(3\,n^2 - 5\,n + 2)a^2bx^n}{2\,(3\,n^3 - 11\,n^2 + 12\,n - 4)x^2}$$

input
```
integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x^3,x, algorithm="maxima")
```

output
```
1/2*(2*(n^2 - 3*n + 2)*b^3*x^(3*n) + 3*(3*n^2 - 8*n + 4)*a*b^2*x^(2*n) + 6
*(3*n^2 - 5*n + 2)*a^2*b*x^n - (3*n^3 - 11*n^2 + 12*n - 4)*a^3)/((3*n^3 -
11*n^2 + 12*n - 4)*x^2)
```

3.528.8 Giac [F]

$$\int \frac{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}{x^3}\,dx = \int \frac{(b^2x^{2n} + 2\,abx^n + a^2)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x^3,x, algorithm="giac")
```

output
```
integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)/x^3, x)
```

3.528.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}{x^3}\,dx = \int \frac{(a^2 + b^2\,x^{2\,n} + 2\,a\,b\,x^n)^{3/2}}{x^3}\,dx$$

input
```
int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)/x^3,x)
```

output
```
int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)/x^3, x)
```

3.529 $\int \frac{(dx)^m}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx$

3.529.1 Optimal result

Integrand size = 30, antiderivative size = 76

$$\int \frac{(dx)^m}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx=\frac{(dx)^{1+m}(a+bx^n)\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{bx^n}{a}\right)}{ad(1+m)\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

output
```
(d*x)^(1+m)*(a+b*x^n)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-b*x^n/a)/a/d/(1+
m)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)
```

3.529.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.82

$$\int \frac{(dx)^m}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx=\frac{x(dx)^m(a+bx^n)\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},1+\frac{1+m}{n},-\frac{bx^n}{a}\right)}{a(1+m)\sqrt{(a+bx^n)^2}}$$

input
```
Integrate[(d*x)^m/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

output
```
(x*(d*x)^m*(a + b*x^n)*Hypergeometric2F1[1, (1 + m)/n, 1 + (1 + m)/n, -((b
*x^n)/a)])/(a*(1 + m)*Sqrt[(a + b*x^n)^2])
```

3.529.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.09, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1384, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{\sqrt{a^2+2abx^n+b^2x^{2n}}}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{(ab+b^2x^n)\int \frac{(dx)^m}{b^2x^n+ab}dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{(dx)^{m+1}(ab+b^2x^n)\text{Hypergeometric2F1}\left(1,\frac{m+1}{n},\frac{m+n+1}{n},-\frac{bx^n}{a}\right)}{abd(m+1)\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

input
```
Int[(d*x)^m/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

output
```
((d*x)^(1 + m)*(a*b + b^2*x^n)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)
/n, -((b*x^n)/a)])/(a*b*d*(1 + m)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

3.529.3.1 Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

3.529.4 Maple [F]

$$\int \frac{(dx)^m}{\sqrt{a^2 + 2ab\,x^n + b^2x^{2n}}}dx$$

input
```
int((d*x)^m/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

output
```
int((d*x)^m/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

3.529.5 Fricas [F]

$$\int \frac{(dx)^m}{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}\,dx = \int \frac{(dx)^m}{\sqrt{b^2x^{2\,n} + 2\,abx^n + a^2}}\,dx$$

input
```
integrate((d*x)^m/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")
```

output
```
integral((d*x)^m/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

3.529.6 Sympy [F]

$$\int \frac{(dx)^m}{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}\,dx = \int \frac{(dx)^m}{\sqrt{(a + bx^n)^2}}\,dx$$

input
```
integrate((d*x)**m/(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

output
```
Integral((d*x)**m/sqrt((a + b*x**n)**2), x)
```

3.529.7 Maxima [F]

$$\int \frac{(dx)^m}{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}\,dx = \int \frac{(dx)^m}{\sqrt{b^2x^{2n} + 2\,abx^n + a^2}}\,dx$$

input
```
integrate((d*x)^m/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x)^m/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

3.529.8 Giac [F]

$$\int \frac{(dx)^m}{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}\,dx = \int \frac{(dx)^m}{\sqrt{b^2x^{2n} + 2\,abx^n + a^2}}\,dx$$

input
```
integrate((d*x)^m/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x)^m/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

3.529.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}\,dx = \int \frac{(d\,x)^m}{\sqrt{a^2 + b^2\,x^{2\,n} + 2\,a\,b\,x^n}}\,dx$$

input
```
int((d*x)^m/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2),x)
```

output
```
int((d*x)^m/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2), x)
```

3.530 $\int \frac{x^2}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx$

3.530.1 Optimal result

Integrand size = 28, antiderivative size = 64

$$\int \frac{x^2}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx=\frac{x^3(a+bx^n)\,\text{Hypergeometric2F1}\left(1,\frac{3}{n},\frac{3+n}{n},-\frac{bx^n}{a}\right)}{3a\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

output
```
1/3*x^3*(a+b*x^n)*hypergeom([1, 3/n],[(3+n)/n],-b*x^n/a)/a/(a^2+2*a*b*x^n+
b^2*x^(2*n))^(1/2)
```

3.530.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.83

$$\int \frac{x^2}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx=\frac{x^3(a+bx^n)\,\text{Hypergeometric2F1}\left(1,\frac{3}{n},1+\frac{3}{n},-\frac{bx^n}{a}\right)}{3a\sqrt{(a+bx^n)^2}}$$

input
```
Integrate[x^2/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

output
```
(x^3*(a + b*x^n)*Hypergeometric2F1[1, 3/n, 1 + 3/n, -((b*x^n)/a)])/(3*a*Sq
rt[(a + b*x^n)^2])
```

3.530.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.11, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1384, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{(ab+b^2x^n)\int \frac{x^2}{b^2x^n+ab}dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{x^3(ab+b^2x^n)\,\text{Hypergeometric2F1}\left(1,\frac{3}{n},\frac{n+3}{n},-\frac{bx^n}{a}\right)}{3ab\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

input
```
Int[x^2/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

output
```
(x^3*(a*b + b^2*x^n)*Hypergeometric2F1[1, 3/n, (3 + n)/n, -((b*x^n)/a)])/(
3*a*b*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

3.530.3.1 Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

3.530.4 Maple [F]

$$\int \frac{x^2}{\sqrt{a^2 + 2ab\, x^n + b^2 x^{2n}}} dx$$

input
```
int(x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

output
```
int(x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

3.530.5 Fricas [F]

$$\int \frac{x^2}{\sqrt{a^2 + 2abx^n + b^2x^{2n}}} \, dx = \int \frac{x^2}{\sqrt{b^2x^{2\,n} + 2\,abx^n + a^2}} \, dx$$

input
```
integrate(x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")
```

output
```
integral(x^2/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

3.530.6 Sympy [F]

$$\int \frac{x^2}{\sqrt{a^2 + 2abx^n + b^2x^{2n}}} \, dx = \int \frac{x^2}{\sqrt{(a + bx^n)^2}} \, dx$$

input
```
integrate(x**2/(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

output
```
Integral(x**2/sqrt((a + b*x**n)**2), x)
```

3.530.7 Maxima [F]

$$\int \frac{x^2}{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}\,dx = \int \frac{x^2}{\sqrt{b^2x^{2\,n} + 2\,abx^n + a^2}}\,dx$$

input
```
integrate(x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

3.530.8 Giac [F]

$$\int \frac{x^2}{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}\,dx = \int \frac{x^2}{\sqrt{b^2x^{2\,n} + 2\,abx^n + a^2}}\,dx$$

input
```
integrate(x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

3.530.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}\,dx = \int \frac{x^2}{\sqrt{a^2 + b^2\,x^{2\,n} + 2\,a\,b\,x^n}}\,dx$$

input
```
int(x^2/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2),x)
```

output
```
int(x^2/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2), x)
```

3.531 $\int \frac{x}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx$

3.531.1 Optimal result

Integrand size = 26, antiderivative size = 64

$$\int \frac{x}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx=\frac{x^2(a+bx^n)\,\text{Hypergeometric2F1}\left(1,\frac{2}{n},\frac{2+n}{n},-\frac{bx^n}{a}\right)}{2a\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

output
```
1/2*x^2*(a+b*x^n)*hypergeom([1, 2/n],[(2+n)/n],-b*x^n/a)/a/(a^2+2*a*b*x^n+
b^2*x^(2*n))^(1/2)
```

3.531.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.83

$$\int \frac{x}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx=\frac{x^2(a+bx^n)\,\text{Hypergeometric2F1}\left(1,\frac{2}{n},1+\frac{2}{n},-\frac{bx^n}{a}\right)}{2a\sqrt{(a+bx^n)^2}}$$

input
```
Integrate[x/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

output
```
(x^2*(a + b*x^n)*Hypergeometric2F1[1, 2/n, 1 + 2/n, -((b*x^n)/a)])/(2*a*Sq
rt[(a + b*x^n)^2])
```

3.531.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.11, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1384, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a^2+2abx^n+b^2x^{2n}}}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{(ab+b^2x^n)\int \frac{x}{b^2x^n+ab}dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{x^2(ab+b^2x^n)\,\text{Hypergeometric2F1}\left(1,\frac{2}{n},\frac{n+2}{n},-\frac{bx^n}{a}\right)}{2ab\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

input
```
Int[x/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

output
```
(x^2*(a*b + b^2*x^n)*Hypergeometric2F1[1, 2/n, (2 + n)/n, -((b*x^n)/a)])/(
2*a*b*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

3.531.3.1 Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

3.531.4 Maple [F]

$$\int \frac{x}{\sqrt{a^2+2ab\,x^n+b^2x^{2n}}}dx$$

input
```
int(x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

output
```
int(x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

3.531.5 Fricas [F]

$$\int \frac{x}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \int \frac{x}{\sqrt{b^2x^{2\,n}+2\,abx^n+a^2}}\,dx$$

input
```
integrate(x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")
```

output
```
integral(x/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

3.531.6 Sympy [F]

$$\int \frac{x}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \int \frac{x}{\sqrt{(a+bx^n)^2}}\,dx$$

input
```
integrate(x/(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

output
```
Integral(x/sqrt((a + b*x**n)**2), x)
```

3.531.7 Maxima [F]

$$\int \frac{x}{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}\,dx = \int \frac{x}{\sqrt{b^2x^{2\,n} + 2\,abx^n + a^2}}\,dx$$

input
```
integrate(x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

3.531.8 Giac [F]

$$\int \frac{x}{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}\,dx = \int \frac{x}{\sqrt{b^2x^{2\,n} + 2\,abx^n + a^2}}\,dx$$

input
```
integrate(x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
```

output
```
integrate(x/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

3.531.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}\,dx = \int \frac{x}{\sqrt{a^2 + b^2\,x^{2\,n} + 2\,a\,b\,x^n}}\,dx$$

input
```
int(x/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2),x)
```

output
```
int(x/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2), x)
```

3.532 $\int \frac{1}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx$

3.532.1 Optimal result

Integrand size = 24, antiderivative size = 55

$$\int \frac{1}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx=\frac{x(a+bx^n)\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

output
```
x*(a+b*x^n)*hypergeom([1, 1/n],[1+1/n],-b*x^n/a)/a/(a^2+2*a*b*x^n+b^2*x^(2
*n))^(1/2)
```

3.532.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.80

$$\int \frac{1}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx=\frac{x(a+bx^n)\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a\sqrt{(a+bx^n)^2}}$$

input
```
Integrate[1/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

output
```
(x*(a + b*x^n)*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a*
Sqrt[(a + b*x^n)^2])
```

3.532.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.13, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1384, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a^2+2abx^n+b^2x^{2n}}}dx$$
$$\Big\downarrow \text{ 1384}$$
$$\frac{(ab+b^2x^n)\int \frac{1}{b^2x^n+ab}dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}$$
$$\Big\downarrow \text{ 778}$$
$$\frac{x(ab+b^2x^n)\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{ab\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

input
```
Int[1/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

output
```
(x*(a*b + b^2*x^n)*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])
/(a*b*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

3.532.3.1 Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

3.532.4 Maple [F]

$$\int \frac{1}{\sqrt{a^2+2ab\,x^n+b^2x^{2n}}}dx$$

input
```
int(1/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

output
```
int(1/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

3.532.5 Fricas [F]

$$\int \frac{1}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \int \frac{1}{\sqrt{b^2x^{2\,n}+2\,abx^n+a^2}}\,dx$$

input
```
integrate(1/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")
```

output
```
integral(1/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

3.532.6 Sympy [F]

$$\int \frac{1}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \int \frac{1}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx$$

input
```
integrate(1/(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

output
```
Integral(1/sqrt(a**2 + 2*a*b*x**n + b**2*x**(2*n)), x)
```

3.532.7 Maxima [F]

$$\int \frac{1}{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}\,dx = \int \frac{1}{\sqrt{b^2x^{2n} + 2\,abx^n + a^2}}\,dx$$

input
```
integrate(1/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

3.532.8 Giac [F]

$$\int \frac{1}{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}\,dx = \int \frac{1}{\sqrt{b^2x^{2n} + 2\,abx^n + a^2}}\,dx$$

input
```
integrate(1/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

3.532.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}\,dx = \int \frac{1}{\sqrt{a^2 + b^2\,x^{2\,n} + 2\,a\,b\,x^n}}\,dx$$

input
```
int(1/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2),x)
```

output
```
int(1/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2), x)
```

3.533 $\int \frac{1}{x\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx$

3.533.1 Optimal result

Integrand size = 28, antiderivative size = 85

$$\int \frac{1}{x\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx=\frac{(a+bx^n)\log(x)}{a\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{(a+bx^n)\log(a+bx^n)}{an\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

output
```
(a+b*x^n)*ln(x)/a/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)-(a+b*x^n)*ln(a+b*x^n)/
a/n/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)
```

3.533.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.53

$$\int \frac{1}{x\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx=\frac{(a+bx^n)\left(\log(x^n)-\log(an(a+bx^n))\right)}{an\sqrt{(a+bx^n)^2}}$$

input
```
Integrate[1/(x*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]),x]
```

output
```
((a + b*x^n)*(Log[x^n] - Log[a*n*(a + b*x^n)]))/(a*n*Sqrt[(a + b*x^n)^2])
```

3.533.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.75, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1384, 798, 27, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a^2+2abx^n+b^2x^{2n}}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{(ab+b^2x^n)\int \frac{1}{x(b^2x^n+ab)}dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

$$\Big\downarrow \text{798}$$

$$\frac{(ab+b^2x^n)\int \frac{x^{-n}}{b(bx^n+a)}dx^n}{n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(ab+b^2x^n)\int \frac{x^{-n}}{bx^n+a}dx^n}{bn\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

$$\Big\downarrow \text{47}$$

$$\frac{(ab+b^2x^n)\left(\frac{\int x^{-n}dx^n}{a}-\frac{b\int \frac{1}{bx^n+a}dx^n}{a}\right)}{bn\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

$$\Big\downarrow \text{14}$$

$$\frac{(ab+b^2x^n)\left(\frac{\log(x^n)}{a}-\frac{b\int \frac{1}{bx^n+a}dx^n}{a}\right)}{bn\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

$$\Big\downarrow \text{16}$$

$$\frac{(ab+b^2x^n)\left(\frac{\log(x^n)}{a}-\frac{\log(a+bx^n)}{a}\right)}{bn\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

input
```
Int[1/(x*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]),x]
```

output
```
((a*b + b^2*x^n)*(Log[x^n]/a - Log[a + b*x^n]/a))/(b*n*Sqrt[a^2 + 2*a*b*x^
n + b^2*x^(2*n)])
```

3.533.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

3.533. $\int \frac{1}{x\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx$

3.533.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.78

method	result	size
risch	$\frac{\sqrt{(a+b\,x^{n})^{2}}\,\ln(x)}{(a+b\,x^{n})a}-\frac{\sqrt{(a+b\,x^{n})^{2}}\,\ln\left(x^{n}+\frac{a}{b}\right)}{(a+b\,x^{n})an}$	66

input
```
int(1/x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((a+b*x^n)^2)^(1/2)/(a+b*x^n)*ln(x)/a-((a+b*x^n)^2)^(1/2)/(a+b*x^n)/a/n*ln
(x^n+a/b)
```

3.533.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.26

$$\int \frac{1}{x\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx=\frac{n\log(x)-\log(bx^n+a)}{an}$$

input
```
integrate(1/x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")
```

output
```
(n*log(x) - log(b*x^n + a))/(a*n)
```

3.533.6 Sympy [F]

$$\int \frac{1}{x\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx=\int \frac{1}{x\sqrt{(a+bx^n)^2}}\,dx$$

input
```
integrate(1/x/(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

output
```
Integral(1/(x*sqrt((a + b*x**n)**2)), x)
```

3.533.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.32

$$\int \frac{1}{x\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \frac{\log(x)}{a} - \frac{\log\left(\frac{bx^n+a}{b}\right)}{an}$$

input
```
integrate(1/x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")
```

output
```
log(x)/a - log((b*x^n + a)/b)/(a*n)
```

3.533.8 Giac [F]

$$\int \frac{1}{x\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \int \frac{1}{\sqrt{b^2x^{2n}+2\,abx^n+a^2}\,x}\,dx$$

input
```
integrate(1/x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x), x)
```

3.533.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \int \frac{1}{x\,\sqrt{a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n}}\,dx$$

input
```
int(1/(x*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)),x)
```

output
```
int(1/(x*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)), x)
```

3.534 $\int \frac{1}{x^2\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx$

3.534.1 Optimal result

Integrand size = 28, antiderivative size = 65

$$\int \frac{1}{x^2\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = -\frac{(a+bx^n)\,\text{Hypergeometric2F1}\left(1,-\frac{1}{n},-\frac{1-n}{n},-\frac{bx^n}{a}\right)}{ax\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

output
```
-(a+b*x^n)*hypergeom([1, -1/n],[(-1+n)/n],-b*x^n/a)/a/x/(a^2+2*a*b*x^n+b^2
*x^(2*n))^(1/2)
```

3.534.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^2\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = -\frac{(a+bx^n)\,\text{Hypergeometric2F1}\left(1,-\frac{1}{n},1-\frac{1}{n},-\frac{bx^n}{a}\right)}{ax\sqrt{(a+bx^n)^2}}$$

input
```
Integrate[1/(x^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]),x]
```

output
```
-(((a + b*x^n)*Hypergeometric2F1[1, -n^(-1), 1 - n^(-1), -((b*x^n)/a)])/(a
*x*Sqrt[(a + b*x^n)^2]))
```

3.534.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.11, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1384, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{a^2+2abx^n+b^2x^{2n}}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{(ab+b^2x^n)\int \frac{1}{x^2(b^2x^n+ab)}dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

$$\Big\downarrow \text{888}$$

$$-\frac{(ab+b^2x^n)\,\text{Hypergeometric2F1}\left(1,-\frac{1}{n},-\frac{1-n}{n},-\frac{bx^n}{a}\right)}{abx\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

input
```
Int[1/(x^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]),x]
```

output
```
-(((a*b + b^2*x^n)*Hypergeometric2F1[1, -n^(-1), -((1 - n)/n), -((b*x^n)/a
)])/(a*b*x*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]))
```

3.534.3.1 Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

3.534.4 Maple [F]

$$\int \frac{1}{x^2\sqrt{a^2+2ab\,x^n+b^2x^{2n}}}dx$$

input
```
int(1/x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

output
```
int(1/x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

3.534.5 Fricas [F]

$$\int \frac{1}{x^2\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \int \frac{1}{\sqrt{b^2x^{2\,n}+2\,abx^n+a^2}x^2}\,dx$$

input
```
integrate(1/x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)/(b^2*x^2*x^(2*n) + 2*a*b*x^2*
x^n + a^2*x^2), x)
```

3.534.6 Sympy [F]

$$\int \frac{1}{x^2\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \int \frac{1}{x^2\sqrt{(a+bx^n)^2}}\,dx$$

input
```
integrate(1/x**2/(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt((a + b*x**n)**2)), x)
```

3.534.7 Maxima [F]

$$\int \frac{1}{x^2\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \int \frac{1}{\sqrt{b^2x^{2\,n}+2\,abx^n+a^2}x^2}\,dx$$

input
```
integrate(1/x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x^2), x)
```

3.534.8 Giac [F]

$$\int \frac{1}{x^2\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \int \frac{1}{\sqrt{b^2x^{2\,n}+2\,abx^n+a^2}x^2}\,dx$$

input
```
integrate(1/x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x^2), x)
```

3.534.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \int \frac{1}{x^2\,\sqrt{a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n}}\,dx$$

input
```
int(1/(x^2*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)),x)
```

output
```
int(1/(x^2*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)), x)
```

3.535 $\int \frac{1}{x^3\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx$

3.535.1 Optimal result

Integrand size = 28, antiderivative size = 67

$$\int \frac{1}{x^3\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx=-\frac{(a+bx^n)\,\text{Hypergeometric2F1}\left(1,-\frac{2}{n},-\frac{2-n}{n},-\frac{bx^n}{a}\right)}{2ax^2\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

output
```
-1/2*(a+b*x^n)*hypergeom([1, -2/n],[(-2+n)/n],-b*x^n/a)/a/x^2/(a^2+2*a*b*x
^n+b^2*x^(2*n))^(1/2)
```

3.535.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^3\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx=-\frac{(a+bx^n)\,\text{Hypergeometric2F1}\left(1,-\frac{2}{n},1-\frac{2}{n},-\frac{bx^n}{a}\right)}{2ax^2\sqrt{(a+bx^n)^2}}$$

input
```
Integrate[1/(x^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]),x]
```

output
```
-1/2*((a + b*x^n)*Hypergeometric2F1[1, -2/n, 1 - 2/n, -((b*x^n)/a)])/(a*x^
2*Sqrt[(a + b*x^n)^2])
```

3.535.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.10, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1384, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{a^2+2abx^n+b^2x^{2n}}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{(ab+b^2x^n)\int \frac{1}{x^3(b^2x^n+ab)}dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

$$\Big\downarrow \text{888}$$

$$-\frac{(ab+b^2x^n)\,\text{Hypergeometric2F1}\left(1,-\frac{2}{n},-\frac{2-n}{n},-\frac{bx^n}{a}\right)}{2abx^2\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

input
```
Int[1/(x^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]),x]
```

output
```
-1/2*((a*b + b^2*x^n)*Hypergeometric2F1[1, -2/n, -((2 - n)/n), -((b*x^n)/a
)])/(a*b*x^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

3.535.3.1 Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

3.535.4 Maple [F]

$$\int \frac{1}{x^3\sqrt{a^2+2ab\,x^n+b^2x^{2n}}}dx$$

input
```
int(1/x^3/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

output
```
int(1/x^3/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

3.535.5 Fricas [F]

$$\int \frac{1}{x^3\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \int \frac{1}{\sqrt{b^2x^{2\,n}+2\,abx^n+a^2}x^3}\,dx$$

input
```
integrate(1/x^3/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)/(b^2*x^3*x^(2*n) + 2*a*b*x^3*
x^n + a^2*x^3), x)
```

3.535.6 Sympy [F]

$$\int \frac{1}{x^3\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \int \frac{1}{x^3\sqrt{(a+bx^n)^2}}\,dx$$

input
```
integrate(1/x**3/(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

output
```
Integral(1/(x**3*sqrt((a + b*x**n)**2)), x)
```

3.535.7 Maxima [F]

$$\int \frac{1}{x^3\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \int \frac{1}{\sqrt{b^2x^{2\,n}+2\,abx^n+a^2}x^3}\,dx$$

input
```
integrate(1/x^3/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x^3), x)
```

3.535.8 Giac [F]

$$\int \frac{1}{x^3\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \int \frac{1}{\sqrt{b^2x^{2\,n}+2\,abx^n+a^2}x^3}\,dx$$

input
```
integrate(1/x^3/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x^3), x)
```

3.535.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \int \frac{1}{x^3\,\sqrt{a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n}}\,dx$$

input
```
int(1/(x^3*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)),x)
```

output
```
int(1/(x^3*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)), x)
```

3.536 $\int \frac{(dx)^m}{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx$

3.536.1 Optimal result

Integrand size = 30, antiderivative size = 76

$$\int \frac{(dx)^m}{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx=\frac{(dx)^{1+m}(a+bx^n)\,\text{Hypergeometric2F1}\left(3,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{bx^n}{a}\right)}{a^3d(1+m)\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

output
```
(d*x)^(1+m)*(a+b*x^n)*hypergeom([3, (1+m)/n],[(1+m+n)/n],-b*x^n/a)/a^3/d/(
1+m)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)
```

3.536.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.80

$$\int \frac{(dx)^m}{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx=\frac{x(dx)^m(a+bx^n)\,\text{Hypergeometric2F1}\left(3,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{bx^n}{a}\right)}{a^3(1+m)\sqrt{(a+bx^n)^2}}$$

input
```
Integrate[(d*x)^m/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

output
```
(x*(d*x)^m*(a + b*x^n)*Hypergeometric2F1[3, (1 + m)/n, (1 + m + n)/n, -((b
*x^n)/a)])/(a^3*(1 + m)*Sqrt[(a + b*x^n)^2])
```

3.536.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.12, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1384, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{(a^2+2abx^n+b^2x^{2n})^{3/2}}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{(ab^3+b^4x^n)\int \frac{(dx)^m}{(b^2x^n+ab)^3}dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{(dx)^{m+1}(ab^3+b^4x^n)\,\text{Hypergeometric2F1}\left(3,\frac{m+1}{n},\frac{m+n+1}{n},-\frac{bx^n}{a}\right)}{a^3b^3d(m+1)\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

input
```
Int[(d*x)^m/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

output
```
((d*x)^(1 + m)*(a*b^3 + b^4*x^n)*Hypergeometric2F1[3, (1 + m)/n, (1 + m +
n)/n, -((b*x^n)/a)])/(a^3*b^3*d*(1 + m)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)
])
```

3.536.3.1 Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] && !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

3.536.4 Maple [F]

$$\int \frac{(dx)^m}{(a^2+2ab\,x^n+b^2x^{2n})^{\frac{3}{2}}}dx$$

input
```
int((d*x)^m/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

output
```
int((d*x)^m/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

3.536.5 Fricas [F]

$$\int \frac{(dx)^m}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx = \int \frac{(dx)^m}{(b^2x^{2\,n}+2\,abx^n+a^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)^m/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*(d*x)^m/(b^4*x^(4*n) + 4*a^2*
b^2*x^(2*n) + 4*a^3*b*x^n + a^4 + 2*(2*a*b^3*x^n + a^2*b^2)*x^(2*n)), x)
```

3.536.6 Sympy [F]

$$\int \frac{(dx)^m}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx = \int \frac{(dx)^m}{((a+bx^n)^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)**m/(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

output
```
Integral((d*x)**m/((a + b*x**n)**2)**(3/2), x)
```

3.536.7 Maxima [F]

$$\int \frac{(dx)^m}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx = \int \frac{(dx)^m}{(b^2x^{2\,n}+2\,abx^n+a^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)^m/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")
```

output
```
(m^2 - m*(3*n - 2) + 2*n^2 - 3*n + 1)*d^m*integrate(1/2*x^m/(a^2*b*n^2*x^n
 + a^3*n^2), x) - 1/2*(a*d^m*(m - 3*n + 1)*x*x^m + b*d^m*(m - 2*n + 1)*x*e
^(m*log(x) + n*log(x)))/(a^2*b^2*n^2*x^(2*n) + 2*a^3*b*n^2*x^n + a^4*n^2)
```

3.536.8 Giac [F]

$$\int \frac{(dx)^m}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx = \int \frac{(dx)^m}{(b^2x^{2\,n}+2\,abx^n+a^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)^m/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x)^m/(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2), x)
```

3.536.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx = \int \frac{(d\,x)^m}{(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n)^{3/2}}\,dx$$

input
```
int((d*x)^m/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2),x)
```

output
```
int((d*x)^m/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2), x)
```

3.537 $\int \frac{x^2}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx$

3.537.1 Optimal result

Integrand size = 28, antiderivative size = 64

$$\int \frac{x^2}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx=\frac{x^3(a+bx^n)\,\text{Hypergeometric2F1}\left(3,\frac{3}{n},\frac{3+n}{n},-\frac{bx^n}{a}\right)}{3a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

output
```
1/3*x^3*(a+b*x^n)*hypergeom([3, 3/n],[(3+n)/n],-b*x^n/a)/a^3/(a^2+2*a*b*x^
n+b^2*x^(2*n))^(1/2)
```

3.537.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.86

$$\int \frac{x^2}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx=\frac{x^3(a+bx^n)^3\,\text{Hypergeometric2F1}\left(3,\frac{3}{n},1+\frac{3}{n},-\frac{bx^n}{a}\right)}{3a^3\left((a+bx^n)^2\right)^{3/2}}$$

input
```
Integrate[x^2/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

output
```
(x^3*(a + b*x^n)^3*Hypergeometric2F1[3, 3/n, 1 + 3/n, -((b*x^n)/a)])/(3*a^
3*((a + b*x^n)^2)^(3/2))
```

3.537.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.14, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1384, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{(ab^3+b^4x^n)\int \frac{x^2}{(b^2x^n+ab)^3}dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

$$\Big\downarrow \text{888}$$

$$\frac{x^3(ab^3+b^4x^n)\,\text{Hypergeometric2F1}\left(3,\frac{3}{n},\frac{n+3}{n},-\frac{bx^n}{a}\right)}{3a^3b^3\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

input
```
Int[x^2/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

output
```
(x^3*(a*b^3 + b^4*x^n)*Hypergeometric2F1[3, 3/n, (3 + n)/n, -((b*x^n)/a)])
/(3*a^3*b^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

3.537.3.1 Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] && !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

3.537.4 Maple [F]

$$\int \frac{x^2}{(a^2 + 2ab\,x^n + b^2 x^{2n})^{\frac{3}{2}}} dx$$

input
```
int(x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

output
```
int(x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

3.537.5 Fricas [F]

$$\int \frac{x^2}{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}\, dx = \int \frac{x^2}{(b^2x^{2\,n} + 2\,abx^n + a^2)^{\frac{3}{2}}}\, dx$$

input
```
integrate(x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x^2/(b^4*x^(4*n) + 4*a^2*b^2*
x^(2*n) + 4*a^3*b*x^n + a^4 + 2*(2*a*b^3*x^n + a^2*b^2)*x^(2*n)), x)
```

3.537.6 Sympy [F]

$$\int \frac{x^2}{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}\, dx = \int \frac{x^2}{\left((a + bx^n)^2\right)^{\frac{3}{2}}}\, dx$$

input
```
integrate(x**2/(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

output
```
Integral(x**2/((a + b*x**n)**2)**(3/2), x)
```

3.537.7 Maxima [F]

$$\int \frac{x^2}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx = \int \frac{x^2}{(b^2x^{2\,n}+2\,abx^n+a^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")
```

output
```
(2*n^2 - 9*n + 9)*integrate(1/2*x^2/(a^2*b*n^2*x^n + a^3*n^2), x) + 1/2*(b
*(2*n - 3)*x^3*x^n + 3*a*(n - 1)*x^3)/(a^2*b^2*n^2*x^(2*n) + 2*a^3*b*n^2*x
^n + a^4*n^2)
```

3.537.8 Giac [F]

$$\int \frac{x^2}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx = \int \frac{x^2}{(b^2x^{2\,n}+2\,abx^n+a^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
```

output
```
integrate(x^2/(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2), x)
```

3.537.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx = \int \frac{x^2}{(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n)^{3/2}}\,dx$$

input
```
int(x^2/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2),x)
```

output
```
int(x^2/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2), x)
```

3.538 $\int \frac{x}{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx$

3.538.1 Optimal result

Integrand size = 26, antiderivative size = 64

$$\int \frac{x}{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx=\frac{x^2(a+bx^n)\,\text{Hypergeometric2F1}\left(3,\frac{2}{n},\frac{2+n}{n},-\frac{bx^n}{a}\right)}{2a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

output
```
1/2*x^2*(a+b*x^n)*hypergeom([3, 2/n],[(2+n)/n],-b*x^n/a)/a^3/(a^2+2*a*b*x^
n+b^2*x^(2*n))^(1/2)
```

3.538.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.86

$$\int \frac{x}{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx=\frac{x^2(a+bx^n)^3\,\text{Hypergeometric2F1}\left(3,\frac{2}{n},1+\frac{2}{n},-\frac{bx^n}{a}\right)}{2a^3\left((a+bx^n)^2\right)^{3/2}}$$

input
```
Integrate[x/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

output
```
(x^2*(a + b*x^n)^3*Hypergeometric2F1[3, 2/n, 1 + 2/n, -((b*x^n)/a)])/(2*a^
3*((a + b*x^n)^2)^(3/2))
```

3.538.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.14, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1384, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx$$

$$\Big\downarrow \ 1384$$

$$\frac{(ab^3+b^4x^n)\int \frac{x}{(b^2x^n+ab)^3}dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

$$\Big\downarrow \ 888$$

$$\frac{x^2(ab^3+b^4x^n)\,\text{Hypergeometric2F1}\left(3,\frac{2}{n},\frac{n+2}{n},-\frac{bx^n}{a}\right)}{2a^3b^3\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

input
```
Int[x/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

output
```
(x^2*(a*b^3 + b^4*x^n)*Hypergeometric2F1[3, 2/n, (2 + n)/n, -((b*x^n)/a)])
/(2*a^3*b^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

3.538.3.1 Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

3.538.4 Maple [F]

$$\int \frac{x}{(a^2 + 2ab\,x^n + b^2x^{2n})^{\frac{3}{2}}}dx$$

input
```
int(x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

output
```
int(x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

3.538.5 Fricas [F]

$$\int \frac{x}{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}\,dx = \int \frac{x}{(b^2x^{2\,n} + 2\,abx^n + a^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x/(b^4*x^(4*n) + 4*a^2*b^2*x^
(2*n) + 4*a^3*b*x^n + a^4 + 2*(2*a*b^3*x^n + a^2*b^2)*x^(2*n)), x)
```

3.538.6 Sympy [F]

$$\int \frac{x}{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}\,dx = \int \frac{x}{\left((a + bx^n)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

output
```
Integral(x/((a + b*x**n)**2)**(3/2), x)
```

3.538.7 Maxima [F]

$$\int \frac{x}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx=\int \frac{x}{(b^2x^{2\,n}+2\,abx^n+a^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")
```

output
```
(n^2 - 3*n + 2)*integrate(x/(a^2*b*n^2*x^n + a^3*n^2), x) + 1/2*(2*b*(n -
1)*x^2*x^n + a*(3*n - 2)*x^2)/(a^2*b^2*n^2*x^(2*n) + 2*a^3*b*n^2*x^n + a^4
*n^2)
```

3.538.8 Giac [F]

$$\int \frac{x}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx=\int \frac{x}{(b^2x^{2\,n}+2\,abx^n+a^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
```

output
```
integrate(x/(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2), x)
```

3.538.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx=\int \frac{x}{(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n)^{3/2}}\,dx$$

input
```
int(x/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2),x)
```

output
```
int(x/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2), x)
```

3.539 $\int \frac{1}{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx$

3.539.1 Optimal result

Integrand size = 24, antiderivative size = 57

$$\int \frac{1}{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx=\frac{x(a+bx^n)^3\,\text{Hypergeometric2F1}\left(3,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^3\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}$$

output
```
x*(a+b*x^n)^3*hypergeom([3, 1/n],[1+1/n],-b*x^n/a)/a^3/(a^2+2*a*b*x^n+b^2*
x^(2*n))^(3/2)
```

3.539.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.81

$$\int \frac{1}{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx=\frac{x(a+bx^n)^3\,\text{Hypergeometric2F1}\left(3,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^3\left((a+bx^n)^2\right)^{3/2}}$$

input
```
Integrate[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(-3/2),x]
```

output
```
(x*(a + b*x^n)^3*Hypergeometric2F1[3, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(
a^3*((a + b*x^n)^2)^(3/2))
```

3.539.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.12, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1384, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{(ab^3+b^4x^n)\int \frac{1}{(b^2x^n+ab)^3}dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

$$\Big\downarrow \text{778}$$

$$\frac{x(ab^3+b^4x^n)\,\text{Hypergeometric2F1}\left(3,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^3b^3\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

input
```
Int[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(-3/2),x]
```

output
```
(x*(a*b^3 + b^4*x^n)*Hypergeometric2F1[3, n^(-1), 1 + n^(-1), -((b*x^n)/a)
])/(a^3*b^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

3.539.3.1 Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] && !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

3.539.4 Maple [F]

$$\int \frac{1}{\left(a^{2}+2 a b\, x^{n}+b^{2} x^{2 n}\right)^{\frac{3}{2}}}d x$$

input
```
int(1/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

output
```
int(1/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

3.539.5 Fricas [F]

$$\int \frac{1}{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}\, dx = \int \frac{1}{\left(b^{2} x^{2\, n} + 2\, a b x^{n} + a^{2}\right)^{\frac{3}{2}}}\, dx$$

input
```
integrate(1/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)/(b^4*x^(4*n) + 4*a^2*b^2*x^(2
*n) + 4*a^3*b*x^n + a^4 + 2*(2*a*b^3*x^n + a^2*b^2)*x^(2*n)), x)
```

3.539.6 Sympy [F]

$$\int \frac{1}{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}\, dx = \int \frac{1}{\left(a^{2} + 2 a b x^{n} + b^{2} x^{2 n}\right)^{\frac{3}{2}}}\, dx$$

input
```
integrate(1/(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

output
```
Integral((a**2 + 2*a*b*x**n + b**2*x**(2*n))**(-3/2), x)
```

3.539.7 Maxima [F]

$$\int \frac{1}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx = \int \frac{1}{(b^2x^{2n}+2\,abx^n+a^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")
```

output
```
(2*n^2 - 3*n + 1)*integrate(1/2/(a^2*b*n^2*x^n + a^3*n^2), x) + 1/2*(b*(2*
n - 1)*x*x^n + a*(3*n - 1)*x)/(a^2*b^2*n^2*x^(2*n) + 2*a^3*b*n^2*x^n + a^4
*n^2)
```

3.539.8 Giac [F]

$$\int \frac{1}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx = \int \frac{1}{(b^2x^{2n}+2\,abx^n+a^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
```

output
```
integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(-3/2), x)
```

3.539.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx = \int \frac{1}{(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n)^{3/2}}\,dx$$

input
```
int(1/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2),x)
```

output
```
int(1/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2), x)
```

3.540 $\int \frac{1}{x(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx$

3.540.1 Optimal result

Integrand size = 28, antiderivative size = 159

$$\int \frac{1}{x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx=\frac{1}{a^2n\sqrt{a^2+2abx^n+b^2x^{2n}}}+\frac{1}{2an\left(a+bx^n\right)\sqrt{a^2+2abx^n+b^2x^{2n}}}+\frac{(a+bx^n)\log(x)}{a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{(a+bx^n)\log\left(a+bx^n\right)}{a^3n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

output
```
1/a^2/n/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)+1/2/a/n/(a+b*x^n)/(a^2+2*a*b*x^n
+b^2*x^(2*n))^(1/2)+(a+b*x^n)*ln(x)/a^3/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)-
(a+b*x^n)*ln(a+b*x^n)/a^3/n/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)
```

3.540.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.50

$$\int \frac{1}{x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx=\frac{a(3a+2bx^n)+2(a+bx^n)^2\log\left(x^n\right)-2(a+bx^n)^2\log\left(a+bx^n\right)}{2a^3n\left(a+bx^n\right)\sqrt{\left(a+bx^n\right)^2}}$$

input
```
Integrate[1/(x*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)),x]
```

output
```
(a*(3*a + 2*b*x^n) + 2*(a + b*x^n)^2*Log[x^n] - 2*(a + b*x^n)^2*Log[a + b*
x^n])/(2*a^3*n*(a + b*x^n)*Sqrt[(a + b*x^n)^2])
```

3.540.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.60, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {1384, 798, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\left(ab^3+b^4x^n\right)\int \frac{1}{x\left(b^2x^n+ab\right)^3}dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

$$\Big\downarrow \text{ 798}$$

$$\frac{\left(ab^3+b^4x^n\right)\int \frac{x^{-n}}{b^3\left(bx^n+a\right)^3}dx^n}{n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\left(ab^3+b^4x^n\right)\int \frac{x^{-n}}{\left(bx^n+a\right)^3}dx^n}{b^3n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

$$\Big\downarrow \text{ 54}$$

$$\frac{\left(ab^3+b^4x^n\right)\int \left(\frac{x^{-n}}{a^3}-\frac{b}{a^3\left(bx^n+a\right)}-\frac{b}{a^2\left(bx^n+a\right)^2}-\frac{b}{a\left(bx^n+a\right)^3}\right)dx^n}{b^3n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\left(ab^3+b^4x^n\right)\left(-\frac{\log\left(a+bx^n\right)}{a^3}+\frac{\log\left(x^n\right)}{a^3}+\frac{1}{a^2\left(a+bx^n\right)}+\frac{1}{2a\left(a+bx^n\right)^2}\right)}{b^3n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

input
```
Int[1/(x*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)),x]
```

output
```
((a*b^3 + b^4*x^n)*(1/(2*a*(a + b*x^n)^2) + 1/(a^2*(a + b*x^n)) + Log[x^n]
/a^3 - Log[a + b*x^n]/a^3))/(b^3*n*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

3.540.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.540.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.65

method	result	size
risch	$\frac{\sqrt{(a+b\,x^n)^2}\,\ln(x)}{(a+b\,x^n)a^3}+\frac{\sqrt{(a+b\,x^n)^2}\,(2b\,x^n+3a)}{2(a+b\,x^n)^3a^2n}-\frac{\sqrt{(a+b\,x^n)^2}\,\ln(x^n+\frac{a}{b})}{(a+b\,x^n)a^3n}$	104

input
```
int(1/x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((a+b*x^n)^2)^(1/2)/(a+b*x^n)*ln(x)/a^3+1/2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)^
3*(2*b*x^n+3*a)/a^2/n-((a+b*x^n)^2)^(1/2)/(a+b*x^n)/a^3/n*ln(x^n+a/b)
```

3.540.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.67

$$\int \frac{1}{x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx = \frac{2\,b^2nx^{2\,n}\log\left(x\right)+2\,a^2n\log\left(x\right)+3\,a^2+2\,(2\,abn\log\left(x\right)+ab)x^n-2\,(b^2x^2}{2\,(a^3b^2nx^{2\,n}+2\,a^4bnx^n+a^5n)}$$

input
```
integrate(1/x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")
```

output
```
1/2*(2*b^2*n*x^(2*n)*log(x) + 2*a^2*n*log(x) + 3*a^2 + 2*(2*a*b*n*log(x) +
 a*b)*x^n - 2*(b^2*x^(2*n) + 2*a*b*x^n + a^2)*log(b*x^n + a))/(a^3*b^2*n*x
^(2*n) + 2*a^4*b*n*x^n + a^5*n)
```

3.540.6 Sympy [F]

$$\int \frac{1}{x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx = \int \frac{1}{x\left((a+bx^n)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

output
```
Integral(1/(x*((a + b*x**n)**2)**(3/2)), x)
```

3.540.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.44

$$\int \frac{1}{x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx = \frac{2\,bx^n+3\,a}{2\,(a^2b^2nx^{2\,n}+2\,a^3bnx^n+a^4n)}+\frac{\log\left(x\right)}{a^3}-\frac{\log\left(\frac{bx^n+a}{b}\right)}{a^3n}$$

input
```
integrate(1/x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")
```

output
```
1/2*(2*b*x^n + 3*a)/(a^2*b^2*n*x^(2*n) + 2*a^3*b*n*x^n + a^4*n) + log(x)/a
^3 - log((b*x^n + a)/b)/(a^3*n)
```

3.540.8 Giac [F]

$$\int \frac{1}{x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx=\int \frac{1}{\left(b^2x^{2\,n}+2\,abx^n+a^2\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(1/x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)*x), x)
```

3.540.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx=\int \frac{1}{x\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)^{3/2}}\,dx$$

input
```
int(1/(x*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)),x)
```

output
```
int(1/(x*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)), x)
```

3.541 $\int \frac{1}{x^2(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx$

3.541.1 Optimal result

Integrand size = 28, antiderivative size = 65

$$\int \frac{1}{x^2\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx=-\frac{(a+bx^n)\,\text{Hypergeometric2F1}\left(3,-\frac{1}{n},-\frac{1-n}{n},-\frac{bx^n}{a}\right)}{a^3x\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

output
```
-(a+b*x^n)*hypergeom([3, -1/n],[(-1+n)/n],-b*x^n/a)/a^3/x/(a^2+2*a*b*x^n+b
^2*x^(2*n))^(1/2)
```

3.541.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^2\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx=-\frac{(a+bx^n)^3\,\text{Hypergeometric2F1}\left(3,-\frac{1}{n},1-\frac{1}{n},-\frac{bx^n}{a}\right)}{a^3x\left((a+bx^n)^2\right)^{3/2}}$$

input
```
Integrate[1/(x^2*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)),x]
```

output
```
-(((a + b*x^n)^3*Hypergeometric2F1[3, -n^(-1), 1 - n^(-1), -((b*x^n)/a)])/
(a^3*x*((a + b*x^n)^2)^(3/2)))
```

3.541.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.14, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1384, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\left(ab^3+b^4x^n\right)\int \frac{1}{x^2(b^2x^n+ab)^3}dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

$$\Big\downarrow \text{888}$$

$$-\frac{\left(ab^3+b^4x^n\right)\text{Hypergeometric2F1}\left(3,-\frac{1}{n},-\frac{1-n}{n},-\frac{bx^n}{a}\right)}{a^3b^3x\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

input
```
Int[1/(x^2*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)),x]
```

output
```
-(((a*b^3 + b^4*x^n)*Hypergeometric2F1[3, -n^(-1), -((1 - n)/n), -((b*x^n)
/a)])/(a^3*b^3*x*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]))
```

3.541.3.1 Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] && !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

3.541.4 Maple [F]

$$\int \frac{1}{x^2 \left(a^2 + 2ab\,x^n + b^2 x^{2n}\right)^{\frac{3}{2}}}dx$$

input
```
int(1/x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

output
```
int(1/x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

3.541.5 Fricas [F]

$$\int \frac{1}{x^2 \left(a^2 + 2abx^n + b^2 x^{2n}\right)^{3/2}}\,dx = \int \frac{1}{\left(b^2 x^{2\,n} + 2\,abx^n + a^2\right)^{\frac{3}{2}} x^2}\,dx$$

input
```
integrate(1/x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)/(b^4*x^2*x^(4*n) + 4*a^2*b^2*
x^2*x^(2*n) + 4*a^3*b*x^2*x^n + a^4*x^2 + 2*(2*a*b^3*x^2*x^n + a^2*b^2*x^2
)*x^(2*n)), x)
```

3.541.6 Sympy [F]

$$\int \frac{1}{x^2 \left(a^2 + 2abx^n + b^2 x^{2n}\right)^{3/2}}\,dx = \int \frac{1}{x^2 \left(\left(a + bx^n\right)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**2/(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

output
```
Integral(1/(x**2*((a + b*x**n)**2)**(3/2)), x)
```

3.541.7 Maxima [F]

$$\int \frac{1}{x^2 (a^2 + 2abx^n + b^2x^{2n})^{3/2}}\, dx = \int \frac{1}{(b^2x^{2\,n} + 2\,abx^n + a^2)^{\frac{3}{2}}x^2}\, dx$$

input
```
integrate(1/x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")
```

output
```
(2*n^2 + 3*n + 1)*integrate(1/2/(a^2*b*n^2*x^2*x^n + a^3*n^2*x^2), x) + 1/
2*(b*(2*n + 1)*x^n + a*(3*n + 1))/(a^2*b^2*n^2*x*x^(2*n) + 2*a^3*b*n^2*x*x
^n + a^4*n^2*x)
```

3.541.8 Giac [F]

$$\int \frac{1}{x^2 (a^2 + 2abx^n + b^2x^{2n})^{3/2}}\, dx = \int \frac{1}{(b^2x^{2\,n} + 2\,abx^n + a^2)^{\frac{3}{2}}x^2}\, dx$$

input
```
integrate(1/x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)*x^2), x)
```

3.541.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a^2 + 2abx^n + b^2x^{2n})^{3/2}}\, dx = \int \frac{1}{x^2\,(a^2 + b^2\,x^{2\,n} + 2\,a\,b\,x^n)^{3/2}}\, dx$$

input
```
int(1/(x^2*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)),x)
```

output
```
int(1/(x^2*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)), x)
```

3.542 $\int \frac{1}{x^3(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx$

3.542.1 Optimal result

Integrand size = 28, antiderivative size = 67

$$\int \frac{1}{x^3(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx = -\frac{(a+bx^n)\,\text{Hypergeometric2F1}\left(3,-\frac{2}{n},-\frac{2-n}{n},-\frac{bx^n}{a}\right)}{2a^3x^2\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

output
```
-1/2*(a+b*x^n)*hypergeom([3, -2/n],[(-2+n)/n],-b*x^n/a)/a^3/x^2/(a^2+2*a*b
*x^n+b^2*x^(2*n))^(1/2)
```

3.542.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^3(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx = -\frac{(a+bx^n)^3\,\text{Hypergeometric2F1}\left(3,-\frac{2}{n},1-\frac{2}{n},-\frac{bx^n}{a}\right)}{2a^3x^2\left((a+bx^n)^2\right)^{3/2}}$$

input
```
Integrate[1/(x^3*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)),x]
```

output
```
-1/2*((a + b*x^n)^3*Hypergeometric2F1[3, -2/n, 1 - 2/n, -((b*x^n)/a)])/(a^
3*x^2*((a + b*x^n)^2)^(3/2))
```

3.542.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.13, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1384, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a^2+2abx^n+b^2x^{2n})^{3/2}} \, dx$$
$$\Big\downarrow \text{1384}$$
$$\frac{(ab^3+b^4x^n)\int \frac{1}{x^3(b^2x^n+ab)^3}dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}$$
$$\Big\downarrow \text{888}$$
$$-\frac{(ab^3+b^4x^n)\,\text{Hypergeometric2F1}\left(3,-\frac{2}{n},-\frac{2-n}{n},-\frac{bx^n}{a}\right)}{2a^3b^3x^2\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

input
```
Int[1/(x^3*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)),x]
```

output
```
-1/2*((a*b^3 + b^4*x^n)*Hypergeometric2F1[3, -2/n, -((2 - n)/n), -((b*x^n)
/a)])/(a^3*b^3*x^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

3.542.3.1 Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

3.542.4 Maple [F]

$$\int \frac{1}{x^3 \left(a^2+2ab\,x^n+b^2x^{2n}\right)^{\frac{3}{2}}}dx$$

input
```
int(1/x^3/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

output
```
int(1/x^3/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

3.542.5 Fricas [F]

$$\int \frac{1}{x^3 \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx = \int \frac{1}{\left(b^2x^{2\,n}+2\,abx^n+a^2\right)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate(1/x^3/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)/(b^4*x^3*x^(4*n) + 4*a^2*b^2*
x^3*x^(2*n) + 4*a^3*b*x^3*x^n + a^4*x^3 + 2*(2*a*b^3*x^3*x^n + a^2*b^2*x^3
)*x^(2*n)), x)
```

3.542.6 Sympy [F]

$$\int \frac{1}{x^3 \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx = \int \frac{1}{x^3\left(\left(a+bx^n\right)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**3/(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

output
```
Integral(1/(x**3*((a + b*x**n)**2)**(3/2)), x)
```

3.542.7 Maxima [F]

$$\int \frac{1}{x^3 (a^2 + 2abx^n + b^2x^{2n})^{3/2}}\, dx = \int \frac{1}{(b^2x^{2\,n} + 2\,abx^n + a^2)^{\frac{3}{2}}x^3}\, dx$$

input
```
integrate(1/x^3/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")
```

output
```
(n^2 + 3*n + 2)*integrate(1/(a^2*b*n^2*x^3*x^n + a^3*n^2*x^3), x) + 1/2*(2
*b*(n + 1)*x^n + a*(3*n + 2))/(a^2*b^2*n^2*x^2*x^(2*n) + 2*a^3*b*n^2*x^2*x
^n + a^4*n^2*x^2)
```

3.542.8 Giac [F]

$$\int \frac{1}{x^3 (a^2 + 2abx^n + b^2x^{2n})^{3/2}}\, dx = \int \frac{1}{(b^2x^{2\,n} + 2\,abx^n + a^2)^{\frac{3}{2}}x^3}\, dx$$

input
```
integrate(1/x^3/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)*x^3), x)
```

3.542.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a^2 + 2abx^n + b^2x^{2n})^{3/2}}\, dx = \int \frac{1}{x^3\,(a^2 + b^2\,x^{2\,n} + 2\,a\,b\,x^n)^{3/2}}\, dx$$

input
```
int(1/(x^3*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)),x)
```

output
```
int(1/(x^3*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)), x)
```

3.543 $\int \left(a^2+b^2x^{-\frac{2}{1+2p}}+2abx^{-\frac{1}{1+2p}}\right)^p dx$

3.543.1 Optimal result

Integrand size = 36, antiderivative size = 52

$$\int \left(a^2+b^2x^{-\frac{2}{1+2p}}+2abx^{-\frac{1}{1+2p}}\right)^p dx=\frac{x\left(a+bx^{\frac{1}{-1-2p}}\right)\left(a^2+2abx^{\frac{1}{-1-2p}}+b^2x^{-\frac{2}{1+2p}}\right)^p}{a}$$

output
```
x*(a+b*x^(1/(-1-2*p)))*(a^2+2*a*b*x^(1/(-1-2*p))+b^2/(x^(2/(1+2*p))))^p/a
```

3.543.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.12

$$\int \left(a^2+b^2x^{-\frac{2}{1+2p}}+2abx^{-\frac{1}{1+2p}}\right)^p dx=\frac{x^{\frac{2p}{1+2p}}\left(b+ax^{\frac{1}{1+2p}}\right)\left(x^{-\frac{2}{1+2p}}\left(b+ax^{\frac{1}{1+2p}}\right)^2\right)^p}{a}$$

input
```
Integrate[(a^2 + b^2/x^(2/(1 + 2*p)) + (2*a*b)/x^(1 + 2*p)^(-1))^p,x]
```

output
```
(x^((2*p)/(1 + 2*p))*(b + a*x^(1 + 2*p)^(-1))*((b + a*x^(1 + 2*p)^(-1))^2/
x^(2/(1 + 2*p)))^p)/a
```

3.543.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1385, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+2abx^{-\frac{1}{2p+1}}+b^2x^{-\frac{2}{2p+1}}\right)^p\,dx$$

$$\Big\downarrow \text{1385}$$

$$\left(\frac{bx^{\frac{1}{-2p-1}}}{a}+1\right)^{-2p}\left(a^2+2abx^{\frac{1}{-2p-1}}+b^2x^{-\frac{2}{2p+1}}\right)^p\int \left(\frac{bx^{\frac{1}{-2p-1}}}{a}+1\right)^{2p}dx$$

$$\Big\downarrow \text{746}$$

$$x\left(\frac{bx^{\frac{1}{-2p-1}}}{a}+1\right)\left(a^2+2abx^{\frac{1}{-2p-1}}+b^2x^{-\frac{2}{2p+1}}\right)^p$$

input
```
Int[(a^2 + b^2/x^(2/(1 + 2*p)) + (2*a*b)/x^(1 + 2*p)^(-1))^p,x]
```

output
```
x*(1 + (b*x^(-1 - 2*p)^(-1))/a)*(a^2 + 2*a*b*x^(-1 - 2*p)^(-1) + b^2/x^(2/
(1 + 2*p)))^p
```

3.543.3.1 Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

3.543.4 Maple [F]

$$\int \left(a^2+b^2x^{-\frac{2}{1+2p}}+2ab\,x^{-\frac{1}{1+2p}}\right)^p dx$$

input
```
int((a^2+b^2/(x^(2/(1+2*p)))+2*a*b/(x^(1/(1+2*p))))^p,x)
```

output
```
int((a^2+b^2/(x^(2/(1+2*p)))+2*a*b/(x^(1/(1+2*p))))^p,x)
```

3.543.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.52

$$\int \left(a^2+b^2x^{-\frac{2}{1+2p}}+2abx^{-\frac{1}{1+2p}}\right)^p dx=\frac{\left(axx^{\left(\frac{1}{2\,p+1}\right)}+bx\right)\left(\frac{a^2x^{\frac{2}{2\,p+1}}+2\,abx^{\left(\frac{1}{2\,p+1}\right)}+b^2}{x^{\frac{2}{2\,p+1}}}\right)^p}{ax^{\left(\frac{1}{2\,p+1}\right)}}$$

input
```
integrate((a^2+b^2/(x^(2/(1+2*p)))+2*a*b/(x^(1/(1+2*p))))^p,x, algorithm="
fricas")
```

output
```
(a*x*x^(1/(2*p + 1)) + b*x)*((a^2*x^(2/(2*p + 1)) + 2*a*b*x^(1/(2*p + 1))
+ b^2)/x^(2/(2*p + 1)))^p/(a*x^(1/(2*p + 1)))
```

3.543.6 Sympy [F(-1)]

Timed out.

$$\int \left(a^2+b^2x^{-\frac{2}{1+2p}}+2abx^{-\frac{1}{1+2p}}\right)^p dx=\text{Timed out}$$

input
```
integrate((a**2+b**2/(x**(2/(1+2*p)))+2*a*b/(x**(1/(1+2*p))))**p,x)
```

output
```
Timed out
```

3.543.7 Maxima [F]

$$\int \left(a^2+b^2x^{-\frac{2}{1+2p}}+2abx^{-\frac{1}{1+2p}}\right)^p\,dx=\int \left(a^2+\frac{b^2}{x^{\frac{2}{2\,p+1}}}+\frac{2\,ab}{x^{\left(\frac{1}{2\,p+1}\right)}}\right)^p\,dx$$

input
```
integrate((a^2+b^2/(x^(2/(1+2*p)))+2*a*b/(x^(1/(1+2*p))))^p,x, algorithm="
maxima")
```

output
```
integrate((a^2 + b^2/x^(2/(2*p + 1)) + 2*a*b/x^(1/(2*p + 1)))^p, x)
```

3.543.8 Giac [F]

$$\int \left(a^2+b^2x^{-\frac{2}{1+2p}}+2abx^{-\frac{1}{1+2p}}\right)^p\,dx=\int \left(a^2+\frac{b^2}{x^{\frac{2}{2\,p+1}}}+\frac{2\,ab}{x^{\left(\frac{1}{2\,p+1}\right)}}\right)^p\,dx$$

input
```
integrate((a^2+b^2/(x^(2/(1+2*p)))+2*a*b/(x^(1/(1+2*p))))^p,x, algorithm="
giac")
```

output
```
integrate((a^2 + b^2/x^(2/(2*p + 1)) + 2*a*b/x^(1/(2*p + 1)))^p, x)
```

3.543.9 Mupad [F(-1)]

Timed out.

$$\int \left(a^2+b^2x^{-\frac{2}{1+2p}}+2abx^{-\frac{1}{1+2p}}\right)^p\,dx=\int \left(a^2+\frac{b^2}{x^{\frac{2}{2\,p+1}}}+\frac{2\,a\,b}{x^{\frac{1}{2\,p+1}}}\right)^p\,dx$$

input
```
int((a^2 + b^2/x^(2/(2*p + 1)) + (2*a*b)/x^(1/(2*p + 1)))^p,x)
```

output
```
int((a^2 + b^2/x^(2/(2*p + 1)) + (2*a*b)/x^(1/(2*p + 1)))^p, x)
```

3.544 $\int \left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{-1-n}{2n}}\,dx$

3.544.1 Optimal result

Integrand size = 33, antiderivative size = 43

$$\int \left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{-1-n}{2n}}\,dx=\frac{x(a+bx^n)\left(a^2+2abx^n+b^2x^{2n}\right)^{-\frac{1+n}{2n}}}{a}$$

output
```
x*(a+b*x^n)/a/((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2*(1+n)/n))
```

3.544.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.74

$$\int \left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{-1-n}{2n}}\,dx=\frac{x(a+bx^n)\left((a+bx^n)^2\right)^{-\frac{1+n}{2n}}}{a}$$

input
```
Integrate[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^((-1 - n)/(2*n)),x]
```

output
```
(x*(a + b*x^n))/(a*((a + b*x^n)^2)^((1 + n)/(2*n)))
```

3.544.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {1385, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{-n-1}{2n}}dx$$

$$\Big\downarrow \text{1385}$$

$$\left(\frac{bx^n}{a}+1\right)^{\frac{1}{n}+1}\left(a^2+2abx^n+b^2x^{2n}\right)^{-\frac{n+1}{2n}}\int \left(\frac{bx^n}{a}+1\right)^{-\frac{n+1}{n}}dx$$

$$\Big\downarrow \text{746}$$

$$x\left(\frac{bx^n}{a}+1\right)\left(a^2+2abx^n+b^2x^{2n}\right)^{-\frac{n+1}{2n}}$$

input
```
Int[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^((-1 - n)/(2*n)),x]
```

output
```
(x*(1 + (b*x^n)/a))/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^((1 + n)/(2*n))
```

3.544.3.1 Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

3.544.4 Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.19

method	result	size
norman	$\left(x+\frac{bx\,\mathrm{e}^{n\ln(x)}}{a}\right)\mathrm{e}^{\frac{(1+n)\ln\left(\frac{1}{\sqrt{a^2+2ab\,\mathrm{e}^{n\ln(x)}+b^2\mathrm{e}^{2n\ln(x)}}}\right)}{n}}$	51

input
```
int(1/((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2*(1+n)/n)),x,method=_RETURNVERBOSE)
```

output
```
(x+b/a*x*exp(n*ln(x)))/exp(1/2*(1+n)/n*ln(a^2+2*a*b*exp(n*ln(x))+b^2*exp(n
*ln(x))^2))
```

3.544.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05

$$\int \left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{-1-n}{2n}}\,dx=\frac{bxx^n+ax}{\left(b^2x^{2\,n}+2\,abx^n+a^2\right)^{\frac{n+1}{2\,n}}a}$$

input
```
integrate(1/((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2*(1+n)/n)),x, algorithm="fric
as")
```

output
```
(b*x*x^n + a*x)/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(1/2*(n + 1)/n)*a)
```

3.544.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. 2(37) = 74.

Time = 4.98 (sec) , antiderivative size = 160, normalized size of antiderivative = 3.72

$$\int \left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{-1-n}{2n}}\,dx$$
$$=\begin{cases} x(a^2+2abx^n+b^2x^{2n})^{-\frac{1}{2}-\frac{1}{2n}}+\frac{bxx^n(a^2+2abx^n+b^2x^{2n})^{-\frac{1}{2}-\frac{1}{2n}}}{a} & \text{for } a\neq 0 \\ -x(b^2x^{2n})^{-1-\frac{1}{n}}(b^2x^{2n})^{\frac{1}{2}+\frac{1}{2n}}+x(b^2x^{2n})^{-\frac{1}{2}-\frac{1}{2n}}-\frac{x(b^2x^{2n})^{-1-\frac{1}{n}}(b^2x^{2n})^{\frac{1}{2}+\frac{1}{2n}}}{n} & \text{otherwise} \end{cases}$$

input
```
integrate(1/((a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2*(1+n)/n)),x)
```

output
```
Piecewise((x*(a**2 + 2*a*b*x**n + b**2*x**(2*n))**(-1/2 - 1/(2*n)) + b*x*x
**n*(a**2 + 2*a*b*x**n + b**2*x**(2*n))**(-1/2 - 1/(2*n))/a, Ne(a, 0)), (-
x*(b**2*x**(2*n))**(-1 - 1/n)*(b**2*x**(2*n))**(1/2 + 1/(2*n)) + x*(b**2*x
**(2*n))**(-1/2 - 1/(2*n)) - x*(b**2*x**(2*n))**(-1 - 1/n)*(b**2*x**(2*n))
**(1/2 + 1/(2*n))/n, True))
```

3.544.7 Maxima [F]

$$\int \left(a^2 + 2abx^n + b^2x^{2n}\right)^{\frac{-1-n}{2n}} dx = \int \frac{1}{\left(b^2x^{2n} + 2\,abx^n + a^2\right)^{\frac{n+1}{2n}}}\, dx$$

input
```
integrate(1/((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2*(1+n)/n)),x, algorithm="maxi
ma")
```

output
```
integrate(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(1/2*(n + 1)/n)), x)
```

3.544.8 Giac [F]

$$\int \left(a^2 + 2abx^n + b^2x^{2n}\right)^{\frac{-1-n}{2n}} dx = \int \frac{1}{\left(b^2x^{2n} + 2\,abx^n + a^2\right)^{\frac{n+1}{2n}}}\, dx$$

input
```
integrate(1/((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2*(1+n)/n)),x, algorithm="giac
")
```

output
```
integrate(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(1/2*(n + 1)/n)), x)
```

3.544.9 Mupad [F(-1)]

Timed out.

$$\int \left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{-1-n}{2n}}\,dx=\int \frac{1}{\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)^{\frac{\frac{n}{2}+\frac{1}{2}}{n}}}\,dx$$

input
```
int(1/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^((n/2 + 1/2)/n),x)
```

output
```
int(1/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^((n/2 + 1/2)/n), x)
```

3.545 $\int \left(a^2+b^2 x^{-\frac{1}{1+p}}+2 a b x^{-\frac{1}{2(1+p)}}\right)^p \, dx$

3.545.1 Optimal result

Integrand size = 34, antiderivative size = 130

$$\int \left(a^2+b^2 x^{-\frac{1}{1+p}}+2 a b x^{-\frac{1}{2(1+p)}}\right)^p \, dx = \frac{2(1+p)x\left(a+bx^{-\frac{1}{2(1+p)}}\right)\left(a^2+b^2 x^{-\frac{1}{1+p}}+2 a b x^{-\frac{1}{2(1+p)}}\right)^p}{a(1+2p)} - \frac{x\left(a+bx^{-\frac{1}{2(1+p)}}\right)^2\left(a^2+b^2 x^{-\frac{1}{1+p}}+2 a b x^{-\frac{1}{2(1+p)}}\right)^p}{a^2(1+2p)}$$

output
```
2*(p+1)*x*(a+b/(x^(1/2/(p+1))))*(a^2+b^2/(x^(1/(p+1)))+2*a*b/(x^(1/2/(p+1)
)))^p/a/(1+2*p)-x*(a+b/(x^(1/2/(p+1))))^2*(a^2+b^2/(x^(1/(p+1)))+2*a*b/(x^
(1/2/(p+1))))^p/a^2/(1+2*p)
```

3.545.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.62

$$\int \left(a^2+b^2 x^{-\frac{1}{1+p}}+2 a b x^{-\frac{1}{2(1+p)}}\right)^p \, dx = \frac{x^{\frac{p}{1+p}}\left(b+ax^{\frac{1}{2+2p}}\right)\left(x^{-\frac{1}{1+p}}\left(b+ax^{\frac{1}{2+2p}}\right)^2\right)^p\left(-b+a(1+2p)x^{\frac{1}{2+2p}}\right)}{a^2(1+2p)}$$

input
```
Integrate[(a^2 + b^2/x^(1 + p)^(-1) + (2*a*b)/x^(1/(2*(1 + p))))^p,x]
```

output
```
(x^(p/(1 + p))*(b + a*x^(2 + 2*p)^(-1))*((b + a*x^(2 + 2*p)^(-1))^2/x^(1 +
 p)^(-1))^p*(-b + a*(1 + 2*p)*x^(2 + 2*p)^(-1)))/(a^2*(1 + 2*p))
```

3.545.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {1385, 777, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+2abx^{-\frac{1}{2(p+1)}}+b^2x^{-\frac{1}{p+1}}\right)^p dx$$

$$\Big\downarrow \ 1385$$

$$\left(\frac{bx^{-\frac{1}{2(p+1)}}}{a}+1\right)^{-2p}\left(a^2+2abx^{-\frac{1}{2(p+1)}}+b^2x^{-\frac{1}{p+1}}\right)^p\int \left(\frac{bx^{-\frac{1}{2(p+1)}}}{a}+1\right)^{2p}dx$$

$$\Big\downarrow \ 777$$

$$\left(\frac{bx^{-\frac{1}{2(p+1)}}}{a}+1\right)^{-2p}\left(a^2+2abx^{-\frac{1}{2(p+1)}}+b^2x^{-\frac{1}{p+1}}\right)^p\left(\frac{2(p+1)x\left(\frac{bx^{-\frac{1}{2(p+1)}}}{a}+1\right)^{2p+1}}{2p+1}-\frac{\int \left(\frac{bx^{-\frac{1}{2(p+1)}}}{a}+1\right)^{2p+1}d}{2p+1}\right.$$

$$\Big\downarrow \ 746$$

$$\left(\frac{bx^{-\frac{1}{2(p+1)}}}{a}+1\right)^{-2p}\left(a^2+2abx^{-\frac{1}{2(p+1)}}+b^2x^{-\frac{1}{p+1}}\right)^p\left(\frac{2(p+1)x\left(\frac{bx^{-\frac{1}{2(p+1)}}}{a}+1\right)^{2p+1}}{2p+1}-\frac{x\left(\frac{bx^{-\frac{1}{2(p+1)}}}{a}+1\right)^{2(p+1)}}{2p+1}\right.$$

input
```
Int[(a^2 + b^2/x^(1 + p)^(-1) + (2*a*b)/x^(1/(2*(1 + p))))^p,x]
```

output
```
((a^2 + b^2/x^(1 + p)^(-1) + (2*a*b)/x^(1/(2*(1 + p))))^p*(-((x*(1 + b/(a*
x^(1/(2*(1 + p)))))^(2*(1 + p)))/(1 + 2*p)) + (2*(1 + p)*x*(1 + b/(a*x^(1/
(2*(1 + p)))))^(1 + 2*p))/(1 + 2*p)))/(1 + b/(a*x^(1/(2*(1 + p)))))^(2*p)
```

3.545.3.1 Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 777
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
 0] && NeQ[p, -1]
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

3.545.4 Maple [F]

$$\int \left(a^2 + b^2 x^{-\frac{1}{1+p}} + 2ab\, x^{-\frac{1}{2(1+p)}}\right)^p dx$$

input
```
int((a^2+b^2/(x^(1/(1+p)))+2*a*b/(x^(1/2/(1+p))))^p,x)
```

output
```
int((a^2+b^2/(x^(1/(1+p)))+2*a*b/(x^(1/2/(1+p))))^p,x)
```

3.545.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.79

$$\int \left(a^2+b^2x^{-\frac{1}{1+p}}+2abx^{-\frac{1}{2(1+p)}}\right)^p dx = \frac{\left(2\,abpxx^{\frac{1}{2\,(p+1)}}-b^2x+(2\,a^2p+a^2)xx^{\left(\frac{1}{p+1}\right)}\right)\left(\frac{2\,abx^{\frac{1}{2\,(p+1)}}+a^2x^{\left(\frac{1}{p+1}\right)}+b^2}{x^{\left(\frac{1}{p+1}\right)}}\right)^p}{(2\,a^2p+a^2)x^{\left(\frac{1}{p+1}\right)}}$$

input
```
integrate((a^2+b^2/(x^(1/(1+p)))+2*a*b/(x^(1/2/(1+p))))^p,x, algorithm="fr
icas")
```

output
```
(2*a*b*p*x*x^(1/2/(p + 1)) - b^2*x + (2*a^2*p + a^2)*x*x^(1/(p + 1)))*((2*
a*b*x^(1/2/(p + 1)) + a^2*x^(1/(p + 1)) + b^2)/x^(1/(p + 1)))^p/((2*a^2*p
+ a^2)*x^(1/(p + 1)))
```

3.545.6 Sympy [F(-1)]

Timed out.

$$\int \left(a^2+b^2x^{-\frac{1}{1+p}}+2abx^{-\frac{1}{2(1+p)}}\right)^p dx = \text{Timed out}$$

input
```
integrate((a**2+b**2/(x**(1/(1+p)))+2*a*b/(x**(1/2/(1+p))))**p,x)
```

output
```
Timed out
```

3.545.7 Maxima [F]

$$\int \left(a^2+b^2x^{-\frac{1}{1+p}}+2abx^{-\frac{1}{2(1+p)}}\right)^p dx = \int \left(a^2+\frac{2\,ab}{x^{\frac{1}{2\,(p+1)}}}+\frac{b^2}{x^{\left(\frac{1}{p+1}\right)}}\right)^p dx$$

input
```
integrate((a^2+b^2/(x^(1/(1+p)))+2*a*b/(x^(1/2/(1+p))))^p,x, algorithm="ma
xima")
```

output
```
integrate((a^2 + 2*a*b/x^(1/2/(p + 1)) + b^2/x^(1/(p + 1)))^p, x)
```

3.545.8 Giac [F]

$$\int \left(a^2+b^2x^{-\frac{1}{1+p}}+2abx^{-\frac{1}{2(1+p)}}\right)^p\,dx=\int \left(a^2+\frac{2\,ab}{x^{\frac{1}{2\,(p+1)}}}+\frac{b^2}{x^{\left(\frac{1}{p+1}\right)}}\right)^p\,dx$$

input
```
integrate((a^2+b^2/(x^(1/(1+p)))+2*a*b/(x^(1/2/(1+p))))^p,x, algorithm="gi
ac")
```

output
```
integrate((a^2 + 2*a*b/x^(1/2/(p + 1)) + b^2/x^(1/(p + 1)))^p, x)
```

3.545.9 Mupad [F(-1)]

Timed out.

$$\int \left(a^2+b^2x^{-\frac{1}{1+p}}+2abx^{-\frac{1}{2(1+p)}}\right)^p\,dx=\int \left(\frac{b^2}{x^{\frac{1}{p+1}}}+a^2+\frac{2\,a\,b}{x^{\frac{1}{2\,(p+1)}}}\right)^p\,dx$$

input
```
int((b^2/x^(1/(p + 1)) + a^2 + (2*a*b)/x^(1/(2*(p + 1))))^p,x)
```

output
```
int((b^2/x^(1/(p + 1)) + a^2 + (2*a*b)/x^(1/(2*(p + 1))))^p, x)
```

3.546 $\int \left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{-1-2n}{2n}}\,dx$

3.546.1 Optimal result

Integrand size = 33, antiderivative size = 102

$$\int \left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{-1-2n}{2n}}\,dx=\frac{x(a+bx^n)\left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{1}{2}\left(-2-\frac{1}{n}\right)}}{a(1+n)}+\frac{nx(a+bx^n)^2\left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{1}{2}\left(-2-\frac{1}{n}\right)}}{a^2(1+n)}$$

output
```
x*(a+b*x^n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(-1-1/2/n)/a/(1+n)+n*x*(a+b*x^n)^2
*(a^2+2*a*b*x^n+b^2*x^(2*n))^(-1-1/2/n)/a^2/(1+n)
```

3.546.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.58

$$\int \left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{-1-2n}{2n}}\,dx=\frac{x\left((a+bx^n)^2\right)^{-\frac{1}{2}/n}\left(1+\frac{bx^n}{a}\right)^{\frac{1}{n}}\text{Hypergeometric2F1}\left(2+\frac{1}{n},\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2}$$

input
```
Integrate[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^((-1 - 2*n)/(2*n)),x]
```

output
```
(x*(1 + (b*x^n)/a)^n^(-1)*Hypergeometric2F1[2 + n^(-1), n^(-1), 1 + n^(-1)
, -((b*x^n)/a)])/(a^2*((a + b*x^n)^2)^(1/(2*n)))
```

3.546.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1385, 777, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{-2n-1}{2n}}dx$$

$$\Big\downarrow \text{1385}$$

$$\left(\frac{bx^n}{a}+1\right)^{\frac{1}{n}+2}\left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{1}{2}\left(-\frac{1}{n}-2\right)}\int \left(\frac{bx^n}{a}+1\right)^{-2-\frac{1}{n}}dx$$

$$\Big\downarrow \text{777}$$

$$\left(\frac{bx^n}{a}+1\right)^{\frac{1}{n}+2}\left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{1}{2}\left(-\frac{1}{n}-2\right)}\left(\frac{n\int \left(\frac{bx^n}{a}+1\right)^{-1-\frac{1}{n}}dx}{n+1}+\frac{x\left(\frac{bx^n}{a}+1\right)^{-\frac{1}{n}-1}}{n+1}\right)$$

$$\Big\downarrow \text{746}$$

$$\left(\frac{bx^n}{a}+1\right)^{\frac{1}{n}+2}\left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{1}{2}\left(-\frac{1}{n}-2\right)}\left(\frac{x\left(\frac{bx^n}{a}+1\right)^{-\frac{1}{n}-1}}{n+1}+\frac{nx\left(\frac{bx^n}{a}+1\right)^{-1/n}}{n+1}\right)$$

input
```
Int[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^((-1 - 2*n)/(2*n)),x]
```

output
```
(1 + (b*x^n)/a)^(2 + n^(-1))*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^((-2 - n^(-1)
)/2)*((x*(1 + (b*x^n)/a)^(-1 - n^(-1)))/(1 + n) + (n*x)/((1 + n)*(1 + (b*x
^n)/a)^n^(-1)))
```

3.546.3.1 Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 777
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
 0] && NeQ[p, -1]
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

3.546.4 Maple [F]

$$\int \left(a^2 + 2ab\,x^n + b^2x^{2n}\right)^{-\frac{1+2n}{2n}}\,dx$$

input
```
int(1/((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2*(1+2*n)/n)),x)
```

output
```
int(1/((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2*(1+2*n)/n)),x)
```

3.546.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.80

$$\int \left(a^2 + 2abx^n + b^2x^{2n}\right)^{\frac{-1-2n}{2n}}\,dx = \frac{b^2nxx^{2\,n} + (2\,abn + ab)xx^n + (a^2n + a^2)x}{(a^2n + a^2)(b^2x^{2\,n} + 2\,abx^n + a^2)^{\frac{2\,n+1}{2\,n}}}$$

input
```
integrate(1/((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2*(1+2*n)/n)),x, algorithm="fr
icas")
```

output
```
(b^2*n*x*x^(2*n) + (2*a*b*n + a*b)*x*x^n + (a^2*n + a^2)*x)/((a^2*n + a^2)
*(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(1/2*(2*n + 1)/n))
```

3.546.6 Sympy [F]

$$\int \left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{-1-2n}{2n}}\,dx$$
$$=\begin{cases} \frac{x}{2\sqrt{\frac{b^2}{x^2}}} \\ -x\left(b^2x^{2n}\right)^{-2-\frac{1}{n}}\left(b^2x^{2n}\right)^{1+\frac{1}{2n}}+x\left(b^2x^{2n}\right)^{-1-\frac{1}{2n}}-\frac{x\left(b^2x^{2n}\right)^{-2-\frac{1}{n}}\left(b^2x^{2n}\right)^{1+\frac{1}{2n}}}{2n} \\ \int \frac{1}{\sqrt{a^2+\frac{2ab}{x}+\frac{b^2}{x^2}}}\,dx \\ \frac{a^2nx\left(a^2+2abx^n+b^2x^{2n}\right)^{-1-\frac{1}{2n}}}{a^2n+a^2}+\frac{a^2x\left(a^2+2abx^n+b^2x^{2n}\right)^{-1-\frac{1}{2n}}}{a^2n+a^2}+\frac{2abnxx^n\left(a^2+2abx^n+b^2x^{2n}\right)^{-1-\frac{1}{2n}}}{a^2n+a^2}+\frac{abxx^n\left(a^2+2abx^n+b^2x^{2n}\right)}{a^2n+a^2} \end{cases}$$

input
```
integrate(1/((a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2*(1+2*n)/n)),x)
```

output
```
Piecewise((x/(2*sqrt(b**2/x**2)), Eq(a, 0) & Eq(n, -1)), (-x*(b**2*x**(2*n
))**(-2 - 1/n)*(b**2*x**(2*n))**(1 + 1/(2*n)) + x*(b**2*x**(2*n))**(-1 - 1
/(2*n)) - x*(b**2*x**(2*n))**(-2 - 1/n)*(b**2*x**(2*n))**(1 + 1/(2*n))/(2*
n), Eq(a, 0)), (Integral(1/sqrt(a**2 + 2*a*b/x + b**2/x**2), x), Eq(n, -1)
), (a**2*n*x*(a**2 + 2*a*b*x**n + b**2*x**(2*n))**(-1 - 1/(2*n))/(a**2*n +
 a**2) + a**2*x*(a**2 + 2*a*b*x**n + b**2*x**(2*n))**(-1 - 1/(2*n))/(a**2*
n + a**2) + 2*a*b*n*x*x**n*(a**2 + 2*a*b*x**n + b**2*x**(2*n))**(-1 - 1/(2
*n))/(a**2*n + a**2) + a*b*x*x**n*(a**2 + 2*a*b*x**n + b**2*x**(2*n))**(-1
 - 1/(2*n))/(a**2*n + a**2) + b**2*n*x*x**(2*n)*(a**2 + 2*a*b*x**n + b**2*
x**(2*n))**(-1 - 1/(2*n))/(a**2*n + a**2), True))
```

3.546.7 Maxima [F]

$$\int \left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{-1-2n}{2n}}\,dx=\int \frac{1}{\left(b^2x^{2n}+2\,abx^n+a^2\right)^{\frac{2n+1}{2n}}}\,dx$$

input
```
integrate(1/((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2*(1+2*n)/n)),x, algorithm="ma
xima")
```

output
```
integrate(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(1/2*(2*n + 1)/n)), x)
```

3.546.8 Giac [F]

$$\int \left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{-1-2n}{2n}}\,dx=\int \frac{1}{\left(b^2x^{2\,n}+2\,abx^n+a^2\right)^{\frac{2\,n+1}{2\,n}}}\,dx$$

input
```
integrate(1/((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2*(1+2*n)/n)),x, algorithm="gi
ac")
```

output
```
integrate(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(1/2*(2*n + 1)/n)), x)
```

3.546.9 Mupad [F(-1)]

Timed out.

$$\int \left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{-1-2n}{2n}}\,dx=\int \frac{1}{\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)^{\frac{n+\frac{1}{2}}{n}}}\,dx$$

input
```
int(1/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^((n + 1/2)/n),x)
```

output
```
int(1/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^((n + 1/2)/n), x)
```

3.547 $\int (dx)^{-1-2n(1+p)} \left(a^2+2abx^n+b^2x^{2n}\right)^p \, dx$

3.547.1 Optimal result

Integrand size = 35, antiderivative size = 117

$$\int (dx)^{-1-2n(1+p)} \left(a^2+2abx^n+b^2x^{2n}\right)^p \, dx = -\frac{(dx)^{-2n(1+p)}(a+bx^n)\left(a^2+2abx^n+b^2x^{2n}\right)^p}{adn(1+2p)} + \frac{(dx)^{-2n(1+p)}\left(a^2+2abx^n+b^2x^{2n}\right)^{1+p}}{2a^2dn(1+p)(1+2p)}$$

output
```
-(a+b*x^n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^p/a/d/n/(1+2*p)/((d*x)^(2*n*(p+1)))
+1/2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(p+1)/a^2/d/n/(p+1)/(1+2*p)/((d*x)^(2*n*(
p+1)))
```

3.547.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.10 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.64

$$\int (dx)^{-1-2n(1+p)} \left(a^2+2abx^n+b^2x^{2n}\right)^p \, dx = -\frac{x(dx)^{-1-2n(1+p)}\left((a+bx^n)^2\right)^p\left(1+\frac{bx^n}{a}\right)^{-2p}\text{Hypergeometric2F1}\left(-2p,-2(1+p),1-2(1+p),-\frac{bx^n}{a}\right)}{2n(1+p)}$$

input
```
Integrate[(d*x)^(-1 - 2*n*(1 + p))*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p,x]
```

output
```
-1/2*(x*(d*x)^(-1 - 2*n*(1 + p))*((a + b*x^n)^2)^p*Hypergeometric2F1[-2*p,
 -2*(1 + p), 1 - 2*(1 + p), -((b*x^n)/a)])/(n*(1 + p)*(1 + (b*x^n)/a)^(2*p
))
```

3.547.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {1385, 805, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^{-2n(p+1)-1} \left(a^2+2abx^n+b^2x^{2n}\right)^p \, dx$$

$$\Big\downarrow \text{1385}$$

$$\left(\frac{bx^n}{a}+1\right)^{-2p} \left(a^2+2abx^n+b^2x^{2n}\right)^p \int (dx)^{-2n(p+1)-1} \left(\frac{bx^n}{a}+1\right)^{2p} dx$$

$$\Big\downarrow \text{805}$$

$$\left(\frac{bx^n}{a}+1\right)^{-2p} \left(a^2+2abx^n+b^2x^{2n}\right)^p \left(-\frac{\int (dx)^{-2n(p+1)-1} \left(\frac{bx^n}{a}+1\right)^{2p+1} dx}{2p+1}-\frac{(dx)^{-2n(p+1)} \left(\frac{bx^n}{a}+1\right)^{2p+1}}{dn(2p+1)}\right)$$

$$\Big\downarrow \text{796}$$

$$\left(\frac{bx^n}{a}+1\right)^{-2p} \left(a^2+2abx^n+b^2x^{2n}\right)^p \left(\frac{(dx)^{-2n(p+1)} \left(\frac{bx^n}{a}+1\right)^{2(p+1)}}{2dn(p+1)(2p+1)}-\frac{(dx)^{-2n(p+1)} \left(\frac{bx^n}{a}+1\right)^{2p+1}}{dn(2p+1)}\right)$$

input
```
Int[(d*x)^(-1 - 2*n*(1 + p))*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p,x]
```

output
```
((a^2 + 2*a*b*x^n + b^2*x^(2*n))^p*((1 + (b*x^n)/a)^(2*(1 + p))/(2*d*n*(1
+ p)*(1 + 2*p)*(d*x)^(2*n*(1 + p))) - (1 + (b*x^n)/a)^(1 + 2*p)/(d*n*(1 +
2*p)*(d*x)^(2*n*(1 + p)))))/(1 + (b*x^n)/a)^(2*p)
```

3.547.3.1 Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 805
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

3.547.4 Maple [F]

$$\int (dx)^{-1-2n(1+p)} \left(a^2+2ab\,x^n+b^2x^{2n}\right)^p dx$$

input
```
int((d*x)^(-1-2*n*(1+p))*(a^2+2*a*b*x^n+b^2*x^(2*n))^p,x)
```

output
```
int((d*x)^(-1-2*n*(1+p))*(a^2+2*a*b*x^n+b^2*x^(2*n))^p,x)
```

3.547.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.41

$$\int (dx)^{-1-2n(1+p)} \left(a^2+2abx^n+b^2x^{2n}\right)^p dx = -\frac{\left(2\,abpxx^n e^{\left(-(2\,np+2\,n+1)\log(d)-(2\,np+2\,n+1)\log(x)\right)} - b^2xx^{2\,n}e^{\left(-(2\,np+2\,n+1)\log(d)-(2\,np+2\,n+1)\log(x)\right)} + \left(2\,a^2p + a^2\right.\right.}{2\left(2\,a^2np^2+3\,a^2np+a^2n\right)}$$

input
```
integrate((d*x)^(-1-2*n*(1+p))*(a^2+2*a*b*x^n+b^2*x^(2*n))^p,x, algorithm=
"fricas")
```

output
```
-1/2*(2*a*b*p*x*x^n*e^(-(2*n*p + 2*n + 1)*log(d) - (2*n*p + 2*n + 1)*log(x
)) - b^2*x*x^(2*n)*e^(-(2*n*p + 2*n + 1)*log(d) - (2*n*p + 2*n + 1)*log(x)
) + (2*a^2*p + a^2)*x*e^(-(2*n*p + 2*n + 1)*log(d) - (2*n*p + 2*n + 1)*log
(x)))*(b^2*x^(2*n) + 2*a*b*x^n + a^2)^p/(2*a^2*n*p^2 + 3*a^2*n*p + a^2*n)
```

3.547.6 Sympy [F]

$$\int (dx)^{-1-2n(1+p)}\left(a^2+2abx^n+b^2x^{2n}\right)^p\,dx=\int (dx)^{-2n(p+1)-1}\left((a+bx^n)^2\right)^p\,dx$$

input
```
integrate((d*x)**(-1-2*n*(1+p))*(a**2+2*a*b*x**n+b**2*x**(2*n))**p,x)
```

output
```
Integral((d*x)**(-2*n*(p + 1) - 1)*((a + b*x**n)**2)**p, x)
```

3.547.7 Maxima [F]

$$\int (dx)^{-1-2n(1+p)}\left(a^2+2abx^n+b^2x^{2n}\right)^p\,dx=\int \left(b^2x^{2n}+2\,abx^n+a^2\right)^p(dx)^{-2\,n(p+1)-1}\,dx$$

input
```
integrate((d*x)^(-1-2*n*(1+p))*(a^2+2*a*b*x^n+b^2*x^(2*n))^p,x, algorithm=
"maxima")
```

output
```
integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^p*(d*x)^(-2*n*(p + 1) - 1), x)
```

3.547.8 Giac [F]

$$\int (dx)^{-1-2n(1+p)}\left(a^2+2abx^n+b^2x^{2n}\right)^p\,dx=\int \left(b^2x^{2n}+2\,abx^n+a^2\right)^p(dx)^{-2\,n(p+1)-1}\,dx$$

input
```
integrate((d*x)^(-1-2*n*(1+p))*(a^2+2*a*b*x^n+b^2*x^(2*n))^p,x, algorithm=
"giac")
```

output
```
integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^p*(d*x)^(-2*n*(p + 1) - 1), x)
```

3.547.9 Mupad [F(-1)]

Timed out.

$$\int (dx)^{-1-2n(1+p)} \left(a^2+2abx^n+b^2x^{2n}\right)^p \, dx = \int \frac{\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)^p}{\left(d\,x\right)^{2\,n\,(p+1)+1}} \, dx$$

input
```
int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^p/(d*x)^(2*n*(p + 1) + 1),x)
```

output
```
int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^p/(d*x)^(2*n*(p + 1) + 1), x)
```

3.548 $\int x^{-1+2n}(a^2+2abx^n+b^2x^{2n})^p\,dx$

3.548.1 Optimal result

Integrand size = 30, antiderivative size = 103

$$\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^p\,dx=-\frac{a^2\left(1+\frac{bx^n}{a}\right)\left(a^2+2abx^n+b^2x^{2n}\right)^p}{b^2n(1+2p)}+\frac{a^2\left(1+\frac{bx^n}{a}\right)^2\left(a^2+2abx^n+b^2x^{2n}\right)^p}{2b^2n(1+p)}$$

output
```
-a^2*(1+b*x^n/a)*(a^2+2*a*b*x^n+b^2*x^(2*n))^p/b^2/n/(1+2*p)+1/2*a^2*(1+b*
x^n/a)^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^p/b^2/n/(p+1)
```

3.548.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.52

$$\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^p\,dx=\frac{(a+bx^n)\left((a+bx^n)^2\right)^p(-a+b(1+2p)x^n)}{2b^2n(1+p)(1+2p)}$$

input
```
Integrate[x^(-1 + 2*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p,x]
```

output
```
((a + b*x^n)*((a + b*x^n)^2)^p*(-a + b*(1 + 2*p)*x^n))/(2*b^2*n*(1 + p)*(1
 + 2*p))
```

3.548.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.84, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1693, 1100, 1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{2n-1}\left(a^2+2abx^n+b^2x^{2n}\right)^p\,dx$$

$$\Big\downarrow \text{ 1693}$$

$$\frac{\int x^n\left(2abx^n+b^2x^{2n}+a^2\right)^p\,dx^n}{n}$$

$$\Big\downarrow \text{ 1100}$$

$$\frac{\frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{p+1}}{2b^2(p+1)}-\frac{a\int\left(2abx^n+b^2x^{2n}+a^2\right)^p dx^n}{b}}{n}$$

$$\Big\downarrow \text{ 1079}$$

$$\frac{\frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{p+1}}{2b^2(p+1)}-\frac{a\left(ab+b^2x^n\right)^{-2p}\left(a^2+2abx^n+b^2x^{2n}\right)^p\int\left(b^2x^n+ab\right)^{2p}dx^n}{b}}{n}$$

$$\Big\downarrow \text{ 17}$$

$$\frac{\frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{p+1}}{2b^2(p+1)}-\frac{a\left(ab+b^2x^n\right)\left(a^2+2abx^n+b^2x^{2n}\right)^p}{b^3(2p+1)}}{n}$$

input
```
Int[x^(-1 + 2*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p,x]
```

output
```
(-((a*(a*b + b^2*x^n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p)/(b^3*(1 + 2*p)))
+ (a^2 + 2*a*b*x^n + b^2*x^(2*n))^(1 + p)/(2*b^2*(1 + p)))/n
```

3.548.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1100
```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.548.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 14.74 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.10

method	result	size
risch	$-\frac{\left(-2 b^{2} p \,x^{2 n}-2 a p \,x^{n} b -b^{2} x^{2 n}+a^{2}\right) \left(a +b \,x^{n}\right)^{2 p} {\mathrm e}^{-\frac{i \operatorname{csgn}\left(i \left(a +b \,x^{n}\right)^{2}\right) \pi p \left(-\operatorname{csgn}\left(i \left(a +b \,x^{n}\right)^{2}\right)+\operatorname{csgn}\left(i \left(a +b \,x^{n}\right)\right)\right)^{2}}{2}}}{2 \left(1+2 p \right) \left(1+p \right) n \,b^{2}}$	113

input
```
int(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^p,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-2*b^2*p*(x^n)^2-2*a*p*x^n*b-b^2*(x^n)^2+a^2)/(1+2*p)/(1+p)/n/b^2*(a
+b*x^n)^(2*p)*exp(-1/2*I*csgn(I*(a+b*x^n)^2)*Pi*p*(-csgn(I*(a+b*x^n)^2)+cs
gn(I*(a+b*x^n)))^2)
```

3.548.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.76

$$\int x^{-1+2n}(a^2+2abx^n+b^2x^{2n})^p\,dx=\frac{(2\,abpx^n-a^2+(2\,b^2p+b^2)x^{2\,n})(b^2x^{2\,n}+2\,abx^n+a^2)^p}{2\,(2\,b^2np^2+3\,b^2np+b^2n)}$$

input
```
integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^p,x, algorithm="fricas")
```

output
```
1/2*(2*a*b*p*x^n - a^2 + (2*b^2*p + b^2)*x^(2*n))*(b^2*x^(2*n) + 2*a*b*x^n
 + a^2)^p/(2*b^2*n*p^2 + 3*b^2*n*p + b^2*n)
```

3.548.6 Sympy [F(-1)]

Timed out.

$$\int x^{-1+2n}(a^2+2abx^n+b^2x^{2n})^p\,dx=\text{Timed out}$$

input
```
integrate(x**(-1+2*n)*(a**2+2*a*b*x**n+b**2*x**(2*n))**p,x)
```

output
```
Timed out
```

3.548.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.57

$$\int x^{-1+2n}(a^2+2abx^n+b^2x^{2n})^p\,dx=\frac{(b^2(2\,p+1)x^{2\,n}+2\,abpx^n-a^2)(bx^n+a)^{2\,p}}{2\,(2\,p^2+3\,p+1)b^2n}$$

input
```
integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^p,x, algorithm="maxima")
```

output
```
1/2*(b^2*(2*p + 1)*x^(2*n) + 2*a*b*p*x^n - a^2)*(b*x^n + a)^(2*p)/((2*p^2
+ 3*p + 1)*b^2*n)
```

3.548.8 Giac [F]

$$\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^p\,dx=\int \left(b^2x^{2\,n}+2\,abx^n+a^2\right)^px^{2\,n-1}\,dx$$

input
```
integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^p,x, algorithm="giac")
```

output
```
integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^p*x^(2*n - 1), x)
```

3.548.9 Mupad [F(-1)]

Timed out.

$$\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^p\,dx=\int x^{2\,n-1}\,\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)^p\,dx$$

input
```
int(x^(2*n - 1)*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^p,x)
```

output
```
int(x^(2*n - 1)*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^p, x)
```

3.549 $\int \frac{x^{-1+4n}}{a+bx^n+cx^{2n}}\,dx$

3.549.1 Optimal result

Integrand size = 24, antiderivative size = 111

$$\int \frac{x^{-1+4n}}{a+bx^n+cx^{2n}}\,dx = -\frac{bx^n}{c^2n}+\frac{x^{2n}}{2cn}+\frac{b(b^2-3ac)\operatorname{arctanh}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}\,n}+\frac{(b^2-ac)\log\left(a+bx^n+cx^{2n}\right)}{2c^3n}$$

output
```
-b*x^n/c^2/n+1/2*x^(2*n)/c/n+1/2*(-a*c+b^2)*ln(a+b*x^n+c*x^(2*n))/c^3/n+b*
(-3*a*c+b^2)*arctanh((b+2*c*x^n)/(-4*a*c+b^2)^(1/2))/c^3/n/(-4*a*c+b^2)^(1
/2)
```

3.549.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.87

$$\int \frac{x^{-1+4n}}{a+bx^n+cx^{2n}}\,dx = \frac{cx^n(-2b+cx^n)-\frac{2b(b^2-3ac)\arctan\left(\frac{b+2cx^n}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+(b^2-ac)\log\left(a+x^n(b+cx^n)\right)}{2c^3n}$$

input
```
Integrate[x^(-1 + 4*n)/(a + b*x^n + c*x^(2*n)),x]
```

output
```
(c*x^n*(-2*b + c*x^n) - (2*b*(b^2 - 3*a*c)*ArcTan[(b + 2*c*x^n)/Sqrt[-b^2
+ 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (b^2 - a*c)*Log[a + x^n*(b + c*x^n)])/(2*c
^3*n)
```

3.549.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1693, 1143, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{4n-1}}{a+bx^n+cx^{2n}}dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{\int \frac{x^{3n}}{bx^n+cx^{2n}+a}dx^n}{n}$$

$$\Big\downarrow \text{1143}$$

$$\frac{\int \left(\frac{x^n}{c}+\frac{(b^2-ac)x^n+ab}{c^2(bx^n+cx^{2n}+a)}-\frac{b}{c^2}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{b(b^2-3ac)\text{arctanh}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}}+\frac{(b^2-ac)\log(a+bx^n+cx^{2n})}{2c^3}-\frac{bx^n}{c^2}+\frac{x^{2n}}{2c}}{n}$$

input
```
Int[x^(-1 + 4*n)/(a + b*x^n + c*x^(2*n)),x]
```

output
```
(-((b*x^n)/c^2) + x^(2*n)/(2*c) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x^n)/S
qrt[b^2 - 4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - a*c)*Log[a + b*x^n +
c*x^(2*n)])/(2*c^3))/n
```

3.549.3.1 Defintions of rubi rules used

rule 1143
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x)^m/(a + b*x + c*x^2), x], x] /; FreeQ[{a,
b, c, d, e}, x] && IGtQ[m, 1]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.549.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 972 vs. $2(103) = 206$.

Time = 0.35 (sec) , antiderivative size = 973, normalized size of antiderivative = 8.77

method	result
risch	$-\frac{\ln(x)a}{c^2}+\frac{\ln(x)b^2}{c^3}+\frac{x^{2n}}{2cn}-\frac{b\,x^n}{c^2n}+\frac{4n^2\ln(x)a^2c^2}{4a\,c^4n^2-b^2c^3n^2}-\frac{5n^2\ln(x)a\,b^2c}{4a\,c^4n^2-b^2c^3n^2}+\frac{n^2\ln(x)b^4}{4a\,c^4n^2-b^2c^3n^2}-\frac{2\ln\left(x^n+\frac{3a\,b^2c-b^4+\sqrt{-3}}{}\right.}{c(}$

input
```
int(x^(-1+4*n)/(a+b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

output
```
-1/c^2*ln(x)*a+1/c^3*ln(x)*b^2+1/2/c/n*(x^n)^2-b*x^n/c^2/n+4/(4*a*c^4*n^2-
b^2*c^3*n^2)*n^2*ln(x)*a^2*c^2-5/(4*a*c^4*n^2-b^2*c^3*n^2)*n^2*ln(x)*a*b^2
*c+1/(4*a*c^4*n^2-b^2*c^3*n^2)*n^2*ln(x)*b^4-2/c/(4*a*c-b^2)/n*ln(x^n+1/2*
(3*a*b^2*c-b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/c/b/
(3*a*c-b^2))*a^2+5/2/c^2/(4*a*c-b^2)/n*ln(x^n+1/2*(3*a*b^2*c-b^4+(-36*a^3*
b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/c/b/(3*a*c-b^2))*a*b^2-1/2/c
^3/(4*a*c-b^2)/n*ln(x^n+1/2*(3*a*b^2*c-b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2
-10*a*b^6*c+b^8)^(1/2))/c/b/(3*a*c-b^2))*b^4+1/2/c^3/(4*a*c-b^2)/n*ln(x^n+
1/2*(3*a*b^2*c-b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/
c/b/(3*a*c-b^2))*(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2)-2/c
/(4*a*c-b^2)/n*ln(x^n-1/2*(-3*a*b^2*c+b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-
10*a*b^6*c+b^8)^(1/2))/c/b/(3*a*c-b^2))*a^2+5/2/c^2/(4*a*c-b^2)/n*ln(x^n-1
/2*(-3*a*b^2*c+b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/
c/b/(3*a*c-b^2))*a*b^2-1/2/c^3/(4*a*c-b^2)/n*ln(x^n-1/2*(-3*a*b^2*c+b^4+(-
36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/c/b/(3*a*c-b^2))*b^4-
1/2/c^3/(4*a*c-b^2)/n*ln(x^n-1/2*(-3*a*b^2*c+b^4+(-36*a^3*b^2*c^3+33*a^2*b
^4*c^2-10*a*b^6*c+b^8)^(1/2))/c/b/(3*a*c-b^2))*(-36*a^3*b^2*c^3+33*a^2*b^4
*c^2-10*a*b^6*c+b^8)^(1/2)
```

3.549.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 353, normalized size of antiderivative = 3.18

$$\int \frac{x^{-1+4n}}{a+bx^n+cx^{2n}}\,dx = \left[-\frac{(b^3-3\,abc)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^{2\,n}+b^2-2\,ac+2\left(bc-\sqrt{b^2-4\,ac}c\right)x^n-\sqrt{b^2-4\,ac}b}{cx^{2\,n}+bx^n+a}\right)-(b^2c^2-4\,ac^3)x^{2\,n}+2\,(b^3c-4\cdots}{2\,(b^2c^3-4\,ac^4)n}\right.$$

input
```
integrate(x^(-1+4*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
[-1/2*((b^3 - 3*a*b*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^(2*n) + b^2 - 2*a*c
+ 2*(b*c - sqrt(b^2 - 4*a*c)*c)*x^n - sqrt(b^2 - 4*a*c)*b)/(c*x^(2*n) + b*
x^n + a)) - (b^2*c^2 - 4*a*c^3)*x^(2*n) + 2*(b^3*c - 4*a*b*c^2)*x^n - (b^4
 - 5*a*b^2*c + 4*a^2*c^2)*log(c*x^(2*n) + b*x^n + a))/((b^2*c^3 - 4*a*c^4)
*n), 1/2*(2*(b^3 - 3*a*b*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*sqrt(-b^2 + 4*a*
c)*c*x^n + sqrt(-b^2 + 4*a*c)*b)/(b^2 - 4*a*c)) + (b^2*c^2 - 4*a*c^3)*x^(2
*n) - 2*(b^3*c - 4*a*b*c^2)*x^n + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*log(c*x^(2
*n) + b*x^n + a))/((b^2*c^3 - 4*a*c^4)*n)]
```

3.549.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1+4n}}{a+bx^n+cx^{2n}}\,dx = \text{Timed out}$$

input
```
integrate(x**(-1+4*n)/(a+b*x**n+c*x**(2*n)),x)
```

output
```
Timed out
```

3.549.7 Maxima [F]

$$\int \frac{x^{-1+4n}}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^{4\,n-1}}{cx^{2\,n}+bx^n+a}\,dx$$

input
```
integrate(x^(-1+4*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
(b^2 - a*c)*log(x)/c^3 + 1/2*(c*x^(2*n) - 2*b*x^n)/(c^2*n) + integrate(-(a
*b^2 - a^2*c + (b^3 - 2*a*b*c)*x^n)/(c^4*x*x^(2*n) + b*c^3*x*x^n + a*c^3*x
), x)
```

3.549.8 Giac [F]

$$\int \frac{x^{-1+4n}}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^{4\,n-1}}{cx^{2\,n}+bx^n+a}\,dx$$

input
```
integrate(x^(-1+4*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate(x^(4*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

3.549.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+4n}}{a+bx^n+cx^{2n}}\,dx=\int \frac{x^{4\,n-1}}{a+b\,x^n+c\,x^{2\,n}}\,dx$$

input
```
int(x^(4*n - 1)/(a + b*x^n + c*x^(2*n)),x)
```

output
```
int(x^(4*n - 1)/(a + b*x^n + c*x^(2*n)), x)
```

3.550 $\int \frac{x^{-1+3n}}{a+bx^n+cx^{2n}}\,dx$

3.550.1 Optimal result

Integrand size = 24, antiderivative size = 87

$$\int \frac{x^{-1+3n}}{a+bx^n+cx^{2n}}\,dx = \frac{x^n}{cn} - \frac{(b^2-2ac)\,\text{arctanh}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}\,n} - \frac{b\log\left(a+bx^n+cx^{2n}\right)}{2c^2n}$$

output
```
x^n/c/n-1/2*b*ln(a+b*x^n+c*x^(2*n))/c^2/n-(-2*a*c+b^2)*arctanh((b+2*c*x^n)
/(-4*a*c+b^2)^(1/2))/c^2/n/(-4*a*c+b^2)^(1/2)
```

3.550.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.94

$$\int \frac{x^{-1+3n}}{a+bx^n+cx^{2n}}\,dx = \frac{2cx^n + \frac{2(b^2-2ac)\arctan\left(\frac{b+2cx^n}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}} - b\log\left(a+x^n(b+cx^n)\right)}{2c^2n}$$

input
```
Integrate[x^(-1 + 3*n)/(a + b*x^n + c*x^(2*n)),x]
```

output
```
(2*c*x^n + (2*(b^2 - 2*a*c)*ArcTan[(b + 2*c*x^n)/Sqrt[-b^2 + 4*a*c]])/Sqrt
[-b^2 + 4*a*c] - b*Log[a + x^n*(b + c*x^n)])/(2*c^2*n)
```

3.550.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1693, 1143, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3n-1}}{a+bx^n+cx^{2n}}dx$$
$$\Big\downarrow \text{1693}$$
$$\frac{\int \frac{x^{2n}}{bx^n+cx^{2n}+a}dx^n}{n}$$
$$\Big\downarrow \text{1143}$$
$$\frac{\int \left(\frac{1}{c}-\frac{bx^n+a}{c(bx^n+cx^{2n}+a)}\right)dx^n}{n}$$
$$\Big\downarrow \text{2009}$$
$$\frac{-\frac{(b^2-2ac)\text{arctanh}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{b\log(a+bx^n+cx^{2n})}{2c^2}+\frac{x^n}{c}}{n}$$

input
```
Int[x^(-1 + 3*n)/(a + b*x^n + c*x^(2*n)),x]
```

output
```
(x^n/c - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/(c^2*Sqr
t[b^2 - 4*a*c]) - (b*Log[a + b*x^n + c*x^(2*n)])/(2*c^2))/n
```

3.550.3.1 Defintions of rubi rules used

rule 1143
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x)^m/(a + b*x + c*x^2), x], x] /; FreeQ[{a,
b, c, d, e}, x] && IGtQ[m, 1]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.550.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 663 vs. 2(81) = 162.

Time = 0.29 (sec) , antiderivative size = 664, normalized size of antiderivative = 7.63

method	result
risch	$-\frac{b\ln(x)}{c^2}+\frac{x^n}{cn}+\frac{4n^2\ln(x)abc}{4a\,c^3n^2-b^2c^2n^2}-\frac{n^2\ln(x)b^3}{4a\,c^3n^2-b^2c^2n^2}-\frac{2\ln\left(x^n-\frac{-2abc+b^3+\sqrt{-16c^3a^3+20a^2b^2c^2-8a\,b^4c+b^6}}{2c\left(2ac-b^2\right)}\right)ab}{(4ac-b^2)cn}+\frac{\ln\left(x^n-\right.}{}$

input
```
int(x^(-1+3*n)/(a+b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

output
```
-b/c^2*ln(x)+x^n/c/n+4/(4*a*c^3*n^2-b^2*c^2*n^2)*n^2*ln(x)*a*b*c-1/(4*a*c^
3*n^2-b^2*c^2*n^2)*n^2*ln(x)*b^3-2/(4*a*c-b^2)/c/n*ln(x^n-1/2*(-2*a*b*c+b^
3+(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2))/c/(2*a*c-b^2))*a*b+1/2
/(4*a*c-b^2)/c^2/n*ln(x^n-1/2*(-2*a*b*c+b^3+(-16*a^3*c^3+20*a^2*b^2*c^2-8*
a*b^4*c+b^6)^(1/2))/c/(2*a*c-b^2))*b^3+1/2/(4*a*c-b^2)/c^2/n*ln(x^n-1/2*(-
2*a*b*c+b^3+(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2))/c/(2*a*c-b^2
))*(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2)-2/(4*a*c-b^2)/c/n*ln(x
^n+1/2*(2*a*b*c-b^3+(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2))/c/(2
*a*c-b^2))*a*b+1/2/(4*a*c-b^2)/c^2/n*ln(x^n+1/2*(2*a*b*c-b^3+(-16*a^3*c^3+
20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2))/c/(2*a*c-b^2))*b^3-1/2/(4*a*c-b^2)/c^
2/n*ln(x^n+1/2*(2*a*b*c-b^3+(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/
2))/c/(2*a*c-b^2))*(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2)
```

3.550.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 285, normalized size of antiderivative = 3.28

$$\int \frac{x^{-1+3n}}{a+bx^n+cx^{2n}}\,dx$$
$$= \left[-\frac{(b^2-2\,ac)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^{2\,n}+b^2-2\,ac+2\left(bc+\sqrt{b^2-4\,ac}c\right)x^n+\sqrt{b^2-4\,ac}b}{cx^{2\,n}+bx^n+a}\right)-2\,(b^2c-4\,ac^2)x^n+(b^3-4\,abc}{2\,(b^2c^2-4\,ac^3)n} \right.$$
$$-\frac{2\,(b^2-2\,ac)\sqrt{-b^2+4\,ac}\arctan\left(-\frac{2\sqrt{-b^2+4\,ac}cx^n+\sqrt{-b^2+4\,ac}b}{b^2-4\,ac}\right)-2\,(b^2c-4\,ac^2)x^n+(b^3-4\,abc)\log\left(cx^2\right.}{2\,(b^2c^2-4\,ac^3)n}$$

input
```
integrate(x^(-1+3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
[-1/2*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^(2*n) + b^2 - 2*a*c +
2*(b*c + sqrt(b^2 - 4*a*c)*c)*x^n + sqrt(b^2 - 4*a*c)*b)/(c*x^(2*n) + b*x^
n + a)) - 2*(b^2*c - 4*a*c^2)*x^n + (b^3 - 4*a*b*c)*log(c*x^(2*n) + b*x^n
+ a))/((b^2*c^2 - 4*a*c^3)*n), -1/2*(2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*ar
ctan(-(2*sqrt(-b^2 + 4*a*c)*c*x^n + sqrt(-b^2 + 4*a*c)*b)/(b^2 - 4*a*c)) -
 2*(b^2*c - 4*a*c^2)*x^n + (b^3 - 4*a*b*c)*log(c*x^(2*n) + b*x^n + a))/((b
^2*c^2 - 4*a*c^3)*n)]
```

3.550.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1+3n}}{a+bx^n+cx^{2n}}\,dx = \text{Timed out}$$

input
```
integrate(x**(-1+3*n)/(a+b*x**n+c*x**(2*n)),x)
```

output
```
Timed out
```

3.550.7 Maxima [F]

$$\int \frac{x^{-1+3n}}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^{3\,n-1}}{cx^{2\,n}+bx^n+a}\,dx$$

input
```
integrate(x^(-1+3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
-b*log(x)/c^2 + x^n/(c*n) - integrate(-(a*b + (b^2 - a*c)*x^n)/(c^3*x*x^(2
*n) + b*c^2*x*x^n + a*c^2*x), x)
```

3.550.8 Giac [F]

$$\int \frac{x^{-1+3n}}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^{3\,n-1}}{cx^{2\,n}+bx^n+a}\,dx$$

input
```
integrate(x^(-1+3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate(x^(3*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

3.550.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+3n}}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^{3\,n-1}}{a+b\,x^n+c\,x^{2\,n}}\,dx$$

input
```
int(x^(3*n - 1)/(a + b*x^n + c*x^(2*n)),x)
```

output
```
int(x^(3*n - 1)/(a + b*x^n + c*x^(2*n)), x)
```

3.551 $\int \frac{x^{-1+2n}}{a+bx^n+cx^{2n}}\,dx$

3.551.1 Optimal result

Integrand size = 24, antiderivative size = 68

$$\int \frac{x^{-1+2n}}{a+bx^n+cx^{2n}}\,dx=\frac{b\operatorname{arctanh}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}n}+\frac{\log\left(a+bx^n+cx^{2n}\right)}{2cn}$$

output
```
1/2*ln(a+b*x^n+c*x^(2*n))/c/n+b*arctanh((b+2*c*x^n)/(-4*a*c+b^2)^(1/2))/c/
n/(-4*a*c+b^2)^(1/2)
```

3.551.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.97

$$\int \frac{x^{-1+2n}}{a+bx^n+cx^{2n}}\,dx=\frac{-\frac{2b\arctan\left(\frac{b+2cx^n}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+\log\left(a+x^n(b+cx^n)\right)}{2cn}$$

input
```
Integrate[x^(-1 + 2*n)/(a + b*x^n + c*x^(2*n)),x]
```

output
```
((-2*b*ArcTan[(b + 2*c*x^n)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + Log[
a + x^n*(b + c*x^n)])/(2*c*n)
```

3.551.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1693, 1142, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{2n-1}}{a+bx^n+cx^{2n}} \, dx$$

$$\Big\downarrow \; 1693$$

$$\frac{\int \frac{x^n}{bx^n+cx^{2n}+a}dx^n}{n}$$

$$\Big\downarrow \; 1142$$

$$\frac{\frac{\int \frac{2cx^n+b}{bx^n+cx^{2n}+a}dx^n}{2c}-\frac{b\int \frac{1}{bx^n+cx^{2n}+a}dx^n}{2c}}{n}$$

$$\Big\downarrow \; 1083$$

$$\frac{\frac{b\int \frac{1}{-x^{2n}+b^2-4ac}d(2cx^n+b)}{c}+\frac{\int \frac{2cx^n+b}{bx^n+cx^{2n}+a}dx^n}{2c}}{n}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\int \frac{2cx^n+b}{bx^n+cx^{2n}+a}dx^n}{2c}+\frac{b\text{arctanh}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}}{n}$$

$$\Big\downarrow \; 1103$$

$$\frac{\frac{b\text{arctanh}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}+\frac{\log(a+bx^n+cx^{2n})}{2c}}{n}$$

input
```
Int[x^(-1 + 2*n)/(a + b*x^n + c*x^(2*n)),x]
```

output
```
((b*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 - 4*a*c]) + Log[
a + b*x^n + c*x^(2*n)]/(2*c))/n
```

3.551.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.551.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 401 vs. $2(62) = 124$.

Time = 0.24 (sec) , antiderivative size = 402, normalized size of antiderivative = 5.91

method	result
risch	$\frac{\ln(x)}{c} - \frac{4n^2\ln(x)ac}{4a\,c^2n^2-b^2c\,n^2} + \frac{n^2\ln(x)b^2}{4a\,c^2n^2-b^2c\,n^2} + \frac{2\ln\left(x^n-\frac{-b^2+\sqrt{-4a\,b^2c+b^4}}{2bc}\right)a}{(4ac-b^2)n} - \frac{\ln\left(x^n-\frac{-b^2+\sqrt{-4a\,b^2c+b^4}}{2bc}\right)b^2}{2c(4ac-b^2)n} + \frac{\ln\left(x^n-\right.}{}$

input
```
int(x^(-1+2*n)/(a+b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

output
```
1/c*ln(x)-4/(4*a*c^2*n^2-b^2*c*n^2)*n^2*ln(x)*a*c+1/(4*a*c^2*n^2-b^2*c*n^2
)*n^2*ln(x)*b^2+2/(4*a*c-b^2)/n*ln(x^n-1/2*(-b^2+(-4*a*b^2*c+b^4)^(1/2))/b
/c)*a-1/2/c/(4*a*c-b^2)/n*ln(x^n-1/2*(-b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*b^
2+1/2/c/(4*a*c-b^2)/n*ln(x^n-1/2*(-b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*(-4*a*
b^2*c+b^4)^(1/2)+2/(4*a*c-b^2)/n*ln(x^n+1/2*(b^2+(-4*a*b^2*c+b^4)^(1/2))/b
/c)*a-1/2/c/(4*a*c-b^2)/n*ln(x^n+1/2*(b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*b^2
-1/2/c/(4*a*c-b^2)/n*ln(x^n+1/2*(b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*(-4*a*b^
2*c+b^4)^(1/2)
```

3.551.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 231, normalized size of antiderivative = 3.40

$$\int \frac{x^{-1+2n}}{a+bx^n+cx^{2n}}\,dx$$
$$= \left[\frac{\sqrt{b^2-4\,ac}b\log\left(\frac{2\,c^2x^{2\,n}+b^2-2\,ac+2\,\left(bc+\sqrt{b^2-4\,ac}c\right)x^n+\sqrt{b^2-4\,ac}b}{cx^{2\,n}+bx^n+a}\right)+\left(b^2-4\,ac\right)\log\left(cx^{2\,n}+bx^n+a\right)}{2\,\left(b^2c-4\,ac^2\right)n}, \frac{2\sqrt{-b^2+\cdots}}{\cdots}\right.$$

input
```
integrate(x^(-1+2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^(2*n) + b^2 - 2*a*c + 2*(b*c + sqrt
(b^2 - 4*a*c)*c)*x^n + sqrt(b^2 - 4*a*c)*b)/(c*x^(2*n) + b*x^n + a)) + (b^
2 - 4*a*c)*log(c*x^(2*n) + b*x^n + a))/((b^2*c - 4*a*c^2)*n), 1/2*(2*sqrt(
-b^2 + 4*a*c)*b*arctan(-(2*sqrt(-b^2 + 4*a*c)*c*x^n + sqrt(-b^2 + 4*a*c)*b
)/(b^2 - 4*a*c)) + (b^2 - 4*a*c)*log(c*x^(2*n) + b*x^n + a))/((b^2*c - 4*a
*c^2)*n)]
```

3.551.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}}{a+bx^n+cx^{2n}}\,dx = \text{Timed out}$$

input
```
integrate(x**(-1+2*n)/(a+b*x**n+c*x**(2*n)),x)
```

output
```
Timed out
```

3.551.7 Maxima [F]

$$\int \frac{x^{-1+2n}}{a+bx^n+cx^{2n}}\,dx=\int \frac{x^{2\,n-1}}{cx^{2\,n}+bx^n+a}\,dx$$

input
```
integrate(x^(-1+2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
log(x)/c - integrate((b*x^n + a)/(c^2*x*x^(2*n) + b*c*x*x^n + a*c*x), x)
```

3.551.8 Giac [F]

$$\int \frac{x^{-1+2n}}{a+bx^n+cx^{2n}}\,dx=\int \frac{x^{2\,n-1}}{cx^{2\,n}+bx^n+a}\,dx$$

input
```
integrate(x^(-1+2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate(x^(2*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

3.551.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}}{a+bx^n+cx^{2n}}\,dx=\int \frac{x^{2\,n-1}}{a+b\,x^n+c\,x^{2\,n}}\,dx$$

input
```
int(x^(2*n - 1)/(a + b*x^n + c*x^(2*n)),x)
```

output
```
int(x^(2*n - 1)/(a + b*x^n + c*x^(2*n)), x)
```

3.552 $\int \frac{x^{-1+n}}{a+bx^n+cx^{2n}}\,dx$

3.552.1 Optimal result

Integrand size = 22, antiderivative size = 39

$$\int \frac{x^{-1+n}}{a+bx^n+cx^{2n}}\,dx=-\frac{2\text{arctanh}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}n}$$

output
```
-2*arctanh((b+2*c*x^n)/(-4*a*c+b^2)^(1/2))/n/(-4*a*c+b^2)^(1/2)
```

3.552.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10

$$\int \frac{x^{-1+n}}{a+bx^n+cx^{2n}}\,dx=\frac{2\arctan\left(\frac{b+2cx^n}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}n}$$

input
```
Integrate[x^(-1 + n)/(a + b*x^n + c*x^(2*n)),x]
```

output
```
(2*ArcTan[(b + 2*c*x^n)/Sqrt[-b^2 + 4*a*c]])/(Sqrt[-b^2 + 4*a*c]*n)
```

3.552.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1690, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{n-1}}{a+bx^n+cx^{2n}}\,dx$$
$$\Big\downarrow \ 1690$$
$$\frac{\int \frac{1}{bx^n+cx^{2n}+a}dx^n}{n}$$
$$\Big\downarrow \ 1083$$
$$-\frac{2\int \frac{1}{-x^{2n}+b^2-4ac}d(2cx^n+b)}{n}$$
$$\Big\downarrow \ 219$$
$$-\frac{2\text{arctanh}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{n\sqrt{b^2-4ac}}$$

input
```
Int[x^(-1 + n)/(a + b*x^n + c*x^(2*n)),x]
```

output
```
(-2*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*n)
```

3.552.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

3.552.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(35) = 70.

Time = 0.20 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.90

method	result	size
risch	$-\frac{\ln\left(x^n+\frac{b^2-4ac+b\sqrt{-4ac+b^2}}{2c\sqrt{-4ac+b^2}}\right)}{\sqrt{-4ac+b^2}\,n}+\frac{\ln\left(x^n+\frac{b\sqrt{-4ac+b^2}+4ac-b^2}{2c\sqrt{-4ac+b^2}}\right)}{\sqrt{-4ac+b^2}\,n}$	113

input
```
int(x^(-1+n)/(a+b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

output
```
-1/(-4*a*c+b^2)^(1/2)/n*ln(x^n+1/2*(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))/c/(-4*
a*c+b^2)^(1/2))+1/(-4*a*c+b^2)^(1/2)/n*ln(x^n+1/2*(b*(-4*a*c+b^2)^(1/2)+4*
a*c-b^2)/c/(-4*a*c+b^2)^(1/2))
```

3.552.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(35) = 70.

Time = 0.27 (sec) , antiderivative size = 159, normalized size of antiderivative = 4.08

$$\int \frac{x^{-1+n}}{a+bx^n+cx^{2n}}\,dx = \left[\frac{\log\left(\frac{2\,c^2x^{2\,n}+b^2-2\,ac+2\,\left(bc-\sqrt{b^2-4\,ac}c\right)x^n-\sqrt{b^2-4\,ac}b}{cx^{2\,n}+bx^n+a}\right)}{\sqrt{b^2-4\,ac}n},\ -\frac{2\,\sqrt{-b^2+4\,ac}\arctan\left(-\frac{2\,\sqrt{-b^2+4\,ac}cx^n+\sqrt{-b^2+4\,ac}b}{b^2-4\,ac}\right)}{(b^2-4\,ac)n}\right]$$

input
```
integrate(x^(-1+n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
[log((2*c^2*x^(2*n) + b^2 - 2*a*c + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*x^n - sq
rt(b^2 - 4*a*c)*b)/(c*x^(2*n) + b*x^n + a))/(sqrt(b^2 - 4*a*c)*n), -2*sqrt
(-b^2 + 4*a*c)*arctan(-(2*sqrt(-b^2 + 4*a*c)*c*x^n + sqrt(-b^2 + 4*a*c)*b)
/(b^2 - 4*a*c))/((b^2 - 4*a*c)*n)]
```

3.552.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1+n}}{a+bx^n+cx^{2n}}\,dx = \text{Timed out}$$

input
```
integrate(x**(-1+n)/(a+b*x**n+c*x**(2*n)),x)
```

output
```
Timed out
```

3.552.7 Maxima [F]

$$\int \frac{x^{-1+n}}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^{n-1}}{cx^{2n}+bx^n+a}\,dx$$

input
```
integrate(x^(-1+n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
integrate(x^(n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

3.552.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1+n}}{a+bx^n+cx^{2n}}\,dx = \frac{2\arctan\left(\frac{2\,cx^n+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}n}$$

input
```
integrate(x^(-1+n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
2*arctan((2*c*x^n + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*n)
```

3.552.9 Mupad [B] (verification not implemented)

Time = 8.38 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1+n}}{a+bx^n+cx^{2n}}\,dx=\frac{2\,\mathrm{atan}\left(\frac{b+2\,c\,x^n}{\sqrt{4\,a\,c-b^2}}\right)}{n\,\sqrt{4\,a\,c-b^2}}$$

input
```
int(x^(n - 1)/(a + b*x^n + c*x^(2*n)),x)
```

output
```
(2*atan((b + 2*c*x^n)/(4*a*c - b^2)^(1/2)))/(n*(4*a*c - b^2)^(1/2))
```

3.553 $\int \frac{x^{-1-n}}{a+bx^n+cx^{2n}}\,dx$

3.553.1 Optimal result

Integrand size = 24, antiderivative size = 98

$$\int \frac{x^{-1-n}}{a+bx^n+cx^{2n}}\,dx=-\frac{x^{-n}}{an}-\frac{(b^2-2ac)\operatorname{arctanh}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}\,n}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx^n+cx^{2n}\right)}{2a^2n}$$

output
```
-1/a/n/(x^n)-b*ln(x)/a^2+1/2*b*ln(a+b*x^n+c*x^(2*n))/a^2/n-(-2*a*c+b^2)*ar
ctanh((b+2*c*x^n)/(-4*a*c+b^2)^(1/2))/a^2/n/(-4*a*c+b^2)^(1/2)
```

3.553.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.92

$$\int \frac{x^{-1-n}}{a+bx^n+cx^{2n}}\,dx=\frac{-2ax^{-n}+\frac{2(b^2-2ac)\arctan\left(\frac{b+2cx^n}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}-2b\log\left(x^n\right)+b\log\left(a+x^n(b+cx^n)\right)}{2a^2n}$$

input
```
Integrate[x^(-1 - n)/(a + b*x^n + c*x^(2*n)),x]
```

output
```
((-2*a)/x^n + (2*(b^2 - 2*a*c)*ArcTan[(b + 2*c*x^n)/Sqrt[-b^2 + 4*a*c]])/S
qrt[-b^2 + 4*a*c] - 2*b*Log[x^n] + b*Log[a + x^n*(b + c*x^n)])/(2*a^2*n)
```

3.553.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1693, 1145, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-n-1}}{a+bx^n+cx^{2n}}\,dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{\int \frac{x^{-2n}}{bx^n+cx^{2n}+a}dx^n}{n}$$

$$\Big\downarrow \text{1145}$$

$$\frac{\frac{\int -\frac{x^{-n}(cx^n+b)}{bx^n+cx^{2n}+a}dx^n}{a}-\frac{x^{-n}}{a}}{n}$$

$$\Big\downarrow \text{25}$$

$$\frac{-\frac{\int \frac{x^{-n}(cx^n+b)}{bx^n+cx^{2n}+a}dx^n}{a}-\frac{x^{-n}}{a}}{n}$$

$$\Big\downarrow \text{1200}$$

$$\frac{-\frac{\int \left(\frac{bx^{-n}}{a}+\frac{-bcx^n-b^2+ac}{a\left(bx^n+cx^{2n}+a\right)}\right)dx^n}{a}-\frac{x^{-n}}{a}}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{\frac{\left(b^2-2ac\right)\text{arctanh}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx^n+cx^{2n}\right)}{2a}+\frac{b\log\left(x^n\right)}{a}}{a}-\frac{x^{-n}}{a}}{n}$$

input
```
Int[x^(-1 - n)/(a + b*x^n + c*x^(2*n)),x]
```

output
```
(-(1/(a*x^n)) - (((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/
(a*Sqrt[b^2 - 4*a*c]) + (b*Log[x^n])/a - (b*Log[a + b*x^n + c*x^(2*n)])/(2
*a))/a)/n
```

3.553.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1145
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp
[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[m, -1]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.553.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 657 vs. $2(92) = 184$.

Time = 0.25 (sec) , antiderivative size = 658, normalized size of antiderivative = 6.71

method	result
risch	$-\frac{x^{-n}}{an}-\frac{4n^2\ln(x)abc}{4a^3cn^2-a^2b^2n^2}+\frac{n^2\ln(x)b^3}{4a^3cn^2-a^2b^2n^2}+\frac{2\ln\left(x^n-\frac{-2abc+b^3+\sqrt{-16c^3a^3+20a^2b^2c^2-8ab^4c+b^6}}{2c(2ac-b^2)}\right)bc}{a(4ac-b^2)n}-\frac{\ln\left(x^n-\frac{-2abc+b^3}{}\right.}{}$

input
```
int(x^(-1-n)/(a+b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

output
```
-1/a/n/(x^n)-4/(4*a^3*c*n^2-a^2*b^2*n^2)*n^2*ln(x)*a*b*c+1/(4*a^3*c*n^2-a^
2*b^2*n^2)*n^2*ln(x)*b^3+2/a/(4*a*c-b^2)/n*ln(x^n-1/2*(-2*a*b*c+b^3+(-16*a
^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2))/c/(2*a*c-b^2))*b*c-1/2/a^2/(4*
a*c-b^2)/n*ln(x^n-1/2*(-2*a*b*c+b^3+(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+
b^6)^(1/2))/c/(2*a*c-b^2))*b^3+1/2/a^2/(4*a*c-b^2)/n*ln(x^n-1/2*(-2*a*b*c+
b^3+(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2))/c/(2*a*c-b^2))*(-16*
a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2)+2/a/(4*a*c-b^2)/n*ln(x^n+1/2*(
2*a*b*c-b^3+(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2))/c/(2*a*c-b^2
))*b*c-1/2/a^2/(4*a*c-b^2)/n*ln(x^n+1/2*(2*a*b*c-b^3+(-16*a^3*c^3+20*a^2*b
^2*c^2-8*a*b^4*c+b^6)^(1/2))/c/(2*a*c-b^2))*b^3-1/2/a^2/(4*a*c-b^2)/n*ln(x
^n+1/2*(2*a*b*c-b^3+(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2))/c/(2
*a*c-b^2))*(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2)
```

3.553.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 333, normalized size of antiderivative = 3.40

$$\int \frac{x^{-1-n}}{a+bx^n+cx^{2n}}\,dx$$
$$= \left[-\frac{2\,(b^3-4\,abc)nx^n\log{(x)}+(b^2-2\,ac)\sqrt{b^2-4\,ac}x^n\log\left(\frac{2\,c^2x^{2\,n}+b^2-2\,ac+2\left(bc+\sqrt{b^2-4\,ac}c\right)x^n+\sqrt{b^2-4\,ac}b}{cx^{2\,n}+bx^n+a}\right)+2}{2\,(a^2b^2-4\,a^3c)nx^n}\right.$$
$$-\frac{2\,(b^3-4\,abc)nx^n\log{(x)}+2\,(b^2-2\,ac)\sqrt{-b^2+4\,ac}x^n\arctan\left(-\frac{2\sqrt{-b^2+4\,ac}cx^n+\sqrt{-b^2+4\,ac}b}{b^2-4\,ac}\right)+2\,ab^2-8\,}{2\,(a^2b^2-4\,a^3c)nx^n}$$

input
```
integrate(x^(-1-n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
[-1/2*(2*(b^3 - 4*a*b*c)*n*x^n*log(x) + (b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*x^
n*log((2*c^2*x^(2*n) + b^2 - 2*a*c + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*x^n + s
qrt(b^2 - 4*a*c)*b)/(c*x^(2*n) + b*x^n + a)) + 2*a*b^2 - 8*a^2*c - (b^3 -
4*a*b*c)*x^n*log(c*x^(2*n) + b*x^n + a))/((a^2*b^2 - 4*a^3*c)*n*x^n), -1/2
*(2*(b^3 - 4*a*b*c)*n*x^n*log(x) + 2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*x^n*
arctan(-(2*sqrt(-b^2 + 4*a*c)*c*x^n + sqrt(-b^2 + 4*a*c)*b)/(b^2 - 4*a*c))
 + 2*a*b^2 - 8*a^2*c - (b^3 - 4*a*b*c)*x^n*log(c*x^(2*n) + b*x^n + a))/((a
^2*b^2 - 4*a^3*c)*n*x^n)]
```

3.553.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1-n}}{a+bx^n+cx^{2n}}\,dx = \text{Timed out}$$

input
```
integrate(x**(-1-n)/(a+b*x**n+c*x**(2*n)),x)
```

output
```
Timed out
```

3.553.7 Maxima [F]

$$\int \frac{x^{-1-n}}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^{-n-1}}{cx^{2n}+bx^n+a}\,dx$$

input
```
integrate(x^(-1-n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
-1/(a*n*x^n) - integrate((c*x^n + b)/(a*c*x*x^(2*n) + a*b*x*x^n + a^2*x),
x)
```

3.553.8 Giac [F]

$$\int \frac{x^{-1-n}}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^{-n-1}}{cx^{2n}+bx^n+a}\,dx$$

input
```
integrate(x^(-1-n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate(x^(-n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

3.553.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-n}}{a+bx^n+cx^{2n}}\,dx = \int \frac{1}{x^{n+1}\,\left(a+b\,x^n+c\,x^{2\,n}\right)}\,dx$$

input
```
int(1/(x^(n + 1)*(a + b*x^n + c*x^(2*n))),x)
```

output
```
int(1/(x^(n + 1)*(a + b*x^n + c*x^(2*n))), x)
```

3.554 $\int \frac{x^{-1-2n}}{a+bx^n+cx^{2n}}\,dx$

3.554.1 Optimal result

Integrand size = 24, antiderivative size = 126

$$\int \frac{x^{-1-2n}}{a+bx^n+cx^{2n}}\,dx = -\frac{x^{-2n}}{2an}+\frac{bx^{-n}}{a^2n}+\frac{b(b^2-3ac)\operatorname{arctanh}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}n}+\frac{(b^2-ac)\log(x)}{a^3}-\frac{(b^2-ac)\log\left(a+bx^n+cx^{2n}\right)}{2a^3n}$$

output
```
-1/2/a/n/(x^(2*n))+b/a^2/n/(x^n)+(-a*c+b^2)*ln(x)/a^3-1/2*(-a*c+b^2)*ln(a+
b*x^n+c*x^(2*n))/a^3/n+b*(-3*a*c+b^2)*arctanh((b+2*c*x^n)/(-4*a*c+b^2)^(1/
2))/a^3/n/(-4*a*c+b^2)^(1/2)
```

3.554.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.91

$$\int \frac{x^{-1-2n}}{a+bx^n+cx^{2n}}\,dx = \frac{ax^{-2n}(-a+2bx^n)-\frac{2b(b^2-3ac)\arctan\left(\frac{b+2cx^n}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+2(b^2-ac)\log(x^n)-(b^2-ac)\log\left(a+x^n(b+cx^n)\right)}{2a^3n}$$

input
```
Integrate[x^(-1 - 2*n)/(a + b*x^n + c*x^(2*n)),x]
```

output
```
((a*(-a + 2*b*x^n))/x^(2*n) - (2*b*(b^2 - 3*a*c)*ArcTan[(b + 2*c*x^n)/Sqrt
[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + 2*(b^2 - a*c)*Log[x^n] - (b^2 - a*c)
*Log[a + x^n*(b + c*x^n)])/(2*a^3*n)
```

3.554.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1693, 1145, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-2n-1}}{a+bx^n+cx^{2n}}dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{\int \frac{x^{-3n}}{bx^n+cx^{2n}+a}dx^n}{n}$$

$$\Big\downarrow \text{1145}$$

$$\frac{\frac{\int -\frac{x^{-2n}(cx^n+b)}{bx^n+cx^{2n}+a}dx^n}{a}-\frac{x^{-2n}}{2a}}{n}$$

$$\Big\downarrow \text{25}$$

$$\frac{-\frac{\int \frac{x^{-2n}(cx^n+b)}{bx^n+cx^{2n}+a}dx^n}{a}-\frac{x^{-2n}}{2a}}{n}$$

$$\Big\downarrow \text{1200}$$

$$\frac{-\frac{\int \left(\frac{bx^{-2n}}{a}+\frac{(ac-b^2)x^{-n}}{a^2}+\frac{c(b^2-ac)x^n+b(b^2-2ac)}{a^2(bx^n+cx^{2n}+a)}\right)dx^n}{a}-\frac{x^{-2n}}{2a}}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{-\frac{b(b^2-3ac)\text{arctanh}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}}-\frac{(b^2-ac)\log(x^n)}{a^2}+\frac{(b^2-ac)\log(a+bx^n+cx^{2n})}{2a^2}-\frac{bx^{-n}}{a}}{a}-\frac{x^{-2n}}{2a}}{n}$$

input
```
Int[x^(-1 - 2*n)/(a + b*x^n + c*x^(2*n)),x]
```

output
```
(-1/2*1/(a*x^(2*n)) - (-(b/(a*x^n)) - (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x^
n)/Sqrt[b^2 - 4*a*c]])/(a^2*Sqrt[b^2 - 4*a*c]) - ((b^2 - a*c)*Log[x^n])/a^
2 + ((b^2 - a*c)*Log[a + b*x^n + c*x^(2*n)])/(2*a^2))/a)/n
```

3.554.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1145
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp
[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[m, -1]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.554.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 957 vs. $2(120) = 240$.

Time = 0.29 (sec) , antiderivative size = 958, normalized size of antiderivative = 7.60

method	result
risch	$\frac{b\,x^{-n}}{a^{2} n}-\frac{x^{-2 n}}{2 a n}-\frac{4 n^{2} \ln \left(x \right) a^{2} c^{2}}{4 a^{4} c\,n^{2}-a^{3} b^{2} n^{2}}+\frac{5 n^{2} \ln \left(x \right) a\,b^{2} c}{4 a^{4} c\,n^{2}-a^{3} b^{2} n^{2}}-\frac{n^{2} \ln \left(x \right) b^{4}}{4 a^{4} c\,n^{2}-a^{3} b^{2} n^{2}}+\frac{2 \ln \left(x^{n}+\frac{3 a\,b^{2} c-b^{4}+\sqrt{-36 a^{3} b^{2} c^{3}+33 a^{2} b^{4} c^{2}-10 a}}{2 c b \left(3 a c-b^{2}\right)}\right)}{a \left(4 a c-b^{2}\right) n}$

input
```
int(x^(-1-2*n)/(a+b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

output
```
b/a^2/n/(x^n)-1/2/a/n/(x^n)^2-4/(4*a^4*c*n^2-a^3*b^2*n^2)*n^2*ln(x)*a^2*c^
2+5/(4*a^4*c*n^2-a^3*b^2*n^2)*n^2*ln(x)*a*b^2*c-1/(4*a^4*c*n^2-a^3*b^2*n^2
)*n^2*ln(x)*b^4+2/a/(4*a*c-b^2)/n*ln(x^n+1/2*(3*a*b^2*c-b^4+(-36*a^3*b^2*c
^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/c/b/(3*a*c-b^2))*c^2-5/2/a^2/(4*a
*c-b^2)/n*ln(x^n+1/2*(3*a*b^2*c-b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b
^6*c+b^8)^(1/2))/c/b/(3*a*c-b^2))*b^2*c+1/2/a^3/(4*a*c-b^2)/n*ln(x^n+1/2*(
3*a*b^2*c-b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/c/b/(
3*a*c-b^2))*b^4+1/2/a^3/(4*a*c-b^2)/n*ln(x^n+1/2*(3*a*b^2*c-b^4+(-36*a^3*b
^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/c/b/(3*a*c-b^2))*(-36*a^3*b^2
*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2)+2/a/(4*a*c-b^2)/n*ln(x^n-1/2*(-3
*a*b^2*c+b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/c/b/(3
*a*c-b^2))*c^2-5/2/a^2/(4*a*c-b^2)/n*ln(x^n-1/2*(-3*a*b^2*c+b^4+(-36*a^3*b
^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/c/b/(3*a*c-b^2))*b^2*c+1/2/a^
3/(4*a*c-b^2)/n*ln(x^n-1/2*(-3*a*b^2*c+b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2
-10*a*b^6*c+b^8)^(1/2))/c/b/(3*a*c-b^2))*b^4-1/2/a^3/(4*a*c-b^2)/n*ln(x^n-
1/2*(-3*a*b^2*c+b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))
/c/b/(3*a*c-b^2))*(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2)
```

3.554.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 429, normalized size of antiderivative = 3.40

$$\int \frac{x^{-1-2n}}{a+bx^n+cx^{2n}}\,dx$$

$$= \left[-\frac{a^2b^2 - 4\,a^3c - 2\,(b^4 - 5\,ab^2c + 4\,a^2c^2)nx^{2\,n}\log\left(x\right) + (b^3 - 3\,abc)\sqrt{b^2 - 4\,ac}x^{2\,n}\log\left(\frac{2\,c^2x^{2\,n} + b^2 - 2\,ac + 2\,(b \cdots}{cx^2 \cdots}\right. \cdots}{2\,(a^3b^2 - 4\,a^4c \cdots} \right.$$

$$- \frac{a^2b^2 - 4\,a^3c - 2\,(b^4 - 5\,ab^2c + 4\,a^2c^2)nx^{2\,n}\log\left(x\right) - 2\,(b^3 - 3\,abc)\sqrt{-b^2 + 4\,ac}x^{2\,n}\arctan\left(-\frac{2\,\sqrt{-b^2 + 4\,ac \cdots}}{b \cdots}\right. \cdots}{2\,(a^3b^2 - 4\,a^4c)nx^2 \cdots}$$

input
```
integrate(x^(-1-2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
[-1/2*(a^2*b^2 - 4*a^3*c - 2*(b^4 - 5*a*b^2*c + 4*a^2*c^2)*n*x^(2*n)*log(x
) + (b^3 - 3*a*b*c)*sqrt(b^2 - 4*a*c)*x^(2*n)*log((2*c^2*x^(2*n) + b^2 - 2
*a*c + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*x^n - sqrt(b^2 - 4*a*c)*b)/(c*x^(2*n)
 + b*x^n + a)) + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*x^(2*n)*log(c*x^(2*n) + b*x
^n + a) - 2*(a*b^3 - 4*a^2*b*c)*x^n)/((a^3*b^2 - 4*a^4*c)*n*x^(2*n)), -1/2
*(a^2*b^2 - 4*a^3*c - 2*(b^4 - 5*a*b^2*c + 4*a^2*c^2)*n*x^(2*n)*log(x) - 2
*(b^3 - 3*a*b*c)*sqrt(-b^2 + 4*a*c)*x^(2*n)*arctan(-(2*sqrt(-b^2 + 4*a*c)*
c*x^n + sqrt(-b^2 + 4*a*c)*b)/(b^2 - 4*a*c)) + (b^4 - 5*a*b^2*c + 4*a^2*c^
2)*x^(2*n)*log(c*x^(2*n) + b*x^n + a) - 2*(a*b^3 - 4*a^2*b*c)*x^n)/((a^3*b
^2 - 4*a^4*c)*n*x^(2*n))]
```

3.554.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1-2n}}{a+bx^n+cx^{2n}}\,dx = \text{Timed out}$$

input
```
integrate(x**(-1-2*n)/(a+b*x**n+c*x**(2*n)),x)
```

output
```
Timed out
```

3.554.7 Maxima [F]

$$\int \frac{x^{-1-2n}}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^{-2\,n-1}}{cx^{2\,n}+bx^n+a}\,dx$$

input
```
integrate(x^(-1-2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
1/2*(2*b*x^n - a)/(a^2*n*x^(2*n)) + integrate((b*c*x^n + b^2 - a*c)/(a^2*c
*x*x^(2*n) + a^2*b*x*x^n + a^3*x), x)
```

3.554.8 Giac [F]

$$\int \frac{x^{-1-2n}}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^{-2\,n-1}}{cx^{2\,n}+bx^n+a}\,dx$$

input
```
integrate(x^(-1-2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate(x^(-2*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

3.554.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-2n}}{a+bx^n+cx^{2n}}\,dx = \int \frac{1}{x^{2\,n+1}\,\left(a+b\,x^n+c\,x^{2\,n}\right)}\,dx$$

input
```
int(1/(x^(2*n + 1)*(a + b*x^n + c*x^(2*n))),x)
```

output
```
int(1/(x^(2*n + 1)*(a + b*x^n + c*x^(2*n))), x)
```

3.555 $\int \frac{x^{-1-3n}}{a+bx^n+cx^{2n}}\,dx$

3.555.1 Optimal result

Integrand size = 24, antiderivative size = 164

$$\int \frac{x^{-1-3n}}{a+bx^n+cx^{2n}}\,dx = -\frac{x^{-3n}}{3an}+\frac{bx^{-2n}}{2a^2n}-\frac{(b^2-ac)\,x^{-n}}{a^3n} - \frac{(b^4-4ab^2c+2a^2c^2)\,\text{arctanh}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a^4\sqrt{b^2-4ac}n} - \frac{b(b^2-2ac)\log(x)}{a^4}+\frac{b(b^2-2ac)\log\left(a+bx^n+cx^{2n}\right)}{2a^4n}$$

output
```
-1/3/a/n/(x^(3*n))+1/2*b/a^2/n/(x^(2*n))+(a*c-b^2)/a^3/n/(x^n)-b*(-2*a*c+b
^2)*ln(x)/a^4+1/2*b*(-2*a*c+b^2)*ln(a+b*x^n+c*x^(2*n))/a^4/n-(2*a^2*c^2-4*
a*b^2*c+b^4)*arctanh((b+2*c*x^n)/(-4*a*c+b^2)^(1/2))/a^4/n/(-4*a*c+b^2)^(1
/2)
```

3.555.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.90

$$\int \frac{x^{-1-3n}}{a+bx^n+cx^{2n}}\,dx = \frac{ax^{-3n}(-2a^2-6b^2x^{2n}+3ax^n(b+2cx^n))+\frac{6(b^4-4ab^2c+2a^2c^2)\arctan\left(\frac{b+2cx^n}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}-6(b^3-2abc)\log\left(x^n\right)+3(b^3}{6a^4n}$$

input
```
Integrate[x^(-1 - 3*n)/(a + b*x^n + c*x^(2*n)),x]
```

output
```
((a*(-2*a^2 - 6*b^2*x^(2*n) + 3*a*x^n*(b + 2*c*x^n)))/x^(3*n) + (6*(b^4 -
4*a*b^2*c + 2*a^2*c^2)*ArcTan[(b + 2*c*x^n)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2
 + 4*a*c] - 6*(b^3 - 2*a*b*c)*Log[x^n] + 3*(b^3 - 2*a*b*c)*Log[a + x^n*(b
+ c*x^n)])/(6*a^4*n)
```

3.555.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1693, 1145, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-3n-1}}{a+bx^n+cx^{2n}}dx$$

$$\Big\downarrow \text{ 1693}$$

$$\frac{\int \frac{x^{-4n}}{bx^n+cx^{2n}+a}dx^n}{n}$$

$$\Big\downarrow \text{ 1145}$$

$$\frac{\frac{\int -\frac{x^{-3n}(cx^n+b)}{bx^n+cx^{2n}+a}dx^n}{a}-\frac{x^{-3n}}{3a}}{n}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{-\frac{\int \frac{x^{-3n}(cx^n+b)}{bx^n+cx^{2n}+a}dx^n}{a}-\frac{x^{-3n}}{3a}}{n}$$

$$\Big\downarrow \text{ 1200}$$

$$\frac{-\frac{\int \left(\frac{bx^{-3n}}{a}+\frac{(ac-b^2)x^{-2n}}{a^2}+\frac{(b^3-2abc)x^{-n}}{a^3}+\frac{-bc(b^2-2ac)x^n-b^4-a^2c^2+3ab^2c}{a^3(bx^n+cx^{2n}+a)}\right)dx^n}{a}-\frac{x^{-3n}}{3a}}{n}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{\frac{b(b^2-2ac)\log(x^n)}{a^3}-\frac{b(b^2-2ac)\log(a+bx^n+cx^{2n})}{2a^3}+\frac{x^{-n}(b^2-ac)}{a^2}+\frac{(2a^2c^2-4ab^2c+b^4)\text{arctanh}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}}-\frac{bx^{-2n}}{2a}}{a}-\frac{x^{-3n}}{3a}}{n}$$

input
```
Int[x^(-1 - 3*n)/(a + b*x^n + c*x^(2*n)),x]
```

output
```
(-1/3*1/(a*x^(3*n)) - (-1/2*b/(a*x^(2*n)) + (b^2 - a*c)/(a^2*x^n) + ((b^4
- 4*a*b^2*c + 2*a^2*c^2)*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/(a^3*Sq
rt[b^2 - 4*a*c]) + (b*(b^2 - 2*a*c)*Log[x^n])/a^3 - (b*(b^2 - 2*a*c)*Log[a
 + b*x^n + c*x^(2*n)])/(2*a^3))/a)/n
```

3.555.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1145
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp
[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[m, -1]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.555.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1299 vs. 2(158) = 316.

Time = 0.37 (sec) , antiderivative size = 1300, normalized size of antiderivative = 7.93

method	result	size
risch	Expression too large to display	1300

input
```
int(x^(-1-3*n)/(a+b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

output
```
1/a^2/n/(x^n)*c-1/a^3/n/(x^n)*b^2+1/2*b/a^2/n/(x^n)^2-1/3/a/n/(x^n)^3+8/(4
*a^5*c*n^2-a^4*b^2*n^2)*n^2*ln(x)*a^2*b*c^2-6/(4*a^5*c*n^2-a^4*b^2*n^2)*n^
2*ln(x)*a*b^3*c+1/(4*a^5*c*n^2-a^4*b^2*n^2)*n^2*ln(x)*b^5-4/a^2/(4*a*c-b^2
)/n*ln(x^n+1/2*(2*a^2*b*c^2-4*a*b^3*c+b^5+(-16*a^5*c^5+68*a^4*b^2*c^4-96*a
^3*b^4*c^3+52*a^2*b^6*c^2-12*a*b^8*c+b^10)^(1/2))/c/(2*a^2*c^2-4*a*b^2*c+b
^4))*b*c^2+3/a^3/(4*a*c-b^2)/n*ln(x^n+1/2*(2*a^2*b*c^2-4*a*b^3*c+b^5+(-16*
a^5*c^5+68*a^4*b^2*c^4-96*a^3*b^4*c^3+52*a^2*b^6*c^2-12*a*b^8*c+b^10)^(1/2
))/c/(2*a^2*c^2-4*a*b^2*c+b^4))*b^3*c-1/2/a^4/(4*a*c-b^2)/n*ln(x^n+1/2*(2*
a^2*b*c^2-4*a*b^3*c+b^5+(-16*a^5*c^5+68*a^4*b^2*c^4-96*a^3*b^4*c^3+52*a^2*
b^6*c^2-12*a*b^8*c+b^10)^(1/2))/c/(2*a^2*c^2-4*a*b^2*c+b^4))*b^5+1/2/a^4/(
4*a*c-b^2)/n*ln(x^n+1/2*(2*a^2*b*c^2-4*a*b^3*c+b^5+(-16*a^5*c^5+68*a^4*b^2
*c^4-96*a^3*b^4*c^3+52*a^2*b^6*c^2-12*a*b^8*c+b^10)^(1/2))/c/(2*a^2*c^2-4*
a*b^2*c+b^4))*(-16*a^5*c^5+68*a^4*b^2*c^4-96*a^3*b^4*c^3+52*a^2*b^6*c^2-12
*a*b^8*c+b^10)^(1/2)-4/a^2/(4*a*c-b^2)/n*ln(x^n-1/2*(-2*a^2*b*c^2+4*a*b^3*
c-b^5+(-16*a^5*c^5+68*a^4*b^2*c^4-96*a^3*b^4*c^3+52*a^2*b^6*c^2-12*a*b^8*c
+b^10)^(1/2))/c/(2*a^2*c^2-4*a*b^2*c+b^4))*b*c^2+3/a^3/(4*a*c-b^2)/n*ln(x^
n-1/2*(-2*a^2*b*c^2+4*a*b^3*c-b^5+(-16*a^5*c^5+68*a^4*b^2*c^4-96*a^3*b^4*c
^3+52*a^2*b^6*c^2-12*a*b^8*c+b^10)^(1/2))/c/(2*a^2*c^2-4*a*b^2*c+b^4))*b^3
*c-1/2/a^4/(4*a*c-b^2)/n*ln(x^n-1/2*(-2*a^2*b*c^2+4*a*b^3*c-b^5+(-16*a^5*c
^5+68*a^4*b^2*c^4-96*a^3*b^4*c^3+52*a^2*b^6*c^2-12*a*b^8*c+b^10)^(1/2))...
```

3.555.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 522, normalized size of antiderivative = 3.18

$$\int \frac{x^{-1-3n}}{a+bx^n+cx^{2n}}\,dx$$

$$= \left[-\frac{2\,a^3b^2 - 8\,a^4c + 6\,(b^5 - 6\,ab^3c + 8\,a^2bc^2)nx^{3\,n}\log{(x)} - 3\,(b^4 - 4\,ab^2c + 2\,a^2c^2)\sqrt{b^2 - 4\,ac}x^{3\,n}\log\left(\frac{2\,c^2x}{}\right.}{}\right.$$

$$-\frac{2\,a^3b^2 - 8\,a^4c + 6\,(b^5 - 6\,ab^3c + 8\,a^2bc^2)nx^{3\,n}\log{(x)} + 6\,(b^4 - 4\,ab^2c + 2\,a^2c^2)\sqrt{-b^2 + 4\,ac}x^{3\,n}\arctan\left(\right.}{}$$

input
```
integrate(x^(-1-3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
[-1/6*(2*a^3*b^2 - 8*a^4*c + 6*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*n*x^(3*n)*l
og(x) - 3*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*sqrt(b^2 - 4*a*c)*x^(3*n)*log((2*c
^2*x^(2*n) + b^2 - 2*a*c + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*x^n - sqrt(b^2 -
4*a*c)*b)/(c*x^(2*n) + b*x^n + a)) - 3*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*x^(
3*n)*log(c*x^(2*n) + b*x^n + a) + 6*(a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*x^(2
*n) - 3*(a^2*b^3 - 4*a^3*b*c)*x^n)/((a^4*b^2 - 4*a^5*c)*n*x^(3*n)), -1/6*(
2*a^3*b^2 - 8*a^4*c + 6*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*n*x^(3*n)*log(x) +
 6*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*sqrt(-b^2 + 4*a*c)*x^(3*n)*arctan(-(2*sqr
t(-b^2 + 4*a*c)*c*x^n + sqrt(-b^2 + 4*a*c)*b)/(b^2 - 4*a*c)) - 3*(b^5 - 6*
a*b^3*c + 8*a^2*b*c^2)*x^(3*n)*log(c*x^(2*n) + b*x^n + a) + 6*(a*b^4 - 5*a
^2*b^2*c + 4*a^3*c^2)*x^(2*n) - 3*(a^2*b^3 - 4*a^3*b*c)*x^n)/((a^4*b^2 - 4
*a^5*c)*n*x^(3*n))]
```

3.555.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1-3n}}{a+bx^n+cx^{2n}}\,dx = \text{Timed out}$$

input
```
integrate(x**(-1-3*n)/(a+b*x**n+c*x**(2*n)),x)
```

output
```
Timed out
```

3.555.7 Maxima [F]

$$\int \frac{x^{-1-3n}}{a+bx^n+cx^{2n}}\,dx=\int \frac{x^{-3\,n-1}}{cx^{2\,n}+bx^n+a}\,dx$$

input
```
integrate(x^(-1-3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
1/6*(3*a*b*x^n - 2*a^2 - 6*(b^2 - a*c)*x^(2*n))/(a^3*n*x^(3*n)) + integrat
e(-(b^3 - 2*a*b*c + (b^2*c - a*c^2)*x^n)/(a^3*c*x*x^(2*n) + a^3*b*x*x^n +
a^4*x), x)
```

3.555.8 Giac [F]

$$\int \frac{x^{-1-3n}}{a+bx^n+cx^{2n}}\,dx=\int \frac{x^{-3\,n-1}}{cx^{2\,n}+bx^n+a}\,dx$$

input
```
integrate(x^(-1-3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate(x^(-3*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

3.555.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-3n}}{a+bx^n+cx^{2n}}\,dx=\int \frac{1}{x^{3\,n+1}\,\left(a+b\,x^n+c\,x^{2\,n}\right)}\,dx$$

input
```
int(1/(x^(3*n + 1)*(a + b*x^n + c*x^(2*n))),x)
```

output
```
int(1/(x^(3*n + 1)*(a + b*x^n + c*x^(2*n))), x)
```

3.556 $\int \frac{x^{-1+\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx$

3.556.1 Optimal result

Integrand size = 26, antiderivative size = 353

$$\int \frac{x^{-1+\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx = \frac{2\,2^{3/4}c^{3/4}\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}n} - \frac{2\,2^{3/4}c^{3/4}\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}n} + \frac{2\,2^{3/4}c^{3/4}\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}n} - \frac{2\,2^{3/4}c^{3/4}\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}n}$$

output
```
2*2^(3/4)*c^(3/4)*arctan(2^(1/4)*c^(1/4)*x^(1/4*n)/(-b-(-4*a*c+b^2)^(1/2))
^(1/4))/n/(-b-(-4*a*c+b^2)^(1/2))^(3/4)/(-4*a*c+b^2)^(1/2)+2*2^(3/4)*c^(3/
4)*arctanh(2^(1/4)*c^(1/4)*x^(1/4*n)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))/n/(-b-
(-4*a*c+b^2)^(1/2))^(3/4)/(-4*a*c+b^2)^(1/2)-2*2^(3/4)*c^(3/4)*arctan(2^(1
/4)*c^(1/4)*x^(1/4*n)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))/n/(-4*a*c+b^2)^(1/2)/
(-b+(-4*a*c+b^2)^(1/2))^(3/4)-2*2^(3/4)*c^(3/4)*arctanh(2^(1/4)*c^(1/4)*x^
(1/4*n)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))/n/(-4*a*c+b^2)^(1/2)/(-b+(-4*a*c+b^
2)^(1/2))^(3/4)
```

3.556.2 Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 340, normalized size of antiderivative = 0.96

$$\int \frac{x^{-1+\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx = \frac{2\,2^{3/4}c^{3/4}\left(-\frac{\sqrt[4]{-b-\sqrt{b^2-4ac}}\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{b^2-4ac+b\sqrt{b^2-4ac}}-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}-\frac{\sqrt[4]{-b-\sqrt{b^2-4ac}}\cdots}{\cdots}\cdots\right)}{n}$$

input
```
Integrate[x^(-1 + n/4)/(a + b*x^n + c*x^(2*n)),x]
```

output
```
(2*2^(3/4)*c^(3/4)*(-(((-b - Sqrt[b^2 - 4*a*c])^(1/4)*ArcTan[(2^(1/4)*c^(1
/4)*x^(n/4))/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(b^2 - 4*a*c + b*Sqrt[b^2 -
4*a*c])) - ArcTan[(2^(1/4)*c^(1/4)*x^(n/4))/(-b + Sqrt[b^2 - 4*a*c])^(1/4)
]/(Sqrt[b^2 - 4*a*c]*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) - ((-b - Sqrt[b^2 - 4
*a*c])^(1/4)*ArcTanh[(2^(1/4)*c^(1/4)*x^(n/4))/(-b - Sqrt[b^2 - 4*a*c])^(1
/4)])/(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c]) - ArcTanh[(2^(1/4)*c^(1/4)*x^(n/
4))/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(Sqrt[b^2 - 4*a*c]*(-b + Sqrt[b^2 - 4*
a*c])^(3/4))))/n
```

3.556.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 329, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1717, 1685, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{\frac{n}{4}-1}}{a+bx^n+cx^{2n}}\,dx$$

$$\Big\downarrow \text{1717}$$

$$\frac{4\int \frac{1}{bx^n+cx^{2n}+a}dx^{n/4}}{n}$$

$$\Big\downarrow \text{1685}$$

$$\frac{4\left(\frac{c\int \frac{1}{cx^n+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx^{n/4}}{\sqrt{b^2-4ac}}-\frac{c\int \frac{1}{cx^n+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx^{n/4}}{\sqrt{b^2-4ac}}\right)}{n}$$

$$\Big\downarrow \text{756}$$

$$\frac{4\left(\frac{c\left(-\frac{\int \frac{1}{\sqrt{\sqrt{b^2-4ac}-b-\sqrt{2}\sqrt{c}x^{n/2}}}dx^{n/4}}{\sqrt{\sqrt{b^2-4ac}-b}}-\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x^{n/2}+\sqrt{\sqrt{b^2-4ac}-b}}dx^{n/4}}{\sqrt{\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}}-\frac{c\left(-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}-\sqrt{2}\sqrt{c}x^{n/2}}}dx^{n/4}}{\sqrt{-\sqrt{b^2-4ac}-b}}-\cdots\right.}{\sqrt{b^2-4ac}}\cdots}{n}$$

$$\Big\downarrow \text{218}$$

$$\frac{4\left(\frac{c\left(-\frac{\int \frac{1}{\sqrt{\sqrt{b^2-4ac}-b-\sqrt{2}\sqrt{c}x^{n/2}}}dx^{n/4}}{\sqrt{\sqrt{b^2-4ac}-b}}-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}\right)}{\sqrt{b^2-4ac}}-\frac{c\left(-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}-\sqrt{2}\sqrt{c}x^{n/2}}}dx^{n/4}}{\sqrt{-\sqrt{b^2-4ac}-b}}-\cdots\right.}{\sqrt{b^2-4ac}}\cdots}{n}$$

$$\Big\downarrow 221$$

$$\frac{4\left(\frac{c\left(-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}\right)}{\sqrt{b^2-4ac}}-\frac{c\left(-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{-\cdots}}\right)}{\sqrt[4]{2}\sqrt[4]{c}(-\cdots}\right.}{\sqrt{b^2-4ac}}\right.}{n}$$

input
```
Int[x^(-1 + n/4)/(a + b*x^n + c*x^(2*n)),x]
```

output
```
(4*(-((c*(-(ArcTan[(2^(1/4)*c^(1/4)*x^(n/4))/(-b - Sqrt[b^2 - 4*a*c])^(1/4
)]/(2^(1/4)*c^(1/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4))) - ArcTanh[(2^(1/4)*c^
(1/4)*x^(n/4))/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b - Sqrt
[b^2 - 4*a*c])^(3/4))))/Sqrt[b^2 - 4*a*c]) + (c*(-(ArcTan[(2^(1/4)*c^(1/4)
*x^(n/4))/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b + Sqrt[b^2
- 4*a*c])^(3/4))) - ArcTanh[(2^(1/4)*c^(1/4)*x^(n/4))/(-b + Sqrt[b^2 - 4*a
*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))))/Sqrt[b^2 -
4*a*c]))/n
```

3.556.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.556. $\int \frac{x^{-1+\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx$

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 1685
```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^n), x], x] - Simp[
c/q   Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2,
2*n] && NeQ[b^2 - 4*a*c, 0]
```

rule 1717
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Simp[1/(m + 1)   Subst[Int[(a + b*x^Simplify[n/(m + 1)] + c*x^Simplify[
2*(n/(m + 1))])^p, x], x, x^(m + 1)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&
 EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[n/(m + 1)]] &&  !
IntegerQ[n]
```

3.556.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.04 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.79

method	result
risch	$\sum_{\_R=\text{RootOf}\left(\left(256a^7c^4n^8-256a^6b^2c^3n^8+96a^5b^4c^2n^8-16a^4b^6c\,n^8+a^3b^8n^8\right)\_Z^8+\left(-48a^3b\,c^3n^4+40a^2b^3c^2n^4-11a\,b^5c\,n^4+b^7n^4\right)\_Z\right.}$

input
```
int(x^(-1+1/4*n)/(a+b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

output
```
sum(_R*ln(x^(1/4*n)+(16/(a*c^2-b^2*c)*n^5*b*a^5*c^2-8/(a*c^2-b^2*c)*n^5*b^
3*a^4*c+1/(a*c^2-b^2*c)*n^5*b^5*a^3)*_R^5+(2/(a*c^2-b^2*c)*n*a^2*c^2-4/(a*
c^2-b^2*c)*n*b^2*a*c+1/(a*c^2-b^2*c)*n*b^4)*_R),_R=RootOf((256*a^7*c^4*n^8
-256*a^6*b^2*c^3*n^8+96*a^5*b^4*c^2*n^8-16*a^4*b^6*c*n^8+a^3*b^8*n^8)*_Z^8
+(-48*a^3*b*c^3*n^4+40*a^2*b^3*c^2*n^4-11*a*b^5*c*n^4+b^7*n^4)*_Z^4+c^3))
```

3.556.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3481 vs. $2(273) = 546$.

Time = 0.40 (sec) , antiderivative size = 3481, normalized size of antiderivative = 9.86

$$\int \frac{x^{-1+\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx = \text{Too large to display}$$

input
```
integrate(x^(-1+1/4*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*sqrt(sqrt(2)*sqrt(-((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4*s
qrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2
- 64*a^9*c^3)*n^8)) + b^3 - 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)
*n^4))*log(-(4*(b^2*c - a*c^2)*x*x^(1/4*n - 1) + sqrt(2)*((a^3*b^5 - 8*a^
4*b^3*c + 16*a^5*b*c^2)*n^5*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 1
2*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) - (b^4 - 5*a*b^2*c + 4*a^
2*c^2)*n)*sqrt(sqrt(2)*sqrt(-((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4*sqr
t((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 -
64*a^9*c^3)*n^8)) + b^3 - 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n
^4))))/x) - 1/2*sqrt(2)*sqrt(sqrt(2)*sqrt(-((a^3*b^4 - 8*a^4*b^2*c + 16*a^
5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*
a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) + b^3 - 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2*c +
 16*a^5*c^2)*n^4)))*log(-(4*(b^2*c - a*c^2)*x*x^(1/4*n - 1) - sqrt(2)*((a^
3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*n^5*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(
(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) - (b^4 - 5*a*
b^2*c + 4*a^2*c^2)*n)*sqrt(sqrt(2)*sqrt(-((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*
c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^
8*b^2*c^2 - 64*a^9*c^3)*n^8)) + b^3 - 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2*c + 1
6*a^5*c^2)*n^4))))/x) + 1/2*sqrt(2)*sqrt(-sqrt(2)*sqrt(-((a^3*b^4 - 8*a^4*
b^2*c + 16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12...
```

3.556.6 Sympy [F]

$$\int \frac{x^{-1+\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^{\frac{n}{4}-1}}{a+bx^n+cx^{2n}}\,dx$$

input
```
integrate(x**(-1+1/4*n)/(a+b*x**n+c*x**(2*n)),x)
```

output
```
Integral(x**(n/4 - 1)/(a + b*x**n + c*x**(2*n)), x)
```

3.556.7 Maxima [F]

$$\int \frac{x^{-1+\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^{\frac{1}{4}\,n-1}}{cx^{2\,n}+bx^n+a}\,dx$$

input
```
integrate(x^(-1+1/4*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
integrate(x^(1/4*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

3.556.8 Giac [F]

$$\int \frac{x^{-1+\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^{\frac{1}{4}\,n-1}}{cx^{2\,n}+bx^n+a}\,dx$$

input
```
integrate(x^(-1+1/4*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate(x^(1/4*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

3.556.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^{\frac{n}{4}-1}}{a+b\,x^n+c\,x^{2\,n}}\,dx$$

input
```
int(x^(n/4 - 1)/(a + b*x^n + c*x^(2*n)),x)
```

output
```
int(x^(n/4 - 1)/(a + b*x^n + c*x^(2*n)), x)
```

3.557 $\int \frac{x^{-1+\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx$

3.557.1 Optimal result

Integrand size = 26, antiderivative size = 610

$$\int \frac{x^{-1+\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx = -\frac{2^{2/3}\sqrt{3}c^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x^{n/3}}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}n} + \frac{2^{2/3}\sqrt{3}c^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x^{n/3}}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)^{2/3}n} + \frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x^{n/3}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}n} - \frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x^{n/3}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)^{2/3}n} - \frac{c^{2/3}\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x^{n/3}+2^{2/3}c^{2/3}x^{2n/3}\right)}{\sqrt[3]{2}\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}n} + \frac{c^{2/3}\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x^{n/3}+2^{2/3}c^{2/3}x^{2n/3}\right)}{\sqrt[3]{2}\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)^{2/3}n}$$

output
```
2^(2/3)*c^(2/3)*ln(2^(1/3)*c^(1/3)*x^(1/3*n)+(b-(-4*a*c+b^2)^(1/2))^(1/3))
/n/(b-(-4*a*c+b^2)^(1/2))^(2/3)/(-4*a*c+b^2)^(1/2)-1/2*c^(2/3)*ln(2^(2/3)*
c^(2/3)*x^(2/3*n)-2^(1/3)*c^(1/3)*x^(1/3*n)*(b-(-4*a*c+b^2)^(1/2))^(1/3)+(
b-(-4*a*c+b^2)^(1/2))^(2/3))*2^(2/3)/n/(b-(-4*a*c+b^2)^(1/2))^(2/3)/(-4*a*
c+b^2)^(1/2)-2^(2/3)*c^(2/3)*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x^(1/3*n)/(b-
(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*3^(1/2)/n/(b-(-4*a*c+b^2)^(1/2))^(2/3)
/(-4*a*c+b^2)^(1/2)-2^(2/3)*c^(2/3)*ln(2^(1/3)*c^(1/3)*x^(1/3*n)+(b+(-4*a*
c+b^2)^(1/2))^(1/3))/n/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(2/3)+1/2
*c^(2/3)*ln(2^(2/3)*c^(2/3)*x^(2/3*n)-2^(1/3)*c^(1/3)*x^(1/3*n)*(b+(-4*a*c
+b^2)^(1/2))^(1/3)+(b+(-4*a*c+b^2)^(1/2))^(2/3))*2^(2/3)/n/(-4*a*c+b^2)^(1
/2)/(b+(-4*a*c+b^2)^(1/2))^(2/3)+2^(2/3)*c^(2/3)*arctan(1/3*(1-2*2^(1/3)*c
^(1/3)*x^(1/3*n)/(b+(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*3^(1/2)/n/(-4*a*c+
b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(2/3)
```

3.557.2 Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 526, normalized size of antiderivative = 0.86

$$\int \frac{x^{-1+\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx$$

$$= \frac{c^{2/3}\left(-2\sqrt{3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x^{n/3}}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)+2\sqrt{3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}\arctan\left(\frac{1-\frac{2\cdots}{\sqrt[3]{b\cdots}}}{\cdots}\right.\cdots}{\cdots}$$

input
```
Integrate[x^(-1 + n/3)/(a + b*x^n + c*x^(2*n)),x]
```

output
```
(c^(2/3)*(-2*Sqrt[3]*(b + Sqrt[b^2 - 4*a*c])^(2/3)*ArcTan[(1 - (2*2^(1/3)*
c^(1/3)*x^(n/3))/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]] + 2*Sqrt[3]*(b -
Sqrt[b^2 - 4*a*c])^(2/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x^(n/3))/(b + Sqrt
[b^2 - 4*a*c])^(1/3))/Sqrt[3]] + 2*(b + Sqrt[b^2 - 4*a*c])^(2/3)*Log[(b -
Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x^(n/3)] - 2*(b - Sqrt[b^2 - 4*
a*c])^(2/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x^(n/3)] -
 (b + Sqrt[b^2 - 4*a*c])^(2/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)
*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x^(n/3) + 2^(2/3)*c^(2/3)*x^((2*n)/
3)] + (b - Sqrt[b^2 - 4*a*c])^(2/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^
(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x^(n/3) + 2^(2/3)*c^(2/3)*x^((
2*n)/3)]))/(2^(1/3)*Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])^(2/3)*(b + S
qrt[b^2 - 4*a*c])^(2/3)*n)
```

3.557.3 Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 533, normalized size of antiderivative = 0.87, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {1717, 1685, 750, 16, 27, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{\frac{n}{3}-1}}{a+bx^n+cx^{2n}}dx$$

$$\Big\downarrow \text{ 1717}$$

$$\frac{3\int \frac{1}{bx^n+cx^{2n}+a}dx^{n/3}}{n}$$

$$\Big\downarrow \text{ 1685}$$

$$\frac{3\left(\frac{c\int \frac{1}{cx^n+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx^{n/3}}{\sqrt{b^2-4ac}}-\frac{c\int \frac{1}{cx^n+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx^{n/3}}{\sqrt{b^2-4ac}}\right)}{n}$$

$$\Big\downarrow \text{ 750}$$

$$3\left(\frac{c\left(\frac{2^{2/3}\int \frac{2\left(2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}x^{n/3}\right)}{-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x^{n/3}+2c^{2/3}x^{2n/3}+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx^{n/3}}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\int \frac{1}{\sqrt[3]{c}x^{n/3}+\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}}dx^{n/3}}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right)}{\sqrt{b^2-4ac}} -\right.$$

$$n$$

$\downarrow$ 16

$$3\left(\frac{c\left(\frac{2^{2/3}\int \frac{2\left(2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}x^{n/3}\right)}{-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x^{n/3}+2c^{2/3}x^{2n/3}+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx^{n/3}}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x^{n/3}\right)}{3\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right)}{\sqrt{b^2-4ac}}\right.$$

$\downarrow$ 27

3.557. $\int \frac{x^{-1+\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx$

$$3\left(\frac{c\left(\frac{2\, 2^{2/3}\int \frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}x^{n/3}}{-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x^{n/3}+2c^{2/3}x^{2n/3}+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx^{n/3}}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x^{n/3}\right)}{3\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right)}{\sqrt{b^2-4ac}}\right.$$

$$\Big\downarrow \text{1142}$$

$$3\left(\frac{c\left(\frac{2\, 2^{2/3}\left(\frac{3\sqrt[3]{b-\sqrt{b^2-4ac}}\int \frac{1}{-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x^{n/3}+2c^{2/3}x^{2n/3}+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx^{n/3}}{2\sqrt[3]{2}}-\int -\frac{\sqrt[3]{c}\left(2\ldots\right.}{-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\ldots}}\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right.}{\sqrt{b^2-4ac}}\right.$$

3.557. $\int \frac{x^{-1+\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx$

$\downarrow$ 25

$$3\left(\frac{c\left(\frac{2\,2^{2/3}\left(\frac{3\sqrt[3]{b-\sqrt{b^2-4ac}}\int\frac{1}{-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x^{n/3}+2c^{2/3}x^{2n/3}+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx^{n/3}}{2\sqrt[3]{2}}+\frac{\int\frac{\sqrt[3]{c}\left(2^{2/\cdots}\right.}{-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{\cdots}}}}{\cdots}\right)}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\cdots\right)}{\sqrt{b^2-4ac}}\cdots\right)$$

$\downarrow$ 27

3.557. $\int \frac{x^{-1+\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx$

$$3\left(\frac{c\left(\frac{2\ 2^{2/3}\left(\frac{3\sqrt[3]{b-\sqrt{b^2-4ac}}\int \frac{1}{-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x^{n/3}+2c^{2/3}x^{2n/3}+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx^{n/3}}{2\sqrt[3]{2}}+\frac{1}{4}\int \frac{2^{2/3}\cdots}{-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{\cdots}}\cdots}\cdots\right)}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\cdots\right)}{\sqrt{b^2-4ac}\cdots}\cdots\right)$$

$\downarrow$ 1082

3.557. $\int \frac{x^{-1+\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx$

$$3\left(\frac{c\left(\frac{2\,2^{2/3}\left(\frac14\int\frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-4\sqrt[3]{c}x^{n/3}}{-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x^{n/3}+2c^{2/3}x^{2n/3}+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx^{n/3}+\frac{3\int\frac{1}{-x^{2n/3}-3}d\left(1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x^{n/3}}{\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[3]{c}}\right)}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\cdots\right)}{\sqrt{b^2-4ac}}\cdots\right)$$

$\downarrow$ 217

3.557. $\int \frac{x^{-1+\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx$

$$3\left(\frac{c\left(\frac{2\,2^{2/3}\left(\frac{1}{4}\int\frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-4\sqrt[3]{c}x^{n/3}}{-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x^{n/3}+2c^{2/3}x^{2n/3}+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx^{n/3}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x^{n/3}}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2\sqrt[3]{c}}\right)}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\cdots}{\cdots}\right.}{\sqrt{b^2-4ac}}\right.$$

$\downarrow$ 1103

3.557. $\int\frac{x^{-1+\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx$

$$3\left(\frac{c\left(\frac{2\ 2^{2/3}\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x^{n/3}}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2\sqrt[3]{c}}-\frac{\log\left(-\sqrt[3]{2}\sqrt[3]{c}x^{n/3}\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^{2n/3}\right)}{4\sqrt[3]{c}}\right)}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\log \ldots}{\ldots}\right.}{\sqrt{b^2-4ac}}\right.$$

input
```
Int[x^(-1 + n/3)/(a + b*x^n + c*x^(2*n)),x]
```

3.557. $\int \frac{x^{-1+\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx$

output
```
(3*((c*((2^(2/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x^(n/
3)])/(3*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + (2*2^(2/3)*(-1/2*(Sqrt[3]
*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x^(n/3))/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sq
rt[3]])/c^(1/3) - Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b -
 Sqrt[b^2 - 4*a*c])^(1/3)*x^(n/3) + 2^(2/3)*c^(2/3)*x^((2*n)/3)]/(4*c^(1/3
))))/(3*(b - Sqrt[b^2 - 4*a*c])^(2/3))))/Sqrt[b^2 - 4*a*c] - (c*((2^(2/3)*
Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x^(n/3)])/(3*c^(1/3)*(
b + Sqrt[b^2 - 4*a*c])^(2/3)) + (2*2^(2/3)*(-1/2*(Sqrt[3]*ArcTan[(1 - (2*2
^(1/3)*c^(1/3)*x^(n/3))/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/c^(1/3) -
 Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c
])^(1/3)*x^(n/3) + 2^(2/3)*c^(2/3)*x^((2*n)/3)]/(4*c^(1/3))))/(3*(b + Sqrt
[b^2 - 4*a*c])^(2/3))))/Sqrt[b^2 - 4*a*c]))/n
```

3.557.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.557. $\int \frac{x^{-1+\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx$

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1685
```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^n), x], x] - Simp[
c/q   Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2,
2*n] && NeQ[b^2 - 4*a*c, 0]
```

rule 1717
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Simp[1/(m + 1)   Subst[Int[(a + b*x^Simplify[n/(m + 1)] + c*x^Simplify[
2*(n/(m + 1))])^p, x], x, x^(m + 1)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&
 EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[n/(m + 1)]] &&  !
IntegerQ[n]
```

3.557.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.70 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.43

method	result
risch	$\sum_{\_R=\operatorname{RootOf}\left(\left(64a^5c^3n^6-48a^4b^2c^2n^6+12a^3b^4c\,n^6-a^2b^6n^6\right)\_Z^6+\left(16a^2b\,c^2n^3-8a\,b^3c\,n^3+b^5n^3\right)\_Z^3+c^2\right)} \_R\ln\left(x^{\frac{n}{3}}+\left(-\frac{16}{2}\right.\right.$

input
```
int(x^(-1+1/3*n)/(a+b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

output
```
sum(_R*ln(x^(1/3*n)+(-16/(2*a*c^2-b^2*c)*n^4*b*a^4*c^2+8/(2*a*c^2-b^2*c)*n
^4*b^3*a^3*c-1/(2*a*c^2-b^2*c)*n^4*b^5*a^2)*_R^4+(4/(2*a*c^2-b^2*c)*n*a^2*
c^2-5/(2*a*c^2-b^2*c)*n*b^2*a*c+1/(2*a*c^2-b^2*c)*n*b^4)*_R),_R=RootOf((64
*a^5*c^3*n^6-48*a^4*b^2*c^2*n^6+12*a^3*b^4*c*n^6-a^2*b^6*n^6)*_Z^6+(16*a^2
*b*c^2*n^3-8*a*b^3*c*n^3+b^5*n^3)*_Z^3+c^2))
```

3.557.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2461 vs. $2(465) = 930$.

Time = 0.35 (sec) , antiderivative size = 2461, normalized size of antiderivative = 4.03

$$\int \frac{x^{-1+\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx = \text{Too large to display}$$

input
```
integrate(x^(-1+1/3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
-1/2*(1/2)^(1/3)*(sqrt(-3) + 1)*(((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*
b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)
*n^6)) + b)/((a^2*b^2 - 4*a^3*c)*n^3))^(1/3)*log(-(4*(b^2*c - 2*a*c^2)*x*x
^(1/3*n - 1) + (1/2)^(1/3)*(sqrt(-3)*(b^4 - 6*a*b^2*c + 8*a^2*c^2)*n + (b^
4 - 6*a*b^2*c + 8*a^2*c^2)*n - (sqrt(-3)*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b
*c^2)*n^4 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*n^4)*sqrt((b^4 - 4*a*b^
2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n
^6)))*(((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b
^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) + b)/((a^2*b^2 - 4*
a^3*c)*n^3))^(1/3))/x) + 1/2*(1/2)^(1/3)*(sqrt(-3) - 1)*(((a^2*b^2 - 4*a^3
*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a
^6*b^2*c^2 - 64*a^7*c^3)*n^6)) + b)/((a^2*b^2 - 4*a^3*c)*n^3))^(1/3)*log(-
(4*(b^2*c - 2*a*c^2)*x*x^(1/3*n - 1) - (1/2)^(1/3)*(sqrt(-3)*(b^4 - 6*a*b^
2*c + 8*a^2*c^2)*n - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*n - (sqrt(-3)*(a^2*b^5
- 8*a^3*b^3*c + 16*a^4*b*c^2)*n^4 - (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)
*n^4)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6
*b^2*c^2 - 64*a^7*c^3)*n^6)))*(((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^
2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n
^6)) + b)/((a^2*b^2 - 4*a^3*c)*n^3))^(1/3))/x) - 1/2*(1/2)^(1/3)*(sqrt(-3)
 + 1)*(-((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a...
```

3.557.6 Sympy [F]

$$\int \frac{x^{-1+\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^{\frac{n}{3}-1}}{a+bx^n+cx^{2n}}\,dx$$

input
```
integrate(x**(-1+1/3*n)/(a+b*x**n+c*x**(2*n)),x)
```

output
```
Integral(x**(n/3 - 1)/(a + b*x**n + c*x**(2*n)), x)
```

3.557.7 Maxima [F]

$$\int \frac{x^{-1+\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx=\int \frac{x^{\frac{1}{3}\,n-1}}{cx^{2\,n}+bx^n+a}\,dx$$

input
```
integrate(x^(-1+1/3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
integrate(x^(1/3*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

3.557.8 Giac [F]

$$\int \frac{x^{-1+\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx=\int \frac{x^{\frac{1}{3}\,n-1}}{cx^{2\,n}+bx^n+a}\,dx$$

input
```
integrate(x^(-1+1/3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate(x^(1/3*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

3.557.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx=\int \frac{x^{\frac{n}{3}-1}}{a+b\,x^n+c\,x^{2\,n}}\,dx$$

input
```
int(x^(n/3 - 1)/(a + b*x^n + c*x^(2*n)),x)
```

output
```
int(x^(n/3 - 1)/(a + b*x^n + c*x^(2*n)), x)
```

3.558 $\int \frac{x^{-1+\frac{n}{2}}}{a+bx^n+cx^{2n}}\,dx$

3.558.1 Optimal result

Integrand size = 26, antiderivative size = 169

$$\int \frac{x^{-1+\frac{n}{2}}}{a+bx^n+cx^{2n}}\,dx=\frac{2\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x^{n/2}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}n}-\frac{2\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x^{n/2}}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}n}$$

output
```
2*arctan(x^(1/2*n)*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)*c
^(1/2)/n/(-4*a*c+b^2)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-2*arctan(x^(1/2*n
)*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)*c^(1/2)/n/(-4*a*c+
b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.558.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.86

$$\int \frac{x^{-1+\frac{n}{2}}}{a+bx^n+cx^{2n}}\,dx=\frac{2\sqrt{2}\sqrt{c}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x^{n/2}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x^{n/2}}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}n}$$

input
```
Integrate[x^(-1 + n/2)/(a + b*x^n + c*x^(2*n)),x]
```

output
```
(2*Sqrt[2]*Sqrt[c]*(ArcTan[(Sqrt[2]*Sqrt[c]*x^(n/2))/Sqrt[b - Sqrt[b^2 - 4
*a*c]]]/Sqrt[b - Sqrt[b^2 - 4*a*c]] - ArcTan[(Sqrt[2]*Sqrt[c]*x^(n/2))/Sqr
t[b + Sqrt[b^2 - 4*a*c]]]/Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(Sqrt[b^2 - 4*a*c]
*n)
```

3.558.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1717, 1406, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{\frac{n}{2}-1}}{a+bx^n+cx^{2n}}dx$$

$$\Big\downarrow \text{1717}$$

$$\frac{2\int \frac{1}{bx^n+cx^{2n}+a}dx^{n/2}}{n}$$

$$\Big\downarrow \text{1406}$$

$$\frac{2\left(\frac{c\int \frac{1}{cx^n+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx^{n/2}}{\sqrt{b^2-4ac}}-\frac{c\int \frac{1}{cx^n+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx^{n/2}}{\sqrt{b^2-4ac}}\right)}{n}$$

$$\Big\downarrow \text{218}$$

$$\frac{2\left(\frac{\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x^{n/2}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x^{n/2}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}\right)}{n}$$

input
```
Int[x^(-1 + n/2)/(a + b*x^n + c*x^(2*n)),x]
```

output
```
(2*((Sqrt[2]*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x^(n/2))/Sqrt[b - Sqrt[b^2 -
4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*Sqrt[
c]*ArcTan[(Sqrt[2]*Sqrt[c]*x^(n/2))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^
2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])))/n
```

3.558.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

rule 1717
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Simp[1/(m + 1)   Subst[Int[(a + b*x^Simplify[n/(m + 1)] + c*x^Simplify[
2*(n/(m + 1))])^p, x], x, x^(m + 1)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&
 EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[n/(m + 1)]] &&  !
IntegerQ[n]
```

3.558.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.37 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.67

method	result
risch	$\sum_{\_R=\mathrm{RootOf}\left(\left(16a^3c^2n^4-8a^2b^2c\,n^4+a\,b^4n^4\right)\_Z^4+\left(-4abc\,n^2+b^3n^2\right)\_Z^2+c\right)} \_R\ln\left(x^{\frac{n}{2}}+\left(4n^3b\,a^2-\frac{n^3b^3a}{c}\right)\_R^3+\right.$

input
```
int(x^(-1+1/2*n)/(a+b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

output
```
sum(_R*ln(x^(1/2*n)+(4*n^3*b*a^2-1/c*n^3*b^3*a)*_R^3+(2*a*n-1/c*n*b^2)*_R)
,_R=RootOf((16*a^3*c^2*n^4-8*a^2*b^2*c*n^4+a*b^4*n^4)*_Z^4+(-4*a*b*c*n^2+b
^3*n^2)*_Z^2+c))
```

3.558.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 801 vs. $2(129) = 258$.

Time = 0.29 (sec) , antiderivative size = 801, normalized size of antiderivative = 4.74

$$\int \frac{x^{-1+\frac{n}{2}}}{a+bx^n+cx^{2n}}\,dx$$

$$= \frac{1}{2}\sqrt{2}\sqrt{-\frac{(ab^2-4a^2c)n^2\sqrt{\frac{1}{(a^2b^2-4a^3c)n^4}}+b}{(ab^2-4a^2c)n^2}}\log\left(\frac{4cxx^{\frac{1}{2}n-1}+\sqrt{2}\Big((ab^3-4a^2bc)n^3\sqrt{\frac{1}{(a^2b^2-4a^3c)n^4}}-(b^2-}{x}\right.$$

$$-\frac{1}{2}\sqrt{2}\sqrt{-\frac{(ab^2-4a^2c)n^2\sqrt{\frac{1}{(a^2b^2-4a^3c)n^4}}+b}{(ab^2-4a^2c)n^2}}\log\left(\frac{4cxx^{\frac{1}{2}n-1}-\sqrt{2}\Big((ab^3-4a^2bc)n^3\sqrt{\frac{1}{(a^2b^2-4a^3c)n^4}}-(b}{x}\right.$$

$$-\frac{1}{2}\sqrt{2}\sqrt{\frac{(ab^2-4a^2c)n^2\sqrt{\frac{1}{(a^2b^2-4a^3c)n^4}}-b}{(ab^2-4a^2c)n^2}}\log\left(\frac{4cxx^{\frac{1}{2}n-1}+\sqrt{2}\Big((ab^3-4a^2bc)n^3\sqrt{\frac{1}{(a^2b^2-4a^3c)n^4}}+(b^2-}{x}\right.$$

$$+\frac{1}{2}\sqrt{2}\sqrt{\frac{(ab^2-4a^2c)n^2\sqrt{\frac{1}{(a^2b^2-4a^3c)n^4}}-b}{(ab^2-4a^2c)n^2}}\log\left(\frac{4cxx^{\frac{1}{2}n-1}-\sqrt{2}\Big((ab^3-4a^2bc)n^3\sqrt{\frac{1}{(a^2b^2-4a^3c)n^4}}+(b^2-}{x}\right.$$

input
```
integrate(x^(-1+1/2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*sqrt(-((a*b^2 - 4*a^2*c)*n^2*sqrt(1/((a^2*b^2 - 4*a^3*c)*n^4))
 + b)/((a*b^2 - 4*a^2*c)*n^2))*log((4*c*x*x^(1/2*n - 1) + sqrt(2)*((a*b^3
- 4*a^2*b*c)*n^3*sqrt(1/((a^2*b^2 - 4*a^3*c)*n^4)) - (b^2 - 4*a*c)*n)*sqrt
(-((a*b^2 - 4*a^2*c)*n^2*sqrt(1/((a^2*b^2 - 4*a^3*c)*n^4)) + b)/((a*b^2 -
4*a^2*c)*n^2)))/x) - 1/2*sqrt(2)*sqrt(-((a*b^2 - 4*a^2*c)*n^2*sqrt(1/((a^2
*b^2 - 4*a^3*c)*n^4)) + b)/((a*b^2 - 4*a^2*c)*n^2))*log((4*c*x*x^(1/2*n -
1) - sqrt(2)*((a*b^3 - 4*a^2*b*c)*n^3*sqrt(1/((a^2*b^2 - 4*a^3*c)*n^4)) -
(b^2 - 4*a*c)*n)*sqrt(-((a*b^2 - 4*a^2*c)*n^2*sqrt(1/((a^2*b^2 - 4*a^3*c)*
n^4)) + b)/((a*b^2 - 4*a^2*c)*n^2)))/x) - 1/2*sqrt(2)*sqrt(((a*b^2 - 4*a^2
*c)*n^2*sqrt(1/((a^2*b^2 - 4*a^3*c)*n^4)) - b)/((a*b^2 - 4*a^2*c)*n^2))*lo
g((4*c*x*x^(1/2*n - 1) + sqrt(2)*((a*b^3 - 4*a^2*b*c)*n^3*sqrt(1/((a^2*b^2
 - 4*a^3*c)*n^4)) + (b^2 - 4*a*c)*n)*sqrt(((a*b^2 - 4*a^2*c)*n^2*sqrt(1/((
a^2*b^2 - 4*a^3*c)*n^4)) - b)/((a*b^2 - 4*a^2*c)*n^2)))/x) + 1/2*sqrt(2)*s
qrt(((a*b^2 - 4*a^2*c)*n^2*sqrt(1/((a^2*b^2 - 4*a^3*c)*n^4)) - b)/((a*b^2
- 4*a^2*c)*n^2))*log((4*c*x*x^(1/2*n - 1) - sqrt(2)*((a*b^3 - 4*a^2*b*c)*n
^3*sqrt(1/((a^2*b^2 - 4*a^3*c)*n^4)) + (b^2 - 4*a*c)*n)*sqrt(((a*b^2 - 4*a
^2*c)*n^2*sqrt(1/((a^2*b^2 - 4*a^3*c)*n^4)) - b)/((a*b^2 - 4*a^2*c)*n^2)))
/x)
```

3.558.6 Sympy [F]

$$\int \frac{x^{-1+\frac{n}{2}}}{a+bx^n+cx^{2n}}\,dx=\int \frac{x^{\frac{n}{2}-1}}{a+bx^n+cx^{2n}}\,dx$$

input
```
integrate(x**(-1+1/2*n)/(a+b*x**n+c*x**(2*n)),x)
```

output
```
Integral(x**(n/2 - 1)/(a + b*x**n + c*x**(2*n)), x)
```

3.558.7 Maxima [F]

$$\int \frac{x^{-1+\frac{n}{2}}}{a+bx^n+cx^{2n}}\,dx=\int \frac{x^{\frac{1}{2}\,n-1}}{cx^{2\,n}+bx^n+a}\,dx$$

input
```
integrate(x^(-1+1/2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
integrate(x^(1/2*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

3.558.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1037 vs. $2(129) = 258$.

Time = 0.68 (sec) , antiderivative size = 1037, normalized size of antiderivative = 6.14

$$\int \frac{x^{-1+\frac{n}{2}}}{a+bx^n+cx^{2n}}\,dx$$

$$= \frac{\left(\sqrt{2}\sqrt{bc+\sqrt{b^2-4\,ac}c}b^4-8\,\sqrt{2}\sqrt{bc+\sqrt{b^2-4\,ac}c}ab^2c-2\,\sqrt{2}\sqrt{bc+\sqrt{b^2-4\,ac}c}b^3c-2\,b^4c+16\,\sqrt{2}\sqrt{bc+\sqrt{b^2-4\,ac}c}a^2c^2+8\,\sqrt{2}\sqrt{bc+\sqrt{b^2-4\,ac}c}abc^2+\cdots}{\cdots}$$

input

```
integrate(x^(-1+1/2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output

```
1/2*((sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^
3*c - 2*b^4*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b^2*c^2 + 16*a*b^2*c^2 + 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*c^3 - 32*a^2*c^3 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b*c^2 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2 - 2*(b^
2 - 4*a*c)*b*c^2)*arctan(2*sqrt(1/2)*sqrt(x^n)/sqrt((b + sqrt(b^2 - 4*a*c)
)/c))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2
*c^2 - 4*a^2*c^3)*abs(c)) + (sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 -
 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*b^3*c + 2*b^4*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a^2*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt
(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - 16*a*b^2*c^2 - 2*b^3*c^2 - 4
*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 + 32*a^2*c^3 + 8*a*b*c^3 +
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 - 4*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sqr...
```

3.558.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+\frac{n}{2}}}{a+bx^n+cx^{2n}}\,dx=\int \frac{x^{\frac{n}{2}-1}}{a+b\,x^n+c\,x^{2\,n}}\,dx$$

input
```
int(x^(n/2 - 1)/(a + b*x^n + c*x^(2*n)),x)
```

output
```
int(x^(n/2 - 1)/(a + b*x^n + c*x^(2*n)), x)
```

3.559 $\int \frac{x^{-1-\frac{n}{2}}}{a+bx^n+cx^{2n}}\,dx$

3.559.1 Optimal result

Integrand size = 26, antiderivative size = 205

$$\int \frac{x^{-1-\frac{n}{2}}}{a+bx^n+cx^{2n}}\,dx = -\frac{2x^{-n/2}}{an}+\frac{\sqrt{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{a}x^{-n/2}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a^{3/2}\sqrt{b-\sqrt{b^2-4ac}}n}+\frac{\sqrt{2}\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{a}x^{-n/2}}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{a^{3/2}\sqrt{b+\sqrt{b^2-4ac}}n}$$

output
```
-2/a/n/(x^(1/2*n))+arctan(2^(1/2)*a^(1/2)/(x^(1/2*n))/(b-(-4*a*c+b^2)^(1/2
))^(1/2))*2^(1/2)*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))/a^(3/2)/n/(b-(-4*a*c+
b^2)^(1/2))^(1/2)+arctan(2^(1/2)*a^(1/2)/(x^(1/2*n))/(b+(-4*a*c+b^2)^(1/2)
)^(1/2))*2^(1/2)*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))/a^(3/2)/n/(b+(-4*a*c+
b^2)^(1/2))^(1/2)
```

3.559.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.22 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.62

$$\int \frac{x^{-1-\frac{n}{2}}}{a+bx^n+cx^{2n}}\,dx = \frac{4cx^{-n/2}\left(\frac{\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)}{b^2-4ac-b\sqrt{b^2-4ac}}+\frac{\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{b^2-4ac+b\sqrt{b^2-4ac}}\right)}{n}$$

input
```
Integrate[x^(-1 - n/2)/(a + b*x^n + c*x^(2*n)),x]
```

output
```
(4*c*(Hypergeometric2F1[-1/2, 1, 1/2, (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]/
(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c]) + Hypergeometric2F1[-1/2, 1, 1/2, (-2*
c*x^n)/(b + Sqrt[b^2 - 4*a*c])]/(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])))/(n*x
^(n/2))
```

3.559.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1717, 1679, 1442, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-\frac{n}{2}-1}}{a+bx^n+cx^{2n}}dx$$

$$\Big\downarrow \text{ 1717}$$

$$-\frac{2\int \frac{1}{bx^n+cx^{2n}+a}dx^{-n/2}}{n}$$

$$\Big\downarrow \text{ 1679}$$

$$-\frac{2\int \frac{x^{-2n}}{ax^{-2n}+bx^{-n}+c}dx^{-n/2}}{n}$$

$$\Big\downarrow \text{ 1442}$$

$$-\frac{2\left(\frac{x^{-n/2}}{a}-\frac{\int \frac{bx^{-n}+c}{ax^{-2n}+bx^{-n}+c}dx^{-n/2}}{a}\right)}{n}$$

$$\Big\downarrow \text{ 1480}$$

$$-\frac{2\left(\frac{x^{-n/2}}{a}-\frac{\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{ax^{-n}+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx^{-n/2}+\frac{1}{2}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\int \frac{1}{ax^{-n}+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx^{-n/2}}{a}\right)}{n}$$

$$\Big\downarrow \text{ 218}$$

$$-\frac{2\left(\frac{x^{-n/2}}{a}-\frac{\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{a}x^{-n/2}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{a}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{\sqrt{2}\sqrt{a}x^{-n/2}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{a}\sqrt{\sqrt{b^2-4ac}+b}}}{a}\right)}{n}$$

input
```
Int[x^(-1 - n/2)/(a + b*x^n + c*x^(2*n)),x]
```

output
```
(-2*(1/(a*x^(n/2)) - (((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
2]*Sqrt[a])/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*x^(n/2))])/(Sqrt[2]*Sqrt[a]*Sqrt[
b - Sqrt[b^2 - 4*a*c]]) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(S
qrt[2]*Sqrt[a])/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*x^(n/2))])/(Sqrt[2]*Sqrt[a]*S
qrt[b + Sqrt[b^2 - 4*a*c]]))/a))/n
```

3.559.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1442
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
x] - Simp[d^4/(c*(m + 4*p + 1))   Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1679
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(
2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n]
 && LtQ[n, 0] && IntegerQ[p]
```

3.559. $\int \frac{x^{-1-\frac{n}{2}}}{a+bx^n+cx^{2n}}\,dx$

rule 1717
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Simp[1/(m + 1)   Subst[Int[(a + b*x^Simplify[n/(m + 1)] + c*x^Simplify[
2*(n/(m + 1))])^p, x], x, x^(m + 1)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&
 EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[n/(m + 1)]] &&  !
IntegerQ[n]
```

3.559.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.53 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.31

method	result
risch	$-\frac{2x^{-\frac{n}{2}}}{an}+\left(\sum_{\_R=\text{RootOf}\left(\left(16a^5c^2n^4-8a^4b^2c\,n^4+a^3b^4n^4\right)\_Z^4+\left(12a^2b\,c^2n^2-7a\,b^3c\,n^2+b^5n^2\right)\_Z^2+c^3\right)}\_R\ln\left(x^{\frac{n}{2}}+\left(-\right.\right.\right.$

input
```
int(x^(-1-1/2*n)/(a+b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

output
```
-2/a/n/(x^(1/2*n))+sum(_R*ln(x^(1/2*n)+(-8/(a*c^3-b^2*c^2)*n^3*a^5*c^2+6/(
a*c^3-b^2*c^2)*n^3*b^2*a^4*c-1/(a*c^3-b^2*c^2)*n^3*b^4*a^3)*_R^3+(-5/(a*c^
3-b^2*c^2)*n*b*a^2*c^2+5/(a*c^3-b^2*c^2)*n*b^3*a*c-1/(a*c^3-b^2*c^2)*n*b^5
)*_R),_R=RootOf((16*a^5*c^2*n^4-8*a^4*b^2*c*n^4+a^3*b^4*n^4)*_Z^4+(12*a^2*
b*c^2*n^2-7*a*b^3*c*n^2+b^5*n^2)*_Z^2+c^3))
```

3.559.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1229 vs. 2(169) = 338.

Time = 0.30 (sec) , antiderivative size = 1229, normalized size of antiderivative = 6.00

$$\int \frac{x^{-1-\frac{n}{2}}}{a+bx^n+cx^{2n}}\,dx = \text{Too large to display}$$

input
```
integrate(x^(-1-1/2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
1/2*(sqrt(2)*a*n*sqrt(-((a^3*b^2 - 4*a^4*c)*n^2*sqrt((b^4 - 2*a*b^2*c + a^
2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) + b^3 - 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*n^
2))*log(-(4*(b^2*c - a*c^2)*x*x^(-1/2*n - 1) + sqrt(2)*((a^3*b^3 - 4*a^4*b
*c)*n^3*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) - (b^4
 - 5*a*b^2*c + 4*a^2*c^2)*n)*sqrt(-((a^3*b^2 - 4*a^4*c)*n^2*sqrt((b^4 - 2*
a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) + b^3 - 3*a*b*c)/((a^3*b^2 -
 4*a^4*c)*n^2)))/x) - sqrt(2)*a*n*sqrt(-((a^3*b^2 - 4*a^4*c)*n^2*sqrt((b^4
 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) + b^3 - 3*a*b*c)/((a^3*
b^2 - 4*a^4*c)*n^2))*log(-(4*(b^2*c - a*c^2)*x*x^(-1/2*n - 1) - sqrt(2)*((
a^3*b^3 - 4*a^4*b*c)*n^3*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^
7*c)*n^4)) - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*n)*sqrt(-((a^3*b^2 - 4*a^4*c)*n
^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) + b^3 - 3*a
*b*c)/((a^3*b^2 - 4*a^4*c)*n^2)))/x) - sqrt(2)*a*n*sqrt(((a^3*b^2 - 4*a^4*
c)*n^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) - b^3 +
 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*n^2))*log(-(4*(b^2*c - a*c^2)*x*x^(-1/2*n -
 1) + sqrt(2)*((a^3*b^3 - 4*a^4*b*c)*n^3*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/
((a^6*b^2 - 4*a^7*c)*n^4)) + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*n)*sqrt(((a^3*b
^2 - 4*a^4*c)*n^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^
4)) - b^3 + 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*n^2)))/x) + sqrt(2)*a*n*sqrt(((a
^3*b^2 - 4*a^4*c)*n^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^...
```

3.559.6 Sympy [F]

$$\int \frac{x^{-1-\frac{n}{2}}}{a + bx^n + cx^{2n}}\,dx = \int \frac{x^{-\frac{n}{2}-1}}{a + bx^n + cx^{2n}}\,dx$$

input
```
integrate(x**(-1-1/2*n)/(a+b*x**n+c*x**(2*n)),x)
```

output
```
Integral(x**(-n/2 - 1)/(a + b*x**n + c*x**(2*n)), x)
```

3.559.7 Maxima [F]

$$\int \frac{x^{-1-\frac{n}{2}}}{a+bx^n+cx^{2n}}\,dx=\int \frac{x^{-\frac{1}{2}\,n-1}}{cx^{2\,n}+bx^n+a}\,dx$$

input
```
integrate(x^(-1-1/2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
-2/(a*n*x^(1/2*n)) - integrate((c*x^(3/2*n) + b*x^(1/2*n))/(a*c*x*x^(2*n)
+ a*b*x*x^n + a^2*x), x)
```

3.559.8 Giac [F]

$$\int \frac{x^{-1-\frac{n}{2}}}{a+bx^n+cx^{2n}}\,dx=\int \frac{x^{-\frac{1}{2}\,n-1}}{cx^{2\,n}+bx^n+a}\,dx$$

input
```
integrate(x^(-1-1/2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate(x^(-1/2*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

3.559.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-\frac{n}{2}}}{a+bx^n+cx^{2n}}\,dx=\int \frac{1}{x^{\frac{n}{2}+1}\,\left(a+b\,x^n+c\,x^{2\,n}\right)}\,dx$$

input
```
int(1/(x^(n/2 + 1)*(a + b*x^n + c*x^(2*n))),x)
```

output
```
int(1/(x^(n/2 + 1)*(a + b*x^n + c*x^(2*n))), x)
```

3.560 $\int \frac{x^{-1-\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx$

3.560.1 Optimal result

Integrand size = 26, antiderivative size = 699

$$\int \frac{x^{-1-\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx = -\frac{3x^{-n/3}}{an} - \frac{\sqrt{3}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{a}x^{-n/3}}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}a^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}n} - \frac{\sqrt{3}\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{a}x^{-n/3}}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}a^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}n} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{a}x^{-n/3}\right)}{\sqrt[3]{2}a^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}n} + \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{a}x^{-n/3}\right)}{\sqrt[3]{2}a^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}n} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}a^{2/3}x^{-2n/3}-\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{b-\sqrt{b^2-4ac}}x^{-n/3}\right)}{2\sqrt[3]{2}a^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}n} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}a^{2/3}x^{-2n/3}-\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{b+\sqrt{b^2-4ac}}x^{-n/3}\right)}{2\sqrt[3]{2}a^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}n}$$

output
```
-3/a/n/(x^(1/3*n))+1/2*ln(2^(1/3)*a^(1/3)/(x^(1/3*n))+(b-(-4*a*c+b^2)^(1/2
))^(1/3))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))*2^(2/3)/a^(4/3)/n/(b-(-4*a*c+
b^2)^(1/2))^(2/3)-1/4*ln(2^(2/3)*a^(2/3)/(x^(2/3*n))-2^(1/3)*a^(1/3)*(b-(-
4*a*c+b^2)^(1/2))^(1/3)/(x^(1/3*n))+(b-(-4*a*c+b^2)^(1/2))^(2/3))*(b+(2*a*
c-b^2)/(-4*a*c+b^2)^(1/2))*2^(2/3)/a^(4/3)/n/(b-(-4*a*c+b^2)^(1/2))^(2/3)-
1/2*arctan(1/3*(1-2*2^(1/3)*a^(1/3)/(x^(1/3*n))/(b-(-4*a*c+b^2)^(1/2))^(1/
3))*3^(1/2))*3^(1/2)*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))*2^(2/3)/a^(4/3)/n/
(b-(-4*a*c+b^2)^(1/2))^(2/3)+1/2*ln(2^(1/3)*a^(1/3)/(x^(1/3*n))+(b+(-4*a*c
+b^2)^(1/2))^(1/3))*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*2^(2/3)/a^(4/3)/n/
(b+(-4*a*c+b^2)^(1/2))^(2/3)-1/4*ln(2^(2/3)*a^(2/3)/(x^(2/3*n))-2^(1/3)*a^
(1/3)*(b+(-4*a*c+b^2)^(1/2))^(1/3)/(x^(1/3*n))+(b+(-4*a*c+b^2)^(1/2))^(2/3
))*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*2^(2/3)/a^(4/3)/n/(b+(-4*a*c+b^2)^(
1/2))^(2/3)-1/2*arctan(1/3*(1-2*2^(1/3)*a^(1/3)/(x^(1/3*n))/(b+(-4*a*c+b^2
)^(1/2))^(1/3))*3^(1/2))*3^(1/2)*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*2^(2/
3)/a^(4/3)/n/(b+(-4*a*c+b^2)^(1/2))^(2/3)
```

3.560.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.18 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.18

$$\int \frac{x^{-1-\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx = \frac{6cx^{-n/3}\left(\frac{\text{Hypergeometric2F1}\left(-\frac{1}{3},1,\frac{2}{3},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)}{b^2-4ac-b\sqrt{b^2-4ac}}+\frac{\text{Hypergeometric2F1}\left(-\frac{1}{3},1,\frac{2}{3},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{b^2-4ac+b\sqrt{b^2-4ac}}\right)}{n}$$

input
```
Integrate[x^(-1 - n/3)/(a + b*x^n + c*x^(2*n)),x]
```

output
```
(6*c*(Hypergeometric2F1[-1/3, 1, 2/3, (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]/
(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c]) + Hypergeometric2F1[-1/3, 1, 2/3, (-2*
c*x^n)/(b + Sqrt[b^2 - 4*a*c])]/(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])))/(n*x
^(n/3))
```

3.560.3 Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 576, normalized size of antiderivative = 0.82, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1717, 1679, 1703, 1752, 750, 16, 27, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-\frac{n}{3}-1}}{a+bx^n+cx^{2n}}\,dx$$

$$\Big\downarrow \text{1717}$$

$$-\frac{3\int \frac{1}{bx^n+cx^{2n}+a}dx^{-n/3}}{n}$$

$$\Big\downarrow \text{1679}$$

$$-\frac{3\int \frac{x^{-2n}}{ax^{-2n}+bx^{-n}+c}dx^{-n/3}}{n}$$

$$\Big\downarrow \text{1703}$$

$$-\frac{3\left(\frac{x^{-n/3}}{a}-\frac{\int \frac{bx^{-n}+c}{ax^{-2n}+bx^{-n}+c}dx^{-n/3}}{a}\right)}{n}$$

$$\Big\downarrow \text{1752}$$

$$-\frac{3\left(\frac{x^{-n/3}}{a}-\frac{\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{ax^{-n}+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx^{-n/3}+\frac{1}{2}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\int \frac{1}{ax^{-n}+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx^{-n/3}}{a}\right)}{n}$$

$$\Big\downarrow \text{750}$$

$$-\frac{3\left(\frac{x^{-n/3}}{a}-\frac{\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{2^{2/3}\int\frac{2\left(2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{a}x^{-n/3}\right)}{2a^{2/3}x^{-2n/3}-2^{2/3}\sqrt[3]{a}\sqrt[3]{b-\sqrt{b^2-4ac}}x^{-n/3}+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx^{-n/3}}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\int\frac{\cdots}{\sqrt[3]{a}x^{-n/3}\cdots}\cdots}{3\left(\cdots\right)}\cdots\right)}{\cdots}\right)}{\cdots}$$

$\downarrow$ 16

$$-\frac{3\left(\frac{x^{-n/3}}{a}-\frac{\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{2^{2/3}\int\frac{2\left(2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{a}x^{-n/3}\right)}{2a^{2/3}x^{-2n/3}-2^{2/3}\sqrt[3]{a}\sqrt[3]{b-\sqrt{b^2-4ac}}x^{-n/3}+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx^{-n/3}}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\log\left(\sqrt[3]{b-\cdots}\cdots\right)}{3\sqrt[3]{\cdots}}\cdots\right)}{\cdots}\right)}{\cdots}$$

$\downarrow$ 27

3.560. $\int \frac{x^{-1-\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx$

$$-\frac{3\left(\frac{x^{-n/3}}{a}-\frac{\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{2\ 2^{2/3}\int \frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{a}x^{-n/3}}{2a^{2/3}x^{-2n/3}-2^{2/3}\sqrt[3]{a}\sqrt[3]{b-\sqrt{b^2-4ac}}x^{-n/3}+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx^{-n/3}}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\log\left(\sqrt[3]{b-\cdots}\right.}{3\sqrt[3]{\cdots}}\right.}{\cdots}\right.}{\cdots}$$

$\downarrow$ 1142

$$-\frac{3\left(\frac{x^{-n/3}}{a}-\frac{\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{2^{2/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}x^{-n/3}+\sqrt[3]{b-\sqrt{b^2-4ac}}\right)}{3\sqrt[3]{a}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2\ 2^{2/3}\left(3\sqrt[3]{b-\sqrt{b^2-4ac}}\int \frac{\cdots}{2a^{2/3}x^{-2n/3}-2^{2/3}\sqrt[3]{a}\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

3.560. $\int \frac{x^{-1-\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx$

↓ 25

$$-\frac{3\left(\frac{x^{-n/3}}{a}-\frac{\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{2^{2/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}x^{-n/3}+\sqrt[3]{b-\sqrt{b^2-4ac}}\right)}{3\sqrt[3]{a}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{\left(3\sqrt[3]{b-\sqrt{b^2-4ac}}\int\frac{\cdots}{2a^{2/3}x^{-2n/3}-2^{2/3}\sqrt[3]{a}\cdots}\cdots\right)}{2\;2^{2/3}\cdots}\cdots\right)}{\cdots}\cdots\right)}{\cdots}$$

↓ 27

3.560. $\int \frac{x^{-1-\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx$

$$-\frac{3\left(\frac{x^{-n/3}}{a}-\frac{\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{2\,2^{2/3}\left(\frac{3\sqrt[3]{b-\sqrt{b^2-4ac}}\int\frac{1}{2a^{2/3}x^{-2n/3}-2^{2/3}\sqrt[3]{a}\sqrt[3]{b-\sqrt{b^2-4ac}}\,x^{-n/3}+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\cdots}{2\sqrt[3]{2}}\cdots\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2}\cdots}\cdots\right)}{\cdots}\cdots\right)}{\cdots}$$

↓ 1082

3.560. $\int \frac{x^{-1-\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx$

$$-\frac{3\left(\frac{x^{-n/3}}{a}-\frac{\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{2\,2^{2/3}\left(\frac{1}{4}\int\frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-4\sqrt[3]{a}\,x^{-n/3}}{2a^{2/3}x^{-2n/3}-2^{2/3}\sqrt[3]{a}\sqrt[3]{b-\sqrt{b^2-4ac}}\,x^{-n/3}+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\,dx^{-n/3}+\frac{3\int\frac{1}{-x^{-2n/3}-\cdots}\cdots}{\cdots}\cdots\right)}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\cdots\right)}{\cdots}\cdots\right)}{\cdots}$$

$\downarrow$ 217

3.560. $\int\frac{x^{-1-\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx$

$$-\frac{3\left(\frac{x^{-n/3}}{a}-\frac{\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{2\,2^{2/3}\left(\frac{1}{4}\int\frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-4\sqrt[3]{a}x^{-n/3}}{2a^{2/3}x^{-2n/3}-2^{2/3}\sqrt[3]{a}\sqrt[3]{b-\sqrt{b^2-4ac}}x^{-n/3}+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx^{-n/3}-\frac{\sqrt{3}\arctan\left(\frac{1-\cdots}{\cdots}\right)\cdots}{\cdots}\right)}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\cdots\right)}{\cdots}\cdots\right)}{\cdots}$$

↓ 1103

3.560. $\int\frac{x^{-1-\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx$

$$-\frac{3\left(\frac{x^{-n/3}}{a}-\frac{\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{2\,2^{2/3}\left(-\frac{\log\left(2^{2/3}a^{2/3}x^{-2n/3}-\sqrt[3]{2}\sqrt[3]{a}x^{-n/3}\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}\right)}{4\sqrt[3]{a}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\cdots}{\sqrt[3]{b\cdots}}}{\cdots}\right)}{2\cdots}\cdots\right)}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}\cdots}\cdots}{\cdots}\cdots\right)}{\cdots}$$

input
```
Int[x^(-1 - n/3)/(a + b*x^n + c*x^(2*n)),x]
```

3.560. $\int \frac{x^{-1-\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx$

output
```
(-3*(1/(a*x^(n/3)) - (((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*((2^(2/3)*Log
[(b - Sqrt[b^2 - 4*a*c])^(1/3) + (2^(1/3)*a^(1/3))/x^(n/3)])/(3*a^(1/3)*(b
 - Sqrt[b^2 - 4*a*c])^(2/3)) + (2*2^(2/3)*(-1/2*(Sqrt[3]*ArcTan[(1 - (2*2^
(1/3)*a^(1/3))/((b - Sqrt[b^2 - 4*a*c])^(1/3)*x^(n/3)))/Sqrt[3]])/a^(1/3)
- Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) + (2^(2/3)*a^(2/3))/x^((2*n)/3) - (2^(
1/3)*a^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3))/x^(n/3)]/(4*a^(1/3))))/(3*(b -
 Sqrt[b^2 - 4*a*c])^(2/3))))/2 + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*((
2^(2/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + (2^(1/3)*a^(1/3))/x^(n/3)])/(3
*a^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)) + (2*2^(2/3)*(-1/2*(Sqrt[3]*ArcTan
[(1 - (2*2^(1/3)*a^(1/3))/((b + Sqrt[b^2 - 4*a*c])^(1/3)*x^(n/3)))/Sqrt[3]
])/a^(1/3) - Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) + (2^(2/3)*a^(2/3))/x^((2*n
)/3) - (2^(1/3)*a^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3))/x^(n/3)]/(4*a^(1/3)
)))/(3*(b + Sqrt[b^2 - 4*a*c])^(2/3))))/2)/a))/n
```

3.560.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

3.560. $\int \frac{x^{-1-\frac{n}{3}}}{a+bx^n+cx^{2n}} \, dx$

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1679
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(
2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n]
 && LtQ[n, 0] && IntegerQ[p]
```

rule 1703
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(2*n - 1)*(d*x)^(m - 2*n + 1)*((a + b*x^n + c*x^(2*n))^(
p + 1)/(c*(m + 2*n*p + 1))), x] - Simp[d^(2*n)/(c*(m + 2*n*p + 1))   Int[(d
*x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x
^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && N
eQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0
] && IntegerQ[p]
```

rule 1717
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Simp[1/(m + 1)   Subst[Int[(a + b*x^Simplify[n/(m + 1)] + c*x^Simplify[
2*(n/(m + 1))])^p, x], x, x^(m + 1)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&
 EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[n/(m + 1)]] &&  !
IntegerQ[n]
```

3.560. $\int \frac{x^{-1-\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx$

rule 1752
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

3.560.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.04 (sec) , antiderivative size = 534, normalized size of antiderivative = 0.76

method	result
risch	$-\frac{3x^{-\frac{n}{3}}}{an}+\left(\sum_{\_R=\text{RootOf}\left((64a^7c^3n^6-48a^6b^2c^2n^6+12a^5b^4c\,n^6-a^4b^6n^6)\_Z^6+(-32a^3b\,c^3n^3+32a^2b^3c^2n^3-10a\,b^5c\,n^3+b^7n^3)\_Z\right.}\right.$

input
```
int(x^(-1-1/3*n)/(a+b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

output
```
-3/a/n/(x^(1/3*n))+sum(_R*ln(x^(1/3*n)+(-64/(2*a^2*c^5-4*a*b^2*c^4+b^4*c^3
)*n^5*a^8*c^4+112/(2*a^2*c^5-4*a*b^2*c^4+b^4*c^3)*n^5*b^2*a^7*c^3-60/(2*a^
2*c^5-4*a*b^2*c^4+b^4*c^3)*n^5*b^4*a^6*c^2+13/(2*a^2*c^5-4*a*b^2*c^4+b^4*c
^3)*n^5*b^6*a^5*c-1/(2*a^2*c^5-4*a*b^2*c^4+b^4*c^3)*n^5*b^8*a^4)*_R^5+(28/
(2*a^2*c^5-4*a*b^2*c^4+b^4*c^3)*n^2*b*a^4*c^4-63/(2*a^2*c^5-4*a*b^2*c^4+b^
4*c^3)*n^2*b^3*a^3*c^3+42/(2*a^2*c^5-4*a*b^2*c^4+b^4*c^3)*n^2*b^5*a^2*c^2-
11/(2*a^2*c^5-4*a*b^2*c^4+b^4*c^3)*n^2*b^7*a*c+1/(2*a^2*c^5-4*a*b^2*c^4+b^
4*c^3)*n^2*b^9)*_R^2),_R=RootOf((64*a^7*c^3*n^6-48*a^6*b^2*c^2*n^6+12*a^5*
b^4*c*n^6-a^4*b^6*n^6)*_Z^6+(-32*a^3*b*c^3*n^3+32*a^2*b^3*c^2*n^3-10*a*b^5
*c*n^3+b^7*n^3)*_Z^3+c^4))
```

3.560.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3155 vs. 2(567) = 1134.

Time = 0.37 (sec) , antiderivative size = 3155, normalized size of antiderivative = 4.51

$$\int \frac{x^{-1-\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx = \text{Too large to display}$$

input
```
integrate(x^(-1-1/3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
1/2*(2*(1/2)^(1/3)*a*n*(((a^4*b^2 - 4*a^5*c)*n^3*sqrt((b^8 - 8*a*b^6*c + 2
0*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*
a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) + b^3 - 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*n^
3))^(1/3)*log((2*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*x*x^(-1/3*n - 1) + (1/2
)^(1/3)*((a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*n^4*sqrt((b^8 - 8*a*b^6*c
+ 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c +
48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) - (b^6 - 8*a*b^4*c + 18*a^2*b^2*c^2 -
 8*a^3*c^3)*n)*(((a^4*b^2 - 4*a^5*c)*n^3*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^
4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2
*c^2 - 64*a^11*c^3)*n^6)) + b^3 - 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*n^3))^(1/3
))/x) + 2*(1/2)^(1/3)*a*n*(-((a^4*b^2 - 4*a^5*c)*n^3*sqrt((b^8 - 8*a*b^6*c
 + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c +
 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) - b^3 + 2*a*b*c)/((a^4*b^2 - 4*a^5*c
)*n^3))^(1/3)*log((2*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*x*x^(-1/3*n - 1) -
(1/2)^(1/3)*((a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*n^4*sqrt((b^8 - 8*a*b^
6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*
c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) + (b^6 - 8*a*b^4*c + 18*a^2*b^2*c
^2 - 8*a^3*c^3)*n)*(-((a^4*b^2 - 4*a^5*c)*n^3*sqrt((b^8 - 8*a*b^6*c + 20*a
^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^1
0*b^2*c^2 - 64*a^11*c^3)*n^6)) - b^3 + 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*n^...
```

3.560.6 Sympy [F]

$$\int \frac{x^{-1-\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^{-\frac{n}{3}-1}}{a+bx^n+cx^{2n}}\,dx$$

input
```
integrate(x**(-1-1/3*n)/(a+b*x**n+c*x**(2*n)),x)
```

output
```
Integral(x**(-n/3 - 1)/(a + b*x**n + c*x**(2*n)), x)
```

3.560.7 Maxima [F]

$$\int \frac{x^{-1-\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^{-\frac{1}{3}\,n-1}}{cx^{2\,n}+bx^n+a}\,dx$$

input
```
integrate(x^(-1-1/3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
-3/(a*n*x^(1/3*n)) - integrate((c*x^(5/3*n) + b*x^(2/3*n))/(a*c*x*x^(2*n)
+ a*b*x*x^n + a^2*x), x)
```

3.560.8 Giac [F]

$$\int \frac{x^{-1-\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^{-\frac{1}{3}\,n-1}}{cx^{2\,n}+bx^n+a}\,dx$$

input
```
integrate(x^(-1-1/3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate(x^(-1/3*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

3.560.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx = \int \frac{1}{x^{\frac{n}{3}+1}\,\left(a+b\,x^n+c\,x^{2\,n}\right)}\,dx$$

input
```
int(1/(x^(n/3 + 1)*(a + b*x^n + c*x^(2*n))),x)
```

output
```
int(1/(x^(n/3 + 1)*(a + b*x^n + c*x^(2*n))), x)
```

3.561 $\int \frac{x^{-1-\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx$

3.561.1 Optimal result

Integrand size = 26, antiderivative size = 414

$$\int \frac{x^{-1-\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx = -\frac{4x^{-n/4}}{an} - \frac{2^{3/4}\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{a^{5/4}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}n} - \frac{2^{3/4}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{a^{5/4}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}n} - \frac{2^{3/4}\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{a^{5/4}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}n} - \frac{2^{3/4}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{a^{5/4}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}n}$$

output
```
-4/a/n/(x^(1/4*n))-2^(3/4)*arctan(2^(1/4)*a^(1/4)/(x^(1/4*n))/(-b-(-4*a*c+
b^2)^(1/2))^(1/4))*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))/a^(5/4)/n/(-b-(-4*a
*c+b^2)^(1/2))^(3/4)-2^(3/4)*arctanh(2^(1/4)*a^(1/4)/(x^(1/4*n))/(-b-(-4*a
*c+b^2)^(1/2))^(1/4))*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))/a^(5/4)/n/(-b-(-
4*a*c+b^2)^(1/2))^(3/4)-2^(3/4)*arctan(2^(1/4)*a^(1/4)/(x^(1/4*n))/(-b+(-4
*a*c+b^2)^(1/2))^(1/4))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))/a^(5/4)/n/(-b+(
-4*a*c+b^2)^(1/2))^(3/4)-2^(3/4)*arctanh(2^(1/4)*a^(1/4)/(x^(1/4*n))/(-b+(
-4*a*c+b^2)^(1/2))^(1/4))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))/a^(5/4)/n/(-b
+(-4*a*c+b^2)^(1/2))^(3/4)
```

3.561.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.17 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.31

$$\int \frac{x^{-1-\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx = \frac{8cx^{-n/4}\left(\frac{\text{Hypergeometric2F1}\left(-\frac{1}{4},1,\frac{3}{4},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)}{b^2-4ac-b\sqrt{b^2-4ac}}+\frac{\text{Hypergeometric2F1}\left(-\frac{1}{4},1,\frac{3}{4},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{b^2-4ac+b\sqrt{b^2-4ac}}\right)}{n}$$

input
```
Integrate[x^(-1 - n/4)/(a + b*x^n + c*x^(2*n)),x]
```

output
```
(8*c*(Hypergeometric2F1[-1/4, 1, 3/4, (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]/
(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c]) + Hypergeometric2F1[-1/4, 1, 3/4, (-2*
c*x^n)/(b + Sqrt[b^2 - 4*a*c])]/(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])))/(n*x
^(n/4))
```

3.561.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 372, normalized size of antiderivative = 0.90, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {1717, 1679, 1703, 1752, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-\frac{n}{4}-1}}{a+bx^n+cx^{2n}}\,dx$$

$$\Big\downarrow \; 1717$$

$$-\frac{4\int \frac{1}{bx^n+cx^{2n}+a}dx^{-n/4}}{n}$$

$$\Big\downarrow \; 1679$$

$$-\frac{4\int \frac{x^{-2n}}{ax^{-2n}+bx^{-n}+c}dx^{-n/4}}{n}$$

$$\Big\downarrow \; 1703$$

$$-\frac{4\left(\frac{x^{-n/4}}{a}-\frac{\int \frac{bx^{-n}+c}{ax^{-2n}+bx^{-n}+c}dx^{-n/4}}{a}\right)}{n}$$

$$\Big\downarrow \; 1752$$

$$-\frac{4\left(\frac{x^{-n/4}}{a}-\frac{\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{ax^{-n}+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx^{-n/4}+\frac{1}{2}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\int \frac{1}{ax^{-n}+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx^{-n/4}}{a}\right)}{n}$$

$$\Big\downarrow \; 756$$

$$-\frac{4\left(\frac{x^{-n/4}}{a}-\frac{\frac{1}{2}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\left(-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{a}x^{-n/2}}dx^{-n/4}}{\sqrt{-\sqrt{b^2-4ac}-b}}-\frac{\int \frac{1}{\sqrt{2}\sqrt{a}x^{-n/2}+\sqrt{-b-\sqrt{b^2-4ac}}}dx^{-n/4}}{\sqrt{-\sqrt{b^2-4ac}-b}}\right)+\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(-\frac{\int \cdots}{\sqrt{\cdots}}\right.}{a}\right.}{n}$$

$$\Big\downarrow \; 218$$

3.561. $\int \frac{x^{-1-\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx$

$$-\frac{4\left(\frac{x^{-n/4}}{a}-\frac{\frac{1}{2}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\left(-\frac{\int\frac{1}{\sqrt{-b-\sqrt{b^2-4ac}-\sqrt{2}\sqrt{a}x^{-n/2}}}dx^{-n/4}}{\sqrt{-\sqrt{b^2-4ac}-b}}-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{a}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}\right)+\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(-\frac{\int\frac{1}{\sqrt{\sqrt{b^2}\cdots}}\cdots\right.}{a}\right.}{n}$$

$\Big\downarrow$ 221

$$-\frac{4\left(\frac{x^{-n/4}}{a}-\frac{\frac{1}{2}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\left(-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{a}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{a}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}\right)+\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(-\frac{\arctan\cdots}{\sqrt[4]{\cdots}}\right.}{a}\right.}{n}$$

input
```
Int[x^(-1 - n/4)/(a + b*x^n + c*x^(2*n)),x]
```

output
```
(-4*(1/(a*x^(n/4)) - (((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*(-(ArcTan[(2^
(1/4)*a^(1/4))/((-b - Sqrt[b^2 - 4*a*c])^(1/4)*x^(n/4))]/(2^(1/4)*a^(1/4)*
(-b - Sqrt[b^2 - 4*a*c])^(3/4))) - ArcTanh[(2^(1/4)*a^(1/4))/((-b - Sqrt[b
^2 - 4*a*c])^(1/4)*x^(n/4))]/(2^(1/4)*a^(1/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/
4))))/2 + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*(-(ArcTan[(2^(1/4)*a^(1/4
))/((-b + Sqrt[b^2 - 4*a*c])^(1/4)*x^(n/4))]/(2^(1/4)*a^(1/4)*(-b + Sqrt[b
^2 - 4*a*c])^(3/4))) - ArcTanh[(2^(1/4)*a^(1/4))/((-b + Sqrt[b^2 - 4*a*c])
^(1/4)*x^(n/4))]/(2^(1/4)*a^(1/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))))/2)/a))
/n
```

3.561. $\int \frac{x^{-1-\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx$

3.561.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 1679
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(
2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n]
 && LtQ[n, 0] && IntegerQ[p]
```

rule 1703
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(2*n - 1)*(d*x)^(m - 2*n + 1)*((a + b*x^n + c*x^(2*n))^(
p + 1)/(c*(m + 2*n*p + 1))), x] - Simp[d^(2*n)/(c*(m + 2*n*p + 1))   Int[(d
*x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x
^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && N
eQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0
] && IntegerQ[p]
```

rule 1717
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Simp[1/(m + 1)   Subst[Int[(a + b*x^Simplify[n/(m + 1)] + c*x^Simplify[
2*(n/(m + 1))])^p, x], x, x^(m + 1)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&
 EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[n/(m + 1)]] &&  !
IntegerQ[n]
```

rule 1752
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

3.561. $\int \frac{x^{-1-\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx$

3.561.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.54 (sec) , antiderivative size = 630, normalized size of antiderivative = 1.52

method	result
risch	$-\frac{4x^{-\frac{n}{4}}}{an}+\left(\sum_{\_R=\text{RootOf}\left(\left(256a^9c^4n^8-256a^8b^2c^3n^8+96a^7b^4c^2n^8-16a^6b^6c\,n^8+a^5b^8n^8\right)\_Z^8+\left(80a^4b\,c^4n^4-120a^3b^3c^3n^4+61a^2\right.\right.}\right.$

input
```
int(x^(-1-1/4*n)/(a+b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

output
```
-4/a/n/(x^(1/4*n))+sum(_R*ln(x^(1/4*n)+(-128/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)
*n^7*a^10*c^5+352/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)*n^7*b^2*a^9*c^4-280/(a^2*c
^6-3*a*b^2*c^5+b^4*c^4)*n^7*b^4*a^8*c^3+98/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)*n
^7*b^6*a^7*c^2-16/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)*n^7*b^8*a^6*c+1/(a^2*c^6-3
*a*b^2*c^5+b^4*c^4)*n^7*b^10*a^5)*_R^7+(-36/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)*
n^3*b*a^5*c^5+129/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)*n^3*b^3*a^4*c^4-138/(a^2*c
^6-3*a*b^2*c^5+b^4*c^4)*n^3*b^5*a^3*c^3+63/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)*n
^3*b^7*a^2*c^2-13/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)*n^3*b^9*a*c+1/(a^2*c^6-3*a
*b^2*c^5+b^4*c^4)*n^3*b^11)*_R^3),_R=RootOf((256*a^9*c^4*n^8-256*a^8*b^2*c
^3*n^8+96*a^7*b^4*c^2*n^8-16*a^6*b^6*c*n^8+a^5*b^8*n^8)*_Z^8+(80*a^4*b*c^4
*n^4-120*a^3*b^3*c^3*n^4+61*a^2*b^5*c^2*n^4-13*a*b^7*c*n^4+b^9*n^4)*_Z^4+c
^5))
```

3.561.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4375 vs. 2(342) = 684.

Time = 0.44 (sec) , antiderivative size = 4375, normalized size of antiderivative = 10.57

$$\int \frac{x^{-1-\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx = \text{Too large to display}$$

input
```
integrate(x^(-1-1/4*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
1/2*(sqrt(2)*a*n*sqrt(sqrt(2)*sqrt(-((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*
n^4*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^
10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) + b^5 - 5*a*
b^3*c + 5*a^2*b*c^2)/((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4)))*log((4*(
b^4*c - 3*a*b^2*c^2 + a^2*c^3)*x*x^(-1/4*n - 1) + sqrt(2)*((a^5*b^5 - 8*a^
6*b^3*c + 16*a^7*b*c^2)*n^5*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3
*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13
*c^3)*n^8)) - (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*n)*sqrt(sqrt(
2)*sqrt(-((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4*sqrt((b^8 - 6*a*b^6*c +
 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48
*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) + b^5 - 5*a*b^3*c + 5*a^2*b*c^2)/((a^5*
b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4))))/x) - sqrt(2)*a*n*sqrt(sqrt(2)*sqrt
(-((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4*sqrt((b^8 - 6*a*b^6*c + 11*a^2
*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b
^2*c^2 - 64*a^13*c^3)*n^8)) + b^5 - 5*a*b^3*c + 5*a^2*b*c^2)/((a^5*b^4 - 8
*a^6*b^2*c + 16*a^7*c^2)*n^4)))*log((4*(b^4*c - 3*a*b^2*c^2 + a^2*c^3)*x*x
^(-1/4*n - 1) - sqrt(2)*((a^5*b^5 - 8*a^6*b^3*c + 16*a^7*b*c^2)*n^5*sqrt((
b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 1
2*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) - (b^6 - 7*a*b^4*c + 1
3*a^2*b^2*c^2 - 4*a^3*c^3)*n)*sqrt(sqrt(2)*sqrt(-((a^5*b^4 - 8*a^6*b^2*...
```

3.561.6 Sympy [F]

$$\int \frac{x^{-1-\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^{-\frac{n}{4}-1}}{a+bx^n+cx^{2n}}\,dx$$

input
```
integrate(x**(-1-1/4*n)/(a+b*x**n+c*x**(2*n)),x)
```

output
```
Integral(x**(-n/4 - 1)/(a + b*x**n + c*x**(2*n)), x)
```

3.561.7 Maxima [F]

$$\int \frac{x^{-1-\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^{-\frac{1}{4}\,n-1}}{cx^{2\,n}+bx^n+a}\,dx$$

input
```
integrate(x^(-1-1/4*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
-4/(a*n*x^(1/4*n)) - integrate((c*x^(7/4*n) + b*x^(3/4*n))/(a*c*x*x^(2*n)
+ a*b*x*x^n + a^2*x), x)
```

3.561.8 Giac [F]

$$\int \frac{x^{-1-\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^{-\frac{1}{4}\,n-1}}{cx^{2\,n}+bx^n+a}\,dx$$

input
```
integrate(x^(-1-1/4*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate(x^(-1/4*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

3.561.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx = \int \frac{1}{x^{\frac{n}{4}+1}\,\left(a+b\,x^n+c\,x^{2\,n}\right)}\,dx$$

input
```
int(1/(x^(n/4 + 1)*(a + b*x^n + c*x^(2*n))),x)
```

output
```
int(1/(x^(n/4 + 1)*(a + b*x^n + c*x^(2*n))), x)
```

3.562 $\int \frac{x^2}{a+bx^n+cx^{2n}}\,dx$

3.562.1 Optimal result

Integrand size = 20, antiderivative size = 140

$$\int \frac{x^2}{a+bx^n+cx^{2n}}\,dx = -\frac{2cx^3\,\text{Hypergeometric2F1}\left(1,\frac{3}{n},\frac{3+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{3\left(b^2-4ac-b\sqrt{b^2-4ac}\right)} - \frac{2cx^3\,\text{Hypergeometric2F1}\left(1,\frac{3}{n},\frac{3+n}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\left(b^2-4ac+b\sqrt{b^2-4ac}\right)}$$

output
```
-2/3*c*x^3*hypergeom([1, 3/n],[(3+n)/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))/(
b^2-4*a*c-b*(-4*a*c+b^2)^(1/2))-2/3*c*x^3*hypergeom([1, 3/n],[(3+n)/n],-2*
c*x^n/(b+(-4*a*c+b^2)^(1/2)))/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))
```

3.562.2 Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.89

$$\int \frac{x^2}{a+bx^n+cx^{2n}}\,dx$$
$$= -\frac{2}{3}cx^3\left(\frac{1-\left(\frac{x^n}{-\frac{-b+\sqrt{b^2-4ac}}{2c}+x^n}\right)^{-3/n}\text{Hypergeometric2F1}\left(-\frac{3}{n},-\frac{3}{n},\frac{-3+n}{n},\frac{b-\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}+2cx^n}\right)}{b^2-4ac-b\sqrt{b^2-4ac}}\right.$$
$$\left.+\frac{1-8^{-1/n}\left(\frac{cx^n}{b+\sqrt{b^2-4ac}+2cx^n}\right)^{-3/n}\text{Hypergeometric2F1}\left(-\frac{3}{n},-\frac{3}{n},\frac{-3+n}{n},\frac{b+\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}+2cx^n}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)}\right)$$

input
```
Integrate[x^2/(a + b*x^n + c*x^(2*n)),x]
```

output
```
(-2*c*x^3*((1 - Hypergeometric2F1[-3/n, -3/n, (-3 + n)/n, (b - Sqrt[b^2 -
4*a*c])/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)]/(x^n/(-1/2*(-b + Sqrt[b^2 - 4*a
*c])/c + x^n))^(3/n))/(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c]) + (1 - Hypergeom
etric2F1[-3/n, -3/n, (-3 + n)/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4
*a*c] + 2*c*x^n)]/(8^n^(-1)*((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^(3
/n)))/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c]))))/3
```

3.562.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1719, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{a+bx^n+cx^{2n}}\,dx$$
$$\Big\downarrow \; 1719$$
$$\frac{2c\int \frac{x^2}{2cx^n+b-\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}-\frac{2c\int \frac{x^2}{2cx^n+b+\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{888}$$

$$\frac{2cx^3 \text{Hypergeometric2F1}\left(1,\frac{3}{n},\frac{n+3}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{3\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2cx^3 \text{Hypergeometric2F1}\left(1,\frac{3}{n},\frac{n+3}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}$$

input
```
Int[x^2/(a + b*x^n + c*x^(2*n)),x]
```

output
```
(2*c*x^3*Hypergeometric2F1[1, 3/n, (3 + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4
*a*c])])/(3*Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])) - (2*c*x^3*Hypergeo
metric2F1[1, 3/n, (3 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(3*Sqrt[
b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c]))
```

3.562.3.1 Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1719
```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int[(d*x)^m/(b - q + 2
*c*x^n), x], x] - Simp[2*(c/q)   Int[(d*x)^m/(b + q + 2*c*x^n), x], x]] /;
FreeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

3.562.4 Maple [F]

$$\int \frac{x^2}{a+b\,x^n+c\,x^{2n}}dx$$

input
```
int(x^2/(a+b*x^n+c*x^(2*n)),x)
```

output
```
int(x^2/(a+b*x^n+c*x^(2*n)),x)
```

3.562.5 Fricas [F]

$$\int \frac{x^2}{a + bx^n + cx^{2n}} \, dx = \int \frac{x^2}{cx^{2n} + bx^n + a} \, dx$$

input
```
integrate(x^2/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
integral(x^2/(c*x^(2*n) + b*x^n + a), x)
```

3.562.6 Sympy [F]

$$\int \frac{x^2}{a + bx^n + cx^{2n}} \, dx = \int \frac{x^2}{a + bx^n + cx^{2n}} \, dx$$

input
```
integrate(x**2/(a+b*x**n+c*x**(2*n)),x)
```

output
```
Integral(x**2/(a + b*x**n + c*x**(2*n)), x)
```

3.562.7 Maxima [F]

$$\int \frac{x^2}{a + bx^n + cx^{2n}} \, dx = \int \frac{x^2}{cx^{2n} + bx^n + a} \, dx$$

input
```
integrate(x^2/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
integrate(x^2/(c*x^(2*n) + b*x^n + a), x)
```

3.562.8 Giac [F]

$$\int \frac{x^2}{a + bx^n + cx^{2n}}\, dx = \int \frac{x^2}{cx^{2n} + bx^n + a}\, dx$$

input
```
integrate(x^2/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate(x^2/(c*x^(2*n) + b*x^n + a), x)
```

3.562.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{a + bx^n + cx^{2n}}\, dx = \int \frac{x^2}{a + b\,x^n + c\,x^{2\,n}}\, dx$$

input
```
int(x^2/(a + b*x^n + c*x^(2*n)),x)
```

output
```
int(x^2/(a + b*x^n + c*x^(2*n)), x)
```

3.563 $\int \frac{x}{a+bx^n+cx^{2n}}\,dx$

3.563.1 Optimal result

Integrand size = 18, antiderivative size = 136

$$\int \frac{x}{a+bx^n+cx^{2n}}\,dx = -\frac{cx^2\,\text{Hypergeometric2F1}\left(1,\frac{2}{n},\frac{2+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{b^2-4ac-b\sqrt{b^2-4ac}} - \frac{cx^2\,\text{Hypergeometric2F1}\left(1,\frac{2}{n},\frac{2+n}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{b^2-4ac+b\sqrt{b^2-4ac}}$$

output
```
-c*x^2*hypergeom([1, 2/n],[(2+n)/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))/(b^2-
4*a*c-b*(-4*a*c+b^2)^(1/2))-c*x^2*hypergeom([1, 2/n],[(2+n)/n],-2*c*x^n/(b
+(-4*a*c+b^2)^(1/2)))/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))
```

3.563.2 Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.93

$$\int \frac{x}{a+bx^n+cx^{2n}}\,dx = -cx^2\left(\frac{1-\left(\frac{x^n}{-\frac{-b+\sqrt{b^2-4ac}}{2c}+x^n}\right)^{-2/n}\text{Hypergeometric2F1}\left(-\frac{2}{n},-\frac{2}{n},\frac{-2+n}{n},\frac{b-\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}+2cx^n}\right)}{b^2-4ac-b\sqrt{b^2-4ac}} + \frac{1-4^{-1/n}\left(\frac{cx^n}{b+\sqrt{b^2-4ac}+2cx^n}\right)^{-2/n}\text{Hypergeometric2F1}\left(-\frac{2}{n},-\frac{2}{n},\frac{-2+n}{n},\frac{b+\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}+2cx^n}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)}\right)$$

input
```
Integrate[x/(a + b*x^n + c*x^(2*n)),x]
```

output
```
-(c*x^2*((1 - Hypergeometric2F1[-2/n, -2/n, (-2 + n)/n, (b - Sqrt[b^2 - 4*
a*c])/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)]/(x^n/(-1/2*(-b + Sqrt[b^2 - 4*a*c
])/c + x^n))^(2/n))/(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c]) + (1 - Hypergeomet
ric2F1[-2/n, -2/n, (-2 + n)/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a
*c] + 2*c*x^n)]/(4^n^(-1)*((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^(2/n
)))/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c]))))
```

3.563.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1719, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{a+bx^n+cx^{2n}} \, dx$$

$$\Big\downarrow \text{ 1719}$$

$$\frac{2c\int \frac{x}{2cx^n+b-\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}-\frac{2c\int \frac{x}{2cx^n+b+\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{cx^2 \text{Hypergeometric2F1}\left(1,\frac{2}{n},\frac{n+2}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{cx^2 \text{Hypergeometric2F1}\left(1,\frac{2}{n},\frac{n+2}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}$$

input
```
Int[x/(a + b*x^n + c*x^(2*n)),x]
```

output
```
(c*x^2*Hypergeometric2F1[1, 2/n, (2 + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a
*c])])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])) - (c*x^2*Hypergeometric
2F1[1, 2/n, (2 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*
a*c]*(b + Sqrt[b^2 - 4*a*c]))
```

3.563.3.1 Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1719
```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int[(d*x)^m/(b - q + 2
*c*x^n), x], x] - Simp[2*(c/q)   Int[(d*x)^m/(b + q + 2*c*x^n), x], x]] /;
FreeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

3.563.4 Maple [F]

$$\int \frac{x}{a+b\,x^{n}+c\,x^{2n}}dx$$

input
```
int(x/(a+b*x^n+c*x^(2*n)),x)
```

output
```
int(x/(a+b*x^n+c*x^(2*n)),x)
```

3.563.5 Fricas [F]

$$\int \frac{x}{a+bx^n+cx^{2n}}\,dx=\int \frac{x}{cx^{2\,n}+bx^n+a}\,dx$$

input
```
integrate(x/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
integral(x/(c*x^(2*n) + b*x^n + a), x)
```

3.563.6 Sympy [F]

$$\int \frac{x}{a+bx^n+cx^{2n}}\,dx=\int \frac{x}{a+bx^n+cx^{2n}}\,dx$$

input
```
integrate(x/(a+b*x**n+c*x**(2*n)),x)
```

output
```
Integral(x/(a + b*x**n + c*x**(2*n)), x)
```

3.563.7 Maxima [F]

$$\int \frac{x}{a+bx^n+cx^{2n}}\,dx=\int \frac{x}{cx^{2n}+bx^n+a}\,dx$$

input
```
integrate(x/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
integrate(x/(c*x^(2*n) + b*x^n + a), x)
```

3.563.8 Giac [F]

$$\int \frac{x}{a+bx^n+cx^{2n}}\,dx=\int \frac{x}{cx^{2n}+bx^n+a}\,dx$$

input
```
integrate(x/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate(x/(c*x^(2*n) + b*x^n + a), x)
```

3.563.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{a + bx^n + cx^{2n}} \, dx = \int \frac{x}{a + b\,x^n + c\,x^{2\,n}} \, dx$$

input
```
int(x/(a + b*x^n + c*x^(2*n)),x)
```

output
```
int(x/(a + b*x^n + c*x^(2*n)), x)
```

3.564 $\int \frac{1}{a+bx^n+cx^{2n}}\,dx$

3.564.1 Optimal result

Integrand size = 16, antiderivative size = 124

$$\int \frac{1}{a+bx^n+cx^{2n}}\,dx=-\frac{2cx \,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{b^2-4ac-b\sqrt{b^2-4ac}}-\frac{2cx \,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{b^2-4ac+b\sqrt{b^2-4ac}}$$

output
```
-2*c*x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))/(b^2-4*
a*c-b*(-4*a*c+b^2)^(1/2))-2*c*x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b+(-4
*a*c+b^2)^(1/2)))/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))
```

3.564.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 261 vs. 2(124) = 248.

Time = 0.46 (sec) , antiderivative size = 261, normalized size of antiderivative = 2.10

$$\int \frac{1}{a+bx^n+cx^{2n}}\,dx$$
$$= -2cx\left(\frac{1-\left(\frac{x^n}{-\frac{-b+\sqrt{b^2-4ac}}{2c}+x^n}\right)^{-1/n}\text{Hypergeometric2F1}\left(-\frac{1}{n},-\frac{1}{n},\frac{-1+n}{n},\frac{b-\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}+2cx^n}\right)}{b^2-4ac-b\sqrt{b^2-4ac}}\right.$$
$$\left.+\frac{1-2^{-1/n}\left(\frac{cx^n}{b+\sqrt{b^2-4ac}+2cx^n}\right)^{-1/n}\text{Hypergeometric2F1}\left(-\frac{1}{n},-\frac{1}{n},\frac{-1+n}{n},\frac{b+\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}+2cx^n}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)}\right)$$

input
```
Integrate[(a + b*x^n + c*x^(2*n))^(-1),x]
```

output
```
-2*c*x*((1 - Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, (b - Sqrt[b^2
 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)]/(x^n/(-1/2*(-b + Sqrt[b^2 -
4*a*c])/c + x^n))^n^(-1))/(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c]) + (1 - Hyper
geometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sq
rt[b^2 - 4*a*c] + 2*c*x^n)]/(2^n^(-1)*((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*
c*x^n))^n^(-1)))/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])))
```

3.564.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.07, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1685, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+bx^n+cx^{2n}}\,dx$$
$$\Big\downarrow \ 1685$$

$$\frac{c\int \frac{1}{cx^n+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx}{\sqrt{b^2-4ac}}-\frac{c\int \frac{1}{cx^n+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{778}$$

$$\frac{2cx\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2cx\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}$$

input
```
Int[(a + b*x^n + c*x^(2*n))^(-1),x]
```

output
```
(2*c*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 -
 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])) - (2*c*x*Hypergeome
tric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt[
b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c]))
```

3.564.3.1 Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 1685
```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^n), x], x] - Simp[
c/q   Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2,
2*n] && NeQ[b^2 - 4*a*c, 0]
```

3.564.4 Maple [F]

$$\int \frac{1}{a+b\,x^{n}+c\,x^{2n}}dx$$

input
```
int(1/(a+b*x^n+c*x^(2*n)),x)
```

output
```
int(1/(a+b*x^n+c*x^(2*n)),x)
```

3.564.5 Fricas [F]

$$\int \frac{1}{a+bx^n+cx^{2n}}\,dx = \int \frac{1}{cx^{2\,n}+bx^n+a}\,dx$$

input
```
integrate(1/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
integral(1/(c*x^(2*n) + b*x^n + a), x)
```

3.564.6 Sympy [F]

$$\int \frac{1}{a+bx^n+cx^{2n}}\,dx = \int \frac{1}{a+bx^n+cx^{2n}}\,dx$$

input
```
integrate(1/(a+b*x**n+c*x**(2*n)),x)
```

output
```
Integral(1/(a + b*x**n + c*x**(2*n)), x)
```

3.564.7 Maxima [F]

$$\int \frac{1}{a+bx^n+cx^{2n}}\,dx = \int \frac{1}{cx^{2\,n}+bx^n+a}\,dx$$

input
```
integrate(1/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
integrate(1/(c*x^(2*n) + b*x^n + a), x)
```

3.564.8 Giac [F]

$$\int \frac{1}{a+bx^n+cx^{2n}}\,dx = \int \frac{1}{cx^{2\,n}+bx^n+a}\,dx$$

input
```
integrate(1/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate(1/(c*x^(2*n) + b*x^n + a), x)
```

3.564.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{a+bx^n+cx^{2n}}\,dx = \int \frac{1}{a+b\,x^n+c\,x^{2\,n}}\,dx$$

input
```
int(1/(a + b*x^n + c*x^(2*n)),x)
```

output
```
int(1/(a + b*x^n + c*x^(2*n)), x)
```

3.565 $\int \frac{1}{x(a+bx^n+cx^{2n})} dx$

3.565.1 Optimal result

Integrand size = 20, antiderivative size = 74

$$\int \frac{1}{x(a+bx^n+cx^{2n})} dx = \frac{b\,\text{arctanh}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}\,n} + \frac{\log(x)}{a} - \frac{\log(a+bx^n+cx^{2n})}{2an}$$

output
```
ln(x)/a-1/2*ln(a+b*x^n+c*x^(2*n))/a/n+b*arctanh((b+2*c*x^n)/(-4*a*c+b^2)^(
1/2))/a/n/(-4*a*c+b^2)^(1/2)
```

3.565.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.97

$$\int \frac{1}{x(a+bx^n+cx^{2n})} dx = -\frac{\frac{2b\arctan\left(\frac{b+2cx^n}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}} - 2\log(x^n) + \log(a+x^n(b+cx^n))}{2an}$$

input
```
Integrate[1/(x*(a + b*x^n + c*x^(2*n))),x]
```

output
```
-1/2*((2*b*ArcTan[(b + 2*c*x^n)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] -
2*Log[x^n] + Log[a + x^n*(b + c*x^n)])/(a*n)
```

3.565.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1693, 1144, 25, 1142, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+bx^n+cx^{2n}\right)}dx$$

$$\Big\downarrow \text{ 1693}$$

$$\frac{\int \frac{x^{-n}}{bx^n+cx^{2n}+a}dx^n}{n}$$

$$\Big\downarrow \text{ 1144}$$

$$\frac{\frac{\int -\frac{cx^n+b}{bx^n+cx^{2n}+a}dx^n}{a}+\frac{\log(x^n)}{a}}{n}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{\log(x^n)}{a}-\frac{\int \frac{cx^n+b}{bx^n+cx^{2n}+a}dx^n}{a}}{n}$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{\frac{\log(x^n)}{a}-\frac{\frac{1}{2}b\int \frac{1}{bx^n+cx^{2n}+a}dx^n+\frac{1}{2}\int \frac{2cx^n+b}{bx^n+cx^{2n}+a}dx^n}{a}}{n}$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{\frac{\log(x^n)}{a}-\frac{\frac{1}{2}\int \frac{2cx^n+b}{bx^n+cx^{2n}+a}dx^n-b\int \frac{1}{-x^{2n}+b^2-4ac}d(2cx^n+b)}{a}}{n}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{\log(x^n)}{a}-\frac{\frac{1}{2}\int \frac{2cx^n+b}{bx^n+cx^{2n}+a}dx^n-\frac{b\,\text{arctanh}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}}{a}}{n}$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{\frac{\log(x^n)}{a}-\frac{\frac{1}{2}\log\left(a+bx^n+cx^{2n}\right)-\frac{b\,\text{arctanh}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}}{a}}{n}$$

input
```
Int[1/(x*(a + b*x^n + c*x^(2*n))),x]
```

output
```
(Log[x^n]/a - (-((b*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4
*a*c]) + Log[a + b*x^n + c*x^(2*n)]/2)/a)/n
```

3.565.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1144
```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Simp[e*(Log[RemoveContent[d + e*x, x]]/(c*d^2 - b*d*e + a*e^2)), x] + S
imp[1/(c*d^2 - b*d*e + a*e^2)   Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2),
x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.565.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 396 vs. 2(68) = 136.

Time = 0.19 (sec) , antiderivative size = 397, normalized size of antiderivative = 5.36

method	result
risch	$\frac{4n^2\ln(x)ac}{4a^2cn^2-ab^2n^2}-\frac{n^2\ln(x)b^2}{4a^2cn^2-ab^2n^2}-\frac{2\ln\left(x^n-\frac{-b^2+\sqrt{-4ab^2c+b^4}}{2bc}\right)c}{(4ac-b^2)n}+\frac{\ln\left(x^n-\frac{-b^2+\sqrt{-4ab^2c+b^4}}{2bc}\right)b^2}{2a(4ac-b^2)n}+\frac{\ln\left(x^n-\frac{-b^2+\sqrt{-4a}}{2bc}\right.}{2a(4c}$

input
```
int(1/x/(a+b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

output
```
4/(4*a^2*c*n^2-a*b^2*n^2)*n^2*ln(x)*a*c-1/(4*a^2*c*n^2-a*b^2*n^2)*n^2*ln(x
)*b^2-2/(4*a*c-b^2)/n*ln(x^n-1/2*(-b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*c+1/2/
a/(4*a*c-b^2)/n*ln(x^n-1/2*(-b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*b^2+1/2/a/(4
*a*c-b^2)/n*ln(x^n-1/2*(-b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*(-4*a*b^2*c+b^4)
^(1/2)-2/(4*a*c-b^2)/n*ln(x^n+1/2*(b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*c+1/2/
a/(4*a*c-b^2)/n*ln(x^n+1/2*(b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*b^2-1/2/a/(4*
a*c-b^2)/n*ln(x^n+1/2*(b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*(-4*a*b^2*c+b^4)^(
1/2)
```

3.565.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 259, normalized size of antiderivative = 3.50

$$\int \frac{1}{x(a+bx^n+cx^{2n})}\,dx = \left[\frac{2\,(b^2-4\,ac)n\log(x)+\sqrt{b^2-4\,ac}b\log\left(\frac{2\,c^2x^{2\,n}+b^2-2\,ac+2\left(bc+\sqrt{b^2-4\,ac}c\right)x^n+\sqrt{b^2-4\,ac}b}{cx^{2\,n}+bx^n+a}\right)-(b^2-4\,ac)\log(cx^2}{2\,(ab^2-4\,a^2c)n}\right.$$

input
```
integrate(1/x/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
[1/2*(2*(b^2 - 4*a*c)*n*log(x) + sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^(2*n) +
b^2 - 2*a*c + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*x^n + sqrt(b^2 - 4*a*c)*b)/(c*
x^(2*n) + b*x^n + a)) - (b^2 - 4*a*c)*log(c*x^(2*n) + b*x^n + a))/((a*b^2
- 4*a^2*c)*n), 1/2*(2*(b^2 - 4*a*c)*n*log(x) + 2*sqrt(-b^2 + 4*a*c)*b*arct
an(-(2*sqrt(-b^2 + 4*a*c)*c*x^n + sqrt(-b^2 + 4*a*c)*b)/(b^2 - 4*a*c)) - (
b^2 - 4*a*c)*log(c*x^(2*n) + b*x^n + a))/((a*b^2 - 4*a^2*c)*n)]
```

3.565.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x(a+bx^n+cx^{2n})}\,dx = \text{Timed out}$$

input
```
integrate(1/x/(a+b*x**n+c*x**(2*n)),x)
```

output
```
Timed out
```

3.565.7 Maxima [F]

$$\int \frac{1}{x(a+bx^n+cx^{2n})}\,dx = \int \frac{1}{(cx^{2\,n}+bx^n+a)x}\,dx$$

input
```
integrate(1/x/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^(2*n) + b*x^n + a)*x), x)
```

3.565.8 Giac [F]

$$\int \frac{1}{x(a+bx^n+cx^{2n})}\,dx = \int \frac{1}{(cx^{2\,n}+bx^n+a)x}\,dx$$

input
```
integrate(1/x/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate(1/((c*x^(2*n) + b*x^n + a)*x), x)
```

3.565.9 Mupad [B] (verification not implemented)

Time = 8.49 (sec) , antiderivative size = 224, normalized size of antiderivative = 3.03

$$\int \frac{1}{x(a+bx^n+cx^{2n})}\,dx = \frac{\ln\left(-\frac{1}{c\,x}-\frac{(2\,a\,n+b\,n\,x^n)\,\left(4\,a\,c+b\,\sqrt{b^2-4\,a\,c}-b^2\right)}{2\,c\,x\,(a\,b^2\,n-4\,a^2\,c\,n)}\right)\,\left(4\,a\,c+b\,\sqrt{b^2-4\,a\,c}-b^2\right)}{2\,\left(a\,b^2\,n-4\,a^2\,c\,n\right)} - \frac{\ln\left(\frac{(2\,a\,n+b\,n\,x^n)\,\left(b\,\sqrt{b^2-4\,a\,c}-4\,a\,c+b^2\right)}{2\,c\,x\,(a\,b^2\,n-4\,a^2\,c\,n)}-\frac{1}{c\,x}\right)\,\left(b\,\sqrt{b^2-4\,a\,c}-4\,a\,c+b^2\right)}{2\,\left(a\,b^2\,n-4\,a^2\,c\,n\right)} + \frac{\ln(x)\,(n-1)}{a\,n}$$

input
```
int(1/(x*(a + b*x^n + c*x^(2*n))),x)
```

output
```
(log(- 1/(c*x) - ((2*a*n + b*n*x^n)*(4*a*c + b*(b^2 - 4*a*c)^(1/2) - b^2))
/(2*c*x*(a*b^2*n - 4*a^2*c*n)))*(4*a*c + b*(b^2 - 4*a*c)^(1/2) - b^2))/(2*
(a*b^2*n - 4*a^2*c*n)) - (log(((2*a*n + b*n*x^n)*(b*(b^2 - 4*a*c)^(1/2) -
4*a*c + b^2))/(2*c*x*(a*b^2*n - 4*a^2*c*n)) - 1/(c*x))*(b*(b^2 - 4*a*c)^(1
/2) - 4*a*c + b^2))/(2*(a*b^2*n - 4*a^2*c*n)) + (log(x)*(n - 1))/(a*n)
```

3.566 $\int \frac{1}{x^2(a+bx^n+cx^{2n})}\,dx$

3.566.1 Optimal result

Integrand size = 20, antiderivative size = 142

$$\int \frac{1}{x^2\left(a+bx^n+cx^{2n}\right)}\,dx=\frac{2c\,\text{Hypergeometric2F1}\left(1,-\frac{1}{n},-\frac{1-n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac-b\sqrt{b^2-4ac}\right)x}+\frac{2c\,\text{Hypergeometric2F1}\left(1,-\frac{1}{n},-\frac{1-n}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac+b\sqrt{b^2-4ac}\right)x}$$

output
```
2*c*hypergeom([1, -1/n],[(-1+n)/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))/x/(b^2
-4*a*c-b*(-4*a*c+b^2)^(1/2))+2*c*hypergeom([1, -1/n],[(-1+n)/n],-2*c*x^n/(
b+(-4*a*c+b^2)^(1/2)))/x/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))
```

3.566.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.69

$$\int \frac{1}{x^2\left(a+bx^n+cx^{2n}\right)}\,dx$$
$$=\frac{2^{1+\frac{1}{n}}c\left(\frac{\left(\frac{cx^n}{b-\sqrt{b^2-4ac}+2cx^n}\right)^{\frac{1}{n}}\text{Hypergeometric2F1}\left(1+\frac{1}{n},1+\frac{1}{n},2+\frac{1}{n},\frac{b-\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}+2cx^n}\right)}{-b+\sqrt{b^2-4ac}-2cx^n}+\frac{x^{-n}\left(\frac{cx^n}{b+\sqrt{b^2-4ac}+2cx^n}\right)^{1+\frac{1}{n}}\text{Hypergeometric}}{c}\right.}{\sqrt{b^2-4ac}(1+n)x}$$

input
```
Integrate[1/(x^2*(a + b*x^n + c*x^(2*n))),x]
```

output
```
(2^(1 + n^(-1))*c*((((c*x^n)/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n))^n^(-1)*Hyp
ergeometric2F1[1 + n^(-1), 1 + n^(-1), 2 + n^(-1), (b - Sqrt[b^2 - 4*a*c])
/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)])/(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^n) +
(((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^(1 + n^(-1))*Hypergeometric2F
1[1 + n^(-1), 1 + n^(-1), 2 + n^(-1), (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^
2 - 4*a*c] + 2*c*x^n)])/(c*x^n)))/(Sqrt[b^2 - 4*a*c]*(1 + n)*x)
```

3.566.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1719, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^n+cx^{2n})} \, dx$$

$$\Big\downarrow \text{ 1719}$$

$$\frac{2c\int \frac{1}{x^2\left(2cx^n+b-\sqrt{b^2-4ac}\right)}dx}{\sqrt{b^2-4ac}}-\frac{2c\int \frac{1}{x^2\left(2cx^n+b+\sqrt{b^2-4ac}\right)}dx}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{2c\,\text{Hypergeometric2F1}\left(1,-\frac{1}{n},-\frac{1-n}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}-\frac{2c\,\text{Hypergeometric2F1}\left(1,-\frac{1}{n},-\frac{1-n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{x\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}$$

input
```
Int[1/(x^2*(a + b*x^n + c*x^(2*n))),x]
```

output
```
(-2*c*Hypergeometric2F1[1, -n^(-1), -((1 - n)/n), (-2*c*x^n)/(b - Sqrt[b^2
 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])*x) + (2*c*Hypergeo
metric2F1[1, -n^(-1), -((1 - n)/n), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(
Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*x)
```

3.566.3.1 Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1719
```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int[(d*x)^m/(b - q + 2
*c*x^n), x], x] - Simp[2*(c/q)   Int[(d*x)^m/(b + q + 2*c*x^n), x], x]] /;
FreeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

3.566.4 Maple [F]

$$\int \frac{1}{x^2\left(a+b\,x^n+c\,x^{2n}\right)}dx$$

input
```
int(1/x^2/(a+b*x^n+c*x^(2*n)),x)
```

output
```
int(1/x^2/(a+b*x^n+c*x^(2*n)),x)
```

3.566.5 Fricas [F]

$$\int \frac{1}{x^2\left(a+bx^n+cx^{2n}\right)}\,dx=\int \frac{1}{(cx^{2\,n}+bx^n+a)x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
integral(1/(c*x^2*x^(2*n) + b*x^2*x^n + a*x^2), x)
```

3.566.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2(a+bx^n+cx^{2n})}\,dx = \text{Timed out}$$

input
```
integrate(1/x**2/(a+b*x**n+c*x**(2*n)),x)
```

output
```
Timed out
```

3.566.7 Maxima [F]

$$\int \frac{1}{x^2(a+bx^n+cx^{2n})}\,dx = \int \frac{1}{(cx^{2n}+bx^n+a)x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^(2*n) + b*x^n + a)*x^2), x)
```

3.566.8 Giac [F]

$$\int \frac{1}{x^2(a+bx^n+cx^{2n})}\,dx = \int \frac{1}{(cx^{2n}+bx^n+a)x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate(1/((c*x^(2*n) + b*x^n + a)*x^2), x)
```

3.566.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a + bx^n + cx^{2n})} \, dx = \int \frac{1}{x^2 \left(a + b\,x^n + c\,x^{2\,n}\right)} \, dx$$

input
```
int(1/(x^2*(a + b*x^n + c*x^(2*n))),x)
```

output
```
int(1/(x^2*(a + b*x^n + c*x^(2*n))), x)
```

3.567 $\int \frac{1}{x^3(a+bx^n+cx^{2n})}\,dx$

3.567.1 Optimal result

Integrand size = 20, antiderivative size = 140

$$\int \frac{1}{x^3\left(a+bx^n+cx^{2n}\right)}\,dx=\frac{c\,\text{Hypergeometric2F1}\left(1,-\frac{2}{n},-\frac{2-n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac-b\sqrt{b^2-4ac}\right)x^2}+\frac{c\,\text{Hypergeometric2F1}\left(1,-\frac{2}{n},-\frac{2-n}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac+b\sqrt{b^2-4ac}\right)x^2}$$

output
```
c*hypergeom([1, -2/n],[(-2+n)/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))/x^2/(b^2
-4*a*c-b*(-4*a*c+b^2)^(1/2))+c*hypergeom([1, -2/n],[(-2+n)/n],-2*c*x^n/(b+
(-4*a*c+b^2)^(1/2)))/x^2/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))
```

3.567.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.84

$$\int \frac{1}{x^3\left(a+bx^n+cx^{2n}\right)}\,dx = \frac{2^{\frac{2+n}{n}}c\left(\frac{\left(\frac{cx^n}{b-\sqrt{b^2-4ac}+2cx^n}\right)^{2/n}\text{Hypergeometric2F1}\left(\frac{2+n}{n},\frac{2+n}{n},2+\frac{2}{n},\frac{b-\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}+2cx^n}\right)}{-b+\sqrt{b^2-4ac}-2cx^n}+\frac{x^{-n}\left(\frac{cx^n}{b+\sqrt{b^2-4ac}+2cx^n}\right)^{\frac{2+n}{n}}\text{Hypergeometric}}{c}\right.}{\sqrt{b^2-4ac}(2+n)x^2}$$

input
```
Integrate[1/(x^3*(a + b*x^n + c*x^(2*n))),x]
```

output
```
(2^((2 + n)/n)*c*((((c*x^n)/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n))^(2/n)*Hyper
geometric2F1[(2 + n)/n, (2 + n)/n, 2 + 2/n, (b - Sqrt[b^2 - 4*a*c])/(b - S
qrt[b^2 - 4*a*c] + 2*c*x^n)])/(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^n) + (((c*x^
n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^((2 + n)/n)*Hypergeometric2F1[(2 + n
)/n, (2 + n)/n, 2 + 2/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c] +
2*c*x^n)])/(c*x^n)))/(Sqrt[b^2 - 4*a*c]*(2 + n)*x^2)
```

3.567.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.07, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1719, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a+bx^n+cx^{2n})} \, dx$$

$$\Big\downarrow \text{ 1719}$$

$$\frac{2c\int \frac{1}{x^3\left(2cx^n+b-\sqrt{b^2-4ac}\right)}dx}{\sqrt{b^2-4ac}}-\frac{2c\int \frac{1}{x^3\left(2cx^n+b+\sqrt{b^2-4ac}\right)}dx}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{c\,\text{Hypergeometric2F1}\left(1,-\frac{2}{n},-\frac{2-n}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x^2\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}-\frac{c\,\text{Hypergeometric2F1}\left(1,-\frac{2}{n},-\frac{2-n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{x^2\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}$$

input
```
Int[1/(x^3*(a + b*x^n + c*x^(2*n))),x]
```

output
```
-((c*Hypergeometric2F1[1, -2/n, -((2 - n)/n), (-2*c*x^n)/(b - Sqrt[b^2 - 4
*a*c])])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])*x^2)) + (c*Hypergeomet
ric2F1[1, -2/n, -((2 - n)/n), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt[b
^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*x^2)
```

3.567.3.1 Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1719
```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int[(d*x)^m/(b - q + 2
*c*x^n), x], x] - Simp[2*(c/q)   Int[(d*x)^m/(b + q + 2*c*x^n), x], x]] /;
FreeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

3.567.4 Maple [F]

$$\int \frac{1}{x^{3} \left(a +b \,x^{n}+c \,x^{2 n}\right)}d x$$

input
```
int(1/x^3/(a+b*x^n+c*x^(2*n)),x)
```

output
```
int(1/x^3/(a+b*x^n+c*x^(2*n)),x)
```

3.567.5 Fricas [F]

$$\int \frac{1}{x^3 (a+bx^n+cx^{2n})} \, dx = \int \frac{1}{(cx^{2\,n} + bx^{n} + a)x^{3}} \, dx$$

input
```
integrate(1/x^3/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
integral(1/(c*x^3*x^(2*n) + b*x^3*x^n + a*x^3), x)
```

3.567.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a + bx^n + cx^{2n})} \, dx = \text{Timed out}$$

input
```
integrate(1/x**3/(a+b*x**n+c*x**(2*n)),x)
```

output
```
Timed out
```

3.567.7 Maxima [F]

$$\int \frac{1}{x^3 (a + bx^n + cx^{2n})} \, dx = \int \frac{1}{(cx^{2n} + bx^n + a)x^3} \, dx$$

input
```
integrate(1/x^3/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^(2*n) + b*x^n + a)*x^3), x)
```

3.567.8 Giac [F]

$$\int \frac{1}{x^3 (a + bx^n + cx^{2n})} \, dx = \int \frac{1}{(cx^{2n} + bx^n + a)x^3} \, dx$$

input
```
integrate(1/x^3/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate(1/((c*x^(2*n) + b*x^n + a)*x^3), x)
```

3.567.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a + bx^n + cx^{2n})} \, dx = \int \frac{1}{x^3 \, (a + b\, x^n + c\, x^{2\, n})} \, dx$$

input
```
int(1/(x^3*(a + b*x^n + c*x^(2*n))),x)
```

output
```
int(1/(x^3*(a + b*x^n + c*x^(2*n))), x)
```

3.568 $\int x^3\sqrt{a+bx^n+cx^{2n}}\,dx$

3.568.1 Optimal result

Integrand size = 22, antiderivative size = 148

$$\int x^3\sqrt{a+bx^n+cx^{2n}}\,dx = \frac{x^4\sqrt{a+bx^n+cx^{2n}}\operatorname{AppellF1}\left(\frac{4}{n},-\frac{1}{2},-\frac{1}{2},\frac{4+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

output
```
1/4*x^4*AppellF1(4/n,-1/2,-1/2,(4+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*
c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(a+b*x^n+c*x^(2*n))^(1/2)/(1+2*c*x^n/(b-(-4*
a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.568.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 365 vs. 2(148) = 296.

Time = 0.61 (sec) , antiderivative size = 365, normalized size of antiderivative = 2.47

$$\int x^3\sqrt{a+bx^n+cx^{2n}}\,dx = \frac{x^4\left(4(4+n)\left(a+x^n(b+cx^n)\right)+an(4+n)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(\frac{4}{n},\frac{1}{2},\frac{1}{2},\frac{4+n}{n},-\frac{}{b+}\right.\right.}{4(4+n)}$$

input
```
Integrate[x^3*Sqrt[a + b*x^n + c*x^(2*n)],x]
```

output
```
(x^4*(4*(4 + n)*(a + x^n*(b + c*x^n)) + a*n*(4 + n)*Sqrt[(b - Sqrt[b^2 - 4
*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*
c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[4/n, 1/2, 1/2, (4 + n)/n, (-2*c*x
^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + 2*b*n*x
^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b
 + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(4 + n)/
n, 1/2, 1/2, 2 + 4/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b +
Sqrt[b^2 - 4*a*c])]))/(4*(4 + n)^2*Sqrt[a + x^n*(b + c*x^n)])
```

3.568.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \sqrt{a+bx^n+cx^{2n}}\,dx$$

$$\Big\downarrow \text{1721}$$

$$\frac{\sqrt{a+bx^n+cx^{2n}}\int x^3 \sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{b+\sqrt{b^2-4ac}}+1}dx}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^4\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(\frac{4}{n},-\frac{1}{2},-\frac{1}{2},\frac{n+4}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[x^3*Sqrt[a + b*x^n + c*x^(2*n)],x]
```

output
```
(x^4*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[4/n, -1/2, -1/2, (4 + n)/n, (-2*
c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(4*Sq
rt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2
 - 4*a*c])])
```

3.568.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

3.568.4 Maple [F]

$$\int x^3\sqrt{a+b\,x^n+c\,x^{2n}}dx$$

input
```
int(x^3*(a+b*x^n+c*x^(2*n))^(1/2),x)
```

output
```
int(x^3*(a+b*x^n+c*x^(2*n))^(1/2),x)
```

3.568.5 Fricas [F(-2)]

Exception generated.

$$\int x^3\sqrt{a+bx^n+cx^{2n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.568.6 Sympy [F]

$$\int x^3\sqrt{a+bx^n+cx^{2n}}\,dx=\int x^3\sqrt{a+bx^n+cx^{2n}}\,dx$$

input
```
integrate(x**3*(a+b*x**n+c*x**(2*n))**(1/2),x)
```

output
```
Integral(x**3*sqrt(a + b*x**n + c*x**(2*n)), x)
```

3.568.7 Maxima [F]

$$\int x^3\sqrt{a+bx^n+cx^{2n}}\,dx=\int \sqrt{cx^{2n}+bx^n+a}\,x^3\,dx$$

input
```
integrate(x^3*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^(2*n) + b*x^n + a)*x^3, x)
```

3.568.8 Giac [F]

$$\int x^3\sqrt{a+bx^n+cx^{2n}}\,dx=\int \sqrt{cx^{2n}+bx^n+a}\,x^3\,dx$$

input
```
integrate(x^3*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^(2*n) + b*x^n + a)*x^3, x)
```

3.568.9 Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{a+bx^n+cx^{2n}}\,dx=\int x^3\,\sqrt{a+b\,x^n+c\,x^{2\,n}}\,dx$$

input
```
int(x^3*(a + b*x^n + c*x^(2*n))^(1/2),x)
```

output
```
int(x^3*(a + b*x^n + c*x^(2*n))^(1/2), x)
```

3.569 $\int x^2\sqrt{a+bx^n+cx^{2n}}\,dx$

3.569.1 Optimal result

Integrand size = 22, antiderivative size = 148

$$\int x^2\sqrt{a+bx^n+cx^{2n}}\,dx = \frac{x^3\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(\frac{3}{n},-\frac{1}{2},-\frac{1}{2},\frac{3+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

output
```
1/3*x^3*AppellF1(3/n,-1/2,-1/2,(3+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*
c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(a+b*x^n+c*x^(2*n))^(1/2)/(1+2*c*x^n/(b-(-4*
a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.569.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 366 vs. 2(148) = 296.

Time = 0.57 (sec) , antiderivative size = 366, normalized size of antiderivative = 2.47

$$\int x^2\sqrt{a+bx^n+cx^{2n}}\,dx = \frac{x^3\left(6(3+n)\left(a+x^n(b+cx^n)\right)+2an(3+n)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{3}{n},\frac{1}{2},\frac{1}{2},\frac{3+n}{n},-\frac{}{b-}\right.\right.}{6(3+n)}$$

input
```
Integrate[x^2*Sqrt[a + b*x^n + c*x^(2*n)],x]
```

output
```
(x^3*(6*(3 + n)*(a + x^n*(b + c*x^n)) + 2*a*n*(3 + n)*Sqrt[(b - Sqrt[b^2 -
 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] +
2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/n, 1/2, 1/2, (3 + n)/n, (-2*c
*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + 3*b*n
*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[
(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(3 + n
)/n, 1/2, 1/2, 2 + 3/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b
+ Sqrt[b^2 - 4*a*c])]))/(6*(3 + n)^2*Sqrt[a + x^n*(b + c*x^n)])
```

3.569.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sqrt{a+bx^n+cx^{2n}}\,dx$$

$$\Big\downarrow \text{ 1721}$$

$$\frac{\sqrt{a+bx^n+cx^{2n}} \int x^2 \sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{b+\sqrt{b^2-4ac}}+1}dx}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

$$\Big\downarrow \text{ 1012}$$

$$\frac{x^3\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(\frac{3}{n},-\frac{1}{2},-\frac{1}{2},\frac{n+3}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[x^2*Sqrt[a + b*x^n + c*x^(2*n)],x]
```

output
```
(x^3*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[3/n, -1/2, -1/2, (3 + n)/n, (-2*
c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(3*Sq
rt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2
 - 4*a*c])])
```

3.569.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

3.569.4 Maple [F]

$$\int x^2 \sqrt{a + b\,x^n + c\,x^{2n}}dx$$

input
```
int(x^2*(a+b*x^n+c*x^(2*n))^(1/2),x)
```

output
```
int(x^2*(a+b*x^n+c*x^(2*n))^(1/2),x)
```

3.569.5 Fricas [F(-2)]

Exception generated.

$$\int x^2 \sqrt{a + bx^n + cx^{2n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.569.6 Sympy [F]

$$\int x^2\sqrt{a+bx^n+cx^{2n}}\,dx=\int x^2\sqrt{a+bx^n+cx^{2n}}\,dx$$

input
```
integrate(x**2*(a+b*x**n+c*x**(2*n))**(1/2),x)
```

output
```
Integral(x**2*sqrt(a + b*x**n + c*x**(2*n)), x)
```

3.569.7 Maxima [F]

$$\int x^2\sqrt{a+bx^n+cx^{2n}}\,dx=\int \sqrt{cx^{2\,n}+bx^n+a}x^2\,dx$$

input
```
integrate(x^2*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^(2*n) + b*x^n + a)*x^2, x)
```

3.569.8 Giac [F]

$$\int x^2\sqrt{a+bx^n+cx^{2n}}\,dx=\int \sqrt{cx^{2\,n}+bx^n+a}x^2\,dx$$

input
```
integrate(x^2*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^(2*n) + b*x^n + a)*x^2, x)
```

3.569.9 Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{a+bx^n+cx^{2n}}\,dx = \int x^2\,\sqrt{a+b\,x^n+c\,x^{2\,n}}\,dx$$

input
```
int(x^2*(a + b*x^n + c*x^(2*n))^(1/2),x)
```

output
```
int(x^2*(a + b*x^n + c*x^(2*n))^(1/2), x)
```

3.570 $\int x\sqrt{a+bx^n+cx^{2n}}\,dx$

3.570.1 Optimal result

Integrand size = 20, antiderivative size = 148

$$\int x\sqrt{a+bx^n+cx^{2n}}\,dx = \frac{x^2\sqrt{a+bx^n+cx^{2n}}\,\operatorname{AppellF1}\left(\frac{2}{n},-\frac{1}{2},-\frac{1}{2},\frac{2+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

output
```
1/2*x^2*AppellF1(2/n,-1/2,-1/2,(2+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*
c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(a+b*x^n+c*x^(2*n))^(1/2)/(1+2*c*x^n/(b-(-4*
a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.570.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 364 vs. 2(148) = 296.

Time = 0.51 (sec) , antiderivative size = 364, normalized size of antiderivative = 2.46

$$\int x\sqrt{a+bx^n+cx^{2n}}\,dx = \frac{x^2\left(2(2+n)\left(a+x^n(b+cx^n)\right)+an(2+n)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(\frac{2}{n},\frac{1}{2},\frac{1}{2},\frac{2+n}{n},-\frac{}{b+}\right.\right.}{2(2+n)^2}$$

input
```
Integrate[x*Sqrt[a + b*x^n + c*x^(2*n)],x]
```

output
```
(x^2*(2*(2 + n)*(a + x^n*(b + c*x^n)) + a*n*(2 + n)*Sqrt[(b - Sqrt[b^2 - 4
*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*
c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[2/n, 1/2, 1/2, (2 + n)/n, (-2*c*x
^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + b*n*x^n
*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b +
 Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(2 + n)/n,
 1/2, 1/2, 2 + 2/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sq
rt[b^2 - 4*a*c])]))/(2*(2 + n)^2*Sqrt[a + x^n*(b + c*x^n)])
```

3.570.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{a+bx^n+cx^{2n}}\,dx$$

$$\Big\downarrow \text{1721}$$

$$\frac{\sqrt{a+bx^n+cx^{2n}}\int x\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{b+\sqrt{b^2-4ac}}+1}dx}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^2\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(\frac{2}{n},-\frac{1}{2},-\frac{1}{2},\frac{n+2}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[x*Sqrt[a + b*x^n + c*x^(2*n)],x]
```

output
```
(x^2*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[2/n, -1/2, -1/2, (2 + n)/n, (-2*
c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(2*Sq
rt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2
 - 4*a*c])])
```

3.570.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

3.570.4 Maple [F]

$$\int x\sqrt{a+b\,x^{n}+c\,x^{2n}}dx$$

input
```
int(x*(a+b*x^n+c*x^(2*n))^(1/2),x)
```

output
```
int(x*(a+b*x^n+c*x^(2*n))^(1/2),x)
```

3.570.5 Fricas [F(-2)]

Exception generated.

$$\int x\sqrt{a+bx^n+cx^{2n}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.570.6 Sympy [F]

$$\int x\sqrt{a+bx^n+cx^{2n}}\,dx=\int x\sqrt{a+bx^n+cx^{2n}}\,dx$$

input
```
integrate(x*(a+b*x**n+c*x**(2*n))**(1/2),x)
```

output
```
Integral(x*sqrt(a + b*x**n + c*x**(2*n)), x)
```

3.570.7 Maxima [F]

$$\int x\sqrt{a+bx^n+cx^{2n}}\,dx=\int \sqrt{cx^{2\,n}+bx^n+a}x\,dx$$

input
```
integrate(x*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^(2*n) + b*x^n + a)*x, x)
```

3.570.8 Giac [F]

$$\int x\sqrt{a+bx^n+cx^{2n}}\,dx=\int \sqrt{cx^{2\,n}+bx^n+a}x\,dx$$

input
```
integrate(x*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^(2*n) + b*x^n + a)*x, x)
```

3.570.9 Mupad [F(-1)]

Timed out.

$$\int x\sqrt{a+bx^n+cx^{2n}}\,dx=\int x\,\sqrt{a+b\,x^n+c\,x^{2\,n}}\,dx$$

input
```
int(x*(a + b*x^n + c*x^(2*n))^(1/2),x)
```

output
```
int(x*(a + b*x^n + c*x^(2*n))^(1/2), x)
```

3.571 $\int \sqrt{a+bx^n+cx^{2n}}\,dx$

3.571.1 Optimal result

Integrand size = 18, antiderivative size = 139

$$\int \sqrt{a+bx^n+cx^{2n}}\,dx = \frac{x\sqrt{a+bx^n+cx^{2n}}\operatorname{AppellF1}\left(\frac{1}{n},-\frac{1}{2},-\frac{1}{2},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

output
```
x*AppellF1(1/n,-1/2,-1/2,1+1/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b
+(-4*a*c+b^2)^(1/2)))*(a+b*x^n+c*x^(2*n))^(1/2)/(1+2*c*x^n/(b-(-4*a*c+b^2)
^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.571.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 351 vs. 2(139) = 278.

Time = 0.51 (sec) , antiderivative size = 351, normalized size of antiderivative = 2.53

$$\int \sqrt{a+bx^n+cx^{2n}}\,dx = \frac{x\left(bnx^n\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(1+\frac{1}{n},\frac{1}{2},\frac{1}{2},2+\frac{1}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)+2(1+n\right.}{2(1+n)^2\sqrt{}}$$

input
```
Integrate[Sqrt[a + b*x^n + c*x^(2*n)],x]
```

output
```
(x*(b*n*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])
]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1
[1 + n^(-1), 1/2, 1/2, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*
c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + 2*(1 + n)*(a + x^n*(b + c*x^n) + a*n*Sq
rt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sq
rt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[n^(-1), 1/2,
1/2, 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[
b^2 - 4*a*c])])))/(2*(1 + n)^2*Sqrt[a + x^n*(b + c*x^n)])
```

3.571.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1686, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^n+cx^{2n}}\,dx$$

$$\Big\downarrow \; 1686$$

$$\frac{\sqrt{a+bx^n+cx^{2n}}\int \sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{b+\sqrt{b^2-4ac}}+1}dx}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

$$\Big\downarrow \; 936$$

$$\frac{x\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(\frac{1}{n},-\frac{1}{2},-\frac{1}{2},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[Sqrt[a + b*x^n + c*x^(2*n)],x]
```

output
```
(x*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[n^(-1), -1/2, -1/2, 1 + n^(-1), (-
2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(Sq
rt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2
 - 4*a*c])])
```

3.571.3.1 Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1686
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^
IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*c*(x^n/(b + Rt[b^2
- 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4*a*c, 2])))^FracPar
t[p]))   Int[(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2*c*(x^n/(b - S
qrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n]
&& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

3.571.4 Maple [F]

$$\int \sqrt{a+b\,x^{n}+c\,x^{2n}}dx$$

input
```
int((a+b*x^n+c*x^(2*n))^(1/2),x)
```

output
```
int((a+b*x^n+c*x^(2*n))^(1/2),x)
```

3.571.5 Fricas [F(-2)]

Exception generated.

$$\int \sqrt{a+bx^n+cx^{2n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.571.6 Sympy [F]

$$\int \sqrt{a+bx^n+cx^{2n}}\,dx=\int \sqrt{a+bx^n+cx^{2n}}\,dx$$

input
```
integrate((a+b*x**n+c*x**(2*n))**(1/2),x)
```

output
```
Integral(sqrt(a + b*x**n + c*x**(2*n)), x)
```

3.571.7 Maxima [F]

$$\int \sqrt{a+bx^n+cx^{2n}}\,dx=\int \sqrt{cx^{2n}+bx^n+a}\,dx$$

input
```
integrate((a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^(2*n) + b*x^n + a), x)
```

3.571.8 Giac [F]

$$\int \sqrt{a+bx^n+cx^{2n}}\,dx=\int \sqrt{cx^{2n}+bx^n+a}\,dx$$

input
```
integrate((a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^(2*n) + b*x^n + a), x)
```

3.571.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx^n+cx^{2n}}\,dx=\int \sqrt{a+b\,x^n+c\,x^{2\,n}}\,dx$$

input
```
int((a + b*x^n + c*x^(2*n))^(1/2),x)
```

output
```
int((a + b*x^n + c*x^(2*n))^(1/2), x)
```

3.572 $\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x}\,dx$

3.572.1 Optimal result

Integrand size = 22, antiderivative size = 119

$$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x}\,dx=\frac{\sqrt{a+bx^n+cx^{2n}}}{n}-\frac{\sqrt{a}\,\text{arctanh}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{n}+\frac{b\,\text{arctanh}\left(\frac{b+2cx^n}{2\sqrt{c}\sqrt{a+bx^n+cx^{2n}}}\right)}{2\sqrt{c}n}$$

output
```
-arctanh(1/2*(2*a+b*x^n)/a^(1/2)/(a+b*x^n+c*x^(2*n))^(1/2))*a^(1/2)/n+1/2*
b*arctanh(1/2*(b+2*c*x^n)/c^(1/2)/(a+b*x^n+c*x^(2*n))^(1/2))/n/c^(1/2)+(a+
b*x^n+c*x^(2*n))^(1/2)/n
```

3.572.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x}\,dx=\frac{2\sqrt{a+x^n(b+cx^n)}+4\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{c}x^n-\sqrt{a+x^n(b+cx^n)}}{\sqrt{a}}\right)-\frac{b\log\left(n\left(b+2cx^n-2\sqrt{c}\sqrt{a+x^n(b+cx^n)}\right)\right)}{\sqrt{c}}}{2n}$$

input
```
Integrate[Sqrt[a + b*x^n + c*x^(2*n)]/x,x]
```

output
```
(2*Sqrt[a + x^n*(b + c*x^n)] + 4*Sqrt[a]*ArcTanh[(Sqrt[c]*x^n - Sqrt[a + x
^n*(b + c*x^n)])/Sqrt[a]] - (b*Log[n*(b + 2*c*x^n - 2*Sqrt[c]*Sqrt[a + x^n
*(b + c*x^n)])])/Sqrt[c])/(2*n)
```

3.572.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {1693, 1162, 25, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x} \, dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{\int x^{-n}\sqrt{bx^n+cx^{2n}+a}dx^n}{n}$$

$$\Big\downarrow \text{1162}$$

$$\frac{\sqrt{a+bx^n+cx^{2n}}-\frac{1}{2}\int -\frac{x^{-n}(bx^n+2a)}{\sqrt{bx^n+cx^{2n}+a}}dx^n}{n}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{1}{2}\int \frac{x^{-n}(bx^n+2a)}{\sqrt{bx^n+cx^{2n}+a}}dx^n+\sqrt{a+bx^n+cx^{2n}}}{n}$$

$$\Big\downarrow \text{1269}$$

$$\frac{\frac{1}{2}\left(b\int \frac{1}{\sqrt{bx^n+cx^{2n}+a}}dx^n+2a\int \frac{x^{-n}}{\sqrt{bx^n+cx^{2n}+a}}dx^n\right)+\sqrt{a+bx^n+cx^{2n}}}{n}$$

$$\Big\downarrow \text{1092}$$

$$\frac{\frac{1}{2}\left(2b\int \frac{1}{4c-x^{2n}}d\frac{2cx^n+b}{\sqrt{bx^n+cx^{2n}+a}}+2a\int \frac{x^{-n}}{\sqrt{bx^n+cx^{2n}+a}}dx^n\right)+\sqrt{a+bx^n+cx^{2n}}}{n}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{1}{2}\left(2a\int \frac{x^{-n}}{\sqrt{bx^n+cx^{2n}+a}}dx^n+\frac{b\text{arctanh}\left(\frac{b+2cx^n}{2\sqrt{c}\sqrt{a+bx^n+cx^{2n}}}\right)}{\sqrt{c}}\right)+\sqrt{a+bx^n+cx^{2n}}}{n}$$

$$\Big\downarrow \; 1154$$

$$\frac{\frac{1}{2}\left(\frac{b\text{arctanh}\left(\frac{b+2cx^n}{2\sqrt{c}\sqrt{a+bx^n+cx^{2n}}}\right)}{\sqrt{c}}-4a\int\frac{1}{4a-x^{2n}}d\frac{bx^n+2a}{\sqrt{bx^n+cx^{2n}+a}}\right)+\sqrt{a+bx^n+cx^{2n}}}{n}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{1}{2}\left(\frac{b\text{arctanh}\left(\frac{b+2cx^n}{2\sqrt{c}\sqrt{a+bx^n+cx^{2n}}}\right)}{\sqrt{c}}-2\sqrt{a}\text{arctanh}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)\right)+\sqrt{a+bx^n+cx^{2n}}}{n}$$

input
```
Int[Sqrt[a + b*x^n + c*x^(2*n)]/x,x]
```

output
```
(Sqrt[a + b*x^n + c*x^(2*n)] + (-2*Sqrt[a]*ArcTanh[(2*a + b*x^n)/(2*Sqrt[a
]*Sqrt[a + b*x^n + c*x^(2*n)])] + (b*ArcTanh[(b + 2*c*x^n)/(2*Sqrt[c]*Sqrt
[a + b*x^n + c*x^(2*n)])])/Sqrt[c])/2)/n
```

3.572.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1162
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p/(e*(m + 2*p + 1))   Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d -
b*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&
!ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.572.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.05

method	result
risch	$\frac{\sqrt{a+b\,e^{n\ln(x)}+c\,e^{2n\ln(x)}}}{n}+\frac{b\ln\left(\frac{\frac{b}{2}+c\,e^{n\ln(x)}}{\sqrt{c}}+\sqrt{a+b\,e^{n\ln(x)}+c\,e^{2n\ln(x)}}\right)}{2n\sqrt{c}}-\frac{\sqrt{a}\,\ln\left(\left(2a+b\,e^{n\ln(x)}+2\sqrt{a}\,\sqrt{a+b\,e^{n\ln(x)}+c\,e^{2n\ln}}\right.\right.}{n}$

input
```
int((a+b*x^n+c*x^(2*n))^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/n*(a+b*exp(n*ln(x))+c*exp(n*ln(x))^2)^(1/2)+1/2/n*b*ln((1/2*b+c*exp(n*ln
(x)))/c^(1/2)+(a+b*exp(n*ln(x))+c*exp(n*ln(x))^2)^(1/2))/c^(1/2)-1/n*a^(1/
2)*ln((2*a+b*exp(n*ln(x))+2*a^(1/2)*(a+b*exp(n*ln(x))+c*exp(n*ln(x))^2)^(1
/2))/exp(n*ln(x)))
```

3.572.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 658, normalized size of antiderivative = 5.53

$$\int \frac{\sqrt{a + bx^n + cx^{2n}}}{x} \, dx$$
$$= \left[\frac{b\sqrt{c}\log\left(-8\,c^2x^{2\,n} - 8\,bcx^n - b^2 - 4\,ac - 4\left(2\,c^{\frac{3}{2}}x^n + b\sqrt{c}\right)\sqrt{cx^{2\,n} + bx^n + a}\right) + 2\,\sqrt{a}c\log\left(-\frac{8\,abx^n + 8\,a^2 \cdots}{\cdots}\right) \cdots}{4\,cn} \right.$$
$$-\frac{b\sqrt{-c}\arctan\left(\frac{\left(2\,\sqrt{-c}cx^n + b\sqrt{-c}\right)\sqrt{cx^{2\,n} + bx^n + a}}{2\,(c^2x^{2\,n} + bcx^n + ac)}\right) - \sqrt{a}c\log\left(-\frac{8\,abx^n + 8\,a^2 + (b^2 + 4\,ac)x^{2\,n} - 4\left(\sqrt{a}bx^n + 2\,a^{\frac{3}{2}}\right)\sqrt{cx^{2\,n} + bx^n + \cdots}}{x^{2\,n}}\right) \cdots}{2\,cn}$$

input
```
integrate((a+b*x^n+c*x^(2*n))^(1/2)/x,x, algorithm="fricas")
```

output
```
[1/4*(b*sqrt(c)*log(-8*c^2*x^(2*n) - 8*b*c*x^n - b^2 - 4*a*c - 4*(2*c^(3/2
)*x^n + b*sqrt(c))*sqrt(c*x^(2*n) + b*x^n + a)) + 2*sqrt(a)*c*log(-(8*a*b*
x^n + 8*a^2 + (b^2 + 4*a*c)*x^(2*n) - 4*(sqrt(a)*b*x^n + 2*a^(3/2))*sqrt(c
*x^(2*n) + b*x^n + a))/x^(2*n)) + 4*sqrt(c*x^(2*n) + b*x^n + a)*c)/(c*n),
-1/2*(b*sqrt(-c)*arctan(1/2*(2*sqrt(-c)*c*x^n + b*sqrt(-c))*sqrt(c*x^(2*n)
 + b*x^n + a)/(c^2*x^(2*n) + b*c*x^n + a*c)) - sqrt(a)*c*log(-(8*a*b*x^n +
 8*a^2 + (b^2 + 4*a*c)*x^(2*n) - 4*(sqrt(a)*b*x^n + 2*a^(3/2))*sqrt(c*x^(2
*n) + b*x^n + a))/x^(2*n)) - 2*sqrt(c*x^(2*n) + b*x^n + a)*c)/(c*n), 1/4*(
4*sqrt(-a)*c*arctan(1/2*(sqrt(-a)*b*x^n + 2*sqrt(-a)*a)*sqrt(c*x^(2*n) + b
*x^n + a)/(a*c*x^(2*n) + a*b*x^n + a^2)) + b*sqrt(c)*log(-8*c^2*x^(2*n) -
8*b*c*x^n - b^2 - 4*a*c - 4*(2*c^(3/2)*x^n + b*sqrt(c))*sqrt(c*x^(2*n) + b
*x^n + a)) + 4*sqrt(c*x^(2*n) + b*x^n + a)*c)/(c*n), 1/2*(2*sqrt(-a)*c*arc
tan(1/2*(sqrt(-a)*b*x^n + 2*sqrt(-a)*a)*sqrt(c*x^(2*n) + b*x^n + a)/(a*c*x
^(2*n) + a*b*x^n + a^2)) - b*sqrt(-c)*arctan(1/2*(2*sqrt(-c)*c*x^n + b*sqr
t(-c))*sqrt(c*x^(2*n) + b*x^n + a)/(c^2*x^(2*n) + b*c*x^n + a*c)) + 2*sqrt
(c*x^(2*n) + b*x^n + a)*c)/(c*n)]
```

3.572.6 Sympy [F]

$$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x}\,dx=\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x}\,dx$$

input
```
integrate((a+b*x**n+c*x**(2*n))**(1/2)/x,x)
```

output
```
Integral(sqrt(a + b*x**n + c*x**(2*n))/x, x)
```

3.572.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x}\,dx=\text{Timed out}$$

input
```
integrate((a+b*x^n+c*x^(2*n))^(1/2)/x,x, algorithm="maxima")
```

output
```
Timed out
```

3.572.8 Giac [F]

$$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x}\,dx=\int \frac{\sqrt{cx^{2n}+bx^n+a}}{x}\,dx$$

input
```
integrate((a+b*x^n+c*x^(2*n))^(1/2)/x,x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^(2*n) + b*x^n + a)/x, x)
```

3.572.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x}\,dx = \int \frac{\sqrt{a+b\,x^n+c\,x^{2\,n}}}{x}\,dx$$

input
```
int((a + b*x^n + c*x^(2*n))^(1/2)/x,x)
```

output
```
int((a + b*x^n + c*x^(2*n))^(1/2)/x, x)
```

3.573 $\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x^2}\,dx$

3.573.1 Optimal result

Integrand size = 22, antiderivative size = 149

$$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x^2}\,dx = -\frac{\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(-\frac{1}{n},-\frac{1}{2},-\frac{1}{2},-\frac{1-n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

output

```
-AppellF1(-1/n,-1/2,-1/2,(-1+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n
/(b+(-4*a*c+b^2)^(1/2)))*(a+b*x^n+c*x^(2*n))^(1/2)/x/(1+2*c*x^n/(b-(-4*a*c
+b^2)^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.573.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 365 vs. 2(149) = 298.

Time = 0.48 (sec) , antiderivative size = 365, normalized size of antiderivative = 2.45

$$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x^2}\,dx = \frac{2(-1+n)\left(a+x^n(b+cx^n)\right)-2a(-1+n)n\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(-\frac{1}{n},\frac{1}{2},\frac{1}{2},\frac{-1+n}{n},\right.}{2(-1+}$$

input
```
Integrate[Sqrt[a + b*x^n + c*x^(2*n)]/x^2,x]
```

output
```
(2*(-1 + n)*(a + x^n*(b + c*x^n)) - 2*a*(-1 + n)*n*Sqrt[(b - Sqrt[b^2 - 4*
a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c
*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[-n^(-1), 1/2, 1/2, (-1 + n)/n, (-2
*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + b*n
*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[
(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(-1 +
n)/n, 1/2, 1/2, 2 - n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/
(-b + Sqrt[b^2 - 4*a*c])])/(2*(-1 + n)^2*x*Sqrt[a + x^n*(b + c*x^n)])
```

3.573.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x^2} \, dx$$

$$\Big\downarrow \text{1721}$$

$$\frac{\sqrt{a+bx^n+cx^{2n}} \int \frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{b+\sqrt{b^2-4ac}}+1}}{x^2}dx}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(-\frac{1}{n},-\frac{1}{2},-\frac{1}{2},-\frac{1-n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[Sqrt[a + b*x^n + c*x^(2*n)]/x^2,x]
```

output
```
-((Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[-n^(-1), -1/2, -1/2, -((1 - n)/n),
 (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/
(x*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqr
t[b^2 - 4*a*c])]))
```

3.573.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

3.573.4 Maple [F]

$$\int \frac{\sqrt{a + b\,x^n + c\,x^{2n}}}{x^2}dx$$

input
```
int((a+b*x^n+c*x^(2*n))^(1/2)/x^2,x)
```

output
```
int((a+b*x^n+c*x^(2*n))^(1/2)/x^2,x)
```

3.573.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a + bx^n + cx^{2n}}}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^n+c*x^(2*n))^(1/2)/x^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.573.6 Sympy [F]

$$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x^2}\,dx=\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x^2}\,dx$$

input
```
integrate((a+b*x**n+c*x**(2*n))**(1/2)/x**2,x)
```

output
```
Integral(sqrt(a + b*x**n + c*x**(2*n))/x**2, x)
```

3.573.7 Maxima [F]

$$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x^2}\,dx=\int \frac{\sqrt{cx^{2\,n}+bx^n+a}}{x^2}\,dx$$

input
```
integrate((a+b*x^n+c*x^(2*n))^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^(2*n) + b*x^n + a)/x^2, x)
```

3.573.8 Giac [F]

$$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x^2}\,dx=\int \frac{\sqrt{cx^{2\,n}+bx^n+a}}{x^2}\,dx$$

input
```
integrate((a+b*x^n+c*x^(2*n))^(1/2)/x^2,x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^(2*n) + b*x^n + a)/x^2, x)
```

3.573.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x^2}\,dx = \int \frac{\sqrt{a+b\,x^n+c\,x^{2\,n}}}{x^2}\,dx$$

input
```
int((a + b*x^n + c*x^(2*n))^(1/2)/x^2,x)
```

output
```
int((a + b*x^n + c*x^(2*n))^(1/2)/x^2, x)
```

3.574 $\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x^3}\,dx$

3.574.1 Optimal result

Integrand size = 22, antiderivative size = 151

$$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x^3}\,dx = -\frac{\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(-\frac{2}{n},-\frac{1}{2},-\frac{1}{2},-\frac{2-n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

output
```
-1/2*AppellF1(-2/n,-1/2,-1/2,(-2+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c
*x^n/(b+(-4*a*c+b^2)^(1/2)))*(a+b*x^n+c*x^(2*n))^(1/2)/x^2/(1+2*c*x^n/(b-(
-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.574.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 365 vs. 2(151) = 302.

Time = 0.49 (sec) , antiderivative size = 365, normalized size of antiderivative = 2.42

$$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x^3}\,dx = \frac{2(-2+n)\left(a+x^n(b+cx^n)\right)-a(-2+n)n\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(-\frac{2}{n},\frac{1}{2},\frac{1}{2},\frac{-2+n}{n},-\right.}{2(-2+n}$$

input
```
Integrate[Sqrt[a + b*x^n + c*x^(2*n)]/x^3,x]
```

output
```
(2*(-2 + n)*(a + x^n*(b + c*x^n)) - a*(-2 + n)*n*Sqrt[(b - Sqrt[b^2 - 4*a*
c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x
^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[-2/n, 1/2, 1/2, (-2 + n)/n, (-2*c*x^
n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + b*n*x^n*
Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b +
Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(-2 + n)/n,
 1/2, 1/2, 2 - 2/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sq
rt[b^2 - 4*a*c])])/(2*(-2 + n)^2*x^2*Sqrt[a + x^n*(b + c*x^n)])
```

3.574.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x^3} \, dx$$

$$\Big\downarrow \; 1721$$

$$\frac{\sqrt{a+bx^n+cx^{2n}} \int \frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{b+\sqrt{b^2-4ac}}+1}}{x^3}dx}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

$$\Big\downarrow \; 1012$$

$$-\frac{\sqrt{a+bx^n+cx^{2n}} \operatorname{AppellF1}\left(-\frac{2}{n},-\frac{1}{2},-\frac{1}{2},-\frac{2-n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[Sqrt[a + b*x^n + c*x^(2*n)]/x^3,x]
```

output
```
-1/2*(Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[-2/n, -1/2, -1/2, -((2 - n)/n),
 (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/
(x^2*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + S
qrt[b^2 - 4*a*c])])
```

3.574.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

3.574.4 Maple [F]

$$\int \frac{\sqrt{a+b\,x^{n}+c\,x^{2n}}}{x^{3}}dx$$

input
```
int((a+b*x^n+c*x^(2*n))^(1/2)/x^3,x)
```

output
```
int((a+b*x^n+c*x^(2*n))^(1/2)/x^3,x)
```

3.574.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^n+c*x^(2*n))^(1/2)/x^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.574.6 Sympy [F]

$$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x^3}\,dx=\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x^3}\,dx$$

input
```
integrate((a+b*x**n+c*x**(2*n))**(1/2)/x**3,x)
```

output
```
Integral(sqrt(a + b*x**n + c*x**(2*n))/x**3, x)
```

3.574.7 Maxima [F]

$$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x^3}\,dx=\int \frac{\sqrt{cx^{2\,n}+bx^n+a}}{x^3}\,dx$$

input
```
integrate((a+b*x^n+c*x^(2*n))^(1/2)/x^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^(2*n) + b*x^n + a)/x^3, x)
```

3.574.8 Giac [F]

$$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x^3}\,dx=\int \frac{\sqrt{cx^{2\,n}+bx^n+a}}{x^3}\,dx$$

input
```
integrate((a+b*x^n+c*x^(2*n))^(1/2)/x^3,x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^(2*n) + b*x^n + a)/x^3, x)
```

3.574.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x^3}\,dx = \int \frac{\sqrt{a+b\,x^n+c\,x^{2\,n}}}{x^3}\,dx$$

input
```
int((a + b*x^n + c*x^(2*n))^(1/2)/x^3,x)
```

output
```
int((a + b*x^n + c*x^(2*n))^(1/2)/x^3, x)
```

3.575 $\int x^3(a+bx^n+cx^{2n})^{3/2}\,dx$

3.575.1 Optimal result

Integrand size = 22, antiderivative size = 149

$$\int x^3\left(a+bx^n+cx^{2n}\right)^{3/2}\,dx=\frac{ax^4\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(\frac{4}{n},-\frac{3}{2},-\frac{3}{2},\frac{4+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

output
```
1/4*a*x^4*AppellF1(4/n,-3/2,-3/2,(4+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-
2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(a+b*x^n+c*x^(2*n))^(1/2)/(1+2*c*x^n/(b-(-
4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.575.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 469 vs. 2(149) = 298.

Time = 1.10 (sec) , antiderivative size = 469, normalized size of antiderivative = 3.15

$$\int x^3\left(a+bx^n+cx^{2n}\right)^{3/2}\,dx=\frac{x^4\Big(2(4+n)\left(3b^2n^2+32ac(2+3n+n^2)+2bc(32+36n+7n^2)\,x^n+8c^2(8+6n+n^2)\,x^{2n}\right)}{}$$

input
```
Integrate[x^3*(a + b*x^n + c*x^(2*n))^(3/2),x]
```

output
```
(x^4*(2*(4 + n)*(3*b^2*n^2 + 32*a*c*(2 + 3*n + n^2) + 2*b*c*(32 + 36*n + 7
*n^2)*x^n + 8*c^2*(8 + 6*n + n^2)*x^(2*n))*(a + x^n*(b + c*x^n)) - 6*a*n^2
*(4 + n)*(b^2 - 2*a*c*(2 + n))*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b -
 Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2
- 4*a*c])]*AppellF1[4/n, 1/2, 1/2, (4 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4
*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - 3*b*n^2*(b^2*(8 + n) - 4*a*c
*(8 + 3*n))*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a
*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*Appe
llF1[(4 + n)/n, 1/2, 1/2, 2 + 4/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*
c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))/(16*c*(2 + n)*(4 + n)^2*(4 + 3*n)*Sqrt[
a + x^n*(b + c*x^n)])
```

3.575.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+bx^n+cx^{2n}\right)^{3/2}\,dx$$

$$\Big\downarrow \text{1721}$$

$$\frac{a\sqrt{a+bx^n+cx^{2n}}\int x^3\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{3/2}\left(\frac{2cx^n}{b+\sqrt{b^2-4ac}}+1\right)^{3/2}dx}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{ax^4\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(\frac{4}{n},-\frac{3}{2},-\frac{3}{2},\frac{n+4}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[x^3*(a + b*x^n + c*x^(2*n))^(3/2),x]
```

output
```
(a*x^4*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[4/n, -3/2, -3/2, (4 + n)/n, (-
2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(4*
Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b
^2 - 4*a*c])])
```

3.575.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

3.575.4 Maple [F]

$$\int x^{3} \left(a +b \,x^{n}+c \,x^{2 n}\right)^{\frac{3}{2}}d x$$

input
```
int(x^3*(a+b*x^n+c*x^(2*n))^(3/2),x)
```

output
```
int(x^3*(a+b*x^n+c*x^(2*n))^(3/2),x)
```

3.575.5 Fricas [F(-2)]

Exception generated.

$$\int x^3 \left(a+bx^n+cx^{2n}\right)^{3/2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.575.6 Sympy [F]

$$\int x^3 \left(a+bx^n+cx^{2n}\right)^{3/2} \, dx = \int x^3 \left(a+bx^n+cx^{2n}\right)^{\frac{3}{2}} \, dx$$

input
```
integrate(x**3*(a+b*x**n+c*x**(2*n))**(3/2),x)
```

output
```
Integral(x**3*(a + b*x**n + c*x**(2*n))**(3/2), x)
```

3.575.7 Maxima [F]

$$\int x^3 \left(a+bx^n+cx^{2n}\right)^{3/2} \, dx = \int \left(cx^{2n}+bx^n+a\right)^{\frac{3}{2}} x^3 \, dx$$

input
```
integrate(x^3*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^(3/2)*x^3, x)
```

3.575.8 Giac [F]

$$\int x^3 \left(a+bx^n+cx^{2n}\right)^{3/2} dx = \int \left(cx^{2n}+bx^n+a\right)^{\frac{3}{2}} x^3 \, dx$$

input
```
integrate(x^3*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^(3/2)*x^3, x)
```

3.575.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \left(a+bx^n+cx^{2n}\right)^{3/2} dx = \int x^3 \left(a+b\,x^n+c\,x^{2\,n}\right)^{3/2} dx$$

input
```
int(x^3*(a + b*x^n + c*x^(2*n))^(3/2),x)
```

output
```
int(x^3*(a + b*x^n + c*x^(2*n))^(3/2), x)
```

3.576 $\int x^2(a+bx^n+cx^{2n})^{3/2}\,dx$

3.576.1 Optimal result

Integrand size = 22, antiderivative size = 149

$$\int x^2\left(a+bx^n+cx^{2n}\right)^{3/2}\,dx=\frac{ax^3\sqrt{a+bx^n+cx^{2n}}\operatorname{AppellF1}\left(\frac{3}{n},-\frac{3}{2},-\frac{3}{2},\frac{3+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

output
```
1/3*a*x^3*AppellF1(3/n,-3/2,-3/2,(3+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-
2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(a+b*x^n+c*x^(2*n))^(1/2)/(1+2*c*x^n/(b-(-
4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.576.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 475 vs. 2(149) = 298.

Time = 1.05 (sec) , antiderivative size = 475, normalized size of antiderivative = 3.19

$$\int x^2\left(a+bx^n+cx^{2n}\right)^{3/2}\,dx=\frac{x^3\left(2(3+n)\left(3b^2n^2+4ac(9+18n+8n^2)+2bc(18+27n+7n^2)x^n+4c^2(9+9n+2n^2)x^{2n}\right.\right.}{}$$

input
```
Integrate[x^2*(a + b*x^n + c*x^(2*n))^(3/2),x]
```

output
```
(x^3*(2*(3 + n)*(3*b^2*n^2 + 4*a*c*(9 + 18*n + 8*n^2) + 2*b*c*(18 + 27*n +
 7*n^2)*x^n + 4*c^2*(9 + 9*n + 2*n^2)*x^(2*n))*(a + x^n*(b + c*x^n)) + 2*a
*n^2*(3 + n)*(-3*b^2 + 4*a*c*(3 + 2*n))*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*
x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b +
Sqrt[b^2 - 4*a*c])]*AppellF1[3/n, 1/2, 1/2, (3 + n)/n, (-2*c*x^n)/(b + Sqr
t[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - 3*b*n^2*(-12*a*c*(2
 + n) + b^2*(6 + n))*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[
b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*
c])]*AppellF1[(3 + n)/n, 1/2, 1/2, 2 + 3/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a
*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))/(24*c*(1 + n)*(3 + n)^2*(3 + 2
*n)*Sqrt[a + x^n*(b + c*x^n)])
```

3.576.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+bx^n+cx^{2n}\right)^{3/2}\,dx$$

$$\Big\downarrow \text{1721}$$

$$\frac{a\sqrt{a+bx^n+cx^{2n}}\int x^2\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{3/2}\left(\frac{2cx^n}{b+\sqrt{b^2-4ac}}+1\right)^{3/2}dx}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{ax^3\sqrt{a+bx^n+cx^{2n}}\operatorname{AppellF1}\left(\frac{3}{n},-\frac{3}{2},-\frac{3}{2},\frac{n+3}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[x^2*(a + b*x^n + c*x^(2*n))^(3/2),x]
```

output
```
(a*x^3*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[3/n, -3/2, -3/2, (3 + n)/n, (-
2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(3*
Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b
^2 - 4*a*c])])
```

3.576.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

3.576.4 Maple [F]

$$\int x^{2} \left(a +b \,x^{n}+c \,x^{2 n}\right)^{\frac{3}{2}}d x$$

input
```
int(x^2*(a+b*x^n+c*x^(2*n))^(3/2),x)
```

output
```
int(x^2*(a+b*x^n+c*x^(2*n))^(3/2),x)
```

3.576.5 Fricas [F(-2)]

Exception generated.

$$\int x^2(a+bx^n+cx^{2n})^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.576.6 Sympy [F]

$$\int x^2(a+bx^n+cx^{2n})^{3/2}\,dx = \int x^2(a+bx^n+cx^{2n})^{\frac{3}{2}}\,dx$$

input
```
integrate(x**2*(a+b*x**n+c*x**(2*n))**(3/2),x)
```

output
```
Integral(x**2*(a + b*x**n + c*x**(2*n))**(3/2), x)
```

3.576.7 Maxima [F]

$$\int x^2(a+bx^n+cx^{2n})^{3/2}\,dx = \int (cx^{2\,n}+bx^n+a)^{\frac{3}{2}}x^2\,dx$$

input
```
integrate(x^2*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^(3/2)*x^2, x)
```

3.576.8 Giac [F]

$$\int x^2 \left(a+bx^n+cx^{2n}\right)^{3/2} dx = \int \left(cx^{2n}+bx^n+a\right)^{\frac{3}{2}} x^2 dx$$

input
```
integrate(x^2*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^(3/2)*x^2, x)
```

3.576.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \left(a+bx^n+cx^{2n}\right)^{3/2} dx = \int x^2 \left(a+b\,x^n+c\,x^{2n}\right)^{3/2} dx$$

input
```
int(x^2*(a + b*x^n + c*x^(2*n))^(3/2),x)
```

output
```
int(x^2*(a + b*x^n + c*x^(2*n))^(3/2), x)
```

3.577 $\int x(a+bx^n+cx^{2n})^{3/2}\,dx$

3.577.1 Optimal result

Integrand size = 20, antiderivative size = 149

$$\int x\left(a+bx^n+cx^{2n}\right)^{3/2}\,dx=\frac{ax^2\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(\frac{2}{n},-\frac{3}{2},-\frac{3}{2},\frac{2+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

output

```
1/2*a*x^2*AppellF1(2/n,-3/2,-3/2,(2+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-
2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(a+b*x^n+c*x^(2*n))^(1/2)/(1+2*c*x^n/(b-(-
4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.577.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 471 vs. 2(149) = 298.

Time = 1.05 (sec) , antiderivative size = 471, normalized size of antiderivative = 3.16

$$\int x\left(a+bx^n+cx^{2n}\right)^{3/2}\,dx=\frac{x^2\Big(2(2+n)\left(3b^2n^2+16ac(1+3n+2n^2)+2bc(8+18n+7n^2)\,x^n+8c^2(2+3n+n^2)\,x^{2n}\right)}{}$$

input
```
Integrate[x*(a + b*x^n + c*x^(2*n))^(3/2),x]
```

output
```
(x^2*(2*(2 + n)*(3*b^2*n^2 + 16*a*c*(1 + 3*n + 2*n^2) + 2*b*c*(8 + 18*n +
7*n^2)*x^n + 8*c^2*(2 + 3*n + n^2)*x^(2*n))*(a + x^n*(b + c*x^n)) - 6*a*n^
2*(2 + n)*(b^2 - 4*a*c*(1 + n))*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b
- Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2
 - 4*a*c])]*AppellF1[2/n, 1/2, 1/2, (2 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 -
4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - 3*b*n^2*(b^2*(4 + n) - 4*a*
c*(4 + 3*n))*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*
a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*App
ellF1[(2 + n)/n, 1/2, 1/2, 2 + 2/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2
*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))/(16*c*(1 + n)*(2 + n)^2*(2 + 3*n)*Sqrt
[a + x^n*(b + c*x^n)])
```

3.577.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+bx^n+cx^{2n}\right)^{3/2}\,dx$$

$$\Big\downarrow \text{ 1721}$$

$$\frac{a\sqrt{a+bx^n+cx^{2n}}\int x\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{3/2}\left(\frac{2cx^n}{b+\sqrt{b^2-4ac}}+1\right)^{3/2}dx}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

$$\Big\downarrow \text{ 1012}$$

$$\frac{ax^2\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(\frac{2}{n},-\frac{3}{2},-\frac{3}{2},\frac{n+2}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[x*(a + b*x^n + c*x^(2*n))^(3/2),x]
```

output
```
(a*x^2*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[2/n, -3/2, -3/2, (2 + n)/n, (-
2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(2*
Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b
^2 - 4*a*c])])
```

3.577.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

3.577.4 Maple [F]

$$\int x\left(a+b\,x^{n}+c\,x^{2n}\right)^{\frac{3}{2}}dx$$

input
```
int(x*(a+b*x^n+c*x^(2*n))^(3/2),x)
```

output
```
int(x*(a+b*x^n+c*x^(2*n))^(3/2),x)
```

3.577.5 Fricas [F(-2)]

Exception generated.

$$\int x(a + bx^n + cx^{2n})^{3/2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.577.6 Sympy [F]

$$\int x(a + bx^n + cx^{2n})^{3/2} \, dx = \int x(a + bx^n + cx^{2n})^{\frac{3}{2}} \, dx$$

input
```
integrate(x*(a+b*x**n+c*x**(2*n))**(3/2),x)
```

output
```
Integral(x*(a + b*x**n + c*x**(2*n))**(3/2), x)
```

3.577.7 Maxima [F]

$$\int x(a + bx^n + cx^{2n})^{3/2} \, dx = \int (cx^{2n} + bx^n + a)^{\frac{3}{2}} x \, dx$$

input
```
integrate(x*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^(3/2)*x, x)
```

3.577.8 Giac [F]

$$\int x\left(a+bx^n+cx^{2n}\right)^{3/2}\,dx=\int\left(cx^{2\,n}+bx^n+a\right)^{\frac{3}{2}}x\,dx$$

input
```
integrate(x*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^(3/2)*x, x)
```

3.577.9 Mupad [F(-1)]

Timed out.

$$\int x\left(a+bx^n+cx^{2n}\right)^{3/2}\,dx=\int x\,\left(a+b\,x^n+c\,x^{2\,n}\right)^{3/2}\,dx$$

input
```
int(x*(a + b*x^n + c*x^(2*n))^(3/2),x)
```

output
```
int(x*(a + b*x^n + c*x^(2*n))^(3/2), x)
```

3.578 $\int (a + bx^n + cx^{2n})^{3/2} \, dx$

3.578.1 Optimal result

Integrand size = 18, antiderivative size = 140

$$\int (a + bx^n + cx^{2n})^{3/2} \, dx = \frac{ax\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(\frac{1}{n},-\frac{3}{2},-\frac{3}{2},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

output
```
a*x*AppellF1(1/n,-3/2,-3/2,1+1/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/
(b+(-4*a*c+b^2)^(1/2)))*(a+b*x^n+c*x^(2*n))^(1/2)/(1+2*c*x^n/(b-(-4*a*c+b^
2)^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.578.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 466 vs. 2(140) = 280.

Time = 1.06 (sec) , antiderivative size = 466, normalized size of antiderivative = 3.33

$$\int (a + bx^n + cx^{2n})^{3/2} \, dx = \frac{x\left(-3bn^2(b^2(2+n)-4ac(2+3n))\,x^n\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(1+\frac{1}{n},\frac{1}{2},\right.\right.}{}$$

input
```
Integrate[(a + b*x^n + c*x^(2*n))^(3/2),x]
```

output
```
(x*(-3*b*n^2*(b^2*(2 + n) - 4*a*c*(2 + 3*n))*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*
c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x
^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1 + n^(-1), 1/2, 1/2, 2 + n^(-1), (-
2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + 2*
(1 + n)*((3*b^2*n^2 + 4*a*c*(1 + 6*n + 8*n^2) + 2*b*c*(2 + 9*n + 7*n^2)*x^
n + 4*c^2*(1 + 3*n + 2*n^2)*x^(2*n))*(a + x^n*(b + c*x^n)) - 3*a*n^2*(b^2
- 4*a*c*(1 + 2*n))*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 -
4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*A
ppellF1[n^(-1), 1/2, 1/2, 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]),
(2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])))/(8*c*(1 + n)^2*(1 + 2*n)*(1 + 3*n)*
Sqrt[a + x^n*(b + c*x^n)])
```

3.578.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1686, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^n+cx^{2n}\right)^{3/2}\,dx$$

$$\Big\downarrow \text{1686}$$

$$\frac{a\sqrt{a+bx^n+cx^{2n}}\int \left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{3/2}\left(\frac{2cx^n}{b+\sqrt{b^2-4ac}}+1\right)^{3/2}dx}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

$$\Big\downarrow \text{936}$$

$$\frac{ax\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(\frac{1}{n},-\frac{3}{2},-\frac{3}{2},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[(a + b*x^n + c*x^(2*n))^(3/2),x]
```

output
```
(a*x*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[n^(-1), -3/2, -3/2, 1 + n^(-1),
(-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(
Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b
^2 - 4*a*c])])
```

3.578.3.1 Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1686
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^
IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*c*(x^n/(b + Rt[b^2
- 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4*a*c, 2])))^FracPar
t[p]))   Int[(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2*c*(x^n/(b - S
qrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n]
&& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

3.578.4 Maple [F]

$$\int \left(a+b\,x^{n}+c\,x^{2n}\right)^{\frac{3}{2}}dx$$

input
```
int((a+b*x^n+c*x^(2*n))^(3/2),x)
```

output
```
int((a+b*x^n+c*x^(2*n))^(3/2),x)
```

3.578.5 Fricas [F(-2)]

Exception generated.

$$\int \left(a+bx^n+cx^{2n}\right)^{3/2} dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (has polynomial part)
```

3.578.6 Sympy [F]

$$\int \left(a+bx^n+cx^{2n}\right)^{3/2} dx = \int \left(a+bx^n+cx^{2n}\right)^{\frac{3}{2}} dx$$

input
```
integrate((a+b*x**n+c*x**(2*n))**(3/2),x)
```

output
```
Integral((a + b*x**n + c*x**(2*n))**(3/2), x)
```

3.578.7 Maxima [F]

$$\int \left(a+bx^n+cx^{2n}\right)^{3/2} dx = \int \left(cx^{2n}+bx^n+a\right)^{\frac{3}{2}} dx$$

input
```
integrate((a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^(3/2), x)
```

3.578.8 Giac [F]

$$\int \left(a+bx^n+cx^{2n}\right)^{3/2} dx = \int \left(cx^{2n}+bx^n+a\right)^{\frac{3}{2}} dx$$

input
```
integrate((a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^(3/2), x)
```

3.578.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^n+cx^{2n}\right)^{3/2}\,dx=\int \left(a+b\,x^n+c\,x^{2\,n}\right)^{3/2}\,dx$$

input
```
int((a + b*x^n + c*x^(2*n))^(3/2),x)
```

output
```
int((a + b*x^n + c*x^(2*n))^(3/2), x)
```

3.579 $\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x}\,dx$

3.579.1 Optimal result

Integrand size = 22, antiderivative size = 173

$$\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x}\,dx = \frac{(b^2+8ac+2bcx^n)\sqrt{a+bx^n+cx^{2n}}}{8cn} + \frac{(a+bx^n+cx^{2n})^{3/2}}{3n} - \frac{a^{3/2}\operatorname{arctanh}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{n} - \frac{b(b^2-12ac)\operatorname{arctanh}\left(\frac{b+2cx^n}{2\sqrt{c}\sqrt{a+bx^n+cx^{2n}}}\right)}{16c^{3/2}n}$$

output
```
1/3*(a+b*x^n+c*x^(2*n))^(3/2)/n-a^(3/2)*arctanh(1/2*(2*a+b*x^n)/a^(1/2)/(a
+b*x^n+c*x^(2*n))^(1/2))/n-1/16*b*(-12*a*c+b^2)*arctanh(1/2*(b+2*c*x^n)/c^
(1/2)/(a+b*x^n+c*x^(2*n))^(1/2))/c^(3/2)/n+1/8*(b^2+8*a*c+2*b*c*x^n)*(a+b*
x^n+c*x^(2*n))^(1/2)/c/n
```

3.579.2 Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x}\,dx = \frac{2\sqrt{c}\sqrt{a+x^n\,(b+cx^n)}(3b^2+14bcx^n+8c(4a+cx^{2n}))+96a^{3/2}c^{3/2}\operatorname{arctanh}\left(\sqrt{c}\ldots\right.}{48c\ldots}$$

input
```
Integrate[(a + b*x^n + c*x^(2*n))^(3/2)/x,x]
```

output
```
(2*Sqrt[c]*Sqrt[a + x^n*(b + c*x^n)]*(3*b^2 + 14*b*c*x^n + 8*c*(4*a + c*x^
(2*n))) + 96*a^(3/2)*c^(3/2)*ArcTanh[(Sqrt[c]*x^n - Sqrt[a + x^n*(b + c*x^
n)])/Sqrt[a]] + 3*(b^3 - 12*a*b*c)*Log[c*n*(b + 2*c*x^n - 2*Sqrt[c]*Sqrt[a
 + x^n*(b + c*x^n)])])/(48*c^(3/2)*n)
```

3.579.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {1693, 1162, 25, 1231, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^n+cx^{2n}\right)^{3/2}}{x}dx$$

$$\Big\downarrow \text{ 1693}$$

$$\frac{\int x^{-n}\left(bx^n+cx^{2n}+a\right)^{3/2}dx^n}{n}$$

$$\Big\downarrow \text{ 1162}$$

$$\frac{\frac{1}{3}\left(a+bx^n+cx^{2n}\right)^{3/2}-\frac{1}{2}\int -x^{-n}(bx^n+2a)\sqrt{bx^n+cx^{2n}+a}dx^n}{n}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{1}{2}\int x^{-n}(bx^n+2a)\sqrt{bx^n+cx^{2n}+a}dx^n+\frac{1}{3}\left(a+bx^n+cx^{2n}\right)^{3/2}}{n}$$

$$\Big\downarrow \text{ 1231}$$

$$\frac{\frac{1}{2}\left(\frac{(8ac+b^2+2bcx^n)\sqrt{a+bx^n+cx^{2n}}}{4c}-\frac{\int -\frac{x^{-n}\left(16a^2c-b\left(b^2-12ac\right)x^n\right)}{2\sqrt{bx^n+cx^{2n}+a}}dx^n}{4c}\right)+\frac{1}{3}\left(a+bx^n+cx^{2n}\right)^{3/2}}{n}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{1}{2}\left(\frac{\int \frac{x^{-n}\left(16a^2c-b\left(b^2-12ac\right)x^n\right)}{\sqrt{bx^n+cx^{2n}+a}}dx^n}{8c}+\frac{\sqrt{a+bx^n+cx^{2n}}(8ac+b^2+2bcx^n)}{4c}\right)+\frac{1}{3}\left(a+bx^n+cx^{2n}\right)^{3/2}}{n}$$

$$\Big\downarrow \text{ 1269}$$

$$\frac{\frac{1}{2}\left(\frac{16a^2c\int \frac{x^{-n}}{\sqrt{bx^n+cx^{2n}+a}}dx^n-b(b^2-12ac)\int \frac{1}{\sqrt{bx^n+cx^{2n}+a}}dx^n}{8c}+\frac{\sqrt{a+bx^n+cx^{2n}}(8ac+b^2+2bcx^n)}{4c}\right)+\frac{1}{3}\left(a+bx^n+cx^{2n}\right)^{3/2}}{n}$$

$$\Big\downarrow \text{1092}$$

$$\frac{\frac{1}{2}\left(\frac{16a^2c\int \frac{x^{-n}}{\sqrt{bx^n+cx^{2n}+a}}dx^n-2b(b^2-12ac)\int \frac{1}{4c-x^{2n}}d\frac{2cx^n+b}{\sqrt{bx^n+cx^{2n}+a}}}{8c}+\frac{\sqrt{a+bx^n+cx^{2n}}(8ac+b^2+2bcx^n)}{4c}\right)+\frac{1}{3}\left(a+bx^n+cx^{2n}\right)^{3/2}}{n}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{1}{2}\left(\frac{16a^2c\int \frac{x^{-n}}{\sqrt{bx^n+cx^{2n}+a}}dx^n-\frac{b\left(b^2-12ac\right)\text{arctanh}\left(\frac{b+2cx^n}{2\sqrt{c}\sqrt{a+bx^n+cx^{2n}}}\right)}{\sqrt{c}}}{8c}+\frac{\sqrt{a+bx^n+cx^{2n}}(8ac+b^2+2bcx^n)}{4c}\right)+\frac{1}{3}\left(a+bx^n+cx^{2n}\right)^{3/}}{n}$$

$$\Big\downarrow \text{1154}$$

$$\frac{\frac{1}{2}\left(\frac{-32a^2c\int \frac{1}{4a-x^{2n}}d\frac{bx^n+2a}{\sqrt{bx^n+cx^{2n}+a}}-\frac{b\left(b^2-12ac\right)\text{arctanh}\left(\frac{b+2cx^n}{2\sqrt{c}\sqrt{a+bx^n+cx^{2n}}}\right)}{\sqrt{c}}}{8c}+\frac{\sqrt{a+bx^n+cx^{2n}}(8ac+b^2+2bcx^n)}{4c}\right)+\frac{1}{3}\left(a+bx^n+c\right.}{n}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{1}{2}\left(\frac{-16a^{3/2}c\text{arctanh}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)-\frac{b\left(b^2-12ac\right)\text{arctanh}\left(\frac{b+2cx^n}{2\sqrt{c}\sqrt{a+bx^n+cx^{2n}}}\right)}{\sqrt{c}}}{8c}+\frac{\sqrt{a+bx^n+cx^{2n}}(8ac+b^2+2bcx^n)}{4c}\right)+\frac{1}{3}\left(a+b\right.}{n}$$

input
```
Int[(a + b*x^n + c*x^(2*n))^(3/2)/x,x]
```

output
```
((a + b*x^n + c*x^(2*n))^(3/2)/3 + (((b^2 + 8*a*c + 2*b*c*x^n)*Sqrt[a + b*
x^n + c*x^(2*n)])/(4*c) + (-16*a^(3/2)*c*ArcTanh[(2*a + b*x^n)/(2*Sqrt[a]*
Sqrt[a + b*x^n + c*x^(2*n)])] - (b*(b^2 - 12*a*c)*ArcTanh[(b + 2*c*x^n)/(2
*Sqrt[c]*Sqrt[a + b*x^n + c*x^(2*n)])])/Sqrt[c])/(8*c))/2)/n
```

3.579. $\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x} dx$

3.579.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1162
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p/(e*(m + 2*p + 1))   Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d -
b*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&
!ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

3.579. $\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x} \, dx$

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.579.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.21

method	result
risch	$\frac{\left(8c^2 e^{2n\ln(x)}+14b\, e^{n\ln(x)} c+32ac+3b^2\right)\sqrt{a+b\,e^{n\ln(x)}+c\,e^{2n\ln(x)}}}{24cn}-\frac{b^3\ln\left(\frac{\frac{b}{2}+c\,e^{n\ln(x)}}{\sqrt{c}}+\sqrt{a+b\,e^{n\ln(x)}+c\,e^{2n\ln(x)}}\right)}{16c^{\frac{3}{2}}n}+\frac{3ab\ln\left(\frac{b}{2}\right.}{}$

input
```
int((a+b*x^n+c*x^(2*n))^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/24*(8*c^2*exp(n*ln(x))^2+14*b*exp(n*ln(x))*c+32*a*c+3*b^2)*(a+b*exp(n*ln
(x))+c*exp(n*ln(x))^2)^(1/2)/c/n-1/16/c^(3/2)/n*b^3*ln((1/2*b+c*exp(n*ln(x
)))/c^(1/2)+(a+b*exp(n*ln(x))+c*exp(n*ln(x))^2)^(1/2))+3/4/c^(1/2)/n*a*b*l
n((1/2*b+c*exp(n*ln(x)))/c^(1/2)+(a+b*exp(n*ln(x))+c*exp(n*ln(x))^2)^(1/2)
)-1/n*a^(3/2)*ln((2*a+b*exp(n*ln(x))+2*a^(1/2)*(a+b*exp(n*ln(x))+c*exp(n*l
n(x))^2)^(1/2))/exp(n*ln(x)))
```

3.579.5 Fricas [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 827, normalized size of antiderivative = 4.78

$$\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x}\,dx = \left[\frac{48\,a^{\frac{3}{2}}c^2\log\left(-\frac{8\,abx^n+8\,a^2+(b^2+4\,ac)x^{2\,n}-4\left(\sqrt{a}bx^n+2\,a^{\frac{3}{2}}\right)\sqrt{cx^{2\,n}+bx^n+a}}{x^{2\,n}}\right)-3\,(b^3-12\,a\right.$$

input
```
integrate((a+b*x^n+c*x^(2*n))^(3/2)/x,x, algorithm="fricas")
```

output
```
[1/96*(48*a^(3/2)*c^2*log(-(8*a*b*x^n + 8*a^2 + (b^2 + 4*a*c)*x^(2*n) - 4*
(sqrt(a)*b*x^n + 2*a^(3/2))*sqrt(c*x^(2*n) + b*x^n + a))/x^(2*n)) - 3*(b^3
 - 12*a*b*c)*sqrt(c)*log(-8*c^2*x^(2*n) - 8*b*c*x^n - b^2 - 4*a*c - 4*(2*c
^(3/2)*x^n + b*sqrt(c))*sqrt(c*x^(2*n) + b*x^n + a)) + 4*(8*c^3*x^(2*n) +
14*b*c^2*x^n + 3*b^2*c + 32*a*c^2)*sqrt(c*x^(2*n) + b*x^n + a))/(c^2*n), 1
/48*(24*a^(3/2)*c^2*log(-(8*a*b*x^n + 8*a^2 + (b^2 + 4*a*c)*x^(2*n) - 4*(s
qrt(a)*b*x^n + 2*a^(3/2))*sqrt(c*x^(2*n) + b*x^n + a))/x^(2*n)) + 3*(b^3 -
 12*a*b*c)*sqrt(-c)*arctan(1/2*(2*sqrt(-c)*c*x^n + b*sqrt(-c))*sqrt(c*x^(2
*n) + b*x^n + a)/(c^2*x^(2*n) + b*c*x^n + a*c)) + 2*(8*c^3*x^(2*n) + 14*b*
c^2*x^n + 3*b^2*c + 32*a*c^2)*sqrt(c*x^(2*n) + b*x^n + a))/(c^2*n), 1/96*(
96*sqrt(-a)*a*c^2*arctan(1/2*(sqrt(-a)*b*x^n + 2*sqrt(-a)*a)*sqrt(c*x^(2*n
) + b*x^n + a)/(a*c*x^(2*n) + a*b*x^n + a^2)) - 3*(b^3 - 12*a*b*c)*sqrt(c)
*log(-8*c^2*x^(2*n) - 8*b*c*x^n - b^2 - 4*a*c - 4*(2*c^(3/2)*x^n + b*sqrt(
c))*sqrt(c*x^(2*n) + b*x^n + a)) + 4*(8*c^3*x^(2*n) + 14*b*c^2*x^n + 3*b^2
*c + 32*a*c^2)*sqrt(c*x^(2*n) + b*x^n + a))/(c^2*n), 1/48*(48*sqrt(-a)*a*c
^2*arctan(1/2*(sqrt(-a)*b*x^n + 2*sqrt(-a)*a)*sqrt(c*x^(2*n) + b*x^n + a)/
(a*c*x^(2*n) + a*b*x^n + a^2)) + 3*(b^3 - 12*a*b*c)*sqrt(-c)*arctan(1/2*(2
*sqrt(-c)*c*x^n + b*sqrt(-c))*sqrt(c*x^(2*n) + b*x^n + a)/(c^2*x^(2*n) + b
*c*x^n + a*c)) + 2*(8*c^3*x^(2*n) + 14*b*c^2*x^n + 3*b^2*c + 32*a*c^2)*sqr
t(c*x^(2*n) + b*x^n + a))/(c^2*n)]
```

3.579.6 Sympy [F]

$$\int \frac{(a + bx^n + cx^{2n})^{3/2}}{x}\, dx = \int \frac{(a + bx^n + cx^{2n})^{\frac{3}{2}}}{x}\, dx$$

input
```
integrate((a+b*x**n+c*x**(2*n))**(3/2)/x,x)
```

output
```
Integral((a + b*x**n + c*x**(2*n))**(3/2)/x, x)
```

3.579.7 Maxima [F]

$$\int \frac{(a + bx^n + cx^{2n})^{3/2}}{x}\, dx = \int \frac{(cx^{2n} + bx^n + a)^{\frac{3}{2}}}{x}\, dx$$

input
```
integrate((a+b*x^n+c*x^(2*n))^(3/2)/x,x, algorithm="maxima")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^(3/2)/x, x)
```

3.579.8 Giac [F]

$$\int \frac{(a + bx^n + cx^{2n})^{3/2}}{x}\, dx = \int \frac{(cx^{2n} + bx^n + a)^{\frac{3}{2}}}{x}\, dx$$

input
```
integrate((a+b*x^n+c*x^(2*n))^(3/2)/x,x, algorithm="giac")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^(3/2)/x, x)
```

3.579.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x}\,dx=\int \frac{(a+b\,x^n+c\,x^{2\,n})^{3/2}}{x}\,dx$$

input
```
int((a + b*x^n + c*x^(2*n))^(3/2)/x,x)
```

output
```
int((a + b*x^n + c*x^(2*n))^(3/2)/x, x)
```

3.580 $\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x^2}\,dx$

3.580.1 Optimal result

Integrand size = 22, antiderivative size = 150

$$\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x^2}\,dx = -\frac{a\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(-\frac{1}{n},-\frac{3}{2},-\frac{3}{2},-\frac{1-n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

output
```
-a*AppellF1(-1/n,-3/2,-3/2,(-1+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x
^n/(b+(-4*a*c+b^2)^(1/2)))*(a+b*x^n+c*x^(2*n))^(1/2)/x/(1+2*c*x^n/(b-(-4*a
*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.580.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 477 vs. 2(150) = 300.

Time = 0.98 (sec) , antiderivative size = 477, normalized size of antiderivative = 3.18

$$\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x^2}\,dx = \frac{2(-1+n)\left(3b^2n^2+4ac(1-6n+8n^2)+2bc(2-9n+7n^2)\,x^n+4c^2(1-3n+\right.}{}$$

input
```
Integrate[(a + b*x^n + c*x^(2*n))^(3/2)/x^2,x]
```

output
```
(2*(-1 + n)*(3*b^2*n^2 + 4*a*c*(1 - 6*n + 8*n^2) + 2*b*c*(2 - 9*n + 7*n^2)
*x^n + 4*c^2*(1 - 3*n + 2*n^2)*x^(2*n))*(a + x^n*(b + c*x^n)) - 6*a*(-1 +
n)*n^2*(b^2 + 4*a*c*(-1 + 2*n))*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b
- Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2
 - 4*a*c])]*AppellF1[-n^(-1), 1/2, 1/2, (-1 + n)/n, (-2*c*x^n)/(b + Sqrt[b
^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - 3*b*(4*a*c*(2 - 3*n) +
 b^2*(-2 + n))*n^2*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^
2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]
)]*AppellF1[(-1 + n)/n, 1/2, 1/2, 2 - n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4
*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/(8*c*(-1 + n)^2*(-1 + 2*n)*(-
1 + 3*n)*x*Sqrt[a + x^n*(b + c*x^n)])
```

3.580.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^n+cx^{2n}\right)^{3/2}}{x^2}\,dx$$

$$\Big\downarrow \text{1721}$$

$$\frac{a\sqrt{a+bx^n+cx^{2n}}\int \frac{\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{3/2}\left(\frac{2cx^n}{b+\sqrt{b^2-4ac}}+1\right)^{3/2}}{x^2}dx}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{a\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(-\frac{1}{n},-\frac{3}{2},-\frac{3}{2},-\frac{1-n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[(a + b*x^n + c*x^(2*n))^(3/2)/x^2,x]
```

output
```
-((a*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[-n^(-1), -3/2, -3/2, -((1 - n)/n
), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]
)/(x*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + S
qrt[b^2 - 4*a*c])]))
```

3.580.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

3.580.4 Maple [F]

$$\int \frac{(a+b\,x^{n}+c\,x^{2n})^{\frac{3}{2}}}{x^{2}}dx$$

input
```
int((a+b*x^n+c*x^(2*n))^(3/2)/x^2,x)
```

output
```
int((a+b*x^n+c*x^(2*n))^(3/2)/x^2,x)
```

3.580.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^n+c*x^(2*n))^(3/2)/x^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.580.6 Sympy [F]

$$\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x^2}\,dx = \int \frac{(a+bx^n+cx^{2n})^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((a+b*x**n+c*x**(2*n))**(3/2)/x**2,x)
```

output
```
Integral((a + b*x**n + c*x**(2*n))**(3/2)/x**2, x)
```

3.580.7 Maxima [F]

$$\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x^2}\,dx = \int \frac{(cx^{2n}+bx^n+a)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((a+b*x^n+c*x^(2*n))^(3/2)/x^2,x, algorithm="maxima")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^(3/2)/x^2, x)
```

3.580.8 Giac [F]

$$\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x^2}\,dx = \int \frac{(cx^{2n}+bx^n+a)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((a+b*x^n+c*x^(2*n))^(3/2)/x^2,x, algorithm="giac")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^(3/2)/x^2, x)
```

3.580.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x^2}\,dx = \int \frac{(a+b\,x^n+c\,x^{2\,n})^{3/2}}{x^2}\,dx$$

input
```
int((a + b*x^n + c*x^(2*n))^(3/2)/x^2,x)
```

output
```
int((a + b*x^n + c*x^(2*n))^(3/2)/x^2, x)
```

3.581 $\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x^3}\,dx$

3.581.1 Optimal result

Integrand size = 22, antiderivative size = 152

$$\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x^3}\,dx = -\frac{a\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(-\frac{2}{n},-\frac{3}{2},-\frac{3}{2},-\frac{2-n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

output
```
-1/2*a*AppellF1(-2/n,-3/2,-3/2,(-2+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2
*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(a+b*x^n+c*x^(2*n))^(1/2)/x^2/(1+2*c*x^n/(b
-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.581.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 471 vs. $2(152) = 304$.

Time = 0.97 (sec) , antiderivative size = 471, normalized size of antiderivative = 3.10

$$\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x^3}\,dx = \frac{2(-2+n)\left(3b^2n^2+16ac(1-3n+2n^2)+2bc(8-18n+7n^2)x^n+8c^2(2-3n\right.}{}$$

input
```
Integrate[(a + b*x^n + c*x^(2*n))^(3/2)/x^3,x]
```

output
```
(2*(-2 + n)*(3*b^2*n^2 + 16*a*c*(1 - 3*n + 2*n^2) + 2*b*c*(8 - 18*n + 7*n^
2)*x^n + 8*c^2*(2 - 3*n + n^2)*x^(2*n))*(a + x^n*(b + c*x^n)) - 6*a*(b^2 +
 4*a*c*(-1 + n))*(-2 + n)*n^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b -
Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 -
 4*a*c])]*AppellF1[-2/n, 1/2, 1/2, (-2 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 -
4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - 3*b*(4*a*c*(4 - 3*n) + b^2*
(-4 + n))*n^2*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4
*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*Ap
pellF1[(-2 + n)/n, 1/2, 1/2, 2 - 2/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]),
(2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/(16*c*(-2 + n)^2*(-1 + n)*(-2 + 3*n)*
x^2*Sqrt[a + x^n*(b + c*x^n)])
```

3.581.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^n+cx^{2n}\right)^{3/2}}{x^3}dx$$

$$\Big\downarrow \text{1721}$$

$$\frac{a\sqrt{a+bx^n+cx^{2n}}\int \frac{\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{3/2}\left(\frac{2cx^n}{b+\sqrt{b^2-4ac}}+1\right)^{3/2}}{x^3}dx}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{a\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(-\frac{2}{n},-\frac{3}{2},-\frac{3}{2},-\frac{2-n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[(a + b*x^n + c*x^(2*n))^(3/2)/x^3,x]
```

output
```
-1/2*(a*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[-2/n, -3/2, -3/2, -((2 - n)/n
), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]
)/(x^2*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b +
 Sqrt[b^2 - 4*a*c])])
```

3.581.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

3.581.4 Maple [F]

$$\int \frac{\left(a+b\,x^{n}+c\,x^{2n}\right)^{\frac{3}{2}}}{x^{3}}dx$$

input
```
int((a+b*x^n+c*x^(2*n))^(3/2)/x^3,x)
```

output
```
int((a+b*x^n+c*x^(2*n))^(3/2)/x^3,x)
```

3.581.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^n+c*x^(2*n))^(3/2)/x^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.581.6 Sympy [F]

$$\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x^3}\,dx = \int \frac{(a+bx^n+cx^{2n})^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((a+b*x**n+c*x**(2*n))**(3/2)/x**3,x)
```

output
```
Integral((a + b*x**n + c*x**(2*n))**(3/2)/x**3, x)
```

3.581.7 Maxima [F]

$$\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x^3}\,dx = \int \frac{(cx^{2n}+bx^n+a)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((a+b*x^n+c*x^(2*n))^(3/2)/x^3,x, algorithm="maxima")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^(3/2)/x^3, x)
```

3.581.8 Giac [F]

$$\int \frac{(a + bx^n + cx^{2n})^{3/2}}{x^3}\,dx = \int \frac{(cx^{2n} + bx^n + a)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((a+b*x^n+c*x^(2*n))^(3/2)/x^3,x, algorithm="giac")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^(3/2)/x^3, x)
```

3.581.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a + bx^n + cx^{2n})^{3/2}}{x^3}\,dx = \int \frac{(a + b\,x^n + c\,x^{2\,n})^{3/2}}{x^3}\,dx$$

input
```
int((a + b*x^n + c*x^(2*n))^(3/2)/x^3,x)
```

output
```
int((a + b*x^n + c*x^(2*n))^(3/2)/x^3, x)
```

3.582 $\int \frac{x^3}{\sqrt{a+bx^n+cx^{2n}}}\,dx$

3.582.1 Optimal result

Integrand size = 22, antiderivative size = 148

$$\int \frac{x^3}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \frac{x^4\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{4}{n},\frac{1}{2},\frac{1}{2},\frac{4+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{a+bx^n+cx^{2n}}}$$

output
```
1/4*x^4*AppellF1(4/n,1/2,1/2,(4+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*
x^n/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*
c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(a+b*x^n+c*x^(2*n))^(1/2)
```

3.582.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.18

$$\int \frac{x^3}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \frac{x^4\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{4}{n},\frac{1}{2},\frac{1}{2},\frac{4+n}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)}{4\sqrt{a+x^n(b+cx^n)}}$$

input
```
Integrate[x^3/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

output
```
(x^4*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[
(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[4/n, 1
/2, 1/2, (4 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sq
rt[b^2 - 4*a*c])])/(4*Sqrt[a + x^n*(b + c*x^n)])
```

3.582.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{a+bx^n+cx^{2n}}}\,dx$$

$$\Big\downarrow \text{1721}$$

$$\frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\int \frac{x^3}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{b+\sqrt{b^2-4ac}}+1}}dx}{\sqrt{a+bx^n+cx^{2n}}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^4\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{4}{n},\frac{1}{2},\frac{1}{2},\frac{n+4}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{a+bx^n+cx^{2n}}}$$

input
```
Int[x^3/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

output
```
(x^4*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + S
qrt[b^2 - 4*a*c])]*AppellF1[4/n, 1/2, 1/2, (4 + n)/n, (-2*c*x^n)/(b - Sqrt
[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(4*Sqrt[a + b*x^n + c
*x^(2*n)])
```

3.582.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

3.582.4 Maple [F]

$$\int \frac{x^3}{\sqrt{a+b\,x^n+c\,x^{2n}}}dx$$

input
```
int(x^3/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

output
```
int(x^3/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

3.582.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.582.6 Sympy [F]

$$\int \frac{x^3}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{x^3}{\sqrt{a+bx^n+cx^{2n}}}\,dx$$

input
```
integrate(x**3/(a+b*x**n+c*x**(2*n))**(1/2),x)
```

output
```
Integral(x**3/sqrt(a + b*x**n + c*x**(2*n)), x)
```

3.582.7 Maxima [F]

$$\int \frac{x^3}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{x^3}{\sqrt{cx^{2n}+bx^n+a}}\,dx$$

input
```
integrate(x^3/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3/sqrt(c*x^(2*n) + b*x^n + a), x)
```

3.582.8 Giac [F]

$$\int \frac{x^3}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{x^3}{\sqrt{cx^{2n}+bx^n+a}}\,dx$$

input
```
integrate(x^3/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^3/sqrt(c*x^(2*n) + b*x^n + a), x)
```

3.582.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{a+bx^n+cx^{2n}}}\,dx=\int \frac{x^3}{\sqrt{a+b\,x^n+c\,x^{2\,n}}}\,dx$$

input
```
int(x^3/(a + b*x^n + c*x^(2*n))^(1/2),x)
```

output
```
int(x^3/(a + b*x^n + c*x^(2*n))^(1/2), x)
```

3.583 $\int \frac{x^2}{\sqrt{a+bx^n+cx^{2n}}}\,dx$

3.583.1 Optimal result

Integrand size = 22, antiderivative size = 148

$$\int \frac{x^2}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \frac{x^3\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,\mathrm{AppellF1}\left(\frac{3}{n},\frac{1}{2},\frac{1}{2},\frac{3+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{a+bx^n+cx^{2n}}}$$

output
```
1/3*x^3*AppellF1(3/n,1/2,1/2,(3+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*
x^n/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*
c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(a+b*x^n+c*x^(2*n))^(1/2)
```

3.583.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.18

$$\int \frac{x^2}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \frac{x^3\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\,\mathrm{AppellF1}\left(\frac{3}{n},\frac{1}{2},\frac{1}{2},\frac{3+n}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)}{3\sqrt{a+x^n(b+cx^n)}}$$

input
```
Integrate[x^2/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

output
```
(x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[
(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/n, 1
/2, 1/2, (3 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sq
rt[b^2 - 4*a*c])])/(3*Sqrt[a + x^n*(b + c*x^n)])
```

3.583.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a+bx^n+cx^{2n}}}dx$$

$$\Big\downarrow \text{ 1721}$$

$$\frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\int \frac{x^2}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{b+\sqrt{b^2-4ac}}+1}}dx}{\sqrt{a+bx^n+cx^{2n}}}$$

$$\Big\downarrow \text{ 1012}$$

$$\frac{x^3\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\text{AppellF1}\left(\frac{3}{n},\frac{1}{2},\frac{1}{2},\frac{n+3}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{a+bx^n+cx^{2n}}}$$

input
```
Int[x^2/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

output
```
(x^3*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + S
qrt[b^2 - 4*a*c])]*AppellF1[3/n, 1/2, 1/2, (3 + n)/n, (-2*c*x^n)/(b - Sqrt
[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(3*Sqrt[a + b*x^n + c
*x^(2*n)])
```

3.583.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

3.583.4 Maple [F]

$$\int \frac{x^2}{\sqrt{a + b\,x^n + c\,x^{2n}}}dx$$

input
```
int(x^2/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

output
```
int(x^2/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

3.583.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2}{\sqrt{a + bx^n + cx^{2n}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.583.6 Sympy [F]

$$\int \frac{x^2}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{x^2}{\sqrt{a+bx^n+cx^{2n}}}\,dx$$

input
```
integrate(x**2/(a+b*x**n+c*x**(2*n))**(1/2),x)
```

output
```
Integral(x**2/sqrt(a + b*x**n + c*x**(2*n)), x)
```

3.583.7 Maxima [F]

$$\int \frac{x^2}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{x^2}{\sqrt{cx^{2n}+bx^n+a}}\,dx$$

input
```
integrate(x^2/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/sqrt(c*x^(2*n) + b*x^n + a), x)
```

3.583.8 Giac [F]

$$\int \frac{x^2}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{x^2}{\sqrt{cx^{2n}+bx^n+a}}\,dx$$

input
```
integrate(x^2/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/sqrt(c*x^(2*n) + b*x^n + a), x)
```

3.583.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a+bx^n+cx^{2n}}}\,dx=\int \frac{x^2}{\sqrt{a+b\,x^n+c\,x^{2\,n}}}\,dx$$

input
```
int(x^2/(a + b*x^n + c*x^(2*n))^(1/2),x)
```

output
```
int(x^2/(a + b*x^n + c*x^(2*n))^(1/2), x)
```

3.584 $\int \frac{x}{\sqrt{a+bx^n+cx^{2n}}}\,dx$

3.584.1 Optimal result

Integrand size = 20, antiderivative size = 148

$$\int \frac{x}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \frac{x^2\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{2}{n},\frac{1}{2},\frac{1}{2},\frac{2+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{a+bx^n+cx^{2n}}}$$

output
```
1/2*x^2*AppellF1(2/n,1/2,1/2,(2+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*
x^n/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*
c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(a+b*x^n+c*x^(2*n))^(1/2)
```

3.584.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.18

$$\int \frac{x}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \frac{x^2\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{2}{n},\frac{1}{2},\frac{1}{2},\frac{2+n}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)}{2\sqrt{a+x^n(b+cx^n)}}$$

input
```
Integrate[x/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

output
```
(x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[
(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[2/n, 1
/2, 1/2, (2 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sq
rt[b^2 - 4*a*c])])/(2*Sqrt[a + x^n*(b + c*x^n)])
```

3.584.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a+bx^n+cx^{2n}}}dx$$

$$\Big\downarrow \text{ 1721}$$

$$\frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\int \frac{x}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{b+\sqrt{b^2-4ac}}+1}}dx}{\sqrt{a+bx^n+cx^{2n}}}$$

$$\Big\downarrow \text{ 1012}$$

$$\frac{x^2\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{2}{n},\frac{1}{2},\frac{1}{2},\frac{n+2}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{a+bx^n+cx^{2n}}}$$

input
```
Int[x/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

output
```
(x^2*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + S
qrt[b^2 - 4*a*c])]*AppellF1[2/n, 1/2, 1/2, (2 + n)/n, (-2*c*x^n)/(b - Sqrt
[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(2*Sqrt[a + b*x^n + c
*x^(2*n)])
```

3.584.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

3.584.4 Maple [F]

$$\int \frac{x}{\sqrt{a+b\,x^{n}+c\,x^{2n}}}dx$$

input
```
int(x/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

output
```
int(x/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

3.584.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.584.6 Sympy [F]

$$\int \frac{x}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{x}{\sqrt{a+bx^n+cx^{2n}}}\,dx$$

input
```
integrate(x/(a+b*x**n+c*x**(2*n))**(1/2),x)
```

output
```
Integral(x/sqrt(a + b*x**n + c*x**(2*n)), x)
```

3.584.7 Maxima [F]

$$\int \frac{x}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{x}{\sqrt{cx^{2n}+bx^n+a}}\,dx$$

input
```
integrate(x/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/sqrt(c*x^(2*n) + b*x^n + a), x)
```

3.584.8 Giac [F]

$$\int \frac{x}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{x}{\sqrt{cx^{2n}+bx^n+a}}\,dx$$

input
```
integrate(x/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
```

output
```
integrate(x/sqrt(c*x^(2*n) + b*x^n + a), x)
```

3.584.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{x}{\sqrt{a+b\,x^n+c\,x^{2\,n}}}\,dx$$

input
```
int(x/(a + b*x^n + c*x^(2*n))^(1/2),x)
```

output
```
int(x/(a + b*x^n + c*x^(2*n))^(1/2), x)
```

3.585 $\int \frac{1}{\sqrt{a+bx^n+cx^{2n}}}\,dx$

3.585.1 Optimal result

Integrand size = 18, antiderivative size = 139

$$\int \frac{1}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \frac{x\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{1}{n},\frac{1}{2},\frac{1}{2},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{a+bx^n+cx^{2n}}}$$

output
```
x*AppellF1(1/n,1/2,1/2,1+1/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(
-4*a*c+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^n/(b
+(-4*a*c+b^2)^(1/2)))^(1/2)/(a+b*x^n+c*x^(2*n))^(1/2)
```

3.585.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.19

$$\int \frac{1}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \frac{x\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{1}{n},\frac{1}{2},\frac{1}{2},1+\frac{1}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)}{\sqrt{a+x^n\left(b+cx^n\right)}}$$

input
```
Integrate[1/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

output
```
(x*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b
 + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[n^(-1),
1/2, 1/2, 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b +
Sqrt[b^2 - 4*a*c])])/Sqrt[a + x^n*(b + c*x^n)]
```

3.585.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1686, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^n+cx^{2n}}}dx$$

$$\Big\downarrow \text{1686}$$

$$\frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\int \frac{1}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{b+\sqrt{b^2-4ac}}+1}}dx}{\sqrt{a+bx^n+cx^{2n}}}$$

$$\Big\downarrow \text{936}$$

$$\frac{x\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{1}{n},\frac{1}{2},\frac{1}{2},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{a+bx^n+cx^{2n}}}$$

input
```
Int[1/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

output
```
(x*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqr
t[b^2 - 4*a*c])]*AppellF1[n^(-1), 1/2, 1/2, 1 + n^(-1), (-2*c*x^n)/(b - Sq
rt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/Sqrt[a + b*x^n + c*
x^(2*n)]
```

3.585.3.1 Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1686
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^
IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*c*(x^n/(b + Rt[b^2
- 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4*a*c, 2])))^FracPar
t[p]))   Int[(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2*c*(x^n/(b - S
qrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n]
&& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

3.585.4 Maple [F]

$$\int \frac{1}{\sqrt{a+b\,x^{n}+c\,x^{2n}}}dx$$

input
```
int(1/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

output
```
int(1/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

3.585.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.585.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{1}{\sqrt{a+bx^n+cx^{2n}}}\,dx$$

input
```
integrate(1/(a+b*x**n+c*x**(2*n))**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*x**n + c*x**(2*n)), x)
```

3.585.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{1}{\sqrt{cx^{2n}+bx^n+a}}\,dx$$

input
```
integrate(1/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(c*x^(2*n) + b*x^n + a), x)
```

3.585.8 Giac [F]

$$\int \frac{1}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{1}{\sqrt{cx^{2n}+bx^n+a}}\,dx$$

input
```
integrate(1/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(c*x^(2*n) + b*x^n + a), x)
```

3.585.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{1}{\sqrt{a+b\,x^n+c\,x^{2\,n}}}\,dx$$

input
```
int(1/(a + b*x^n + c*x^(2*n))^(1/2),x)
```

output
```
int(1/(a + b*x^n + c*x^(2*n))^(1/2), x)
```

3.586 $\int \frac{1}{x\sqrt{a+bx^n+cx^{2n}}}\,dx$

3.586.1 Optimal result

Integrand size = 22, antiderivative size = 47

$$\int \frac{1}{x\sqrt{a+bx^n+cx^{2n}}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{\sqrt{a}n}$$

output
```
-arctanh(1/2*(2*a+b*x^n)/a^(1/2)/(a+b*x^n+c*x^(2*n))^(1/2))/n/a^(1/2)
```

3.586.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

$$\int \frac{1}{x\sqrt{a+bx^n+cx^{2n}}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{c}x^n-\sqrt{a+x^n(b+cx^n)}}{\sqrt{a}}\right)}{\sqrt{a}n}$$

input
```
Integrate[1/(x*Sqrt[a + b*x^n + c*x^(2*n)]),x]
```

output
```
(2*ArcTanh[(Sqrt[c]*x^n - Sqrt[a + x^n*(b + c*x^n)])/Sqrt[a]])/(Sqrt[a]*n)
```

3.586.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1693, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a+bx^n+cx^{2n}}}dx$$
$$\Big\downarrow \text{1693}$$
$$\frac{\int \frac{x^{-n}}{\sqrt{bx^n+cx^{2n}+a}}dx^n}{n}$$
$$\Big\downarrow \text{1154}$$
$$-\frac{2\int \frac{1}{4a-x^{2n}}d\frac{bx^n+2a}{\sqrt{bx^n+cx^{2n}+a}}}{n}$$
$$\Big\downarrow \text{219}$$
$$-\frac{\text{arctanh}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{\sqrt{a}n}$$

input
```
Int[1/(x*Sqrt[a + b*x^n + c*x^(2*n)]),x]
```

output
```
-(ArcTanh[(2*a + b*x^n)/(2*Sqrt[a]*Sqrt[a + b*x^n + c*x^(2*n)])]/(Sqrt[a]*
n))
```

3.586.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.586.4 Maple [F]

$$\int \frac{1}{x\sqrt{a+b\,x^{n}+c\,x^{2n}}}dx$$

input
```
int(1/x/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

output
```
int(1/x/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

3.586.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 148, normalized size of antiderivative = 3.15

$$\int \frac{1}{x\sqrt{a+bx^n+cx^{2n}}}\,dx = \left[\frac{\log\left(-\frac{8\,abx^n+8\,a^2+(b^2+4\,ac)x^{2\,n}-4\left(\sqrt{a}bx^n+2\,a^{\frac{3}{2}}\right)\sqrt{cx^{2\,n}+bx^n+a}}{x^{2\,n}}\right)}{2\,\sqrt{a}n}, \frac{\sqrt{-a}\arctan\left(\frac{\left(\sqrt{-a}bx^n+2\,\sqrt{-a}a\right)\sqrt{cx^{2\,n}+bx^n+a}}{2\,(acx^{2\,n}+abx^n+a^2)}\right)}{an}\right]$$

input
```
integrate(1/x/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
```

output
```
[1/2*log(-(8*a*b*x^n + 8*a^2 + (b^2 + 4*a*c)*x^(2*n) - 4*(sqrt(a)*b*x^n +
2*a^(3/2))*sqrt(c*x^(2*n) + b*x^n + a))/x^(2*n))/(sqrt(a)*n), sqrt(-a)*arc
tan(1/2*(sqrt(-a)*b*x^n + 2*sqrt(-a)*a)*sqrt(c*x^(2*n) + b*x^n + a)/(a*c*x
^(2*n) + a*b*x^n + a^2))/(a*n)]
```

3.586.6 Sympy [F]

$$\int \frac{1}{x\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{1}{x\sqrt{a+bx^n+cx^{2n}}}\,dx$$

input
```
integrate(1/x/(a+b*x**n+c*x**(2*n))**(1/2),x)
```

output
```
Integral(1/(x*sqrt(a + b*x**n + c*x**(2*n))), x)
```

3.586.7 Maxima [F]

$$\int \frac{1}{x\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{1}{\sqrt{cx^{2n}+bx^n+a}\,x}\,dx$$

input
```
integrate(1/x/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^(2*n) + b*x^n + a)*x), x)
```

3.586.8 Giac [F]

$$\int \frac{1}{x\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{1}{\sqrt{cx^{2n}+bx^n+a}\,x}\,dx$$

input
```
integrate(1/x/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^(2*n) + b*x^n + a)*x), x)
```

3.586.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{a+bx^n+cx^{2n}}}\,dx=\int \frac{1}{x\,\sqrt{a+b\,x^n+c\,x^{2\,n}}}\,dx$$

input
```
int(1/(x*(a + b*x^n + c*x^(2*n))^(1/2)),x)
```

output
```
int(1/(x*(a + b*x^n + c*x^(2*n))^(1/2)), x)
```

3.587 $\int \frac{1}{x^2\sqrt{a+bx^n+cx^{2n}}}\,dx$

3.587.1 Optimal result

Integrand size = 22, antiderivative size = 149

$$\int \frac{1}{x^2\sqrt{a+bx^n+cx^{2n}}}\,dx = -\frac{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(-\frac{1}{n},\frac{1}{2},\frac{1}{2},-\frac{1-n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{a+bx^n+cx^{2n}}}$$

output
```
-AppellF1(-1/n,1/2,1/2,(-1+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(
b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^n
/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/x/(a+b*x^n+c*x^(2*n))^(1/2)
```

3.587.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.16

$$\int \frac{1}{x^2\sqrt{a+bx^n+cx^{2n}}}\,dx = -\frac{\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(-\frac{1}{n},\frac{1}{2},\frac{1}{2},\frac{-1+n}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)}{x\sqrt{a+x^n(b+cx^n)}}$$

input
```
Integrate[1/(x^2*Sqrt[a + b*x^n + c*x^(2*n)]),x]
```

output
```
-((Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b
 + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[-n^(-1),
 1/2, 1/2, (-1 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b +
 Sqrt[b^2 - 4*a*c])])/(x*Sqrt[a + x^n*(b + c*x^n)]))
```

3.587.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{a+bx^n+cx^{2n}}}dx$$

$$\Big\downarrow \text{1721}$$

$$\frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\int \frac{1}{x^2\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{b+\sqrt{b^2-4ac}}+1}}dx}{\sqrt{a+bx^n+cx^{2n}}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\operatorname{AppellF1}\left(-\frac{1}{n},\frac{1}{2},\frac{1}{2},-\frac{1-n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{a+bx^n+cx^{2n}}}$$

input
```
Int[1/(x^2*Sqrt[a + b*x^n + c*x^(2*n)]),x]
```

output
```
-((Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqr
t[b^2 - 4*a*c])]*AppellF1[-n^(-1), 1/2, 1/2, -((1 - n)/n), (-2*c*x^n)/(b -
 Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(x*Sqrt[a + b*x^
n + c*x^(2*n)]))
```

3.587.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

3.587.4 Maple [F]

$$\int \frac{1}{x^{2}\sqrt{a+b\,x^{n}+c\,x^{2 n}}}dx$$

input
```
int(1/x^2/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

output
```
int(1/x^2/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

3.587.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^2\sqrt{a+bx^n+cx^{2n}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^2/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.587.6 Sympy [F]

$$\int \frac{1}{x^2\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{1}{x^2\sqrt{a+bx^n+cx^{2n}}}\,dx$$

input
```
integrate(1/x**2/(a+b*x**n+c*x**(2*n))**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(a + b*x**n + c*x**(2*n))), x)
```

3.587.7 Maxima [F]

$$\int \frac{1}{x^2\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{1}{\sqrt{cx^{2n}+bx^n+a}\,x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^(2*n) + b*x^n + a)*x^2), x)
```

3.587.8 Giac [F]

$$\int \frac{1}{x^2\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{1}{\sqrt{cx^{2n}+bx^n+a}\,x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^(2*n) + b*x^n + a)*x^2), x)
```

3.587.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{a+bx^n+cx^{2n}}}\,dx=\int \frac{1}{x^2\,\sqrt{a+b\,x^n+c\,x^{2\,n}}}\,dx$$

input
```
int(1/(x^2*(a + b*x^n + c*x^(2*n))^(1/2)),x)
```

output
```
int(1/(x^2*(a + b*x^n + c*x^(2*n))^(1/2)), x)
```

3.588 $\int \frac{1}{x^3\sqrt{a+bx^n+cx^{2n}}}\,dx$

3.588.1 Optimal result

Integrand size = 22, antiderivative size = 151

$$\int \frac{1}{x^3\sqrt{a+bx^n+cx^{2n}}}\,dx = -\frac{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(-\frac{2}{n},\frac{1}{2},\frac{1}{2},-\frac{2-n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{a+bx^n+cx^{2n}}}$$

output
```
-1/2*AppellF1(-2/n,1/2,1/2,(-2+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x
^n/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c
*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/x^2/(a+b*x^n+c*x^(2*n))^(1/2)
```

3.588.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.16

$$\int \frac{1}{x^3\sqrt{a+bx^n+cx^{2n}}}\,dx = -\frac{\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(-\frac{2}{n},\frac{1}{2},\frac{1}{2},\frac{-2+n}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{a+x^n(b+cx^n)}}$$

input
```
Integrate[1/(x^3*Sqrt[a + b*x^n + c*x^(2*n)]),x]
```

output
```
-1/2*(Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt
[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[-2/n,
 1/2, 1/2, (-2 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b +
 Sqrt[b^2 - 4*a*c])])/(x^2*Sqrt[a + x^n*(b + c*x^n)])
```

3.588.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{a+bx^n+cx^{2n}}}dx$$

$$\Big\downarrow \text{ 1721}$$

$$\frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\int \frac{1}{x^3\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{b+\sqrt{b^2-4ac}}+1}}dx}{\sqrt{a+bx^n+cx^{2n}}}$$

$$\Big\downarrow \text{ 1012}$$

$$-\frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\operatorname{AppellF1}\left(-\frac{2}{n},\frac{1}{2},\frac{1}{2},-\frac{2-n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{a+bx^n+cx^{2n}}}$$

input
```
Int[1/(x^3*Sqrt[a + b*x^n + c*x^(2*n)]),x]
```

output
```
-1/2*(Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b +
Sqrt[b^2 - 4*a*c])]*AppellF1[-2/n, 1/2, 1/2, -((2 - n)/n), (-2*c*x^n)/(b -
 Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(x^2*Sqrt[a + b*
x^n + c*x^(2*n)])
```

3.588.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

3.588.4 Maple [F]

$$\int \frac{1}{x^3\sqrt{a+b\,x^n+c\,x^{2n}}}dx$$

input
```
int(1/x^3/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

output
```
int(1/x^3/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

3.588.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^3\sqrt{a+bx^n+cx^{2n}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^3/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.588.6 Sympy [F]

$$\int \frac{1}{x^3\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{1}{x^3\sqrt{a+bx^n+cx^{2n}}}\,dx$$

input
```
integrate(1/x**3/(a+b*x**n+c*x**(2*n))**(1/2),x)
```

output
```
Integral(1/(x**3*sqrt(a + b*x**n + c*x**(2*n))), x)
```

3.588.7 Maxima [F]

$$\int \frac{1}{x^3\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{1}{\sqrt{cx^{2n}+bx^n+a}\,x^3}\,dx$$

input
```
integrate(1/x^3/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^(2*n) + b*x^n + a)*x^3), x)
```

3.588.8 Giac [F]

$$\int \frac{1}{x^3\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{1}{\sqrt{cx^{2n}+bx^n+a}\,x^3}\,dx$$

input
```
integrate(1/x^3/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^(2*n) + b*x^n + a)*x^3), x)
```

3.588.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{a+bx^n+cx^{2n}}}\,dx=\int \frac{1}{x^3\,\sqrt{a+b\,x^n+c\,x^{2\,n}}}\,dx$$

input
```
int(1/(x^3*(a + b*x^n + c*x^(2*n))^(1/2)),x)
```

output
```
int(1/(x^3*(a + b*x^n + c*x^(2*n))^(1/2)), x)
```

3.589 $\int \frac{x^3}{(a+bx^n+cx^{2n})^{3/2}} dx$

3.589.1 Optimal result

Integrand size = 22, antiderivative size = 151

$$\int \frac{x^3}{(a+bx^n+cx^{2n})^{3/2}} dx = \frac{x^4\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\text{AppellF1}\left(\frac{4}{n},\frac{3}{2},\frac{3}{2},\frac{4+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{4a\sqrt{a+bx^n+cx^{2n}}}$$

output
```
1/4*x^4*AppellF1(4/n,3/2,3/2,(4+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*
x^n/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*
c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/(a+b*x^n+c*x^(2*n))^(1/2)
```

3.589.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 398 vs. 2(151) = 302.

Time = 0.65 (sec) , antiderivative size = 398, normalized size of antiderivative = 2.64

$$\int \frac{x^3}{(a+bx^n+cx^{2n})^{3/2}} dx = \frac{x^4\left(-8(4+n)\left(b^2-2ac+bcx^n\right)-\left(b^2(-8+n)-4ac(-4+n)\right)(4+n)\sqrt{\frac{b-\sqrt{b^2}}{b-}}\right.}{}$$

input
```
Integrate[x^3/(a + b*x^n + c*x^(2*n))^(3/2),x]
```

output
```
(x^4*(-8*(4 + n)*(b^2 - 2*a*c + b*c*x^n) - (b^2*(-8 + n) - 4*a*c*(-4 + n))
*(4 + n)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*S
qrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[4/
n, 1/2, 1/2, (4 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b
+ Sqrt[b^2 - 4*a*c])] + 32*b*c*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/
(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[
b^2 - 4*a*c])]*AppellF1[(4 + n)/n, 1/2, 1/2, 2 + 4/n, (-2*c*x^n)/(b + Sqrt
[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))/(4*a*(-b^2 + 4*a*c)*
n*(4 + n)*Sqrt[a + x^n*(b + c*x^n)])
```

3.589.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx^n+cx^{2n})^{3/2}}\,dx$$

$$\Big\downarrow \; 1721$$

$$\frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\int \frac{x^3}{\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{3/2}\left(\frac{2cx^n}{b+\sqrt{b^2-4ac}}+1\right)^{3/2}}dx}{a\sqrt{a+bx^n+cx^{2n}}}$$

$$\Big\downarrow \; 1012$$

$$\frac{x^4\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{4}{n},\frac{3}{2},\frac{3}{2},\frac{n+4}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{4a\sqrt{a+bx^n+cx^{2n}}}$$

input
```
Int[x^3/(a + b*x^n + c*x^(2*n))^(3/2),x]
```

output
```
(x^4*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + S
qrt[b^2 - 4*a*c])]*AppellF1[4/n, 3/2, 3/2, (4 + n)/n, (-2*c*x^n)/(b - Sqrt
[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(4*a*Sqrt[a + b*x^n +
 c*x^(2*n)])
```

3.589.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

3.589.4 Maple [F]

$$\int \frac{x^3}{(a+b\,x^n+c\,x^{2n})^{\frac{3}{2}}}dx$$

input
```
int(x^3/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

output
```
int(x^3/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

3.589.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.589.6 Sympy [F]

$$\int \frac{x^3}{\left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx = \int \frac{x^3}{\left(a+bx^n+cx^{2n}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3/(a+b*x**n+c*x**(2*n))**(3/2),x)
```

output
```
Integral(x**3/(a + b*x**n + c*x**(2*n))**(3/2), x)
```

3.589.7 Maxima [F]

$$\int \frac{x^3}{\left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx = \int \frac{x^3}{\left(cx^{2n}+bx^n+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^3/(c*x^(2*n) + b*x^n + a)^(3/2), x)
```

3.589.8 Giac [F]

$$\int \frac{x^3}{\left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx = \int \frac{x^3}{\left(cx^{2n}+bx^n+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
```

output
```
integrate(x^3/(c*x^(2*n) + b*x^n + a)^(3/2), x)
```

3.589.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+bx^n+cx^{2n})^{3/2}}\,dx=\int \frac{x^3}{\left(a+b\,x^n+c\,x^{2\,n}\right)^{3/2}}\,dx$$

input
```
int(x^3/(a + b*x^n + c*x^(2*n))^(3/2),x)
```

output
```
int(x^3/(a + b*x^n + c*x^(2*n))^(3/2), x)
```

3.590 $\int \frac{x^2}{(a+bx^n+cx^{2n})^{3/2}}\,dx$

3.590.1 Optimal result

Integrand size = 22, antiderivative size = 151

$$\int \frac{x^2}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \frac{x^3\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,\mathrm{AppellF1}\left(\frac{3}{n},\frac{3}{2},\frac{3}{2},\frac{3+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-\cdots}}\right.}{3a\sqrt{a+bx^n+cx^{2n}}}$$

output
```
1/3*x^3*AppellF1(3/n,3/2,3/2,(3+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*
x^n/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*
c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/(a+b*x^n+c*x^(2*n))^(1/2)
```

3.590.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 398 vs. 2(151) = 302.

Time = 0.68 (sec) , antiderivative size = 398, normalized size of antiderivative = 2.64

$$\int \frac{x^2}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \frac{x^3\left(-6(3+n)\left(b^2-2ac+bcx^n\right)-\left(b^2(-6+n)-4ac(-3+n)\right)(3+n)\sqrt{\frac{b-\sqrt{b^2\cdots}}{b-\cdots}}\right.}{\cdots}$$

input
```
Integrate[x^2/(a + b*x^n + c*x^(2*n))^(3/2),x]
```

output
```
(x^3*(-6*(3 + n)*(b^2 - 2*a*c + b*c*x^n) - (b^2*(-6 + n) - 4*a*c*(-3 + n))
*(3 + n)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*S
qrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/
n, 1/2, 1/2, (3 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b
+ Sqrt[b^2 - 4*a*c])] + 18*b*c*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/
(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[
b^2 - 4*a*c])]*AppellF1[(3 + n)/n, 1/2, 1/2, 2 + 3/n, (-2*c*x^n)/(b + Sqrt
[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))/(3*a*(-b^2 + 4*a*c)*
n*(3 + n)*Sqrt[a + x^n*(b + c*x^n)])
```

3.590.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^n+cx^{2n})^{3/2}} \, dx$$

$$\Big\downarrow \text{ 1721}$$

$$\frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\int \frac{x^2}{\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{3/2}\left(\frac{2cx^n}{b+\sqrt{b^2-4ac}}+1\right)^{3/2}}dx}{a\sqrt{a+bx^n+cx^{2n}}}$$

$$\Big\downarrow \text{ 1012}$$

$$\frac{x^3\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{3}{n},\frac{3}{2},\frac{3}{2},\frac{n+3}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3a\sqrt{a+bx^n+cx^{2n}}}$$

input
```
Int[x^2/(a + b*x^n + c*x^(2*n))^(3/2),x]
```

output
```
(x^3*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + S
qrt[b^2 - 4*a*c])]*AppellF1[3/n, 3/2, 3/2, (3 + n)/n, (-2*c*x^n)/(b - Sqrt
[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(3*a*Sqrt[a + b*x^n +
 c*x^(2*n)])
```

3.590.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

3.590.4 Maple [F]

$$\int \frac{x^2}{(a+b\,x^n+c\,x^{2n})^{\frac{3}{2}}}dx$$

input
```
int(x^2/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

output
```
int(x^2/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

3.590.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.590.6 Sympy [F]

$$\int \frac{x^2}{(a + bx^n + cx^{2n})^{3/2}}\,dx = \int \frac{x^2}{(a + bx^n + cx^{2n})^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2/(a+b*x**n+c*x**(2*n))**(3/2),x)
```

output
```
Integral(x**2/(a + b*x**n + c*x**(2*n))**(3/2), x)
```

3.590.7 Maxima [F]

$$\int \frac{x^2}{(a + bx^n + cx^{2n})^{3/2}}\,dx = \int \frac{x^2}{(cx^{2\,n} + bx^n + a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^2/(c*x^(2*n) + b*x^n + a)^(3/2), x)
```

3.590.8 Giac [F]

$$\int \frac{x^2}{(a + bx^n + cx^{2n})^{3/2}}\,dx = \int \frac{x^2}{(cx^{2\,n} + bx^n + a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
```

output
```
integrate(x^2/(c*x^(2*n) + b*x^n + a)^(3/2), x)
```

3.590.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^n+cx^{2n})^{3/2}}\,dx=\int \frac{x^2}{(a+b\,x^n+c\,x^{2\,n})^{3/2}}\,dx$$

input
```
int(x^2/(a + b*x^n + c*x^(2*n))^(3/2),x)
```

output
```
int(x^2/(a + b*x^n + c*x^(2*n))^(3/2), x)
```

3.591 $\int \frac{x}{(a+bx^n+cx^{2n})^{3/2}}\,dx$

3.591.1 Optimal result

Integrand size = 20, antiderivative size = 151

$$\int \frac{x}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \frac{x^2\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{2}{n},\frac{3}{2},\frac{3}{2},\frac{2+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2a\sqrt{a+bx^n+cx^{2n}}}$$

output
```
1/2*x^2*AppellF1(2/n,3/2,3/2,(2+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*
x^n/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*
c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/(a+b*x^n+c*x^(2*n))^(1/2)
```

3.591.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 398 vs. $2(151) = 302$.

Time = 0.63 (sec) , antiderivative size = 398, normalized size of antiderivative = 2.64

$$\int \frac{x}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \frac{x^2\left(-4(2+n)\left(b^2-2ac+bcx^n\right)-\left(b^2(-4+n)-4ac(-2+n)\right)(2+n)\sqrt{\frac{b-\sqrt{b^2}}{b-}}\right.}{}$$

input
```
Integrate[x/(a + b*x^n + c*x^(2*n))^(3/2),x]
```

output
```
(x^2*(-4*(2 + n)*(b^2 - 2*a*c + b*c*x^n) - (b^2*(-4 + n) - 4*a*c*(-2 + n))
*(2 + n)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*S
qrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[2/
n, 1/2, 1/2, (2 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b
+ Sqrt[b^2 - 4*a*c])] + 8*b*c*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(
b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b
^2 - 4*a*c])]*AppellF1[(2 + n)/n, 1/2, 1/2, 2 + 2/n, (-2*c*x^n)/(b + Sqrt[
b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))/(2*a*(-b^2 + 4*a*c)*n
*(2 + n)*Sqrt[a + x^n*(b + c*x^n)])
```

3.591.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx^n+cx^{2n})^{3/2}}\,dx$$

$$\Big\downarrow \text{1721}$$

$$\frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\int \frac{x}{\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{3/2}\left(\frac{2cx^n}{b+\sqrt{b^2-4ac}}+1\right)^{3/2}}dx}{a\sqrt{a+bx^n+cx^{2n}}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^2\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{2}{n},\frac{3}{2},\frac{3}{2},\frac{n+2}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2a\sqrt{a+bx^n+cx^{2n}}}$$

input
```
Int[x/(a + b*x^n + c*x^(2*n))^(3/2),x]
```

output
```
(x^2*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + S
qrt[b^2 - 4*a*c])]*AppellF1[2/n, 3/2, 3/2, (2 + n)/n, (-2*c*x^n)/(b - Sqrt
[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(2*a*Sqrt[a + b*x^n +
 c*x^(2*n)])
```

3.591.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

3.591.4 Maple [F]

$$\int \frac{x}{\left(a+b\,x^{n}+c\,x^{2 n}\right)^{\frac{3}{2}}}d x$$

input
```
int(x/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

output
```
int(x/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

3.591.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.591.6 Sympy [F]

$$\int \frac{x}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \int \frac{x}{(a+bx^n+cx^{2n})^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(a+b*x**n+c*x**(2*n))**(3/2),x)
```

output
```
Integral(x/(a + b*x**n + c*x**(2*n))**(3/2), x)
```

3.591.7 Maxima [F]

$$\int \frac{x}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \int \frac{x}{(cx^{2n}+bx^n+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
```

output
```
integrate(x/(c*x^(2*n) + b*x^n + a)^(3/2), x)
```

3.591.8 Giac [F]

$$\int \frac{x}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \int \frac{x}{(cx^{2n}+bx^n+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
```

output
```
integrate(x/(c*x^(2*n) + b*x^n + a)^(3/2), x)
```

3.591.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \int \frac{x}{{\left(a+b\,x^n+c\,x^{2\,n}\right)}^{3/2}}\,dx$$

input
```
int(x/(a + b*x^n + c*x^(2*n))^(3/2),x)
```

output
```
int(x/(a + b*x^n + c*x^(2*n))^(3/2), x)
```

3.592 $\int \frac{1}{(a+bx^n+cx^{2n})^{3/2}}\,dx$

3.592.1 Optimal result

Integrand size = 18, antiderivative size = 142

$$\int \frac{1}{(a+bx^n+cx^{2n})^{3/2}}\,dx=\frac{x\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(\frac{1}{n},\frac{3}{2},\frac{3}{2},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-}}\right.}{a\sqrt{a+bx^n+cx^{2n}}}$$

output
```
x*AppellF1(1/n,3/2,3/2,1+1/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(
-4*a*c+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^n/(b
+(-4*a*c+b^2)^(1/2)))^(1/2)/a/(a+b*x^n+c*x^(2*n))^(1/2)
```

3.592.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 384 vs. 2(142) = 284.

Time = 0.70 (sec) , antiderivative size = 384, normalized size of antiderivative = 2.70

$$\int \frac{1}{(a+bx^n+cx^{2n})^{3/2}}\,dx=\frac{x\left(2bcx^n\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(1+\frac{1}{n},\frac{1}{2},\frac{1}{2},2+\frac{1}{n},-\frac{2cx}{b+\sqrt{b^2}}\right.\right.}{}$$

input
```
Integrate[(a + b*x^n + c*x^(2*n))^(-3/2),x]
```

output
```
(x*(2*b*c*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c
])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*Appell
F1[1 + n^(-1), 1/2, 1/2, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (
2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - (1 + n)*(2*(b^2 - 2*a*c + b*c*x^n) +
(b^2*(-2 + n) - 4*a*c*(-1 + n))*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b
- Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2
 - 4*a*c])]*AppellF1[n^(-1), 1/2, 1/2, 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^
2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])))/(a*(-b^2 + 4*a*c)*n*(1
 + n)*Sqrt[a + x^n*(b + c*x^n)])
```

3.592.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1686, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^n+cx^{2n})^{3/2}}\,dx$$

$$\Big\downarrow \text{1686}$$

$$\frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\int \frac{1}{\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{3/2}\left(\frac{2cx^n}{b+\sqrt{b^2-4ac}}+1\right)^{3/2}}dx}{a\sqrt{a+bx^n+cx^{2n}}}$$

$$\Big\downarrow \text{936}$$

$$\frac{x\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{1}{n},\frac{3}{2},\frac{3}{2},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{a\sqrt{a+bx^n+cx^{2n}}}$$

input
```
Int[(a + b*x^n + c*x^(2*n))^(-3/2),x]
```

output
```
(x*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqr
t[b^2 - 4*a*c])]*AppellF1[n^(-1), 3/2, 3/2, 1 + n^(-1), (-2*c*x^n)/(b - Sq
rt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*Sqrt[a + b*x^n +
 c*x^(2*n)])
```

3.592.3.1 Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1686
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^
IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*c*(x^n/(b + Rt[b^2
- 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4*a*c, 2])))^FracPar
t[p]))   Int[(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2*c*(x^n/(b - S
qrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n]
&& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

3.592.4 Maple [F]

$$\int \frac{1}{\left(a+b\,x^{n}+c\,x^{2n}\right)^{\frac{3}{2}}}dx$$

input
```
int(1/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

output
```
int(1/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

3.592.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.592.6 Sympy [F]

$$\int \frac{1}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \int \frac{1}{(a+bx^n+cx^{2n})^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*x**n+c*x**(2*n))**(3/2),x)
```

output
```
Integral((a + b*x**n + c*x**(2*n))**(-3/2), x)
```

3.592.7 Maxima [F]

$$\int \frac{1}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \int \frac{1}{(cx^{2n}+bx^n+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^(-3/2), x)
```

3.592.8 Giac [F]

$$\int \frac{1}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \int \frac{1}{(cx^{2n}+bx^n+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^(-3/2), x)
```

3.592.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^n+cx^{2n})^{3/2}}\,dx=\int \frac{1}{(a+b\,x^n+c\,x^{2\,n})^{3/2}}\,dx$$

input
```
int(1/(a + b*x^n + c*x^(2*n))^(3/2),x)
```

output
```
int(1/(a + b*x^n + c*x^(2*n))^(3/2), x)
```

3.593 $\int \frac{1}{x(a+bx^n+cx^{2n})^{3/2}}\,dx$

3.593.1 Optimal result

Integrand size = 22, antiderivative size = 98

$$\int \frac{1}{x\left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx=\frac{2(b^2-2ac+bcx^n)}{a\left(b^2-4ac\right)n\sqrt{a+bx^n+cx^{2n}}}-\frac{\operatorname{arctanh}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{a^{3/2}n}$$

output
```
-arctanh(1/2*(2*a+b*x^n)/a^(1/2)/(a+b*x^n+c*x^(2*n))^(1/2))/a^(3/2)/n+2*(b
^2-2*a*c+b*c*x^n)/a/(-4*a*c+b^2)/n/(a+b*x^n+c*x^(2*n))^(1/2)
```

3.593.2 Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx=\frac{2\left(-\frac{\sqrt{a}(-b^2+2ac-bcx^n)}{(b^2-4ac)\sqrt{a+x^n(b+cx^n)}}+\operatorname{arctanh}\left(\frac{\sqrt{c}x^n-\sqrt{a+x^n(b+cx^n)}}{\sqrt{a}}\right)\right)}{a^{3/2}n}$$

input
```
Integrate[1/(x*(a + b*x^n + c*x^(2*n))^(3/2)),x]
```

output
```
(2*(-((Sqrt[a]*(-b^2 + 2*a*c - b*c*x^n))/((b^2 - 4*a*c)*Sqrt[a + x^n*(b +
c*x^n)])) + ArcTanh[(Sqrt[c]*x^n - Sqrt[a + x^n*(b + c*x^n)])/Sqrt[a]]))/(
a^(3/2)*n)
```

3.593.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1693, 1165, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+bx^n+cx^{2n}\right)^{3/2}}dx$$
$$\Big\downarrow \text{1693}$$
$$\frac{\int \frac{x^{-n}}{\left(bx^n+cx^{2n}+a\right)^{3/2}}dx^n}{n}$$
$$\Big\downarrow \text{1165}$$
$$\frac{\frac{2\left(-2ac+b^2+bcx^n\right)}{a(b^2-4ac)\sqrt{a+bx^n+cx^{2n}}}-\frac{2\int -\frac{\left(b^2-4ac\right)x^{-n}}{2\sqrt{bx^n+cx^{2n}+a}}dx^n}{a(b^2-4ac)}}{n}$$
$$\Big\downarrow \text{27}$$
$$\frac{\frac{\int \frac{x^{-n}}{\sqrt{bx^n+cx^{2n}+a}}dx^n}{a}+\frac{2\left(-2ac+b^2+bcx^n\right)}{a(b^2-4ac)\sqrt{a+bx^n+cx^{2n}}}}{n}$$
$$\Big\downarrow \text{1154}$$
$$\frac{\frac{2\left(-2ac+b^2+bcx^n\right)}{a(b^2-4ac)\sqrt{a+bx^n+cx^{2n}}}-\frac{2\int \frac{1}{4a-x^{2n}}d\frac{bx^n+2a}{\sqrt{bx^n+cx^{2n}+a}}}{a}}{n}$$
$$\Big\downarrow \text{219}$$
$$\frac{\frac{2\left(-2ac+b^2+bcx^n\right)}{a(b^2-4ac)\sqrt{a+bx^n+cx^{2n}}}-\frac{\text{arctanh}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{a^{3/2}}}{n}$$

input
```
Int[1/(x*(a + b*x^n + c*x^(2*n))^(3/2)),x]
```

output
```
((2*(b^2 - 2*a*c + b*c*x^n))/(a*(b^2 - 4*a*c)*Sqrt[a + b*x^n + c*x^(2*n)])
 - ArcTanh[(2*a + b*x^n)/(2*Sqrt[a]*Sqrt[a + b*x^n + c*x^(2*n)])]/a^(3/2))
/n
```

3.593.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.593.4 Maple [F]

$$\int \frac{1}{x\,(a+b\,x^{n}+c\,x^{2n})^{\frac{3}{2}}}dx$$

input
```
int(1/x/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

output
```
int(1/x/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

3.593.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(88) = 176$.

Time = 0.34 (sec) , antiderivative size = 449, normalized size of antiderivative = 4.58

$$\int \frac{1}{x\left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx = \left[\frac{\left((b^2c-4\,ac^2)\sqrt{a}x^{2\,n}+(b^3-4\,abc)\sqrt{a}x^n+(ab^2-4\,a^2c)\sqrt{a}\right)\log\left(-\frac{8\,abx^n+8}{}\right.}{2\left((a^2b^2c-4\,a^3c^2)nx^{2\,n}+(\right.}\right.$$

input
```
integrate(1/x/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
```

output
```
[1/2*(((b^2*c - 4*a*c^2)*sqrt(a)*x^(2*n) + (b^3 - 4*a*b*c)*sqrt(a)*x^n + (
a*b^2 - 4*a^2*c)*sqrt(a))*log(-(8*a*b*x^n + 8*a^2 + (b^2 + 4*a*c)*x^(2*n)
- 4*(sqrt(a)*b*x^n + 2*a^(3/2))*sqrt(c*x^(2*n) + b*x^n + a))/x^(2*n)) + 4*
(a*b*c*x^n + a*b^2 - 2*a^2*c)*sqrt(c*x^(2*n) + b*x^n + a))/((a^2*b^2*c - 4
*a^3*c^2)*n*x^(2*n) + (a^2*b^3 - 4*a^3*b*c)*n*x^n + (a^3*b^2 - 4*a^4*c)*n)
, (((b^2*c - 4*a*c^2)*sqrt(-a)*x^(2*n) + (b^3 - 4*a*b*c)*sqrt(-a)*x^n + (a
*b^2 - 4*a^2*c)*sqrt(-a))*arctan(1/2*(sqrt(-a)*b*x^n + 2*sqrt(-a)*a)*sqrt(
c*x^(2*n) + b*x^n + a)/(a*c*x^(2*n) + a*b*x^n + a^2)) + 2*(a*b*c*x^n + a*b
^2 - 2*a^2*c)*sqrt(c*x^(2*n) + b*x^n + a))/((a^2*b^2*c - 4*a^3*c^2)*n*x^(2
*n) + (a^2*b^3 - 4*a^3*b*c)*n*x^n + (a^3*b^2 - 4*a^4*c)*n)]
```

3.593.6 Sympy [F]

$$\int \frac{1}{x\left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx = \int \frac{1}{x\left(a+bx^n+cx^{2n}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/(a+b*x**n+c*x**(2*n))**(3/2),x)
```

output
```
Integral(1/(x*(a + b*x**n + c*x**(2*n))**(3/2)), x)
```

3.593.7 Maxima [F]

$$\int \frac{1}{x\,(a+bx^n+cx^{2n})^{3/2}}\,dx = \int \frac{1}{(cx^{2\,n}+bx^n+a)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(1/x/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^(2*n) + b*x^n + a)^(3/2)*x), x)
```

3.593.8 Giac [F]

$$\int \frac{1}{x\,(a+bx^n+cx^{2n})^{3/2}}\,dx = \int \frac{1}{(cx^{2\,n}+bx^n+a)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(1/x/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((c*x^(2*n) + b*x^n + a)^(3/2)*x), x)
```

3.593.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\,(a+bx^n+cx^{2n})^{3/2}}\,dx = \int \frac{1}{x\,(a+b\,x^n+c\,x^{2\,n})^{3/2}}\,dx$$

input
```
int(1/(x*(a + b*x^n + c*x^(2*n))^(3/2)),x)
```

output
```
int(1/(x*(a + b*x^n + c*x^(2*n))^(3/2)), x)
```

3.594 $\int \frac{1}{x^2(a+bx^n+cx^{2n})^{3/2}}\,dx$

3.594.1 Optimal result

Integrand size = 22, antiderivative size = 152

$$\int \frac{1}{x^2(a+bx^n+cx^{2n})^{3/2}}\,dx = -\frac{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(-\frac{1}{n},\frac{3}{2},\frac{3}{2},-\frac{1-n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{ax\sqrt{a+bx^n+cx^{2n}}}$$

output
```
-AppellF1(-1/n,3/2,3/2,(-1+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(
b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^n
/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/x/(a+b*x^n+c*x^(2*n))^(1/2)
```

3.594.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 395 vs. $2(152) = 304$.

Time = 0.60 (sec) , antiderivative size = 395, normalized size of antiderivative = 2.60

$$\int \frac{1}{x^2(a+bx^n+cx^{2n})^{3/2}}\,dx = \frac{(-1+n)\left(-4ac(1+n)+b^2(2+n)\right)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{Appel}}{}$$

input
```
Integrate[1/(x^2*(a + b*x^n + c*x^(2*n))^(3/2)),x]
```

output
```
((-1 + n)*(-4*a*c*(1 + n) + b^2*(2 + n))*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c
*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b +
 Sqrt[b^2 - 4*a*c])]*AppellF1[-n^(-1), 1/2, 1/2, (-1 + n)/n, (-2*c*x^n)/(b
 + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - 2*((-1 + n)*(
b^2 - 2*a*c + b*c*x^n) + b*c*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b
 - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^
2 - 4*a*c])]*AppellF1[(-1 + n)/n, 1/2, 1/2, 2 - n^(-1), (-2*c*x^n)/(b + Sq
rt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))/(a*(-b^2 + 4*a*c)*
(-1 + n)*n*x*Sqrt[a + x^n*(b + c*x^n)])
```

3.594.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^n+cx^{2n})^{3/2}} \, dx$$

$$\Big\downarrow \text{1721}$$

$$\frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\int \frac{1}{x^2\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{3/2}\left(\frac{2cx^n}{b+\sqrt{b^2-4ac}}+1\right)^{3/2}}dx}{a\sqrt{a+bx^n+cx^{2n}}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(-\frac{1}{n},\frac{3}{2},\frac{3}{2},-\frac{1-n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{ax\sqrt{a+bx^n+cx^{2n}}}$$

input
```
Int[1/(x^2*(a + b*x^n + c*x^(2*n))^(3/2)),x]
```

output
```
-((Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqr
t[b^2 - 4*a*c])]*AppellF1[-n^(-1), 3/2, 3/2, -((1 - n)/n), (-2*c*x^n)/(b -
 Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*x*Sqrt[a + b*
x^n + c*x^(2*n)]))
```

3.594.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

3.594.4 Maple [F]

$$\int \frac{1}{x^{2} \left(a +b \,x^{n}+c \,x^{2 n}\right)^{\frac{3}{2}}}d x$$

input
```
int(1/x^2/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

output
```
int(1/x^2/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

3.594.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^2 (a+bx^n+cx^{2n})^{3/2}} \, dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/x^2/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.594.6 Sympy [F]

$$\int \frac{1}{x^2 \left(a + bx^n + cx^{2n}\right)^{3/2}} \, dx = \int \frac{1}{x^2 \left(a + bx^n + cx^{2n}\right)^{\frac{3}{2}}} \, dx$$

input
```
integrate(1/x**2/(a+b*x**n+c*x**(2*n))**(3/2),x)
```

output
```
Integral(1/(x**2*(a + b*x**n + c*x**(2*n))**(3/2)), x)
```

3.594.7 Maxima [F]

$$\int \frac{1}{x^2 \left(a + bx^n + cx^{2n}\right)^{3/2}} \, dx = \int \frac{1}{\left(cx^{2n} + bx^n + a\right)^{\frac{3}{2}} x^2} \, dx$$

input
```
integrate(1/x^2/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^(2*n) + b*x^n + a)^(3/2)*x^2), x)
```

3.594.8 Giac [F]

$$\int \frac{1}{x^2 \left(a + bx^n + cx^{2n}\right)^{3/2}} \, dx = \int \frac{1}{\left(cx^{2n} + bx^n + a\right)^{\frac{3}{2}} x^2} \, dx$$

input
```
integrate(1/x^2/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((c*x^(2*n) + b*x^n + a)^(3/2)*x^2), x)
```

3.594.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 \left(a + bx^n + cx^{2n}\right)^{3/2}} \, dx = \int \frac{1}{x^2 \left(a + b\, x^n + c\, x^{2\,n}\right)^{3/2}} \, dx$$

input
```
int(1/(x^2*(a + b*x^n + c*x^(2*n))^(3/2)),x)
```

output
```
int(1/(x^2*(a + b*x^n + c*x^(2*n))^(3/2)), x)
```

3.595 $\int \frac{1}{x^3(a+bx^n+cx^{2n})^{3/2}}\,dx$

3.595.1 Optimal result

Integrand size = 22, antiderivative size = 154

$$\int \frac{1}{x^3\left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx = -\frac{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(-\frac{2}{n},\frac{3}{2},\frac{3}{2},-\frac{2-n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2ax^2\sqrt{a+bx^n+cx^{2n}}}$$

output
```
-1/2*AppellF1(-2/n,3/2,3/2,(-2+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x
^n/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c
*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/x^2/(a+b*x^n+c*x^(2*n))^(1/2)
```

3.595.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 399 vs. 2(154) = 308.

Time = 0.61 (sec) , antiderivative size = 399, normalized size of antiderivative = 2.59

$$\int \frac{1}{x^3\left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx = \frac{(-2+n)\left(-4ac(2+n)+b^2(4+n)\right)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\operatorname{Appel}}{}$$

input
```
Integrate[1/(x^3*(a + b*x^n + c*x^(2*n))^(3/2)),x]
```

output
```
((-2 + n)*(-4*a*c*(2 + n) + b^2*(4 + n))*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c
*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b +
 Sqrt[b^2 - 4*a*c])]*AppellF1[-2/n, 1/2, 1/2, (-2 + n)/n, (-2*c*x^n)/(b +
Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - 4*((-2 + n)*(b^2
 - 2*a*c + b*c*x^n) + 2*b*c*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b
- Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2
 - 4*a*c])]*AppellF1[(-2 + n)/n, 1/2, 1/2, 2 - 2/n, (-2*c*x^n)/(b + Sqrt[b
^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))/(2*a*(-b^2 + 4*a*c)*(-
2 + n)*n*x^2*Sqrt[a + x^n*(b + c*x^n)])
```

3.595.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \left(a+b x^n+c x^{2 n}\right)^{3/2}} \, dx$$

$$\Big\downarrow \text{1721}$$

$$\frac{\sqrt{\frac{2 c x^n}{b-\sqrt{b^2-4 a c}}+1} \sqrt{\frac{2 c x^n}{\sqrt{b^2-4 a c}+b}+1} \int \frac{1}{x^3 \left(\frac{2 c x^n}{b-\sqrt{b^2-4 a c}}+1\right)^{3/2} \left(\frac{2 c x^n}{b+\sqrt{b^2-4 a c}}+1\right)^{3/2}}dx}{a \sqrt{a+b x^n+c x^{2 n}}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt{\frac{2 c x^n}{b-\sqrt{b^2-4 a c}}+1} \sqrt{\frac{2 c x^n}{\sqrt{b^2-4 a c}+b}+1} \text{AppellF1}\left(-\frac{2}{n},\frac{3}{2},\frac{3}{2},-\frac{2-n}{n},-\frac{2 c x^n}{b-\sqrt{b^2-4 a c}},-\frac{2 c x^n}{b+\sqrt{b^2-4 a c}}\right)}{2 a x^2 \sqrt{a+b x^n+c x^{2 n}}}$$

input
```
Int[1/(x^3*(a + b*x^n + c*x^(2*n))^(3/2)),x]
```

output
```
-1/2*(Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b +
Sqrt[b^2 - 4*a*c])]*AppellF1[-2/n, 3/2, 3/2, -((2 - n)/n), (-2*c*x^n)/(b -
 Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*x^2*Sqrt[a +
b*x^n + c*x^(2*n)])
```

3.595.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

3.595.4 Maple [F]

$$\int \frac{1}{x^{3} \left(a +b \,x^{n}+c \,x^{2 n}\right)^{\frac{3}{2}}}d x$$

input
```
int(1/x^3/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

output
```
int(1/x^3/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

3.595.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^3 (a+bx^n+cx^{2n})^{3/2}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^3/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.595.6 Sympy [F]

$$\int \frac{1}{x^3 (a + bx^n + cx^{2n})^{3/2}} \, dx = \int \frac{1}{x^3 (a + bx^n + cx^{2n})^{\frac{3}{2}}} \, dx$$

input
```
integrate(1/x**3/(a+b*x**n+c*x**(2*n))**(3/2),x)
```

output
```
Integral(1/(x**3*(a + b*x**n + c*x**(2*n))**(3/2)), x)
```

3.595.7 Maxima [F]

$$\int \frac{1}{x^3 (a + bx^n + cx^{2n})^{3/2}} \, dx = \int \frac{1}{(cx^{2n} + bx^n + a)^{\frac{3}{2}} x^3} \, dx$$

input
```
integrate(1/x^3/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^(2*n) + b*x^n + a)^(3/2)*x^3), x)
```

3.595.8 Giac [F]

$$\int \frac{1}{x^3 (a + bx^n + cx^{2n})^{3/2}} \, dx = \int \frac{1}{(cx^{2n} + bx^n + a)^{\frac{3}{2}} x^3} \, dx$$

input
```
integrate(1/x^3/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((c*x^(2*n) + b*x^n + a)^(3/2)*x^3), x)
```

3.595.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a + bx^n + cx^{2n})^{3/2}} \, dx = \int \frac{1}{x^3 (a + b\,x^n + c\,x^{2\,n})^{3/2}} \, dx$$

input
```
int(1/(x^3*(a + b*x^n + c*x^(2*n))^(3/2)),x)
```

output
```
int(1/(x^3*(a + b*x^n + c*x^(2*n))^(3/2)), x)
```

3.596 $\int (dx)^m (a+bx^n+cx^{2n})^3\,dx$

3.596.1 Optimal result

Integrand size = 22, antiderivative size = 182

$$\int (dx)^m (a+bx^n+cx^{2n})^3\,dx = \frac{3a^2bx^{1+n}(dx)^m}{1+m+n}+\frac{3a(b^2+ac)\,x^{1+2n}(dx)^m}{1+m+2n}+\frac{b(b^2+6ac)\,x^{1+3n}(dx)^m}{1+m+3n}+\frac{3c(b^2+ac)\,x^{1+4n}(dx)^m}{1+m+4n}+\frac{3bc^2x^{1+5n}(dx)^m}{1+m+5n}+\frac{c^3x^{1+6n}(dx)^m}{1+m+6n}+\frac{a^3(dx)^{1+m}}{d(1+m)}$$

output
```
3*a^2*b*x^(1+n)*(d*x)^m/(1+m+n)+3*a*(a*c+b^2)*x^(1+2*n)*(d*x)^m/(1+m+2*n)+
b*(6*a*c+b^2)*x^(1+3*n)*(d*x)^m/(1+m+3*n)+3*c*(a*c+b^2)*x^(1+4*n)*(d*x)^m/
(1+m+4*n)+3*b*c^2*x^(1+5*n)*(d*x)^m/(1+m+5*n)+c^3*x^(1+6*n)*(d*x)^m/(1+m+6
*n)+a^3*(d*x)^(1+m)/d/(1+m)
```

3.596.2 Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.75

$$\int (dx)^m (a+bx^n+cx^{2n})^3\,dx = x(dx)^m\left(\frac{a^3}{1+m}+\frac{3a^2bx^n}{1+m+n}+\frac{3a(b^2+ac)\,x^{2n}}{1+m+2n}+\frac{b(b^2+6ac)\,x^{3n}}{1+m+3n}+\frac{3c(b^2+ac)\,x^{4n}}{1+m+4n}+\frac{3bc^2x^{5n}}{1+m+5n}+\frac{c^3x^{6n}}{1+m+6n}\right)$$

input
```
Integrate[(d*x)^m*(a + b*x^n + c*x^(2*n))^3,x]
```

output
```
x*(d*x)^m*(a^3/(1 + m) + (3*a^2*b*x^n)/(1 + m + n) + (3*a*(b^2 + a*c)*x^(2
*n))/(1 + m + 2*n) + (b*(b^2 + 6*a*c)*x^(3*n))/(1 + m + 3*n) + (3*c*(b^2 +
 a*c)*x^(4*n))/(1 + m + 4*n) + (3*b*c^2*x^(5*n))/(1 + m + 5*n) + (c^3*x^(6
*n))/(1 + m + 6*n))
```

3.596.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1691, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^3 dx$$

$$\Big\downarrow \text{ 1691}$$

$$\int \left(a^3(dx)^m+3a^2bx^n(dx)^m+3ab^2x^{2n}\left(\frac{ac}{b^2}+1\right)(dx)^m+3b^2cx^{4n}\left(\frac{ac}{b^2}+1\right)(dx)^m+b^3x^{3n}\left(\frac{6ac}{b^2}+1\right)(dx)^m+3b$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^3(dx)^{m+1}}{d(m+1)}+\frac{3a^2bx^{n+1}(dx)^m}{m+n+1}+\frac{3ax^{2n+1}\left(ac+b^2\right)(dx)^m}{m+2n+1}+\frac{bx^{3n+1}\left(6ac+b^2\right)(dx)^m}{m+3n+1}+\frac{3cx^{4n+1}\left(ac+b^2\right)(dx)^m}{m+4n+1}+\frac{3bc^2x^{5n+1}(dx)^m}{m+5n+1}+\frac{c^3x^{6n+1}(dx)^m}{m+6n+1}$$

input
```
Int[(d*x)^m*(a + b*x^n + c*x^(2*n))^3,x]
```

output
```
(3*a^2*b*x^(1 + n)*(d*x)^m)/(1 + m + n) + (3*a*(b^2 + a*c)*x^(1 + 2*n)*(d*
x)^m)/(1 + m + 2*n) + (b*(b^2 + 6*a*c)*x^(1 + 3*n)*(d*x)^m)/(1 + m + 3*n)
 + (3*c*(b^2 + a*c)*x^(1 + 4*n)*(d*x)^m)/(1 + m + 4*n) + (3*b*c^2*x^(1 + 5*
n)*(d*x)^m)/(1 + m + 5*n) + (c^3*x^(1 + 6*n)*(d*x)^m)/(1 + m + 6*n) + (a^3
*(d*x)^(1 + m))/(d*(1 + m))
```

3.596.3.1 Defintions of rubi rules used

rule 1691
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^n + c*x^(2*n))^p, x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && IGtQ[p, 0] &&  !IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.596.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.56 (sec) , antiderivative size = 3765, normalized size of antiderivative = 20.69

method	result	size
risch	Expression too large to display	3765
parallelrisch	Expression too large to display	5804

input
```
int((d*x)^m*(a+b*x^n+c*x^(2*n))^3,x,method=_RETURNVERBOSE)
```

output
```
x*(3*a*c^2*m^6*(x^n)^4+1284*b^2*c*m*n^2*(x^n)^4+240*b*c^2*m*n*(x^n)^5+300*
a^2*b*m^4*n*x^n+1860*a^2*b*m^3*n^2*x^n+5220*a^2*b*m^2*n^3*x^n+6264*a^2*b*m
*n^4*x^n+570*a^2*c*m^3*n*(x^n)^2+2466*a^2*c*m^2*n^2*(x^n)^2+4149*a^2*c*m*n
^3*(x^n)^2+570*a*b^2*m^3*n*(x^n)^2+2466*a*b^2*m^2*n^2*(x^n)^2+4149*a*b^2*m
*n^3*(x^n)^2+120*a*b*c*m^3*(x^n)^3+2904*a*b*c*m*n^2*(x^n)^3+540*a*b*c*m*n*
(x^n)^3+6*(x^n)^3*c*a*b+3*b^2*c*m^6*(x^n)^4+a^3+3*(x^n)^4*c^2*a+3*(x^n)^4*
c*b^2+3*(x^n)^2*c*a^2+3*(x^n)^5*c^2*b+15*c^3*m^5*n*(x^n)^6+1383*a^2*c*m^3*
n^3*(x^n)^2+2106*a^2*c*m^2*n^4*(x^n)^2+1080*a^2*c*m*n^5*(x^n)^2+57*a*b^2*m
^5*n*(x^n)^2+411*a*b^2*m^4*n^2*(x^n)^2+1383*a*b^2*m^3*n^3*(x^n)^2+2106*a*b
^2*m^2*n^4*(x^n)^2+1080*a*b^2*m*n^5*(x^n)^2+36*a*b*c*m^5*(x^n)^3+1440*a*b*
c*n^5*(x^n)^3+2106*a^2*c*n^4*(x^n)^2+45*a*b^2*m^4*(x^n)^2+2106*a*b^2*n^4*(
x^n)^2+45*a*c^2*m^2*(x^n)^4+321*a*c^2*n^2*(x^n)^4+540*a*c^2*m*n^5*(x^n)^4+
51*b^2*c*m^5*n*(x^n)^4+321*b^2*c*m^4*n^2*(x^n)^4+921*b^2*c*m^3*n^3*(x^n)^4
+1188*b^2*c*m^2*n^4*(x^n)^4+180*b^3*m^3*n*(x^n)^3+726*b^3*m^2*n^2*(x^n)^3+
1116*b^3*m*n^3*(x^n)^3+60*b^2*c*m^3*(x^n)^4+921*b^2*c*n^3*(x^n)^4+6*b^3*m^
5*(x^n)^3+240*b^3*n^5*(x^n)^3+15*c^3*m^2*(x^n)^6+85*c^3*n^2*(x^n)^6+15*b^3
*m^4*(x^n)^3+508*b^3*n^4*(x^n)^3+675*c^3*m*n^3*(x^n)^6+18*a*c^2*m^5*(x^n)^
4+540*a*c^2*n^5*(x^n)^4+6096*a*b*c*m*n^4*(x^n)^3+1080*a*b*c*m^3*n*(x^n)^3+
4356*a*b*c*m^2*n^2*(x^n)^3+85*c^3*m^4*n^2*(x^n)^6+2763*b^2*c*m^2*n^3*(x^n)
^4+2376*b^2*c*m*n^4*(x^n)^4+480*b*c^2*m^3*n*(x^n)^5+1710*b*c^2*m^2*n^2*...
```

3.596.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2303 vs. $2(182) = 364$.

Time = 0.33 (sec) , antiderivative size = 2303, normalized size of antiderivative = 12.65

$$\int (dx)^m \left(a + bx^n + cx^{2n}\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^3,x, algorithm="fricas")
```

output
```
((c^3*m^6 + 6*c^3*m^5 + 15*c^3*m^4 + 20*c^3*m^3 + 120*(c^3*m + c^3)*n^5 +
15*c^3*m^2 + 274*(c^3*m^2 + 2*c^3*m + c^3)*n^4 + 6*c^3*m + 225*(c^3*m^3 +
3*c^3*m^2 + 3*c^3*m + c^3)*n^3 + c^3 + 85*(c^3*m^4 + 4*c^3*m^3 + 6*c^3*m^2
 + 4*c^3*m + c^3)*n^2 + 15*(c^3*m^5 + 5*c^3*m^4 + 10*c^3*m^3 + 10*c^3*m^2
+ 5*c^3*m + c^3)*n)*x*x^(6*n)*e^(m*log(d) + m*log(x)) + 3*(b*c^2*m^6 + 6*b
*c^2*m^5 + 15*b*c^2*m^4 + 20*b*c^2*m^3 + 144*(b*c^2*m + b*c^2)*n^5 + 15*b*
c^2*m^2 + 324*(b*c^2*m^2 + 2*b*c^2*m + b*c^2)*n^4 + 6*b*c^2*m + 260*(b*c^2
*m^3 + 3*b*c^2*m^2 + 3*b*c^2*m + b*c^2)*n^3 + b*c^2 + 95*(b*c^2*m^4 + 4*b*
c^2*m^3 + 6*b*c^2*m^2 + 4*b*c^2*m + b*c^2)*n^2 + 16*(b*c^2*m^5 + 5*b*c^2*m
^4 + 10*b*c^2*m^3 + 10*b*c^2*m^2 + 5*b*c^2*m + b*c^2)*n)*x*x^(5*n)*e^(m*lo
g(d) + m*log(x)) + 3*((b^2*c + a*c^2)*m^6 + 6*(b^2*c + a*c^2)*m^5 + 180*(b
^2*c + a*c^2 + (b^2*c + a*c^2)*m)*n^5 + 15*(b^2*c + a*c^2)*m^4 + 396*(b^2*
c + a*c^2 + (b^2*c + a*c^2)*m^2 + 2*(b^2*c + a*c^2)*m)*n^4 + 20*(b^2*c + a
*c^2)*m^3 + 307*((b^2*c + a*c^2)*m^3 + b^2*c + a*c^2 + 3*(b^2*c + a*c^2)*m
^2 + 3*(b^2*c + a*c^2)*m)*n^3 + b^2*c + a*c^2 + 15*(b^2*c + a*c^2)*m^2 + 1
07*((b^2*c + a*c^2)*m^4 + 4*(b^2*c + a*c^2)*m^3 + b^2*c + a*c^2 + 6*(b^2*c
 + a*c^2)*m^2 + 4*(b^2*c + a*c^2)*m)*n^2 + 6*(b^2*c + a*c^2)*m + 17*((b^2*
c + a*c^2)*m^5 + 5*(b^2*c + a*c^2)*m^4 + 10*(b^2*c + a*c^2)*m^3 + b^2*c +
a*c^2 + 10*(b^2*c + a*c^2)*m^2 + 5*(b^2*c + a*c^2)*m)*n)*x*x^(4*n)*e^(m*lo
g(d) + m*log(x)) + ((b^3 + 6*a*b*c)*m^6 + 6*(b^3 + 6*a*b*c)*m^5 + 240*(...
```

3.596.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68491 vs. $2(170) = 340$.

Time = 65.90 (sec) , antiderivative size = 68491, normalized size of antiderivative = 376.32

$$\int (dx)^m \left(a + bx^n + cx^{2n}\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate((d*x)**m*(a+b*x**n+c*x**(2*n))**3,x)
```

output
```
Piecewise(((a + b + c)**3*log(x)/d, Eq(m, -1) & Eq(n, 0)), ((a**3*log(x) +
 3*a**2*b*x**n/n + 3*a**2*c*x**(2*n)/(2*n) + 3*a*b**2*x**(2*n)/(2*n) + 2*a
*b*c*x**(3*n)/n + 3*a*c**2*x**(4*n)/(4*n) + b**3*x**(3*n)/(3*n) + 3*b**2*c
*x**(4*n)/(4*n) + 3*b*c**2*x**(5*n)/(5*n) + c**3*x**(6*n)/(6*n))/d, Eq(m,
-1)), (a**3*Piecewise((0**(-6*n - 1)*x, Eq(d, 0)), (Piecewise((-1/(6*n*(d*
x)**(6*n)), Ne(n, 0)), (log(d*x), True))/d, True)) + 3*a**2*b*Piecewise((-
x*x**n*(d*x)**(-6*n - 1)/(5*n), Ne(n, 0)), (x*x**n*(d*x)**(-6*n - 1)*log(x
), True)) + 3*a**2*c*Piecewise((-x*x**(2*n)*(d*x)**(-6*n - 1)/(4*n), Ne(n,
 0)), (x*x**(2*n)*(d*x)**(-6*n - 1)*log(x), True)) + 3*a*b**2*Piecewise((-
x*x**(2*n)*(d*x)**(-6*n - 1)/(4*n), Ne(n, 0)), (x*x**(2*n)*(d*x)**(-6*n -
1)*log(x), True)) + 6*a*b*c*Piecewise((-x*x**(3*n)*(d*x)**(-6*n - 1)/(3*n)
, Ne(n, 0)), (x*x**(3*n)*(d*x)**(-6*n - 1)*log(x), True)) + 3*a*c**2*Piece
wise((-x*x**(4*n)*(d*x)**(-6*n - 1)/(2*n), Ne(n, 0)), (x*x**(4*n)*(d*x)**(
-6*n - 1)*log(x), True)) + b**3*Piecewise((-x*x**(3*n)*(d*x)**(-6*n - 1)/(
3*n), Ne(n, 0)), (x*x**(3*n)*(d*x)**(-6*n - 1)*log(x), True)) + 3*b**2*c*P
iecewise((-x*x**(4*n)*(d*x)**(-6*n - 1)/(2*n), Ne(n, 0)), (x*x**(4*n)*(d*x
)**(-6*n - 1)*log(x), True)) + 3*b*c**2*Piecewise((-x*x**(5*n)*(d*x)**(-6*
n - 1)/n, Ne(n, 0)), (x*x**(5*n)*(d*x)**(-6*n - 1)*log(x), True)) + c**3*x
*x**(6*n)*(d*x)**(-6*n - 1)*log(x), Eq(m, -6*n - 1)), (a**3*Piecewise((0**
(-5*n - 1)*x, Eq(d, 0)), (Piecewise((-1/(5*n*(d*x)**(5*n)), Ne(n, 0)), ...
```

3.596.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.50

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^3 \,dx = \frac{c^3 d^m x e^{(m\log(x)+6\,n\log(x))}}{m+6\,n+1} + \frac{3\,bc^2 d^m x e^{(m\log(x)+5\,n\log(x))}}{m+5\,n+1} + \frac{3\,b^2 c d^m x e^{(m\log(x)+4\,n\log(x))}}{m+4\,n+1} + \frac{3\,ac^2 d^m x e^{(m\log(x)+4\,n\log(x))}}{m+4\,n+1} + \frac{b^3 d^m x e^{(m\log(x)+3\,n\log(x))}}{m+3\,n+1} + \frac{6\,abcd^m x e^{(m\log(x)+3\,n\log(x))}}{m+3\,n+1} + \frac{3\,ab^2 d^m x e^{(m\log(x)+2\,n\log(x))}}{m+2\,n+1} + \frac{3\,a^2 c d^m x e^{(m\log(x)+2\,n\log(x))}}{m+2\,n+1} + \frac{3\,a^2 b d^m x e^{(m\log(x)+n\log(x))}}{m+n+1} + \frac{(dx)^{m+1}\,a^3}{d(m+1)}$$

input
```
integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^3,x, algorithm="maxima")
```

output
```
c^3*d^m*x*e^(m*log(x) + 6*n*log(x))/(m + 6*n + 1) + 3*b*c^2*d^m*x*e^(m*log
(x) + 5*n*log(x))/(m + 5*n + 1) + 3*b^2*c*d^m*x*e^(m*log(x) + 4*n*log(x))/
(m + 4*n + 1) + 3*a*c^2*d^m*x*e^(m*log(x) + 4*n*log(x))/(m + 4*n + 1) + b^
3*d^m*x*e^(m*log(x) + 3*n*log(x))/(m + 3*n + 1) + 6*a*b*c*d^m*x*e^(m*log(x
) + 3*n*log(x))/(m + 3*n + 1) + 3*a*b^2*d^m*x*e^(m*log(x) + 2*n*log(x))/(m
 + 2*n + 1) + 3*a^2*c*d^m*x*e^(m*log(x) + 2*n*log(x))/(m + 2*n + 1) + 3*a^
2*b*d^m*x*e^(m*log(x) + n*log(x))/(m + n + 1) + (d*x)^(m + 1)*a^3/(d*(m +
1))
```

3.596.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25656 vs. 2(182) = 364.

Time = 0.49 (sec) , antiderivative size = 25656, normalized size of antiderivative = 140.97

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^3,x, algorithm="giac")
```

output
```
(c^3*m^6*x*x^(6*n)*e^(m*log(d) + m*log(x)) + 15*c^3*m^5*n*x*x^(6*n)*e^(m*l
og(d) + m*log(x)) + 85*c^3*m^4*n^2*x*x^(6*n)*e^(m*log(d) + m*log(x)) + 225
*c^3*m^3*n^3*x*x^(6*n)*e^(m*log(d) + m*log(x)) + 274*c^3*m^2*n^4*x*x^(6*n)
*e^(m*log(d) + m*log(x)) + 120*c^3*m*n^5*x*x^(6*n)*e^(m*log(d) + m*log(x))
 + 3*b*c^2*m^6*x*x^(5*n)*e^(m*log(d) + m*log(x)) + c^3*m^6*x*x^(5*n)*e^(m*
log(d) + m*log(x)) + 48*b*c^2*m^5*n*x*x^(5*n)*e^(m*log(d) + m*log(x)) + 15
*c^3*m^5*n*x*x^(5*n)*e^(m*log(d) + m*log(x)) + 285*b*c^2*m^4*n^2*x*x^(5*n)
*e^(m*log(d) + m*log(x)) + 85*c^3*m^4*n^2*x*x^(5*n)*e^(m*log(d) + m*log(x)
) + 780*b*c^2*m^3*n^3*x*x^(5*n)*e^(m*log(d) + m*log(x)) + 225*c^3*m^3*n^3*
x*x^(5*n)*e^(m*log(d) + m*log(x)) + 972*b*c^2*m^2*n^4*x*x^(5*n)*e^(m*log(d
) + m*log(x)) + 274*c^3*m^2*n^4*x*x^(5*n)*e^(m*log(d) + m*log(x)) + 432*b*
c^2*m*n^5*x*x^(5*n)*e^(m*log(d) + m*log(x)) + 120*c^3*m*n^5*x*x^(5*n)*e^(m
*log(d) + m*log(x)) + 3*b^2*c*m^6*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 3*a*
c^2*m^6*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 3*b*c^2*m^6*x*x^(4*n)*e^(m*log
(d) + m*log(x)) + c^3*m^6*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 51*b^2*c*m^5
*n*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 51*a*c^2*m^5*n*x*x^(4*n)*e^(m*log(d
) + m*log(x)) + 48*b*c^2*m^5*n*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 15*c^3*
m^5*n*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 321*b^2*c*m^4*n^2*x*x^(4*n)*e^(m
*log(d) + m*log(x)) + 321*a*c^2*m^4*n^2*x*x^(4*n)*e^(m*log(d) + m*log(x))
+ 285*b*c^2*m^4*n^2*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 85*c^3*m^4*n^2*...
```

3.596.9 Mupad [B] (verification not implemented)

Time = 9.43 (sec) , antiderivative size = 1734, normalized size of antiderivative = 9.53

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^3 \, dx = \text{Too large to display}$$

input
```
int((d*x)^m*(a + b*x^n + c*x^(2*n))^3,x)
```

output
```
(a^3*x*(d*x)^m)/(m + 1) + (c^3*x*x^(6*n)*(d*x)^m*(5*m + 15*n + 60*m*n + 25
5*m*n^2 + 90*m^2*n + 450*m*n^3 + 60*m^3*n + 274*m*n^4 + 15*m^4*n + 10*m^2
+ 10*m^3 + 5*m^4 + m^5 + 85*n^2 + 225*n^3 + 274*n^4 + 120*n^5 + 255*m^2*n^
2 + 225*m^2*n^3 + 85*m^3*n^2 + 1))/(6*m + 21*n + 105*m*n + 700*m*n^2 + 210
*m^2*n + 2205*m*n^3 + 210*m^3*n + 3248*m*n^4 + 105*m^4*n + 1764*m*n^5 + 21
*m^5*n + 15*m^2 + 20*m^3 + 15*m^4 + 6*m^5 + m^6 + 175*n^2 + 735*n^3 + 1624
*n^4 + 1764*n^5 + 720*n^6 + 1050*m^2*n^2 + 2205*m^2*n^3 + 700*m^3*n^2 + 16
24*m^2*n^4 + 735*m^3*n^3 + 175*m^4*n^2 + 1) + (3*a*x*x^(2*n)*(d*x)^m*(a*c
+ b^2)*(5*m + 19*n + 76*m*n + 411*m*n^2 + 114*m^2*n + 922*m*n^3 + 76*m^3*n
 + 702*m*n^4 + 19*m^4*n + 10*m^2 + 10*m^3 + 5*m^4 + m^5 + 137*n^2 + 461*n^
3 + 702*n^4 + 360*n^5 + 411*m^2*n^2 + 461*m^2*n^3 + 137*m^3*n^2 + 1))/(6*m
 + 21*n + 105*m*n + 700*m*n^2 + 210*m^2*n + 2205*m*n^3 + 210*m^3*n + 3248*
m*n^4 + 105*m^4*n + 1764*m*n^5 + 21*m^5*n + 15*m^2 + 20*m^3 + 15*m^4 + 6*m
^5 + m^6 + 175*n^2 + 735*n^3 + 1624*n^4 + 1764*n^5 + 720*n^6 + 1050*m^2*n^
2 + 2205*m^2*n^3 + 700*m^3*n^2 + 1624*m^2*n^4 + 735*m^3*n^3 + 175*m^4*n^2
+ 1) + (b*x*x^(3*n)*(d*x)^m*(6*a*c + b^2)*(5*m + 18*n + 72*m*n + 363*m*n^2
 + 108*m^2*n + 744*m*n^3 + 72*m^3*n + 508*m*n^4 + 18*m^4*n + 10*m^2 + 10*m
^3 + 5*m^4 + m^5 + 121*n^2 + 372*n^3 + 508*n^4 + 240*n^5 + 363*m^2*n^2 + 3
72*m^2*n^3 + 121*m^3*n^2 + 1))/(6*m + 21*n + 105*m*n + 700*m*n^2 + 210*m^2
*n + 2205*m*n^3 + 210*m^3*n + 3248*m*n^4 + 105*m^4*n + 1764*m*n^5 + 21*...
```

3.597 $\int (dx)^m \left(a+bx^n+cx^{2n}\right)^2 \, dx$

3.597.1 Optimal result

Integrand size = 22, antiderivative size = 117

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^2 \, dx = \frac{2abx^{1+n}(dx)^m}{1+m+n}+\frac{\left(b^2+2ac\right)x^{1+2n}(dx)^m}{1+m+2n}+\frac{2bcx^{1+3n}(dx)^m}{1+m+3n}+\frac{c^2x^{1+4n}(dx)^m}{1+m+4n}+\frac{a^2(dx)^{1+m}}{d(1+m)}$$

output
```
2*a*b*x^(1+n)*(d*x)^m/(1+m+n)+(2*a*c+b^2)*x^(1+2*n)*(d*x)^m/(1+m+2*n)+2*b*
c*x^(1+3*n)*(d*x)^m/(1+m+3*n)+c^2*x^(1+4*n)*(d*x)^m/(1+m+4*n)+a^2*(d*x)^(1
+m)/d/(1+m)
```

3.597.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.74

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^2 \, dx = x(dx)^m\left(\frac{a^2}{1+m}+\frac{2abx^n}{1+m+n}+\frac{\left(b^2+2ac\right)x^{2n}}{1+m+2n}+\frac{2bcx^{3n}}{1+m+3n}+\frac{c^2x^{4n}}{1+m+4n}\right)$$

input
```
Integrate[(d*x)^m*(a + b*x^n + c*x^(2*n))^2,x]
```

output
```
x*(d*x)^m*(a^2/(1 + m) + (2*a*b*x^n)/(1 + m + n) + ((b^2 + 2*a*c)*x^(2*n))
/(1 + m + 2*n) + (2*b*c*x^(3*n))/(1 + m + 3*n) + (c^2*x^(4*n))/(1 + m + 4*
n))
```

3.597.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1691, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^2 \, dx$$

$$\Big\downarrow \text{ 1691}$$

$$\int \left(a^2(dx)^m+b^2x^{2n}\left(\frac{2ac}{b^2}+1\right)(dx)^m+2abx^n(dx)^m+2bcx^{3n}(dx)^m+c^2x^{4n}(dx)^m\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^2(dx)^{m+1}}{d(m+1)}+\frac{x^{2n+1}\left(2ac+b^2\right)(dx)^m}{m+2n+1}+\frac{2abx^{n+1}(dx)^m}{m+n+1}+\frac{2bcx^{3n+1}(dx)^m}{m+3n+1}+\frac{c^2x^{4n+1}(dx)^m}{m+4n+1}$$

input
```
Int[(d*x)^m*(a + b*x^n + c*x^(2*n))^2,x]
```

output
```
(2*a*b*x^(1 + n)*(d*x)^m)/(1 + m + n) + ((b^2 + 2*a*c)*x^(1 + 2*n)*(d*x)^m
)/(1 + m + 2*n) + (2*b*c*x^(1 + 3*n)*(d*x)^m)/(1 + m + 3*n) + (c^2*x^(1 +
4*n)*(d*x)^m)/(1 + m + 4*n) + (a^2*(d*x)^(1 + m))/(d*(1 + m))
```

3.597.3.1 Defintions of rubi rules used

rule 1691
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^n + c*x^(2*n))^p, x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && IGtQ[p, 0] &&  !IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.597.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.67 (sec) , antiderivative size = 1032, normalized size of antiderivative = 8.82

method	result	size
risch	Expression too large to display	1032
parallelrisch	Expression too large to display	1566

input
```
int((d*x)^m*(a+b*x^n+c*x^(2*n))^2,x,method=_RETURNVERBOSE)
```

output
```
x*(a^2+54*a*b*m*n*x^n+18*a*b*m^3*n*x^n+24*a*c*m*n^3*(x^n)^2+2*(x^n)^3*b*c+
2*(x^n)^2*a*c+8*m*b*c*(x^n)^3+14*b*c*(x^n)^3*n+12*a*b*m^2*x^n+52*a*b*n^2*x
^n+8*a*c*(x^n)^2*m+16*a*c*(x^n)^2*n+24*b^2*m^2*n*(x^n)^2+18*a*b*x^n*n+8*a*
b*x^n*m+38*a*c*n^2*(x^n)^2+24*b^2*m*n*(x^n)^2+14*b*c*m^3*n*(x^n)^3+28*b*c*
m^2*n^2*(x^n)^3+16*b*c*m*n^3*(x^n)^3+42*b*c*m^2*n*(x^n)^3+56*b*c*m*n^2*(x^
n)^3+19*b^2*m^2*n^2*(x^n)^2+22*c^2*m*n^2*(x^n)^4+2*a*c*m^4*(x^n)^2+8*b^2*m
^3*n*(x^n)^2+30*a^2*m*n+a^2*m^4+4*a^2*m^3+50*a^2*n^3+6*a^2*m^2+35*a^2*n^2+
10*a^2*m^3*n+52*a*b*m^2*n^2*x^n+48*a*b*m*n^3*x^n+48*a*c*m^2*n*(x^n)^2+76*a
*c*m*n^2*(x^n)^2+42*b*c*m*n*(x^n)^3+54*a*b*m^2*n*x^n+104*a*b*m*n^2*x^n+48*
a*c*m*n*(x^n)^2+18*c^2*m^2*n*(x^n)^4+38*b^2*m*n^2*(x^n)^2+12*b*c*m^2*(x^n)
^3+28*b*c*n^2*(x^n)^3+8*a*b*m^3*x^n+48*a*b*n^3*x^n+12*a*c*m^2*(x^n)^2+12*b
^2*m*n^3*(x^n)^2+8*b*c*m^3*(x^n)^3+16*b*c*n^3*(x^n)^3+35*a^2*m^2*n^2+50*a^
2*m*n^3+11*c^2*m^2*n^2*(x^n)^4+6*c^2*m*n^3*(x^n)^4+2*b*c*m^4*(x^n)^3+6*c^2
*m^3*n*(x^n)^4+6*c^2*(x^n)^4*n+6*b^2*m^2*(x^n)^2+19*b^2*n^2*(x^n)^2+12*b^2
*n^3*(x^n)^2+4*m*c^2*(x^n)^4+16*a*c*m^3*n*(x^n)^2+38*a*c*m^2*n^2*(x^n)^2+3
0*a^2*m^2*n+70*a^2*m*n^2+24*a^2*n^4+b^2*(x^n)^2+24*a*c*n^3*(x^n)^2+4*a^2*m
+10*a^2*n+6*c^2*n^3*(x^n)^4+b^2*m^4*(x^n)^2+6*c^2*m^2*(x^n)^4+c^2*m^4*(x^n
)^4+4*c^2*m^3*(x^n)^4+2*a*b*m^4*x^n+8*a*c*m^3*(x^n)^2+4*b^2*(x^n)^2*m+18*c
^2*m*n*(x^n)^4+c^2*(x^n)^4+8*b^2*(x^n)^2*n+11*c^2*n^2*(x^n)^4+4*b^2*m^3*(x
^n)^2+2*a*b*x^n)/(1+m)/(1+m+n)/(1+m+2*n)/(1+m+3*n)/(1+m+4*n)*x^m*d^m*ex...
```

3.597.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 706 vs. 2(117) = 234.

Time = 0.31 (sec) , antiderivative size = 706, normalized size of antiderivative = 6.03

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^2 dx = \frac{(c^2m^4+4\,c^2m^3+6\,c^2m^2+6\,(c^2m+c^2)n^3+4\,c^2m+11\,(c^2m^2+2\,c^2m+c^2)n^2+c^2+6\,(c^2m^3+3\,c^2m^2}{}$$

input
```
integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^2,x, algorithm="fricas")
```

output
```
((c^2*m^4 + 4*c^2*m^3 + 6*c^2*m^2 + 6*(c^2*m + c^2)*n^3 + 4*c^2*m + 11*(c^
2*m^2 + 2*c^2*m + c^2)*n^2 + c^2 + 6*(c^2*m^3 + 3*c^2*m^2 + 3*c^2*m + c^2)
*n)*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 2*(b*c*m^4 + 4*b*c*m^3 + 6*b*c*m^2
 + 8*(b*c*m + b*c)*n^3 + 4*b*c*m + 14*(b*c*m^2 + 2*b*c*m + b*c)*n^2 + b*c
+ 7*(b*c*m^3 + 3*b*c*m^2 + 3*b*c*m + b*c)*n)*x*x^(3*n)*e^(m*log(d) + m*log
(x)) + ((b^2 + 2*a*c)*m^4 + 4*(b^2 + 2*a*c)*m^3 + 12*(b^2 + 2*a*c + (b^2 +
 2*a*c)*m)*n^3 + 6*(b^2 + 2*a*c)*m^2 + 19*((b^2 + 2*a*c)*m^2 + b^2 + 2*a*c
 + 2*(b^2 + 2*a*c)*m)*n^2 + b^2 + 2*a*c + 4*(b^2 + 2*a*c)*m + 8*((b^2 + 2*
a*c)*m^3 + 3*(b^2 + 2*a*c)*m^2 + b^2 + 2*a*c + 3*(b^2 + 2*a*c)*m)*n)*x*x^(
2*n)*e^(m*log(d) + m*log(x)) + 2*(a*b*m^4 + 4*a*b*m^3 + 6*a*b*m^2 + 24*(a*
b*m + a*b)*n^3 + 4*a*b*m + 26*(a*b*m^2 + 2*a*b*m + a*b)*n^2 + a*b + 9*(a*b
*m^3 + 3*a*b*m^2 + 3*a*b*m + a*b)*n)*x*x^n*e^(m*log(d) + m*log(x)) + (a^2*
m^4 + 24*a^2*n^4 + 4*a^2*m^3 + 6*a^2*m^2 + 50*(a^2*m + a^2)*n^3 + 4*a^2*m
+ 35*(a^2*m^2 + 2*a^2*m + a^2)*n^2 + a^2 + 10*(a^2*m^3 + 3*a^2*m^2 + 3*a^2
*m + a^2)*n)*x*e^(m*log(d) + m*log(x)))/(m^5 + 24*(m + 1)*n^4 + 5*m^4 + 50
*(m^2 + 2*m + 1)*n^3 + 10*m^3 + 35*(m^3 + 3*m^2 + 3*m + 1)*n^2 + 10*m^2 +
10*(m^4 + 4*m^3 + 6*m^2 + 4*m + 1)*n + 5*m + 1)
```

3.597.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12323 vs. 2(107) = 214.

Time = 16.76 (sec) , antiderivative size = 12323, normalized size of antiderivative = 105.32

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^2 dx = \text{Too large to display}$$

input
```
integrate((d*x)**m*(a+b*x**n+c*x**(2*n))**2,x)
```

output
```
Piecewise(((a + b + c)**2*log(x)/d, Eq(m, -1) & Eq(n, 0)), ((a**2*log(x) +
 2*a*b*x**n/n + a*c*x**(2*n)/n + b**2*x**(2*n)/(2*n) + 2*b*c*x**(3*n)/(3*n
) + c**2*x**(4*n)/(4*n))/d, Eq(m, -1)), (a**2*Piecewise((0**(-4*n - 1)*x,
Eq(d, 0)), (Piecewise((-1/(4*n*(d*x)**(4*n)), Ne(n, 0)), (log(d*x), True))
/d, True)) + 2*a*b*Piecewise((-x*x**n*(d*x)**(-4*n - 1)/(3*n), Ne(n, 0)),
(x*x**n*(d*x)**(-4*n - 1)*log(x), True)) + 2*a*c*Piecewise((-x*x**(2*n)*(d
*x)**(-4*n - 1)/(2*n), Ne(n, 0)), (x*x**(2*n)*(d*x)**(-4*n - 1)*log(x), Tr
ue)) + b**2*Piecewise((-x*x**(2*n)*(d*x)**(-4*n - 1)/(2*n), Ne(n, 0)), (x*
x**(2*n)*(d*x)**(-4*n - 1)*log(x), True)) + 2*b*c*Piecewise((-x*x**(3*n)*(
d*x)**(-4*n - 1)/n, Ne(n, 0)), (x*x**(3*n)*(d*x)**(-4*n - 1)*log(x), True)
) + c**2*x*x**(4*n)*(d*x)**(-4*n - 1)*log(x), Eq(m, -4*n - 1)), (a**2*Piec
ewise((0**(-3*n - 1)*x, Eq(d, 0)), (Piecewise((-1/(3*n*(d*x)**(3*n)), Ne(n
, 0)), (log(d*x), True))/d, True)) + 2*a*b*Piecewise((-x*x**n*(d*x)**(-3*n
 - 1)/(2*n), Ne(n, 0)), (x*x**n*(d*x)**(-3*n - 1)*log(x), True)) + 2*a*c*P
iecewise((-x*x**(2*n)*(d*x)**(-3*n - 1)/n, Ne(n, 0)), (x*x**(2*n)*(d*x)**(
-3*n - 1)*log(x), True)) + b**2*Piecewise((-x*x**(2*n)*(d*x)**(-3*n - 1)/n
, Ne(n, 0)), (x*x**(2*n)*(d*x)**(-3*n - 1)*log(x), True)) + 2*b*c*x*x**(3*
n)*(d*x)**(-3*n - 1)*log(x) + c**2*Piecewise((x*x**(4*n)*(d*x)**(-3*n - 1)
/n, Ne(n, 0)), (x*x**(4*n)*(d*x)**(-3*n - 1)*log(x), True)), Eq(m, -3*n -
1)), (a**2*Piecewise((0**(-2*n - 1)*x, Eq(d, 0)), (Piecewise((-1/(2*n*(...
```

3.597.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.30

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^2 \, dx = \frac{c^2 d^m x e^{(m\log(x)+4\,n\log(x))}}{m+4\,n+1} + \frac{2\,bcd^m x e^{(m\log(x)+3\,n\log(x))}}{m+3\,n+1} + \frac{b^2 d^m x e^{(m\log(x)+2\,n\log(x))}}{m+2\,n+1} + \frac{2\,acd^m x e^{(m\log(x)+2\,n\log(x))}}{m+2\,n+1} + \frac{2\,abd^m x e^{(m\log(x)+n\log(x))}}{m+n+1} + \frac{(dx)^{m+1}\,a^2}{d(m+1)}$$

input
```
integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^2,x, algorithm="maxima")
```

output
```
c^2*d^m*x*e^(m*log(x) + 4*n*log(x))/(m + 4*n + 1) + 2*b*c*d^m*x*e^(m*log(x
) + 3*n*log(x))/(m + 3*n + 1) + b^2*d^m*x*e^(m*log(x) + 2*n*log(x))/(m + 2
*n + 1) + 2*a*c*d^m*x*e^(m*log(x) + 2*n*log(x))/(m + 2*n + 1) + 2*a*b*d^m*
x*e^(m*log(x) + n*log(x))/(m + n + 1) + (d*x)^(m + 1)*a^2/(d*(m + 1))
```

3.597.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5454 vs. 2(117) = 234.

Time = 0.33 (sec) , antiderivative size = 5454, normalized size of antiderivative = 46.62

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^2,x, algorithm="giac")
```

output
```
(c^2*m^4*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 6*c^2*m^3*n*x*x^(4*n)*e^(m*lo
g(d) + m*log(x)) + 11*c^2*m^2*n^2*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 6*c^
2*m*n^3*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 2*b*c*m^4*x*x^(3*n)*e^(m*log(d
) + m*log(x)) + c^2*m^4*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 14*b*c*m^3*n*x
*x^(3*n)*e^(m*log(d) + m*log(x)) + 6*c^2*m^3*n*x*x^(3*n)*e^(m*log(d) + m*l
og(x)) + 28*b*c*m^2*n^2*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 11*c^2*m^2*n^2
*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 16*b*c*m*n^3*x*x^(3*n)*e^(m*log(d) +
m*log(x)) + 6*c^2*m*n^3*x*x^(3*n)*e^(m*log(d) + m*log(x)) + b^2*m^4*x*x^(2
*n)*e^(m*log(d) + m*log(x)) + 2*a*c*m^4*x*x^(2*n)*e^(m*log(d) + m*log(x))
+ 2*b*c*m^4*x*x^(2*n)*e^(m*log(d) + m*log(x)) + c^2*m^4*x*x^(2*n)*e^(m*log
(d) + m*log(x)) + 8*b^2*m^3*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 16*a*c*m
^3*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 14*b*c*m^3*n*x*x^(2*n)*e^(m*log(d
) + m*log(x)) + 6*c^2*m^3*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 19*b^2*m^2
*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 38*a*c*m^2*n^2*x*x^(2*n)*e^(m*log
(d) + m*log(x)) + 28*b*c*m^2*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 11*c^
2*m^2*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 12*b^2*m*n^3*x*x^(2*n)*e^(m*
log(d) + m*log(x)) + 24*a*c*m*n^3*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 16*b
*c*m*n^3*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 6*c^2*m*n^3*x*x^(2*n)*e^(m*lo
g(d) + m*log(x)) + 2*a*b*m^4*x*x^n*e^(m*log(d) + m*log(x)) + b^2*m^4*x*x^n
*e^(m*log(d) + m*log(x)) + 2*a*c*m^4*x*x^n*e^(m*log(d) + m*log(x)) + 2*...
```

3.597.9 Mupad [B] (verification not implemented)

Time = 9.14 (sec) , antiderivative size = 543, normalized size of antiderivative = 4.64

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^2 \, dx = \frac{a^2\,x\,(d\,x)^m}{m+1}$$
$$+\frac{x\,x^{2\,n}\,(d\,x)^m\,(b^2+2\,a\,c)\,\left(m^3+8\,m^2\,n+3\,m^2+19\,m\,n^2+16\,m\,n+3\,m+12\,n^3+19\,n^2+\right.}{m^4+10\,m^3\,n+4\,m^3+35\,m^2\,n^2+30\,m^2\,n+6\,m^2+50\,m\,n^3+70\,m\,n^2+30\,m\,n+4\,m+24\,n^4+50\,n}$$
$$+\frac{c^2\,x\,x^{4\,n}\,(d\,x)^m\,\left(m^3+6\,m^2\,n+3\,m^2+11\,m\,n^2+12\,m\,n+3\,m+6\,n^3+11\,n^2+6\,n+\right.}{m^4+10\,m^3\,n+4\,m^3+35\,m^2\,n^2+30\,m^2\,n+6\,m^2+50\,m\,n^3+70\,m\,n^2+30\,m\,n+4\,m+24\,n^4+50\,n}$$
$$+\frac{2\,a\,b\,x\,x^n\,(d\,x)^m\,\left(m^3+9\,m^2\,n+3\,m^2+26\,m\,n^2+18\,m\,n+3\,m+24\,n^3+26\,n^2+9\,n\right.}{m^4+10\,m^3\,n+4\,m^3+35\,m^2\,n^2+30\,m^2\,n+6\,m^2+50\,m\,n^3+70\,m\,n^2+30\,m\,n+4\,m+24\,n^4+50\,n}$$
$$+\frac{2\,b\,c\,x\,x^{3\,n}\,(d\,x)^m\,\left(m^3+7\,m^2\,n+3\,m^2+14\,m\,n^2+14\,m\,n+3\,m+8\,n^3+14\,n^2+7\,n\right.}{m^4+10\,m^3\,n+4\,m^3+35\,m^2\,n^2+30\,m^2\,n+6\,m^2+50\,m\,n^3+70\,m\,n^2+30\,m\,n+4\,m+24\,n^4+50\,n}$$

input
```
int((d*x)^m*(a + b*x^n + c*x^(2*n))^2,x)
```

output
```
(a^2*x*(d*x)^m)/(m + 1) + (x*x^(2*n)*(d*x)^m*(2*a*c + b^2)*(3*m + 8*n + 16
*m*n + 19*m*n^2 + 8*m^2*n + 3*m^2 + m^3 + 19*n^2 + 12*n^3 + 1))/(4*m + 10*
n + 30*m*n + 70*m*n^2 + 30*m^2*n + 50*m*n^3 + 10*m^3*n + 6*m^2 + 4*m^3 + m
^4 + 35*n^2 + 50*n^3 + 24*n^4 + 35*m^2*n^2 + 1) + (c^2*x*x^(4*n)*(d*x)^m*(
3*m + 6*n + 12*m*n + 11*m*n^2 + 6*m^2*n + 3*m^2 + m^3 + 11*n^2 + 6*n^3 + 1
))/(4*m + 10*n + 30*m*n + 70*m*n^2 + 30*m^2*n + 50*m*n^3 + 10*m^3*n + 6*m^
2 + 4*m^3 + m^4 + 35*n^2 + 50*n^3 + 24*n^4 + 35*m^2*n^2 + 1) + (2*a*b*x*x^
n*(d*x)^m*(3*m + 9*n + 18*m*n + 26*m*n^2 + 9*m^2*n + 3*m^2 + m^3 + 26*n^2
+ 24*n^3 + 1))/(4*m + 10*n + 30*m*n + 70*m*n^2 + 30*m^2*n + 50*m*n^3 + 10*
m^3*n + 6*m^2 + 4*m^3 + m^4 + 35*n^2 + 50*n^3 + 24*n^4 + 35*m^2*n^2 + 1) +
 (2*b*c*x*x^(3*n)*(d*x)^m*(3*m + 7*n + 14*m*n + 14*m*n^2 + 7*m^2*n + 3*m^2
 + m^3 + 14*n^2 + 8*n^3 + 1))/(4*m + 10*n + 30*m*n + 70*m*n^2 + 30*m^2*n +
 50*m*n^3 + 10*m^3*n + 6*m^2 + 4*m^3 + m^4 + 35*n^2 + 50*n^3 + 24*n^4 + 35
*m^2*n^2 + 1)
```

3.598 $\int (dx)^m (a + bx^n + cx^{2n}) \, dx$

3.598.1 Optimal result

Integrand size = 20, antiderivative size = 58

$$\int (dx)^m (a + bx^n + cx^{2n}) \, dx = \frac{bx^{1+n}(dx)^m}{1+m+n} + \frac{cx^{1+2n}(dx)^m}{1+m+2n} + \frac{a(dx)^{1+m}}{d(1+m)}$$

output
```
b*x^(1+n)*(d*x)^m/(1+m+n)+c*x^(1+2*n)*(d*x)^m/(1+m+2*n)+a*(d*x)^(1+m)/d/(1
+m)
```

3.598.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.71

$$\int (dx)^m (a + bx^n + cx^{2n}) \, dx = x(dx)^m \left(\frac{a}{1+m} + x^n \left(\frac{b}{1+m+n} + \frac{cx^n}{1+m+2n}\right)\right)$$

input
```
Integrate[(d*x)^m*(a + b*x^n + c*x^(2*n)),x]
```

output
```
x*(d*x)^m*(a/(1 + m) + x^n*(b/(1 + m + n) + (c*x^n)/(1 + m + 2*n)))
```

3.598.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1691, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right) dx$$

$$\Big\downarrow \ 1691$$

$$\int \left(a(dx)^m+bx^n(dx)^m+cx^{2n}(dx)^m\right) dx$$

$$\Big\downarrow \ 2009$$

$$\frac{a(dx)^{m+1}}{d(m+1)}+\frac{bx^{n+1}(dx)^m}{m+n+1}+\frac{cx^{2n+1}(dx)^m}{m+2n+1}$$

input
```
Int[(d*x)^m*(a + b*x^n + c*x^(2*n)),x]
```

output
```
(b*x^(1 + n)*(d*x)^m)/(1 + m + n) + (c*x^(1 + 2*n)*(d*x)^m)/(1 + m + 2*n)
+ (a*(d*x)^(1 + m))/(d*(1 + m))
```

3.598.3.1 Defintions of rubi rules used

rule 1691
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^n + c*x^(2*n))^p, x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && IGtQ[p, 0] &&  !IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.598.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.16 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.97

method	result
risch	$\frac{x\left(c\,m^{2}x^{2n}+cmn\,x^{2n}+x^{n}b\,m^{2}+2mb\,x^{n}n+2x^{2n}cm+c\,x^{2n}n+a\,m^{2}+3amn+2a\,n^{2}+2mb\,x^{n}+2b\,x^{n}n+c\,x^{2n}+2am+3an+b\,x^{n}+\right.}{(1+m)(1+m+n)(1+m+2n)}$
parallelrisch	$\frac{x\,x^{2n}(dx)^{m}c+3x(dx)^{m}an+x(dx)^{m}a+2x(dx)^{m}a\,n^{2}+x\,x^{n}(dx)^{m}b+2x\,x^{n}(dx)^{m}bmn+x\,x^{2n}(dx)^{m}cmn+x\,x^{n}(dx)^{m}b\,m^{2}+x\,x^{2n}}{(1+m)(1+m+n}$

input

```
int((d*x)^m*(a+b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

output

```
x*(c*m^2*(x^n)^2+c*m*n*(x^n)^2+x^n*b*m^2+2*m*b*x^n*n+2*m*c*(x^n)^2+c*(x^n)
^2*n+a*m^2+3*a*m*n+2*a*n^2+2*m*b*x^n+2*b*x^n*n+c*(x^n)^2+2*a*m+3*a*n+b*x^n
+a)/(1+m)/(1+m+n)/(1+m+2*n)*d^m*x^m*exp(1/2*I*Pi*csgn(I*d*x)*m*(csgn(I*d*x
)-csgn(I*x))*(-csgn(I*d*x)+csgn(I*d)))
```

3.598.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. 2(58) = 116.

Time = 0.28 (sec) , antiderivative size = 142, normalized size of antiderivative = 2.45

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)\,dx$$
$$= \frac{(cm^2+2\,cm+(cm+c)n+c)xx^{2\,n}e^{(m\log(d)+m\log(x))}+(bm^2+2\,bm+2\,(bm+b)n+b)xx^{n}e^{(m\log(d)+m\log(x}}{m^3+2\,(m+1)n^2+3\,m^2+3\,(m^2+2\,m+1)n+}$$

input

```
integrate((d*x)^m*(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output

```
((c*m^2 + 2*c*m + (c*m + c)*n + c)*x*x^(2*n)*e^(m*log(d) + m*log(x)) + (b*
m^2 + 2*b*m + 2*(b*m + b)*n + b)*x*x^n*e^(m*log(d) + m*log(x)) + (a*m^2 +
2*a*n^2 + 2*a*m + 3*(a*m + a)*n + a)*x*e^(m*log(d) + m*log(x)))/(m^3 + 2*(
m + 1)*n^2 + 3*m^2 + 3*(m^2 + 2*m + 1)*n + 3*m + 1)
```

3.598.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1096 vs. $2(49) = 98$.

Time = 3.36 (sec) , antiderivative size = 1096, normalized size of antiderivative = 18.90

$$\int (dx)^m \left(a + bx^n + cx^{2n}\right) \, dx = \text{Too large to display}$$

input
```
integrate((d*x)**m*(a+b*x**n+c*x**(2*n)),x)
```

output
```
Piecewise(((a + b + c)*log(x)/d, Eq(m, -1) & Eq(n, 0)), ((a*log(x) + b*x**
n/n + c*x**(2*n)/(2*n))/d, Eq(m, -1)), (a*Piecewise((0**(-2*n - 1)*x, Eq(d
, 0)), (Piecewise((-1/(2*n*(d*x)**(2*n)), Ne(n, 0)), (log(d*x), True))/d,
True)) + b*Piecewise((-x*x**n*(d*x)**(-2*n - 1)/n, Ne(n, 0)), (x*x**n*(d*x
)**(-2*n - 1)*log(x), True)) + c*x*x**(2*n)*(d*x)**(-2*n - 1)*log(x), Eq(m
, -2*n - 1)), (a*Piecewise((0**(-n - 1)*x, Eq(d, 0)), (Piecewise((-1/(n*(d
*x)**n), Ne(n, 0)), (log(d*x), True))/d, True)) + b*x*x**n*(d*x)**(-n - 1)
*log(x) + c*Piecewise((x*x**(2*n)*(d*x)**(-n - 1)/n, Ne(n, 0)), (x*x**(2*n
)*(d*x)**(-n - 1)*log(x), True)), Eq(m, -n - 1)), (a*m**2*x*(d*x)**m/(m**3
 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + 3*a*m*
n*x*(d*x)**m/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 +
 3*n + 1) + 2*a*m*x*(d*x)**m/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n
+ 3*m + 2*n**2 + 3*n + 1) + 2*a*n**2*x*(d*x)**m/(m**3 + 3*m**2*n + 3*m**2
+ 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + 3*a*n*x*(d*x)**m/(m**3 + 3*
m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + a*x*(d*x)**
m/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) +
 b*m**2*x*x**n*(d*x)**m/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m
 + 2*n**2 + 3*n + 1) + 2*b*m*n*x*x**n*(d*x)**m/(m**3 + 3*m**2*n + 3*m**2 +
 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + 2*b*m*x*x**n*(d*x)**m/(m**3
+ 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + 2*b*...
```

3.598.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.12

$$\int (dx)^m \left(a + bx^n + cx^{2n}\right) \, dx = \frac{cd^m xe^{(m \log(x) + 2 n \log(x))}}{m + 2 n + 1} + \frac{bd^m xe^{(m \log(x) + n \log(x))}}{m + n + 1} + \frac{(dx)^{m+1} a}{d(m + 1)}$$

input
```
integrate((d*x)^m*(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
c*d^m*x*e^(m*log(x) + 2*n*log(x))/(m + 2*n + 1) + b*d^m*x*e^(m*log(x) + n*
log(x))/(m + n + 1) + (d*x)^(m + 1)*a/(d*(m + 1))
```

3.598.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 557 vs. $2(58) = 116$.

Time = 0.28 (sec) , antiderivative size = 557, normalized size of antiderivative = 9.60

$$\int (dx)^m \left(a + bx^n + cx^{2n}\right) dx$$
$$= \frac{cm^2 x x^{2n} e^{(m\log(d)+m\log(x))} + cmnx x^{2n} e^{(m\log(d)+m\log(x))} + bm^2 x x^n e^{(m\log(d)+m\log(x))} + cm^2 x x^n e^{(m\log(d)+m\log}}{}$$

input
```
integrate((d*x)^m*(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
(c*m^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + c*m*n*x*x^(2*n)*e^(m*log(d) + m
*log(x)) + b*m^2*x*x^n*e^(m*log(d) + m*log(x)) + c*m^2*x*x^n*e^(m*log(d) +
 m*log(x)) + 2*b*m*n*x*x^n*e^(m*log(d) + m*log(x)) + c*m*n*x*x^n*e^(m*log(
d) + m*log(x)) + a*m^2*x*e^(m*log(d) + m*log(x)) + b*m^2*x*e^(m*log(d) + m
*log(x)) + c*m^2*x*e^(m*log(d) + m*log(x)) + 3*a*m*n*x*e^(m*log(d) + m*log
(x)) + 2*b*m*n*x*e^(m*log(d) + m*log(x)) + c*m*n*x*e^(m*log(d) + m*log(x))
 + 2*a*n^2*x*e^(m*log(d) + m*log(x)) + 2*c*m*x*x^(2*n)*e^(m*log(d) + m*log
(x)) + c*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 2*b*m*x*x^n*e^(m*log(d) + m
*log(x)) + 2*c*m*x*x^n*e^(m*log(d) + m*log(x)) + 2*b*n*x*x^n*e^(m*log(d) +
 m*log(x)) + c*n*x*x^n*e^(m*log(d) + m*log(x)) + 2*a*m*x*e^(m*log(d) + m*l
og(x)) + 2*b*m*x*e^(m*log(d) + m*log(x)) + 2*c*m*x*e^(m*log(d) + m*log(x))
 + 3*a*n*x*e^(m*log(d) + m*log(x)) + 2*b*n*x*e^(m*log(d) + m*log(x)) + c*n
*x*e^(m*log(d) + m*log(x)) + c*x*x^(2*n)*e^(m*log(d) + m*log(x)) + b*x*x^n
*e^(m*log(d) + m*log(x)) + c*x*x^n*e^(m*log(d) + m*log(x)) + a*x*e^(m*log(
d) + m*log(x)) + b*x*e^(m*log(d) + m*log(x)) + c*x*e^(m*log(d) + m*log(x))
)/(m^3 + 3*m^2*n + 2*m*n^2 + 3*m^2 + 6*m*n + 2*n^2 + 3*m + 3*n + 1)
```

3.598.9 Mupad [B] (verification not implemented)

Time = 8.93 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.43

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right) dx = (d\,x)^m \left(\frac{a\,x}{m+1}+\frac{b\,x\,x^n\,(m+2\,n+1)}{m^2+3\,m\,n+2\,m+2\,n^2+3\,n+1} + \frac{c\,x\,x^{2\,n}\,(m+n+1)}{m^2+3\,m\,n+2\,m+2\,n^2+3\,n+1}\right)$$

input
```
int((d*x)^m*(a + b*x^n + c*x^(2*n)),x)
```

output
```
(d*x)^m*((a*x)/(m + 1) + (b*x*x^n*(m + 2*n + 1))/(2*m + 3*n + 3*m*n + m^2
+ 2*n^2 + 1) + (c*x*x^(2*n)*(m + n + 1))/(2*m + 3*n + 3*m*n + m^2 + 2*n^2
+ 1))
```

3.599 $\int \frac{(dx)^m}{a+bx^n+cx^{2n}}\,dx$

3.599.1 Optimal result

Integrand size = 22, antiderivative size = 175

$$\int \frac{(dx)^m}{a+bx^n+cx^{2n}}\,dx = \frac{2c(dx)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)d(1+m)} - \frac{2c(dx)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)d(1+m)}$$

output
```
2*c*(d*x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-2*c*x^n/(b-(-4*a*c+b^2
)^(1/2)))/d/(1+m)/(b-(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)-2*c*(d*x)^(1+m
)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/d/(1
+m)/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))
```

3.599.2 Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.75

$$\int \frac{(dx)^m}{a+bx^n+cx^{2n}}\,dx = -\frac{x(dx)^m\left(\frac{2c\left(1-2^{-\frac{1+m}{n}}\left(\frac{cx^n}{b-\sqrt{b^2-4ac}+2cx^n}\right)^{-\frac{1+m}{n}}\text{Hypergeometric2F1}\left(-\frac{1+m}{n},-\frac{1+m}{n},1-\frac{1+m}{n},\frac{b-\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}+2cx^n}\right)\right)}{b^2-4ac-b\sqrt{b^2-4ac}} + \frac{2c\left(1-2^{-\frac{1}{}}\right.}{}\right.}{1+m}$$

input
```
Integrate[(d*x)^m/(a + b*x^n + c*x^(2*n)),x]
```

output
```
-((x*(d*x)^m*((2*c*(1 - Hypergeometric2F1[-((1 + m)/n), -((1 + m)/n), 1 -
(1 + m)/n, (b - Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)]/(2^(
(1 + m)/n)*((c*x^n)/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n))^((1 + m)/n))))/(b^2
 - 4*a*c - b*Sqrt[b^2 - 4*a*c]) + (2*c*(1 - Hypergeometric2F1[-((1 + m)/n)
, -((1 + m)/n), (-1 - m + n)/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*
a*c] + 2*c*x^n)]/(2^((1 + m)/n)*((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)
)^((1 + m)/n))))/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c]))))/(1 + m))
```

3.599.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1719, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{a+bx^n+cx^{2n}}dx$$

$$\Big\downarrow \text{1719}$$

$$\frac{2c\int \frac{(dx)^m}{2cx^n+b-\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}-\frac{2c\int \frac{(dx)^m}{2cx^n+b+\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{888}$$

$$\frac{2c(dx)^{m+1}\text{Hypergeometric2F1}\left(1,\frac{m+1}{n},\frac{m+n+1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c(dx)^{m+1}\text{Hypergeometric2F1}\left(1,\frac{m+1}{n},\frac{m+n+1}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}$$

input
```
Int[(d*x)^m/(a + b*x^n + c*x^(2*n)),x]
```

output
```
(2*c*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, (-2*c*x^
n)/(b - Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])*d*
(1 + m)) - (2*c*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/
n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 -
 4*a*c])*d*(1 + m))
```

3.599.3.1 Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1719
```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int[(d*x)^m/(b - q + 2
*c*x^n), x], x] - Simp[2*(c/q)   Int[(d*x)^m/(b + q + 2*c*x^n), x], x]] /;
FreeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

3.599.4 Maple [F]

$$\int \frac{(dx)^m}{a+b\,x^n+c\,x^{2n}}dx$$

input
```
int((d*x)^m/(a+b*x^n+c*x^(2*n)),x)
```

output
```
int((d*x)^m/(a+b*x^n+c*x^(2*n)),x)
```

3.599.5 Fricas [F]

$$\int \frac{(dx)^m}{a+bx^n+cx^{2n}}\,dx = \int \frac{(dx)^m}{cx^{2\,n}+bx^n+a}\,dx$$

input
```
integrate((d*x)^m/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
integral((d*x)^m/(c*x^(2*n) + b*x^n + a), x)
```

3.599.6 Sympy [F]

$$\int \frac{(dx)^m}{a+bx^n+cx^{2n}}\,dx=\int \frac{(dx)^m}{a+bx^n+cx^{2n}}\,dx$$

input
```
integrate((d*x)**m/(a+b*x**n+c*x**(2*n)),x)
```

output
```
Integral((d*x)**m/(a + b*x**n + c*x**(2*n)), x)
```

3.599.7 Maxima [F]

$$\int \frac{(dx)^m}{a+bx^n+cx^{2n}}\,dx=\int \frac{(dx)^m}{cx^{2\,n}+bx^n+a}\,dx$$

input
```
integrate((d*x)^m/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
integrate((d*x)^m/(c*x^(2*n) + b*x^n + a), x)
```

3.599.8 Giac [F]

$$\int \frac{(dx)^m}{a+bx^n+cx^{2n}}\,dx=\int \frac{(dx)^m}{cx^{2\,n}+bx^n+a}\,dx$$

input
```
integrate((d*x)^m/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate((d*x)^m/(c*x^(2*n) + b*x^n + a), x)
```

3.599.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{a+bx^n+cx^{2n}}\,dx=\int \frac{{\left(d\,x\right)}^m}{a+b\,x^n+c\,x^{2\,n}} \,d x$$

input
```
int((d*x)^m/(a + b*x^n + c*x^(2*n)),x)
```

output
```
int((d*x)^m/(a + b*x^n + c*x^(2*n)), x)
```

3.600 $\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^2} dx$

3.600.1 Optimal result

Integrand size = 22, antiderivative size = 328

$$\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^2} dx = \frac{(dx)^{1+m}(b^2-2ac+bcx^n)}{a(b^2-4ac)dn(a+bx^n+cx^{2n})} + \frac{c\left(\frac{4ac(1+m-2n)-b^2(1+m-n)}{\sqrt{b^2-4ac}}-b(1+m-n)\right)(dx)^{1+m}\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{a(b^2-4ac)\left(b-\sqrt{b^2-4ac}\right)d(1+m)n} - \frac{c\left(4ac(1+m-2n)-b^2(1+m-n)+b\sqrt{b^2-4ac}(1+m-n)\right)(dx)^{1+m}\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\right.}{a(b^2-4ac)^{3/2}\left(b+\sqrt{b^2-4ac}\right)d(1+m)n}$$

output
```
(d*x)^(1+m)*(b^2-2*a*c+b*c*x^n)/a/(-4*a*c+b^2)/d/n/(a+b*x^n+c*x^(2*n))+c*(
d*x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/
2)))*(-b*(1+m-n)+(4*a*c*(1+m-2*n)-b^2*(1+m-n))/(-4*a*c+b^2)^(1/2))/a/(-4*a
*c+b^2)/d/(1+m)/n/(b-(-4*a*c+b^2)^(1/2))-c*(d*x)^(1+m)*hypergeom([1, (1+m)
/n],[(1+m+n)/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(4*a*c*(1+m-2*n)-b^2*(1+m
-n)+b*(1+m-n)*(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(3/2)/d/(1+m)/n/(b+(-4*a*
c+b^2)^(1/2))
```

3.600.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1890 vs. $2(328) = 656$.

Time = 4.41 (sec) , antiderivative size = 1890, normalized size of antiderivative = 5.76

$$\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(d*x)^m/(a + b*x^n + c*x^(2*n))^2,x]
```

output
```
(x*(d*x)^m*((2*b^2*c)/(a*Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*(1 + m)
) - (8*c^2)/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*(1 + m)) + (2*b^2*c
)/(a*(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(1 + m)) + (8*c^2)/((-b^2 + 4*a*c
 + b*Sqrt[b^2 - 4*a*c])*(1 + m)) - (2*b^2*c)/(a*Sqrt[b^2 - 4*a*c]*(b + Sqr
t[b^2 - 4*a*c])*(1 + m)*n) + (4*c^2)/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*
a*c])*(1 + m)*n) + (4*c^2)/((b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(1 + m)*n)
 + (2*b^2*c)/(a*(-b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*(1 + m)*n) - (2*b^2*c
*m)/(a*Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*(1 + m)*n) + (4*c^2*m)/(S
qrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*(1 + m)*n) + (4*c^2*m)/((b^2 - 4*
a*c - b*Sqrt[b^2 - 4*a*c])*(1 + m)*n) + (2*b^2*c*m)/(a*(-b^2 + 4*a*c + b*S
qrt[b^2 - 4*a*c])*(1 + m)*n) - b^2/(a*n*(a + x^n*(b + c*x^n))) + (2*c)/(n*
(a + x^n*(b + c*x^n))) - (b*c*x^n)/(a*n*(a + x^n*(b + c*x^n))) + (c*(4*a*c
*Sqrt[b^2 - 4*a*c]*(1 + m - 2*n) + 4*a*b*c*(1 + m - n) - b^2*Sqrt[b^2 - 4*
a*c]*(1 + m - n) + b^3*(-1 - m + n))*Hypergeometric2F1[-((1 + m)/n), -((1
+ m)/n), 1 - (1 + m)/n, (b - Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c] + 2
*c*x^n)])/(2^((1 + m)/n)*a*Sqrt[b^2 - 4*a*c]*(-b^2 + 4*a*c + b*Sqrt[b^2 -
4*a*c])*(1 + m)*n*((c*x^n)/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n))^((1 + m)/n))
 + (b*c*(-1 - m + n)*Hypergeometric2F1[-((1 + m)/n), -((1 + m)/n), 1 - (1
+ m)/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)])/(2^((1
 + m)/n)*a*Sqrt[b^2 - 4*a*c]*(1 + m)*n*((c*x^n)/(b + Sqrt[b^2 - 4*a*c] ...
```

3.600.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 314, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1720, 25, 1884, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^2}\,dx$$

$$\Big\downarrow \; 1720$$

$$\frac{(dx)^{m+1}\left(-2ac+b^2+bcx^n\right)}{adn\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)}-\frac{\int -\frac{(dx)^m\left(-bc(m-n+1)x^n+2ac(m-2n+1)-b^2(m-n+1)\right)}{bx^n+cx^{2n}+a}dx}{an\left(b^2-4ac\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{(dx)^m\left(-bc(m-n+1)x^n+2ac(m-2n+1)-b^2(m-n+1)\right)}{bx^n+cx^{2n}+a}dx}{an\left(b^2-4ac\right)}+\frac{(dx)^{m+1}\left(-2ac+b^2+bcx^n\right)}{adn\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)}$$

$$\Big\downarrow \; 1884$$

$$\frac{\int \left(\frac{\left(-bc(m-n+1)-\frac{c\left(mb^2-nb^2+b^2-4ac-4acm+8acn\right)}{\sqrt{b^2-4ac}}\right)(dx)^m}{2cx^n+b-\sqrt{b^2-4ac}}+\frac{\left(\frac{c\left(mb^2-nb^2+b^2-4ac-4acm+8acn\right)}{\sqrt{b^2-4ac}}-bc(m-n+1)\right)(dx)^m}{2cx^n+b+\sqrt{b^2-4ac}}\right)dx}{an\left(b^2-4ac\right)}+\frac{(dx)^{m+1}\left(-2ac+b^2+bcx^n\right)}{adn\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{c(dx)^{m+1}\left(\frac{4ac(m-2n+1)-b^2(m-n+1)}{\sqrt{b^2-4ac}}-b(m-n+1)\right)\text{Hypergeometric2F1}\left(1,\frac{m+1}{n},\frac{m+n+1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{d(m+1)\left(b-\sqrt{b^2-4ac}\right)}-\frac{c(dx)^{m+1}\left(\frac{4ac(m-2n+1)-b^2(m-n+\right.}{\sqrt{b^2-4ac}}}{}}{an\left(b^2-4ac\right)}$$

$$\frac{(dx)^{m+1}\left(-2ac+b^2+bcx^n\right)}{adn\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)}$$

input
```
Int[(d*x)^m/(a + b*x^n + c*x^(2*n))^2,x]
```

output
```
((d*x)^(1 + m)*(b^2 - 2*a*c + b*c*x^n))/(a*(b^2 - 4*a*c)*d*n*(a + b*x^n +
c*x^(2*n))) + ((c*((4*a*c*(1 + m - 2*n) - b^2*(1 + m - n))/Sqrt[b^2 - 4*a*
c] - b*(1 + m - n))*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m +
 n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/((b - Sqrt[b^2 - 4*a*c])*d*(1
+ m)) - (c*((4*a*c*(1 + m - 2*n) - b^2*(1 + m - n))/Sqrt[b^2 - 4*a*c] + b*
(1 + m - n))*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n,
(-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((b + Sqrt[b^2 - 4*a*c])*d*(1 + m)))/
(a*(b^2 - 4*a*c)*n)
```

3.600. $\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^2}\,dx$

3.600.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1720
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(-(d*x)^(m + 1))*(b^2 - 2*a*c + b*c*x^n)*((a + b*x^n + c*x
^(2*n))^(p + 1)/(a*d*n*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(a*n*(p + 1)*(b
^2 - 4*a*c))   Int[(d*x)^m*(a + b*x^n + c*x^(2*n))^(p + 1)*Simp[b^2*(n*(p +
 1) + m + 1) - 2*a*c*(m + 2*n*(p + 1) + 1) + b*c*(2*n*p + 3*n + m + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*
a*c, 0] && ILtQ[p + 1, 0]
```

rule 1884
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] &&  !RationalQ[n] && (
IGtQ[p, 0] || IGtQ[q, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.600.4 Maple [F]

$$\int \frac{(dx)^m}{(a+b\,x^n+c\,x^{2n})^2}dx$$

input
```
int((d*x)^m/(a+b*x^n+c*x^(2*n))^2,x)
```

output
```
int((d*x)^m/(a+b*x^n+c*x^(2*n))^2,x)
```

3.600.5 Fricas [F]

$$\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^2}\,dx = \int \frac{(dx)^m}{(cx^{2n}+bx^n+a)^2}\,dx$$

input
```
integrate((d*x)^m/(a+b*x^n+c*x^(2*n))^2,x, algorithm="fricas")
```

output
```
integral((d*x)^m/(c^2*x^(4*n) + b^2*x^(2*n) + 2*a*b*x^n + a^2 + 2*(b*c*x^n
 + a*c)*x^(2*n)), x)
```

3.600.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x)**m/(a+b*x**n+c*x**(2*n))**2,x)
```

output
```
Timed out
```

3.600.7 Maxima [F]

$$\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^2}\,dx = \int \frac{(dx)^m}{(cx^{2n}+bx^n+a)^2}\,dx$$

input
```
integrate((d*x)^m/(a+b*x^n+c*x^(2*n))^2,x, algorithm="maxima")
```

output
```
(b*c*d^m*x*e^(m*log(x) + n*log(x)) + (b^2*d^m - 2*a*c*d^m)*x*x^m)/(a^2*b^2
*n - 4*a^3*c*n + (a*b^2*c*n - 4*a^2*c^2*n)*x^(2*n) + (a*b^3*n - 4*a^2*b*c*
n)*x^n) + integrate(-(b*c*d^m*(m - n + 1)*e^(m*log(x) + n*log(x)) + (b^2*d
^m*(m - n + 1) - 2*a*c*d^m*(m - 2*n + 1))*x^m)/(a^2*b^2*n - 4*a^3*c*n + (a
*b^2*c*n - 4*a^2*c^2*n)*x^(2*n) + (a*b^3*n - 4*a^2*b*c*n)*x^n), x)
```

3.600.8 Giac [F]

$$\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^2}\,dx = \int \frac{(dx)^m}{(cx^{2n}+bx^n+a)^2}\,dx$$

input
```
integrate((d*x)^m/(a+b*x^n+c*x^(2*n))^2,x, algorithm="giac")
```

output
```
integrate((d*x)^m/(c*x^(2*n) + b*x^n + a)^2, x)
```

3.600.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^2}\,dx = \int \frac{(d\,x)^m}{(a+b\,x^n+c\,x^{2\,n})^2}\,dx$$

input
```
int((d*x)^m/(a + b*x^n + c*x^(2*n))^2,x)
```

output
```
int((d*x)^m/(a + b*x^n + c*x^(2*n))^2, x)
```

3.601 $\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^3}\,dx$

3.601.1 Optimal result

Integrand size = 22, antiderivative size = 615

$$\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^3}\,dx = \frac{(dx)^{1+m}(b^2-2ac+bcx^n)}{2a(b^2-4ac)\,dn\,(a+bx^n+cx^{2n})^2}$$
$$-\frac{(dx)^{1+m}\left(4a^2c^2(1+m-4n)-5ab^2c(1+m-3n)+b^4(1+m-2n)-bc(2ac(2+2m-7n)-b^2(1+\right.}{2a^2(b^2-4ac)^2\,dn^2\,(a+bx^n+cx^{2n})}$$
$$-\frac{c\left(b\sqrt{b^2-4ac}(2ac(2+2m-7n)-b^2(1+m-2n))\,(1+m-n)-b^4(1+m^2+m(2-3n)-3n+2n^2\right.}{2}$$
$$-\frac{c\left(b\sqrt{b^2-4ac}(2ac(2+2m-7n)-b^2(1+m-2n))\,(1+m-n)+b^4(1+m^2+m(2-3n)-3n+2n^2\right.}{2}$$

output
```
1/2*(d*x)^(1+m)*(b^2-2*a*c+b*c*x^n)/a/(-4*a*c+b^2)/d/n/(a+b*x^n+c*x^(2*n))
^2-1/2*(d*x)^(1+m)*(4*a^2*c^2*(1+m-4*n)-5*a*b^2*c*(1+m-3*n)+b^4*(1+m-2*n)-
b*c*(2*a*c*(2+2*m-7*n)-b^2*(1+m-2*n))*x^n)/a^2/(-4*a*c+b^2)^2/d/n^2/(a+b*x
^n+c*x^(2*n))-1/2*c*(d*x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-2*c*x^
n/(b-(-4*a*c+b^2)^(1/2)))*(-b^4*(1+m^2+m*(2-3*n)-3*n+2*n^2)+6*a*b^2*c*(1+m
^2+m*(2-4*n)-4*n+3*n^2)-8*a^2*c^2*(1+m^2+m*(2-6*n)-6*n+8*n^2)+b*(2*a*c*(2+
2*m-7*n)-b^2*(1+m-2*n))*(1+m-n)*(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(5/2)
/d/(1+m)/n^2/(b-(-4*a*c+b^2)^(1/2))-1/2*c*(d*x)^(1+m)*hypergeom([1, (1+m)/
n],[(1+m+n)/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(b^4*(1+m^2+m*(2-3*n)-3*n+
2*n^2)-6*a*b^2*c*(1+m^2+m*(2-4*n)-4*n+3*n^2)+8*a^2*c^2*(1+m^2+m*(2-6*n)-6*
n+8*n^2)+b*(2*a*c*(2+2*m-7*n)-b^2*(1+m-2*n))*(1+m-n)*(-4*a*c+b^2)^(1/2))/a
^2/(-4*a*c+b^2)^(5/2)/d/(1+m)/n^2/(b+(-4*a*c+b^2)^(1/2))
```

3.601.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 12289 vs. 2(615) = 1230.

Time = 7.24 (sec) , antiderivative size = 12289, normalized size of antiderivative = 19.98

$$\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[(d*x)^m/(a + b*x^n + c*x^(2*n))^3,x]
```

output
```
Result too large to show
```

3.601.3 Rubi [A] (verified)

Time = 5.19 (sec) , antiderivative size = 677, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1720, 25, 1882, 25, 1884, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^3}\,dx$$

$$\Big\downarrow \; 1720$$

$$\frac{(dx)^{m+1}\left(-2ac+b^2+bcx^n\right)}{2adn\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)^2}-\frac{\int -\frac{(dx)^m\left(-bc(m-3n+1)x^n+2ac(m-4n+1)-b^2(m-2n+1)\right)}{\left(bx^n+cx^{2n}+a\right)^2}dx}{2an\left(b^2-4ac\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{(dx)^m\left(-bc(m-3n+1)x^n+2ac(m-4n+1)-b^2(m-2n+1)\right)}{\left(bx^n+cx^{2n}+a\right)^2}dx}{2an\left(b^2-4ac\right)}+\frac{(dx)^{m+1}\left(-2ac+b^2+bcx^n\right)}{2adn\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)^2}$$

$$\Big\downarrow \; 1882$$

$$\frac{-\frac{\int -\frac{(dx)^m\left(-bc\left(2ac(2m-7n+2)-b^2(m-2n+1)\right)(m-n+1)x^n+\left(2ac(m-4n+1)-b^2(m-2n+1)\right)\left(2ac(m-2n+1)-b^2(m-n+1)\right)-ab^2c(m+1)(m-3n+1)\right)}{bx^n+cx^{2n}+a}dx}{an(b^2-4ac)}-}{2an\left(b^2-4ac\right)}$$

$$\frac{(dx)^{m+1}\left(-2ac+b^2+bcx^n\right)}{2adn\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)^2}$$

$\downarrow$ 25

$$\frac{\int \frac{(dx)^m\left(-bc\left(2ac(2m-7n+2)-b^2(m-2n+1)\right)(m-n+1)x^n+\left(2ac(m-4n+1)-b^2(m-2n+1)\right)\left(2ac(m-2n+1)-b^2(m-n+1)\right)-ab^2c(m+1)(m-3n+1)\right)}{bx^n+cx^{2n}+a}dx}{an(b^2-4ac)} - \frac{(dx)}{2an\left(b^2-4ac\right)}$$

$$\frac{(dx)^{m+1}\left(-2ac+b^2+bcx^n\right)}{2adn\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)^2}$$

$\downarrow$ 1884

$$\int\left(\frac{\left(\frac{c\left(m^2b^4+2n^2b^4+2mb^4-3mnb^4-3nb^4+b^4-6acm^2b^2-18acn^2b^2-6acb^2-12acmb^2+24acnb^2+24acmnb^2+8a^2c^2+8a^2c^2m^2+64a^2c^2n^2+16a^2c^2m-48a^2c^2n\right.}{\sqrt{b^2-4ac}}}{2cx^n+b-\sqrt{b^2-4ac}}\right.$$

$$\frac{(dx)^{m+1}\left(-2ac+b^2+bcx^n\right)}{2adn\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)^2}$$

$\downarrow$ 2009

$$\frac{c(dx)^{m+1}\left(8a^2c^2\left(m^2+m(2-6n)+8n^2-6n+1\right)-6ab^2c\left(m^2+m(2-4n)+3n^2-4n+1\right)+b\sqrt{b^2-4ac}\left(b^2\left(m^2+m(2-3n)+2n^2-3n+1\right)-2ac\left(2m^2+m(4-9n)+7n^2-9n\right.\right.\right.}{d(m+1)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}$$

$$\frac{(dx)^{m+1}\left(-2ac+b^2+bcx^n\right)}{2adn\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)^2}$$

input
```
Int[(d*x)^m/(a + b*x^n + c*x^(2*n))^3,x]
```

3.601. $\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^3}\,dx$

output
```
((d*x)^(1 + m)*(b^2 - 2*a*c + b*c*x^n))/(2*a*(b^2 - 4*a*c)*d*n*(a + b*x^n
+ c*x^(2*n))^2) + (-(((d*x)^(1 + m)*(4*a^2*c^2*(1 + m - 4*n) - 5*a*b^2*c*(
1 + m - 3*n) + b^4*(1 + m - 2*n) - b*c*(2*a*c*(2 + 2*m - 7*n) - b^2*(1 + m
 - 2*n))*x^n))/(a*(b^2 - 4*a*c)*d*n*(a + b*x^n + c*x^(2*n)))) + ((c*(b^4*(
1 + m^2 + m*(2 - 3*n) - 3*n + 2*n^2) - 6*a*b^2*c*(1 + m^2 + m*(2 - 4*n) -
4*n + 3*n^2) + 8*a^2*c^2*(1 + m^2 + m*(2 - 6*n) - 6*n + 8*n^2) + b*Sqrt[b^
2 - 4*a*c]*(b^2*(1 + m^2 + m*(2 - 3*n) - 3*n + 2*n^2) - 2*a*c*(2 + 2*m^2 +
 m*(4 - 9*n) - 9*n + 7*n^2)))*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n
, (1 + m + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(
b - Sqrt[b^2 - 4*a*c])*d*(1 + m)) - (c*(b^4*(1 + m^2 + m*(2 - 3*n) - 3*n +
 2*n^2) - 6*a*b^2*c*(1 + m^2 + m*(2 - 4*n) - 4*n + 3*n^2) + 8*a^2*c^2*(1 +
 m^2 + m*(2 - 6*n) - 6*n + 8*n^2) - b*Sqrt[b^2 - 4*a*c]*(b^2*(1 + m^2 + m*
(2 - 3*n) - 3*n + 2*n^2) - 2*a*c*(2 + 2*m^2 + m*(4 - 9*n) - 9*n + 7*n^2)))
*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, (-2*c*x^n)/(
b + Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*d*(1 +
 m)))/(a*(b^2 - 4*a*c)*n))/(2*a*(b^2 - 4*a*c)*n)
```

3.601.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1720
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(-(d*x)^(m + 1))*(b^2 - 2*a*c + b*c*x^n)*((a + b*x^n + c*x
^(2*n))^(p + 1)/(a*d*n*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(a*n*(p + 1)*(b
^2 - 4*a*c))   Int[(d*x)^m*(a + b*x^n + c*x^(2*n))^(p + 1)*Simp[b^2*(n*(p +
 1) + m + 1) - 2*a*c*(m + 2*n*(p + 1) + 1) + b*c*(2*n*p + 3*n + m + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*
a*c, 0] && ILtQ[p + 1, 0]
```

rule 1882
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(a + b*x^n + c*x^
(2*n))^(p + 1)*((d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^n)/(a*f*n*(p +
 1)*(b^2 - 4*a*c))), x] + Simp[1/(a*n*(p + 1)*(b^2 - 4*a*c))   Int[(f*x)^m*
(a + b*x^n + c*x^(2*n))^(p + 1)*Simp[d*(b^2*(m + n*(p + 1) + 1) - 2*a*c*(m
+ 2*n*(p + 1) + 1)) - a*b*e*(m + 1) + (m + n*(2*p + 3) + 1)*(b*d - 2*a*e)*c
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[n2, 2*n] && N
eQ[b^2 - 4*a*c, 0] &&  !RationalQ[n] && ILtQ[p + 1, 0]
```

3.601. $\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^3}\,dx$

rule 1884
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] &&  !RationalQ[n] && (
IGtQ[p, 0] || IGtQ[q, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.601.4 Maple [F]

$$\int \frac{(dx)^m}{(a + b\,x^n + c\,x^{2n})^3}dx$$

input
```
int((d*x)^m/(a+b*x^n+c*x^(2*n))^3,x)
```

output
```
int((d*x)^m/(a+b*x^n+c*x^(2*n))^3,x)
```

3.601.5 Fricas [F]

$$\int \frac{(dx)^m}{(a + bx^n + cx^{2n})^3}\,dx = \int \frac{(dx)^m}{(cx^{2\,n} + bx^n + a)^3}\,dx$$

input
```
integrate((d*x)^m/(a+b*x^n+c*x^(2*n))^3,x, algorithm="fricas")
```

output
```
integral((d*x)^m/(c^3*x^(6*n) + b^3*x^(3*n) + 3*a*b^2*x^(2*n) + 3*a^2*b*x^
n + a^3 + 3*(b*c^2*x^n + a*c^2)*x^(4*n) + 3*(b^2*c*x^(2*n) + 2*a*b*c*x^n +
 a^2*c)*x^(2*n)), x)
```

3.601.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^3}\,dx = \text{Timed out}$$

input
```
integrate((d*x)**m/(a+b*x**n+c*x**(2*n))**3,x)
```

output
```
Timed out
```

3.601.7 Maxima [F]

$$\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^3}\,dx = \int \frac{(dx)^m}{(cx^{2n}+bx^n+a)^3}\,dx$$

input
```
integrate((d*x)^m/(a+b*x^n+c*x^(2*n))^3,x, algorithm="maxima")
```

output
```
1/2*((a^2*b^2*c*d^m*(5*m - 21*n + 5) - a*b^4*d^m*(m - 3*n + 1) - 4*a^3*c^2
*d^m*(m - 6*n + 1))*x*x^m + (2*a*b*c^3*d^m*(2*m - 7*n + 2) - b^3*c^2*d^m*(
m - 2*n + 1))*x*e^(m*log(x) + 3*n*log(x)) + (a*b^2*c^2*d^m*(9*m - 29*n + 9
) - 2*b^4*c*d^m*(m - 2*n + 1) - 4*a^2*c^3*d^m*(m - 4*n + 1))*x*e^(m*log(x)
 + 2*n*log(x)) - (b^5*d^m*(m - 2*n + 1) - 4*a*b^3*c*d^m*(m - 3*n + 1) + 2*
a^2*b*c^2*d^m*n)*x*e^(m*log(x) + n*log(x)))/(a^4*b^4*n^2 - 8*a^5*b^2*c*n^2
 + 16*a^6*c^2*n^2 + (a^2*b^4*c^2*n^2 - 8*a^3*b^2*c^3*n^2 + 16*a^4*c^4*n^2)
*x^(4*n) + 2*(a^2*b^5*c*n^2 - 8*a^3*b^3*c^2*n^2 + 16*a^4*b*c^3*n^2)*x^(3*n
) + (a^2*b^6*n^2 - 6*a^3*b^4*c*n^2 + 32*a^5*c^3*n^2)*x^(2*n) + 2*(a^3*b^5*
n^2 - 8*a^4*b^3*c*n^2 + 16*a^5*b*c^2*n^2)*x^n) - integrate(-1/2*(((m^2 - m
*(3*n - 2) + 2*n^2 - 3*n + 1)*b^4*d^m - (5*m^2 - m*(21*n - 10) + 16*n^2 -
21*n + 5)*a*b^2*c*d^m + 4*(m^2 - 2*m*(3*n - 1) + 8*n^2 - 6*n + 1)*a^2*c^2*
d^m)*x^m + ((m^2 - m*(3*n - 2) + 2*n^2 - 3*n + 1)*b^3*c*d^m - 2*(2*m^2 - m
*(9*n - 4) + 7*n^2 - 9*n + 2)*a*b*c^2*d^m)*e^(m*log(x) + n*log(x)))/(a^3*b
^4*n^2 - 8*a^4*b^2*c*n^2 + 16*a^5*c^2*n^2 + (a^2*b^4*c*n^2 - 8*a^3*b^2*c^2
*n^2 + 16*a^4*c^3*n^2)*x^(2*n) + (a^2*b^5*n^2 - 8*a^3*b^3*c*n^2 + 16*a^4*b
*c^2*n^2)*x^n), x)
```

3.601.8 Giac [F]

$$\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^3}\,dx = \int \frac{(dx)^m}{(cx^{2n}+bx^n+a)^3}\,dx$$

input
```
integrate((d*x)^m/(a+b*x^n+c*x^(2*n))^3,x, algorithm="giac")
```

output
```
integrate((d*x)^m/(c*x^(2*n) + b*x^n + a)^3, x)
```

3.601.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^3}\,dx = \int \frac{(d\,x)^m}{(a+b\,x^n+c\,x^{2\,n})^3}\,dx$$

input
```
int((d*x)^m/(a + b*x^n + c*x^(2*n))^3,x)
```

output
```
int((d*x)^m/(a + b*x^n + c*x^(2*n))^3, x)
```

3.602 $\int (dx)^m (a+bx^n+cx^{2n})^{3/2} \, dx$

3.602.1 Optimal result

Integrand size = 24, antiderivative size = 161

$$\int (dx)^m (a+bx^n+cx^{2n})^{3/2} \, dx = \frac{a(dx)^{1+m}\sqrt{a+bx^n+cx^{2n}} \operatorname{AppellF1}\left(\frac{1+m}{n},-\frac{3}{2},-\frac{3}{2},\frac{1+m+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{d(1+m)\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

output
```
a*(d*x)^(1+m)*AppellF1((1+m)/n,-3/2,-3/2,(1+m+n)/n,-2*c*x^n/(b-(-4*a*c+b^2
)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(a+b*x^n+c*x^(2*n))^(1/2)/d/(1+m
)/(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2
)))^(1/2)
```

3.602.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 545 vs. 2(161) = 322.

Time = 2.56 (sec) , antiderivative size = 545, normalized size of antiderivative = 3.39

$$\int (dx)^m (a+bx^n+cx^{2n})^{3/2} \, dx = \frac{x(dx)^m \left(-6an^2(1+m+n)\left(b^2(1+m)-4ac(1+m+2n)\right)\right)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2c}{b+\sqrt{b^2-4ac}}}}{}$$

input
```
Integrate[(d*x)^m*(a + b*x^n + c*x^(2*n))^(3/2),x]
```

output
```
(x*(d*x)^m*(-6*a*n^2*(1 + m + n)*(b^2*(1 + m) - 4*a*c*(1 + m + 2*n))*Sqrt[
(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[
b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/n, 1/2,
1/2, (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sq
rt[b^2 - 4*a*c])] + (1 + m)*(2*(1 + m + n)*(3*b^2*n^2 + 4*a*c*(1 + m^2 + 6
*n + 8*n^2 + m*(2 + 6*n)) + 2*b*c*(2 + 4*m + 2*m^2 + 9*n + 9*m*n + 7*n^2)*
x^n + 4*c^2*(1 + 2*m + m^2 + 3*n + 3*m*n + 2*n^2)*x^(2*n))*(a + x^n*(b + c
*x^n)) - 3*b*n^2*(b^2*(2 + 2*m + n) - 4*a*c*(2 + 2*m + 3*n))*x^n*Sqrt[(b -
 Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2
- 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m + n)/n, 1/2,
1/2, (1 + m + 2*n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b +
Sqrt[b^2 - 4*a*c])])))/(8*c*(1 + m)*(1 + m + n)^2*(1 + m + 2*n)*(1 + m + 3
*n)*Sqrt[a + x^n*(b + c*x^n)])
```

3.602.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^{3/2} dx$$

$$\Big\downarrow \text{ 1721}$$

$$\frac{a\sqrt{a+bx^n+cx^{2n}} \int (dx)^m \left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{3/2} \left(\frac{2cx^n}{b+\sqrt{b^2-4ac}}+1\right)^{3/2} dx}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

$$\Big\downarrow \text{ 1012}$$

$$\frac{a(dx)^{m+1}\sqrt{a+bx^n+cx^{2n}} \text{AppellF1}\left(\frac{m+1}{n},-\frac{3}{2},-\frac{3}{2},\frac{m+n+1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[(d*x)^m*(a + b*x^n + c*x^(2*n))^(3/2),x]
```

output
```
(a*(d*x)^(1 + m)*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[(1 + m)/n, -3/2, -3/
2, (1 + m + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt
[b^2 - 4*a*c])])/(d*(1 + m)*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sq
rt[1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])
```

3.602.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

3.602.4 Maple [F]

$$\int (dx)^m \left(a + b\,x^n + c\,x^{2n}\right)^{\frac{3}{2}} dx$$

input
```
int((d*x)^m*(a+b*x^n+c*x^(2*n))^(3/2),x)
```

output
```
int((d*x)^m*(a+b*x^n+c*x^(2*n))^(3/2),x)
```

3.602.5 Fricas [F(-2)]

Exception generated.

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^{3/2} dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.602.6 Sympy [F]

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^{3/2} dx = \int (dx)^m \left(a+bx^n+cx^{2n}\right)^{\frac{3}{2}} dx$$

input
```
integrate((d*x)**m*(a+b*x**n+c*x**(2*n))**(3/2),x)
```

output
```
Integral((d*x)**m*(a + b*x**n + c*x**(2*n))**(3/2), x)
```

3.602.7 Maxima [F]

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^{3/2} dx = \int \left(cx^{2n}+bx^n+a\right)^{\frac{3}{2}} (dx)^m dx$$

input
```
integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^(3/2)*(d*x)^m, x)
```

3.602.8 Giac [F]

$$\int (dx)^m \left(a + bx^n + cx^{2n}\right)^{3/2} dx = \int \left(cx^{2n} + bx^n + a\right)^{\frac{3}{2}} (dx)^m dx$$

input
```
integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^(3/2)*(d*x)^m, x)
```

3.602.9 Mupad [F(-1)]

Timed out.

$$\int (dx)^m \left(a + bx^n + cx^{2n}\right)^{3/2} dx = \int (d\,x)^m \left(a + b\,x^n + c\,x^{2\,n}\right)^{3/2} dx$$

input
```
int((d*x)^m*(a + b*x^n + c*x^(2*n))^(3/2),x)
```

output
```
int((d*x)^m*(a + b*x^n + c*x^(2*n))^(3/2), x)
```

3.603 $\int (dx)^m \sqrt{a+bx^n+cx^{2n}}\,dx$

3.603.1 Optimal result

Integrand size = 24, antiderivative size = 160

$$\int (dx)^m \sqrt{a+bx^n+cx^{2n}}\,dx = \frac{(dx)^{1+m}\sqrt{a+bx^n+cx^{2n}}\operatorname{AppellF1}\left(\frac{1+m}{n},-\frac{1}{2},-\frac{1}{2},\frac{1+m+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{d(1+m)\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

output
```
(d*x)^(1+m)*AppellF1((1+m)/n,-1/2,-1/2,(1+m+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^
(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(a+b*x^n+c*x^(2*n))^(1/2)/d/(1+m)/
(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2))
)^(1/2)
```

3.603.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 388 vs. 2(160) = 320.

Time = 0.71 (sec) , antiderivative size = 388, normalized size of antiderivative = 2.42

$$\int (dx)^m \sqrt{a+bx^n+cx^{2n}}\,dx = \frac{x(dx)^m\left(2an(1+m+n)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(\frac{1+m}{n},\frac{1}{2},\frac{1}{2},\frac{1+m+n}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2\ldots}{-b+\ldots}\right.\right.}{}$$

input
```
Integrate[(d*x)^m*Sqrt[a + b*x^n + c*x^(2*n)],x]
```

output
```
(x*(d*x)^m*(2*a*n*(1 + m + n)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b -
Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 -
 4*a*c])]*AppellF1[(1 + m)/n, 1/2, 1/2, (1 + m + n)/n, (-2*c*x^n)/(b + Sqr
t[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + (1 + m)*(2*(1 + m +
 n)*(a + x^n*(b + c*x^n)) + b*n*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)
/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt
[b^2 - 4*a*c])]*AppellF1[(1 + m + n)/n, 1/2, 1/2, (1 + m + 2*n)/n, (-2*c*x
^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])))/(2*(1
+ m)*(1 + m + n)^2*Sqrt[a + x^n*(b + c*x^n)])
```

3.603.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m\sqrt{a+bx^n+cx^{2n}}\,dx$$

$$\Big\downarrow \text{ 1721}$$

$$\frac{\sqrt{a+bx^n+cx^{2n}}\int (dx)^m\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{b+\sqrt{b^2-4ac}}+1}dx}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

$$\Big\downarrow \text{ 1012}$$

$$\frac{(dx)^{m+1}\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(\frac{m+1}{n},-\frac{1}{2},-\frac{1}{2},\frac{m+n+1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[(d*x)^m*Sqrt[a + b*x^n + c*x^(2*n)],x]
```

output
```
((d*x)^(1 + m)*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[(1 + m)/n, -1/2, -1/2,
 (1 + m + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b
^2 - 4*a*c])])/(d*(1 + m)*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt
[1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])
```

3.603.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

3.603.4 Maple [F]

$$\int (dx)^m \sqrt{a + b\,x^n + c\,x^{2n}}dx$$

input
```
int((d*x)^m*(a+b*x^n+c*x^(2*n))^(1/2),x)
```

output
```
int((d*x)^m*(a+b*x^n+c*x^(2*n))^(1/2),x)
```

3.603.5 Fricas [F(-2)]

Exception generated.

$$\int (dx)^m\sqrt{a+bx^n+cx^{2n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.603.6 Sympy [F]

$$\int (dx)^m \sqrt{a+bx^n+cx^{2n}}\,dx = \int (dx)^m \sqrt{a+bx^n+cx^{2n}}\,dx$$

input
```
integrate((d*x)**m*(a+b*x**n+c*x**(2*n))**(1/2),x)
```

output
```
Integral((d*x)**m*sqrt(a + b*x**n + c*x**(2*n)), x)
```

3.603.7 Maxima [F]

$$\int (dx)^m \sqrt{a+bx^n+cx^{2n}}\,dx = \int \sqrt{cx^{2n}+bx^n+a}(dx)^m\,dx$$

input
```
integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^(2*n) + b*x^n + a)*(d*x)^m, x)
```

3.603.8 Giac [F]

$$\int (dx)^m \sqrt{a+bx^n+cx^{2n}}\,dx = \int \sqrt{cx^{2n}+bx^n+a}(dx)^m\,dx$$

input
```
integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^(2*n) + b*x^n + a)*(d*x)^m, x)
```

3.603.9 Mupad [F(-1)]

Timed out.

$$\int (dx)^m \sqrt{a+bx^n+cx^{2n}}\,dx = \int (d\,x)^m\,\sqrt{a+b\,x^n+c\,x^{2\,n}}\,dx$$

input
```
int((d*x)^m*(a + b*x^n + c*x^(2*n))^(1/2),x)
```

output
```
int((d*x)^m*(a + b*x^n + c*x^(2*n))^(1/2), x)
```

3.604 $\int \frac{(dx)^m}{\sqrt{a+bx^n+cx^{2n}}}\,dx$

3.604.1 Optimal result

Integrand size = 24, antiderivative size = 160

$$\int \frac{(dx)^m}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \frac{(dx)^{1+m}\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{1+m}{n},\frac{1}{2},\frac{1}{2},\frac{1+m+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{d(1+m)\sqrt{a+bx^n+cx^{2n}}}$$

output
```
(d*x)^(1+m)*AppellF1((1+m)/n,1/2,1/2,(1+m+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1
/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(
1/2)*(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/d/(1+m)/(a+b*x^n+c*x^(2*n))^
(1/2)
```

3.604.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.14

$$\int \frac{(dx)^m}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \frac{x(dx)^m\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{1+m}{n},\frac{1}{2},\frac{1}{2},\frac{1+m+n}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)}{(1+m)\sqrt{a+x^n\,(b+cx^n)}}$$

input
```
Integrate[(d*x)^m/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

output
```
(x*(d*x)^m*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]
*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[
(1 + m)/n, 1/2, 1/2, (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2
*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/((1 + m)*Sqrt[a + x^n*(b + c*x^n)])
```

3.604.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{\sqrt{a+bx^n+cx^{2n}}}dx$$

$$\Big\downarrow \text{1721}$$

$$\frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\int \frac{(dx)^m}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{b+\sqrt{b^2-4ac}}+1}}dx}{\sqrt{a+bx^n+cx^{2n}}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{(dx)^{m+1}\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{m+1}{n},\frac{1}{2},\frac{1}{2},\frac{m+n+1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{a+bx^n+cx^{2n}}}$$

input
```
Int[(d*x)^m/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

output
```
((d*x)^(1 + m)*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x
^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/n, 1/2, 1/2, (1 + m + n)/n,
(-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(
d*(1 + m)*Sqrt[a + b*x^n + c*x^(2*n)])
```

3.604.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

3.604.4 Maple [F]

$$\int \frac{(dx)^m}{\sqrt{a+b\,x^n+c\,x^{2n}}}dx$$

input
```
int((d*x)^m/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

output
```
int((d*x)^m/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

3.604.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(dx)^m}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x)^m/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.604.6 Sympy [F]

$$\int \frac{(dx)^m}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{(dx)^m}{\sqrt{a+bx^n+cx^{2n}}}\,dx$$

input
```
integrate((d*x)**m/(a+b*x**n+c*x**(2*n))**(1/2),x)
```

output
```
Integral((d*x)**m/sqrt(a + b*x**n + c*x**(2*n)), x)
```

3.604.7 Maxima [F]

$$\int \frac{(dx)^m}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{(dx)^m}{\sqrt{cx^{2n}+bx^n+a}}\,dx$$

input
```
integrate((d*x)^m/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x)^m/sqrt(c*x^(2*n) + b*x^n + a), x)
```

3.604.8 Giac [F]

$$\int \frac{(dx)^m}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{(dx)^m}{\sqrt{cx^{2n}+bx^n+a}}\,dx$$

input
```
integrate((d*x)^m/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x)^m/sqrt(c*x^(2*n) + b*x^n + a), x)
```

3.604.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{\sqrt{a+bx^n+cx^{2n}}}\,dx=\int \frac{(d\,x)^m}{\sqrt{a+b\,x^n+c\,x^{2\,n}}}\,dx$$

input
```
int((d*x)^m/(a + b*x^n + c*x^(2*n))^(1/2),x)
```

output
```
int((d*x)^m/(a + b*x^n + c*x^(2*n))^(1/2), x)
```

3.605 $\int \frac{(dx)^m}{\left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx$

3.605.1 Optimal result

Integrand size = 24, antiderivative size = 163

$$\int \frac{(dx)^m}{\left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx=\frac{(dx)^{1+m}\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(\frac{1+m}{n},\frac{3}{2},\frac{3}{2},\frac{1+m+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4a}}\right.}{ad(1+m)\sqrt{a+bx^n+cx^{2n}}}$$

output
```
(d*x)^(1+m)*AppellF1((1+m)/n,3/2,3/2,(1+m+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1
/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(
1/2)*(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/d/(1+m)/(a+b*x^n+c*x^(2*n)
)^(1/2)
```

3.605.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 428 vs. 2(163) = 326.

Time = 1.45 (sec) , antiderivative size = 428, normalized size of antiderivative = 2.63

$$\int \frac{(dx)^m}{\left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx=\frac{x(dx)^m\left(\left(-4ac(1+m-n)+b^2(2+2m-n)\right)(1+m+n)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\cdots}\right.}{}$$

input
```
Integrate[(d*x)^m/(a + b*x^n + c*x^(2*n))^(3/2),x]
```

output
```
(x*(d*x)^m*((-4*a*c*(1 + m - n) + b^2*(2 + 2*m - n))*(1 + m + n)*Sqrt[(b -
 Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2
- 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/n, 1/2, 1/2,
 (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b
^2 - 4*a*c])] - 2*(1 + m)*((1 + m + n)*(b^2 - 2*a*c + b*c*x^n) - b*c*(1 +
m)*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqr
t[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 +
 m + n)/n, 1/2, 1/2, (1 + m + 2*n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]),
(2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])))/(a*(-b^2 + 4*a*c)*(1 + m)*n*(1 + m
+ n)*Sqrt[a + x^n*(b + c*x^n)])
```

3.605.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^{3/2}}\,dx$$

$$\Big\downarrow \text{1721}$$

$$\frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\int \frac{(dx)^m}{\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{3/2}\left(\frac{2cx^n}{b+\sqrt{b^2-4ac}}+1\right)^{3/2}}dx}{a\sqrt{a+bx^n+cx^{2n}}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{(dx)^{m+1}\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{m+1}{n},\frac{3}{2},\frac{3}{2},\frac{m+n+1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{ad(m+1)\sqrt{a+bx^n+cx^{2n}}}$$

input
```
Int[(d*x)^m/(a + b*x^n + c*x^(2*n))^(3/2),x]
```

output
```
((d*x)^(1 + m)*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x
^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/n, 3/2, 3/2, (1 + m + n)/n,
(-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(
a*d*(1 + m)*Sqrt[a + b*x^n + c*x^(2*n)])
```

3.605.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

3.605.4 Maple [F]

$$\int \frac{(dx)^m}{(a+b\,x^n+c\,x^{2n})^{\frac{3}{2}}}dx$$

input
```
int((d*x)^m/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

output
```
int((d*x)^m/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

3.605.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x)^m/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.605.6 Sympy [F]

$$\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \int \frac{(dx)^m}{(a+bx^n+cx^{2n})^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)**m/(a+b*x**n+c*x**(2*n))**(3/2),x)
```

output
```
Integral((d*x)**m/(a + b*x**n + c*x**(2*n))**(3/2), x)
```

3.605.7 Maxima [F]

$$\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \int \frac{(dx)^m}{(cx^{2n}+bx^n+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)^m/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x)^m/(c*x^(2*n) + b*x^n + a)^(3/2), x)
```

3.605.8 Giac [F]

$$\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \int \frac{(dx)^m}{(cx^{2n}+bx^n+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)^m/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x)^m/(c*x^(2*n) + b*x^n + a)^(3/2), x)
```

3.605.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \int \frac{(d\,x)^m}{(a+b\,x^n+c\,x^{2\,n})^{3/2}}\,dx$$

input
```
int((d*x)^m/(a + b*x^n + c*x^(2*n))^(3/2),x)
```

output
```
int((d*x)^m/(a + b*x^n + c*x^(2*n))^(3/2), x)
```

3.606 $\int (dx)^m (a+bx^n+cx^{2n})^p \, dx$

3.606.1 Optimal result

Integrand size = 22, antiderivative size = 158

$$\int (dx)^m (a+bx^n+cx^{2n})^p \, dx = \frac{(dx)^{1+m} \left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p} \left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p} (a+bx^n+cx^{2n})^p \operatorname{AppellF1}\left(\frac{1+m}{n},-p,-p,\frac{1+m+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-}}\right.}{d(1+m)}$$

output
```
(d*x)^(1+m)*(a+b*x^n+c*x^(2*n))^p*AppellF1((1+m)/n,-p,-p,(1+m+n)/n,-2*c*x^
n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/d/(1+m)/((1+2*c*
x^n/(b-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^p)
```

3.606.2 Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.15

$$\int (dx)^m (a+bx^n+cx^{2n})^p \, dx = \frac{x(dx)^m \left(\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p} \left(\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p} (a+x^n(b+cx^n))^p \operatorname{AppellF1}\left(\frac{1+m}{n},-p,-p,\frac{1+m+n}{n},-\frac{2c}{b+\sqrt{b}}\right.}{1+m}$$

input
```
Integrate[(d*x)^m*(a + b*x^n + c*x^(2*n))^p,x]
```

output
```
(x*(d*x)^m*(a + x^n*(b + c*x^n))^p*AppellF1[(1 + m)/n, -p, -p, (1 + m + n)
/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])
])/((1 + m)*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*
((b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

3.606.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^p dx$$

$$\Big\downarrow \text{ 1721}$$

$$\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p\int (dx)^m\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^p\left(\frac{2cx^n}{b+\sqrt{b^2-}\right.$$

$$\Big\downarrow \text{ 1012}$$

$$\frac{(dx)^{m+1}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p\text{AppellF1}\left(\frac{m+1}{n},-p,-p,\frac{m+n+1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},\right.}{d(m+1)}$$

input
```
Int[(d*x)^m*(a + b*x^n + c*x^(2*n))^p,x]
```

output
```
((d*x)^(1 + m)*(a + b*x^n + c*x^(2*n))^p*AppellF1[(1 + m)/n, -p, -p, (1 +
m + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4
*a*c])])/(d*(1 + m)*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^
n)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

3.606.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

3.606.4 Maple [F]

$$\int (dx)^m \left(a+b\,x^n+c\,x^{2n}\right)^p dx$$

input
```
int((d*x)^m*(a+b*x^n+c*x^(2*n))^p,x)
```

output
```
int((d*x)^m*(a+b*x^n+c*x^(2*n))^p,x)
```

3.606.5 Fricas [F]

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^p\,dx=\int \left(cx^{2\,n}+bx^n+a\right)^p(dx)^m\,dx$$

input
```
integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^p,x, algorithm="fricas")
```

output
```
integral((c*x^(2*n) + b*x^n + a)^p*(d*x)^m, x)
```

3.606.6 Sympy [F(-1)]

Timed out.

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^p \, dx = \text{Timed out}$$

input
```
integrate((d*x)**m*(a+b*x**n+c*x**(2*n))**p,x)
```

output
```
Timed out
```

3.606.7 Maxima [F]

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^p \, dx = \int \left(cx^{2n}+bx^n+a\right)^p (dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^p,x, algorithm="maxima")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^p*(d*x)^m, x)
```

3.606.8 Giac [F]

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^p \, dx = \int \left(cx^{2n}+bx^n+a\right)^p (dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^p,x, algorithm="giac")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^p*(d*x)^m, x)
```

3.606.9 Mupad [F(-1)]

Timed out.

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^p \, dx=\int (d\,x)^m\,\left(a+b\,x^n+c\,x^{2\,n}\right)^p\,dx$$

input
```
int((d*x)^m*(a + b*x^n + c*x^(2*n))^p,x)
```

output
```
int((d*x)^m*(a + b*x^n + c*x^(2*n))^p, x)
```

3.607 $\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right) dx$

3.607.1 Optimal result

Integrand size = 28, antiderivative size = 46

$$\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right) dx=\frac{a(d+ex)^4}{4e}+\frac{b(d+ex)^6}{6e}+\frac{c(d+ex)^8}{8e}$$

output
```
1/4*a*(e*x+d)^4/e+1/6*b*(e*x+d)^6/e+1/8*c*(e*x+d)^8/e
```

3.607.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 150 vs. 2(46) = 92.

Time = 0.05 (sec) , antiderivative size = 150, normalized size of antiderivative = 3.26

$$\begin{aligned}\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right) dx &= d^3\left(a+bd^2+cd^4\right)x+\frac{1}{2}d^2\left(3a+5bd^2+7cd^4\right)ex^2\\ &+\frac{1}{3}d\left(3a+10bd^2+21cd^4\right)e^2x^3\\ &+\frac{1}{4}\left(a+10bd^2+35cd^4\right)e^3x^4\\ &+d\left(b+7cd^2\right)e^4x^5+\frac{1}{6}\left(b+21cd^2\right)e^5x^6\\ &+cde^6x^7+\frac{1}{8}ce^7x^8\end{aligned}$$

input
```
Integrate[(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

output
```
d^3*(a + b*d^2 + c*d^4)*x + (d^2*(3*a + 5*b*d^2 + 7*c*d^4)*e*x^2)/2 + (d*(
3*a + 10*b*d^2 + 21*c*d^4)*e^2*x^3)/3 + ((a + 10*b*d^2 + 35*c*d^4)*e^3*x^4
)/4 + d*(b + 7*c*d^2)*e^4*x^5 + ((b + 21*c*d^2)*e^5*x^6)/6 + c*d*e^6*x^7 +
 (c*e^7*x^8)/8
```

3.607.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {1462, 1433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right) \, dx$$

$$\Big\downarrow \text{ 1462}$$

$$\frac{\int (d+ex)^3 \left(c(d+ex)^4+b(d+ex)^2+a\right) d(d+ex)}{e}$$

$$\Big\downarrow \text{ 1433}$$

$$\frac{\int \left(c(d+ex)^7+b(d+ex)^5+a(d+ex)^3\right) d(d+ex)}{e}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{1}{4}a(d+ex)^4+\frac{1}{6}b(d+ex)^6+\frac{1}{8}c(d+ex)^8}{e}$$

input
```
Int[(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

output
```
((a*(d + e*x)^4)/4 + (b*(d + e*x)^6)/6 + (c*(d + e*x)^8)/8)/e
```

3.607.3.1 Defintions of rubi rules used

rule 1433
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] ||  !IntegerQ[(m + 1)/2])
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.607.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 161 vs. $2(40) = 80$.

Time = 0.60 (sec) , antiderivative size = 162, normalized size of antiderivative = 3.52

method	result
norman	$\frac{e^7 c x^8}{8} + d e^6 c x^7 + \left(\frac{7}{2} d^2 e^5 c + \frac{1}{6} b e^5\right) x^6 + \left(7 d^3 c e^4 + b d e^4\right) x^5 + \left(\frac{35}{4} d^4 c e^3 + \frac{5}{2} b d^2 e^3 + \frac{1}{4} a e^3\right) x$
gosper	$\frac{x\left(3 e^7 c x^7 + 24 d e^6 c x^6 + 84 x^5 d^2 e^5 c + 168 c d^3 e^4 x^4 + 4 x^5 b e^5 + 210 x^3 d^4 c e^3 + 24 b d e^4 x^4 + 168 x^2 c d^5 e^2 + 60 x^3 b d^2 e^3 + 84 x c d^6 e + 80 x^2\right.}{24}$
risch	$\frac{1}{8} e^7 c x^8 + d e^6 c x^7 + \frac{7}{2} x^6 d^2 e^5 c + \frac{1}{6} x^6 b e^5 + 7 c d^3 e^4 x^5 + b d e^4 x^5 + \frac{35}{4} x^4 d^4 c e^3 + \frac{5}{2} x^4 b d^2 e^3 + \frac{1}{4} x$
parallelrisch	$\frac{1}{8} e^7 c x^8 + d e^6 c x^7 + \frac{7}{2} x^6 d^2 e^5 c + \frac{1}{6} x^6 b e^5 + 7 c d^3 e^4 x^5 + b d e^4 x^5 + \frac{35}{4} x^4 d^4 c e^3 + \frac{5}{2} x^4 b d^2 e^3 + \frac{1}{4} x$
default	$\frac{e^7 c x^8}{8} + d e^6 c x^7 + \frac{\left(15 d^2 e^5 c + e^3\left(6 c d^2 e^2 + b e^2\right)\right) x^6}{6} + \frac{\left(13 d^3 c e^4 + 3 d e^2\left(6 c d^2 e^2 + b e^2\right) + e^3\left(4 d^3 e c + 2 b d e\right)\right) x^5}{5} + \frac{(4 d^4 c e}{}$

input
```
int((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4),x,method=_RETURNVERBOSE)
```

output
```
1/8*e^7*c*x^8+d*e^6*c*x^7+(7/2*d^2*e^5*c+1/6*b*e^5)*x^6+(7*c*d^3*e^4+b*d*e
^4)*x^5+(35/4*d^4*c*e^3+5/2*b*d^2*e^3+1/4*a*e^3)*x^4+(7*c*d^5*e^2+10/3*b*d
^3*e^2+d*e^2*a)*x^3+(7/2*c*d^6*e+5/2*b*d^4*e+3/2*a*d^2*e)*x^2+(c*d^7+b*d^5
+a*d^3)*x
```

3.607.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(40) = 80$.

Time = 0.27 (sec) , antiderivative size = 142, normalized size of antiderivative = 3.09

$$\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right) dx$$
$$= \frac{1}{8} ce^7x^8 + cde^6x^7 + \frac{1}{6}\left(21\,cd^2+b\right)e^5x^6 + \left(7\,cd^3+bd\right)e^4x^5 + \frac{1}{4}\left(35\,cd^4+10\,bd^2+a\right)e^3x^4 + \frac{1}{3}\left(21\,cd^5+10\,bd^3+3\,ad\right)e^2x^3 + \frac{1}{2}\left(7\,cd^6+5\,bd^4+3\,ad^2\right)ex^2 + \left(cd^7+bd^5+ad^3\right)x$$

input
```
integrate((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

output
```
1/8*c*e^7*x^8 + c*d*e^6*x^7 + 1/6*(21*c*d^2 + b)*e^5*x^6 + (7*c*d^3 + b*d)
*e^4*x^5 + 1/4*(35*c*d^4 + 10*b*d^2 + a)*e^3*x^4 + 1/3*(21*c*d^5 + 10*b*d^
3 + 3*a*d)*e^2*x^3 + 1/2*(7*c*d^6 + 5*b*d^4 + 3*a*d^2)*e*x^2 + (c*d^7 + b*
d^5 + a*d^3)*x
```

3.607.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(34) = 68$.

Time = 0.04 (sec) , antiderivative size = 178, normalized size of antiderivative = 3.87

$$\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right) dx$$
$$= cde^6x^7 + \frac{ce^7x^8}{8} + x^6\left(\frac{be^5}{6}+\frac{7cd^2e^5}{2}\right) + x^5\left(bde^4+7cd^3e^4\right) + x^4\left(\frac{ae^3}{4}+\frac{5bd^2e^3}{2}+\frac{35cd^4e^3}{4}\right) + x^3\left(ade^2+\frac{10bd^3e^2}{3}+7cd^5e^2\right) + x^2\cdot\left(\frac{3ad^2e}{2}+\frac{5bd^4e}{2}+\frac{7cd^6e}{2}\right) + x\left(ad^3+bd^5+cd^7\right)$$

input
```
integrate((e*x+d)**3*(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

output
```
c*d*e**6*x**7 + c*e**7*x**8/8 + x**6*(b*e**5/6 + 7*c*d**2*e**5/2) + x**5*(
b*d*e**4 + 7*c*d**3*e**4) + x**4*(a*e**3/4 + 5*b*d**2*e**3/2 + 35*c*d**4*e
**3/4) + x**3*(a*d*e**2 + 10*b*d**3*e**2/3 + 7*c*d**5*e**2) + x**2*(3*a*d*
*2*e/2 + 5*b*d**4*e/2 + 7*c*d**6*e/2) + x*(a*d**3 + b*d**5 + c*d**7)
```

3.607.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(40) = 80$.

Time = 0.20 (sec) , antiderivative size = 142, normalized size of antiderivative = 3.09

$$\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right) dx$$
$$= \frac{1}{8}ce^7x^8 + cde^6x^7 + \frac{1}{6}\left(21\,cd^2+b\right)e^5x^6 + \left(7\,cd^3+bd\right)e^4x^5 + \frac{1}{4}\left(35\,cd^4+10\,bd^2+a\right)e^3x^4 + \frac{1}{3}\left(21\,cd^5+10\,bd^3+3\,ad\right)e^2x^3 + \frac{1}{2}\left(7\,cd^6+5\,bd^4+3\,ad^2\right)ex^2 + \left(cd^7+bd^5+ad^3\right)x$$

input
```
integrate((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

output
```
1/8*c*e^7*x^8 + c*d*e^6*x^7 + 1/6*(21*c*d^2 + b)*e^5*x^6 + (7*c*d^3 + b*d)
*e^4*x^5 + 1/4*(35*c*d^4 + 10*b*d^2 + a)*e^3*x^4 + 1/3*(21*c*d^5 + 10*b*d^
3 + 3*a*d)*e^2*x^3 + 1/2*(7*c*d^6 + 5*b*d^4 + 3*a*d^2)*e*x^2 + (c*d^7 + b*
d^5 + a*d^3)*x
```

3.607.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(40) = 80$.

Time = 0.27 (sec) , antiderivative size = 160, normalized size of antiderivative = 3.48

$$\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right) dx = \frac{1}{2}\left(ex^2+2\,dx\right)cd^6 + \frac{3}{4}\left(ex^2+2\,dx\right)^2cd^4e + \frac{1}{2}\left(ex^2+2\,dx\right)^3cd^2e^2 + \frac{1}{8}\left(ex^2+2\,dx\right)^4ce^3 + \frac{1}{2}\left(ex^2+2\,dx\right)bd^4 + \frac{1}{2}\left(ex^2+2\,dx\right)^2bd^2e + \frac{1}{6}\left(ex^2+2\,dx\right)^3be^2 + \frac{1}{2}\left(ex^2+2\,dx\right)ad^2 + \frac{1}{4}\left(ex^2+2\,dx\right)^2ae$$

input
```
integrate((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

output
```
1/2*(e*x^2 + 2*d*x)*c*d^6 + 3/4*(e*x^2 + 2*d*x)^2*c*d^4*e + 1/2*(e*x^2 + 2
*d*x)^3*c*d^2*e^2 + 1/8*(e*x^2 + 2*d*x)^4*c*e^3 + 1/2*(e*x^2 + 2*d*x)*b*d^
4 + 1/2*(e*x^2 + 2*d*x)^2*b*d^2*e + 1/6*(e*x^2 + 2*d*x)^3*b*e^2 + 1/2*(e*x
^2 + 2*d*x)*a*d^2 + 1/4*(e*x^2 + 2*d*x)^2*a*e
```

3.607.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 141, normalized size of antiderivative = 3.07

$$\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)\,dx = x\left(c\,d^7+b\,d^5+a\,d^3\right)+\frac{e^5\,x^6\,\left(21\,c\,d^2+b\right)}{6}+\frac{c\,e^7\,x^8}{8}+\frac{e^3\,x^4\,\left(35\,c\,d^4+10\,b\,d^2+a\right)}{4}+\frac{d^2\,e\,x^2\,\left(7\,c\,d^4+5\,b\,d^2+3\,a\right)}{2}+\frac{d\,e^2\,x^3\,\left(21\,c\,d^4+10\,b\,d^2+3\,a\right)}{3}+d\,e^4\,x^5\,\left(7\,c\,d^2+b\right)+c\,d\,e^6\,x^7$$

input
```
int((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4),x)
```

output
```
x*(a*d^3 + b*d^5 + c*d^7) + (e^5*x^6*(b + 21*c*d^2))/6 + (c*e^7*x^8)/8 + (
e^3*x^4*(a + 10*b*d^2 + 35*c*d^4))/4 + (d^2*e*x^2*(3*a + 5*b*d^2 + 7*c*d^4
))/2 + (d*e^2*x^3*(3*a + 10*b*d^2 + 21*c*d^4))/3 + d*e^4*x^5*(b + 7*c*d^2)
 + c*d*e^6*x^7
```

3.608 $\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2 \, dx$

3.608.1 Optimal result

Integrand size = 30, antiderivative size = 89

$$\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2 \, dx = \frac{a^2(d+ex)^4}{4e}+\frac{ab(d+ex)^6}{3e}+\frac{(b^2+2ac)\,(d+ex)^8}{8e}+\frac{bc(d+ex)^{10}}{5e}+\frac{c^2(d+ex)^{12}}{12e}$$

output
```
1/4*a^2*(e*x+d)^4/e+1/3*a*b*(e*x+d)^6/e+1/8*(2*a*c+b^2)*(e*x+d)^8/e+1/5*b*
c*(e*x+d)^10/e+1/12*c^2*(e*x+d)^12/e
```

3.608.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 401 vs. 2(89) = 178.

Time = 0.08 (sec) , antiderivative size = 401, normalized size of antiderivative = 4.51

$$\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2 dx$$
$$= d^3\left(a+bd^2+cd^4\right)^2 x+\frac{1}{2}d^2\left(3a^2+10abd^2+7b^2d^4+14acd^4+18bcd^6+11c^2d^8\right)ex^2$$
$$+\frac{1}{3}d\left(3a^2+20abd^2+21b^2d^4+42acd^4+72bcd^6+55c^2d^8\right)e^2x^3$$
$$+\frac{1}{4}\left(a^2+20abd^2+35b^2d^4+70acd^4+168bcd^6+165c^2d^8\right)e^3x^4$$
$$+\frac{1}{5}d\left(10ab+35b^2d^2+70acd^2+252bcd^4+330c^2d^6\right)e^4x^5$$
$$+\frac{1}{6}\left(2ab+21b^2d^2+42acd^2+252bcd^4+462c^2d^6\right)e^5x^6$$
$$+d\left(b^2+2ac+24bcd^2+66c^2d^4\right)e^6x^7+\frac{1}{8}\left(b^2+2ac+72bcd^2+330c^2d^4\right)e^7x^8$$
$$+\frac{1}{3}cd\left(6b+55cd^2\right)e^8x^9+\frac{1}{10}c\left(2b+55cd^2\right)e^9x^{10}+c^2de^{10}x^{11}+\frac{1}{12}c^2e^{11}x^{12}$$

input
```
Integrate[(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

output
```
d^3*(a + b*d^2 + c*d^4)^2*x + (d^2*(3*a^2 + 10*a*b*d^2 + 7*b^2*d^4 + 14*a*
c*d^4 + 18*b*c*d^6 + 11*c^2*d^8)*e*x^2)/2 + (d*(3*a^2 + 20*a*b*d^2 + 21*b^
2*d^4 + 42*a*c*d^4 + 72*b*c*d^6 + 55*c^2*d^8)*e^2*x^3)/3 + ((a^2 + 20*a*b*
d^2 + 35*b^2*d^4 + 70*a*c*d^4 + 168*b*c*d^6 + 165*c^2*d^8)*e^3*x^4)/4 + (d
*(10*a*b + 35*b^2*d^2 + 70*a*c*d^2 + 252*b*c*d^4 + 330*c^2*d^6)*e^4*x^5)/5
 + ((2*a*b + 21*b^2*d^2 + 42*a*c*d^2 + 252*b*c*d^4 + 462*c^2*d^6)*e^5*x^6)
/6 + d*(b^2 + 2*a*c + 24*b*c*d^2 + 66*c^2*d^4)*e^6*x^7 + ((b^2 + 2*a*c + 7
2*b*c*d^2 + 330*c^2*d^4)*e^7*x^8)/8 + (c*d*(6*b + 55*c*d^2)*e^8*x^9)/3 + (
c*(2*b + 55*c*d^2)*e^9*x^10)/10 + c^2*d*e^10*x^11 + (c^2*e^11*x^12)/12
```

3.608.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1462, 1434, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2 dx$$

$$\Big\downarrow \text{1462}$$

$$\frac{\int (d+ex)^3 \left(c(d+ex)^4+b(d+ex)^2+a\right)^2 d(d+ex)}{e}$$

$$\Big\downarrow \text{1434}$$

$$\frac{\int (d+ex)^2 \left(c(d+ex)^4+b(d+ex)^2+a\right)^2 d(d+ex)^2}{2e}$$

$$\Big\downarrow \text{1140}$$

$$\frac{\int \left(c^2(d+ex)^{10}+2bc(d+ex)^8+\left(b^2+2ac\right)(d+ex)^6+2ab(d+ex)^4+a^2(d+ex)^2\right) d(d+ex)^2}{2e}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{2}a^2(d+ex)^4+\frac{1}{4}\left(2ac+b^2\right)(d+ex)^8+\frac{2}{3}ab(d+ex)^6+\frac{2}{5}bc(d+ex)^{10}+\frac{1}{6}c^2(d+ex)^{12}}{2e}$$

input
```
Int[(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

output
```
((a^2*(d + e*x)^4)/2 + (2*a*b*(d + e*x)^6)/3 + ((b^2 + 2*a*c)*(d + e*x)^8)
/4 + (2*b*c*(d + e*x)^10)/5 + (c^2*(d + e*x)^12)/6)/(2*e)
```

3.608.3.1 Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.608.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 493 vs. $2(79) = 158$.

Time = 0.63 (sec) , antiderivative size = 494, normalized size of antiderivative = 5.55

method	result
norman	$\frac{e^{11}c^2x^{12}}{12}+d\,e^{10}c^2x^{11}+\left(\frac{11}{2}d^2e^9c^2+\frac{1}{5}bc\,e^9\right)x^{10}+\left(\frac{55}{3}d^3c^2e^8+2bcd\,e^8\right)x^9+\left(\frac{165}{4}c^2d^4e^7+9bc\right.$
gosper	$\frac{x\left(10e^{11}c^2x^{11}+120d\,e^{10}c^2x^{10}+660x^9d^2e^9c^2+2200x^8d^3c^2e^8+24x^9bc\,e^9+4950x^7c^2d^4e^7+240x^8bcd\,e^8+7920c^2d^5e^6x^6+1080x^7\right.}{}$
risch	$24bc\,d^3e^6x^7+2x^9bcd\,e^8+9x^8bc\,d^2e^7+42x^6bc\,d^4e^5+7x^6ac\,d^2e^5+\frac{11}{2}x^{10}d^2e^9c^2+\frac{1}{5}x^{10}bc\,e^9$
parallelrisch	$24bc\,d^3e^6x^7+2x^9bcd\,e^8+9x^8bc\,d^2e^7+42x^6bc\,d^4e^5+7x^6ac\,d^2e^5+\frac{11}{2}x^{10}d^2e^9c^2+\frac{1}{5}x^{10}bc\,e^9$
default	Expression too large to display

input
```
int((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x,method=_RETURNVERBOSE)
```

output
```
1/12*e^11*c^2*x^12+d*e^10*c^2*x^11+(11/2*d^2*e^9*c^2+1/5*b*c*e^9)*x^10+(55
/3*d^3*c^2*e^8+2*b*c*d*e^8)*x^9+(165/4*c^2*d^4*e^7+9*b*c*d^2*e^7+1/4*a*c*e
^7+1/8*b^2*e^7)*x^8+(66*c^2*d^5*e^6+24*b*c*d^3*e^6+2*a*c*d*e^6+b^2*d*e^6)*
x^7+(77*c^2*d^6*e^5+42*b*c*d^4*e^5+7*a*c*d^2*e^5+7/2*b^2*d^2*e^5+1/3*a*b*e
^5)*x^6+(66*c^2*d^7*e^4+252/5*b*c*d^5*e^4+14*a*c*d^3*e^4+7*b^2*d^3*e^4+2*a
*b*d*e^4)*x^5+(165/4*c^2*d^8*e^3+42*b*c*d^6*e^3+35/2*a*c*d^4*e^3+35/4*b^2*
d^4*e^3+5*e^3*a*b*d^2+1/4*e^3*a^2)*x^4+(55/3*c^2*d^9*e^2+24*b*c*d^7*e^2+14
*a*c*d^5*e^2+7*b^2*d^5*e^2+20/3*a*b*d^3*e^2+d*e^2*a^2)*x^3+(11/2*c^2*d^10*
e+9*b*c*d^8*e+7*a*c*d^6*e+7/2*b^2*d^6*e+5*a*b*d^4*e+3/2*a^2*d^2*e)*x^2+(c^
2*d^11+2*b*c*d^9+2*a*c*d^7+b^2*d^7+2*a*b*d^5+a^2*d^3)*x
```

3.608.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 403 vs. $2(79) = 158$.

Time = 0.25 (sec) , antiderivative size = 403, normalized size of antiderivative = 4.53

$$
\begin{aligned}
&\int (d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^2\,dx \\
&= \frac{1}{12}c^2e^{11}x^{12}+c^2de^{10}x^{11}+\frac{1}{10}\left(55\,c^2d^2+2\,bc\right)e^9x^{10}+\frac{1}{3}\left(55\,c^2d^3+6\,bcd\right)e^8x^9 \\
&+\frac{1}{8}\left(330\,c^2d^4+72\,bcd^2+b^2+2\,ac\right)e^7x^8+\left(66\,c^2d^5+24\,bcd^3+\left(b^2+2\,ac\right)d\right)e^6x^7 \\
&+\frac{1}{6}\left(462\,c^2d^6+252\,bcd^4+21\left(b^2+2\,ac\right)d^2+2\,ab\right)e^5x^6 \\
&+\frac{1}{5}\left(330\,c^2d^7+252\,bcd^5+35\left(b^2+2\,ac\right)d^3+10\,abd\right)e^4x^5 \\
&+\frac{1}{4}\left(165\,c^2d^8+168\,bcd^6+35\left(b^2+2\,ac\right)d^4+20\,abd^2+a^2\right)e^3x^4 \\
&+\frac{1}{3}\left(55\,c^2d^9+72\,bcd^7+21\left(b^2+2\,ac\right)d^5+20\,abd^3+3\,a^2d\right)e^2x^3 \\
&+\frac{1}{2}\left(11\,c^2d^{10}+18\,bcd^8+7\left(b^2+2\,ac\right)d^6+10\,abd^4+3\,a^2d^2\right)ex^2 \\
&+\left(c^2d^{11}+2\,bcd^9+\left(b^2+2\,ac\right)d^7+2\,abd^5+a^2d^3\right)x
\end{aligned}
$$

input
```
integrate((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")
```

output
```
1/12*c^2*e^11*x^12 + c^2*d*e^10*x^11 + 1/10*(55*c^2*d^2 + 2*b*c)*e^9*x^10
+ 1/3*(55*c^2*d^3 + 6*b*c*d)*e^8*x^9 + 1/8*(330*c^2*d^4 + 72*b*c*d^2 + b^2
 + 2*a*c)*e^7*x^8 + (66*c^2*d^5 + 24*b*c*d^3 + (b^2 + 2*a*c)*d)*e^6*x^7 +
1/6*(462*c^2*d^6 + 252*b*c*d^4 + 21*(b^2 + 2*a*c)*d^2 + 2*a*b)*e^5*x^6 + 1
/5*(330*c^2*d^7 + 252*b*c*d^5 + 35*(b^2 + 2*a*c)*d^3 + 10*a*b*d)*e^4*x^5 +
 1/4*(165*c^2*d^8 + 168*b*c*d^6 + 35*(b^2 + 2*a*c)*d^4 + 20*a*b*d^2 + a^2)
*e^3*x^4 + 1/3*(55*c^2*d^9 + 72*b*c*d^7 + 21*(b^2 + 2*a*c)*d^5 + 20*a*b*d^
3 + 3*a^2*d)*e^2*x^3 + 1/2*(11*c^2*d^10 + 18*b*c*d^8 + 7*(b^2 + 2*a*c)*d^6
 + 10*a*b*d^4 + 3*a^2*d^2)*e*x^2 + (c^2*d^11 + 2*b*c*d^9 + (b^2 + 2*a*c)*d
^7 + 2*a*b*d^5 + a^2*d^3)*x
```

3.608.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 559 vs. $2(71) = 142$.

Time = 0.07 (sec) , antiderivative size = 559, normalized size of antiderivative = 6.28

$$\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2 dx$$
$$= c^2de^{10}x^{11} + \frac{c^2e^{11}x^{12}}{12} + x^{10}\left(\frac{bce^9}{5} + \frac{11c^2d^2e^9}{2}\right) + x^9\cdot\left(2bcde^8 + \frac{55c^2d^3e^8}{3}\right)$$
$$+ x^8\left(\frac{ace^7}{4} + \frac{b^2e^7}{8} + 9bcd^2e^7 + \frac{165c^2d^4e^7}{4}\right) + x^7\cdot\left(2acde^6 + b^2de^6 + 24bcd^3e^6 + 66c^2d^5e^6\right)$$
$$+ x^6\left(\frac{abe^5}{3} + 7acd^2e^5 + \frac{7b^2d^2e^5}{2} + 42bcd^4e^5 + 77c^2d^6e^5\right) + x^5$$
$$\cdot\left(2abde^4 + 14acd^3e^4 + 7b^2d^3e^4 + \frac{252bcd^5e^4}{5} + 66c^2d^7e^4\right)$$
$$+ x^4\left(\frac{a^2e^3}{4} + 5abd^2e^3 + \frac{35acd^4e^3}{2} + \frac{35b^2d^4e^3}{4} + 42bcd^6e^3 + \frac{165c^2d^8e^3}{4}\right)$$
$$+ x^3\left(a^2de^2 + \frac{20abd^3e^2}{3} + 14acd^5e^2 + 7b^2d^5e^2 + 24bcd^7e^2 + \frac{55c^2d^9e^2}{3}\right)$$
$$+ x^2\cdot\left(\frac{3a^2d^2e}{2} + 5abd^4e + 7acd^6e + \frac{7b^2d^6e}{2} + 9bcd^8e + \frac{11c^2d^{10}e}{2}\right)$$
$$+ x\left(a^2d^3 + 2abd^5 + 2acd^7 + b^2d^7 + 2bcd^9 + c^2d^{11}\right)$$

input
```
integrate((e*x+d)**3*(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

output
```
c**2*d*e**10*x**11 + c**2*e**11*x**12/12 + x**10*(b*c*e**9/5 + 11*c**2*d**
2*e**9/2) + x**9*(2*b*c*d*e**8 + 55*c**2*d**3*e**8/3) + x**8*(a*c*e**7/4 +
 b**2*e**7/8 + 9*b*c*d**2*e**7 + 165*c**2*d**4*e**7/4) + x**7*(2*a*c*d*e**
6 + b**2*d*e**6 + 24*b*c*d**3*e**6 + 66*c**2*d**5*e**6) + x**6*(a*b*e**5/3
 + 7*a*c*d**2*e**5 + 7*b**2*d**2*e**5/2 + 42*b*c*d**4*e**5 + 77*c**2*d**6*
e**5) + x**5*(2*a*b*d*e**4 + 14*a*c*d**3*e**4 + 7*b**2*d**3*e**4 + 252*b*c
*d**5*e**4/5 + 66*c**2*d**7*e**4) + x**4*(a**2*e**3/4 + 5*a*b*d**2*e**3 +
35*a*c*d**4*e**3/2 + 35*b**2*d**4*e**3/4 + 42*b*c*d**6*e**3 + 165*c**2*d**
8*e**3/4) + x**3*(a**2*d*e**2 + 20*a*b*d**3*e**2/3 + 14*a*c*d**5*e**2 + 7*
b**2*d**5*e**2 + 24*b*c*d**7*e**2 + 55*c**2*d**9*e**2/3) + x**2*(3*a**2*d*
*2*e/2 + 5*a*b*d**4*e + 7*a*c*d**6*e + 7*b**2*d**6*e/2 + 9*b*c*d**8*e + 11
*c**2*d**10*e/2) + x*(a**2*d**3 + 2*a*b*d**5 + 2*a*c*d**7 + b**2*d**7 + 2*
b*c*d**9 + c**2*d**11)
```

3.608.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 403 vs. 2(79) = 158.

Time = 0.21 (sec) , antiderivative size = 403, normalized size of antiderivative = 4.53

$$\int (d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^2\,dx$$
$$= \frac{1}{12}c^2e^{11}x^{12}+c^2de^{10}x^{11}+\frac{1}{10}\left(55\,c^2d^2+2\,bc\right)e^9x^{10}+\frac{1}{3}\left(55\,c^2d^3+6\,bcd\right)e^8x^9$$
$$+\frac{1}{8}\left(330\,c^2d^4+72\,bcd^2+b^2+2\,ac\right)e^7x^8+\left(66\,c^2d^5+24\,bcd^3+\left(b^2+2\,ac\right)d\right)e^6x^7$$
$$+\frac{1}{6}\left(462\,c^2d^6+252\,bcd^4+21\left(b^2+2\,ac\right)d^2+2\,ab\right)e^5x^6$$
$$+\frac{1}{5}\left(330\,c^2d^7+252\,bcd^5+35\left(b^2+2\,ac\right)d^3+10\,abd\right)e^4x^5$$
$$+\frac{1}{4}\left(165\,c^2d^8+168\,bcd^6+35\left(b^2+2\,ac\right)d^4+20\,abd^2+a^2\right)e^3x^4$$
$$+\frac{1}{3}\left(55\,c^2d^9+72\,bcd^7+21\left(b^2+2\,ac\right)d^5+20\,abd^3+3\,a^2d\right)e^2x^3$$
$$+\frac{1}{2}\left(11\,c^2d^{10}+18\,bcd^8+7\left(b^2+2\,ac\right)d^6+10\,abd^4+3\,a^2d^2\right)ex^2$$
$$+\left(c^2d^{11}+2\,bcd^9+\left(b^2+2\,ac\right)d^7+2\,abd^5+a^2d^3\right)x$$

input
```
integrate((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")
```

output
```
1/12*c^2*e^11*x^12 + c^2*d*e^10*x^11 + 1/10*(55*c^2*d^2 + 2*b*c)*e^9*x^10
+ 1/3*(55*c^2*d^3 + 6*b*c*d)*e^8*x^9 + 1/8*(330*c^2*d^4 + 72*b*c*d^2 + b^2
 + 2*a*c)*e^7*x^8 + (66*c^2*d^5 + 24*b*c*d^3 + (b^2 + 2*a*c)*d)*e^6*x^7 +
1/6*(462*c^2*d^6 + 252*b*c*d^4 + 21*(b^2 + 2*a*c)*d^2 + 2*a*b)*e^5*x^6 + 1
/5*(330*c^2*d^7 + 252*b*c*d^5 + 35*(b^2 + 2*a*c)*d^3 + 10*a*b*d)*e^4*x^5 +
 1/4*(165*c^2*d^8 + 168*b*c*d^6 + 35*(b^2 + 2*a*c)*d^4 + 20*a*b*d^2 + a^2)
*e^3*x^4 + 1/3*(55*c^2*d^9 + 72*b*c*d^7 + 21*(b^2 + 2*a*c)*d^5 + 20*a*b*d^
3 + 3*a^2*d)*e^2*x^3 + 1/2*(11*c^2*d^10 + 18*b*c*d^8 + 7*(b^2 + 2*a*c)*d^6
 + 10*a*b*d^4 + 3*a^2*d^2)*e*x^2 + (c^2*d^11 + 2*b*c*d^9 + (b^2 + 2*a*c)*d
^7 + 2*a*b*d^5 + a^2*d^3)*x
```

3.608.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 475 vs. $2(79) = 158$.

Time = 0.30 (sec) , antiderivative size = 475, normalized size of antiderivative = 5.34

$$\begin{aligned}
&\int (d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^2\,dx \\
&= \frac{1}{2}\left(ex^2+2\,dx\right)c^2d^{10} + \frac{5}{4}\left(ex^2+2\,dx\right)^2c^2d^8e + \frac{5}{3}\left(ex^2+2\,dx\right)^3c^2d^6e^2 \\
&+ \frac{5}{4}\left(ex^2+2\,dx\right)^4c^2d^4e^3 + \frac{1}{2}\left(ex^2+2\,dx\right)^5c^2d^2e^4 + \frac{1}{12}\left(ex^2+2\,dx\right)^6c^2e^5 \\
&+ \left(ex^2+2\,dx\right)bcd^8 + 2\left(ex^2+2\,dx\right)^2bcd^6e + 2\left(ex^2+2\,dx\right)^3bcd^4e^2 \\
&+ \left(ex^2+2\,dx\right)^4bcd^2e^3 + \frac{1}{5}\left(ex^2+2\,dx\right)^5bce^4 + \frac{1}{2}\left(ex^2+2\,dx\right)b^2d^6 + \left(ex^2+2\,dx\right)acd^6 \\
&+ \frac{3}{4}\left(ex^2+2\,dx\right)^2b^2d^4e + \frac{3}{2}\left(ex^2+2\,dx\right)^2acd^4e + \frac{1}{2}\left(ex^2+2\,dx\right)^3b^2d^2e^2 \\
&+ \left(ex^2+2\,dx\right)^3acd^2e^2 + \frac{1}{8}\left(ex^2+2\,dx\right)^4b^2e^3 + \frac{1}{4}\left(ex^2+2\,dx\right)^4ace^3 + \left(ex^2+2\,dx\right)abd^4 \\
&+ \left(ex^2+2\,dx\right)^2abd^2e + \frac{1}{3}\left(ex^2+2\,dx\right)^3abe^2 + \frac{1}{2}\left(ex^2+2\,dx\right)a^2d^2 + \frac{1}{4}\left(ex^2+2\,dx\right)^2a^2e
\end{aligned}$$

input
```
integrate((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

output
```
1/2*(e*x^2 + 2*d*x)*c^2*d^10 + 5/4*(e*x^2 + 2*d*x)^2*c^2*d^8*e + 5/3*(e*x^
2 + 2*d*x)^3*c^2*d^6*e^2 + 5/4*(e*x^2 + 2*d*x)^4*c^2*d^4*e^3 + 1/2*(e*x^2
+ 2*d*x)^5*c^2*d^2*e^4 + 1/12*(e*x^2 + 2*d*x)^6*c^2*e^5 + (e*x^2 + 2*d*x)*
b*c*d^8 + 2*(e*x^2 + 2*d*x)^2*b*c*d^6*e + 2*(e*x^2 + 2*d*x)^3*b*c*d^4*e^2
+ (e*x^2 + 2*d*x)^4*b*c*d^2*e^3 + 1/5*(e*x^2 + 2*d*x)^5*b*c*e^4 + 1/2*(e*x
^2 + 2*d*x)*b^2*d^6 + (e*x^2 + 2*d*x)*a*c*d^6 + 3/4*(e*x^2 + 2*d*x)^2*b^2*
d^4*e + 3/2*(e*x^2 + 2*d*x)^2*a*c*d^4*e + 1/2*(e*x^2 + 2*d*x)^3*b^2*d^2*e^
2 + (e*x^2 + 2*d*x)^3*a*c*d^2*e^2 + 1/8*(e*x^2 + 2*d*x)^4*b^2*e^3 + 1/4*(e
*x^2 + 2*d*x)^4*a*c*e^3 + (e*x^2 + 2*d*x)*a*b*d^4 + (e*x^2 + 2*d*x)^2*a*b*
d^2*e + 1/3*(e*x^2 + 2*d*x)^3*a*b*e^2 + 1/2*(e*x^2 + 2*d*x)*a^2*d^2 + 1/4*
(e*x^2 + 2*d*x)^2*a^2*e
```

3.608.9 Mupad [B] (verification not implemented)

Time = 8.88 (sec) , antiderivative size = 383, normalized size of antiderivative = 4.30

$$\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2 \, dx = \frac{e^7\,x^8\,\left(b^2+72\,b\,c\,d^2+330\,c^2\,d^4+2\,a\,c\right)}{8} + \frac{e^5\,x^6\,\left(21\,b^2\,d^2+252\,b\,c\,d^4+2\,a\,b+462\,c^2\,d^6+42\,a\,c\,d^2\right)}{6} + \frac{e^3\,x^4\,\left(a^2+20\,a\,b\,d^2+70\,a\,c\,d^4+35\,b^2\,d^4+168\,b\,c\,d^6+165\,c^2\,d^8\right)}{4} + \frac{c^2\,e^{11}\,x^{12}}{12} + d^3\,x\,{\left(c\,d^4+b\,d^2+a\right)}^2 + \frac{c\,e^9\,x^{10}\,\left(55\,c\,d^2+2\,b\right)}{10} + c^2\,d\,e^{10}\,x^{11} + \frac{d^2\,e\,x^2\,\left(3\,a^2+10\,a\,b\,d^2+14\,a\,c\,d^4+7\,b^2\,d^4+18\,b\,c\,d^6+11\,c^2\,d^8\right)}{2} + \frac{d\,e^2\,x^3\,\left(3\,a^2+20\,a\,b\,d^2+42\,a\,c\,d^4+21\,b^2\,d^4+72\,b\,c\,d^6+55\,c^2\,d^8\right)}{3} + d\,e^6\,x^7\,\left(b^2+24\,b\,c\,d^2+66\,c^2\,d^4+2\,a\,c\right) + \frac{d\,e^4\,x^5\,\left(35\,b^2\,d^2+252\,b\,c\,d^4+10\,a\,b+330\,c^2\,d^6+70\,a\,c\,d^2\right)}{5} + \frac{c\,d\,e^8\,x^9\,\left(55\,c\,d^2+6\,b\right)}{3}$$

input
```
int((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x)
```

output
```
(e^7*x^8*(2*a*c + b^2 + 330*c^2*d^4 + 72*b*c*d^2))/8 + (e^5*x^6*(2*a*b + 2
1*b^2*d^2 + 462*c^2*d^6 + 42*a*c*d^2 + 252*b*c*d^4))/6 + (e^3*x^4*(a^2 + 3
5*b^2*d^4 + 165*c^2*d^8 + 20*a*b*d^2 + 70*a*c*d^4 + 168*b*c*d^6))/4 + (c^2
*e^11*x^12)/12 + d^3*x*(a + b*d^2 + c*d^4)^2 + (c*e^9*x^10*(2*b + 55*c*d^2
))/10 + c^2*d*e^10*x^11 + (d^2*e*x^2*(3*a^2 + 7*b^2*d^4 + 11*c^2*d^8 + 10*
a*b*d^2 + 14*a*c*d^4 + 18*b*c*d^6))/2 + (d*e^2*x^3*(3*a^2 + 21*b^2*d^4 + 5
5*c^2*d^8 + 20*a*b*d^2 + 42*a*c*d^4 + 72*b*c*d^6))/3 + d*e^6*x^7*(2*a*c +
b^2 + 66*c^2*d^4 + 24*b*c*d^2) + (d*e^4*x^5*(10*a*b + 35*b^2*d^2 + 330*c^2
*d^6 + 70*a*c*d^2 + 252*b*c*d^4))/5 + (c*d*e^8*x^9*(6*b + 55*c*d^2))/3
```

3.609 $\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^3 \, dx$

3.609.1 Optimal result

Integrand size = 30, antiderivative size = 138

$$\begin{aligned}\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^3 \, dx &= \frac{a^3(d+ex)^4}{4e}+\frac{a^2b(d+ex)^6}{2e}+\frac{3a(b^2+ac)\,(d+ex)^8}{8e}+\frac{b(b^2+6ac)\,(d+ex)^{10}}{10e}\\&\quad+\frac{c(b^2+ac)\,(d+ex)^{12}}{4e}+\frac{3bc^2(d+ex)^{14}}{14e}+\frac{c^3(d+ex)^{16}}{16e}\end{aligned}$$

output
```
1/4*a^3*(e*x+d)^4/e+1/2*a^2*b*(e*x+d)^6/e+3/8*a*(a*c+b^2)*(e*x+d)^8/e+1/10
*b*(6*a*c+b^2)*(e*x+d)^10/e+1/4*c*(a*c+b^2)*(e*x+d)^12/e+3/14*b*c^2*(e*x+d
)^14/e+1/16*c^3*(e*x+d)^16/e
```

3.609.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 797 vs. $2(138) = 276$.

Time = 0.20 (sec) , antiderivative size = 797, normalized size of antiderivative = 5.78

$$\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^3 dx = d^3\left(a+bd^2+cd^4\right)^3x+\frac{3}{2}d^2\left(a+bd^2+cd^4\right)^2\left(a+3bd^2+5cd^4\right)ex^2+d\left(a^3+10a^2bd^2+21ab^2d^4+21a^2cd^4+12b^3d^6+72abcd^6+55b^2cd^8+55ac^2d^8+78bc^2d^{10}+35c^3d^{12}\right)e^2x^3+\frac{1}{4}\left(a^3+30a^2bd^2+105ab^2d^4+105a^2cd^4+84b^3d^6+504abcd^6+495b^2cd^8+495ac^2d^8+858bc^2d^{10}+455c^3d^{12}\right)e^3x^4+\frac{3}{5}d\left(5a^2b+35ab^2d^2+35a^2cd^2+42b^3d^4+252abcd^4+330b^2cd^6+330ac^2d^6+715bc^2d^8+455c^3d^{10}\right)e^4x^5+\frac{1}{2}\left(a^2b+21ab^2d^2+21a^2cd^2+42b^3d^4+252abcd^4+462b^2cd^6+462ac^2d^6+1287bc^2d^8+1001c^3d^{10}\right)e^5x^6+\frac{1}{7}d\left(21ab^2+21a^2c+84b^3d^2+504abcd^2+1386b^2cd^4+1386ac^2d^4+5148bc^2d^6+5005c^3d^8\right)e^6x^7+\frac{3}{8}\left(ab^2+a^2c+12b^3d^2+72abcd^2+330b^2cd^4+330ac^2d^4+1716bc^2d^6+2145c^3d^8\right)e^7x^8+d\left(b^3+6abc+55b^2cd^2+55ac^2d^2+429bc^2d^4+715c^3d^6\right)e^8x^9+\frac{1}{10}\left(b^3+6abc+165b^2cd^2+165ac^2d^2+2145bc^2d^4+5005c^3d^6\right)e^9x^{10}+3cd\left(b^2+ac+26bcd^2+91c^2d^4\right)e^{10}x^{11}+\frac{1}{4}c\left(b^2+ac+78bcd^2+455c^2d^4\right)e^{11}x^{12}+c^2d\left(3b+35cd^2\right)e^{12}x^{13}+\frac{3}{14}c^2\left(b+35cd^2\right)e^{13}x^{14}+c^3de^{14}x^{15}+\frac{1}{16}c^3e^{15}x^{16}$$

input
```
Integrate[(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

output
```
d^3*(a + b*d^2 + c*d^4)^3*x + (3*d^2*(a + b*d^2 + c*d^4)^2*(a + 3*b*d^2 +
5*c*d^4)*e*x^2)/2 + d*(a^3 + 10*a^2*b*d^2 + 21*a*b^2*d^4 + 21*a^2*c*d^4 +
12*b^3*d^6 + 72*a*b*c*d^6 + 55*b^2*c*d^8 + 55*a*c^2*d^8 + 78*b*c^2*d^10 +
35*c^3*d^12)*e^2*x^3 + ((a^3 + 30*a^2*b*d^2 + 105*a*b^2*d^4 + 105*a^2*c*d^
4 + 84*b^3*d^6 + 504*a*b*c*d^6 + 495*b^2*c*d^8 + 495*a*c^2*d^8 + 858*b*c^2
*d^10 + 455*c^3*d^12)*e^3*x^4)/4 + (3*d*(5*a^2*b + 35*a*b^2*d^2 + 35*a^2*c
*d^2 + 42*b^3*d^4 + 252*a*b*c*d^4 + 330*b^2*c*d^6 + 330*a*c^2*d^6 + 715*b*
c^2*d^8 + 455*c^3*d^10)*e^4*x^5)/5 + ((a^2*b + 21*a*b^2*d^2 + 21*a^2*c*d^2
 + 42*b^3*d^4 + 252*a*b*c*d^4 + 462*b^2*c*d^6 + 462*a*c^2*d^6 + 1287*b*c^2
*d^8 + 1001*c^3*d^10)*e^5*x^6)/2 + (d*(21*a*b^2 + 21*a^2*c + 84*b^3*d^2 +
504*a*b*c*d^2 + 1386*b^2*c*d^4 + 1386*a*c^2*d^4 + 5148*b*c^2*d^6 + 5005*c^
3*d^8)*e^6*x^7)/7 + (3*(a*b^2 + a^2*c + 12*b^3*d^2 + 72*a*b*c*d^2 + 330*b^
2*c*d^4 + 330*a*c^2*d^4 + 1716*b*c^2*d^6 + 2145*c^3*d^8)*e^7*x^8)/8 + d*(b
^3 + 6*a*b*c + 55*b^2*c*d^2 + 55*a*c^2*d^2 + 429*b*c^2*d^4 + 715*c^3*d^6)*
e^8*x^9 + ((b^3 + 6*a*b*c + 165*b^2*c*d^2 + 165*a*c^2*d^2 + 2145*b*c^2*d^4
 + 5005*c^3*d^6)*e^9*x^10)/10 + 3*c*d*(b^2 + a*c + 26*b*c*d^2 + 91*c^2*d^4
)*e^10*x^11 + (c*(b^2 + a*c + 78*b*c*d^2 + 455*c^2*d^4)*e^11*x^12)/4 + c^2
*d*(3*b + 35*c*d^2)*e^12*x^13 + (3*c^2*(b + 35*c*d^2)*e^13*x^14)/14 + c^3*
d*e^14*x^15 + (c^3*e^15*x^16)/16
```

3.609.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1462, 1434, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^3 dx$$

$$\Big\downarrow \text{1462}$$

$$\frac{\int (d+ex)^3 \left(c(d+ex)^4+b(d+ex)^2+a\right)^3 d(d+ex)}{e}$$

$$\Big\downarrow \text{1434}$$

$$\frac{\int (d+ex)^2 \left(c(d+ex)^4+b(d+ex)^2+a\right)^3 d(d+ex)^2}{2e}$$

$$\Big\downarrow \text{1140}$$

$$\frac{\int \left(c^3(d+ex)^{14}+3bc^2(d+ex)^{12}+3c\left(b^2+ac\right)(d+ex)^{10}+b\left(b^2+6ac\right)(d+ex)^8+3a\left(b^2+ac\right)(d+ex)^6+3a^2b\right.}{2e}$$

$\big\downarrow$ 2009

$$\frac{\frac{1}{2}a^3(d+ex)^4+a^2b(d+ex)^6+\frac{1}{2}c\left(ac+b^2\right)(d+ex)^{12}+\frac{1}{5}b\left(6ac+b^2\right)(d+ex)^{10}+\frac{3}{4}a\left(ac+b^2\right)(d+ex)^8+\frac{3}{7}bc^2(}{2e}$$

input
```
Int[(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

output
```
((a^3*(d + e*x)^4)/2 + a^2*b*(d + e*x)^6 + (3*a*(b^2 + a*c)*(d + e*x)^8)/4
 + (b*(b^2 + 6*a*c)*(d + e*x)^10)/5 + (c*(b^2 + a*c)*(d + e*x)^12)/2 + (3*
b*c^2*(d + e*x)^14)/7 + (c^3*(d + e*x)^16)/8)/(2*e)
```

3.609.3.1 Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.609.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1129 vs. $2(124) = 248$.

Time = 0.64 (sec) , antiderivative size = 1130, normalized size of antiderivative = 8.19

method	result	size
norman	Expression too large to display	1130
gosper	Expression too large to display	1315
risch	Expression too large to display	1336
parallelrisch	Expression too large to display	1336
default	Expression too large to display	7550

input
```
int((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x,method=_RETURNVERBOSE)
```

output
```
(c^3*d^15+3*b*c^2*d^13+3*a*c^2*d^11+3*b^2*c*d^11+6*a*b*c*d^9+b^3*d^9+3*a^2
*c*d^7+3*a*b^2*d^7+3*a^2*b*d^5+a^3*d^3)*x+(15/2*c^3*d^14*e+39/2*b*c^2*d^12
*e+33/2*a*c^2*d^10*e+33/2*b^2*c*d^10*e+27*a*b*c*d^8*e+9/2*b^3*d^8*e+21/2*a
^2*c*d^6*e+21/2*a*b^2*d^6*e+15/2*a^2*b*d^4*e+3/2*a^3*d^2*e)*x^2+(35*c^3*d^
13*e^2+78*b*c^2*d^11*e^2+55*a*c^2*d^9*e^2+55*b^2*c*d^9*e^2+72*a*b*c*d^7*e^
2+12*b^3*d^7*e^2+21*a^2*c*d^5*e^2+21*a*b^2*d^5*e^2+10*a^2*b*d^3*e^2+a^3*d*
e^2)*x^3+(455/4*c^3*d^12*e^3+429/2*b*c^2*d^10*e^3+495/4*a*c^2*d^8*e^3+495/
4*b^2*c*d^8*e^3+126*a*b*c*d^6*e^3+21*b^3*d^6*e^3+105/4*a^2*c*d^4*e^3+105/4
*a*b^2*d^4*e^3+15/2*a^2*b*d^2*e^3+1/4*a^3*e^3)*x^4+(273*c^3*d^11*e^4+429*b
*c^2*d^9*e^4+198*a*c^2*d^7*e^4+198*b^2*c*d^7*e^4+756/5*a*b*c*d^5*e^4+126/5
*b^3*d^5*e^4+21*a^2*c*d^3*e^4+21*a*b^2*d^3*e^4+3*a^2*b*d*e^4)*x^5+(1001/2*
c^3*d^10*e^5+1287/2*b*c^2*d^8*e^5+231*a*c^2*d^6*e^5+231*b^2*c*d^6*e^5+126*
a*b*c*d^4*e^5+21*b^3*d^4*e^5+21/2*a^2*c*d^2*e^5+21/2*a*b^2*d^2*e^5+1/2*a^2
*b*e^5)*x^6+(715*c^3*d^9*e^6+5148/7*b*c^2*d^7*e^6+198*a*c^2*d^5*e^6+198*b^
2*c*d^5*e^6+72*a*b*c*d^3*e^6+12*b^3*d^3*e^6+3*a^2*c*d*e^6+3*a*b^2*d*e^6)*x
^7+(6435/8*c^3*d^8*e^7+1287/2*b*c^2*d^6*e^7+495/4*a*c^2*d^4*e^7+495/4*b^2*
c*d^4*e^7+27*a*b*c*d^2*e^7+9/2*b^3*d^2*e^7+3/8*a^2*c*e^7+3/8*a*b^2*e^7)*x^
8+(715*c^3*d^7*e^8+429*b*c^2*d^5*e^8+55*a*c^2*d^3*e^8+55*b^2*c*d^3*e^8+6*a
*b*c*d*e^8+b^3*d*e^8)*x^9+(1001/2*c^3*d^6*e^9+429/2*b*c^2*d^4*e^9+33/2*a*c
^2*d^2*e^9+33/2*b^2*c*d^2*e^9+3/5*a*b*c*e^9+1/10*b^3*e^9)*x^10+(273*c^3...
```

3.609.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 872 vs. $2(124) = 248$.

Time = 0.26 (sec) , antiderivative size = 872, normalized size of antiderivative = 6.32

$$\begin{aligned}
&\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^3 dx \\
&= \frac{1}{16}c^3e^{15}x^{16} + c^3de^{14}x^{15} + \frac{3}{14}\left(35\,c^3d^2+bc^2\right)e^{13}x^{14} + \left(35\,c^3d^3+3\,bc^2d\right)e^{12}x^{13} \\
&\quad + \frac{1}{4}\left(455\,c^3d^4+78\,bc^2d^2+b^2c+ac^2\right)e^{11}x^{12} + 3\left(91\,c^3d^5+26\,bc^2d^3+\left(b^2c+ac^2\right)d\right)e^{10}x^{11} \\
&\quad + \frac{1}{10}\left(5005\,c^3d^6+2145\,bc^2d^4+b^3+6\,abc+165\left(b^2c+ac^2\right)d^2\right)e^9x^{10} \\
&\quad + \left(715\,c^3d^7+429\,bc^2d^5+55\left(b^2c+ac^2\right)d^3+\left(b^3+6\,abc\right)d\right)e^8x^9 \\
&\quad + \frac{3}{8}\left(2145\,c^3d^8+1716\,bc^2d^6+330\left(b^2c+ac^2\right)d^4+ab^2+a^2c+12\left(b^3+6\,abc\right)d^2\right)e^7x^8 \\
&\quad + \frac{1}{7}\left(5005\,c^3d^9+5148\,bc^2d^7+1386\left(b^2c+ac^2\right)d^5+84\left(b^3+6\,abc\right)d^3+21\left(ab^2+a^2c\right)d\right)e^6x^7 \\
&\quad + \frac{1}{2}\left(1001\,c^3d^{10}+1287\,bc^2d^8+462\left(b^2c+ac^2\right)d^6+42\left(b^3+6\,abc\right)d^4+a^2b+21\left(ab^2+a^2c\right)d^2\right)e^5x^6 \\
&\quad + \frac{3}{5}\left(455\,c^3d^{11}+715\,bc^2d^9+330\left(b^2c+ac^2\right)d^7+42\left(b^3+6\,abc\right)d^5+5\,a^2bd+35\left(ab^2+a^2c\right)d^3\right)e^4x^5 \\
&\quad + \frac{1}{4}\left(455\,c^3d^{12}+858\,bc^2d^{10}+495\left(b^2c+ac^2\right)d^8+84\left(b^3+6\,abc\right)d^6+30\,a^2bd^2+105\left(ab^2+a^2c\right)d^4+a^3\right) \\
&\quad + \left(35\,c^3d^{13}+78\,bc^2d^{11}+55\left(b^2c+ac^2\right)d^9+12\left(b^3+6\,abc\right)d^7+10\,a^2bd^3+21\left(ab^2+a^2c\right)d^5+a^3d\right)e^2x^3 \\
&\quad + \frac{3}{2}\left(5\,c^3d^{14}+13\,bc^2d^{12}+11\left(b^2c+ac^2\right)d^{10}+3\left(b^3+6\,abc\right)d^8+5\,a^2bd^4+7\left(ab^2+a^2c\right)d^6+a^3d^2\right)ex^2 \\
&\quad + \left(c^3d^{15}+3\,bc^2d^{13}+3\left(b^2c+ac^2\right)d^{11}+\left(b^3+6\,abc\right)d^9+3\,a^2bd^5+3\left(ab^2+a^2c\right)d^7+a^3d^3\right)x
\end{aligned}$$

input
```
integrate((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")
```

output
```
1/16*c^3*e^15*x^16 + c^3*d*e^14*x^15 + 3/14*(35*c^3*d^2 + b*c^2)*e^13*x^14
 + (35*c^3*d^3 + 3*b*c^2*d)*e^12*x^13 + 1/4*(455*c^3*d^4 + 78*b*c^2*d^2 +
b^2*c + a*c^2)*e^11*x^12 + 3*(91*c^3*d^5 + 26*b*c^2*d^3 + (b^2*c + a*c^2)*
d)*e^10*x^11 + 1/10*(5005*c^3*d^6 + 2145*b*c^2*d^4 + b^3 + 6*a*b*c + 165*(
b^2*c + a*c^2)*d^2)*e^9*x^10 + (715*c^3*d^7 + 429*b*c^2*d^5 + 55*(b^2*c +
a*c^2)*d^3 + (b^3 + 6*a*b*c)*d)*e^8*x^9 + 3/8*(2145*c^3*d^8 + 1716*b*c^2*d
^6 + 330*(b^2*c + a*c^2)*d^4 + a*b^2 + a^2*c + 12*(b^3 + 6*a*b*c)*d^2)*e^7
*x^8 + 1/7*(5005*c^3*d^9 + 5148*b*c^2*d^7 + 1386*(b^2*c + a*c^2)*d^5 + 84*
(b^3 + 6*a*b*c)*d^3 + 21*(a*b^2 + a^2*c)*d)*e^6*x^7 + 1/2*(1001*c^3*d^10 +
 1287*b*c^2*d^8 + 462*(b^2*c + a*c^2)*d^6 + 42*(b^3 + 6*a*b*c)*d^4 + a^2*b
 + 21*(a*b^2 + a^2*c)*d^2)*e^5*x^6 + 3/5*(455*c^3*d^11 + 715*b*c^2*d^9 + 3
30*(b^2*c + a*c^2)*d^7 + 42*(b^3 + 6*a*b*c)*d^5 + 5*a^2*b*d + 35*(a*b^2 +
a^2*c)*d^3)*e^4*x^5 + 1/4*(455*c^3*d^12 + 858*b*c^2*d^10 + 495*(b^2*c + a*
c^2)*d^8 + 84*(b^3 + 6*a*b*c)*d^6 + 30*a^2*b*d^2 + 105*(a*b^2 + a^2*c)*d^4
 + a^3)*e^3*x^4 + (35*c^3*d^13 + 78*b*c^2*d^11 + 55*(b^2*c + a*c^2)*d^9 +
12*(b^3 + 6*a*b*c)*d^7 + 10*a^2*b*d^3 + 21*(a*b^2 + a^2*c)*d^5 + a^3*d)*e^
2*x^3 + 3/2*(5*c^3*d^14 + 13*b*c^2*d^12 + 11*(b^2*c + a*c^2)*d^10 + 3*(b^3
 + 6*a*b*c)*d^8 + 5*a^2*b*d^4 + 7*(a*b^2 + a^2*c)*d^6 + a^3*d^2)*e*x^2 + (
c^3*d^15 + 3*b*c^2*d^13 + 3*(b^2*c + a*c^2)*d^11 + (b^3 + 6*a*b*c)*d^9 + 3
*a^2*b*d^5 + 3*(a*b^2 + a^2*c)*d^7 + a^3*d^3)*x
```

3.609.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1314 vs. $2(117) = 234$.

Time = 0.13 (sec) , antiderivative size = 1314, normalized size of antiderivative = 9.52

$$\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^3 dx = \text{Too large to display}$$

input
```
integrate((e*x+d)**3*(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

output
```
c**3*d*e**14*x**15 + c**3*e**15*x**16/16 + x**14*(3*b*c**2*e**13/14 + 15*c
**3*d**2*e**13/2) + x**13*(3*b*c**2*d*e**12 + 35*c**3*d**3*e**12) + x**12*
(a*c**2*e**11/4 + b**2*c*e**11/4 + 39*b*c**2*d**2*e**11/2 + 455*c**3*d**4*
e**11/4) + x**11*(3*a*c**2*d*e**10 + 3*b**2*c*d*e**10 + 78*b*c**2*d**3*e**
10 + 273*c**3*d**5*e**10) + x**10*(3*a*b*c*e**9/5 + 33*a*c**2*d**2*e**9/2
+ b**3*e**9/10 + 33*b**2*c*d**2*e**9/2 + 429*b*c**2*d**4*e**9/2 + 1001*c**
3*d**6*e**9/2) + x**9*(6*a*b*c*d*e**8 + 55*a*c**2*d**3*e**8 + b**3*d*e**8
+ 55*b**2*c*d**3*e**8 + 429*b*c**2*d**5*e**8 + 715*c**3*d**7*e**8) + x**8*
(3*a**2*c*e**7/8 + 3*a*b**2*e**7/8 + 27*a*b*c*d**2*e**7 + 495*a*c**2*d**4*
e**7/4 + 9*b**3*d**2*e**7/2 + 495*b**2*c*d**4*e**7/4 + 1287*b*c**2*d**6*e*
*7/2 + 6435*c**3*d**8*e**7/8) + x**7*(3*a**2*c*d*e**6 + 3*a*b**2*d*e**6 +
72*a*b*c*d**3*e**6 + 198*a*c**2*d**5*e**6 + 12*b**3*d**3*e**6 + 198*b**2*c
*d**5*e**6 + 5148*b*c**2*d**7*e**6/7 + 715*c**3*d**9*e**6) + x**6*(a**2*b*
e**5/2 + 21*a**2*c*d**2*e**5/2 + 21*a*b**2*d**2*e**5/2 + 126*a*b*c*d**4*e*
*5 + 231*a*c**2*d**6*e**5 + 21*b**3*d**4*e**5 + 231*b**2*c*d**6*e**5 + 128
7*b*c**2*d**8*e**5/2 + 1001*c**3*d**10*e**5/2) + x**5*(3*a**2*b*d*e**4 + 2
1*a**2*c*d**3*e**4 + 21*a*b**2*d**3*e**4 + 756*a*b*c*d**5*e**4/5 + 198*a*c
**2*d**7*e**4 + 126*b**3*d**5*e**4/5 + 198*b**2*c*d**7*e**4 + 429*b*c**2*d
**9*e**4 + 273*c**3*d**11*e**4) + x**4*(a**3*e**3/4 + 15*a**2*b*d**2*e**3/
2 + 105*a**2*c*d**4*e**3/4 + 105*a*b**2*d**4*e**3/4 + 126*a*b*c*d**6*e*...
```

3.609.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 872 vs. $2(124) = 248$.

Time = 0.22 (sec) , antiderivative size = 872, normalized size of antiderivative = 6.32

$$\int (d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^3\,dx$$
$$= \frac{1}{16}c^3e^{15}x^{16} + c^3de^{14}x^{15} + \frac{3}{14}\left(35\,c^3d^2+bc^2\right)e^{13}x^{14} + \left(35\,c^3d^3+3\,bc^2d\right)e^{12}x^{13}$$
$$+ \frac{1}{4}\left(455\,c^3d^4+78\,bc^2d^2+b^2c+ac^2\right)e^{11}x^{12} + 3\left(91\,c^3d^5+26\,bc^2d^3+\left(b^2c+ac^2\right)d\right)e^{10}x^{11}$$
$$+ \frac{1}{10}\left(5005\,c^3d^6+2145\,bc^2d^4+b^3+6\,abc+165\left(b^2c+ac^2\right)d^2\right)e^9x^{10}$$
$$+ \left(715\,c^3d^7+429\,bc^2d^5+55\left(b^2c+ac^2\right)d^3+\left(b^3+6\,abc\right)d\right)e^8x^9$$
$$+ \frac{3}{8}\left(2145\,c^3d^8+1716\,bc^2d^6+330\left(b^2c+ac^2\right)d^4+ab^2+a^2c+12\left(b^3+6\,abc\right)d^2\right)e^7x^8$$
$$+ \frac{1}{7}\left(5005\,c^3d^9+5148\,bc^2d^7+1386\left(b^2c+ac^2\right)d^5+84\left(b^3+6\,abc\right)d^3+21\left(ab^2+a^2c\right)d\right)e^6x^7$$
$$+ \frac{1}{2}\left(1001\,c^3d^{10}+1287\,bc^2d^8+462\left(b^2c+ac^2\right)d^6+42\left(b^3+6\,abc\right)d^4+a^2b+21\left(ab^2+a^2c\right)d^2\right)e^5x^6$$
$$+ \frac{3}{5}\left(455\,c^3d^{11}+715\,bc^2d^9+330\left(b^2c+ac^2\right)d^7+42\left(b^3+6\,abc\right)d^5+5\,a^2bd+35\left(ab^2+a^2c\right)d^3\right)e^4x^5$$
$$+ \frac{1}{4}\left(455\,c^3d^{12}+858\,bc^2d^{10}+495\left(b^2c+ac^2\right)d^8+84\left(b^3+6\,abc\right)d^6+30\,a^2bd^2+105\left(ab^2+a^2c\right)d^4+a^3\right)$$
$$+ \left(35\,c^3d^{13}+78\,bc^2d^{11}+55\left(b^2c+ac^2\right)d^9+12\left(b^3+6\,abc\right)d^7+10\,a^2bd^3+21\left(ab^2+a^2c\right)d^5+a^3d\right)e^2x^3$$
$$+ \frac{3}{2}\left(5\,c^3d^{14}+13\,bc^2d^{12}+11\left(b^2c+ac^2\right)d^{10}+3\left(b^3+6\,abc\right)d^8+5\,a^2bd^4+7\left(ab^2+a^2c\right)d^6+a^3d^2\right)ex^2$$
$$+ \left(c^3d^{15}+3\,bc^2d^{13}+3\left(b^2c+ac^2\right)d^{11}+\left(b^3+6\,abc\right)d^9+3\,a^2bd^5+3\left(ab^2+a^2c\right)d^7+a^3d^3\right)x$$

input
```
integrate((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")
```

3.609. $\int (d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^3\,dx$

output
```
1/16*c^3*e^15*x^16 + c^3*d*e^14*x^15 + 3/14*(35*c^3*d^2 + b*c^2)*e^13*x^14
 + (35*c^3*d^3 + 3*b*c^2*d)*e^12*x^13 + 1/4*(455*c^3*d^4 + 78*b*c^2*d^2 +
b^2*c + a*c^2)*e^11*x^12 + 3*(91*c^3*d^5 + 26*b*c^2*d^3 + (b^2*c + a*c^2)*
d)*e^10*x^11 + 1/10*(5005*c^3*d^6 + 2145*b*c^2*d^4 + b^3 + 6*a*b*c + 165*(
b^2*c + a*c^2)*d^2)*e^9*x^10 + (715*c^3*d^7 + 429*b*c^2*d^5 + 55*(b^2*c +
a*c^2)*d^3 + (b^3 + 6*a*b*c)*d)*e^8*x^9 + 3/8*(2145*c^3*d^8 + 1716*b*c^2*d
^6 + 330*(b^2*c + a*c^2)*d^4 + a*b^2 + a^2*c + 12*(b^3 + 6*a*b*c)*d^2)*e^7
*x^8 + 1/7*(5005*c^3*d^9 + 5148*b*c^2*d^7 + 1386*(b^2*c + a*c^2)*d^5 + 84*
(b^3 + 6*a*b*c)*d^3 + 21*(a*b^2 + a^2*c)*d)*e^6*x^7 + 1/2*(1001*c^3*d^10 +
 1287*b*c^2*d^8 + 462*(b^2*c + a*c^2)*d^6 + 42*(b^3 + 6*a*b*c)*d^4 + a^2*b
 + 21*(a*b^2 + a^2*c)*d^2)*e^5*x^6 + 3/5*(455*c^3*d^11 + 715*b*c^2*d^9 + 3
30*(b^2*c + a*c^2)*d^7 + 42*(b^3 + 6*a*b*c)*d^5 + 5*a^2*b*d + 35*(a*b^2 +
a^2*c)*d^3)*e^4*x^5 + 1/4*(455*c^3*d^12 + 858*b*c^2*d^10 + 495*(b^2*c + a*
c^2)*d^8 + 84*(b^3 + 6*a*b*c)*d^6 + 30*a^2*b*d^2 + 105*(a*b^2 + a^2*c)*d^4
 + a^3)*e^3*x^4 + (35*c^3*d^13 + 78*b*c^2*d^11 + 55*(b^2*c + a*c^2)*d^9 +
12*(b^3 + 6*a*b*c)*d^7 + 10*a^2*b*d^3 + 21*(a*b^2 + a^2*c)*d^5 + a^3*d)*e^
2*x^3 + 3/2*(5*c^3*d^14 + 13*b*c^2*d^12 + 11*(b^2*c + a*c^2)*d^10 + 3*(b^3
 + 6*a*b*c)*d^8 + 5*a^2*b*d^4 + 7*(a*b^2 + a^2*c)*d^6 + a^3*d^2)*e*x^2 + (
c^3*d^15 + 3*b*c^2*d^13 + 3*(b^2*c + a*c^2)*d^11 + (b^3 + 6*a*b*c)*d^9 + 3
*a^2*b*d^5 + 3*(a*b^2 + a^2*c)*d^7 + a^3*d^3)*x
```

3.609.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1079 vs. $2(124) = 248$.

Time = 0.31 (sec) , antiderivative size = 1079, normalized size of antiderivative = 7.82

$$\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

output
```
1/2*(e*x^2 + 2*d*x)*c^3*d^14 + 7/4*(e*x^2 + 2*d*x)^2*c^3*d^12*e + 7/2*(e*x
^2 + 2*d*x)^3*c^3*d^10*e^2 + 35/8*(e*x^2 + 2*d*x)^4*c^3*d^8*e^3 + 7/2*(e*x
^2 + 2*d*x)^5*c^3*d^6*e^4 + 7/4*(e*x^2 + 2*d*x)^6*c^3*d^4*e^5 + 1/2*(e*x^2
 + 2*d*x)^7*c^3*d^2*e^6 + 1/16*(e*x^2 + 2*d*x)^8*c^3*e^7 + 3/2*(e*x^2 + 2*
d*x)*b*c^2*d^12 + 9/2*(e*x^2 + 2*d*x)^2*b*c^2*d^10*e + 15/2*(e*x^2 + 2*d*x
)^3*b*c^2*d^8*e^2 + 15/2*(e*x^2 + 2*d*x)^4*b*c^2*d^6*e^3 + 9/2*(e*x^2 + 2*
d*x)^5*b*c^2*d^4*e^4 + 3/2*(e*x^2 + 2*d*x)^6*b*c^2*d^2*e^5 + 3/14*(e*x^2 +
 2*d*x)^7*b*c^2*e^6 + 3/2*(e*x^2 + 2*d*x)*b^2*c*d^10 + 3/2*(e*x^2 + 2*d*x)
*a*c^2*d^10 + 15/4*(e*x^2 + 2*d*x)^2*b^2*c*d^8*e + 15/4*(e*x^2 + 2*d*x)^2*
a*c^2*d^8*e + 5*(e*x^2 + 2*d*x)^3*b^2*c*d^6*e^2 + 5*(e*x^2 + 2*d*x)^3*a*c^
2*d^6*e^2 + 15/4*(e*x^2 + 2*d*x)^4*b^2*c*d^4*e^3 + 15/4*(e*x^2 + 2*d*x)^4*
a*c^2*d^4*e^3 + 3/2*(e*x^2 + 2*d*x)^5*b^2*c*d^2*e^4 + 3/2*(e*x^2 + 2*d*x)^
5*a*c^2*d^2*e^4 + 1/4*(e*x^2 + 2*d*x)^6*b^2*c*e^5 + 1/4*(e*x^2 + 2*d*x)^6*
a*c^2*e^5 + 1/2*(e*x^2 + 2*d*x)*b^3*d^8 + 3*(e*x^2 + 2*d*x)*a*b*c*d^8 + (e
*x^2 + 2*d*x)^2*b^3*d^6*e + 6*(e*x^2 + 2*d*x)^2*a*b*c*d^6*e + (e*x^2 + 2*d
*x)^3*b^3*d^4*e^2 + 6*(e*x^2 + 2*d*x)^3*a*b*c*d^4*e^2 + 1/2*(e*x^2 + 2*d*x
)^4*b^3*d^2*e^3 + 3*(e*x^2 + 2*d*x)^4*a*b*c*d^2*e^3 + 1/10*(e*x^2 + 2*d*x)
^5*b^3*e^4 + 3/5*(e*x^2 + 2*d*x)^5*a*b*c*e^4 + 3/2*(e*x^2 + 2*d*x)*a*b^2*d
^6 + 3/2*(e*x^2 + 2*d*x)*a^2*c*d^6 + 9/4*(e*x^2 + 2*d*x)^2*a*b^2*d^4*e + 9
/4*(e*x^2 + 2*d*x)^2*a^2*c*d^4*e + 3/2*(e*x^2 + 2*d*x)^3*a*b^2*d^2*e^2 ...
```

3.609. $\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^3 \, dx$

3.609.9 Mupad [B] (verification not implemented)

Time = 9.02 (sec) , antiderivative size = 777, normalized size of antiderivative = 5.63

$$\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^3 \, dx$$
$$= \frac{3\,e^7\,x^8\,(a^2\,c+a\,b^2+72\,a\,b\,c\,d^2+330\,a\,c^2\,d^4+12\,b^3\,d^2+330\,b^2\,c\,d^4+1716\,b\,c^2\,d^6+2145\,c^3\,d^8)}{8}$$
$$+\frac{e^5\,x^6\,(a^2\,b+21\,a^2\,c\,d^2+21\,a\,b^2\,d^2+252\,a\,b\,c\,d^4+462\,a\,c^2\,d^6+42\,b^3\,d^4+462\,b^2\,c\,d^6+1287\,b\,c^2\,d^8+100}{2}$$
$$+\frac{e^9\,x^{10}\,(b^3+165\,b^2\,c\,d^2+2145\,b\,c^2\,d^4+6\,a\,b\,c+5005\,c^3\,d^6+165\,a\,c^2\,d^2)}{10}$$
$$+\frac{c^3\,e^{15}\,x^{16}}{16}+d^3\,x\,\left(c\,d^4+b\,d^2+a\right)^3$$
$$+\frac{e^3\,x^4\,(a^3+30\,a^2\,b\,d^2+105\,a^2\,c\,d^4+105\,a\,b^2\,d^4+504\,a\,b\,c\,d^6+495\,a\,c^2\,d^8+84\,b^3\,d^6+495\,b^2\,c\,d^8+858}{4}$$
$$+\frac{3\,c^2\,e^{13}\,x^{14}\,(35\,c\,d^2+b)}{14}+c^3\,d\,e^{14}\,x^{15}+d\,e^2\,x^3\,\left(a^3+10\,a^2\,b\,d^2+21\,a^2\,c\,d^4+21\,a\,b^2\,d^4\right.$$
$$\left.+72\,a\,b\,c\,d^6+55\,a\,c^2\,d^8+12\,b^3\,d^6+55\,b^2\,c\,d^8+78\,b\,c^2\,d^{10}+35\,c^3\,d^{12}\right)$$
$$+\frac{c\,e^{11}\,x^{12}\,(b^2+78\,b\,c\,d^2+455\,c^2\,d^4+a\,c)}{4}$$
$$+\frac{d\,e^6\,x^7\,(21\,a^2\,c+21\,a\,b^2+504\,a\,b\,c\,d^2+1386\,a\,c^2\,d^4+84\,b^3\,d^2+1386\,b^2\,c\,d^4+5148\,b\,c^2\,d^6+5005\,c^3\,d^8)}{7}$$
$$+\frac{3\,d\,e^4\,x^5\,(5\,a^2\,b+35\,a^2\,c\,d^2+35\,a\,b^2\,d^2+252\,a\,b\,c\,d^4+330\,a\,c^2\,d^6+42\,b^3\,d^4+330\,b^2\,c\,d^6+715\,b\,c^2\,d^8+}{5}$$
$$+d\,e^8\,x^9\,\left(b^3+55\,b^2\,c\,d^2+429\,b\,c^2\,d^4+6\,a\,b\,c+715\,c^3\,d^6+55\,a\,c^2\,d^2\right)$$
$$+\frac{3\,d^2\,e\,x^2\,\left(c\,d^4+b\,d^2+a\right)^2\,(5\,c\,d^4+3\,b\,d^2+a)}{2}$$
$$+c^2\,d\,e^{12}\,x^{13}\,\left(35\,c\,d^2+3\,b\right)+3\,c\,d\,e^{10}\,x^{11}\,\left(b^2+26\,b\,c\,d^2+91\,c^2\,d^4+a\,c\right)$$

input
```
int((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x)
```

output
```
(3*e^7*x^8*(a*b^2 + a^2*c + 12*b^3*d^2 + 2145*c^3*d^8 + 330*a*c^2*d^4 + 33
0*b^2*c*d^4 + 1716*b*c^2*d^6 + 72*a*b*c*d^2))/8 + (e^5*x^6*(a^2*b + 42*b^3
*d^4 + 1001*c^3*d^10 + 21*a*b^2*d^2 + 21*a^2*c*d^2 + 462*a*c^2*d^6 + 462*b
^2*c*d^6 + 1287*b*c^2*d^8 + 252*a*b*c*d^4))/2 + (e^9*x^10*(b^3 + 5005*c^3*
d^6 + 165*a*c^2*d^2 + 165*b^2*c*d^2 + 2145*b*c^2*d^4 + 6*a*b*c))/10 + (c^3
*e^15*x^16)/16 + d^3*x*(a + b*d^2 + c*d^4)^3 + (e^3*x^4*(a^3 + 84*b^3*d^6
+ 455*c^3*d^12 + 30*a^2*b*d^2 + 105*a*b^2*d^4 + 105*a^2*c*d^4 + 495*a*c^2*
d^8 + 495*b^2*c*d^8 + 858*b*c^2*d^10 + 504*a*b*c*d^6))/4 + (3*c^2*e^13*x^1
4*(b + 35*c*d^2))/14 + c^3*d*e^14*x^15 + d*e^2*x^3*(a^3 + 12*b^3*d^6 + 35*
c^3*d^12 + 10*a^2*b*d^2 + 21*a*b^2*d^4 + 21*a^2*c*d^4 + 55*a*c^2*d^8 + 55*
b^2*c*d^8 + 78*b*c^2*d^10 + 72*a*b*c*d^6) + (c*e^11*x^12*(a*c + b^2 + 455*
c^2*d^4 + 78*b*c*d^2))/4 + (d*e^6*x^7*(21*a*b^2 + 21*a^2*c + 84*b^3*d^2 +
5005*c^3*d^8 + 1386*a*c^2*d^4 + 1386*b^2*c*d^4 + 5148*b*c^2*d^6 + 504*a*b*
c*d^2))/7 + (3*d*e^4*x^5*(5*a^2*b + 42*b^3*d^4 + 455*c^3*d^10 + 35*a*b^2*d
^2 + 35*a^2*c*d^2 + 330*a*c^2*d^6 + 330*b^2*c*d^6 + 715*b*c^2*d^8 + 252*a*
b*c*d^4))/5 + d*e^8*x^9*(b^3 + 715*c^3*d^6 + 55*a*c^2*d^2 + 55*b^2*c*d^2 +
 429*b*c^2*d^4 + 6*a*b*c) + (3*d^2*e*x^2*(a + b*d^2 + c*d^4)^2*(a + 3*b*d^
2 + 5*c*d^4))/2 + c^2*d*e^12*x^13*(3*b + 35*c*d^2) + 3*c*d*e^10*x^11*(a*c
+ b^2 + 91*c^2*d^4 + 26*b*c*d^2)
```

3.609. $\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^3 \, dx$

3.610 $\int (df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right) dx$

3.610.1 Optimal result

Integrand size = 31, antiderivative size = 55

$$\int (df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right) dx = \frac{af^3(d+ex)^4}{4e}+\frac{bf^3(d+ex)^6}{6e}+\frac{cf^3(d+ex)^8}{8e}$$

output
```
1/4*a*f^3*(e*x+d)^4/e+1/6*b*f^3*(e*x+d)^6/e+1/8*c*f^3*(e*x+d)^8/e
```

3.610.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 154 vs. 2(55) = 110.

Time = 0.02 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.80

$$\int (df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right) dx = f^3\left(d^3\left(a+bd^2+cd^4\right)x+\frac{1}{2}d^2\left(3a+5bd^2+7cd^4\right)ex^2+\frac{1}{3}d\left(3a+10bd^2+21cd^4\right)e^2x^3+\frac{1}{4}\left(a+10bd^2+35cd^4\right)e^3x^4+d\left(b+7cd^2\right)e^4x^5+\frac{1}{6}\left(b+21cd^2\right)e^5x^6+cde^6x^7+\frac{1}{8}ce^7x^8\right)$$

input
```
Integrate[(d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

output
```
f^3*(d^3*(a + b*d^2 + c*d^4)*x + (d^2*(3*a + 5*b*d^2 + 7*c*d^4)*e*x^2)/2 +
 (d*(3*a + 10*b*d^2 + 21*c*d^4)*e^2*x^3)/3 + ((a + 10*b*d^2 + 35*c*d^4)*e^
3*x^4)/4 + d*(b + 7*c*d^2)*e^4*x^5 + ((b + 21*c*d^2)*e^5*x^6)/6 + c*d*e^6*
x^7 + (c*e^7*x^8)/8)
```

3.610.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.80, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1462, 1433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right) \, dx$$

$$\Big\downarrow \ 1462$$

$$\frac{f^3 \int (d+ex)^3 \left(c(d+ex)^4+b(d+ex)^2+a\right) d(d+ex)}{e}$$

$$\Big\downarrow \ 1433$$

$$\frac{f^3 \int \left(c(d+ex)^7+b(d+ex)^5+a(d+ex)^3\right) d(d+ex)}{e}$$

$$\Big\downarrow \ 2009$$

$$\frac{f^3 \left(\frac{1}{4}a(d+ex)^4+\frac{1}{6}b(d+ex)^6+\frac{1}{8}c(d+ex)^8\right)}{e}$$

input
```
Int[(d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

output
```
(f^3*((a*(d + e*x)^4)/4 + (b*(d + e*x)^6)/6 + (c*(d + e*x)^8)/8))/e
```

3.610.3.1 Defintions of rubi rules used

rule 1433
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] ||  !IntegerQ[(m + 1)/2])
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.610.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(49) = 98$.

Time = 0.63 (sec) , antiderivative size = 179, normalized size of antiderivative = 3.25

method	result
gosper	$\frac{f^3x(3e^7cx^7+24de^6cx^6+84x^5d^2e^5c+168cd^3e^4x^4+4x^5be^5+210x^3d^4ce^3+24bde^4x^4+168x^2cd^5e^2+60x^3bd^2e^3+84xcd^6e+8}{24}$
norman	$\left(\frac{7}{2}d^2f^3e^5c+\frac{1}{6}be^5f^3\right)x^6+\left(7cd^5e^2f^3+\frac{10}{3}bd^3e^2f^3+ade^2f^3\right)x^3+\left(\frac{7}{2}cd^6ef^3+\frac{5}{2}bd^4ef^3+\right.$
risch	$\frac{1}{8}e^7f^3cx^8+df^3e^6cx^7+\frac{7}{2}f^3x^6d^2e^5c+\frac{1}{6}f^3x^6be^5+7f^3cd^3e^4x^5+f^3bde^4x^5+\frac{35}{4}f^3x^4d^4ce^3$
parallelrisch	$\frac{1}{8}e^7f^3cx^8+df^3e^6cx^7+\frac{7}{2}f^3x^6d^2e^5c+\frac{1}{6}f^3x^6be^5+7f^3cd^3e^4x^5+f^3bde^4x^5+\frac{35}{4}f^3x^4d^4ce^3$
default	$\frac{e^7f^3cx^8}{8}+df^3e^6cx^7+\frac{(15d^2f^3e^5c+e^3f^3(6cd^2e^2+be^2))x^6}{6}+\frac{(13d^3f^3ce^4+3df^3e^2(6cd^2e^2+be^2)+e^3f^3(4d^3ec+2b}{5}$

input
```
int((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4),x,method=_RETURNVERBOSE)
```

output
```
1/24*f^3*x*(3*c*e^7*x^7+24*c*d*e^6*x^6+84*c*d^2*e^5*x^5+168*c*d^3*e^4*x^4+
4*b*e^5*x^5+210*c*d^4*e^3*x^3+24*b*d*e^4*x^4+168*c*d^5*e^2*x^2+60*b*d^2*e^
3*x^3+84*c*d^6*e*x+80*b*d^3*e^2*x^2+24*c*d^7+6*a*e^3*x^3+60*b*d^4*e*x+24*a
*d*e^2*x^2+24*b*d^5+36*a*d^2*e*x+24*a*d^3)
```

3.610.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(49) = 98$.

Time = 0.27 (sec) , antiderivative size = 166, normalized size of antiderivative = 3.02

$$\begin{aligned}
&\int (df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)\,dx \\
&= \frac{1}{8}ce^7f^3x^8+cde^6f^3x^7+\frac{1}{6}\left(21\,cd^2+b\right)e^5f^3x^6+\left(7\,cd^3+bd\right)e^4f^3x^5 \\
&\quad+\frac{1}{4}\left(35\,cd^4+10\,bd^2+a\right)e^3f^3x^4+\frac{1}{3}\left(21\,cd^5+10\,bd^3+3\,ad\right)e^2f^3x^3 \\
&\quad+\frac{1}{2}\left(7\,cd^6+5\,bd^4+3\,ad^2\right)ef^3x^2+\left(cd^7+bd^5+ad^3\right)f^3x
\end{aligned}$$

input
```
integrate((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

output
```
1/8*c*e^7*f^3*x^8 + c*d*e^6*f^3*x^7 + 1/6*(21*c*d^2 + b)*e^5*f^3*x^6 + (7*
c*d^3 + b*d)*e^4*f^3*x^5 + 1/4*(35*c*d^4 + 10*b*d^2 + a)*e^3*f^3*x^4 + 1/3
*(21*c*d^5 + 10*b*d^3 + 3*a*d)*e^2*f^3*x^3 + 1/2*(7*c*d^6 + 5*b*d^4 + 3*a*
d^2)*e*f^3*x^2 + (c*d^7 + b*d^5 + a*d^3)*f^3*x
```

3.610.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(44) = 88$.

Time = 0.05 (sec) , antiderivative size = 240, normalized size of antiderivative = 4.36

$$\int (df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)\,dx$$
$$= cde^6f^3x^7+\frac{ce^7f^3x^8}{8}+x^6\left(\frac{be^5f^3}{6}+\frac{7cd^2e^5f^3}{2}\right)+x^5\left(bde^4f^3+7cd^3e^4f^3\right)$$
$$+x^4\left(\frac{ae^3f^3}{4}+\frac{5bd^2e^3f^3}{2}+\frac{35cd^4e^3f^3}{4}\right)+x^3\left(ade^2f^3+\frac{10bd^3e^2f^3}{3}+7cd^5e^2f^3\right)$$
$$+x^2\cdot\left(\frac{3ad^2ef^3}{2}+\frac{5bd^4ef^3}{2}+\frac{7cd^6ef^3}{2}\right)+x\left(ad^3f^3+bd^5f^3+cd^7f^3\right)$$

input
```
integrate((e*f*x+d*f)**3*(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

output
```
c*d*e**6*f**3*x**7 + c*e**7*f**3*x**8/8 + x**6*(b*e**5*f**3/6 + 7*c*d**2*e
**5*f**3/2) + x**5*(b*d*e**4*f**3 + 7*c*d**3*e**4*f**3) + x**4*(a*e**3*f**
3/4 + 5*b*d**2*e**3*f**3/2 + 35*c*d**4*e**3*f**3/4) + x**3*(a*d*e**2*f**3
+ 10*b*d**3*e**2*f**3/3 + 7*c*d**5*e**2*f**3) + x**2*(3*a*d**2*e*f**3/2 +
5*b*d**4*e*f**3/2 + 7*c*d**6*e*f**3/2) + x*(a*d**3*f**3 + b*d**5*f**3 + c*
d**7*f**3)
```

3.610.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(49) = 98$.

Time = 0.21 (sec) , antiderivative size = 166, normalized size of antiderivative = 3.02

$$\int (df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)\,dx$$
$$=\frac{1}{8}ce^7f^3x^8+cde^6f^3x^7+\frac{1}{6}\left(21\,cd^2+b\right)e^5f^3x^6+\left(7\,cd^3+bd\right)e^4f^3x^5$$
$$+\frac{1}{4}\left(35\,cd^4+10\,bd^2+a\right)e^3f^3x^4+\frac{1}{3}\left(21\,cd^5+10\,bd^3+3\,ad\right)e^2f^3x^3$$
$$+\frac{1}{2}\left(7\,cd^6+5\,bd^4+3\,ad^2\right)ef^3x^2+\left(cd^7+bd^5+ad^3\right)f^3x$$

input
```
integrate((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

output
```
1/8*c*e^7*f^3*x^8 + c*d*e^6*f^3*x^7 + 1/6*(21*c*d^2 + b)*e^5*f^3*x^6 + (7*
c*d^3 + b*d)*e^4*f^3*x^5 + 1/4*(35*c*d^4 + 10*b*d^2 + a)*e^3*f^3*x^4 + 1/3
*(21*c*d^5 + 10*b*d^3 + 3*a*d)*e^2*f^3*x^3 + 1/2*(7*c*d^6 + 5*b*d^4 + 3*a*
d^2)*e*f^3*x^2 + (c*d^7 + b*d^5 + a*d^3)*f^3*x
```

3.610.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(49) = 98$.

Time = 0.30 (sec) , antiderivative size = 204, normalized size of antiderivative = 3.71

$$\int (df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right) dx$$
$$= \frac{1}{2}\left(efx^2+2\,dfx\right)cd^6f^2+\frac{1}{2}\left(efx^2+2\,dfx\right)bd^4f^2+\frac{1}{2}\left(efx^2+2\,dfx\right)ad^2f^2$$
$$+\frac{18\left(efx^2+2\,dfx\right)^2cd^4ef^2+12\left(efx^2+2\,dfx\right)^3cd^2e^2f+3\left(efx^2+2\,dfx\right)^4ce^3+12\left(efx^2+2\,dfx\right)^2bd^2ej}{24\,f}$$

input
```
integrate((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

output
```
1/2*(e*f*x^2 + 2*d*f*x)*c*d^6*f^2 + 1/2*(e*f*x^2 + 2*d*f*x)*b*d^4*f^2 + 1/
2*(e*f*x^2 + 2*d*f*x)*a*d^2*f^2 + 1/24*(18*(e*f*x^2 + 2*d*f*x)^2*c*d^4*e*f
^2 + 12*(e*f*x^2 + 2*d*f*x)^3*c*d^2*e^2*f + 3*(e*f*x^2 + 2*d*f*x)^4*c*e^3
+ 12*(e*f*x^2 + 2*d*f*x)^2*b*d^2*e*f^2 + 4*(e*f*x^2 + 2*d*f*x)^3*b*e^2*f +
 6*(e*f*x^2 + 2*d*f*x)^2*a*e*f^2)/f
```

3.610.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.98

$$\int (df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right) dx$$
$$= \frac{e^5\,f^3\,x^6\,\left(21\,c\,d^2+b\right)}{6}+\frac{c\,e^7\,f^3\,x^8}{8}+d^3\,f^3\,x\,\left(c\,d^4+b\,d^2+a\right)$$
$$+\frac{e^3\,f^3\,x^4\,\left(35\,c\,d^4+10\,b\,d^2+a\right)}{4}+\frac{d^2\,e\,f^3\,x^2\,\left(7\,c\,d^4+5\,b\,d^2+3\,a\right)}{2}$$
$$+\frac{d\,e^2\,f^3\,x^3\,\left(21\,c\,d^4+10\,b\,d^2+3\,a\right)}{3}+d\,e^4\,f^3\,x^5\,\left(7\,c\,d^2+b\right)+c\,d\,e^6\,f^3\,x^7$$

input
```
int((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4),x)
```

output
```
(e^5*f^3*x^6*(b + 21*c*d^2))/6 + (c*e^7*f^3*x^8)/8 + d^3*f^3*x*(a + b*d^2
+ c*d^4) + (e^3*f^3*x^4*(a + 10*b*d^2 + 35*c*d^4))/4 + (d^2*e*f^3*x^2*(3*a
 + 5*b*d^2 + 7*c*d^4))/2 + (d*e^2*f^3*x^3*(3*a + 10*b*d^2 + 21*c*d^4))/3 +
 d*e^4*f^3*x^5*(b + 7*c*d^2) + c*d*e^6*f^3*x^7
```

3.610. $\int (df + efx)^3 \left(a + b(d + ex)^2 + c(d + ex)^4\right) dx$

3.611 $\int (df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2 \, dx$

3.611.1 Optimal result

Integrand size = 33, antiderivative size = 104

$$\int (df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2 \, dx = \frac{a^2f^3(d+ex)^4}{4e}+\frac{abf^3(d+ex)^6}{3e}+\frac{(b^2+2ac)\,f^3(d+ex)^8}{8e}+\frac{bcf^3(d+ex)^{10}}{5e}+\frac{c^2f^3(d+ex)^{12}}{12e}$$

output
```
1/4*a^2*f^3*(e*x+d)^4/e+1/3*a*b*f^3*(e*x+d)^6/e+1/8*(2*a*c+b^2)*f^3*(e*x+d
)^8/e+1/5*b*c*f^3*(e*x+d)^10/e+1/12*c^2*f^3*(e*x+d)^12/e
```

3.611.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 405 vs. $2(104) = 208$.

Time = 0.06 (sec) , antiderivative size = 405, normalized size of antiderivative = 3.89

$$\int (df + efx)^3 \left(a + b(d + ex)^2 + c(d + ex)^4\right)^2 dx$$
$$= f^3\Bigg(d^3\left(a + bd^2 + cd^4\right)^2 x + \frac{1}{2}d^2\left(3a^2 + 10abd^2 + 7b^2d^4 + 14acd^4 + 18bcd^6 + 11c^2d^8\right)ex^2$$
$$+ \frac{1}{3}d\left(3a^2 + 20abd^2 + 21b^2d^4 + 42acd^4 + 72bcd^6 + 55c^2d^8\right)e^2x^3$$
$$+ \frac{1}{4}\left(a^2 + 20abd^2 + 35b^2d^4 + 70acd^4 + 168bcd^6 + 165c^2d^8\right)e^3x^4$$
$$+ \frac{1}{5}d\left(10ab + 35b^2d^2 + 70acd^2 + 252bcd^4 + 330c^2d^6\right)e^4x^5$$
$$+ \frac{1}{6}\left(2ab + 21b^2d^2 + 42acd^2 + 252bcd^4 + 462c^2d^6\right)e^5x^6$$
$$+ d\left(b^2 + 2ac + 24bcd^2 + 66c^2d^4\right)e^6x^7 + \frac{1}{8}\left(b^2 + 2ac + 72bcd^2 + 330c^2d^4\right)e^7x^8$$
$$+ \frac{1}{3}cd\left(6b + 55cd^2\right)e^8x^9 + \frac{1}{10}c\left(2b + 55cd^2\right)e^9x^{10} + c^2de^{10}x^{11} + \frac{1}{12}c^2e^{11}x^{12}\Bigg)$$

input
```
Integrate[(d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

output
```
f^3*(d^3*(a + b*d^2 + c*d^4)^2*x + (d^2*(3*a^2 + 10*a*b*d^2 + 7*b^2*d^4 +
14*a*c*d^4 + 18*b*c*d^6 + 11*c^2*d^8)*e*x^2)/2 + (d*(3*a^2 + 20*a*b*d^2 +
21*b^2*d^4 + 42*a*c*d^4 + 72*b*c*d^6 + 55*c^2*d^8)*e^2*x^3)/3 + ((a^2 + 20
*a*b*d^2 + 35*b^2*d^4 + 70*a*c*d^4 + 168*b*c*d^6 + 165*c^2*d^8)*e^3*x^4)/4
 + (d*(10*a*b + 35*b^2*d^2 + 70*a*c*d^2 + 252*b*c*d^4 + 330*c^2*d^6)*e^4*x
^5)/5 + ((2*a*b + 21*b^2*d^2 + 42*a*c*d^2 + 252*b*c*d^4 + 462*c^2*d^6)*e^5
*x^6)/6 + d*(b^2 + 2*a*c + 24*b*c*d^2 + 66*c^2*d^4)*e^6*x^7 + ((b^2 + 2*a*
c + 72*b*c*d^2 + 330*c^2*d^4)*e^7*x^8)/8 + (c*d*(6*b + 55*c*d^2)*e^8*x^9)/
3 + (c*(2*b + 55*c*d^2)*e^9*x^10)/10 + c^2*d*e^10*x^11 + (c^2*e^11*x^12)/1
2)
```

3.611.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.81, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {1462, 1434, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2 dx$$

$$\Big\downarrow \text{1462}$$

$$\frac{f^3 \int (d+ex)^3 \left(c(d+ex)^4+b(d+ex)^2+a\right)^2 d(d+ex)}{e}$$

$$\Big\downarrow \text{1434}$$

$$\frac{f^3 \int (d+ex)^2 \left(c(d+ex)^4+b(d+ex)^2+a\right)^2 d(d+ex)^2}{2e}$$

$$\Big\downarrow \text{1140}$$

$$\frac{f^3 \int \left(c^2(d+ex)^{10}+2bc(d+ex)^8+\left(b^2+2ac\right)(d+ex)^6+2ab(d+ex)^4+a^2(d+ex)^2\right) d(d+ex)^2}{2e}$$

$$\Big\downarrow \text{2009}$$

$$\frac{f^3 \left(\frac{1}{2}a^2(d+ex)^4+\frac{1}{4}\left(2ac+b^2\right)(d+ex)^8+\frac{2}{3}ab(d+ex)^6+\frac{2}{5}bc(d+ex)^{10}+\frac{1}{6}c^2(d+ex)^{12}\right)}{2e}$$

input
```
Int[(d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

output
```
(f^3*((a^2*(d + e*x)^4)/2 + (2*a*b*(d + e*x)^6)/3 + ((b^2 + 2*a*c)*(d + e*
x)^8)/4 + (2*b*c*(d + e*x)^10)/5 + (c^2*(d + e*x)^12)/6))/(2*e)
```

3.611.3.1 Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.611.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 565 vs. $2(94) = 188$.

Time = 0.60 (sec) , antiderivative size = 566, normalized size of antiderivative = 5.44

method	result
gosper	$\frac{f^3x(10e^{11}c^2x^{11}+120d\,e^{10}c^2x^{10}+660x^9d^2e^9c^2+2200x^8d^3c^2e^8+24x^9bc\,e^9+4950x^7c^2d^4e^7+240x^8bcd\,e^8+7920c^2d^5e^6x^6+1080$
norman	$\left(\frac{11}{2}d^2f^3e^9c^2+\frac{1}{5}bc\,e^9f^3\right)x^{10}+\left(\frac{55}{3}d^3f^3c^2e^8+2bcd\,e^8f^3\right)x^9+\left(\frac{165}{4}d^4f^3c^2e^7+9bc\,d^2e^7f^3+\frac{1}{4}\right.$
risch	$\frac{11}{2}f^3x^{10}d^2e^9c^2+\frac{1}{5}f^3x^{10}bc\,e^9+d\,f^3e^{10}c^2x^{11}+\frac{55}{3}f^3x^9d^3c^2e^8+\frac{165}{4}f^3x^8c^2d^4e^7+\frac{1}{4}f^3x^8ac\,e^7+$
parallelrisch	$\frac{11}{2}f^3x^{10}d^2e^9c^2+\frac{1}{5}f^3x^{10}bc\,e^9+d\,f^3e^{10}c^2x^{11}+\frac{55}{3}f^3x^9d^3c^2e^8+\frac{165}{4}f^3x^8c^2d^4e^7+\frac{1}{4}f^3x^8ac\,e^7+$
default	Expression too large to display

input
```
int((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x,method=_RETURNVERBOSE)
```

output
```
1/120*f^3*x*(10*c^2*e^11*x^11+120*c^2*d*e^10*x^10+660*c^2*d^2*e^9*x^9+2200
*c^2*d^3*e^8*x^8+24*b*c*e^9*x^9+4950*c^2*d^4*e^7*x^7+240*b*c*d*e^8*x^8+792
0*c^2*d^5*e^6*x^6+1080*b*c*d^2*e^7*x^7+9240*c^2*d^6*e^5*x^5+2880*b*c*d^3*e
^6*x^6+7920*c^2*d^7*e^4*x^4+30*a*c*e^7*x^7+15*b^2*e^7*x^7+5040*b*c*d^4*e^5
*x^5+4950*c^2*d^8*e^3*x^3+240*a*c*d*e^6*x^6+120*b^2*d*e^6*x^6+6048*b*c*d^5
*e^4*x^4+2200*c^2*d^9*e^2*x^2+840*a*c*d^2*e^5*x^5+420*b^2*d^2*e^5*x^5+5040
*b*c*d^6*e^3*x^3+660*c^2*d^10*e*x+1680*a*c*d^3*e^4*x^4+840*b^2*d^3*e^4*x^4
+2880*b*c*d^7*e^2*x^2+120*c^2*d^11+40*a*b*e^5*x^5+2100*a*c*d^4*e^3*x^3+105
0*b^2*d^4*e^3*x^3+1080*b*c*d^8*e*x+240*a*b*d*e^4*x^4+1680*a*c*d^5*e^2*x^2+
840*b^2*d^5*e^2*x^2+240*b*c*d^9+600*a*b*d^2*e^3*x^3+840*a*c*d^6*e*x+420*b^
2*d^6*e*x+800*a*b*d^3*e^2*x^2+240*a*c*d^7+120*b^2*d^7+30*a^2*e^3*x^3+600*a
*b*d^4*e*x+120*a^2*d*e^2*x^2+240*a*b*d^5+180*a^2*d^2*e*x+120*a^2*d^3)
```

3.611.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 439 vs. $2(94) = 188$.

Time = 0.26 (sec) , antiderivative size = 439, normalized size of antiderivative = 4.22

$$\begin{aligned}
&\int (df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^2\,dx\\
&=\frac{1}{12}c^2e^{11}f^3x^{12}+c^2de^{10}f^3x^{11}+\frac{1}{10}\left(55\,c^2d^2+2\,bc\right)e^9f^3x^{10}+\frac{1}{3}\left(55\,c^2d^3+6\,bcd\right)e^8f^3x^9\\
&\quad+\frac{1}{8}\left(330\,c^2d^4+72\,bcd^2+b^2+2\,ac\right)e^7f^3x^8+\left(66\,c^2d^5+24\,bcd^3+\left(b^2+2\,ac\right)d\right)e^6f^3x^7\\
&\quad+\frac{1}{6}\left(462\,c^2d^6+252\,bcd^4+21\left(b^2+2\,ac\right)d^2+2\,ab\right)e^5f^3x^6\\
&\quad+\frac{1}{5}\left(330\,c^2d^7+252\,bcd^5+35\left(b^2+2\,ac\right)d^3+10\,abd\right)e^4f^3x^5\\
&\quad+\frac{1}{4}\left(165\,c^2d^8+168\,bcd^6+35\left(b^2+2\,ac\right)d^4+20\,abd^2+a^2\right)e^3f^3x^4\\
&\quad+\frac{1}{3}\left(55\,c^2d^9+72\,bcd^7+21\left(b^2+2\,ac\right)d^5+20\,abd^3+3\,a^2d\right)e^2f^3x^3\\
&\quad+\frac{1}{2}\left(11\,c^2d^{10}+18\,bcd^8+7\left(b^2+2\,ac\right)d^6+10\,abd^4+3\,a^2d^2\right)ef^3x^2\\
&\quad+\left(c^2d^{11}+2\,bcd^9+\left(b^2+2\,ac\right)d^7+2\,abd^5+a^2d^3\right)f^3x
\end{aligned}$$

input
```
integrate((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas
")
```

output
```
1/12*c^2*e^11*f^3*x^12 + c^2*d*e^10*f^3*x^11 + 1/10*(55*c^2*d^2 + 2*b*c)*e
^9*f^3*x^10 + 1/3*(55*c^2*d^3 + 6*b*c*d)*e^8*f^3*x^9 + 1/8*(330*c^2*d^4 +
72*b*c*d^2 + b^2 + 2*a*c)*e^7*f^3*x^8 + (66*c^2*d^5 + 24*b*c*d^3 + (b^2 +
2*a*c)*d)*e^6*f^3*x^7 + 1/6*(462*c^2*d^6 + 252*b*c*d^4 + 21*(b^2 + 2*a*c)*
d^2 + 2*a*b)*e^5*f^3*x^6 + 1/5*(330*c^2*d^7 + 252*b*c*d^5 + 35*(b^2 + 2*a*
c)*d^3 + 10*a*b*d)*e^4*f^3*x^5 + 1/4*(165*c^2*d^8 + 168*b*c*d^6 + 35*(b^2
+ 2*a*c)*d^4 + 20*a*b*d^2 + a^2)*e^3*f^3*x^4 + 1/3*(55*c^2*d^9 + 72*b*c*d^
7 + 21*(b^2 + 2*a*c)*d^5 + 20*a*b*d^3 + 3*a^2*d)*e^2*f^3*x^3 + 1/2*(11*c^2
*d^10 + 18*b*c*d^8 + 7*(b^2 + 2*a*c)*d^6 + 10*a*b*d^4 + 3*a^2*d^2)*e*f^3*x
^2 + (c^2*d^11 + 2*b*c*d^9 + (b^2 + 2*a*c)*d^7 + 2*a*b*d^5 + a^2*d^3)*f^3*
x
```

3.611.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 722 vs. $2(88) = 176$.

Time = 0.07 (sec) , antiderivative size = 722, normalized size of antiderivative = 6.94

$$\begin{aligned}
&\int (df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^2\,dx\\
&= c^2de^{10}f^3x^{11}+\frac{c^2e^{11}f^3x^{12}}{12}+x^{10}\left(\frac{bce^9f^3}{5}+\frac{11c^2d^2e^9f^3}{2}\right)+x^9\\
&\quad\cdot\left(2bcde^8f^3+\frac{55c^2d^3e^8f^3}{3}\right)+x^8\left(\frac{ace^7f^3}{4}+\frac{b^2e^7f^3}{8}+9bcd^2e^7f^3+\frac{165c^2d^4e^7f^3}{4}\right)\\
&\quad+x^7\cdot\left(2acde^6f^3+b^2de^6f^3+24bcd^3e^6f^3+66c^2d^5e^6f^3\right)\\
&\quad+x^6\left(\frac{abe^5f^3}{3}+7acd^2e^5f^3+\frac{7b^2d^2e^5f^3}{2}+42bcd^4e^5f^3+77c^2d^6e^5f^3\right)+x^5\\
&\quad\cdot\left(2abde^4f^3+14acd^3e^4f^3+7b^2d^3e^4f^3+\frac{252bcd^5e^4f^3}{5}+66c^2d^7e^4f^3\right)\\
&\quad+x^4\left(\frac{a^2e^3f^3}{4}+5abd^2e^3f^3+\frac{35acd^4e^3f^3}{2}+\frac{35b^2d^4e^3f^3}{4}+42bcd^6e^3f^3+\frac{165c^2d^8e^3f^3}{4}\right)\\
&\quad+x^3\left(a^2de^2f^3+\frac{20abd^3e^2f^3}{3}+14acd^5e^2f^3+7b^2d^5e^2f^3+24bcd^7e^2f^3+\frac{55c^2d^9e^2f^3}{3}\right)\\
&\quad+x^2\cdot\left(\frac{3a^2d^2ef^3}{2}+5abd^4ef^3+7acd^6ef^3+\frac{7b^2d^6ef^3}{2}+9bcd^8ef^3+\frac{11c^2d^{10}ef^3}{2}\right)\\
&\quad+x\left(a^2d^3f^3+2abd^5f^3+2acd^7f^3+b^2d^7f^3+2bcd^9f^3+c^2d^{11}f^3\right)
\end{aligned}$$

input
```
integrate((e*f*x+d*f)**3*(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

output
```
c**2*d*e**10*f**3*x**11 + c**2*e**11*f**3*x**12/12 + x**10*(b*c*e**9*f**3/
5 + 11*c**2*d**2*e**9*f**3/2) + x**9*(2*b*c*d*e**8*f**3 + 55*c**2*d**3*e**
8*f**3/3) + x**8*(a*c*e**7*f**3/4 + b**2*e**7*f**3/8 + 9*b*c*d**2*e**7*f**
3 + 165*c**2*d**4*e**7*f**3/4) + x**7*(2*a*c*d*e**6*f**3 + b**2*d*e**6*f**
3 + 24*b*c*d**3*e**6*f**3 + 66*c**2*d**5*e**6*f**3) + x**6*(a*b*e**5*f**3/
3 + 7*a*c*d**2*e**5*f**3 + 7*b**2*d**2*e**5*f**3/2 + 42*b*c*d**4*e**5*f**3
 + 77*c**2*d**6*e**5*f**3) + x**5*(2*a*b*d*e**4*f**3 + 14*a*c*d**3*e**4*f*
*3 + 7*b**2*d**3*e**4*f**3 + 252*b*c*d**5*e**4*f**3/5 + 66*c**2*d**7*e**4*
f**3) + x**4*(a**2*e**3*f**3/4 + 5*a*b*d**2*e**3*f**3 + 35*a*c*d**4*e**3*f
**3/2 + 35*b**2*d**4*e**3*f**3/4 + 42*b*c*d**6*e**3*f**3 + 165*c**2*d**8*e
**3*f**3/4) + x**3*(a**2*d*e**2*f**3 + 20*a*b*d**3*e**2*f**3/3 + 14*a*c*d*
*5*e**2*f**3 + 7*b**2*d**5*e**2*f**3 + 24*b*c*d**7*e**2*f**3 + 55*c**2*d**
9*e**2*f**3/3) + x**2*(3*a**2*d**2*e*f**3/2 + 5*a*b*d**4*e*f**3 + 7*a*c*d*
*6*e*f**3 + 7*b**2*d**6*e*f**3/2 + 9*b*c*d**8*e*f**3 + 11*c**2*d**10*e*f**
3/2) + x*(a**2*d**3*f**3 + 2*a*b*d**5*f**3 + 2*a*c*d**7*f**3 + b**2*d**7*f
**3 + 2*b*c*d**9*f**3 + c**2*d**11*f**3)
```

3.611.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 439 vs. $2(94) = 188$.

Time = 0.22 (sec) , antiderivative size = 439, normalized size of antiderivative = 4.22

$$\begin{aligned}
&\int (df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^2\,dx \\
&= \frac{1}{12}c^2e^{11}f^3x^{12}+c^2de^{10}f^3x^{11}+\frac{1}{10}\left(55\,c^2d^2+2\,bc\right)e^9f^3x^{10}+\frac{1}{3}\left(55\,c^2d^3+6\,bcd\right)e^8f^3x^9 \\
&\quad+\frac{1}{8}\left(330\,c^2d^4+72\,bcd^2+b^2+2\,ac\right)e^7f^3x^8+\left(66\,c^2d^5+24\,bcd^3+\left(b^2+2\,ac\right)d\right)e^6f^3x^7 \\
&\quad+\frac{1}{6}\left(462\,c^2d^6+252\,bcd^4+21\left(b^2+2\,ac\right)d^2+2\,ab\right)e^5f^3x^6 \\
&\quad+\frac{1}{5}\left(330\,c^2d^7+252\,bcd^5+35\left(b^2+2\,ac\right)d^3+10\,abd\right)e^4f^3x^5 \\
&\quad+\frac{1}{4}\left(165\,c^2d^8+168\,bcd^6+35\left(b^2+2\,ac\right)d^4+20\,abd^2+a^2\right)e^3f^3x^4 \\
&\quad+\frac{1}{3}\left(55\,c^2d^9+72\,bcd^7+21\left(b^2+2\,ac\right)d^5+20\,abd^3+3\,a^2d\right)e^2f^3x^3 \\
&\quad+\frac{1}{2}\left(11\,c^2d^{10}+18\,bcd^8+7\left(b^2+2\,ac\right)d^6+10\,abd^4+3\,a^2d^2\right)ef^3x^2 \\
&\quad+\left(c^2d^{11}+2\,bcd^9+\left(b^2+2\,ac\right)d^7+2\,abd^5+a^2d^3\right)f^3x
\end{aligned}$$

input
```
integrate((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima
")
```

output
```
1/12*c^2*e^11*f^3*x^12 + c^2*d*e^10*f^3*x^11 + 1/10*(55*c^2*d^2 + 2*b*c)*e
^9*f^3*x^10 + 1/3*(55*c^2*d^3 + 6*b*c*d)*e^8*f^3*x^9 + 1/8*(330*c^2*d^4 +
72*b*c*d^2 + b^2 + 2*a*c)*e^7*f^3*x^8 + (66*c^2*d^5 + 24*b*c*d^3 + (b^2 +
2*a*c)*d)*e^6*f^3*x^7 + 1/6*(462*c^2*d^6 + 252*b*c*d^4 + 21*(b^2 + 2*a*c)*
d^2 + 2*a*b)*e^5*f^3*x^6 + 1/5*(330*c^2*d^7 + 252*b*c*d^5 + 35*(b^2 + 2*a*
c)*d^3 + 10*a*b*d)*e^4*f^3*x^5 + 1/4*(165*c^2*d^8 + 168*b*c*d^6 + 35*(b^2
+ 2*a*c)*d^4 + 20*a*b*d^2 + a^2)*e^3*f^3*x^4 + 1/3*(55*c^2*d^9 + 72*b*c*d^
7 + 21*(b^2 + 2*a*c)*d^5 + 20*a*b*d^3 + 3*a^2*d)*e^2*f^3*x^3 + 1/2*(11*c^2
*d^10 + 18*b*c*d^8 + 7*(b^2 + 2*a*c)*d^6 + 10*a*b*d^4 + 3*a^2*d^2)*e*f^3*x
^2 + (c^2*d^11 + 2*b*c*d^9 + (b^2 + 2*a*c)*d^7 + 2*a*b*d^5 + a^2*d^3)*f^3*
x
```

3.611.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 597 vs. $2(94) = 188$.

Time = 0.30 (sec) , antiderivative size = 597, normalized size of antiderivative = 5.74

$$\int (df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2 dx$$
$$= \frac{1}{2}\left(efx^2+2\,dfx\right)c^2d^{10}f^2 + \left(efx^2+2\,dfx\right)bcd^8f^2 + \frac{1}{2}\left(efx^2+2\,dfx\right)b^2d^6f^2 + \left(efx^2+2\,dfx\right)acd^6f^2 + \left(efx^2+2\,dfx\right)abd^4f^2 + \frac{1}{2}\left(efx^2+2\,dfx\right)a^2d^2f^2 + \frac{150\left(efx^2+2\,dfx\right)^2c^2d^8ef^4 + 200\left(efx^2+2\,dfx\right)^3c^2d^6e^2f^3 + 150\left(efx^2+2\,dfx\right)^4c^2d^4e^3f^2 + 240\left(efx^2\right.}{}$$

input
```
integrate((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

output
```
1/2*(e*f*x^2 + 2*d*f*x)*c^2*d^10*f^2 + (e*f*x^2 + 2*d*f*x)*b*c*d^8*f^2 + 1
/2*(e*f*x^2 + 2*d*f*x)*b^2*d^6*f^2 + (e*f*x^2 + 2*d*f*x)*a*c*d^6*f^2 + (e*
f*x^2 + 2*d*f*x)*a*b*d^4*f^2 + 1/2*(e*f*x^2 + 2*d*f*x)*a^2*d^2*f^2 + 1/120
*(150*(e*f*x^2 + 2*d*f*x)^2*c^2*d^8*e*f^4 + 200*(e*f*x^2 + 2*d*f*x)^3*c^2*
d^6*e^2*f^3 + 150*(e*f*x^2 + 2*d*f*x)^4*c^2*d^4*e^3*f^2 + 240*(e*f*x^2 + 2
*d*f*x)^2*b*c*d^6*e*f^4 + 60*(e*f*x^2 + 2*d*f*x)^5*c^2*d^2*e^4*f + 240*(e*
f*x^2 + 2*d*f*x)^3*b*c*d^4*e^2*f^3 + 10*(e*f*x^2 + 2*d*f*x)^6*c^2*e^5 + 12
0*(e*f*x^2 + 2*d*f*x)^4*b*c*d^2*e^3*f^2 + 90*(e*f*x^2 + 2*d*f*x)^2*b^2*d^4
*e*f^4 + 180*(e*f*x^2 + 2*d*f*x)^2*a*c*d^4*e*f^4 + 24*(e*f*x^2 + 2*d*f*x)^
5*b*c*e^4*f + 60*(e*f*x^2 + 2*d*f*x)^3*b^2*d^2*e^2*f^3 + 120*(e*f*x^2 + 2*
d*f*x)^3*a*c*d^2*e^2*f^3 + 15*(e*f*x^2 + 2*d*f*x)^4*b^2*e^3*f^2 + 30*(e*f*
x^2 + 2*d*f*x)^4*a*c*e^3*f^2 + 120*(e*f*x^2 + 2*d*f*x)^2*a*b*d^2*e*f^4 + 4
0*(e*f*x^2 + 2*d*f*x)^3*a*b*e^2*f^3 + 30*(e*f*x^2 + 2*d*f*x)^2*a^2*e*f^4)/
f^3
```

3.611.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 419, normalized size of antiderivative = 4.03

$$\int (df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^2\,dx = \frac{e^3\,f^3\,x^4\,\left(a^2+20\,a\,b\,d^2+70\,a\,c\,d^4+35\,b^2\,d^4+168\,b\,c\,d^6+165\,c^2\,d^8\right)}{4}+\frac{c^2\,e^{11}\,f^3\,x^{12}}{12}+d^3\,f^3\,x\,{\left(c\,d^4+b\,d^2+a\right)}^2+\frac{e^7\,f^3\,x^8\,\left(b^2+72\,b\,c\,d^2+330\,c^2\,d^4+2\,a\,c\right)}{8}+\frac{e^5\,f^3\,x^6\,\left(21\,b^2\,d^2+252\,b\,c\,d^4+2\,a\,b+462\,c^2\,d^6+42\,a\,c\,d^2\right)}{6}+\frac{d^2\,e\,f^3\,x^2\,\left(3\,a^2+10\,a\,b\,d^2+14\,a\,c\,d^4+7\,b^2\,d^4+18\,b\,c\,d^6+11\,c^2\,d^8\right)}{2}+\frac{d\,e^2\,f^3\,x^3\,\left(3\,a^2+20\,a\,b\,d^2+42\,a\,c\,d^4+21\,b^2\,d^4+72\,b\,c\,d^6+55\,c^2\,d^8\right)}{3}+d\,e^6\,f^3\,x^7\,\left(b^2+24\,b\,c\,d^2+66\,c^2\,d^4+2\,a\,c\right)+\frac{d\,e^4\,f^3\,x^5\,\left(35\,b^2\,d^2+252\,b\,c\,d^4+10\,a\,b+330\,c^2\,d^6+70\,a\,c\,d^2\right)}{5}+\frac{c\,e^9\,f^3\,x^{10}\,\left(55\,c\,d^2+2\,b\right)}{10}+c^2\,d\,e^{10}\,f^3\,x^{11}+\frac{c\,d\,e^8\,f^3\,x^9\,\left(55\,c\,d^2+6\,b\right)}{3}$$

input
```
int((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x)
```

output
```
(e^3*f^3*x^4*(a^2 + 35*b^2*d^4 + 165*c^2*d^8 + 20*a*b*d^2 + 70*a*c*d^4 + 1
68*b*c*d^6))/4 + (c^2*e^11*f^3*x^12)/12 + d^3*f^3*x*(a + b*d^2 + c*d^4)^2
+ (e^7*f^3*x^8*(2*a*c + b^2 + 330*c^2*d^4 + 72*b*c*d^2))/8 + (e^5*f^3*x^6*
(2*a*b + 21*b^2*d^2 + 462*c^2*d^6 + 42*a*c*d^2 + 252*b*c*d^4))/6 + (d^2*e*
f^3*x^2*(3*a^2 + 7*b^2*d^4 + 11*c^2*d^8 + 10*a*b*d^2 + 14*a*c*d^4 + 18*b*c
*d^6))/2 + (d*e^2*f^3*x^3*(3*a^2 + 21*b^2*d^4 + 55*c^2*d^8 + 20*a*b*d^2 +
42*a*c*d^4 + 72*b*c*d^6))/3 + d*e^6*f^3*x^7*(2*a*c + b^2 + 66*c^2*d^4 + 24
*b*c*d^2) + (d*e^4*f^3*x^5*(10*a*b + 35*b^2*d^2 + 330*c^2*d^6 + 70*a*c*d^2
 + 252*b*c*d^4))/5 + (c*e^9*f^3*x^10*(2*b + 55*c*d^2))/10 + c^2*d*e^10*f^3
*x^11 + (c*d*e^8*f^3*x^9*(6*b + 55*c*d^2))/3
```

3.611. $\int (df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2 \, dx$

3.612 $\int (df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^3 \, dx$

3.612.1 Optimal result

Integrand size = 33, antiderivative size = 159

$$\begin{aligned}\int (df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^3 \, dx &= \frac{a^3f^3(d+ex)^4}{4e}+\frac{a^2bf^3(d+ex)^6}{2e}+\frac{3a(b^2+ac)\,f^3(d+ex)^8}{8e}+\frac{b(b^2+6ac)\,f^3(d+ex)^{10}}{10e}\\&\quad+\frac{c(b^2+ac)\,f^3(d+ex)^{12}}{4e}+\frac{3bc^2f^3(d+ex)^{14}}{14e}+\frac{c^3f^3(d+ex)^{16}}{16e}\end{aligned}$$

output
```
1/4*a^3*f^3*(e*x+d)^4/e+1/2*a^2*b*f^3*(e*x+d)^6/e+3/8*a*(a*c+b^2)*f^3*(e*x
+d)^8/e+1/10*b*(6*a*c+b^2)*f^3*(e*x+d)^10/e+1/4*c*(a*c+b^2)*f^3*(e*x+d)^12
/e+3/14*b*c^2*f^3*(e*x+d)^14/e+1/16*c^3*f^3*(e*x+d)^16/e
```

3.612.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 801 vs. $2(159) = 318$.

Time = 0.03 (sec) , antiderivative size = 801, normalized size of antiderivative = 5.04

$$\int (df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^3\,dx$$
$$= f^3\left(d^3\left(a+bd^2+cd^4\right)^3x+\frac{3}{2}d^2\left(a+bd^2+cd^4\right)^2\left(a+3bd^2+5cd^4\right)ex^2+d\left(a^3+10a^2bd^2+21ab^2d^4+21a^2cd^4+12b^3d^6+72abcd^6+55b^2cd^8+55ac^2d^8+78bc^2d^{10}+35c^3d^{12}\right)e^2x^3+\frac{1}{4}\left(a^3+30a^2bd^2+105ab^2d^4+105a^2cd^4+84b^3d^6+504abcd^6+495b^2cd^8+495ac^2d^8+858bc^2d^{10}+455c^3d^{12}\right)e^3x^4+\frac{3}{5}d\left(5a^2b+35ab^2d^2+35a^2cd^2+42b^3d^4+252abcd^4+330b^2cd^6+330ac^2d^6+715bc^2d^8+455c^3d^{10}\right)e^4x^5+\frac{1}{2}\left(a^2b+21ab^2d^2+21a^2cd^2+42b^3d^4+252abcd^4+462b^2cd^6+462ac^2d^6+1287bc^2d^8+1001c^3d^{10}\right)e^5x^6+\frac{1}{7}d\left(21ab^2+21a^2c+84b^3d^2+504abcd^2+1386b^2cd^4+1386ac^2d^4+5148bc^2d^6+5005c^3d^8\right)e^6x^7+\frac{3}{8}\left(ab^2+a^2c+12b^3d^2+72abcd^2+330b^2cd^4+330ac^2d^4+1716bc^2d^6+2145c^3d^8\right)e^7x^8+d\left(b^3+6abc+55b^2cd^2+55ac^2d^2+429bc^2d^4+715c^3d^6\right)e^8x^9+\frac{1}{10}\left(b^3+6abc+165b^2cd^2+165ac^2d^2+2145bc^2d^4+5005c^3d^6\right)e^9x^{10}+3cd\left(b^2+ac+26bcd^2+91c^2d^4\right)e^{10}x^{11}+\frac{1}{4}c\left(b^2+ac+78bcd^2+455c^2d^4\right)e^{11}x^{12}+c^2d\left(3b+35cd^2\right)e^{12}x^{13}+\frac{3}{14}c^2\left(b+35cd^2\right)e^{13}x^{14}+c^3de^{14}x^{15}+\frac{1}{16}c^3e^{15}x^{16}\right)$$

input
```
Integrate[(d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

output
```
f^3*(d^3*(a + b*d^2 + c*d^4)^3*x + (3*d^2*(a + b*d^2 + c*d^4)^2*(a + 3*b*d
^2 + 5*c*d^4)*e*x^2)/2 + d*(a^3 + 10*a^2*b*d^2 + 21*a*b^2*d^4 + 21*a^2*c*d
^4 + 12*b^3*d^6 + 72*a*b*c*d^6 + 55*b^2*c*d^8 + 55*a*c^2*d^8 + 78*b*c^2*d^
10 + 35*c^3*d^12)*e^2*x^3 + ((a^3 + 30*a^2*b*d^2 + 105*a*b^2*d^4 + 105*a^2
*c*d^4 + 84*b^3*d^6 + 504*a*b*c*d^6 + 495*b^2*c*d^8 + 495*a*c^2*d^8 + 858*
b*c^2*d^10 + 455*c^3*d^12)*e^3*x^4)/4 + (3*d*(5*a^2*b + 35*a*b^2*d^2 + 35*
a^2*c*d^2 + 42*b^3*d^4 + 252*a*b*c*d^4 + 330*b^2*c*d^6 + 330*a*c^2*d^6 + 7
15*b*c^2*d^8 + 455*c^3*d^10)*e^4*x^5)/5 + ((a^2*b + 21*a*b^2*d^2 + 21*a^2*
c*d^2 + 42*b^3*d^4 + 252*a*b*c*d^4 + 462*b^2*c*d^6 + 462*a*c^2*d^6 + 1287*
b*c^2*d^8 + 1001*c^3*d^10)*e^5*x^6)/2 + (d*(21*a*b^2 + 21*a^2*c + 84*b^3*d
^2 + 504*a*b*c*d^2 + 1386*b^2*c*d^4 + 1386*a*c^2*d^4 + 5148*b*c^2*d^6 + 50
05*c^3*d^8)*e^6*x^7)/7 + (3*(a*b^2 + a^2*c + 12*b^3*d^2 + 72*a*b*c*d^2 + 3
30*b^2*c*d^4 + 330*a*c^2*d^4 + 1716*b*c^2*d^6 + 2145*c^3*d^8)*e^7*x^8)/8 +
 d*(b^3 + 6*a*b*c + 55*b^2*c*d^2 + 55*a*c^2*d^2 + 429*b*c^2*d^4 + 715*c^3*
d^6)*e^8*x^9 + ((b^3 + 6*a*b*c + 165*b^2*c*d^2 + 165*a*c^2*d^2 + 2145*b*c^
2*d^4 + 5005*c^3*d^6)*e^9*x^10)/10 + 3*c*d*(b^2 + a*c + 26*b*c*d^2 + 91*c^
2*d^4)*e^10*x^11 + (c*(b^2 + a*c + 78*b*c*d^2 + 455*c^2*d^4)*e^11*x^12)/4
+ c^2*d*(3*b + 35*c*d^2)*e^12*x^13 + (3*c^2*(b + 35*c*d^2)*e^13*x^14)/14 +
 c^3*d*e^14*x^15 + (c^3*e^15*x^16)/16)
```

3.612.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.78, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {1462, 1434, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^3 \, dx$$

$$\Big\downarrow \text{ 1462}$$

$$\frac{f^3 \int (d+ex)^3 \left(c(d+ex)^4+b(d+ex)^2+a\right)^3 d(d+ex)}{e}$$

$$\Big\downarrow \text{ 1434}$$

$$\frac{f^3 \int (d+ex)^2 \left(c(d+ex)^4+b(d+ex)^2+a\right)^3 d(d+ex)^2}{2e}$$

$$\Big\downarrow \text{ 1140}$$

$$\frac{f^3 \int \left(c^3(d+ex)^{14} + 3bc^2(d+ex)^{12} + 3c\left(b^2+ac\right)(d+ex)^{10} + b\left(b^2+6ac\right)(d+ex)^8 + 3a\left(b^2+ac\right)(d+ex)^6 + 3}{2e}$$

$\Big\downarrow$ 2009

$$\frac{f^3 \left(\frac{1}{2}a^3(d+ex)^4 + a^2b(d+ex)^6 + \frac{1}{2}c\left(ac+b^2\right)(d+ex)^{12} + \frac{1}{5}b\left(6ac+b^2\right)(d+ex)^{10} + \frac{3}{4}a\left(ac+b^2\right)(d+ex)^8 + \frac{3}{7}}{2e}$$

input
```
Int[(d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

output
```
(f^3*((a^3*(d + e*x)^4)/2 + a^2*b*(d + e*x)^6 + (3*a*(b^2 + a*c)*(d + e*x)
^8)/4 + (b*(b^2 + 6*a*c)*(d + e*x)^10)/5 + (c*(b^2 + a*c)*(d + e*x)^12)/2
+ (3*b*c^2*(d + e*x)^14)/7 + (c^3*(d + e*x)^16)/8))/(2*e)
```

3.612.3.1 Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.612.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1317 vs. 2(145) = 290.

Time = 0.64 (sec) , antiderivative size = 1318, normalized size of antiderivative = 8.29

method	result	size
gosper	Expression too large to display	1318
norman	Expression too large to display	1430
risch	Expression too large to display	1636
parallelrisch	Expression too large to display	1636
default	Expression too large to display	7697

input
```
int((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x,method=_RETURNVERBOSE)
```

output
```
1/560*f^3*x*(35*c^3*e^15*x^15+560*c^3*d*e^14*x^14+4200*c^3*d^2*e^13*x^13+1
9600*c^3*d^3*e^12*x^12+120*b*c^2*e^13*x^13+63700*c^3*d^4*e^11*x^11+1680*b*
c^2*d*e^12*x^12+152880*c^3*d^5*e^10*x^10+10920*b*c^2*d^2*e^11*x^11+280280*
c^3*d^6*e^9*x^9+43680*b*c^2*d^3*e^10*x^10+400400*c^3*d^7*e^8*x^8+140*a*c^2
*e^11*x^11+140*b^2*c*e^11*x^11+120120*b*c^2*d^4*e^9*x^9+450450*c^3*d^8*e^7
*x^7+1680*a*c^2*d*e^10*x^10+1680*b^2*c*d*e^10*x^10+240240*b*c^2*d^5*e^8*x^
8+400400*c^3*d^9*e^6*x^6+9240*a*c^2*d^2*e^9*x^9+9240*b^2*c*d^2*e^9*x^9+360
360*b*c^2*d^6*e^7*x^7+280280*c^3*d^10*e^5*x^5+30800*a*c^2*d^3*e^8*x^8+3080
0*b^2*c*d^3*e^8*x^8+411840*b*c^2*d^7*e^6*x^6+152880*c^3*d^11*e^4*x^4+336*a
*b*c*e^9*x^9+69300*a*c^2*d^4*e^7*x^7+56*b^3*e^9*x^9+69300*b^2*c*d^4*e^7*x^
7+360360*b*c^2*d^8*e^5*x^5+63700*c^3*d^12*e^3*x^3+3360*a*b*c*d*e^8*x^8+110
880*a*c^2*d^5*e^6*x^6+560*b^3*d*e^8*x^8+110880*b^2*c*d^5*e^6*x^6+240240*b*
c^2*d^9*e^4*x^4+19600*c^3*d^13*e^2*x^2+15120*a*b*c*d^2*e^7*x^7+129360*a*c^
2*d^6*e^5*x^5+2520*b^3*d^2*e^7*x^7+129360*b^2*c*d^6*e^5*x^5+120120*b*c^2*d
^10*e^3*x^3+4200*c^3*d^14*e*x+40320*a*b*c*d^3*e^6*x^6+110880*a*c^2*d^7*e^4
*x^4+6720*b^3*d^3*e^6*x^6+110880*b^2*c*d^7*e^4*x^4+43680*b*c^2*d^11*e^2*x^
2+560*c^3*d^15+210*a^2*c*e^7*x^7+210*a*b^2*e^7*x^7+70560*a*b*c*d^4*e^5*x^5
+69300*a*c^2*d^8*e^3*x^3+11760*b^3*d^4*e^5*x^5+69300*b^2*c*d^8*e^3*x^3+109
20*b*c^2*d^12*e*x+1680*a^2*c*d*e^6*x^6+1680*a*b^2*d*e^6*x^6+84672*a*b*c*d^
5*e^4*x^4+30800*a*c^2*d^9*e^2*x^2+14112*b^3*d^5*e^4*x^4+30800*b^2*c*d^9...
```

3.612.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 920 vs. $2(145) = 290$.

Time = 0.26 (sec) , antiderivative size = 920, normalized size of antiderivative = 5.79

$$
\begin{aligned}
&\int (df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^3 \, dx \\
&= \frac{1}{16} c^3 e^{15} f^3 x^{16} + c^3 d e^{14} f^3 x^{15} + \frac{3}{14} \left(35 \, c^3 d^2 + b c^2\right) e^{13} f^3 x^{14} \\
&\quad + \left(35 \, c^3 d^3 + 3 \, b c^2 d\right) e^{12} f^3 x^{13} + \frac{1}{4} \left(455 \, c^3 d^4 + 78 \, b c^2 d^2 + b^2 c + a c^2\right) e^{11} f^3 x^{12} \\
&\quad + 3 \left(91 \, c^3 d^5 + 26 \, b c^2 d^3 + \left(b^2 c + a c^2\right) d\right) e^{10} f^3 x^{11} \\
&\quad + \frac{1}{10} \left(5005 \, c^3 d^6 + 2145 \, b c^2 d^4 + b^3 + 6 \, a b c + 165 \left(b^2 c + a c^2\right) d^2\right) e^9 f^3 x^{10} \\
&\quad + \left(715 \, c^3 d^7 + 429 \, b c^2 d^5 + 55 \left(b^2 c + a c^2\right) d^3 + \left(b^3 + 6 \, a b c\right) d\right) e^8 f^3 x^9 \\
&\quad + \frac{3}{8} \left(2145 \, c^3 d^8 + 1716 \, b c^2 d^6 + 330 \left(b^2 c + a c^2\right) d^4 + a b^2 + a^2 c + 12 \left(b^3 + 6 \, a b c\right) d^2\right) e^7 f^3 x^8 \\
&\quad + \frac{1}{7} \left(5005 \, c^3 d^9 + 5148 \, b c^2 d^7 + 1386 \left(b^2 c + a c^2\right) d^5 + 84 \left(b^3 + 6 \, a b c\right) d^3 + 21 \left(a b^2 + a^2 c\right) d\right) e^6 f^3 x^7 \\
&\quad + \frac{1}{2} \left(1001 \, c^3 d^{10} + 1287 \, b c^2 d^8 + 462 \left(b^2 c + a c^2\right) d^6 + 42 \left(b^3 + 6 \, a b c\right) d^4 + a^2 b + 21 \left(a b^2 + a^2 c\right) d^2\right) e^5 f^3 x^6 \\
&\quad + \frac{3}{5} \left(455 \, c^3 d^{11} + 715 \, b c^2 d^9 + 330 \left(b^2 c + a c^2\right) d^7 + 42 \left(b^3 + 6 \, a b c\right) d^5 + 5 \, a^2 b d + 35 \left(a b^2 + a^2 c\right) d^3\right) e^4 f^3 x^5 \\
&\quad + \frac{1}{4} \left(455 \, c^3 d^{12} + 858 \, b c^2 d^{10} + 495 \left(b^2 c + a c^2\right) d^8 + 84 \left(b^3 + 6 \, a b c\right) d^6 + 30 \, a^2 b d^2 + 105 \left(a b^2 + a^2 c\right) d^4 + a^3\right) \\
&\quad + \left(35 \, c^3 d^{13} + 78 \, b c^2 d^{11} + 55 \left(b^2 c + a c^2\right) d^9 + 12 \left(b^3 + 6 \, a b c\right) d^7 + 10 \, a^2 b d^3 + 21 \left(a b^2 + a^2 c\right) d^5 + a^3 d\right) e^2 f^3 x \\
&\quad + \frac{3}{2} \left(5 \, c^3 d^{14} + 13 \, b c^2 d^{12} + 11 \left(b^2 c + a c^2\right) d^{10} + 3 \left(b^3 + 6 \, a b c\right) d^8 + 5 \, a^2 b d^4 + 7 \left(a b^2 + a^2 c\right) d^6 + a^3 d^2\right) e f^3 x^2 \\
&\quad + \left(c^3 d^{15} + 3 \, b c^2 d^{13} + 3 \left(b^2 c + a c^2\right) d^{11} + \left(b^3 + 6 \, a b c\right) d^9 + 3 \, a^2 b d^5 + 3 \left(a b^2 + a^2 c\right) d^7 + a^3 d^3\right) f^3 x
\end{aligned}
$$

input
```
integrate((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas
")
```

output
```
1/16*c^3*e^15*f^3*x^16 + c^3*d*e^14*f^3*x^15 + 3/14*(35*c^3*d^2 + b*c^2)*e
^13*f^3*x^14 + (35*c^3*d^3 + 3*b*c^2*d)*e^12*f^3*x^13 + 1/4*(455*c^3*d^4 +
 78*b*c^2*d^2 + b^2*c + a*c^2)*e^11*f^3*x^12 + 3*(91*c^3*d^5 + 26*b*c^2*d^
3 + (b^2*c + a*c^2)*d)*e^10*f^3*x^11 + 1/10*(5005*c^3*d^6 + 2145*b*c^2*d^4
 + b^3 + 6*a*b*c + 165*(b^2*c + a*c^2)*d^2)*e^9*f^3*x^10 + (715*c^3*d^7 +
429*b*c^2*d^5 + 55*(b^2*c + a*c^2)*d^3 + (b^3 + 6*a*b*c)*d)*e^8*f^3*x^9 +
3/8*(2145*c^3*d^8 + 1716*b*c^2*d^6 + 330*(b^2*c + a*c^2)*d^4 + a*b^2 + a^2
*c + 12*(b^3 + 6*a*b*c)*d^2)*e^7*f^3*x^8 + 1/7*(5005*c^3*d^9 + 5148*b*c^2*
d^7 + 1386*(b^2*c + a*c^2)*d^5 + 84*(b^3 + 6*a*b*c)*d^3 + 21*(a*b^2 + a^2*
c)*d)*e^6*f^3*x^7 + 1/2*(1001*c^3*d^10 + 1287*b*c^2*d^8 + 462*(b^2*c + a*c
^2)*d^6 + 42*(b^3 + 6*a*b*c)*d^4 + a^2*b + 21*(a*b^2 + a^2*c)*d^2)*e^5*f^3
*x^6 + 3/5*(455*c^3*d^11 + 715*b*c^2*d^9 + 330*(b^2*c + a*c^2)*d^7 + 42*(b
^3 + 6*a*b*c)*d^5 + 5*a^2*b*d + 35*(a*b^2 + a^2*c)*d^3)*e^4*f^3*x^5 + 1/4*
(455*c^3*d^12 + 858*b*c^2*d^10 + 495*(b^2*c + a*c^2)*d^8 + 84*(b^3 + 6*a*b
*c)*d^6 + 30*a^2*b*d^2 + 105*(a*b^2 + a^2*c)*d^4 + a^3)*e^3*f^3*x^4 + (35*
c^3*d^13 + 78*b*c^2*d^11 + 55*(b^2*c + a*c^2)*d^9 + 12*(b^3 + 6*a*b*c)*d^7
 + 10*a^2*b*d^3 + 21*(a*b^2 + a^2*c)*d^5 + a^3*d)*e^2*f^3*x^3 + 3/2*(5*c^3
*d^14 + 13*b*c^2*d^12 + 11*(b^2*c + a*c^2)*d^10 + 3*(b^3 + 6*a*b*c)*d^8 +
5*a^2*b*d^4 + 7*(a*b^2 + a^2*c)*d^6 + a^3*d^2)*e*f^3*x^2 + (c^3*d^15 + 3*b
*c^2*d^13 + 3*(b^2*c + a*c^2)*d^11 + (b^3 + 6*a*b*c)*d^9 + 3*a^2*b*d^5 ...
```

3.612.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1654 vs. $2(141) = 282$.

Time = 0.15 (sec) , antiderivative size = 1654, normalized size of antiderivative = 10.40

$$\int (df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate((e*f*x+d*f)**3*(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

output
```
c**3*d*e**14*f**3*x**15 + c**3*e**15*f**3*x**16/16 + x**14*(3*b*c**2*e**13
*f**3/14 + 15*c**3*d**2*e**13*f**3/2) + x**13*(3*b*c**2*d*e**12*f**3 + 35*
c**3*d**3*e**12*f**3) + x**12*(a*c**2*e**11*f**3/4 + b**2*c*e**11*f**3/4 +
 39*b*c**2*d**2*e**11*f**3/2 + 455*c**3*d**4*e**11*f**3/4) + x**11*(3*a*c*
*2*d*e**10*f**3 + 3*b**2*c*d*e**10*f**3 + 78*b*c**2*d**3*e**10*f**3 + 273*
c**3*d**5*e**10*f**3) + x**10*(3*a*b*c*e**9*f**3/5 + 33*a*c**2*d**2*e**9*f
**3/2 + b**3*e**9*f**3/10 + 33*b**2*c*d**2*e**9*f**3/2 + 429*b*c**2*d**4*e
**9*f**3/2 + 1001*c**3*d**6*e**9*f**3/2) + x**9*(6*a*b*c*d*e**8*f**3 + 55*
a*c**2*d**3*e**8*f**3 + b**3*d*e**8*f**3 + 55*b**2*c*d**3*e**8*f**3 + 429*
b*c**2*d**5*e**8*f**3 + 715*c**3*d**7*e**8*f**3) + x**8*(3*a**2*c*e**7*f**
3/8 + 3*a*b**2*e**7*f**3/8 + 27*a*b*c*d**2*e**7*f**3 + 495*a*c**2*d**4*e**
7*f**3/4 + 9*b**3*d**2*e**7*f**3/2 + 495*b**2*c*d**4*e**7*f**3/4 + 1287*b*
c**2*d**6*e**7*f**3/2 + 6435*c**3*d**8*e**7*f**3/8) + x**7*(3*a**2*c*d*e**
6*f**3 + 3*a*b**2*d*e**6*f**3 + 72*a*b*c*d**3*e**6*f**3 + 198*a*c**2*d**5*
e**6*f**3 + 12*b**3*d**3*e**6*f**3 + 198*b**2*c*d**5*e**6*f**3 + 5148*b*c*
*2*d**7*e**6*f**3/7 + 715*c**3*d**9*e**6*f**3) + x**6*(a**2*b*e**5*f**3/2
+ 21*a**2*c*d**2*e**5*f**3/2 + 21*a*b**2*d**2*e**5*f**3/2 + 126*a*b*c*d**4
*e**5*f**3 + 231*a*c**2*d**6*e**5*f**3 + 21*b**3*d**4*e**5*f**3 + 231*b**2
*c*d**6*e**5*f**3 + 1287*b*c**2*d**8*e**5*f**3/2 + 1001*c**3*d**10*e**5*f*
*3/2) + x**5*(3*a**2*b*d*e**4*f**3 + 21*a**2*c*d**3*e**4*f**3 + 21*a*b*...
```

3.612.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 920 vs. $2(145) = 290$.

Time = 0.22 (sec) , antiderivative size = 920, normalized size of antiderivative = 5.79

$$\int (df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^3\,dx$$
$$=\frac{1}{16}c^3e^{15}f^3x^{16}+c^3de^{14}f^3x^{15}+\frac{3}{14}\left(35\,c^3d^2+bc^2\right)e^{13}f^3x^{14}$$
$$+\left(35\,c^3d^3+3\,bc^2d\right)e^{12}f^3x^{13}+\frac{1}{4}\left(455\,c^3d^4+78\,bc^2d^2+b^2c+ac^2\right)e^{11}f^3x^{12}$$
$$+3\left(91\,c^3d^5+26\,bc^2d^3+\left(b^2c+ac^2\right)d\right)e^{10}f^3x^{11}$$
$$+\frac{1}{10}\left(5005\,c^3d^6+2145\,bc^2d^4+b^3+6\,abc+165\left(b^2c+ac^2\right)d^2\right)e^9f^3x^{10}$$
$$+\left(715\,c^3d^7+429\,bc^2d^5+55\left(b^2c+ac^2\right)d^3+\left(b^3+6\,abc\right)d\right)e^8f^3x^9$$
$$+\frac{3}{8}\left(2145\,c^3d^8+1716\,bc^2d^6+330\left(b^2c+ac^2\right)d^4+ab^2+a^2c+12\left(b^3+6\,abc\right)d^2\right)e^7f^3x^8$$
$$+\frac{1}{7}\left(5005\,c^3d^9+5148\,bc^2d^7+1386\left(b^2c+ac^2\right)d^5+84\left(b^3+6\,abc\right)d^3+21\left(ab^2+a^2c\right)d\right)e^6f^3x^7$$
$$+\frac{1}{2}\left(1001\,c^3d^{10}+1287\,bc^2d^8+462\left(b^2c+ac^2\right)d^6+42\left(b^3+6\,abc\right)d^4+a^2b+21\left(ab^2+a^2c\right)d^2\right)e^5f^3x^6$$
$$+\frac{3}{5}\left(455\,c^3d^{11}+715\,bc^2d^9+330\left(b^2c+ac^2\right)d^7+42\left(b^3+6\,abc\right)d^5+5\,a^2bd+35\left(ab^2+a^2c\right)d^3\right)e^4f^3x^5$$
$$+\frac{1}{4}\left(455\,c^3d^{12}+858\,bc^2d^{10}+495\left(b^2c+ac^2\right)d^8+84\left(b^3+6\,abc\right)d^6+30\,a^2bd^2+105\left(ab^2+a^2c\right)d^4+a^3\right)$$
$$+\left(35\,c^3d^{13}+78\,bc^2d^{11}+55\left(b^2c+ac^2\right)d^9+12\left(b^3+6\,abc\right)d^7+10\,a^2bd^3+21\left(ab^2+a^2c\right)d^5+a^3d\right)e^2f^3x$$
$$+\frac{3}{2}\left(5\,c^3d^{14}+13\,bc^2d^{12}+11\left(b^2c+ac^2\right)d^{10}+3\left(b^3+6\,abc\right)d^8+5\,a^2bd^4+7\left(ab^2+a^2c\right)d^6+a^3d^2\right)ef^3x^2$$
$$+\left(c^3d^{15}+3\,bc^2d^{13}+3\left(b^2c+ac^2\right)d^{11}+\left(b^3+6\,abc\right)d^9+3\,a^2bd^5+3\left(ab^2+a^2c\right)d^7+a^3d^3\right)f^3x$$

input
```
integrate((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima
")
```

3.612. $\int (df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^3\,dx$

output

```
1/16*c^3*e^15*f^3*x^16 + c^3*d*e^14*f^3*x^15 + 3/14*(35*c^3*d^2 + b*c^2)*e
^13*f^3*x^14 + (35*c^3*d^3 + 3*b*c^2*d)*e^12*f^3*x^13 + 1/4*(455*c^3*d^4 +
 78*b*c^2*d^2 + b^2*c + a*c^2)*e^11*f^3*x^12 + 3*(91*c^3*d^5 + 26*b*c^2*d^
3 + (b^2*c + a*c^2)*d)*e^10*f^3*x^11 + 1/10*(5005*c^3*d^6 + 2145*b*c^2*d^4
 + b^3 + 6*a*b*c + 165*(b^2*c + a*c^2)*d^2)*e^9*f^3*x^10 + (715*c^3*d^7 +
429*b*c^2*d^5 + 55*(b^2*c + a*c^2)*d^3 + (b^3 + 6*a*b*c)*d)*e^8*f^3*x^9 +
3/8*(2145*c^3*d^8 + 1716*b*c^2*d^6 + 330*(b^2*c + a*c^2)*d^4 + a*b^2 + a^2
*c + 12*(b^3 + 6*a*b*c)*d^2)*e^7*f^3*x^8 + 1/7*(5005*c^3*d^9 + 5148*b*c^2*
d^7 + 1386*(b^2*c + a*c^2)*d^5 + 84*(b^3 + 6*a*b*c)*d^3 + 21*(a*b^2 + a^2*
c)*d)*e^6*f^3*x^7 + 1/2*(1001*c^3*d^10 + 1287*b*c^2*d^8 + 462*(b^2*c + a*c
^2)*d^6 + 42*(b^3 + 6*a*b*c)*d^4 + a^2*b + 21*(a*b^2 + a^2*c)*d^2)*e^5*f^3
*x^6 + 3/5*(455*c^3*d^11 + 715*b*c^2*d^9 + 330*(b^2*c + a*c^2)*d^7 + 42*(b
^3 + 6*a*b*c)*d^5 + 5*a^2*b*d + 35*(a*b^2 + a^2*c)*d^3)*e^4*f^3*x^5 + 1/4*
(455*c^3*d^12 + 858*b*c^2*d^10 + 495*(b^2*c + a*c^2)*d^8 + 84*(b^3 + 6*a*b
*c)*d^6 + 30*a^2*b*d^2 + 105*(a*b^2 + a^2*c)*d^4 + a^3)*e^3*f^3*x^4 + (35*
c^3*d^13 + 78*b*c^2*d^11 + 55*(b^2*c + a*c^2)*d^9 + 12*(b^3 + 6*a*b*c)*d^7
 + 10*a^2*b*d^3 + 21*(a*b^2 + a^2*c)*d^5 + a^3*d)*e^2*f^3*x^3 + 3/2*(5*c^3
*d^14 + 13*b*c^2*d^12 + 11*(b^2*c + a*c^2)*d^10 + 3*(b^3 + 6*a*b*c)*d^8 +
5*a^2*b*d^4 + 7*(a*b^2 + a^2*c)*d^6 + a^3*d^2)*e*f^3*x^2 + (c^3*d^15 + 3*b
*c^2*d^13 + 3*(b^2*c + a*c^2)*d^11 + (b^3 + 6*a*b*c)*d^9 + 3*a^2*b*d^5 ...
```

3.612.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1330 vs. $2(145) = 290$.

Time = 0.31 (sec) , antiderivative size = 1330, normalized size of antiderivative = 8.36

$$\int (df + efx)^3 \left(a + b(d+ex)^2 + c(d+ex)^4\right)^3 \, dx = \text{Too large to display}$$

input

```
integrate((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

output
```
1/2*(e*f*x^2 + 2*d*f*x)*c^3*d^14*f^2 + 3/2*(e*f*x^2 + 2*d*f*x)*b*c^2*d^12*
f^2 + 3/2*(e*f*x^2 + 2*d*f*x)*b^2*c*d^10*f^2 + 3/2*(e*f*x^2 + 2*d*f*x)*a*c
^2*d^10*f^2 + 1/2*(e*f*x^2 + 2*d*f*x)*b^3*d^8*f^2 + 3*(e*f*x^2 + 2*d*f*x)*
a*b*c*d^8*f^2 + 3/2*(e*f*x^2 + 2*d*f*x)*a*b^2*d^6*f^2 + 3/2*(e*f*x^2 + 2*d
*f*x)*a^2*c*d^6*f^2 + 3/2*(e*f*x^2 + 2*d*f*x)*a^2*b*d^4*f^2 + 1/2*(e*f*x^2
 + 2*d*f*x)*a^3*d^2*f^2 + 1/560*(980*(e*f*x^2 + 2*d*f*x)^2*c^3*d^12*e*f^6
+ 1960*(e*f*x^2 + 2*d*f*x)^3*c^3*d^10*e^2*f^5 + 2450*(e*f*x^2 + 2*d*f*x)^4
*c^3*d^8*e^3*f^4 + 2520*(e*f*x^2 + 2*d*f*x)^2*b*c^2*d^10*e*f^6 + 1960*(e*f
*x^2 + 2*d*f*x)^5*c^3*d^6*e^4*f^3 + 4200*(e*f*x^2 + 2*d*f*x)^3*b*c^2*d^8*e
^2*f^5 + 980*(e*f*x^2 + 2*d*f*x)^6*c^3*d^4*e^5*f^2 + 4200*(e*f*x^2 + 2*d*f
*x)^4*b*c^2*d^6*e^3*f^4 + 2100*(e*f*x^2 + 2*d*f*x)^2*b^2*c*d^8*e*f^6 + 210
0*(e*f*x^2 + 2*d*f*x)^2*a*c^2*d^8*e*f^6 + 280*(e*f*x^2 + 2*d*f*x)^7*c^3*d^
2*e^6*f + 2520*(e*f*x^2 + 2*d*f*x)^5*b*c^2*d^4*e^4*f^3 + 2800*(e*f*x^2 + 2
*d*f*x)^3*b^2*c*d^6*e^2*f^5 + 2800*(e*f*x^2 + 2*d*f*x)^3*a*c^2*d^6*e^2*f^5
 + 35*(e*f*x^2 + 2*d*f*x)^8*c^3*e^7 + 840*(e*f*x^2 + 2*d*f*x)^6*b*c^2*d^2*
e^5*f^2 + 2100*(e*f*x^2 + 2*d*f*x)^4*b^2*c*d^4*e^3*f^4 + 2100*(e*f*x^2 + 2
*d*f*x)^4*a*c^2*d^4*e^3*f^4 + 560*(e*f*x^2 + 2*d*f*x)^2*b^3*d^6*e*f^6 + 33
60*(e*f*x^2 + 2*d*f*x)^2*a*b*c*d^6*e*f^6 + 120*(e*f*x^2 + 2*d*f*x)^7*b*c^2
*e^6*f + 840*(e*f*x^2 + 2*d*f*x)^5*b^2*c*d^2*e^4*f^3 + 840*(e*f*x^2 + 2*d*
f*x)^5*a*c^2*d^2*e^4*f^3 + 560*(e*f*x^2 + 2*d*f*x)^3*b^3*d^4*e^2*f^5 + ...
```

3.612. $\int (df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^3 \, dx$

3.612.9 Mupad [B] (verification not implemented)

Time = 8.91 (sec) , antiderivative size = 825, normalized size of antiderivative = 5.19

$$\int (df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^3\,dx$$

$$= \frac{3\,e^7\,f^3\,x^8\,(a^2\,c+a\,b^2+72\,a\,b\,c\,d^2+330\,a\,c^2\,d^4+12\,b^3\,d^2+330\,b^2\,c\,d^4+1716\,b\,c^2\,d^6+2145\,c^3\,d^8)}{8}$$
$$+\frac{e^5\,f^3\,x^6\,(a^2\,b+21\,a^2\,c\,d^2+21\,a\,b^2\,d^2+252\,a\,b\,c\,d^4+462\,a\,c^2\,d^6+42\,b^3\,d^4+462\,b^2\,c\,d^6+1287\,b\,c^2\,d^8+}{2}$$
$$+\frac{e^9\,f^3\,x^{10}\,(b^3+165\,b^2\,c\,d^2+2145\,b\,c^2\,d^4+6\,a\,b\,c+5005\,c^3\,d^6+165\,a\,c^2\,d^2)}{10}$$
$$+\frac{c^3\,e^{15}\,f^3\,x^{16}}{16}+d^3\,f^3\,x\left(c\,d^4+b\,d^2+a\right)^3$$
$$+\frac{e^3\,f^3\,x^4\,(a^3+30\,a^2\,b\,d^2+105\,a^2\,c\,d^4+105\,a\,b^2\,d^4+504\,a\,b\,c\,d^6+495\,a\,c^2\,d^8+84\,b^3\,d^6+495\,b^2\,c\,d^8+8}{4}$$
$$+\frac{c\,e^{11}\,f^3\,x^{12}\,(b^2+78\,b\,c\,d^2+455\,c^2\,d^4+a\,c)}{4}$$
$$+\frac{d\,e^6\,f^3\,x^7\,(21\,a^2\,c+21\,a\,b^2+504\,a\,b\,c\,d^2+1386\,a\,c^2\,d^4+84\,b^3\,d^2+1386\,b^2\,c\,d^4+5148\,b\,c^2\,d^6+5005\,c^3}{7}$$
$$+\frac{3\,d\,e^4\,f^3\,x^5\,(5\,a^2\,b+35\,a^2\,c\,d^2+35\,a\,b^2\,d^2+252\,a\,b\,c\,d^4+330\,a\,c^2\,d^6+42\,b^3\,d^4+330\,b^2\,c\,d^6+715\,b\,c^2\,d}{5}$$
$$+d\,e^8\,f^3\,x^9\left(b^3+55\,b^2\,c\,d^2+429\,b\,c^2\,d^4+6\,a\,b\,c+715\,c^3\,d^6+55\,a\,c^2\,d^2\right)$$
$$+\frac{3\,c^2\,e^{13}\,f^3\,x^{14}\,(35\,c\,d^2+b)}{14}+c^3\,d\,e^{14}\,f^3\,x^{15}+d\,e^2\,f^3\,x^3\left(a^3+10\,a^2\,b\,d^2+21\,a^2\,c\,d^4\right.$$
$$\left.+21\,a\,b^2\,d^4+72\,a\,b\,c\,d^6+55\,a\,c^2\,d^8+12\,b^3\,d^6+55\,b^2\,c\,d^8+78\,b\,c^2\,d^{10}+35\,c^3\,d^{12}\right)$$
$$+\frac{3\,d^2\,e\,f^3\,x^2\,\left(c\,d^4+b\,d^2+a\right)^2\,(5\,c\,d^4+3\,b\,d^2+a)}{2}$$
$$+c^2\,d\,e^{12}\,f^3\,x^{13}\left(35\,c\,d^2+3\,b\right)+3\,c\,d\,e^{10}\,f^3\,x^{11}\left(b^2+26\,b\,c\,d^2+91\,c^2\,d^4+a\,c\right)$$

input
```
int((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x)
```

output
```
(3*e^7*f^3*x^8*(a*b^2 + a^2*c + 12*b^3*d^2 + 2145*c^3*d^8 + 330*a*c^2*d^4
+ 330*b^2*c*d^4 + 1716*b*c^2*d^6 + 72*a*b*c*d^2))/8 + (e^5*f^3*x^6*(a^2*b
+ 42*b^3*d^4 + 1001*c^3*d^10 + 21*a*b^2*d^2 + 21*a^2*c*d^2 + 462*a*c^2*d^6
 + 462*b^2*c*d^6 + 1287*b*c^2*d^8 + 252*a*b*c*d^4))/2 + (e^9*f^3*x^10*(b^3
 + 5005*c^3*d^6 + 165*a*c^2*d^2 + 165*b^2*c*d^2 + 2145*b*c^2*d^4 + 6*a*b*c
))/10 + (c^3*e^15*f^3*x^16)/16 + d^3*f^3*x*(a + b*d^2 + c*d^4)^3 + (e^3*f^
3*x^4*(a^3 + 84*b^3*d^6 + 455*c^3*d^12 + 30*a^2*b*d^2 + 105*a*b^2*d^4 + 10
5*a^2*c*d^4 + 495*a*c^2*d^8 + 495*b^2*c*d^8 + 858*b*c^2*d^10 + 504*a*b*c*d
^6))/4 + (c*e^11*f^3*x^12*(a*c + b^2 + 455*c^2*d^4 + 78*b*c*d^2))/4 + (d*e
^6*f^3*x^7*(21*a*b^2 + 21*a^2*c + 84*b^3*d^2 + 5005*c^3*d^8 + 1386*a*c^2*d
^4 + 1386*b^2*c*d^4 + 5148*b*c^2*d^6 + 504*a*b*c*d^2))/7 + (3*d*e^4*f^3*x^
5*(5*a^2*b + 42*b^3*d^4 + 455*c^3*d^10 + 35*a*b^2*d^2 + 35*a^2*c*d^2 + 330
*a*c^2*d^6 + 330*b^2*c*d^6 + 715*b*c^2*d^8 + 252*a*b*c*d^4))/5 + d*e^8*f^3
*x^9*(b^3 + 715*c^3*d^6 + 55*a*c^2*d^2 + 55*b^2*c*d^2 + 429*b*c^2*d^4 + 6*
a*b*c) + (3*c^2*e^13*f^3*x^14*(b + 35*c*d^2))/14 + c^3*d*e^14*f^3*x^15 + d
*e^2*f^3*x^3*(a^3 + 12*b^3*d^6 + 35*c^3*d^12 + 10*a^2*b*d^2 + 21*a*b^2*d^4
 + 21*a^2*c*d^4 + 55*a*c^2*d^8 + 55*b^2*c*d^8 + 78*b*c^2*d^10 + 72*a*b*c*d
^6) + (3*d^2*e*f^3*x^2*(a + b*d^2 + c*d^4)^2*(a + 3*b*d^2 + 5*c*d^4))/2 +
c^2*d*e^12*f^3*x^13*(3*b + 35*c*d^2) + 3*c*d*e^10*f^3*x^11*(a*c + b^2 + 91
*c^2*d^4 + 26*b*c*d^2)
```

3.612. $\int (df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^3 \, dx$

3.613 $\int \frac{(d+ex)^4}{a+b(d+ex)^2+c(d+ex)^4} dx$

3.613.1 Optimal result

Integrand size = 30, antiderivative size = 193

$$\int \frac{(d+ex)^4}{a+b(d+ex)^2+c(d+ex)^4} dx = \frac{x}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}e} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}e}$$

output
```
x/c-1/2*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b+(2
*a*c-b^2)/(-4*a*c+b^2)^(1/2))/c^(3/2)/e*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/
2)-1/2*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-2
*a*c+b^2)/(-4*a*c+b^2)^(1/2))/c^(3/2)/e*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/
2)
```

3.613.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.13

$$\int \frac{(d+ex)^4}{a+b(d+ex)^2+c(d+ex)^4} dx = \frac{2\sqrt{c}(d+ex) - \frac{\sqrt{2}\left(-b^2+2ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}\left(b^2-2ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}}{2c^{3/2}e}$$

input
```
Integrate[(d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

output
```
(2*Sqrt[c]*(d + e*x) - (Sqrt[2]*(-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTa
n[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*
a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*(b^2 - 2*a*c + b*Sqrt[b^2 - 4
*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(S
qrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*c^(3/2)*e)
```

3.613.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1462, 1442, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^4}{a+b(d+ex)^2+c(d+ex)^4}\,dx$$

$$\Big\downarrow \ 1462$$

$$\frac{\int \frac{(d+ex)^4}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)}{e}$$

$$\Big\downarrow \ 1442$$

$$\frac{\frac{d+ex}{c}-\frac{\int \frac{b(d+ex)^2+a}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)}{c}}{e}$$

$$\Big\downarrow \ 1480$$

$$\frac{\frac{d+ex}{c}-\frac{\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{c(d+ex)^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}d(d+ex)+\frac{1}{2}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\int \frac{1}{c(d+ex)^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}d(d+ex)}{c}}{e}$$

$$\Big\downarrow \ 218$$

$$\frac{\frac{d+ex}{c}-\frac{\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{c}}{e}$$

input
```
Int[(d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

output
```
((d + e*x)/c - (((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqr
t[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sq
rt[b^2 - 4*a*c]]) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]
*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b
+ Sqrt[b^2 - 4*a*c]]))/c)/e
```

3.613.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1442
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
x] - Simp[d^4/(c*(m + 4*p + 1))   Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

3.613.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.60 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.82

method	result
default	$\frac{x}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\,e^{4}\_Z^{4}+4cd\,e^{3}\_Z^{3}+\left(6c\,d^{2}e^{2}+b\,e^{2}\right)\_Z^{2}+\left(4d^{3}ec+2bde\right)\_Z+d^{4}c+b\,d^{2}+a\right)}\frac{\left(-\_R^{2}b\,e^{2}-2\_Rbde-b\,d^{2}-a\right)\ln\left(x-\right.}{2e^{3}c\_R^{3}+6cd\,e^{2}\_R^{2}+6c\,d^{2}e\_R+2d^{3}c+}}{2ce}$
risch	$\frac{x}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\,e^{4}\_Z^{4}+4cd\,e^{3}\_Z^{3}+\left(6c\,d^{2}e^{2}+b\,e^{2}\right)\_Z^{2}+\left(4d^{3}ec+2bde\right)\_Z+d^{4}c+b\,d^{2}+a\right)}\frac{\left(-\_R^{2}b\,e^{2}-2\_Rbde-b\,d^{2}-a\right)\ln\left(x-\right.}{2e^{3}c\_R^{3}+6cd\,e^{2}\_R^{2}+6c\,d^{2}e\_R+2d^{3}c+}}{2ce}$

input
```
int((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x,method=_RETURNVERBOSE)
```

output
```
x/c+1/2/c/e*sum((-_R^2*b*e^2-2*_R*b*d*e-b*d^2-a)/(2*_R^3*c*e^3+6*_R^2*c*d*
e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e
^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a))
```

3.613.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1231 vs. 2(157) = 314.

Time = 0.28 (sec) , antiderivative size = 1231, normalized size of antiderivative = 6.38

$$\int \frac{(d+ex)^4}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

output
```
1/2*(sqrt(1/2)*c*sqrt(-((b^2*c^3 - 4*a*c^4)*e^2*sqrt((b^4 - 2*a*b^2*c + a^
2*c^2)/((b^2*c^6 - 4*a*c^7)*e^4)) + b^3 - 3*a*b*c)/((b^2*c^3 - 4*a*c^4)*e^
2))*log(-2*(a*b^2 - a^2*c)*e*x - 2*(a*b^2 - a^2*c)*d + sqrt(1/2)*((b^3*c^3
 - 4*a*b*c^4)*e^3*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((b^2*c^6 - 4*a*c^7)*e^
4)) - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e)*sqrt(-((b^2*c^3 - 4*a*c^4)*e^2*sqrt
((b^4 - 2*a*b^2*c + a^2*c^2)/((b^2*c^6 - 4*a*c^7)*e^4)) + b^3 - 3*a*b*c)/(
(b^2*c^3 - 4*a*c^4)*e^2))) - sqrt(1/2)*c*sqrt(-((b^2*c^3 - 4*a*c^4)*e^2*sq
rt((b^4 - 2*a*b^2*c + a^2*c^2)/((b^2*c^6 - 4*a*c^7)*e^4)) + b^3 - 3*a*b*c)
/((b^2*c^3 - 4*a*c^4)*e^2))*log(-2*(a*b^2 - a^2*c)*e*x - 2*(a*b^2 - a^2*c)
*d - sqrt(1/2)*((b^3*c^3 - 4*a*b*c^4)*e^3*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)
/((b^2*c^6 - 4*a*c^7)*e^4)) - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e)*sqrt(-((b^2
*c^3 - 4*a*c^4)*e^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((b^2*c^6 - 4*a*c^7)*
e^4)) + b^3 - 3*a*b*c)/((b^2*c^3 - 4*a*c^4)*e^2))) - sqrt(1/2)*c*sqrt(((b^
2*c^3 - 4*a*c^4)*e^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((b^2*c^6 - 4*a*c^7)
*e^4)) - b^3 + 3*a*b*c)/((b^2*c^3 - 4*a*c^4)*e^2))*log(-2*(a*b^2 - a^2*c)*
e*x - 2*(a*b^2 - a^2*c)*d + sqrt(1/2)*((b^3*c^3 - 4*a*b*c^4)*e^3*sqrt((b^4
 - 2*a*b^2*c + a^2*c^2)/((b^2*c^6 - 4*a*c^7)*e^4)) + (b^4 - 5*a*b^2*c + 4*
a^2*c^2)*e)*sqrt(((b^2*c^3 - 4*a*c^4)*e^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)
/((b^2*c^6 - 4*a*c^7)*e^4)) - b^3 + 3*a*b*c)/((b^2*c^3 - 4*a*c^4)*e^2))) +
 sqrt(1/2)*c*sqrt(((b^2*c^3 - 4*a*c^4)*e^2*sqrt((b^4 - 2*a*b^2*c + a^2*...
```

3.613.6 Sympy [A] (verification not implemented)

Time = 1.73 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.92

$$\int \frac{(d+ex)^4}{a+b(d+ex)^2+c(d+ex)^4}\,dx$$
$$= \operatorname{RootSum}\left(t^4\cdot\left(256a^2c^5e^4-128ab^2c^4e^4+16b^4c^3e^4\right)+t^2\cdot\left(48a^2bc^2e^2-28ab^3ce^2+4b^5e^2\right)+a^3,\left(t\mapsto t\,\mathrm{lo}\right.\right.$$
$$+\frac{x}{c}$$

input
```
integrate((e*x+d)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

output
```
RootSum(_t**4*(256*a**2*c**5*e**4 - 128*a*b**2*c**4*e**4 + 16*b**4*c**3*e*
*4) + _t**2*(48*a**2*b*c**2*e**2 - 28*a*b**3*c*e**2 + 4*b**5*e**2) + a**3,
 Lambda(_t, _t*log(x + (32*_t**3*a*b*c**4*e**3 - 8*_t**3*b**3*c**3*e**3 -
4*_t*a**2*c**2*e + 8*_t*a*b**2*c*e - 2*_t*b**4*e + a**2*c*d - a*b**2*d)/(a
**2*c*e - a*b**2*e)))) + x/c
```

3.613.7 Maxima [F]

$$\int \frac{(d+ex)^4}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \int \frac{(ex+d)^4}{(ex+d)^4c+(ex+d)^2b+a}\,dx$$

input
```
integrate((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

output
```
x/c - integrate((b*e^2*x^2 + 2*b*d*e*x + b*d^2 + a)/(c*e^4*x^4 + 4*c*d*e^3
*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a),
 x)/c
```

3.613.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1315 vs. $2(157) = 314$.

Time = 0.30 (sec) , antiderivative size = 1315, normalized size of antiderivative = 6.81

$$\int \frac{(d+ex)^4}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

output
```
x/c + 1/2*((b*e^6*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)
) + d/e)^2 - 2*b*d*e^5*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c
*e^4)) + d/e) + b*d^2*e^4 + a*e^4)*log(x + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b
^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b
^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^3 - 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 + s
qrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2
 + b*e^2)*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)
) - (b*e^6*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e
)^2 + 2*b*d*e^5*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4))
- d/e) + b*d^2*e^4 + a*e^4)*log(x - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*
a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*
a*c)*e^2)/(c*e^4)) - d/e)^3 + 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2
 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2 + 2*c*d^3*e + b*d*e + (6*c*d^2*e^2 + b*e^
2)*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)) + (b*
e^6*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 - 2
*b*d*e^5*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)
+ b*d^2*e^4 + a*e^4)*log(x + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^
2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^
2)/(c*e^4)) + d/e)^3 - 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*
c)*e^2)/(c*e^4)) + d/e)^2 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(...
```

3.613.9 Mupad [B] (verification not implemented)

Time = 9.18 (sec) , antiderivative size = 3988, normalized size of antiderivative = 20.66

$$\int \frac{(d+ex)^4}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \text{Too large to display}$$

input
```
int((d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x)
```

output
```
atan(((-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a
*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^
4*e^2)))^(1/2)*(((16*a^2*c^3*e^12 - 4*a*b^2*c^2*e^12)/c + ((8*b^3*c^3*d*e^
13 - 32*a*b*c^4*d*e^13)/c + (2*x*(4*b^3*c^3*e^14 - 16*a*b*c^4*e^14))/c)*(-
(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*
a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))
^(1/2))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c -
 a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*
c^4*e^2)))^(1/2) + (2*b^4*d*e^11 + 4*a^2*c^2*d*e^11 - 8*a*b^2*c*d*e^11)/c
+ (2*x*(b^4*e^12 + 2*a^2*c^2*e^12 - 4*a*b^2*c*e^12))/c)*1i + (-(b^5 + b^2*
(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^
3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2)*((2*
b^4*d*e^11 + 4*a^2*c^2*d*e^11 - 8*a*b^2*c*d*e^11)/c - ((16*a^2*c^3*e^12 -
4*a*b^2*c^2*e^12)/c - ((8*b^3*c^3*d*e^13 - 32*a*b*c^4*d*e^13)/c + (2*x*(4*
b^3*c^3*e^14 - 16*a*b*c^4*e^14))/c)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2)
+ 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*
e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2))*(-(b^5 + b^2*(-(4*a*c - b^2)
^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(1
6*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2) + (2*x*(b^4*e^12 +
2*a^2*c^2*e^12 - 4*a*b^2*c*e^12))/c)*1i)/((-(b^5 + b^2*(-(4*a*c - b^2)^...
```

3.613. $\int \frac{(d+ex)^4}{a+b(d+ex)^2+c(d+ex)^4}\,dx$

3.614 $\int \frac{(d+ex)^3}{a+b(d+ex)^2+c(d+ex)^4}\,dx$

3.614.1 Optimal result

Integrand size = 30, antiderivative size = 81

$$\int \frac{(d+ex)^3}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \frac{b\,\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}e}+\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4ce}$$

output
```
1/4*ln(a+b*(e*x+d)^2+c*(e*x+d)^4)/c/e+1/2*b*arctanh((b+2*c*(e*x+d)^2)/(-4*
a*c+b^2)^(1/2))/c/e/(-4*a*c+b^2)^(1/2)
```

3.614.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.95

$$\int \frac{(d+ex)^3}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \frac{-\frac{2b\arctan\left(\frac{b+2c(d+ex)^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4ce}$$

input
```
Integrate[(d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

output
```
((-2*b*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c
] + Log[a + b*(d + e*x)^2 + c*(d + e*x)^4])/(4*c*e)
```

3.614.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1462, 1434, 1142, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3}{a+b(d+ex)^2+c(d+ex)^4}\,dx$$

$$\Big\downarrow \text{1462}$$

$$\frac{\int \frac{(d+ex)^3}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)}{e}$$

$$\Big\downarrow \text{1434}$$

$$\frac{\int \frac{(d+ex)^2}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)^2}{2e}$$

$$\Big\downarrow \text{1142}$$

$$\frac{\frac{\int \frac{2c(d+ex)^2+b}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)^2}{2c}-\frac{b\int \frac{1}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)^2}{2c}}{2e}$$

$$\Big\downarrow \text{1083}$$

$$\frac{\frac{b\int \frac{1}{-(d+ex)^4+b^2-4ac}d\left(2c(d+ex)^2+b\right)}{c}+\frac{\int \frac{2c(d+ex)^2+b}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)^2}{2c}}{2e}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\int \frac{2c(d+ex)^2+b}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)^2}{2c}+\frac{b\,\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}}{2e}$$

$$\Big\downarrow \text{1103}$$

$$\frac{\frac{b\,\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}+\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{2c}}{2e}$$

input
```
Int[(d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

output
```
((b*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 - 4*a*c]
) + Log[a + b*(d + e*x)^2 + c*(d + e*x)^4]/(2*c))/(2*e)
```

3.614.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

3.614. $\int \frac{(d+ex)^3}{a+b(d+ex)^2+c(d+ex)^4}\,dx$

3.614.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.64 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.86

method	result
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(c\,e^4\_Z^4+4cd\,e^3\_Z^3+\left(6c\,d^2e^2+b\,e^2\right)\_Z^2+\left(4d^3ec+2bde\right)\_Z+d^4c+b\,d^2+a\right)}\frac{\left(\_R^3e^3+3\_R^2d\,e^2+3\_R\,d^2e+d^3\right)\ln\left(x-\_\right.}{2e^3c\_R^3+6cd\,e^2\_R^2+6c\,d^2e\_R+2d^3c+be\_}}{2e}$
risch	Expression too large to display

input
```
int((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x,method=_RETURNVERBOSE)
```

output
```
1/2/e*sum((_R^3*e^3+3*_R^2*d*e^2+3*_R*d^2*e+d^3)/(2*_R^3*c*e^3+6*_R^2*c*d*
e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e
^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a))
```

3.614.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 152 vs. 2(73) = 146.

Time = 0.29 (sec) , antiderivative size = 434, normalized size of antiderivative = 5.36

$$\int \frac{(d+ex)^3}{a+b(d+ex)^2+c(d+ex)^4}\,dx$$
$$=\left[\frac{\sqrt{b^2-4\,ac}b\log\left(\frac{2\,c^2e^4x^4+8\,c^2de^3x^3+2\,c^2d^4+2\,(6\,c^2d^2+bc)e^2x^2+2\,bcd^2+4\,(2\,c^2d^3+bcd)ex+b^2-2\,ac+(2\,ce^2x^2+4\,cdex+2\,cd^2+b)\sqrt{b^2-}}{ce^4x^4+4\,cde^3x^3+cd^4+(6\,cd^2+b)e^2x^2+bd^2+2\,(2\,cd^3+bd)ex+a}\right.}{4\,(b^2c-4\,ac}\right.$$

input
```
integrate((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4
 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b
^2 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c
*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*
d^3 + b*d)*e*x + a)) + (b^2 - 4*a*c)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4
 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a))/((b^2*c - 4
*a*c^2)*e), 1/4*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x +
 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^2 - 4*a*c)*log(c*e^4*
x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 +
 b*d)*e*x + a))/((b^2*c - 4*a*c^2)*e)]
```

3.614.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(68) = 136$.

Time = 0.92 (sec) , antiderivative size = 280, normalized size of antiderivative = 3.46

$$\int \frac{(d+ex)^3}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \left(-\frac{b\sqrt{-4ac+b^2}}{4ce\,(4ac-b^2)} + \frac{1}{4ce}\right)\log\left(\frac{2dx}{e}+x^2+\frac{-8ace\left(-\frac{b\sqrt{-4ac+b^2}}{4ce(4ac-b^2)}+\frac{1}{4ce}\right)+2a+2b^2e\left(-\frac{b\sqrt{-4ac+b^2}}{4ce(4ac-b^2)}+\frac{1}{4ce}\right)+bd^2}{be^2}\right) + \left(\frac{b\sqrt{-4ac+b^2}}{4ce\,(4ac-b^2)} + \frac{1}{4ce}\right)\log\left(\frac{2dx}{e}+x^2+\frac{-8ace\left(\frac{b\sqrt{-4ac+b^2}}{4ce(4ac-b^2)}+\frac{1}{4ce}\right)+2a+2b^2e\left(\frac{b\sqrt{-4ac+b^2}}{4ce(4ac-b^2)}+\frac{1}{4ce}\right)+bd^2}{be^2}\right)$$

input
```
integrate((e*x+d)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

output
```
(-b*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*c - b**2)) + 1/(4*c*e))*log(2*d*x/e +
x**2 + (-8*a*c*e*(-b*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*c - b**2)) + 1/(4*c*e
)) + 2*a + 2*b**2*e*(-b*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*c - b**2)) + 1/(4*
c*e)) + b*d**2)/(b*e**2)) + (b*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*c - b**2))
+ 1/(4*c*e))*log(2*d*x/e + x**2 + (-8*a*c*e*(b*sqrt(-4*a*c + b**2)/(4*c*e*
(4*a*c - b**2)) + 1/(4*c*e)) + 2*a + 2*b**2*e*(b*sqrt(-4*a*c + b**2)/(4*c*
e*(4*a*c - b**2)) + 1/(4*c*e)) + b*d**2)/(b*e**2))
```

3.614.7 Maxima [F]

$$\int \frac{(d+ex)^3}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \int \frac{(ex+d)^3}{(ex+d)^4c+(ex+d)^2b+a}\,dx$$

input
```
integrate((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^3/((e*x + d)^4*c + (e*x + d)^2*b + a), x)
```

3.614.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.56

$$\int \frac{(d+ex)^3}{a+b(d+ex)^2+c(d+ex)^4}\,dx = -\frac{b\arctan\left(\frac{2\,cd^2+2\,(ex^2+2\,dx)ce+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}ce} + \frac{\log\left(cd^4+2\,(ex^2+2\,dx)cd^2e+(ex^2+2\,dx)^2ce^2+bd^2+(ex^2+2\,dx)be+a\right)}{4\,ce}$$

input
```
integrate((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

output
```
-1/2*b*arctan((2*c*d^2 + 2*(e*x^2 + 2*d*x)*c*e + b)/sqrt(-b^2 + 4*a*c))/(s
qrt(-b^2 + 4*a*c)*c*e) + 1/4*log(c*d^4 + 2*(e*x^2 + 2*d*x)*c*d^2*e + (e*x^
2 + 2*d*x)^2*c*e^2 + b*d^2 + (e*x^2 + 2*d*x)*b*e + a)/(c*e)
```

3.614.9 Mupad [B] (verification not implemented)

Time = 8.88 (sec) , antiderivative size = 278, normalized size of antiderivative = 3.43

$$\int \frac{(d+ex)^3}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \frac{4\,a\,c\,e\,\ln\left(c\,d^4+4\,c\,d^3\,e\,x+6\,c\,d^2\,e^2\,x^2+b\,d^2+4\,c\,d\,e^3\,x^3+2\,b\,d\,e\,x+c\,e^4\,x^4+b\,e^2\,x^2+a\right)}{16\,a\,c^2\,e^2-4\,b^2\,c\,e^2} - \frac{b^2\,e\,\ln\left(c\,d^4+4\,c\,d^3\,e\,x+6\,c\,d^2\,e^2\,x^2+b\,d^2+4\,c\,d\,e^3\,x^3+2\,b\,d\,e\,x+c\,e^4\,x^4+b\,e^2\,x^2+a\right)}{16\,a\,c^2\,e^2-4\,b^2\,c\,e^2} - \frac{b\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,d^2}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,e^2\,x^2}{\sqrt{4\,a\,c-b^2}}+\frac{4\,c\,d\,e\,x}{\sqrt{4\,a\,c-b^2}}\right)}{2\,c\,e\,\sqrt{4\,a\,c-b^2}}$$

input
```
int((d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x)
```

output
```
(4*a*c*e*log(a + b*d^2 + c*d^4 + b*e^2*x^2 + c*e^4*x^4 + 2*b*d*e*x + 6*c*d
^2*e^2*x^2 + 4*c*d^3*e*x + 4*c*d*e^3*x^3))/(16*a*c^2*e^2 - 4*b^2*c*e^2) -
(b^2*e*log(a + b*d^2 + c*d^4 + b*e^2*x^2 + c*e^4*x^4 + 2*b*d*e*x + 6*c*d^2
*e^2*x^2 + 4*c*d^3*e*x + 4*c*d*e^3*x^3))/(16*a*c^2*e^2 - 4*b^2*c*e^2) - (b
*atan(b/(4*a*c - b^2)^(1/2) + (2*c*d^2)/(4*a*c - b^2)^(1/2) + (2*c*e^2*x^2
)/(4*a*c - b^2)^(1/2) + (4*c*d*e*x)/(4*a*c - b^2)^(1/2)))/(2*c*e*(4*a*c -
b^2)^(1/2))
```

3.614. $\int \frac{(d+ex)^3}{a+b(d+ex)^2+c(d+ex)^4}\,dx$

3.615 $\int \frac{(d+ex)^2}{a+b(d+ex)^2+c(d+ex)^4} \, dx$

3.615.1 Optimal result

Integrand size = 30, antiderivative size = 164

$$\int \frac{(d+ex)^2}{a+b(d+ex)^2+c(d+ex)^4} \, dx = -\frac{\sqrt{b-\sqrt{b^2-4ac}}\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}e} + \frac{\sqrt{b+\sqrt{b^2-4ac}}\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}e}$$

output
```
-1/2*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b-(-4*a
*c+b^2)^(1/2))^(1/2)/e*2^(1/2)/c^(1/2)/(-4*a*c+b^2)^(1/2)+1/2*arctan((e*x+
d)*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-4*a*c+b^2)^(1/2))^(1
/2)/e*2^(1/2)/c^(1/2)/(-4*a*c+b^2)^(1/2)
```

3.615.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.07

$$\int \frac{(d+ex)^2}{a+b(d+ex)^2+c(d+ex)^4} \, dx = \frac{\left(-b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)+\sqrt{b-\sqrt{b^2-4ac}}\sqrt{b+\sqrt{b^2-4ac}}\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}e}$$

input
```
Integrate[(d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

output
```
((-b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt
[b^2 - 4*a*c]]] + Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*
ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*
Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e)
```

3.615.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1462, 1450, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{a+b(d+ex)^2+c(d+ex)^4}dx$$

$$\Big\downarrow \text{ 1462}$$

$$\frac{\int \frac{(d+ex)^2}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)}{e}$$

$$\Big\downarrow \text{ 1450}$$

$$\frac{\frac{1}{2}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{c(d+ex)^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}d(d+ex)+\frac{1}{2}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\int \frac{1}{c(d+ex)^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}d(d+ex)}{e}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{e}$$

input
```
Int[(d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

output
```
(((1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sq
rt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((1 + b
/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 -
 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/e
```

3.615.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1450
```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^2/2)*(b/q + 1)   Int[(d*x)^(m - 2)/(b/
2 + q/2 + c*x^2), x], x] - Simp[(d^2/2)*(b/q - 1)   Int[(d*x)^(m - 2)/(b/2
- q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] &&
 GeQ[m, 2]
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

3.615.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.59 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.85

method	result
default	$\frac{\sum_{\_R=\operatorname{RootOf}\left(c\,e^4\_Z^4+4cd\,e^3\_Z^3+\left(6c\,d^2e^2+b\,e^2\right)\_Z^2+\left(4d^3ec+2bde\right)\_Z+d^4c+b\,d^2+a\right)}\frac{\left(\_R^2e^2+2\_Rde+d^2\right)\ln\left(x-\_R\right)}{2e^3c\_R^3+6cd\,e^2\_R^2+6c\,d^2e\_R+2d^3c+be\_R}}{2e}$
risch	$\frac{\sum_{\_R=\operatorname{RootOf}\left(c\,e^4\_Z^4+4cd\,e^3\_Z^3+\left(6c\,d^2e^2+b\,e^2\right)\_Z^2+\left(4d^3ec+2bde\right)\_Z+d^4c+b\,d^2+a\right)}\frac{\left(\_R^2e^2+2\_Rde+d^2\right)\ln\left(x-\_R\right)}{2e^3c\_R^3+6cd\,e^2\_R^2+6c\,d^2e\_R+2d^3c+be\_R}}{2e}$

input
```
int((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x,method=_RETURNVERBOSE)
```

output
```
1/2/e*sum((_R^2*e^2+2*_R*d*e+d^2)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*
e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^
2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a))
```

3.615.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 703 vs. $2(129) = 258$.

Time = 0.25 (sec) , antiderivative size = 703, normalized size of antiderivative = 4.29

$$
\begin{aligned}
&\int \frac{(d+ex)^2}{a+b(d+ex)^2+c(d+ex)^4}\,dx \\
&= \frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{(b^2c-4\,ac^2)e^2\sqrt{\frac{1}{(b^2c^2-4\,ac^3)e^4}}+b}{(b^2c-4\,ac^2)e^2}}\log\left(\sqrt{\frac{1}{2}}(b^2c-4\,ac^2)e^3\sqrt{-\frac{(b^2c-4\,ac^2)e^2\sqrt{\frac{1}{(b^2c^2-4\,ac^3)e^4}}+b\cdots}{(b^2c-4\,ac^2)e^2}}\cdots\right. \\
&\qquad \left. + ex + d\right) \\
&\quad -\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{(b^2c-4\,ac^2)e^2\sqrt{\frac{1}{(b^2c^2-4\,ac^3)e^4}}+b}{(b^2c-4\,ac^2)e^2}}\log\left(-\sqrt{\frac{1}{2}}(b^2c-4\,ac^2)e^3\sqrt{-\frac{(b^2c-4\,ac^2)e^2\sqrt{\frac{1}{(b^2c^2-4\,ac^3)e^4}}\cdots}{(b^2c-4\,ac^2)e^2}}\cdots\right. \\
&\qquad \left. + ex + d\right) \\
&\quad -\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{(b^2c-4\,ac^2)e^2\sqrt{\frac{1}{(b^2c^2-4\,ac^3)e^4}}-b}{(b^2c-4\,ac^2)e^2}}\log\left(\sqrt{\frac{1}{2}}(b^2c-4\,ac^2)e^3\sqrt{\frac{(b^2c-4\,ac^2)e^2\sqrt{\frac{1}{(b^2c^2-4\,ac^3)e^4}}-b}{(b^2c-4\,ac^2)e^2}}\cdots\right. \\
&\qquad \left. + ex + d\right) \\
&\quad +\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{(b^2c-4\,ac^2)e^2\sqrt{\frac{1}{(b^2c^2-4\,ac^3)e^4}}-b}{(b^2c-4\,ac^2)e^2}}\log\left(-\sqrt{\frac{1}{2}}(b^2c-4\,ac^2)e^3\sqrt{\frac{(b^2c-4\,ac^2)e^2\sqrt{\frac{1}{(b^2c^2-4\,ac^3)e^4}}-\cdots}{(b^2c-4\,ac^2)e^2}}\cdots\right. \\
&\qquad \left. + ex + d\right)
\end{aligned}
$$

input
```
integrate((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

output
```
1/2*sqrt(1/2)*sqrt(-((b^2*c - 4*a*c^2)*e^2*sqrt(1/((b^2*c^2 - 4*a*c^3)*e^4
)) + b)/((b^2*c - 4*a*c^2)*e^2))*log(sqrt(1/2)*(b^2*c - 4*a*c^2)*e^3*sqrt(
-((b^2*c - 4*a*c^2)*e^2*sqrt(1/((b^2*c^2 - 4*a*c^3)*e^4)) + b)/((b^2*c - 4
*a*c^2)*e^2))*sqrt(1/((b^2*c^2 - 4*a*c^3)*e^4)) + e*x + d) - 1/2*sqrt(1/2)
*sqrt(-((b^2*c - 4*a*c^2)*e^2*sqrt(1/((b^2*c^2 - 4*a*c^3)*e^4)) + b)/((b^2
*c - 4*a*c^2)*e^2))*log(-sqrt(1/2)*(b^2*c - 4*a*c^2)*e^3*sqrt(-((b^2*c - 4
*a*c^2)*e^2*sqrt(1/((b^2*c^2 - 4*a*c^3)*e^4)) + b)/((b^2*c - 4*a*c^2)*e^2)
)*sqrt(1/((b^2*c^2 - 4*a*c^3)*e^4)) + e*x + d) - 1/2*sqrt(1/2)*sqrt(((b^2*
c - 4*a*c^2)*e^2*sqrt(1/((b^2*c^2 - 4*a*c^3)*e^4)) - b)/((b^2*c - 4*a*c^2)
*e^2))*log(sqrt(1/2)*(b^2*c - 4*a*c^2)*e^3*sqrt(((b^2*c - 4*a*c^2)*e^2*sqr
t(1/((b^2*c^2 - 4*a*c^3)*e^4)) - b)/((b^2*c - 4*a*c^2)*e^2))*sqrt(1/((b^2*
c^2 - 4*a*c^3)*e^4)) + e*x + d) + 1/2*sqrt(1/2)*sqrt(((b^2*c - 4*a*c^2)*e^
2*sqrt(1/((b^2*c^2 - 4*a*c^3)*e^4)) - b)/((b^2*c - 4*a*c^2)*e^2))*log(-sqr
t(1/2)*(b^2*c - 4*a*c^2)*e^3*sqrt(((b^2*c - 4*a*c^2)*e^2*sqrt(1/((b^2*c^2
- 4*a*c^3)*e^4)) - b)/((b^2*c - 4*a*c^2)*e^2))*sqrt(1/((b^2*c^2 - 4*a*c^3)
*e^4)) + e*x + d)
```

3.615.6 Sympy [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.63

$$\int \frac{(d+ex)^2}{a+b(d+ex)^2+c(d+ex)^4}\,dx$$
$$=\text{RootSum}\left(t^4\cdot\left(256a^2c^3e^4-128ab^2c^2e^4+16b^4ce^4\right)+t^2\left(-16abce^2+4b^3e^2\right)+a,\left(t\mapsto t\log\left(x+\frac{64t^3ac^2\right.\right.\right.$$

input
```
integrate((e*x+d)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

output
```
RootSum(_t**4*(256*a**2*c**3*e**4 - 128*a*b**2*c**2*e**4 + 16*b**4*c*e**4)
 + _t**2*(-16*a*b*c*e**2 + 4*b**3*e**2) + a, Lambda(_t, _t*log(x + (64*_t*
*3*a*c**2*e**3 - 16*_t**3*b**2*c*e**3 - 2*_t*b*e + d)/e)))
```

3.615.7 Maxima [F]

$$\int \frac{(d+ex)^2}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \int \frac{(ex+d)^2}{(ex+d)^4c+(ex+d)^2b+a}\,dx$$

input
```
integrate((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^2/((e*x + d)^4*c + (e*x + d)^2*b + a), x)
```

3.615.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1403 vs. 2(129) = 258.

Time = 0.29 (sec) , antiderivative size = 1403, normalized size of antiderivative = 8.55

$$\int \frac{(d+ex)^2}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

output
```
-1/2*(e^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)
^2 - 2*d*e*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e
) + d^2)*log(x + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4))
+ d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4))
+ d/e)^3 - 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e
^4)) + d/e)^2 + 6*c*d^2*e^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^
2)/(c*e^4)) + d/e) - 2*c*d^3*e + b*e^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2
- 4*a*c)*e^2)/(c*e^4)) + d/e) - b*d*e) + 1/2*(e^2*(sqrt(1/2)*sqrt(-(b*e^2
+ sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2 + 2*d*e*(sqrt(1/2)*sqrt(-(b*e^2
 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e) + d^2)*log(x - sqrt(1/2)*sqrt(-(
b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(
b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^3 + 6*c*d*e^3*(sqrt(1/2)*sq
rt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2 + 6*c*d^2*e^2*(sqrt(
1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e) + 2*c*d^3*e + b
*e^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e) + b*
d*e) - 1/2*(e^2*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4))
+ d/e)^2 - 2*d*e*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4))
 + d/e) + d^2)*log(x + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*
e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*
e^4)) + d/e)^3 - 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*...
```

3.615.9 Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 590, normalized size of antiderivative = 3.60

$$\int \frac{(d+ex)^2}{a+b(d+ex)^2+c(d+ex)^4}\,dx = -2\,\mathrm{atanh}\left(\frac{\sqrt{-\frac{b^3+\sqrt{-(4\,a\,c-b^2)^3}-4\,a\,b\,c}{8\,(16\,a^2\,c^3\,e^2-8\,a\,b^2\,c^2\,e^2+b^4\,c\,e^2)}}\left(x\,(4\,a\,c^2\,e^{12}-2\,b^2\,c\,e^{12})+\frac{(x\,(8\,b^3\,c^2\,e^{14}-32\,a\,b\,c^3\,e^{14})+8\,b^3\,c^2\,d\,e^{13}-32\,a}{8\,(16\,a^2\,c^3\,e^2-8\,a\,b^2}\right.}{a\,c\,e^{10}}\right.$$

$$-2\,\mathrm{atanh}\left(\frac{\sqrt{\frac{\sqrt{-(4\,a\,c-b^2)^3}-b^3+4\,a\,b\,c}{8\,(16\,a^2\,c^3\,e^2-8\,a\,b^2\,c^2\,e^2+b^4\,c\,e^2)}}\left(x\,(4\,a\,c^2\,e^{12}-2\,b^2\,c\,e^{12})-\frac{(x\,(8\,b^3\,c^2\,e^{14}-32\,a\,b\,c^3\,e^{14})+8\,b^3\,c^2\,d\,e^{13}-32\,a\,b\,c}{8\,(16\,a^2\,c^3\,e^2-8\,a\,b^2\,c^2}\right.}{a\,c\,e^{10}}\right.$$

input
```
int((d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x)
```

output
```
- 2*atanh(((-(b^3 + (-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c)/(8*(b^4*c*e^2 + 16
*a^2*c^3*e^2 - 8*a*b^2*c^2*e^2)))^(1/2)*(x*(4*a*c^2*e^12 - 2*b^2*c*e^12) +
 ((x*(8*b^3*c^2*e^14 - 32*a*b*c^3*e^14) + 8*b^3*c^2*d*e^13 - 32*a*b*c^3*d*
e^13)*(b^3 + (-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c))/(8*(b^4*c*e^2 + 16*a^2*c
^3*e^2 - 8*a*b^2*c^2*e^2)) + 4*a*c^2*d*e^11 - 2*b^2*c*d*e^11))/(a*c*e^10))
*(-(b^3 + (-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c)/(8*(b^4*c*e^2 + 16*a^2*c^3*e
^2 - 8*a*b^2*c^2*e^2)))^(1/2) - 2*atanh(((((-(4*a*c - b^2)^3)^(1/2) - b^3
+ 4*a*b*c)/(8*(b^4*c*e^2 + 16*a^2*c^3*e^2 - 8*a*b^2*c^2*e^2)))^(1/2)*(x*(4
*a*c^2*e^12 - 2*b^2*c*e^12) - ((x*(8*b^3*c^2*e^14 - 32*a*b*c^3*e^14) + 8*b
^3*c^2*d*e^13 - 32*a*b*c^3*d*e^13)*((-(4*a*c - b^2)^3)^(1/2) - b^3 + 4*a*b
*c))/(8*(b^4*c*e^2 + 16*a^2*c^3*e^2 - 8*a*b^2*c^2*e^2)) + 4*a*c^2*d*e^11 -
 2*b^2*c*d*e^11))/(a*c*e^10))*(((-(4*a*c - b^2)^3)^(1/2) - b^3 + 4*a*b*c)/
(8*(b^4*c*e^2 + 16*a^2*c^3*e^2 - 8*a*b^2*c^2*e^2)))^(1/2)
```

3.616 $\int \frac{d+ex}{a+b(d+ex)^2+c(d+ex)^4} dx$

3.616.1 Optimal result

Integrand size = 28, antiderivative size = 43

$$\int \frac{d+ex}{a+b(d+ex)^2+c(d+ex)^4} \, dx = -\frac{\operatorname{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}e}$$

output
```
-arctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/e/(-4*a*c+b^2)^(1/2)
```

3.616.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.07

$$\int \frac{d+ex}{a+b(d+ex)^2+c(d+ex)^4} \, dx = \frac{\arctan\left(\frac{b+2c(d+ex)^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}e}$$

input
```
Integrate[(d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

output
```
ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]]/(Sqrt[-b^2 + 4*a*c]*e)
```

3.616.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1462, 1432, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{a+b(d+ex)^2+c(d+ex)^4} \, dx$$
$$\Big\downarrow \; 1462$$
$$\frac{\int \frac{d+ex}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)}{e}$$
$$\Big\downarrow \; 1432$$
$$\frac{\int \frac{1}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)^2}{2e}$$
$$\Big\downarrow \; 1083$$
$$-\frac{\int \frac{1}{-(d+ex)^4+b^2-4ac}d\left(2c(d+ex)^2+b\right)}{e}$$
$$\Big\downarrow \; 219$$
$$-\frac{\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\sqrt{b^2-4ac}}$$

input
```
Int[(d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

output
```
-(ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]]/(Sqrt[b^2 - 4*a*c]*e))
```

3.616.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

3.616.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 129, normalized size of antiderivative = 3.00

method	result
default	$\frac{\sum_{\_R=\operatorname{RootOf}\left(c e^4 \_Z^4+4cd e^3 \_Z^3+\left(6c d^2 e^2+b e^2\right)\_Z^2+\left(4d^3 ec+2bde\right)\_Z+d^4 c+b d^2+a\right)} \frac{\left(\_R e+d\right)\ln\left(x-\_R\right)}{2e^3 c \_R^3+6cd e^2 \_R^2+6c d^2 e \_R+2d^3 c+be \_R}}{2e}$
risch	$-\frac{\ln\left(\left(e^2\sqrt{-4ac+b^2}-b e^2\right)x^2+\left(2ed\sqrt{-4ac+b^2}-2bde\right)x+d^2\sqrt{-4ac+b^2}-b d^2-2a\right)}{2\sqrt{-4ac+b^2}\, e}+\frac{\ln\left(\left(e^2\sqrt{-4ac+b^2}+b e^2\right)x^2+\left(2ed\sqrt{-4ac+\right.\right.}{2\sqrt{-4ac+}}$

input
```
int((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x,method=_RETURNVERBOSE)
```

output
```
1/2/e*sum((_R*e+d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*
e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z
^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a))
```

3.616.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 272, normalized size of antiderivative = 6.33

$$\int \frac{d+ex}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \left[\frac{\log\left(\frac{2\,c^2e^4x^4+8\,c^2de^3x^3+2\,c^2d^4+2\,(6\,c^2d^2+bc)e^2x^2+2\,bcd^2+4\,(2\,c^2d^3+bcd)ex+b^2-2\,ac-(2\,ce^2x^2+4\,cdex+2\,cd^2+b)\sqrt{b^2-4\,ac}}{ce^4x^4+4\,cde^3x^3+cd^4+(6\,cd^2+b)e^2x^2+bd^2+2\,(2\,cd^3+bd)ex+a}\right)}{2\,\sqrt{b^2-4\,ac}e}, -\frac{\sqrt{-b^2+4\,ac}\arctan\left(-\frac{(2\,ce^2x^2+4\,cdex+2\,cd^2+b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)}{(b^2-4\,ac)e}\right]$$

input
```
integrate((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

output
```
[1/2*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c
)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c - (2*c*e^2
*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*
x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a))/
(sqrt(b^2 - 4*a*c)*e), -sqrt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 + 4*c*d*e*
x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c))/((b^2 - 4*a*c)*e)]
```

3.616.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(39) = 78.

Time = 0.57 (sec) , antiderivative size = 168, normalized size of antiderivative = 3.91

$$\int \frac{d+ex}{a+b(d+ex)^2+c(d+ex)^4}\,dx = -\frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(\frac{2dx}{e}+x^2+\frac{-4ac\sqrt{-\frac{1}{4ac-b^2}}+b^2\sqrt{-\frac{1}{4ac-b^2}}+b+2cd^2}{2ce^2}\right)}{2e} + \frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(\frac{2dx}{e}+x^2+\frac{4ac\sqrt{-\frac{1}{4ac-b^2}}-b^2\sqrt{-\frac{1}{4ac-b^2}}+b+2cd^2}{2ce^2}\right)}{2e}$$

input
```
integrate((e*x+d)/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

output
```
-sqrt(-1/(4*a*c - b**2))*log(2*d*x/e + x**2 + (-4*a*c*sqrt(-1/(4*a*c - b**
2)) + b**2*sqrt(-1/(4*a*c - b**2)) + b + 2*c*d**2)/(2*c*e**2))/(2*e) + sqr
t(-1/(4*a*c - b**2))*log(2*d*x/e + x**2 + (4*a*c*sqrt(-1/(4*a*c - b**2)) -
 b**2*sqrt(-1/(4*a*c - b**2)) + b + 2*c*d**2)/(2*c*e**2))/(2*e)
```

3.616.7 Maxima [F]

$$\int \frac{d+ex}{a+b(d+ex)^2+c(d+ex)^4}\,dx=\int \frac{ex+d}{(ex+d)^4c+(ex+d)^2b+a}\,dx$$

input
```
integrate((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

output
```
integrate((e*x + d)/((e*x + d)^4*c + (e*x + d)^2*b + a), x)
```

3.616.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.21

$$\int \frac{d+ex}{a+b(d+ex)^2+c(d+ex)^4}\,dx=\frac{\arctan\left(\frac{2\,cd^2+2\,(ex^2+2\,dx)ce+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}e}$$

input
```
integrate((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

output
```
arctan((2*c*d^2 + 2*(e*x^2 + 2*d*x)*c*e + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^
2 + 4*a*c)*e)
```

3.616.9 Mupad [B] (verification not implemented)

Time = 8.68 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.42

$$\int \frac{d+ex}{a+b(d+ex)^2+c(d+ex)^4}\,dx=\frac{\mathrm{atan}\left(\frac{2\,a\,c\,d^2+4\,a\,c\,d\,e\,x+2\,a\,c\,e^2\,x^2+a\,b}{a\,\sqrt{4\,a\,c-b^2}}\right)}{e\,\sqrt{4\,a\,c-b^2}}$$

input
```
int((d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x)
```

output
```
atan((a*b + 2*a*c*d^2 + 2*a*c*e^2*x^2 + 4*a*c*d*e*x)/(a*(4*a*c - b^2)^(1/2
)))/(e*(4*a*c - b^2)^(1/2))
```

3.616. $\int \frac{d+ex}{a+b(d+ex)^2+c(d+ex)^4} \, dx$

3.617 $\int \frac{1}{(d+ex)(a+b(d+ex)^2+c(d+ex)^4)}\,dx$

3.617.1 Optimal result

Integrand size = 30, antiderivative size = 94

$$\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx = \frac{b\operatorname{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}e}+\frac{\log(d+ex)}{ae}-\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4ae}$$

output
```
ln(e*x+d)/a/e-1/4*ln(a+b*(e*x+d)^2+c*(e*x+d)^4)/a/e+1/2*b*arctanh((b+2*c*(
e*x+d)^2)/(-4*a*c+b^2)^(1/2))/a/e/(-4*a*c+b^2)^(1/2)
```

3.617.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.36

$$\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx = \frac{4\sqrt{b^2-4ac}\log(d+ex)-\left(b+\sqrt{b^2-4ac}\right)\log\left(b-\sqrt{b^2-4ac}+2c(d+ex)^2\right)+\left(b-\sqrt{b^2-4ac}\right)\log\left(b+\right.}{4a\sqrt{b^2-4ac}e}$$

input
```
Integrate[1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

output
```
(4*Sqrt[b^2 - 4*a*c]*Log[d + e*x] - (b + Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b
^2 - 4*a*c] + 2*c*(d + e*x)^2] + (b - Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2
- 4*a*c] + 2*c*(d + e*x)^2])/(4*a*Sqrt[b^2 - 4*a*c]*e)
```

3.617.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1462, 1434, 1144, 25, 1142, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)}dx$$

$$\Big\downarrow \text{ 1462}$$

$$\frac{\int \frac{1}{(d+ex)(c(d+ex)^4+b(d+ex)^2+a)}d(d+ex)}{e}$$

$$\Big\downarrow \text{ 1434}$$

$$\frac{\int \frac{1}{(d+ex)^2(c(d+ex)^4+b(d+ex)^2+a)}d(d+ex)^2}{2e}$$

$$\Big\downarrow \text{ 1144}$$

$$\frac{\frac{\int -\frac{c(d+ex)^2+b}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)^2}{a}+\frac{\log\left((d+ex)^2\right)}{a}}{2e}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{\log\left((d+ex)^2\right)}{a}-\frac{\int \frac{c(d+ex)^2+b}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)^2}{a}}{2e}$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{\frac{\log\left((d+ex)^2\right)}{a}-\frac{\frac{1}{2}b\int \frac{1}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)^2+\frac{1}{2}\int \frac{2c(d+ex)^2+b}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)^2}{a}}{2e}$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{\frac{\log\left((d+ex)^2\right)}{a}-\frac{\frac{1}{2}\int \frac{2c(d+ex)^2+b}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)^2-b\int \frac{1}{-(d+ex)^4+b^2-4ac}d\left(2c(d+ex)^2+b\right)}{a}}{2e}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{\log\left((d+ex)^2\right)}{a}-\frac{\frac{1}{2}\int \frac{2c(d+ex)^2+b}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)^2-\frac{b\,\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}}{a}}{2e}$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{\frac{\log((d+ex)^2)}{a}-\frac{\frac{1}{2}\log(a+b(d+ex)^2+c(d+ex)^4)-\frac{b\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}}{a}}{2e}$$

input
```
Int[1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

output
```
(Log[(d + e*x)^2]/a - (-((b*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c
]])/Sqrt[b^2 - 4*a*c]) + Log[a + b*(d + e*x)^2 + c*(d + e*x)^4]/2)/a)/(2*e
)
```

3.617.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1144
```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Simp[e*(Log[RemoveContent[d + e*x, x]]/(c*d^2 - b*d*e + a*e^2)), x] + S
imp[1/(c*d^2 - b*d*e + a*e^2)   Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2),
x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

3.617.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.63 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.65

method	result
risch	$\frac{\ln(ex+d)}{ae} + \frac{\left(\sum_{\_R=\text{RootOf}\left(\left(4a^2c\,e^2-a\,b^2e^2\right)\_Z^2+\left(4ace-b^2e\right)\_Z+c\right)} \_R\ln\left(\left(\left(10e^3ac-3b^2e^3\right)\_R+5c\,e^2\right)x^2+\left(\left(20acd\,e^2-6b^2d\,e\right.\right.\right.\right.}{2}$
default	$\frac{\sum_{\_R=\text{RootOf}\left(c\,e^4\_Z^4+4cd\,e^3\_Z^3+\left(6c\,d^2e^2+b\,e^2\right)\_Z^2+\left(4d^3ec+2bde\right)\_Z+d^4c+b\,d^2+a\right)} \frac{\left(-e^3c\_R^3-3cd\,e^2\_R^2+e\left(-3c\,d^2-b\right)\_R-d^3\right.}{2e^3c\_R^3+6cd\,e^2\_R^2+6c\,d^2e\_R+2d^3c}}{2ae}$

input
```
int(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x,method=_RETURNVERBOSE)
```

output
```
ln(e*x+d)/a/e+1/2*sum(_R*ln(((10*a*c*e^3-3*b^2*e^3)*_R+5*c*e^2)*x^2+((20*a
*c*d*e^2-6*b^2*d*e^2)*_R+10*d*c*e)*x+(10*a*c*d^2*e-3*b^2*d^2*e-a*b*e)*_R+5
*c*d^2+2*b),_R=RootOf((4*a^2*c*e^2-a*b^2*e^2)*_Z^2+(4*a*c*e-b^2*e)*_Z+c))
```

3.617.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 468, normalized size of antiderivative = 4.98

$$\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx$$
$$= \left[\frac{\sqrt{b^2-4\,ac}b\log\left(\frac{2\,c^2e^4x^4+8\,c^2de^3x^3+2\,c^2d^4+2\,(6\,c^2d^2+bc)e^2x^2+2\,bcd^2+4\,(2\,c^2d^3+bcd)ex+b^2-2\,ac+(2\,ce^2x^2+4\,cdex+2\,cd^2+b)\sqrt{b^2-}}{ce^4x^4+4\,cde^3x^3+cd^4+(6\,cd^2+b)e^2x^2+bd^2+2\,(2\,cd^3+bd)ex+a}\right.}{}\right.$$

input
```
integrate(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4
 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b
^2 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c
*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*
d^3 + b*d)*e*x + a)) - (b^2 - 4*a*c)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4
 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*(b^2 - 4
*a*c)*log(e*x + d))/((a*b^2 - 4*a^2*c)*e), 1/4*(2*sqrt(-b^2 + 4*a*c)*b*arc
tan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a
*c)) - (b^2 - 4*a*c)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)
*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*(b^2 - 4*a*c)*log(e*x +
d))/((a*b^2 - 4*a^2*c)*e)]
```

3.617.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(78) = 156$.

Time = 13.83 (sec) , antiderivative size = 320, normalized size of antiderivative = 3.40

$$\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx = \left(-\frac{b\sqrt{-4ac+b^2}}{4ae\left(4ac-b^2\right)} - \frac{1}{4ae}\right)\log\left(\frac{2dx}{e}+x^2+\frac{-8a^2ce\left(-\frac{b\sqrt{-4ac+b^2}}{4ae(4ac-b^2)}-\frac{1}{4ae}\right)+2ab^2e\left(-\frac{b\sqrt{-4ac+b^2}}{4ae(4ac-b^2)}-\frac{1}{4ae}\right)-2ac+b^2+bcd^2}{bce^2}\right) + \left(\frac{b\sqrt{-4ac+b^2}}{4ae\left(4ac-b^2\right)} - \frac{1}{4ae}\right)\log\left(\frac{2dx}{e}+x^2+\frac{-8a^2ce\left(\frac{b\sqrt{-4ac+b^2}}{4ae(4ac-b^2)}-\frac{1}{4ae}\right)+2ab^2e\left(\frac{b\sqrt{-4ac+b^2}}{4ae(4ac-b^2)}-\frac{1}{4ae}\right)-2ac+b^2+bcd^2}{bce^2}\right) + \frac{\log\left(\frac{d}{e}+x\right)}{ae}$$

input
```
integrate(1/(e*x+d)/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

output
```
(-b*sqrt(-4*a*c + b**2)/(4*a*e*(4*a*c - b**2)) - 1/(4*a*e))*log(2*d*x/e +
x**2 + (-8*a**2*c*e*(-b*sqrt(-4*a*c + b**2)/(4*a*e*(4*a*c - b**2)) - 1/(4*
a*e)) + 2*a*b**2*e*(-b*sqrt(-4*a*c + b**2)/(4*a*e*(4*a*c - b**2)) - 1/(4*a
*e)) - 2*a*c + b**2 + b*c*d**2)/(b*c*e**2)) + (b*sqrt(-4*a*c + b**2)/(4*a*
e*(4*a*c - b**2)) - 1/(4*a*e))*log(2*d*x/e + x**2 + (-8*a**2*c*e*(b*sqrt(-
4*a*c + b**2)/(4*a*e*(4*a*c - b**2)) - 1/(4*a*e)) + 2*a*b**2*e*(b*sqrt(-4*
a*c + b**2)/(4*a*e*(4*a*c - b**2)) - 1/(4*a*e)) - 2*a*c + b**2 + b*c*d**2)
/(b*c*e**2)) + log(d/e + x)/(a*e)
```

3.617.7 Maxima [F]

$$\int \frac{1}{(d+ex)\,(a+b(d+ex)^2+c(d+ex)^4)}\,dx = \int \frac{1}{\left((ex+d)^4c+(ex+d)^2b+a\right)(ex+d)}\,dx$$

input
```
integrate(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

output
```
-integrate((c*e^3*x^3 + 3*c*d*e^2*x^2 + c*d^3 + (3*c*d^2 + b)*e*x + b*d)/(
c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c
*d^3 + b*d)*e*x + a), x)/a + log(e*x + d)/(a*e)
```

3.617.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(86) = 172$.

Time = 0.36 (sec) , antiderivative size = 280, normalized size of antiderivative = 2.98

$$\int \frac{1}{(d+ex)\,(a+b(d+ex)^2+c(d+ex)^4)}\,dx$$
$$= -\frac{\log\left(\left|ce^4x^4+4\,cde^3x^3+6\,cd^2e^2x^2+4\,cd^3ex+cd^4+be^2x^2+2\,bdex+bd^2+a\right|\right)}{4\,ae} + \frac{\log\left(\left|ex+d\right|\right)}{ae} - \frac{\frac{abce^3\log\left(\left|be^2x^2+\sqrt{b^2-4\,ac}e^2x^2+2\,bdex+2\,\sqrt{b^2-4\,ac}dex+bd^2+\sqrt{b^2-4\,ac}d^2+2\,a\right|\right)}{\sqrt{b^2-4\,ac}} - \frac{abce^3\log\left(\left|-be^2x^2+\sqrt{b^2-4\,ac}e^2x^2-2\,bdex+2\,\sqrt{b^2-\cdots}\right.\right.}{\sqrt{b^2-4\,ac}}}{4\,a^2ce^4}$$

input
```
integrate(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

output
```
-1/4*log(abs(c*e^4*x^4 + 4*c*d*e^3*x^3 + 6*c*d^2*e^2*x^2 + 4*c*d^3*e*x + c
*d^4 + b*e^2*x^2 + 2*b*d*e*x + b*d^2 + a))/(a*e) + log(abs(e*x + d))/(a*e)
 - 1/4*(a*b*c*e^3*log(abs(b*e^2*x^2 + sqrt(b^2 - 4*a*c)*e^2*x^2 + 2*b*d*e*
x + 2*sqrt(b^2 - 4*a*c)*d*e*x + b*d^2 + sqrt(b^2 - 4*a*c)*d^2 + 2*a))/sqrt
(b^2 - 4*a*c) - a*b*c*e^3*log(abs(-b*e^2*x^2 + sqrt(b^2 - 4*a*c)*e^2*x^2 -
 2*b*d*e*x + 2*sqrt(b^2 - 4*a*c)*d*e*x - b*d^2 + sqrt(b^2 - 4*a*c)*d^2 - 2
*a))/sqrt(b^2 - 4*a*c))/(a^2*c*e^4)
```

3.617.9 Mupad [B] (verification not implemented)

Time = 9.56 (sec) , antiderivative size = 2173, normalized size of antiderivative = 23.12

$$\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx = \text{Too large to display}$$

input
```
int(1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x)
```

output
```
log(d + e*x)/(a*e) - (log(a + b*d^2 + c*d^4 + b*e^2*x^2 + c*e^4*x^4 + 2*b*
d*e*x + 6*c*d^2*e^2*x^2 + 4*c*d^3*e*x + 4*c*d*e^3*x^3)*(2*b^2*e - 8*a*c*e)
)/(2*(4*a*b^2*e^2 - 16*a^2*c*e^2)) - (b*atan((16*a^3*x^2*(((3*b^3 - 8*a*b*
c)*((b^2*(10*b*c^3*e^18 + ((2*b^2*e - 8*a*c*e)*(12*b^3*c^2*e^19 - 40*a*b*c
^3*e^19))/(2*(4*a*b^2*e^2 - 16*a^2*c*e^2))))/(16*a^2*e^2*(4*a*c - b^2)) -
((2*b^2*e - 8*a*c*e)^2*(10*b*c^3*e^18 + ((2*b^2*e - 8*a*c*e)*(12*b^3*c^2*e
^19 - 40*a*b*c^3*e^19))/(2*(4*a*b^2*e^2 - 16*a^2*c*e^2))))/(4*(4*a*b^2*e^2
 - 16*a^2*c*e^2)^2) + (b^2*(2*b^2*e - 8*a*c*e)*(12*b^3*c^2*e^19 - 40*a*b*c
^3*e^19))/(16*a^2*e^2*(4*a*c - b^2)*(4*a*b^2*e^2 - 16*a^2*c*e^2))))/(8*a^3
*c^2*(25*a*c - 6*b^2)) - (((b*(2*b^2*e - 8*a*c*e)^2*(12*b^3*c^2*e^19 - 40*
a*b*c^3*e^19))/(16*a*e*(4*a*c - b^2)^(1/2)*(4*a*b^2*e^2 - 16*a^2*c*e^2)^2)
 - (b^3*(12*b^3*c^2*e^19 - 40*a*b*c^3*e^19))/(64*a^3*e^3*(4*a*c - b^2)^(3/
2)) + (b*(2*b^2*e - 8*a*c*e)*(10*b*c^3*e^18 + ((2*b^2*e - 8*a*c*e)*(12*b^3
*c^2*e^19 - 40*a*b*c^3*e^19))/(2*(4*a*b^2*e^2 - 16*a^2*c*e^2))))/(4*a*e*(4
*a*c - b^2)^(1/2)*(4*a*b^2*e^2 - 16*a^2*c*e^2)))*(3*b^4 + 10*a^2*c^2 - 14*
a*b^2*c))/(8*a^3*c^2*(4*a*c - b^2)^(1/2)*(25*a*c - 6*b^2)))*(4*a*c - b^2)^
(3/2))/(b^2*c^2*e^14) + (2*(3*b^3 - 8*a*b*c)*(4*a*c - b^2)^(3/2)*((b^2*(((
2*b^2*e - 8*a*c*e)*(4*a*b^2*c^2*e^17 + 12*b^3*c^2*d^2*e^17 - 40*a*b*c^3*d^
2*e^17))/(2*(4*a*b^2*e^2 - 16*a^2*c*e^2)) + 4*b^2*c^2*e^16 + 10*b*c^3*d^2*
e^16))/(16*a^2*e^2*(4*a*c - b^2)) - ((2*b^2*e - 8*a*c*e)^2*(((2*b^2*e -...
```

3.618 $\int \frac{1}{(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)} dx$

3.618.1 Optimal result

Integrand size = 30, antiderivative size = 195

$$\int \frac{1}{(d+ex)^2 (a+b(d+ex)^2+c(d+ex)^4)} dx = -\frac{1}{ae(d+ex)} - \frac{\sqrt{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}e} - \frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b+\sqrt{b^2-4ac}}e}$$

output
```
-1/a/e/(e*x+d)-1/2*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(
1/2))*c^(1/2)*(1+b/(-4*a*c+b^2)^(1/2))/a/e*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^
(1/2)-1/2*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(
1/2)*(1-b/(-4*a*c+b^2)^(1/2))/a/e*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.618.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.06

$$\int \frac{1}{(d+ex)^2 (a+b(d+ex)^2+c(d+ex)^4)} dx = -\frac{\frac{2}{d+ex} + \frac{\sqrt{2}\sqrt{c}\left(b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\sqrt{c}\left(-b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}}{2ae}$$

input
```
Integrate[1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

output
```
-1/2*(2/(d + e*x) + (Sqrt[2]*Sqrt[c]*(b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqr
t[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-b + Sqrt[b^2 - 4*a*c])*ArcT
an[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4
*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(a*e)
```

3.618.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1462, 1443, 25, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^2 (a+b(d+ex)^2+c(d+ex)^4)} dx$$

$$\Big\downarrow \text{1462}$$

$$\frac{\int \frac{1}{(d+ex)^2(c(d+ex)^4+b(d+ex)^2+a)}d(d+ex)}{e}$$

$$\Big\downarrow \text{1443}$$

$$\frac{\frac{\int -\frac{c(d+ex)^2+b}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)}{a}-\frac{1}{a(d+ex)}}{e}$$

$$\Big\downarrow \text{25}$$

$$\frac{-\frac{\int \frac{c(d+ex)^2+b}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)}{a}-\frac{1}{a(d+ex)}}{e}$$

$$\Big\downarrow \text{1480}$$

$$\frac{-\frac{\frac{1}{2}c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\int \frac{1}{c(d+ex)^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}d(d+ex)+\frac{1}{2}c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{c(d+ex)^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}d(d+ex)}{a}-\frac{1}{a(d+ex)}}{e}$$

$$\Big\downarrow \text{218}$$

$$-\frac{\frac{\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}}{a}-\frac{1}{a(d+ex)}}{e}$$

input
```
Int[1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

output
```
(-(1/(a*(d + e*x))) - ((Sqrt[c]*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*
Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^
2 - 4*a*c]]) + (Sqrt[c]*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*
(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 - 4*a*
c]]))/a)/e
```

3.618.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1443
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Sim
p[1/(a*d^2*(m + 1))   Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*
x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

3.618.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.64 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.86

method	result
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(c\,e^4\_Z^4+4cd\,e^3\_Z^3+\left(6c\,d^2e^2+b\,e^2\right)\_Z^2+\left(4d^3ec+2bde\right)\_Z+d^4c+b\,d^2+a\right)} \frac{\left(-\_R^2c\,e^2-2\_Rcde-c\,d^2-b\right)\ln\left(x-\_R\right)}{2e^3c\_R^3+6cd\,e^2\_R^2+6c\,d^2e\_R+2d^3c+be\_R}}{2ae}$
risch	$-\frac{1}{ae(ex+d)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(16a^5c^2e^4-8b^2e^4c\,a^4+b^4e^4a^3\right)\_Z^4+\left(12a^2b\,c^2e^2-7a\,b^3c\,e^2+b^5e^2\right)\_Z^2+c^3\right)} \_R\ln\left(\left(\left((40a^5c^2e^5-22a^4b\right.\right.\right.}{}$

input
```
int(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x,method=_RETURNVERBOSE)
```

output
```
1/2/a/e*sum((-_R^2*c*e^2-2*_R*c*d*e-c*d^2-b)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+
6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_
Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a))-1/a/e/
(e*x+d)
```

3.618.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1339 vs. 2(158) = 316.

Time = 0.28 (sec) , antiderivative size = 1339, normalized size of antiderivative = 6.87

$$\int \frac{1}{(d+ex)^2\,(a+b(d+ex)^2+c(d+ex)^4)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

output
```
1/2*(sqrt(1/2)*(a*e^2*x + a*d*e)*sqrt(-((a^3*b^2 - 4*a^4*c)*e^2*sqrt((b^4
- 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4)) + b^3 - 3*a*b*c)/((a^3*b
^2 - 4*a^4*c)*e^2))*log(-2*(b^2*c^2 - a*c^3)*e*x - 2*(b^2*c^2 - a*c^3)*d +
 sqrt(1/2)*((a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*e^3*sqrt((b^4 - 2*a*b^2*c
+ a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4)) - (b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*e)
*sqrt(-((a^3*b^2 - 4*a^4*c)*e^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2
 - 4*a^7*c)*e^4)) + b^3 - 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*e^2))) - sqrt(1/2)
*(a*e^2*x + a*d*e)*sqrt(-((a^3*b^2 - 4*a^4*c)*e^2*sqrt((b^4 - 2*a*b^2*c +
a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4)) + b^3 - 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*
e^2))*log(-2*(b^2*c^2 - a*c^3)*e*x - 2*(b^2*c^2 - a*c^3)*d - sqrt(1/2)*((a
^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*e^3*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a
^6*b^2 - 4*a^7*c)*e^4)) - (b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*e)*sqrt(-((a^3*b
^2 - 4*a^4*c)*e^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^
4)) + b^3 - 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*e^2))) - sqrt(1/2)*(a*e^2*x + a*
d*e)*sqrt(((a^3*b^2 - 4*a^4*c)*e^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*
b^2 - 4*a^7*c)*e^4)) - b^3 + 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*e^2))*log(-2*(b
^2*c^2 - a*c^3)*e*x - 2*(b^2*c^2 - a*c^3)*d + sqrt(1/2)*((a^3*b^4 - 6*a^4*
b^2*c + 8*a^5*c^2)*e^3*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*
c)*e^4)) + (b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*e)*sqrt(((a^3*b^2 - 4*a^4*c)*e^
2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4)) - b^3 + 3...
```

3.618.6 Sympy [A] (verification not implemented)

Time = 2.96 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.08

$$\int \frac{1}{(d+ex)^2\,(a+b(d+ex)^2+c(d+ex)^4)}\,dx$$
$$=\text{RootSum}\left(t^4\cdot\left(256a^5c^2e^4-128a^4b^2ce^4+16a^3b^4e^4\right)+t^2\cdot\left(48a^2bc^2e^2-28ab^3ce^2+4b^5e^2\right)+c^3,\left(t\mapsto t\,\text{lo}\right.\right.$$
$$-\frac{1}{ade+ae^2x}$$

input
```
integrate(1/(e*x+d)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

output
```
RootSum(_t**4*(256*a**5*c**2*e**4 - 128*a**4*b**2*c*e**4 + 16*a**3*b**4*e*
*4) + _t**2*(48*a**2*b*c**2*e**2 - 28*a*b**3*c*e**2 + 4*b**5*e**2) + c**3,
 Lambda(_t, _t*log(x + (-64*_t**3*a**5*c**2*e**3 + 48*_t**3*a**4*b**2*c*e*
*3 - 8*_t**3*a**3*b**4*e**3 - 10*_t*a**2*b*c**2*e + 10*_t*a*b**3*c*e - 2*_
t*b**5*e + a*c**3*d - b**2*c**2*d)/(a*c**3*e - b**2*c**2*e))) - 1/(a*d*e
+ a*e**2*x)
```

3.618.7 Maxima [F]

$$\int \frac{1}{(d+ex)^2 (a+b(d+ex)^2+c(d+ex)^4)} dx = \int \frac{1}{\left((ex+d)^4 c + (ex+d)^2 b + a\right)(ex+d)^2} dx$$

input
```
integrate(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

output
```
-integrate((c*e^2*x^2 + 2*c*d*e*x + c*d^2 + b)/(c*e^4*x^4 + 4*c*d*e^3*x^3
+ c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a), x)/a
 - 1/(a*e^2*x + a*d*e)
```

3.618.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 932 vs. $2(158) = 316$.

Time = 0.31 (sec) , antiderivative size = 932, normalized size of antiderivative = 4.78

$$\int \frac{1}{(d+ex)^2 (a+b(d+ex)^2+c(d+ex)^4)} dx =$$

$$-\frac{\left(\left(b^8 - 9\,ab^6c + 25\,a^2b^4c^2 - 20\,a^3b^2c^3 + (b^7 - 7\,ab^5c + 13\,a^2b^3c^2 - 4\,a^3bc^3)\sqrt{b^2-4\,ac}\right)\sqrt{2\,ab + 2\,\sqrt{b^2 -}}\right.}{}$$

$$-\frac{\left(\left(b^8 - 9\,ab^6c + 25\,a^2b^4c^2 - 20\,a^3b^2c^3 - (b^7 - 7\,ab^5c + 13\,a^2b^3c^2 - 4\,a^3bc^3)\sqrt{b^2-4\,ac}\right)\sqrt{2\,ab - 2\,\sqrt{b^2 -}}\right.}{}$$

$$-\frac{1}{(ex+d)ae}$$

input
```
integrate(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

output
```
-1/8*((b^8 - 9*a*b^6*c + 25*a^2*b^4*c^2 - 20*a^3*b^2*c^3 + (b^7 - 7*a*b^5*
c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3)*sqrt(b^2 - 4*a*c))*sqrt(2*a*b + 2*sqrt(b
^2 - 4*a*c)*a)*a^2 - 2*(a^2*b^6*c - 7*a^3*b^4*c^2 + 13*a^4*b^2*c^3 - 4*a^5
*c^4 + (a^2*b^5*c - 5*a^3*b^3*c^2 + 5*a^4*b*c^3)*sqrt(b^2 - 4*a*c))*sqrt(2
*a*b + 2*sqrt(b^2 - 4*a*c)*a)*abs(a) - (a^2*b^8 - 7*a^3*b^6*c + 15*a^4*b^4
*c^2 - 10*a^5*b^2*c^3 + (a^2*b^7 - 5*a^3*b^5*c + 7*a^4*b^3*c^2 - 2*a^5*b*c
^3)*sqrt(b^2 - 4*a*c))*sqrt(2*a*b + 2*sqrt(b^2 - 4*a*c)*a))*arctan(2*sqrt(
1/2)/((e*x + d)*e*sqrt((a*b*e^6 + sqrt(a^2*b^2*e^12 - 4*a^3*c*e^12))/(a^2*
e^8))))/((a^3*b^6*c - 7*a^4*b^4*c^2 + 13*a^5*b^2*c^3 - 4*a^6*c^4 + (a^3*b^
5*c - 5*a^4*b^3*c^2 + 5*a^5*b*c^3)*sqrt(b^2 - 4*a*c))*a^2*e) - 1/8*((b^8 -
 9*a*b^6*c + 25*a^2*b^4*c^2 - 20*a^3*b^2*c^3 - (b^7 - 7*a*b^5*c + 13*a^2*b
^3*c^2 - 4*a^3*b*c^3)*sqrt(b^2 - 4*a*c))*sqrt(2*a*b - 2*sqrt(b^2 - 4*a*c)*
a)*a^2 - 2*(a^2*b^6*c - 7*a^3*b^4*c^2 + 13*a^4*b^2*c^3 - 4*a^5*c^4 - (a^2*
b^5*c - 5*a^3*b^3*c^2 + 5*a^4*b*c^3)*sqrt(b^2 - 4*a*c))*sqrt(2*a*b - 2*sqr
t(b^2 - 4*a*c)*a)*abs(a) - (a^2*b^8 - 7*a^3*b^6*c + 15*a^4*b^4*c^2 - 10*a^
5*b^2*c^3 - (a^2*b^7 - 5*a^3*b^5*c + 7*a^4*b^3*c^2 - 2*a^5*b*c^3)*sqrt(b^2
 - 4*a*c))*sqrt(2*a*b - 2*sqrt(b^2 - 4*a*c)*a))*arctan(2*sqrt(1/2)/((e*x +
 d)*e*sqrt((a*b*e^6 - sqrt(a^2*b^2*e^12 - 4*a^3*c*e^12))/(a^2*e^8))))/((a^
3*b^6*c - 7*a^4*b^4*c^2 + 13*a^5*b^2*c^3 - 4*a^6*c^4 - (a^3*b^5*c - 5*a^4*
b^3*c^2 + 5*a^5*b*c^3)*sqrt(b^2 - 4*a*c))*a^2*e) - 1/((e*x + d)*a*e)
```

3.618.9 Mupad [B] (verification not implemented)

Time = 9.22 (sec) , antiderivative size = 3844, normalized size of antiderivative = 19.71

$$\int \frac{1}{(d+ex)^2\,(a+b(d+ex)^2+c(d+ex)^4)}\,dx = \text{Too large to display}$$

input
```
int(1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x)
```

output
```
- atan(((-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c -
 a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^
2*c*e^2)))^(1/2)*(x*(4*a^4*c^4*e^12 - 2*a^3*b^2*c^3*e^12) + ((x*(32*a^6*b*
c^3*e^14 - 8*a^5*b^3*c^2*e^14) + 32*a^6*b*c^3*d*e^13 - 8*a^5*b^3*c^2*d*e^1
3)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*
(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c*e
^2)))^(1/2) - 16*a^5*b*c^3*e^12 + 4*a^4*b^3*c^2*e^12)*(-(b^5 + b^2*(-(4*a*
c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2
))/(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2)))^(1/2) + 4*a^4*c^4
*d*e^11 - 2*a^3*b^2*c^3*d*e^11)*1i + (-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2)
 + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^
2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2)))^(1/2)*(x*(4*a^4*c^4*e^12 - 2*a^3*b
^2*c^3*e^12) + ((x*(32*a^6*b*c^3*e^14 - 8*a^5*b^3*c^2*e^14) + 32*a^6*b*c^3
*d*e^13 - 8*a^5*b^3*c^2*d*e^13)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12
*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2 + 1
6*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2)))^(1/2) + 16*a^5*b*c^3*e^12 - 4*a^4*b^3*c
^2*e^12)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c
- a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b
^2*c*e^2)))^(1/2) + 4*a^4*c^4*d*e^11 - 2*a^3*b^2*c^3*d*e^11)*1i)/((-(b^5 +
 b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c...
```

3.618. $\int \frac{1}{(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)}\,dx$

3.619 $\int \frac{1}{(d+ex)^3(a+b(d+ex)^2+c(d+ex)^4)} dx$

3.619.1 Optimal result

Integrand size = 30, antiderivative size = 121

$$\int \frac{1}{(d+ex)^3 (a+b(d+ex)^2+c(d+ex)^4)} dx = -\frac{1}{2ae(d+ex)^2}-\frac{(b^2-2ac)\,\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}e} - \frac{b\log(d+ex)}{a^2e}+\frac{b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^2e}$$

output
```
-1/2/a/e/(e*x+d)^2-b*ln(e*x+d)/a^2/e+1/4*b*ln(a+b*(e*x+d)^2+c*(e*x+d)^4)/a
^2/e-1/2*(-2*a*c+b^2)*arctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/a^2/e/
(-4*a*c+b^2)^(1/2)
```

3.619.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.27

$$\int \frac{1}{(d+ex)^3 (a+b(d+ex)^2+c(d+ex)^4)} dx = \frac{-\frac{2a}{(d+ex)^2}-4b\log(d+ex)+\frac{\left(b^2-2ac+b\sqrt{b^2-4ac}\right)\log\left(b-\sqrt{b^2-4ac}+2c(d+ex)^2\right)}{\sqrt{b^2-4ac}}+\frac{\left(-b^2+2ac+b\sqrt{b^2-4ac}\right)\log\left(b+\sqrt{b^2-4ac}+2c(d\right.}{\sqrt{b^2-4ac}}}{4a^2e}$$

input
```
Integrate[1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

output
```
((-2*a)/(d + e*x)^2 - 4*b*Log[d + e*x] + ((b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*
c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/Sqrt[b^2 - 4*a*c] + ((-b
^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x
)^2])/Sqrt[b^2 - 4*a*c])/(4*a^2*e)
```

3.619.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1462, 1434, 1145, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)}dx$$

$$\Big\downarrow \text{ 1462}$$

$$\frac{\int \frac{1}{(d+ex)^3(c(d+ex)^4+b(d+ex)^2+a)}d(d+ex)}{e}$$

$$\Big\downarrow \text{ 1434}$$

$$\frac{\int \frac{1}{(d+ex)^4(c(d+ex)^4+b(d+ex)^2+a)}d(d+ex)^2}{2e}$$

$$\Big\downarrow \text{ 1145}$$

$$\frac{\frac{\int -\frac{c(d+ex)^2+b}{(d+ex)^2\left(c(d+ex)^4+b(d+ex)^2+a\right)}d(d+ex)^2}{a}-\frac{1}{a(d+ex)^2}}{2e}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{-\frac{\int \frac{c(d+ex)^2+b}{(d+ex)^2\left(c(d+ex)^4+b(d+ex)^2+a\right)}d(d+ex)^2}{a}-\frac{1}{a(d+ex)^2}}{2e}$$

$$\Big\downarrow \text{ 1200}$$

$$\frac{-\frac{\int \left(\frac{b}{a(d+ex)^2}+\frac{-b^2-c(d+ex)^2b+ac}{a\left(c(d+ex)^4+b(d+ex)^2+a\right)}\right)d(d+ex)^2}{a}-\frac{1}{a(d+ex)^2}}{2e}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{-\frac{(b^2-2ac)\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}-\frac{b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{2a}+\frac{b\log\left((d+ex)^2\right)}{a}}{a}-\frac{1}{a(d+ex)^2}}{2e}$$

input
```
Int[1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

output
```
(-(1/(a*(d + e*x)^2)) - (((b^2 - 2*a*c)*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt
[b^2 - 4*a*c]])/(a*Sqrt[b^2 - 4*a*c]) + (b*Log[(d + e*x)^2])/a - (b*Log[a
+ b*(d + e*x)^2 + c*(d + e*x)^4])/(2*a))/a)/(2*e)
```

3.619.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1145
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_^2), x_Symbol]
 :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp
[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[m, -1]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.619. $\int \frac{1}{(d+ex)^3(a+b(d+ex)^2+c(d+ex)^4)}\,dx$

3.619.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.66 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.76

method	result
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(c\,e^4\_Z^4+4cd\,e^3\_Z^3+\left(6c\,d^2e^2+b\,e^2\right)\_Z^2+\left(4d^3ec+2bde\right)\_Z+d^4c+b\,d^2+a\right)}\frac{\left(bc\,e^3\_R^3+3bcd\,e^2\_R^2+e\left(3bc\,d^2-ac+b^2\right)\_R\right.}{2e^3c\_R^3+6cd\,e^2\_R^2+6c\,d^2e\_R}}{2a^2e}$
risch	$-\frac{1}{2ae(ex+d)^2}-\frac{b\ln(ex+d)}{a^2e}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4a^3c\,e^2-a^2b^2e^2\right)\_Z^2+\left(-4abce+b^3e\right)\_Z+c^2\right)}\_R\ln\left(\left(\left(10a^3c\,e^4-3a^2b^2e^4\right)\_R^2-\right.\right.\right.}{}$

input
```
int(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x,method=_RETURNVERBOSE)
```

output
```
1/2/a^2/e*sum((b*c*e^3*_R^3+3*b*c*d*e^2*_R^2+e*(3*b*c*d^2-a*c+b^2)*_R+b*c*
d^3-a*c*d+b^2*d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+
b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2
+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a))-1/2/a/e/(e*x+d)^2-b*ln(e*x+d)/a^2/
e
```

3.619.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 340 vs. 2(111) = 222.

Time = 0.29 (sec) , antiderivative size = 810, normalized size of antiderivative = 6.69

$$\int \frac{1}{(d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx$$
$$=\left[-\frac{2\,ab^2-8\,a^2c+\left((b^2-2\,ac)e^2x^2+2\,(b^2-2\,ac)dex+(b^2-2\,ac)d^2\right)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2e^4x^4+8\,c^2de^3x^3+2\,c^2\cdots}{\cdots}\right.}{\cdots}\right.$$
$$-\frac{2\,ab^2-8\,a^2c+2\left((b^2-2\,ac)e^2x^2+2\,(b^2-2\,ac)dex+(b^2-2\,ac)d^2\right)\sqrt{-b^2+4\,ac}\arctan\left(-\frac{(2\,ce^2x^2+4\,cde\cdots}{\cdots}\right.}{\cdots}$$

input
```
integrate(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

output
```
[-1/4*(2*a*b^2 - 8*a^2*c + ((b^2 - 2*a*c)*e^2*x^2 + 2*(b^2 - 2*a*c)*d*e*x
+ (b^2 - 2*a*c)*d^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^
3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b
*c*d)*e*x + b^2 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2
 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*
d^2 + 2*(2*c*d^3 + b*d)*e*x + a)) - ((b^3 - 4*a*b*c)*e^2*x^2 + 2*(b^3 - 4*
a*b*c)*d*e*x + (b^3 - 4*a*b*c)*d^2)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4
+ (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*((b^3 - 4
*a*b*c)*e^2*x^2 + 2*(b^3 - 4*a*b*c)*d*e*x + (b^3 - 4*a*b*c)*d^2)*log(e*x +
 d))/((a^2*b^2 - 4*a^3*c)*e^3*x^2 + 2*(a^2*b^2 - 4*a^3*c)*d*e^2*x + (a^2*b
^2 - 4*a^3*c)*d^2*e), -1/4*(2*a*b^2 - 8*a^2*c + 2*((b^2 - 2*a*c)*e^2*x^2 +
 2*(b^2 - 2*a*c)*d*e*x + (b^2 - 2*a*c)*d^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*
c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (
(b^3 - 4*a*b*c)*e^2*x^2 + 2*(b^3 - 4*a*b*c)*d*e*x + (b^3 - 4*a*b*c)*d^2)*l
og(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(
2*c*d^3 + b*d)*e*x + a) + 4*((b^3 - 4*a*b*c)*e^2*x^2 + 2*(b^3 - 4*a*b*c)*d
*e*x + (b^3 - 4*a*b*c)*d^2)*log(e*x + d))/((a^2*b^2 - 4*a^3*c)*e^3*x^2 + 2
*(a^2*b^2 - 4*a^3*c)*d*e^2*x + (a^2*b^2 - 4*a^3*c)*d^2*e)]
```

3.619.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^3 (a+b(d+ex)^2+c(d+ex)^4)} \, dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

output
```
Timed out
```

3.619.7 Maxima [F]

$$\int \frac{1}{(d+ex)^3 (a+b(d+ex)^2+c(d+ex)^4)} \, dx = \int \frac{1}{\left((ex+d)^4c+(ex+d)^2b+a\right)(ex+d)^3} \, dx$$

input
```
integrate(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

output
```
-1/2/(a*e^3*x^2 + 2*a*d*e^2*x + a*d^2*e) + integrate((b*c*e^3*x^3 + 3*b*c*
d*e^2*x^2 + b*c*d^3 + (3*b*c*d^2 + b^2 - a*c)*e*x + (b^2 - a*c)*d)/(c*e^4*
x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 +
 b*d)*e*x + a), x)/a^2 - b*log(e*x + d)/(a^2*e)
```

3.619.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.83

$$\int \frac{1}{(d+ex)^3(a+b(d+ex)^2+c(d+ex)^4)}\,dx = \frac{b\log\left(c+\frac{b}{(ex+d)^2}+\frac{a}{(ex+d)^4}\right)}{4\,a^2e} + \frac{(b^2-2\,ac)\arctan\left(-\frac{b+\frac{2\,a}{(ex+d)^2}}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}a^2e} - \frac{1}{2\,(ex+d)^2ae}$$

input
```
integrate(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

output
```
1/4*b*log(c + b/(e*x + d)^2 + a/(e*x + d)^4)/(a^2*e) + 1/2*(b^2 - 2*a*c)*a
rctan(-(b + 2*a/(e*x + d)^2)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^2*e
) - 1/2/((e*x + d)^2*a*e)
```

3.619.9 Mupad [B] (verification not implemented)

Time = 12.24 (sec) , antiderivative size = 4950, normalized size of antiderivative = 40.91

$$\int \frac{1}{(d+ex)^3(a+b(d+ex)^2+c(d+ex)^4)}\,dx = \text{Too large to display}$$

input
```
int(1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x)
```

output
```
(atan((16*a^6*x^2*(4*a*c - b^2)^(3/2)*(((3*b^4 + a^2*c^2 - 9*a*b^2*c)*((((
((20*a^3*c^4*e^18 + 2*a^2*b^2*c^3*e^18)/a^3 + ((2*b^3*e - 8*a*b*c*e)*(40*a
^4*b*c^3*e^19 - 12*a^3*b^3*c^2*e^19))/(2*a^3*(16*a^3*c*e^2 - 4*a^2*b^2*e^2
)))*(2*b^3*e - 8*a*b*c*e))/(2*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)) + (6*b*c^4*e
^17)/a^2)*(2*b^3*e - 8*a*b*c*e))/(2*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)) + (c^5
*e^16)/a^3 - (((((20*a^3*c^4*e^18 + 2*a^2*b^2*c^3*e^18)/a^3 + ((2*b^3*e -
8*a*b*c*e)*(40*a^4*b*c^3*e^19 - 12*a^3*b^3*c^2*e^19))/(2*a^3*(16*a^3*c*e^2
 - 4*a^2*b^2*e^2)))*(2*a*c - b^2))/(4*a^2*e*(4*a*c - b^2)^(1/2)) + ((2*a*c
 - b^2)*(2*b^3*e - 8*a*b*c*e)*(40*a^4*b*c^3*e^19 - 12*a^3*b^3*c^2*e^19))/(
8*a^5*e*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)*(4*a*c - b^2)^(1/2)))*(2*a*c - b^2)
)/(4*a^2*e*(4*a*c - b^2)^(1/2)) - ((2*a*c - b^2)^2*(2*b^3*e - 8*a*b*c*e)*(
40*a^4*b*c^3*e^19 - 12*a^3*b^3*c^2*e^19))/(32*a^7*e^2*(16*a^3*c*e^2 - 4*a^
2*b^2*e^2)*(4*a*c - b^2))))/(8*a^3*c^2*(a^2*c^2 - 6*b^4 + 24*a*b^2*c)) + (
((((((20*a^3*c^4*e^18 + 2*a^2*b^2*c^3*e^18)/a^3 + ((2*b^3*e - 8*a*b*c*e)*(
40*a^4*b*c^3*e^19 - 12*a^3*b^3*c^2*e^19))/(2*a^3*(16*a^3*c*e^2 - 4*a^2*b^2
*e^2)))*(2*a*c - b^2))/(4*a^2*e*(4*a*c - b^2)^(1/2)) + ((2*a*c - b^2)*(2*b
^3*e - 8*a*b*c*e)*(40*a^4*b*c^3*e^19 - 12*a^3*b^3*c^2*e^19))/(8*a^5*e*(16*
a^3*c*e^2 - 4*a^2*b^2*e^2)*(4*a*c - b^2)^(1/2)))*(2*b^3*e - 8*a*b*c*e))/(2
*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)) + (((((20*a^3*c^4*e^18 + 2*a^2*b^2*c^3*e^
18)/a^3 + ((2*b^3*e - 8*a*b*c*e)*(40*a^4*b*c^3*e^19 - 12*a^3*b^3*c^2*e^...
```

3.619. $\int \frac{1}{(d+ex)^3(a+b(d+ex)^2+c(d+ex)^4)}\,dx$

3.620 $\int \frac{1}{(d+ex)^4(a+b(d+ex)^2+c(d+ex)^4)}\,dx$

3.620.1 Optimal result

Integrand size = 30, antiderivative size = 224

$$\int \frac{1}{(d+ex)^4\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx = -\frac{1}{3ae(d+ex)^3}+\frac{b}{a^2e(d+ex)} + \frac{\sqrt{c}\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b-\sqrt{b^2-4ac}}e} + \frac{\sqrt{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b+\sqrt{b^2-4ac}}e}$$

output
```
-1/3/a/e/(e*x+d)^3+b/a^2/e/(e*x+d)+1/2*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b-(
-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))/a^2/
e*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/2*arctan((e*x+d)*2^(1/2)*c^(1/2)/
(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))/a
^2/e*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.620.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.05

$$\int \frac{1}{(d+ex)^4 (a+b(d+ex)^2+c(d+ex)^4)}\,dx = \frac{-\frac{2a}{(d+ex)^3}+\frac{6b}{d+ex}+\frac{3\sqrt{2}\sqrt{c}\left(b^2-2ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{2}\sqrt{c}\left(-b^2+2ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}}{6a^2e}$$

input
```
Integrate[1/((d + e*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

output
```
((-2*a)/(d + e*x)^3 + (6*b)/(d + e*x) + (3*Sqrt[2]*Sqrt[c]*(b^2 - 2*a*c +
b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2
- 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*S
qrt[c]*(-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e
*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 -
4*a*c]]))/(6*a^2*e)
```

3.620.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1462, 1443, 27, 1604, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^4 (a+b(d+ex)^2+c(d+ex)^4)}\,dx$$

$$\Big\downarrow \text{1462}$$

$$\frac{\int \frac{1}{(d+ex)^4(c(d+ex)^4+b(d+ex)^2+a)}d(d+ex)}{e}$$

$$\Big\downarrow \text{1443}$$

$$\frac{\frac{\int -\frac{3\left(c(d+ex)^2+b\right)}{(d+ex)^2\left(c(d+ex)^4+b(d+ex)^2+a\right)}d(d+ex)}{3a}-\frac{1}{3a(d+ex)^3}}{e}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{\int \frac{c(d+ex)^2+b}{(d+ex)^2\left(c(d+ex)^4+b(d+ex)^2+a\right)}d(d+ex)}{a}-\frac{1}{3a(d+ex)^3}}{e}$$

$$\Big\downarrow \; 1604$$

$$\frac{-\frac{-\frac{\int \frac{b^2+c(d+ex)^2b-ac}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)}{a}-\frac{b}{a(d+ex)}}{a}-\frac{1}{3a(d+ex)^3}}{e}$$

$$\Big\downarrow \; 1480$$

$$\frac{-\frac{-\frac{\frac{1}{2}c\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\int \frac{1}{c(d+ex)^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}d(d+ex)+\frac{1}{2}c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{c(d+ex)^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}d(d+ex)}{a}-\frac{b}{a(d+ex)}}{a}-\frac{1}{3a(d+ex)^3}}{e}$$

$$\Big\downarrow \; 218$$

$$\frac{-\frac{-\frac{\frac{\sqrt{c}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}}{a}-\frac{b}{a(d+ex)}}{a}-\frac{1}{3a(d+ex)^3}}{e}$$

input

```
Int[1/((d + e*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

output

```
(-1/3*1/(a*(d + e*x)^3) - (-(b/(a*(d + e*x))) - ((Sqrt[c]*(b + (b^2 - 2*a*
c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2
 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(b - (b^2 -
2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt
[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/a)/a)/e
```

3.620.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

3.620. $\int \frac{1}{(d+ex)^4(a+b(d+ex)^2+c(d+ex)^4)}\,dx$

rule 1443
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Sim
p[1/(a*d^2*(m + 1))   Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*
x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

3.620.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.70 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.84

method	result
default	$\frac{\sum_{\_R=\operatorname{RootOf}\left(c\,e^{4}\_Z^{4}+4cd\,e^{3}\_Z^{3}+\left(6c\,d^{2}e^{2}+b\,e^{2}\right)\_Z^{2}+\left(4d^{3}ec+2bde\right)\_Z+d^{4}c+b\,d^{2}+a\right)}\frac{\left(\_R^{2}bc\,e^{2}+2\_Rbcde+bc\,d^{2}-ac+b^{2}\right)\ln\left(x-\_\right.}{2e^{3}c\_R^{3}+6cd\,e^{2}\_R^{2}+6c\,d^{2}e\_R+2d^{3}c+be\_}}{2a^{2}e}$
risch	$\frac{\frac{be\,x^{2}}{a^{2}}+\frac{2bdx}{a^{2}}-\frac{-3b\,d^{2}+a}{3e\,a^{2}}}{\left(ex+d\right)^{3}}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\left(16e^{4}c^{2}a^{7}-8a^{6}b^{2}c\,e^{4}+a^{5}b^{4}e^{4}\right)\_Z^{4}+\left(-20b\,e^{2}c^{3}a^{3}+25b^{3}e^{2}c^{2}a^{2}-9b^{5}e^{2}ca+b^{7}e^{2}\right)\_Z^{2}+c^{5}\right)}\_R\right.}{}$

3.620. $\int \frac{1}{(d+ex)^4(a+b(d+ex)^2+c(d+ex)^4)}\,dx$

input
```
int(1/(e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x,method=_RETURNVERBOSE)
```

output
```
1/2/a^2/e*sum((_R^2*b*c*e^2+2*_R*b*c*d*e+b*c*d^2-a*c+b^2)/(2*_R^3*c*e^3+6*
_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^
4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d
^2+a))-1/3/a/e/(e*x+d)^3+b/a^2/e/(e*x+d)
```

3.620.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2044 vs. $2(188) = 376$.

Time = 0.33 (sec) , antiderivative size = 2044, normalized size of antiderivative = 9.12

$$\int \frac{1}{(d+ex)^4\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

output
```
1/6*(6*b*e^2*x^2 + 12*b*d*e*x + 6*b*d^2 + 3*sqrt(1/2)*(a^2*e^4*x^3 + 3*a^2
*d*e^3*x^2 + 3*a^2*d^2*e^2*x + a^2*d^3*e)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b
*c^2 + (a^5*b^2 - 4*a^6*c)*e^2*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*
a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4)))/((a^5*b^2 - 4*a^6*c)*
e^2))*log(2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*e*x + 2*(b^4*c^3 - 3*a*b^2*c
^4 + a^2*c^5)*d + sqrt(1/2)*((a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^2)*e^3*sq
rt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2
 - 4*a^11*c)*e^4)) - (b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*b^2*c^3 +
4*a^4*c^4)*e)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^2 - 4*a^6*c)*e
^2*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^1
0*b^2 - 4*a^11*c)*e^4)))/((a^5*b^2 - 4*a^6*c)*e^2))) - 3*sqrt(1/2)*(a^2*e^
4*x^3 + 3*a^2*d*e^3*x^2 + 3*a^2*d^2*e^2*x + a^2*d^3*e)*sqrt(-(b^5 - 5*a*b^
3*c + 5*a^2*b*c^2 + (a^5*b^2 - 4*a^6*c)*e^2*sqrt((b^8 - 6*a*b^6*c + 11*a^2
*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4)))/((a^5*b^
2 - 4*a^6*c)*e^2))*log(2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*e*x + 2*(b^4*c^
3 - 3*a*b^2*c^4 + a^2*c^5)*d - sqrt(1/2)*((a^5*b^5 - 7*a^6*b^3*c + 12*a^7*
b*c^2)*e^3*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^
4)/((a^10*b^2 - 4*a^11*c)*e^4)) - (b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a
^3*b^2*c^3 + 4*a^4*c^4)*e)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^2
 - 4*a^6*c)*e^2*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 ...
```

3.620.6 Sympy [A] (verification not implemented)

Time = 104.66 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.55

$$\int \frac{1}{(d+ex)^4(a+b(d+ex)^2+c(d+ex)^4)}\,dx = \frac{-a+3bd^2+6bdex+3be^2x^2}{3a^2d^3e+9a^2d^2e^2x+9a^2de^3x^2+3a^2e^4x^3} + \mathrm{RootSum}\Big(t^4\cdot\left(256a^7c^2e^4-128a^6b^2ce^4+16a^5b^4e^4\right)+t^2\left(-80a^3bc^3e^2+100a^2b^3c^2e^2-36ab^5ce^2+4b^7e\right.$$

input
```
integrate(1/(e*x+d)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

output
```
(-a + 3*b*d**2 + 6*b*d*e*x + 3*b*e**2*x**2)/(3*a**2*d**3*e + 9*a**2*d**2*e
**2*x + 9*a**2*d*e**3*x**2 + 3*a**2*e**4*x**3) + RootSum(_t**4*(256*a**7*c
**2*e**4 - 128*a**6*b**2*c*e**4 + 16*a**5*b**4*e**4) + _t**2*(-80*a**3*b*c
**3*e**2 + 100*a**2*b**3*c**2*e**2 - 36*a*b**5*c*e**2 + 4*b**7*e**2) + c**
5, Lambda(_t, _t*log(x + (-96*_t**3*a**7*b*c**2*e**3 + 56*_t**3*a**6*b**3*
c*e**3 - 8*_t**3*a**5*b**5*e**3 - 4*_t*a**4*c**4*e + 32*_t*a**3*b**2*c**3*
e - 40*_t*a**2*b**4*c**2*e + 16*_t*a*b**6*c*e - 2*_t*b**8*e + a**2*c**5*d
- 3*a*b**2*c**4*d + b**4*c**3*d)/(a**2*c**5*e - 3*a*b**2*c**4*e + b**4*c**
3*e)))
```

3.620.7 Maxima [F]

$$\int \frac{1}{(d+ex)^4(a+b(d+ex)^2+c(d+ex)^4)}\,dx = \int \frac{1}{\left((ex+d)^4c+(ex+d)^2b+a\right)(ex+d)^4}\,dx$$

input
```
integrate(1/(e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

output
```
1/3*(3*b*e^2*x^2 + 6*b*d*e*x + 3*b*d^2 - a)/(a^2*e^4*x^3 + 3*a^2*d*e^3*x^2
 + 3*a^2*d^2*e^2*x + a^2*d^3*e) + integrate((b*c*e^2*x^2 + 2*b*c*d*e*x + b
*c*d^2 + b^2 - a*c)/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2
*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a), x)/a^2
```

3.620.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1347 vs. $2(188) = 376$.

Time = 0.30 (sec) , antiderivative size = 1347, normalized size of antiderivative = 6.01

$$\int \frac{1}{(d+ex)^4 (a+b(d+ex)^2+c(d+ex)^4)} \, dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

output
```
-1/2*((b*c*e^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) +
 d/e)^2 - 2*b*c*d*e*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^
4)) + d/e) + b*c*d^2 + b^2 - a*c)*log(x + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^
2 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^
2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^3 - 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 + sq
rt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2
+ b*e^2)*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e))
 - (b*c*e^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/
e)^2 + 2*b*c*d*e*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4))
 - d/e) + b*c*d^2 + b^2 - a*c)*log(x - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 -
 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 -
 4*a*c)*e^2)/(c*e^4)) - d/e)^3 + 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(
b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2 + 2*c*d^3*e + b*d*e + (6*c*d^2*e^2 + b
*e^2)*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)) +
(b*c*e^2*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^
2 - 2*b*c*d*e*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) +
d/e) + b*c*d^2 + b^2 - a*c)*log(x + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*
a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*
a*c)*e^2)/(c*e^4)) + d/e)^3 - 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2
 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b...
```

3.620.9 Mupad [B] (verification not implemented)

Time = 9.48 (sec) , antiderivative size = 5214, normalized size of antiderivative = 23.28

$$\int \frac{1}{(d+ex)^4 (a+b(d+ex)^2+c(d+ex)^4)} \, dx = \text{Too large to display}$$

input
```
int(1/((d + e*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x)
```

output
```
((2*b*d*x)/a^2 - (a - 3*b*d^2)/(3*a^2*e) + (b*e*x^2)/a^2)/(d^3 + e^3*x^3 +
 3*d*e^2*x^2 + 3*d^2*e*x) - atan((((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 2
0*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*
c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 -
 8*a^6*b^2*c*e^2)))^(1/2)*(x*(4*a^8*c^5*e^12 + 2*a^6*b^4*c^3*e^12 - 8*a^7*
b^2*c^4*e^12) - ((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a
^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4
*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2))
)^(1/2)*((x*(32*a^11*b*c^3*e^14 - 8*a^10*b^3*c^2*e^14) + 32*a^11*b*c^3*d*e
^13 - 8*a^10*b^3*c^2*d*e^13)*((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3
*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3
*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^
6*b^2*c*e^2)))^(1/2) - 16*a^10*c^4*e^12 - 4*a^8*b^4*c^2*e^12 + 20*a^9*b^2*
c^3*e^12) + 4*a^8*c^5*d*e^11 + 2*a^6*b^4*c^3*d*e^11 - 8*a^7*b^2*c^4*d*e^11
)*1i + ((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^
2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^
2)^3)^(1/2))/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2)))^(1/2)*(
x*(4*a^8*c^5*e^12 + 2*a^6*b^4*c^3*e^12 - 8*a^7*b^2*c^4*e^12) - ((b^4*(-(4*
a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a
*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8...
```

3.620. $\int \frac{1}{(d+ex)^4(a+b(d+ex)^2+c(d+ex)^4)}\,dx$

3.621 $\int \frac{(d+ex)^4}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$

3.621.1 Optimal result

Integrand size = 30, antiderivative size = 270

$$\int \frac{(d+ex)^4}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = \frac{(d+ex)\left(2a+b(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\left(b-\frac{b^2+4ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}e} + \frac{\left(b^2+4ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}e}$$

output
```
1/2*(e*x+d)*(2*a+b*(e*x+d)^2)/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)+1
/4*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-4*a*c
-b^2)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)/e*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(
1/2))^(1/2)+1/4*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2
))*(b^2+4*a*c+b*(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)/e*2^(1/2)/c^(1/2)/(
b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.621.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.97

$$\int \frac{(d+ex)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$$
$$= \frac{-\frac{2(-2a(d+ex)-b(d+ex)^3)}{(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)} + \frac{\sqrt{2}\left(-b^2-4ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\left(b^2+4ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex}{\sqrt{b+\sqrt{b^2-4}}}\right.}{\sqrt{c}(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}}{4e}$$

input
```
Integrate[(d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

output
```
((-2*(-2*a*(d + e*x) - b*(d + e*x)^3))/((b^2 - 4*a*c)*(a + b*(d + e*x)^2 +
 c*(d + e*x)^4)) + (Sqrt[2]*(-b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(S
qrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*
a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b^2 + 4*a*c + b*Sqrt[b
^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]
]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*e)
```

3.621.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1462, 1440, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$$
$$\Big\downarrow \ 1462$$
$$\frac{\int \frac{(d+ex)^4}{\left(c(d+ex)^4+b(d+ex)^2+a\right)^2}d(d+ex)}{e}$$
$$\Big\downarrow \ 1440$$
$$\frac{\frac{(d+ex)\left(2a+b(d+ex)^2\right)}{2(b^2-4ac)\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{\int \frac{2a-b(d+ex)^2}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)}{2(b^2-4ac)}}{e}$$
$$\Big\downarrow \ 1480$$

$$\frac{\frac{(d+ex)(2a+b(d+ex)^2)}{2(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)}-\frac{-\frac{1}{2}\left(b-\frac{4ac+b^2}{\sqrt{b^2-4ac}}\right)\int\frac{1}{c(d+ex)^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}d(d+ex)-\frac{1}{2}\left(\frac{4ac+b^2}{\sqrt{b^2-4ac}}+b\right)\int\frac{1}{c(d+ex)^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}d(}{2(b^2-4ac)}}{e}$$

$\downarrow$ 218

$$\frac{\frac{(d+ex)(2a+b(d+ex)^2)}{2(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)}-\frac{-\frac{\left(b-\frac{4ac+b^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{4ac+b^2}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{2(b^2-4ac)}}{e}$$

input
```
Int[(d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

output
```
(((d + e*x)*(2*a + b*(d + e*x)^2))/(2*(b^2 - 4*a*c)*(a + b*(d + e*x)^2 + c
*(d + e*x)^4)) - (-(((b - (b^2 + 4*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]
*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b
- Sqrt[b^2 - 4*a*c]])) - ((b + (b^2 + 4*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sq
rt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sq
rt[b + Sqrt[b^2 - 4*a*c]]))/(2*(b^2 - 4*a*c)))/e
```

3.621.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1440
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(-d^3)*(d*x)^(m - 3)*(2*a + b*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*
(p + 1)*(b^2 - 4*a*c))), x] + Simp[d^4/(2*(p + 1)*(b^2 - 4*a*c))   Int[(d*x
)^(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1),
x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Gt
Q[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

3.621. $\int \frac{(d+ex)^4}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

3.621.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.63 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.20

method	result
default	$\frac{-\frac{b e^2 x^3}{2(4ac-b^2)}-\frac{3bde\,x^2}{2(4ac-b^2)}-\frac{(3b\,d^2+2a)x}{2(4ac-b^2)}-\frac{d(b\,d^2+2a)}{2e(4ac-b^2)}}{c\,x^4e^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+d^4c+2bdex+b\,d^2+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\,e^4\_Z^4+4cd\,e^3\_Z^3+\left(6c\,d^2e^2+b\,e^2\right)\_Z^2+\left(4d^3ec-\right.\right.}}{}$
risch	$\frac{-\frac{b e^2 x^3}{2(4ac-b^2)}-\frac{3bde\,x^2}{2(4ac-b^2)}-\frac{(3b\,d^2+2a)x}{2(4ac-b^2)}-\frac{d(b\,d^2+2a)}{2e(4ac-b^2)}}{c\,x^4e^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+d^4c+2bdex+b\,d^2+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\,e^4\_Z^4+4cd\,e^3\_Z^3+\left(6c\,d^2e^2+b\,e^2\right)\_Z^2+\left(4d^3ec-\right.\right.}}{}$

input
```
int((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x,method=_RETURNVERBOSE)
```

output
```
(-1/2*b*e^2/(4*a*c-b^2)*x^3-3/2/(4*a*c-b^2)*b*d*e*x^2-1/2*(3*b*d^2+2*a)/(4
*a*c-b^2)*x-1/2*d/e*(b*d^2+2*a)/(4*a*c-b^2))/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*
d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)+1/4/(4*a*c-b^2)
/e*sum((-_R^2*b*e^2-2*_R*b*d*e-b*d^2+2*a)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_
R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3
+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a))
```

3.621.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2454 vs. 2(226) = 452.

Time = 0.32 (sec) , antiderivative size = 2454, normalized size of antiderivative = 9.09

$$\int \frac{(d+ex)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")
```

output
```
1/4*(2*b*e^3*x^3 + 6*b*d*e^2*x^2 + 2*b*d^3 + 2*(3*b*d^2 + 2*a)*e*x + sqrt(
1/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4
*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (
b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3
- 4*a*b*c)*d^2)*e)*sqrt(-((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*
c^4)*e^2*sqrt(1/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^
4)) + b^3 + 12*a*b*c)/((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4
)*e^2))*log((3*b^2 + 4*a*c)*e*x + (3*b^2 + 4*a*c)*d + sqrt(1/2)*(2*(b^7*c
- 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*e^3*sqrt(1/((b^6*c^2 - 12*
a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4)) + (b^4 - 8*a*b^2*c + 16*a^2
*c^2)*e)*sqrt(-((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2*s
qrt(1/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4)) + b^3
+ 12*a*b*c)/((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))) -
 sqrt(1/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b
^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d
^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c +
 (b^3 - 4*a*b*c)*d^2)*e)*sqrt(-((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 6
4*a^3*c^4)*e^2*sqrt(1/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c
^5)*e^4)) + b^3 + 12*a*b*c)/((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a
^3*c^4)*e^2))*log((3*b^2 + 4*a*c)*e*x + (3*b^2 + 4*a*c)*d - sqrt(1/2)*(...
```

3.621.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 573 vs. $2(243) = 486$.

Time = 11.78 (sec) , antiderivative size = 573, normalized size of antiderivative = 2.12

$$\int \frac{(d+ex)^4}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$$

$$= \frac{-2ad-bd^3-3bde^2x^2-be^3}{8a^2ce-2ab^2e+8abcd^2e+8ac^2d^4e-2b^3d^2e-2b^2cd^4e+x^4\cdot(8ac^2e^5-2b^2ce^5)+x^3\cdot(32ac^2de^4-8b^2cde}$$

$$+\mathrm{RootSum}\Big(t^4\cdot(1048576a^6c^7e^4-1572864a^5b^2c^6e^4+983040a^4b^4c^5e^4-327680a^3b^6c^4e^4+61440a^2b^8c^3e$$

input
```
integrate((e*x+d)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

output
```
(-2*a*d - b*d**3 - 3*b*d*e**2*x**2 - b*e**3*x**3 + x*(-2*a*e - 3*b*d**2*e)
)/(8*a**2*c*e - 2*a*b**2*e + 8*a*b*c*d**2*e + 8*a*c**2*d**4*e - 2*b**3*d**
2*e - 2*b**2*c*d**4*e + x**4*(8*a*c**2*e**5 - 2*b**2*c*e**5) + x**3*(32*a*
c**2*d*e**4 - 8*b**2*c*d*e**4) + x**2*(8*a*b*c*e**3 + 48*a*c**2*d**2*e**3
- 2*b**3*e**3 - 12*b**2*c*d**2*e**3) + x*(16*a*b*c*d*e**2 + 32*a*c**2*d**3
*e**2 - 4*b**3*d*e**2 - 8*b**2*c*d**3*e**2)) + RootSum(_t**4*(1048576*a**6
*c**7*e**4 - 1572864*a**5*b**2*c**6*e**4 + 983040*a**4*b**4*c**5*e**4 - 32
7680*a**3*b**6*c**4*e**4 + 61440*a**2*b**8*c**3*e**4 - 6144*a*b**10*c**2*e
**4 + 256*b**12*c*e**4) + _t**2*(-12288*a**4*b*c**4*e**2 + 8192*a**3*b**3*
c**3*e**2 - 1536*a**2*b**5*c**2*e**2 + 16*b**9*e**2) + 16*a**3*c**2 + 24*a
**2*b**2*c + 9*a*b**4, Lambda(_t, _t*log(x + (16384*_t**3*a**3*b*c**4*e**3
 - 12288*_t**3*a**2*b**3*c**3*e**3 + 3072*_t**3*a*b**5*c**2*e**3 - 256*_t*
*3*b**7*c*e**3 + 64*_t*a**2*c**2*e - 128*_t*a*b**2*c*e - 4*_t*b**4*e + 4*a
*c*d + 3*b**2*d)/(4*a*c*e + 3*b**2*e))))
```

3.621.7 Maxima [F]

$$\int \frac{(d+ex)^4}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = \int \frac{(ex+d)^4}{\left((ex+d)^4c+(ex+d)^2b+a\right)^2}\,dx$$

input
```
integrate((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")
```

output
```
1/2*(b*e^3*x^3 + 3*b*d*e^2*x^2 + b*d^3 + (3*b*d^2 + 2*a)*e*x + 2*a*d)/((b^
2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c +
6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a
*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c
)*d^2)*e) - 1/2*integrate(-(b*e^2*x^2 + 2*b*d*e*x + b*d^2 - 2*a)/((b^2*c -
 4*a*c^2)*e^4*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4
+ (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b
^3 - 4*a*b*c)*d^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e*x),
x)
```

3.621.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1408 vs. 2(226) = 452.

Time = 0.37 (sec) , antiderivative size = 1408, normalized size of antiderivative = 5.21

$$\int \frac{(d+ex)^4}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

output
```
-1/4*((b*e^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d
/e)^2 - 2*b*d*e*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4))
+ d/e) + b*d^2 - 2*a)*log(x + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e
^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e
^2)/(c*e^4)) + d/e)^3 - 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a
*c)*e^2)/(c*e^4)) + d/e)^2 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(sq
rt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)) - (b*e^2*(s
qrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2 + 2*b*d*e
*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e) + b*d^2
- 2*a)*log(x - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) +
d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) -
d/e)^3 + 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4
)) - d/e)^2 + 2*c*d^3*e + b*d*e + (6*c*d^2*e^2 + b*e^2)*(sqrt(1/2)*sqrt(-(
b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)) + (b*e^2*(sqrt(1/2)*sqrt(-
(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 - 2*b*d*e*(sqrt(1/2)*sqr
t(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e) + b*d^2 - 2*a)*log(x +
sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(
sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^3 - 6*c*d*
e^3*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 - 2
*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt...
```

3.621.9 Mupad [B] (verification not implemented)

Time = 10.48 (sec) , antiderivative size = 7327, normalized size of antiderivative = 27.14

$$\int \frac{(d+ex)^4}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = \text{Too large to display}$$

input
```
int((d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x)
```

output
```
atan(-(((((64*b^9*c^2*d*e^13 - 1024*a*b^7*c^3*d*e^13 + 16384*a^4*b*c^6*d*e
^13 + 6144*a^2*b^5*c^4*d*e^13 - 16384*a^3*b^3*c^5*d*e^13)/(8*(b^6 - 64*a^3
*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(16*b^7*c^2*e^14 - 192*a*b^5*c^3
*e^14 - 1024*a^3*b*c^5*e^14 + 768*a^2*b^3*c^4*e^14))/(2*(b^4 + 16*a^2*c^2
- 8*a*b^2*c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*
b^5*c^2 + 512*a^3*b^3*c^3)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*
c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^
2 - 6144*a^5*b^2*c^6*e^2)))^(1/2) - (2048*a^4*c^5*e^12 - 32*a*b^6*c^2*e^12
 + 384*a^2*b^4*c^3*e^12 - 1536*a^3*b^2*c^4*e^12)/(8*(b^6 - 64*a^3*c^3 + 48
*a^2*b^2*c^2 - 12*a*b^4*c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b
*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^
2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*
a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2) - (128*a^3*c^4*d*e^11 - 4*
b^6*c*d*e^11 + 8*a*b^4*c^2*d*e^11)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 -
 12*a*b^4*c)) + (x*(b^4*c*e^12 + 8*a^2*c^3*e^12 + 2*a*b^2*c^2*e^12))/(2*(b
^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4
*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(b^12*c*e^2 + 4096*a^6*c^7*
e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 384
0*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2)*1i + ((((64*b^9*c^2*d*e^
13 - 1024*a*b^7*c^3*d*e^13 + 16384*a^4*b*c^6*d*e^13 + 6144*a^2*b^5*c^4*...
```

3.621. $\int \frac{(d+ex)^4}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$

3.622 $\int \frac{(d+ex)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

3.622.1 Optimal result

Integrand size = 30, antiderivative size = 97

$$\int \frac{(d+ex)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{2a+b(d+ex)^2}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{b\,\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}e}$$

output
```
1/2*(2*a+b*(e*x+d)^2)/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)-b*arctanh
((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)/e
```

3.622.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.03

$$\int \frac{(d+ex)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{\frac{2a+b(d+ex)^2}{(b^2-4ac)(a+(d+ex)^2(b+c(d+ex)^2))} - \frac{2b\arctan\left(\frac{b+2c(d+ex)^2}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}}{2e}$$

input
```
Integrate[(d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

output
```
((2*a + b*(d + e*x)^2)/((b^2 - 4*a*c)*(a + (d + e*x)^2*(b + c*(d + e*x)^2)
)) - (2*b*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)
^(3/2))/(2*e)
```

3.622.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1462, 1434, 1159, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$$

$$\Big\downarrow \; 1462$$

$$\frac{\int \frac{(d+ex)^3}{(c(d+ex)^4+b(d+ex)^2+a)^2}d(d+ex)}{e}$$

$$\Big\downarrow \; 1434$$

$$\frac{\int \frac{(d+ex)^2}{(c(d+ex)^4+b(d+ex)^2+a)^2}d(d+ex)^2}{2e}$$

$$\Big\downarrow \; 1159$$

$$\frac{\frac{b\int \frac{1}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)^2}{b^2-4ac}+\frac{2a+b(d+ex)^2}{(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)}}{2e}$$

$$\Big\downarrow \; 1083$$

$$\frac{\frac{2a+b(d+ex)^2}{(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)}-\frac{2b\int \frac{1}{-(d+ex)^4+b^2-4ac}d(2c(d+ex)^2+b)}{b^2-4ac}}{2e}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{2a+b(d+ex)^2}{(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)}-\frac{2b\,\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}}{2e}$$

input
```
Int[(d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

output
```
((2*a + b*(d + e*x)^2)/((b^2 - 4*a*c)*(a + b*(d + e*x)^2 + c*(d + e*x)^4))
 - (2*b*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3
/2))/(2*e)
```

3.622.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

3.622.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.66 (sec) , antiderivative size = 276, normalized size of antiderivative = 2.85

method	result
default	$\frac{-\frac{x^{2} e b}{2\left(4 a c-b^{2}\right)}-\frac{x b d}{4 a c-b^{2}}-\frac{b\,d^{2}+2 a}{2 e\left(4 a c-b^{2}\right)}}{c\,x^{4} e^{4}+4 c d\,e^{3} x^{3}+6 c\,d^{2} e^{2} x^{2}+4 c\,d^{3} e x+b\,e^{2} x^{2}+d^{4} c+2 b d e x+b\,d^{2}+a}+\frac{b \left(\sum_{\_R=\operatorname{RootOf}\left(c\,e^{4}\_Z^{4}+4 c d\,e^{3}\_Z^{3}+\left(6 c\,d^{2} e^{2}+b\,e^{2}\right)\_Z^{2}+\left(4 d^{3}\right.\right.}\right.}{}$
risch	$\frac{-\frac{x^{2} e b}{2\left(4 a c-b^{2}\right)}-\frac{x b d}{4 a c-b^{2}}-\frac{b\,d^{2}+2 a}{2 e\left(4 a c-b^{2}\right)}}{c\,x^{4} e^{4}+4 c d\,e^{3} x^{3}+6 c\,d^{2} e^{2} x^{2}+4 c\,d^{3} e x+b\,e^{2} x^{2}+d^{4} c+2 b d e x+b\,d^{2}+a}+\frac{b \ln\left(\left(-\left(-4 a c+b^{2}\right)^{\frac{3}{2}} e^{2}+4 a b\,e^{2} c-b^{3} e^{2}\right) x^{2}+\left(-2\left(-4 a c+b^{2}\right.\right.\right.}{2 e}$

3.622. $\int \frac{(d+ex)^3}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$

input
```
int((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x,method=_RETURNVERBOSE)
```

output
```
(-1/2/(4*a*c-b^2)*x^2*e*b-1/(4*a*c-b^2)*x*b*d-1/2/e*(b*d^2+2*a)/(4*a*c-b^2
))/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*
b*d*e*x+b*d^2+a)+1/2*b/(4*a*c-b^2)/e*sum((-_R*e-d)/(2*_R^3*c*e^3+6*_R^2*c*
d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d
*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a))
```

3.622.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 445 vs. $2(91) = 182$.

Time = 0.31 (sec) , antiderivative size = 1021, normalized size of antiderivative = 10.53

$$\int \frac{(d+ex)^3}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$$
$$= \left[\frac{(b^3-4\,abc)e^2x^2+2\,(b^3-4\,abc)dex+2\,ab^2-8\,a^2c+(b^3-4\,abc)d^2-(bce^4x^4+4\,bcde^3x^3+}{2\,((b^4c-8\,ab^2c^2+16\,a^2c^3)e^5x^4+4\,(b^4c-8\,ab^2c^2+16\,a^2c^3)de^4x^3+(b^5-8\,ab^3c+16\,a^2bc^2+6\,(b^4c-8\right.$$

input
```
integrate((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")
```

output
```
[1/2*((b^3 - 4*a*b*c)*e^2*x^2 + 2*(b^3 - 4*a*b*c)*d*e*x + 2*a*b^2 - 8*a^2*
c + (b^3 - 4*a*b*c)*d^2 - (b*c*e^4*x^4 + 4*b*c*d*e^3*x^3 + b*c*d^4 + (6*b*
c*d^2 + b^2)*e^2*x^2 + b^2*d^2 + 2*(2*b*c*d^3 + b^2*d)*e*x + a*b)*sqrt(b^2
 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2
+ b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c + (2*
c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^4*x^4 + 4*c*d
*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x +
 a)))/((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e^5*x^4 + 4*(b^4*c - 8*a*b^2*c^2
 + 16*a^2*c^3)*d*e^4*x^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + 6*(b^4*c - 8*
a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^3*x^2 + 2*(2*(b^4*c - 8*a*b^2*c^2 + 16*a^2*
c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*e^2*x + (a*b^4 - 8*a^2*b^2*
c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^4 + (b^5 - 8*a*b^3*c
 + 16*a^2*b*c^2)*d^2)*e), 1/2*((b^3 - 4*a*b*c)*e^2*x^2 + 2*(b^3 - 4*a*b*c)
*d*e*x + 2*a*b^2 - 8*a^2*c + (b^3 - 4*a*b*c)*d^2 - 2*(b*c*e^4*x^4 + 4*b*c*
d*e^3*x^3 + b*c*d^4 + (6*b*c*d^2 + b^2)*e^2*x^2 + b^2*d^2 + 2*(2*b*c*d^3 +
 b^2*d)*e*x + a*b)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2
*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/((b^4*c - 8*a*b^2*c^2 + 16*
a^2*c^3)*e^5*x^4 + 4*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*e^4*x^3 + (b^5 -
 8*a*b^3*c + 16*a^2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^3*
x^2 + 2*(2*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + ...
```

3.622.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 495 vs. $2(80) = 160$.

Time = 2.65 (sec) , antiderivative size = 495, normalized size of antiderivative = 5.10

$$\int \frac{(d+ex)^3}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = \frac{b\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(\frac{2dx}{e}+x^2+\frac{-16a^2bc^2\sqrt{-\frac{1}{(4ac-b^2)^3}}+8ab^3c\sqrt{-\frac{1}{(4ac-b^2)^3}}-b^5\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^2+2bcd^2}{2bce^2}\right)}{2e} - \frac{b\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(\frac{2dx}{e}+x^2+\frac{16a^2bc^2\sqrt{-\frac{1}{(4ac-b^2)^3}}-8ab^3c\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^5\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^2+2bcd^2}{2bce^2}\right)}{2e} + \frac{-2a-bd^2-2}{8a^2ce-2ab^2e+8abcd^2e+8ac^2d^4e-2b^3d^2e-2b^2cd^4e+x^4\cdot(8ac^2e^5-2b^2ce^5)+x^3\cdot(32ac^2de^4-8b^2c}$$

input
```
integrate((e*x+d)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

output
```
b*sqrt(-1/(4*a*c - b**2)**3)*log(2*d*x/e + x**2 + (-16*a**2*b*c**2*sqrt(-1
/(4*a*c - b**2)**3) + 8*a*b**3*c*sqrt(-1/(4*a*c - b**2)**3) - b**5*sqrt(-1
/(4*a*c - b**2)**3) + b**2 + 2*b*c*d**2)/(2*b*c*e**2))/(2*e) - b*sqrt(-1/(
4*a*c - b**2)**3)*log(2*d*x/e + x**2 + (16*a**2*b*c**2*sqrt(-1/(4*a*c - b*
*2)**3) - 8*a*b**3*c*sqrt(-1/(4*a*c - b**2)**3) + b**5*sqrt(-1/(4*a*c - b*
*2)**3) + b**2 + 2*b*c*d**2)/(2*b*c*e**2))/(2*e) + (-2*a - b*d**2 - 2*b*d*
e*x - b*e**2*x**2)/(8*a**2*c*e - 2*a*b**2*e + 8*a*b*c*d**2*e + 8*a*c**2*d*
*4*e - 2*b**3*d**2*e - 2*b**2*c*d**4*e + x**4*(8*a*c**2*e**5 - 2*b**2*c*e*
*5) + x**3*(32*a*c**2*d*e**4 - 8*b**2*c*d*e**4) + x**2*(8*a*b*c*e**3 + 48*
a*c**2*d**2*e**3 - 2*b**3*e**3 - 12*b**2*c*d**2*e**3) + x*(16*a*b*c*d*e**2
 + 32*a*c**2*d**3*e**2 - 4*b**3*d*e**2 - 8*b**2*c*d**3*e**2))
```

3.622.7 Maxima [F]

$$\int \frac{(d+ex)^3}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = \int \frac{(ex+d)^3}{\left((ex+d)^4c+(ex+d)^2b+a\right)^2}\,dx$$

input
```
integrate((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")
```

output
```
-b*integrate(-(e*x + d)/((b^2*c - 4*a*c^2)*e^4*x^4 + 4*(b^2*c - 4*a*c^2)*d
*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^
2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2 + 2*(2*(b^2*c - 4*a*c^2
)*d^3 + (b^3 - 4*a*b*c)*d)*e*x), x) + 1/2*(b*e^2*x^2 + 2*b*d*e*x + b*d^2 +
 2*a)/((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 -
4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 +
(b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3
 - 4*a*b*c)*d^2)*e)
```

3.622.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.67

$$\int \frac{(d+ex)^3}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = \frac{b\arctan\left(\frac{2\,cd^2+2\,(ex^2+2\,dx)ce+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}e} + \frac{bd^2+(ex^2+2\,dx)be+2\,a}{2\,\left(cd^4+2\,(ex^2+2\,dx)cd^2e+(ex^2+2\,dx)^2ce^2+bd^2+(ex^2+2\,dx)be+a\right)(b^2e-4\,ace)}$$

input
```
integrate((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

output
```
b*arctan((2*c*d^2 + 2*(e*x^2 + 2*d*x)*c*e + b)/sqrt(-b^2 + 4*a*c))/((b^2 -
 4*a*c)*sqrt(-b^2 + 4*a*c)*e) + 1/2*(b*d^2 + (e*x^2 + 2*d*x)*b*e + 2*a)/((
c*d^4 + 2*(e*x^2 + 2*d*x)*c*d^2*e + (e*x^2 + 2*d*x)^2*c*e^2 + b*d^2 + (e*x
^2 + 2*d*x)*b*e + a)*(b^2*e - 4*a*c*e))
```

3.622.9 Mupad [B] (verification not implemented)

Time = 8.60 (sec) , antiderivative size = 427, normalized size of antiderivative = 4.40

$$\int \frac{(d+ex)^3}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$$
$$= \frac{b\,\mathrm{atan}\left(\frac{(4\,a\,c-b^2)^4\left(x\left(\frac{b^3\,\left(2\,b^3\,c^2\,d\,e^9-8\,a\,b\,c^3\,d\,e^9\right)}{a\,e^2\,\left(4\,a\,c-b^2\right)^{11/2}}-\frac{2\,b^2\,c^2\,d\,e^7}{a\,\left(4\,a\,c-b^2\right)^{7/2}}\right)+x^2\left(\frac{b^3\,\left(2\,b^3\,c^2\,e^{10}-8\,a\,b\,c^3\,e^{10}\right)}{2\,a\,e^2\,\left(4\,a\,c-b^2\right)^{11/2}}-\frac{b^2\,c^2\,e^8}{a\,\left(4\,a\,c-b^2\right)^{7/2}}\right)-\frac{b^3\,\left(16\,a^2\,c^3\,e^8-4\cdots\right.}{\cdots}\cdots}{2\,b^2\,c^2\,e^6}\right)}{e\,\left(4\,a\,c-b^2\right)^{3/2}}$$
$$-\frac{\frac{b\,d^2+2\,a}{2\,e\,\left(4\,a\,c-b^2\right)}+\frac{b\,e\,x^2}{2\,\left(4\,a\,c-b^2\right)}+\frac{b\,d\,x}{4\,a\,c-b^2}}{a+x^2\,\left(6\,c\,d^2\,e^2+b\,e^2\right)+b\,d^2+c\,d^4+x\,\left(4\,c\,e\,d^3+2\,b\,e\,d\right)+c\,e^4\,x^4+4\,c\,d\,e^3\,x^3}$$

input
```
int((d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x)
```

output
```
(b*atan(((4*a*c - b^2)^4*(x*((b^3*(2*b^3*c^2*d*e^9 - 8*a*b*c^3*d*e^9))/(a*
e^2*(4*a*c - b^2)^(11/2)) - (2*b^2*c^2*d*e^7)/(a*(4*a*c - b^2)^(7/2))) + x
^2*((b^3*(2*b^3*c^2*e^10 - 8*a*b*c^3*e^10))/(2*a*e^2*(4*a*c - b^2)^(11/2))
 - (b^2*c^2*e^8)/(a*(4*a*c - b^2)^(7/2))) - (b^3*(16*a^2*c^3*e^8 - 4*a*b^2
*c^2*e^8 - 2*b^3*c^2*d^2*e^8 + 8*a*b*c^3*d^2*e^8))/(2*a*e^2*(4*a*c - b^2)^
(11/2)) - (b^2*c^2*d^2*e^6)/(a*(4*a*c - b^2)^(7/2))))/(2*b^2*c^2*e^6)))/(e
*(4*a*c - b^2)^(3/2)) - ((2*a + b*d^2)/(2*e*(4*a*c - b^2)) + (b*e*x^2)/(2*
(4*a*c - b^2)) + (b*d*x)/(4*a*c - b^2))/(a + x^2*(b*e^2 + 6*c*d^2*e^2) + b
*d^2 + c*d^4 + x*(2*b*d*e + 4*c*d^3*e) + c*e^4*x^4 + 4*c*d*e^3*x^3)
```

3.623 $\int \frac{(d+ex)^2}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

3.623.1 Optimal result

Integrand size = 30, antiderivative size = 254

$$\int \frac{(d+ex)^2}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = -\frac{(d+ex)\left(b+2c(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\sqrt{c}\left(2b-\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}e} - \frac{\sqrt{c}\left(2b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}e}$$

output
```
-1/2*(e*x+d)*(b+2*c*(e*x+d)^2)/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)+
1/2*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(
2*b-(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)/e*2^(1/2)/(b-(-4*a*c+b^2)^(1/2)
)^(1/2)-1/2*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c
^(1/2)*(2*b+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)/e*2^(1/2)/(b+(-4*a*c+b^
2)^(1/2))^(1/2)
```

3.623.2 Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.97

$$\int \frac{(d+ex)^2}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx =$$

$$-\frac{\frac{b(d+ex)+2c(d+ex)^3}{(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)}+\frac{\sqrt{2}\sqrt{c}\left(-2b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(2b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}}{2e}$$

input
```
Integrate[(d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

output
```
-1/2*((b*(d + e*x) + 2*c*(d + e*x)^3)/((b^2 - 4*a*c)*(a + b*(d + e*x)^2 +
c*(d + e*x)^4)) + (Sqrt[2]*Sqrt[c]*(-2*b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt
[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*
Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(2*b + Sqrt[b^2 - 4*a*c])*
ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4
*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/e
```

3.623.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1462, 1439, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$$

$$\Big\downarrow \text{1462}$$

$$\frac{\int \frac{(d+ex)^2}{\left(c(d+ex)^4+b(d+ex)^2+a\right)^2}d(d+ex)}{e}$$

$$\Big\downarrow \text{1439}$$

$$\frac{\frac{\int \frac{b-2c(d+ex)^2}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)}{2(b^2-4ac)}-\frac{(d+ex)\left(b+2c(d+ex)^2\right)}{2(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)}}{e}$$

$$\Big\downarrow \text{1480}$$

$$\frac{\frac{-c\left(1-\frac{2b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{c(d+ex)^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}d(d+ex)-c\left(\frac{2b}{\sqrt{b^2-4ac}}+1\right)\int \frac{1}{c(d+ex)^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}d(d+ex)}{2(b^2-4ac)}-\frac{(d+ex)(b+2c(d+ex)^2)}{2(b^2-4ac)(a+b(d+ex)^2+c(d+e}}{e}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{-\frac{\sqrt{2}\sqrt{c}\left(1-\frac{2b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\left(\frac{2b}{\sqrt{b^2-4ac}}+1\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}}}{2(b^2-4ac)}-\frac{(d+ex)(b+2c(d+ex)^2)}{2(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)}}{e}$$

input
```
Int[(d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

output
```
(-1/2*((d + e*x)*(b + 2*c*(d + e*x)^2))/((b^2 - 4*a*c)*(a + b*(d + e*x)^2
+ c*(d + e*x)^4)) + (-((Sqrt[2]*Sqrt[c]*(1 - (2*b)/Sqrt[b^2 - 4*a*c])*ArcT
an[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/Sqrt[b - Sqrt
[b^2 - 4*a*c]]) - (Sqrt[2]*Sqrt[c]*(1 + (2*b)/Sqrt[b^2 - 4*a*c])*ArcTan[(S
qrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/Sqrt[b + Sqrt[b^2
- 4*a*c]])/(2*(b^2 - 4*a*c)))/e
```

3.623.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1439
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d*(d*x)^(m - 1)*(b + 2*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*(p +
1)*(b^2 - 4*a*c))), x] - Simp[d^2/(2*(p + 1)*(b^2 - 4*a*c))   Int[(d*x)^(m
- 2)*(b*(m - 1) + 2*c*(m + 4*p + 5)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x
] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m,
1] && LeQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

3.623. $\int \frac{(d+ex)^2}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

3.623.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.62 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.26

method	result
default	$\frac{\frac{c\,e^2x^3}{4ac-b^2}+\frac{3x^2cde}{4ac-b^2}+\frac{\left(6c\,d^2+b\right)x}{8ac-2b^2}+\frac{d\left(2c\,d^2+b\right)}{2e\left(4ac-b^2\right)}}{c\,x^4e^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+d^4c+2bdex+b\,d^2+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\,e^4\_Z^4+4cd\,e^3\_Z^3+\left(6c\,d^2e^2+b\,e^2\right)\_Z^2+\left(4d^3ec-\right.\right.}}{}$
risch	$\frac{\frac{c\,e^2x^3}{4ac-b^2}+\frac{3x^2cde}{4ac-b^2}+\frac{\left(6c\,d^2+b\right)x}{8ac-2b^2}+\frac{d\left(2c\,d^2+b\right)}{2e\left(4ac-b^2\right)}}{c\,x^4e^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+d^4c+2bdex+b\,d^2+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\,e^4\_Z^4+4cd\,e^3\_Z^3+\left(6c\,d^2e^2+b\,e^2\right)\_Z^2+\left(4d^3ec-\right.\right.}}{}$

input
```
int((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x,method=_RETURNVERBOSE)
```

output
```
(c*e^2/(4*a*c-b^2)*x^3+3/(4*a*c-b^2)*x^2*c*d*e+1/2*(6*c*d^2+b)/(4*a*c-b^2)
*x+1/2*d/e*(2*c*d^2+b)/(4*a*c-b^2))/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x
^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)+1/4/(4*a*c-b^2)/e*sum((2
*_R^2*c*e^2+4*_R*c*d*e+2*c*d^2-b)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*
e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^
2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a))
```

3.623.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2474 vs. 2(213) = 426.

Time = 0.35 (sec) , antiderivative size = 2474, normalized size of antiderivative = 9.74

$$\int \frac{(d+ex)^2}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")
```

output
```
-1/4*(4*c*e^3*x^3 + 12*c*d*e^2*x^2 + 4*c*d^3 + 2*(6*c*d^2 + b)*e*x - sqrt(
1/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4
*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (
b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3
- 4*a*b*c)*d^2)*e)*sqrt(-((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*
c^3)*e^2*sqrt(1/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^
4)) + b^3 + 12*a*b*c)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3
)*e^2))*log((3*b^2*c + 4*a*c^2)*e*x + (3*b^2*c + 4*a*c^2)*d + 1/2*sqrt(1/2
)*((a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 256*a^5*c^4)*e^3*sqrt(1/((a^2*
b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4)) - (b^5 - 8*a*b^3*c
 + 16*a^2*b*c^2)*e)*sqrt(-((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4
*c^3)*e^2*sqrt(1/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e
^4)) + b^3 + 12*a*b*c)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^
3)*e^2))) + sqrt(1/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e
^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c -
 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2
- 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)*sqrt(-((a*b^6 - 12*a^2*b^4*c + 48*a^3*
b^2*c^2 - 64*a^4*c^3)*e^2*sqrt(1/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2
 - 64*a^5*c^3)*e^4)) + b^3 + 12*a*b*c)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2
*c^2 - 64*a^4*c^3)*e^2))*log((3*b^2*c + 4*a*c^2)*e*x + (3*b^2*c + 4*a*c...
```

3.623.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

output
```
Timed out
```

3.623.7 Maxima [F]

$$\int \frac{(d+ex)^2}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = \int \frac{(ex+d)^2}{\left((ex+d)^4c+(ex+d)^2b+a\right)^2}\,dx$$

input
```
integrate((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")
```

output
```
-1/2*(2*c*e^3*x^3 + 6*c*d*e^2*x^2 + 2*c*d^3 + (6*c*d^2 + b)*e*x + b*d)/((b
^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c +
 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*
a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*
c)*d^2)*e) + 1/2*integrate(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 - b)/((b^2*
c - 4*a*c^2)*e^4*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d
^4 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^2*x^2 + a*b^2 - 4*a^2*c +
 (b^3 - 4*a*b*c)*d^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e*x
), x)
```

3.623.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1416 vs. $2(213) = 426$.

Time = 0.32 (sec) , antiderivative size = 1416, normalized size of antiderivative = 5.57

$$\int \frac{(d+ex)^2}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

output
```
1/4*((2*c*e^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) +
d/e)^2 - 4*c*d*e*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4))
 + d/e) + 2*c*d^2 - b)*log(x + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*
e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*
e^2)/(c*e^4)) + d/e)^3 - 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*
a*c)*e^2)/(c*e^4)) + d/e)^2 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(s
qrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)) - (2*c*e^2
*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2 + 4*c*
d*e*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e) + 2*c
*d^2 - b)*log(x - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4))
 + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4))
 - d/e)^3 + 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*
e^4)) - d/e)^2 + 2*c*d^3*e + b*d*e + (6*c*d^2*e^2 + b*e^2)*(sqrt(1/2)*sqrt
(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)) + (2*c*e^2*(sqrt(1/2)*s
qrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 - 4*c*d*e*(sqrt(1/2
)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e) + 2*c*d^2 - b)*log
(x + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*
e^4*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^3 - 6
*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^
2 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(sqrt(1/2)*sqrt(-(b*e^2 -...
```

3.623.9 Mupad [B] (verification not implemented)

Time = 9.93 (sec) , antiderivative size = 7200, normalized size of antiderivative = 28.35

$$\int \frac{(d+ex)^2}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \text{Too large to display}$$

input
```
int((d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x)
```

output
```
atan(((((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 -
512*a^3*b^3*c^3)/(32*(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 +
240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a
^6*b^2*c^5*e^2)))^(1/2)*((64*a^2*c^5*d*e^11 + 20*b^4*c^3*d*e^11 - 96*a*b^2
*c^4*d*e^11)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (((32*
b^9*c^2*d*e^13 - 512*a*b^7*c^3*d*e^13 + 8192*a^4*b*c^6*d*e^13 + 3072*a^2*b
^5*c^4*d*e^13 - 8192*a^3*b^3*c^5*d*e^13)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2
*c^2 - 12*a*b^4*c)) + (x*(8*b^7*c^2*e^14 - 96*a*b^5*c^3*e^14 - 512*a^3*b*c
^5*e^14 + 384*a^2*b^3*c^4*e^14))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(((-(4*a*
c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3
)/(32*(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2
*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)
))^(1/2) - (8*b^7*c^2*e^12 - 96*a*b^5*c^3*e^12 - 512*a^3*b*c^5*e^12 + 384*
a^2*b^3*c^4*e^12)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)))*((
(-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*
b^3*c^3)/(32*(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*
b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c
^5*e^2)))^(1/2) - (x*(4*a*c^4*e^12 - 5*b^2*c^3*e^12))/(b^4 + 16*a^2*c^2 -
8*a*b^2*c))*1i + (((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2
*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2...
```

3.623. $\int \frac{(d+ex)^2}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$

3.624 $\int \frac{d+ex}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

3.624.1 Optimal result

Integrand size = 28, antiderivative size = 98

$$\int \frac{d+ex}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{-b-2c(d+ex)^2}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{2c\,\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}e}$$

output
```
1/2*(-b-2*c*(e*x+d)^2)/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)+2*c*arct
anh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)/e
```

3.624.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00

$$\int \frac{d+ex}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = -\frac{\frac{b+2c(d+ex)^2}{a+b(d+ex)^2+c(d+ex)^4} + \frac{4c\arctan\left(\frac{b+2c(d+ex)^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}}{2\left(b^2-4ac\right)e}$$

input
```
Integrate[(d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

output
```
-1/2*((b + 2*c*(d + e*x)^2)/(a + b*(d + e*x)^2 + c*(d + e*x)^4) + (4*c*Arc
Tan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/((b^2 -
 4*a*c)*e)
```

3.624.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1462, 1432, 1086, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}dx$$

$$\Big\downarrow \text{1462}$$

$$\frac{\int \frac{d+ex}{\left(c(d+ex)^4+b(d+ex)^2+a\right)^2}d(d+ex)}{e}$$

$$\Big\downarrow \text{1432}$$

$$\frac{\int \frac{1}{\left(c(d+ex)^4+b(d+ex)^2+a\right)^2}d(d+ex)^2}{2e}$$

$$\Big\downarrow \text{1086}$$

$$\frac{-\frac{2c\int \frac{1}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)^2}{b^2-4ac}-\frac{b+2c(d+ex)^2}{(b^2-4ac)\left(a+b(d+ex)^2+c(d+ex)^4\right)}}{2e}$$

$$\Big\downarrow \text{1083}$$

$$\frac{\frac{4c\int \frac{1}{-(d+ex)^4+b^2-4ac}d\left(2c(d+ex)^2+b\right)}{b^2-4ac}-\frac{b+2c(d+ex)^2}{(b^2-4ac)\left(a+b(d+ex)^2+c(d+ex)^4\right)}}{2e}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{4c\,\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2c(d+ex)^2}{(b^2-4ac)\left(a+b(d+ex)^2+c(d+ex)^4\right)}}{2e}$$

input
```
Int[(d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

output
```
(-((b + 2*c*(d + e*x)^2)/((b^2 - 4*a*c)*(a + b*(d + e*x)^2 + c*(d + e*x)^4
))) + (4*c*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)
^(3/2))/(2*e)
```

3.624.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

3.624.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.17 (sec) , antiderivative size = 270, normalized size of antiderivative = 2.76

method	result
default	$\frac{\frac{c\,x^2 e}{4ac-b^2}+\frac{2xcd}{4ac-b^2}+\frac{2c\,d^2+b}{2e\left(4ac-b^2\right)}}{c\,x^4 e^4+4cd\,e^3 x^3+6c\,d^2 e^2 x^2+4c\,d^3 ex+b\,e^2 x^2+d^4 c+2bdex+b\,d^2+a}+\frac{c\left(\sum_{\_R=\text{RootOf}\left(c\,e^4\_Z^4+4cd\,e^3\_Z^3+\left(6c\,d^2 e^2+b\,e^2\right)\_Z^2+\left(4d^3\right.\right.}\right.}{}$
risch	$\frac{\frac{c\,x^2 e}{4ac-b^2}+\frac{2xcd}{4ac-b^2}+\frac{2c\,d^2+b}{2e\left(4ac-b^2\right)}}{c\,x^4 e^4+4cd\,e^3 x^3+6c\,d^2 e^2 x^2+4c\,d^3 ex+b\,e^2 x^2+d^4 c+2bdex+b\,d^2+a}+\frac{c\ln\left(\left(\left(\left(-4ac+b^2\right)^{\frac{3}{2}} e^2+4ab\,e^2 c-b^3 e^2\right)x^2+\left(2\left(-4ac+b^2\right)^{\frac{3}{2}}\right.\right.\right.}{e(-}$

input
```
int((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x,method=_RETURNVERBOSE)
```

output
```
(c/(4*a*c-b^2)*x^2*e+2/(4*a*c-b^2)*x*c*d+1/2/e*(2*c*d^2+b)/(4*a*c-b^2))/(c
*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e
*x+b*d^2+a)+c/(4*a*c-b^2)/e*sum((_R*e+d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R
*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+
(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a))
```

3.624.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 455 vs. $2(90) = 180$.

Time = 0.30 (sec) , antiderivative size = 1042, normalized size of antiderivative = 10.63

$$\int \frac{d+ex}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$$

$$= \Bigg[-\frac{2\,(b^2c-4\,ac^2)e^2x^2+4\,(b^2c-4\,ac^2)dex+b^3-4\,abc+2\,(b^2c-4\,ac^2)d^2+2\,(c^2e^4x^4+4\,c^2de^3x}{2\,((b^4c-8\,ab^2c^2+16\,a^2c^3)e^5x^4+4\,(b^4c-8\,ab^2c^2+16\,a^2c^3)de^4x^3+(b^5-8\,ab^3c+16\,a^2bc^2+6\,(b^4c-}$$

$$-\frac{2\,(b^2c-4\,ac^2)e^2x^2+4\,(b^2c-4\,ac^2)dex+b^3-4\,abc+2\,(b^2c-4\,ac}{2\,((b^4c-8\,ab^2c^2+16\,a^2c^3)e^5x^4+4\,(b^4c-8\,ab^2c^2+16\,a^2c^3)de^4x^3+(b^5-8\,ab^3c+16\,a^2bc^2+6\,(b^4c-}$$

input
```
integrate((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")
```

output
```
[-1/2*(2*(b^2*c - 4*a*c^2)*e^2*x^2 + 4*(b^2*c - 4*a*c^2)*d*e*x + b^3 - 4*a
*b*c + 2*(b^2*c - 4*a*c^2)*d^2 + 2*(c^2*e^4*x^4 + 4*c^2*d*e^3*x^3 + c^2*d^
4 + (6*c^2*d^2 + b*c)*e^2*x^2 + b*c*d^2 + 2*(2*c^2*d^3 + b*c*d)*e*x + a*c)
*sqrt(b^2 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6
*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*
a*c - (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^4*x^
4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b
*d)*e*x + a)))/((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e^5*x^4 + 4*(b^4*c - 8*
a*b^2*c^2 + 16*a^2*c^3)*d*e^4*x^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + 6*(b
^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^3*x^2 + 2*(2*(b^4*c - 8*a*b^2*c^2
+ 16*a^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*e^2*x + (a*b^4 - 8
*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^4 + (b^5 -
8*a*b^3*c + 16*a^2*b*c^2)*d^2)*e), -1/2*(2*(b^2*c - 4*a*c^2)*e^2*x^2 + 4*(
b^2*c - 4*a*c^2)*d*e*x + b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*d^2 - 4*(c^2*
e^4*x^4 + 4*c^2*d*e^3*x^3 + c^2*d^4 + (6*c^2*d^2 + b*c)*e^2*x^2 + b*c*d^2
+ 2*(2*c^2*d^3 + b*c*d)*e*x + a*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2
 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/((b^4*c - 8
*a*b^2*c^2 + 16*a^2*c^3)*e^5*x^4 + 4*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*
e^4*x^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^
2*c^3)*d^2)*e^3*x^2 + 2*(2*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^...
```

3.624.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 495 vs. $2(83) = 166$.

Time = 2.52 (sec) , antiderivative size = 495, normalized size of antiderivative = 5.05

$$\int \frac{d+ex}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx =$$

$$-\frac{c\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(\frac{2dx}{e}+x^2+\frac{-16a^2c^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+8ab^2c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}-b^4c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+bc+2c^2d^2}{2c^2e^2}\right)}{e}$$
$$+\frac{c\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(\frac{2dx}{e}+x^2+\frac{16a^2c^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}-8ab^2c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+b^4c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+bc+2c^2d^2}{2c^2e^2}\right)}{e}$$
$$+\frac{b+2cd^2+4c\cdots}{8a^2ce-2ab^2e+8abcd^2e+8ac^2d^4e-2b^3d^2e-2b^2cd^4e+x^4\cdot(8ac^2e^5-2b^2ce^5)+x^3\cdot(32ac^2de^4-8b^2c\cdots}$$

input
```
integrate((e*x+d)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

output
```
-c*sqrt(-1/(4*a*c - b**2)**3)*log(2*d*x/e + x**2 + (-16*a**2*c**3*sqrt(-1/
(4*a*c - b**2)**3) + 8*a*b**2*c**2*sqrt(-1/(4*a*c - b**2)**3) - b**4*c*sqr
t(-1/(4*a*c - b**2)**3) + b*c + 2*c**2*d**2)/(2*c**2*e**2))/e + c*sqrt(-1/
(4*a*c - b**2)**3)*log(2*d*x/e + x**2 + (16*a**2*c**3*sqrt(-1/(4*a*c - b**
2)**3) - 8*a*b**2*c**2*sqrt(-1/(4*a*c - b**2)**3) + b**4*c*sqrt(-1/(4*a*c
- b**2)**3) + b*c + 2*c**2*d**2)/(2*c**2*e**2))/e + (b + 2*c*d**2 + 4*c*d*
e*x + 2*c*e**2*x**2)/(8*a**2*c*e - 2*a*b**2*e + 8*a*b*c*d**2*e + 8*a*c**2*
d**4*e - 2*b**3*d**2*e - 2*b**2*c*d**4*e + x**4*(8*a*c**2*e**5 - 2*b**2*c*
e**5) + x**3*(32*a*c**2*d*e**4 - 8*b**2*c*d*e**4) + x**2*(8*a*b*c*e**3 + 4
8*a*c**2*d**2*e**3 - 2*b**3*e**3 - 12*b**2*c*d**2*e**3) + x*(16*a*b*c*d*e*
*2 + 32*a*c**2*d**3*e**2 - 4*b**3*d*e**2 - 8*b**2*c*d**3*e**2))
```

3.624.7 Maxima [F]

$$\int \frac{d+ex}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx=\int \frac{ex+d}{\left((ex+d)^4c+(ex+d)^2b+a\right)^2}\,dx$$

input
```
integrate((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")
```

output
```
2*c*integrate(-(e*x + d)/((b^2*c - 4*a*c^2)*e^4*x^4 + 4*(b^2*c - 4*a*c^2)*
d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d
^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2 + 2*(2*(b^2*c - 4*a*c^
2)*d^3 + (b^3 - 4*a*b*c)*d)*e*x), x) - 1/2*(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*
d^2 + b)/((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3
 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3
 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (
b^3 - 4*a*b*c)*d^2)*e)
```

3.624.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.66

$$\int \frac{d+ex}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx=-\frac{2\,c\arctan\left(\frac{2\,cd^2+2\,(ex^2+2\,dx)ce+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}e}-\frac{2\,cd^2+2\,(ex^2+2\,dx)ce+b}{2\,\left(cd^4+2\,(ex^2+2\,dx)cd^2e+(ex^2+2\,dx)^2ce^2+bd^2+(ex^2+2\,dx)be+a\right)(b^2e-4\,ace)}$$

input
```
integrate((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

output
```
-2*c*arctan((2*c*d^2 + 2*(e*x^2 + 2*d*x)*c*e + b)/sqrt(-b^2 + 4*a*c))/((b^
2 - 4*a*c)*sqrt(-b^2 + 4*a*c)*e) - 1/2*(2*c*d^2 + 2*(e*x^2 + 2*d*x)*c*e +
b)/((c*d^4 + 2*(e*x^2 + 2*d*x)*c*d^2*e + (e*x^2 + 2*d*x)^2*c*e^2 + b*d^2 +
 (e*x^2 + 2*d*x)*b*e + a)*(b^2*e - 4*a*c*e))
```

3.624.9 Mupad [B] (verification not implemented)

Time = 8.62 (sec) , antiderivative size = 417, normalized size of antiderivative = 4.26

$$\int \frac{d+ex}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$$

$$= \frac{\frac{2\,c\,d^2+b}{2\,e\,(4\,a\,c-b^2)}+\frac{c\,e\,x^2}{4\,a\,c-b^2}+\frac{2\,c\,d\,x}{4\,a\,c-b^2}}{a+x^2\,\left(6\,c\,d^2\,e^2+b\,e^2\right)+b\,d^2+c\,d^4+x\,\left(4\,c\,e\,d^3+2\,b\,e\,d\right)+c\,e^4\,x^4+4\,c\,d\,e^3\,x^3}$$
$$+ \frac{2\,c\,\mathrm{atan}\left(\frac{\left(4\,a\,c-b^2\right)^4\,\left(x\,\left(\frac{8\,c^4\,d\,e^7}{a\,\left(4\,a\,c-b^2\right)^{7/2}}-\frac{8\,b\,c^2\,\left(b^3\,c^2\,d\,e^9-4\,a\,b\,c^3\,d\,e^9\right)}{a\,e^2\,\left(4\,a\,c-b^2\right)^{11/2}}\right)+x^2\,\left(\frac{4\,c^4\,e^8}{a\,\left(4\,a\,c-b^2\right)^{7/2}}-\frac{4\,b\,c^2\,\left(b^3\,c^2\,e^{10}-4\,a\,b\,c^3\,e^{10}\right)}{a\,e^2\,\left(4\,a\,c-b^2\right)^{11/2}}\right)+\frac{4\,c^4\,d}{a\,\left(4\,a\,c-\right.}\right.}{8\,c^4\,e^6}\right.}{e\,\left(4\,a\,c-b^2\right)^{3/2}}$$

input
```
int((d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x)
```

output
```
((b + 2*c*d^2)/(2*e*(4*a*c - b^2)) + (c*e*x^2)/(4*a*c - b^2) + (2*c*d*x)/(
4*a*c - b^2))/(a + x^2*(b*e^2 + 6*c*d^2*e^2) + b*d^2 + c*d^4 + x*(2*b*d*e
+ 4*c*d^3*e) + c*e^4*x^4 + 4*c*d*e^3*x^3) + (2*c*atan(((4*a*c - b^2)^4*(x*
((8*c^4*d*e^7)/(a*(4*a*c - b^2)^(7/2)) - (8*b*c^2*(b^3*c^2*d*e^9 - 4*a*b*c
^3*d*e^9))/(a*e^2*(4*a*c - b^2)^(11/2))) + x^2*((4*c^4*e^8)/(a*(4*a*c - b^
2)^(7/2)) - (4*b*c^2*(b^3*c^2*e^10 - 4*a*b*c^3*e^10))/(a*e^2*(4*a*c - b^2)
^(11/2))) + (4*c^4*d^2*e^6)/(a*(4*a*c - b^2)^(7/2)) + (4*b*c^2*(8*a^2*c^3*
e^8 - 2*a*b^2*c^2*e^8 - b^3*c^2*d^2*e^8 + 4*a*b*c^3*d^2*e^8))/(a*e^2*(4*a*
c - b^2)^(11/2))))/(8*c^4*e^6)))/(e*(4*a*c - b^2)^(3/2))
```

3.625 $\int \frac{1}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

3.625.1 Optimal result

Integrand size = 22, antiderivative size = 299

$$\int \frac{1}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{\left(\frac{d}{e}+x\right)\left(b^2-2ac+bce^2\left(\frac{d}{e}+x\right)^2\right)}{2a\left(b^2-4ac\right)\left(a+be^2\left(\frac{d}{e}+x\right)^2+ce^4\left(\frac{d}{e}+x\right)^4\right)} + \frac{\sqrt{c}\left(b^2-12ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}\,e} - \frac{\sqrt{c}\left(b^2-12ac-b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}\,e}$$

output
```
1/2*(d/e+x)*(b^2-2*a*c+b*c*e^2*(d/e+x)^2)/a/(-4*a*c+b^2)/(a+b*e^2*(d/e+x)^
2+c*e^4*(d/e+x)^4)+1/4*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2
))^(1/2))*c^(1/2)*(b^2-12*a*c+b*(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(3/2)/e
*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*arctan((e*x+d)*2^(1/2)*c^(1/2)/(
b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b^2-12*a*c-b*(-4*a*c+b^2)^(1/2))/a/(
-4*a*c+b^2)^(3/2)/e*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.625.2 Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.91

$$\int \frac{1}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$$
$$= \frac{\frac{2(d+ex)(b^2-2ac+bc(d+ex)^2)}{(b^2-4ac)(a+(d+ex)^2(b+c(d+ex)^2))}+\frac{\sqrt{2}\sqrt{c}\left(b^2-12ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(-b^2+12ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\ldots}{\sqrt{b-\ldots}}\right.}{(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}}{4ae}$$

input
```
Integrate[(a + b*(d + e*x)^2 + c*(d + e*x)^4)^(-2),x]
```

output
```
((2*(d + e*x)*(b^2 - 2*a*c + b*c*(d + e*x)^2))/((b^2 - 4*a*c)*(a + (d + e*
x)^2*(b + c*(d + e*x)^2))) + (Sqrt[2]*Sqrt[c]*(b^2 - 12*a*c + b*Sqrt[b^2 -
 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/
((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-b^2
 + 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b
 + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))
/(4*a*e)
```

3.625.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.88, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1687, 1405, 25, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$$
$$\Big\downarrow \text{1687}$$
$$\frac{\int \frac{1}{(c(d+ex)^4+b(d+ex)^2+a)^2}d(d+ex)}{e}$$
$$\Big\downarrow \text{1405}$$
$$\frac{\frac{(d+ex)(-2ac+b^2+bc(d+ex)^2)}{2a(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)}-\frac{\int -\frac{b^2+c(d+ex)^2b-6ac}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)}{2a(b^2-4ac)}}{e}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{b^2+c(d+ex)^2b-6ac}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)}{2a(b^2-4ac)}+\frac{(d+ex)(-2ac+b^2+bc(d+ex)^2)}{2a(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)}}{e}$$

$$\Big\downarrow \; 1480$$

$$\frac{\frac{\frac{1}{2}c\left(\frac{b^2-12ac}{\sqrt{b^2-4ac}}+b\right)\int \frac{1}{c(d+ex)^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}d(d+ex)+\frac{1}{2}c\left(b-\frac{b^2-12ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{c(d+ex)^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}d(d+ex)}{2a(b^2-4ac)}+\frac{(d+ex)(-2ac+b^2+bc(d+e}{2a(b^2-4ac)(a+b(d+ex)^2+c(d}}{e}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{\frac{\sqrt{c}\left(\frac{b^2-12ac}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(b-\frac{b^2-12ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}}{2a(b^2-4ac)}+\frac{(d+ex)(-2ac+b^2+bc(d+ex)^2)}{2a(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)}}{e}$$

input
```
Int[(a + b*(d + e*x)^2 + c*(d + e*x)^4)^(-2),x]
```

output
```
(((d + e*x)*(b^2 - 2*a*c + b*c*(d + e*x)^2))/(2*a*(b^2 - 4*a*c)*(a + b*(d
+ e*x)^2 + c*(d + e*x)^4)) + ((Sqrt[c]*(b + (b^2 - 12*a*c)/Sqrt[b^2 - 4*a*
c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt
[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(b - (b^2 - 12*a*c)/Sqrt[b^2 -
 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/
(Sqrt[2]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*a*(b^2 - 4*a*c)))/e
```

3.625.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1687
```
Int[((a_) + (c_.)*(u_)^(n2_.) + (b_.)*(u_)^(n_))^(p_), x_Symbol] :> Simp[1/
Coefficient[u, x, 1]   Subst[Int[(a + b*x^n + c*x^(2*n))^p, x], x, u], x] /
; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && LinearQ[u, x] && NeQ[u, x]
```

3.625.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.13 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.22

method	result
default	$\frac{-\frac{bc\,e^{2}x^{3}}{2a(4ac-b^{2})}-\frac{3dbce\,x^{2}}{2a(4ac-b^{2})}+\frac{(-3bc\,d^{2}+2ac-b^{2})x}{2a(4ac-b^{2})}+\frac{d(-bc\,d^{2}+2ac-b^{2})}{2ea(4ac-b^{2})}}{c\,x^{4}e^{4}+4cd\,e^{3}x^{3}+6c\,d^{2}e^{2}x^{2}+4c\,d^{3}ex+b\,e^{2}x^{2}+d^{4}c+2bdex+b\,d^{2}+a}+\frac{\sum\limits_{\_R=\operatorname{RootOf}\left(c\,e^{4}\_Z^{4}+4cd\,e^{3}\_Z^{3}+\left(6c\,d^{2}e^{2}+b\,e^{2}\right)\_Z^{2}+\left(4d^{3}e\right.\right.}}{}$
risch	$\frac{-\frac{bc\,e^{2}x^{3}}{2a(4ac-b^{2})}-\frac{3dbce\,x^{2}}{2a(4ac-b^{2})}+\frac{(-3bc\,d^{2}+2ac-b^{2})x}{2a(4ac-b^{2})}+\frac{d(-bc\,d^{2}+2ac-b^{2})}{2ea(4ac-b^{2})}}{c\,x^{4}e^{4}+4cd\,e^{3}x^{3}+6c\,d^{2}e^{2}x^{2}+4c\,d^{3}ex+b\,e^{2}x^{2}+d^{4}c+2bdex+b\,d^{2}+a}+\frac{\sum\limits_{\_R=\operatorname{RootOf}\left(c\,e^{4}\_Z^{4}+4cd\,e^{3}\_Z^{3}+\left(6c\,d^{2}e^{2}+b\,e^{2}\right)\_Z^{2}+\left(4d^{3}e\right.\right.}}{}$

input
```
int(1/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x,method=_RETURNVERBOSE)
```

output
```
(-1/2*b*c*e^2/a/(4*a*c-b^2)*x^3-3/2*d*b*c*e/a/(4*a*c-b^2)*x^2+1/2*(-3*b*c*
d^2+2*a*c-b^2)/a/(4*a*c-b^2)*x+1/2*d/e*(-b*c*d^2+2*a*c-b^2)/a/(4*a*c-b^2))
/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*
d*e*x+b*d^2+a)+1/4/a/(4*a*c-b^2)/e*sum((-_R^2*b*c*e^2-2*_R*b*c*d*e-b*c*d^2
+6*a*c-b^2)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*
ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c
*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a))
```

3.625.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3228 vs. $2(253) = 506$.

Time = 0.35 (sec) , antiderivative size = 3228, normalized size of antiderivative = 10.80

$$\int \frac{1}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")
```

output
```
1/4*(2*b*c*e^3*x^3 + 6*b*c*d*e^2*x^2 + 2*b*c*d^3 + 2*(3*b*c*d^2 + b^2 - 2*
a*c)*e*x - sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*e^5*x^4 + 4*(a*b^2*c - 4*a^2*c
^2)*d*e^4*x^3 + (a*b^3 - 4*a^2*b*c + 6*(a*b^2*c - 4*a^2*c^2)*d^2)*e^3*x^2
+ 2*(2*(a*b^2*c - 4*a^2*c^2)*d^3 + (a*b^3 - 4*a^2*b*c)*d)*e^2*x + ((a*b^2*
c - 4*a^2*c^2)*d^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*d^2)*e)*sqrt(
-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c
^2 - 64*a^6*c^3)*e^2*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/((a^6*b^6 - 12*a
^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*e^4)))/((a^3*b^6 - 12*a^4*b^4*c +
48*a^5*b^2*c^2 - 64*a^6*c^3)*e^2))*log((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2
*c^4)*e*x + (5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*d + 1/2*sqrt(1/2)*((a
^3*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)
*e^3*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^
8*b^2*c^2 - 64*a^9*c^3)*e^4)) - (b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 672*
a^3*b^2*c^3 + 864*a^4*c^4)*e)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (a^
3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*e^2*sqrt((b^4 - 18*a*b
^2*c + 81*a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)
*e^4)))/((a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*e^2))) + s
qrt(1/2)*((a*b^2*c - 4*a^2*c^2)*e^5*x^4 + 4*(a*b^2*c - 4*a^2*c^2)*d*e^4*x^
3 + (a*b^3 - 4*a^2*b*c + 6*(a*b^2*c - 4*a^2*c^2)*d^2)*e^3*x^2 + 2*(2*(a*b^
2*c - 4*a^2*c^2)*d^3 + (a*b^3 - 4*a^2*b*c)*d)*e^2*x + ((a*b^2*c - 4*a^2...
```

3.625.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

output
```
Timed out
```

3.625.7 Maxima [F]

$$\int \frac{1}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = \int \frac{1}{\left((ex+d)^4c+(ex+d)^2b+a\right)^2}\,dx$$

input
```
integrate(1/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")
```

output
```
1/2*(b*c*e^3*x^3 + 3*b*c*d*e^2*x^2 + b*c*d^3 + (3*b*c*d^2 + b^2 - 2*a*c)*e
*x + (b^2 - 2*a*c)*d)/((a*b^2*c - 4*a^2*c^2)*e^5*x^4 + 4*(a*b^2*c - 4*a^2*
c^2)*d*e^4*x^3 + (a*b^3 - 4*a^2*b*c + 6*(a*b^2*c - 4*a^2*c^2)*d^2)*e^3*x^2
 + 2*(2*(a*b^2*c - 4*a^2*c^2)*d^3 + (a*b^3 - 4*a^2*b*c)*d)*e^2*x + ((a*b^2
*c - 4*a^2*c^2)*d^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*d^2)*e) - 1/
2*integrate(-(b*c*e^2*x^2 + 2*b*c*d*e*x + b*c*d^2 + b^2 - 6*a*c)/((b^2*c -
 4*a*c^2)*e^4*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4
+ (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b
^3 - 4*a*b*c)*d^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e*x),
x)/a
```

3.625.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1460 vs. $2(253) = 506$.

Time = 0.30 (sec) , antiderivative size = 1460, normalized size of antiderivative = 4.88

$$\int \frac{1}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

output
```
-1/4*((b*c*e^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) +
 d/e)^2 - 2*b*c*d*e*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^
4)) + d/e) + b*c*d^2 + b^2 - 6*a*c)*log(x + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(
b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(
b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^3 - 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 +
sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^
2 + b*e^2)*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e
)) - (b*c*e^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) -
d/e)^2 + 2*b*c*d*e*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4
)) - d/e) + b*c*d^2 + b^2 - 6*a*c)*log(x - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b
^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b
^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^3 + 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 + s
qrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2 + 2*c*d^3*e + b*d*e + (6*c*d^2*e^2
 + b*e^2)*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)
) + (b*c*e^2*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d
/e)^2 - 2*b*c*d*e*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)
) + d/e) + b*c*d^2 + b^2 - 6*a*c)*log(x + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^
2 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^
2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^3 - 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 - sq
rt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 - 2*c*d^3*e - b*d*e + (6*c*d^2*e...
```

3.625.9 Mupad [B] (verification not implemented)

Time = 10.57 (sec) , antiderivative size = 9056, normalized size of antiderivative = 30.29

$$\int \frac{1}{(a+b(d+ex)^2+c(d+ex)^4)^2} \, dx = \text{Too large to display}$$

input
```
int(1/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x)
```

output
```
atan(((-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^
7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c
- b^2)^9)^(1/2))/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2
+ 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144
*a^8*b^2*c^5*e^2)))^(1/2)*(((6144*a^5*c^6*e^12 + 16*a*b^8*c^2*e^12 - 288*a
^2*b^6*c^3*e^12 + 1920*a^3*b^4*c^4*e^12 - 5632*a^4*b^2*c^5*e^12)/(8*(a^2*b
^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + ((16384*a^6*b*c^6*d*e^
13 + 64*a^2*b^9*c^2*d*e^13 - 1024*a^3*b^7*c^3*d*e^13 + 6144*a^4*b^5*c^4*d*
e^13 - 16384*a^5*b^3*c^5*d*e^13)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c +
 48*a^4*b^2*c^2)) - (x*(1024*a^5*b*c^5*e^14 - 16*a^2*b^7*c^2*e^14 + 192*a^
3*b^5*c^3*e^14 - 768*a^4*b^3*c^4*e^14))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b
^2*c)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*
b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*
c - b^2)^9)^(1/2))/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^
2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 61
44*a^8*b^2*c^5*e^2)))^(1/2))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840
*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*
b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e
^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840
*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2)))^(1/2) - (1152*a^3*c^6*d*e^11...
```

3.625. $\int \frac{1}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$

3.626 $\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

3.626.1 Optimal result

Integrand size = 30, antiderivative size = 162

$$\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{b\left(b^2-6ac\right)\operatorname{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2\left(b^2-4ac\right)^{3/2}e} + \frac{\log(d+ex)}{a^2e} - \frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^2e}$$

output
```
1/2*(b^2-2*a*c+b*c*(e*x+d)^2)/a/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)
+1/2*b*(-6*a*c+b^2)*arctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/a^2/(-4*
a*c+b^2)^(3/2)/e+ln(e*x+d)/a^2/e-1/4*ln(a+b*(e*x+d)^2+c*(e*x+d)^4)/a^2/e
```

3.626.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.45

$$\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{\frac{2a(b^2-2ac+bc(d+ex)^2)}{(b^2-4ac)(a+(d+ex)^2(b+c(d+ex)^2))}+4\log(d+ex)-\frac{\left(b^3-6abc+b^2\sqrt{b^2-4ac}-4ac\sqrt{b^2-4ac}\right)\log\left(b-\sqrt{b^2-4ac}+2c(d+ex)^2\right)}{(b^2-4ac)^{3/2}}+\frac{\left(b^3-6\right.}{}}{4a^2e}$$

input
```
Integrate[1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

output
```
((2*a*(b^2 - 2*a*c + b*c*(d + e*x)^2))/((b^2 - 4*a*c)*(a + (d + e*x)^2*(b
+ c*(d + e*x)^2))) + 4*Log[d + e*x] - ((b^3 - 6*a*b*c + b^2*Sqrt[b^2 - 4*a
*c] - 4*a*c*Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2
])/(b^2 - 4*a*c)^(3/2) + ((b^3 - 6*a*b*c - b^2*Sqrt[b^2 - 4*a*c] + 4*a*c*S
qrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(b^2 - 4*a
*c)^(3/2))/(4*a^2*e)
```

3.626.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1462, 1434, 1165, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$$

$$\Big\downarrow \; 1462$$

$$\frac{\int \frac{1}{(d+ex)\left(c(d+ex)^4+b(d+ex)^2+a\right)^2}d(d+ex)}{e}$$

$$\Big\downarrow \; 1434$$

$$\frac{\int \frac{1}{(d+ex)^2\left(c(d+ex)^4+b(d+ex)^2+a\right)^2}d(d+ex)^2}{2e}$$

$$\Big\downarrow \; 1165$$

$$\frac{\frac{-2ac+b^2+bc(d+ex)^2}{a(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)}-\frac{\int -\frac{b^2+c(d+ex)^2b-4ac}{(d+ex)^2\left(c(d+ex)^4+b(d+ex)^2+a\right)}d(d+ex)^2}{a(b^2-4ac)}}{2e}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{b^2+c(d+ex)^2b-4ac}{(d+ex)^2\left(c(d+ex)^4+b(d+ex)^2+a\right)}d(d+ex)^2}{a(b^2-4ac)}+\frac{-2ac+b^2+bc(d+ex)^2}{a(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)}}{2e}$$

$$\Big\downarrow \; 1200$$

$$\frac{\frac{\int \left(\frac{b^2-4ac}{a(d+ex)^2}+\frac{-c\left(b^2-4ac\right)(d+ex)^2-b\left(b^2-5ac\right)}{a\left(c(d+ex)^4+b(d+ex)^2+a\right)}\right)d(d+ex)^2}{a(b^2-4ac)}+\frac{-2ac+b^2+bc(d+ex)^2}{a(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)}}{2e}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{\frac{b\left(b^2-6ac\right)\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}+\frac{\left(b^2-4ac\right)\log\left((d+ex)^2\right)}{a}-\frac{\left(b^2-4ac\right)\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{2a}}{a(b^2-4ac)}+\frac{-2ac+b^2+bc(d+ex)^2}{a(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)}}{2e}$$

input
```
Int[1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

output
```
((b^2 - 2*a*c + b*c*(d + e*x)^2)/(a*(b^2 - 4*a*c)*(a + b*(d + e*x)^2 + c*(
d + e*x)^4)) + ((b*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 -
4*a*c]])/(a*Sqrt[b^2 - 4*a*c]) + ((b^2 - 4*a*c)*Log[(d + e*x)^2])/a - ((b^
2 - 4*a*c)*Log[a + b*(d + e*x)^2 + c*(d + e*x)^4])/(2*a))/(a*(b^2 - 4*a*c)
))/(2*e)
```

3.626.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

3.626. $\int \frac{1}{(d+ex)(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.626.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.74 (sec) , antiderivative size = 399, normalized size of antiderivative = 2.46

method	result
default	$-\frac{\frac{aebc\,x^2}{8ac-2b^2}+\frac{bcdax}{4ac-b^2}-\frac{a\left(-bc\,d^2+2ac-b^2\right)}{2e\left(4ac-b^2\right)}}{c\,x^4e^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+d^4c+2bdex+b\,d^2+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\,e^4\_Z^4+4cd\,e^3\_Z^3+\left(6c\,d^2e^2+b\,e^2\right)\_Z^2+\left(4d^3ec+2bde\right)\right.}}{}$
risch	$\frac{-\frac{c\,x^2be}{2a\left(4ac-b^2\right)}-\frac{xbcd}{\left(4ac-b^2\right)a}+\frac{-bc\,d^2+2ac-b^2}{2ea\left(4ac-b^2\right)}}{c\,x^4e^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+d^4c+2bdex+b\,d^2+a}+\frac{\ln(ex+d)}{a^2e}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(64a^5c^3e^2-48a^4b^2c^2e^2+12a^3b^4c\,e\right.\right.}\right.}{}$

input
```
int(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/a^2*((1/2*a/(4*a*c-b^2)*e*b*c*x^2+b*c*d*a/(4*a*c-b^2)*x-1/2/e*a*(-b*c*d
^2+2*a*c-b^2)/(4*a*c-b^2))/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^
3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)+1/2/(4*a*c-b^2)/e*sum((e^3*c*(4*a
*c-b^2)*_R^3+3*c*d*e^2*(4*a*c-b^2)*_R^2+e*(12*a*c^2*d^2-3*b^2*c*d^2+5*a*b*
c-b^3)*_R+4*a*c^2*d^3-b^2*c*d^3+5*a*b*c*d-b^3*d)/(2*_R^3*c*e^3+6*_R^2*c*d*
e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e
^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a))+l
n(e*x+d)/a^2/e
```

3.626.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1173 vs. $2(152) = 304$.

Time = 0.45 (sec) , antiderivative size = 2476, normalized size of antiderivative = 15.28

$$\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")
```

output
```
[1/4*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(a*b^3*c - 4*a^2*b*c^2)*e^2*
x^2 + 4*(a*b^3*c - 4*a^2*b*c^2)*d*e*x + 2*(a*b^3*c - 4*a^2*b*c^2)*d^2 + ((
b^3*c - 6*a*b*c^2)*e^4*x^4 + 4*(b^3*c - 6*a*b*c^2)*d*e^3*x^3 + (b^3*c - 6*
a*b*c^2)*d^4 + (b^4 - 6*a*b^2*c + 6*(b^3*c - 6*a*b*c^2)*d^2)*e^2*x^2 + a*b
^3 - 6*a^2*b*c + (b^4 - 6*a*b^2*c)*d^2 + 2*(2*(b^3*c - 6*a*b*c^2)*d^3 + (b
^4 - 6*a*b^2*c)*d)*e*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3
*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3
+ b*c*d)*e*x + b^2 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(
b^2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 +
 b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a)) - ((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)
*e^4*x^4 + 4*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*e^3*x^3 + a*b^4 - 8*a^2*
b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^4 + (b^5 - 8*a*b
^3*c + 16*a^2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^2*x^2 +
(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d^2 + 2*(2*(b^4*c - 8*a*b^2*c^2 + 16*a^2*
c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*e*x)*log(c*e^4*x^4 + 4*c*d*
e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x +
a) + 4*((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e^4*x^4 + 4*(b^4*c - 8*a*b^2*c^
2 + 16*a^2*c^3)*d*e^3*x^3 + a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*
a*b^2*c^2 + 16*a^2*c^3)*d^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + 6*(b^4*c -
 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^2*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c...
```

3.626.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx=\text{Timed out}$$

input
```
integrate(1/(e*x+d)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

output
```
Timed out
```

3.626.7 Maxima [F]

$$\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \int \frac{1}{\left((ex+d)^4c+(ex+d)^2b+a\right)^2(ex+d)}\,dx$$

input
```
integrate(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")
```

output
```
1/2*(b*c*e^2*x^2 + 2*b*c*d*e*x + b*c*d^2 + b^2 - 2*a*c)/((a*b^2*c - 4*a^2*
c^2)*e^5*x^4 + 4*(a*b^2*c - 4*a^2*c^2)*d*e^4*x^3 + (a*b^3 - 4*a^2*b*c + 6*
(a*b^2*c - 4*a^2*c^2)*d^2)*e^3*x^2 + 2*(2*(a*b^2*c - 4*a^2*c^2)*d^3 + (a*b
^3 - 4*a^2*b*c)*d)*e^2*x + ((a*b^2*c - 4*a^2*c^2)*d^4 + a^2*b^2 - 4*a^3*c
+ (a*b^3 - 4*a^2*b*c)*d^2)*e) - integrate(((b^2*c - 4*a*c^2)*e^3*x^3 + 3*(
b^2*c - 4*a*c^2)*d*e^2*x^2 + (b^2*c - 4*a*c^2)*d^3 + (b^3 - 5*a*b*c + 3*(b
^2*c - 4*a*c^2)*d^2)*e*x + (b^3 - 5*a*b*c)*d)/((b^2*c - 4*a*c^2)*e^4*x^4 +
 4*(b^2*c - 4*a*c^2)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3 - 4*a*b*c +
6*(b^2*c - 4*a*c^2)*d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2 +
 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e*x), x)/a^2 + log(e*x +
d)/(a^2*e)
```

3.626.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 467 vs. $2(152) = 304$.

Time = 0.40 (sec) , antiderivative size = 467, normalized size of antiderivative = 2.88

$$\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx =$$
$$-\frac{(a^2b^3ce^3-6a^3bc^2e^3)\sqrt{b^2-4ac}\log\left(\left|be^2x^2+\sqrt{b^2-4ac}e^2x^2+2bdex+2\sqrt{b^2-4ac}dex+bd^2+\sqrt{b^2 \cdots\right.\right.}{\cdots}$$
$$-\frac{\log\left(\left|ce^4x^4+4cde^3x^3+6cd^2e^2x^2+4cd^3ex+cd^4+be^2x^2+2bdex+bd^2+a\right|\right)}{4a^2e}$$
$$+\frac{\log\left(\left|ex+d\right|\right)}{a^2e}$$
$$+\frac{abce^2x^2+2abcdex+abcd^2+ab^2-2a^2c}{2\left(ce^4x^4+4cde^3x^3+6cd^2e^2x^2+4cd^3ex+cd^4+be^2x^2+2bdex+bd^2+a\right)(b^2-4ac)a^2e}$$

input
```
integrate(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

output
```
-1/4*((a^2*b^3*c*e^3 - 6*a^3*b*c^2*e^3)*sqrt(b^2 - 4*a*c)*log(abs(b*e^2*x^
2 + sqrt(b^2 - 4*a*c)*e^2*x^2 + 2*b*d*e*x + 2*sqrt(b^2 - 4*a*c)*d*e*x + b*
d^2 + sqrt(b^2 - 4*a*c)*d^2 + 2*a)) - (a^2*b^3*c*e^3 - 6*a^3*b*c^2*e^3)*sq
rt(b^2 - 4*a*c)*log(abs(-b*e^2*x^2 + sqrt(b^2 - 4*a*c)*e^2*x^2 - 2*b*d*e*x
 + 2*sqrt(b^2 - 4*a*c)*d*e*x - b*d^2 + sqrt(b^2 - 4*a*c)*d^2 - 2*a)))/(a^4
*b^4*c*e^4 - 8*a^5*b^2*c^2*e^4 + 16*a^6*c^3*e^4) - 1/4*log(abs(c*e^4*x^4 +
 4*c*d*e^3*x^3 + 6*c*d^2*e^2*x^2 + 4*c*d^3*e*x + c*d^4 + b*e^2*x^2 + 2*b*d
*e*x + b*d^2 + a))/(a^2*e) + log(abs(e*x + d))/(a^2*e) + 1/2*(a*b*c*e^2*x^
2 + 2*a*b*c*d*e*x + a*b*c*d^2 + a*b^2 - 2*a^2*c)/((c*e^4*x^4 + 4*c*d*e^3*x
^3 + 6*c*d^2*e^2*x^2 + 4*c*d^3*e*x + c*d^4 + b*e^2*x^2 + 2*b*d*e*x + b*d^2
 + a)*(b^2 - 4*a*c)*a^2*e)
```

3.626.9 Mupad [B] (verification not implemented)

Time = 14.69 (sec) , antiderivative size = 11072, normalized size of antiderivative = 68.35

$$\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x)
```

output
```
((b^2 - 2*a*c + b*c*d^2)/(2*e*(a*b^2 - 4*a^2*c)) + (b*c*e*x^2)/(2*(a*b^2 -
 4*a^2*c)) + (b*c*d*x)/(a*b^2 - 4*a^2*c))/(a + x^2*(b*e^2 + 6*c*d^2*e^2) +
 b*d^2 + c*d^4 + x*(2*b*d*e + 4*c*d^3*e) + c*e^4*x^4 + 4*c*d*e^3*x^3) + lo
g(d + e*x)/(a^2*e) - (log((((a^2*e*(-(b^2*(6*a*c - b^2)^2)/(a^4*e^2*(4*a*c
 - b^2)^3))^(1/2) - 1)*(((a^2*e*(-(b^2*(6*a*c - b^2)^2)/(a^4*e^2*(4*a*c -
b^2)^3))^(1/2) - 1)*((b*c^2*e^16*(a^2*e*(-(b^2*(6*a*c - b^2)^2)/(a^4*e^2*(
4*a*c - b^2)^3))^(1/2) - 1)*(a*b + 3*b^2*d^2 + 3*b^2*e^2*x^2 - 10*a*c*d^2
+ 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*a*c*d*e*x))/a^2 + (2*b*c^2*e^16*(2*b^3
 - 10*a*c^2*d^2 + b^2*c*d^2 - 10*a*b*c))/(a*(4*a*c - b^2)) - (2*b*c^3*e^18
*x^2*(10*a*c - b^2))/(a*(4*a*c - b^2)) - (4*b*c^3*d*e^17*x*(10*a*c - b^2))
/(a*(4*a*c - b^2))))/(4*a^2*e) - (b*c^3*e^15*(4*b^3 - 20*a*c^2*d^2 + 6*b^2
*c*d^2 - 17*a*b*c))/(a^2*(4*a*c - b^2)^2) + (2*b*c^4*e^17*x^2*(10*a*c - 3*
b^2))/(a^2*(4*a*c - b^2)^2) + (4*b*c^4*d*e^16*x*(10*a*c - 3*b^2))/(a^2*(4*
a*c - b^2)^2)))/(4*a^2*e) + (b^3*c^5*e^16*x^2)/(a^3*(4*a*c - b^2)^3) + (b^
2*c^4*e^14*(b^2 - 4*a*c + b*c*d^2))/(a^3*(4*a*c - b^2)^3) + (2*b^3*c^5*d*e
^15*x)/(a^3*(4*a*c - b^2)^3))*((b^3*c^5*e^16*x^2)/(a^3*(4*a*c - b^2)^3) -
((a^2*e*(-(b^2*(6*a*c - b^2)^2)/(a^4*e^2*(4*a*c - b^2)^3))^(1/2) + 1)*(((a
^2*e*(-(b^2*(6*a*c - b^2)^2)/(a^4*e^2*(4*a*c - b^2)^3))^(1/2) + 1)*((b*c^2
*e^16*(a^2*e*(-(b^2*(6*a*c - b^2)^2)/(a^4*e^2*(4*a*c - b^2)^3))^(1/2) + 1)
*(a*b + 3*b^2*d^2 + 3*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e...
```

3.626. $\int \frac{1}{(d+ex)(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$

3.627 $\int \frac{1}{(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

3.627.1 Optimal result

Integrand size = 30, antiderivative size = 348

$$\begin{aligned}
&\int \frac{1}{(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx\\
&=-\frac{3b^2-10ac}{2a^2(b^2-4ac)\,e(d+ex)}+\frac{b^2-2ac+bc(d+ex)^2}{2a(b^2-4ac)\,e(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)}\\
&\quad-\frac{\sqrt{c}\left(3b^3-16abc+(3b^2-10ac)\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}\,e}\\
&\quad+\frac{\sqrt{c}\left(3b^3-16abc-(3b^2-10ac)\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}\,e}
\end{aligned}$$

output
```
1/2*(10*a*c-3*b^2)/a^2/(-4*a*c+b^2)/e/(e*x+d)+1/2*(b^2-2*a*c+b*c*(e*x+d)^2
)/a/(-4*a*c+b^2)/e/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)-1/4*arctan((e*x+d)*
2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(3*b^3-16*a*b*c+(-10
*a*c+3*b^2)*(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(3/2)/e*2^(1/2)/(b-(-4*a*
c+b^2)^(1/2))^(1/2)+1/4*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/
2))^(1/2))*c^(1/2)*(3*b^3-16*a*b*c-(-10*a*c+3*b^2)*(-4*a*c+b^2)^(1/2))/a^2
/(-4*a*c+b^2)^(3/2)/e*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.627.2 Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 339, normalized size of antiderivative = 0.97

$$\int \frac{1}{(d+ex)^2 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2} \, dx = \frac{-\frac{4}{d+ex}+\frac{2(d+ex)\left(b^3-3abc+b^2c(d+ex)^2-2ac^2(d+ex)^2\right)}{(-b^2+4ac)(a+b(d+ex)^2+c(d+ex)^4)}+\frac{\sqrt{2}\sqrt{c}\left(-3b^3+16abc-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+}{4a^2e}$$

input
```
Integrate[1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

output
```
(-4/(d + e*x) + (2*(d + e*x)*(b^3 - 3*a*b*c + b^2*c*(d + e*x)^2 - 2*a*c^2*
(d + e*x)^2))/((-b^2 + 4*a*c)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (Sqrt
[2]*Sqrt[c]*(-3*b^3 + 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2
 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]]
)/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(3*
b^3 - 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c])*ArcTa
n[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)
^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*a^2*e)
```

3.627.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 339, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1462, 1441, 25, 1604, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^2 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2} \, dx$$

$$\Big\downarrow \text{1462}$$

$$\frac{\int \frac{1}{(d+ex)^2(c(d+ex)^4+b(d+ex)^2+a)^2}d(d+ex)}{e}$$

$$\Big\downarrow \text{1441}$$

$$\frac{\frac{-2ac+b^2+bc(d+ex)^2}{2a(b^2-4ac)(d+ex)(a+b(d+ex)^2+c(d+ex)^4)}-\frac{\int -\frac{3b^2+3c(d+ex)^2b-10ac}{(d+ex)^2\left(c(d+ex)^4+b(d+ex)^2+a\right)}d(d+ex)}{2a(b^2-4ac)}}{e}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{3b^2+3c(d+ex)^2b-10ac}{(d+ex)^2\left(c(d+ex)^4+b(d+ex)^2+a\right)}d(d+ex)}{2a(b^2-4ac)}+\frac{-2ac+b^2+bc(d+ex)^2}{2a(b^2-4ac)(d+ex)(a+b(d+ex)^2+c(d+ex)^4)}}{e}$$

$$\Big\downarrow \ 1604$$

$$\frac{\frac{-\frac{\int \frac{c\left(3b^2-10ac\right)(d+ex)^2+b\left(3b^2-13ac\right)}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)}{a}-\frac{3b^2-10ac}{a(d+ex)}}{2a(b^2-4ac)}+\frac{-2ac+b^2+bc(d+ex)^2}{2a(b^2-4ac)(d+ex)(a+b(d+ex)^2+c(d+ex)^4)}}{e}$$

$$\Big\downarrow \ 1480$$

$$\frac{-\frac{\frac{\frac{1}{2}c\left(-\frac{16abc}{\sqrt{b^2-4ac}}+\frac{3b^3}{\sqrt{b^2-4ac}}-10ac+3b^2\right)\int\frac{1}{c(d+ex)^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}d(d+ex)-\frac{c\left(-\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\int\frac{1}{c(d+ex)^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}d(d+e}{2\sqrt{b^2-4ac}}}{a}}{2a(b^2-4ac)}}{e}$$

$$\Big\downarrow \ 218$$

$$\frac{\frac{-\frac{\frac{\sqrt{c}\left(-\frac{16abc}{\sqrt{b^2-4ac}}+\frac{3b^3}{\sqrt{b^2-4ac}}-10ac+3b^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(-\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}}{a}-\frac{3b^2-10ac}{a(d+ex)}}{2a(b^2-4ac)}+\frac{}{2a(b}}{e}$$

input
```
Int[1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

output
```
((b^2 - 2*a*c + b*c*(d + e*x)^2)/(2*a*(b^2 - 4*a*c)*(d + e*x)*(a + b*(d +
e*x)^2 + c*(d + e*x)^4)) + (-((3*b^2 - 10*a*c)/(a*(d + e*x))) - ((Sqrt[c]*
(3*b^2 - 10*a*c + (3*b^3)/Sqrt[b^2 - 4*a*c] - (16*a*b*c)/Sqrt[b^2 - 4*a*c]
)*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2
]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(3*b^3 - 16*a*b*c - (3*b^2 - 10*
a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b
^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/a)
/(2*a*(b^2 - 4*a*c)))/e
```

3.627. $\int \frac{1}{(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$

3.627.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1441
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(-(d*x)^(m + 1))*(b^2 - 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p +
1)/(2*a*d*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))
  Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m
 + 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x
] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] ||
IntegerQ[m])
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

3.627. $\int \frac{1}{(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$

3.627.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.73 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.27

method	result
default	$-\frac{\frac{c e^{2}\left(2ac-b^{2}\right)x^{3}}{8ac-2b^{2}}+\frac{3dce\left(2ac-b^{2}\right)x^{2}}{2\left(4ac-b^{2}\right)}+\frac{\left(6a c^{2}d^{2}-3b^{2}c d^{2}+3abc-b^{3}\right)x}{8ac-2b^{2}}+\frac{d\left(2a c^{2}d^{2}-b^{2}c d^{2}+3abc-b^{3}\right)}{2e\left(4ac-b^{2}\right)}}{c x^{4}e^{4}+4cd e^{3}x^{3}+6c d^{2}e^{2}x^{2}+4c d^{3}ex+b e^{2}x^{2}+d^{4}c+2bdex+b d^{2}+a}+\_R\text{=RootOf}\left(c e^{4}\_Z^{4}+4cd e^{3}\_Z^{3}+(\right.$
risch	Expression too large to display

input
```
int(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/a^2*((1/2*c*e^2*(2*a*c-b^2)/(4*a*c-b^2)*x^3+3/2*d*c*e*(2*a*c-b^2)/(4*a*
c-b^2)*x^2+1/2*(6*a*c^2*d^2-3*b^2*c*d^2+3*a*b*c-b^3)/(4*a*c-b^2)*x+1/2*d/e
*(2*a*c^2*d^2-b^2*c*d^2+3*a*b*c-b^3)/(4*a*c-b^2))/(c*e^4*x^4+4*c*d*e^3*x^3
+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)+1/4/(4*a*c
-b^2)/e*sum((c*e^2*(10*a*c-3*b^2)*_R^2+2*d*c*e*(10*a*c-3*b^2)*_R+10*a*c^2*
d^2-3*b^2*c*d^2+13*a*b*c-3*b^3)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+
2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*
e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a))-1/a^2/e/(e*x+d)
```

3.627.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4330 vs. 2(300) = 600.

Time = 0.45 (sec) , antiderivative size = 4330, normalized size of antiderivative = 12.44

$$\int \frac{1}{(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")
```

output
```
-1/4*(2*(3*b^2*c - 10*a*c^2)*e^4*x^4 + 8*(3*b^2*c - 10*a*c^2)*d*e^3*x^3 +
2*(3*b^2*c - 10*a*c^2)*d^4 + 2*(3*b^3 - 11*a*b*c + 6*(3*b^2*c - 10*a*c^2)*
d^2)*e^2*x^2 + 4*a*b^2 - 16*a^2*c + 2*(3*b^3 - 11*a*b*c)*d^2 + 4*(2*(3*b^2
*c - 10*a*c^2)*d^3 + (3*b^3 - 11*a*b*c)*d)*e*x + sqrt(1/2)*((a^2*b^2*c - 4
*a^3*c^2)*e^6*x^5 + 5*(a^2*b^2*c - 4*a^3*c^2)*d*e^5*x^4 + (a^2*b^3 - 4*a^3
*b*c + 10*(a^2*b^2*c - 4*a^3*c^2)*d^2)*e^4*x^3 + (10*(a^2*b^2*c - 4*a^3*c^
2)*d^3 + 3*(a^2*b^3 - 4*a^3*b*c)*d)*e^3*x^2 + (a^3*b^2 - 4*a^4*c + 5*(a^2*
b^2*c - 4*a^3*c^2)*d^4 + 3*(a^2*b^3 - 4*a^3*b*c)*d^2)*e^2*x + ((a^2*b^2*c
- 4*a^3*c^2)*d^5 + (a^2*b^3 - 4*a^3*b*c)*d^3 + (a^3*b^2 - 4*a^4*c)*d)*e)*s
qrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (a^5*b^6 - 1
2*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*sqrt((81*b^8 - 918*a*b^6*c
+ 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*a^11*
b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4)))/((a^5*b^6 - 12*a^6*b^4*c + 4
8*a^7*b^2*c^2 - 64*a^8*c^3)*e^2))*log(-(189*b^6*c^3 - 1971*a*b^4*c^4 + 562
5*a^2*b^2*c^5 - 2500*a^3*c^6)*e*x - (189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a
^2*b^2*c^5 - 2500*a^3*c^6)*d + 1/2*sqrt(1/2)*((3*a^5*b^10 - 55*a^6*b^8*c +
 392*a^7*b^6*c^2 - 1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4 - 1280*a^10*c^5)*e^
3*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a
^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4))
- (27*b^11 - 486*a*b^9*c + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 14408...
```

3.627.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

output
```
Timed out
```

3.627.7 Maxima [F]

$$\int \frac{1}{(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$$
$$=\int \frac{1}{\left((ex+d)^4c+(ex+d)^2b+a\right)^2(ex+d)^2}\,dx$$

input
```
integrate(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")
```

output
```
-1/2*((3*b^2*c - 10*a*c^2)*e^4*x^4 + 4*(3*b^2*c - 10*a*c^2)*d*e^3*x^3 + (3
*b^2*c - 10*a*c^2)*d^4 + (3*b^3 - 11*a*b*c + 6*(3*b^2*c - 10*a*c^2)*d^2)*e
^2*x^2 + 2*a*b^2 - 8*a^2*c + (3*b^3 - 11*a*b*c)*d^2 + 2*(2*(3*b^2*c - 10*a
*c^2)*d^3 + (3*b^3 - 11*a*b*c)*d)*e*x)/((a^2*b^2*c - 4*a^3*c^2)*e^6*x^5 +
5*(a^2*b^2*c - 4*a^3*c^2)*d*e^5*x^4 + (a^2*b^3 - 4*a^3*b*c + 10*(a^2*b^2*c
 - 4*a^3*c^2)*d^2)*e^4*x^3 + (10*(a^2*b^2*c - 4*a^3*c^2)*d^3 + 3*(a^2*b^3
- 4*a^3*b*c)*d)*e^3*x^2 + (a^3*b^2 - 4*a^4*c + 5*(a^2*b^2*c - 4*a^3*c^2)*d
^4 + 3*(a^2*b^3 - 4*a^3*b*c)*d^2)*e^2*x + ((a^2*b^2*c - 4*a^3*c^2)*d^5 + (
a^2*b^3 - 4*a^3*b*c)*d^3 + (a^3*b^2 - 4*a^4*c)*d)*e) - 1/2*integrate(((3*b
^2*c - 10*a*c^2)*e^2*x^2 + 2*(3*b^2*c - 10*a*c^2)*d*e*x + 3*b^3 - 13*a*b*c
 + (3*b^2*c - 10*a*c^2)*d^2)/((b^2*c - 4*a*c^2)*e^4*x^4 + 4*(b^2*c - 4*a*c
^2)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^
2)*d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2 + 2*(2*(b^2*c - 4*
a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e*x), x)/a^2
```

3.627.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 879 vs. $2(300) = 600$.

Time = 0.35 (sec) , antiderivative size = 879, normalized size of antiderivative = 2.53

$$\int \frac{1}{(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$$
$$= -\frac{\frac{b^2c}{(ex+d)e}-\frac{2\,ac^2}{(ex+d)e}+\frac{b^3}{(ex+d)^3e}-\frac{3\,abc}{(ex+d)^3e}}{2\,(a^2b^2-4\,a^3c)\left(c+\frac{b}{(ex+d)^2}+\frac{a}{(ex+d)^4}\right)}-\frac{1}{(ex+d)a^2e}$$
$$+\frac{\left((3\,a^4b^7-31\,a^5b^5c+96\,a^6b^3c^2-80\,a^7bc^3)\sqrt{2\,ab+2\,\sqrt{b^2-4\,ac}a}e^4+2\,(3\,a^3b^2c-10\,a^4c^2)\sqrt{2\,ab+2\,\sqrt{}}\right.}{}$$
$$-\frac{\left((3\,a^4b^7-31\,a^5b^5c+96\,a^6b^3c^2-80\,a^7bc^3)\sqrt{2\,ab-2\,\sqrt{b^2-4\,ac}a}e^4-2\,(3\,a^3b^2c-10\,a^4c^2)\sqrt{2\,ab-2\,\sqrt{}}\right.}{}$$

input
```
integrate(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

output
```
-1/2*(b^2*c/((e*x + d)*e) - 2*a*c^2/((e*x + d)*e) + b^3/((e*x + d)^3*e) -
3*a*b*c/((e*x + d)^3*e))/((a^2*b^2 - 4*a^3*c)*(c + b/(e*x + d)^2 + a/(e*x
+ d)^4)) - 1/((e*x + d)*a^2*e) + 1/16*((3*a^4*b^7 - 31*a^5*b^5*c + 96*a^6*
b^3*c^2 - 80*a^7*b*c^3)*sqrt(2*a*b + 2*sqrt(b^2 - 4*a*c)*a)*e^4 + 2*(3*a^3
*b^2*c - 10*a^4*c^2)*sqrt(2*a*b + 2*sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)
*e^2*abs(a^2*b^2*e^2 - 4*a^3*c*e^2) - (a^2*b^2*e^2 - 4*a^3*c*e^2)^2*(3*b^3
 - 13*a*b*c)*sqrt(2*a*b + 2*sqrt(b^2 - 4*a*c)*a))*arctan(2*sqrt(1/2)/((e*x
 + d)*e*sqrt((a^2*b^3*e^2 - 4*a^3*b*c*e^2 + sqrt((a^2*b^3*e^2 - 4*a^3*b*c*
e^2)^2 - 4*(a^3*b^2*e^4 - 4*a^4*c*e^4)*(a^2*b^2*c - 4*a^3*c^2)))/(a^3*b^2*
e^4 - 4*a^4*c*e^4))))/((a^5*b^2*c - 4*a^6*c^2)*sqrt(b^2 - 4*a*c)*e^3*abs(a
^2*b^2*e^2 - 4*a^3*c*e^2)*abs(a)) - 1/16*((3*a^4*b^7 - 31*a^5*b^5*c + 96*a
^6*b^3*c^2 - 80*a^7*b*c^3)*sqrt(2*a*b - 2*sqrt(b^2 - 4*a*c)*a)*e^4 - 2*(3*
a^3*b^2*c - 10*a^4*c^2)*sqrt(2*a*b - 2*sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a
*c)*e^2*abs(a^2*b^2*e^2 - 4*a^3*c*e^2) - (a^2*b^2*e^2 - 4*a^3*c*e^2)^2*(3*
b^3 - 13*a*b*c)*sqrt(2*a*b - 2*sqrt(b^2 - 4*a*c)*a))*arctan(2*sqrt(1/2)/((
e*x + d)*e*sqrt((a^2*b^3*e^2 - 4*a^3*b*c*e^2 - sqrt((a^2*b^3*e^2 - 4*a^3*b
*c*e^2)^2 - 4*(a^3*b^2*e^4 - 4*a^4*c*e^4)*(a^2*b^2*c - 4*a^3*c^2)))/(a^3*b
^2*e^4 - 4*a^4*c*e^4))))/((a^5*b^2*c - 4*a^6*c^2)*sqrt(b^2 - 4*a*c)*e^3*ab
s(a^2*b^2*e^2 - 4*a^3*c*e^2)*abs(a))
```

3.627. $\int \frac{1}{(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$

3.627.9 Mupad [B] (verification not implemented)

Time = 11.53 (sec) , antiderivative size = 10556, normalized size of antiderivative = 30.33

$$\int \frac{1}{(d+ex)^2 (a+b(d+ex)^2+c(d+ex)^4)^2} \, dx = \text{Too large to display}$$

input
```
int(1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x)
```

output
```
- atan(((-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 207
7*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5
- 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c
 - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2 + 4096*a^11*c^6*e^2 - 24*a^6*b^10*c*e^
2 + 240*a^7*b^8*c^2*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 - 61
44*a^10*b^2*c^5*e^2)))^(1/2)*(x*(204800*a^12*c^9*e^12 + 144*a^6*b^12*c^3*e
^12 - 3264*a^7*b^10*c^4*e^12 + 30112*a^8*b^8*c^5*e^12 - 143360*a^9*b^6*c^6
*e^12 + 365568*a^10*b^4*c^7*e^12 - 458752*a^11*b^2*c^8*e^12) + (-(9*b^13 -
 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 106
56*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a
*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(
32*(a^5*b^12*e^2 + 4096*a^11*c^6*e^2 - 24*a^6*b^10*c*e^2 + 240*a^7*b^8*c^2
*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^10*b^2*c^5*e^2
)))^(1/2)*((-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 +
2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c
^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*
a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2 + 4096*a^11*c^6*e^2 - 24*a^6*b^10*c
*e^2 + 240*a^7*b^8*c^2*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 -
 6144*a^10*b^2*c^5*e^2)))^(1/2)*(x*(1048576*a^16*b*c^8*e^14 + 256*a^10*b^1
3*c^2*e^14 - 6144*a^11*b^11*c^3*e^14 + 61440*a^12*b^9*c^4*e^14 - 327680...
```

3.628 $\int \frac{1}{(d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

3.628.1 Optimal result

Integrand size = 30, antiderivative size = 213

$$\begin{aligned}&\int \frac{1}{(d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx\\&=-\frac{b^2-3ac}{a^2\left(b^2-4ac\right)e(d+ex)^2}+\frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)e(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}\\&\quad-\frac{\left(b^4-6ab^2c+6a^2c^2\right)\operatorname{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{a^3\left(b^2-4ac\right)^{3/2}e}\\&\quad-\frac{2b\log(d+ex)}{a^3e}+\frac{b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{2a^3e}\end{aligned}$$

output
```
(3*a*c-b^2)/a^2/(-4*a*c+b^2)/e/(e*x+d)^2+1/2*(b^2-2*a*c+b*c*(e*x+d)^2)/a/(
-4*a*c+b^2)/e/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)-(6*a^2*c^2-6*a*b^2*c+b
^4)*arctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)^(3/2)/e
-2*b*ln(e*x+d)/a^3/e+1/2*b*ln(a+b*(e*x+d)^2+c*(e*x+d)^4)/a^3/e
```

3.628.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.33

$$\int \frac{1}{(d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$$
$$= \frac{-\frac{a}{(d+ex)^2}+\frac{a\left(b^3-3abc+b^2c(d+ex)^2-2ac^2(d+ex)^2\right)}{(-b^2+4ac)(a+b(d+ex)^2+c(d+ex)^4)}-4b\log(d+ex)+\frac{\left(b^4-6ab^2c+6a^2c^2+b^3\sqrt{b^2-4ac}-4abc\sqrt{b^2-4ac}\right)\log\left(b-\sqrt{b^2}\right.}{(b^2-4ac)^{3/2}}}{2a^3e}$$

input
```
Integrate[1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

output
```
(-(a/(d + e*x)^2) + (a*(b^3 - 3*a*b*c + b^2*c*(d + e*x)^2 - 2*a*c^2*(d + e
*x)^2))/((-b^2 + 4*a*c)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) - 4*b*Log[d +
 e*x] + ((b^4 - 6*a*b^2*c + 6*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 4*a*b*c*Sq
rt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(b^2 - 4*a*
c)^(3/2) + ((-b^4 + 6*a*b^2*c - 6*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 4*a*b*
c*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(b^2 -
4*a*c)^(3/2))/(2*a^3*e)
```

3.628.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1462, 1434, 1165, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$$
$$\Big\downarrow \; 1462$$
$$\frac{\int \frac{1}{(d+ex)^3(c(d+ex)^4+b(d+ex)^2+a)^2}d(d+ex)}{e}$$
$$\Big\downarrow \; 1434$$
$$\frac{\int \frac{1}{(d+ex)^4(c(d+ex)^4+b(d+ex)^2+a)^2}d(d+ex)^2}{2e}$$
$$\Big\downarrow \; 1165$$

$$\frac{\frac{-2ac+b^2+bc(d+ex)^2}{a(b^2-4ac)(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)}-\frac{\int -\frac{2\left(b^2+c(d+ex)^2b-3ac\right)}{(d+ex)^4\left(c(d+ex)^4+b(d+ex)^2+a\right)}d(d+ex)^2}{a(b^2-4ac)}}{2e}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{2\int \frac{b^2+c(d+ex)^2b-3ac}{(d+ex)^4\left(c(d+ex)^4+b(d+ex)^2+a\right)}d(d+ex)^2}{a(b^2-4ac)}+\frac{-2ac+b^2+bc(d+ex)^2}{a(b^2-4ac)(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)}}{2e}$$

$$\Big\downarrow \text{ 1200}$$

$$\frac{\frac{2\int \left(\frac{b^2-3ac}{a(d+ex)^4}+\frac{b^4-5acb^2+c\left(b^2-4ac\right)(d+ex)^2b+3a^2c^2}{a^2\left(c(d+ex)^4+b(d+ex)^2+a\right)}+\frac{4abc-b^3}{a^2(d+ex)^2}\right)d(d+ex)^2}{a(b^2-4ac)}+\frac{-2ac+b^2+bc(d+ex)^2}{a(b^2-4ac)(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)}}{2e}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{2\left(-\frac{\left(6a^2c^2-6ab^2c+b^4\right)\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}}-\frac{b\left(b^2-4ac\right)\log\left((d+ex)^2\right)}{a^2}+\frac{b\left(b^2-4ac\right)\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{2a^2}-\frac{b^2-3ac}{a(d+ex)^2}\right)}{a(b^2-4ac)}+\frac{-2\cdots}{a(b^2-4ac)(d+\cdots}}{2e}$$

input
```
Int[1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

output
```
((b^2 - 2*a*c + b*c*(d + e*x)^2)/(a*(b^2 - 4*a*c)*(d + e*x)^2*(a + b*(d +
e*x)^2 + c*(d + e*x)^4)) + (2*(-((b^2 - 3*a*c)/(a*(d + e*x)^2)) - ((b^4 -
6*a*b^2*c + 6*a^2*c^2)*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(
a^2*Sqrt[b^2 - 4*a*c]) - (b*(b^2 - 4*a*c)*Log[(d + e*x)^2])/a^2 + (b*(b^2
- 4*a*c)*Log[a + b*(d + e*x)^2 + c*(d + e*x)^4])/(2*a^2)))/(a*(b^2 - 4*a*c
)))/(2*e)
```

3.628. $\int \frac{1}{(d+ex)^3(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$

3.628.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.628. $\int \frac{1}{(d+ex)^3(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$

3.628.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.75 (sec) , antiderivative size = 462, normalized size of antiderivative = 2.17

method	result
default	$-\frac{\frac{\frac{eac\left(2ac-b^{2}\right)x^{2}}{8ac-2b^{2}}+\frac{cda\left(2ac-b^{2}\right)x}{4ac-b^{2}}+\frac{a\left(2a\,c^{2}d^{2}-b^{2}c\,d^{2}+3abc-b^{3}\right)}{2e\left(4ac-b^{2}\right)}}{c\,x^{4}e^{4}+4cd\,e^{3}x^{3}+6c\,d^{2}e^{2}x^{2}+4c\,d^{3}ex+b\,e^{2}x^{2}+d^{4}c+2bdex+b\,d^{2}+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\,e^{4}\_Z^{4}+4cd\,e^{3}\_Z^{3}+\left(6c\,d^{2}e^{2}+b\,e^{2}\right)\_Z^{2}+\left(4d^{3}ec+2bde\right)\right.}}{}}{}$
risch	$\frac{-\frac{\left(3ac-b^{2}\right)c\,e^{3}x^{4}}{\left(4ac-b^{2}\right)a^{2}}-\frac{4\left(3ac-b^{2}\right)cd\,e^{2}x^{3}}{\left(4ac-b^{2}\right)a^{2}}-\frac{\left(36a\,c^{2}d^{2}-12b^{2}c\,d^{2}+7abc-2b^{3}\right)e\,x^{2}}{2a^{2}\left(4ac-b^{2}\right)}-\frac{d\left(12a\,c^{2}d^{2}-4b^{2}c\,d^{2}+7abc-2b^{3}\right)x}{a^{2}\left(4ac-b^{2}\right)}-\frac{6a\,c^{2}d^{4}-2b^{2}c\,d^{4}+7b\,d^{2}ca-}{2e\,a^{2}\left(4ac-\right.}}{(ex+d)^{2}(c\,x^{4}e^{4}+4cd\,e^{3}x^{3}+6c\,d^{2}e^{2}x^{2}+4c\,d^{3}ex+b\,e^{2}x^{2}+d^{4}c+2bdex+b\,d^{2}+a)}$

input
```
int(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/a^3*((1/2*e*a*c*(2*a*c-b^2)/(4*a*c-b^2)*x^2+c*d*a*(2*a*c-b^2)/(4*a*c-b^
2)*x+1/2/e*a*(2*a*c^2*d^2-b^2*c*d^2+3*a*b*c-b^3)/(4*a*c-b^2))/(c*e^4*x^4+4
*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a
)+1/(4*a*c-b^2)/e*sum((e^3*b*c*(-4*a*c+b^2)*_R^3+3*d*e^2*b*c*(-4*a*c+b^2)*
_R^2+e*(-12*a*b*c^2*d^2+3*b^3*c*d^2+3*a^2*c^2-5*a*b^2*c+b^4)*_R-4*a*b*c^2*
d^3+b^3*c*d^3+3*a^2*c^2*d-5*a*b^2*c*d+d*b^4)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+
6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_
Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a))-1/2/a
^2/e/(e*x+d)^2-2*b*ln(e*x+d)/a^3/e
```

3.628.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2216 vs. 2(205) = 410.

Time = 0.61 (sec) , antiderivative size = 4562, normalized size of antiderivative = 21.42

$$\int \frac{1}{(d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")
```

output
```
[-1/2*(2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*e^4*x^4 + 8*(a*b^4*c - 7*a
^2*b^2*c^2 + 12*a^3*c^3)*d*e^3*x^3 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 +
2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d^4 + (2*a*b^5 - 15*a^2*b^3*c + 2
8*a^3*b*c^2 + 12*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d^2)*e^2*x^2 + (2*
a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*d^2 + 2*(4*(a*b^4*c - 7*a^2*b^2*c^2 +
 12*a^3*c^3)*d^3 + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*d)*e*x + ((b^4*
c - 6*a*b^2*c^2 + 6*a^2*c^3)*e^6*x^6 + 6*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)
*d*e^5*x^5 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2 + 15*(b^4*c - 6*a*b^2*c^2 + 6*
a^2*c^3)*d^2)*e^4*x^4 + (b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^6 + 4*(5*(b^4*
c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^3 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d)*e^3*
x^3 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d^4 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c
^2 + 15*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^4 + 6*(b^5 - 6*a*b^3*c + 6*a^2
*b*c^2)*d^2)*e^2*x^2 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*d^2 + 2*(3*(b^4*c
 - 6*a*b^2*c^2 + 6*a^2*c^3)*d^5 + 2*(b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d^3 +
(a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*d)*e*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*e^4
*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d
^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x +
2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*
d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a)) - ((b^5*c - 8*a*b^3
*c^2 + 16*a^2*b*c^3)*e^6*x^6 + 6*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d...
```

3.628.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

output
```
Timed out
```

3.628.7 Maxima [F]

$$\int \frac{1}{(d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \int \frac{1}{\left((ex+d)^4c+(ex+d)^2b+a\right)^2(ex+d)^3}\,dx$$

input
```
integrate(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")
```

output
```
-1/2*(2*(b^2*c - 3*a*c^2)*e^4*x^4 + 8*(b^2*c - 3*a*c^2)*d*e^3*x^3 + 2*(b^2
*c - 3*a*c^2)*d^4 + (2*b^3 - 7*a*b*c + 12*(b^2*c - 3*a*c^2)*d^2)*e^2*x^2 +
 a*b^2 - 4*a^2*c + (2*b^3 - 7*a*b*c)*d^2 + 2*(4*(b^2*c - 3*a*c^2)*d^3 + (2
*b^3 - 7*a*b*c)*d)*e*x)/((a^2*b^2*c - 4*a^3*c^2)*e^7*x^6 + 6*(a^2*b^2*c -
4*a^3*c^2)*d*e^6*x^5 + (a^2*b^3 - 4*a^3*b*c + 15*(a^2*b^2*c - 4*a^3*c^2)*d
^2)*e^5*x^4 + 4*(5*(a^2*b^2*c - 4*a^3*c^2)*d^3 + (a^2*b^3 - 4*a^3*b*c)*d)*
e^4*x^3 + (a^3*b^2 - 4*a^4*c + 15*(a^2*b^2*c - 4*a^3*c^2)*d^4 + 6*(a^2*b^3
 - 4*a^3*b*c)*d^2)*e^3*x^2 + 2*(3*(a^2*b^2*c - 4*a^3*c^2)*d^5 + 2*(a^2*b^3
 - 4*a^3*b*c)*d^3 + (a^3*b^2 - 4*a^4*c)*d)*e^2*x + ((a^2*b^2*c - 4*a^3*c^2
)*d^6 + (a^2*b^3 - 4*a^3*b*c)*d^4 + (a^3*b^2 - 4*a^4*c)*d^2)*e) + 2*integr
ate(((b^3*c - 4*a*b*c^2)*e^3*x^3 + 3*(b^3*c - 4*a*b*c^2)*d*e^2*x^2 + (b^3*
c - 4*a*b*c^2)*d^3 + (b^4 - 5*a*b^2*c + 3*a^2*c^2 + 3*(b^3*c - 4*a*b*c^2)*
d^2)*e*x + (b^4 - 5*a*b^2*c + 3*a^2*c^2)*d)/((b^2*c - 4*a*c^2)*e^4*x^4 + 4
*(b^2*c - 4*a*c^2)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3 - 4*a*b*c + 6*
(b^2*c - 4*a*c^2)*d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2 + 2
*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e*x), x)/a^3 - 2*b*log(e*x
+ d)/(a^3*e)
```

3.628.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.03

$$\int \frac{1}{(d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{(b^4-6\,ab^2c+6\,a^2c^2)\arctan\left(-\frac{b+\frac{2\,a}{(ex+d)^2}}{\sqrt{-b^2+4\,ac}}\right)}{(a^3b^2-4\,a^4c)\sqrt{-b^2+4\,ac}e} + \frac{b\log\left(c+\frac{b}{(ex+d)^2}+\frac{a}{(ex+d)^4}\right)}{2\,a^3e} + \frac{\frac{b^3c-3\,abc^2}{a}+\frac{b^4e-4\,ab^2ce+2\,a^2c^2e}{(ex+d)^2ae}}{2\,(b^2-4\,ac)a^2\left(c+\frac{b}{(ex+d)^2}+\frac{a}{(ex+d)^4}\right)e} - \frac{1}{2\,(ex+d)^2a^2e}$$

input
```
integrate(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

output
```
(b^4 - 6*a*b^2*c + 6*a^2*c^2)*arctan(-(b + 2*a/(e*x + d)^2)/sqrt(-b^2 + 4*
a*c))/((a^3*b^2 - 4*a^4*c)*sqrt(-b^2 + 4*a*c)*e) + 1/2*b*log(c + b/(e*x +
d)^2 + a/(e*x + d)^4)/(a^3*e) + 1/2*((b^3*c - 3*a*b*c^2)/a + (b^4*e - 4*a*
b^2*c*e + 2*a^2*c^2*e)/((e*x + d)^2*a*e))/((b^2 - 4*a*c)*a^2*(c + b/(e*x +
 d)^2 + a/(e*x + d)^4)*e) - 1/2/((e*x + d)^2*a^2*e)
```

3.628.9 Mupad [B] (verification not implemented)

Time = 15.32 (sec) , antiderivative size = 12436, normalized size of antiderivative = 58.38

$$\int \frac{1}{(d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2} \, dx = \text{Too large to display}$$

input
```
int(1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x)
```

output
```
((x*(2*b^3*d - 12*a*c^2*d^3 + 4*b^2*c*d^3 - 7*a*b*c*d))/(4*a^3*c - a^2*b^2
) - (x^4*(3*a*c^2*e^3 - b^2*c*e^3))/(4*a^3*c - a^2*b^2) - (4*x^3*(3*a*c^2*
d*e^2 - b^2*c*d*e^2))/(4*a^3*c - a^2*b^2) + (a*b^2 - 4*a^2*c + 2*b^3*d^2 -
 6*a*c^2*d^4 + 2*b^2*c*d^4 - 7*a*b*c*d^2)/(2*e*(4*a^3*c - a^2*b^2)) + (x^2
*(2*b^3*e - 36*a*c^2*d^2*e + 12*b^2*c*d^2*e - 7*a*b*c*e))/(2*(4*a^3*c - a^
2*b^2)))/(x^4*(b*e^4 + 15*c*d^2*e^4) + a*d^2 + b*d^4 + c*d^6 + x*(2*a*d*e
+ 4*b*d^3*e + 6*c*d^5*e) + x^2*(a*e^2 + 6*b*d^2*e^2 + 15*c*d^4*e^2) + x^3*
(20*c*d^3*e^3 + 4*b*d*e^3) + c*e^6*x^6 + 6*c*d*e^5*x^5) + (log((((b + a^3*
e*(-(b^4 + 6*a^2*c^2 - 6*a*b^2*c)^2/(a^6*e^2*(4*a*c - b^2)^3))^(1/2))*(((b
 + a^3*e*(-(b^4 + 6*a^2*c^2 - 6*a*b^2*c)^2/(a^6*e^2*(4*a*c - b^2)^3))^(1/2
))*((4*c^2*e^16*(2*b^5 + 6*a^2*b*c^2 + b^4*c*d^2 - 30*a^2*c^3*d^2 - 10*a*b
^3*c + 2*a*b^2*c^2*d^2))/(a^2*(4*a*c - b^2)) + (4*c^3*e^18*x^2*(b^4 - 30*a
^2*c^2 + 2*a*b^2*c))/(a^2*(4*a*c - b^2)) - (2*b*c^2*e^16*(b + a^3*e*(-(b^4
 + 6*a^2*c^2 - 6*a*b^2*c)^2/(a^6*e^2*(4*a*c - b^2)^3))^(1/2))*(a*b + 3*b^2
*d^2 + 3*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*a*c*
d*e*x))/a^3 + (8*c^3*d*e^17*x*(b^4 - 30*a^2*c^2 + 2*a*b^2*c))/(a^2*(4*a*c
- b^2))))/(2*a^3*e) - (4*c^3*e^15*(3*a*c - b^2)*(4*b^4 + 3*a^2*c^2 + 6*b^3
*c*d^2 - 17*a*b^2*c - 23*a*b*c^2*d^2))/(a^4*(4*a*c - b^2)^2) + (4*b*c^4*e^
17*x^2*(6*b^4 + 69*a^2*c^2 - 41*a*b^2*c))/(a^4*(4*a*c - b^2)^2) + (8*b*c^4
*d*e^16*x*(6*b^4 + 69*a^2*c^2 - 41*a*b^2*c))/(a^4*(4*a*c - b^2)^2)))/(2...
```

3.629 $\int \frac{1}{(d+ex)^4(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$

3.629.1 Optimal result

Integrand size = 30, antiderivative size = 408

$$\begin{aligned}\int \frac{1}{(d+ex)^4\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx &= -\frac{5b^2-14ac}{6a^2(b^2-4ac)\,e(d+ex)^3}+\frac{b(5b^2-19ac)}{2a^3(b^2-4ac)\,e(d+ex)} \\ &+\frac{b^2-2ac+bc(d+ex)^2}{2a(b^2-4ac)\,e(d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)} \\ &+\frac{\sqrt{c}\left(5b^4-29ab^2c+28a^2c^2+b(5b^2-19ac)\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^3(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}e} \\ &-\frac{\sqrt{c}\left(5b^4-29ab^2c+28a^2c^2-b(5b^2-19ac)\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^3(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}e}\end{aligned}$$

output
```
1/6*(14*a*c-5*b^2)/a^2/(-4*a*c+b^2)/e/(e*x+d)^3+1/2*b*(-19*a*c+5*b^2)/a^3/
(-4*a*c+b^2)/e/(e*x+d)+1/2*(b^2-2*a*c+b*c*(e*x+d)^2)/a/(-4*a*c+b^2)/e/(e*x
+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)+1/4*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b-(-
4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(5*b^4-29*a*b^2*c+28*a^2*c^2+b*(-19*a*c+5
*b^2)*(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)^(3/2)/e*2^(1/2)/(b-(-4*a*c+b^2)
^(1/2))^(1/2)-1/4*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1
/2))*c^(1/2)*(5*b^4-29*a*b^2*c+28*a^2*c^2-b*(-19*a*c+5*b^2)*(-4*a*c+b^2)^(
1/2))/a^3/(-4*a*c+b^2)^(3/2)/e*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.629.2 Mathematica [A] (verified)

Time = 1.78 (sec) , antiderivative size = 384, normalized size of antiderivative = 0.94

$$\int \frac{1}{(d+ex)^4 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$$

$$= \frac{-\frac{4a}{(d+ex)^3}+\frac{24b}{d+ex}+\frac{6(d+ex)\left(b^4-4ab^2c+2a^2c^2+b^3c(d+ex)^2-3abc^2(d+ex)^2\right)}{(b^2-4ac)\left(a+(d+ex)^2(b+c(d+ex)^2)\right)}+\frac{3\sqrt{2}\sqrt{c}\left(5b^4-29ab^2c+28a^2c^2+5b^3\sqrt{b^2-4ac}-19abc\sqrt{b^2-\right.}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}}{12a^3e}$$

input
```
Integrate[1/((d + e*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

output
```
((-4*a)/(d + e*x)^3 + (24*b)/(d + e*x) + (6*(d + e*x)*(b^4 - 4*a*b^2*c + 2
*a^2*c^2 + b^3*c*(d + e*x)^2 - 3*a*b*c^2*(d + e*x)^2))/((b^2 - 4*a*c)*(a +
 (d + e*x)^2*(b + c*(d + e*x)^2))) + (3*Sqrt[2]*Sqrt[c]*(5*b^4 - 29*a*b^2*
c + 28*a^2*c^2 + 5*b^3*Sqrt[b^2 - 4*a*c] - 19*a*b*c*Sqrt[b^2 - 4*a*c])*Arc
Tan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*
c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt[c]*(-5*b^4 + 29*a*
b^2*c - 28*a^2*c^2 + 5*b^3*Sqrt[b^2 - 4*a*c] - 19*a*b*c*Sqrt[b^2 - 4*a*c])
*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 -
4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(12*a^3*e)
```

3.629.3 Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 394, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1462, 1441, 25, 1604, 27, 1604, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^4 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$$

$$\Big\downarrow \ 1462$$

$$\frac{\int \frac{1}{(d+ex)^4\left(c(d+ex)^4+b(d+ex)^2+a\right)^2}d(d+ex)}{e}$$

$$\Big\downarrow \ 1441$$

$$\frac{\frac{-2ac+b^2+bc(d+ex)^2}{2a(b^2-4ac)(d+ex)^3(a+b(d+ex)^2+c(d+ex)^4)}-\frac{\int -\frac{5b^2+5c(d+ex)^2b-14ac}{(d+ex)^4\left(c(d+ex)^4+b(d+ex)^2+a\right)}d(d+ex)}{2a(b^2-4ac)}}{e}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{5b^2+5c(d+ex)^2b-14ac}{(d+ex)^4\left(c(d+ex)^4+b(d+ex)^2+a\right)}d(d+ex)}{2a(b^2-4ac)}+\frac{-2ac+b^2+bc(d+ex)^2}{2a(b^2-4ac)(d+ex)^3(a+b(d+ex)^2+c(d+ex)^4)}}{e}$$

$$\Big\downarrow \; 1604$$

$$\frac{\frac{-\frac{\int \frac{3\left(c\left(5b^2-14ac\right)(d+ex)^2+b\left(5b^2-19ac\right)\right)}{(d+ex)^2\left(c(d+ex)^4+b(d+ex)^2+a\right)}d(d+ex)}{3a}-\frac{5b^2-14ac}{3a(d+ex)^3}}{2a(b^2-4ac)}+\frac{-2ac+b^2+bc(d+ex)^2}{2a(b^2-4ac)(d+ex)^3(a+b(d+ex)^2+c(d+ex)^4)}}{e}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{-\frac{\int \frac{c\left(5b^2-14ac\right)(d+ex)^2+b\left(5b^2-19ac\right)}{(d+ex)^2\left(c(d+ex)^4+b(d+ex)^2+a\right)}d(d+ex)}{a}-\frac{5b^2-14ac}{3a(d+ex)^3}}{2a(b^2-4ac)}+\frac{-2ac+b^2+bc(d+ex)^2}{2a(b^2-4ac)(d+ex)^3(a+b(d+ex)^2+c(d+ex)^4)}}{e}$$

$$\Big\downarrow \; 1604$$

$$\frac{\frac{-\frac{-\frac{\int \frac{5b^4-24acb^2+c\left(5b^2-19ac\right)(d+ex)^2b+14a^2c^2}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)}{a}-\frac{b\left(5b^2-19ac\right)}{a(d+ex)}}{a}-\frac{5b^2-14ac}{3a(d+ex)^3}}{2a(b^2-4ac)}+\frac{-2ac+b^2+bc(d+ex)^2}{2a(b^2-4ac)(d+ex)^3(a+b(d+ex)^2+c(d+ex)^4)}}{e}$$

$$\Big\downarrow \; 1480$$

$$\frac{\frac{-\frac{-\frac{\frac{c\left(28a^2c^2-29ab^2c+b\left(5b^2-19ac\right)\sqrt{b^2-4ac}+5b^4\right)\int \frac{1}{c(d+ex)^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}d(d+ex)}{2\sqrt{b^2-4ac}}-\frac{c\left(28a^2c^2-29ab^2c-b\left(5b^2-19ac\right)\sqrt{b^2-4ac}+5b^4\right)\int \frac{1}{c(d+ex)^2+\frac{1}{2}\left(b\right.}}{2\sqrt{b^2-4ac}}}{a}}{a}}{2a(b^2-4ac)}}{e}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{-\frac{-\frac{\frac{\sqrt{c}\left(28a^2c^2-29ab^2c+b\left(5b^2-19ac\right)\sqrt{b^2-4ac}+5b^4\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(28a^2c^2-29ab^2c-b\left(5b^2-19ac\right)\sqrt{b^2-4ac}+5b^4\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}}{a}-}{a}}{2a(b^2-4ac)}}{e}$$

3.629. $\int \frac{1}{(d+ex)^4(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$

input
```
Int[1/((d + e*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

output
```
((b^2 - 2*a*c + b*c*(d + e*x)^2)/(2*a*(b^2 - 4*a*c)*(d + e*x)^3*(a + b*(d
+ e*x)^2 + c*(d + e*x)^4)) + (-1/3*(5*b^2 - 14*a*c)/(a*(d + e*x)^3) - (-((
b*(5*b^2 - 19*a*c))/(a*(d + e*x))) - ((Sqrt[c]*(5*b^4 - 29*a*b^2*c + 28*a^
2*c^2 + b*(5*b^2 - 19*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d +
 e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[b - S
qrt[b^2 - 4*a*c]]) - (Sqrt[c]*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 - b*(5*b^2
- 19*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + S
qrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]
))/a)/a)/(2*a*(b^2 - 4*a*c)))/e
```

3.629.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1441
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(-(d*x)^(m + 1))*(b^2 - 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p +
1)/(2*a*d*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))
  Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m
 + 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x
] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] ||
IntegerQ[m])
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

3.629. $\int \frac{1}{(d+ex)^4(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

3.629.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.78 (sec) , antiderivative size = 489, normalized size of antiderivative = 1.20

method	result
default	$-\frac{\frac{-\frac{bc\,e^{2}\left(3ac-b^{2}\right)x^{3}}{2\left(4ac-b^{2}\right)}-\frac{3dbce\left(3ac-b^{2}\right)x^{2}}{2\left(4ac-b^{2}\right)}+\frac{\left(-9b\,c^{2}d^{2}a+3b^{3}c\,d^{2}+2a^{2}c^{2}-4a\,b^{2}c+b^{4}\right)x}{8ac-2b^{2}}+\frac{d\left(-3b\,c^{2}d^{2}a+b^{3}c\,d^{2}+2a^{2}c^{2}-4a\,b^{2}c+b^{4}\right)}{2e\left(4ac-b^{2}\right)}}{c\,x^{4}e^{4}+4cd\,e^{3}x^{3}+6c\,d^{2}e^{2}x^{2}+4c\,d^{3}ex+b\,e^{2}x^{2}+d^{4}c+2bdex+b\,d^{2}+a}+\frac{\_R=\text{Root}}{}}{}$
risch	Expression too large to display

input
```
int(1/(e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/a^3*((-1/2*b*c*e^2*(3*a*c-b^2)/(4*a*c-b^2)*x^3-3/2*d*b*c*e*(3*a*c-b^2)/
(4*a*c-b^2)*x^2+1/2*(-9*a*b*c^2*d^2+3*b^3*c*d^2+2*a^2*c^2-4*a*b^2*c+b^4)/(
4*a*c-b^2)*x+1/2*d/e*(-3*a*b*c^2*d^2+b^3*c*d^2+2*a^2*c^2-4*a*b^2*c+b^4)/(4
*a*c-b^2))/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+
c*d^4+2*b*d*e*x+b*d^2+a)+1/4/(4*a*c-b^2)/e*sum((b*c*e^2*(-19*a*c+5*b^2)*_R
^2+2*b*c*d*e*(-19*a*c+5*b^2)*_R-19*b*c^2*d^2*a+5*b^3*c*d^2+14*a^2*c^2-24*a
*b^2*c+5*b^4)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d
)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4
*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a))-1/3/a^2/e/(e*x+d)^3+2/a^3*b/e/(e*x+d
)
```

3.629.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5734 vs. 2(358) = 716.

Time = 0.64 (sec) , antiderivative size = 5734, normalized size of antiderivative = 14.05

$$\int \frac{1}{(d+ex)^4 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2} \, dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.629.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^4 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2} \, dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

output
```
Timed out
```

3.629.7 Maxima [F]

$$\int \frac{1}{(d+ex)^4 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2} \, dx = \int \frac{1}{\left((ex+d)^4c+(ex+d)^2b+a\right)^2 (ex+d)^4} \, dx$$

input
```
integrate(1/(e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")
```

output
```
1/6*(3*(5*b^3*c - 19*a*b*c^2)*e^6*x^6 + 18*(5*b^3*c - 19*a*b*c^2)*d*e^5*x^
5 + (15*b^4 - 62*a*b^2*c + 14*a^2*c^2 + 45*(5*b^3*c - 19*a*b*c^2)*d^2)*e^4
*x^4 + 3*(5*b^3*c - 19*a*b*c^2)*d^6 + 4*(15*(5*b^3*c - 19*a*b*c^2)*d^3 + (
15*b^4 - 62*a*b^2*c + 14*a^2*c^2)*d)*e^3*x^3 + (15*b^4 - 62*a*b^2*c + 14*a
^2*c^2)*d^4 + (45*(5*b^3*c - 19*a*b*c^2)*d^4 + 10*a*b^3 - 40*a^2*b*c + 6*(
15*b^4 - 62*a*b^2*c + 14*a^2*c^2)*d^2)*e^2*x^2 - 2*a^2*b^2 + 8*a^3*c + 10*
(a*b^3 - 4*a^2*b*c)*d^2 + 2*(9*(5*b^3*c - 19*a*b*c^2)*d^5 + 2*(15*b^4 - 62
*a*b^2*c + 14*a^2*c^2)*d^3 + 10*(a*b^3 - 4*a^2*b*c)*d)*e*x)/((a^3*b^2*c -
4*a^4*c^2)*e^8*x^7 + 7*(a^3*b^2*c - 4*a^4*c^2)*d*e^7*x^6 + (a^3*b^3 - 4*a^
4*b*c + 21*(a^3*b^2*c - 4*a^4*c^2)*d^2)*e^6*x^5 + 5*(7*(a^3*b^2*c - 4*a^4*
c^2)*d^3 + (a^3*b^3 - 4*a^4*b*c)*d)*e^5*x^4 + (a^4*b^2 - 4*a^5*c + 35*(a^3
*b^2*c - 4*a^4*c^2)*d^4 + 10*(a^3*b^3 - 4*a^4*b*c)*d^2)*e^4*x^3 + (21*(a^3
*b^2*c - 4*a^4*c^2)*d^5 + 10*(a^3*b^3 - 4*a^4*b*c)*d^3 + 3*(a^4*b^2 - 4*a^
5*c)*d)*e^3*x^2 + (7*(a^3*b^2*c - 4*a^4*c^2)*d^6 + 5*(a^3*b^3 - 4*a^4*b*c)
*d^4 + 3*(a^4*b^2 - 4*a^5*c)*d^2)*e^2*x + ((a^3*b^2*c - 4*a^4*c^2)*d^7 + (
a^3*b^3 - 4*a^4*b*c)*d^5 + (a^4*b^2 - 4*a^5*c)*d^3)*e) + 1/2*integrate(((5
*b^3*c - 19*a*b*c^2)*e^2*x^2 + 5*b^4 - 24*a*b^2*c + 14*a^2*c^2 + 2*(5*b^3*
c - 19*a*b*c^2)*d*e*x + (5*b^3*c - 19*a*b*c^2)*d^2)/((b^2*c - 4*a*c^2)*e^4
*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3 - 4*a*
b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b...
```

3.629.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2122 vs. $2(358) = 716$.

Time = 0.36 (sec) , antiderivative size = 2122, normalized size of antiderivative = 5.20

$$\int \frac{1}{(d+ex)^4 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

output
```
-1/4*((5*b^3*c*e^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4
)) + d/e)^2 - 19*a*b*c^2*e^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e
^2)/(c*e^4)) + d/e)^2 - 10*b^3*c*d*e*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 -
4*a*c)*e^2)/(c*e^4)) + d/e) + 38*a*b*c^2*d*e*(sqrt(1/2)*sqrt(-(b*e^2 + sqr
t(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e) + 5*b^3*c*d^2 - 19*a*b*c^2*d^2 + 5*b^4
 - 24*a*b^2*c + 14*a^2*c^2)*log(x + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*
a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*
a*c)*e^2)/(c*e^4)) + d/e)^3 - 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2
 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^
2)*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)) - (5*
b^3*c*e^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)
^2 - 19*a*b*c^2*e^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^
4)) - d/e)^2 + 10*b^3*c*d*e*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^
2)/(c*e^4)) - d/e) - 38*a*b*c^2*d*e*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4
*a*c)*e^2)/(c*e^4)) - d/e) + 5*b^3*c*d^2 - 19*a*b*c^2*d^2 + 5*b^4 - 24*a*b
^2*c + 14*a^2*c^2)*log(x - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)
/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)
/(c*e^4)) - d/e)^3 + 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)
*e^2)/(c*e^4)) - d/e)^2 + 2*c*d^3*e + b*d*e + (6*c*d^2*e^2 + b*e^2)*(sqrt(
1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)) + (5*b^3*c*...
```

3.629.9 Mupad [B] (verification not implemented)

Time = 13.05 (sec) , antiderivative size = 12239, normalized size of antiderivative = 30.00

$$\int \frac{1}{(d+ex)^4 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2} \, dx = \text{Too large to display}$$

input
```
int(1/((d + e*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x)
```

output
```
atan(((-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 636
6*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c
^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13
*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2
)^9)^(1/2))/(32*(a^7*b^12*e^2 + 4096*a^13*c^6*e^2 - 24*a^8*b^10*c*e^2 + 24
0*a^9*b^8*c^2*e^2 - 1280*a^10*b^6*c^3*e^2 + 3840*a^11*b^4*c^4*e^2 - 6144*a
^12*b^2*c^5*e^2)))^(1/2)*((-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 8
0640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^
4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9
)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*
b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2 + 4096*a^13*c^6*e^2 - 24
*a^8*b^10*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^10*b^6*c^3*e^2 + 3840*a^11*
b^4*c^4*e^2 - 6144*a^12*b^2*c^5*e^2)))^(1/2)*((-(25*b^15 - 25*b^6*(-(4*a*c
 - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3
 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c
^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^
2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2 + 40
96*a^13*c^6*e^2 - 24*a^8*b^10*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^10*b^6*
c^3*e^2 + 3840*a^11*b^4*c^4*e^2 - 6144*a^12*b^2*c^5*e^2)))^(1/2)*(x*(10485
76*a^21*b*c^8*e^14 + 256*a^15*b^13*c^2*e^14 - 6144*a^16*b^11*c^3*e^14 +...
```

3.629. $\int \frac{1}{(d+ex)^4(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$

3.630 $\int \frac{(d+ex)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$

3.630.1 Optimal result

Integrand size = 30, antiderivative size = 341

$$\begin{aligned}\int \frac{(d+ex)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx &= \frac{(d+ex)\left(2a+b(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} \\ &\quad - \frac{(d+ex)\left(7b^2-4ac+12bc(d+ex)^2\right)}{8\left(b^2-4ac\right)^2 e\left(a+b(d+ex)^2+c(d+ex)^4\right)} \\ &\quad + \frac{3\sqrt{c}\left(3b^2+4ac-2b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{4\sqrt{2}\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}\,e} \\ &\quad - \frac{3\sqrt{c}\left(3b^2+4ac+2b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{4\sqrt{2}\left(b^2-4ac\right)^{5/2}\sqrt{b+\sqrt{b^2-4ac}}\,e}\end{aligned}$$

output
```
1/4*(e*x+d)*(2*a+b*(e*x+d)^2)/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2
-1/8*(e*x+d)*(7*b^2-4*a*c+12*b*c*(e*x+d)^2)/(-4*a*c+b^2)^2/e/(a+b*(e*x+d)^
2+c*(e*x+d)^4)+3/8*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(
1/2))*c^(1/2)*(3*b^2+4*a*c-2*b*(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(5/2)/e*2^
(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-3/8*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b+(
-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(3*b^2+4*a*c+2*b*(-4*a*c+b^2)^(1/2))/(-4
*a*c+b^2)^(5/2)/e*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.630.2 Mathematica [A] (verified)

Time = 2.60 (sec) , antiderivative size = 328, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex)^4}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx$$

$$= \frac{-\frac{2(-2a(d+ex)-b(d+ex)^3)}{(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}+\frac{(d+ex)(-7b^2+4ac-12bc(d+ex)^2)}{(b^2-4ac)^2(a+b(d+ex)^2+c(d+ex)^4)}+\frac{3\sqrt{2}\sqrt{c}\left(3b^2+4ac-2b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}-}{8e}$$

input
```
Integrate[(d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

output
```
((-2*(-2*a*(d + e*x) - b*(d + e*x)^3))/((b^2 - 4*a*c)*(a + b*(d + e*x)^2 +
 c*(d + e*x)^4)^2) + ((d + e*x)*(-7*b^2 + 4*a*c - 12*b*c*(d + e*x)^2))/((b
^2 - 4*a*c)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (3*Sqrt[2]*Sqrt[c]*(3
*b^2 + 4*a*c - 2*b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/S
qrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*
c]]) - (3*Sqrt[2]*Sqrt[c]*(3*b^2 + 4*a*c + 2*b*Sqrt[b^2 - 4*a*c])*ArcTan[(
Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5
/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(8*e)
```

3.630.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1462, 1440, 1492, 27, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^4}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx$$

$$\Big\downarrow \text{1462}$$

$$\frac{\int \frac{(d+ex)^4}{(c(d+ex)^4+b(d+ex)^2+a)^3}d(d+ex)}{e}$$

$$\Big\downarrow \text{1440}$$

$$\frac{\frac{(d+ex)(2a+b(d+ex)^2)}{4(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}-\frac{\int \frac{2a-5b(d+ex)^2}{\left(c(d+ex)^4+b(d+ex)^2+a\right)^2}d(d+ex)}{4(b^2-4ac)}}{e}$$

$\Big\downarrow$ 1492

$$\frac{\frac{(d+ex)(2a+b(d+ex)^2)}{4(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}-\frac{\frac{(d+ex)\left(-4ac+7b^2+12bc(d+ex)^2\right)}{2\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{\int \frac{3a\left(b^2-4c(d+ex)^2b+4ac\right)}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)}{2a\left(b^2-4ac\right)}}{4(b^2-4ac)}}{e}$$

$\Big\downarrow$ 27

$$\frac{\frac{(d+ex)(2a+b(d+ex)^2)}{4(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}-\frac{\frac{(d+ex)\left(-4ac+7b^2+12bc(d+ex)^2\right)}{2\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{3\int \frac{b^2-4c(d+ex)^2b+4ac}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)}{2\left(b^2-4ac\right)}}{4(b^2-4ac)}}{e}$$

$\Big\downarrow$ 1480

$$\frac{\frac{(d+ex)(2a+b(d+ex)^2)}{4(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}-\frac{\frac{(d+ex)\left(-4ac+7b^2+12bc(d+ex)^2\right)}{2\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{3\left(-c\left(2b-\frac{4ac+3b^2}{\sqrt{b^2-4ac}}\right)\int \frac{1}{c(d+ex)^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}d(d+ex)-c\left(\frac{4ac+3b^2}{\sqrt{b^2-4a}}\right.\cdots\right.}{2\left(b^2-4ac\right)}}{4(b^2-4ac)}}{e}$$

$\Big\downarrow$ 218

$$\frac{\frac{(d+ex)(2a+b(d+ex)^2)}{4(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}-\frac{\frac{(d+ex)\left(-4ac+7b^2+12bc(d+ex)^2\right)}{2\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{3\left(-\frac{\sqrt{2}\sqrt{c}\left(2b-\frac{4ac+3b^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\left(\frac{4ac+3b^2}{\sqrt{b^2-4ac}}+\cdots\right.}{\sqrt{\cdots}}\right.}{2\left(b^2-4ac\right)}}{4(b^2-4ac)}}{e}$$

input
```
Int[(d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

output
```
(((d + e*x)*(2*a + b*(d + e*x)^2))/(4*(b^2 - 4*a*c)*(a + b*(d + e*x)^2 + c
*(d + e*x)^4)^2) - (((d + e*x)*(7*b^2 - 4*a*c + 12*b*c*(d + e*x)^2))/(2*(b
^2 - 4*a*c)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) - (3*(-((Sqrt[2]*Sqrt[c]*
(2*b - (3*b^2 + 4*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x
))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*S
qrt[c]*(2*b + (3*b^2 + 4*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(
d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*(
b^2 - 4*a*c)))/(4*(b^2 - 4*a*c)))/e
```

3.630. $\int \frac{(d+ex)^4}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx$

3.630.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1440
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(-d^3)*(d*x)^(m - 3)*(2*a + b*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*
(p + 1)*(b^2 - 4*a*c))), x] + Simp[d^4/(2*(p + 1)*(b^2 - 4*a*c))   Int[(d*x
)^(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1),
x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Gt
Q[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

3.630. $\int \frac{(d+ex)^4}{(a+b(d+ex)^2+c(d+ex)^4)^3} \, dx$

3.630.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.69 (sec) , antiderivative size = 704, normalized size of antiderivative = 2.06

method	result
default	$\frac{-\frac{3c^2e^6b\,x^7}{2(16a^2c^2-8a\,b^2c+b^4)}-\frac{21c^2d\,e^5b\,x^6}{2(16a^2c^2-8a\,b^2c+b^4)}+\frac{(-252bc\,d^2+4ac-19b^2)c\,e^4x^5}{128a^2c^2-64a\,b^2c+8b^4}+\frac{5cd\,e^3(-84bc\,d^2+4ac-19b^2)x^4}{8(16a^2c^2-8a\,b^2c+b^4)}-\frac{e^2(420b\,c^2d^4-40a\,c^2d^2+}{8(16a^2c^2-}}{(cx}$
risch	$\frac{-\frac{3c^2e^6b\,x^7}{2(16a^2c^2-8a\,b^2c+b^4)}-\frac{21c^2d\,e^5b\,x^6}{2(16a^2c^2-8a\,b^2c+b^4)}+\frac{(-252bc\,d^2+4ac-19b^2)c\,e^4x^5}{128a^2c^2-64a\,b^2c+8b^4}+\frac{5cd\,e^3(-84bc\,d^2+4ac-19b^2)x^4}{8(16a^2c^2-8a\,b^2c+b^4)}-\frac{e^2(420b\,c^2d^4-40a\,c^2d^2+}{8(16a^2c^2-}}{(cx}$

input
```
int((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x,method=_RETURNVERBOSE)
```

output
```
(-3/2*c^2*e^6*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x^7-21/2*c^2*d*e^5*b/(16*a^2*c^
2-8*a*b^2*c+b^4)*x^6+1/8*(-252*b*c*d^2+4*a*c-19*b^2)*c*e^4/(16*a^2*c^2-8*a
*b^2*c+b^4)*x^5+5/8*c*d*e^3*(-84*b*c*d^2+4*a*c-19*b^2)/(16*a^2*c^2-8*a*b^2
*c+b^4)*x^4-1/8*e^2*(420*b*c^2*d^4-40*a*c^2*d^2+190*b^2*c*d^2+16*a*b*c+5*b
^3)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3-1/8*d*e*(252*b*c^2*d^4-40*a*c^2*d^2+190
*b^2*c*d^2+48*a*b*c+15*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2-1/8*(84*b*c^2*d
^6-20*a*c^2*d^4+95*b^2*c*d^4+48*a*b*c*d^2+15*b^3*d^2+12*a^2*c+3*a*b^2)/(16
*a^2*c^2-8*a*b^2*c+b^4)*x-1/8*d/e*(12*b*c^2*d^6-4*a*c^2*d^4+19*b^2*c*d^4+1
6*a*b*c*d^2+5*b^3*d^2+12*a^2*c+3*a*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*e^4
*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b
*d^2+a)^2+3/16/(16*a^2*c^2-8*a*b^2*c+b^4)/e*sum((-4*_R^2*b*c*e^2-8*_R*b*c*
d*e-4*b*c*d^2+4*a*c+b^2)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3
+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e
^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a))
```

3.630.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6633 vs. 2(293) = 586.

Time = 0.45 (sec) , antiderivative size = 6633, normalized size of antiderivative = 19.45

$$\int \frac{(d+ex)^4}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.630.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx=\text{Timed out}$$

input
```
integrate((e*x+d)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

output
```
Timed out
```

3.630.7 Maxima [F]

$$\int \frac{(d+ex)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx=\int \frac{(ex+d)^4}{\left((ex+d)^4c+(ex+d)^2b+a\right)^3}\,dx$$

input
```
integrate((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")
```

output
```
-1/8*(12*b*c^2*e^7*x^7 + 84*b*c^2*d*e^6*x^6 + (252*b*c^2*d^2 + 19*b^2*c -
4*a*c^2)*e^5*x^5 + 12*b*c^2*d^7 + 5*(84*b*c^2*d^3 + (19*b^2*c - 4*a*c^2)*d
)*e^4*x^4 + (420*b*c^2*d^4 + 5*b^3 + 16*a*b*c + 10*(19*b^2*c - 4*a*c^2)*d^
2)*e^3*x^3 + (19*b^2*c - 4*a*c^2)*d^5 + (252*b*c^2*d^5 + 10*(19*b^2*c - 4*
a*c^2)*d^3 + 3*(5*b^3 + 16*a*b*c)*d)*e^2*x^2 + (5*b^3 + 16*a*b*c)*d^3 + (8
4*b*c^2*d^6 + 5*(19*b^2*c - 4*a*c^2)*d^4 + 3*a*b^2 + 12*a^2*c + 3*(5*b^3 +
 16*a*b*c)*d^2)*e*x + 3*(a*b^2 + 4*a^2*c)*d)/((b^4*c^2 - 8*a*b^2*c^3 + 16*
a^2*c^4)*e^9*x^8 + 8*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*e^8*x^7 + 2*(b
^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3 + 14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4
)*d^2)*e^7*x^6 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^3 + 3*(b^5*c
 - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d)*e^6*x^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3
+ 70*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^4 + 30*(b^5*c - 8*a*b^3*c^2 +
16*a^2*b*c^3)*d^2)*e^5*x^4 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^
5 + 10*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 6*a*b^4*c + 32*a^
3*c^3)*d)*e^4*x^3 + 2*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^6 + a*b^5
 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^
4 + 3*(b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^2)*e^3*x^2 + 4*(2*(b^4*c^2 - 8*a*b^
2*c^3 + 16*a^2*c^4)*d^7 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + (b^
6 - 6*a*b^4*c + 32*a^3*c^3)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*
e^2*x + ((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^8 + 2*(b^5*c - 8*a*b^3*...
```

3.630.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1802 vs. $2(293) = 586$.

Time = 0.37 (sec) , antiderivative size = 1802, normalized size of antiderivative = 5.28

$$\int \frac{(d+ex)^4}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

output
```
3/16*((4*b*c*e^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4))
 + d/e)^2 - 8*b*c*d*e*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*
e^4)) + d/e) + 4*b*c*d^2 - b^2 - 4*a*c)*log(x + sqrt(1/2)*sqrt(-(b*e^2 + s
qrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 + s
qrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^3 - 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^
2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 - 2*c*d^3*e - b*d*e + (6*c*d^
2*e^2 + b*e^2)*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) +
 d/e)) - (4*b*c*e^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^
4)) - d/e)^2 + 8*b*c*d*e*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/
(c*e^4)) - d/e) + 4*b*c*d^2 - b^2 - 4*a*c)*log(x - sqrt(1/2)*sqrt(-(b*e^2
+ sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2
+ sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^3 + 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b
*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2 + 2*c*d^3*e + b*d*e + (6*c
*d^2*e^2 + b*e^2)*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)
) - d/e)) + (4*b*c*e^2*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c
*e^4)) + d/e)^2 - 8*b*c*d*e*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^
2)/(c*e^4)) + d/e) + 4*b*c*d^2 - b^2 - 4*a*c)*log(x + sqrt(1/2)*sqrt(-(b*e
^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e
^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^3 - 6*c*d*e^3*(sqrt(1/2)*sqrt(
-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 - 2*c*d^3*e - b*d*e ...
```

3.630.9 Mupad [B] (verification not implemented)

Time = 12.05 (sec) , antiderivative size = 12677, normalized size of antiderivative = 37.18

$$\int \frac{(d+ex)^4}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = \text{Too large to display}$$

input
```
int((d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x)
```

output
```
atan(((((786432*a^6*c^8*e^12 - 192*b^12*c^2*e^12 + 3072*a*b^10*c^3*e^12 -
15360*a^2*b^8*c^4*e^12 + 245760*a^4*b^4*c^6*e^12 - 786432*a^5*b^2*c^7*e^12
)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^
4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + ((1024*b^15*c^2*d*e^13 - 28
672*a*b^13*c^3*d*e^13 - 16777216*a^7*b*c^9*d*e^13 + 344064*a^2*b^11*c^4*d*
e^13 - 2293760*a^3*b^9*c^5*d*e^13 + 9175040*a^4*b^7*c^6*d*e^13 - 22020096*
a^5*b^5*c^7*d*e^13 + 29360128*a^6*b^3*c^8*d*e^13)/(128*(b^12 + 4096*a^6*c^
6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c
^5 - 24*a*b^10*c)) + (x*(128*b^11*c^2*e^14 - 2560*a*b^9*c^3*e^14 - 131072*
a^5*b*c^7*e^14 + 20480*a^2*b^7*c^4*e^14 - 81920*a^3*b^5*c^5*e^14 + 163840*
a^4*b^3*c^6*e^14))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c
^3 - 16*a*b^6*c)))*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c^7
 + 560*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*
c^5 - 61440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(a*b^20*e^2 + 1048576*a^11*c^
10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2
+ 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^
2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c
^9*e^2)))^(1/2))*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c^7 +
 560*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*c^
5 - 61440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(a*b^20*e^2 + 1048576*a^11*c...
```

3.630. $\int \frac{(d+ex)^4}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx$

3.631 $\int \frac{(d+ex)^3}{(a+b(d+ex)^2+c(d+ex)^4)^3} dx$

3.631.1 Optimal result

Integrand size = 30, antiderivative size = 150

$$\int \frac{(d+ex)^3}{(a+b(d+ex)^2+c(d+ex)^4)^3} dx = \frac{2a+b(d+ex)^2}{4(b^2-4ac)e(a+b(d+ex)^2+c(d+ex)^4)^2} - \frac{3b(b+2c(d+ex)^2)}{4(b^2-4ac)^2e(a+b(d+ex)^2+c(d+ex)^4)} + \frac{3b\text{carctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}e}$$

output
```
1/4*(2*a+b*(e*x+d)^2)/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2-3/4*b*(
b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^2/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)+3*b*c*arctan
h((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(5/2)/e
```

3.631.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.97

$$\int \frac{(d+ex)^3}{(a+b(d+ex)^2+c(d+ex)^4)^3} dx = \frac{-\frac{3b(b+2c(d+ex)^2)}{a+b(d+ex)^2+c(d+ex)^4} + \frac{(b^2-4ac)(2a+b(d+ex)^2)}{(a+(d+ex)^2(b+c(d+ex)^2))^2} - \frac{12bc\arctan\left(\frac{b+2c(d+ex)^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}}{4(b^2-4ac)^2e}$$

input
```
Integrate[(d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

output
```
((-3*b*(b + 2*c*(d + e*x)^2))/(a + b*(d + e*x)^2 + c*(d + e*x)^4) + ((b^2
- 4*a*c)*(2*a + b*(d + e*x)^2))/(a + (d + e*x)^2*(b + c*(d + e*x)^2))^2 -
(12*b*c*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*
c])/(4*(b^2 - 4*a*c)^2*e)
```

3.631.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1462, 1434, 1159, 1086, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}dx$$

$$\Big\downarrow \text{1462}$$

$$\frac{\int \frac{(d+ex)^3}{\left(c(d+ex)^4+b(d+ex)^2+a\right)^3}d(d+ex)}{e}$$

$$\Big\downarrow \text{1434}$$

$$\frac{\int \frac{(d+ex)^2}{\left(c(d+ex)^4+b(d+ex)^2+a\right)^3}d(d+ex)^2}{2e}$$

$$\Big\downarrow \text{1159}$$

$$\frac{\frac{3b\int \frac{1}{\left(c(d+ex)^4+b(d+ex)^2+a\right)^2}d(d+ex)^2}{2(b^2-4ac)}+\frac{2a+b(d+ex)^2}{2(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}}{2e}$$

$$\Big\downarrow \text{1086}$$

$$\frac{\frac{3b\left(-\frac{2c\int \frac{1}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)^2}{b^2-4ac}-\frac{b+2c(d+ex)^2}{\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}\right)}{2(b^2-4ac)}+\frac{2a+b(d+ex)^2}{2(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}}{2e}$$

$$\Big\downarrow \text{1083}$$

$$\frac{\frac{3b\left(\frac{4c\int \frac{1}{-(d+ex)^4+b^2-4ac}d\left(2c(d+ex)^2+b\right)}{b^2-4ac}-\frac{b+2c(d+ex)^2}{\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}\right)}{2(b^2-4ac)}+\frac{2a+b(d+ex)^2}{2(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}}{2e}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{3b\left(\frac{4c\,\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{b+2c(d+ex)^2}{\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}\right)}{2(b^2-4ac)}+\frac{2a+b(d+ex)^2}{2(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}}{2e}$$

input
```
Int[(d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

output
```
((2*a + b*(d + e*x)^2)/(2*(b^2 - 4*a*c)*(a + b*(d + e*x)^2 + c*(d + e*x)^4
)^2) + (3*b*(-((b + 2*c*(d + e*x)^2)/((b^2 - 4*a*c)*(a + b*(d + e*x)^2 + c
*(d + e*x)^4))) + (4*c*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(
b^2 - 4*a*c)^(3/2)))/(2*(b^2 - 4*a*c)))/(2*e)
```

3.631.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

3.631. $\int \frac{(d+ex)^3}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx$

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

3.631.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.73 (sec) , antiderivative size = 544, normalized size of antiderivative = 3.63

method	result
default	$\frac{-\frac{3c^2e^5bx^6}{2(16a^2c^2-8ab^2c+b^4)}-\frac{9e^4bc^2dx^5}{16a^2c^2-8ab^2c+b^4}-\frac{9bce^3(10cd^2+b)x^4}{4(16a^2c^2-8ab^2c+b^4)}-\frac{3cde^2b(10cd^2+3b)x^3}{16a^2c^2-8ab^2c+b^4}-\frac{be(45c^2d^4+27bcd^2+5ac+b^2)x^2}{2(16a^2c^2-8ab^2c+b^4)}-\frac{db(9c^2d^4+}{16a^2c^2}}{(cx^4e^4+4cde^3x^3+6cd^2e^2x^2+4cd^3ex+be^2x^2+d^4c+2bdex+bd^2+a)^2}$
risch	$\frac{-\frac{3c^2e^5bx^6}{2(16a^2c^2-8ab^2c+b^4)}-\frac{9e^4bc^2dx^5}{16a^2c^2-8ab^2c+b^4}-\frac{9bce^3(10cd^2+b)x^4}{4(16a^2c^2-8ab^2c+b^4)}-\frac{3cde^2b(10cd^2+3b)x^3}{16a^2c^2-8ab^2c+b^4}-\frac{be(45c^2d^4+27bcd^2+5ac+b^2)x^2}{2(16a^2c^2-8ab^2c+b^4)}-\frac{db(9c^2d^4+}{16a^2c^2}}{(cx^4e^4+4cde^3x^3+6cd^2e^2x^2+4cd^3ex+be^2x^2+d^4c+2bdex+bd^2+a)^2}$

input
```
int((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x,method=_RETURNVERBOSE)
```

output
```
(-3/2*c^2*e^5*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6-9*e^4*b*c^2*d/(16*a^2*c^2-8
*a*b^2*c+b^4)*x^5-9/4*b*c*e^3*(10*c*d^2+b)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4-
3*c*d*e^2*b*(10*c*d^2+3*b)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3-1/2*b*e*(45*c^2*
d^4+27*b*c*d^2+5*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2-d*b*(9*c^2*d^4+9*
b*c*d^2+5*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x-1/4/e*(6*b*c^2*d^6+9*b^2*c
*d^4+10*a*b*c*d^2+2*b^3*d^2+8*a^2*c+a*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*
e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*
x+b*d^2+a)^2+3/2*b*c/(16*a^2*c^2-8*a*b^2*c+b^4)/e*sum((-_R*e-d)/(2*_R^3*c*
e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e
^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4
*c+b*d^2+a))
```

3.631.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1804 vs. $2(142) = 284$.

Time = 0.46 (sec) , antiderivative size = 3739, normalized size of antiderivative = 24.93

$$\int \frac{(d+ex)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")
```

output
```
[-1/4*(6*(b^3*c^2 - 4*a*b*c^3)*e^6*x^6 + 36*(b^3*c^2 - 4*a*b*c^3)*d*e^5*x^
5 + 9*(b^4*c - 4*a*b^2*c^2 + 10*(b^3*c^2 - 4*a*b*c^3)*d^2)*e^4*x^4 + 6*(b^
3*c^2 - 4*a*b*c^3)*d^6 + 12*(10*(b^3*c^2 - 4*a*b*c^3)*d^3 + 3*(b^4*c - 4*a
*b^2*c^2)*d)*e^3*x^3 + a*b^4 + 4*a^2*b^2*c - 32*a^3*c^2 + 9*(b^4*c - 4*a*b
^2*c^2)*d^4 + 2*(b^5 + a*b^3*c - 20*a^2*b*c^2 + 45*(b^3*c^2 - 4*a*b*c^3)*d
^4 + 27*(b^4*c - 4*a*b^2*c^2)*d^2)*e^2*x^2 + 2*(b^5 + a*b^3*c - 20*a^2*b*c
^2)*d^2 + 4*(9*(b^3*c^2 - 4*a*b*c^3)*d^5 + 9*(b^4*c - 4*a*b^2*c^2)*d^3 + (
b^5 + a*b^3*c - 20*a^2*b*c^2)*d)*e*x - 6*(b*c^3*e^8*x^8 + 8*b*c^3*d*e^7*x^
7 + 2*(14*b*c^3*d^2 + b^2*c^2)*e^6*x^6 + b*c^3*d^8 + 4*(14*b*c^3*d^3 + 3*b
^2*c^2*d)*e^5*x^5 + 2*b^2*c^2*d^6 + (70*b*c^3*d^4 + 30*b^2*c^2*d^2 + b^3*c
 + 2*a*b*c^2)*e^4*x^4 + 4*(14*b*c^3*d^5 + 10*b^2*c^2*d^3 + (b^3*c + 2*a*b*
c^2)*d)*e^3*x^3 + 2*a*b^2*c*d^2 + (b^3*c + 2*a*b*c^2)*d^4 + 2*(14*b*c^3*d^
6 + 15*b^2*c^2*d^4 + a*b^2*c + 3*(b^3*c + 2*a*b*c^2)*d^2)*e^2*x^2 + a^2*b*
c + 4*(2*b*c^3*d^7 + 3*b^2*c^2*d^5 + a*b^2*c*d + (b^3*c + 2*a*b*c^2)*d^3)*
e*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 +
2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2
- 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^
4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3
 + b*d)*e*x + a)))/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)
*e^9*x^8 + 8*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d*e...
```

3.631.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1671 vs. $2(134) = 268$.

Time = 7.15 (sec) , antiderivative size = 1671, normalized size of antiderivative = 11.14

$$\int \frac{(d+ex)^3}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

output
```
3*b*c*sqrt(-1/(4*a*c - b**2)**5)*log(2*d*x/e + x**2 + (-192*a**3*b*c**4*sq
rt(-1/(4*a*c - b**2)**5) + 144*a**2*b**3*c**3*sqrt(-1/(4*a*c - b**2)**5) -
 36*a*b**5*c**2*sqrt(-1/(4*a*c - b**2)**5) + 3*b**7*c*sqrt(-1/(4*a*c - b**
2)**5) + 3*b**2*c + 6*b*c**2*d**2)/(6*b*c**2*e**2))/(2*e) - 3*b*c*sqrt(-1/
(4*a*c - b**2)**5)*log(2*d*x/e + x**2 + (192*a**3*b*c**4*sqrt(-1/(4*a*c -
b**2)**5) - 144*a**2*b**3*c**3*sqrt(-1/(4*a*c - b**2)**5) + 36*a*b**5*c**2
*sqrt(-1/(4*a*c - b**2)**5) - 3*b**7*c*sqrt(-1/(4*a*c - b**2)**5) + 3*b**2
*c + 6*b*c**2*d**2)/(6*b*c**2*e**2))/(2*e) + (-8*a**2*c - a*b**2 - 10*a*b*
c*d**2 - 2*b**3*d**2 - 9*b**2*c*d**4 - 6*b*c**2*d**6 - 36*b*c**2*d*e**5*x*
*5 - 6*b*c**2*e**6*x**6 + x**4*(-9*b**2*c*e**4 - 90*b*c**2*d**2*e**4) + x*
*3*(-36*b**2*c*d*e**3 - 120*b*c**2*d**3*e**3) + x**2*(-10*a*b*c*e**2 - 2*b
**3*e**2 - 54*b**2*c*d**2*e**2 - 90*b*c**2*d**4*e**2) + x*(-20*a*b*c*d*e -
 4*b**3*d*e - 36*b**2*c*d**3*e - 36*b*c**2*d**5*e))/(64*a**4*c**2*e - 32*a
**3*b**2*c*e + 128*a**3*b*c**2*d**2*e + 128*a**3*c**3*d**4*e + 4*a**2*b**4
*e - 64*a**2*b**3*c*d**2*e + 128*a**2*b*c**3*d**6*e + 64*a**2*c**4*d**8*e
+ 8*a*b**5*d**2*e - 24*a*b**4*c*d**4*e - 64*a*b**3*c**2*d**6*e - 32*a*b**2
*c**3*d**8*e + 4*b**6*d**4*e + 8*b**5*c*d**6*e + 4*b**4*c**2*d**8*e + x**8
*(64*a**2*c**4*e**9 - 32*a*b**2*c**3*e**9 + 4*b**4*c**2*e**9) + x**7*(512*
a**2*c**4*d*e**8 - 256*a*b**2*c**3*d*e**8 + 32*b**4*c**2*d*e**8) + x**6*(1
28*a**2*b*c**3*e**7 + 1792*a**2*c**4*d**2*e**7 - 64*a*b**3*c**2*e**7 - ...
```

3.631.7 Maxima [F]

$$\int \frac{(d+ex)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx=\int \frac{(ex+d)^3}{\left((ex+d)^4c+(ex+d)^2b+a\right)^3}\,dx$$

input
```
integrate((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")
```

output
```
-3*b*c*integrate((e*x + d)/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 +
 b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a), x)/(b^4 - 8*a*b^2*c + 16
*a^2*c^2) - 1/4*(6*b*c^2*e^6*x^6 + 36*b*c^2*d*e^5*x^5 + 6*b*c^2*d^6 + 9*(1
0*b*c^2*d^2 + b^2*c)*e^4*x^4 + 9*b^2*c*d^4 + 12*(10*b*c^2*d^3 + 3*b^2*c*d)
*e^3*x^3 + 2*(45*b*c^2*d^4 + 27*b^2*c*d^2 + b^3 + 5*a*b*c)*e^2*x^2 + a*b^2
 + 8*a^2*c + 2*(b^3 + 5*a*b*c)*d^2 + 4*(9*b*c^2*d^5 + 9*b^2*c*d^3 + (b^3 +
 5*a*b*c)*d)*e*x)/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*e^9*x^8 + 8*(b^4*c
^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*e^8*x^7 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2
*b*c^3 + 14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^2)*e^7*x^6 + 4*(14*(b^4
*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^3 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c
^3)*d)*e^6*x^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3 + 70*(b^4*c^2 - 8*a*b^2*c^3
 + 16*a^2*c^4)*d^4 + 30*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e^5*x^4
+ 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^5 + 10*(b^5*c - 8*a*b^3*c^2
 + 16*a^2*b*c^3)*d^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d)*e^4*x^3 + 2*(14*(
b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^6 + a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c
^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 3*(b^6 - 6*a*b^4*c + 32
*a^3*c^3)*d^2)*e^3*x^2 + 4*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^7 + 3
*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)
*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e^2*x + ((b^4*c^2 - 8*a*b^2
*c^3 + 16*a^2*c^4)*d^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + a...
```

3.631.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 349 vs. $2(142) = 284$.

Time = 0.35 (sec) , antiderivative size = 349, normalized size of antiderivative = 2.33

$$\int \frac{(d+ex)^3}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = -\frac{3\,bc\arctan\left(\frac{2\,cd^2+2\,(ex^2+2\,dx)ce+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-b^2+4\,ac}e} - \frac{6\,bc^2d^6+18\,(ex^2+2\,dx)bc^2d^4e+18\,(ex^2+2\,dx)^2bc^2d^2e^2+6\,(ex^2+2\,dx)^3bc^2e^3+9\,b^2cd^4+18\,(ex^2+2}{4\,\left(cd^4+2\,(ex^2+2\,dx)cd^2e+(ex^2+2\,dx)^2ce^2}$$

input
```
integrate((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

output
```
-3*b*c*arctan((2*c*d^2 + 2*(e*x^2 + 2*d*x)*c*e + b)/sqrt(-b^2 + 4*a*c))/((
b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)*e) - 1/4*(6*b*c^2*d^6 + 1
8*(e*x^2 + 2*d*x)*b*c^2*d^4*e + 18*(e*x^2 + 2*d*x)^2*b*c^2*d^2*e^2 + 6*(e*
x^2 + 2*d*x)^3*b*c^2*e^3 + 9*b^2*c*d^4 + 18*(e*x^2 + 2*d*x)*b^2*c*d^2*e +
9*(e*x^2 + 2*d*x)^2*b^2*c*e^2 + 2*b^3*d^2 + 10*a*b*c*d^2 + 2*(e*x^2 + 2*d*
x)*b^3*e + 10*(e*x^2 + 2*d*x)*a*b*c*e + a*b^2 + 8*a^2*c)/((c*d^4 + 2*(e*x^
2 + 2*d*x)*c*d^2*e + (e*x^2 + 2*d*x)^2*c*e^2 + b*d^2 + (e*x^2 + 2*d*x)*b*e
 + a)^2*(b^4*e - 8*a*b^2*c*e + 16*a^2*c^2*e))
```

3.631.9 Mupad [B] (verification not implemented)

Time = 10.00 (sec) , antiderivative size = 1182, normalized size of antiderivative = 7.88

$$\int \frac{(d+ex)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx =$$

$$-\frac{\frac{9\,x^4\,\left(b^2\,c\,e^3+10\,b\,c^2\,d^2\,e^3\right)}{4\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}}{x^2\,\left(6\,b^2\,d^2\,e^2+30\,b\,c\,d^4\,e^2+2\,a\,b\,e^2+28\,c^2\,d^6\,e^2+12\,a\,c\,d^2\,e^2\right)+x^6\,\left(28\,c^2\,d^2\,e^6+2\,b\,c\,e^6\right)+x\,\left(4\,e\,b^2\,d^3\right.}$$

$$-\frac{3\,b\,c\,\mathrm{atan}\left(\frac{\left(b^4\,\left(4\,a\,c-b^2\right)^5+16\,a^2\,c^2\,\left(4\,a\,c-b^2\right)^5-8\,a\,b^2\,c\,\left(4\,a\,c-b^2\right)^5\right)\,\left(x^2\,\left(\frac{9\,b^2\,c^4\,e^8}{a\,\left(4\,a\,c-b^2\right)^{9/2}\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{9\,b^3\,c^2\,\left(32\,a^2\,b\,c^4\,e^{10}-\right.}{2\,a\,e^2\,\left(4\,a\,c-b^2\right)^{15/2}}\right.\right.}{}\right.}{}$$

input
```
int((d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x)
```

output
```
- ((9*x^4*(b^2*c*e^3 + 10*b*c^2*d^2*e^3))/(4*(b^4 + 16*a^2*c^2 - 8*a*b^2*c
)) + (a*b^2 + 8*a^2*c + 2*b^3*d^2 + 9*b^2*c*d^4 + 6*b*c^2*d^6 + 10*a*b*c*d
^2)/(4*e*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^2*(b^3*e + 27*b^2*c*d^2*e +
45*b*c^2*d^4*e + 5*a*b*c*e))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*d*x^3
*(3*b^2*c*e^2 + 10*b*c^2*d^2*e^2))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (d*x*(
b^3 + 9*b^2*c*d^2 + 9*b*c^2*d^4 + 5*a*b*c))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c)
 + (3*b*c^2*e^5*x^6)/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (9*b*c^2*d*e^4*x
^5)/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(x^2*(6*b^2*d^2*e^2 + 28*c^2*d^6*e^2 +
 2*a*b*e^2 + 12*a*c*d^2*e^2 + 30*b*c*d^4*e^2) + x^6*(28*c^2*d^2*e^6 + 2*b*
c*e^6) + x*(4*b^2*d^3*e + 8*c^2*d^7*e + 8*a*c*d^3*e + 12*b*c*d^5*e + 4*a*b
*d*e) + x^3*(4*b^2*d*e^3 + 56*c^2*d^5*e^3 + 8*a*c*d*e^3 + 40*b*c*d^3*e^3)
+ x^5*(56*c^2*d^3*e^5 + 12*b*c*d*e^5) + x^4*(b^2*e^4 + 70*c^2*d^4*e^4 + 2*
a*c*e^4 + 30*b*c*d^2*e^4) + a^2 + b^2*d^4 + c^2*d^8 + c^2*e^8*x^8 + 2*a*b*
d^2 + 2*a*c*d^4 + 2*b*c*d^6 + 8*c^2*d*e^7*x^7) - (3*b*c*atan(((b^4*(4*a*c
- b^2)^5 + 16*a^2*c^2*(4*a*c - b^2)^5 - 8*a*b^2*c*(4*a*c - b^2)^5)*(x^2*((
9*b^2*c^4*e^8)/(a*(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (9
*b^3*c^2*(2*b^5*c^2*e^10 - 16*a*b^3*c^3*e^10 + 32*a^2*b*c^4*e^10))/(2*a*e^
2*(4*a*c - b^2)^(15/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) + x*((18*b^2*c^4*d
*e^7)/(a*(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (9*b^3*c^2*
(2*b^5*c^2*d*e^9 - 16*a*b^3*c^3*d*e^9 + 32*a^2*b*c^4*d*e^9))/(a*e^2*(4*...
```

3.631. $\int \frac{(d+ex)^3}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx$

3.632 $\int \frac{(d+ex)^2}{(a+b(d+ex)^2+c(d+ex)^4)^3} dx$

3.632.1 Optimal result

Integrand size = 30, antiderivative size = 363

$$\int \frac{(d+ex)^2}{(a+b(d+ex)^2+c(d+ex)^4)^3} dx = -\frac{(d+ex)(b+2c(d+ex)^2)}{4(b^2-4ac)e(a+b(d+ex)^2+c(d+ex)^4)^2} + \frac{(d+ex)(b(b^2+8ac)+c(b^2+20ac)(d+ex)^2)}{8a(b^2-4ac)^2e(a+b(d+ex)^2+c(d+ex)^4)} + \frac{\sqrt{c}\left(b^2+20ac+\frac{b(b^2-52ac)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a(b^2-4ac)^2\sqrt{b-\sqrt{b^2-4ac}}e} + \frac{\sqrt{c}\left(b^2+20ac-\frac{b(b^2-52ac)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a(b^2-4ac)^2\sqrt{b+\sqrt{b^2-4ac}}e}$$

output
```
-1/4*(e*x+d)*(b+2*c*(e*x+d)^2)/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)^
2+1/8*(e*x+d)*(b*(8*a*c+b^2)+c*(20*a*c+b^2)*(e*x+d)^2)/a/(-4*a*c+b^2)^2/e/
(a+b*(e*x+d)^2+c*(e*x+d)^4)+1/16*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b-(-4*a*c
+b^2)^(1/2))^(1/2))*c^(1/2)*(b^2+20*a*c+b*(-52*a*c+b^2)/(-4*a*c+b^2)^(1/2)
)/a/(-4*a*c+b^2)^2/e*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/16*arctan((e*x
+d)*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b^2+20*a*c-b*(-
52*a*c+b^2)/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^2/e*2^(1/2)/(b+(-4*a*c+b^2)
^(1/2))^(1/2)
```

3.632.2 Mathematica [A] (verified)

Time = 2.86 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.05

$$\int \frac{(d+ex)^2}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$$

$$= \frac{-\frac{4(b(d+ex)+2c(d+ex)^3)}{(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}+\frac{2(d+ex)\left(b^3+8abc+b^2c(d+ex)^2+20ac^2(d+ex)^2\right)}{a(b^2-4ac)^2(a+b(d+ex)^2+c(d+ex)^4)}+\frac{\sqrt{2}\sqrt{c}\left(b^3-52abc+b^2\sqrt{b^2-4ac}+20ac\sqrt{b^2-4ac}\right)}{a(b^2-4ac)^{5/2}\sqrt{b-\sqrt{b^2-4}}}}{16e}$$

input
```
Integrate[(d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

output
```
((-4*(b*(d + e*x) + 2*c*(d + e*x)^3))/((b^2 - 4*a*c)*(a + b*(d + e*x)^2 +
c*(d + e*x)^4)^2) + (2*(d + e*x)*(b^3 + 8*a*b*c + b^2*c*(d + e*x)^2 + 20*a
*c^2*(d + e*x)^2))/(a*(b^2 - 4*a*c)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4))
 + (Sqrt[2]*Sqrt[c]*(b^3 - 52*a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*Sqrt[
b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c
]]])/(a*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c
]*(-b^3 + 52*a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*Sqrt[b^2 - 4*a*c])*Arc
Tan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*
a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(16*e)
```

3.632.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1462, 1439, 1492, 25, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$$

$$\Big\downarrow \ 1462$$

$$\frac{\int \frac{(d+ex)^2}{\left(c(d+ex)^4+b(d+ex)^2+a\right)^3}d(d+ex)}{e}$$

$$\Big\downarrow \ 1439$$

$$\frac{\frac{\int \frac{b-10c(d+ex)^2}{\left(c(d+ex)^4+b(d+ex)^2+a\right)^2}d(d+ex)}{4(b^2-4ac)} - \frac{(d+ex)(b+2c(d+ex)^2)}{4(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}}{e}$$

$\Big\downarrow$ 1492

$$\frac{\frac{\frac{(d+ex)\left(c\left(20ac+b^2\right)(d+ex)^2+b\left(8ac+b^2\right)\right)}{2a\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{\int -\frac{c\left(b^2+20ac\right)(d+ex)^2+b\left(b^2-16ac\right)}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)}{2a\left(b^2-4ac\right)}}{4(b^2-4ac)} - \frac{(d+ex)(b+2c(d+ex)^2)}{4(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}}{e}$$

$\Big\downarrow$ 25

$$\frac{\frac{\frac{\int \frac{c\left(b^2+20ac\right)(d+ex)^2+b\left(b^2-16ac\right)}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)}{2a\left(b^2-4ac\right)} + \frac{(d+ex)\left(c\left(20ac+b^2\right)(d+ex)^2+b\left(8ac+b^2\right)\right)}{2a\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}}{4(b^2-4ac)} - \frac{(d+ex)(b+2c(d+ex)^2)}{4(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}}{e}$$

$\Big\downarrow$ 1480

$$\frac{\frac{\frac{\frac{1}{2}c\left(-\frac{52abc}{\sqrt{b^2-4ac}}+\frac{b^3}{\sqrt{b^2-4ac}}+20ac+b^2\right)\int \frac{1}{c(d+ex)^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}d(d+ex)+\frac{1}{2}c\left(\frac{52abc}{\sqrt{b^2-4ac}}-\frac{b^3}{\sqrt{b^2-4ac}}+20ac+b^2\right)\int \frac{1}{c(d+ex)^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}d(d+ex)}{2a\left(b^2-4ac\right)}+}{4(b^2-4ac)}}{e}$$

$\Big\downarrow$ 218

$$\frac{\frac{\frac{\frac{\sqrt{c}\left(-\frac{52abc}{\sqrt{b^2-4ac}}+\frac{b^3}{\sqrt{b^2-4ac}}+20ac+b^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(\frac{52abc}{\sqrt{b^2-4ac}}-\frac{b^3}{\sqrt{b^2-4ac}}+20ac+b^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}}{2a\left(b^2-4ac\right)}+\frac{(d+ex)\left(c\left(20ac+b^2\right)(d+e\ldots}{2a\left(b^2-4ac\right)\left(a+b(d+ex\ldots}}{4(b^2-4ac)}}{e}$$

input
```
Int[(d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

output
```
(-1/4*((d + e*x)*(b + 2*c*(d + e*x)^2))/((b^2 - 4*a*c)*(a + b*(d + e*x)^2
+ c*(d + e*x)^4)^2) + (((d + e*x)*(b*(b^2 + 8*a*c) + c*(b^2 + 20*a*c)*(d +
 e*x)^2))/(2*a*(b^2 - 4*a*c)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + ((Sqrt
[c]*(b^2 + 20*a*c + b^3/Sqrt[b^2 - 4*a*c] - (52*a*b*c)/Sqrt[b^2 - 4*a*c])*
ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*
Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(b^2 + 20*a*c - b^3/Sqrt[b^2 - 4*a
*c] + (52*a*b*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqr
t[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*a*(b^
2 - 4*a*c)))/(4*(b^2 - 4*a*c)))/e
```

3.632. $\int \frac{(d+ex)^2}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx$

3.632.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1439
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d*(d*x)^(m - 1)*(b + 2*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*(p +
1)*(b^2 - 4*a*c))), x] - Simp[d^2/(2*(p + 1)*(b^2 - 4*a*c))   Int[(d*x)^(m
- 2)*(b*(m - 1) + 2*c*(m + 4*p + 5)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x
] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m,
1] && LeQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

3.632. $\int \frac{(d+ex)^2}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx$

3.632.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.69 (sec) , antiderivative size = 885, normalized size of antiderivative = 2.44

method	result
default	$\frac{c^2 e^6 (20ac+b^2) x^7}{8(16a^2c^2-8ab^2c+b^4)a}+\frac{7c^2 d e^5 (20ac+b^2) x^6}{8(16a^2c^2-8ab^2c+b^4)a}+\frac{(420a c^2 d^2+21b^2 c d^2+28abc+2b^3) c e^4 x^5}{8(16a^2c^2-8ab^2c+b^4)a}+\frac{5cd e^3 (140a c^2 d^2+7b^2 c d^2+28abc+2b^3) x^4}{8(16a^2c^2-8ab^2c+b^4)a}+\frac{e^2(}{}$
risch	$\frac{c^2 e^6 (20ac+b^2) x^7}{8(16a^2c^2-8ab^2c+b^4)a}+\frac{7c^2 d e^5 (20ac+b^2) x^6}{8(16a^2c^2-8ab^2c+b^4)a}+\frac{(420a c^2 d^2+21b^2 c d^2+28abc+2b^3) c e^4 x^5}{8(16a^2c^2-8ab^2c+b^4)a}+\frac{5cd e^3 (140a c^2 d^2+7b^2 c d^2+28abc+2b^3) x^4}{8(16a^2c^2-8ab^2c+b^4)a}+\frac{e^2(}{}$

input
```
int((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/8*c^2*e^6*(20*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^7+7/8*c^2*d*e^5*(
20*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^6+1/8*(420*a*c^2*d^2+21*b^2*c*d
^2+28*a*b*c+2*b^3)*c*e^4/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^5+5/8*c*d*e^3*(140
*a*c^2*d^2+7*b^2*c*d^2+28*a*b*c+2*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^4+1/
8*e^2*(700*a*c^3*d^4+35*b^2*c^2*d^4+280*a*b*c^2*d^2+20*b^3*c*d^2+36*a^2*c^
2+5*a*b^2*c+b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^3+1/8*d*e*(420*a*c^3*d^4+2
1*b^2*c^2*d^4+280*a*b*c^2*d^2+20*b^3*c*d^2+108*a^2*c^2+15*a*b^2*c+3*b^4)/(
16*a^2*c^2-8*a*b^2*c+b^4)/a*x^2+1/8*(140*a*c^3*d^6+7*b^2*c^2*d^6+140*a*b*c
^2*d^4+10*b^3*c*d^4+108*a^2*c^2*d^2+15*a*b^2*c*d^2+3*b^4*d^2+16*a^2*b*c-a*
b^3)/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x+1/8*d/e*(20*a*c^3*d^6+b^2*c^2*d^6+28*a
*b*c^2*d^4+2*b^3*c*d^4+36*a^2*c^2*d^2+5*a*b^2*c*d^2+b^4*d^2+16*a^2*b*c-a*b
^3)/(16*a^2*c^2-8*a*b^2*c+b^4)/a)/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2
+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2+1/16/(16*a^2*c^2-8*a*b^2
*c+b^4)/a/e*sum((c*e^2*(20*a*c+b^2)*_R^2+2*d*c*e*(20*a*c+b^2)*_R+20*a*c^2*
d^2+b^2*c*d^2-16*a*b*c+b^3)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*
d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+
b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a))
```

3.632.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7701 vs. 2(319) = 638.

Time = 0.62 (sec) , antiderivative size = 7701, normalized size of antiderivative = 21.21

$$\int \frac{(d+ex)^2}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.632.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

output
```
Timed out
```

3.632.7 Maxima [F]

$$\int \frac{(d+ex)^2}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = \int \frac{(ex+d)^2}{\left((ex+d)^4c+(ex+d)^2b+a\right)^3}\,dx$$

input
```
integrate((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")
```

output
```
1/8*((b^2*c^2 + 20*a*c^3)*e^7*x^7 + 7*(b^2*c^2 + 20*a*c^3)*d*e^6*x^6 + (2*
b^3*c + 28*a*b*c^2 + 21*(b^2*c^2 + 20*a*c^3)*d^2)*e^5*x^5 + 5*(7*(b^2*c^2
+ 20*a*c^3)*d^3 + 2*(b^3*c + 14*a*b*c^2)*d)*e^4*x^4 + (b^2*c^2 + 20*a*c^3)
*d^7 + (35*(b^2*c^2 + 20*a*c^3)*d^4 + b^4 + 5*a*b^2*c + 36*a^2*c^2 + 20*(b
^3*c + 14*a*b*c^2)*d^2)*e^3*x^3 + 2*(b^3*c + 14*a*b*c^2)*d^5 + (21*(b^2*c^
2 + 20*a*c^3)*d^5 + 20*(b^3*c + 14*a*b*c^2)*d^3 + 3*(b^4 + 5*a*b^2*c + 36*
a^2*c^2)*d)*e^2*x^2 + (b^4 + 5*a*b^2*c + 36*a^2*c^2)*d^3 + (7*(b^2*c^2 + 2
0*a*c^3)*d^6 + 10*(b^3*c + 14*a*b*c^2)*d^4 - a*b^3 + 16*a^2*b*c + 3*(b^4 +
 5*a*b^2*c + 36*a^2*c^2)*d^2)*e*x - (a*b^3 - 16*a^2*b*c)*d)/((a*b^4*c^2 -
8*a^2*b^2*c^3 + 16*a^3*c^4)*e^9*x^8 + 8*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^
3*c^4)*d*e^8*x^7 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3 + 14*(a*b^4*c
^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^2)*e^7*x^6 + 4*(14*(a*b^4*c^2 - 8*a^2*b
^2*c^3 + 16*a^3*c^4)*d^3 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d)*e
^6*x^5 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3 + 70*(a*b^4*c^2 - 8*a^2*b^2*c^3
 + 16*a^3*c^4)*d^4 + 30*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^2)*e^5*
x^4 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^5 + 10*(a*b^5*c - 8
*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^3 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d)*e
^4*x^3 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 + 14*(a*b^4*c^2 - 8*a^2*b
^2*c^3 + 16*a^3*c^4)*d^6 + 15*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^4
 + 3*(a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^2)*e^3*x^2 + 4*(2*(a*b^4*c^2 ...
```

3.632.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2447 vs. $2(319) = 638$.

Time = 0.34 (sec) , antiderivative size = 2447, normalized size of antiderivative = 6.74

$$\int \frac{(d+ex)^2}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

output
```
-1/16*((b^2*c*e^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)
) + d/e)^2 + 20*a*c^2*e^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)
/(c*e^4)) + d/e)^2 - 2*b^2*c*d*e*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*
c)*e^2)/(c*e^4)) + d/e) - 40*a*c^2*d*e*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2
- 4*a*c)*e^2)/(c*e^4)) + d/e) + b^2*c*d^2 + 20*a*c^2*d^2 + b^3 - 16*a*b*c)
*log(x + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(
2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^3
 - 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d
/e)^2 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(sqrt(1/2)*sqrt(-(b*e^2
+ sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)) - (b^2*c*e^2*(sqrt(1/2)*sqrt(-(b
*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2 + 20*a*c^2*e^2*(sqrt(1/2)*
sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2 + 2*b^2*c*d*e*(sqr
t(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e) + 40*a*c^2*d*
e*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e) + b^2*c
*d^2 + 20*a*c^2*d^2 + b^3 - 16*a*b*c)*log(x - sqrt(1/2)*sqrt(-(b*e^2 + sqr
t(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqr
t(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^3 + 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2
+ sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2 + 2*c*d^3*e + b*d*e + (6*c*d^2*
e^2 + b*e^2)*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d
/e)) + (b^2*c*e^2*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e...
```

3.632.9 Mupad [B] (verification not implemented)

Time = 12.33 (sec) , antiderivative size = 14584, normalized size of antiderivative = 40.18

$$\int \frac{(d+ex)^2}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = \text{Too large to display}$$

input
```
int((d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x)
```

output
```
((x^5*(2*b^3*c*e^4 + 420*a*c^3*d^2*e^4 + 21*b^2*c^2*d^2*e^4 + 28*a*b*c^2*e
^4))/(8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^2*(3*b^4*d*e + 21*b^2*c^2*d
^5*e + 108*a^2*c^2*d*e + 420*a*c^3*d^5*e + 20*b^3*c*d^3*e + 280*a*b*c^2*d^
3*e + 15*a*b^2*c*d*e))/(8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (7*x^6*(b^2*
c^2*d*e^5 + 20*a*c^3*d*e^5))/(8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^7*(
20*a*c^3*e^6 + b^2*c^2*e^6))/(8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x*(3*
b^4*d^2 - a*b^3 + 140*a*c^3*d^6 + 10*b^3*c*d^4 + 108*a^2*c^2*d^2 + 7*b^2*c
^2*d^6 + 16*a^2*b*c + 15*a*b^2*c*d^2 + 140*a*b*c^2*d^4))/(8*a*(b^4 + 16*a^
2*c^2 - 8*a*b^2*c)) + (x^3*(b^4*e^2 + 36*a^2*c^2*e^2 + 700*a*c^3*d^4*e^2 +
 20*b^3*c*d^2*e^2 + 35*b^2*c^2*d^4*e^2 + 5*a*b^2*c*e^2 + 280*a*b*c^2*d^2*e
^2))/(8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (b^4*d^3 + 20*a*c^3*d^7 + 2*b^
3*c*d^5 + 36*a^2*c^2*d^3 + b^2*c^2*d^7 - a*b^3*d + 16*a^2*b*c*d + 5*a*b^2*
c*d^3 + 28*a*b*c^2*d^5)/(8*a*e*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (5*x^4*(1
40*a*c^3*d^3*e^3 + 7*b^2*c^2*d^3*e^3 + 2*b^3*c*d*e^3 + 28*a*b*c^2*d*e^3))/
(8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^2*(6*b^2*d^2*e^2 + 28*c^2*d^6*e^2
 + 2*a*b*e^2 + 12*a*c*d^2*e^2 + 30*b*c*d^4*e^2) + x^6*(28*c^2*d^2*e^6 + 2*
b*c*e^6) + x*(4*b^2*d^3*e + 8*c^2*d^7*e + 8*a*c*d^3*e + 12*b*c*d^5*e + 4*a
*b*d*e) + x^3*(4*b^2*d*e^3 + 56*c^2*d^5*e^3 + 8*a*c*d*e^3 + 40*b*c*d^3*e^3
) + x^5*(56*c^2*d^3*e^5 + 12*b*c*d*e^5) + x^4*(b^2*e^4 + 70*c^2*d^4*e^4 +
2*a*c*e^4 + 30*b*c*d^2*e^4) + a^2 + b^2*d^4 + c^2*d^8 + c^2*e^8*x^8 + 2...
```

3.632. $\int \frac{(d+ex)^2}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx$

3.633 $\int \frac{d+ex}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$

3.633.1 Optimal result

Integrand size = 28, antiderivative size = 152

$$\int \frac{d+ex}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \frac{-b-2c(d+ex)^2}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{3c\left(b+2c(d+ex)^2\right)}{2\left(b^2-4ac\right)^2e\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{6c^2\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}e}$$

output
```
1/4*(-b-2*c*(e*x+d)^2)/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2+3/2*c*
(b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^2/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)-6*c^2*arcta
nh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(5/2)/e
```

3.633.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.97

$$\int \frac{d+ex}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \frac{\frac{\left(b^2-4ac\right)\left(-b-2c(d+ex)^2\right)}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{6c\left(b+2c(d+ex)^2\right)}{a+b(d+ex)^2+c(d+ex)^4}+\frac{24c^2\arctan\left(\frac{b+2c(d+ex)^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}}{4\left(b^2-4ac\right)^2e}$$

input
```
Integrate[(d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

output
```
(((b^2 - 4*a*c)*(-b - 2*c*(d + e*x)^2))/(a + b*(d + e*x)^2 + c*(d + e*x)^4
)^2 + (6*c*(b + 2*c*(d + e*x)^2))/(a + b*(d + e*x)^2 + c*(d + e*x)^4) + (2
4*c^2*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c]
)/(4*(b^2 - 4*a*c)^2*e)
```

3.633.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1462, 1432, 1086, 1086, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{(a+b(d+ex)^2+c(d+ex)^4)^3}dx$$

$$\Big\downarrow \text{1462}$$

$$\frac{\int \frac{d+ex}{(c(d+ex)^4+b(d+ex)^2+a)^3}d(d+ex)}{e}$$

$$\Big\downarrow \text{1432}$$

$$\frac{\int \frac{1}{(c(d+ex)^4+b(d+ex)^2+a)^3}d(d+ex)^2}{2e}$$

$$\Big\downarrow \text{1086}$$

$$\frac{-\frac{3c\int \frac{1}{\left(c(d+ex)^4+b(d+ex)^2+a\right)^2}d(d+ex)^2}{b^2-4ac}-\frac{b+2c(d+ex)^2}{2(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}}{2e}$$

$$\Big\downarrow \text{1086}$$

$$\frac{-\frac{3c\left(-\frac{2c\int \frac{1}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)^2}{b^2-4ac}-\frac{b+2c(d+ex)^2}{\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}\right)}{b^2-4ac}-\frac{b+2c(d+ex)^2}{2(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}}{2e}$$

$$\Big\downarrow \text{1083}$$

$$-\frac{\frac{3c\left(\frac{4c\int \frac{1}{-(d+ex)^4+b^2-4ac}d\left(2c(d+ex)^2+b\right)}{b^2-4ac}-\frac{b+2c(d+ex)^2}{\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}\right)}{b^2-4ac}-\frac{b+2c(d+ex)^2}{2(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}}{2e}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\frac{3c\left(\frac{4c\,\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{b+2c(d+ex)^2}{\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}\right)}{b^2-4ac}-\frac{b+2c(d+ex)^2}{2(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}}{2e}$$

input
```
Int[(d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

output
```
(-1/2*(b + 2*c*(d + e*x)^2)/((b^2 - 4*a*c)*(a + b*(d + e*x)^2 + c*(d + e*x
)^4)^2) - (3*c*(-((b + 2*c*(d + e*x)^2)/((b^2 - 4*a*c)*(a + b*(d + e*x)^2
+ c*(d + e*x)^4))) + (4*c*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]]
)/(b^2 - 4*a*c)^(3/2)))/(b^2 - 4*a*c))/(2*e)
```

3.633.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

3.633.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.35 (sec) , antiderivative size = 541, normalized size of antiderivative = 3.56

method	result
default	$\frac{\frac{3c^3e^5x^6}{16a^2c^2-8a\,b^2c+b^4}+\frac{18e^4c^3d\,x^5}{16a^2c^2-8a\,b^2c+b^4}+\frac{9c^2e^3\left(10c\,d^2+b\right)x^4}{2\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{6c^2d\,e^2\left(10c\,d^2+3b\right)x^3}{16a^2c^2-8a\,b^2c+b^4}+\frac{ce\left(45c^2d^4+27bc\,d^2+5ac+b^2\right)x^2}{16a^2c^2-8a\,b^2c+b^4}+\frac{2cd\left(9c^2d^4+9bc\,d \cdots\right.}{16a^2c^2-8a \cdots}}{\left(c\,x^4e^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+d^4c+2bdex+b\,d^2+a\right)^2}$
risch	$\frac{\frac{3c^3e^5x^6}{16a^2c^2-8a\,b^2c+b^4}+\frac{18e^4c^3d\,x^5}{16a^2c^2-8a\,b^2c+b^4}+\frac{9c^2e^3\left(10c\,d^2+b\right)x^4}{2\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{6c^2d\,e^2\left(10c\,d^2+3b\right)x^3}{16a^2c^2-8a\,b^2c+b^4}+\frac{ce\left(45c^2d^4+27bc\,d^2+5ac+b^2\right)x^2}{16a^2c^2-8a\,b^2c+b^4}+\frac{2cd\left(9c^2d^4+9bc\,d \cdots\right.}{16a^2c^2-8a \cdots}}{\left(c\,x^4e^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+d^4c+2bdex+b\,d^2+a\right)^2}$

input
```
int((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x,method=_RETURNVERBOSE)
```

output
```
(3*c^3*e^5/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+18*e^4*c^3*d/(16*a^2*c^2-8*a*b^2
*c+b^4)*x^5+9/2*c^2*e^3*(10*c*d^2+b)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4+6*c^2*
d*e^2*(10*c*d^2+3*b)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+c*e*(45*c^2*d^4+27*b*c
*d^2+5*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+2*c*d*(9*c^2*d^4+9*b*c*d^2+
5*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x+1/4/e*(12*c^3*d^6+18*b*c^2*d^4+20*
a*c^2*d^2+4*b^2*c*d^2+10*a*b*c-b^3)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*e^4*x^4
+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2
+a)^2+3*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)/e*sum((_R*e+d)/(2*_R^3*c*e^3+6*_R^2
*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*
c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a
))
```

3.633.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1788 vs. 2(142) = 284.

Time = 0.43 (sec) , antiderivative size = 3708, normalized size of antiderivative = 24.39

$$\int \frac{d+ex}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")
```

output
```
[1/4*(12*(b^2*c^3 - 4*a*c^4)*e^6*x^6 + 72*(b^2*c^3 - 4*a*c^4)*d*e^5*x^5 +
18*(b^3*c^2 - 4*a*b*c^3 + 10*(b^2*c^3 - 4*a*c^4)*d^2)*e^4*x^4 + 12*(b^2*c^
3 - 4*a*c^4)*d^6 + 24*(10*(b^2*c^3 - 4*a*c^4)*d^3 + 3*(b^3*c^2 - 4*a*b*c^3
)*d)*e^3*x^3 - b^5 + 14*a*b^3*c - 40*a^2*b*c^2 + 18*(b^3*c^2 - 4*a*b*c^3)*
d^4 + 4*(b^4*c + a*b^2*c^2 - 20*a^2*c^3 + 45*(b^2*c^3 - 4*a*c^4)*d^4 + 27*
(b^3*c^2 - 4*a*b*c^3)*d^2)*e^2*x^2 + 4*(b^4*c + a*b^2*c^2 - 20*a^2*c^3)*d^
2 + 8*(9*(b^2*c^3 - 4*a*c^4)*d^5 + 9*(b^3*c^2 - 4*a*b*c^3)*d^3 + (b^4*c +
a*b^2*c^2 - 20*a^2*c^3)*d)*e*x + 12*(c^4*e^8*x^8 + 8*c^4*d*e^7*x^7 + 2*(14
*c^4*d^2 + b*c^3)*e^6*x^6 + c^4*d^8 + 4*(14*c^4*d^3 + 3*b*c^3*d)*e^5*x^5 +
 2*b*c^3*d^6 + (70*c^4*d^4 + 30*b*c^3*d^2 + b^2*c^2 + 2*a*c^3)*e^4*x^4 + 4
*(14*c^4*d^5 + 10*b*c^3*d^3 + (b^2*c^2 + 2*a*c^3)*d)*e^3*x^3 + 2*a*b*c^2*d
^2 + (b^2*c^2 + 2*a*c^3)*d^4 + 2*(14*c^4*d^6 + 15*b*c^3*d^4 + a*b*c^2 + 3*
(b^2*c^2 + 2*a*c^3)*d^2)*e^2*x^2 + a^2*c^2 + 4*(2*c^4*d^7 + 3*b*c^3*d^5 +
a*b*c^2*d + (b^2*c^2 + 2*a*c^3)*d^3)*e*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*e^4
*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d
^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c - (2*c*e^2*x^2 + 4*c*d*e*x +
2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*
d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a)))/((b^6*c^2 - 12*a*b
^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^9*x^8 + 8*(b^6*c^2 - 12*a*b^4*c^3
+ 48*a^2*b^2*c^4 - 64*a^3*c^5)*d*e^8*x^7 + 2*(b^7*c - 12*a*b^5*c^2 + 48...
```

3.633.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1646 vs. 2(136) = 272.

Time = 6.93 (sec) , antiderivative size = 1646, normalized size of antiderivative = 10.83

$$\int \frac{d+ex}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

output
```
-3*c**2*sqrt(-1/(4*a*c - b**2)**5)*log(2*d*x/e + x**2 + (-192*a**3*c**5*sq
rt(-1/(4*a*c - b**2)**5) + 144*a**2*b**2*c**4*sqrt(-1/(4*a*c - b**2)**5) -
 36*a*b**4*c**3*sqrt(-1/(4*a*c - b**2)**5) + 3*b**6*c**2*sqrt(-1/(4*a*c -
b**2)**5) + 3*b*c**2 + 6*c**3*d**2)/(6*c**3*e**2))/e + 3*c**2*sqrt(-1/(4*a
*c - b**2)**5)*log(2*d*x/e + x**2 + (192*a**3*c**5*sqrt(-1/(4*a*c - b**2)*
*5) - 144*a**2*b**2*c**4*sqrt(-1/(4*a*c - b**2)**5) + 36*a*b**4*c**3*sqrt(
-1/(4*a*c - b**2)**5) - 3*b**6*c**2*sqrt(-1/(4*a*c - b**2)**5) + 3*b*c**2
+ 6*c**3*d**2)/(6*c**3*e**2))/e + (10*a*b*c + 20*a*c**2*d**2 - b**3 + 4*b*
*2*c*d**2 + 18*b*c**2*d**4 + 12*c**3*d**6 + 72*c**3*d*e**5*x**5 + 12*c**3*
e**6*x**6 + x**4*(18*b*c**2*e**4 + 180*c**3*d**2*e**4) + x**3*(72*b*c**2*d
*e**3 + 240*c**3*d**3*e**3) + x**2*(20*a*c**2*e**2 + 4*b**2*c*e**2 + 108*b
*c**2*d**2*e**2 + 180*c**3*d**4*e**2) + x*(40*a*c**2*d*e + 8*b**2*c*d*e +
72*b*c**2*d**3*e + 72*c**3*d**5*e))/(64*a**4*c**2*e - 32*a**3*b**2*c*e + 1
28*a**3*b*c**2*d**2*e + 128*a**3*c**3*d**4*e + 4*a**2*b**4*e - 64*a**2*b**
3*c*d**2*e + 128*a**2*b*c**3*d**6*e + 64*a**2*c**4*d**8*e + 8*a*b**5*d**2*
e - 24*a*b**4*c*d**4*e - 64*a*b**3*c**2*d**6*e - 32*a*b**2*c**3*d**8*e + 4
*b**6*d**4*e + 8*b**5*c*d**6*e + 4*b**4*c**2*d**8*e + x**8*(64*a**2*c**4*e
**9 - 32*a*b**2*c**3*e**9 + 4*b**4*c**2*e**9) + x**7*(512*a**2*c**4*d*e**8
 - 256*a*b**2*c**3*d*e**8 + 32*b**4*c**2*d*e**8) + x**6*(128*a**2*b*c**3*e
**7 + 1792*a**2*c**4*d**2*e**7 - 64*a*b**3*c**2*e**7 - 896*a*b**2*c**3*...
```

3.633.7 Maxima [F]

$$\int \frac{d+ex}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = \int \frac{ex+d}{\left((ex+d)^4c+(ex+d)^2b+a\right)^3}\,dx$$

input
```
integrate((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")
```

output
```
6*c^2*integrate((e*x + d)/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 +
b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a), x)/(b^4 - 8*a*b^2*c + 16*
a^2*c^2) + 1/4*(12*c^3*e^6*x^6 + 72*c^3*d*e^5*x^5 + 12*c^3*d^6 + 18*(10*c^
3*d^2 + b*c^2)*e^4*x^4 + 18*b*c^2*d^4 + 24*(10*c^3*d^3 + 3*b*c^2*d)*e^3*x^
3 + 4*(45*c^3*d^4 + 27*b*c^2*d^2 + b^2*c + 5*a*c^2)*e^2*x^2 - b^3 + 10*a*b
*c + 4*(b^2*c + 5*a*c^2)*d^2 + 8*(9*c^3*d^5 + 9*b*c^2*d^3 + (b^2*c + 5*a*c
^2)*d)*e*x)/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*e^9*x^8 + 8*(b^4*c^2 - 8
*a*b^2*c^3 + 16*a^2*c^4)*d*e^8*x^7 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3
 + 14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^2)*e^7*x^6 + 4*(14*(b^4*c^2 -
 8*a*b^2*c^3 + 16*a^2*c^4)*d^3 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d)
*e^6*x^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3 + 70*(b^4*c^2 - 8*a*b^2*c^3 + 16*
a^2*c^4)*d^4 + 30*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e^5*x^4 + 4*(1
4*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^5 + 10*(b^5*c - 8*a*b^3*c^2 + 16*
a^2*b*c^3)*d^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d)*e^4*x^3 + 2*(14*(b^4*c^
2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^6 + a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 1
5*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 3*(b^6 - 6*a*b^4*c + 32*a^3*c
^3)*d^2)*e^3*x^2 + 4*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^7 + 3*(b^5*
c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^3 +
 (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e^2*x + ((b^4*c^2 - 8*a*b^2*c^3 +
 16*a^2*c^4)*d^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + a^2*b^4...
```

3.633.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 349 vs. $2(142) = 284$.

Time = 0.31 (sec) , antiderivative size = 349, normalized size of antiderivative = 2.30

$$\int \frac{d+ex}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \frac{6\,c^2\arctan\left(\frac{2\,cd^2+2\,(ex^2+2\,dx)ce+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-b^2+4\,ace}} + \frac{12\,c^3d^6+36\,(ex^2+2\,dx)c^3d^4e+36\,(ex^2+2\,dx)^2c^3d^2e^2+12\,(ex^2+2\,dx)^3c^3e^3+18\,bc^2d^4+36\,(ex^2+2}{4\left(cd^4+2\,(ex^2+2\,dx)cd^2e+(ex^2+2\,dx)^2ce^2\right.}$$

input
```
integrate((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

output
```
6*c^2*arctan((2*c*d^2 + 2*(e*x^2 + 2*d*x)*c*e + b)/sqrt(-b^2 + 4*a*c))/((b
^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)*e) + 1/4*(12*c^3*d^6 + 36*
(e*x^2 + 2*d*x)*c^3*d^4*e + 36*(e*x^2 + 2*d*x)^2*c^3*d^2*e^2 + 12*(e*x^2 +
 2*d*x)^3*c^3*e^3 + 18*b*c^2*d^4 + 36*(e*x^2 + 2*d*x)*b*c^2*d^2*e + 18*(e*
x^2 + 2*d*x)^2*b*c^2*e^2 + 4*b^2*c*d^2 + 20*a*c^2*d^2 + 4*(e*x^2 + 2*d*x)*
b^2*c*e + 20*(e*x^2 + 2*d*x)*a*c^2*e - b^3 + 10*a*b*c)/((c*d^4 + 2*(e*x^2
+ 2*d*x)*c*d^2*e + (e*x^2 + 2*d*x)^2*c*e^2 + b*d^2 + (e*x^2 + 2*d*x)*b*e +
 a)^2*(b^4*e - 8*a*b^2*c*e + 16*a^2*c^2*e))
```

3.633.9 Mupad [B] (verification not implemented)

Time = 9.81 (sec) , antiderivative size = 1157, normalized size of antiderivative = 7.61

$$\int \frac{d+ex}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$$

$$= \frac{\frac{-b^3+4b^2cd^2+18bc^2d^4+10a\cdots}{4e\left(16a^2c^2-8a\cdots\right.}}{x^2\left(6b^2d^2e^2+30bcd^4e^2+2abe^2+28c^2d^6e^2+12acd^2e^2\right)+x^6\left(28c^2d^2e^6+2bce^6\right)+x\left(4eb^2d^3+\cdots\right.}$$

$$+\frac{6c^2\operatorname{atan}\left(\frac{\left(b^4\left(4ac-b^2\right)^5+16a^2c^2\left(4ac-b^2\right)^5-8ab^2c\left(4ac-b^2\right)^5\right)\left(x\left(\frac{72c^6de^7}{a\left(4ac-b^2\right)^{9/2}\left(16a^2c^2-8ab^2c+b^4\right)}+\frac{72bc^4\left(16da^2bc^4e^9-8\cdots\right.}{ae^2\left(4ac-b^2\right)^{15/2}\left(1\cdots\right.}\cdots\right.\right.}{\cdots}\right.}{\cdots}$$

input
```
int((d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x)
```

output
```
((12*c^3*d^6 - b^3 + 20*a*c^2*d^2 + 4*b^2*c*d^2 + 18*b*c^2*d^4 + 10*a*b*c)
/(4*e*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^2*(45*c^3*d^4*e + 5*a*c^2*e + b
^2*c*e + 27*b*c^2*d^2*e))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (9*x^4*(b*c^2*e
^3 + 10*c^3*d^2*e^3))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*c^3*e^5*x^6)
/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (2*d*x*(5*a*c^2 + b^2*c + 9*c^3*d^4 + 9*
b*c^2*d^2))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (6*d*x^3*(3*b*c^2*e^2 + 10*c^
3*d^2*e^2))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (18*c^3*d*e^4*x^5)/(b^4 + 16*
a^2*c^2 - 8*a*b^2*c))/(x^2*(6*b^2*d^2*e^2 + 28*c^2*d^6*e^2 + 2*a*b*e^2 + 1
2*a*c*d^2*e^2 + 30*b*c*d^4*e^2) + x^6*(28*c^2*d^2*e^6 + 2*b*c*e^6) + x*(4*
b^2*d^3*e + 8*c^2*d^7*e + 8*a*c*d^3*e + 12*b*c*d^5*e + 4*a*b*d*e) + x^3*(4
*b^2*d*e^3 + 56*c^2*d^5*e^3 + 8*a*c*d*e^3 + 40*b*c*d^3*e^3) + x^5*(56*c^2*
d^3*e^5 + 12*b*c*d*e^5) + x^4*(b^2*e^4 + 70*c^2*d^4*e^4 + 2*a*c*e^4 + 30*b
*c*d^2*e^4) + a^2 + b^2*d^4 + c^2*d^8 + c^2*e^8*x^8 + 2*a*b*d^2 + 2*a*c*d^
4 + 2*b*c*d^6 + 8*c^2*d*e^7*x^7) + (6*c^2*atan(((b^4*(4*a*c - b^2)^5 + 16*
a^2*c^2*(4*a*c - b^2)^5 - 8*a*b^2*c*(4*a*c - b^2)^5)*(x*((72*c^6*d*e^7)/(a
*(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (72*b*c^4*(b^5*c^2*
d*e^9 - 8*a*b^3*c^3*d*e^9 + 16*a^2*b*c^4*d*e^9))/(a*e^2*(4*a*c - b^2)^(15/
2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) + x^2*((36*c^6*e^8)/(a*(4*a*c - b^2)^(
9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (36*b*c^4*(b^5*c^2*e^10 - 8*a*b^3*c
^3*e^10 + 16*a^2*b*c^4*e^10))/(a*e^2*(4*a*c - b^2)^(15/2)*(b^4 + 16*a^2...
```

3.633. $\int \frac{d+ex}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$

3.634 $\int \frac{1}{(a+b(d+ex)^2+c(d+ex)^4)^3} dx$

3.634.1 Optimal result

Integrand size = 22, antiderivative size = 437

$$\begin{aligned}
&\int \frac{1}{(a+b(d+ex)^2+c(d+ex)^4)^3} dx \\
&= \frac{\left(\frac{d}{e}+x\right)\left(b^2-2ac+bce^2\left(\frac{d}{e}+x\right)^2\right)}{4a\,(b^2-4ac)\left(a+be^2\left(\frac{d}{e}+x\right)^2+ce^4\left(\frac{d}{e}+x\right)^4\right)^2} \\
&\quad+\frac{\left(\frac{d}{e}+x\right)\left((b^2-7ac)\,(3b^2-4ac)+3bc(b^2-8ac)\,e^2\left(\frac{d}{e}+x\right)^2\right)}{8a^2\,(b^2-4ac)^2\left(a+be^2\left(\frac{d}{e}+x\right)^2+ce^4\left(\frac{d}{e}+x\right)^4\right)} \\
&\quad+\frac{3\sqrt{c}\left(b^4-10ab^2c+56a^2c^2+b(b^2-8ac)\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a^2\,(b^2-4ac)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}\,e} \\
&\quad-\frac{3\sqrt{c}\left(b^4-10ab^2c+56a^2c^2-b(b^2-8ac)\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a^2\,(b^2-4ac)^{5/2}\sqrt{b+\sqrt{b^2-4ac}}\,e}
\end{aligned}$$

output
```
1/4*(d/e+x)*(b^2-2*a*c+b*c*e^2*(d/e+x)^2)/a/(-4*a*c+b^2)/(a+b*e^2*(d/e+x)^
2+c*e^4*(d/e+x)^4)^2+1/8*(d/e+x)*((-7*a*c+b^2)*(-4*a*c+3*b^2)+3*b*c*(-8*a*
c+b^2)*e^2*(d/e+x)^2)/a^2/(-4*a*c+b^2)^2/(a+b*e^2*(d/e+x)^2+c*e^4*(d/e+x)^
4)+3/16*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/
2)*(b^4-10*a*b^2*c+56*a^2*c^2+b*(-8*a*c+b^2)*(-4*a*c+b^2)^(1/2))/a^2/(-4*a
*c+b^2)^(5/2)/e*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-3/16*arctan((e*x+d)*2
^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b^4-10*a*b^2*c+56*a^
2*c^2-b*(-8*a*c+b^2)*(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(5/2)/e*2^(1/2)/
(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.634.2 Mathematica [A] (verified)

Time = 4.49 (sec) , antiderivative size = 424, normalized size of antiderivative = 0.97

$$\int \frac{1}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$$

$$= \frac{\frac{4a(d+ex)\left(-b^2+2ac-bc(d+ex)^2\right)}{(-b^2+4ac)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{2(d+ex)\left(3b^4-25ab^2c+28a^2c^2+3b^3c(d+ex)^2-24abc^2(d+ex)^2\right)}{(b^2-4ac)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{3\sqrt{2}\sqrt{c}\left(b^4-10ab^2c+56a^2c^2+b^3\sqrt{\cdots}\right.}{(b^2-4\cdots}}{16a^2e\cdots}$$

input
```
Integrate[(a + b*(d + e*x)^2 + c*(d + e*x)^4)^(-3),x]
```

output
```
((4*a*(d + e*x)*(-b^2 + 2*a*c - b*c*(d + e*x)^2))/((-b^2 + 4*a*c)*(a + b*(
d + e*x)^2 + c*(d + e*x)^4)^2) + (2*(d + e*x)*(3*b^4 - 25*a*b^2*c + 28*a^2
*c^2 + 3*b^3*c*(d + e*x)^2 - 24*a*b*c^2*(d + e*x)^2))/((b^2 - 4*a*c)^2*(a
+ b*(d + e*x)^2 + c*(d + e*x)^4)) + (3*Sqrt[2]*Sqrt[c]*(b^4 - 10*a*b^2*c +
 56*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 8*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan[(S
qrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/
2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (3*Sqrt[2]*Sqrt[c]*(b^4 - 10*a*b^2*c + 5
6*a^2*c^2 - b^3*Sqrt[b^2 - 4*a*c] + 8*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqr
t[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)
*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(16*a^2*e)
```

3.634.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 405, normalized size of antiderivative = 0.93, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {1687, 1405, 25, 1492, 27, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b(d+ex)^2+c(d+ex)^4)^3}dx$$

$$\Big\downarrow \text{1687}$$

$$\frac{\int \frac{1}{(c(d+ex)^4+b(d+ex)^2+a)^3}d(d+ex)}{e}$$

$$\Big\downarrow \text{1405}$$

$$\frac{\frac{(d+ex)(-2ac+b^2+bc(d+ex)^2)}{4a(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}-\frac{\int -\frac{3b^2+5c(d+ex)^2b-14ac}{\left(c(d+ex)^4+b(d+ex)^2+a\right)^2}d(d+ex)}{4a(b^2-4ac)}}{e}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{3b^2+5c(d+ex)^2b-14ac}{\left(c(d+ex)^4+b(d+ex)^2+a\right)^2}d(d+ex)}{4a(b^2-4ac)}+\frac{(d+ex)(-2ac+b^2+bc(d+ex)^2)}{4a(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}}{e}$$

$$\Big\downarrow \text{1492}$$

$$\frac{\frac{\frac{(d+ex)\left(3bc\left(b^2-8ac\right)(d+ex)^2+\left(b^2-7ac\right)\left(3b^2-4ac\right)\right)}{2a\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{\int -\frac{3\left(b^4-9acb^2+c\left(b^2-8ac\right)(d+ex)^2b+28a^2c^2\right)}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)}{2a\left(b^2-4ac\right)}}{4a(b^2-4ac)}+\frac{(d+ex)(-2ac+b^2+bc(d+ex)^2)}{4a(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}}{e}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{3\int \frac{b^4-9acb^2+c\left(b^2-8ac\right)(d+ex)^2b+28a^2c^2}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)}{2a\left(b^2-4ac\right)}+\frac{(d+ex)\left(3bc\left(b^2-8ac\right)(d+ex)^2+\left(b^2-7ac\right)\left(3b^2-4ac\right)\right)}{2a\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}}{4a(b^2-4ac)}+\frac{(d+ex)(-2ac+b^2+bc(d+ex)^2)}{4a(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}}{e}$$

$$\Big\downarrow \text{1480}$$

$$\frac{\frac{3\left(\frac{1}{2}c\left(\frac{56a^2c^2-10ab^2c+b^4}{\sqrt{b^2-4ac}}+b\left(b^2-8ac\right)\right)\int\frac{1}{c(d+ex)^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}d(d+ex)+\frac{1}{2}c\left(b\left(b^2-8ac\right)-\frac{56a^2c^2-10ab^2c+b^4}{\sqrt{b^2-4ac}}\right)\int\frac{1}{c(d+ex)^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}d(d+ex)}{2a\left(b^2-4ac\right)}}{4a(b^2-4ac)}}{e}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{3\left(\frac{\sqrt{c}\left(\frac{56a^2c^2-10ab^2c+b^4}{\sqrt{b^2-4ac}}+b\left(b^2-8ac\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(b\left(b^2-8ac\right)-\frac{56a^2c^2-10ab^2c+b^4}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2a\left(b^2-4ac\right)}+\frac{(d+ex)\left(3bc\left(b^2-8ac\right)\right.}{2a\left(b^2-4ac\right)\left(\right.}}{4a(b^2-4ac)}}{e}$$

input
```
Int[(a + b*(d + e*x)^2 + c*(d + e*x)^4)^(-3),x]
```

output
```
(((d + e*x)*(b^2 - 2*a*c + b*c*(d + e*x)^2))/(4*a*(b^2 - 4*a*c)*(a + b*(d
+ e*x)^2 + c*(d + e*x)^4)^2) + (((d + e*x)*((b^2 - 7*a*c)*(3*b^2 - 4*a*c)
+ 3*b*c*(b^2 - 8*a*c)*(d + e*x)^2))/(2*a*(b^2 - 4*a*c)*(a + b*(d + e*x)^2
+ c*(d + e*x)^4)) + (3*((Sqrt[c]*(b*(b^2 - 8*a*c) + (b^4 - 10*a*b^2*c + 56
*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - S
qrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(b*(b
^2 - 8*a*c) - (b^4 - 10*a*b^2*c + 56*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(S
qrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b +
Sqrt[b^2 - 4*a*c]])))/(2*a*(b^2 - 4*a*c)))/(4*a*(b^2 - 4*a*c)))/e
```

3.634.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

3.634. $\int \frac{1}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx$

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 1687
```
Int[((a_) + (c_.)*(u_)^(n2_.) + (b_.)*(u_)^(n_))^(p_), x_Symbol] :> Simp[1/
Coefficient[u, x, 1]   Subst[Int[(a + b*x^n + c*x^(2*n))^p, x], x, u], x] /
; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && LinearQ[u, x] && NeQ[u, x]
```

3.634.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.33 (sec) , antiderivative size = 1010, normalized size of antiderivative = 2.31

method	result	size
default	Expression too large to display	1010
risch	Expression too large to display	1059

input
```
int(1/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x,method=_RETURNVERBOSE)
```

output
```
(-3/8*c^2*e^6*b*(8*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)/a^2*x^7-21/8*c^2*d*
e^5*b*(8*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)/a^2*x^6+1/8*(-504*a*b*c^2*d^2
+63*b^3*c*d^2+28*a^2*c^2-49*a*b^2*c+6*b^4)*c*e^4/(16*a^2*c^2-8*a*b^2*c+b^4
)/a^2*x^5+5/8*c*d*e^3*(-168*a*b*c^2*d^2+21*b^3*c*d^2+28*a^2*c^2-49*a*b^2*c
+6*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/a^2*x^4-1/8*e^2*(840*a*b*c^3*d^4-105*b^
3*c^2*d^4-280*a^2*c^3*d^2+490*a*b^2*c^2*d^2-60*b^4*c*d^2+4*a^2*b*c^2+20*a*
b^3*c-3*b^5)/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3-1/8*d*e*(504*a*b*c^3*d^4-6
3*b^3*c^2*d^4-280*a^2*c^3*d^2+490*a*b^2*c^2*d^2-60*b^4*c*d^2+12*a^2*b*c^2+
60*a*b^3*c-9*b^5)/(16*a^2*c^2-8*a*b^2*c+b^4)/a^2*x^2+1/8*(-168*a*b*c^3*d^6
+21*b^3*c^2*d^6+140*a^2*c^3*d^4-245*a*b^2*c^2*d^4+30*b^4*c*d^4-12*a^2*b*c^
2*d^2-60*a*b^3*c*d^2+9*b^5*d^2+44*a^3*c^2-37*a^2*b^2*c+5*a*b^4)/(16*a^2*c^
2-8*a*b^2*c+b^4)/a^2*x+1/8*d/e*(-24*a*b*c^3*d^6+3*b^3*c^2*d^6+28*a^2*c^3*d
^4-49*a*b^2*c^2*d^4+6*b^4*c*d^4-4*a^2*b*c^2*d^2-20*a*b^3*c*d^2+3*b^5*d^2+4
4*a^3*c^2-37*a^2*b^2*c+5*a*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/a^2)/(c*e^4*x^4
+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2
+a)^2+3/16/(16*a^2*c^2-8*a*b^2*c+b^4)/a^2/e*sum((b*c*e^2*(-8*a*c+b^2)*_R^2
+2*b*c*d*e*(-8*a*c+b^2)*_R-8*b*c^2*d^2*a+b^3*c*d^2+28*a^2*c^2-9*a*b^2*c+b^
4)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),
_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*
b*d*e)*_Z+d^4*c+b*d^2+a))
```

3.634.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8554 vs. $2(389) = 778$.

Time = 0.77 (sec) , antiderivative size = 8554, normalized size of antiderivative = 19.57

$$\int \frac{1}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.634.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

output
```
Timed out
```

3.634.7 Maxima [F]

$$\int \frac{1}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \int \frac{1}{\left((ex+d)^4c+(ex+d)^2b+a\right)^3}\,dx$$

input
```
integrate(1/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")
```

output
```
1/8*(3*(b^3*c^2 - 8*a*b*c^3)*e^7*x^7 + 21*(b^3*c^2 - 8*a*b*c^3)*d*e^6*x^6
+ (6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3 + 63*(b^3*c^2 - 8*a*b*c^3)*d^2)*e^5
*x^5 + 5*(21*(b^3*c^2 - 8*a*b*c^3)*d^3 + (6*b^4*c - 49*a*b^2*c^2 + 28*a^2*
c^3)*d)*e^4*x^4 + 3*(b^3*c^2 - 8*a*b*c^3)*d^7 + (3*b^5 - 20*a*b^3*c - 4*a^
2*b*c^2 + 105*(b^3*c^2 - 8*a*b*c^3)*d^4 + 10*(6*b^4*c - 49*a*b^2*c^2 + 28*
a^2*c^3)*d^2)*e^3*x^3 + (6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3)*d^5 + (63*(b
^3*c^2 - 8*a*b*c^3)*d^5 + 10*(6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3)*d^3 + 3
*(3*b^5 - 20*a*b^3*c - 4*a^2*b*c^2)*d)*e^2*x^2 + (3*b^5 - 20*a*b^3*c - 4*a
^2*b*c^2)*d^3 + (21*(b^3*c^2 - 8*a*b*c^3)*d^6 + 5*a*b^4 - 37*a^2*b^2*c + 4
4*a^3*c^2 + 5*(6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3)*d^4 + 3*(3*b^5 - 20*a*
b^3*c - 4*a^2*b*c^2)*d^2)*e*x + (5*a*b^4 - 37*a^2*b^2*c + 44*a^3*c^2)*d)/(
(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*e^9*x^8 + 8*(a^2*b^4*c^2 - 8*a^
3*b^2*c^3 + 16*a^4*c^4)*d*e^8*x^7 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*
b*c^3 + 14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^2)*e^7*x^6 + 4*(14
*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^3 + 3*(a^2*b^5*c - 8*a^3*b^3
*c^2 + 16*a^4*b*c^3)*d)*e^6*x^5 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3 + 70
*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^4 + 30*(a^2*b^5*c - 8*a^3*b^
3*c^2 + 16*a^4*b*c^3)*d^2)*e^5*x^4 + 4*(14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 +
16*a^4*c^4)*d^5 + 10*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^3 + (a^2
*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d)*e^4*x^3 + 2*(a^3*b^5 - 8*a^4*b^3*c ...
```

3.634.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2641 vs. $2(389) = 778$.

Time = 0.31 (sec) , antiderivative size = 2641, normalized size of antiderivative = 6.04

$$\int \frac{1}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

output
```
-3/16*((b^3*c*e^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)
) + d/e)^2 - 8*a*b*c^2*e^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2
)/(c*e^4)) + d/e)^2 - 2*b^3*c*d*e*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a
*c)*e^2)/(c*e^4)) + d/e) + 16*a*b*c^2*d*e*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b
^2 - 4*a*c)*e^2)/(c*e^4)) + d/e) + b^3*c*d^2 - 8*a*b*c^2*d^2 + b^4 - 9*a*b
^2*c + 28*a^2*c^2)*log(x + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)
/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)
/(c*e^4)) + d/e)^3 - 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)
*e^2)/(c*e^4)) + d/e)^2 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(sqrt(
1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)) - (b^3*c*e^2*(
sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2 - 8*a*b*
c^2*e^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2
 + 2*b^3*c*d*e*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) -
 d/e) - 16*a*b*c^2*d*e*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c
*e^4)) - d/e) + b^3*c*d^2 - 8*a*b*c^2*d^2 + b^4 - 9*a*b^2*c + 28*a^2*c^2)*
log(x - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(2
*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^3
+ 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/
e)^2 + 2*c*d^3*e + b*d*e + (6*c*d^2*e^2 + b*e^2)*(sqrt(1/2)*sqrt(-(b*e^2 +
 sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)) + (b^3*c*e^2*(sqrt(1/2)*sqrt(-...
```

3.634.9 Mupad [B] (verification not implemented)

Time = 12.26 (sec) , antiderivative size = 16086, normalized size of antiderivative = 36.81

$$\int \frac{1}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \text{Too large to display}$$

input
```
int(1/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x)
```

output
```
((3*b^5*d^3 + 44*a^3*c^2*d + 6*b^4*c*d^5 + 28*a^2*c^3*d^5 + 3*b^3*c^2*d^7
+ 5*a*b^4*d - 4*a^2*b*c^2*d^3 - 49*a*b^2*c^2*d^5 - 37*a^2*b^2*c*d - 20*a*b
^3*c*d^3 - 24*a*b*c^3*d^7)/(8*a^2*e*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^3
*(3*b^5*e^2 - 4*a^2*b*c^2*e^2 + 60*b^4*c*d^2*e^2 + 280*a^2*c^3*d^2*e^2 + 1
05*b^3*c^2*d^4*e^2 - 20*a*b^3*c*e^2 - 840*a*b*c^3*d^4*e^2 - 490*a*b^2*c^2*
d^2*e^2))/(8*a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^5*(6*b^4*c*e^4 + 28*
a^2*c^3*e^4 - 49*a*b^2*c^2*e^4 + 63*b^3*c^2*d^2*e^4 - 504*a*b*c^3*d^2*e^4)
)/(8*a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^2*(9*b^5*d*e + 280*a^2*c^3*d
^3*e + 63*b^3*c^2*d^5*e + 60*b^4*c*d^3*e - 12*a^2*b*c^2*d*e - 504*a*b*c^3*
d^5*e - 490*a*b^2*c^2*d^3*e - 60*a*b^3*c*d*e))/(8*a^2*(b^4 + 16*a^2*c^2 -
8*a*b^2*c)) + (5*x^4*(28*a^2*c^3*d*e^3 + 21*b^3*c^2*d^3*e^3 + 6*b^4*c*d*e^
3 - 49*a*b^2*c^2*d*e^3 - 168*a*b*c^3*d^3*e^3))/(8*a^2*(b^4 + 16*a^2*c^2 -
8*a*b^2*c)) + (21*x^6*(b^3*c^2*d*e^5 - 8*a*b*c^3*d*e^5))/(8*a^2*(b^4 + 16*
a^2*c^2 - 8*a*b^2*c)) + (x*(5*a*b^4 + 44*a^3*c^2 + 9*b^5*d^2 - 37*a^2*b^2*
c + 30*b^4*c*d^4 + 140*a^2*c^3*d^4 + 21*b^3*c^2*d^6 - 12*a^2*b*c^2*d^2 - 2
45*a*b^2*c^2*d^4 - 60*a*b^3*c*d^2 - 168*a*b*c^3*d^6))/(8*a^2*(b^4 + 16*a^2
*c^2 - 8*a*b^2*c)) + (3*x^7*(b^3*c^2*e^6 - 8*a*b*c^3*e^6))/(8*a^2*(b^4 + 1
6*a^2*c^2 - 8*a*b^2*c)))/(x^2*(6*b^2*d^2*e^2 + 28*c^2*d^6*e^2 + 2*a*b*e^2
+ 12*a*c*d^2*e^2 + 30*b*c*d^4*e^2) + x^6*(28*c^2*d^2*e^6 + 2*b*c*e^6) + x*
(4*b^2*d^3*e + 8*c^2*d^7*e + 8*a*c*d^3*e + 12*b*c*d^5*e + 4*a*b*d*e) + ...
```

3.634. $\int \frac{1}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx$

3.635 $\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$

3.635.1 Optimal result

Integrand size = 30, antiderivative size = 255

$$\begin{aligned}\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx &= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} \\ &\quad+\frac{2b^4-15ab^2c+16a^2c^2+2bc(b^2-7ac)\,(d+ex)^2}{4a^2\left(b^2-4ac\right)^2e\left(a+b(d+ex)^2+c(d+ex)^4\right)} \\ &\quad+\frac{b(b^4-10ab^2c+30a^2c^2)\,\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^3\left(b^2-4ac\right)^{5/2}e} \\ &\quad+\frac{\log(d+ex)}{a^3e}-\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^3e}\end{aligned}$$

output
```
1/4*(b^2-2*a*c+b*c*(e*x+d)^2)/a/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)
^2+1/4*(2*b^4-15*a*b^2*c+16*a^2*c^2+2*b*c*(-7*a*c+b^2)*(e*x+d)^2)/a^2/(-4*
a*c+b^2)^2/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)+1/2*b*(30*a^2*c^2-10*a*b^2*c+b^4)
*arctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)^(5/2)/e+ln
(e*x+d)/a^3/e-1/4*ln(a+b*(e*x+d)^2+c*(e*x+d)^4)/a^3/e
```

3.635.2 Mathematica [A] (verified)

Time = 2.66 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.53

$$\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$$

$$= \frac{\frac{a^2(-b^2+2ac-bc(d+ex)^2)}{(-b^2+4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}+\frac{a(2b^4-15ab^2c+16a^2c^2+2b^3c(d+ex)^2-14abc^2(d+ex)^2)}{(b^2-4ac)^2(a+(d+ex)^2(b+c(d+ex)^2))}+4\log(d+ex)-\frac{(b^5-10ab^3c+30a^2b \cdots}{\cdots}}{\cdots}$$

input
```
Integrate[1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

output
```
((a^2*(-b^2 + 2*a*c - b*c*(d + e*x)^2))/((-b^2 + 4*a*c)*(a + b*(d + e*x)^2
 + c*(d + e*x)^4)^2) + (a*(2*b^4 - 15*a*b^2*c + 16*a^2*c^2 + 2*b^3*c*(d +
e*x)^2 - 14*a*b*c^2*(d + e*x)^2))/((b^2 - 4*a*c)^2*(a + (d + e*x)^2*(b + c
*(d + e*x)^2))) + 4*Log[d + e*x] - ((b^5 - 10*a*b^3*c + 30*a^2*b*c^2 + b^4
*Sqrt[b^2 - 4*a*c] - 8*a*b^2*c*Sqrt[b^2 - 4*a*c] + 16*a^2*c^2*Sqrt[b^2 - 4
*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(b^2 - 4*a*c)^(5/2) +
 ((b^5 - 10*a*b^3*c + 30*a^2*b*c^2 - b^4*Sqrt[b^2 - 4*a*c] + 8*a*b^2*c*Sqr
t[b^2 - 4*a*c] - 16*a^2*c^2*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] +
 2*c*(d + e*x)^2])/(b^2 - 4*a*c)^(5/2))/(4*a^3*e)
```

3.635.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.16, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1462, 1434, 1165, 25, 1235, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$$

$$\Big\downarrow \text{1462}$$

$$\frac{\int \frac{1}{(d+ex)(c(d+ex)^4+b(d+ex)^2+a)^3}d(d+ex)}{e}$$

$$\Big\downarrow \text{1434}$$

$$\frac{\int \frac{1}{(d+ex)^2(c(d+ex)^4+b(d+ex)^2+a)^3}d(d+ex)^2}{2e}$$

$\downarrow$ 1165

$$\frac{\frac{-2ac+b^2+bc(d+ex)^2}{2a(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}-\frac{\int -\frac{3bc(d+ex)^2+2\left(b^2-4ac\right)}{(d+ex)^2\left(c(d+ex)^4+b(d+ex)^2+a\right)^2}d(d+ex)^2}{2a(b^2-4ac)}}{2e}$$

$\downarrow$ 25

$$\frac{\frac{\int \frac{3bc(d+ex)^2+2\left(b^2-4ac\right)}{(d+ex)^2\left(c(d+ex)^4+b(d+ex)^2+a\right)^2}d(d+ex)^2}{2a(b^2-4ac)}+\frac{-2ac+b^2+bc(d+ex)^2}{2a(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}}{2e}$$

$\downarrow$ 1235

$$\frac{\frac{\frac{16a^2c^2+2bc\left(b^2-7ac\right)(d+ex)^2-15ab^2c+2b^4}{a\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{\int -\frac{2\left(\left(b^2-4ac\right)^2+bc\left(b^2-7ac\right)(d+ex)^2\right)}{(d+ex)^2\left(c(d+ex)^4+b(d+ex)^2+a\right)}d(d+ex)^2}{a\left(b^2-4ac\right)}}{2a(b^2-4ac)}+\frac{-2ac+b^2+bc(d+ex)^2}{2a(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}}{2e}$$

$\downarrow$ 27

$$\frac{\frac{\frac{2\int \frac{\left(b^2-4ac\right)^2+bc\left(b^2-7ac\right)(d+ex)^2}{(d+ex)^2\left(c(d+ex)^4+b(d+ex)^2+a\right)}d(d+ex)^2}{a\left(b^2-4ac\right)}+\frac{16a^2c^2+2bc\left(b^2-7ac\right)(d+ex)^2-15ab^2c+2b^4}{a\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}}{2a(b^2-4ac)}+\frac{-2ac+b^2+bc(d+ex)^2}{2a(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}}{2e}$$

$\downarrow$ 1200

$$\frac{\frac{\frac{2\int \left(\frac{\left(4ac-b^2\right)^2}{a(d+ex)^2}+\frac{-c\left(b^2-4ac\right)^2(d+ex)^2-b\left(b^4-9acb^2+23a^2c^2\right)}{a\left(c(d+ex)^4+b(d+ex)^2+a\right)}\right)d(d+ex)^2}{a\left(b^2-4ac\right)}+\frac{16a^2c^2+2bc\left(b^2-7ac\right)(d+ex)^2-15ab^2c+2b^4}{a\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}}{2a(b^2-4ac)}+\frac{-2ac+b^2+bc(d+ex)^2}{2a(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}}{2e}$$

$\downarrow$ 2009

$$\frac{\frac{\frac{2\left(\frac{b\left(30a^2c^2-10ab^2c+b^4\right)\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}+\frac{\left(b^2-4ac\right)^2\log\left((d+ex)^2\right)}{a}-\frac{\left(b^2-4ac\right)^2\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{2a}\right)}{a\left(b^2-4ac\right)}+\frac{16a^2c^2+2bc\left(b^2-7ac\right)(d+ex)^2-15ab^2c+2b^4}{a\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}}{2a(b^2-4ac)}+\frac{-2ac+b^2+bc(d+ex)^2}{2a(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}}{2e}$$

input
```
Int[1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

3.635. $\int \frac{1}{(d+ex)(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx$

output
```
((b^2 - 2*a*c + b*c*(d + e*x)^2)/(2*a*(b^2 - 4*a*c)*(a + b*(d + e*x)^2 + c
*(d + e*x)^4)^2) + ((2*b^4 - 15*a*b^2*c + 16*a^2*c^2 + 2*b*c*(b^2 - 7*a*c)
*(d + e*x)^2)/(a*(b^2 - 4*a*c)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (2*(
(b*(b^4 - 10*a*b^2*c + 30*a^2*c^2)*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2
- 4*a*c]])/(a*Sqrt[b^2 - 4*a*c]) + ((b^2 - 4*a*c)^2*Log[(d + e*x)^2])/a -
((b^2 - 4*a*c)^2*Log[a + b*(d + e*x)^2 + c*(d + e*x)^4])/(2*a)))/(a*(b^2 -
 4*a*c)))/(2*a*(b^2 - 4*a*c)))/(2*e)
```

3.635.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.635.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.43 (sec) , antiderivative size = 966, normalized size of antiderivative = 3.79

method	result
default	$-\frac{\frac{c^{2} e^{5} \left(7 a c-b^{2}\right) a b \,x^{6}}{32 a^{2} c^{2}-16 a \,b^{2} c+2 b^{4}}+\frac{3 \left(7 a c-b^{2}\right) a b \,c^{2} d \,e^{4} x^{5}}{16 a^{2} c^{2}-8 a \,b^{2} c+b^{4}}-\frac{e^{3} a c \left(-210 b \,c^{2} d^{2} a+30 b^{3} c \,d^{2}+16 a^{2} c^{2}-29 a \,b^{2} c+4 b^{4}\right) x^{4}}{4 \left(16 a^{2} c^{2}-8 a \,b^{2} c+b^{4}\right)}-\frac{c d \,e^{2} a \left(-70 b \,c^{2} d^{2} a+10 b^{3} c \,d^{2}+16 a^{2}\right.}{16 a^{2} c^{2}-8 a \,b^{2} c+b}}{}$
risch	Expression too large to display

input
```
int(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x,method=_RETURNVERBOSE)
```

3.635. $\int \frac{1}{(d+ex)(a+b(d+ex)^2+c(d+ex)^4)^3} \, dx$

output
```
-1/a^3*((1/2*c^2*e^5*(7*a*c-b^2)*a*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+3*(7*a
*c-b^2)*a*b*c^2*d*e^4/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5-1/4*e^3*a*c*(-210*a*b
*c^2*d^2+30*b^3*c*d^2+16*a^2*c^2-29*a*b^2*c+4*b^4)/(16*a^2*c^2-8*a*b^2*c+b
^4)*x^4-c*d*e^2*a*(-70*a*b*c^2*d^2+10*b^3*c*d^2+16*a^2*c^2-29*a*b^2*c+4*b^
4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+1/2*e*a*(105*a*b*c^3*d^4-15*b^3*c^2*d^4-
48*a^2*c^3*d^2+87*a*b^2*c^2*d^2-12*b^4*c*d^2+a^2*b*c^2+6*a*b^3*c-b^5)/(16*
a^2*c^2-8*a*b^2*c+b^4)*x^2+d*a*(21*a*b*c^3*d^4-3*b^3*c^2*d^4-16*a^2*c^3*d^
2+29*a*b^2*c^2*d^2-4*b^4*c*d^2+a^2*b*c^2+6*a*b^3*c-b^5)/(16*a^2*c^2-8*a*b^
2*c+b^4)*x-1/4/e*a*(-14*a*b*c^3*d^6+2*b^3*c^2*d^6+16*a^2*c^3*d^4-29*a*b^2*
c^2*d^4+4*b^4*c*d^4-2*a^2*b*c^2*d^2-12*a*b^3*c*d^2+2*b^5*d^2+24*a^3*c^2-21
*a^2*b^2*c+3*a*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*e^4*x^4+4*c*d*e^3*x^3+6
*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2+1/2/(16*a^
2*c^2-8*a*b^2*c+b^4)/e*sum((e^3*c*(16*a^2*c^2-8*a*b^2*c+b^4)*_R^3+3*c*d*e^
2*(16*a^2*c^2-8*a*b^2*c+b^4)*_R^2+e*(48*a^2*c^3*d^2-24*a*b^2*c^2*d^2+3*b^4
*c*d^2+23*a^2*b*c^2-9*a*b^3*c+b^5)*_R+16*a^2*c^3*d^3-8*b^2*a*c^2*d^3+b^4*c
*d^3+23*a^2*b*c^2*d-9*a*b^3*c*d+b^5*d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c
*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6
*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a))+ln(e*x+d)/a
^3/e
```

3.635.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4889 vs. $2(243) = 486$.

Time = 0.95 (sec) , antiderivative size = 9908, normalized size of antiderivative = 38.85

$$\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.635.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx=\text{Timed out}$$

input
```
integrate(1/(e*x+d)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

output
```
Timed out
```

3.635.7 Maxima [F]

$$\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx=\int \frac{1}{\left((ex+d)^4c+(ex+d)^2b+a\right)^3(ex+d)}\,dx$$

input
```
integrate(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")
```

output
```
1/4*(2*(b^3*c^2 - 7*a*b*c^3)*e^6*x^6 + 12*(b^3*c^2 - 7*a*b*c^3)*d*e^5*x^5
+ (4*b^4*c - 29*a*b^2*c^2 + 16*a^2*c^3 + 30*(b^3*c^2 - 7*a*b*c^3)*d^2)*e^4
*x^4 + 2*(b^3*c^2 - 7*a*b*c^3)*d^6 + 4*(10*(b^3*c^2 - 7*a*b*c^3)*d^3 + (4*
b^4*c - 29*a*b^2*c^2 + 16*a^2*c^3)*d)*e^3*x^3 + 3*a*b^4 - 21*a^2*b^2*c + 2
4*a^3*c^2 + (4*b^4*c - 29*a*b^2*c^2 + 16*a^2*c^3)*d^4 + 2*(b^5 - 6*a*b^3*c
 - a^2*b*c^2 + 15*(b^3*c^2 - 7*a*b*c^3)*d^4 + 3*(4*b^4*c - 29*a*b^2*c^2 +
16*a^2*c^3)*d^2)*e^2*x^2 + 2*(b^5 - 6*a*b^3*c - a^2*b*c^2)*d^2 + 4*(3*(b^3
*c^2 - 7*a*b*c^3)*d^5 + (4*b^4*c - 29*a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 -
 6*a*b^3*c - a^2*b*c^2)*d)*e*x)/((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4
)*e^9*x^8 + 8*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d*e^8*x^7 + 2*(a^
2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3 + 14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 +
 16*a^4*c^4)*d^2)*e^7*x^6 + 4*(14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^
4)*d^3 + 3*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d)*e^6*x^5 + (a^2*b^
6 - 6*a^3*b^4*c + 32*a^5*c^3 + 70*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^
4)*d^4 + 30*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^2)*e^5*x^4 + 4*(1
4*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^5 + 10*(a^2*b^5*c - 8*a^3*b
^3*c^2 + 16*a^4*b*c^3)*d^3 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d)*e^4*x
^3 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2 + 14*(a^2*b^4*c^2 - 8*a^3*b^2
*c^3 + 16*a^4*c^4)*d^6 + 15*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^4
 + 3*(a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d^2)*e^3*x^2 + 4*(2*(a^2*b^4*...
```

3.635.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1045 vs. 2(243) = 486.

Time = 0.45 (sec) , antiderivative size = 1045, normalized size of antiderivative = 4.10

$$\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx =$$
$$-\frac{(a^3b^7ce^3-14\,a^4b^5c^2e^3+70\,a^5b^3c^3e^3-120\,a^6bc^4e^3)\sqrt{b^2-4\,ac}\log\left(\left|be^2x^2+\sqrt{b^2-4\,ac}e^2x^2+2\,bdex+\right.\right.}{}$$
$$-\frac{\log\left(\left|ce^4x^4+4\,cde^3x^3+6\,cd^2e^2x^2+4\,cd^3ex+cd^4+be^2x^2+2\,bdex+bd^2+a\right|\right)}{4\,a^3e}$$
$$+\frac{\log\left(\left|ex+d\right|\right)}{a^3e}$$
$$+\frac{2\,ab^3c^2d^6-14\,a^2bc^3d^6+4\,ab^4cd^4-29\,a^2b^2c^2d^4+16\,a^3c^3d^4+2\,ab^5d^2-12\,a^2b^3cd^2-2\,a^3bc^2d^2+2\,(ab^3c}{}$$

input
```
integrate(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

output
```
-1/4*((a^3*b^7*c*e^3 - 14*a^4*b^5*c^2*e^3 + 70*a^5*b^3*c^3*e^3 - 120*a^6*b
*c^4*e^3)*sqrt(b^2 - 4*a*c)*log(abs(b*e^2*x^2 + sqrt(b^2 - 4*a*c)*e^2*x^2
+ 2*b*d*e*x + 2*sqrt(b^2 - 4*a*c)*d*e*x + b*d^2 + sqrt(b^2 - 4*a*c)*d^2 +
2*a)) - (a^3*b^7*c*e^3 - 14*a^4*b^5*c^2*e^3 + 70*a^5*b^3*c^3*e^3 - 120*a^6
*b*c^4*e^3)*sqrt(b^2 - 4*a*c)*log(abs(-b*e^2*x^2 + sqrt(b^2 - 4*a*c)*e^2*x
^2 - 2*b*d*e*x + 2*sqrt(b^2 - 4*a*c)*d*e*x - b*d^2 + sqrt(b^2 - 4*a*c)*d^2
 - 2*a)))/(a^6*b^8*c*e^4 - 16*a^7*b^6*c^2*e^4 + 96*a^8*b^4*c^3*e^4 - 256*a
^9*b^2*c^4*e^4 + 256*a^10*c^5*e^4) - 1/4*log(abs(c*e^4*x^4 + 4*c*d*e^3*x^3
 + 6*c*d^2*e^2*x^2 + 4*c*d^3*e*x + c*d^4 + b*e^2*x^2 + 2*b*d*e*x + b*d^2 +
 a))/(a^3*e) + log(abs(e*x + d))/(a^3*e) + 1/4*(2*a*b^3*c^2*d^6 - 14*a^2*b
*c^3*d^6 + 4*a*b^4*c*d^4 - 29*a^2*b^2*c^2*d^4 + 16*a^3*c^3*d^4 + 2*a*b^5*d
^2 - 12*a^2*b^3*c*d^2 - 2*a^3*b*c^2*d^2 + 2*(a*b^3*c^2*e^6 - 7*a^2*b*c^3*e
^6)*x^6 + 3*a^2*b^4 - 21*a^3*b^2*c + 24*a^4*c^2 + 12*(a*b^3*c^2*d*e^5 - 7*
a^2*b*c^3*d*e^5)*x^5 + (30*a*b^3*c^2*d^2*e^4 - 210*a^2*b*c^3*d^2*e^4 + 4*a
*b^4*c*e^4 - 29*a^2*b^2*c^2*e^4 + 16*a^3*c^3*e^4)*x^4 + 4*(10*a*b^3*c^2*d^
3*e^3 - 70*a^2*b*c^3*d^3*e^3 + 4*a*b^4*c*d*e^3 - 29*a^2*b^2*c^2*d*e^3 + 16
*a^3*c^3*d*e^3)*x^3 + 2*(15*a*b^3*c^2*d^4*e^2 - 105*a^2*b*c^3*d^4*e^2 + 12
*a*b^4*c*d^2*e^2 - 87*a^2*b^2*c^2*d^2*e^2 + 48*a^3*c^3*d^2*e^2 + a*b^5*e^2
 - 6*a^2*b^3*c*e^2 - a^3*b*c^2*e^2)*x^2 + 4*(3*a*b^3*c^2*d^5*e - 21*a^2*b*
c^3*d^5*e + 4*a*b^4*c*d^3*e - 29*a^2*b^2*c^2*d^3*e + 16*a^3*c^3*d^3*e +...
```

3.635.9 Mupad [B] (verification not implemented)

Time = 18.81 (sec) , antiderivative size = 19440, normalized size of antiderivative = 76.24

$$\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \text{Too large to display}$$

input
```
int(1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x)
```

output
```
((x^2*(b^5*e + 48*a^2*c^3*d^2*e + 15*b^3*c^2*d^4*e - 6*a*b^3*c*e - a^2*b*c
^2*e + 12*b^4*c*d^2*e - 105*a*b*c^3*d^4*e - 87*a*b^2*c^2*d^2*e))/(2*(a^2*b
^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (x^4*(4*b^4*c*e^3 + 16*a^2*c^3*e^3 - 29*
a*b^2*c^2*e^3 + 30*b^3*c^2*d^2*e^3 - 210*a*b*c^3*d^2*e^3))/(4*(a^2*b^4 + 1
6*a^4*c^2 - 8*a^3*b^2*c)) + (x^3*(16*a^2*c^3*d*e^2 + 10*b^3*c^2*d^3*e^2 +
4*b^4*c*d*e^2 - 29*a*b^2*c^2*d*e^2 - 70*a*b*c^3*d^3*e^2))/(a^2*b^4 + 16*a^
4*c^2 - 8*a^3*b^2*c) + (3*x^5*(b^3*c^2*d*e^4 - 7*a*b*c^3*d*e^4))/(a^2*b^4
+ 16*a^4*c^2 - 8*a^3*b^2*c) + (x^6*(b^3*c^2*e^5 - 7*a*b*c^3*e^5))/(2*(a^2*
b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (x*(b^5*d + 4*b^4*c*d^3 + 16*a^2*c^3*d^
3 + 3*b^3*c^2*d^5 - 29*a*b^2*c^2*d^3 - 6*a*b^3*c*d - a^2*b*c^2*d - 21*a*b*
c^3*d^5))/(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c) + (3*a*b^4 + 24*a^3*c^2 + 2
*b^5*d^2 - 21*a^2*b^2*c + 4*b^4*c*d^4 + 16*a^2*c^3*d^4 + 2*b^3*c^2*d^6 - 2
*a^2*b*c^2*d^2 - 29*a*b^2*c^2*d^4 - 12*a*b^3*c*d^2 - 14*a*b*c^3*d^6)/(4*e*
(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))/(x^2*(6*b^2*d^2*e^2 + 28*c^2*d^6*e^
2 + 2*a*b*e^2 + 12*a*c*d^2*e^2 + 30*b*c*d^4*e^2) + x^6*(28*c^2*d^2*e^6 + 2
*b*c*e^6) + x*(4*b^2*d^3*e + 8*c^2*d^7*e + 8*a*c*d^3*e + 12*b*c*d^5*e + 4*
a*b*d*e) + x^3*(4*b^2*d*e^3 + 56*c^2*d^5*e^3 + 8*a*c*d*e^3 + 40*b*c*d^3*e^
3) + x^5*(56*c^2*d^3*e^5 + 12*b*c*d*e^5) + x^4*(b^2*e^4 + 70*c^2*d^4*e^4 +
 2*a*c*e^4 + 30*b*c*d^2*e^4) + a^2 + b^2*d^4 + c^2*d^8 + c^2*e^8*x^8 + 2*a
*b*d^2 + 2*a*c*d^4 + 2*b*c*d^6 + 8*c^2*d*e^7*x^7) + log(d + e*x)/(a^3*e...
```

3.636 $\int \frac{1}{(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$

3.636.1 Optimal result

Integrand size = 30, antiderivative size = 484

$$\begin{aligned}\int \frac{1}{(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx &= -\frac{3(5b^2-12ac)\left(b^2-5ac\right)}{8a^3\left(b^2-4ac\right)^2e(d+ex)} + \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)e(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} \\ &\quad + \frac{5b^4-35ab^2c+36a^2c^2+bc(5b^2-32ac)\,(d+ex)^2}{8a^2\left(b^2-4ac\right)^2e(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)} \\ &\quad - \frac{3\sqrt{c}\left((5b^2-12ac)\left(b^2-5ac\right)+\frac{b(5b^4-47ab^2c+124a^2c^2)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a^3\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}e} \\ &\quad - \frac{3\sqrt{c}\left((5b^2-12ac)\left(b^2-5ac\right)-\frac{5b^5-47ab^3c+124a^2bc^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a^3\left(b^2-4ac\right)^2\sqrt{b+\sqrt{b^2-4ac}}e}\end{aligned}$$

output
```
-3/8*(-12*a*c+5*b^2)*(-5*a*c+b^2)/a^3/(-4*a*c+b^2)^2/e/(e*x+d)+1/4*(b^2-2*
a*c+b*c*(e*x+d)^2)/a/(-4*a*c+b^2)/e/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2+
1/8*(5*b^4-35*a*b^2*c+36*a^2*c^2+b*c*(-32*a*c+5*b^2)*(e*x+d)^2)/a^2/(-4*a*
c+b^2)^2/e/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)-3/16*arctan((e*x+d)*2^(1/2)
*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*((-12*a*c+5*b^2)*(-5*a*c+b^
2)+b*(124*a^2*c^2-47*a*b^2*c+5*b^4)/(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)^2
/e*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-3/16*arctan((e*x+d)*2^(1/2)*c^(1/2
)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*((-12*a*c+5*b^2)*(-5*a*c+b^2)+(-12
4*a^2*b*c^2+47*a*b^3*c-5*b^5)/(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)^2/e*2^(
1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.636.2 Mathematica [A] (verified)

Time = 6.18 (sec) , antiderivative size = 560, normalized size of antiderivative = 1.16

$$\int \frac{1}{(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$$
$$= -\frac{1}{a^3e(d+ex)} + \frac{b^3(d+ex)-3abc(d+ex)+b^2c(d+ex)^3-2ac^2(d+ex)^3}{4a^2\left(-b^2+4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}$$
$$+ \frac{-7b^5(d+ex)+52ab^3c(d+ex)-84a^2bc^2(d+ex)-7b^4c(d+ex)^3+47ab^2c^2(d+ex)^3-52a^2c^3(d+ex \ldots}{8a^3\left(-b^2+4ac\right)^2e\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$
$$- \frac{3\sqrt{c}\left(5b^5-47ab^3c+124a^2bc^2+5b^4\sqrt{b^2-4ac}-37ab^2c\sqrt{b^2-4ac}+60a^2c^2\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d \ldots}{\sqrt{b-\sqrt{b^2 \ldots}}\right.}{8\sqrt{2}a^3\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}e}$$
$$- \frac{3\sqrt{c}\left(-5b^5+47ab^3c-124a^2bc^2+5b^4\sqrt{b^2-4ac}-37ab^2c\sqrt{b^2-4ac}+60a^2c^2\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c} \ldots}{\sqrt{b+\sqrt{\ldots}}}\right.}{8\sqrt{2}a^3\left(b^2-4ac\right)^{5/2}\sqrt{b+\sqrt{b^2-4ac}}e}$$

input
```
Integrate[1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

output
```
-(1/(a^3*e*(d + e*x))) + (b^3*(d + e*x) - 3*a*b*c*(d + e*x) + b^2*c*(d + e
*x)^3 - 2*a*c^2*(d + e*x)^3)/(4*a^2*(-b^2 + 4*a*c)*e*(a + b*(d + e*x)^2 +
c*(d + e*x)^4)^2) + (-7*b^5*(d + e*x) + 52*a*b^3*c*(d + e*x) - 84*a^2*b*c^
2*(d + e*x) - 7*b^4*c*(d + e*x)^3 + 47*a*b^2*c^2*(d + e*x)^3 - 52*a^2*c^3*
(d + e*x)^3)/(8*a^3*(-b^2 + 4*a*c)^2*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)
) - (3*Sqrt[c]*(5*b^5 - 47*a*b^3*c + 124*a^2*b*c^2 + 5*b^4*Sqrt[b^2 - 4*a*
c] - 37*a*b^2*c*Sqrt[b^2 - 4*a*c] + 60*a^2*c^2*Sqrt[b^2 - 4*a*c])*ArcTan[(
Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^3*(b
^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) - (3*Sqrt[c]*(-5*b^5 + 47
*a*b^3*c - 124*a^2*b*c^2 + 5*b^4*Sqrt[b^2 - 4*a*c] - 37*a*b^2*c*Sqrt[b^2 -
 4*a*c] + 60*a^2*c^2*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))
/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^3*(b^2 - 4*a*c)^(5/2)*Sqrt[b +
 Sqrt[b^2 - 4*a*c]]*e)
```

3.636.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 471, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1462, 1441, 25, 1600, 27, 1604, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}dx$$

$$\Big\downarrow \text{1462}$$

$$\frac{\int \frac{1}{(d+ex)^2(c(d+ex)^4+b(d+ex)^2+a)^3}d(d+ex)}{e}$$

$$\Big\downarrow \text{1441}$$

$$\frac{\frac{-2ac+b^2+bc(d+ex)^2}{4a(b^2-4ac)(d+ex)(a+b(d+ex)^2+c(d+ex)^4)^2}-\frac{\int -\frac{5b^2+7c(d+ex)^2b-18ac}{(d+ex)^2\left(c(d+ex)^4+b(d+ex)^2+a\right)^2}d(d+ex)}{4a(b^2-4ac)}}{e}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{5b^2+7c(d+ex)^2b-18ac}{(d+ex)^2\left(c(d+ex)^4+b(d+ex)^2+a\right)^2}d(d+ex)}{4a(b^2-4ac)}+\frac{-2ac+b^2+bc(d+ex)^2}{4a(b^2-4ac)(d+ex)(a+b(d+ex)^2+c(d+ex)^4)^2}}{e}$$

$$\Big\downarrow \text{1600}$$

$$\frac{\frac{\frac{36a^2c^2+bc\left(5b^2-32ac\right)(d+ex)^2-35ab^2c+5b^4}{2a\left(b^2-4ac\right)(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{\int -\frac{3\left(bc\left(5b^2-32ac\right)(d+ex)^2+\left(5b^2-12ac\right)\left(b^2-5ac\right)\right)}{(d+ex)^2\left(c(d+ex)^4+b(d+ex)^2+a\right)}d(d+ex)}{2a\left(b^2-4ac\right)}}{4a(b^2-4ac)}+\frac{-2ac+b^2+bc(d+ex)^2}{4a(b^2-4ac)(d+ex)(a+b(d+ex)^2+c(d+e}}{e}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{3\int \frac{bc\left(5b^2-32ac\right)(d+ex)^2+\left(5b^2-12ac\right)\left(b^2-5ac\right)}{(d+ex)^2\left(c(d+ex)^4+b(d+ex)^2+a\right)}d(d+ex)}{2a\left(b^2-4ac\right)}+\frac{36a^2c^2+bc\left(5b^2-32ac\right)(d+ex)^2-35ab^2c+5b^4}{2a\left(b^2-4ac\right)(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)}}{4a(b^2-4ac)}+\frac{-2ac+b^2+bc(d+ex)^2}{4a(b^2-4ac)(d+ex)(a+b(d+ex)^2+c(d+ex)^4}}{e}$$

$$\Big\downarrow \text{1604}$$

$$\frac{\frac{3\left(-\frac{\int \frac{c\left(5b^2-12ac\right)\left(b^2-5ac\right)(d+ex)^2+b\left(5b^4-42acb^2+92a^2c^2\right)}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)}{a}-\frac{\left(5b^2-12ac\right)\left(b^2-5ac\right)}{a(d+ex)}\right)}{2a\left(b^2-4ac\right)}+\frac{36a^2c^2+bc\left(5b^2-32ac\right)(d+ex)^2-35ab^2c+5b^4}{2a\left(b^2-4ac\right)(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)}}{4a(b^2-4ac)}+\frac{}{4a(b^2-}}{e}$$

$\downarrow$ 1480

$$\frac{3\left(-\frac{\frac{1}{2}c\left(\frac{b\left(124a^2c^2-47ab^2c+5b^4\right)}{\sqrt{b^2-4ac}}+\left(5b^2-12ac\right)\left(b^2-5ac\right)\right)\int \frac{1}{c(d+ex)^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}d(d+ex)+\frac{1}{2}c\left(\left(5b^2-12ac\right)\left(b^2-5ac\right)-\frac{b\left(124a^2c^2-47ab^2c+5b^4\right)}{\sqrt{b^2-4ac}}\right)\int}{a}\right.}{2a\left(b^2-4ac\right)} \Bigg/ 4a(b^2-4ac)$$

$\downarrow$ 218

$$\frac{3\left(-\frac{\frac{\sqrt{c}\left(\frac{b\left(124a^2c^2-47ab^2c+5b^4\right)}{\sqrt{b^2-4ac}}+\left(5b^2-12ac\right)\left(b^2-5ac\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(\left(5b^2-12ac\right)\left(b^2-5ac\right)-\frac{b\left(124a^2c^2-47ab^2c+5b^4\right)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{}}{\sqrt{\sqrt{b}}}\right.}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}}{a}\right.}{2a\left(b^2-4ac\right)} \Bigg/ 4a(b^2-4ac) \Bigg/ e$$

input
```
Int[1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

output
```
((b^2 - 2*a*c + b*c*(d + e*x)^2)/(4*a*(b^2 - 4*a*c)*(d + e*x)*(a + b*(d +
e*x)^2 + c*(d + e*x)^4)^2) + ((5*b^4 - 35*a*b^2*c + 36*a^2*c^2 + b*c*(5*b^
2 - 32*a*c)*(d + e*x)^2)/(2*a*(b^2 - 4*a*c)*(d + e*x)*(a + b*(d + e*x)^2 +
 c*(d + e*x)^4)) + (3*(-(((5*b^2 - 12*a*c)*(b^2 - 5*a*c))/(a*(d + e*x))) -
 ((Sqrt[c]*((5*b^2 - 12*a*c)*(b^2 - 5*a*c) + (b*(5*b^4 - 47*a*b^2*c + 124*
a^2*c^2))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - S
qrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*((5*b
^2 - 12*a*c)*(b^2 - 5*a*c) - (b*(5*b^4 - 47*a*b^2*c + 124*a^2*c^2))/Sqrt[b
^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]
]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/a))/(2*a*(b^2 - 4*a*c)))/(4*a*(
b^2 - 4*a*c)))/e
```

3.636. $\int \frac{1}{(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx$

3.636.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1441
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(-(d*x)^(m + 1))*(b^2 - 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p +
1)/(2*a*d*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))
  Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m
 + 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x
] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] ||
IntegerQ[m])
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1600
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(a + b*x^2 + c*x^4)^(p + 1)
*((d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^2)/(2*a*f*(p + 1)*(b^2 - 4*a
*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(f*x)^m*(a + b*x^2 + c
*x^4)^(p + 1)*Simp[d*(b^2*(m + 2*(p + 1) + 1) - 2*a*c*(m + 4*(p + 1) + 1))
- a*b*e*(m + 1) + c*(m + 2*(2*p + 3) + 1)*(b*d - 2*a*e)*x^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Int
egerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

3.636. $\int \frac{1}{(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx$

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

3.636.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.90 (sec) , antiderivative size = 1197, normalized size of antiderivative = 2.47

method	result	size
default	Expression too large to display	1197
risch	Expression too large to display	2458

input
```
int(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/a^3*((1/8*c^2*e^6*(52*a^2*c^2-47*a*b^2*c+7*b^4)/(16*a^2*c^2-8*a*b^2*c+b
^4)*x^7+7/8*c^2*d*e^5*(52*a^2*c^2-47*a*b^2*c+7*b^4)/(16*a^2*c^2-8*a*b^2*c+
b^4)*x^6+1/8*(1092*a^2*c^3*d^2-987*a*b^2*c^2*d^2+147*b^4*c*d^2+136*a^2*b*c
^2-99*a*b^3*c+14*b^5)*c*e^4/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5+5/8*c*d*e^3*(36
4*a^2*c^3*d^2-329*a*b^2*c^2*d^2+49*b^4*c*d^2+136*a^2*b*c^2-99*a*b^3*c+14*b
^5)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4+1/8*e^2*(1820*a^2*c^4*d^4-1645*a*b^2*c^
3*d^4+245*b^4*c^2*d^4+1360*a^2*b*c^3*d^2-990*a*b^3*c^2*d^2+140*b^5*c*d^2+6
8*a^3*c^3+25*a^2*b^2*c^2-43*a*b^4*c+7*b^6)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+
1/8*d*e*(1092*a^2*c^4*d^4-987*a*b^2*c^3*d^4+147*b^4*c^2*d^4+1360*a^2*b*c^3
*d^2-990*a*b^3*c^2*d^2+140*b^5*c*d^2+204*a^3*c^3+75*a^2*b^2*c^2-129*a*b^4*
c+21*b^6)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+1/8*(364*a^2*c^4*d^6-329*a*b^2*c^
3*d^6+49*b^4*c^2*d^6+680*a^2*b*c^3*d^4-495*a*b^3*c^2*d^4+70*b^5*c*d^4+204*
a^3*c^3*d^2+75*a^2*b^2*c^2*d^2-129*a*b^4*c*d^2+21*b^6*d^2+108*a^3*b*c^2-66
*a^2*b^3*c+9*a*b^5)/(16*a^2*c^2-8*a*b^2*c+b^4)*x+1/8*d/e*(52*a^2*c^4*d^6-4
7*a*b^2*c^3*d^6+7*b^4*c^2*d^6+136*a^2*b*c^3*d^4-99*a*b^3*c^2*d^4+14*b^5*c*
d^4+68*a^3*c^3*d^2+25*a^2*b^2*c^2*d^2-43*a*b^4*c*d^2+7*b^6*d^2+108*a^3*b*c
^2-66*a^2*b^3*c+9*a*b^5)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*e^4*x^4+4*c*d*e^3*
x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2+3/16/
(16*a^2*c^2-8*a*b^2*c+b^4)/e*sum((c*e^2*(60*a^2*c^2-37*a*b^2*c+5*b^4)*_R^2
+2*d*c*e*(60*a^2*c^2-37*a*b^2*c+5*b^4)*_R+60*a^2*c^3*d^2-37*a*b^2*c^2*d...
```

3.636.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10260 vs. $2(438) = 876$.

Time = 1.11 (sec) , antiderivative size = 10260, normalized size of antiderivative = 21.20

$$\int \frac{1}{(d+ex)^2 (a+b(d+ex)^2+c(d+ex)^4)^3} \, dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.636.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^2 (a+b(d+ex)^2+c(d+ex)^4)^3} \, dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

output
```
Timed out
```

3.636.7 Maxima [F]

$$\int \frac{1}{(d+ex)^2 (a+b(d+ex)^2+c(d+ex)^4)^3} \, dx = \int \frac{1}{\left((ex+d)^4 c+(ex+d)^2 b+a\right)^3 (ex+d)^2} \, dx$$

input
```
integrate(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")
```

output
```
-1/8*(3*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*e^8*x^8 + 24*(5*b^4*c^2 -
37*a*b^2*c^3 + 60*a^2*c^4)*d*e^7*x^7 + (30*b^5*c - 227*a*b^3*c^2 + 392*a^2
*b*c^3 + 84*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^2)*e^6*x^6 + 6*(28*(
5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^3 + (30*b^5*c - 227*a*b^3*c^2 + 3
92*a^2*b*c^3)*d)*e^5*x^5 + 3*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^8 +
 (15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c^2 + 324*a^3*c^3 + 210*(5*b^4*c^2 - 37
*a*b^2*c^3 + 60*a^2*c^4)*d^4 + 15*(30*b^5*c - 227*a*b^3*c^2 + 392*a^2*b*c^
3)*d^2)*e^4*x^4 + (30*b^5*c - 227*a*b^3*c^2 + 392*a^2*b*c^3)*d^6 + 4*(42*(
5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^5 + 5*(30*b^5*c - 227*a*b^3*c^2 +
 392*a^2*b*c^3)*d^3 + (15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c^2 + 324*a^3*c^3)
*d)*e^3*x^3 + 8*a^2*b^4 - 64*a^3*b^2*c + 128*a^4*c^2 + (15*b^6 - 91*a*b^4*
c + 25*a^2*b^2*c^2 + 324*a^3*c^3)*d^4 + (84*(5*b^4*c^2 - 37*a*b^2*c^3 + 60
*a^2*c^4)*d^6 + 25*a*b^5 - 194*a^2*b^3*c + 364*a^3*b*c^2 + 15*(30*b^5*c -
227*a*b^3*c^2 + 392*a^2*b*c^3)*d^4 + 6*(15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c
^2 + 324*a^3*c^3)*d^2)*e^2*x^2 + (25*a*b^5 - 194*a^2*b^3*c + 364*a^3*b*c^2
)*d^2 + 2*(12*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^7 + 3*(30*b^5*c -
227*a*b^3*c^2 + 392*a^2*b*c^3)*d^5 + 2*(15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c
^2 + 324*a^3*c^3)*d^3 + (25*a*b^5 - 194*a^2*b^3*c + 364*a^3*b*c^2)*d)*e*x)
/((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e^10*x^9 + 9*(a^3*b^4*c^2 - 8
*a^4*b^2*c^3 + 16*a^5*c^4)*d*e^9*x^8 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 1...
```

3.636.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1458 vs. $2(438) = 876$.

Time = 0.40 (sec) , antiderivative size = 1458, normalized size of antiderivative = 3.01

$$\int \frac{1}{(d+ex)^2 \left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

output
```
-1/8*(7*b^4*c^2/((e*x + d)*e) - 47*a*b^2*c^3/((e*x + d)*e) + 52*a^2*c^4/((
e*x + d)*e) + 14*b^5*c/((e*x + d)^3*e) - 99*a*b^3*c^2/((e*x + d)^3*e) + 13
6*a^2*b*c^3/((e*x + d)^3*e) + 7*b^6/((e*x + d)^5*e) - 43*a*b^4*c/((e*x + d
)^5*e) + 25*a^2*b^2*c^2/((e*x + d)^5*e) + 68*a^3*c^3/((e*x + d)^5*e) + 9*a
*b^5/((e*x + d)^7*e) - 66*a^2*b^3*c/((e*x + d)^7*e) + 108*a^3*b*c^2/((e*x
+ d)^7*e))/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*(c + b/(e*x + d)^2 + a/(e
*x + d)^4)^2) + 3/64*((5*a^6*b^13 - 112*a^7*b^11*c + 1030*a^8*b^9*c^2 - 49
28*a^9*b^7*c^3 + 12736*a^10*b^5*c^4 - 16384*a^11*b^3*c^5 + 7680*a^12*b*c^6
)*sqrt(2*a*b + 2*sqrt(b^2 - 4*a*c)*a)*e^4 + 2*(5*a^4*b^6*c - 57*a^5*b^4*c^
2 + 208*a^6*b^2*c^3 - 240*a^7*c^4)*sqrt(2*a*b + 2*sqrt(b^2 - 4*a*c)*a)*sqr
t(b^2 - 4*a*c)*e^2*abs(a^3*b^4*e^2 - 8*a^4*b^2*c*e^2 + 16*a^5*c^2*e^2) - (
a^3*b^4*e^2 - 8*a^4*b^2*c*e^2 + 16*a^5*c^2*e^2)^2*(5*b^5 - 42*a*b^3*c + 92
*a^2*b*c^2)*sqrt(2*a*b + 2*sqrt(b^2 - 4*a*c)*a))*arctan(2*sqrt(1/2)/((e*x
+ d)*e*sqrt((a^3*b^5*e^2 - 8*a^4*b^3*c*e^2 + 16*a^5*b*c^2*e^2 + sqrt((a^3*
b^5*e^2 - 8*a^4*b^3*c*e^2 + 16*a^5*b*c^2*e^2)^2 - 4*(a^4*b^4*e^4 - 8*a^5*b
^2*c*e^4 + 16*a^6*c^2*e^4)*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)))/(a^4
*b^4*e^4 - 8*a^5*b^2*c*e^4 + 16*a^6*c^2*e^4))))/((a^7*b^6*c - 12*a^8*b^4*c
^2 + 48*a^9*b^2*c^3 - 64*a^10*c^4)*sqrt(b^2 - 4*a*c)*e^3*abs(a^3*b^4*e^2 -
 8*a^4*b^2*c*e^2 + 16*a^5*c^2*e^2)*abs(a)) - 3/64*((5*a^6*b^13 - 112*a^7*b
^11*c + 1030*a^8*b^9*c^2 - 4928*a^9*b^7*c^3 + 12736*a^10*b^5*c^4 - 1638...
```

3.636.9 Mupad [B] (verification not implemented)

Time = 16.58 (sec) , antiderivative size = 18112, normalized size of antiderivative = 37.42

$$\int \frac{1}{(d+ex)^2 (a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = \text{Too large to display}$$

input
```
int(1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x)
```

output
```
- ((x^4*(15*b^6*e^3 + 324*a^3*c^3*e^3 + 450*b^5*c*d^2*e^3 + 25*a^2*b^2*c^2
*e^3 + 12600*a^2*c^4*d^4*e^3 + 1050*b^4*c^2*d^4*e^3 - 91*a*b^4*c*e^3 - 340
5*a*b^3*c^2*d^2*e^3 + 5880*a^2*b*c^3*d^2*e^3 - 7770*a*b^2*c^3*d^4*e^3))/(8
*a*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (x^6*(30*b^5*c*e^5 - 227*a*b^3*
c^2*e^5 + 392*a^2*b*c^3*e^5 + 5040*a^2*c^4*d^2*e^5 + 420*b^4*c^2*d^2*e^5 -
 3108*a*b^2*c^3*d^2*e^5))/(8*a*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (x*
(30*b^6*d^3 + 90*b^5*c*d^5 + 648*a^3*c^3*d^3 + 720*a^2*c^4*d^7 + 60*b^4*c^
2*d^7 + 25*a*b^5*d - 681*a*b^3*c^2*d^5 + 1176*a^2*b*c^3*d^5 - 444*a*b^2*c^
3*d^7 + 50*a^2*b^2*c^2*d^3 - 194*a^2*b^3*c*d + 364*a^3*b*c^2*d - 182*a*b^4
*c*d^3))/(4*a*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (3*x^5*(1680*a^2*c^4
*d^3*e^4 + 140*b^4*c^2*d^3*e^4 + 30*b^5*c*d*e^4 - 227*a*b^3*c^2*d*e^4 + 39
2*a^2*b*c^3*d*e^4 - 1036*a*b^2*c^3*d^3*e^4))/(4*a*(a^2*b^4 + 16*a^4*c^2 -
8*a^3*b^2*c)) + (3*x^8*(60*a^2*c^4*e^7 + 5*b^4*c^2*e^7 - 37*a*b^2*c^3*e^7)
)/(8*a*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (x^2*(90*b^6*d^2*e + 25*a*b
^5*e + 1944*a^3*c^3*d^2*e + 5040*a^2*c^4*d^6*e + 420*b^4*c^2*d^6*e - 194*a
^2*b^3*c*e + 364*a^3*b*c^2*e + 450*b^5*c*d^4*e - 546*a*b^4*c*d^2*e - 3405*
a*b^3*c^2*d^4*e + 5880*a^2*b*c^3*d^4*e - 3108*a*b^2*c^3*d^6*e + 150*a^2*b^
2*c^2*d^2*e))/(8*a*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (x^3*(15*b^6*d*
e^2 + 324*a^3*c^3*d*e^2 + 150*b^5*c*d^3*e^2 + 2520*a^2*c^4*d^5*e^2 + 210*b
^4*c^2*d^5*e^2 - 91*a*b^4*c*d*e^2 + 25*a^2*b^2*c^2*d*e^2 - 1135*a*b^3*c...
```

3.636. $\int \frac{1}{(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx$

3.637 $\int \frac{1}{(d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$

3.637.1 Optimal result

Integrand size = 30, antiderivative size = 325

$$\int \frac{1}{(d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = -\frac{3(b^2-5ac)(b^2-2ac)}{2a^3(b^2-4ac)^2e(d+ex)^2}+\frac{b^2-2ac+bc(d+ex)^2}{4a(b^2-4ac)e(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{3b^4-20ab^2c+20a^2c^2+3bc(b^2-6ac)(d+ex)^2}{4a^2(b^2-4ac)^2e(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{3(b^6-10ab^4c+30a^2b^2c^2-20a^3c^3)\,\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^4(b^2-4ac)^{5/2}e}-\frac{3b\log(d+ex)}{a^4e}+\frac{3b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^4e}$$

output
```
-3/2*(-5*a*c+b^2)*(-2*a*c+b^2)/a^3/(-4*a*c+b^2)^2/e/(e*x+d)^2+1/4*(b^2-2*a
*c+b*c*(e*x+d)^2)/a/(-4*a*c+b^2)/e/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2
+1/4*(3*b^4-20*a*b^2*c+20*a^2*c^2+3*b*c*(-6*a*c+b^2)*(e*x+d)^2)/a^2/(-4*a*
c+b^2)^2/e/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)-3/2*(-20*a^3*c^3+30*a^2*b
^2*c^2-10*a*b^4*c+b^6)*arctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/a^4/(
-4*a*c+b^2)^(5/2)/e-3*b*ln(e*x+d)/a^4/e+3/4*b*ln(a+b*(e*x+d)^2+c*(e*x+d)^4
)/a^4/e
```

3.637.2 Mathematica [A] (verified)

Time = 6.16 (sec) , antiderivative size = 491, normalized size of antiderivative = 1.51

$$\int \frac{1}{(d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$$
$$= -\frac{1}{2a^3e(d+ex)^2} + \frac{b^3-3abc+b^2c(d+ex)^2-2ac^2(d+ex)^2}{4a^2\left(-b^2+4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}$$
$$+ \frac{-4b^5+29ab^3c-46a^2bc^2-4b^4c(d+ex)^2+26ab^2c^2(d+ex)^2-28a^2c^3(d+ex)^2}{4a^3\left(-b^2+4ac\right)^2e\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$
$$- \frac{3b\log(d+ex)}{a^4e}$$
$$+ \frac{3\left(b^6-10ab^4c+30a^2b^2c^2-20a^3c^3+b^5\sqrt{b^2-4ac}-8ab^3c\sqrt{b^2-4ac}+16a^2bc^2\sqrt{b^2-4ac}\right)\log\left(b-\sqrt{b^2\right.}}{4a^4\left(b^2-4ac\right)^{5/2}e}$$
$$+ \frac{3\left(-b^6+10ab^4c-30a^2b^2c^2+20a^3c^3+b^5\sqrt{b^2-4ac}-8ab^3c\sqrt{b^2-4ac}+16a^2bc^2\sqrt{b^2-4ac}\right)\log\left(b+\sqrt{\right.}}{4a^4\left(b^2-4ac\right)^{5/2}e}$$

input
```
Integrate[1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

output
```
-1/2*1/(a^3*e*(d + e*x)^2) + (b^3 - 3*a*b*c + b^2*c*(d + e*x)^2 - 2*a*c^2*
(d + e*x)^2)/(4*a^2*(-b^2 + 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2
) + (-4*b^5 + 29*a*b^3*c - 46*a^2*b*c^2 - 4*b^4*c*(d + e*x)^2 + 26*a*b^2*c
^2*(d + e*x)^2 - 28*a^2*c^3*(d + e*x)^2)/(4*a^3*(-b^2 + 4*a*c)^2*e*(a + b*
(d + e*x)^2 + c*(d + e*x)^4)) - (3*b*Log[d + e*x])/(a^4*e) + (3*(b^6 - 10*
a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3 + b^5*Sqrt[b^2 - 4*a*c] - 8*a*b^3*c*
Sqrt[b^2 - 4*a*c] + 16*a^2*b*c^2*Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a
*c] + 2*c*(d + e*x)^2])/(4*a^4*(b^2 - 4*a*c)^(5/2)*e) + (3*(-b^6 + 10*a*b^
4*c - 30*a^2*b^2*c^2 + 20*a^3*c^3 + b^5*Sqrt[b^2 - 4*a*c] - 8*a*b^3*c*Sqrt
[b^2 - 4*a*c] + 16*a^2*b*c^2*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c]
+ 2*c*(d + e*x)^2])/(4*a^4*(b^2 - 4*a*c)^(5/2)*e)
```

3.637.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1462, 1434, 1165, 25, 1235, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}dx$$

$$\Big\downarrow \; 1462$$

$$\frac{\int \frac{1}{(d+ex)^3(c(d+ex)^4+b(d+ex)^2+a)^3}d(d+ex)}{e}$$

$$\Big\downarrow \; 1434$$

$$\frac{\int \frac{1}{(d+ex)^4(c(d+ex)^4+b(d+ex)^2+a)^3}d(d+ex)^2}{2e}$$

$$\Big\downarrow \; 1165$$

$$\frac{\frac{-2ac+b^2+bc(d+ex)^2}{2a(b^2-4ac)(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)^2}-\frac{\int -\frac{3b^2+4c(d+ex)^2b-10ac}{(d+ex)^4\left(c(d+ex)^4+b(d+ex)^2+a\right)^2}d(d+ex)^2}{2a(b^2-4ac)}}{2e}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{3b^2+4c(d+ex)^2b-10ac}{(d+ex)^4\left(c(d+ex)^4+b(d+ex)^2+a\right)^2}d(d+ex)^2}{2a(b^2-4ac)}+\frac{-2ac+b^2+bc(d+ex)^2}{2a(b^2-4ac)(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)^2}}{2e}$$

$$\Big\downarrow \; 1235$$

$$\frac{\frac{\frac{20a^2c^2+3bc\left(b^2-6ac\right)(d+ex)^2-20ab^2c+3b^4}{a\left(b^2-4ac\right)(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{\int -\frac{6\left(bc\left(b^2-6ac\right)(d+ex)^2+\left(b^2-5ac\right)\left(b^2-2ac\right)\right)}{(d+ex)^4\left(c(d+ex)^4+b(d+ex)^2+a\right)}d(d+ex)^2}{a\left(b^2-4ac\right)}}{2a(b^2-4ac)}+\frac{-2ac+b^2+bc(d+ex)^2}{2a(b^2-4ac)(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)^2}}{2e}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{6\int \frac{bc\left(b^2-6ac\right)(d+ex)^2+\left(b^2-5ac\right)\left(b^2-2ac\right)}{(d+ex)^4\left(c(d+ex)^4+b(d+ex)^2+a\right)}d(d+ex)^2}{a\left(b^2-4ac\right)}+\frac{20a^2c^2+3bc\left(b^2-6ac\right)(d+ex)^2-20ab^2c+3b^4}{a\left(b^2-4ac\right)(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}}{2a(b^2-4ac)}+\frac{-2ac+b^2+bc(d+ex)^2}{2a(b^2-4ac)(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)^2}}{2e}$$

$$\Big\downarrow \textbf{1200}$$

$$\frac{\frac{6\int \left(-\frac{b\left(4ac-b^2\right)^2}{a^2(d+ex)^2}+\frac{b^6-9acb^4+23a^2c^2b^2+c\left(b^2-4ac\right)^2(d+ex)^2b-10a^3c^3}{a^2\left(c(d+ex)^4+b(d+ex)^2+a\right)}+\frac{\left(b^2-5ac\right)\left(b^2-2ac\right)}{a(d+ex)^4}\right)d(d+ex)^2}{a\left(b^2-4ac\right)}+\frac{20a^2c^2+3bc\left(b^2-6ac\right)(d+ex)^2-20ab^2c+3b^4}{a\left(b^2-4ac\right)(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}}{2a(b^2-4ac)}+}{2e}$$

$$\Big\downarrow \textbf{2009}$$

$$\frac{\frac{\frac{20a^2c^2+3bc\left(b^2-6ac\right)(d+ex)^2-20ab^2c+3b^4}{a\left(b^2-4ac\right)(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{6\left(-\frac{b\left(b^2-4ac\right)^2\log\left((d+ex)^2\right)}{a^2}+\frac{b\left(b^2-4ac\right)^2\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{2a^2}-\frac{\left(-20a^3c^3+30a^2b^2c^2-10ab^4c+b^6\right)}{a^2\sqrt{b^2-4}}\right.}{a\left(b^2-4ac\right)}}{2a(b^2-4ac)}}{2e}$$

input
```
Int[1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

output
```
((b^2 - 2*a*c + b*c*(d + e*x)^2)/(2*a*(b^2 - 4*a*c)*(d + e*x)^2*(a + b*(d
+ e*x)^2 + c*(d + e*x)^4)^2) + ((3*b^4 - 20*a*b^2*c + 20*a^2*c^2 + 3*b*c*(
b^2 - 6*a*c)*(d + e*x)^2)/(a*(b^2 - 4*a*c)*(d + e*x)^2*(a + b*(d + e*x)^2
+ c*(d + e*x)^4)) + (6*(-(((b^2 - 5*a*c)*(b^2 - 2*a*c))/(a*(d + e*x)^2)) -
 ((b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*ArcTanh[(b + 2*c*(d + e
*x)^2)/Sqrt[b^2 - 4*a*c]])/(a^2*Sqrt[b^2 - 4*a*c]) - (b*(b^2 - 4*a*c)^2*Lo
g[(d + e*x)^2])/a^2 + (b*(b^2 - 4*a*c)^2*Log[a + b*(d + e*x)^2 + c*(d + e*
x)^4])/(2*a^2)))/(a*(b^2 - 4*a*c)))/(2*a*(b^2 - 4*a*c)))/(2*e)
```

3.637.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.637. $\int \frac{1}{(d+ex)^3(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx$

3.637.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.94 (sec) , antiderivative size = 1141, normalized size of antiderivative = 3.51

method	result	size
default	Expression too large to display	1141
risch	Expression too large to display	2190

input
```
int(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/a^4*((1/2*c^2*e^5*(14*a^2*c^2-13*a*b^2*c+2*b^4)*a/(16*a^2*c^2-8*a*b^2*c
+b^4)*x^6+3*(14*a^2*c^2-13*a*b^2*c+2*b^4)*a*c^2*d*e^4/(16*a^2*c^2-8*a*b^2*
c+b^4)*x^5+1/4*e^3*a*c*(420*a^2*c^3*d^2-390*a*b^2*c^2*d^2+60*b^4*c*d^2+74*
a^2*b*c^2-55*a*b^3*c+8*b^5)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4+c*d*e^2*a*(140*
a^2*c^3*d^2-130*a*b^2*c^2*d^2+20*b^4*c*d^2+74*a^2*b*c^2-55*a*b^3*c+8*b^5)/
(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+1/2*e*a*(210*a^2*c^4*d^4-195*a*b^2*c^3*d^4+
30*b^4*c^2*d^4+222*a^2*b*c^3*d^2-165*a*b^3*c^2*d^2+24*b^5*c*d^2+18*a^3*c^3
+7*a^2*b^2*c^2-12*a*b^4*c+2*b^6)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+d*a*(42*a^
2*c^4*d^4-39*a*b^2*c^3*d^4+6*b^4*c^2*d^4+74*a^2*b*c^3*d^2-55*a*b^3*c^2*d^2
+8*b^5*c*d^2+18*a^3*c^3+7*a^2*b^2*c^2-12*a*b^4*c+2*b^6)/(16*a^2*c^2-8*a*b^
2*c+b^4)*x+1/4/e*a*(28*a^2*c^4*d^6-26*a*b^2*c^3*d^6+4*b^4*c^2*d^6+74*a^2*b
*c^3*d^4-55*a*b^3*c^2*d^4+8*b^5*c*d^4+36*a^3*c^3*d^2+14*a^2*b^2*c^2*d^2-24
*a*b^4*c*d^2+4*b^6*d^2+58*a^3*b*c^2-36*a^2*b^3*c+5*a*b^5)/(16*a^2*c^2-8*a*
b^2*c+b^4))/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2
+c*d^4+2*b*d*e*x+b*d^2+a)^2+3/2/(16*a^2*c^2-8*a*b^2*c+b^4)/e*sum((e^3*b*c*
(-16*a^2*c^2+8*a*b^2*c-b^4)*_R^3+3*d*e^2*b*c*(-16*a^2*c^2+8*a*b^2*c-b^4)*_
R^2+e*(-48*a^2*b*c^3*d^2+24*a*b^3*c^2*d^2-3*b^5*c*d^2+10*a^3*c^3-23*a^2*b^
2*c^2+9*a*b^4*c-b^6)*_R-16*a^2*b*c^3*d^3+8*a*b^3*c^2*d^3-b^5*c*d^3+10*a^3*
c^3*d-23*a^2*b^2*c^2*d+9*a*b^4*c*d-b^6*d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_
R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_...
```

3.637.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7517 vs. $2(311) = 622$.

Time = 1.61 (sec) , antiderivative size = 15165, normalized size of antiderivative = 46.66

$$\int \frac{1}{(d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^3} \, dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.637.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^3} \, dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

output
```
Timed out
```

3.637.7 Maxima [F]

$$\int \frac{1}{(d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^3} \, dx = \int \frac{1}{\left((ex+d)^4 c+(ex+d)^2 b+a\right)^3 (ex+d)^3} \, dx$$

input
```
integrate(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")
```

output
```
-1/4*(6*(b^4*c^2 - 7*a*b^2*c^3 + 10*a^2*c^4)*e^8*x^8 + 48*(b^4*c^2 - 7*a*b
^2*c^3 + 10*a^2*c^4)*d*e^7*x^7 + 3*(4*b^5*c - 29*a*b^3*c^2 + 46*a^2*b*c^3
+ 56*(b^4*c^2 - 7*a*b^2*c^3 + 10*a^2*c^4)*d^2)*e^6*x^6 + 6*(56*(b^4*c^2 -
7*a*b^2*c^3 + 10*a^2*c^4)*d^3 + 3*(4*b^5*c - 29*a*b^3*c^2 + 46*a^2*b*c^3)*
d)*e^5*x^5 + 6*(b^4*c^2 - 7*a*b^2*c^3 + 10*a^2*c^4)*d^8 + (6*b^6 - 36*a*b^
4*c + 14*a^2*b^2*c^2 + 100*a^3*c^3 + 420*(b^4*c^2 - 7*a*b^2*c^3 + 10*a^2*c
^4)*d^4 + 45*(4*b^5*c - 29*a*b^3*c^2 + 46*a^2*b*c^3)*d^2)*e^4*x^4 + 3*(4*b
^5*c - 29*a*b^3*c^2 + 46*a^2*b*c^3)*d^6 + 4*(84*(b^4*c^2 - 7*a*b^2*c^3 + 1
0*a^2*c^4)*d^5 + 15*(4*b^5*c - 29*a*b^3*c^2 + 46*a^2*b*c^3)*d^3 + 2*(3*b^6
 - 18*a*b^4*c + 7*a^2*b^2*c^2 + 50*a^3*c^3)*d)*e^3*x^3 + 2*a^2*b^4 - 16*a^
3*b^2*c + 32*a^4*c^2 + 2*(3*b^6 - 18*a*b^4*c + 7*a^2*b^2*c^2 + 50*a^3*c^3)
*d^4 + (168*(b^4*c^2 - 7*a*b^2*c^3 + 10*a^2*c^4)*d^6 + 9*a*b^5 - 68*a^2*b^
3*c + 122*a^3*b*c^2 + 45*(4*b^5*c - 29*a*b^3*c^2 + 46*a^2*b*c^3)*d^4 + 12*
(3*b^6 - 18*a*b^4*c + 7*a^2*b^2*c^2 + 50*a^3*c^3)*d^2)*e^2*x^2 + (9*a*b^5
- 68*a^2*b^3*c + 122*a^3*b*c^2)*d^2 + 2*(24*(b^4*c^2 - 7*a*b^2*c^3 + 10*a^
2*c^4)*d^7 + 9*(4*b^5*c - 29*a*b^3*c^2 + 46*a^2*b*c^3)*d^5 + 4*(3*b^6 - 18
*a*b^4*c + 7*a^2*b^2*c^2 + 50*a^3*c^3)*d^3 + (9*a*b^5 - 68*a^2*b^3*c + 122
*a^3*b*c^2)*d)*e*x)/((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e^11*x^10
+ 10*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d*e^10*x^9 + (2*a^3*b^5*c
- 16*a^4*b^3*c^2 + 32*a^5*b*c^3 + 45*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*...
```

3.637.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.17

$$\int \frac{1}{(d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$$
$$= \frac{3\left(b^6-10\,ab^4c+30\,a^2b^2c^2-20\,a^3c^3\right)\arctan\left(-\frac{b+\frac{2\,a}{(ex+d)^2}}{\sqrt{-b^2+4\,ac}}\right)}{2\left(a^4b^4-8\,a^5b^2c+16\,a^6c^2\right)\sqrt{-b^2+4\,ac}e}$$
$$+\frac{3\,b\log\left(c+\frac{b}{(ex+d)^2}+\frac{a}{(ex+d)^4}\right)}{4\,a^4e}-\frac{1}{2\,(ex+d)^2a^3e}$$
$$+\frac{5\,b^5c^2-36\,ab^3c^3+58\,a^2bc^4+\frac{2\left(5\,b^6ce-38\,ab^4c^2e+71\,a^2b^2c^3e-14\,a^3c^4e\right)}{(ex+d)^2e}+\frac{5\,b^7e^2-34\,ab^5ce^2+41\,a^2b^3c^2e^2+42\,a^3bc^3e^2}{(ex+d)^4e^2}+\frac{6\,(a}{}}{4\left(b^2-4\,ac\right)^2a^4\left(c+\frac{b}{(ex+d)^2}+\frac{a}{(ex+d)^4}\right)^2e}$$

input
```
integrate(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

output
```
3/2*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*arctan(-(b + 2*a/(e*x
 + d)^2)/sqrt(-b^2 + 4*a*c))/((a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*sqrt(-b
^2 + 4*a*c)*e) + 3/4*b*log(c + b/(e*x + d)^2 + a/(e*x + d)^4)/(a^4*e) - 1/
2/((e*x + d)^2*a^3*e) + 1/4*(5*b^5*c^2 - 36*a*b^3*c^3 + 58*a^2*b*c^4 + 2*(
5*b^6*c*e - 38*a*b^4*c^2*e + 71*a^2*b^2*c^3*e - 14*a^3*c^4*e)/((e*x + d)^2
*e) + (5*b^7*e^2 - 34*a*b^5*c*e^2 + 41*a^2*b^3*c^2*e^2 + 42*a^3*b*c^3*e^2)
/((e*x + d)^4*e^2) + 6*(a*b^6*e^3 - 8*a^2*b^4*c*e^3 + 17*a^3*b^2*c^2*e^3 -
 6*a^4*c^3*e^3)/((e*x + d)^6*e^3))/((b^2 - 4*a*c)^2*a^4*(c + b/(e*x + d)^2
 + a/(e*x + d)^4)^2*e)
```

3.637.9 Mupad [B] (verification not implemented)

Time = 22.78 (sec) , antiderivative size = 21465, normalized size of antiderivative = 66.05

$$\int \frac{1}{(d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \text{Too large to display}$$

input
```
int(1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x)
```

output
```
(log(((27*c^4*e^14*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)^2*(b^5 + 16*a^2*b*c^2 +
b^4*c*d^2 + 10*a^2*c^3*d^2 - 8*a*b^3*c - 7*a*b^2*c^2*d^2))/(a^9*(4*a*c - b
^2)^6) - ((3*b - 3*a^4*e*(-(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c
)^2/(a^8*e^2*(4*a*c - b^2)^5))^(1/2))*((9*c^3*e^15*(b^4 + 10*a^2*c^2 - 7*a
*b^2*c)*(4*b^6 - 10*a^3*c^3 + 6*b^5*c*d^2 + 71*a^2*b^2*c^2 - 33*a*b^4*c -
47*a*b^3*c^2*d^2 + 90*a^2*b*c^3*d^2))/(a^6*(4*a*c - b^2)^4) - ((3*b - 3*a^
4*e*(-(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2/(a^8*e^2*(4*a*c -
 b^2)^5))^(1/2))*((6*c^2*e^16*(2*b^7 - 20*a^3*b*c^3 + b^6*c*d^2 + 46*a^2*b
^3*c^2 + 100*a^3*c^4*d^2 - 18*a*b^5*c - 2*a*b^4*c^2*d^2 - 30*a^2*b^2*c^3*d
^2))/(a^3*(4*a*c - b^2)^2) + (6*c^3*e^18*x^2*(b^6 + 100*a^3*c^3 - 30*a^2*b
^2*c^2 - 2*a*b^4*c))/(a^3*(4*a*c - b^2)^2) + (b*c^2*e^16*(3*b - 3*a^4*e*(-
(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2/(a^8*e^2*(4*a*c - b^2)^
5))^(1/2))*(a*b + 3*b^2*d^2 + 3*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 1
0*a*c*e^2*x^2 - 20*a*c*d*e*x))/a^4 + (12*c^3*d*e^17*x*(b^6 + 100*a^3*c^3 -
 30*a^2*b^2*c^2 - 2*a*b^4*c))/(a^3*(4*a*c - b^2)^2)))/(4*a^4*e) + (9*b*c^4
*e^17*x^2*(6*b^8 + 900*a^4*c^4 + 479*a^2*b^4*c^2 - 1100*a^3*b^2*c^3 - 89*a
*b^6*c))/(a^6*(4*a*c - b^2)^4) + (18*b*c^4*d*e^16*x*(6*b^8 + 900*a^4*c^4 +
 479*a^2*b^4*c^2 - 1100*a^3*b^2*c^3 - 89*a*b^6*c))/(a^6*(4*a*c - b^2)^4)))
/(4*a^4*e) + (27*c^5*e^16*x^2*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)^3)/(a^9*(4*a*
c - b^2)^6) + (54*c^5*d*e^15*x*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)^3)/(a^9*(...
```

3.638 $\int \frac{(df+efx)^4}{a+b(d+ex)^2+c(d+ex)^4}\,dx$

3.638.1 Optimal result

Integrand size = 33, antiderivative size = 202

$$\int \frac{(df+efx)^4}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \frac{f^4x}{c}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)f^4\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}e}-\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)f^4\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}e}$$

output
```
f^4*x/c-1/2*f^4*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2
))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))/c^(3/2)/e*2^(1/2)/(b-(-4*a*c+b^2)^(1
/2))^(1/2)-1/2*f^4*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(
1/2))*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))/c^(3/2)/e*2^(1/2)/(b+(-4*a*c+b^2
)^(1/2))^(1/2)
```

3.638.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.10

$$\int \frac{(df+efx)^4}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \frac{f^4\left(2\sqrt{c}(d+ex)-\frac{\sqrt{2}\left(-b^2+2ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\left(b^2-2ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2c^{3/2}e}$$

input
```
Integrate[(d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

output
```
(f^4*(2*Sqrt[c]*(d + e*x) - (Sqrt[2]*(-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*
ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2
 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*(b^2 - 2*a*c + b*Sqrt[b^
2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]
])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])))/(2*c^(3/2)*e)
```

3.638.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {1462, 1442, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(df+efx)^4}{a+b(d+ex)^2+c(d+ex)^4} \, dx$$

$$\Big\downarrow \text{1462}$$

$$\frac{f^4 \int \frac{(d+ex)^4}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)}{e}$$

$$\Big\downarrow \text{1442}$$

$$\frac{f^4 \left(\frac{d+ex}{c}-\frac{\int \frac{b(d+ex)^2+a}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)}{c}\right)}{e}$$

$$\Big\downarrow \text{1480}$$

$$\frac{f^4 \left(\frac{d+ex}{c}-\frac{\frac{1}{2} \left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \int \frac{1}{c(d+ex)^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}d(d+ex)+\frac{1}{2} \left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right) \int \frac{1}{c(d+ex)^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}d(d+ex)}{c}\right)}{e}$$

$$\Big\downarrow \text{218}$$

$$\frac{f^4\left(\frac{d+ex}{c}-\frac{\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{c}\right)}{e}$$

input
```
Int[(d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

output
```
(f^4*((d + e*x)/c - (((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2
]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b
 - Sqrt[b^2 - 4*a*c]]) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sq
rt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sq
rt[b + Sqrt[b^2 - 4*a*c]]))/c))/e
```

3.638.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1442
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
x] - Simp[d^4/(c*(m + 4*p + 1))   Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

3.638. $\int \frac{(df+efx)^4}{a+b(d+ex)^2+c(d+ex)^4}\,dx$

3.638.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.59 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.80

method	result
default	$f^4\left(\frac{x}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\,e^4\_Z^4+4cd\,e^3\_Z^3+\left(6c\,d^2e^2+b\,e^2\right)\_Z^2+\left(4d^3ec+2bde\right)\_Z+d^4c+b\,d^2+a\right)}\frac{\left(-b\,e^2\_R^2-2bde\_R-b\,d^2-a\right)}{2e^3c\_R^3+6cd\,e^2\_R^2+6c\,d^2e\_R}}{2ce}\right.$
risch	$\frac{f^4x}{c}+\frac{f^4\left(\sum_{\_R=\mathrm{RootOf}\left(c\,e^4\_Z^4+4cd\,e^3\_Z^3+\left(6c\,d^2e^2+b\,e^2\right)\_Z^2+\left(4d^3ec+2bde\right)\_Z+d^4c+b\,d^2+a\right)}\frac{\left(-b\,e^2\_R^2-2bde\_R-b\,d^2-a\right.}{2e^3c\_R^3+6cd\,e^2\_R^2+6c\,d^2e\_R}\right.}{2ce}$

input
```
int((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x,method=_RETURNVERBOSE)
```

output
```
f^4*(x/c+1/2/c/e*sum((-_R^2*b*e^2-2*_R*b*d*e-b*d^2-a)/(2*_R^3*c*e^3+6*_R^2
*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*
c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a
)))
```

3.638.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1346 vs. 2(166) = 332.

Time = 0.29 (sec) , antiderivative size = 1346, normalized size of antiderivative = 6.66

$$\int \frac{(df+efx)^4}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \text{Too large to display}$$

input
```
integrate((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

output
```
1/2*(2*f^4*x - sqrt(1/2)*c*sqrt(-((b^3 - 3*a*b*c)*f^8 + sqrt((b^4 - 2*a*b^
2*c + a^2*c^2)*f^16/((b^2*c^6 - 4*a*c^7)*e^4))*(b^2*c^3 - 4*a*c^4)*e^2)/((
b^2*c^3 - 4*a*c^4)*e^2))*log(-2*(a*b^2 - a^2*c)*e*f^12*x - 2*(a*b^2 - a^2*
c)*d*f^12 + sqrt(1/2)*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*e*f^8 - sqrt((b^4 - 2
*a*b^2*c + a^2*c^2)*f^16/((b^2*c^6 - 4*a*c^7)*e^4))*(b^3*c^3 - 4*a*b*c^4)*
e^3)*sqrt(-((b^3 - 3*a*b*c)*f^8 + sqrt((b^4 - 2*a*b^2*c + a^2*c^2)*f^16/((
b^2*c^6 - 4*a*c^7)*e^4))*(b^2*c^3 - 4*a*c^4)*e^2)/((b^2*c^3 - 4*a*c^4)*e^2
))) + sqrt(1/2)*c*sqrt(-((b^3 - 3*a*b*c)*f^8 + sqrt((b^4 - 2*a*b^2*c + a^2
*c^2)*f^16/((b^2*c^6 - 4*a*c^7)*e^4))*(b^2*c^3 - 4*a*c^4)*e^2)/((b^2*c^3 -
 4*a*c^4)*e^2))*log(-2*(a*b^2 - a^2*c)*e*f^12*x - 2*(a*b^2 - a^2*c)*d*f^12
 - sqrt(1/2)*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*e*f^8 - sqrt((b^4 - 2*a*b^2*c
+ a^2*c^2)*f^16/((b^2*c^6 - 4*a*c^7)*e^4))*(b^3*c^3 - 4*a*b*c^4)*e^3)*sqrt
(-((b^3 - 3*a*b*c)*f^8 + sqrt((b^4 - 2*a*b^2*c + a^2*c^2)*f^16/((b^2*c^6 -
 4*a*c^7)*e^4))*(b^2*c^3 - 4*a*c^4)*e^2)/((b^2*c^3 - 4*a*c^4)*e^2))) - sqr
t(1/2)*c*sqrt(-((b^3 - 3*a*b*c)*f^8 - sqrt((b^4 - 2*a*b^2*c + a^2*c^2)*f^1
6/((b^2*c^6 - 4*a*c^7)*e^4))*(b^2*c^3 - 4*a*c^4)*e^2)/((b^2*c^3 - 4*a*c^4)
*e^2))*log(-2*(a*b^2 - a^2*c)*e*f^12*x - 2*(a*b^2 - a^2*c)*d*f^12 + sqrt(1
/2)*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*e*f^8 + sqrt((b^4 - 2*a*b^2*c + a^2*c^2
)*f^16/((b^2*c^6 - 4*a*c^7)*e^4))*(b^3*c^3 - 4*a*b*c^4)*e^3)*sqrt(-((b^3 -
 3*a*b*c)*f^8 - sqrt((b^4 - 2*a*b^2*c + a^2*c^2)*f^16/((b^2*c^6 - 4*a*c...
```

3.638.6 Sympy [A] (verification not implemented)

Time = 1.93 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.08

$$\int \frac{(df+efx)^4}{a+b(d+ex)^2+c(d+ex)^4}\,dx$$
$$=\text{RootSum}\left(t^4\cdot\left(256a^2c^5e^4-128ab^2c^4e^4+16b^4c^3e^4\right)+t^2\cdot\left(48a^2bc^2e^2f^8-28ab^3ce^2f^8+4b^5e^2f^8\right)+a^3f^{16}\right.$$
$$+\frac{f^4x}{c}$$

input
```
integrate((e*f*x+d*f)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

output
```
RootSum(_t**4*(256*a**2*c**5*e**4 - 128*a*b**2*c**4*e**4 + 16*b**4*c**3*e*
*4) + _t**2*(48*a**2*b*c**2*e**2*f**8 - 28*a*b**3*c*e**2*f**8 + 4*b**5*e**
2*f**8) + a**3*f**16, Lambda(_t, _t*log(x + (32*_t**3*a*b*c**4*e**3 - 8*_t
**3*b**3*c**3*e**3 - 4*_t*a**2*c**2*e*f**8 + 8*_t*a*b**2*c*e*f**8 - 2*_t*b
**4*e*f**8 + a**2*c*d*f**12 - a*b**2*d*f**12)/(a**2*c*e*f**12 - a*b**2*e*f
**12)))) + f**4*x/c
```

3.638.7 Maxima [F]

$$\int \frac{(df+efx)^4}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \int \frac{(efx+df)^4}{(ex+d)^4c+(ex+d)^2b+a}\,dx$$

input
```
integrate((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

output
```
f^4*x/c - f^4*integrate((b*e^2*x^2 + 2*b*d*e*x + b*d^2 + a)/(c*e^4*x^4 + 4
*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e
*x + a), x)/c
```

3.638.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1366 vs. $2(166) = 332$.

Time = 0.30 (sec) , antiderivative size = 1366, normalized size of antiderivative = 6.76

$$\int \frac{(df+efx)^4}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \text{Too large to display}$$

input
```
integrate((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

output
```
f^4*x/c + 1/2*((b*e^6*f^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)
/(c*e^4)) + d/e)^2 - 2*b*d*e^5*f^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*
a*c)*e^2)/(c*e^4)) + d/e) + b*d^2*e^4*f^4 + a*e^4*f^4)*log(x + sqrt(1/2)*s
qrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*s
qrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^3 - 6*c*d*e^3*(sqrt(1
/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 - 2*c*d^3*e -
b*d*e + (6*c*d^2*e^2 + b*e^2)*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*
e^2)/(c*e^4)) + d/e)) - (b*e^6*f^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*
a*c)*e^2)/(c*e^4)) - d/e)^2 + 2*b*d*e^5*f^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt
(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e) + b*d^2*e^4*f^4 + a*e^4*f^4)*log(x - sq
rt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sq
rt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^3 + 6*c*d*e^
3*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2 + 2*c
*d^3*e + b*d*e + (6*c*d^2*e^2 + b*e^2)*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2
- 4*a*c)*e^2)/(c*e^4)) - d/e)) + (b*e^6*f^4*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt
(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 - 2*b*d*e^5*f^4*(sqrt(1/2)*sqrt(-(b*e
^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e) + b*d^2*e^4*f^4 + a*e^4*f^4)*l
og(x + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*
c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^3 -
 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + ...
```

3.638.9 Mupad [B] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 4605, normalized size of antiderivative = 22.80

$$\int \frac{(df+efx)^4}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \text{Too large to display}$$

input
```
int((d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x)
```

output
```
atan((((2*b^4*d*e^11*f^8 + 4*a^2*c^2*d*e^11*f^8 - 8*a*b^2*c*d*e^11*f^8)/c
+ ((16*a^2*c^3*e^12*f^4 - 4*a*b^2*c^2*e^12*f^4)/c + ((8*b^3*c^3*d*e^13 - 3
2*a*b*c^4*d*e^13)/c + (2*x*(4*b^3*c^3*e^14 - 16*a*b*c^4*e^14))/c)*(-(b^5*f
^8 + b^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 -
 a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*
b^2*c^4*e^2)))^(1/2))*(-(b^5*f^8 + b^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12*a
^2*b*c^2*f^8 - 7*a*b^3*c*f^8 - a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^
2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2) + (2*x*(b^4*e^12*f^8 +
2*a^2*c^2*e^12*f^8 - 4*a*b^2*c*e^12*f^8))/c)*(-(b^5*f^8 + b^2*f^8*(-(4*a*c
 - b^2)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 - a*c*f^8*(-(4*a*c - b
^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2)*
1i + ((2*b^4*d*e^11*f^8 + 4*a^2*c^2*d*e^11*f^8 - 8*a*b^2*c*d*e^11*f^8)/c -
 ((16*a^2*c^3*e^12*f^4 - 4*a*b^2*c^2*e^12*f^4)/c - ((8*b^3*c^3*d*e^13 - 32
*a*b*c^4*d*e^13)/c + (2*x*(4*b^3*c^3*e^14 - 16*a*b*c^4*e^14))/c)*(-(b^5*f^
8 + b^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 -
a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b
^2*c^4*e^2)))^(1/2))*(-(b^5*f^8 + b^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12*a^
2*b*c^2*f^8 - 7*a*b^3*c*f^8 - a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2
*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2) + (2*x*(b^4*e^12*f^8 + 2
*a^2*c^2*e^12*f^8 - 4*a*b^2*c*e^12*f^8))/c)*(-(b^5*f^8 + b^2*f^8*(-(4*a...
```

3.638. $\int \frac{(df+efx)^4}{a+b(d+ex)^2+c(d+ex)^4}\,dx$

3.639 $\int \frac{(df+efx)^3}{a+b(d+ex)^2+c(d+ex)^4}\,dx$

3.639.1 Optimal result

Integrand size = 33, antiderivative size = 87

$$\int \frac{(df+efx)^3}{a+b(d+ex)^2+c(d+ex)^4}\,dx=\frac{bf^3\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}e}+\frac{f^3\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4ce}$$

output
```
1/4*f^3*ln(a+b*(e*x+d)^2+c*(e*x+d)^4)/c/e+1/2*b*f^3*arctanh((b+2*c*(e*x+d)
^2)/(-4*a*c+b^2)^(1/2))/c/e/(-4*a*c+b^2)^(1/2)
```

3.639.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.92

$$\int \frac{(df+efx)^3}{a+b(d+ex)^2+c(d+ex)^4}\,dx=\frac{f^3\left(-\frac{2b\arctan\left(\frac{b+2c(d+ex)^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+\log\left(a+b(d+ex)^2+c(d+ex)^4\right)\right)}{4ce}$$

input
```
Integrate[(d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

output
```
(f^3*((-2*b*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 +
4*a*c] + Log[a + b*(d + e*x)^2 + c*(d + e*x)^4]))/(4*c*e)
```

3.639.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1462, 1434, 1142, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(df+efx)^3}{a+b(d+ex)^2+c(d+ex)^4}\,dx$$

$$\Big\downarrow \text{1462}$$

$$\frac{f^3\int \frac{(d+ex)^3}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)}{e}$$

$$\Big\downarrow \text{1434}$$

$$\frac{f^3\int \frac{(d+ex)^2}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)^2}{2e}$$

$$\Big\downarrow \text{1142}$$

$$\frac{f^3\left(\frac{\int \frac{2c(d+ex)^2+b}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)^2}{2c}-\frac{b\int \frac{1}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)^2}{2c}\right)}{2e}$$

$$\Big\downarrow \text{1083}$$

$$\frac{f^3\left(\frac{b\int \frac{1}{-(d+ex)^4+b^2-4ac}d(2c(d+ex)^2+b)}{c}+\frac{\int \frac{2c(d+ex)^2+b}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)^2}{2c}\right)}{2e}$$

$$\Big\downarrow \text{219}$$

$$\frac{f^3\left(\frac{\int \frac{2c(d+ex)^2+b}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)^2}{2c}+\frac{b\,\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}\right)}{2e}$$

$$\Big\downarrow \text{1103}$$

$$\frac{f^3\left(\frac{b\,\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}+\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{2c}\right)}{2e}$$

input
```
Int[(d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

output
```
(f^3*((b*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 - 4
*a*c]) + Log[a + b*(d + e*x)^2 + c*(d + e*x)^4]/(2*c)))/(2*e)
```

3.639.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

3.639. $\int \frac{(df+efx)^3}{a+b(d+ex)^2+c(d+ex)^4}\,dx$

3.639.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.62 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.77

method	result
default	$\frac{f^3\left(\sum_{\_R=\mathrm{RootOf}\left(c\,e^4\_Z^4+4cd\,e^3\_Z^3+\left(6c\,d^2e^2+b\,e^2\right)\_Z^2+\left(4d^3ec+2bde\right)\_Z+d^4c+b\,d^2+a\right)}\frac{\left(\_R^3e^3+3\_R^2d\,e^2+3\_R\,d^2e+d^3\right)\ln\left(x-\_R\right)}{2e^3c\_R^3+6cd\,e^2\_R^2+6c\,d^2e\_R+2d^3c+b\ldots}\right)}{2e}$
risch	Expression too large to display

input
```
int((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x,method=_RETURNVERBOSE)
```

output
```
1/2*f^3/e*sum((_R^3*e^3+3*_R^2*d*e^2+3*_R*d^2*e+d^3)/(2*_R^3*c*e^3+6*_R^2*
c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c
*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a)
)
```

3.639.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 446, normalized size of antiderivative = 5.13

$$\int \frac{(df+efx)^3}{a+b(d+ex)^2+c(d+ex)^4}\,dx$$
$$=\left[\frac{\sqrt{b^2-4\,ac}bf^3\log\left(\frac{2\,c^2e^4x^4+8\,c^2de^3x^3+2\,c^2d^4+2\,(6\,c^2d^2+bc)e^2x^2+2\,bcd^2+4\,(2\,c^2d^3+bcd)ex+b^2-2\,ac+(2\,ce^2x^2+4\,cdex+2\,cd^2+b)\sqrt{\ldots}}{ce^4x^4+4\,cde^3x^3+cd^4+(6\,cd^2+b)e^2x^2+bd^2+2\,(2\,cd^3+bd)ex+a}\right.}{4\,(b^2c-4\,\ldots}\right.$$

input
```
integrate((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(b^2 - 4*a*c)*b*f^3*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2
*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x
 + b^2 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c)
)/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(
2*c*d^3 + b*d)*e*x + a)) + (b^2 - 4*a*c)*f^3*log(c*e^4*x^4 + 4*c*d*e^3*x^3
 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a))/((b
^2*c - 4*a*c^2)*e), 1/4*(2*sqrt(-b^2 + 4*a*c)*b*f^3*arctan(-(2*c*e^2*x^2 +
 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^2 - 4*a*c
)*f^3*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^
2 + 2*(2*c*d^3 + b*d)*e*x + a))/((b^2*c - 4*a*c^2)*e)]
```

3.639.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 332 vs. $2(75) = 150$.

Time = 1.01 (sec) , antiderivative size = 332, normalized size of antiderivative = 3.82

$$\int \frac{(df+efx)^3}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \left(-\frac{bf^3\sqrt{-4ac+b^2}}{4ce\,(4ac-b^2)} + \frac{f^3}{4ce}\right)\log\left(\frac{2dx}{e}+x^2+\frac{-8ace\left(-\frac{bf^3\sqrt{-4ac+b^2}}{4ce(4ac-b^2)}+\frac{f^3}{4ce}\right)+2af^3+2b^2e\left(-\frac{bf^3\sqrt{-4ac+b^2}}{4ce(4ac-b^2)}+\frac{f^3}{4ce}\right)+bd^2f^3}{be^2f^3}\right) + \left(\frac{bf^3\sqrt{-4ac+b^2}}{4ce\,(4ac-b^2)} + \frac{f^3}{4ce}\right)\log\left(\frac{2dx}{e}+x^2+\frac{-8ace\left(\frac{bf^3\sqrt{-4ac+b^2}}{4ce(4ac-b^2)}+\frac{f^3}{4ce}\right)+2af^3+2b^2e\left(\frac{bf^3\sqrt{-4ac+b^2}}{4ce(4ac-b^2)}+\frac{f^3}{4ce}\right)+bd^2f^3}{be^2f^3}\right)$$

input
```
integrate((e*f*x+d*f)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

output
```
(-b*f**3*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*c - b**2)) + f**3/(4*c*e))*log(2*
d*x/e + x**2 + (-8*a*c*e*(-b*f**3*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*c - b**2
)) + f**3/(4*c*e)) + 2*a*f**3 + 2*b**2*e*(-b*f**3*sqrt(-4*a*c + b**2)/(4*c
*e*(4*a*c - b**2)) + f**3/(4*c*e)) + b*d**2*f**3)/(b*e**2*f**3)) + (b*f**3
*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*c - b**2)) + f**3/(4*c*e))*log(2*d*x/e +
x**2 + (-8*a*c*e*(b*f**3*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*c - b**2)) + f**3
/(4*c*e)) + 2*a*f**3 + 2*b**2*e*(b*f**3*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*c
- b**2)) + f**3/(4*c*e)) + b*d**2*f**3)/(b*e**2*f**3))
```

3.639.7 Maxima [F]

$$\int \frac{(df+efx)^3}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \int \frac{(efx+df)^3}{(ex+d)^4c+(ex+d)^2b+a}\,dx$$

input
```
integrate((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

output
```
integrate((e*f*x + d*f)^3/((e*x + d)^4*c + (e*x + d)^2*b + a), x)
```

3.639.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.82

$$\int \frac{(df+efx)^3}{a+b(d+ex)^2+c(d+ex)^4}\,dx = -\frac{bf^3\arctan\left(\frac{2\,cd^2f+2\,(efx^2+2\,dfx)ce+bf}{\sqrt{-b^2+4\,ac}f}\right)}{2\,\sqrt{-b^2+4\,ac}ce} + \frac{f^3\log\left(cd^4f^2+2\,(efx^2+2\,dfx)cd^2ef+(efx^2+2\,dfx)^2ce^2+bd^2f^2+(efx^2+2\,dfx)bef+af^2\right)}{4\,ce}$$

input
```
integrate((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

output
```
-1/2*b*f^3*arctan((2*c*d^2*f + 2*(e*f*x^2 + 2*d*f*x)*c*e + b*f)/(sqrt(-b^2
 + 4*a*c)*f))/(sqrt(-b^2 + 4*a*c)*c*e) + 1/4*f^3*log(c*d^4*f^2 + 2*(e*f*x^
2 + 2*d*f*x)*c*d^2*e*f + (e*f*x^2 + 2*d*f*x)^2*c*e^2 + b*d^2*f^2 + (e*f*x^
2 + 2*d*f*x)*b*e*f + a*f^2)/(c*e)
```

3.639.9 Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 287, normalized size of antiderivative = 3.30

$$\int \frac{(df+efx)^3}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \frac{4\,a\,c\,e\,f^3\,\ln\left(c\,d^4+4\,c\,d^3\,e\,x+6\,c\,d^2\,e^2\,x^2+b\,d^2+4\,c\,d\,e^3\,x^3+2\,b\,d\,e\,x+c\,e^4\,x^4+b\,e^2\,x^2+a\right)}{16\,a\,c^2\,e^2-4\,b^2\,c\,e^2} - \frac{b\,f^3\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,d^2}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,e^2\,x^2}{\sqrt{4\,a\,c-b^2}}+\frac{4\,c\,d\,e\,x}{\sqrt{4\,a\,c-b^2}}\right)}{2\,c\,e\,\sqrt{4\,a\,c-b^2}} - \frac{b^2\,e\,f^3\,\ln\left(c\,d^4+4\,c\,d^3\,e\,x+6\,c\,d^2\,e^2\,x^2+b\,d^2+4\,c\,d\,e^3\,x^3+2\,b\,d\,e\,x+c\,e^4\,x^4+b\,e^2\,x^2+a\right)}{16\,a\,c^2\,e^2-4\,b^2\,c\,e^2}$$

input
```
int((d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x)
```

output
```
(4*a*c*e*f^3*log(a + b*d^2 + c*d^4 + b*e^2*x^2 + c*e^4*x^4 + 2*b*d*e*x + 6
*c*d^2*e^2*x^2 + 4*c*d^3*e*x + 4*c*d*e^3*x^3))/(16*a*c^2*e^2 - 4*b^2*c*e^2
) - (b*f^3*atan(b/(4*a*c - b^2)^(1/2) + (2*c*d^2)/(4*a*c - b^2)^(1/2) + (2
*c*e^2*x^2)/(4*a*c - b^2)^(1/2) + (4*c*d*e*x)/(4*a*c - b^2)^(1/2)))/(2*c*e
*(4*a*c - b^2)^(1/2)) - (b^2*e*f^3*log(a + b*d^2 + c*d^4 + b*e^2*x^2 + c*e
^4*x^4 + 2*b*d*e*x + 6*c*d^2*e^2*x^2 + 4*c*d^3*e*x + 4*c*d*e^3*x^3))/(16*a
*c^2*e^2 - 4*b^2*c*e^2)
```

3.639. $\int \frac{(df+efx)^3}{a+b(d+ex)^2+c(d+ex)^4} \, dx$

3.640 $\int \frac{(df+efx)^2}{a+b(d+ex)^2+c(d+ex)^4}\,dx$

3.640.1 Optimal result

Integrand size = 33, antiderivative size = 170

$$\int \frac{(df+efx)^2}{a+b(d+ex)^2+c(d+ex)^4}\,dx=-\frac{\sqrt{b-\sqrt{b^2-4ac}}f^2\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}e}+\frac{\sqrt{b+\sqrt{b^2-4ac}}f^2\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}e}$$

output
```
-1/2*f^2*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b-(
-4*a*c+b^2)^(1/2))^(1/2)/e*2^(1/2)/c^(1/2)/(-4*a*c+b^2)^(1/2)+1/2*f^2*arct
an((e*x+d)*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-4*a*c+b^2)^(
1/2))^(1/2)/e*2^(1/2)/c^(1/2)/(-4*a*c+b^2)^(1/2)
```

3.640.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.05

$$\int \frac{(df+efx)^2}{a+b(d+ex)^2+c(d+ex)^4}\,dx=\frac{f^2\left(\left(-b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)+\sqrt{b-\sqrt{b^2-4ac}}\sqrt{b+\sqrt{b^2-4ac}}\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}e}$$

input
```
Integrate[(d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

output
```
(f^2*((-b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b -
 Sqrt[b^2 - 4*a*c]]] + Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[b + Sqrt[b^2 - 4*a
*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]]))/(Sq
rt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e)
```

3.640.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1462, 1450, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(df+efx)^2}{a+b(d+ex)^2+c(d+ex)^4}dx$$

$$\Big\downarrow \text{ 1462}$$

$$\frac{f^2\int \frac{(d+ex)^2}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)}{e}$$

$$\Big\downarrow \text{ 1450}$$

$$\frac{f^2\left(\frac{1}{2}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{c(d+ex)^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}d(d+ex)+\frac{1}{2}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\int \frac{1}{c(d+ex)^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}d(d+ex)\right)}{e}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{f^2\left(\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}\right)}{e}$$

input
```
Int[(d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

output
```
(f^2*(((1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b
 - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((
1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[
b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])))/e
```

3.640.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1450
```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^2/2)*(b/q + 1)   Int[(d*x)^(m - 2)/(b/
2 + q/2 + c*x^2), x], x] - Simp[(d^2/2)*(b/q - 1)   Int[(d*x)^(m - 2)/(b/2
- q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] &&
 GeQ[m, 2]
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

3.640.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.59 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.84

method	result
default	$\frac{f^2\left(\sum_{\_R=\text{RootOf}\left(c\,e^4\_Z^4+4cd\,e^3\_Z^3+\left(6c\,d^2e^2+b\,e^2\right)\_Z^2+\left(4d^3ec+2bde\right)\_Z+d^4c+b\,d^2+a\right)}\frac{\left(e^2\_R^2+2ed\_R+d^2\right)\ln\left(x-\_R\right.}{2e^3c\_R^3+6cd\,e^2\_R^2+6c\,d^2e\_R+2d^3c+b}\right.}{2e}$
risch	$\frac{f^2\left(\sum_{\_R=\text{RootOf}\left(c\,e^4\_Z^4+4cd\,e^3\_Z^3+\left(6c\,d^2e^2+b\,e^2\right)\_Z^2+\left(4d^3ec+2bde\right)\_Z+d^4c+b\,d^2+a\right)}\frac{\left(e^2\_R^2+2ed\_R+d^2\right)\ln\left(x-\_R\right.}{2e^3c\_R^3+6cd\,e^2\_R^2+6c\,d^2e\_R+2d^3c+b}\right.}{2e}$

input
```
int((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x,method=_RETURNVERBOSE)
```

output
```
1/2*f^2/e*sum((_R^2*e^2+2*_R*d*e+d^2)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*
d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*
c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a))
```

3.640.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 799 vs. $2(135) = 270$.

Time = 0.28 (sec) , antiderivative size = 799, normalized size of antiderivative = 4.70

$$\int \frac{(df+efx)^2}{a+b(d+ex)^2+c(d+ex)^4}\,dx$$
$$= \frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{bf^4+(b^2c-4\,ac^2)\sqrt{\frac{f^8}{(b^2c^2-4\,ac^3)e^4}}e^2}{(b^2c-4\,ac^2)e^2}}\log\left(ef^6x+df^6\right.$$
$$\left.+\sqrt{\frac{1}{2}}(b^2c-4\,ac^2)\sqrt{\frac{f^8}{(b^2c^2-4\,ac^3)e^4}}e^3\sqrt{-\frac{bf^4+(b^2c-4\,ac^2)\sqrt{\frac{f^8}{(b^2c^2-4\,ac^3)e^4}}e^2}{(b^2c-4\,ac^2)e^2}}\right)$$
$$-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{bf^4+(b^2c-4\,ac^2)\sqrt{\frac{f^8}{(b^2c^2-4\,ac^3)e^4}}e^2}{(b^2c-4\,ac^2)e^2}}\log\left(ef^6x+df^6\right.$$
$$\left.-\sqrt{\frac{1}{2}}(b^2c-4\,ac^2)\sqrt{\frac{f^8}{(b^2c^2-4\,ac^3)e^4}}e^3\sqrt{-\frac{bf^4+(b^2c-4\,ac^2)\sqrt{\frac{f^8}{(b^2c^2-4\,ac^3)e^4}}e^2}{(b^2c-4\,ac^2)e^2}}\right)$$
$$-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{bf^4-(b^2c-4\,ac^2)\sqrt{\frac{f^8}{(b^2c^2-4\,ac^3)e^4}}e^2}{(b^2c-4\,ac^2)e^2}}\log\left(ef^6x+df^6\right.$$
$$\left.+\sqrt{\frac{1}{2}}(b^2c-4\,ac^2)\sqrt{\frac{f^8}{(b^2c^2-4\,ac^3)e^4}}e^3\sqrt{-\frac{bf^4-(b^2c-4\,ac^2)\sqrt{\frac{f^8}{(b^2c^2-4\,ac^3)e^4}}e^2}{(b^2c-4\,ac^2)e^2}}\right)$$
$$+\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{bf^4-(b^2c-4\,ac^2)\sqrt{\frac{f^8}{(b^2c^2-4\,ac^3)e^4}}e^2}{(b^2c-4\,ac^2)e^2}}\log\left(ef^6x+df^6\right.$$
$$\left.-\sqrt{\frac{1}{2}}(b^2c-4\,ac^2)\sqrt{\frac{f^8}{(b^2c^2-4\,ac^3)e^4}}e^3\sqrt{-\frac{bf^4-(b^2c-4\,ac^2)\sqrt{\frac{f^8}{(b^2c^2-4\,ac^3)e^4}}e^2}{(b^2c-4\,ac^2)e^2}}\right)$$

input
```
integrate((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

3.640. $\int \frac{(df+efx)^2}{a+b(d+ex)^2+c(d+ex)^4}\,dx$

output
```
1/2*sqrt(1/2)*sqrt(-(b*f^4 + (b^2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a*c^
3)*e^4))*e^2)/((b^2*c - 4*a*c^2)*e^2))*log(e*f^6*x + d*f^6 + sqrt(1/2)*(b^
2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a*c^3)*e^4))*e^3*sqrt(-(b*f^4 + (b^2
*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a*c^3)*e^4))*e^2)/((b^2*c - 4*a*c^2)*
e^2))) - 1/2*sqrt(1/2)*sqrt(-(b*f^4 + (b^2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2
 - 4*a*c^3)*e^4))*e^2)/((b^2*c - 4*a*c^2)*e^2))*log(e*f^6*x + d*f^6 - sqrt
(1/2)*(b^2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a*c^3)*e^4))*e^3*sqrt(-(b*f
^4 + (b^2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a*c^3)*e^4))*e^2)/((b^2*c -
4*a*c^2)*e^2))) - 1/2*sqrt(1/2)*sqrt(-(b*f^4 - (b^2*c - 4*a*c^2)*sqrt(f^8/
((b^2*c^2 - 4*a*c^3)*e^4))*e^2)/((b^2*c - 4*a*c^2)*e^2))*log(e*f^6*x + d*f
^6 + sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a*c^3)*e^4))*e^3*s
qrt(-(b*f^4 - (b^2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a*c^3)*e^4))*e^2)/(
(b^2*c - 4*a*c^2)*e^2))) + 1/2*sqrt(1/2)*sqrt(-(b*f^4 - (b^2*c - 4*a*c^2)*
sqrt(f^8/((b^2*c^2 - 4*a*c^3)*e^4))*e^2)/((b^2*c - 4*a*c^2)*e^2))*log(e*f^
6*x + d*f^6 - sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a*c^3)*e^
4))*e^3*sqrt(-(b*f^4 - (b^2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a*c^3)*e^4
))*e^2)/((b^2*c - 4*a*c^2)*e^2)))
```

3.640.6 Sympy [A] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.73

$$\int \frac{(df+efx)^2}{a+b(d+ex)^2+c(d+ex)^4}\,dx$$
$$= \operatorname{RootSum}\left(t^4\cdot\left(256a^2c^3e^4-128ab^2c^2e^4+16b^4ce^4\right)+t^2\left(-16abce^2f^4+4b^3e^2f^4\right)+af^8,\left(t\mapsto t\log\left(x+\right.\right.\right.$$

input
```
integrate((e*f*x+d*f)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

output
```
RootSum(_t**4*(256*a**2*c**3*e**4 - 128*a*b**2*c**2*e**4 + 16*b**4*c*e**4)
 + _t**2*(-16*a*b*c*e**2*f**4 + 4*b**3*e**2*f**4) + a*f**8, Lambda(_t, _t*
log(x + (64*_t**3*a*c**2*e**3 - 16*_t**3*b**2*c*e**3 - 2*_t*b*e*f**4 + d*f
**6)/(e*f**6))))
```

3.640.7 Maxima [F]

$$\int \frac{(df+efx)^2}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \int \frac{(efx+df)^2}{(ex+d)^4c+(ex+d)^2b+a}\,dx$$

input
```
integrate((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

output
```
integrate((e*f*x + d*f)^2/((e*x + d)^4*c + (e*x + d)^2*b + a), x)
```

3.640.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1443 vs. $2(135) = 270$.

Time = 0.33 (sec) , antiderivative size = 1443, normalized size of antiderivative = 8.49

$$\int \frac{(df+efx)^2}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \text{Too large to display}$$

input
```
integrate((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

output
```
-1/2*(e^2*f^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) +
d/e)^2 - 2*d*e*f^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4
)) + d/e) + d^2*f^2)*log(x + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^
2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^
2)/(c*e^4)) + d/e)^3 - 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*
c)*e^2)/(c*e^4)) + d/e)^2 + 6*c*d^2*e^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2
 - 4*a*c)*e^2)/(c*e^4)) + d/e) - 2*c*d^3*e + b*e^2*(sqrt(1/2)*sqrt(-(b*e^2
 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e) - b*d*e) + 1/2*(e^2*f^2*(sqrt(1/
2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2 + 2*d*e*f^2*(sq
rt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e) + d^2*f^2)*l
og(x - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*
c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^3 +
 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e
)^2 + 6*c*d^2*e^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)
) - d/e) + 2*c*d^3*e + b*e^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e
^2)/(c*e^4)) - d/e) + b*d*e) - 1/2*(e^2*f^2*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt
(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 - 2*d*e*f^2*(sqrt(1/2)*sqrt(-(b*e^2 -
 sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e) + d^2*f^2)*log(x + sqrt(1/2)*sqrt(
-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(
-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^3 - 6*c*d*e^3*(sqrt(1/...
```

3.640.9 Mupad [B] (verification not implemented)

Time = 8.55 (sec) , antiderivative size = 683, normalized size of antiderivative = 4.02

$$\int \frac{(df+efx)^2}{a+b(d+ex)^2+c(d+ex)^4}\,dx = -2\,\mathrm{atanh}\left(\frac{\sqrt{-\frac{b^3\,f^4+f^4\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-4\,a\,b\,c\,f^4}{8\,\left(16\,a^2\,c^3\,e^2-8\,a\,b^2\,c^2\,e^2+b^4\,c\,e^2\right)}}\,\left(x\,\left(4\,a\,c^2\,e^{12}\,f^4-2\,b^2\,c\,e^{12}\,f^4\right)+\frac{\left(x\,\left(8\,b^3\,c^2\,e^{14}-32\,a\,b\,c^3\,e^{14}\right)+8\,b^3\,c^2\,d\right.}{8\,\left(16\,a^2\right.}\right.}{a\,c\,e^{10}\,f^6}\right. - 2\,\mathrm{atanh}\left(\frac{\sqrt{\frac{f^4\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-b^3\,f^4+4\,a\,b\,c\,f^4}{8\,\left(16\,a^2\,c^3\,e^2-8\,a\,b^2\,c^2\,e^2+b^4\,c\,e^2\right)}}\,\left(x\,\left(4\,a\,c^2\,e^{12}\,f^4-2\,b^2\,c\,e^{12}\,f^4\right)-\frac{\left(x\,\left(8\,b^3\,c^2\,e^{14}-32\,a\,b\,c^3\,e^{14}\right)+8\,b^3\,c^2\,d\,e^{13}\right.}{8\,\left(16\,a^2\,c^3\right.}\right.}{a\,c\,e^{10}\,f^6}\right.$$

input
```
int((d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x)
```

output
```
- 2*atanh(((-(b^3*f^4 + f^4*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c*f^4)/(8*(b^
4*c*e^2 + 16*a^2*c^3*e^2 - 8*a*b^2*c^2*e^2)))^(1/2)*(x*(4*a*c^2*e^12*f^4 -
 2*b^2*c*e^12*f^4) + ((x*(8*b^3*c^2*e^14 - 32*a*b*c^3*e^14) + 8*b^3*c^2*d*
e^13 - 32*a*b*c^3*d*e^13)*(b^3*f^4 + f^4*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*
c*f^4))/(8*(b^4*c*e^2 + 16*a^2*c^3*e^2 - 8*a*b^2*c^2*e^2)) + 4*a*c^2*d*e^1
1*f^4 - 2*b^2*c*d*e^11*f^4))/(a*c*e^10*f^6))*(-(b^3*f^4 + f^4*(-(4*a*c - b
^2)^3)^(1/2) - 4*a*b*c*f^4)/(8*(b^4*c*e^2 + 16*a^2*c^3*e^2 - 8*a*b^2*c^2*e
^2)))^(1/2) - 2*atanh((((f^4*(-(4*a*c - b^2)^3)^(1/2) - b^3*f^4 + 4*a*b*c*
f^4)/(8*(b^4*c*e^2 + 16*a^2*c^3*e^2 - 8*a*b^2*c^2*e^2)))^(1/2)*(x*(4*a*c^2
*e^12*f^4 - 2*b^2*c*e^12*f^4) - ((x*(8*b^3*c^2*e^14 - 32*a*b*c^3*e^14) + 8
*b^3*c^2*d*e^13 - 32*a*b*c^3*d*e^13)*(f^4*(-(4*a*c - b^2)^3)^(1/2) - b^3*f
^4 + 4*a*b*c*f^4))/(8*(b^4*c*e^2 + 16*a^2*c^3*e^2 - 8*a*b^2*c^2*e^2)) + 4*
a*c^2*d*e^11*f^4 - 2*b^2*c*d*e^11*f^4))/(a*c*e^10*f^6))*((f^4*(-(4*a*c - b
^2)^3)^(1/2) - b^3*f^4 + 4*a*b*c*f^4)/(8*(b^4*c*e^2 + 16*a^2*c^3*e^2 - 8*a
*b^2*c^2*e^2)))^(1/2)
```

3.641 $\int \frac{df+efx}{a+b(d+ex)^2+c(d+ex)^4}\,dx$

3.641.1 Optimal result

Integrand size = 31, antiderivative size = 44

$$\int \frac{df+efx}{a+b(d+ex)^2+c(d+ex)^4}\,dx = -\frac{f\operatorname{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\,e}$$

output
```
-f*arctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/e/(-4*a*c+b^2)^(1/2)
```

3.641.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.07

$$\int \frac{df+efx}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \frac{f\arctan\left(\frac{b+2c(d+ex)^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}\,e}$$

input
```
Integrate[(d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

output
```
(f*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/(Sqrt[-b^2 + 4*a*c]*e
)
```

3.641.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1462, 1432, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{df+efx}{a+b(d+ex)^2+c(d+ex)^4}\,dx$$

$$\Big\downarrow \text{1462}$$

$$\frac{f\int \frac{d+ex}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)}{e}$$

$$\Big\downarrow \text{1432}$$

$$\frac{f\int \frac{1}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)^2}{2e}$$

$$\Big\downarrow \text{1083}$$

$$-\frac{f\int \frac{1}{-(d+ex)^4+b^2-4ac}d\left(2c(d+ex)^2+b\right)}{e}$$

$$\Big\downarrow \text{219}$$

$$-\frac{f\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\sqrt{b^2-4ac}}$$

input
```
Int[(d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

output
```
-((f*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*
e))
```

3.641.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

3.641.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.95

method	result
default	$\frac{f\left(\sum_{\_R=\mathrm{RootOf}\left(c\,e^4\_Z^4+4cd\,e^3\_Z^3+\left(6c\,d^2e^2+b\,e^2\right)\_Z^2+\left(4d^3ec+2bde\right)\_Z+d^4c+b\,d^2+a\right)}\frac{\left(\_R e+d\right)\ln\left(x-\_R\right)}{2e^3c\_R^3+6cd\,e^2\_R^2+6c\,d^2e\_R+2d^3c+be\ldots}\right)}{2e}$
risch	$-\frac{f\ln\left(\left(e^2\sqrt{-4ac+b^2}-b\,e^2\right)x^2+\left(2ed\sqrt{-4ac+b^2}-2bde\right)x+d^2\sqrt{-4ac+b^2}-b\,d^2-2a\right)}{2\sqrt{-4ac+b^2}\,e}+\frac{f\ln\left(\left(e^2\sqrt{-4ac+b^2}+b\,e^2\right)x^2+\left(2ed\sqrt{-4a\ldots}\right.\right.}{2\sqrt{-4a\ldots}}$

input
```
int((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x,method=_RETURNVERBOSE)
```

output
```
1/2*f/e*sum((_R*e+d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*
b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*
_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a))
```

3.641.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 274, normalized size of antiderivative = 6.23

$$\int \frac{df + efx}{a + b(d + ex)^2 + c(d + ex)^4} dx = \left[\frac{f \log \left(\frac{2 c^2 e^4 x^4 + 8 c^2 d e^3 x^3 + 2 c^2 d^4 + 2 (6 c^2 d^2 + bc) e^2 x^2 + 2 b c d^2 + 4 (2 c^2 d^3 + bcd) e x + b^2 - 2 ac - (2 c e^2 x^2 + 4 c d e x + 2 c d^2 + b) \sqrt{b^2 - 4 ac}}{c e^4 x^4 + 4 c d e^3 x^3 + c d^4 + (6 c d^2 + b) e^2 x^2 + b d^2 + 2 (2 c d^3 + bd) e x + a} \right)}{2 \sqrt{b^2 - 4 ac} e}, - \frac{\sqrt{-b^2 + 4 ac} f \arctan \left(- \frac{(2 c e^2 x^2 + 4 c d e x + 2 c d^2 + b) \sqrt{-b^2 + 4 ac}}{b^2 - 4 ac} \right)}{(b^2 - 4 ac) e} \right]$$

input
```
integrate((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

output
```
[1/2*f*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b
*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c - (2*c*e
^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^
3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a)
)/(sqrt(b^2 - 4*a*c)*e), -sqrt(-b^2 + 4*a*c)*f*arctan(-(2*c*e^2*x^2 + 4*c*
d*e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c))/((b^2 - 4*a*c)*e)]
```

3.641.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(41) = 82.

Time = 0.59 (sec) , antiderivative size = 189, normalized size of antiderivative = 4.30

$$\int \frac{df + efx}{a + b(d + ex)^2 + c(d + ex)^4} dx = - \frac{f \sqrt{- \frac{1}{4ac - b^2}} \log \left(\frac{2dx}{e} + x^2 + \frac{-4acf \sqrt{- \frac{1}{4ac - b^2}} + b^2 f \sqrt{- \frac{1}{4ac - b^2}} + bf + 2cd^2 f}{2ce^2 f} \right)}{2e} + \frac{f \sqrt{- \frac{1}{4ac - b^2}} \log \left(\frac{2dx}{e} + x^2 + \frac{4acf \sqrt{- \frac{1}{4ac - b^2}} - b^2 f \sqrt{- \frac{1}{4ac - b^2}} + bf + 2cd^2 f}{2ce^2 f} \right)}{2e}$$

input
```
integrate((e*f*x+d*f)/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

output
```
-f*sqrt(-1/(4*a*c - b**2))*log(2*d*x/e + x**2 + (-4*a*c*f*sqrt(-1/(4*a*c -
 b**2)) + b**2*f*sqrt(-1/(4*a*c - b**2)) + b*f + 2*c*d**2*f)/(2*c*e**2*f))
/(2*e) + f*sqrt(-1/(4*a*c - b**2))*log(2*d*x/e + x**2 + (4*a*c*f*sqrt(-1/(
4*a*c - b**2)) - b**2*f*sqrt(-1/(4*a*c - b**2)) + b*f + 2*c*d**2*f)/(2*c*e
**2*f))/(2*e)
```

3.641.7 Maxima [F]

$$\int \frac{df + efx}{a + b(d + ex)^2 + c(d + ex)^4} \, dx = \int \frac{efx + df}{(ex + d)^4 c + (ex + d)^2 b + a} \, dx$$

input
```
integrate((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

output
```
integrate((e*f*x + d*f)/((e*x + d)^4*c + (e*x + d)^2*b + a), x)
```

3.641.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.39

$$\int \frac{df + efx}{a + b(d + ex)^2 + c(d + ex)^4} \, dx = \frac{f \arctan\left(\frac{2\,cd^2f + 2\,(efx^2 + 2\,dfx)ce + bf}{\sqrt{-b^2 + 4\,ac}f}\right)}{\sqrt{-b^2 + 4\,ac}e}$$

input
```
integrate((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

output
```
f*arctan((2*c*d^2*f + 2*(e*f*x^2 + 2*d*f*x)*c*e + b*f)/(sqrt(-b^2 + 4*a*c)
*f))/(sqrt(-b^2 + 4*a*c)*e)
```

3.641.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 477, normalized size of antiderivative = 10.84

$$\int \frac{df+efx}{a+b(d+ex)^2+c(d+ex)^4}\,dx$$

$$= \frac{f\,\mathrm{atan}\left(\frac{\frac{f\left(4\,c^2\,d^2\,e^7\,f+4\,c^2\,e^9\,f\,x^2-\frac{f\left(8\,b\,c^2\,d^2\,e^8+16\,b\,c^2\,d\,e^9\,x+8\,b\,c^2\,e^{10}\,x^2+16\,a\,c^2\,e^8\right)}{2\,e\,\sqrt{b^2-4\,a\,c}}+8\,c^2\,d\,e^8\,f\,x\right)\,1\mathrm{i}}{2\,e\,\sqrt{b^2-4\,a\,c}}+\frac{f\left(4\,c^2\,d^2\,e^7\,f+4\,c^2\,e^9\,f\,x^2+\frac{f\left(8\,b\,c^2\,d^2\,e\right.}{}\right.}{}}{\frac{f\left(4\,c^2\,d^2\,e^7\,f+4\,c^2\,e^9\,f\,x^2-\frac{f\left(8\,b\,c^2\,d^2\,e^8+16\,b\,c^2\,d\,e^9\,x+8\,b\,c^2\,e^{10}\,x^2+16\,a\,c^2\,e^8\right)}{2\,e\,\sqrt{b^2-4\,a\,c}}+8\,c^2\,d\,e^8\,f\,x\right)}{2\,e\,\sqrt{b^2-4\,a\,c}}-\frac{f\left(4\,c^2\,d^2\,e^7\,f+4\,c^2\,e^9\,f\,x^2+\frac{f\left(8\,b\,c^2\,d^2\,e\right.}{}\right.}{}}\right.}{e\,\sqrt{b^2-4\,a\,c}}$$

input
```
int((d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x)
```

output
```
(f*atan(((f*(4*c^2*d^2*e^7*f + 4*c^2*e^9*f*x^2 - (f*(16*a*c^2*e^8 + 8*b*c^
2*d^2*e^8 + 8*b*c^2*e^10*x^2 + 16*b*c^2*d*e^9*x))/(2*e*(b^2 - 4*a*c)^(1/2)
) + 8*c^2*d*e^8*f*x)*1i)/(2*e*(b^2 - 4*a*c)^(1/2)) + (f*(4*c^2*d^2*e^7*f +
 4*c^2*e^9*f*x^2 + (f*(16*a*c^2*e^8 + 8*b*c^2*d^2*e^8 + 8*b*c^2*e^10*x^2 +
 16*b*c^2*d*e^9*x))/(2*e*(b^2 - 4*a*c)^(1/2)) + 8*c^2*d*e^8*f*x)*1i)/(2*e*
(b^2 - 4*a*c)^(1/2)))/((f*(4*c^2*d^2*e^7*f + 4*c^2*e^9*f*x^2 - (f*(16*a*c^
2*e^8 + 8*b*c^2*d^2*e^8 + 8*b*c^2*e^10*x^2 + 16*b*c^2*d*e^9*x))/(2*e*(b^2
- 4*a*c)^(1/2)) + 8*c^2*d*e^8*f*x))/(2*e*(b^2 - 4*a*c)^(1/2)) - (f*(4*c^2*
d^2*e^7*f + 4*c^2*e^9*f*x^2 + (f*(16*a*c^2*e^8 + 8*b*c^2*d^2*e^8 + 8*b*c^2
*e^10*x^2 + 16*b*c^2*d*e^9*x))/(2*e*(b^2 - 4*a*c)^(1/2)) + 8*c^2*d*e^8*f*x
))/(2*e*(b^2 - 4*a*c)^(1/2))))*1i)/(e*(b^2 - 4*a*c)^(1/2))
```

3.642 $\int \frac{1}{(df+efx)(a+b(d+ex)^2+c(d+ex)^4)} dx$

3.642.1 Optimal result

Integrand size = 33, antiderivative size = 103

$$\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)} dx = \frac{b\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}ef} + \frac{\log(d+ex)}{aef} - \frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4aef}$$

output
```
ln(e*x+d)/a/e/f-1/4*ln(a+b*(e*x+d)^2+c*(e*x+d)^4)/a/e/f+1/2*b*arctanh((b+2
*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/a/e/f/(-4*a*c+b^2)^(1/2)
```

3.642.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.27

$$\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)} dx = \frac{4\sqrt{b^2-4ac}\log(d+ex)-\left(b+\sqrt{b^2-4ac}\right)\log\left(b-\sqrt{b^2-4ac}+2c(d+ex)^2\right)+\left(b-\sqrt{b^2-4ac}\right)\log\left(b+\right.}{4a\sqrt{b^2-4ac}ef}$$

input
```
Integrate[1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

output
```
(4*Sqrt[b^2 - 4*a*c]*Log[d + e*x] - (b + Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b
^2 - 4*a*c] + 2*c*(d + e*x)^2] + (b - Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2
- 4*a*c] + 2*c*(d + e*x)^2])/(4*a*Sqrt[b^2 - 4*a*c]*e*f)
```

3.642.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.92, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {1462, 1434, 1144, 25, 1142, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)}dx$$

$$\Big\downarrow \text{ 1462}$$

$$\frac{\int \frac{1}{(d+ex)(c(d+ex)^4+b(d+ex)^2+a)}d(d+ex)}{ef}$$

$$\Big\downarrow \text{ 1434}$$

$$\frac{\int \frac{1}{(d+ex)^2(c(d+ex)^4+b(d+ex)^2+a)}d(d+ex)^2}{2ef}$$

$$\Big\downarrow \text{ 1144}$$

$$\frac{\frac{\int -\frac{c(d+ex)^2+b}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)^2}{a}+\frac{\log\left((d+ex)^2\right)}{a}}{2ef}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{\log\left((d+ex)^2\right)}{a}-\frac{\int \frac{c(d+ex)^2+b}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)^2}{a}}{2ef}$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{\frac{\log\left((d+ex)^2\right)}{a}-\frac{\frac{1}{2}b\int \frac{1}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)^2+\frac{1}{2}\int \frac{2c(d+ex)^2+b}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)^2}{a}}{2ef}$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{\frac{\log\left((d+ex)^2\right)}{a}-\frac{\frac{1}{2}\int \frac{2c(d+ex)^2+b}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)^2-b\int \frac{1}{-(d+ex)^4+b^2-4ac}d\left(2c(d+ex)^2+b\right)}{a}}{2ef}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{\log\left((d+ex)^2\right)}{a}-\frac{\frac{1}{2}\int \frac{2c(d+ex)^2+b}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)^2-\frac{b\,\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}}{a}}{2ef}$$

$$\Big\downarrow \text{1103}$$

$$\frac{\frac{\log\left((d+ex)^2\right)}{a}-\frac{\frac{1}{2}\log\left(a+b(d+ex)^2+c(d+ex)^4\right)-\frac{b\,\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}}{a}}{2ef}$$

input
```
Int[1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

output
```
(Log[(d + e*x)^2]/a - (-((b*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c
]])/Sqrt[b^2 - 4*a*c]) + Log[a + b*(d + e*x)^2 + c*(d + e*x)^4]/2)/a)/(2*e
*f)
```

3.642.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1144
```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Simp[e*(Log[RemoveContent[d + e*x, x]]/(c*d^2 - b*d*e + a*e^2)), x] + S
imp[1/(c*d^2 - b*d*e + a*e^2)   Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2),
x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

3.642.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.66 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.68

method	result
risch	$\frac{\ln(ex+d)}{aef} + \frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\left(4a^2f^2e^2c-a\,b^2f^2e^2\right)\_Z^2+\left(4acef-b^2ef\right)\_Z+c\right)} \_R\ln\left(\left(\left(10ac\,e^3f-3b^2e^3f\right)\_R+5c\,e^2\right)x^2+\left(\left(20acd\,e \cdots\right.\right.\right.\right.}{2}$
default	$\frac{\frac{\sum_{\_R=\operatorname{RootOf}\left(c\,e^4\_Z^4+4cd\,e^3\_Z^3+\left(6c\,d^2e^2+b\,e^2\right)\_Z^2+\left(4d^3ec+2bde\right)\_Z+d^4c+b\,d^2+a\right)} \frac{\left(-e^3c\_R^3-3cd\,e^2\_R^2+e\left(-3c\,d^2-b\right)\_R-d^3 \cdots\right.}{2e^3c\_R^3+6cd\,e^2\_R^2+6c\,d^2e\_R+2d^3c \cdots}}{2ae}}{f}$

input
```
int(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x,method=_RETURNVERBOSE)
```

output
```
ln(e*x+d)/a/e/f+1/2*sum(_R*ln(((10*a*c*e^3*f-3*b^2*e^3*f)*_R+5*c*e^2)*x^2+
((20*a*c*d*e^2*f-6*b^2*d*e^2*f)*_R+10*d*c*e)*x+(10*a*c*d^2*e*f-3*b^2*d^2*e
*f-a*b*e*f)*_R+5*c*d^2+2*b),_R=RootOf((4*a^2*c*e^2*f^2-a*b^2*e^2*f^2)*_Z^2
+(4*a*c*e*f-b^2*e*f)*_Z+c))
```

3.642.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 474, normalized size of antiderivative = 4.60

$$\int \frac{1}{(df+efx)\,(a+b(d+ex)^2+c(d+ex)^4)}\,dx$$
$$= \left[\frac{\sqrt{b^2-4\,ac}b\log\left(\frac{2\,c^2e^4x^4+8\,c^2de^3x^3+2\,c^2d^4+2\,(6\,c^2d^2+bc)e^2x^2+2\,bcd^2+4\,(2\,c^2d^3+bcd)ex+b^2-2\,ac+(2\,ce^2x^2+4\,cdex+2\,cd^2+b)\sqrt{b^2-}}{ce^4x^4+4\,cde^3x^3+cd^4+(6\,cd^2+b)e^2x^2+bd^2+2\,(2\,cd^3+bd)ex+a}\right.}{} \right.$$

input
```
integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4
 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b
^2 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c
*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*
d^3 + b*d)*e*x + a)) - (b^2 - 4*a*c)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4
 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*(b^2 - 4
*a*c)*log(e*x + d))/((a*b^2 - 4*a^2*c)*e*f), 1/4*(2*sqrt(-b^2 + 4*a*c)*b*a
rctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4
*a*c)) - (b^2 - 4*a*c)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 +
b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*(b^2 - 4*a*c)*log(e*x
+ d))/((a*b^2 - 4*a^2*c)*e*f)]
```

3.642.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. 2(83) = 166.

Time = 17.68 (sec) , antiderivative size = 348, normalized size of antiderivative = 3.38

$$\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx = \left(-\frac{b\sqrt{-4ac+b^2}}{4aef\left(4ac-b^2\right)} - \frac{1}{4aef}\right)\log\left(\frac{2dx}{e}+x^2+\frac{-8a^2cef\left(-\frac{b\sqrt{-4ac+b^2}}{4aef(4ac-b^2)}-\frac{1}{4aef}\right)+2ab^2ef\left(-\frac{b\sqrt{-4ac+b^2}}{4aef(4ac-b^2)}-\frac{1}{4aef}\right)-2ac+b^2+}{bce^2}\right) + \left(\frac{b\sqrt{-4ac+b^2}}{4aef\left(4ac-b^2\right)} - \frac{1}{4aef}\right)\log\left(\frac{2dx}{e}+x^2+\frac{-8a^2cef\left(\frac{b\sqrt{-4ac+b^2}}{4aef(4ac-b^2)}-\frac{1}{4aef}\right)+2ab^2ef\left(\frac{b\sqrt{-4ac+b^2}}{4aef(4ac-b^2)}-\frac{1}{4aef}\right)-2ac+b^2+bcd}{bce^2}\right) + \frac{\log\left(\frac{d}{e}+x\right)}{aef}$$

input
```
integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

output
```
(-b*sqrt(-4*a*c + b**2)/(4*a*e*f*(4*a*c - b**2)) - 1/(4*a*e*f))*log(2*d*x/
e + x**2 + (-8*a**2*c*e*f*(-b*sqrt(-4*a*c + b**2)/(4*a*e*f*(4*a*c - b**2))
 - 1/(4*a*e*f)) + 2*a*b**2*e*f*(-b*sqrt(-4*a*c + b**2)/(4*a*e*f*(4*a*c - b
**2)) - 1/(4*a*e*f)) - 2*a*c + b**2 + b*c*d**2)/(b*c*e**2)) + (b*sqrt(-4*a
*c + b**2)/(4*a*e*f*(4*a*c - b**2)) - 1/(4*a*e*f))*log(2*d*x/e + x**2 + (-
8*a**2*c*e*f*(b*sqrt(-4*a*c + b**2)/(4*a*e*f*(4*a*c - b**2)) - 1/(4*a*e*f)
) + 2*a*b**2*e*f*(b*sqrt(-4*a*c + b**2)/(4*a*e*f*(4*a*c - b**2)) - 1/(4*a*
e*f)) - 2*a*c + b**2 + b*c*d**2)/(b*c*e**2)) + log(d/e + x)/(a*e*f)
```

3.642.7 Maxima [F]

$$\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx = \int \frac{1}{\left((ex+d)^4c+(ex+d)^2b+a\right)(efx+df)}\,dx$$

input
```
integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

output
```
-integrate((c*e^3*x^3 + 3*c*d*e^2*x^2 + c*d^3 + (3*c*d^2 + b)*e*x + b*d)/(
c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c
*d^3 + b*d)*e*x + a), x)/(a*f) + log(e*x + d)/(a*e*f)
```

3.642.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 291 vs. $2(95) = 190$.

Time = 0.40 (sec) , antiderivative size = 291, normalized size of antiderivative = 2.83

$$\int \frac{1}{(df + efx)\left(a + b(d + ex)^2 + c(d + ex)^4\right)}\,dx$$
$$= -\frac{\log\left(\left|ce^4x^4 + 4\,cde^3x^3 + 6\,cd^2e^2x^2 + 4\,cd^3ex + cd^4 + be^2x^2 + 2\,bdex + bd^2 + a\right|\right)}{4\,aef}$$
$$+ \frac{\log\left(\left|ex + d\right|\right)}{aef}$$
$$- \frac{\frac{abce^3f\log\left(\left|be^2x^2+\sqrt{b^2-4\,ac}e^2x^2+2\,bdex+2\,\sqrt{b^2-4\,ac}dex+bd^2+\sqrt{b^2-4\,ac}d^2+2\,a\right|\right)}{\sqrt{b^2-4\,ac}} - \frac{abce^3f\log\left(\left|-be^2x^2+\sqrt{b^2-4\,ac}e^2x^2-2\,bdex+2\,\sqrt{b}\right.\right.}{\sqrt{b^2-4\,ac}}}{4\,a^2ce^4f^2}$$

input
```
integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

output
```
-1/4*log(abs(c*e^4*x^4 + 4*c*d*e^3*x^3 + 6*c*d^2*e^2*x^2 + 4*c*d^3*e*x + c
*d^4 + b*e^2*x^2 + 2*b*d*e*x + b*d^2 + a))/(a*e*f) + log(abs(e*x + d))/(a*
e*f) - 1/4*(a*b*c*e^3*f*log(abs(b*e^2*x^2 + sqrt(b^2 - 4*a*c)*e^2*x^2 + 2*
b*d*e*x + 2*sqrt(b^2 - 4*a*c)*d*e*x + b*d^2 + sqrt(b^2 - 4*a*c)*d^2 + 2*a)
)/sqrt(b^2 - 4*a*c) - a*b*c*e^3*f*log(abs(-b*e^2*x^2 + sqrt(b^2 - 4*a*c)*e
^2*x^2 - 2*b*d*e*x + 2*sqrt(b^2 - 4*a*c)*d*e*x - b*d^2 + sqrt(b^2 - 4*a*c)
*d^2 - 2*a))/sqrt(b^2 - 4*a*c))/(a^2*c*e^4*f^2)
```

3.642.9 Mupad [B] (verification not implemented)

Time = 9.53 (sec) , antiderivative size = 2520, normalized size of antiderivative = 24.47

$$\int \frac{1}{(df + efx)\left(a + b(d + ex)^2 + c(d + ex)^4\right)}\,dx = \text{Too large to display}$$

input
```
int(1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x)
```

output
```
log(d + e*x)/(a*e*f) - (log(a + b*d^2 + c*d^4 + b*e^2*x^2 + c*e^4*x^4 + 2*
b*d*e*x + 6*c*d^2*e^2*x^2 + 4*c*d^3*e*x + 4*c*d*e^3*x^3)*(2*b^2*e*f - 8*a*
c*e*f))/(2*(4*a*b^2*e^2*f^2 - 16*a^2*c*e^2*f^2)) - (b*atan((16*a^3*f^3*x*(
4*a*c - b^2)^(3/2)*(((3*b^3 - 8*a*b*c)*((b^2*((2*(2*b^2*e*f - 8*a*c*e*f)*(
6*b^3*c^2*d*e^18*f - 20*a*b*c^3*d*e^18*f))/(f*(4*a*b^2*e^2*f^2 - 16*a^2*c*
e^2*f^2)) + (20*b*c^3*d*e^17)/f))/(16*a^2*e^2*f^2*(4*a*c - b^2)) - ((2*b^2
*e*f - 8*a*c*e*f)^2*((2*(2*b^2*e*f - 8*a*c*e*f)*(6*b^3*c^2*d*e^18*f - 20*a
*b*c^3*d*e^18*f))/(f*(4*a*b^2*e^2*f^2 - 16*a^2*c*e^2*f^2)) + (20*b*c^3*d*e
^17)/f))/(4*(4*a*b^2*e^2*f^2 - 16*a^2*c*e^2*f^2)^2) + (b^2*(2*b^2*e*f - 8*
a*c*e*f)*(6*b^3*c^2*d*e^18*f - 20*a*b*c^3*d*e^18*f))/(4*a^2*e^2*f^3*(4*a*b
^2*e^2*f^2 - 16*a^2*c*e^2*f^2)*(4*a*c - b^2))))/(8*a^3*c^2*(25*a*c - 6*b^2
)) - (((b*(2*b^2*e*f - 8*a*c*e*f)*((2*(2*b^2*e*f - 8*a*c*e*f)*(6*b^3*c^2*d
*e^18*f - 20*a*b*c^3*d*e^18*f))/(f*(4*a*b^2*e^2*f^2 - 16*a^2*c*e^2*f^2)) +
 (20*b*c^3*d*e^17)/f))/(4*a*e*f*(4*a*b^2*e^2*f^2 - 16*a^2*c*e^2*f^2)*(4*a*
c - b^2)^(1/2)) - (b^3*(6*b^3*c^2*d*e^18*f - 20*a*b*c^3*d*e^18*f))/(16*a^3
*e^3*f^4*(4*a*c - b^2)^(3/2)) + (b*(2*b^2*e*f - 8*a*c*e*f)^2*(6*b^3*c^2*d*
e^18*f - 20*a*b*c^3*d*e^18*f))/(4*a*e*f^2*(4*a*b^2*e^2*f^2 - 16*a^2*c*e^2*
f^2)^2*(4*a*c - b^2)^(1/2)))*(3*b^4 + 10*a^2*c^2 - 14*a*b^2*c))/(8*a^3*c^2
*(4*a*c - b^2)^(1/2)*(25*a*c - 6*b^2))))/(b^2*c^2*e^14) + (2*f^3*(3*b^3 -
8*a*b*c)*(4*a*c - b^2)^(3/2)*((b^2*((2*(2*b^2*c^2*e^16 + 5*b*c^3*d^2*e^...
```

3.642. $\int \frac{1}{(df+efx)(a+b(d+ex)^2+c(d+ex)^4)}\,dx$

3.643 $\int \frac{1}{(df+efx)^2(a+b(d+ex)^2+c(d+ex)^4)}\,dx$

3.643.1 Optimal result

Integrand size = 33, antiderivative size = 204

$$\int \frac{1}{(df+efx)^2\,(a+b(d+ex)^2+c(d+ex)^4)}\,dx = -\frac{1}{aef^2(d+ex)} - \frac{\sqrt{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}ef^2} - \frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b+\sqrt{b^2-4ac}}ef^2}$$

output
```
-1/a/e/f^2/(e*x+d)-1/2*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2
))^(1/2))*c^(1/2)*(1+b/(-4*a*c+b^2)^(1/2))/a/e/f^2*2^(1/2)/(b-(-4*a*c+b^2)
^(1/2))^(1/2)-1/2*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1
/2))*c^(1/2)*(1-b/(-4*a*c+b^2)^(1/2))/a/e/f^2*2^(1/2)/(b+(-4*a*c+b^2)^(1/2
))^(1/2)
```

3.643.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.02

$$\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx$$
$$=-\frac{\frac{2}{d+ex}+\frac{\sqrt{2}\sqrt{c}\left(b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(-b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}}{2aef^2}$$

input
```
Integrate[1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

output
```
-1/2*(2/(d + e*x) + (Sqrt[2]*Sqrt[c]*(b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqr
t[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-b + Sqrt[b^2 - 4*a*c])*ArcT
an[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4
*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(a*e*f^2)
```

3.643.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1462, 1443, 25, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx$$
$$\Big\downarrow \text{1462}$$
$$\frac{\int \frac{1}{(d+ex)^2\left(c(d+ex)^4+b(d+ex)^2+a\right)}d(d+ex)}{ef^2}$$
$$\Big\downarrow \text{1443}$$
$$\frac{\frac{\int -\frac{c(d+ex)^2+b}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)}{a}-\frac{1}{a(d+ex)}}{ef^2}$$
$$\Big\downarrow \text{25}$$

$$\frac{-\frac{\int \frac{c(d+ex)^2+b}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)}{a}-\frac{1}{a(d+ex)}}{ef^2}$$

$\downarrow$ 1480

$$\frac{-\frac{\frac{1}{2}c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\int \frac{1}{c(d+ex)^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}d(d+ex)+\frac{1}{2}c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{c(d+ex)^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}d(d+ex)}{a}-\frac{1}{a(d+ex)}}{ef^2}$$

$\downarrow$ 218

$$\frac{-\frac{\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}}{a}-\frac{1}{a(d+ex)}}{ef^2}$$

input
```
Int[1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

output
```
(-(1/(a*(d + e*x))) - ((Sqrt[c]*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*
Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^
2 - 4*a*c]]) + (Sqrt[c]*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*
(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 - 4*a*
c]]))/a)/(e*f^2)
```

3.643.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1443
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Sim
p[1/(a*d^2*(m + 1))   Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*
x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

3.643.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.66 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.84

method	result
default	$\frac{\frac{\sum_{\_R=\mathrm{RootOf}\left(c e^4 \_Z^4+4cd e^3 \_Z^3+\left(6c d^2 e^2+b e^2\right)\_Z^2+\left(4d^3 ec+2bde\right)\_Z+d^4 c+b d^2+a\right)} \frac{\left(-\_R^2 c e^2-2\_Rcde-c d^2-b\right)\ln\left(x-\_R\right)}{2e^3 c\_R^3+6cd e^2\_R^2+6c d^2 e\_R+2d^3 c+be\_}}{2ae}}{f^2}$
risch	$-\frac{1}{ae f^2(ex+d)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(16f^8 e^4 c^2 a^5-8b^2 f^8 e^4 c a^4+b^4 f^8 e^4 a^3\right)\_Z^4+\left(12a^2 b c^2 e^2 f^4-7a b^3 c e^2 f^4+b^5 e^2 f^4\right)\_Z^2+c^3\right)} \_R\ln(((\right.}{}$

input
```
int(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x,method=_RETURNVERBOSE)
```

output
```
1/f^2*(1/2/a/e*sum((-_R^2*c*e^2-2*_R*c*d*e-c*d^2-b)/(2*_R^3*c*e^3+6*_R^2*c
*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*
d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a))
-1/a/e/(e*x+d))
```

3.643.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1477 vs. $2(167) = 334$.

Time = 0.31 (sec) , antiderivative size = 1477, normalized size of antiderivative = 7.24

$$\int \frac{1}{(df+efx)^2\,(a+b(d+ex)^2+c(d+ex)^4)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas
")
```

output
```
1/2*(sqrt(1/2)*(a*e^2*f^2*x + a*d*e*f^2)*sqrt(-((a^3*b^2 - 4*a^4*c)*e^2*f^
4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4*f^8)) + b^3 -
3*a*b*c)/((a^3*b^2 - 4*a^4*c)*e^2*f^4))*log(-2*(b^2*c^2 - a*c^3)*e*x - 2*(
b^2*c^2 - a*c^3)*d + sqrt(1/2)*((a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*e^3*f^
6*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4*f^8)) - (b^5 -
 5*a*b^3*c + 4*a^2*b*c^2)*e*f^2)*sqrt(-((a^3*b^2 - 4*a^4*c)*e^2*f^4*sqrt((
b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4*f^8)) + b^3 - 3*a*b*c)
/((a^3*b^2 - 4*a^4*c)*e^2*f^4))) - sqrt(1/2)*(a*e^2*f^2*x + a*d*e*f^2)*sqr
t(-((a^3*b^2 - 4*a^4*c)*e^2*f^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2
 - 4*a^7*c)*e^4*f^8)) + b^3 - 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*e^2*f^4))*log(
-2*(b^2*c^2 - a*c^3)*e*x - 2*(b^2*c^2 - a*c^3)*d - sqrt(1/2)*((a^3*b^4 - 6
*a^4*b^2*c + 8*a^5*c^2)*e^3*f^6*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2
 - 4*a^7*c)*e^4*f^8)) - (b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*e*f^2)*sqrt(-((a^3
*b^2 - 4*a^4*c)*e^2*f^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7
*c)*e^4*f^8)) + b^3 - 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*e^2*f^4))) - sqrt(1/2)
*(a*e^2*f^2*x + a*d*e*f^2)*sqrt(((a^3*b^2 - 4*a^4*c)*e^2*f^4*sqrt((b^4 - 2
*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4*f^8)) - b^3 + 3*a*b*c)/((a^3*
b^2 - 4*a^4*c)*e^2*f^4))*log(-2*(b^2*c^2 - a*c^3)*e*x - 2*(b^2*c^2 - a*c^3
)*d + sqrt(1/2)*((a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*e^3*f^6*sqrt((b^4 - 2
*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4*f^8)) + (b^5 - 5*a*b^3*c +...
```

3.643.6 Sympy [A] (verification not implemented)

Time = 3.20 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.26

$$\int \frac{1}{(df+efx)^2\,(a+b(d+ex)^2+c(d+ex)^4)}\,dx$$
$$= \operatorname{RootSum}\Big(t^4\cdot\left(256a^5c^2e^4f^8 - 128a^4b^2ce^4f^8 + 16a^3b^4e^4f^8\right) + t^2\cdot\left(48a^2bc^2e^2f^4 - 28ab^3ce^2f^4 + 4b^5e^2f^4\right)$$
$$- \frac{1}{adef^2 + ae^2f^2x}$$

input
```
integrate(1/(e*f*x+d*f)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

output
```
RootSum(_t**4*(256*a**5*c**2*e**4*f**8 - 128*a**4*b**2*c*e**4*f**8 + 16*a*
*3*b**4*e**4*f**8) + _t**2*(48*a**2*b*c**2*e**2*f**4 - 28*a*b**3*c*e**2*f*
*4 + 4*b**5*e**2*f**4) + c**3, Lambda(_t, _t*log(x + (-64*_t**3*a**5*c**2*
e**3*f**6 + 48*_t**3*a**4*b**2*c*e**3*f**6 - 8*_t**3*a**3*b**4*e**3*f**6 -
 10*_t*a**2*b*c**2*e*f**2 + 10*_t*a*b**3*c*e*f**2 - 2*_t*b**5*e*f**2 + a*c
**3*d - b**2*c**2*d)/(a*c**3*e - b**2*c**2*e)))) - 1/(a*d*e*f**2 + a*e**2*
f**2*x)
```

3.643.7 Maxima [F]

$$\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx = \int \frac{1}{\left((ex+d)^4c+(ex+d)^2b+a\right)(efx+df)^2}\,dx$$

input
```
integrate(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima
")
```

output
```
-1/(a*e^2*f^2*x + a*d*e*f^2) - integrate((c*e^2*x^2 + 2*c*d*e*x + c*d^2 +
b)/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*
(2*c*d^3 + b*d)*e*x + a), x)/(a*f^2)
```

3.643.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error index.cc index_gcd Error: Bad
 Argument ValueError index.cc index_gcd Error: Bad Argument ValueDone
```

3.643.9 Mupad [B] (verification not implemented)

Time = 9.83 (sec) , antiderivative size = 4339, normalized size of antiderivative = 21.27

$$\int \frac{1}{(df+efx)^2\,(a+b(d+ex)^2+c(d+ex)^4)}\,dx = \text{Too large to display}$$

input
```
int(1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x)
```

output
```
- atan(((-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c -
 a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 -
8*a^4*b^2*c*e^2*f^4)))^(1/2)*(x*(4*a^4*c^4*e^12*f^6 - 2*a^3*b^2*c^3*e^12*f
^6) - ((x*(8*a^5*b^3*c^2*e^14*f^10 - 32*a^6*b*c^3*e^14*f^10) - 32*a^6*b*c^
3*d*e^13*f^10 + 8*a^5*b^3*c^2*d*e^13*f^10)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)
^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*
b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2) - 4*a^4*b^
3*c^2*e^12*f^8 + 16*a^5*b*c^3*e^12*f^8)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1
/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4
*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2) + 4*a^4*c^4*d
*e^11*f^6 - 2*a^3*b^2*c^3*d*e^11*f^6)*1i + (-(b^5 + b^2*(-(4*a*c - b^2)^3)
^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*
b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2)*(x*(4*a^4*
c^4*e^12*f^6 - 2*a^3*b^2*c^3*e^12*f^6) - ((x*(8*a^5*b^3*c^2*e^14*f^10 - 32
*a^6*b*c^3*e^14*f^10) - 32*a^6*b*c^3*d*e^13*f^10 + 8*a^5*b^3*c^2*d*e^13*f^
10)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c
*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^
4*b^2*c*e^2*f^4)))^(1/2) + 4*a^4*b^3*c^2*e^12*f^8 - 16*a^5*b*c^3*e^12*f^8)
*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-
(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^...
```

3.644 $\int \frac{1}{(df+efx)^3(a+b(d+ex)^2+c(d+ex)^4)}\,dx$

3.644.1 Optimal result

Integrand size = 33, antiderivative size = 133

$$\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx = -\frac{1}{2aef^3(d+ex)^2}-\frac{(b^2-2ac)\operatorname{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}ef^3}-\frac{b\log(d+ex)}{a^2ef^3}+\frac{b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^2ef^3}$$

output
```
-1/2/a/e/f^3/(e*x+d)^2-b*ln(e*x+d)/a^2/e/f^3+1/4*b*ln(a+b*(e*x+d)^2+c*(e*x
+d)^4)/a^2/e/f^3-1/2*(-2*a*c+b^2)*arctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(
1/2))/a^2/e/f^3/(-4*a*c+b^2)^(1/2)
```

3.644.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.18

$$\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx = \frac{-\frac{2a}{(d+ex)^2}-4b\log(d+ex)+\frac{\left(b^2-2ac+b\sqrt{b^2-4ac}\right)\log\left(b-\sqrt{b^2-4ac}+2c(d+ex)^2\right)}{\sqrt{b^2-4ac}}+\frac{\left(-b^2+2ac+b\sqrt{b^2-4ac}\right)\log\left(b+\sqrt{b^2-4ac}+2c(d \ldots\right)}{\sqrt{b^2-4ac}}}{4a^2ef^3}$$

input
```
Integrate[1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

output
```
((-2*a)/(d + e*x)^2 - 4*b*Log[d + e*x] + ((b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*
c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/Sqrt[b^2 - 4*a*c] + ((-b
^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x
)^2])/Sqrt[b^2 - 4*a*c])/(4*a^2*e*f^3)
```

3.644.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1462, 1434, 1145, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)} \, dx$$

$$\Big\downarrow \text{1462}$$

$$\frac{\int \frac{1}{(d+ex)^3 \left(c(d+ex)^4+b(d+ex)^2+a\right)}d(d+ex)}{ef^3}$$

$$\Big\downarrow \text{1434}$$

$$\frac{\int \frac{1}{(d+ex)^4 \left(c(d+ex)^4+b(d+ex)^2+a\right)}d(d+ex)^2}{2ef^3}$$

$$\Big\downarrow \text{1145}$$

$$\frac{\frac{\int -\frac{c(d+ex)^2+b}{(d+ex)^2\left(c(d+ex)^4+b(d+ex)^2+a\right)}d(d+ex)^2}{a}-\frac{1}{a(d+ex)^2}}{2ef^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{-\frac{\int \frac{c(d+ex)^2+b}{(d+ex)^2\left(c(d+ex)^4+b(d+ex)^2+a\right)}d(d+ex)^2}{a}-\frac{1}{a(d+ex)^2}}{2ef^3}$$

$$\Big\downarrow \text{1200}$$

$$\frac{-\frac{\int \left(\frac{b}{a(d+ex)^2}+\frac{-b^2-c(d+ex)^2b+ac}{a\left(c(d+ex)^4+b(d+ex)^2+a\right)}\right)d(d+ex)^2}{a}-\frac{1}{a(d+ex)^2}}{2ef^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{\frac{(b^2-2ac)\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}-\frac{b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{2a}+\frac{b\log\left((d+ex)^2\right)}{a}}{a}-\frac{1}{a(d+ex)^2}}{2ef^3}$$

input
```
Int[1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

output
```
(-(1/(a*(d + e*x)^2)) - (((b^2 - 2*a*c)*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt
[b^2 - 4*a*c]])/(a*Sqrt[b^2 - 4*a*c]) + (b*Log[(d + e*x)^2])/a - (b*Log[a
+ b*(d + e*x)^2 + c*(d + e*x)^4])/(2*a))/a)/(2*e*f^3)
```

3.644.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1145
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp
[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[m, -1]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.644.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.67 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.63

method	result
default	$\frac{\frac{\sum_{\_R=\mathrm{RootOf}\left(c\,e^{4}\_Z^{4}+4cd\,e^{3}\_Z^{3}+\left(6c\,d^{2}e^{2}+b\,e^{2}\right)\_Z^{2}+\left(4d^{3}ec+2bde\right)\_Z+d^{4}c+b\,d^{2}+a\right)}\frac{\left(bc\,e^{3}\_R^{3}+3bcd\,e^{2}\_R^{2}+e\left(3bc\,d^{2}-ac+b^{2}\right)\_R}{2e^{3}c\_R^{3}+6cd\,e^{2}\_R^{2}+6c\,d^{2}e\_R}}{2a^{2}e}}{f^{3}}$
risch	$-\frac{1}{2ae\,f^{3}(ex+d)^{2}}-\frac{b\ln(ex+d)}{a^{2}e\,f^{3}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4a^{3}c\,e^{2}f^{6}-a^{2}b^{2}e^{2}f^{6}\right)\_Z^{2}+\left(-4abce\,f^{3}+b^{3}e\,f^{3}\right)\_Z+c^{2}\right)}\_R\ln\left(\left(\left(10a^{3}c\,e^{4}f^{6}-3\right.\right.\right.}{}$

input
```
int(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x,method=_RETURNVERBOSE)
```

output
```
1/f^3*(1/2/a^2/e*sum((b*c*e^3*_R^3+3*b*c*d*e^2*_R^2+e*(3*b*c*d^2-a*c+b^2)*
_R+b*c*d^3-a*c*d+b^2*d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+
_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^
2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a))-1/2/a/e/(e*x+d)^2-b*ln(e*x+
d)/a^2/e)
```

3.644.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 349 vs. 2(123) = 246.

Time = 0.38 (sec) , antiderivative size = 828, normalized size of antiderivative = 6.23

$$\int \frac{1}{(df+efx)^3\,(a+b(d+ex)^2+c(d+ex)^4)}\,dx$$
$$=\left[-\frac{2\,ab^2-8\,a^2c+((b^2-2\,ac)e^2x^2+2\,(b^2-2\,ac)dex+(b^2-2\,ac)d^2)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2e^4x^4+8\,c^2de^3x^3+2\,c^2}{}\right.}{}\right.$$
$$-\frac{2\,ab^2-8\,a^2c+2\,((b^2-2\,ac)e^2x^2+2\,(b^2-2\,ac)dex+(b^2-2\,ac)d^2)\sqrt{-b^2+4\,ac}\arctan\left(-\frac{(2\,ce^2x^2+4\,cde}{}\right.}{}$$

input
```
integrate(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas
")
```

output
```
[-1/4*(2*a*b^2 - 8*a^2*c + ((b^2 - 2*a*c)*e^2*x^2 + 2*(b^2 - 2*a*c)*d*e*x
+ (b^2 - 2*a*c)*d^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^
3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b
*c*d)*e*x + b^2 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2
 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*
d^2 + 2*(2*c*d^3 + b*d)*e*x + a)) - ((b^3 - 4*a*b*c)*e^2*x^2 + 2*(b^3 - 4*
a*b*c)*d*e*x + (b^3 - 4*a*b*c)*d^2)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4
+ (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*((b^3 - 4
*a*b*c)*e^2*x^2 + 2*(b^3 - 4*a*b*c)*d*e*x + (b^3 - 4*a*b*c)*d^2)*log(e*x +
 d))/((a^2*b^2 - 4*a^3*c)*e^3*f^3*x^2 + 2*(a^2*b^2 - 4*a^3*c)*d*e^2*f^3*x
+ (a^2*b^2 - 4*a^3*c)*d^2*e*f^3), -1/4*(2*a*b^2 - 8*a^2*c + 2*((b^2 - 2*a*
c)*e^2*x^2 + 2*(b^2 - 2*a*c)*d*e*x + (b^2 - 2*a*c)*d^2)*sqrt(-b^2 + 4*a*c)
*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 -
 4*a*c)) - ((b^3 - 4*a*b*c)*e^2*x^2 + 2*(b^3 - 4*a*b*c)*d*e*x + (b^3 - 4*a
*b*c)*d^2)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 +
 b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*((b^3 - 4*a*b*c)*e^2*x^2 + 2*(b^3
- 4*a*b*c)*d*e*x + (b^3 - 4*a*b*c)*d^2)*log(e*x + d))/((a^2*b^2 - 4*a^3*c)
*e^3*f^3*x^2 + 2*(a^2*b^2 - 4*a^3*c)*d*e^2*f^3*x + (a^2*b^2 - 4*a^3*c)*d^2
*e*f^3)]
```

3.644.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(df+efx)^3\,(a+b(d+ex)^2+c(d+ex)^4)}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*f*x+d*f)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

output
```
Timed out
```

3.644.7 Maxima [F]

$$\int \frac{1}{(df+efx)^3\,(a+b(d+ex)^2+c(d+ex)^4)}\,dx = \int \frac{1}{\left((ex+d)^4c+(ex+d)^2b+a\right)(efx+df)^3}\,dx$$

input
```
integrate(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima
")
```

output
```
-1/2/(a*e^3*f^3*x^2 + 2*a*d*e^2*f^3*x + a*d^2*e*f^3) + integrate((b*c*e^3*
x^3 + 3*b*c*d*e^2*x^2 + b*c*d^3 + (3*b*c*d^2 + b^2 - a*c)*e*x + (b^2 - a*c
)*d)/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 +
2*(2*c*d^3 + b*d)*e*x + a), x)/(a^2*f^3) - b*log(e*x + d)/(a^2*e*f^3)
```

3.644.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(123) = 246$.

Time = 0.41 (sec) , antiderivative size = 356, normalized size of antiderivative = 2.68

$$\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx$$
$$= \frac{b\log\left(|ce^4x^4+4\,cde^3x^3+6\,cd^2e^2x^2+4\,cd^3ex+cd^4+be^2x^2+2\,bdex+bd^2+a|\right)}{4\,a^2ef^3}$$
$$-\frac{b\log\left(|ex+d|\right)}{a^2ef^3}-\frac{1}{2\,(ex+d)^2aef^3}$$
$$+\frac{\frac{(a^2b^2ce^3f^3-2\,a^3c^2e^3f^3)\log\left(\left|be^2x^2+\sqrt{b^2-4\,ac}e^2x^2+2\,bdex+2\,\sqrt{b^2-4\,ac}dex+bd^2+\sqrt{b^2-4\,ac}d^2+2\,a\right|\right)}{\sqrt{b^2-4\,ac}}-\frac{(a^2b^2ce^3f^3-2\,a^3c^2e^3f^3)\log\left(\left|-\right.\right.}{}}{4\,a^4ce^4f^6}$$

input
```
integrate(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

output
```
1/4*b*log(abs(c*e^4*x^4 + 4*c*d*e^3*x^3 + 6*c*d^2*e^2*x^2 + 4*c*d^3*e*x +
c*d^4 + b*e^2*x^2 + 2*b*d*e*x + b*d^2 + a))/(a^2*e*f^3) - b*log(abs(e*x +
d))/(a^2*e*f^3) - 1/2/((e*x + d)^2*a*e*f^3) + 1/4*((a^2*b^2*c*e^3*f^3 - 2*
a^3*c^2*e^3*f^3)*log(abs(b*e^2*x^2 + sqrt(b^2 - 4*a*c)*e^2*x^2 + 2*b*d*e*x
 + 2*sqrt(b^2 - 4*a*c)*d*e*x + b*d^2 + sqrt(b^2 - 4*a*c)*d^2 + 2*a))/sqrt(
b^2 - 4*a*c) - (a^2*b^2*c*e^3*f^3 - 2*a^3*c^2*e^3*f^3)*log(abs(-b*e^2*x^2
+ sqrt(b^2 - 4*a*c)*e^2*x^2 - 2*b*d*e*x + 2*sqrt(b^2 - 4*a*c)*d*e*x - b*d^
2 + sqrt(b^2 - 4*a*c)*d^2 - 2*a))/sqrt(b^2 - 4*a*c))/(a^4*c*e^4*f^6)
```

3.644.9 Mupad [B] (verification not implemented)

Time = 12.00 (sec) , antiderivative size = 5947, normalized size of antiderivative = 44.71

$$\int \frac{1}{(df+efx)^3 (a+b(d+ex)^2+c(d+ex)^4)} dx = \text{Too large to display}$$

input
```
int(1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x)
```

output
```
(atan((16*a^6*f^9*x*(((3*b^4 + a^2*c^2 - 9*a*b^2*c)*(((((2*b^3*e*f^3 - 8*a
*b*c*e*f^3)*((2*(20*a^3*c^4*d*e^17*f^6 + 2*a^2*b^2*c^3*d*e^17*f^6))/(a^3*f
^9) + ((40*a^4*b*c^3*d*e^18*f^9 - 12*a^3*b^3*c^2*d*e^18*f^9)*(2*b^3*e*f^3
- 8*a*b*c*e*f^3))/(a^3*f^9*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6))))/(2*(1
6*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)) + (12*b*c^4*d*e^16)/(a^2*f^6))*(2*b^
3*e*f^3 - 8*a*b*c*e*f^3))/(2*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)) + (2*
c^5*d*e^15)/(a^3*f^9) - (((((2*(20*a^3*c^4*d*e^17*f^6 + 2*a^2*b^2*c^3*d*e^
17*f^6))/(a^3*f^9) + ((40*a^4*b*c^3*d*e^18*f^9 - 12*a^3*b^3*c^2*d*e^18*f^9
)*(2*b^3*e*f^3 - 8*a*b*c*e*f^3))/(a^3*f^9*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^
2*f^6)))*(2*a*c - b^2))/(4*a^2*e*f^3*(4*a*c - b^2)^(1/2)) + ((40*a^4*b*c^3
*d*e^18*f^9 - 12*a^3*b^3*c^2*d*e^18*f^9)*(2*b^3*e*f^3 - 8*a*b*c*e*f^3)*(2*
a*c - b^2))/(4*a^5*e*f^12*(4*a*c - b^2)^(1/2)*(16*a^3*c*e^2*f^6 - 4*a^2*b^
2*e^2*f^6)))*(2*a*c - b^2))/(4*a^2*e*f^3*(4*a*c - b^2)^(1/2)) - ((40*a^4*b
*c^3*d*e^18*f^9 - 12*a^3*b^3*c^2*d*e^18*f^9)*(2*b^3*e*f^3 - 8*a*b*c*e*f^3)
*(2*a*c - b^2)^2)/(16*a^7*e^2*f^15*(4*a*c - b^2)*(16*a^3*c*e^2*f^6 - 4*a^2
*b^2*e^2*f^6))))/(8*a^3*c^2*(a^2*c^2 - 6*b^4 + 24*a*b^2*c)) + ((3*b^5 + 13
*a^2*b*c^2 - 15*a*b^3*c)*((((((2*(20*a^3*c^4*d*e^17*f^6 + 2*a^2*b^2*c^3*d*
e^17*f^6))/(a^3*f^9) + ((40*a^4*b*c^3*d*e^18*f^9 - 12*a^3*b^3*c^2*d*e^18*f
^9)*(2*b^3*e*f^3 - 8*a*b*c*e*f^3))/(a^3*f^9*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*
e^2*f^6)))*(2*a*c - b^2))/(4*a^2*e*f^3*(4*a*c - b^2)^(1/2)) + ((40*a^4*...
```

3.645 $\int \frac{1}{(df+efx)^4(a+b(d+ex)^2+c(d+ex)^4)}\,dx$

3.645.1 Optimal result

Integrand size = 33, antiderivative size = 236

$$\int \frac{1}{(df+efx)^4\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx = -\frac{1}{3aef^4(d+ex)^3}+\frac{b}{a^2ef^4(d+ex)}+\frac{\sqrt{c}\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b-\sqrt{b^2-4ac}}ef^4}+\frac{\sqrt{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b+\sqrt{b^2-4ac}}ef^4}$$

output
```
-1/3/a/e/f^4/(e*x+d)^3+b/a^2/e/f^4/(e*x+d)+1/2*arctan((e*x+d)*2^(1/2)*c^(1
/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/
2))/a^2/e/f^4*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/2*arctan((e*x+d)*2^(1
/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b+(2*a*c-b^2)/(-4*a*c+b
^2)^(1/2))/a^2/e/f^4*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.645.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.01

$$\int \frac{1}{(df+efx)^4\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx = \frac{-\frac{2a}{(d+ex)^3}+\frac{6b}{d+ex}+\frac{3\sqrt{2}\sqrt{c}\left(b^2-2ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{2}\sqrt{c}\left(-b^2+2ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}}{6a^2ef^4}$$

input
```
Integrate[1/((d*f + e*f*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

output
```
((-2*a)/(d + e*x)^3 + (6*b)/(d + e*x) + (3*Sqrt[2]*Sqrt[c]*(b^2 - 2*a*c +
b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2
- 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*S
qrt[c]*(-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e
*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 -
4*a*c]]))/(6*a^2*e*f^4)
```

3.645.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1462, 1443, 27, 1604, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(df+efx)^4\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx$$

$$\Big\downarrow \text{1462}$$

$$\frac{\int \frac{1}{(d+ex)^4\left(c(d+ex)^4+b(d+ex)^2+a\right)}d(d+ex)}{ef^4}$$

$$\Big\downarrow \text{1443}$$

$$\frac{\frac{\int -\frac{3\left(c(d+ex)^2+b\right)}{(d+ex)^2\left(c(d+ex)^4+b(d+ex)^2+a\right)}d(d+ex)}{3a}-\frac{1}{3a(d+ex)^3}}{ef^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{\int \frac{c(d+ex)^2+b}{(d+ex)^2\left(c(d+ex)^4+b(d+ex)^2+a\right)}d(d+ex)}{a}-\frac{1}{3a(d+ex)^3}}{ef^4}$$

$$\Big\downarrow \ 1604$$

$$\frac{-\frac{-\frac{\int \frac{b^2+c(d+ex)^2b-ac}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)}{a}-\frac{b}{a(d+ex)}}{a}-\frac{1}{3a(d+ex)^3}}{ef^4}$$

$$\Big\downarrow \ 1480$$

$$\frac{-\frac{-\frac{\frac{1}{2}c\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\int \frac{1}{c(d+ex)^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}d(d+ex)+\frac{1}{2}c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{c(d+ex)^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}d(d+ex)}{a}-\frac{b}{a(d+ex)}}{a}-\frac{1}{3a(d+ex)^3}}{ef^4}$$

$$\Big\downarrow \ 218$$

$$\frac{-\frac{-\frac{\frac{\sqrt{c}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}}{a}-\frac{b}{a(d+ex)}}{a}-\frac{1}{3a(d+ex)^3}}{ef^4}$$

input
```
Int[1/((d*f + e*f*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

output
```
(-1/3*1/(a*(d + e*x)^3) - (-(b/(a*(d + e*x))) - ((Sqrt[c]*(b + (b^2 - 2*a*
c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2
 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(b - (b^2 -
2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt
[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/a)/a)/(e*f^4)
```

3.645.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

3.645. $\int \frac{1}{(df+efx)^4(a+b(d+ex)^2+c(d+ex)^4)}\,dx$

rule 1443
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Sim
p[1/(a*d^2*(m + 1))   Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*
x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

3.645.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.70 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.81

method	result
default	$\frac{\frac{\sum_{\_R=\operatorname{RootOf}\left(c e^4 \_Z^4+4cd e^3 \_Z^3+\left(6c d^2 e^2+b e^2\right)\_Z^2+\left(4d^3 ec+2bde\right)\_Z+d^4 c+b d^2+a\right)} \frac{\left(\_R^2 bc e^2+2\_R bcde+bc d^2-ac+b^2\right)\ln\left(x-\_\right.}{2e^3 c\_R^3+6cd e^2\_R^2+6c d^2 e\_R+2d^3 c+be\_}}{2a^2 e}}{f^4}$
risch	$\frac{\frac{be x^2}{a^2}+\frac{2bdx}{a^2}-\frac{-3b d^2+a}{3e a^2}}{f^4(ex+d)^3}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\left(16f^{16}e^4c^2a^7-8a^6b^2c e^4 f^{16}+a^5b^4e^4f^{16}\right)\_Z^4+\left(-20b f^8e^2c^3a^3+25b^3f^8e^2c^2a^2-9b^5f^8e^2ca+b^7\right.\right.}\right.}{}$

input
```
int(1/(e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x,method=_RETURNVERBOSE)
```

output
```
1/f^4*(1/2/a^2/e*sum((_R^2*b*c*e^2+2*_R*b*c*d*e+b*c*d^2-a*c+b^2)/(2*_R^3*c
*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*
e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^
4*c+b*d^2+a))-1/3/a/e/(e*x+d)^3+b/a^2/e/(e*x+d))
```

3.645.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2212 vs. $2(200) = 400$.

Time = 0.30 (sec) , antiderivative size = 2212, normalized size of antiderivative = 9.37

$$\int \frac{1}{(df+efx)^4\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas
")
```

output
```
1/6*(6*b*e^2*x^2 + 12*b*d*e*x + 6*b*d^2 + 3*sqrt(1/2)*(a^2*e^4*f^4*x^3 + 3
*a^2*d*e^3*f^4*x^2 + 3*a^2*d^2*e^2*f^4*x + a^2*d^3*e*f^4)*sqrt(-((a^5*b^2
- 4*a^6*c)*e^2*f^8*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3
+ a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4*f^16)) + b^5 - 5*a*b^3*c + 5*a^2*b*c
^2)/((a^5*b^2 - 4*a^6*c)*e^2*f^8))*log(2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)
*e*x + 2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*d + sqrt(1/2)*((a^5*b^5 - 7*a^6
*b^3*c + 12*a^7*b*c^2)*e^3*f^12*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6
*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4*f^16)) - (b^8 - 8*a*b^6
*c + 20*a^2*b^4*c^2 - 17*a^3*b^2*c^3 + 4*a^4*c^4)*e*f^4)*sqrt(-((a^5*b^2 -
 4*a^6*c)*e^2*f^8*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 +
 a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4*f^16)) + b^5 - 5*a*b^3*c + 5*a^2*b*c^
2)/((a^5*b^2 - 4*a^6*c)*e^2*f^8))) - 3*sqrt(1/2)*(a^2*e^4*f^4*x^3 + 3*a^2*
d*e^3*f^4*x^2 + 3*a^2*d^2*e^2*f^4*x + a^2*d^3*e*f^4)*sqrt(-((a^5*b^2 - 4*a
^6*c)*e^2*f^8*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4
*c^4)/((a^10*b^2 - 4*a^11*c)*e^4*f^16)) + b^5 - 5*a*b^3*c + 5*a^2*b*c^2)/(
(a^5*b^2 - 4*a^6*c)*e^2*f^8))*log(2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*e*x
+ 2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*d - sqrt(1/2)*((a^5*b^5 - 7*a^6*b^3*
c + 12*a^7*b*c^2)*e^3*f^12*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*
b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4*f^16)) - (b^8 - 8*a*b^6*c +
20*a^2*b^4*c^2 - 17*a^3*b^2*c^3 + 4*a^4*c^4)*e*f^4)*sqrt(-((a^5*b^2 - 4...
```

3.645.6 Sympy [A] (verification not implemented)

Time = 105.92 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.74

$$\int \frac{1}{(df+efx)^4 (a+b(d+ex)^2+c(d+ex)^4)} dx$$
$$= \frac{-a+3bd^2+6bdex+3be^2x^2}{3a^2d^3ef^4+9a^2d^2e^2f^4x+9a^2de^3f^4x^2+3a^2e^4f^4x^3}$$
$$+\text{RootSum}\left(t^4\cdot\left(256a^7c^2e^4f^{16}-128a^6b^2ce^4f^{16}+16a^5b^4e^4f^{16}\right)+t^2\left(-80a^3bc^3e^2f^8+100a^2b^3c^2e^2f^8-3\right.\right.$$

input
```
integrate(1/(e*f*x+d*f)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

output
```
(-a + 3*b*d**2 + 6*b*d*e*x + 3*b*e**2*x**2)/(3*a**2*d**3*e*f**4 + 9*a**2*d
**2*e**2*f**4*x + 9*a**2*d*e**3*f**4*x**2 + 3*a**2*e**4*f**4*x**3) + RootS
um(_t**4*(256*a**7*c**2*e**4*f**16 - 128*a**6*b**2*c*e**4*f**16 + 16*a**5*
b**4*e**4*f**16) + _t**2*(-80*a**3*b*c**3*e**2*f**8 + 100*a**2*b**3*c**2*e
**2*f**8 - 36*a*b**5*c*e**2*f**8 + 4*b**7*e**2*f**8) + c**5, Lambda(_t, _t
*log(x + (-96*_t**3*a**7*b*c**2*e**3*f**12 + 56*_t**3*a**6*b**3*c*e**3*f**
12 - 8*_t**3*a**5*b**5*e**3*f**12 - 4*_t*a**4*c**4*e*f**4 + 32*_t*a**3*b**
2*c**3*e*f**4 - 40*_t*a**2*b**4*c**2*e*f**4 + 16*_t*a*b**6*c*e*f**4 - 2*_t
*b**8*e*f**4 + a**2*c**5*d - 3*a*b**2*c**4*d + b**4*c**3*d)/(a**2*c**5*e -
 3*a*b**2*c**4*e + b**4*c**3*e))))
```

3.645.7 Maxima [F]

$$\int \frac{1}{(df+efx)^4 (a+b(d+ex)^2+c(d+ex)^4)} dx$$
$$= \int \frac{1}{\left((ex+d)^4c+(ex+d)^2b+a\right)(efx+df)^4} dx$$

input
```
integrate(1/(e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima
")
```

output
```
1/3*(3*b*e^2*x^2 + 6*b*d*e*x + 3*b*d^2 - a)/(a^2*e^4*f^4*x^3 + 3*a^2*d*e^3
*f^4*x^2 + 3*a^2*d^2*e^2*f^4*x + a^2*d^3*e*f^4) + integrate((b*c*e^2*x^2 +
 2*b*c*d*e*x + b*c*d^2 + b^2 - a*c)/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (
6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a), x)/(a^2*f^4)
```

3.645.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1353 vs. 2(200) = 400.

Time = 0.29 (sec) , antiderivative size = 1353, normalized size of antiderivative = 5.73

$$\int \frac{1}{(df+efx)^4 (a+b(d+ex)^2+c(d+ex)^4)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

output
```
-1/2*((b*c*e^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) +
 d/e)^2 - 2*b*c*d*e*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^
4)) + d/e) + b*c*d^2 + b^2 - a*c)*log(x + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^
2 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^
2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^3 - 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 + sq
rt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2
+ b*e^2)*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e))
 - (b*c*e^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/
e)^2 + 2*b*c*d*e*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4))
 - d/e) + b*c*d^2 + b^2 - a*c)*log(x - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 -
 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 -
 4*a*c)*e^2)/(c*e^4)) - d/e)^3 + 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(
b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2 + 2*c*d^3*e + b*d*e + (6*c*d^2*e^2 + b
*e^2)*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)) +
(b*c*e^2*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^
2 - 2*b*c*d*e*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) +
d/e) + b*c*d^2 + b^2 - a*c)*log(x + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*
a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*
a*c)*e^2)/(c*e^4)) + d/e)^3 - 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2
 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b...
```

3.645.9 Mupad [B] (verification not implemented)

Time = 9.42 (sec) , antiderivative size = 5771, normalized size of antiderivative = 24.45

$$\int \frac{1}{(df+efx)^4 (a+b(d+ex)^2+c(d+ex)^4)}\,dx = \text{Too large to display}$$

input
```
int(1/((d*f + e*f*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x)
```

output
```
((2*b*d*x)/a^2 - (a - 3*b*d^2)/(3*a^2*e) + (b*e*x^2)/a^2)/(d^3*f^4 + e^3*f
^4*x^3 + 3*d*e^2*f^4*x^2 + 3*d^2*e*f^4*x) - atan((((b^4*(-(4*a*c - b^2)^3)
^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^
(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2*f^
8 + 16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8)))^(1/2)*(((b^4*(-(4*a*c - b^
2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2
)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e
^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8)))^(1/2)*(((b^4*(-(4*a*c
 - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c
- b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*
b^4*e^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8)))^(1/2)*(x*(8*a^10
*b^3*c^2*e^14*f^20 - 32*a^11*b*c^3*e^14*f^20) - 32*a^11*b*c^3*d*e^13*f^20
+ 8*a^10*b^3*c^2*d*e^13*f^20) - 16*a^10*c^4*e^12*f^16 - 4*a^8*b^4*c^2*e^12
*f^16 + 20*a^9*b^2*c^3*e^12*f^16) + x*(4*a^8*c^5*e^12*f^12 + 2*a^6*b^4*c^3
*e^12*f^12 - 8*a^7*b^2*c^4*e^12*f^12) + 4*a^8*c^5*d*e^11*f^12 + 2*a^6*b^4*
c^3*d*e^11*f^12 - 8*a^7*b^2*c^4*d*e^11*f^12)*1i + ((b^4*(-(4*a*c - b^2)^3)
^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^
(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2*f^
8 + 16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8)))^(1/2)*(((b^4*(-(4*a*c - b^
2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - ...
```

3.645. $\int \frac{1}{(df+efx)^4(a+b(d+ex)^2+c(d+ex)^4)}\,dx$

3.646 $\int \frac{(df+efx)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

3.646.1 Optimal result

Integrand size = 33, antiderivative size = 279

$$\int \frac{(df+efx)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{f^4(d+ex)\left(2a+b(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\left(b-\frac{b^2+4ac}{\sqrt{b^2-4ac}}\right)f^4\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}\,e} + \frac{\left(b^2+4ac+b\sqrt{b^2-4ac}\right)f^4\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}\,e}$$

output
```
1/2*f^4*(e*x+d)*(2*a+b*(e*x+d)^2)/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^
4)+1/4*f^4*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b
+(-4*a*c-b^2)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)/e*2^(1/2)/c^(1/2)/(b-(-4*a*
c+b^2)^(1/2))^(1/2)+1/4*f^4*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)
^(1/2))^(1/2))*(b^2+4*a*c+b*(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)/e*2^(1/
2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.646.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.95

$$\int \frac{(df+efx)^4}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$$
$$= \frac{f^4\left(-\frac{2(-2a(d+ex)-b(d+ex)^3)}{(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)}+\frac{\sqrt{2}\left(-b^2-4ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\left(b^2+4ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{c}(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}\right)}{4e}$$

input
```
Integrate[(d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

output
```
(f^4*((-2*(-2*a*(d + e*x) - b*(d + e*x)^3))/((b^2 - 4*a*c)*(a + b*(d + e*x
)^2 + c*(d + e*x)^4)) + (Sqrt[2]*(-b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcT
an[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2
 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b^2 + 4*a*c + b*S
qrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4
*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])))/(4*e)
```

3.646.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {1462, 1440, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(df+efx)^4}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$$
$$\Big\downarrow \text{1462}$$
$$\frac{f^4\int \frac{(d+ex)^4}{(c(d+ex)^4+b(d+ex)^2+a)^2}d(d+ex)}{e}$$
$$\Big\downarrow \text{1440}$$
$$\frac{f^4\left(\frac{(d+ex)(2a+b(d+ex)^2)}{2(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)}-\frac{\int \frac{2a-b(d+ex)^2}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)}{2(b^2-4ac)}\right)}{e}$$

$\Big\downarrow$ 1480

$$f^4\left(\frac{(d+ex)(2a+b(d+ex)^2)}{2(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)}-\frac{-\frac{1}{2}\left(b-\frac{4ac+b^2}{\sqrt{b^2-4ac}}\right)\int\frac{1}{c(d+ex)^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}d(d+ex)-\frac{1}{2}\left(\frac{4ac+b^2}{\sqrt{b^2-4ac}}+b\right)\int\frac{1}{c(d+ex)^2+\frac{1}{2}\left(b+\sqrt{b^2-4a}\right.}}{2(b^2-4ac)}\right.$$
$$\overline{\qquad\qquad e\qquad\qquad}$$

$\Big\downarrow$ 218

$$\frac{f^4\left(\frac{(d+ex)(2a+b(d+ex)^2)}{2(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)}-\frac{-\frac{\left(b-\frac{4ac+b^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{4ac+b^2}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{2(b^2-4ac)}\right)}{e}$$

input
```
Int[(d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

output
```
(f^4*(((d + e*x)*(2*a + b*(d + e*x)^2))/(2*(b^2 - 4*a*c)*(a + b*(d + e*x)^
2 + c*(d + e*x)^4)) - (-(((b - (b^2 + 4*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sq
rt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sq
rt[b - Sqrt[b^2 - 4*a*c]])) - ((b + (b^2 + 4*a*c)/Sqrt[b^2 - 4*a*c])*ArcTa
n[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[
c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*(b^2 - 4*a*c))))/e
```

3.646.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1440
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(-d^3)*(d*x)^(m - 3)*(2*a + b*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*
(p + 1)*(b^2 - 4*a*c))), x] + Simp[d^4/(2*(p + 1)*(b^2 - 4*a*c))   Int[(d*x
)^(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1),
x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Gt
Q[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

3.646. $\int \frac{(df+efx)^4}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

3.646.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.62 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.17

method	result
default	$f^4 \left(\frac{-\frac{b e^2 x^3}{2(4ac-b^2)} - \frac{3bde\,x^2}{2(4ac-b^2)} - \frac{(3b d^2+2a)x}{2(4ac-b^2)} - \frac{d(b d^2+2a)}{2e(4ac-b^2)}}{c x^4 e^4+4cd e^3 x^3+6c d^2 e^2 x^2+4c d^3 ex+b e^2 x^2+d^4 c+2bdex+b d^2+a} + \frac{\sum_{\_R=\mathrm{RootOf}\left(c e^4 \_Z^4+4cd e^3 \_Z^3+\left(6c d^2 e^2+b e^2\right)\_Z^2+\left(\right.} }{} \right.$
risch	$\frac{-\frac{b e^2 f^4 x^3}{2(4ac-b^2)} - \frac{3dbe\,f^4 x^2}{2(4ac-b^2)} - \frac{f^4(3b d^2+2a)x}{2(4ac-b^2)} - \frac{d f^4(b d^2+2a)}{2e(4ac-b^2)}}{c x^4 e^4+4cd e^3 x^3+6c d^2 e^2 x^2+4c d^3 ex+b e^2 x^2+d^4 c+2bdex+b d^2+a} + \frac{f^4 \left(\sum_{\_R=\mathrm{RootOf}\left(c e^4 \_Z^4+4cd e^3 \_Z^3+\left(6c d^2 e^2+b e^2\right)\_Z^2+\left(4\right.} \right.}{}$

input
```
int((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x,method=_RETURNVERBOSE)
```

output
```
f^4*((-1/2*b*e^2/(4*a*c-b^2)*x^3-3/2/(4*a*c-b^2)*b*d*e*x^2-1/2*(3*b*d^2+2*
a)/(4*a*c-b^2)*x-1/2*d/e*(b*d^2+2*a)/(4*a*c-b^2))/(c*e^4*x^4+4*c*d*e^3*x^3
+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)+1/4/(4*a*c
-b^2)/e*sum((-_R^2*b*e^2-2*_R*b*d*e-b*d^2+2*a)/(2*_R^3*c*e^3+6*_R^2*c*d*e^
2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3
*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a))
```

3.646.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2578 vs. 2(235) = 470.

Time = 0.32 (sec) , antiderivative size = 2578, normalized size of antiderivative = 9.24

$$\int \frac{(df+efx)^4}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas
")
```

output
```
1/4*(2*b*e^3*f^4*x^3 + 6*b*d*e^2*f^4*x^2 + 2*(3*b*d^2 + 2*a)*e*f^4*x + 2*(
b*d^3 + 2*a*d)*f^4 + sqrt(1/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a
*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2
*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4
 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)*sqrt(-((b^3 + 12*a*b*c)*f^8 +
 sqrt(f^16/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4))*(
b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2)/((b^6*c - 12*a*b^
4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))*log((3*b^2 + 4*a*c)*e*f^12*x +
(3*b^2 + 4*a*c)*d*f^12 + sqrt(1/2)*((b^4 - 8*a*b^2*c + 16*a^2*c^2)*e*f^8 +
 2*sqrt(f^16/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4))
*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*e^3)*sqrt(-((b^3 +
 12*a*b*c)*f^8 + sqrt(f^16/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*
a^3*c^5)*e^4))*(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2)/(
(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))) - sqrt(1/2)*((
b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c
+ 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4
*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b
*c)*d^2)*e)*sqrt(-((b^3 + 12*a*b*c)*f^8 + sqrt(f^16/((b^6*c^2 - 12*a*b^4*c
^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4))*(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2
*c^3 - 64*a^3*c^4)*e^2)/((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^...
```

3.646.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 641 vs. 2(252) = 504.

Time = 11.98 (sec) , antiderivative size = 641, normalized size of antiderivative = 2.30

$$\int \frac{(df+efx)^4}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$$
$$= \frac{-2adf^4-bd^3f^4-3bde^2f^4x^2-be^3}{8a^2ce-2ab^2e+8abcd^2e+8ac^2d^4e-2b^3d^2e-2b^2cd^4e+x^4\cdot(8ac^2e^5-2b^2ce^5)+x^3\cdot(32ac^2de^4-8b^2cde}$$
$$+\mathrm{RootSum}\Big(t^4\cdot(1048576a^6c^7e^4-1572864a^5b^2c^6e^4+983040a^4b^4c^5e^4-327680a^3b^6c^4e^4+61440a^2b^8c^3e$$

input
```
integrate((e*f*x+d*f)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

output
```
(-2*a*d*f**4 - b*d**3*f**4 - 3*b*d*e**2*f**4*x**2 - b*e**3*f**4*x**3 + x*(
-2*a*e*f**4 - 3*b*d**2*e*f**4))/(8*a**2*c*e - 2*a*b**2*e + 8*a*b*c*d**2*e
+ 8*a*c**2*d**4*e - 2*b**3*d**2*e - 2*b**2*c*d**4*e + x**4*(8*a*c**2*e**5
- 2*b**2*c*e**5) + x**3*(32*a*c**2*d*e**4 - 8*b**2*c*d*e**4) + x**2*(8*a*b
*c*e**3 + 48*a*c**2*d**2*e**3 - 2*b**3*e**3 - 12*b**2*c*d**2*e**3) + x*(16
*a*b*c*d*e**2 + 32*a*c**2*d**3*e**2 - 4*b**3*d*e**2 - 8*b**2*c*d**3*e**2))
 + RootSum(_t**4*(1048576*a**6*c**7*e**4 - 1572864*a**5*b**2*c**6*e**4 + 9
83040*a**4*b**4*c**5*e**4 - 327680*a**3*b**6*c**4*e**4 + 61440*a**2*b**8*c
**3*e**4 - 6144*a*b**10*c**2*e**4 + 256*b**12*c*e**4) + _t**2*(-12288*a**4
*b*c**4*e**2*f**8 + 8192*a**3*b**3*c**3*e**2*f**8 - 1536*a**2*b**5*c**2*e*
*2*f**8 + 16*b**9*e**2*f**8) + 16*a**3*c**2*f**16 + 24*a**2*b**2*c*f**16 +
 9*a*b**4*f**16, Lambda(_t, _t*log(x + (16384*_t**3*a**3*b*c**4*e**3 - 122
88*_t**3*a**2*b**3*c**3*e**3 + 3072*_t**3*a*b**5*c**2*e**3 - 256*_t**3*b**
7*c*e**3 + 64*_t*a**2*c**2*e*f**8 - 128*_t*a*b**2*c*e*f**8 - 4*_t*b**4*e*f
**8 + 4*a*c*d*f**12 + 3*b**2*d*f**12)/(4*a*c*e*f**12 + 3*b**2*e*f**12))))
```

3.646.7 Maxima [F]

$$\int \frac{(df+efx)^4}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = \int \frac{(efx+df)^4}{\left((ex+d)^4c+(ex+d)^2b+a\right)^2}\,dx$$

input
```
integrate((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima
")
```

output
```
-1/2*f^4*integrate(-(b*e^2*x^2 + 2*b*d*e*x + b*d^2 - 2*a)/((b^2*c - 4*a*c^
2)*e^4*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3
- 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 - 4*
a*b*c)*d^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e*x), x) + 1/
2*(b*e^3*f^4*x^3 + 3*b*d*e^2*f^4*x^2 + (3*b*d^2 + 2*a)*e*f^4*x + (b*d^3 +
2*a*d)*f^4)/((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (
b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*
d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c
+ (b^3 - 4*a*b*c)*d^2)*e)
```

3.646.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1474 vs. $2(235) = 470$.

Time = 0.30 (sec) , antiderivative size = 1474, normalized size of antiderivative = 5.28

$$\int \frac{(df + efx)^4}{(a + b(d + ex)^2 + c(d + ex)^4)^2} \, dx = \text{Too large to display}$$

input
```
integrate((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

output
```
-1/4*((b*e^2*f^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4))
 + d/e)^2 - 2*b*d*e*f^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(
c*e^4)) + d/e) + b*d^2*f^4 - 2*a*f^4)*log(x + sqrt(1/2)*sqrt(-(b*e^2 + sqr
t(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqr
t(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^3 - 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2
+ sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 - 2*c*d^3*e - b*d*e + (6*c*d^2*
e^2 + b*e^2)*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d
/e)) - (b*e^2*f^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)
) - d/e)^2 + 2*b*d*e*f^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/
(c*e^4)) - d/e) + b*d^2*f^4 - 2*a*f^4)*log(x - sqrt(1/2)*sqrt(-(b*e^2 + sq
rt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 + sq
rt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^3 + 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2
 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2 + 2*c*d^3*e + b*d*e + (6*c*d^2
*e^2 + b*e^2)*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) -
d/e)) + (b*e^2*f^4*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4
)) + d/e)^2 - 2*b*d*e*f^4*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)
/(c*e^4)) + d/e) + b*d^2*f^4 - 2*a*f^4)*log(x + sqrt(1/2)*sqrt(-(b*e^2 - s
qrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 - s
qrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^3 - 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^
2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 - 2*c*d^3*e - b*d*e + (6*c...
```

3.646.9 Mupad [B] (verification not implemented)

Time = 10.24 (sec) , antiderivative size = 8025, normalized size of antiderivative = 28.76

$$\int \frac{(df+efx)^4}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = \text{Too large to display}$$

input
```
int((d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x)
```

output
```
atan(((((2048*a^4*c^5*e^12*f^4 + 384*a^2*b^4*c^3*e^12*f^4 - 1536*a^3*b^2*c
^4*e^12*f^4 - 32*a*b^6*c^2*e^12*f^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2
 - 12*a*b^4*c)) + ((64*b^9*c^2*d*e^13 - 1024*a*b^7*c^3*d*e^13 + 16384*a^4*
b*c^6*d*e^13 + 6144*a^2*b^5*c^4*d*e^13 - 16384*a^3*b^3*c^5*d*e^13)/(8*(b^6
 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(16*b^7*c^2*e^14 - 192*
a*b^5*c^3*e^14 - 1024*a^3*b*c^5*e^14 + 768*a^2*b^3*c^4*e^14))/(2*(b^4 + 16
*a^2*c^2 - 8*a*b^2*c)))*(-(b^9*f^8 + f^8*(-(4*a*c - b^2)^9)^(1/2) - 768*a^
4*b*c^4*f^8 - 96*a^2*b^5*c^2*f^8 + 512*a^3*b^3*c^3*f^8)/(32*(b^12*c*e^2 +
4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*
c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2))*(-(b^9*f^8
 + f^8*(-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4*f^8 - 96*a^2*b^5*c^2*f^8 +
 512*a^3*b^3*c^3*f^8)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e
^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6
144*a^5*b^2*c^6*e^2)))^(1/2) - (128*a^3*c^4*d*e^11*f^8 - 4*b^6*c*d*e^11*f^
8 + 8*a*b^4*c^2*d*e^11*f^8)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b
^4*c)) + (x*(b^4*c*e^12*f^8 + 8*a^2*c^3*e^12*f^8 + 2*a*b^2*c^2*e^12*f^8))/
(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(b^9*f^8 + f^8*(-(4*a*c - b^2)^9)^(1
/2) - 768*a^4*b*c^4*f^8 - 96*a^2*b^5*c^2*f^8 + 512*a^3*b^3*c^3*f^8)/(32*(b
^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1
280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1...
```

3.647 $\int \frac{(df+efx)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

3.647.1 Optimal result

Integrand size = 33, antiderivative size = 103

$$\int \frac{(df+efx)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx=\frac{f^3(2a+b(d+ex)^2)}{2\,(b^2-4ac)\,e\,(a+b(d+ex)^2+c(d+ex)^4)}-\frac{bf^3\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}\,e}$$

output
```
1/2*f^3*(2*a+b*(e*x+d)^2)/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)-b*f^3
*arctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)/e
```

3.647.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00

$$\int \frac{(df+efx)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx=\frac{f^3\left(\frac{2a+b(d+ex)^2}{(b^2-4ac)(a+(d+ex)^2(b+c(d+ex)^2))}-\frac{2b\arctan\left(\frac{b+2c(d+ex)^2}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}\right)}{2e}$$

input
```
Integrate[(d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

output
```
(f^3*((2*a + b*(d + e*x)^2)/((b^2 - 4*a*c)*(a + (d + e*x)^2*(b + c*(d + e*
x)^2))) - (2*b*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4
*a*c)^(3/2)))/(2*e)
```

3.647.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {1462, 1434, 1159, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(df+efx)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}dx$$

$$\Big\downarrow \text{1462}$$

$$\frac{f^3\int \frac{(d+ex)^3}{\left(c(d+ex)^4+b(d+ex)^2+a\right)^2}d(d+ex)}{e}$$

$$\Big\downarrow \text{1434}$$

$$\frac{f^3\int \frac{(d+ex)^2}{\left(c(d+ex)^4+b(d+ex)^2+a\right)^2}d(d+ex)^2}{2e}$$

$$\Big\downarrow \text{1159}$$

$$\frac{f^3\left(\frac{b\int \frac{1}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)^2}{b^2-4ac}+\frac{2a+b(d+ex)^2}{(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)}\right)}{2e}$$

$$\Big\downarrow \text{1083}$$

$$\frac{f^3\left(\frac{2a+b(d+ex)^2}{(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)}-\frac{2b\int \frac{1}{-(d+ex)^4+b^2-4ac}d\left(2c(d+ex)^2+b\right)}{b^2-4ac}\right)}{2e}$$

$$\Big\downarrow \text{219}$$

$$\frac{f^3\left(\frac{2a+b(d+ex)^2}{(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)}-\frac{2b\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}\right)}{2e}$$

input
```
Int[(d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

output
```
(f^3*((2*a + b*(d + e*x)^2)/((b^2 - 4*a*c)*(a + b*(d + e*x)^2 + c*(d + e*x
)^4)) - (2*b*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*
c)^(3/2)))/(2*e)
```

3.647.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

3.647.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.68 (sec) , antiderivative size = 280, normalized size of antiderivative = 2.72

method	result
default	$f^3\left(\frac{-\frac{x^2eb}{2(4ac-b^2)}-\frac{xbd}{4ac-b^2}-\frac{bd^2+2a}{2e(4ac-b^2)}}{cx^4e^4+4cde^3x^3+6cd^2e^2x^2+4cd^3ex+be^2x^2+d^4c+2bdex+bd^2+a}+\frac{b\left(\sum_{\_R=\mathrm{RootOf}\left(ce^4\_Z^4+4cde^3\_Z^3+\left(6cd^2e^2+be^2\right)\_Z\right.}\right.}{}\right.$
risch	$\frac{-\frac{bef^3x^2}{2(4ac-b^2)}-\frac{bdf^3x}{4ac-b^2}-\frac{f^3(bd^2+2a)}{2e(4ac-b^2)}}{cx^4e^4+4cde^3x^3+6cd^2e^2x^2+4cd^3ex+be^2x^2+d^4c+2bdex+bd^2+a}+\frac{bf^3\ln\left(\left(-(-4ac+b^2)^{\frac{3}{2}}e^2+4abe^2c-b^3e^2\right)x^2+\left(-2(-4ac+\right.\right.}{}$

input
```
int((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x,method=_RETURNVERBOSE)
```

output
```
f^3*((-1/2/(4*a*c-b^2)*x^2*e*b-1/(4*a*c-b^2)*x*b*d-1/2/e*(b*d^2+2*a)/(4*a*
c-b^2))/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d
^4+2*b*d*e*x+b*d^2+a)+1/2*b/(4*a*c-b^2)/e*sum((-_R*e-d)/(2*_R^3*c*e^3+6*_R
^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+
4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2
+a))
```

3.647.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 473 vs. 2(97) = 194.

Time = 0.32 (sec) , antiderivative size = 1077, normalized size of antiderivative = 10.46

$$\int \frac{(df+efx)^3}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$$
$$=\left[\frac{(b^3-4\,abc)e^2f^3x^2+2\,(b^3-4\,abc)def^3x+(2\,ab^2-8\,a^2c+(b^3-4\,abc)d^2)f^3-(bce^4f^3x^4+4\,bcde^3f^3x^3}{2\,((b^4c-8\,ab^2c^2+16\,a^2c^3)e^5x^4+4\,(b^4c-8\,ab^2c^2+16\,a^2c^3)de^4x^3+(b^5-8\,ab^3c+16\,a^2bc^2+6\,(b^4c-8\right.$$

input
```
integrate((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas
")
```

output
```
[1/2*((b^3 - 4*a*b*c)*e^2*f^3*x^2 + 2*(b^3 - 4*a*b*c)*d*e*f^3*x + (2*a*b^2
 - 8*a^2*c + (b^3 - 4*a*b*c)*d^2)*f^3 - (b*c*e^4*f^3*x^4 + 4*b*c*d*e^3*f^3
*x^3 + (6*b*c*d^2 + b^2)*e^2*f^3*x^2 + 2*(2*b*c*d^3 + b^2*d)*e*f^3*x + (b*
c*d^4 + b^2*d^2 + a*b)*f^3)*sqrt(b^2 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d
*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*
d^3 + b*c*d)*e*x + b^2 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*s
qrt(b^2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x
^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a)))/((b^4*c - 8*a*b^2*c^2 + 16*a^2*c
^3)*e^5*x^4 + 4*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*e^4*x^3 + (b^5 - 8*a*
b^3*c + 16*a^2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^3*x^2 +
 2*(2*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b
*c^2)*d)*e^2*x + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2
+ 16*a^2*c^3)*d^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d^2)*e), 1/2*((b^3 -
4*a*b*c)*e^2*f^3*x^2 + 2*(b^3 - 4*a*b*c)*d*e*f^3*x + (2*a*b^2 - 8*a^2*c +
(b^3 - 4*a*b*c)*d^2)*f^3 - 2*(b*c*e^4*f^3*x^4 + 4*b*c*d*e^3*f^3*x^3 + (6*b
*c*d^2 + b^2)*e^2*f^3*x^2 + 2*(2*b*c*d^3 + b^2*d)*e*f^3*x + (b*c*d^4 + b^2
*d^2 + a*b)*f^3)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c
*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/((b^4*c - 8*a*b^2*c^2 + 16*a^
2*c^3)*e^5*x^4 + 4*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*e^4*x^3 + (b^5 - 8
*a*b^3*c + 16*a^2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^3...
```

3.647.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 556 vs. $2(88) = 176$.

Time = 2.77 (sec) , antiderivative size = 556, normalized size of antiderivative = 5.40

$$\int \frac{(df+efx)^3}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = \frac{bf^3\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(\frac{2dx}{e}+x^2+\frac{-16a^2bc^2f^3\sqrt{-\frac{1}{(4ac-b^2)^3}}+8ab^3cf^3\sqrt{-\frac{1}{(4ac-b^2)^3}}-b^5f^3\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^2f^3+2bcd^2f^3}{2bce^2f^3}\right)}{2e} - \frac{bf^3\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(\frac{2dx}{e}+x^2+\frac{16a^2bc^2f^3\sqrt{-\frac{1}{(4ac-b^2)^3}}-8ab^3cf^3\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^5f^3\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^2f^3+2bcd^2f^3}{2bce^2f^3}\right)}{2e} + \frac{-2af^3-bd^2f^3-2}{8a^2ce-2ab^2e+8abcd^2e+8ac^2d^4e-2b^3d^2e-2b^2cd^4e+x^4\cdot(8ac^2e^5-2b^2ce^5)+x^3\cdot(32ac^2de^4-8b^2c}$$

input
```
integrate((e*f*x+d*f)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

output
```
b*f**3*sqrt(-1/(4*a*c - b**2)**3)*log(2*d*x/e + x**2 + (-16*a**2*b*c**2*f*
*3*sqrt(-1/(4*a*c - b**2)**3) + 8*a*b**3*c*f**3*sqrt(-1/(4*a*c - b**2)**3)
 - b**5*f**3*sqrt(-1/(4*a*c - b**2)**3) + b**2*f**3 + 2*b*c*d**2*f**3)/(2*
b*c*e**2*f**3))/(2*e) - b*f**3*sqrt(-1/(4*a*c - b**2)**3)*log(2*d*x/e + x*
*2 + (16*a**2*b*c**2*f**3*sqrt(-1/(4*a*c - b**2)**3) - 8*a*b**3*c*f**3*sqr
t(-1/(4*a*c - b**2)**3) + b**5*f**3*sqrt(-1/(4*a*c - b**2)**3) + b**2*f**3
 + 2*b*c*d**2*f**3)/(2*b*c*e**2*f**3))/(2*e) + (-2*a*f**3 - b*d**2*f**3 -
2*b*d*e*f**3*x - b*e**2*f**3*x**2)/(8*a**2*c*e - 2*a*b**2*e + 8*a*b*c*d**2
*e + 8*a*c**2*d**4*e - 2*b**3*d**2*e - 2*b**2*c*d**4*e + x**4*(8*a*c**2*e*
*5 - 2*b**2*c*e**5) + x**3*(32*a*c**2*d*e**4 - 8*b**2*c*d*e**4) + x**2*(8*
a*b*c*e**3 + 48*a*c**2*d**2*e**3 - 2*b**3*e**3 - 12*b**2*c*d**2*e**3) + x*
(16*a*b*c*d*e**2 + 32*a*c**2*d**3*e**2 - 4*b**3*d*e**2 - 8*b**2*c*d**3*e**
2))
```

3.647.7 Maxima [F]

$$\int \frac{(df+efx)^3}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx=\int \frac{(efx+df)^3}{\left((ex+d)^4c+(ex+d)^2b+a\right)^2}\,dx$$

input
```
integrate((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima
")
```

output
```
-b*f^3*integrate(-(e*x + d)/((b^2*c - 4*a*c^2)*e^4*x^4 + 4*(b^2*c - 4*a*c^
2)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2
)*d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2 + 2*(2*(b^2*c - 4*a
*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e*x), x) + 1/2*(b*e^2*f^3*x^2 + 2*b*d*e*f^3
*x + (b*d^2 + 2*a)*f^3)/((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d
*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c
 - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^
2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)
```

3.647.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(97) = 194$.

Time = 0.29 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.96

$$\int \frac{(df+efx)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{bf^3\arctan\left(\frac{2\,cd^2f+2\,(efx^2+2\,dfx)ce+bf}{\sqrt{-b^2+4\,acf}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ace}} + \frac{bd^2f^5+(efx^2+2\,dfx)bef^4+2\,af^5}{2\left(cd^4f^2+2\,(efx^2+2\,dfx)cd^2ef+(efx^2+2\,dfx)^2ce^2+bd^2f^2+(efx^2+2\,dfx)bef+af^2\right)(b^2e-4\,ac$$

input
```
integrate((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

output
```
b*f^3*arctan((2*c*d^2*f + 2*(e*f*x^2 + 2*d*f*x)*c*e + b*f)/(sqrt(-b^2 + 4*
a*c)*f))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a*c)*e) + 1/2*(b*d^2*f^5 + (e*f*x^2
+ 2*d*f*x)*b*e*f^4 + 2*a*f^5)/((c*d^4*f^2 + 2*(e*f*x^2 + 2*d*f*x)*c*d^2*e*
f + (e*f*x^2 + 2*d*f*x)^2*c*e^2 + b*d^2*f^2 + (e*f*x^2 + 2*d*f*x)*b*e*f +
a*f^2)*(b^2*e - 4*a*c*e))
```

3.647.9 Mupad [B] (verification not implemented)

Time = 8.60 (sec) , antiderivative size = 460, normalized size of antiderivative = 4.47

$$\int \frac{(df+efx)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$$

$$= \frac{b\,f^3\,\mathrm{atan}\left(\frac{(4\,a\,c-b^2)^4\left(x\left(\frac{b^3\,f^6\,\left(2\,b^3\,c^2\,d\,e^9-8\,a\,b\,c^3\,d\,e^9\right)}{a\,e^2\,\left(4\,a\,c-b^2\right)^{11/2}}-\frac{2\,b^2\,c^2\,d\,e^7\,f^6}{a\,\left(4\,a\,c-b^2\right)^{7/2}}\right)+x^2\,\left(\frac{b^3\,f^6\,\left(2\,b^3\,c^2\,e^{10}-8\,a\,b\,c^3\,e^{10}\right)}{2\,a\,e^2\,\left(4\,a\,c-b^2\right)^{11/2}}-\frac{b^2\,c^2\,e^8\,f^6}{a\,\left(4\,a\,c-b^2\right)^{7/2}}\right)-\frac{b^3\,f^6\,(}{}\right.}{2\,b^2\,c^2\,e^6\,f^6}\right.}{e\,(4\,a\,c-b^2)^{3/2}}$$

$$- \frac{\frac{f^3\,(b\,d^2+2\,a)}{2\,e\,(4\,a\,c-b^2)}+\frac{b\,d\,f^3\,x}{4\,a\,c-b^2}+\frac{b\,e\,f^3\,x^2}{2\,(4\,a\,c-b^2)}}{a+x^2\,\left(6\,c\,d^2\,e^2+b\,e^2\right)+b\,d^2+c\,d^4+x\,\left(4\,c\,e\,d^3+2\,b\,e\,d\right)+c\,e^4\,x^4+4\,c\,d\,e^3\,x^3}$$

input
```
int((d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x)
```

output
```
(b*f^3*atan(((4*a*c - b^2)^4*(x*((b^3*f^6*(2*b^3*c^2*d*e^9 - 8*a*b*c^3*d*e
^9))/(a*e^2*(4*a*c - b^2)^(11/2)) - (2*b^2*c^2*d*e^7*f^6)/(a*(4*a*c - b^2)
^(7/2))) + x^2*((b^3*f^6*(2*b^3*c^2*e^10 - 8*a*b*c^3*e^10))/(2*a*e^2*(4*a*
c - b^2)^(11/2)) - (b^2*c^2*e^8*f^6)/(a*(4*a*c - b^2)^(7/2))) - (b^3*f^6*(
16*a^2*c^3*e^8 - 4*a*b^2*c^2*e^8 - 2*b^3*c^2*d^2*e^8 + 8*a*b*c^3*d^2*e^8))
/(2*a*e^2*(4*a*c - b^2)^(11/2)) - (b^2*c^2*d^2*e^6*f^6)/(a*(4*a*c - b^2)^(
7/2))))/(2*b^2*c^2*e^6*f^6)))/(e*(4*a*c - b^2)^(3/2)) - ((f^3*(2*a + b*d^2
))/(2*e*(4*a*c - b^2)) + (b*d*f^3*x)/(4*a*c - b^2) + (b*e*f^3*x^2)/(2*(4*a
*c - b^2)))/(a + x^2*(b*e^2 + 6*c*d^2*e^2) + b*d^2 + c*d^4 + x*(2*b*d*e +
4*c*d^3*e) + c*e^4*x^4 + 4*c*d*e^3*x^3)
```

3.647. $\int \frac{(df+efx)^3}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$

3.648 $\int \frac{(df+efx)^2}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

3.648.1 Optimal result

Integrand size = 33, antiderivative size = 263

$$\int \frac{(df+efx)^2}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = -\frac{f^2(d+ex)\left(b+2c(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\sqrt{c}\left(2b-\sqrt{b^2-4ac}\right)f^2\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}\,e} - \frac{\sqrt{c}\left(2b+\sqrt{b^2-4ac}\right)f^2\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}\,e}$$

output
```
-1/2*f^2*(e*x+d)*(b+2*c*(e*x+d)^2)/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)
^4)+1/2*f^2*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c
^(1/2)*(2*b-(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)/e*2^(1/2)/(b-(-4*a*c+b^
2)^(1/2))^(1/2)-1/2*f^2*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/
2))^(1/2))*c^(1/2)*(2*b+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)/e*2^(1/2)/(
b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.648.2 Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.95

$$\int \frac{(df+efx)^2}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = -\frac{f^2\left(\frac{b(d+ex)+2c(d+ex)^3}{(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)}+\frac{\sqrt{2}\sqrt{c}\left(-2b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(2b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2e}$$

input
```
Integrate[(d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

output
```
-1/2*(f^2*((b*(d + e*x) + 2*c*(d + e*x)^3)/((b^2 - 4*a*c)*(a + b*(d + e*x)
^2 + c*(d + e*x)^4)) + (Sqrt[2]*Sqrt[c]*(-2*b + Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(
3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(2*b + Sqrt[b^2 - 4*a
*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^
2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])))/e
```

3.648.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {1462, 1439, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(df+efx)^2}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$$

$$\Big\downarrow \text{1462}$$

$$\frac{f^2\int \frac{(d+ex)^2}{(c(d+ex)^4+b(d+ex)^2+a)^2}d(d+ex)}{e}$$

$$\Big\downarrow \text{1439}$$

$$\frac{f^2\left(\frac{\int \frac{b-2c(d+ex)^2}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)}{2(b^2-4ac)}-\frac{(d+ex)\left(b+2c(d+ex)^2\right)}{2(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)}\right)}{e}$$

$$\Big\downarrow \; 1480$$

$$\frac{f^2\left(\frac{-c\left(1-\frac{2b}{\sqrt{b^2-4ac}}\right)\int\frac{1}{c(d+ex)^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}d(d+ex)-c\left(\frac{2b}{\sqrt{b^2-4ac}}+1\right)\int\frac{1}{c(d+ex)^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}d(d+ex)}{2(b^2-4ac)}-\frac{(d+ex)(b+2c(d+ex)^2)}{2(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)}\right)}{e}$$

$$\Big\downarrow \; 218$$

$$\frac{f^2\left(\frac{-\frac{\sqrt{2}\sqrt{c}\left(1-\frac{2b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\left(\frac{2b}{\sqrt{b^2-4ac}}+1\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}}}{2(b^2-4ac)}-\frac{(d+ex)(b+2c(d+ex)^2)}{2(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)}\right)}{e}$$

input
```
Int[(d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

output
```
(f^2*(-1/2*((d + e*x)*(b + 2*c*(d + e*x)^2))/((b^2 - 4*a*c)*(a + b*(d + e*
x)^2 + c*(d + e*x)^4)) + (-((Sqrt[2]*Sqrt[c]*(1 - (2*b)/Sqrt[b^2 - 4*a*c])
*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/Sqrt[b -
 Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*Sqrt[c]*(1 + (2*b)/Sqrt[b^2 - 4*a*c])*ArcT
an[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/Sqrt[b + Sqrt
[b^2 - 4*a*c]])/(2*(b^2 - 4*a*c))))/e
```

3.648.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1439
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d*(d*x)^(m - 1)*(b + 2*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*(p +
1)*(b^2 - 4*a*c))), x] - Simp[d^2/(2*(p + 1)*(b^2 - 4*a*c))   Int[(d*x)^(m
- 2)*(b*(m - 1) + 2*c*(m + 4*p + 5)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x
] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m,
1] && LeQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

3.648. $\int \frac{(df+efx)^2}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

3.648.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.63 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.23

method	result
default	$f^2 \Big(\frac{\frac{c\,e^2x^3}{4ac-b^2}+\frac{3x^2cde}{4ac-b^2}+\frac{(6c\,d^2+b)x}{8ac-2b^2}+\frac{d(2c\,d^2+b)}{2e(4ac-b^2)}}{c\,x^4e^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+d^4c+2bdex+b\,d^2+a} + \frac{\sum_{\_R=\text{RootOf}(c\,e^4\_Z^4+4cd\,e^3\_Z^3+(6c\,d^2e^2+b\,e^2)\_Z^2+(}}{}$
risch	$\frac{\frac{c\,e^2f^2x^3}{4ac-b^2}+\frac{3dce\,f^2x^2}{4ac-b^2}+\frac{f^2(6c\,d^2+b)x}{8ac-2b^2}+\frac{d\,f^2(2c\,d^2+b)}{2e(4ac-b^2)}}{c\,x^4e^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+d^4c+2bdex+b\,d^2+a} + \frac{f^2\Big(\sum_{\_R=\text{RootOf}(c\,e^4\_Z^4+4cd\,e^3\_Z^3+(6c\,d^2e^2+b\,e^2)\_Z^2+(4}}{}$

input
```
int((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x,method=_RETURNVERBOSE)
```

output
```
f^2*((c*e^2/(4*a*c-b^2)*x^3+3/(4*a*c-b^2)*x^2*c*d*e+1/2*(6*c*d^2+b)/(4*a*c
-b^2)*x+1/2*d/e*(2*c*d^2+b)/(4*a*c-b^2))/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*
e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)+1/4/(4*a*c-b^2)/e*s
um((2*_R^2*c*e^2+4*_R*c*d*e+2*c*d^2-b)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c
*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6
*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a))
```

3.648.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2600 vs. $2(222) = 444$.

Time = 0.30 (sec) , antiderivative size = 2600, normalized size of antiderivative = 9.89

$$\int \frac{(df + efx)^2}{(a + b(d + ex)^2 + c(d + ex)^4)^2} \, dx = \text{Too large to display}$$

input
```
integrate((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas
")
```

output
```
-1/4*(4*c*e^3*f^2*x^3 + 12*c*d*e^2*f^2*x^2 + 2*(6*c*d^2 + b)*e*f^2*x + 2*(
2*c*d^3 + b*d)*f^2 + sqrt(1/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a
*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2
*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4
 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)*sqrt(-((b^3 + 12*a*b*c)*f^4 +
 (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*sqrt(f^8/((a^2*b^6 -
 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4))*e^2)/((a*b^6 - 12*a^2*b
^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2))*log((3*b^2*c + 4*a*c^2)*e*f^6*x
+ (3*b^2*c + 4*a*c^2)*d*f^6 + 1/2*sqrt(1/2)*((b^5 - 8*a*b^3*c + 16*a^2*b*c
^2)*e*f^4 - (a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 256*a^5*c^4)*sqrt(f^8
/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4))*e^3)*sqrt(-
((b^3 + 12*a*b*c)*f^4 + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^
3)*sqrt(f^8/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4))*
e^2)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2))) - sqrt(1
/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*
a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b
^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 -
 4*a*b*c)*d^2)*e)*sqrt(-((b^3 + 12*a*b*c)*f^4 + (a*b^6 - 12*a^2*b^4*c + 48
*a^3*b^2*c^2 - 64*a^4*c^3)*sqrt(f^8/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*
c^2 - 64*a^5*c^3)*e^4))*e^2)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - ...
```

3.648.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(df + efx)^2}{(a + b(d + ex)^2 + c(d + ex)^4)^2} \, dx = \text{Timed out}$$

input
```
integrate((e*f*x+d*f)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

output
```
Timed out
```

3.648.7 Maxima [F]

$$\int \frac{(df+efx)^2}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = \int \frac{(efx+df)^2}{\left((ex+d)^4c+(ex+d)^2b+a\right)^2}\,dx$$

input
```
integrate((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima
")
```

output
```
1/2*f^2*integrate(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 - b)/((b^2*c - 4*a*c
^2)*e^4*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3
 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 - 4
*a*b*c)*d^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e*x), x) - 1
/2*(2*c*e^3*f^2*x^3 + 6*c*d*e^2*f^2*x^2 + (6*c*d^2 + b)*e*f^2*x + (2*c*d^3
 + b*d)*f^2)/((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 +
(b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)
*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c
 + (b^3 - 4*a*b*c)*d^2)*e)
```

3.648.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1482 vs. $2(222) = 444$.

Time = 0.30 (sec) , antiderivative size = 1482, normalized size of antiderivative = 5.63

$$\int \frac{(df+efx)^2}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

output
```
1/4*((2*c*e^2*f^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)
) + d/e)^2 - 4*c*d*e*f^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/
(c*e^4)) + d/e) + 2*c*d^2*f^2 - b*f^2)*log(x + sqrt(1/2)*sqrt(-(b*e^2 + sq
rt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 + sq
rt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^3 - 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2
 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 - 2*c*d^3*e - b*d*e + (6*c*d^2
*e^2 + b*e^2)*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) +
d/e)) - (2*c*e^2*f^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e
^4)) - d/e)^2 + 4*c*d*e*f^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^
2)/(c*e^4)) - d/e) + 2*c*d^2*f^2 - b*f^2)*log(x - sqrt(1/2)*sqrt(-(b*e^2 +
 sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 +
 sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^3 + 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*
e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2 + 2*c*d^3*e + b*d*e + (6*c*
d^2*e^2 + b*e^2)*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4))
 - d/e)) + (2*c*e^2*f^2*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(
c*e^4)) + d/e)^2 - 4*c*d*e*f^2*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)
*e^2)/(c*e^4)) + d/e) + 2*c*d^2*f^2 - b*f^2)*log(x + sqrt(1/2)*sqrt(-(b*e^
2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^
2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^3 - 6*c*d*e^3*(sqrt(1/2)*sqrt(-
(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 - 2*c*d^3*e - b*d*e +...
```

3.648.9 Mupad [B] (verification not implemented)

Time = 9.98 (sec) , antiderivative size = 7835, normalized size of antiderivative = 29.79

$$\int \frac{(df+efx)^2}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = \text{Too large to display}$$

input
```
int((d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x)
```

output
```
((x*(b*f^2 + 6*c*d^2*f^2))/(2*(4*a*c - b^2)) + (2*c*d^3*f^2 + b*d*f^2)/(2*
e*(4*a*c - b^2)) + (c*e^2*f^2*x^3)/(4*a*c - b^2) + (3*c*d*e*f^2*x^2)/(4*a*
c - b^2))/(a + x^2*(b*e^2 + 6*c*d^2*e^2) + b*d^2 + c*d^4 + x*(2*b*d*e + 4*
c*d^3*e) + c*e^4*x^4 + 4*c*d*e^3*x^3) + atan(((((f^4*(-(4*a*c - b^2)^9)^(1
/2))/32 - (b^9*f^4)/32 + 24*a^4*b*c^4*f^4 + 3*a^2*b^5*c^2*f^4 - 16*a^3*b^3
*c^3*f^4)/(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8
*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*
e^2))^(1/2)*((((f^4*(-(4*a*c - b^2)^9)^(1/2))/32 - (b^9*f^4)/32 + 24*a^4*b
*c^4*f^4 + 3*a^2*b^5*c^2*f^4 - 16*a^3*b^3*c^3*f^4)/(a*b^12*e^2 + 4096*a^7*
c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 +
 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2))^(1/2)*(((8*b^9*c^2*d*e^13 -
 128*a*b^7*c^3*d*e^13 + 2048*a^4*b*c^6*d*e^13 + 768*a^2*b^5*c^4*d*e^13 - 2
048*a^3*b^3*c^5*d*e^13)/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c) +
 (x*(8*b^7*c^2*e^14 - 96*a*b^5*c^3*e^14 - 512*a^3*b*c^5*e^14 + 384*a^2*b^3
*c^4*e^14))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(((f^4*(-(4*a*c - b^2)^9)^(1/2
))/32 - (b^9*f^4)/32 + 24*a^4*b*c^4*f^4 + 3*a^2*b^5*c^2*f^4 - 16*a^3*b^3*c
^3*f^4)/(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c
^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^
2))^(1/2) + (2*b^7*c^2*e^12*f^2 + 96*a^2*b^3*c^4*e^12*f^2 - 24*a*b^5*c^3*e
^12*f^2 - 128*a^3*b*c^5*e^12*f^2)/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - ...
```

3.648. $\int \frac{(df+efx)^2}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$

3.649 $\int \frac{df+efx}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

3.649.1 Optimal result

Integrand size = 31, antiderivative size = 98

$$\int \frac{df+efx}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = -\frac{f\left(b+2c(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{2cf\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}e}$$

output
```
-1/2*f*(b+2*c*(e*x+d)^2)/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)+2*c*f*
arctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)/e
```

3.649.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.01

$$\int \frac{df+efx}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = -\frac{f\left(\frac{b+2c(d+ex)^2}{a+b(d+ex)^2+c(d+ex)^4}+\frac{4c\arctan\left(\frac{b+2c(d+ex)^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}\right)}{2\left(b^2-4ac\right)e}$$

input
```
Integrate[(d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

output
```
-1/2*(f*((b + 2*c*(d + e*x)^2)/(a + b*(d + e*x)^2 + c*(d + e*x)^4) + (4*c*
ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c]))/((b
^2 - 4*a*c)*e)
```

3.649.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1462, 1432, 1086, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{df+efx}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$$

$$\Big\downarrow \text{1462}$$

$$\frac{f\int \frac{d+ex}{(c(d+ex)^4+b(d+ex)^2+a)^2}d(d+ex)}{e}$$

$$\Big\downarrow \text{1432}$$

$$\frac{f\int \frac{1}{(c(d+ex)^4+b(d+ex)^2+a)^2}d(d+ex)^2}{2e}$$

$$\Big\downarrow \text{1086}$$

$$\frac{f\left(-\frac{2c\int \frac{1}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)^2}{b^2-4ac}-\frac{b+2c(d+ex)^2}{(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)}\right)}{2e}$$

$$\Big\downarrow \text{1083}$$

$$\frac{f\left(\frac{4c\int \frac{1}{-(d+ex)^4+b^2-4ac}d(2c(d+ex)^2+b)}{b^2-4ac}-\frac{b+2c(d+ex)^2}{(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)}\right)}{2e}$$

$$\Big\downarrow \text{219}$$

$$\frac{f\left(\frac{4c\,\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2c(d+ex)^2}{(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)}\right)}{2e}$$

input
```
Int[(d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

output
```
(f*(-((b + 2*c*(d + e*x)^2)/((b^2 - 4*a*c)*(a + b*(d + e*x)^2 + c*(d + e*x
)^4))) + (4*c*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a
*c)^(3/2)))/(2*e)
```

3.649.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

3.649.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.18 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.78

method	result
default	$f\left(\frac{\frac{c\,x^{2} e}{4ac-b^{2}}+\frac{2xcd}{4ac-b^{2}}+\frac{2c\,d^{2}+b}{2e\left(4ac-b^{2}\right)}}{c\,x^{4} e^{4}+4cd\,e^{3} x^{3}+6c\,d^{2} e^{2} x^{2}+4c\,d^{3} ex+b\,e^{2} x^{2}+d^{4} c+2bdex+b\,d^{2}+a}+\frac{c\left(\sum_{\_R=\mathrm{RootOf}\left(c\,e^{4} \_Z^{4}+4cd\,e^{3} \_Z^{3}+\left(6c\,d^{2} e^{2}+b\,e^{2}\right)\_Z^{2}\right.}\right.}{}\right.$
risch	$\frac{\frac{cef\,x^{2}}{4ac-b^{2}}+\frac{2cdfx}{4ac-b^{2}}+\frac{f\left(2c\,d^{2}+b\right)}{2e\left(4ac-b^{2}\right)}}{c\,x^{4} e^{4}+4cd\,e^{3} x^{3}+6c\,d^{2} e^{2} x^{2}+4c\,d^{3} ex+b\,e^{2} x^{2}+d^{4} c+2bdex+b\,d^{2}+a}+\frac{cf\ln\left(\left(\left(\left(-4ac+b^{2}\right)^{\frac{3}{2}} e^{2}+4ab\,e^{2} c-b^{3} e^{2}\right)x^{2}+\left(2\left(-4ac+b^{2}\right)^{\frac{3}{2}}\right.\right.\right.}{e(}$

input
```
int((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x,method=_RETURNVERBOSE)
```

output
```
f*((c/(4*a*c-b^2)*x^2*e+2/(4*a*c-b^2)*x*c*d+1/2/e*(2*c*d^2+b)/(4*a*c-b^2))
/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*
d*e*x+b*d^2+a)+c/(4*a*c-b^2)/e*sum((_R*e+d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6
*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z
^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a))
```

3.649.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 467 vs. 2(92) = 184.

Time = 0.30 (sec) , antiderivative size = 1066, normalized size of antiderivative = 10.88

$$\int \frac{df + efx}{(a + b(d + ex)^2 + c(d + ex)^4)^2} dx$$
$$= \left[-\frac{2\,(b^2c - 4\,ac^2)e^2fx^2 + 4\,(b^2c - 4\,ac^2)defx + 2\,(c^2e^4fx^4 + 4\,c^2de^3fx^3 + (6\,c^2d^2 + bc)e^2fx^2 + 2\,(2\,c^2d^3}{2\,((b^4c - 8\,ab^2c^2 + 16\,a^2c^3)e^5x^4 + 4\,(b^4c - 8\,ab^2c^2 + 16\,a^2c^3)de^4x^3 + (b^5 - 8\,ab^3c + 16\,a^2bc^2 + 6\,(b^4c -} \right.$$
$$- \frac{2\,(b^2c - 4\,ac^2)e^2fx^2 + 4\,(b^2c - 4\,ac^2)defx - 4\,(c^2e^4fx^4 + 4\,c^2de^3fx^3 + (6}{2\,((b^4c - 8\,ab^2c^2 + 16\,a^2c^3)e^5x^4 + 4\,(b^4c - 8\,ab^2c^2 + 16\,a^2c^3)de^4x^3 + (b^5 - 8\,ab^3c + 16\,a^2bc^2 + 6\,(b^4c -}$$

input
```
integrate((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")
```

output
```
[-1/2*(2*(b^2*c - 4*a*c^2)*e^2*f*x^2 + 4*(b^2*c - 4*a*c^2)*d*e*f*x + 2*(c^
2*e^4*f*x^4 + 4*c^2*d*e^3*f*x^3 + (6*c^2*d^2 + b*c)*e^2*f*x^2 + 2*(2*c^2*d
^3 + b*c*d)*e*f*x + (c^2*d^4 + b*c*d^2 + a*c)*f)*sqrt(b^2 - 4*a*c)*log((2*
c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 +
2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c - (2*c*e^2*x^2 + 4*c*d
*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4
+ (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a)) + (b^3 - 4*a
*b*c + 2*(b^2*c - 4*a*c^2)*d^2)*f)/((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e^5
*x^4 + 4*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*e^4*x^3 + (b^5 - 8*a*b^3*c +
 16*a^2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^3*x^2 + 2*(2*(
b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d
)*e^2*x + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^
2*c^3)*d^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d^2)*e), -1/2*(2*(b^2*c - 4*
a*c^2)*e^2*f*x^2 + 4*(b^2*c - 4*a*c^2)*d*e*f*x - 4*(c^2*e^4*f*x^4 + 4*c^2*
d*e^3*f*x^3 + (6*c^2*d^2 + b*c)*e^2*f*x^2 + 2*(2*c^2*d^3 + b*c*d)*e*f*x +
(c^2*d^4 + b*c*d^2 + a*c)*f)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 + 4*c
*d*e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^3 - 4*a*b*c +
 2*(b^2*c - 4*a*c^2)*d^2)*f)/((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e^5*x^4 +
 4*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*e^4*x^3 + (b^5 - 8*a*b^3*c + 16*a^
2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^3*x^2 + 2*(2*(b^4...
```

3.649.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 525 vs. $2(87) = 174$.

Time = 2.61 (sec) , antiderivative size = 525, normalized size of antiderivative = 5.36

$$\int \frac{df+efx}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = -\frac{cf\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(\frac{2dx}{e}+x^2+\frac{-16a^2c^3f\sqrt{-\frac{1}{(4ac-b^2)^3}}+8ab^2c^2f\sqrt{-\frac{1}{(4ac-b^2)^3}}-b^4cf\sqrt{-\frac{1}{(4ac-b^2)^3}}+bcf+2c^2d^2f}{2c^2e^2f}\right)}{e} + \frac{cf\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(\frac{2dx}{e}+x^2+\frac{16a^2c^3f\sqrt{-\frac{1}{(4ac-b^2)^3}}-8ab^2c^2f\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^4cf\sqrt{-\frac{1}{(4ac-b^2)^3}}+bcf+2c^2d^2f}{2c^2e^2f}\right)}{e} + \frac{bf+2cd^2f+4c}{8a^2ce-2ab^2e+8abcd^2e+8ac^2d^4e-2b^3d^2e-2b^2cd^4e+x^4\cdot(8ac^2e^5-2b^2ce^5)+x^3\cdot(32ac^2de^4-8b^2c}$$

input
```
integrate((e*f*x+d*f)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

output
```
-c*f*sqrt(-1/(4*a*c - b**2)**3)*log(2*d*x/e + x**2 + (-16*a**2*c**3*f*sqrt
(-1/(4*a*c - b**2)**3) + 8*a*b**2*c**2*f*sqrt(-1/(4*a*c - b**2)**3) - b**4
*c*f*sqrt(-1/(4*a*c - b**2)**3) + b*c*f + 2*c**2*d**2*f)/(2*c**2*e**2*f))/
e + c*f*sqrt(-1/(4*a*c - b**2)**3)*log(2*d*x/e + x**2 + (16*a**2*c**3*f*sq
rt(-1/(4*a*c - b**2)**3) - 8*a*b**2*c**2*f*sqrt(-1/(4*a*c - b**2)**3) + b*
*4*c*f*sqrt(-1/(4*a*c - b**2)**3) + b*c*f + 2*c**2*d**2*f)/(2*c**2*e**2*f)
)/e + (b*f + 2*c*d**2*f + 4*c*d*e*f*x + 2*c*e**2*f*x**2)/(8*a**2*c*e - 2*a
*b**2*e + 8*a*b*c*d**2*e + 8*a*c**2*d**4*e - 2*b**3*d**2*e - 2*b**2*c*d**4
*e + x**4*(8*a*c**2*e**5 - 2*b**2*c*e**5) + x**3*(32*a*c**2*d*e**4 - 8*b**
2*c*d*e**4) + x**2*(8*a*b*c*e**3 + 48*a*c**2*d**2*e**3 - 2*b**3*e**3 - 12*
b**2*c*d**2*e**3) + x*(16*a*b*c*d*e**2 + 32*a*c**2*d**3*e**2 - 4*b**3*d*e*
*2 - 8*b**2*c*d**3*e**2))
```

3.649.7 Maxima [F]

$$\int \frac{df+efx}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = \int \frac{efx+df}{\left((ex+d)^4c+(ex+d)^2b+a\right)^2}\,dx$$

input
```
integrate((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")
```

output
```
2*c*f*integrate(-(e*x + d)/((b^2*c - 4*a*c^2)*e^4*x^4 + 4*(b^2*c - 4*a*c^2
)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)
*d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2 + 2*(2*(b^2*c - 4*a*
c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e*x), x) - 1/2*(2*c*e^2*f*x^2 + 4*c*d*e*f*x
+ (2*c*d^2 + b)*f)/((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*
x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*
a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4
*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)
```

3.649.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(92) = 184$.

Time = 0.30 (sec) , antiderivative size = 202, normalized size of antiderivative = 2.06

$$\int \frac{df+efx}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = -\frac{2\,cf\arctan\left(\frac{2\,cd^2f+2\,(efx^2+2\,dfx)ce+bf}{\sqrt{-b^2+4\,acf}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ace}} - \frac{2\,cd^2f^3+2\,(efx^2+2\,dfx)cef^2+bf^3}{2\,\left(cd^4f^2+2\,(efx^2+2\,dfx)cd^2ef+(efx^2+2\,dfx)^2ce^2+bd^2f^2+(efx^2+2\,dfx)bef+af^2\right)(b^2e-4\,ac}$$

input
```
integrate((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

output
```
-2*c*f*arctan((2*c*d^2*f + 2*(e*f*x^2 + 2*d*f*x)*c*e + b*f)/(sqrt(-b^2 + 4
*a*c)*f))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a*c)*e) - 1/2*(2*c*d^2*f^3 + 2*(e*f
*x^2 + 2*d*f*x)*c*e*f^2 + b*f^3)/((c*d^4*f^2 + 2*(e*f*x^2 + 2*d*f*x)*c*d^2
*e*f + (e*f*x^2 + 2*d*f*x)^2*c*e^2 + b*d^2*f^2 + (e*f*x^2 + 2*d*f*x)*b*e*f
 + a*f^2)*(b^2*e - 4*a*c*e))
```

3.649.9 Mupad [B] (verification not implemented)

Time = 8.55 (sec) , antiderivative size = 442, normalized size of antiderivative = 4.51

$$\int \frac{df+efx}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{\frac{f\,\left(2\,c\,d^2+b\right)}{2\,e\,\left(4\,a\,c-b^2\right)}+\frac{2\,c\,d\,f\,x}{4\,a\,c-b^2}+\frac{c\,e\,f\,x^2}{4\,a\,c-b^2}}{a+x^2\,\left(6\,c\,d^2\,e^2+b\,e^2\right)+b\,d^2+c\,d^4+x\,\left(4\,c\,e\,d^3+2\,b\,e\,d\right)+c\,e^4\,x^4+4\,c\,d\,e^3\,x^3} + \frac{2\,c\,f\,\mathrm{atan}\left(\frac{{\left(4\,a\,c-b^2\right)}^4\,\left(x\,\left(\frac{8\,c^4\,d\,e^7\,f^2}{a\,{\left(4\,a\,c-b^2\right)}^{7/2}}-\frac{8\,b\,c^2\,f^2\,\left(b^3\,c^2\,d\,e^9-4\,a\,b\,c^3\,d\,e^9\right)}{a\,e^2\,{\left(4\,a\,c-b^2\right)}^{11/2}}\right)+x^2\,\left(\frac{4\,c^4\,e^8\,f^2}{a\,{\left(4\,a\,c-b^2\right)}^{7/2}}-\frac{4\,b\,c^2\,f^2\,\left(b^3\,c^2\,e^{10}-4\,a\,b\,c^3\,e^{10}\right)}{a\,e^2\,{\left(4\,a\,c-b^2\right)}^{11/2}}\right)+\dots}{8\,c^4\,e^6\,f^2}\right)}{e\,{\left(4\,a\,c-b^2\right)}^{3/2}}$$

input
```
int((d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x)
```

output
```
((f*(b + 2*c*d^2))/(2*e*(4*a*c - b^2)) + (2*c*d*f*x)/(4*a*c - b^2) + (c*e*
f*x^2)/(4*a*c - b^2))/(a + x^2*(b*e^2 + 6*c*d^2*e^2) + b*d^2 + c*d^4 + x*(
2*b*d*e + 4*c*d^3*e) + c*e^4*x^4 + 4*c*d*e^3*x^3) + (2*c*f*atan(((4*a*c -
b^2)^4*(x*((8*c^4*d*e^7*f^2)/(a*(4*a*c - b^2)^(7/2)) - (8*b*c^2*f^2*(b^3*c
^2*d*e^9 - 4*a*b*c^3*d*e^9))/(a*e^2*(4*a*c - b^2)^(11/2))) + x^2*((4*c^4*e
^8*f^2)/(a*(4*a*c - b^2)^(7/2)) - (4*b*c^2*f^2*(b^3*c^2*e^10 - 4*a*b*c^3*e
^10))/(a*e^2*(4*a*c - b^2)^(11/2))) + (4*c^4*d^2*e^6*f^2)/(a*(4*a*c - b^2)
^(7/2)) + (4*b*c^2*f^2*(8*a^2*c^3*e^8 - 2*a*b^2*c^2*e^8 - b^3*c^2*d^2*e^8
+ 4*a*b*c^3*d^2*e^8))/(a*e^2*(4*a*c - b^2)^(11/2))))/(8*c^4*e^6*f^2)))/(e*
(4*a*c - b^2)^(3/2))
```

3.650 $\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

3.650.1 Optimal result

Integrand size = 33, antiderivative size = 174

$$\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)ef\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{b\left(b^2-6ac\right)\operatorname{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2\left(b^2-4ac\right)^{3/2}ef}+\frac{\log(d+ex)}{a^2ef}-\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^2ef}$$

output
```
1/2*(b^2-2*a*c+b*c*(e*x+d)^2)/a/(-4*a*c+b^2)/e/f/(a+b*(e*x+d)^2+c*(e*x+d)^
4)+1/2*b*(-6*a*c+b^2)*arctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/a^2/(-
4*a*c+b^2)^(3/2)/e/f+ln(e*x+d)/a^2/e/f-1/4*ln(a+b*(e*x+d)^2+c*(e*x+d)^4)/a
^2/e/f
```

3.650.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.37

$$\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{\frac{2a\left(b^2-2ac+bc(d+ex)^2\right)}{\left(b^2-4ac\right)\left(a+(d+ex)^2\left(b+c(d+ex)^2\right)\right)}+4\log(d+ex)-\frac{\left(b^3-6abc+b^2\sqrt{b^2-4ac}-4ac\sqrt{b^2-4ac}\right)\log\left(b-\sqrt{b^2-4ac}+2c(d+ex)^2\right)}{\left(b^2-4ac\right)^{3/2}}+\frac{\left(b^3-6\right.}{}}{4a^2ef}$$

input
```
Integrate[1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

output
```
((2*a*(b^2 - 2*a*c + b*c*(d + e*x)^2))/((b^2 - 4*a*c)*(a + (d + e*x)^2*(b
+ c*(d + e*x)^2))) + 4*Log[d + e*x] - ((b^3 - 6*a*b*c + b^2*Sqrt[b^2 - 4*a
*c] - 4*a*c*Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2
])/(b^2 - 4*a*c)^(3/2) + ((b^3 - 6*a*b*c - b^2*Sqrt[b^2 - 4*a*c] + 4*a*c*S
qrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(b^2 - 4*a
*c)^(3/2))/(4*a^2*e*f)
```

3.650.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1462, 1434, 1165, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}dx$$

$$\Big\downarrow \; 1462$$

$$\frac{\int \frac{1}{(d+ex)\left(c(d+ex)^4+b(d+ex)^2+a\right)^2}d(d+ex)}{ef}$$

$$\Big\downarrow \; 1434$$

$$\frac{\int \frac{1}{(d+ex)^2\left(c(d+ex)^4+b(d+ex)^2+a\right)^2}d(d+ex)^2}{2ef}$$

$$\Big\downarrow \; 1165$$

$$\frac{\frac{-2ac+b^2+bc(d+ex)^2}{a(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)}-\frac{\int -\frac{b^2+c(d+ex)^2b-4ac}{(d+ex)^2\left(c(d+ex)^4+b(d+ex)^2+a\right)}d(d+ex)^2}{a(b^2-4ac)}}{2ef}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{b^2+c(d+ex)^2b-4ac}{(d+ex)^2\left(c(d+ex)^4+b(d+ex)^2+a\right)}d(d+ex)^2}{a(b^2-4ac)}+\frac{-2ac+b^2+bc(d+ex)^2}{a(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)}}{2ef}$$

$$\Big\downarrow \; 1200$$

$$\frac{\frac{\int \left(\frac{b^2-4ac}{a(d+ex)^2}+\frac{-c\left(b^2-4ac\right)(d+ex)^2-b\left(b^2-5ac\right)}{a\left(c(d+ex)^4+b(d+ex)^2+a\right)}\right)d(d+ex)^2}{a(b^2-4ac)}+\frac{-2ac+b^2+bc(d+ex)^2}{a(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)}}{2ef}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\frac{b\left(b^2-6ac\right)\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}+\frac{\left(b^2-4ac\right)\log\left((d+ex)^2\right)}{a}-\frac{\left(b^2-4ac\right)\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{2a}}{a(b^2-4ac)}+\frac{-2ac+b^2+bc(d+ex)^2}{a(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)}}{2ef}$$

input
```
Int[1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

output
```
((b^2 - 2*a*c + b*c*(d + e*x)^2)/(a*(b^2 - 4*a*c)*(a + b*(d + e*x)^2 + c*(
d + e*x)^4)) + ((b*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 -
4*a*c]])/(a*Sqrt[b^2 - 4*a*c]) + ((b^2 - 4*a*c)*Log[(d + e*x)^2])/a - ((b^
2 - 4*a*c)*Log[a + b*(d + e*x)^2 + c*(d + e*x)^4])/(2*a))/(a*(b^2 - 4*a*c)
))/(2*e*f)
```

3.650.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

3.650. $\int \frac{1}{(df+efx)(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.650.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.74 (sec) , antiderivative size = 403, normalized size of antiderivative = 2.32

method	result
default	$-\frac{-\frac{\frac{aebc\,x^2}{8ac-2b^2}+\frac{bcdax}{4ac-b^2}-\frac{a\left(-bc\,d^2+2ac-b^2\right)}{2e\left(4ac-b^2\right)}}{c\,x^4e^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+d^4c+2bdex+b\,d^2+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\,e^4\_Z^4+4cd\,e^3\_Z^3+\left(6c\,d^2e^2+b\,e^2\right)\_Z^2+\left(4d^3ec+2bde\right)\_\right.}}{}}{}$
risch	$\frac{-\frac{c\,x^2be}{2a\left(4ac-b^2\right)}-\frac{xbcd}{\left(4ac-b^2\right)a}+\frac{-bc\,d^2+2ac-b^2}{2ea\left(4ac-b^2\right)}}{f(c\,x^4e^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+d^4c+2bdex+b\,d^2+a)}+\frac{\ln(ex+d)}{a^2ef}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(64a^5c^3e^2f^2-48a^4b^2c^2e^2f^2+\right.\right.}\right.}{}$

input
```
int(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/a^2*((1/2*a/(4*a*c-b^2)*e*b*c*x^2+b*c*d*a/(4*a*c-b^2)*x-1/2/e*a*(-
b*c*d^2+2*a*c-b^2)/(4*a*c-b^2))/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4
*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)+1/2/(4*a*c-b^2)/e*sum((e^3*c
*(4*a*c-b^2)*_R^3+3*c*d*e^2*(4*a*c-b^2)*_R^2+e*(12*a*c^2*d^2-3*b^2*c*d^2+5
*a*b*c-b^3)*_R+4*a*c^2*d^3-b^2*c*d^3+5*a*b*c*d-b^3*d)/(2*_R^3*c*e^3+6*_R^2
*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*
c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a
)))+ln(e*x+d)/a^2/e)
```

3.650.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1178 vs. $2(164) = 328$.

Time = 0.54 (sec) , antiderivative size = 2486, normalized size of antiderivative = 14.29

$$\int \frac{1}{(df + efx)\left(a + b(d + ex)^2 + c(d + ex)^4\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas
")
```

output
```
[1/4*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(a*b^3*c - 4*a^2*b*c^2)*e^2*
x^2 + 4*(a*b^3*c - 4*a^2*b*c^2)*d*e*x + 2*(a*b^3*c - 4*a^2*b*c^2)*d^2 + ((
b^3*c - 6*a*b*c^2)*e^4*x^4 + 4*(b^3*c - 6*a*b*c^2)*d*e^3*x^3 + (b^3*c - 6*
a*b*c^2)*d^4 + (b^4 - 6*a*b^2*c + 6*(b^3*c - 6*a*b*c^2)*d^2)*e^2*x^2 + a*b
^3 - 6*a^2*b*c + (b^4 - 6*a*b^2*c)*d^2 + 2*(2*(b^3*c - 6*a*b*c^2)*d^3 + (b
^4 - 6*a*b^2*c)*d)*e*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3
*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3
+ b*c*d)*e*x + b^2 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(
b^2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 +
 b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a)) - ((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)
*e^4*x^4 + 4*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*e^3*x^3 + a*b^4 - 8*a^2*
b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^4 + (b^5 - 8*a*b
^3*c + 16*a^2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^2*x^2 +
(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d^2 + 2*(2*(b^4*c - 8*a*b^2*c^2 + 16*a^2*
c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*e*x)*log(c*e^4*x^4 + 4*c*d*
e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x +
a) + 4*((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e^4*x^4 + 4*(b^4*c - 8*a*b^2*c^
2 + 16*a^2*c^3)*d*e^3*x^3 + a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*
a*b^2*c^2 + 16*a^2*c^3)*d^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + 6*(b^4*c -
 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^2*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c...
```

3.650.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(df + efx)\left(a + b(d + ex)^2 + c(d + ex)^4\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

output
```
Timed out
```

3.650.7 Maxima [F]

$$\int \frac{1}{(df + efx)\left(a + b(d+ex)^2 + c(d+ex)^4\right)^2}\,dx = \int \frac{1}{\left((ex+d)^4c + (ex+d)^2b + a\right)^2(efx + df)}\,dx$$

input
```
integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima
")
```

output
```
1/2*(b*c*e^2*x^2 + 2*b*c*d*e*x + b*c*d^2 + b^2 - 2*a*c)/((a*b^2*c - 4*a^2*
c^2)*e^5*f*x^4 + 4*(a*b^2*c - 4*a^2*c^2)*d*e^4*f*x^3 + (a*b^3 - 4*a^2*b*c
+ 6*(a*b^2*c - 4*a^2*c^2)*d^2)*e^3*f*x^2 + 2*(2*(a*b^2*c - 4*a^2*c^2)*d^3
+ (a*b^3 - 4*a^2*b*c)*d)*e^2*f*x + ((a*b^2*c - 4*a^2*c^2)*d^4 + a^2*b^2 -
4*a^3*c + (a*b^3 - 4*a^2*b*c)*d^2)*e*f) - integrate(((b^2*c - 4*a*c^2)*e^3
*x^3 + 3*(b^2*c - 4*a*c^2)*d*e^2*x^2 + (b^2*c - 4*a*c^2)*d^3 + (b^3 - 5*a*
b*c + 3*(b^2*c - 4*a*c^2)*d^2)*e*x + (b^3 - 5*a*b*c)*d)/((b^2*c - 4*a*c^2)
*e^4*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3 -
4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 - 4*a*
b*c)*d^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e*x), x)/(a^2*f
) + log(e*x + d)/(a^2*e*f)
```

3.650.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 489 vs. $2(164) = 328$.

Time = 0.40 (sec) , antiderivative size = 489, normalized size of antiderivative = 2.81

$$\int \frac{1}{(df + efx)\left(a + b(d+ex)^2 + c(d+ex)^4\right)^2}\,dx =$$

$$-\frac{(a^2b^3ce^3f - 6\,a^3bc^2e^3f)\sqrt{b^2-4\,ac}\log\left(\left|be^2x^2 + \sqrt{b^2-4\,ac}e^2x^2 + 2\,bdex + 2\,\sqrt{b^2-4\,ac}dex + bd^2 + \sqrt{\right.\right.}{}$$

$$-\frac{\log\left(\left|ce^4x^4 + 4\,cde^3x^3 + 6\,cd^2e^2x^2 + 4\,cd^3ex + cd^4 + be^2x^2 + 2\,bdex + bd^2 + a\right|\right)}{4\,a^2ef}$$

$$+\frac{\log\left(\left|ex+d\right|\right)}{a^2ef}$$

$$+\frac{abce^2x^2 + 2\,abcdex + abcd^2 + ab^2 - 2\,a^2c}{2\,(ce^4x^4 + 4\,cde^3x^3 + 6\,cd^2e^2x^2 + 4\,cd^3ex + cd^4 + be^2x^2 + 2\,bdex + bd^2 + a)(b^2-4\,ac)a^2ef}$$

input
```
integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

output
```
-1/4*((a^2*b^3*c*e^3*f - 6*a^3*b*c^2*e^3*f)*sqrt(b^2 - 4*a*c)*log(abs(b*e^
2*x^2 + sqrt(b^2 - 4*a*c)*e^2*x^2 + 2*b*d*e*x + 2*sqrt(b^2 - 4*a*c)*d*e*x
+ b*d^2 + sqrt(b^2 - 4*a*c)*d^2 + 2*a)) - (a^2*b^3*c*e^3*f - 6*a^3*b*c^2*e
^3*f)*sqrt(b^2 - 4*a*c)*log(abs(-b*e^2*x^2 + sqrt(b^2 - 4*a*c)*e^2*x^2 - 2
*b*d*e*x + 2*sqrt(b^2 - 4*a*c)*d*e*x - b*d^2 + sqrt(b^2 - 4*a*c)*d^2 - 2*a
)))/(a^4*b^4*c*e^4*f^2 - 8*a^5*b^2*c^2*e^4*f^2 + 16*a^6*c^3*e^4*f^2) - 1/4
*log(abs(c*e^4*x^4 + 4*c*d*e^3*x^3 + 6*c*d^2*e^2*x^2 + 4*c*d^3*e*x + c*d^4
 + b*e^2*x^2 + 2*b*d*e*x + b*d^2 + a))/(a^2*e*f) + log(abs(e*x + d))/(a^2*
e*f) + 1/2*(a*b*c*e^2*x^2 + 2*a*b*c*d*e*x + a*b*c*d^2 + a*b^2 - 2*a^2*c)/(
(c*e^4*x^4 + 4*c*d*e^3*x^3 + 6*c*d^2*e^2*x^2 + 4*c*d^3*e*x + c*d^4 + b*e^2
*x^2 + 2*b*d*e*x + b*d^2 + a)*(b^2 - 4*a*c)*a^2*e*f)
```

3.650.9 Mupad [B] (verification not implemented)

Time = 14.69 (sec) , antiderivative size = 13434, normalized size of antiderivative = 77.21

$$\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x)
```

output
```
((b^2 - 2*a*c + b*c*d^2)/(2*e*(a*b^2 - 4*a^2*c)) + (b*c*e*x^2)/(2*(a*b^2 -
 4*a^2*c)) + (b*c*d*x)/(a*b^2 - 4*a^2*c))/(a*f + x^2*(b*e^2*f + 6*c*d^2*e^
2*f) + x*(4*c*d^3*e*f + 2*b*d*e*f) + b*d^2*f + c*d^4*f + c*e^4*f*x^4 + 4*c
*d*e^3*f*x^3) - (log((((a^2*e*f*(-(b^2*(6*a*c - b^2)^2)/(a^4*e^2*f^2*(4*a*
c - b^2)^3))^(1/2) - 1)*(((a^2*e*f*(-(b^2*(6*a*c - b^2)^2)/(a^4*e^2*f^2*(4
*a*c - b^2)^3))^(1/2) - 1)*((2*b*c^2*e^16*(2*b^3 - 10*a*c^2*d^2 + b^2*c*d^
2 - 10*a*b*c))/(a*f*(4*a*c - b^2)) + (b*c^2*e^16*(a^2*e*f*(-(b^2*(6*a*c -
b^2)^2)/(a^4*e^2*f^2*(4*a*c - b^2)^3))^(1/2) - 1)*(a*b + 3*b^2*d^2 + 3*b^2
*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*a*c*d*e*x))/(a^2
*f) - (2*b*c^3*e^18*x^2*(10*a*c - b^2))/(a*f*(4*a*c - b^2)) - (4*b*c^3*d*e
^17*x*(10*a*c - b^2))/(a*f*(4*a*c - b^2))))/(4*a^2*e*f) - (b*c^3*e^15*(4*b
^3 - 20*a*c^2*d^2 + 6*b^2*c*d^2 - 17*a*b*c))/(a^2*f^2*(4*a*c - b^2)^2) + (
2*b*c^4*e^17*x^2*(10*a*c - 3*b^2))/(a^2*f^2*(4*a*c - b^2)^2) + (4*b*c^4*d*
e^16*x*(10*a*c - 3*b^2))/(a^2*f^2*(4*a*c - b^2)^2)))/(4*a^2*e*f) + (b^3*c^
5*e^16*x^2)/(a^3*f^3*(4*a*c - b^2)^3) + (b^2*c^4*e^14*(b^2 - 4*a*c + b*c*d
^2))/(a^3*f^3*(4*a*c - b^2)^3) + (2*b^3*c^5*d*e^15*x)/(a^3*f^3*(4*a*c - b^
2)^3))*((b^3*c^5*e^16*x^2)/(a^3*f^3*(4*a*c - b^2)^3) - ((a^2*e*f*(-(b^2*(6
*a*c - b^2)^2)/(a^4*e^2*f^2*(4*a*c - b^2)^3))^(1/2) + 1)*(((a^2*e*f*(-(b^2
*(6*a*c - b^2)^2)/(a^4*e^2*f^2*(4*a*c - b^2)^3))^(1/2) + 1)*((b*c^2*e^16*(
a^2*e*f*(-(b^2*(6*a*c - b^2)^2)/(a^4*e^2*f^2*(4*a*c - b^2)^3))^(1/2) + ...
```

3.650. $\int \frac{1}{(df+efx)(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$

3.651 $\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

3.651.1 Optimal result

Integrand size = 33, antiderivative size = 360

$$\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = -\frac{3b^2-10ac}{2a^2(b^2-4ac)\,ef^2(d+ex)}+\frac{b^2-2ac+bc(d+ex)^2}{2a(b^2-4ac)\,ef^2(d+ex)\,(a+b(d+ex)^2+c(d+ex)^4)} - \frac{\sqrt{c}\left(3b^3-16abc+(3b^2-10ac)\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}ef^2} + \frac{\sqrt{c}\left(3b^3-16abc-(3b^2-10ac)\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}ef^2}$$

output
```
1/2*(10*a*c-3*b^2)/a^2/(-4*a*c+b^2)/e/f^2/(e*x+d)+1/2*(b^2-2*a*c+b*c*(e*x+
d)^2)/a/(-4*a*c+b^2)/e/f^2/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)-1/4*arctan(
(e*x+d)*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(3*b^3-16*a*
b*c+(-10*a*c+3*b^2)*(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(3/2)/e/f^2*2^(1/
2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b+(-4*
a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(3*b^3-16*a*b*c-(-10*a*c+3*b^2)*(-4*a*c+b^2
)^(1/2))/a^2/(-4*a*c+b^2)^(3/2)/e/f^2*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.651.2 Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 342, normalized size of antiderivative = 0.95

$$\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{-\frac{4}{d+ex}+\frac{2(d+ex)\left(b^3-3abc+b^2c(d+ex)^2-2ac^2(d+ex)^2\right)}{(-b^2+4ac)(a+b(d+ex)^2+c(d+ex)^4)}+\frac{\sqrt{2}\sqrt{c}\left(-3b^3+16abc-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+}{4a^2ef^2}$$

input
```
Integrate[1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

output
```
(-4/(d + e*x) + (2*(d + e*x)*(b^3 - 3*a*b*c + b^2*c*(d + e*x)^2 - 2*a*c^2*
(d + e*x)^2))/((-b^2 + 4*a*c)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (Sqrt
[2]*Sqrt[c]*(-3*b^3 + 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2
 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]]
)/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(3*
b^3 - 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c])*ArcTa
n[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)
^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*a^2*e*f^2)
```

3.651.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 342, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1462, 1441, 25, 1604, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$$

$$\Big\downarrow \text{1462}$$

$$\frac{\int \frac{1}{(d+ex)^2\left(c(d+ex)^4+b(d+ex)^2+a\right)^2}d(d+ex)}{ef^2}$$

$$\Big\downarrow \text{1441}$$

$$\frac{\frac{-2ac+b^2+bc(d+ex)^2}{2a(b^2-4ac)(d+ex)(a+b(d+ex)^2+c(d+ex)^4)}-\frac{\int -\frac{3b^2+3c(d+ex)^2b-10ac}{(d+ex)^2\left(c(d+ex)^4+b(d+ex)^2+a\right)}d(d+ex)}{2a(b^2-4ac)}}{ef^2}$$

$\downarrow$ 25

$$\frac{\frac{\int \frac{3b^2+3c(d+ex)^2b-10ac}{(d+ex)^2\left(c(d+ex)^4+b(d+ex)^2+a\right)}d(d+ex)}{2a(b^2-4ac)}+\frac{-2ac+b^2+bc(d+ex)^2}{2a(b^2-4ac)(d+ex)(a+b(d+ex)^2+c(d+ex)^4)}}{ef^2}$$

$\downarrow$ 1604

$$\frac{\frac{-\frac{\int \frac{c\left(3b^2-10ac\right)(d+ex)^2+b\left(3b^2-13ac\right)}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)}{a}-\frac{3b^2-10ac}{a(d+ex)}}{2a(b^2-4ac)}+\frac{-2ac+b^2+bc(d+ex)^2}{2a(b^2-4ac)(d+ex)(a+b(d+ex)^2+c(d+ex)^4)}}{ef^2}$$

$\downarrow$ 1480

$$\frac{-\frac{\frac{1}{2}c\left(-\frac{16abc}{\sqrt{b^2-4ac}}+\frac{3b^3}{\sqrt{b^2-4ac}}-10ac+3b^2\right)\int \frac{1}{c(d+ex)^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}d(d+ex)-\frac{c\left(-\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\int \frac{1}{c(d+ex)^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}d(d+e}{2\sqrt{b^2-4ac}}}{a}}{2a(b^2-4ac)}}{ef^2}$$

$\downarrow$ 218

$$\frac{-\frac{\frac{\sqrt{c}\left(-\frac{16abc}{\sqrt{b^2-4ac}}+\frac{3b^3}{\sqrt{b^2-4ac}}-10ac+3b^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(-\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}}{a}-\frac{3b^2-10ac}{a(d+ex)}}{2a(b^2-4ac)}+\frac{}{2a(b}}{ef^2}$$

input
```
Int[1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

output
```
((b^2 - 2*a*c + b*c*(d + e*x)^2)/(2*a*(b^2 - 4*a*c)*(d + e*x)*(a + b*(d +
e*x)^2 + c*(d + e*x)^4)) + (-((3*b^2 - 10*a*c)/(a*(d + e*x))) - ((Sqrt[c]*
(3*b^2 - 10*a*c + (3*b^3)/Sqrt[b^2 - 4*a*c] - (16*a*b*c)/Sqrt[b^2 - 4*a*c]
)*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2
]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(3*b^3 - 16*a*b*c - (3*b^2 - 10*
a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b
^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/a)
/(2*a*(b^2 - 4*a*c)))/(e*f^2)
```

3.651. $\int \frac{1}{(df+efx)^2(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$

3.651.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1441
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(-(d*x)^(m + 1))*(b^2 - 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p +
1)/(2*a*d*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))
  Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m
 + 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x
] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] ||
IntegerQ[m])
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

3.651. $\int \frac{1}{(df+efx)^2(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$

3.651.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.75 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.24

method	result
default	$-\frac{\frac{\frac{c\,e^{2}\left(2ac-b^{2}\right)x^{3}}{8ac-2b^{2}}+\frac{3dce\left(2ac-b^{2}\right)x^{2}}{2\left(4ac-b^{2}\right)}+\frac{\left(6a\,c^{2}d^{2}-3b^{2}c\,d^{2}+3abc-b^{3}\right)x}{8ac-2b^{2}}+\frac{d\left(2a\,c^{2}d^{2}-b^{2}c\,d^{2}+3abc-b^{3}\right)}{2e\left(4ac-b^{2}\right)}}{c\,x^{4}e^{4}+4cd\,e^{3}x^{3}+6c\,d^{2}e^{2}x^{2}+4c\,d^{3}ex+b\,e^{2}x^{2}+d^{4}c+2bdex+b\,d^{2}+a}+\frac{\_R=\text{RootOf}\left(c\,e^{4}\_Z^{4}+4cd\,e^{3}\_Z^{3}+\left(6\right.\right.}{}}{}$
risch	Expression too large to display

input
```
int(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f^2*(-1/a^2*((1/2*c*e^2*(2*a*c-b^2)/(4*a*c-b^2)*x^3+3/2*d*c*e*(2*a*c-b^2
)/(4*a*c-b^2)*x^2+1/2*(6*a*c^2*d^2-3*b^2*c*d^2+3*a*b*c-b^3)/(4*a*c-b^2)*x+
1/2*d/e*(2*a*c^2*d^2-b^2*c*d^2+3*a*b*c-b^3)/(4*a*c-b^2))/(c*e^4*x^4+4*c*d*
e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)+1/4
/(4*a*c-b^2)/e*sum((c*e^2*(10*a*c-3*b^2)*_R^2+2*d*c*e*(10*a*c-3*b^2)*_R+10
*a*c^2*d^2-3*b^2*c*d^2+13*a*b*c-3*b^3)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c
*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6
*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a))-1/a^2/e/(e*
x+d))
```

3.651.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4520 vs. $2(312) = 624$.

Time = 0.39 (sec) , antiderivative size = 4520, normalized size of antiderivative = 12.56

$$\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fric
as")
```

output
```
-1/4*(2*(3*b^2*c - 10*a*c^2)*e^4*x^4 + 8*(3*b^2*c - 10*a*c^2)*d*e^3*x^3 +
2*(3*b^2*c - 10*a*c^2)*d^4 + 2*(3*b^3 - 11*a*b*c + 6*(3*b^2*c - 10*a*c^2)*
d^2)*e^2*x^2 + 4*a*b^2 - 16*a^2*c + 2*(3*b^3 - 11*a*b*c)*d^2 + 4*(2*(3*b^2
*c - 10*a*c^2)*d^3 + (3*b^3 - 11*a*b*c)*d)*e*x + sqrt(1/2)*((a^2*b^2*c - 4
*a^3*c^2)*e^6*f^2*x^5 + 5*(a^2*b^2*c - 4*a^3*c^2)*d*e^5*f^2*x^4 + (a^2*b^3
 - 4*a^3*b*c + 10*(a^2*b^2*c - 4*a^3*c^2)*d^2)*e^4*f^2*x^3 + (10*(a^2*b^2*
c - 4*a^3*c^2)*d^3 + 3*(a^2*b^3 - 4*a^3*b*c)*d)*e^3*f^2*x^2 + (a^3*b^2 - 4
*a^4*c + 5*(a^2*b^2*c - 4*a^3*c^2)*d^4 + 3*(a^2*b^3 - 4*a^3*b*c)*d^2)*e^2*
f^2*x + ((a^2*b^2*c - 4*a^3*c^2)*d^5 + (a^2*b^3 - 4*a^3*b*c)*d^3 + (a^3*b^
2 - 4*a^4*c)*d)*e*f^2)*sqrt(-((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 6
4*a^8*c^3)*e^2*f^4*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^
3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64
*a^13*c^3)*e^4*f^8)) + 9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c
^3)/((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*f^4))*log(
-(189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*e*x - (1
89*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*d + 1/2*sqr
t(1/2)*((3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7*b^6*c^2 - 1344*a^8*b^4*c^3 +
2176*a^9*b^2*c^4 - 1280*a^10*c^5)*e^3*f^6*sqrt((81*b^8 - 918*a*b^6*c + 305
1*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c
 + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4*f^8)) - (27*b^11 - 486*a*b^9*c + ...
```

3.651.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*f*x+d*f)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

output
```
Timed out
```

3.651.7 Maxima [F]

$$\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \int \frac{1}{\left((ex+d)^4c+(ex+d)^2b+a\right)^2(efx+df)^2}\,dx$$

input
```
integrate(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxi
ma")
```

output
```
-1/2*((3*b^2*c - 10*a*c^2)*e^4*x^4 + 4*(3*b^2*c - 10*a*c^2)*d*e^3*x^3 + (3
*b^2*c - 10*a*c^2)*d^4 + (3*b^3 - 11*a*b*c + 6*(3*b^2*c - 10*a*c^2)*d^2)*e
^2*x^2 + 2*a*b^2 - 8*a^2*c + (3*b^3 - 11*a*b*c)*d^2 + 2*(2*(3*b^2*c - 10*a
*c^2)*d^3 + (3*b^3 - 11*a*b*c)*d)*e*x)/((a^2*b^2*c - 4*a^3*c^2)*e^6*f^2*x^
5 + 5*(a^2*b^2*c - 4*a^3*c^2)*d*e^5*f^2*x^4 + (a^2*b^3 - 4*a^3*b*c + 10*(a
^2*b^2*c - 4*a^3*c^2)*d^2)*e^4*f^2*x^3 + (10*(a^2*b^2*c - 4*a^3*c^2)*d^3 +
 3*(a^2*b^3 - 4*a^3*b*c)*d)*e^3*f^2*x^2 + (a^3*b^2 - 4*a^4*c + 5*(a^2*b^2*
c - 4*a^3*c^2)*d^4 + 3*(a^2*b^3 - 4*a^3*b*c)*d^2)*e^2*f^2*x + ((a^2*b^2*c
- 4*a^3*c^2)*d^5 + (a^2*b^3 - 4*a^3*b*c)*d^3 + (a^3*b^2 - 4*a^4*c)*d)*e*f^
2) - 1/2*integrate(((3*b^2*c - 10*a*c^2)*e^2*x^2 + 2*(3*b^2*c - 10*a*c^2)*
d*e*x + 3*b^3 - 13*a*b*c + (3*b^2*c - 10*a*c^2)*d^2)/((b^2*c - 4*a*c^2)*e^
4*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3 - 4*a
*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c
)*d^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e*x), x)/(a^2*f^2)
```

3.651.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1031 vs. $2(312) = 624$.

Time = 0.34 (sec) , antiderivative size = 1031, normalized size of antiderivative = 2.86

$$\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$$
$$= -\frac{\frac{b^2c}{(efx+df)ef} - \frac{2\,ac^2}{(efx+df)ef} + \frac{b^3f}{(efx+df)^3e} - \frac{3\,abcf}{(efx+df)^3e}}{2\,(a^2b^2-4\,a^3c)\left(c+\frac{bf^2}{(efx+df)^2}+\frac{af^4}{(efx+df)^4}\right)} - \frac{1}{(efx+df)a^2ef}$$
$$+ \frac{\left((3\,a^4b^7-31\,a^5b^5c+96\,a^6b^3c^2-80\,a^7bc^3)\sqrt{2\,ab+2\,\sqrt{b^2-4\,ac}a}e^4f^8+2\,(3\,a^3b^2c-10\,a^4c^2)\sqrt{2\,ab+2}\right.}{}$$
$$- \frac{\left((3\,a^4b^7-31\,a^5b^5c+96\,a^6b^3c^2-80\,a^7bc^3)\sqrt{2\,ab-2\,\sqrt{b^2-4\,ac}a}e^4f^8-2\,(3\,a^3b^2c-10\,a^4c^2)\sqrt{2\,ab-2}\right.}{}$$

input
```
integrate(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac
")
```

output
```
-1/2*(b^2*c/((e*f*x + d*f)*e*f) - 2*a*c^2/((e*f*x + d*f)*e*f) + b^3*f/((e*
f*x + d*f)^3*e) - 3*a*b*c*f/((e*f*x + d*f)^3*e))/((a^2*b^2 - 4*a^3*c)*(c +
 b*f^2/(e*f*x + d*f)^2 + a*f^4/(e*f*x + d*f)^4)) - 1/((e*f*x + d*f)*a^2*e*
f) + 1/16*((3*a^4*b^7 - 31*a^5*b^5*c + 96*a^6*b^3*c^2 - 80*a^7*b*c^3)*sqrt
(2*a*b + 2*sqrt(b^2 - 4*a*c)*a)*e^4*f^8 + 2*(3*a^3*b^2*c - 10*a^4*c^2)*sqr
t(2*a*b + 2*sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*e^2*f^4*abs(a^2*b^2*e^2
*f^4 - 4*a^3*c*e^2*f^4) - (a^2*b^2*e^2*f^4 - 4*a^3*c*e^2*f^4)^2*(3*b^3 - 1
3*a*b*c)*sqrt(2*a*b + 2*sqrt(b^2 - 4*a*c)*a))*arctan(2*sqrt(1/2)/((e*f*x +
 d*f)*e*f*sqrt((a^2*b^3*e^2*f^4 - 4*a^3*b*c*e^2*f^4 + sqrt((a^2*b^3*e^2*f^
4 - 4*a^3*b*c*e^2*f^4)^2 - 4*(a^3*b^2*e^4*f^8 - 4*a^4*c*e^4*f^8)*(a^2*b^2*
c - 4*a^3*c^2)))/(a^3*b^2*e^4*f^8 - 4*a^4*c*e^4*f^8))))/((a^5*b^2*c - 4*a^
6*c^2)*sqrt(b^2 - 4*a*c)*e^3*f^6*abs(a^2*b^2*e^2*f^4 - 4*a^3*c*e^2*f^4)*ab
s(a)) - 1/16*((3*a^4*b^7 - 31*a^5*b^5*c + 96*a^6*b^3*c^2 - 80*a^7*b*c^3)*s
qrt(2*a*b - 2*sqrt(b^2 - 4*a*c)*a)*e^4*f^8 - 2*(3*a^3*b^2*c - 10*a^4*c^2)*
sqrt(2*a*b - 2*sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*e^2*f^4*abs(a^2*b^2*
e^2*f^4 - 4*a^3*c*e^2*f^4) - (a^2*b^2*e^2*f^4 - 4*a^3*c*e^2*f^4)^2*(3*b^3
 - 13*a*b*c)*sqrt(2*a*b - 2*sqrt(b^2 - 4*a*c)*a))*arctan(2*sqrt(1/2)/((e*f*
x + d*f)*e*f*sqrt((a^2*b^3*e^2*f^4 - 4*a^3*b*c*e^2*f^4 - sqrt((a^2*b^3*e^2
*f^4 - 4*a^3*b*c*e^2*f^4)^2 - 4*(a^3*b^2*e^4*f^8 - 4*a^4*c*e^4*f^8)*(a^2*b
^2*c - 4*a^3*c^2)))/(a^3*b^2*e^4*f^8 - 4*a^4*c*e^4*f^8))))/((a^5*b^2*c ...
```

3.651. $\int \frac{1}{(df+efx)^2(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$

3.651.9 Mupad [B] (verification not implemented)

Time = 11.87 (sec) , antiderivative size = 12008, normalized size of antiderivative = 33.36

$$\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x)
```

output
```
- atan(((-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 207
7*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5
- 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c
 - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2*f^4 + 4096*a^11*c^6*e^2*f^4 + 240*a^7*
b^8*c^2*e^2*f^4 - 1280*a^8*b^6*c^3*e^2*f^4 + 3840*a^9*b^4*c^4*e^2*f^4 - 61
44*a^10*b^2*c^5*e^2*f^4 - 24*a^6*b^10*c*e^2*f^4)))^(1/2)*((-(9*b^13 - 9*b^
4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^
3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c -
b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a
^5*b^12*e^2*f^4 + 4096*a^11*c^6*e^2*f^4 + 240*a^7*b^8*c^2*e^2*f^4 - 1280*a
^8*b^6*c^3*e^2*f^4 + 3840*a^9*b^4*c^4*e^2*f^4 - 6144*a^10*b^2*c^5*e^2*f^4
- 24*a^6*b^10*c*e^2*f^4)))^(1/2)*((-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/
2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^
5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^
11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2*f^4 + 4096*a
^11*c^6*e^2*f^4 + 240*a^7*b^8*c^2*e^2*f^4 - 1280*a^8*b^6*c^3*e^2*f^4 + 384
0*a^9*b^4*c^4*e^2*f^4 - 6144*a^10*b^2*c^5*e^2*f^4 - 24*a^6*b^10*c*e^2*f^4)
))^(1/2)*(x*(256*a^10*b^13*c^2*e^14*f^10 - 6144*a^11*b^11*c^3*e^14*f^10 +
61440*a^12*b^9*c^4*e^14*f^10 - 327680*a^13*b^7*c^5*e^14*f^10 + 983040*a^14
*b^5*c^6*e^14*f^10 - 1572864*a^15*b^3*c^7*e^14*f^10 + 1048576*a^16*b*c^...
```

3.652 $\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

3.652.1 Optimal result

Integrand size = 33, antiderivative size = 228

$$\begin{aligned}\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx &= -\frac{b^2-3ac}{a^2(b^2-4ac)\,ef^3(d+ex)^2}+\frac{b^2-2ac+bc(d+ex)^2}{2a(b^2-4ac)\,ef^3(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}\\ &\quad-\frac{(b^4-6ab^2c+6a^2c^2)\,\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{a^3(b^2-4ac)^{3/2}\,ef^3}\\ &\quad-\frac{2b\log(d+ex)}{a^3ef^3}+\frac{b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{2a^3ef^3}\end{aligned}$$

output
```
(3*a*c-b^2)/a^2/(-4*a*c+b^2)/e/f^3/(e*x+d)^2+1/2*(b^2-2*a*c+b*c*(e*x+d)^2)
/a/(-4*a*c+b^2)/e/f^3/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)-(6*a^2*c^2-6*a
*b^2*c+b^4)*arctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)
^(3/2)/e/f^3-2*b*ln(e*x+d)/a^3/e/f^3+1/2*b*ln(a+b*(e*x+d)^2+c*(e*x+d)^4)/a
^3/e/f^3
```

3.652.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.26

$$\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{-\frac{a}{(d+ex)^2}+\frac{a\left(b^3-3abc+b^2c(d+ex)^2-2ac^2(d+ex)^2\right)}{(-b^2+4ac)(a+b(d+ex)^2+c(d+ex)^4)}-4b\log(d+ex)+\frac{\left(b^4-6ab^2c+6a^2c^2+b^3\sqrt{b^2-4ac}-4abc\sqrt{b^2-4ac}\right)\log\left(b-\sqrt{b^2\right.}{(b^2-4ac)^{3/2}}}{2a^3ef^3}$$

input
```
Integrate[1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

output
```
(-(a/(d + e*x)^2) + (a*(b^3 - 3*a*b*c + b^2*c*(d + e*x)^2 - 2*a*c^2*(d + e
*x)^2))/((-b^2 + 4*a*c)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) - 4*b*Log[d +
 e*x] + ((b^4 - 6*a*b^2*c + 6*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 4*a*b*c*Sq
rt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(b^2 - 4*a*
c)^(3/2) + ((-b^4 + 6*a*b^2*c - 6*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 4*a*b*
c*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(b^2 -
4*a*c)^(3/2))/(2*a^3*e*f^3)
```

3.652.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1462, 1434, 1165, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$$

$$\Big\downarrow \text{1462}$$

$$\frac{\int \frac{1}{(d+ex)^3\left(c(d+ex)^4+b(d+ex)^2+a\right)^2}d(d+ex)}{ef^3}$$

$$\Big\downarrow \text{1434}$$

$$\frac{\int \frac{1}{(d+ex)^4\left(c(d+ex)^4+b(d+ex)^2+a\right)^2}d(d+ex)^2}{2ef^3}$$

$$\Big\downarrow \text{1165}$$

$$\frac{\frac{-2ac+b^2+bc(d+ex)^2}{a(b^2-4ac)(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)}-\frac{\int -\frac{2\left(b^2+c(d+ex)^2b-3ac\right)}{(d+ex)^4\left(c(d+ex)^4+b(d+ex)^2+a\right)}d(d+ex)^2}{a(b^2-4ac)}}{2ef^3}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{2\int \frac{b^2+c(d+ex)^2b-3ac}{(d+ex)^4\left(c(d+ex)^4+b(d+ex)^2+a\right)}d(d+ex)^2}{a(b^2-4ac)}+\frac{-2ac+b^2+bc(d+ex)^2}{a(b^2-4ac)(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)}}{2ef^3}$$

$$\Big\downarrow \; 1200$$

$$\frac{\frac{2\int \left(\frac{b^2-3ac}{a(d+ex)^4}+\frac{b^4-5acb^2+c\left(b^2-4ac\right)(d+ex)^2b+3a^2c^2}{a^2\left(c(d+ex)^4+b(d+ex)^2+a\right)}+\frac{4abc-b^3}{a^2(d+ex)^2}\right)d(d+ex)^2}{a(b^2-4ac)}+\frac{-2ac+b^2+bc(d+ex)^2}{a(b^2-4ac)(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)}}{2ef^3}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{2\left(-\frac{\left(6a^2c^2-6ab^2c+b^4\right)\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}}-\frac{b\left(b^2-4ac\right)\log\left((d+ex)^2\right)}{a^2}+\frac{b\left(b^2-4ac\right)\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{2a^2}-\frac{b^2-3ac}{a(d+ex)^2}\right)}{a(b^2-4ac)}+\frac{-2\dots}{a(b^2-4ac)(d+\dots}}{2ef^3}$$

input
```
Int[1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

output
```
((b^2 - 2*a*c + b*c*(d + e*x)^2)/(a*(b^2 - 4*a*c)*(d + e*x)^2*(a + b*(d +
e*x)^2 + c*(d + e*x)^4)) + (2*(-((b^2 - 3*a*c)/(a*(d + e*x)^2)) - ((b^4 -
6*a*b^2*c + 6*a^2*c^2)*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(
a^2*Sqrt[b^2 - 4*a*c]) - (b*(b^2 - 4*a*c)*Log[(d + e*x)^2])/a^2 + (b*(b^2
- 4*a*c)*Log[a + b*(d + e*x)^2 + c*(d + e*x)^4])/(2*a^2)))/(a*(b^2 - 4*a*c
)))/(2*e*f^3)
```

3.652. $\int \frac{1}{(df+efx)^3(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$

3.652.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.652. $\int \frac{1}{(df+efx)^3(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$

3.652.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.81 (sec) , antiderivative size = 466, normalized size of antiderivative = 2.04

method	result
default	$-\frac{\frac{\frac{eac\left(2ac-b^2\right)x^2}{8ac-2b^2}+\frac{cda\left(2ac-b^2\right)x}{4ac-b^2}+\frac{a\left(2a c^2 d^2-b^2 c d^2+3abc-b^3\right)}{2e\left(4ac-b^2\right)}}{c x^4 e^4+4cd e^3 x^3+6c d^2 e^2 x^2+4c d^3 ex+b e^2 x^2+d^4 c+2bdex+b d^2+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c e^4 \_Z^4+4cd e^3 \_Z^3+\left(6c d^2 e^2+b e^2\right)\_Z^2+\left(4d^3 ec+2bde\right)\_}}{}}{}$
risch	Expression too large to display

input
```
int(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f^3*(-1/a^3*((1/2*e*a*c*(2*a*c-b^2)/(4*a*c-b^2)*x^2+c*d*a*(2*a*c-b^2)/(4
*a*c-b^2)*x+1/2/e*a*(2*a*c^2*d^2-b^2*c*d^2+3*a*b*c-b^3)/(4*a*c-b^2))/(c*e^
4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+
b*d^2+a)+1/(4*a*c-b^2)/e*sum((e^3*b*c*(-4*a*c+b^2)*_R^3+3*d*e^2*b*c*(-4*a*
c+b^2)*_R^2+e*(-12*a*b*c^2*d^2+3*b^3*c*d^2+3*a^2*c^2-5*a*b^2*c+b^4)*_R-4*a
*b*c^2*d^3+b^3*c*d^3+3*a^2*c^2*d-5*a*b^2*c*d+d*b^4)/(2*_R^3*c*e^3+6*_R^2*c
*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*
d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a))
)-1/2/a^2/e/(e*x+d)^2-2*b*ln(e*x+d)/a^3/e)
```

3.652.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2237 vs. 2(220) = 440.

Time = 1.19 (sec) , antiderivative size = 4604, normalized size of antiderivative = 20.19

$$\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fric
as")
```

output
```
[-1/2*(2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*e^4*x^4 + 8*(a*b^4*c - 7*a
^2*b^2*c^2 + 12*a^3*c^3)*d*e^3*x^3 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 +
2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d^4 + (2*a*b^5 - 15*a^2*b^3*c + 2
8*a^3*b*c^2 + 12*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d^2)*e^2*x^2 + (2*
a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*d^2 + 2*(4*(a*b^4*c - 7*a^2*b^2*c^2 +
 12*a^3*c^3)*d^3 + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*d)*e*x + ((b^4*
c - 6*a*b^2*c^2 + 6*a^2*c^3)*e^6*x^6 + 6*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)
*d*e^5*x^5 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2 + 15*(b^4*c - 6*a*b^2*c^2 + 6*
a^2*c^3)*d^2)*e^4*x^4 + (b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^6 + 4*(5*(b^4*
c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^3 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d)*e^3*
x^3 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d^4 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c
^2 + 15*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^4 + 6*(b^5 - 6*a*b^3*c + 6*a^2
*b*c^2)*d^2)*e^2*x^2 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*d^2 + 2*(3*(b^4*c
 - 6*a*b^2*c^2 + 6*a^2*c^3)*d^5 + 2*(b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d^3 +
(a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*d)*e*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*e^4
*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d
^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x +
2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*
d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a)) - ((b^5*c - 8*a*b^3
*c^2 + 16*a^2*b*c^3)*e^6*x^6 + 6*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d...
```

3.652.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx=\text{Timed out}$$

input
```
integrate(1/(e*f*x+d*f)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

output
```
Timed out
```

3.652.7 Maxima [F]

$$\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$$
$$=\int \frac{1}{\left((ex+d)^4c+(ex+d)^2b+a\right)^2(efx+df)^3}\,dx$$

input
```
integrate(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxi
ma")
```

output
```
-1/2*(2*(b^2*c - 3*a*c^2)*e^4*x^4 + 8*(b^2*c - 3*a*c^2)*d*e^3*x^3 + 2*(b^2
*c - 3*a*c^2)*d^4 + (2*b^3 - 7*a*b*c + 12*(b^2*c - 3*a*c^2)*d^2)*e^2*x^2 +
 a*b^2 - 4*a^2*c + (2*b^3 - 7*a*b*c)*d^2 + 2*(4*(b^2*c - 3*a*c^2)*d^3 + (2
*b^3 - 7*a*b*c)*d)*e*x)/((a^2*b^2*c - 4*a^3*c^2)*e^7*f^3*x^6 + 6*(a^2*b^2*
c - 4*a^3*c^2)*d*e^6*f^3*x^5 + (a^2*b^3 - 4*a^3*b*c + 15*(a^2*b^2*c - 4*a^
3*c^2)*d^2)*e^5*f^3*x^4 + 4*(5*(a^2*b^2*c - 4*a^3*c^2)*d^3 + (a^2*b^3 - 4*
a^3*b*c)*d)*e^4*f^3*x^3 + (a^3*b^2 - 4*a^4*c + 15*(a^2*b^2*c - 4*a^3*c^2)*
d^4 + 6*(a^2*b^3 - 4*a^3*b*c)*d^2)*e^3*f^3*x^2 + 2*(3*(a^2*b^2*c - 4*a^3*c
^2)*d^5 + 2*(a^2*b^3 - 4*a^3*b*c)*d^3 + (a^3*b^2 - 4*a^4*c)*d)*e^2*f^3*x +
 ((a^2*b^2*c - 4*a^3*c^2)*d^6 + (a^2*b^3 - 4*a^3*b*c)*d^4 + (a^3*b^2 - 4*a
^4*c)*d^2)*e*f^3) + 2*integrate(((b^3*c - 4*a*b*c^2)*e^3*x^3 + 3*(b^3*c -
4*a*b*c^2)*d*e^2*x^2 + (b^3*c - 4*a*b*c^2)*d^3 + (b^4 - 5*a*b^2*c + 3*a^2*
c^2 + 3*(b^3*c - 4*a*b*c^2)*d^2)*e*x + (b^4 - 5*a*b^2*c + 3*a^2*c^2)*d)/((
b^2*c - 4*a*c^2)*e^4*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^3*x^3 + (b^2*c - 4*a*c^
2)*d^4 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^2*x^2 + a*b^2 - 4*a^2
*c + (b^3 - 4*a*b*c)*d^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)
*e*x), x)/(a^3*f^3) - 2*b*log(e*x + d)/(a^3*e*f^3)
```

3.652.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 705 vs. $2(220) = 440$.

Time = 0.41 (sec) , antiderivative size = 705, normalized size of antiderivative = 3.09

$$\int \frac{1}{(df+efx)^3\,(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$$
$$= \frac{(a^3b^4ce^3f^3 - 6\,a^4b^2c^2e^3f^3 + 6\,a^5c^3e^3f^3)\sqrt{b^2-4\,ac}\log\left(\left|be^2x^2 + \sqrt{b^2-4\,ac}e^2x^2 + 2\,bdex + 2\,\sqrt{b^2-4\,ac}d\right.\right.}{}$$
$$+ \frac{b\log\left(\left|ce^4x^4 + 4\,cde^3x^3 + 6\,cd^2e^2x^2 + 4\,cd^3ex + cd^4 + be^2x^2 + 2\,bdex + bd^2 + a\right|\right)}{2\,a^3ef^3}$$
$$- \frac{2\,b\log\left(\left|ex+d\right|\right)}{a^3ef^3}$$
$$- \frac{2\,ab^2cd^4 - 6\,a^2c^2d^4 + 2\,ab^3d^2 - 7\,a^2bcd^2 + 2\,(ab^2ce^4 - 3\,a^2c^2e^4)x^4 + a^2b^2 - 4\,a^3c + 8\,(ab^2cde^3 - 3\,a^2c^2d}{2\,(ce^4x^4 + 4\,cde^3x^3 + 6\,cd^2e^2x^2 + 4\,cd^3ex + cd^4}$$

input
```
integrate(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac
")
```

output
```
1/2*((a^3*b^4*c*e^3*f^3 - 6*a^4*b^2*c^2*e^3*f^3 + 6*a^5*c^3*e^3*f^3)*sqrt(
b^2 - 4*a*c)*log(abs(b*e^2*x^2 + sqrt(b^2 - 4*a*c)*e^2*x^2 + 2*b*d*e*x + 2
*sqrt(b^2 - 4*a*c)*d*e*x + b*d^2 + sqrt(b^2 - 4*a*c)*d^2 + 2*a)) - (a^3*b^
4*c*e^3*f^3 - 6*a^4*b^2*c^2*e^3*f^3 + 6*a^5*c^3*e^3*f^3)*sqrt(b^2 - 4*a*c)
*log(abs(-b*e^2*x^2 + sqrt(b^2 - 4*a*c)*e^2*x^2 - 2*b*d*e*x + 2*sqrt(b^2 -
 4*a*c)*d*e*x - b*d^2 + sqrt(b^2 - 4*a*c)*d^2 - 2*a)))/(a^6*b^4*c*e^4*f^6
- 8*a^7*b^2*c^2*e^4*f^6 + 16*a^8*c^3*e^4*f^6) + 1/2*b*log(abs(c*e^4*x^4 +
4*c*d*e^3*x^3 + 6*c*d^2*e^2*x^2 + 4*c*d^3*e*x + c*d^4 + b*e^2*x^2 + 2*b*d*
e*x + b*d^2 + a))/(a^3*e*f^3) - 2*b*log(abs(e*x + d))/(a^3*e*f^3) - 1/2*(2
*a*b^2*c*d^4 - 6*a^2*c^2*d^4 + 2*a*b^3*d^2 - 7*a^2*b*c*d^2 + 2*(a*b^2*c*e^
4 - 3*a^2*c^2*e^4)*x^4 + a^2*b^2 - 4*a^3*c + 8*(a*b^2*c*d*e^3 - 3*a^2*c^2*
d*e^3)*x^3 + (12*a*b^2*c*d^2*e^2 - 36*a^2*c^2*d^2*e^2 + 2*a*b^3*e^2 - 7*a^
2*b*c*e^2)*x^2 + 2*(4*a*b^2*c*d^3*e - 12*a^2*c^2*d^3*e + 2*a*b^3*d*e - 7*a
^2*b*c*d*e)*x)/((c*e^4*x^4 + 4*c*d*e^3*x^3 + 6*c*d^2*e^2*x^2 + 4*c*d^3*e*x
 + c*d^4 + b*e^2*x^2 + 2*b*d*e*x + b*d^2 + a)*(b^2 - 4*a*c)*(e*x + d)^2*a^
3*e*f^3)
```

3.652.9 Mupad [B] (verification not implemented)

Time = 15.94 (sec) , antiderivative size = 14830, normalized size of antiderivative = 65.04

$$\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x)
```

output
```
((x*(2*b^3*d - 12*a*c^2*d^3 + 4*b^2*c*d^3 - 7*a*b*c*d))/(4*a^3*c - a^2*b^2
) - (x^4*(3*a*c^2*e^3 - b^2*c*e^3))/(4*a^3*c - a^2*b^2) - (4*x^3*(3*a*c^2*
d*e^2 - b^2*c*d*e^2))/(4*a^3*c - a^2*b^2) + (a*b^2 - 4*a^2*c + 2*b^3*d^2 -
 6*a*c^2*d^4 + 2*b^2*c*d^4 - 7*a*b*c*d^2)/(2*e*(4*a^3*c - a^2*b^2)) + (x^2
*(2*b^3*e - 36*a*c^2*d^2*e + 12*b^2*c*d^2*e - 7*a*b*c*e))/(2*(4*a^3*c - a^
2*b^2)))/(x^3*(20*c*d^3*e^3*f^3 + 4*b*d*e^3*f^3) + x*(2*a*d*e*f^3 + 4*b*d^
3*e*f^3 + 6*c*d^5*e*f^3) + x^4*(b*e^4*f^3 + 15*c*d^2*e^4*f^3) + x^2*(a*e^2
*f^3 + 6*b*d^2*e^2*f^3 + 15*c*d^4*e^2*f^3) + a*d^2*f^3 + b*d^4*f^3 + c*d^6
*f^3 + c*e^6*f^3*x^6 + 6*c*d*e^5*f^3*x^5) + (log((((b + a^3*e*f^3*(-(b^4 +
 6*a^2*c^2 - 6*a*b^2*c)^2/(a^6*e^2*f^6*(4*a*c - b^2)^3))^(1/2))*(((b + a^3
*e*f^3*(-(b^4 + 6*a^2*c^2 - 6*a*b^2*c)^2/(a^6*e^2*f^6*(4*a*c - b^2)^3))^(1
/2))*((4*c^2*e^16*(2*b^5 + 6*a^2*b*c^2 + b^4*c*d^2 - 30*a^2*c^3*d^2 - 10*a
*b^3*c + 2*a*b^2*c^2*d^2))/(a^2*f^3*(4*a*c - b^2)) + (4*c^3*e^18*x^2*(b^4
- 30*a^2*c^2 + 2*a*b^2*c))/(a^2*f^3*(4*a*c - b^2)) - (2*b*c^2*e^16*(b + a^
3*e*f^3*(-(b^4 + 6*a^2*c^2 - 6*a*b^2*c)^2/(a^6*e^2*f^6*(4*a*c - b^2)^3))^(
1/2))*(a*b + 3*b^2*d^2 + 3*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c
*e^2*x^2 - 20*a*c*d*e*x))/(a^3*f^3) + (8*c^3*d*e^17*x*(b^4 - 30*a^2*c^2 +
2*a*b^2*c))/(a^2*f^3*(4*a*c - b^2))))/(2*a^3*e*f^3) - (4*c^3*e^15*(3*a*c -
 b^2)*(4*b^4 + 3*a^2*c^2 + 6*b^3*c*d^2 - 17*a*b^2*c - 23*a*b*c^2*d^2))/(a^
4*f^6*(4*a*c - b^2)^2) + (4*b*c^4*e^17*x^2*(6*b^4 + 69*a^2*c^2 - 41*a*b...
```

3.653 $\int \frac{1}{(df+efx)^4\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

3.653.1 Optimal result

Integrand size = 33, antiderivative size = 423

$$\begin{aligned}&\int \frac{1}{(df+efx)^4\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx\\&=-\frac{5b^2-14ac}{6a^2(b^2-4ac)\,ef^4(d+ex)^3}+\frac{b(5b^2-19ac)}{2a^3(b^2-4ac)\,ef^4(d+ex)}\\&\quad+\frac{b^2-2ac+bc(d+ex)^2}{2a(b^2-4ac)\,ef^4(d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)}\\&\quad+\frac{\sqrt{c}\left(5b^4-29ab^2c+28a^2c^2+b(5b^2-19ac)\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^3(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}ef^4}\\&\quad-\frac{\sqrt{c}\left(5b^4-29ab^2c+28a^2c^2-b(5b^2-19ac)\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^3(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}ef^4}\end{aligned}$$

output
```
1/6*(14*a*c-5*b^2)/a^2/(-4*a*c+b^2)/e/f^4/(e*x+d)^3+1/2*b*(-19*a*c+5*b^2)/
a^3/(-4*a*c+b^2)/e/f^4/(e*x+d)+1/2*(b^2-2*a*c+b*c*(e*x+d)^2)/a/(-4*a*c+b^2
)/e/f^4/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)+1/4*arctan((e*x+d)*2^(1/2)*c
^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(5*b^4-29*a*b^2*c+28*a^2*c^2+
b*(-19*a*c+5*b^2)*(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)^(3/2)/e/f^4*2^(1/2)
/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b+(-4*a*
c+b^2)^(1/2))^(1/2))*c^(1/2)*(5*b^4-29*a*b^2*c+28*a^2*c^2-b*(-19*a*c+5*b^2
)*(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)^(3/2)/e/f^4*2^(1/2)/(b+(-4*a*c+b^2)
^(1/2))^(1/2)
```

3.653.2 Mathematica [A] (verified)

Time = 1.73 (sec) , antiderivative size = 387, normalized size of antiderivative = 0.91

$$\int \frac{1}{(df+efx)^4 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2} \, dx$$
$$= \frac{-\frac{4a}{(d+ex)^3}+\frac{24b}{d+ex}+\frac{6(d+ex)\left(b^4-4ab^2c+2a^2c^2+b^3c(d+ex)^2-3abc^2(d+ex)^2\right)}{(b^2-4ac)\left(a+(d+ex)^2\left(b+c(d+ex)^2\right)\right)}+\frac{3\sqrt{2}\sqrt{c}\left(5b^4-29ab^2c+28a^2c^2+5b^3\sqrt{b^2-4ac}-19abc\sqrt{b^2-\right.}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}}{12a^3ef^4}$$

input

```
Integrate[1/((d*f + e*f*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

output

```
((-4*a)/(d + e*x)^3 + (24*b)/(d + e*x) + (6*(d + e*x)*(b^4 - 4*a*b^2*c + 2
*a^2*c^2 + b^3*c*(d + e*x)^2 - 3*a*b*c^2*(d + e*x)^2))/((b^2 - 4*a*c)*(a +
 (d + e*x)^2*(b + c*(d + e*x)^2))) + (3*Sqrt[2]*Sqrt[c]*(5*b^4 - 29*a*b^2*
c + 28*a^2*c^2 + 5*b^3*Sqrt[b^2 - 4*a*c] - 19*a*b*c*Sqrt[b^2 - 4*a*c])*Arc
Tan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*
c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt[c]*(-5*b^4 + 29*a*
b^2*c - 28*a^2*c^2 + 5*b^3*Sqrt[b^2 - 4*a*c] - 19*a*b*c*Sqrt[b^2 - 4*a*c])
*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 -
4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(12*a^3*e*f^4)
```

3.653.3 Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 397, normalized size of antiderivative = 0.94, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {1462, 1441, 25, 1604, 27, 1604, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(df+efx)^4 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2} \, dx$$
$$\Big\downarrow \ 1462$$
$$\frac{\int \frac{1}{(d+ex)^4\left(c(d+ex)^4+b(d+ex)^2+a\right)^2}d(d+ex)}{ef^4}$$
$$\Big\downarrow \ 1441$$

$$\frac{\frac{-2ac+b^2+bc(d+ex)^2}{2a(b^2-4ac)(d+ex)^3(a+b(d+ex)^2+c(d+ex)^4)}-\frac{\int -\frac{5b^2+5c(d+ex)^2b-14ac}{(d+ex)^4\left(c(d+ex)^4+b(d+ex)^2+a\right)}d(d+ex)}{2a(b^2-4ac)}}{ef^4}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{5b^2+5c(d+ex)^2b-14ac}{(d+ex)^4\left(c(d+ex)^4+b(d+ex)^2+a\right)}d(d+ex)}{2a(b^2-4ac)}+\frac{-2ac+b^2+bc(d+ex)^2}{2a(b^2-4ac)(d+ex)^3(a+b(d+ex)^2+c(d+ex)^4)}}{ef^4}$$

$\Big\downarrow$ 1604

$$\frac{\frac{-\frac{\int \frac{3\left(c\left(5b^2-14ac\right)(d+ex)^2+b\left(5b^2-19ac\right)\right)}{(d+ex)^2\left(c(d+ex)^4+b(d+ex)^2+a\right)}d(d+ex)}{3a}-\frac{5b^2-14ac}{3a(d+ex)^3}}{2a(b^2-4ac)}+\frac{-2ac+b^2+bc(d+ex)^2}{2a(b^2-4ac)(d+ex)^3(a+b(d+ex)^2+c(d+ex)^4)}}{ef^4}$$

$\Big\downarrow$ 27

$$\frac{\frac{-\frac{\int \frac{c\left(5b^2-14ac\right)(d+ex)^2+b\left(5b^2-19ac\right)}{(d+ex)^2\left(c(d+ex)^4+b(d+ex)^2+a\right)}d(d+ex)}{a}-\frac{5b^2-14ac}{3a(d+ex)^3}}{2a(b^2-4ac)}+\frac{-2ac+b^2+bc(d+ex)^2}{2a(b^2-4ac)(d+ex)^3(a+b(d+ex)^2+c(d+ex)^4)}}{ef^4}$$

$\Big\downarrow$ 1604

$$\frac{\frac{-\frac{-\frac{\int \frac{5b^4-24acb^2+c\left(5b^2-19ac\right)(d+ex)^2b+14a^2c^2}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)}{a}-\frac{b\left(5b^2-19ac\right)}{a(d+ex)}}{a}-\frac{5b^2-14ac}{3a(d+ex)^3}}{2a(b^2-4ac)}+\frac{-2ac+b^2+bc(d+ex)^2}{2a(b^2-4ac)(d+ex)^3(a+b(d+ex)^2+c(d+ex)^4)}}{ef^4}$$

$\Big\downarrow$ 1480

$$\frac{\frac{-\frac{-\frac{\frac{c\left(28a^2c^2-29ab^2c+b\left(5b^2-19ac\right)\sqrt{b^2-4ac}+5b^4\right)\int \frac{1}{c(d+ex)^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}d(d+ex)}{2\sqrt{b^2-4ac}}-\frac{c\left(28a^2c^2-29ab^2c-b\left(5b^2-19ac\right)\sqrt{b^2-4ac}+5b^4\right)\int \frac{1}{c(d+ex)^2+\frac{1}{2}\left(b\ldots\right.}}{2\sqrt{b^2-4ac}}}{a}}{a}}{2a(b^2-4ac)}}{ef^4}$$

$\Big\downarrow$ 218

$$\frac{\frac{-\frac{-\frac{\frac{\sqrt{c}\left(28a^2c^2-29ab^2c+b\left(5b^2-19ac\right)\sqrt{b^2-4ac}+5b^4\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(28a^2c^2-29ab^2c-b\left(5b^2-19ac\right)\sqrt{b^2-4ac}+5b^4\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}}{a}}{a}}{2a(b^2-4ac)}}{ef^4}$$

3.653. $\displaystyle\int \frac{1}{(df+efx)^4(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$

input
```
Int[1/((d*f + e*f*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

output
```
((b^2 - 2*a*c + b*c*(d + e*x)^2)/(2*a*(b^2 - 4*a*c)*(d + e*x)^3*(a + b*(d
+ e*x)^2 + c*(d + e*x)^4)) + (-1/3*(5*b^2 - 14*a*c)/(a*(d + e*x)^3) - (-((
b*(5*b^2 - 19*a*c))/(a*(d + e*x))) - ((Sqrt[c]*(5*b^4 - 29*a*b^2*c + 28*a^
2*c^2 + b*(5*b^2 - 19*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d +
 e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[b - S
qrt[b^2 - 4*a*c]]) - (Sqrt[c]*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 - b*(5*b^2
- 19*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + S
qrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]
))/a)/a)/(2*a*(b^2 - 4*a*c)))/(e*f^4)
```

3.653.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1441
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(-(d*x)^(m + 1))*(b^2 - 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p +
1)/(2*a*d*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))
  Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m
 + 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x
] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] ||
IntegerQ[m])
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

3.653. $\int \frac{1}{(df+efx)^4(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

3.653.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.80 (sec) , antiderivative size = 493, normalized size of antiderivative = 1.17

method	result
default	$-\frac{-\frac{\frac{-\frac{b c e^{2}\left(3 a c-b^{2}\right) x^{3}}{2\left(4 a c-b^{2}\right)}-\frac{3 d b c e\left(3 a c-b^{2}\right) x^{2}}{2\left(4 a c-b^{2}\right)}+\frac{\left(-9 b c^{2} d^{2} a+3 b^{3} c d^{2}+2 a^{2} c^{2}-4 a b^{2} c+b^{4}\right) x}{8 a c-2 b^{2}}+\frac{d\left(-3 b c^{2} d^{2} a+b^{3} c d^{2}+2 a^{2} c^{2}-4 a b^{2} c+b^{4}\right)}{2 e\left(4 a c-b^{2}\right)}}{c x^{4} e^{4}+4 c d e^{3} x^{3}+6 c d^{2} e^{2} x^{2}+4 c d^{3} e x+b e^{2} x^{2}+d^{4} c+2 b d e x+b d^{2}+a}+\frac{\_R=\mathrm{RootO}}{}}{}}{}$
risch	Expression too large to display

input
```
int(1/(e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f^4*(-1/a^3*((-1/2*b*c*e^2*(3*a*c-b^2)/(4*a*c-b^2)*x^3-3/2*d*b*c*e*(3*a*
c-b^2)/(4*a*c-b^2)*x^2+1/2*(-9*a*b*c^2*d^2+3*b^3*c*d^2+2*a^2*c^2-4*a*b^2*c
+b^4)/(4*a*c-b^2)*x+1/2*d/e*(-3*a*b*c^2*d^2+b^3*c*d^2+2*a^2*c^2-4*a*b^2*c+
b^4)/(4*a*c-b^2))/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e
^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)+1/4/(4*a*c-b^2)/e*sum((b*c*e^2*(-19*a*c+5*
b^2)*_R^2+2*b*c*d*e*(-19*a*c+5*b^2)*_R-19*b*c^2*d^2a+5*b^3*c*d^2+14*a^2*c
^2-24*a*b^2*c+5*b^4)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*
b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*
_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a))-1/3/a^2/e/(e*x+d)^3+2/a^3*b/e
/(e*x+d))
```

3.653.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5954 vs. 2(373) = 746.

Time = 0.49 (sec) , antiderivative size = 5954, normalized size of antiderivative = 14.08

$$\int \frac{1}{(df+efx)^4\,(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fric
as")
```

output
```
Too large to include
```

3.653.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(df+efx)^4\,(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*f*x+d*f)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

output
```
Timed out
```

3.653.7 Maxima [F]

$$\int \frac{1}{(df+efx)^4\,(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = \int \frac{1}{\left((ex+d)^4c+(ex+d)^2b+a\right)^2(efx+df)^4}\,dx$$

input
```
integrate(1/(e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxi
ma")
```

output
```
1/6*(3*(5*b^3*c - 19*a*b*c^2)*e^6*x^6 + 18*(5*b^3*c - 19*a*b*c^2)*d*e^5*x^
5 + (15*b^4 - 62*a*b^2*c + 14*a^2*c^2 + 45*(5*b^3*c - 19*a*b*c^2)*d^2)*e^4
*x^4 + 3*(5*b^3*c - 19*a*b*c^2)*d^6 + 4*(15*(5*b^3*c - 19*a*b*c^2)*d^3 + (
15*b^4 - 62*a*b^2*c + 14*a^2*c^2)*d)*e^3*x^3 + (15*b^4 - 62*a*b^2*c + 14*a
^2*c^2)*d^4 + (45*(5*b^3*c - 19*a*b*c^2)*d^4 + 10*a*b^3 - 40*a^2*b*c + 6*(
15*b^4 - 62*a*b^2*c + 14*a^2*c^2)*d^2)*e^2*x^2 - 2*a^2*b^2 + 8*a^3*c + 10*
(a*b^3 - 4*a^2*b*c)*d^2 + 2*(9*(5*b^3*c - 19*a*b*c^2)*d^5 + 2*(15*b^4 - 62
*a*b^2*c + 14*a^2*c^2)*d^3 + 10*(a*b^3 - 4*a^2*b*c)*d)*e*x)/((a^3*b^2*c -
4*a^4*c^2)*e^8*f^4*x^7 + 7*(a^3*b^2*c - 4*a^4*c^2)*d*e^7*f^4*x^6 + (a^3*b^
3 - 4*a^4*b*c + 21*(a^3*b^2*c - 4*a^4*c^2)*d^2)*e^6*f^4*x^5 + 5*(7*(a^3*b^
2*c - 4*a^4*c^2)*d^3 + (a^3*b^3 - 4*a^4*b*c)*d)*e^5*f^4*x^4 + (a^4*b^2 - 4
*a^5*c + 35*(a^3*b^2*c - 4*a^4*c^2)*d^4 + 10*(a^3*b^3 - 4*a^4*b*c)*d^2)*e^
4*f^4*x^3 + (21*(a^3*b^2*c - 4*a^4*c^2)*d^5 + 10*(a^3*b^3 - 4*a^4*b*c)*d^3
 + 3*(a^4*b^2 - 4*a^5*c)*d)*e^3*f^4*x^2 + (7*(a^3*b^2*c - 4*a^4*c^2)*d^6 +
 5*(a^3*b^3 - 4*a^4*b*c)*d^4 + 3*(a^4*b^2 - 4*a^5*c)*d^2)*e^2*f^4*x + ((a^
3*b^2*c - 4*a^4*c^2)*d^7 + (a^3*b^3 - 4*a^4*b*c)*d^5 + (a^4*b^2 - 4*a^5*c)
*d^3)*e*f^4) + 1/2*integrate(((5*b^3*c - 19*a*b*c^2)*e^2*x^2 + 5*b^4 - 24*
a*b^2*c + 14*a^2*c^2 + 2*(5*b^3*c - 19*a*b*c^2)*d*e*x + (5*b^3*c - 19*a*b*
c^2)*d^2)/((b^2*c - 4*a*c^2)*e^4*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^3*x^3 + (b^
2*c - 4*a*c^2)*d^4 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^2*x^2 ...
```

3.653.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2137 vs. $2(373) = 746$.

Time = 0.32 (sec) , antiderivative size = 2137, normalized size of antiderivative = 5.05

$$\int \frac{1}{(df+efx)^4\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac
")
```

output
```
-1/4*((5*b^3*c*e^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4
)) + d/e)^2 - 19*a*b*c^2*e^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e
^2)/(c*e^4)) + d/e)^2 - 10*b^3*c*d*e*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 -
4*a*c)*e^2)/(c*e^4)) + d/e) + 38*a*b*c^2*d*e*(sqrt(1/2)*sqrt(-(b*e^2 + sqr
t(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e) + 5*b^3*c*d^2 - 19*a*b*c^2*d^2 + 5*b^4
 - 24*a*b^2*c + 14*a^2*c^2)*log(x + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*
a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*
a*c)*e^2)/(c*e^4)) + d/e)^3 - 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2
 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^
2)*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)) - (5*
b^3*c*e^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)
^2 - 19*a*b*c^2*e^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^
4)) - d/e)^2 + 10*b^3*c*d*e*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^
2)/(c*e^4)) - d/e) - 38*a*b*c^2*d*e*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4
*a*c)*e^2)/(c*e^4)) - d/e) + 5*b^3*c*d^2 - 19*a*b*c^2*d^2 + 5*b^4 - 24*a*b
^2*c + 14*a^2*c^2)*log(x - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)
/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)
/(c*e^4)) - d/e)^3 + 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)
*e^2)/(c*e^4)) - d/e)^2 + 2*c*d^3*e + b*d*e + (6*c*d^2*e^2 + b*e^2)*(sqrt(
1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)) + (5*b^3*c*...
```

3.653.9 Mupad [B] (verification not implemented)

Time = 14.00 (sec) , antiderivative size = 13781, normalized size of antiderivative = 32.58

$$\int \frac{1}{(df+efx)^4\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/((d*f + e*f*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x)
```

output
```
atan(((-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 636
6*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c
^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13
*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2
)^9)^(1/2))/(32*(a^7*b^12*e^2*f^8 + 4096*a^13*c^6*e^2*f^8 + 240*a^9*b^8*c^
2*e^2*f^8 - 1280*a^10*b^6*c^3*e^2*f^8 + 3840*a^11*b^4*c^4*e^2*f^8 - 6144*a
^12*b^2*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2*f^8)))^(1/2)*((-(25*b^15 - 25*b^6*
(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3
*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 +
49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*
a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*
e^2*f^8 + 4096*a^13*c^6*e^2*f^8 + 240*a^9*b^8*c^2*e^2*f^8 - 1280*a^10*b^6*
c^3*e^2*f^8 + 3840*a^11*b^4*c^4*e^2*f^8 - 6144*a^12*b^2*c^5*e^2*f^8 - 24*a
^8*b^10*c*e^2*f^8)))^(1/2)*((-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) -
 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*
c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)
^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*
a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2*f^8 + 4096*a^13*c^6*e^
2*f^8 + 240*a^9*b^8*c^2*e^2*f^8 - 1280*a^10*b^6*c^3*e^2*f^8 + 3840*a^11*b^
4*c^4*e^2*f^8 - 6144*a^12*b^2*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2*f^8)))^(1...
```

3.653. $\int \frac{1}{(df+efx)^4(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$

3.654 $\int \frac{(df+efx)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3} \, dx$

3.654.1 Optimal result

Integrand size = 33, antiderivative size = 353

$$\begin{aligned}
\int \frac{(df+efx)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3} \, dx &= \frac{f^4(d+ex)\left(2a+b(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} \\
&\quad - \frac{f^4(d+ex)\left(7b^2-4ac+12bc(d+ex)^2\right)}{8\left(b^2-4ac\right)^2 e\left(a+b(d+ex)^2+c(d+ex)^4\right)} \\
&\quad + \frac{3\sqrt{c}\left(3b^2+4ac-2b\sqrt{b^2-4ac}\right)f^4\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{4\sqrt{2}\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}e} \\
&\quad - \frac{3\sqrt{c}\left(3b^2+4ac+2b\sqrt{b^2-4ac}\right)f^4\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{4\sqrt{2}\left(b^2-4ac\right)^{5/2}\sqrt{b+\sqrt{b^2-4ac}}e}
\end{aligned}$$

output
```
1/4*f^4*(e*x+d)*(2*a+b*(e*x+d)^2)/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^
4)^2-1/8*f^4*(e*x+d)*(7*b^2-4*a*c+12*b*c*(e*x+d)^2)/(-4*a*c+b^2)^2/e/(a+b*
(e*x+d)^2+c*(e*x+d)^4)+3/8*f^4*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b-(-4*a*c+b
^2)^(1/2))^(1/2))*c^(1/2)*(3*b^2+4*a*c-2*b*(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2
)^(5/2)/e*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-3/8*f^4*arctan((e*x+d)*2^(1
/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(3*b^2+4*a*c+2*b*(-4*a*c
+b^2)^(1/2))/(-4*a*c+b^2)^(5/2)/e*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.654.2 Mathematica [A] (verified)

Time = 2.61 (sec) , antiderivative size = 331, normalized size of antiderivative = 0.94

$$\int \frac{(df+efx)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$$
$$= \frac{f^4\left(-\frac{2(-2a(d+ex)-b(d+ex)^3)}{(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}+\frac{(d+ex)(-7b^2+4ac-12bc(d+ex)^2)}{(b^2-4ac)^2(a+b(d+ex)^2+c(d+ex)^4)}+\frac{3\sqrt{2}\sqrt{c}\left(3b^2+4ac-2b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right.}{(b^2-4ac)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}\right.}{8e}$$

input

```
Integrate[(d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

output

```
(f^4*((-2*(-2*a*(d + e*x) - b*(d + e*x)^3))/((b^2 - 4*a*c)*(a + b*(d + e*x
)^2 + c*(d + e*x)^4)^2) + ((d + e*x)*(-7*b^2 + 4*a*c - 12*b*c*(d + e*x)^2)
)/((b^2 - 4*a*c)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (3*Sqrt[2]*Sqrt[
c]*(3*b^2 + 4*a*c - 2*b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*
x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 -
 4*a*c]]) - (3*Sqrt[2]*Sqrt[c]*(3*b^2 + 4*a*c + 2*b*Sqrt[b^2 - 4*a*c])*Arc
Tan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*
c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])))/(8*e)
```

3.654.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 343, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1462, 1440, 1492, 27, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(df+efx)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$$
$$\Big\downarrow \text{ 1462}$$
$$\frac{f^4\int \frac{(d+ex)^4}{\left(c(d+ex)^4+b(d+ex)^2+a\right)^3}d(d+ex)}{e}$$
$$\Big\downarrow \text{ 1440}$$

$$\frac{f^4\left(\frac{(d+ex)(2a+b(d+ex)^2)}{4(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}-\frac{\int \frac{2a-5b(d+ex)^2}{\left(c(d+ex)^4+b(d+ex)^2+a\right)^2}d(d+ex)}{4(b^2-4ac)}\right)}{e}$$

$\Big\downarrow$ 1492

$$\frac{f^4\left(\frac{(d+ex)(2a+b(d+ex)^2)}{4(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}-\frac{\frac{(d+ex)\left(-4ac+7b^2+12bc(d+ex)^2\right)}{2\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{\int \frac{3a\left(b^2-4c(d+ex)^2b+4ac\right)}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)}{2a\left(b^2-4ac\right)}}{4(b^2-4ac)}\right)}{e}$$

$\Big\downarrow$ 27

$$\frac{f^4\left(\frac{(d+ex)(2a+b(d+ex)^2)}{4(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}-\frac{\frac{(d+ex)\left(-4ac+7b^2+12bc(d+ex)^2\right)}{2\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{3\int \frac{b^2-4c(d+ex)^2b+4ac}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)}{2\left(b^2-4ac\right)}}{4(b^2-4ac)}\right)}{e}$$

$\Big\downarrow$ 1480

$$\frac{f^4\left(\frac{(d+ex)(2a+b(d+ex)^2)}{4(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}-\frac{\frac{(d+ex)\left(-4ac+7b^2+12bc(d+ex)^2\right)}{2\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{3\left(-c\left(2b-\frac{4ac+3b^2}{\sqrt{b^2-4ac}}\right)\int \frac{1}{c(d+ex)^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}d(d+ex)-c\left(\frac{4ac}{\sqrt{\ldots}}\right.\ldots\right)}{2\left(b^2-4ac\right)}}{4(b^2-4ac)}\ldots\right)}{e}$$

$\Big\downarrow$ 218

$$\frac{f^4\left(\frac{(d+ex)(2a+b(d+ex)^2)}{4(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}-\frac{\frac{(d+ex)\left(-4ac+7b^2+12bc(d+ex)^2\right)}{2\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{3\left(-\frac{\sqrt{2}\sqrt{c}\left(2b-\frac{4ac+3b^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\left(\frac{4ac+\ldots}{\sqrt{b^2\ldots}}\right.\ldots}{\ldots}\right)}{2\left(b^2-4ac\right)}}{4(b^2-4ac)}\ldots\right)}{e}$$

input

```
Int[(d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

3.654. $\int \frac{(df+efx)^4}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx$

output
```
(f^4*(((d + e*x)*(2*a + b*(d + e*x)^2))/(4*(b^2 - 4*a*c)*(a + b*(d + e*x)^
2 + c*(d + e*x)^4)^2) - (((d + e*x)*(7*b^2 - 4*a*c + 12*b*c*(d + e*x)^2))/
(2*(b^2 - 4*a*c)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) - (3*(-((Sqrt[2]*Sqr
t[c]*(2*b - (3*b^2 + 4*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d
+ e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt
[2]*Sqrt[c]*(2*b + (3*b^2 + 4*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt
[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/Sqrt[b + Sqrt[b^2 - 4*a*c]]))
/(2*(b^2 - 4*a*c)))/(4*(b^2 - 4*a*c))))/e
```

3.654.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1440
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(-d^3)*(d*x)^(m - 3)*(2*a + b*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*
(p + 1)*(b^2 - 4*a*c))), x] + Simp[d^4/(2*(p + 1)*(b^2 - 4*a*c))   Int[(d*x
)^(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1),
x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Gt
Q[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

3.654. $\int \frac{(df+efx)^4}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx$

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

3.654.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.69 (sec) , antiderivative size = 708, normalized size of antiderivative = 2.01

method	result
default	$f^4 \left(\frac{-\frac{3c^2e^6bx^7}{2(16a^2c^2-8ab^2c+b^4)}-\frac{21c^2de^5bx^6}{2(16a^2c^2-8ab^2c+b^4)}+\frac{(-252bcd^2+4ac-19b^2)ce^4x^5}{128a^2c^2-64ab^2c+8b^4}+\frac{5cde^3(-84bcd^2+4ac-19b^2)x^4}{8(16a^2c^2-8ab^2c+b^4)}-\frac{e^2(420bc^2d^4-40a}{8(16}}{} \right.$
risch	$\frac{-\frac{3c^2e^6bf^4x^7}{2(16a^2c^2-8ab^2c+b^4)}-\frac{21c^2de^5bf^4x^6}{2(16a^2c^2-8ab^2c+b^4)}+\frac{(-252bcd^2+4ac-19b^2)ce^4f^4x^5}{128a^2c^2-64ab^2c+8b^4}+\frac{5cde^3f^4(-84bcd^2+4ac-19b^2)x^4}{8(16a^2c^2-8ab^2c+b^4)}-\frac{e^2f^4(420bc^2d^4-40a}{8(16}}{}$

input
```
int((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x,method=_RETURNVERBOSE)
```

output
```
f^4*((-3/2*c^2*e^6*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x^7-21/2*c^2*d*e^5*b/(16*a
^2*c^2-8*a*b^2*c+b^4)*x^6+1/8*(-252*b*c*d^2+4*a*c-19*b^2)*c*e^4/(16*a^2*c^
2-8*a*b^2*c+b^4)*x^5+5/8*c*d*e^3*(-84*b*c*d^2+4*a*c-19*b^2)/(16*a^2*c^2-8*
a*b^2*c+b^4)*x^4-1/8*e^2*(420*b*c^2*d^4-40*a*c^2*d^2+190*b^2*c*d^2+16*a*b*
c+5*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3-1/8*d*e*(252*b*c^2*d^4-40*a*c^2*d^
2+190*b^2*c*d^2+48*a*b*c+15*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2-1/8*(84*b*
c^2*d^6-20*a*c^2*d^4+95*b^2*c*d^4+48*a*b*c*d^2+15*b^3*d^2+12*a^2*c+3*a*b^2
)/(16*a^2*c^2-8*a*b^2*c+b^4)*x-1/8*d/e*(12*b*c^2*d^6-4*a*c^2*d^4+19*b^2*c*
d^4+16*a*b*c*d^2+5*b^3*d^2+12*a^2*c+3*a*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4))/(
c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*
e*x+b*d^2+a)^2+3/16/(16*a^2*c^2-8*a*b^2*c+b^4)/e*sum((-4*_R^2*b*c*e^2-8*_R
*b*c*d*e-4*b*c*d^2+4*a*c+b^2)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*
c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^
2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a))
```

3.654.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6770 vs. $2(305) = 610$.

Time = 0.46 (sec) , antiderivative size = 6770, normalized size of antiderivative = 19.18

$$\int \frac{(df+efx)^4}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas
")
```

output
```
Too large to include
```

3.654.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(df+efx)^4}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = \text{Timed out}$$

input
```
integrate((e*f*x+d*f)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

output
```
Timed out
```

3.654.7 Maxima [F]

$$\int \frac{(df+efx)^4}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = \int \frac{(efx+df)^4}{\left((ex+d)^4c+(ex+d)^2b+a\right)^3}\,dx$$

input
```
integrate((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima
")
```

output
```
-3/8*f^4*integrate((4*b*c*e^2*x^2 + 8*b*c*d*e*x + 4*b*c*d^2 - b^2 - 4*a*c)
/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2
*c*d^3 + b*d)*e*x + a), x)/(b^4 - 8*a*b^2*c + 16*a^2*c^2) - 1/8*(12*b*c^2*
e^7*f^4*x^7 + 84*b*c^2*d*e^6*f^4*x^6 + (252*b*c^2*d^2 + 19*b^2*c - 4*a*c^2
)*e^5*f^4*x^5 + 5*(84*b*c^2*d^3 + (19*b^2*c - 4*a*c^2)*d)*e^4*f^4*x^4 + (4
20*b*c^2*d^4 + 5*b^3 + 16*a*b*c + 10*(19*b^2*c - 4*a*c^2)*d^2)*e^3*f^4*x^3
 + (252*b*c^2*d^5 + 10*(19*b^2*c - 4*a*c^2)*d^3 + 3*(5*b^3 + 16*a*b*c)*d)*
e^2*f^4*x^2 + (84*b*c^2*d^6 + 5*(19*b^2*c - 4*a*c^2)*d^4 + 3*a*b^2 + 12*a^
2*c + 3*(5*b^3 + 16*a*b*c)*d^2)*e*f^4*x + (12*b*c^2*d^7 + (19*b^2*c - 4*a*
c^2)*d^5 + (5*b^3 + 16*a*b*c)*d^3 + 3*(a*b^2 + 4*a^2*c)*d)*f^4)/((b^4*c^2
- 8*a*b^2*c^3 + 16*a^2*c^4)*e^9*x^8 + 8*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^
4)*d*e^8*x^7 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3 + 14*(b^4*c^2 - 8*a*b
^2*c^3 + 16*a^2*c^4)*d^2)*e^7*x^6 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*
c^4)*d^3 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d)*e^6*x^5 + (b^6 - 6*a*
b^4*c + 32*a^3*c^3 + 70*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^4 + 30*(b^5
*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e^5*x^4 + 4*(14*(b^4*c^2 - 8*a*b^2*c
^3 + 16*a^2*c^4)*d^5 + 10*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6
- 6*a*b^4*c + 32*a^3*c^3)*d)*e^4*x^3 + 2*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a
^2*c^4)*d^6 + a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2
 + 16*a^2*b*c^3)*d^4 + 3*(b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^2)*e^3*x^2 + ...
```

3.654.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1958 vs. $2(305) = 610$.

Time = 0.33 (sec) , antiderivative size = 1958, normalized size of antiderivative = 5.55

$$\int \frac{(df+efx)^4}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

output
```
3/16*((4*b*c*e^2*f^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e
^4)) + d/e)^2 - 8*b*c*d*e*f^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*
e^2)/(c*e^4)) + d/e) + 4*b*c*d^2*f^4 - b^2*f^4 - 4*a*c*f^4)*log(x + sqrt(1
/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1
/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^3 - 6*c*d*e^3*(s
qrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 - 2*c*d^3
*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*
a*c)*e^2)/(c*e^4)) + d/e)) - (4*b*c*e^2*f^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt
(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2 + 8*b*c*d*e*f^4*(sqrt(1/2)*sqrt(-(b*e
^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e) + 4*b*c*d^2*f^4 - b^2*f^4 - 4*
a*c*f^4)*log(x - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4))
+ d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4))
- d/e)^3 + 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e
^4)) - d/e)^2 + 2*c*d^3*e + b*d*e + (6*c*d^2*e^2 + b*e^2)*(sqrt(1/2)*sqrt(
-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)) + (4*b*c*e^2*f^4*(sqrt(1
/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 - 8*b*c*d*e*f^
4*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e) + 4*b*c
*d^2*f^4 - b^2*f^4 - 4*a*c*f^4)*log(x + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2
- 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2
- 4*a*c)*e^2)/(c*e^4)) + d/e)^3 - 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 - s...
```

3.654.9 Mupad [B] (verification not implemented)

Time = 12.21 (sec) , antiderivative size = 13840, normalized size of antiderivative = 39.21

$$\int \frac{(df+efx)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \text{Too large to display}$$

input
```
int((d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x)
```

output
```
atan(((-(9*(b^15*f^8 + f^8*(-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7*f^8
 - 560*a^2*b^11*c^2*f^8 + 4160*a^3*b^9*c^3*f^8 - 11520*a^4*b^7*c^4*f^8 - 1
024*a^5*b^5*c^5*f^8 + 61440*a^6*b^3*c^6*f^8 + 20*a*b^13*c*f^8))/(512*(a*b^
20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2
- 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2
 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*
e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2)*((((1024*b^15*c^2*d*e^13 - 28672*a
*b^13*c^3*d*e^13 - 16777216*a^7*b*c^9*d*e^13 + 344064*a^2*b^11*c^4*d*e^13
- 2293760*a^3*b^9*c^5*d*e^13 + 9175040*a^4*b^7*c^6*d*e^13 - 22020096*a^5*b
^5*c^7*d*e^13 + 29360128*a^6*b^3*c^8*d*e^13)/(128*(b^12 + 4096*a^6*c^6 + 2
40*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 -
24*a*b^10*c)) + (x*(128*b^11*c^2*e^14 - 2560*a*b^9*c^3*e^14 - 131072*a^5*b
*c^7*e^14 + 20480*a^2*b^7*c^4*e^14 - 81920*a^3*b^5*c^5*e^14 + 163840*a^4*b
^3*c^6*e^14))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 -
16*a*b^6*c)))*(-(9*(b^15*f^8 + f^8*(-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b
*c^7*f^8 - 560*a^2*b^11*c^2*f^8 + 4160*a^3*b^9*c^3*f^8 - 11520*a^4*b^7*c^4
*f^8 - 1024*a^5*b^5*c^5*f^8 + 61440*a^6*b^3*c^6*f^8 + 20*a*b^13*c*f^8))/(5
12*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*
c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10
*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a...
```

3.654. $\int \frac{(df+efx)^4}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx$

3.655 $\int \frac{(df+efx)^3}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx$

3.655.1 Optimal result

Integrand size = 33, antiderivative size = 159

$$\int \frac{(df+efx)^3}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = \frac{f^3(2a+b(d+ex)^2)}{4(b^2-4ac)e(a+b(d+ex)^2+c(d+ex)^4)^2} - \frac{3bf^3(b+2c(d+ex)^2)}{4(b^2-4ac)^2e(a+b(d+ex)^2+c(d+ex)^4)} + \frac{3bcf^3\operatorname{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}e}$$

output

```
1/4*f^3*(2*a+b*(e*x+d)^2)/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2-3/4
*b*f^3*(b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^2/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)+3*b*
c*f^3*arctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(5/2)/e
```

3.655.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.94

$$\int \frac{(df+efx)^3}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = \frac{f^3\left(-\frac{3b(b+2c(d+ex)^2)}{a+b(d+ex)^2+c(d+ex)^4} + \frac{(b^2-4ac)(2a+b(d+ex)^2)}{(a+(d+ex)^2(b+c(d+ex)^2))^2} - \frac{12bc\arctan\left(\frac{b+2c(d+ex)^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}\right)}{4(b^2-4ac)^2e}$$

input
```
Integrate[(d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

output
```
(f^3*((-3*b*(b + 2*c*(d + e*x)^2))/(a + b*(d + e*x)^2 + c*(d + e*x)^4) + (
(b^2 - 4*a*c)*(2*a + b*(d + e*x)^2))/(a + (d + e*x)^2*(b + c*(d + e*x)^2))
^2 - (12*b*c*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 +
 4*a*c]))/(4*(b^2 - 4*a*c)^2*e)
```

3.655.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1462, 1434, 1159, 1086, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(df+efx)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}dx$$

$$\Big\downarrow \text{1462}$$

$$\frac{f^3\int \frac{(d+ex)^3}{\left(c(d+ex)^4+b(d+ex)^2+a\right)^3}d(d+ex)}{e}$$

$$\Big\downarrow \text{1434}$$

$$\frac{f^3\int \frac{(d+ex)^2}{\left(c(d+ex)^4+b(d+ex)^2+a\right)^3}d(d+ex)^2}{2e}$$

$$\Big\downarrow \text{1159}$$

$$\frac{f^3\left(\frac{3b\int \frac{1}{\left(c(d+ex)^4+b(d+ex)^2+a\right)^2}d(d+ex)^2}{2(b^2-4ac)}+\frac{2a+b(d+ex)^2}{2(b^2-4ac)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\right)}{2e}$$

$$\Big\downarrow \text{1086}$$

$$\frac{f^3\left(\frac{3b\left(-\frac{2c\int \frac{1}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)^2}{b^2-4ac}-\frac{b+2c(d+ex)^2}{(b^2-4ac)\left(a+b(d+ex)^2+c(d+ex)^4\right)}\right)}{2(b^2-4ac)}+\frac{2a+b(d+ex)^2}{2(b^2-4ac)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\right)}{2e}$$

$$\Big\downarrow \text{1083}$$

$$\frac{f^3\left(\frac{3b\left(\frac{4c\int\frac{1}{-(d+ex)^4+b^2-4ac}d\left(2c(d+ex)^2+b\right)}{b^2-4ac}-\frac{b+2c(d+ex)^2}{\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}\right)}{2(b^2-4ac)}+\frac{2a+b(d+ex)^2}{2(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}\right)}{2e}$$

$$\Big\downarrow \; 219$$

$$\frac{f^3\left(\frac{3b\left(\frac{4c\,\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{b+2c(d+ex)^2}{\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}\right)}{2(b^2-4ac)}+\frac{2a+b(d+ex)^2}{2(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}\right)}{2e}$$

input
```
Int[(d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

output
```
(f^3*((2*a + b*(d + e*x)^2)/(2*(b^2 - 4*a*c)*(a + b*(d + e*x)^2 + c*(d + e
*x)^4)^2) + (3*b*(-((b + 2*c*(d + e*x)^2)/((b^2 - 4*a*c)*(a + b*(d + e*x)^
2 + c*(d + e*x)^4))) + (4*c*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c
]])/(b^2 - 4*a*c)^(3/2)))/(2*(b^2 - 4*a*c))))/(2*e)
```

3.655.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

3.655. $\int \frac{(df+efx)^3}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx$

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

3.655.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.74 (sec) , antiderivative size = 548, normalized size of antiderivative = 3.45

method	result
default	$f^3 \left(\frac{-\frac{3c^2e^5bx^6}{2(16a^2c^2-8ab^2c+b^4)}-\frac{9e^4bc^2dx^5}{16a^2c^2-8ab^2c+b^4}-\frac{9bce^3(10cd^2+b)x^4}{4(16a^2c^2-8ab^2c+b^4)}-\frac{3cde^2b(10cd^2+3b)x^3}{16a^2c^2-8ab^2c+b^4}-\frac{be(45c^2d^4+27bcd^2+5ac+b^2)x^2}{2(16a^2c^2-8ab^2c+b^4)}-\frac{db(9}{}}{(cx^4e^4+4cde^3x^3+6cd^2e^2x^2+4cd^3ex+be^2x^2+d^4c+2bdex+bd^2}\right.$
risch	$\frac{-\frac{3c^2e^5bf^3x^6}{2(16a^2c^2-8ab^2c+b^4)}-\frac{9f^3e^4bc^2dx^5}{16a^2c^2-8ab^2c+b^4}-\frac{9bce^3f^3(10cd^2+b)x^4}{4(16a^2c^2-8ab^2c+b^4)}-\frac{3cde^2bf^3(10cd^2+3b)x^3}{16a^2c^2-8ab^2c+b^4}-\frac{bef^3(45c^2d^4+27bcd^2+5ac+b^2)x^2}{2(16a^2c^2-8ab^2c+b^4)}-\frac{dbf^3}{}}{(cx^4e^4+4cde^3x^3+6cd^2e^2x^2+4cd^3ex+be^2x^2+d^4c+2bdex+bd^2}$

input
```
int((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x,method=_RETURNVERBOSE)
```

output
```
f^3*((-3/2*c^2*e^5*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6-9*e^4*b*c^2*d/(16*a^2*
c^2-8*a*b^2*c+b^4)*x^5-9/4*b*c*e^3*(10*c*d^2+b)/(16*a^2*c^2-8*a*b^2*c+b^4)
*x^4-3*c*d*e^2*b*(10*c*d^2+3*b)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3-1/2*b*e*(45
*c^2*d^4+27*b*c*d^2+5*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2-d*b*(9*c^2*d
^4+9*b*c*d^2+5*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x-1/4/e*(6*b*c^2*d^6+9*
b^2*c*d^4+10*a*b*c*d^2+2*b^3*d^2+8*a^2*c+a*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)
)/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b
*d*e*x+b*d^2+a)^2+3/2*b*c/(16*a^2*c^2-8*a*b^2*c+b^4)/e*sum((-_R*e-d)/(2*_R
^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootO
f(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_
Z+d^4*c+b*d^2+a)))
```

3.655.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1856 vs. $2(151) = 302$.

Time = 0.39 (sec) , antiderivative size = 3843, normalized size of antiderivative = 24.17

$$\int \frac{(df+efx)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas
")
```

output
```
[-1/4*(6*(b^3*c^2 - 4*a*b*c^3)*e^6*f^3*x^6 + 36*(b^3*c^2 - 4*a*b*c^3)*d*e^
5*f^3*x^5 + 9*(b^4*c - 4*a*b^2*c^2 + 10*(b^3*c^2 - 4*a*b*c^3)*d^2)*e^4*f^3
*x^4 + 12*(10*(b^3*c^2 - 4*a*b*c^3)*d^3 + 3*(b^4*c - 4*a*b^2*c^2)*d)*e^3*f
^3*x^3 + 2*(b^5 + a*b^3*c - 20*a^2*b*c^2 + 45*(b^3*c^2 - 4*a*b*c^3)*d^4 +
27*(b^4*c - 4*a*b^2*c^2)*d^2)*e^2*f^3*x^2 + 4*(9*(b^3*c^2 - 4*a*b*c^3)*d^5
 + 9*(b^4*c - 4*a*b^2*c^2)*d^3 + (b^5 + a*b^3*c - 20*a^2*b*c^2)*d)*e*f^3*x
 + (6*(b^3*c^2 - 4*a*b*c^3)*d^6 + a*b^4 + 4*a^2*b^2*c - 32*a^3*c^2 + 9*(b^
4*c - 4*a*b^2*c^2)*d^4 + 2*(b^5 + a*b^3*c - 20*a^2*b*c^2)*d^2)*f^3 - 6*(b*
c^3*e^8*f^3*x^8 + 8*b*c^3*d*e^7*f^3*x^7 + 2*(14*b*c^3*d^2 + b^2*c^2)*e^6*f
^3*x^6 + 4*(14*b*c^3*d^3 + 3*b^2*c^2*d)*e^5*f^3*x^5 + (70*b*c^3*d^4 + 30*b
^2*c^2*d^2 + b^3*c + 2*a*b*c^2)*e^4*f^3*x^4 + 4*(14*b*c^3*d^5 + 10*b^2*c^2
*d^3 + (b^3*c + 2*a*b*c^2)*d)*e^3*f^3*x^3 + 2*(14*b*c^3*d^6 + 15*b^2*c^2*d
^4 + a*b^2*c + 3*(b^3*c + 2*a*b*c^2)*d^2)*e^2*f^3*x^2 + 4*(2*b*c^3*d^7 + 3
*b^2*c^2*d^5 + a*b^2*c*d + (b^3*c + 2*a*b*c^2)*d^3)*e*f^3*x + (b*c^3*d^8 +
 2*b^2*c^2*d^6 + 2*a*b^2*c*d^2 + (b^3*c + 2*a*b*c^2)*d^4 + a^2*b*c)*f^3)*s
qrt(b^2 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c
^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*
c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^4*x^4
+ 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d
)*e*x + a)))/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^...
```

3.655.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1794 vs. $2(144) = 288$.

Time = 7.32 (sec) , antiderivative size = 1794, normalized size of antiderivative = 11.28

$$\int \frac{(df+efx)^3}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*f*x+d*f)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

output
```
3*b*c*f**3*sqrt(-1/(4*a*c - b**2)**5)*log(2*d*x/e + x**2 + (-192*a**3*b*c*
*4*f**3*sqrt(-1/(4*a*c - b**2)**5) + 144*a**2*b**3*c**3*f**3*sqrt(-1/(4*a*
c - b**2)**5) - 36*a*b**5*c**2*f**3*sqrt(-1/(4*a*c - b**2)**5) + 3*b**7*c*
f**3*sqrt(-1/(4*a*c - b**2)**5) + 3*b**2*c*f**3 + 6*b*c**2*d**2*f**3)/(6*b
*c**2*e**2*f**3))/(2*e) - 3*b*c*f**3*sqrt(-1/(4*a*c - b**2)**5)*log(2*d*x/
e + x**2 + (192*a**3*b*c**4*f**3*sqrt(-1/(4*a*c - b**2)**5) - 144*a**2*b**
3*c**3*f**3*sqrt(-1/(4*a*c - b**2)**5) + 36*a*b**5*c**2*f**3*sqrt(-1/(4*a*
c - b**2)**5) - 3*b**7*c*f**3*sqrt(-1/(4*a*c - b**2)**5) + 3*b**2*c*f**3 +
 6*b*c**2*d**2*f**3)/(6*b*c**2*e**2*f**3))/(2*e) + (-8*a**2*c*f**3 - a*b**
2*f**3 - 10*a*b*c*d**2*f**3 - 2*b**3*d**2*f**3 - 9*b**2*c*d**4*f**3 - 6*b*
c**2*d**6*f**3 - 36*b*c**2*d*e**5*f**3*x**5 - 6*b*c**2*e**6*f**3*x**6 + x*
*4*(-9*b**2*c*e**4*f**3 - 90*b*c**2*d**2*e**4*f**3) + x**3*(-36*b**2*c*d*e
**3*f**3 - 120*b*c**2*d**3*e**3*f**3) + x**2*(-10*a*b*c*e**2*f**3 - 2*b**3
*e**2*f**3 - 54*b**2*c*d**2*e**2*f**3 - 90*b*c**2*d**4*e**2*f**3) + x*(-20
*a*b*c*d*e*f**3 - 4*b**3*d*e*f**3 - 36*b**2*c*d**3*e*f**3 - 36*b*c**2*d**5
*e*f**3))/(64*a**4*c**2*e - 32*a**3*b**2*c*e + 128*a**3*b*c**2*d**2*e + 12
8*a**3*c**3*d**4*e + 4*a**2*b**4*e - 64*a**2*b**3*c*d**2*e + 128*a**2*b*c*
*3*d**6*e + 64*a**2*c**4*d**8*e + 8*a*b**5*d**2*e - 24*a*b**4*c*d**4*e - 6
4*a*b**3*c**2*d**6*e - 32*a*b**2*c**3*d**8*e + 4*b**6*d**4*e + 8*b**5*c*d*
*6*e + 4*b**4*c**2*d**8*e + x**8*(64*a**2*c**4*e**9 - 32*a*b**2*c**3*e*...
```

3.655.7 Maxima [F]

$$\int \frac{(df+efx)^3}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = \int \frac{(efx+df)^3}{\left((ex+d)^4c+(ex+d)^2b+a\right)^3}\,dx$$

input
```
integrate((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima
")
```

output
```
-3*b*c*f^3*integrate((e*x + d)/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d
^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a), x)/(b^4 - 8*a*b^2*c
+ 16*a^2*c^2) - 1/4*(6*b*c^2*e^6*f^3*x^6 + 36*b*c^2*d*e^5*f^3*x^5 + 9*(10*
b*c^2*d^2 + b^2*c)*e^4*f^3*x^4 + 12*(10*b*c^2*d^3 + 3*b^2*c*d)*e^3*f^3*x^3
 + 2*(45*b*c^2*d^4 + 27*b^2*c*d^2 + b^3 + 5*a*b*c)*e^2*f^3*x^2 + 4*(9*b*c^
2*d^5 + 9*b^2*c*d^3 + (b^3 + 5*a*b*c)*d)*e*f^3*x + (6*b*c^2*d^6 + 9*b^2*c*
d^4 + a*b^2 + 8*a^2*c + 2*(b^3 + 5*a*b*c)*d^2)*f^3)/((b^4*c^2 - 8*a*b^2*c^
3 + 16*a^2*c^4)*e^9*x^8 + 8*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*e^8*x^7
 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3 + 14*(b^4*c^2 - 8*a*b^2*c^3 + 16*
a^2*c^4)*d^2)*e^7*x^6 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^3 + 3
*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d)*e^6*x^5 + (b^6 - 6*a*b^4*c + 32*a
^3*c^3 + 70*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^4 + 30*(b^5*c - 8*a*b^3
*c^2 + 16*a^2*b*c^3)*d^2)*e^5*x^4 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*
c^4)*d^5 + 10*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 6*a*b^4*c
+ 32*a^3*c^3)*d)*e^4*x^3 + 2*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^6
+ a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*
c^3)*d^4 + 3*(b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^2)*e^3*x^2 + 4*(2*(b^4*c^2 -
 8*a*b^2*c^3 + 16*a^2*c^4)*d^7 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^
5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c
^2)*d)*e^2*x + ((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^8 + 2*(b^5*c - 8...
```

3.655.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 431 vs. $2(151) = 302$.

Time = 0.33 (sec) , antiderivative size = 431, normalized size of antiderivative = 2.71

$$\int \frac{(df+efx)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = -\frac{3\,bcf^3\arctan\left(\frac{2\,cd^2f+2\,(efx^2+2\,dfx)ce+bf}{\sqrt{-b^2+4\,ac}f}\right)}{(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-b^2+4\,ace}} - \frac{6\,bc^2d^6f^7+18\,(efx^2+2\,dfx)bc^2d^4ef^6+18\,(efx^2+2\,dfx)^2bc^2d^2e^2f^5+9\,b^2cd^4f^7+6\,(efx^2+2\,dfx)^3b\cdots}{4\left(cd^4f^2+2\,(efx^2+2\,dfx)cd^2e\cdots\right.}$$

input
```
integrate((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

output
```
-3*b*c*f^3*arctan((2*c*d^2*f + 2*(e*f*x^2 + 2*d*f*x)*c*e + b*f)/(sqrt(-b^2
 + 4*a*c)*f))/((b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)*e) - 1/4*
(6*b*c^2*d^6*f^7 + 18*(e*f*x^2 + 2*d*f*x)*b*c^2*d^4*e*f^6 + 18*(e*f*x^2 +
2*d*f*x)^2*b*c^2*d^2*e^2*f^5 + 9*b^2*c*d^4*f^7 + 6*(e*f*x^2 + 2*d*f*x)^3*b
*c^2*e^3*f^4 + 18*(e*f*x^2 + 2*d*f*x)*b^2*c*d^2*e*f^6 + 9*(e*f*x^2 + 2*d*f
*x)^2*b^2*c*e^2*f^5 + 2*b^3*d^2*f^7 + 10*a*b*c*d^2*f^7 + 2*(e*f*x^2 + 2*d*
f*x)*b^3*e*f^6 + 10*(e*f*x^2 + 2*d*f*x)*a*b*c*e*f^6 + a*b^2*f^7 + 8*a^2*c*
f^7)/((c*d^4*f^2 + 2*(e*f*x^2 + 2*d*f*x)*c*d^2*e*f + (e*f*x^2 + 2*d*f*x)^2
*c*e^2 + b*d^2*f^2 + (e*f*x^2 + 2*d*f*x)*b*e*f + a*f^2)^2*(b^4*e - 8*a*b^2
*c*e + 16*a^2*c^2*e))
```

3.655.9 Mupad [B] (verification not implemented)

Time = 9.80 (sec) , antiderivative size = 1267, normalized size of antiderivative = 7.97

$$\int \frac{(df+efx)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx =$$

$$-\frac{\frac{8\,a^2\,c\,f^3+a\,b^2\,f^3+10\,a\,b\,c\,d^2\,f^3+2\,b^3\,d^2\,f^3+9\,b^2\,c\,d^4\,f}{4\,e\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}}{x^2\,\left(6\,b^2\,d^2\,e^2+30\,b\,c\,d^4\,e^2+2\,a\,b\,e^2+28\,c^2\,d^6\,e^2+12\,a\,c\,d^2\,e^2\right)+x^6\,\left(28\,c^2\,d^2\,e^6+2\,b\,c\,e^6\right)+x\,\left(4\,e\,b^2\,d^3\right.}$$

$$-\frac{3\,b\,c\,f^3\,\mathrm{atan}\left(\frac{\left(b^4\,\left(4\,a\,c-b^2\right)^5+16\,a^2\,c^2\,\left(4\,a\,c-b^2\right)^5-8\,a\,b^2\,c\,\left(4\,a\,c-b^2\right)^5\right)\,\left(x^2\,\left(\frac{9\,b^2\,c^4\,e^8\,f^6}{a\,\left(4\,a\,c-b^2\right)^{9/2}\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{9\,b^3\,c^2\,f^6\,\left(32\,a^2\,b\,c\right.}{2\,a\,e^2\,\left(4\,a\,c-b^2\right.}\right.\right.}{}\right.}{}$$

input
```
int((d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x)
```

output
```
- ((a*b^2*f^3 + 8*a^2*c*f^3 + 2*b^3*d^2*f^3 + 9*b^2*c*d^4*f^3 + 6*b*c^2*d^
6*f^3 + 10*a*b*c*d^2*f^3)/(4*e*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^2*(b^3
*e*f^3 + 27*b^2*c*d^2*e*f^3 + 45*b*c^2*d^4*e*f^3 + 5*a*b*c*e*f^3))/(2*(b^4
 + 16*a^2*c^2 - 8*a*b^2*c)) + (9*x^4*(b^2*c*e^3*f^3 + 10*b*c^2*d^2*e^3*f^3
))/(4*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*d*x^3*(3*b^2*c*e^2*f^3 + 10*b*c
^2*d^2*e^2*f^3))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (d*x*(b^3*f^3 + 9*b^2*c*
d^2*f^3 + 9*b*c^2*d^4*f^3 + 5*a*b*c*f^3))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) +
 (3*b*c^2*e^5*f^3*x^6)/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (9*b*c^2*d*e^4
*f^3*x^5)/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(x^2*(6*b^2*d^2*e^2 + 28*c^2*d^6
*e^2 + 2*a*b*e^2 + 12*a*c*d^2*e^2 + 30*b*c*d^4*e^2) + x^6*(28*c^2*d^2*e^6
+ 2*b*c*e^6) + x*(4*b^2*d^3*e + 8*c^2*d^7*e + 8*a*c*d^3*e + 12*b*c*d^5*e +
 4*a*b*d*e) + x^3*(4*b^2*d*e^3 + 56*c^2*d^5*e^3 + 8*a*c*d*e^3 + 40*b*c*d^3
*e^3) + x^5*(56*c^2*d^3*e^5 + 12*b*c*d*e^5) + x^4*(b^2*e^4 + 70*c^2*d^4*e^
4 + 2*a*c*e^4 + 30*b*c*d^2*e^4) + a^2 + b^2*d^4 + c^2*d^8 + c^2*e^8*x^8 +
2*a*b*d^2 + 2*a*c*d^4 + 2*b*c*d^6 + 8*c^2*d*e^7*x^7) - (3*b*c*f^3*atan(((b
^4*(4*a*c - b^2)^5 + 16*a^2*c^2*(4*a*c - b^2)^5 - 8*a*b^2*c*(4*a*c - b^2)^
5)*(x^2*((9*b^2*c^4*e^8*f^6)/(a*(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*
a*b^2*c)) + (9*b^3*c^2*f^6*(2*b^5*c^2*e^10 - 16*a*b^3*c^3*e^10 + 32*a^2*b*
c^4*e^10))/(2*a*e^2*(4*a*c - b^2)^(15/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))
+ x*((9*b^3*c^2*f^6*(2*b^5*c^2*d*e^9 - 16*a*b^3*c^3*d*e^9 + 32*a^2*b*c^...
```

3.655. $\int \frac{(df+efx)^3}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx$

3.656 $\int \frac{(df+efx)^2}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx$

3.656.1 Optimal result

Integrand size = 33, antiderivative size = 375

$$\begin{aligned}\int \frac{(df+efx)^2}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx &= -\frac{f^2(d+ex)\left(b+2c(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\\ &\quad+\frac{f^2(d+ex)\left(b(b^2+8ac)+c(b^2+20ac)\,(d+ex)^2\right)}{8a\left(b^2-4ac\right)^2e\left(a+b(d+ex)^2+c(d+ex)^4\right)}\\ &\quad+\frac{\sqrt{c}\left(b^2+20ac+\frac{b(b^2-52ac)}{\sqrt{b^2-4ac}}\right)f^2\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}e}\\ &\quad+\frac{\sqrt{c}\left(b^2+20ac-\frac{b(b^2-52ac)}{\sqrt{b^2-4ac}}\right)f^2\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a\left(b^2-4ac\right)^2\sqrt{b+\sqrt{b^2-4ac}}e}\end{aligned}$$

output
```
-1/4*f^2*(e*x+d)*(b+2*c*(e*x+d)^2)/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)
^4)^2+1/8*f^2*(e*x+d)*(b*(8*a*c+b^2)+c*(20*a*c+b^2)*(e*x+d)^2)/a/(-4*a*c+b
^2)^2/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)+1/16*f^2*arctan((e*x+d)*2^(1/2)*c^(1/2
)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b^2+20*a*c+b*(-52*a*c+b^2)/(-4*a*
c+b^2)^(1/2))/a/(-4*a*c+b^2)^2/e*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/16
*f^2*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*
(b^2+20*a*c-b*(-52*a*c+b^2)/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^2/e*2^(1/2)
/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.656.2 Mathematica [A] (verified)

Time = 2.78 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.03

$$\int \frac{(df+efx)^2}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$$

$$= \frac{f^2\left(-\frac{4(b(d+ex)+2c(d+ex)^3)}{(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}+\frac{2(d+ex)(b^3+8abc+b^2c(d+ex)^2+20ac^2(d+ex)^2)}{a(b^2-4ac)^2(a+b(d+ex)^2+c(d+ex)^4)}+\frac{\sqrt{2}\sqrt{c}\left(b^3-52abc+b^2\sqrt{b^2-4ac}+20ac\sqrt{b^2-\cdots}\right.}{a(b^2-4ac)^{5/2}\sqrt{b-\cdots}}\right.}{16e}$$

input
```
Integrate[(d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

output
```
(f^2*((-4*(b*(d + e*x) + 2*c*(d + e*x)^3))/((b^2 - 4*a*c)*(a + b*(d + e*x)
^2 + c*(d + e*x)^4)^2) + (2*(d + e*x)*(b^3 + 8*a*b*c + b^2*c*(d + e*x)^2 +
 20*a*c^2*(d + e*x)^2))/(a*(b^2 - 4*a*c)^2*(a + b*(d + e*x)^2 + c*(d + e*x
)^4)) + (Sqrt[2]*Sqrt[c]*(b^3 - 52*a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*
Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 -
4*a*c]]])/(a*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*S
qrt[c]*(-b^3 + 52*a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*Sqrt[b^2 - 4*a*c]
)*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(a*(b^2
 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])))/(16*e)
```

3.656.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1462, 1439, 1492, 25, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(df+efx)^2}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$$

$$\Big\downarrow \; 1462$$

$$\frac{f^2\int \frac{(d+ex)^2}{\left(c(d+ex)^4+b(d+ex)^2+a\right)^3}d(d+ex)}{e}$$

$$\Big\downarrow \; 1439$$

$$\frac{f^2\left(\frac{\int \frac{b-10c(d+ex)^2}{\left(c(d+ex)^4+b(d+ex)^2+a\right)^2}d(d+ex)}{4(b^2-4ac)}-\frac{(d+ex)(b+2c(d+ex)^2)}{4(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}\right)}{e}$$

$\downarrow$ 1492

$$\frac{f^2\left(\frac{\frac{(d+ex)\left(c\left(20ac+b^2\right)(d+ex)^2+b\left(8ac+b^2\right)\right)}{2a\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{\int -\frac{c\left(b^2+20ac\right)(d+ex)^2+b\left(b^2-16ac\right)}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)}{2a\left(b^2-4ac\right)}}{4(b^2-4ac)}-\frac{(d+ex)(b+2c(d+ex)^2)}{4(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}\right)}{e}$$

$\downarrow$ 25

$$\frac{f^2\left(\frac{\frac{\int \frac{c\left(b^2+20ac\right)(d+ex)^2+b\left(b^2-16ac\right)}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)}{2a\left(b^2-4ac\right)}+\frac{(d+ex)\left(c\left(20ac+b^2\right)(d+ex)^2+b\left(8ac+b^2\right)\right)}{2a\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}}{4(b^2-4ac)}-\frac{(d+ex)(b+2c(d+ex)^2)}{4(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}\right)}{e}$$

$\downarrow$ 1480

$$\frac{f^2\left(\frac{\frac{\frac{1}{2}c\left(-\frac{52abc}{\sqrt{b^2-4ac}}+\frac{b^3}{\sqrt{b^2-4ac}}+20ac+b^2\right)\int \frac{1}{c(d+ex)^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}d(d+ex)+\frac{1}{2}c\left(\frac{52abc}{\sqrt{b^2-4ac}}-\frac{b^3}{\sqrt{b^2-4ac}}+20ac+b^2\right)\int \frac{1}{c(d+ex)^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}d(d+}{2a\left(b^2-4ac\right)}}{4(b^2-4ac)}}{e}$$

$\downarrow$ 218

$$\frac{f^2\left(\frac{\frac{\frac{\sqrt{c}\left(-\frac{52abc}{\sqrt{b^2-4ac}}+\frac{b^3}{\sqrt{b^2-4ac}}+20ac+b^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(\frac{52abc}{\sqrt{b^2-4ac}}-\frac{b^3}{\sqrt{b^2-4ac}}+20ac+b^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}}{2a\left(b^2-4ac\right)}+\frac{(d+ex)\left(c\left(20ac+b^2\right.\right.}{2a\left(b^2-4ac\right)\left(a+\right.}}{4(b^2-4ac)}}{e}$$

input
```
Int[(d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

3.656. $\int \frac{(df+efx)^2}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx$

output
```
(f^2*(-1/4*((d + e*x)*(b + 2*c*(d + e*x)^2))/((b^2 - 4*a*c)*(a + b*(d + e*
x)^2 + c*(d + e*x)^4)^2) + (((d + e*x)*(b*(b^2 + 8*a*c) + c*(b^2 + 20*a*c)
*(d + e*x)^2))/(2*a*(b^2 - 4*a*c)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (
(Sqrt[c]*(b^2 + 20*a*c + b^3/Sqrt[b^2 - 4*a*c] - (52*a*b*c)/Sqrt[b^2 - 4*a
*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqr
t[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(b^2 + 20*a*c - b^3/Sqrt[b^2
- 4*a*c] + (52*a*b*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x)
)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*
a*(b^2 - 4*a*c)))/(4*(b^2 - 4*a*c))))/e
```

3.656.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1439
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d*(d*x)^(m - 1)*(b + 2*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*(p +
1)*(b^2 - 4*a*c))), x] - Simp[d^2/(2*(p + 1)*(b^2 - 4*a*c))   Int[(d*x)^(m
- 2)*(b*(m - 1) + 2*c*(m + 4*p + 5)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x
] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m,
1] && LeQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

3.656. $\int \frac{(df+efx)^2}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx$

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

3.656.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.70 (sec) , antiderivative size = 889, normalized size of antiderivative = 2.37

method	result
default	$f^2\left(\frac{\frac{c^2e^6(20ac+b^2)x^7}{8(16a^2c^2-8ab^2c+b^4)a}+\frac{7c^2de^5(20ac+b^2)x^6}{8(16a^2c^2-8ab^2c+b^4)a}+\frac{(420ac^2d^2+21b^2cd^2+28abc+2b^3)ce^4x^5}{8(16a^2c^2-8ab^2c+b^4)a}+\frac{5cde^3(140ac^2d^2+7b^2cd^2+28abc+2b^3)x^4}{8(16a^2c^2-8ab^2c+b^4)a}\cdots}{\cdots}\right.$
risch	$\frac{\frac{c^2e^6f^2(20ac+b^2)x^7}{8(16a^2c^2-8ab^2c+b^4)a}+\frac{7c^2de^5f^2(20ac+b^2)x^6}{8(16a^2c^2-8ab^2c+b^4)a}+\frac{(420ac^2d^2+21b^2cd^2+28abc+2b^3)ce^4f^2x^5}{8(16a^2c^2-8ab^2c+b^4)a}+\frac{5cde^3f^2(140ac^2d^2+7b^2cd^2+28abc+2b^3)x^4}{8(16a^2c^2-8ab^2c+b^4)a}\cdots}{\cdots}$

input
```
int((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x,method=_RETURNVERBOSE)
```

output
```
f^2*((1/8*c^2*e^6*(20*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^7+7/8*c^2*d*
e^5*(20*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^6+1/8*(420*a*c^2*d^2+21*b^
2*c*d^2+28*a*b*c+2*b^3)*c*e^4/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^5+5/8*c*d*e^3
*(140*a*c^2*d^2+7*b^2*c*d^2+28*a*b*c+2*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x
^4+1/8*e^2*(700*a*c^3*d^4+35*b^2*c^2*d^4+280*a*b*c^2*d^2+20*b^3*c*d^2+36*a
^2*c^2+5*a*b^2*c+b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^3+1/8*d*e*(420*a*c^3*
d^4+21*b^2*c^2*d^4+280*a*b*c^2*d^2+20*b^3*c*d^2+108*a^2*c^2+15*a*b^2*c+3*b
^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^2+1/8*(140*a*c^3*d^6+7*b^2*c^2*d^6+140*
a*b*c^2*d^4+10*b^3*c*d^4+108*a^2*c^2*d^2+15*a*b^2*c*d^2+3*b^4*d^2+16*a^2*b
*c-a*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x+1/8*d/e*(20*a*c^3*d^6+b^2*c^2*d^6
+28*a*b*c^2*d^4+2*b^3*c*d^4+36*a^2*c^2*d^2+5*a*b^2*c*d^2+b^4*d^2+16*a^2*b*
c-a*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4)/a)/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^
2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2+1/16/(16*a^2*c^2-8*
a*b^2*c+b^4)/a/e*sum((c*e^2*(20*a*c+b^2)*_R^2+2*d*c*e*(20*a*c+b^2)*_R+20*a
*c^2*d^2+b^2*c*d^2-16*a*b*c+b^3)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e
+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2
*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a)))
```

3.656.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7838 vs. $2(331) = 662$.

Time = 0.53 (sec) , antiderivative size = 7838, normalized size of antiderivative = 20.90

$$\int \frac{(df+efx)^2}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas
")
```

output
```
Too large to include
```

3.656.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(df+efx)^2}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx=\text{Timed out}$$

input
```
integrate((e*f*x+d*f)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

output
```
Timed out
```

3.656.7 Maxima [F]

$$\int \frac{(df+efx)^2}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx=\int \frac{(efx+df)^2}{\left((ex+d)^4c+(ex+d)^2b+a\right)^3}\,dx$$

input
```
integrate((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima
")
```

output
```
1/8*f^2*integrate(((b^2*c + 20*a*c^2)*e^2*x^2 + 2*(b^2*c + 20*a*c^2)*d*e*x
 + b^3 - 16*a*b*c + (b^2*c + 20*a*c^2)*d^2)/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c
*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a), x)/(a*b
^4 - 8*a^2*b^2*c + 16*a^3*c^2) + 1/8*((b^2*c^2 + 20*a*c^3)*e^7*f^2*x^7 + 7
*(b^2*c^2 + 20*a*c^3)*d*e^6*f^2*x^6 + (2*b^3*c + 28*a*b*c^2 + 21*(b^2*c^2
+ 20*a*c^3)*d^2)*e^5*f^2*x^5 + 5*(7*(b^2*c^2 + 20*a*c^3)*d^3 + 2*(b^3*c +
14*a*b*c^2)*d)*e^4*f^2*x^4 + (35*(b^2*c^2 + 20*a*c^3)*d^4 + b^4 + 5*a*b^2*
c + 36*a^2*c^2 + 20*(b^3*c + 14*a*b*c^2)*d^2)*e^3*f^2*x^3 + (21*(b^2*c^2 +
 20*a*c^3)*d^5 + 20*(b^3*c + 14*a*b*c^2)*d^3 + 3*(b^4 + 5*a*b^2*c + 36*a^2
*c^2)*d)*e^2*f^2*x^2 + (7*(b^2*c^2 + 20*a*c^3)*d^6 + 10*(b^3*c + 14*a*b*c^
2)*d^4 - a*b^3 + 16*a^2*b*c + 3*(b^4 + 5*a*b^2*c + 36*a^2*c^2)*d^2)*e*f^2*
x + ((b^2*c^2 + 20*a*c^3)*d^7 + 2*(b^3*c + 14*a*b*c^2)*d^5 + (b^4 + 5*a*b^
2*c + 36*a^2*c^2)*d^3 - (a*b^3 - 16*a^2*b*c)*d)*f^2)/((a*b^4*c^2 - 8*a^2*b
^2*c^3 + 16*a^3*c^4)*e^9*x^8 + 8*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*
d*e^8*x^7 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3 + 14*(a*b^4*c^2 - 8*
a^2*b^2*c^3 + 16*a^3*c^4)*d^2)*e^7*x^6 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3
+ 16*a^3*c^4)*d^3 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d)*e^6*x^5
+ (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3 + 70*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a
^3*c^4)*d^4 + 30*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^2)*e^5*x^4 + 4
*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^5 + 10*(a*b^5*c - 8*a^2...
```

3.656.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2679 vs. 2(331) = 662.

Time = 0.33 (sec) , antiderivative size = 2679, normalized size of antiderivative = 7.14

$$\int \frac{(df+efx)^2}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

output
```
-1/16*((b^2*c*e^2*f^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*
e^4)) + d/e)^2 + 20*a*c^2*e^2*f^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a
*c)*e^2)/(c*e^4)) + d/e)^2 - 2*b^2*c*d*e*f^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqr
t(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e) - 40*a*c^2*d*e*f^2*(sqrt(1/2)*sqrt(-(b
*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e) + b^2*c*d^2*f^2 + 20*a*c^2*d
^2*f^2 + b^3*f^2 - 16*a*b*c*f^2)*log(x + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2
 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2
 - 4*a*c)*e^2)/(c*e^4)) + d/e)^3 - 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 + sqr
t(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 +
 b*e^2)*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e))
- (b^2*c*e^2*f^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4))
 - d/e)^2 + 20*a*c^2*e^2*f^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e
^2)/(c*e^4)) - d/e)^2 + 2*b^2*c*d*e*f^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2
 - 4*a*c)*e^2)/(c*e^4)) - d/e) + 40*a*c^2*d*e*f^2*(sqrt(1/2)*sqrt(-(b*e^2
+ sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e) + b^2*c*d^2*f^2 + 20*a*c^2*d^2*f^
2 + b^3*f^2 - 16*a*b*c*f^2)*log(x - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*
a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*
a*c)*e^2)/(c*e^4)) - d/e)^3 + 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2
 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2 + 2*c*d^3*e + b*d*e + (6*c*d^2*e^2 + b*e^
2)*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)) + ...
```

3.656.9 Mupad [B] (verification not implemented)

Time = 12.30 (sec) , antiderivative size = 16025, normalized size of antiderivative = 42.73

$$\int \frac{(df+efx)^2}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \text{Too large to display}$$

input
```
int((d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x)
```

output
```
atan((((((67108864*a^9*b*c^9*d*e^13 - 4096*a^2*b^15*c^2*d*e^13 + 114688*a^
3*b^13*c^3*d*e^13 - 1376256*a^4*b^11*c^4*d*e^13 + 9175040*a^5*b^9*c^5*d*e^
13 - 36700160*a^6*b^7*c^6*d*e^13 + 88080384*a^7*b^5*c^7*d*e^13 - 117440512
*a^8*b^3*c^8*d*e^13)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a
^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x
*(262144*a^7*b*c^7*e^14 - 256*a^2*b^11*c^2*e^14 + 5120*a^3*b^9*c^3*e^14 -
40960*a^4*b^7*c^4*e^14 + 163840*a^5*b^5*c^5*e^14 - 327680*a^6*b^3*c^6*e^14
))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^
2*c^3)))*(-(b^17*f^4 + b^2*f^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c
^8*f^4 + 1140*a^2*b^13*c^2*f^4 - 10160*a^3*b^11*c^3*f^4 + 34880*a^4*b^9*c^
4*f^4 + 43776*a^5*b^7*c^5*f^4 - 680960*a^6*b^5*c^6*f^4 + 1863680*a^7*b^3*c
^7*f^4 - 55*a*b^15*c*f^4 - 25*a*c*f^4*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3
*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e
^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*
e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4
*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2) - (122880*a^3*b^9*c^4*e^12*f^
2 - 9216*a^2*b^11*c^3*e^12*f^2 - 819200*a^4*b^7*c^5*e^12*f^2 + 2949120*a^5
*b^5*c^6*e^12*f^2 - 5505024*a^6*b^3*c^7*e^12*f^2 + 256*a*b^13*c^2*e^12*f^2
 + 4194304*a^7*b*c^8*e^12*f^2)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10
*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b...
```

3.656. $\int \frac{(df+efx)^2}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx$

3.657 $\int \frac{df+efx}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3} dx$

3.657.1 Optimal result

Integrand size = 31, antiderivative size = 153

$$\int \frac{df+efx}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3} dx = -\frac{f(b+2c(d+ex)^2)}{4(b^2-4ac)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{3cf(b+2c(d+ex)^2)}{2(b^2-4ac)^2 e\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{6c^2 f \operatorname{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2} e}$$

output

```
-1/4*f*(b+2*c*(e*x+d)^2)/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2+3/2*
c*f*(b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^2/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)-6*c^2*f
*arctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(5/2)/e
```

3.657.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.97

$$\int \frac{df+efx}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3} dx = \frac{f\left(\frac{(b^2-4ac)(-b-2c(d+ex)^2)}{(a+b(d+ex)^2+c(d+ex)^4)^2} + \frac{6c(b+2c(d+ex)^2)}{a+b(d+ex)^2+c(d+ex)^4} + \frac{24c^2 \arctan\left(\frac{b+2c(d+ex)^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}\right)}{4(b^2-4ac)^2 e}$$

input
```
Integrate[(d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

output
```
(f*(((b^2 - 4*a*c)*(-b - 2*c*(d + e*x)^2))/(a + b*(d + e*x)^2 + c*(d + e*x
)^4)^2 + (6*c*(b + 2*c*(d + e*x)^2))/(a + b*(d + e*x)^2 + c*(d + e*x)^4) +
 (24*c^2*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a
*c]))/(4*(b^2 - 4*a*c)^2*e)
```

3.657.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {1462, 1432, 1086, 1086, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{df+efx}{(a+b(d+ex)^2+c(d+ex)^4)^3}dx$$

$$\Big\downarrow \text{ 1462}$$

$$\frac{f\int \frac{d+ex}{(c(d+ex)^4+b(d+ex)^2+a)^3}d(d+ex)}{e}$$

$$\Big\downarrow \text{ 1432}$$

$$\frac{f\int \frac{1}{(c(d+ex)^4+b(d+ex)^2+a)^3}d(d+ex)^2}{2e}$$

$$\Big\downarrow \text{ 1086}$$

$$\frac{f\left(-\frac{3c\int \frac{1}{\left(c(d+ex)^4+b(d+ex)^2+a\right)^2}d(d+ex)^2}{b^2-4ac}-\frac{b+2c(d+ex)^2}{2(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}\right)}{2e}$$

$$\Big\downarrow \text{ 1086}$$

$$\frac{f\left(-\frac{3c\left(-\frac{2c\int \frac{1}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)^2}{b^2-4ac}-\frac{b+2c(d+ex)^2}{\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}\right)}{b^2-4ac}-\frac{b+2c(d+ex)^2}{2(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}\right)}{2e}$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{f\left(-\frac{3c\left(\frac{4c\int\frac{1}{-(d+ex)^4+b^2-4ac}d\left(2c(d+ex)^2+b\right)}{b^2-4ac}-\frac{b+2c(d+ex)^2}{\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}\right)}{b^2-4ac}-\frac{b+2c(d+ex)^2}{2(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}\right)}{2e}$$

$$\Big\downarrow \; 219$$

$$\frac{f\left(-\frac{3c\left(\frac{4c\,\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{b+2c(d+ex)^2}{\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}\right)}{b^2-4ac}-\frac{b+2c(d+ex)^2}{2(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}\right)}{2e}$$

input
```
Int[(d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

output
```
(f*(-1/2*(b + 2*c*(d + e*x)^2)/((b^2 - 4*a*c)*(a + b*(d + e*x)^2 + c*(d +
e*x)^4)^2) - (3*c*(-((b + 2*c*(d + e*x)^2)/((b^2 - 4*a*c)*(a + b*(d + e*x)
^2 + c*(d + e*x)^4))) + (4*c*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*
c]])/(b^2 - 4*a*c)^(3/2)))/(b^2 - 4*a*c)))/(2*e)
```

3.657.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

3.657. $\int \frac{df+efx}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx$

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

3.657.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.38 (sec) , antiderivative size = 543, normalized size of antiderivative = 3.55

method	result
default	$f\left(\frac{\frac{3c^3e^5x^6}{16a^2c^2-8a\,b^2c+b^4}+\frac{18e^4c^3d\,x^5}{16a^2c^2-8a\,b^2c+b^4}+\frac{9c^2e^3\left(10c\,d^2+b\right)x^4}{2\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{6c^2d\,e^2\left(10c\,d^2+3b\right)x^3}{16a^2c^2-8a\,b^2c+b^4}+\frac{ce\left(45c^2d^4+27bc\,d^2+5ac+b^2\right)x^2}{16a^2c^2-8a\,b^2c+b^4}+\frac{2cd\left(9c^2d^4-\right.}{16a^2c}}{\left(c\,x^4e^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+d^4c+2bdex+b\,d^2+\right.}\right.$
risch	$\frac{\frac{3c^3e^5f\,x^6}{16a^2c^2-8a\,b^2c+b^4}+\frac{18f\,e^4c^3d\,x^5}{16a^2c^2-8a\,b^2c+b^4}+\frac{9c^2e^3f\left(10c\,d^2+b\right)x^4}{2\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{6c^2d\,e^2f\left(10c\,d^2+3b\right)x^3}{16a^2c^2-8a\,b^2c+b^4}+\frac{cef\left(45c^2d^4+27bc\,d^2+5ac+b^2\right)x^2}{16a^2c^2-8a\,b^2c+b^4}+\frac{2cdf\left(9c^2d^4+9\right.}{16a^2c^2-}}{\left(c\,x^4e^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+d^4c+2bdex+b\,d^2+a\right)}$

input
```
int((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x,method=_RETURNVERBOSE)
```

output
```
f*((3*c^3*e^5/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+18*e^4*c^3*d/(16*a^2*c^2-8*a*
b^2*c+b^4)*x^5+9/2*c^2*e^3*(10*c*d^2+b)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4+6*c
^2*d*e^2*(10*c*d^2+3*b)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+c*e*(45*c^2*d^4+27*
b*c*d^2+5*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+2*c*d*(9*c^2*d^4+9*b*c*d
^2+5*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x+1/4/e*(12*c^3*d^6+18*b*c^2*d^4+
20*a*c^2*d^2+4*b^2*c*d^2+10*a*b*c-b^3)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*e^4*
x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*
d^2+a)^2+3*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)/e*sum((_R*e+d)/(2*_R^3*c*e^3+6*_
R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4
+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^
2+a))
```

3.657.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1808 vs. $2(145) = 290$.

Time = 0.56 (sec) , antiderivative size = 3748, normalized size of antiderivative = 24.50

$$\int \frac{df + efx}{\left(a + b(d + ex)^2 + c(d + ex)^4\right)^3} \, dx = \text{Too large to display}$$

input
```
integrate((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")
```

output
```
[1/4*(12*(b^2*c^3 - 4*a*c^4)*e^6*f*x^6 + 72*(b^2*c^3 - 4*a*c^4)*d*e^5*f*x^
5 + 18*(b^3*c^2 - 4*a*b*c^3 + 10*(b^2*c^3 - 4*a*c^4)*d^2)*e^4*f*x^4 + 24*(
10*(b^2*c^3 - 4*a*c^4)*d^3 + 3*(b^3*c^2 - 4*a*b*c^3)*d)*e^3*f*x^3 + 4*(b^4
*c + a*b^2*c^2 - 20*a^2*c^3 + 45*(b^2*c^3 - 4*a*c^4)*d^4 + 27*(b^3*c^2 - 4
*a*b*c^3)*d^2)*e^2*f*x^2 + 8*(9*(b^2*c^3 - 4*a*c^4)*d^5 + 9*(b^3*c^2 - 4*a
*b*c^3)*d^3 + (b^4*c + a*b^2*c^2 - 20*a^2*c^3)*d)*e*f*x + 12*(c^4*e^8*f*x^
8 + 8*c^4*d*e^7*f*x^7 + 2*(14*c^4*d^2 + b*c^3)*e^6*f*x^6 + 4*(14*c^4*d^3 +
 3*b*c^3*d)*e^5*f*x^5 + (70*c^4*d^4 + 30*b*c^3*d^2 + b^2*c^2 + 2*a*c^3)*e^
4*f*x^4 + 4*(14*c^4*d^5 + 10*b*c^3*d^3 + (b^2*c^2 + 2*a*c^3)*d)*e^3*f*x^3
+ 2*(14*c^4*d^6 + 15*b*c^3*d^4 + a*b*c^2 + 3*(b^2*c^2 + 2*a*c^3)*d^2)*e^2*
f*x^2 + 4*(2*c^4*d^7 + 3*b*c^3*d^5 + a*b*c^2*d + (b^2*c^2 + 2*a*c^3)*d^3)*
e*f*x + (c^4*d^8 + 2*b*c^3*d^6 + 2*a*b*c^2*d^2 + (b^2*c^2 + 2*a*c^3)*d^4 +
 a^2*c^2)*f)*sqrt(b^2 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^
2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*
x + b^2 - 2*a*c - (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c
))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*
(2*c*d^3 + b*d)*e*x + a)) + (12*(b^2*c^3 - 4*a*c^4)*d^6 - b^5 + 14*a*b^3*c
 - 40*a^2*b*c^2 + 18*(b^3*c^2 - 4*a*b*c^3)*d^4 + 4*(b^4*c + a*b^2*c^2 - 20
*a^2*c^3)*d^2)*f)/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*
e^9*x^8 + 8*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d*e^...
```

3.657.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1707 vs. $2(139) = 278$.

Time = 7.21 (sec) , antiderivative size = 1707, normalized size of antiderivative = 11.16

$$\int \frac{df + efx}{\left(a + b(d + ex)^2 + c(d + ex)^4\right)^3} \, dx = \text{Too large to display}$$

input
```
integrate((e*f*x+d*f)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

output
```
-3*c**2*f*sqrt(-1/(4*a*c - b**2)**5)*log(2*d*x/e + x**2 + (-192*a**3*c**5*
f*sqrt(-1/(4*a*c - b**2)**5) + 144*a**2*b**2*c**4*f*sqrt(-1/(4*a*c - b**2)
**5) - 36*a*b**4*c**3*f*sqrt(-1/(4*a*c - b**2)**5) + 3*b**6*c**2*f*sqrt(-1
/(4*a*c - b**2)**5) + 3*b*c**2*f + 6*c**3*d**2*f)/(6*c**3*e**2*f))/e + 3*c
**2*f*sqrt(-1/(4*a*c - b**2)**5)*log(2*d*x/e + x**2 + (192*a**3*c**5*f*sqr
t(-1/(4*a*c - b**2)**5) - 144*a**2*b**2*c**4*f*sqrt(-1/(4*a*c - b**2)**5)
+ 36*a*b**4*c**3*f*sqrt(-1/(4*a*c - b**2)**5) - 3*b**6*c**2*f*sqrt(-1/(4*a
*c - b**2)**5) + 3*b*c**2*f + 6*c**3*d**2*f)/(6*c**3*e**2*f))/e + (10*a*b*
c*f + 20*a*c**2*d**2*f - b**3*f + 4*b**2*c*d**2*f + 18*b*c**2*d**4*f + 12*
c**3*d**6*f + 72*c**3*d*e**5*f*x**5 + 12*c**3*e**6*f*x**6 + x**4*(18*b*c**
2*e**4*f + 180*c**3*d**2*e**4*f) + x**3*(72*b*c**2*d*e**3*f + 240*c**3*d**
3*e**3*f) + x**2*(20*a*c**2*e**2*f + 4*b**2*c*e**2*f + 108*b*c**2*d**2*e**
2*f + 180*c**3*d**4*e**2*f) + x*(40*a*c**2*d*e*f + 8*b**2*c*d*e*f + 72*b*c
**2*d**3*e*f + 72*c**3*d**5*e*f))/(64*a**4*c**2*e - 32*a**3*b**2*c*e + 128
*a**3*b*c**2*d**2*e + 128*a**3*c**3*d**4*e + 4*a**2*b**4*e - 64*a**2*b**3*
c*d**2*e + 128*a**2*b*c**3*d**6*e + 64*a**2*c**4*d**8*e + 8*a*b**5*d**2*e
- 24*a*b**4*c*d**4*e - 64*a*b**3*c**2*d**6*e - 32*a*b**2*c**3*d**8*e + 4*b
**6*d**4*e + 8*b**5*c*d**6*e + 4*b**4*c**2*d**8*e + x**8*(64*a**2*c**4*e**
9 - 32*a*b**2*c**3*e**9 + 4*b**4*c**2*e**9) + x**7*(512*a**2*c**4*d*e**8 -
 256*a*b**2*c**3*d*e**8 + 32*b**4*c**2*d*e**8) + x**6*(128*a**2*b*c**3*...
```

3.657.7 Maxima [F]

$$\int \frac{df + efx}{(a + b(d + ex)^2 + c(d + ex)^4)^3}\,dx = \int \frac{efx + df}{\left((ex + d)^4c + (ex + d)^2b + a\right)^3}\,dx$$

input
```
integrate((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")
```

output
```
6*c^2*f*integrate((e*x + d)/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2
+ b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a), x)/(b^4 - 8*a*b^2*c + 1
6*a^2*c^2) + 1/4*(12*c^3*e^6*f*x^6 + 72*c^3*d*e^5*f*x^5 + 18*(10*c^3*d^2 +
 b*c^2)*e^4*f*x^4 + 24*(10*c^3*d^3 + 3*b*c^2*d)*e^3*f*x^3 + 4*(45*c^3*d^4
+ 27*b*c^2*d^2 + b^2*c + 5*a*c^2)*e^2*f*x^2 + 8*(9*c^3*d^5 + 9*b*c^2*d^3 +
 (b^2*c + 5*a*c^2)*d)*e*f*x + (12*c^3*d^6 + 18*b*c^2*d^4 - b^3 + 10*a*b*c
+ 4*(b^2*c + 5*a*c^2)*d^2)*f)/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*e^9*x^
8 + 8*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*e^8*x^7 + 2*(b^5*c - 8*a*b^3*
c^2 + 16*a^2*b*c^3 + 14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^2)*e^7*x^6
+ 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^3 + 3*(b^5*c - 8*a*b^3*c^2
+ 16*a^2*b*c^3)*d)*e^6*x^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3 + 70*(b^4*c^2 -
 8*a*b^2*c^3 + 16*a^2*c^4)*d^4 + 30*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d
^2)*e^5*x^4 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^5 + 10*(b^5*c -
 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d)*e^4*x
^3 + 2*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^6 + a*b^5 - 8*a^2*b^3*c
+ 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 3*(b^6 - 6*
a*b^4*c + 32*a^3*c^3)*d^2)*e^3*x^2 + 4*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*
c^4)*d^7 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + (b^6 - 6*a*b^4*c +
 32*a^3*c^3)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e^2*x + ((b^4*c
^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*...
```

3.657.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 429 vs. $2(145) = 290$.

Time = 0.33 (sec) , antiderivative size = 429, normalized size of antiderivative = 2.80

$$\int \frac{df+efx}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = \frac{6\,c^2f\arctan\left(\frac{2\,cd^2f+2\,(efx^2+2\,dfx)ce+bf}{\sqrt{-b^2+4\,ac}f}\right)}{(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-b^2+4\,ac}e} + \frac{12\,c^3d^6f^5+36\,(efx^2+2\,dfx)c^3d^4ef^4+36\,(efx^2+2\,dfx)^2c^3d^2e^2f^3+18\,bc^2d^4f^5+12\,(efx^2+2\,dfx)^3c}{4\,(cd^4f^2+2\,(efx^2+2\,dfx)cd^2}$$

input
```
integrate((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

output
```
6*c^2*f*arctan((2*c*d^2*f + 2*(e*f*x^2 + 2*d*f*x)*c*e + b*f)/(sqrt(-b^2 +
4*a*c)*f))/((b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)*e) + 1/4*(12
*c^3*d^6*f^5 + 36*(e*f*x^2 + 2*d*f*x)*c^3*d^4*e*f^4 + 36*(e*f*x^2 + 2*d*f*
x)^2*c^3*d^2*e^2*f^3 + 18*b*c^2*d^4*f^5 + 12*(e*f*x^2 + 2*d*f*x)^3*c^3*e^3
*f^2 + 36*(e*f*x^2 + 2*d*f*x)*b*c^2*d^2*e*f^4 + 18*(e*f*x^2 + 2*d*f*x)^2*b
*c^2*e^2*f^3 + 4*b^2*c*d^2*f^5 + 20*a*c^2*d^2*f^5 + 4*(e*f*x^2 + 2*d*f*x)*
b^2*c*e*f^4 + 20*(e*f*x^2 + 2*d*f*x)*a*c^2*e*f^4 - b^3*f^5 + 10*a*b*c*f^5)
/((c*d^4*f^2 + 2*(e*f*x^2 + 2*d*f*x)*c*d^2*e*f + (e*f*x^2 + 2*d*f*x)^2*c*e
^2 + b*d^2*f^2 + (e*f*x^2 + 2*d*f*x)*b*e*f + a*f^2)^2*(b^4*e - 8*a*b^2*c*e
 + 16*a^2*c^2*e))
```

3.657.9 Mupad [B] (verification not implemented)

Time = 9.76 (sec) , antiderivative size = 1199, normalized size of antiderivative = 7.84

$$\int \frac{df+efx}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$$

$$= \frac{\frac{x^2\,\left(e\,f\,b^2\,c+27\,e\,f\,b\,c^2\,d^2+45\,e\,f\,c^3\,d^4+5\,a\,e\,f\,\ldots}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}}{x^2\,\left(6\,b^2\,d^2\,e^2+30\,b\,c\,d^4\,e^2+2\,a\,b\,e^2+28\,c^2\,d^6\,e^2+12\,a\,c\,d^2\,e^2\right)+x^6\,\left(28\,c^2\,d^2\,e^6+2\,b\,c\,e^6\right)+x\,\left(4\,e\,b^2\,d^3+\ldots\right.}$$

$$+ \frac{6\,c^2\,f\,\mathrm{atan}\left(\frac{\left(b^4\,\left(4\,a\,c-b^2\right)^5+16\,a^2\,c^2\,\left(4\,a\,c-b^2\right)^5-8\,a\,b^2\,c\,\left(4\,a\,c-b^2\right)^5\right)\,\left(x^2\,\left(\frac{36\,c^6\,e^8\,f^2}{a\,\left(4\,a\,c-b^2\right)^{9/2}\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{36\,b\,c^4\,f^2\,\left(16\,a^2\,b\,c^4\,e\ldots\right.}{a\,e^2\,\left(4\,a\,c-b^2\right)^{15/\ldots}}\ldots\right.\right.}{\ldots}\right.}{\ldots}$$

input
```
int((d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x)
```

output
```
((x^2*(5*a*c^2*e*f + b^2*c*e*f + 45*c^3*d^4*e*f + 27*b*c^2*d^2*e*f))/(b^4
+ 16*a^2*c^2 - 8*a*b^2*c) + (12*c^3*d^6*f - b^3*f + 20*a*c^2*d^2*f + 4*b^2
*c*d^2*f + 18*b*c^2*d^4*f + 10*a*b*c*f)/(4*e*(b^4 + 16*a^2*c^2 - 8*a*b^2*c
)) + (9*x^4*(10*c^3*d^2*e^3*f + b*c^2*e^3*f))/(2*(b^4 + 16*a^2*c^2 - 8*a*b
^2*c)) + (2*d*x*(9*c^3*d^4*f + 5*a*c^2*f + b^2*c*f + 9*b*c^2*d^2*f))/(b^4
+ 16*a^2*c^2 - 8*a*b^2*c) + (6*d*x^3*(10*c^3*d^2*e^2*f + 3*b*c^2*e^2*f))/(
b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (3*c^3*e^5*f*x^6)/(b^4 + 16*a^2*c^2 - 8*a*
b^2*c) + (18*c^3*d*e^4*f*x^5)/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(x^2*(6*b^2*
d^2*e^2 + 28*c^2*d^6*e^2 + 2*a*b*e^2 + 12*a*c*d^2*e^2 + 30*b*c*d^4*e^2) +
x^6*(28*c^2*d^2*e^6 + 2*b*c*e^6) + x*(4*b^2*d^3*e + 8*c^2*d^7*e + 8*a*c*d^
3*e + 12*b*c*d^5*e + 4*a*b*d*e) + x^3*(4*b^2*d*e^3 + 56*c^2*d^5*e^3 + 8*a*
c*d*e^3 + 40*b*c*d^3*e^3) + x^5*(56*c^2*d^3*e^5 + 12*b*c*d*e^5) + x^4*(b^2
*e^4 + 70*c^2*d^4*e^4 + 2*a*c*e^4 + 30*b*c*d^2*e^4) + a^2 + b^2*d^4 + c^2*
d^8 + c^2*e^8*x^8 + 2*a*b*d^2 + 2*a*c*d^4 + 2*b*c*d^6 + 8*c^2*d*e^7*x^7) +
 (6*c^2*f*atan(((b^4*(4*a*c - b^2)^5 + 16*a^2*c^2*(4*a*c - b^2)^5 - 8*a*b^
2*c*(4*a*c - b^2)^5)*(x^2*((36*c^6*e^8*f^2)/(a*(4*a*c - b^2)^(9/2)*(b^4 +
16*a^2*c^2 - 8*a*b^2*c)) + (36*b*c^4*f^2*(b^5*c^2*e^10 - 8*a*b^3*c^3*e^10
+ 16*a^2*b*c^4*e^10))/(a*e^2*(4*a*c - b^2)^(15/2)*(b^4 + 16*a^2*c^2 - 8*a*
b^2*c))) + x*((72*c^6*d*e^7*f^2)/(a*(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2
- 8*a*b^2*c)) + (72*b*c^4*f^2*(b^5*c^2*d*e^9 - 8*a*b^3*c^3*d*e^9 + 16*a...
```

3.657. $\int \frac{df+efx}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx$

3.658 $\int \frac{1}{(df+efx)(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx$

3.658.1 Optimal result

Integrand size = 33, antiderivative size = 270

$$\begin{aligned}\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx &= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)ef\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\\ &\quad+\frac{2b^4-15ab^2c+16a^2c^2+2bc(b^2-7ac)\,(d+ex)^2}{4a^2\left(b^2-4ac\right)^2ef\left(a+b(d+ex)^2+c(d+ex)^4\right)}\\ &\quad+\frac{b(b^4-10ab^2c+30a^2c^2)\operatorname{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^3\left(b^2-4ac\right)^{5/2}ef}\\ &\quad+\frac{\log(d+ex)}{a^3ef}-\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^3ef}\end{aligned}$$

output

```
1/4*(b^2-2*a*c+b*c*(e*x+d)^2)/a/(-4*a*c+b^2)/e/f/(a+b*(e*x+d)^2+c*(e*x+d)^
4)^2+1/4*(2*b^4-15*a*b^2*c+16*a^2*c^2+2*b*c*(-7*a*c+b^2)*(e*x+d)^2)/a^2/(-
4*a*c+b^2)^2/e/f/(a+b*(e*x+d)^2+c*(e*x+d)^4)+1/2*b*(30*a^2*c^2-10*a*b^2*c+
b^4)*arctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)^(5/2)/
e/f+ln(e*x+d)/a^3/e/f-1/4*ln(a+b*(e*x+d)^2+c*(e*x+d)^4)/a^3/e/f
```

3.658.2 Mathematica [A] (verified)

Time = 2.37 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.46

$$\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$$
$$= \frac{\frac{a^2(-b^2+2ac-bc(d+ex)^2)}{(-b^2+4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}+\frac{a(2b^4-15ab^2c+16a^2c^2+2b^3c(d+ex)^2-14abc^2(d+ex)^2)}{(b^2-4ac)^2(a+(d+ex)^2(b+c(d+ex)^2))}+4\log(d+ex)-\frac{\left(b^5-10ab^3c+30a^2b\right.}{}}{}$$

input
```
Integrate[1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

output
```
((a^2*(-b^2 + 2*a*c - b*c*(d + e*x)^2))/((-b^2 + 4*a*c)*(a + b*(d + e*x)^2
 + c*(d + e*x)^4)^2) + (a*(2*b^4 - 15*a*b^2*c + 16*a^2*c^2 + 2*b^3*c*(d +
e*x)^2 - 14*a*b*c^2*(d + e*x)^2))/((b^2 - 4*a*c)^2*(a + (d + e*x)^2*(b + c
*(d + e*x)^2))) + 4*Log[d + e*x] - ((b^5 - 10*a*b^3*c + 30*a^2*b*c^2 + b^4
*Sqrt[b^2 - 4*a*c] - 8*a*b^2*c*Sqrt[b^2 - 4*a*c] + 16*a^2*c^2*Sqrt[b^2 - 4
*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(b^2 - 4*a*c)^(5/2) +
 ((b^5 - 10*a*b^3*c + 30*a^2*b*c^2 - b^4*Sqrt[b^2 - 4*a*c] + 8*a*b^2*c*Sqr
t[b^2 - 4*a*c] - 16*a^2*c^2*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] +
 2*c*(d + e*x)^2])/(b^2 - 4*a*c)^(5/2))/(4*a^3*e*f)
```

3.658.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {1462, 1434, 1165, 25, 1235, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$$
$$\Big\downarrow \ 1462$$
$$\frac{\int \frac{1}{(d+ex)\left(c(d+ex)^4+b(d+ex)^2+a\right)^3}d(d+ex)}{ef}$$
$$\Big\downarrow \ 1434$$
$$\frac{\int \frac{1}{(d+ex)^2\left(c(d+ex)^4+b(d+ex)^2+a\right)^3}d(d+ex)^2}{2ef}$$

$$\Big\downarrow \; 1165$$

$$\frac{\frac{-2ac+b^2+bc(d+ex)^2}{2a(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}-\frac{\int -\frac{3bc(d+ex)^2+2\left(b^2-4ac\right)}{(d+ex)^2\left(c(d+ex)^4+b(d+ex)^2+a\right)^2}d(d+ex)^2}{2a(b^2-4ac)}}{2ef}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{3bc(d+ex)^2+2\left(b^2-4ac\right)}{(d+ex)^2\left(c(d+ex)^4+b(d+ex)^2+a\right)^2}d(d+ex)^2}{2a(b^2-4ac)}+\frac{-2ac+b^2+bc(d+ex)^2}{2a(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}}{2ef}$$

$$\Big\downarrow \; 1235$$

$$\frac{\frac{\frac{16a^2c^2+2bc\left(b^2-7ac\right)(d+ex)^2-15ab^2c+2b^4}{a\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{\int -\frac{2\left(\left(b^2-4ac\right)^2+bc\left(b^2-7ac\right)(d+ex)^2\right)}{(d+ex)^2\left(c(d+ex)^4+b(d+ex)^2+a\right)}d(d+ex)^2}{a\left(b^2-4ac\right)}}{2a(b^2-4ac)}+\frac{-2ac+b^2+bc(d+ex)^2}{2a(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}}{2ef}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{2\int \frac{\left(b^2-4ac\right)^2+bc\left(b^2-7ac\right)(d+ex)^2}{(d+ex)^2\left(c(d+ex)^4+b(d+ex)^2+a\right)}d(d+ex)^2}{a\left(b^2-4ac\right)}+\frac{16a^2c^2+2bc\left(b^2-7ac\right)(d+ex)^2-15ab^2c+2b^4}{a\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}}{2a(b^2-4ac)}+\frac{-2ac+b^2+bc(d+ex)^2}{2a(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}}{2ef}$$

$$\Big\downarrow \; 1200$$

$$\frac{\frac{\frac{2\int \left(\frac{\left(4ac-b^2\right)^2}{a(d+ex)^2}+\frac{-c\left(b^2-4ac\right)^2(d+ex)^2-b\left(b^4-9acb^2+23a^2c^2\right)}{a\left(c(d+ex)^4+b(d+ex)^2+a\right)}\right)d(d+ex)^2}{a\left(b^2-4ac\right)}+\frac{16a^2c^2+2bc\left(b^2-7ac\right)(d+ex)^2-15ab^2c+2b^4}{a\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}}{2a(b^2-4ac)}+\frac{-2ac+b^2+bc(d+ex)^{\cdots}}{2a(b^2-4ac)(a+b(d+ex)^2+c(\cdots}}{2ef}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{\frac{2\left(\frac{b\left(30a^2c^2-10ab^2c+b^4\right)\operatorname{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}+\frac{\left(b^2-4ac\right)^2\log\left((d+ex)^2\right)}{a}-\frac{\left(b^2-4ac\right)^2\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{2a}\right)}{a\left(b^2-4ac\right)}+\frac{16a^2c^2+2bc\left(b^2-7ac\right)(d+ex)^{2\cdots}}{a\left(b^2-4ac\right)\left(a+b(d+ex)^2+\cdots\right.}}{2a(b^2-4ac)}}{2ef}$$

input
```
Int[1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

3.658. $\int \frac{1}{(df+efx)(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx$

output
```
((b^2 - 2*a*c + b*c*(d + e*x)^2)/(2*a*(b^2 - 4*a*c)*(a + b*(d + e*x)^2 + c
*(d + e*x)^4)^2) + ((2*b^4 - 15*a*b^2*c + 16*a^2*c^2 + 2*b*c*(b^2 - 7*a*c)
*(d + e*x)^2)/(a*(b^2 - 4*a*c)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (2*(
(b*(b^4 - 10*a*b^2*c + 30*a^2*c^2)*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2
- 4*a*c]])/(a*Sqrt[b^2 - 4*a*c]) + ((b^2 - 4*a*c)^2*Log[(d + e*x)^2])/a -
((b^2 - 4*a*c)^2*Log[a + b*(d + e*x)^2 + c*(d + e*x)^4])/(2*a)))/(a*(b^2 -
 4*a*c)))/(2*a*(b^2 - 4*a*c)))/(2*e*f)
```

3.658.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

3.658. $\int \frac{1}{(df+efx)(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx$

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.658.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.90 (sec) , antiderivative size = 970, normalized size of antiderivative = 3.59

method	result
default	$-\frac{\frac{c^2 e^5 \left(7ac-b^2\right) ab\, x^6}{32a^2c^2-16a\, b^2c+2b^4}+\frac{3\left(7ac-b^2\right) ab\, c^2 d\, e^4 x^5}{16a^2c^2-8a\, b^2c+b^4}-\frac{e^3 ac\left(-210b\, c^2 d^2 a+30b^3 c\, d^2+16a^2c^2-29a\, b^2c+4b^4\right) x^4}{4\left(16a^2c^2-8a\, b^2c+b^4\right)}-\frac{cd\, e^2 a\left(-70b\, c^2 d^2 a+10b^3 c\, d^2+16a^2 c \cdots\right.}{16a^2c^2-8a\, b^2c+b \cdots}}{\cdots}$
risch	Expression too large to display

input
```
int(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/a^3*((1/2*c^2*e^5*(7*a*c-b^2)*a*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+3
*(7*a*c-b^2)*a*b*c^2*d*e^4/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5-1/4*e^3*a*c*(-21
0*a*b*c^2*d^2+30*b^3*c*d^2+16*a^2*c^2-29*a*b^2*c+4*b^4)/(16*a^2*c^2-8*a*b^
2*c+b^4)*x^4-c*d*e^2*a*(-70*a*b*c^2*d^2+10*b^3*c*d^2+16*a^2*c^2-29*a*b^2*c
+4*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+1/2*e*a*(105*a*b*c^3*d^4-15*b^3*c^2
*d^4-48*a^2*c^3*d^2+87*a*b^2*c^2*d^2-12*b^4*c*d^2+a^2*b*c^2+6*a*b^3*c-b^5)
/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+d*a*(21*a*b*c^3*d^4-3*b^3*c^2*d^4-16*a^2*c
^3*d^2+29*a*b^2*c^2*d^2-4*b^4*c*d^2+a^2*b*c^2+6*a*b^3*c-b^5)/(16*a^2*c^2-8
*a*b^2*c+b^4)*x-1/4/e*a*(-14*a*b*c^3*d^6+2*b^3*c^2*d^6+16*a^2*c^3*d^4-29*a
*b^2*c^2*d^4+4*b^4*c*d^4-2*a^2*b*c^2*d^2-12*a*b^3*c*d^2+2*b^5*d^2+24*a^3*c
^2-21*a^2*b^2*c+3*a*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*e^4*x^4+4*c*d*e^3*
x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2+1/2/(
16*a^2*c^2-8*a*b^2*c+b^4)/e*sum((e^3*c*(16*a^2*c^2-8*a*b^2*c+b^4)*_R^3+3*c
*d*e^2*(16*a^2*c^2-8*a*b^2*c+b^4)*_R^2+e*(48*a^2*c^3*d^2-24*a*b^2*c^2*d^2+
3*b^4*c*d^2+23*a^2*b*c^2-9*a*b^3*c+b^5)*_R+16*a^2*c^3*d^3-8*b^2*a*c^2*d^3+
b^4*c*d^3+23*a^2*b*c^2*d-9*a*b^3*c*d+b^5*d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6
*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z
^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a)))+ln(e*x
+d)/a^3/e)
```

3.658.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4898 vs. $2(258) = 516$.

Time = 1.58 (sec) , antiderivative size = 9926, normalized size of antiderivative = 36.76

$$\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas
")
```

output
```
Too large to include
```

3.658.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx=\text{Timed out}$$

input
```
integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

output
```
Timed out
```

3.658.7 Maxima [F]

$$\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \int \frac{1}{\left((ex+d)^4c+(ex+d)^2b+a\right)^3(efx+df)}\,dx$$

input
```
integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima
")
```

output
```
1/4*(2*(b^3*c^2 - 7*a*b*c^3)*e^6*x^6 + 12*(b^3*c^2 - 7*a*b*c^3)*d*e^5*x^5
+ (4*b^4*c - 29*a*b^2*c^2 + 16*a^2*c^3 + 30*(b^3*c^2 - 7*a*b*c^3)*d^2)*e^4
*x^4 + 2*(b^3*c^2 - 7*a*b*c^3)*d^6 + 4*(10*(b^3*c^2 - 7*a*b*c^3)*d^3 + (4*
b^4*c - 29*a*b^2*c^2 + 16*a^2*c^3)*d)*e^3*x^3 + 3*a*b^4 - 21*a^2*b^2*c + 2
4*a^3*c^2 + (4*b^4*c - 29*a*b^2*c^2 + 16*a^2*c^3)*d^4 + 2*(b^5 - 6*a*b^3*c
 - a^2*b*c^2 + 15*(b^3*c^2 - 7*a*b*c^3)*d^4 + 3*(4*b^4*c - 29*a*b^2*c^2 +
16*a^2*c^3)*d^2)*e^2*x^2 + 2*(b^5 - 6*a*b^3*c - a^2*b*c^2)*d^2 + 4*(3*(b^3
*c^2 - 7*a*b*c^3)*d^5 + (4*b^4*c - 29*a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 -
 6*a*b^3*c - a^2*b*c^2)*d)*e*x)/((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4
)*e^9*f*x^8 + 8*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d*e^8*f*x^7 + 2
*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3 + 14*(a^2*b^4*c^2 - 8*a^3*b^2*c
^3 + 16*a^4*c^4)*d^2)*e^7*f*x^6 + 4*(14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*
a^4*c^4)*d^3 + 3*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d)*e^6*f*x^5 +
 (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3 + 70*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 1
6*a^4*c^4)*d^4 + 30*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^2)*e^5*f*
x^4 + 4*(14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^5 + 10*(a^2*b^5*c
 - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^3 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3
)*d)*e^4*f*x^3 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2 + 14*(a^2*b^4*c^2
 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^6 + 15*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^
4*b*c^3)*d^4 + 3*(a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d^2)*e^3*f*x^2 + ...
```

3.658.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1077 vs. $2(258) = 516$.

Time = 0.43 (sec) , antiderivative size = 1077, normalized size of antiderivative = 3.99

$$\int \frac{1}{(df + efx)\left(a + b(d + ex)^2 + c(d + ex)^4\right)^3}\,dx =$$

$$-\frac{\left(a^3b^7ce^3f - 14\,a^4b^5c^2e^3f + 70\,a^5b^3c^3e^3f - 120\,a^6bc^4e^3f\right)\sqrt{b^2 - 4\,ac}\log\left(\left|be^2x^2 + \sqrt{b^2 - 4\,ac}e^2x^2 + 2\,bc\right.\right.}{}$$

$$-\frac{\log\left(\left|ce^4x^4 + 4\,cde^3x^3 + 6\,cd^2e^2x^2 + 4\,cd^3ex + cd^4 + be^2x^2 + 2\,bdex + bd^2 + a\right|\right)}{4\,a^3ef}$$

$$+\frac{\log\left(|ex + d|\right)}{a^3ef}$$

$$+\frac{2\,ab^3c^2d^6 - 14\,a^2bc^3d^6 + 4\,ab^4cd^4 - 29\,a^2b^2c^2d^4 + 16\,a^3c^3d^4 + 2\,ab^5d^2 - 12\,a^2b^3cd^2 - 2\,a^3bc^2d^2 + 2\,(ab^3c}{}$$

input
```
integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

output
```
-1/4*((a^3*b^7*c*e^3*f - 14*a^4*b^5*c^2*e^3*f + 70*a^5*b^3*c^3*e^3*f - 120
*a^6*b*c^4*e^3*f)*sqrt(b^2 - 4*a*c)*log(abs(b*e^2*x^2 + sqrt(b^2 - 4*a*c)*
e^2*x^2 + 2*b*d*e*x + 2*sqrt(b^2 - 4*a*c)*d*e*x + b*d^2 + sqrt(b^2 - 4*a*c
)*d^2 + 2*a)) - (a^3*b^7*c*e^3*f - 14*a^4*b^5*c^2*e^3*f + 70*a^5*b^3*c^3*e
^3*f - 120*a^6*b*c^4*e^3*f)*sqrt(b^2 - 4*a*c)*log(abs(-b*e^2*x^2 + sqrt(b^
2 - 4*a*c)*e^2*x^2 - 2*b*d*e*x + 2*sqrt(b^2 - 4*a*c)*d*e*x - b*d^2 + sqrt(
b^2 - 4*a*c)*d^2 - 2*a)))/(a^6*b^8*c*e^4*f^2 - 16*a^7*b^6*c^2*e^4*f^2 + 96
*a^8*b^4*c^3*e^4*f^2 - 256*a^9*b^2*c^4*e^4*f^2 + 256*a^10*c^5*e^4*f^2) - 1
/4*log(abs(c*e^4*x^4 + 4*c*d*e^3*x^3 + 6*c*d^2*e^2*x^2 + 4*c*d^3*e*x + c*d
^4 + b*e^2*x^2 + 2*b*d*e*x + b*d^2 + a))/(a^3*e*f) + log(abs(e*x + d))/(a^
3*e*f) + 1/4*(2*a*b^3*c^2*d^6 - 14*a^2*b*c^3*d^6 + 4*a*b^4*c*d^4 - 29*a^2*
b^2*c^2*d^4 + 16*a^3*c^3*d^4 + 2*a*b^5*d^2 - 12*a^2*b^3*c*d^2 - 2*a^3*b*c^
2*d^2 + 2*(a*b^3*c^2*e^6 - 7*a^2*b*c^3*e^6)*x^6 + 3*a^2*b^4 - 21*a^3*b^2*c
 + 24*a^4*c^2 + 12*(a*b^3*c^2*d*e^5 - 7*a^2*b*c^3*d*e^5)*x^5 + (30*a*b^3*c
^2*d^2*e^4 - 210*a^2*b*c^3*d^2*e^4 + 4*a*b^4*c*e^4 - 29*a^2*b^2*c^2*e^4 +
16*a^3*c^3*e^4)*x^4 + 4*(10*a*b^3*c^2*d^3*e^3 - 70*a^2*b*c^3*d^3*e^3 + 4*a
*b^4*c*d*e^3 - 29*a^2*b^2*c^2*d*e^3 + 16*a^3*c^3*d*e^3)*x^3 + 2*(15*a*b^3*
c^2*d^4*e^2 - 105*a^2*b*c^3*d^4*e^2 + 12*a*b^4*c*d^2*e^2 - 87*a^2*b^2*c^2*
d^2*e^2 + 48*a^3*c^3*d^2*e^2 + a*b^5*e^2 - 6*a^2*b^3*c*e^2 - a^3*b*c^2*e^2
)*x^2 + 4*(3*a*b^3*c^2*d^5*e - 21*a^2*b*c^3*d^5*e + 4*a*b^4*c*d^3*e - 2...
```

3.658.9 Mupad [B] (verification not implemented)

Time = 19.42 (sec) , antiderivative size = 22621, normalized size of antiderivative = 83.78

$$\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx=\text{Too large to display}$$

input
```
int(1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x)
```

output
```
((x^2*(b^5*e + 48*a^2*c^3*d^2*e + 15*b^3*c^2*d^4*e - 6*a*b^3*c*e - a^2*b*c
^2*e + 12*b^4*c*d^2*e - 105*a*b*c^3*d^4*e - 87*a*b^2*c^2*d^2*e))/(2*(a^2*b
^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (x^4*(4*b^4*c*e^3 + 16*a^2*c^3*e^3 - 29*
a*b^2*c^2*e^3 + 30*b^3*c^2*d^2*e^3 - 210*a*b*c^3*d^2*e^3))/(4*(a^2*b^4 + 1
6*a^4*c^2 - 8*a^3*b^2*c)) + (x^3*(16*a^2*c^3*d*e^2 + 10*b^3*c^2*d^3*e^2 +
4*b^4*c*d*e^2 - 29*a*b^2*c^2*d*e^2 - 70*a*b*c^3*d^3*e^2))/(a^2*b^4 + 16*a^
4*c^2 - 8*a^3*b^2*c) + (3*x^5*(b^3*c^2*d*e^4 - 7*a*b*c^3*d*e^4))/(a^2*b^4
+ 16*a^4*c^2 - 8*a^3*b^2*c) + (x^6*(b^3*c^2*e^5 - 7*a*b*c^3*e^5))/(2*(a^2*
b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (x*(b^5*d + 4*b^4*c*d^3 + 16*a^2*c^3*d^
3 + 3*b^3*c^2*d^5 - 29*a*b^2*c^2*d^3 - 6*a*b^3*c*d - a^2*b*c^2*d - 21*a*b*
c^3*d^5))/(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c) + (3*a*b^4 + 24*a^3*c^2 + 2
*b^5*d^2 - 21*a^2*b^2*c + 4*b^4*c*d^4 + 16*a^2*c^3*d^4 + 2*b^3*c^2*d^6 - 2
*a^2*b*c^2*d^2 - 29*a*b^2*c^2*d^4 - 12*a*b^3*c*d^2 - 14*a*b*c^3*d^6)/(4*e*
(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))/(x^3*(56*c^2*d^5*e^3*f + 4*b^2*d*e^
3*f + 40*b*c*d^3*e^3*f + 8*a*c*d*e^3*f) + x^2*(6*b^2*d^2*e^2*f + 28*c^2*d^
6*e^2*f + 2*a*b*e^2*f + 12*a*c*d^2*e^2*f + 30*b*c*d^4*e^2*f) + x*(4*b^2*d^
3*e*f + 8*c^2*d^7*e*f + 4*a*b*d*e*f + 8*a*c*d^3*e*f + 12*b*c*d^5*e*f) + x^
4*(b^2*e^4*f + 70*c^2*d^4*e^4*f + 2*a*c*e^4*f + 30*b*c*d^2*e^4*f) + x^5*(5
6*c^2*d^3*e^5*f + 12*b*c*d*e^5*f) + a^2*f + x^6*(28*c^2*d^2*e^6*f + 2*b*c*
e^6*f) + b^2*d^4*f + c^2*d^8*f + c^2*e^8*f*x^8 + 2*a*b*d^2*f + 2*a*c*d^...
```

3.658. $\int \frac{1}{(df+efx)(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx$

3.659 $\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$

3.659.1 Optimal result

Integrand size = 33, antiderivative size = 499

$$\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = -\frac{3(5b^2-12ac)\left(b^2-5ac\right)}{8a^3\left(b^2-4ac\right)^2ef^2(d+ex)}+\frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)ef^2(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{5b^4-35ab^2c+36a^2c^2+bc(5b^2-32ac)(d+ex)^2}{8a^2\left(b^2-4ac\right)^2ef^2(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{3\sqrt{c}\left((5b^2-12ac)\left(b^2-5ac\right)+\frac{b(5b^4-47ab^2c+124a^2c^2)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a^3\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}ef^2}-\frac{3\sqrt{c}\left((5b^2-12ac)\left(b^2-5ac\right)-\frac{5b^5-47ab^3c+124a^2bc^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a^3\left(b^2-4ac\right)^2\sqrt{b+\sqrt{b^2-4ac}}ef^2}$$

output
```
-3/8*(-12*a*c+5*b^2)*(-5*a*c+b^2)/a^3/(-4*a*c+b^2)^2/e/f^2/(e*x+d)+1/4*(b^
2-2*a*c+b*c*(e*x+d)^2)/a/(-4*a*c+b^2)/e/f^2/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+
d)^4)^2+1/8*(5*b^4-35*a*b^2*c+36*a^2*c^2+b*c*(-32*a*c+5*b^2)*(e*x+d)^2)/a^
2/(-4*a*c+b^2)^2/e/f^2/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)-3/16*arctan((e*
x+d)*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*((-12*a*c+5*b^2
)*(-5*a*c+b^2)+b*(124*a^2*c^2-47*a*b^2*c+5*b^4)/(-4*a*c+b^2)^(1/2))/a^3/(-
4*a*c+b^2)^2/e/f^2*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-3/16*arctan((e*x+d
)*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*((-12*a*c+5*b^2)*(
-5*a*c+b^2)+(-124*a^2*b*c^2+47*a*b^3*c-5*b^5)/(-4*a*c+b^2)^(1/2))/a^3/(-4*
a*c+b^2)^2/e/f^2*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.659.2 Mathematica [A] (verified)

Time = 6.14 (sec) , antiderivative size = 575, normalized size of antiderivative = 1.15

$$\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$$
$$= -\frac{1}{a^3ef^2(d+ex)} + \frac{b^3(d+ex)-3abc(d+ex)+b^2c(d+ex)^3-2ac^2(d+ex)^3}{4a^2\left(-b^2+4ac\right)ef^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}$$
$$+ \frac{-7b^5(d+ex)+52ab^3c(d+ex)-84a^2bc^2(d+ex)-7b^4c(d+ex)^3+47ab^2c^2(d+ex)^3-52a^2c^3(d+ex}{8a^3\left(-b^2+4ac\right)^2ef^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$
$$- \frac{3\sqrt{c}\left(5b^5-47ab^3c+124a^2bc^2+5b^4\sqrt{b^2-4ac}-37ab^2c\sqrt{b^2-4ac}+60a^2c^2\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d-}{\sqrt{b-\sqrt{b^2}}}\right.}{8\sqrt{2}a^3\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}ef^2}$$
$$- \frac{3\sqrt{c}\left(-5b^5+47ab^3c-124a^2bc^2+5b^4\sqrt{b^2-4ac}-37ab^2c\sqrt{b^2-4ac}+60a^2c^2\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{b+\sqrt{}}}\right.}{8\sqrt{2}a^3\left(b^2-4ac\right)^{5/2}\sqrt{b+\sqrt{b^2-4ac}}ef^2}$$

input
```
Integrate[1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

output
```
-(1/(a^3*e*f^2*(d + e*x))) + (b^3*(d + e*x) - 3*a*b*c*(d + e*x) + b^2*c*(d
 + e*x)^3 - 2*a*c^2*(d + e*x)^3)/(4*a^2*(-b^2 + 4*a*c)*e*f^2*(a + b*(d + e
*x)^2 + c*(d + e*x)^4)^2) + (-7*b^5*(d + e*x) + 52*a*b^3*c*(d + e*x) - 84*
a^2*b*c^2*(d + e*x) - 7*b^4*c*(d + e*x)^3 + 47*a*b^2*c^2*(d + e*x)^3 - 52*
a^2*c^3*(d + e*x)^3)/(8*a^3*(-b^2 + 4*a*c)^2*e*f^2*(a + b*(d + e*x)^2 + c*
(d + e*x)^4)) - (3*Sqrt[c]*(5*b^5 - 47*a*b^3*c + 124*a^2*b*c^2 + 5*b^4*Sqr
t[b^2 - 4*a*c] - 37*a*b^2*c*Sqrt[b^2 - 4*a*c] + 60*a^2*c^2*Sqrt[b^2 - 4*a*
c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sq
rt[2]*a^3*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e*f^2) - (3*Sqrt
[c]*(-5*b^5 + 47*a*b^3*c - 124*a^2*b*c^2 + 5*b^4*Sqrt[b^2 - 4*a*c] - 37*a*
b^2*c*Sqrt[b^2 - 4*a*c] + 60*a^2*c^2*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sq
rt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^3*(b^2 - 4*a*c
)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e*f^2)
```

3.659.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 474, normalized size of antiderivative = 0.95, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {1462, 1441, 25, 1600, 27, 1604, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}dx$$

$$\Big\downarrow \text{1462}$$

$$\frac{\int \frac{1}{(d+ex)^2(c(d+ex)^4+b(d+ex)^2+a)^3}d(d+ex)}{ef^2}$$

$$\Big\downarrow \text{1441}$$

$$\frac{\frac{-2ac+b^2+bc(d+ex)^2}{4a(b^2-4ac)(d+ex)(a+b(d+ex)^2+c(d+ex)^4)^2}-\frac{\int -\frac{5b^2+7c(d+ex)^2b-18ac}{(d+ex)^2\left(c(d+ex)^4+b(d+ex)^2+a\right)^2}d(d+ex)}{4a(b^2-4ac)}}{ef^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{5b^2+7c(d+ex)^2b-18ac}{(d+ex)^2\left(c(d+ex)^4+b(d+ex)^2+a\right)^2}d(d+ex)}{4a(b^2-4ac)}+\frac{-2ac+b^2+bc(d+ex)^2}{4a(b^2-4ac)(d+ex)(a+b(d+ex)^2+c(d+ex)^4)^2}}{ef^2}$$

$$\Big\downarrow \text{1600}$$

$$\frac{\frac{\frac{36a^2c^2+bc\left(5b^2-32ac\right)(d+ex)^2-35ab^2c+5b^4}{2a\left(b^2-4ac\right)(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{\int -\frac{3\left(bc\left(5b^2-32ac\right)(d+ex)^2+\left(5b^2-12ac\right)\left(b^2-5ac\right)\right)}{(d+ex)^2\left(c(d+ex)^4+b(d+ex)^2+a\right)}d(d+ex)}{2a\left(b^2-4ac\right)}}{4a(b^2-4ac)}+\frac{-2ac+b^2+bc(d+ex)^2}{4a(b^2-4ac)(d+ex)(a+b(d+ex)^2+c(d+e}}{ef^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{3\int \frac{bc\left(5b^2-32ac\right)(d+ex)^2+\left(5b^2-12ac\right)\left(b^2-5ac\right)}{(d+ex)^2\left(c(d+ex)^4+b(d+ex)^2+a\right)}d(d+ex)}{2a\left(b^2-4ac\right)}+\frac{36a^2c^2+bc\left(5b^2-32ac\right)(d+ex)^2-35ab^2c+5b^4}{2a\left(b^2-4ac\right)(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)}}{4a(b^2-4ac)}+\frac{-2ac+b^2+bc(d+ex)^2}{4a(b^2-4ac)(d+ex)(a+b(d+ex)^2+c(d+ex)^4}}{ef^2}$$

$$\Big\downarrow \text{1604}$$

$$\frac{\frac{3\left(-\frac{\int \frac{c\left(5b^2-12ac\right)\left(b^2-5ac\right)(d+ex)^2+b\left(5b^4-42acb^2+92a^2c^2\right)}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)}{a}-\frac{\left(5b^2-12ac\right)\left(b^2-5ac\right)}{a(d+ex)}\right)}{2a\left(b^2-4ac\right)}+\frac{36a^2c^2+bc\left(5b^2-32ac\right)(d+ex)^2-35ab^2c+5b^4}{2a\left(b^2-4ac\right)(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)}}{4a(b^2-4ac)}+\frac{\ldots}{4a(b^2-\ldots}}{ef^2}$$

$\Big\downarrow$ 1480

$$\frac{3\left(-\frac{\frac{1}{2}c\left(\frac{b\left(124a^2c^2-47ab^2c+5b^4\right)}{\sqrt{b^2-4ac}}+\left(5b^2-12ac\right)\left(b^2-5ac\right)\right)\int \frac{1}{c(d+ex)^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}d(d+ex)+\frac{1}{2}c\left(\left(5b^2-12ac\right)\left(b^2-5ac\right)-\frac{b\left(124a^2c^2-47ab^2c+5b^4\right)}{\sqrt{b^2-4ac}}\right)\int \ldots}{a}\ldots\right)}{\frac{2a\left(b^2-4ac\right)}{4a(b^2-4ac)}}$$

$\Big\downarrow$ 218

$$\frac{3\left(-\frac{\frac{\sqrt{c}\left(\frac{b\left(124a^2c^2-47ab^2c+5b^4\right)}{\sqrt{b^2-4ac}}+\left(5b^2-12ac\right)\left(b^2-5ac\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(\left(5b^2-12ac\right)\left(b^2-5ac\right)-\frac{b\left(124a^2c^2-47ab^2c+5b^4\right)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{\ldots}}{\sqrt{\sqrt{b\ldots}}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}}{a}\ldots\right)}{\frac{2a\left(b^2-4ac\right)}{4a(b^2-4ac)}}$$

$$ef^2$$

input
```
Int[1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

output
```
((b^2 - 2*a*c + b*c*(d + e*x)^2)/(4*a*(b^2 - 4*a*c)*(d + e*x)*(a + b*(d +
e*x)^2 + c*(d + e*x)^4)^2) + ((5*b^4 - 35*a*b^2*c + 36*a^2*c^2 + b*c*(5*b^
2 - 32*a*c)*(d + e*x)^2)/(2*a*(b^2 - 4*a*c)*(d + e*x)*(a + b*(d + e*x)^2 +
 c*(d + e*x)^4)) + (3*(-(((5*b^2 - 12*a*c)*(b^2 - 5*a*c))/(a*(d + e*x))) -
 ((Sqrt[c]*((5*b^2 - 12*a*c)*(b^2 - 5*a*c) + (b*(5*b^4 - 47*a*b^2*c + 124*
a^2*c^2))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - S
qrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*((5*b
^2 - 12*a*c)*(b^2 - 5*a*c) - (b*(5*b^4 - 47*a*b^2*c + 124*a^2*c^2))/Sqrt[b
^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]
]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/a))/(2*a*(b^2 - 4*a*c)))/(4*a*(
b^2 - 4*a*c)))/(e*f^2)
```

3.659. $\int \frac{1}{(df+efx)^2(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx$

3.659.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1441
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(-(d*x)^(m + 1))*(b^2 - 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p +
1)/(2*a*d*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))
  Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m
 + 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x
] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] ||
IntegerQ[m])
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1600
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(a + b*x^2 + c*x^4)^(p + 1)
*((d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^2)/(2*a*f*(p + 1)*(b^2 - 4*a
*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(f*x)^m*(a + b*x^2 + c
*x^4)^(p + 1)*Simp[d*(b^2*(m + 2*(p + 1) + 1) - 2*a*c*(m + 4*(p + 1) + 1))
- a*b*e*(m + 1) + c*(m + 2*(2*p + 3) + 1)*(b*d - 2*a*e)*x^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Int
egerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

3.659. $\int \frac{1}{(df+efx)^2(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx$

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

3.659.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.92 (sec) , antiderivative size = 1201, normalized size of antiderivative = 2.41

method	result	size
default	Expression too large to display	1201
risch	Expression too large to display	2710

input
```
int(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f^2*(-1/a^3*((1/8*c^2*e^6*(52*a^2*c^2-47*a*b^2*c+7*b^4)/(16*a^2*c^2-8*a*
b^2*c+b^4)*x^7+7/8*c^2*d*e^5*(52*a^2*c^2-47*a*b^2*c+7*b^4)/(16*a^2*c^2-8*a
*b^2*c+b^4)*x^6+1/8*(1092*a^2*c^3*d^2-987*a*b^2*c^2*d^2+147*b^4*c*d^2+136*
a^2*b*c^2-99*a*b^3*c+14*b^5)*c*e^4/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5+5/8*c*d*
e^3*(364*a^2*c^3*d^2-329*a*b^2*c^2*d^2+49*b^4*c*d^2+136*a^2*b*c^2-99*a*b^3
*c+14*b^5)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4+1/8*e^2*(1820*a^2*c^4*d^4-1645*a
*b^2*c^3*d^4+245*b^4*c^2*d^4+1360*a^2*b*c^3*d^2-990*a*b^3*c^2*d^2+140*b^5*
c*d^2+68*a^3*c^3+25*a^2*b^2*c^2-43*a*b^4*c+7*b^6)/(16*a^2*c^2-8*a*b^2*c+b^
4)*x^3+1/8*d*e*(1092*a^2*c^4*d^4-987*a*b^2*c^3*d^4+147*b^4*c^2*d^4+1360*a^
2*b*c^3*d^2-990*a*b^3*c^2*d^2+140*b^5*c*d^2+204*a^3*c^3+75*a^2*b^2*c^2-129
*a*b^4*c+21*b^6)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+1/8*(364*a^2*c^4*d^6-329*a
*b^2*c^3*d^6+49*b^4*c^2*d^6+680*a^2*b*c^3*d^4-495*a*b^3*c^2*d^4+70*b^5*c*d
^4+204*a^3*c^3*d^2+75*a^2*b^2*c^2*d^2-129*a*b^4*c*d^2+21*b^6*d^2+108*a^3*b
*c^2-66*a^2*b^3*c+9*a*b^5)/(16*a^2*c^2-8*a*b^2*c+b^4)*x+1/8*d/e*(52*a^2*c^
4*d^6-47*a*b^2*c^3*d^6+7*b^4*c^2*d^6+136*a^2*b*c^3*d^4-99*a*b^3*c^2*d^4+14
*b^5*c*d^4+68*a^3*c^3*d^2+25*a^2*b^2*c^2*d^2-43*a*b^4*c*d^2+7*b^6*d^2+108*
a^3*b*c^2-66*a^2*b^3*c+9*a*b^5)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*e^4*x^4+4*c
*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^
2+3/16/(16*a^2*c^2-8*a*b^2*c+b^4)/e*sum((c*e^2*(60*a^2*c^2-37*a*b^2*c+5*b^
4)*_R^2+2*d*c*e*(60*a^2*c^2-37*a*b^2*c+5*b^4)*_R+60*a^2*c^3*d^2-37*a*b^...
```

3.659.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10518 vs. 2(453) = 906.

Time = 1.14 (sec) , antiderivative size = 10518, normalized size of antiderivative = 21.08

$$\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx=\text{Too large to display}$$

input
```
integrate(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fric
as")
```

output
```
Too large to include
```

3.659.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx=\text{Timed out}$$

input
```
integrate(1/(e*f*x+d*f)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

output
```
Timed out
```

3.659.7 Maxima [F]

$$\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \int \frac{1}{\left((ex+d)^4c+(ex+d)^2b+a\right)^3(efx+df)^2}\,dx$$

input
```
integrate(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxi
ma")
```

output
```
-1/8*(3*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*e^8*x^8 + 24*(5*b^4*c^2 -
37*a*b^2*c^3 + 60*a^2*c^4)*d*e^7*x^7 + (30*b^5*c - 227*a*b^3*c^2 + 392*a^2
*b*c^3 + 84*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^2)*e^6*x^6 + 6*(28*(
5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^3 + (30*b^5*c - 227*a*b^3*c^2 + 3
92*a^2*b*c^3)*d)*e^5*x^5 + 3*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^8 +
 (15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c^2 + 324*a^3*c^3 + 210*(5*b^4*c^2 - 37
*a*b^2*c^3 + 60*a^2*c^4)*d^4 + 15*(30*b^5*c - 227*a*b^3*c^2 + 392*a^2*b*c^
3)*d^2)*e^4*x^4 + (30*b^5*c - 227*a*b^3*c^2 + 392*a^2*b*c^3)*d^6 + 4*(42*(
5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^5 + 5*(30*b^5*c - 227*a*b^3*c^2 +
 392*a^2*b*c^3)*d^3 + (15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c^2 + 324*a^3*c^3)
*d)*e^3*x^3 + 8*a^2*b^4 - 64*a^3*b^2*c + 128*a^4*c^2 + (15*b^6 - 91*a*b^4*
c + 25*a^2*b^2*c^2 + 324*a^3*c^3)*d^4 + (84*(5*b^4*c^2 - 37*a*b^2*c^3 + 60
*a^2*c^4)*d^6 + 25*a*b^5 - 194*a^2*b^3*c + 364*a^3*b*c^2 + 15*(30*b^5*c -
227*a*b^3*c^2 + 392*a^2*b*c^3)*d^4 + 6*(15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c
^2 + 324*a^3*c^3)*d^2)*e^2*x^2 + (25*a*b^5 - 194*a^2*b^3*c + 364*a^3*b*c^2
)*d^2 + 2*(12*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^7 + 3*(30*b^5*c -
227*a*b^3*c^2 + 392*a^2*b*c^3)*d^5 + 2*(15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c
^2 + 324*a^3*c^3)*d^3 + (25*a*b^5 - 194*a^2*b^3*c + 364*a^3*b*c^2)*d)*e*x)
/((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e^10*f^2*x^9 + 9*(a^3*b^4*c^2
 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d*e^9*f^2*x^8 + 2*(a^3*b^5*c - 8*a^4*b^3...
```

3.659.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1704 vs. $2(453) = 906$.

Time = 0.40 (sec) , antiderivative size = 1704, normalized size of antiderivative = 3.41

$$\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac
")
```

output
```
-1/8*(7*b^4*c^2/((e*f*x + d*f)*e*f) - 47*a*b^2*c^3/((e*f*x + d*f)*e*f) + 5
2*a^2*c^4/((e*f*x + d*f)*e*f) + 14*b^5*c*f/((e*f*x + d*f)^3*e) - 99*a*b^3*
c^2*f/((e*f*x + d*f)^3*e) + 136*a^2*b*c^3*f/((e*f*x + d*f)^3*e) + 7*b^6*f^
3/((e*f*x + d*f)^5*e) - 43*a*b^4*c*f^3/((e*f*x + d*f)^5*e) + 25*a^2*b^2*c^
2*f^3/((e*f*x + d*f)^5*e) + 68*a^3*c^3*f^3/((e*f*x + d*f)^5*e) + 9*a*b^5*f
^5/((e*f*x + d*f)^7*e) - 66*a^2*b^3*c*f^5/((e*f*x + d*f)^7*e) + 108*a^3*b*
c^2*f^5/((e*f*x + d*f)^7*e))/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*(c + b*
f^2/(e*f*x + d*f)^2 + a*f^4/(e*f*x + d*f)^4)^2) - 1/((e*f*x + d*f)*a^3*e*f
) + 3/64*((5*a^6*b^13 - 112*a^7*b^11*c + 1030*a^8*b^9*c^2 - 4928*a^9*b^7*c
^3 + 12736*a^10*b^5*c^4 - 16384*a^11*b^3*c^5 + 7680*a^12*b*c^6)*sqrt(2*a*b
 + 2*sqrt(b^2 - 4*a*c)*a)*e^4*f^8 + 2*(5*a^4*b^6*c - 57*a^5*b^4*c^2 + 208*
a^6*b^2*c^3 - 240*a^7*c^4)*sqrt(2*a*b + 2*sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 -
4*a*c)*e^2*f^4*abs(a^3*b^4*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4 + 16*a^5*c^2*e^2*
f^4) - (a^3*b^4*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4 + 16*a^5*c^2*e^2*f^4)^2*(5*b
^5 - 42*a*b^3*c + 92*a^2*b*c^2)*sqrt(2*a*b + 2*sqrt(b^2 - 4*a*c)*a))*arcta
n(2*sqrt(1/2)/((e*f*x + d*f)*e*f*sqrt((a^3*b^5*e^2*f^4 - 8*a^4*b^3*c*e^2*f
^4 + 16*a^5*b*c^2*e^2*f^4 + sqrt((a^3*b^5*e^2*f^4 - 8*a^4*b^3*c*e^2*f^4 +
16*a^5*b*c^2*e^2*f^4)^2 - 4*(a^4*b^4*e^4*f^8 - 8*a^5*b^2*c*e^4*f^8 + 16*a^
6*c^2*e^4*f^8)*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)))/(a^4*b^4*e^4*f^8
 - 8*a^5*b^2*c*e^4*f^8 + 16*a^6*c^2*e^4*f^8))))/((a^7*b^6*c - 12*a^8*b^...
```

3.659.9 Mupad [B] (verification not implemented)

Time = 17.02 (sec) , antiderivative size = 20580, normalized size of antiderivative = 41.24

$$\int \frac{1}{(df+efx)^2\,(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = \text{Too large to display}$$

input
```
int(1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x)
```

output
```
- atan(((-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b
*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 -
6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684
160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(
1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^
4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2*f^4 + 1048576*a^17*c^10
*e^2*f^4 + 720*a^9*b^16*c^2*e^2*f^4 - 7680*a^10*b^14*c^3*e^2*f^4 + 53760*a
^11*b^12*c^4*e^2*f^4 - 258048*a^12*b^10*c^5*e^2*f^4 + 860160*a^13*b^8*c^6*
e^2*f^4 - 1966080*a^14*b^6*c^7*e^2*f^4 + 2949120*a^15*b^4*c^8*e^2*f^4 - 26
21440*a^16*b^2*c^9*e^2*f^4 - 40*a^8*b^18*c*e^2*f^4)))^(1/2)*(x*(2717908992
00*a^20*c^14*e^12*f^6 - 230400*a^9*b^22*c^3*e^12*f^6 + 9861120*a^10*b^20*c
^4*e^12*f^6 - 191038464*a^11*b^18*c^5*e^12*f^6 + 2207803392*a^12*b^16*c^6*
e^12*f^6 - 16878108672*a^13*b^14*c^7*e^12*f^6 + 89374851072*a^14*b^12*c^8*
e^12*f^6 - 333226967040*a^15*b^10*c^9*e^12*f^6 + 869815812096*a^16*b^8*c^1
0*e^12*f^6 - 1543847804928*a^17*b^6*c^11*e^12*f^6 + 1747313491968*a^18*b^4
*c^12*e^12*f^6 - 1101055131648*a^19*b^2*c^13*e^12*f^6) - (-(9*(25*b^21 - 2
5*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^
2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19
905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 5203968
0*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - ...
```

3.659. $\int \frac{1}{(df+efx)^2(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx$

3.660 $\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$

3.660.1 Optimal result

Integrand size = 33, antiderivative size = 343

$$\begin{aligned}
&\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx \\
&= -\frac{3(b^2-5ac)\,(b^2-2ac)}{2a^3\,(b^2-4ac)^2\,ef^3(d+ex)^2} \\
&\quad+\frac{b^2-2ac+bc(d+ex)^2}{4a\,(b^2-4ac)\,ef^3(d+ex)^2\,\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} \\
&\quad+\frac{3b^4-20ab^2c+20a^2c^2+3bc(b^2-6ac)\,(d+ex)^2}{4a^2\,(b^2-4ac)^2\,ef^3(d+ex)^2\,\left(a+b(d+ex)^2+c(d+ex)^4\right)} \\
&\quad-\frac{3(b^6-10ab^4c+30a^2b^2c^2-20a^3c^3)\,\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^4\,(b^2-4ac)^{5/2}\,ef^3} \\
&\quad-\frac{3b\log(d+ex)}{a^4ef^3}+\frac{3b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^4ef^3}
\end{aligned}$$

output
```
-3/2*(-5*a*c+b^2)*(-2*a*c+b^2)/a^3/(-4*a*c+b^2)^2/e/f^3/(e*x+d)^2+1/4*(b^2
-2*a*c+b*c*(e*x+d)^2)/a/(-4*a*c+b^2)/e/f^3/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x
+d)^4)^2+1/4*(3*b^4-20*a*b^2*c+20*a^2*c^2+3*b*c*(-6*a*c+b^2)*(e*x+d)^2)/a^
2/(-4*a*c+b^2)^2/e/f^3/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)-3/2*(-20*a^3*
c^3+30*a^2*b^2*c^2-10*a*b^4*c+b^6)*arctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^
(1/2))/a^4/(-4*a*c+b^2)^(5/2)/e/f^3-3*b*ln(e*x+d)/a^4/e/f^3+3/4*b*ln(a+b*(
e*x+d)^2+c*(e*x+d)^4)/a^4/e/f^3
```

3.660.2 Mathematica [A] (verified)

Time = 6.06 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.34

$$\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$$
$$= \frac{-\frac{2a}{(d+ex)^2}+\frac{a^2(b^3-3abc+b^2c(d+ex)^2-2ac^2(d+ex)^2)}{(-b^2+4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}-\frac{a(4b^5-29ab^3c+46a^2bc^2+4b^4c(d+ex)^2-26ab^2c^2(d+ex)^2+28a^2c^3(d+ex)^2)}{(b^2-4ac)^2(a+(d+ex)^2(b+c(d+ex)^2))}-12}{}$$

input
```
Integrate[1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

output
```
((-2*a)/(d + e*x)^2 + (a^2*(b^3 - 3*a*b*c + b^2*c*(d + e*x)^2 - 2*a*c^2*(d
 + e*x)^2))/((-b^2 + 4*a*c)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2) - (a*(4
*b^5 - 29*a*b^3*c + 46*a^2*b*c^2 + 4*b^4*c*(d + e*x)^2 - 26*a*b^2*c^2*(d +
 e*x)^2 + 28*a^2*c^3*(d + e*x)^2))/((b^2 - 4*a*c)^2*(a + (d + e*x)^2*(b +
c*(d + e*x)^2))) - 12*b*Log[d + e*x] + (3*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c
^2 - 20*a^3*c^3 + b^5*Sqrt[b^2 - 4*a*c] - 8*a*b^3*c*Sqrt[b^2 - 4*a*c] + 16
*a^2*b*c^2*Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2]
)/(b^2 - 4*a*c)^(5/2) + (3*(-b^6 + 10*a*b^4*c - 30*a^2*b^2*c^2 + 20*a^3*c^
3 + b^5*Sqrt[b^2 - 4*a*c] - 8*a*b^3*c*Sqrt[b^2 - 4*a*c] + 16*a^2*b*c^2*Sqr
t[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(b^2 - 4*a*c
)^(5/2))/(4*a^4*e*f^3)
```

3.660.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {1462, 1434, 1165, 25, 1235, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$$

$$\Big\downarrow \text{1462}$$

$$\frac{\int \frac{1}{(d+ex)^3\left(c(d+ex)^4+b(d+ex)^2+a\right)^3}d(d+ex)}{ef^3}$$

$$\Big\downarrow \text{1434}$$

$$\frac{\int \frac{1}{(d+ex)^4(c(d+ex)^4+b(d+ex)^2+a)^3}d(d+ex)^2}{2ef^3}$$

$\downarrow$ 1165

$$\frac{\frac{-2ac+b^2+bc(d+ex)^2}{2a(b^2-4ac)(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)^2}-\frac{\int -\frac{3b^2+4c(d+ex)^2b-10ac}{(d+ex)^4\left(c(d+ex)^4+b(d+ex)^2+a\right)^2}d(d+ex)^2}{2a(b^2-4ac)}}{2ef^3}$$

$\downarrow$ 25

$$\frac{\frac{\int \frac{3b^2+4c(d+ex)^2b-10ac}{(d+ex)^4\left(c(d+ex)^4+b(d+ex)^2+a\right)^2}d(d+ex)^2}{2a(b^2-4ac)}+\frac{-2ac+b^2+bc(d+ex)^2}{2a(b^2-4ac)(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)^2}}{2ef^3}$$

$\downarrow$ 1235

$$\frac{\frac{\frac{20a^2c^2+3bc\left(b^2-6ac\right)(d+ex)^2-20ab^2c+3b^4}{a\left(b^2-4ac\right)(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{\int -\frac{6\left(bc\left(b^2-6ac\right)(d+ex)^2+\left(b^2-5ac\right)\left(b^2-2ac\right)\right)}{(d+ex)^4\left(c(d+ex)^4+b(d+ex)^2+a\right)}d(d+ex)^2}{a\left(b^2-4ac\right)}}{2a(b^2-4ac)}+\frac{-2ac+b^2+bc(d+ex)^2}{2a(b^2-4ac)(d+ex)^2(a+b(d+ex)^2+c(d+ex)}}{2ef^3}$$

$\downarrow$ 27

$$\frac{\frac{\frac{6\int \frac{bc\left(b^2-6ac\right)(d+ex)^2+\left(b^2-5ac\right)\left(b^2-2ac\right)}{(d+ex)^4\left(c(d+ex)^4+b(d+ex)^2+a\right)}d(d+ex)^2}{a\left(b^2-4ac\right)}+\frac{20a^2c^2+3bc\left(b^2-6ac\right)(d+ex)^2-20ab^2c+3b^4}{a\left(b^2-4ac\right)(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}}{2a(b^2-4ac)}+\frac{-2ac+b^2+bc(d+ex)^2}{2a(b^2-4ac)(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)^2}}{2ef^3}$$

$\downarrow$ 1200

$$\frac{\frac{\frac{6\int \left(-\frac{b\left(4ac-b^2\right)^2}{a^2(d+ex)^2}+\frac{b^6-9acb^4+23a^2c^2b^2+c\left(b^2-4ac\right)^2(d+ex)^2b-10a^3c^3}{a^2\left(c(d+ex)^4+b(d+ex)^2+a\right)}+\frac{\left(b^2-5ac\right)\left(b^2-2ac\right)}{a(d+ex)^4}\right)d(d+ex)^2}{a\left(b^2-4ac\right)}+\frac{20a^2c^2+3bc\left(b^2-6ac\right)(d+ex)^2-20ab^2c+3b^4}{a\left(b^2-4ac\right)(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}}{2a(b^2-4ac)}+}{2ef^3}$$

$\downarrow$ 2009

$$\frac{\frac{\frac{20a^2c^2+3bc\left(b^2-6ac\right)(d+ex)^2-20ab^2c+3b^4}{a\left(b^2-4ac\right)(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{6\left(-\frac{b\left(b^2-4ac\right)^2\log\left((d+ex)^2\right)}{a^2}+\frac{b\left(b^2-4ac\right)^2\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{2a^2}-\frac{\left(-20a^3c^3+30a^2b^2c^2-10ab^4c+b^6\right)}{a^2\sqrt{b^2-}}\right.}{a\left(b^2-4ac\right)}}{2a(b^2-4ac)}}{2ef^3}$$

3.660. $\int \frac{1}{(df+efx)^3(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx$

input
```
Int[1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

output
```
((b^2 - 2*a*c + b*c*(d + e*x)^2)/(2*a*(b^2 - 4*a*c)*(d + e*x)^2*(a + b*(d
+ e*x)^2 + c*(d + e*x)^4)^2) + ((3*b^4 - 20*a*b^2*c + 20*a^2*c^2 + 3*b*c*(
b^2 - 6*a*c)*(d + e*x)^2)/(a*(b^2 - 4*a*c)*(d + e*x)^2*(a + b*(d + e*x)^2
+ c*(d + e*x)^4)) + (6*(-(((b^2 - 5*a*c)*(b^2 - 2*a*c))/(a*(d + e*x)^2)) -
 ((b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*ArcTanh[(b + 2*c*(d + e
*x)^2)/Sqrt[b^2 - 4*a*c]])/(a^2*Sqrt[b^2 - 4*a*c]) - (b*(b^2 - 4*a*c)^2*Lo
g[(d + e*x)^2])/a^2 + (b*(b^2 - 4*a*c)^2*Log[a + b*(d + e*x)^2 + c*(d + e*
x)^4])/(2*a^2)))/(a*(b^2 - 4*a*c)))/(2*a*(b^2 - 4*a*c)))/(2*e*f^3)
```

3.660.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

3.660. $\int \frac{1}{(df+efx)^3(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx$

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.660.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.00 (sec) , antiderivative size = 1145, normalized size of antiderivative = 3.34

method	result	size
default	Expression too large to display	1145
risch	Expression too large to display	2364

input
```
int(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f^3*(-1/a^4*((1/2*c^2*e^5*(14*a^2*c^2-13*a*b^2*c+2*b^4)*a/(16*a^2*c^2-8*
a*b^2*c+b^4)*x^6+3*(14*a^2*c^2-13*a*b^2*c+2*b^4)*a*c^2*d*e^4/(16*a^2*c^2-8
*a*b^2*c+b^4)*x^5+1/4*e^3*a*c*(420*a^2*c^3*d^2-390*a*b^2*c^2*d^2+60*b^4*c*
d^2+74*a^2*b*c^2-55*a*b^3*c+8*b^5)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4+c*d*e^2*
a*(140*a^2*c^3*d^2-130*a*b^2*c^2*d^2+20*b^4*c*d^2+74*a^2*b*c^2-55*a*b^3*c+
8*b^5)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+1/2*e*a*(210*a^2*c^4*d^4-195*a*b^2*c
^3*d^4+30*b^4*c^2*d^4+222*a^2*b*c^3*d^2-165*a*b^3*c^2*d^2+24*b^5*c*d^2+18*
a^3*c^3+7*a^2*b^2*c^2-12*a*b^4*c+2*b^6)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+d*a
*(42*a^2*c^4*d^4-39*a*b^2*c^3*d^4+6*b^4*c^2*d^4+74*a^2*b*c^3*d^2-55*a*b^3*
c^2*d^2+8*b^5*c*d^2+18*a^3*c^3+7*a^2*b^2*c^2-12*a*b^4*c+2*b^6)/(16*a^2*c^2
-8*a*b^2*c+b^4)*x+1/4/e*a*(28*a^2*c^4*d^6-26*a*b^2*c^3*d^6+4*b^4*c^2*d^6+7
4*a^2*b*c^3*d^4-55*a*b^3*c^2*d^4+8*b^5*c*d^4+36*a^3*c^3*d^2+14*a^2*b^2*c^2
*d^2-24*a*b^4*c*d^2+4*b^6*d^2+58*a^3*b*c^2-36*a^2*b^3*c+5*a*b^5)/(16*a^2*c
^2-8*a*b^2*c+b^4))/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*
e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2+3/2/(16*a^2*c^2-8*a*b^2*c+b^4)/e*sum((e
^3*b*c*(-16*a^2*c^2+8*a*b^2*c-b^4)*_R^3+3*d*e^2*b*c*(-16*a^2*c^2+8*a*b^2*c
-b^4)*_R^2+e*(-48*a^2*b*c^3*d^2+24*a*b^3*c^2*d^2-3*b^5*c*d^2+10*a^3*c^3-23
*a^2*b^2*c^2+9*a*b^4*c-b^6)*_R-16*a^2*b*c^3*d^3+8*a*b^3*c^2*d^3-b^5*c*d^3+
10*a^3*c^3*d-23*a^2*b^2*c^2*d+9*a*b^4*c*d-b^6*d)/(2*_R^3*c*e^3+6*_R^2*c*d*
e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*...
```

3.660.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7550 vs. $2(329) = 658$.

Time = 4.42 (sec) , antiderivative size = 15231, normalized size of antiderivative = 44.41

$$\int \frac{1}{(df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fric
as")
```

output
```
Too large to include
```

3.660.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*f*x+d*f)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

output
```
Timed out
```

3.660.7 Maxima [F]

$$\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \int \frac{1}{\left((ex+d)^4c+(ex+d)^2b+a\right)^3(efx+df)^3}\,dx$$

input
```
integrate(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxi
ma")
```

output
```
-1/4*(6*(b^4*c^2 - 7*a*b^2*c^3 + 10*a^2*c^4)*e^8*x^8 + 48*(b^4*c^2 - 7*a*b
^2*c^3 + 10*a^2*c^4)*d*e^7*x^7 + 3*(4*b^5*c - 29*a*b^3*c^2 + 46*a^2*b*c^3
+ 56*(b^4*c^2 - 7*a*b^2*c^3 + 10*a^2*c^4)*d^2)*e^6*x^6 + 6*(56*(b^4*c^2 -
7*a*b^2*c^3 + 10*a^2*c^4)*d^3 + 3*(4*b^5*c - 29*a*b^3*c^2 + 46*a^2*b*c^3)*
d)*e^5*x^5 + 6*(b^4*c^2 - 7*a*b^2*c^3 + 10*a^2*c^4)*d^8 + (6*b^6 - 36*a*b^
4*c + 14*a^2*b^2*c^2 + 100*a^3*c^3 + 420*(b^4*c^2 - 7*a*b^2*c^3 + 10*a^2*c
^4)*d^4 + 45*(4*b^5*c - 29*a*b^3*c^2 + 46*a^2*b*c^3)*d^2)*e^4*x^4 + 3*(4*b
^5*c - 29*a*b^3*c^2 + 46*a^2*b*c^3)*d^6 + 4*(84*(b^4*c^2 - 7*a*b^2*c^3 + 1
0*a^2*c^4)*d^5 + 15*(4*b^5*c - 29*a*b^3*c^2 + 46*a^2*b*c^3)*d^3 + 2*(3*b^6
 - 18*a*b^4*c + 7*a^2*b^2*c^2 + 50*a^3*c^3)*d)*e^3*x^3 + 2*a^2*b^4 - 16*a^
3*b^2*c + 32*a^4*c^2 + 2*(3*b^6 - 18*a*b^4*c + 7*a^2*b^2*c^2 + 50*a^3*c^3)
*d^4 + (168*(b^4*c^2 - 7*a*b^2*c^3 + 10*a^2*c^4)*d^6 + 9*a*b^5 - 68*a^2*b^
3*c + 122*a^3*b*c^2 + 45*(4*b^5*c - 29*a*b^3*c^2 + 46*a^2*b*c^3)*d^4 + 12*
(3*b^6 - 18*a*b^4*c + 7*a^2*b^2*c^2 + 50*a^3*c^3)*d^2)*e^2*x^2 + (9*a*b^5
- 68*a^2*b^3*c + 122*a^3*b*c^2)*d^2 + 2*(24*(b^4*c^2 - 7*a*b^2*c^3 + 10*a^
2*c^4)*d^7 + 9*(4*b^5*c - 29*a*b^3*c^2 + 46*a^2*b*c^3)*d^5 + 4*(3*b^6 - 18
*a*b^4*c + 7*a^2*b^2*c^2 + 50*a^3*c^3)*d^3 + (9*a*b^5 - 68*a^2*b^3*c + 122
*a^3*b*c^2)*d)*e*x)/((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e^11*f^3*x
^10 + 10*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d*e^10*f^3*x^9 + (2*a^
3*b^5*c - 16*a^4*b^3*c^2 + 32*a^5*b*c^3 + 45*(a^3*b^4*c^2 - 8*a^4*b^2*c...
```

3.660.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1791 vs. $2(329) = 658$.

Time = 0.43 (sec) , antiderivative size = 1791, normalized size of antiderivative = 5.22

$$\int \frac{1}{(df+efx)^3\,(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac
")
```

output
```
3/4*((a^4*b^8*c*e^3*f^3 - 14*a^5*b^6*c^2*e^3*f^3 + 70*a^6*b^4*c^3*e^3*f^3
- 140*a^7*b^2*c^4*e^3*f^3 + 80*a^8*c^5*e^3*f^3)*sqrt(b^2 - 4*a*c)*log(abs(
b*e^2*x^2 + sqrt(b^2 - 4*a*c)*e^2*x^2 + 2*b*d*e*x + 2*sqrt(b^2 - 4*a*c)*d*
e*x + b*d^2 + sqrt(b^2 - 4*a*c)*d^2 + 2*a)) - (a^4*b^8*c*e^3*f^3 - 14*a^5*
b^6*c^2*e^3*f^3 + 70*a^6*b^4*c^3*e^3*f^3 - 140*a^7*b^2*c^4*e^3*f^3 + 80*a^
8*c^5*e^3*f^3)*sqrt(b^2 - 4*a*c)*log(abs(-b*e^2*x^2 + sqrt(b^2 - 4*a*c)*e^
2*x^2 - 2*b*d*e*x + 2*sqrt(b^2 - 4*a*c)*d*e*x - b*d^2 + sqrt(b^2 - 4*a*c)*
d^2 - 2*a)))/(a^8*b^8*c*e^4*f^6 - 16*a^9*b^6*c^2*e^4*f^6 + 96*a^10*b^4*c^3
*e^4*f^6 - 256*a^11*b^2*c^4*e^4*f^6 + 256*a^12*c^5*e^4*f^6) - 1/4*(6*b^4*c
^2*e^8*x^8 - 42*a*b^2*c^3*e^8*x^8 + 60*a^2*c^4*e^8*x^8 + 48*b^4*c^2*d*e^7*
x^7 - 336*a*b^2*c^3*d*e^7*x^7 + 480*a^2*c^4*d*e^7*x^7 + 168*b^4*c^2*d^2*e^
6*x^6 - 1176*a*b^2*c^3*d^2*e^6*x^6 + 1680*a^2*c^4*d^2*e^6*x^6 + 336*b^4*c^
2*d^3*e^5*x^5 - 2352*a*b^2*c^3*d^3*e^5*x^5 + 3360*a^2*c^4*d^3*e^5*x^5 + 42
0*b^4*c^2*d^4*e^4*x^4 - 2940*a*b^2*c^3*d^4*e^4*x^4 + 4200*a^2*c^4*d^4*e^4*
x^4 + 12*b^5*c*e^6*x^6 - 87*a*b^3*c^2*e^6*x^6 + 138*a^2*b*c^3*e^6*x^6 + 33
6*b^4*c^2*d^5*e^3*x^3 - 2352*a*b^2*c^3*d^5*e^3*x^3 + 3360*a^2*c^4*d^5*e^3*
x^3 + 72*b^5*c*d*e^5*x^5 - 522*a*b^3*c^2*d*e^5*x^5 + 828*a^2*b*c^3*d*e^5*x
^5 + 168*b^4*c^2*d^6*e^2*x^2 - 1176*a*b^2*c^3*d^6*e^2*x^2 + 1680*a^2*c^4*d
^6*e^2*x^2 + 180*b^5*c*d^2*e^4*x^4 - 1305*a*b^3*c^2*d^2*e^4*x^4 + 2070*a^2
*b*c^3*d^2*e^4*x^4 + 48*b^4*c^2*d^7*e*x - 336*a*b^2*c^3*d^7*e*x + 480*a...
```

3.660.9 Mupad [B] (verification not implemented)

Time = 24.18 (sec) , antiderivative size = 25334, normalized size of antiderivative = 73.86

$$\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \text{Too large to display}$$

input
```
int(1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x)
```

output
```
(log(((27*c^5*e^16*x^2*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)^3)/(a^9*f^9*(4*a*c -
 b^2)^6) - ((3*b - 3*a^4*e*f^3*(-(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a
*b^4*c)^2/(a^8*e^2*f^6*(4*a*c - b^2)^5))^(1/2))*((9*c^3*e^15*(b^4 + 10*a^2
*c^2 - 7*a*b^2*c)*(4*b^6 - 10*a^3*c^3 + 6*b^5*c*d^2 + 71*a^2*b^2*c^2 - 33*
a*b^4*c - 47*a*b^3*c^2*d^2 + 90*a^2*b*c^3*d^2))/(a^6*f^6*(4*a*c - b^2)^4)
- ((3*b - 3*a^4*e*f^3*(-(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2
/(a^8*e^2*f^6*(4*a*c - b^2)^5))^(1/2))*((6*c^2*e^16*(2*b^7 - 20*a^3*b*c^3
+ b^6*c*d^2 + 46*a^2*b^3*c^2 + 100*a^3*c^4*d^2 - 18*a*b^5*c - 2*a*b^4*c^2*
d^2 - 30*a^2*b^2*c^3*d^2))/(a^3*f^3*(4*a*c - b^2)^2) + (b*c^2*e^16*(3*b -
3*a^4*e*f^3*(-(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2/(a^8*e^2*
f^6*(4*a*c - b^2)^5))^(1/2))*(a*b + 3*b^2*d^2 + 3*b^2*e^2*x^2 - 10*a*c*d^2
 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*a*c*d*e*x))/(a^4*f^3) + (6*c^3*e^18*x
^2*(b^6 + 100*a^3*c^3 - 30*a^2*b^2*c^2 - 2*a*b^4*c))/(a^3*f^3*(4*a*c - b^2
)^2) + (12*c^3*d*e^17*x*(b^6 + 100*a^3*c^3 - 30*a^2*b^2*c^2 - 2*a*b^4*c))/
(a^3*f^3*(4*a*c - b^2)^2)))/(4*a^4*e*f^3) + (9*b*c^4*e^17*x^2*(6*b^8 + 900
*a^4*c^4 + 479*a^2*b^4*c^2 - 1100*a^3*b^2*c^3 - 89*a*b^6*c))/(a^6*f^6*(4*a
*c - b^2)^4) + (18*b*c^4*d*e^16*x*(6*b^8 + 900*a^4*c^4 + 479*a^2*b^4*c^2 -
 1100*a^3*b^2*c^3 - 89*a*b^6*c))/(a^6*f^6*(4*a*c - b^2)^4)))/(4*a^4*e*f^3)
 + (27*c^4*e^14*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)^2*(b^5 + 16*a^2*b*c^2 + b^4
*c*d^2 + 10*a^2*c^3*d^2 - 8*a*b^3*c - 7*a*b^2*c^2*d^2))/(a^9*f^9*(4*a*c...
```

3.660. $\int \frac{1}{(df+efx)^3(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx$

3.661 $\int \frac{x}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx$

3.661.1 Optimal result

Integrand size = 26, antiderivative size = 340

$$\int \frac{x}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx = -\frac{d(d+ex)\sqrt{1+\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}},-\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}\right)}{e^2\sqrt{a+b(d+ex)^3+c(d+ex)^6}} + \frac{(d+ex)^2\sqrt{1+\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{5}{3},-\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}},-\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}\right)}{2e^2\sqrt{a+b(d+ex)^3+c(d+ex)^6}}$$

output
```
-d*(e*x+d)*AppellF1(1/3,1/2,1/2,4/3,-2*c*(e*x+d)^3/(b-(-4*a*c+b^2)^(1/2)),
-2*c*(e*x+d)^3/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*(e*x+d)^3/(b-(-4*a*c+b^2)^(1
/2)))^(1/2)*(1+2*c*(e*x+d)^3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/e^2/(a+b*(e*x+d
)^3+c*(e*x+d)^6)^(1/2)+1/2*(e*x+d)^2*AppellF1(2/3,1/2,1/2,5/3,-2*c*(e*x+d)
^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*(e*x+d)^3/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*(e
*x+d)^3/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*(e*x+d)^3/(b+(-4*a*c+b^2)^(1/
2)))^(1/2)/e^2/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2)
```

3.661.2 Mathematica [F]

$$\int \frac{x}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx=\int \frac{x}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx$$

input
```
Integrate[x/Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6],x]
```

output
```
Integrate[x/Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6], x]
```

3.661.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 338, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1726, 25, 2322, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx$$

$$\Big\downarrow \text{1726}$$

$$\frac{\int \frac{ex}{\sqrt{c(d+ex)^6+b(d+ex)^3+a}}d(d+ex)}{e^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int -\frac{ex}{\sqrt{c(d+ex)^6+b(d+ex)^3+a}}d(d+ex)}{e^2}$$

$$\Big\downarrow \text{2322}$$

$$-\frac{\int \left(\frac{d}{\sqrt{c(d+ex)^6+b(d+ex)^3+a}}-\frac{d+ex}{\sqrt{c(d+ex)^6+b(d+ex)^3+a}}\right)d(d+ex)}{e^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{(d+ex)^2\sqrt{\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2c(d+ex)^3}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{5}{3},-\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}},-\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{a+b(d+ex)^3+c(d+ex)^6}}-\frac{d(d+ex)\sqrt{\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2c(d+ex)^3}{\sqrt{b^2-4ac}+b}+1}\,\text{Appe}}{\sqrt{a+b(d+ex)^3}}}{e^2}$$

input
```
Int[x/Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6],x]
```

output
```
(-((d*(d + e*x)*Sqrt[1 + (2*c*(d + e*x)^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1
 + (2*c*(d + e*x)^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/3, 1/2, 1/2, 4/3,
 (-2*c*(d + e*x)^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*(d + e*x)^3)/(b + Sqrt[
b^2 - 4*a*c])])/Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6]) + ((d + e*x)^2*Sq
rt[1 + (2*c*(d + e*x)^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*(d + e*x)^
3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[2/3, 1/2, 1/2, 5/3, (-2*c*(d + e*x)^3
)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*(d + e*x)^3)/(b + Sqrt[b^2 - 4*a*c])])/(2
*Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6]))/e^2
```

3.661.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1726
```
Int[((a_.) + (c_.)*(v_)^(n2_.) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Simp[1/Coefficient[v, x, 1]^(m + 1)   Subst[Int[SimplifyIntegrand[(x
- Coefficient[v, x, 0])^m*(a + b*x^n + c*x^(2*n))^p, x], x], x, v], x] /; F
reeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && LinearQ[v, x] && IntegerQ[m] &&
 NeQ[v, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2322
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*n]*x^(k*n
), {k, 0, (q - j)/n + 1}]*(a + b*x^n + c*x^(2*n))^p, {j, 0, n - 1}], x]] /;
 FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && IGtQ[n, 0] &&  !PolyQ[Pq, x^n]
```

3.661.4 Maple [F]

$$\int \frac{x}{\sqrt{a+b\left(ex+d\right)^{3}+c\left(ex+d\right)^{6}}}dx$$

input
```
int(x/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2),x)
```

output
```
int(x/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2),x)
```

3.661.5 Fricas [F]

$$\int \frac{x}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx=\int \frac{x}{\sqrt{\left(ex+d\right)^{6}c+\left(ex+d\right)^{3}b+a}}\,dx$$

input
```
integrate(x/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2),x, algorithm="fricas")
```

output
```
integral(x/sqrt(c*e^6*x^6 + 6*c*d*e^5*x^5 + 15*c*d^2*e^4*x^4 + c*d^6 + (20
*c*d^3 + b)*e^3*x^3 + 3*(5*c*d^4 + b*d)*e^2*x^2 + b*d^3 + 3*(2*c*d^5 + b*d
^2)*e*x + a), x)
```

3.661.6 Sympy [F]

$$\int \frac{x}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx$$
$$=\int \frac{x}{\sqrt{a+bd^3+3bd^2ex+3bde^2x^2+be^3x^3+cd^6+6cd^5ex+15cd^4e^2x^2+20cd^3e^3x^3+15cd^2e^4x^4+6cde^5x^5}}$$

input
```
integrate(x/(a+b*(e*x+d)**3+c*(e*x+d)**6)**(1/2),x)
```

output
```
Integral(x/sqrt(a + b*d**3 + 3*b*d**2*e*x + 3*b*d*e**2*x**2 + b*e**3*x**3
+ c*d**6 + 6*c*d**5*e*x + 15*c*d**4*e**2*x**2 + 20*c*d**3*e**3*x**3 + 15*c
*d**2*e**4*x**4 + 6*c*d*e**5*x**5 + c*e**6*x**6), x)
```

3.661.7 Maxima [F]

$$\int \frac{x}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx = \int \frac{x}{\sqrt{(ex+d)^6c+(ex+d)^3b+a}}\,dx$$

input
```
integrate(x/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/sqrt((e*x + d)^6*c + (e*x + d)^3*b + a), x)
```

3.661.8 Giac [F]

$$\int \frac{x}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx = \int \frac{x}{\sqrt{(ex+d)^6c+(ex+d)^3b+a}}\,dx$$

input
```
integrate(x/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/sqrt((e*x + d)^6*c + (e*x + d)^3*b + a), x)
```

3.661.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx = \int \frac{x}{\sqrt{a+b\,(d+e\,x)^3+c\,(d+e\,x)^6}}\,dx$$

input
```
int(x/(a + b*(d + e*x)^3 + c*(d + e*x)^6)^(1/2),x)
```

output
```
int(x/(a + b*(d + e*x)^3 + c*(d + e*x)^6)^(1/2), x)
```

3.662 $\int \frac{x^2}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx$

3.662.1 Optimal result

Integrand size = 28, antiderivative size = 398

$$\int \frac{x^2}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx = \frac{d^2(d+ex)\sqrt{1+\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}},-\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}\right)}{e^3\sqrt{a+b(d+ex)^3+c(d+ex)^6}} - \frac{d(d+ex)^2\sqrt{1+\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{5}{3},-\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}},-\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}\right)}{e^3\sqrt{a+b(d+ex)^3+c(d+ex)^6}} + \frac{\operatorname{arctanh}\left(\frac{b+2c(d+ex)^3}{2\sqrt{c}\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\right)}{3\sqrt{c}e^3}$$

output
```
1/3*arctanh(1/2*(b+2*c*(e*x+d)^3)/c^(1/2)/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2
))/e^3/c^(1/2)+d^2*(e*x+d)*AppellF1(1/3,1/2,1/2,4/3,-2*c*(e*x+d)^3/(b-(-4*
a*c+b^2)^(1/2)),-2*c*(e*x+d)^3/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*(e*x+d)^3/(b
-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*(e*x+d)^3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
/e^3/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2)-d*(e*x+d)^2*AppellF1(2/3,1/2,1/2,5/
3,-2*c*(e*x+d)^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*(e*x+d)^3/(b+(-4*a*c+b^2)^(1/
2)))*(1+2*c*(e*x+d)^3/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*(e*x+d)^3/(b+(-
4*a*c+b^2)^(1/2)))^(1/2)/e^3/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2)
```

3.662.2 Mathematica [F]

$$\int \frac{x^2}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx=\int \frac{x^2}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx$$

input
```
Integrate[x^2/Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6],x]
```

output
```
Integrate[x^2/Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6], x]
```

3.662.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 393, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {1726, 2322, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx$$

$$\Big\downarrow \text{1726}$$

$$\frac{\int \frac{e^2x^2}{\sqrt{c(d+ex)^6+b(d+ex)^3+a}}d(d+ex)}{e^3}$$

$$\Big\downarrow \text{2322}$$

$$\frac{\int \left(\frac{d^2}{\sqrt{c(d+ex)^6+b(d+ex)^3+a}}-\frac{2(d+ex)d}{\sqrt{c(d+ex)^6+b(d+ex)^3+a}}+\frac{(d+ex)^2}{\sqrt{c(d+ex)^6+b(d+ex)^3+a}}\right)d(d+ex)}{e^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{d^2(d+ex)\sqrt{\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2c(d+ex)^3}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}},-\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}-\frac{d(d+ex)^2\sqrt{\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2c(d+ex)^3}{\sqrt{b^2-4ac}+b}+1}\,\text{Ap}}{\sqrt{a+b(d+ex}}}{e^3}$$

input
```
Int[x^2/Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6],x]
```

output
```
((d^2*(d + e*x)*Sqrt[1 + (2*c*(d + e*x)^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1
 + (2*c*(d + e*x)^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/3, 1/2, 1/2, 4/3,
 (-2*c*(d + e*x)^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*(d + e*x)^3)/(b + Sqrt[
b^2 - 4*a*c])])/Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6] - (d*(d + e*x)^2*S
qrt[1 + (2*c*(d + e*x)^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*(d + e*x)
^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[2/3, 1/2, 1/2, 5/3, (-2*c*(d + e*x)^
3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*(d + e*x)^3)/(b + Sqrt[b^2 - 4*a*c])])/S
qrt[a + b*(d + e*x)^3 + c*(d + e*x)^6] + ArcTanh[(b + 2*c*(d + e*x)^3)/(2*
Sqrt[c]*Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6])]/(3*Sqrt[c]))/e^3
```

3.662.3.1 Defintions of rubi rules used

rule 1726
```
Int[((a_.) + (c_.)*(v_)^(n2_.) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Simp[1/Coefficient[v, x, 1]^(m + 1)   Subst[Int[SimplifyIntegrand[(x
- Coefficient[v, x, 0])^m*(a + b*x^n + c*x^(2*n))^p, x], x], x, v], x] /; F
reeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && LinearQ[v, x] && IntegerQ[m] &&
 NeQ[v, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2322
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*n]*x^(k*n
), {k, 0, (q - j)/n + 1}]*(a + b*x^n + c*x^(2*n))^p, {j, 0, n - 1}], x]] /;
 FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && IGtQ[n, 0] &&  !PolyQ[Pq, x^n]
```

3.662.4 Maple [F]

$$\int \frac{x^2}{\sqrt{a+b\,(ex+d)^3+c\,(ex+d)^6}}dx$$

input
```
int(x^2/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2),x)
```

output
```
int(x^2/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2),x)
```

3.662.5 Fricas [F]

$$\int \frac{x^2}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx=\int \frac{x^2}{\sqrt{(ex+d)^6c+(ex+d)^3b+a}}\,dx$$

input
```
integrate(x^2/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^2/sqrt(c*e^6*x^6 + 6*c*d*e^5*x^5 + 15*c*d^2*e^4*x^4 + c*d^6 + (
20*c*d^3 + b)*e^3*x^3 + 3*(5*c*d^4 + b*d)*e^2*x^2 + b*d^3 + 3*(2*c*d^5 + b
*d^2)*e*x + a), x)
```

3.662.6 Sympy [F]

$$\int \frac{x^2}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx$$
$$=\int \frac{x^2}{\sqrt{a+bd^3+3bd^2ex+3bde^2x^2+be^3x^3+cd^6+6cd^5ex+15cd^4e^2x^2+20cd^3e^3x^3+15cd^2e^4x^4+6cde^5x^5}}$$

input
```
integrate(x**2/(a+b*(e*x+d)**3+c*(e*x+d)**6)**(1/2),x)
```

output
```
Integral(x**2/sqrt(a + b*d**3 + 3*b*d**2*e*x + 3*b*d*e**2*x**2 + b*e**3*x*
*3 + c*d**6 + 6*c*d**5*e*x + 15*c*d**4*e**2*x**2 + 20*c*d**3*e**3*x**3 + 1
5*c*d**2*e**4*x**4 + 6*c*d*e**5*x**5 + c*e**6*x**6), x)
```

3.662.7 Maxima [F]

$$\int \frac{x^2}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx=\int \frac{x^2}{\sqrt{(ex+d)^6c+(ex+d)^3b+a}}\,dx$$

input
```
integrate(x^2/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/sqrt((e*x + d)^6*c + (e*x + d)^3*b + a), x)
```

3.662.8 Giac [F]

$$\int \frac{x^2}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx = \int \frac{x^2}{\sqrt{(ex+d)^6c+(ex+d)^3b+a}}\,dx$$

input
```
integrate(x^2/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/sqrt((e*x + d)^6*c + (e*x + d)^3*b + a), x)
```

3.662.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx = \int \frac{x^2}{\sqrt{a+b\,(d+e\,x)^3+c\,(d+e\,x)^6}}\,dx$$

input
```
int(x^2/(a + b*(d + e*x)^3 + c*(d + e*x)^6)^(1/2),x)
```

output
```
int(x^2/(a + b*(d + e*x)^3 + c*(d + e*x)^6)^(1/2), x)
```

3.663 $\int (2+3x)^6 \left(1+(2+3x)^7+(2+3x)^{14}\right) \, dx$

3.663.1 Optimal result

Integrand size = 24, antiderivative size = 34

$$\int (2+3x)^6 \left(1+(2+3x)^7+(2+3x)^{14}\right) \, dx = \frac{1}{21}(2+3x)^7+\frac{1}{42}(2+3x)^{14}+\frac{1}{63}(2+3x)^{21}$$

output
```
1/21*(2+3*x)^7+1/42*(2+3*x)^14+1/63*(2+3*x)^21
```

3.663.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int (2+3x)^6 \left(1+(2+3x)^7+(2+3x)^{14}\right) \, dx = \frac{1}{21}(2+3x)^7+\frac{1}{42}(2+3x)^{14}+\frac{1}{63}(2+3x)^{21}$$

input
```
Integrate[(2 + 3*x)^6*(1 + (2 + 3*x)^7 + (2 + 3*x)^14),x]
```

output
```
(2 + 3*x)^7/21 + (2 + 3*x)^14/42 + (2 + 3*x)^21/63
```

3.663.3 Rubi [A] (warning: unable to verify)

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1725, 1690, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (3x+2)^6 \left((3x+2)^{14}+(3x+2)^7+1\right) dx$$

$$\Big\downarrow \text{1725}$$

$$\frac{1}{3}\int (3x+2)^6 \left((3x+2)^{14}+(3x+2)^7+1\right) d(3x+2)$$

$$\Big\downarrow \text{1690}$$

$$\frac{1}{21}\int \left((3x+2)^{14}+3x+3\right) d(3x+2)^7$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{21}\left((3x+2)^7+\frac{1}{3}(3x+2)^3+\frac{1}{2}(3x+2)^2\right)$$

input
```
Int[(2 + 3*x)^6*(1 + (2 + 3*x)^7 + (2 + 3*x)^14),x]
```

output
```
((2 + 3*x)^2/2 + (2 + 3*x)^3/3 + (2 + 3*x)^7)/21
```

3.663.3.1 Defintions of rubi rules used

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

rule 1725
```
Int[(u_)^(m_.)*((a_.) + (c_.)*(v_)^(n2_.) + (b_.)*(v_)^(n_))^(p_.), x_Symbo
l] :> Simp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n + c*x^
(2*n))^p, x], x, v], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] &&
LinearPairQ[u, v, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.663.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(28) = 56.

Time = 0.56 (sec) , antiderivative size = 104, normalized size of antiderivative = 3.06

method	result
gosper	$\frac{x(2324522934x^{20}+32543321076x^{19}+216955473840x^{18}+916034222880x^{17}+2748102668640x^{16}+6229032715584x^{15}+110738}{}$
default	$1056832x+15808800x^2+149902032x^3+1010576952x^4+5149786572x^5+20588764518x$
norman	$1056832x+15808800x^2+149902032x^3+1010576952x^4+5149786572x^5+20588764518x$
risch	$1056832x+15808800x^2+149902032x^3+1010576952x^4+5149786572x^5+20588764518x$
parallelrisch	$1056832x+15808800x^2+149902032x^3+1010576952x^4+5149786572x^5+20588764518x$

input
```
int((3*x+2)^6*(1+(3*x+2)^7+(3*x+2)^14),x,method=_RETURNVERBOSE)
```

output
```
1/14*x*(2324522934*x^20+32543321076*x^19+216955473840*x^18+916034222880*x^
17+2748102668640*x^16+6229032715584*x^15+11073835938816*x^14+1581976722120
3*x^13+18456408111708*x^12+17772887593188*x^11+14218430440032*x^10+9479154
235824*x^9+5266441986624*x^8+2430891860544*x^7+926214166962*x^6+2882427032
52*x^5+72097012008*x^4+14148077328*x^3+2098628448*x^2+221323200*x+14795648
)
```

3.663.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(28) = 56.

Time = 0.27 (sec) , antiderivative size = 104, normalized size of antiderivative = 3.06

$$\int (2+3x)^6 \left(1+(2+3x)^7+(2+3x)^{14}\right)\,dx = \frac{1162261467}{7}\,x^{21} + 2324522934\,x^{20} + 15496819560\,x^{19} + 65431015920\,x^{18} + 196293047760\,x^{17} + 444930908256\,x^{16} + 790988281344\,x^{15} + \frac{15819767221203}{14}\,x^{14} + 1318314865122\,x^{13} + 1269491970942\,x^{12} + 1015602174288\,x^{11} + 677082445416\,x^{10} + 376174427616\,x^{9} + 173635132896\,x^{8} + 66158154783\,x^{7} + 20588764518\,x^{6} + 5149786572\,x^{5} + 1010576952\,x^{4} + 149902032\,x^{3} + 15808800\,x^{2} + 1056832\,x$$

input
```
integrate((2+3*x)^6*(1+(2+3*x)^7+(2+3*x)^14),x, algorithm="fricas")
```

output
```
1162261467/7*x^21 + 2324522934*x^20 + 15496819560*x^19 + 65431015920*x^18
+ 196293047760*x^17 + 444930908256*x^16 + 790988281344*x^15 + 158197672212
03/14*x^14 + 1318314865122*x^13 + 1269491970942*x^12 + 1015602174288*x^11
+ 677082445416*x^10 + 376174427616*x^9 + 173635132896*x^8 + 66158154783*x^
7 + 20588764518*x^6 + 5149786572*x^5 + 1010576952*x^4 + 149902032*x^3 + 15
808800*x^2 + 1056832*x
```

3.663.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(24) = 48$.

Time = 0.04 (sec) , antiderivative size = 107, normalized size of antiderivative = 3.15

$$\int (2+3x)^6 \left(1+(2+3x)^7+(2+3x)^{14}\right) dx = \frac{1162261467x^{21}}{7} + 2324522934x^{20} + 15496819560x^{19} + 65431015920x^{18} + 196293047760x^{17} + 444930908256x^{16} + 790988281344x^{15} + \frac{15819767221203x^{14}}{14} + 1318314865122x^{13} + 1269491970942x^{12} + 1015602174288x^{11} + 677082445416x^{10} + 376174427616x^{9} + 173635132896x^{8} + 66158154783x^{7} + 20588764518x^{6} + 5149786572x^{5} + 1010576952x^{4} + 149902032x^{3} + 15808800x^{2} + 1056832x$$

input
```
integrate((2+3*x)**6*(1+(2+3*x)**7+(2+3*x)**14),x)
```

output
```
1162261467*x**21/7 + 2324522934*x**20 + 15496819560*x**19 + 65431015920*x*
*18 + 196293047760*x**17 + 444930908256*x**16 + 790988281344*x**15 + 15819
767221203*x**14/14 + 1318314865122*x**13 + 1269491970942*x**12 + 101560217
4288*x**11 + 677082445416*x**10 + 376174427616*x**9 + 173635132896*x**8 +
66158154783*x**7 + 20588764518*x**6 + 5149786572*x**5 + 1010576952*x**4 +
149902032*x**3 + 15808800*x**2 + 1056832*x
```

3.663.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(28) = 56.

Time = 0.21 (sec) , antiderivative size = 104, normalized size of antiderivative = 3.06

$$\int (2+3x)^6 \left(1+(2+3x)^7+(2+3x)^{14}\right)\, dx = \frac{1162261467}{7}\, x^{21} + 2324522934\, x^{20} + 15496819560\, x^{19} + 65431015920\, x^{18} + 196293047760\, x^{17} + 444930908256\, x^{16} + 790988281344\, x^{15} + \frac{15819767221203}{14}\, x^{14} + 1318314865122\, x^{13} + 1269491970942\, x^{12} + 1015602174288\, x^{11} + 677082445416\, x^{10} + 376174427616\, x^{9} + 173635132896\, x^{8} + 66158154783\, x^{7} + 20588764518\, x^{6} + 5149786572\, x^{5} + 1010576952\, x^{4} + 149902032\, x^{3} + 15808800\, x^{2} + 1056832\, x$$

input
```
integrate((2+3*x)^6*(1+(2+3*x)^7+(2+3*x)^14),x, algorithm="maxima")
```

output
```
1162261467/7*x^21 + 2324522934*x^20 + 15496819560*x^19 + 65431015920*x^18
+ 196293047760*x^17 + 444930908256*x^16 + 790988281344*x^15 + 158197672212
03/14*x^14 + 1318314865122*x^13 + 1269491970942*x^12 + 1015602174288*x^11
+ 677082445416*x^10 + 376174427616*x^9 + 173635132896*x^8 + 66158154783*x^
7 + 20588764518*x^6 + 5149786572*x^5 + 1010576952*x^4 + 149902032*x^3 + 15
808800*x^2 + 1056832*x
```

3.663.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int (2+3x)^6 \left(1+(2+3x)^7+(2+3x)^{14}\right)\, dx = \frac{1}{63}\,(3\,x+2)^{21} + \frac{1}{42}\,(3\,x+2)^{14} + \frac{1}{21}\,(3\,x+2)^{7}$$

input
```
integrate((2+3*x)^6*(1+(2+3*x)^7+(2+3*x)^14),x, algorithm="giac")
```

output
```
1/63*(3*x + 2)^21 + 1/42*(3*x + 2)^14 + 1/21*(3*x + 2)^7
```

3.663.9 Mupad [B] (verification not implemented)

Time = 8.50 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int (2+3x)^6 \left(1+(2+3x)^7+(2+3x)^{14}\right) \, dx = \frac{(3\,x+2)^7\,\left(3\,(3\,x+2)^7+2\,(3\,x+2)^{14}+6\right)}{126}$$

input
```
int((3*x + 2)^6*((3*x + 2)^7 + (3*x + 2)^14 + 1),x)
```

output
```
((3*x + 2)^7*(3*(3*x + 2)^7 + 2*(3*x + 2)^14 + 6))/126
```

3.664 $\int (2+3x)^6 \left(1+(2+3x)^7+(2+3x)^{14}\right)^2 \, dx$

3.664.1 Optimal result

Integrand size = 26, antiderivative size = 56

$$\int (2+3x)^6 \left(1+(2+3x)^7+(2+3x)^{14}\right)^2 \, dx = \frac{1}{21}(2+3x)^7 + \frac{1}{21}(2+3x)^{14} + \frac{1}{21}(2+3x)^{21} + \frac{1}{42}(2+3x)^{28} + \frac{1}{105}(2+3x)^{35}$$

output
```
1/21*(2+3*x)^7+1/21*(2+3*x)^14+1/21*(2+3*x)^21+1/42*(2+3*x)^28+1/105*(2+3*
x)^35
```

3.664.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 188 vs. $2(56) = 112$.

Time = 0.01 (sec) , antiderivative size = 188, normalized size of antiderivative = 3.36

$$\begin{aligned}
&\int (2+3x)^6 \left(1+(2+3x)^7+(2+3x)^{14}\right)^2 \, dx \\
&= 17451466816x + 443569828128x^2 + 7299544818384x^3 + 87406679578680x^4 \\
&\quad + \frac{4057390785756924x^5}{5} + 6077684727888102x^6 + 37727143432895007x^7 \\
&\quad + 197897276851452864x^8 + 889942562270387136x^9 + \frac{17344958593049772048x^{10}}{5} \\
&\quad + 11821487501620716192x^{11} + 35454069480572048124x^{12} + 94069263918929616324x^{13} \\
&\quad + 221699757548270194389x^{14} + 465517091041681015296x^{15} \\
&\quad + 872775774067455498528x^{16} + 1463104032160519033200x^{17} \\
&\quad + 2194577166014752240080x^{18} + 2945285062308448290360x^{19} \\
&\quad + 3534290697929473864098x^{20} + \frac{26506949038858918036881x^{21}}{7} \\
&\quad + 3614565944605222108800x^{22} + 3064515076512846852480x^{23} \\
&\quad + 2298383223254096766840x^{24} + \frac{7584660010542711771792x^{25}}{5} \\
&\quad + 875152864622814086340x^{26} + 437576396725285446564x^{27} \\
&\quad + \frac{2625458326972530284475x^{28}}{14} + 67899784121041365504x^{29} \\
&\quad + \frac{101849676181562048256x^{30}}{5} + 4928210137817518464x^{31} + 924039400840784712x^{32} \\
&\quad + 126005372841925188x^{33} + 11118121133111046x^{34} + \frac{16677181699666569x^{35}}{35}
\end{aligned}$$

input
```
Integrate[(2 + 3*x)^6*(1 + (2 + 3*x)^7 + (2 + 3*x)^14)^2,x]
```

output
```
17451466816*x + 443569828128*x^2 + 7299544818384*x^3 + 87406679578680*x^4
+ (4057390785756924*x^5)/5 + 6077684727888102*x^6 + 37727143432895007*x^7
+ 197897276851452864*x^8 + 889942562270387136*x^9 + (17344958593049772048*
x^10)/5 + 11821487501620716192*x^11 + 35454069480572048124*x^12 + 94069263
918929616324*x^13 + 221699757548270194389*x^14 + 465517091041681015296*x^1
5 + 872775774067455498528*x^16 + 1463104032160519033200*x^17 + 21945771660
14752240080*x^18 + 2945285062308448290360*x^19 + 3534290697929473864098*x^
20 + (26506949038858918036881*x^21)/7 + 3614565944605222108800*x^22 + 3064
515076512846852480*x^23 + 2298383223254096766840*x^24 + (7584660010542711
771792*x^25)/5 + 875152864622814086340*x^26 + 437576396725285446564*x^27 +
(2625458326972530284475*x^28)/14 + 67899784121041365504*x^29 + (1018496761
81562048256*x^30)/5 + 4928210137817518464*x^31 + 924039400840784712*x^32 +
 126005372841925188*x^33 + 11118121133111046*x^34 + (1667718169966569*x^3
5)/35
```

3.664.3 Rubi [A] (warning: unable to verify)

Time = 0.22 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1725, 1690, 1085, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (3x+2)^6 \left((3x+2)^{14}+(3x+2)^7+1\right)^2 dx$$

$$\Big\downarrow \ 1725$$

$$\frac{1}{3}\int (3x+2)^6 \left((3x+2)^{14}+(3x+2)^7+1\right)^2 d(3x+2)$$

$$\Big\downarrow \ 1690$$

$$\frac{1}{21}\int \left((3x+2)^{14}+3x+3\right)^2 d(3x+2)^7$$

$$\Big\downarrow \ 1085$$

$$\frac{1}{21}\int \left((3x+2)^{28}+2(3x+2)^{21}+3(3x+2)^{14}+2(3x+2)^7+1\right) d(3x+2)^7$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{21}\left((3x+2)^7+\frac{1}{5}(3x+2)^5+\frac{1}{2}(3x+2)^4+(3x+2)^3+(3x+2)^2\right)$$

input
```
Int[(2 + 3*x)^6*(1 + (2 + 3*x)^7 + (2 + 3*x)^14)^2,x]
```

output
```
((2 + 3*x)^2 + (2 + 3*x)^3 + (2 + 3*x)^4/2 + (2 + 3*x)^5/5 + (2 + 3*x)^7)/
21
```

3.664.3.1 Defintions of rubi rules used

rule 1085
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegr
and[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && IntegerQ[p] && (G
tQ[p, 0] || EqQ[a, 0])
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

rule 1725
```
Int[(u_)^(m_.)*((a_.) + (c_.)*(v_)^(n2_.) + (b_.)*(v_)^(n_))^(p_.), x_Symbo
l] :> Simp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n + c*x^
(2*n))^p, x], x, v], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] &&
LinearPairQ[u, v, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.664.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(46) = 92$.

Time = 0.59 (sec) , antiderivative size = 174, normalized size of antiderivative = 3.11

method	result
gosper	$\frac{x(33354363399333138x^{34}+778268479317773220x^{33}+8820376098934763160x^{32}+64682758058854929840x^{31}+3449747096472}{}$
default	$\frac{17344958593049772048}{5}x^{10} + 11821487501620716192x^{11} + 35454069480572048124x^{12} + 9406926$
norman	$\frac{17344958593049772048}{5}x^{10} + 11821487501620716192x^{11} + 35454069480572048124x^{12} + 9406926$
risch	$\frac{17344958593049772048}{5}x^{10} + 11821487501620716192x^{11} + 35454069480572048124x^{12} + 9406926$
parallelrisch	$\frac{17344958593049772048}{5}x^{10} + 11821487501620716192x^{11} + 35454069480572048124x^{12} + 9406926$

input
```
int((3*x+2)^6*(1+(3*x+2)^7+(3*x+2)^14)^2,x,method=_RETURNVERBOSE)
```

output
```
1/70*x*(33354363399333138*x^34+778268479317773220*x^33+8820376098934763160
*x^32+64682758058854929840*x^31+344974709647226292480*x^30+142589546654186
8675584*x^29+4752984888472895585280*x^28+13127291634862651422375*x^27+3063
0347770769981259480*x^26+61260700523596986043800*x^25+10618524014759796480
5088*x^24+160886825627786773678800*x^23+214516055355899279673600*x^22+2530
19616122365547616000*x^21+265069490388589180368810*x^20+247400348855063170
486860*x^19+206169954361591380325200*x^18+153620401621032656805600*x^17+10
2417282251236332324000*x^16+61094304184721884896960*x^15+32586196372917671
070720*x^14+15518983028378913607230*x^13+6584848474325073142680*x^12+24817
84863640043368680*x^11+827504125113450133440*x^10+242829420302696808672*x^
9+62295979358927099520*x^8+13852809379601700480*x^7+2640900040302650490*x^
6+425437930952167140*x^5+56803471000596936*x^4+6118467570507600*x^3+510968
137286880*x^2+31049887968960*x+1221602677120)
```

3.664.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(46) = 92$.

Time = 0.24 (sec) , antiderivative size = 174, normalized size of antiderivative = 3.11

$$\int (2+3x)^6 \left(1+(2+3x)^7+(2+3x)^{14}\right)^2 \, dx = \frac{16677181699666569}{35} x^{35} + 11118121133111046 x^{34} + 126005372841925188 x^{33} + 924039400840784712 x^{32} + 4928210137817518464 x^{31} + \frac{101849676181562048256}{5} x^{30} + 67899784121041365504 x^{29} + \frac{2625458326972530284475}{14} x^{28} + 437576396725285446564 x^{27} + 875152864622814086340 x^{26} + \frac{7584660010542711771792}{5} x^{25} + 2298383223254096766840 x^{24} + 3064515076512846852480 x^{23} + 3614565944605222108800 x^{22} + \frac{26506949038858918036881}{7} x^{21} + 3534290697929473864098 x^{20} + 2945285062308448290360 x^{19} + 2194577166014752240080 x^{18} + 1463104032160519033200 x^{17} + 872775774067455498528 x^{16} + 465517091041681015296 x^{15} + 221699757548270194389 x^{14} + 94069263918929616324 x^{13} + 35454069480572048124 x^{12} + 11821487501620716192 x^{11} + \frac{17344958593049772048}{5} x^{10} + 889942562270387136 x^{9} + 197897276851452864 x^{8} + 37727143432895007 x^{7} + 6077684727888102 x^{6} + \frac{4057390785756924}{5} x^{5} + 87406679578680 x^{4} + 7299544818384 x^{3} + 443569828128 x^{2} + 17451466816 x$$

input
```
integrate((2+3*x)^6*(1+(2+3*x)^7+(2+3*x)^14)^2,x, algorithm="fricas")
```

output
```
16677181699666569/35*x^35 + 11118121133111046*x^34 + 126005372841925188*x^
33 + 924039400840784712*x^32 + 4928210137817518464*x^31 + 1018496761815620
48256/5*x^30 + 67899784121041365504*x^29 + 2625458326972530284475/14*x^28
+ 437576396725285446564*x^27 + 875152864622814086340*x^26 + 75846600105427
11771792/5*x^25 + 2298383223254096766840*x^24 + 3064515076512846852480*x^2
3 + 3614565944605222108800*x^22 + 26506949038858918036881/7*x^21 + 3534290
697929473864098*x^20 + 2945285062308448290360*x^19 + 219457716601475224008
0*x^18 + 1463104032160519033200*x^17 + 872775774067455498528*x^16 + 465517
091041681015296*x^15 + 221699757548270194389*x^14 + 94069263918929616324*x
^13 + 35454069480572048124*x^12 + 11821487501620716192*x^11 + 173449585930
49772048/5*x^10 + 889942562270387136*x^9 + 197897276851452864*x^8 + 377271
43432895007*x^7 + 6077684727888102*x^6 + 4057390785756924/5*x^5 + 87406679
578680*x^4 + 7299544818384*x^3 + 443569828128*x^2 + 17451466816*x
```

3.664. $\int (2+3x)^6 \left(1+(2+3x)^7+(2+3x)^{14}\right)^2 \, dx$

3.664.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(41) = 82$.

Time = 0.07 (sec) , antiderivative size = 187, normalized size of antiderivative = 3.34

$$\begin{aligned}
&\int (2+3x)^6 \left(1+(2+3x)^7+(2+3x)^{14}\right)^2 \, dx \\
&= \frac{16677181699666569x^{35}}{35} + 11118121133111046x^{34} + 126005372841925188x^{33} \\
&+ 924039400840784712x^{32} + 4928210137817518464x^{31} + \frac{101849676181562048256x^{30}}{5} \\
&+ 67899784121041365504x^{29} + \frac{2625458326972530284475x^{28}}{14} \\
&+ 437576396725285446564x^{27} + 875152864622814086340x^{26} \\
&+ \frac{7584660010542711771792x^{25}}{5} + 2298383223254096766840x^{24} \\
&+ 3064515076512846852480x^{23} + 3614565944605222108800x^{22} \\
&+ \frac{26506949038858918036881x^{21}}{7} + 3534290697929473864098x^{20} \\
&+ 2945285062308448290360x^{19} + 2194577166014752240080x^{18} \\
&+ 1463104032160519033200x^{17} + 872775774067455498528x^{16} \\
&+ 465517091041681015296x^{15} + 221699757548270194389x^{14} \\
&+ 94069263918929616324x^{13} + 35454069480572048124x^{12} + 11821487501620716192x^{11} \\
&+ \frac{17344958593049772048x^{10}}{5} + 889942562270387136x^{9} + 197897276851452864x^{8} \\
&+ 37727143432895007x^{7} + 6077684727888102x^{6} + \frac{4057390785756924x^{5}}{5} \\
&+ 87406679578680x^{4} + 7299544818384x^{3} + 443569828128x^{2} + 17451466816x
\end{aligned}$$

input

```
integrate((2+3*x)**6*(1+(2+3*x)**7+(2+3*x)**14)**2,x)
```

output
```
16677181699666569*x**35/35 + 11118121133111046*x**34 + 126005372841925188*
x**33 + 924039400840784712*x**32 + 4928210137817518464*x**31 + 10184967618
1562048256*x**30/5 + 67899784121041365504*x**29 + 262545832697253028 4475*x
**28/14 + 437576396725285446564*x**27 + 875152864622814086340*x**26 + 7584
66001054271 1771792*x**25/5 + 2298383223254096766840*x**24 + 30645150765128
46852480*x**23 + 3614565944605222108800*x**22 + 26506949038858918036881*x*
*21/7 + 3534290697929473864098*x**20 + 2945285062308448290360*x**19 + 2194
577166014752240080*x**18 + 1463104032160519033200*x**17 + 8727757740674554
98528*x**16 + 465517091041681015296*x**15 + 221699757548270194389*x**14 +
94069263918929616324*x**13 + 35454069480572048124*x**12 + 1182148750162071
6192*x**11 + 17344958593049772048*x**10/5 + 889942562270387136*x**9 + 1978
97276851452864*x**8 + 37727143432895007*x**7 + 6077684727888102*x**6 + 405
7390785756924*x**5/5 + 87406679578680*x**4 + 7299544818384*x**3 + 44356982
8128*x**2 + 17451466816*x
```

3.664.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(46) = 92$.

Time = 0.23 (sec) , antiderivative size = 174, normalized size of antiderivative = 3.11

$$\begin{aligned}\int (2+3x)^6 \left(1+(2+3x)^7+(2+3x)^{14}\right)^2 \, dx &= \frac{16677181699666569}{35}\,x^{35} + 11118121133111046\,x^{34} + 126005372841925188\,x^{33} \\ &+ 924039400840784712\,x^{32} + 4928210137817518464\,x^{31} + \frac{101849676181562048256}{5}\,x^{30} \\ &+ 67899784121041365504\,x^{29} + \frac{2625458326972530284475}{14}\,x^{28} \\ &+ 437576396725285446564\,x^{27} + 875152864622814086340\,x^{26} \\ &+ \frac{7584660010542711771792}{5}\,x^{25} + 2298383223254096766840\,x^{24} \\ &+ 3064515076512846852480\,x^{23} + 3614565944605222108800\,x^{22} \\ &+ \frac{26506949038858918036881}{7}\,x^{21} + 3534290697929473864098\,x^{20} \\ &+ 2945285062308448290360\,x^{19} + 2194577166014752240080\,x^{18} \\ &+ 1463104032160519033200\,x^{17} + 872775774067455498528\,x^{16} \\ &+ 465517091041681015296\,x^{15} + 221699757548270194389\,x^{14} \\ &+ 94069263918929616324\,x^{13} + 35454069480572048124\,x^{12} + 11821487501620716192\,x^{11} \\ &+ \frac{17344958593049772048}{5}\,x^{10} + 889942562270387136\,x^{9} + 197897276851452864\,x^{8} \\ &+ 37727143432895007\,x^{7} + 6077684727888102\,x^{6} + \frac{4057390785756924}{5}\,x^{5} \\ &+ 87406679578680\,x^{4} + 7299544818384\,x^{3} + 443569828128\,x^{2} + 17451466816\,x\end{aligned}$$

input
```
integrate((2+3*x)^6*(1+(2+3*x)^7+(2+3*x)^14)^2,x, algorithm="maxima")
```

output
```
16677181699666569/35*x^35 + 1111812113311046*x^34 + 126005372841925188*x^
33 + 924039400840784712*x^32 + 4928210137817518464*x^31 + 1018496761815620
48256/5*x^30 + 67899784121041365504*x^29 + 262545832697253028475/14*x^28
+ 437576396725285446564*x^27 + 875152864622814086340*x^26 + 75846600105427
11771792/5*x^25 + 2298383223254096766840*x^24 + 3064515076512846852480*x^2
3 + 3614565944605222108800*x^22 + 26506949038858918036881/7*x^21 + 3534290
697929473864098*x^20 + 2945285062308448290360*x^19 + 219457716601475224008
0*x^18 + 1463104032160519033200*x^17 + 872775774067455498528*x^16 + 465517
091041681015296*x^15 + 221699757548270194389*x^14 + 94069263918929616324*x
^13 + 35454069480572048124*x^12 + 11821487501620716192*x^11 + 173449585930
49772048/5*x^10 + 889942562270387136*x^9 + 197897276851452864*x^8 + 377271
43432895007*x^7 + 6077684727888102*x^6 + 4057390785756924/5*x^5 + 87406679
578680*x^4 + 7299544818384*x^3 + 443569828128*x^2 + 17451466816*x
```

3.664.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int (2+3x)^6 \left(1+(2+3x)^7+(2+3x)^{14}\right)^2 \, dx = \frac{1}{105} \, (3\,x+2)^{35} + \frac{1}{42} \, (3\,x+2)^{28} + \frac{1}{21} \, (3\,x+2)^{21} + \frac{1}{21} \, (3\,x+2)^{14} + \frac{1}{21} \, (3\,x+2)^{7}$$

input
```
integrate((2+3*x)^6*(1+(2+3*x)^7+(2+3*x)^14)^2,x, algorithm="giac")
```

output
```
1/105*(3*x + 2)^35 + 1/42*(3*x + 2)^28 + 1/21*(3*x + 2)^21 + 1/21*(3*x + 2
)^14 + 1/21*(3*x + 2)^7
```

3.664.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int (2+3x)^6 \left(1+(2+3x)^7+(2+3x)^{14}\right)^2 \, dx = \frac{(3\,x+2)^7}{21} + \frac{(3\,x+2)^{14}}{21} + \frac{(3\,x+2)^{21}}{21} + \frac{(3\,x+2)^{28}}{42} + \frac{(3\,x+2)^{35}}{105}$$

input
```
int((3*x + 2)^6*((3*x + 2)^7 + (3*x + 2)^14 + 1)^2,x)
```

output
```
(3*x + 2)^7/21 + (3*x + 2)^14/21 + (3*x + 2)^21/21 + (3*x + 2)^28/42 + (3*
x + 2)^35/105
```

3.664. $\int (2+3x)^6 \left(1+(2+3x)^7+(2+3x)^{14}\right)^2 \, dx$

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)



(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
if ExpnType_result<=ExpnType_optimal then
    if debug then
         print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                    print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
               return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
            end if
        else #result contains complex but optimal is not
            if debug then
                    print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
         # this assumes optimal do not as well. No check is needed here.
        if debug then
               print("result do not contain complex, this assumes optimal do not as well"
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A"," ";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B",cat("Leaf count of result is larger than twice the leaf count of o
                                convert(leaf_count_result,string),"$ vs. $2(",
                                convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
        fi;
    fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```python
#Dec 24, 2019. Nasser M. Abbasi:
#            Port of original Maple grading function by
#            Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#            added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
      return 1
  elif isinstance(expn,list):
     return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
  elif  is_sqrt(expn):
     if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
        return 1
     else:
        return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
  elif isinstance(expn,Pow):   #type(expn,'`^`')
     if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
        return expnType(expn.args[0])   #ExpnType(op(1,expn))
     elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
           return 1
        else:
           return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
     else:
        return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
  elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
     m1 = expnType(expn.args[0])
     m2 = expnType(list(expn.args[1:]))
     return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
     return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
  elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
     m1 = max(map(expnType, list(expn.args)))
     return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
  elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
     m1 = max(map(expnType, list(expn.args)))
     return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
  elif is_appell_function(expn.func):
     m1 = max(map(expnType, list(expn.args)))
     return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
  elif isinstance(expn,RootSum):
     m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
     return max(7,m1)
  elif str(expn).find("Integral") != -1:
     m1 = max(map(expnType, list(expn.args)))
     return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
  else:
     return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
        return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinsta
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))

leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation =" "
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
return grade, grade_annotation
```